Homework 11

1 Problem 1:

(a) The average linear expansion is:

$$\Delta l = \alpha l_0 \Delta T \tag{1}$$

Therefore:

$$\pi D_2 - \pi D_1 = \alpha_1 \pi D_1 \Delta T \tag{2}$$

i.e.

$$\Delta T = \frac{D_2 - D_1}{\alpha_1 D1} = 416.7^\circ C \Longrightarrow T = 436.7^\circ C \tag{3}$$

(b) Similarly

$$\pi D_2 + \alpha_2 \pi D_2 \Delta T = \alpha_1 \pi D_1 \Delta T + \pi D_1 \Longrightarrow \Delta T = \frac{D_2 - D_1}{\alpha_1 D_1 - \alpha_2 D_2} = 2079^\circ \mathrm{C} \tag{4}$$

with

$$T = 2099^\circ C \tag{5}$$

This is impossible because aluminum melts at $600^\circ C$ and brass at $1083^\circ C$.

2 Problem 2:

Due to:

$$PV = N k_B T \tag{6}$$

The number of molecules is:

$$N = \frac{PV}{k_B T} = 2.415 \times 10^{11} \tag{7}$$

3 Problem 3:

According the equation of state:

$$(P_0 + \rho g h) V_1 = n R T_1 \tag{8}$$

$$P_0 V_2 = nRT_2 \tag{9}$$

Thus,

$$V_2 = 3.72cm^3 \tag{10}$$

V_2 is the volume of the bubble right before it breaks the surface.

4 Problem 4:

$$v_{rms} = \sqrt{\frac{3k_B T}{m}} = 1350\sqrt{\frac{m_{\mathrm{He}}}{m_{\mathrm{O_2}}}}\mathrm{m/s} = 1350\sqrt{\frac{M_{\mathrm{He}}}{M_{\mathrm{O_2}}}}\mathrm{m/s} = 477\mathrm{m/s}$$

5 Problem 5:

(a) By the equation of state:

$$PV = nRT \tag{11}$$

and

$$\Delta(PV) = P\Delta V + V\Delta P \Longrightarrow V\frac{\Delta P}{\Delta V} = \frac{nR\Delta T}{\Delta V} - P \tag{12}$$

We can derive:

$$B = P - \frac{nR\Delta T}{\Delta V} = P \tag{13}$$

at a constant temperature.

(b) The speed of sound is:

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{nRT}{V\rho}} = \sqrt{\frac{k_B T}{m}} \tag{14}$$

(c)

$$v = \sqrt{\frac{k_B T}{m}} \approx 294.84\mathrm{m/s} \tag{15}$$

where $m = \frac{28.014\times10^{-3}}{6.02\times10^{-23}}\mathrm{kg}$, $T = (20 + 273.15)\mathrm{K} = 293.15\mathrm{K}$.

(d)

$$v \approx \sqrt{\gamma}\sqrt{\frac{Nk_BT}{m}} \approx 348.86m/s \tag{16}$$

this is near the sound speed value 340m/s as we know.

(e) the rms speed is:

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} \approx 510.69\text{m/s} \tag{17}$$