Homework 9

1 Problem 1:

(a) According to the law of gravity:

$$\frac{GMm}{(R+h)^2} = \frac{m \cdot 4\pi^2 (R+h)}{T^2} \Longrightarrow T = 2\pi\sqrt{\tfrac{(R+h)^3}{GM}} \tag{1}$$

Therefore the time difference for each orbit:

$$\Delta T = T - T\sqrt{1 - \frac{v^2}{c^2}} = 1.78 \times 10^{-6}\text{s} \approx 2 \times 10^{-6}\text{s} \tag{2}$$

where $R = 6371\text{km}$, $h = 160\text{km}$, $M = 5.977 \times 10^{24}\text{kg}$.
The total time difference is:

$$\Delta T_{\text{total}} = 22\Delta T = 4.4 \times 10^{-5}\text{s} \tag{3}$$

(b) Because $(2 - 1.78) \times 10^{-6} \approx 0$, the press did report accurate information.

2 Problem 2:

We konw:

$$x_0 = L_0 \cos\theta_0 \tag{4}$$

$$y_0 = L_0 \sin\theta_0 \tag{5}$$

therefore the parameters measured by a stationary observer is:

$$x = L_0 \cos\theta_0 \sqrt{1 - \frac{v^2}{c^2}},$$

$$y = L_0 \sin\theta_0$$

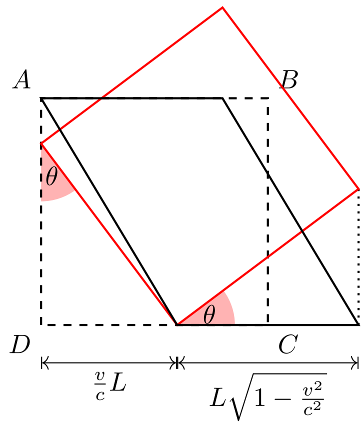
and:

$$L = \sqrt{x^2 + y^2} = L_0\sqrt{1 - \frac{v^2}{c^2}\cos^2\theta_0}$$

(b)

$$\tan\theta = \frac{y}{x} = \frac{\tan\theta_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\tan\theta_0$$

3 Problem 3:



As can be seen from the picture, considering that light which comes from A and B would be $\frac{L}{c}$ later than that comes from C and D, we will observe CD as $\frac{vL}{c}$ farther to the left than AB. Also noting that AB and CD would be observed as of length $L\sqrt{1-\frac{v^2}{c^2}}$, result turns out to be that the square looks like a parallelogram as the black solid one shown in the picture.

Observe that $(\frac{v}{c})^2 + (\sqrt{1-\frac{v^2}{c^2}})^2 = 1$, so let $\sin\theta = \frac{v}{c}, \cos\theta = \sqrt{1-\frac{v^2}{c^2}}$ and we can have a square of side length L as the red square shown in the picture which can be seen as the original square after being rotated!

And in the condition that we are standing far away from the square, the parallelogram just looks like the red square as shown in the picture.

4 Problem 4:

In the frame of the spaceship, B's clock reads $\frac{Lv}{c^2}$ ahead of A's. So the starting time of B's clock is $\frac{Lv}{c^2}$, and the time it takes in B's clock observed by people in the spaceship is s^2L/v, then when the spaceship reaches B, B's clock reads:

$$t_{\mathrm{B}} = \frac{Lv}{c^2} + \frac{s^2L}{v} = \frac{L}{v}$$

5 Problem 5:

(a)

$$t' = \frac{L}{u} = \frac{L}{\frac{c}{3}} = \frac{3L}{c} \tag{6}$$

$$x' = L \tag{7}$$

(b) Using the velocity addition argument:

$$w = \frac{u+v}{1+\frac{u}{c}\cdot\frac{v}{c}} = \frac{\frac{c}{3}+\frac{c}{2}}{1+\frac{\frac{c}{3}}{c}\cdot\frac{\frac{c}{2}}{c}} = \frac{5}{7}c \tag{8}$$

$$L' = \sqrt{1-\frac{v^2}{c^2}}L = \frac{\sqrt{3}}{2}L \tag{9}$$

$$t = \frac{L'}{\frac{5c}{7}-\frac{c}{2}} = \frac{7\sqrt{3}L}{3c} \tag{10}$$

$$x = wt = \frac{5\sqrt{3}}{3}L \tag{11}$$

Using the Lorentz transformation:

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-v^2/c^2}} = \frac{7L}{\sqrt{3}c} \tag{12}$$

$$x = \frac{x' + vt'}{\sqrt{1-v^2/c^2}} = \frac{5L}{\sqrt{3}} \tag{13}$$

(c)

$$L' = \sqrt{1-\frac{u^2}{c^2}}L = \frac{2\sqrt{2}L}{3} \tag{14}$$

$$t = \frac{L'}{u} = \frac{2\sqrt{2}L}{c} \tag{15}$$

$$x = 0 \tag{16}$$

(d)train:

$$c^2t^2 - x^2 = c^2(\frac{3L}{c})^2 - L^2 = 8L^2 \tag{17}$$

ground:

$$c^2t^2 - x^2 = c^2(\frac{7\sqrt{3}}{3c})^2 - (\frac{5\sqrt{3}L}{3})^2 = 8L^2 \tag{18}$$

ball:

$$c^2t^2 - x^2 = c^2(\frac{2\sqrt{2}L}{c})^2 = 8L^2 \tag{19}$$

(e)

$$\frac{t_g}{t_b} = \frac{\frac{7\sqrt{3}L}{3c}}{\frac{2\sqrt{2}L}{c}} = \frac{7}{2\sqrt{6}} \tag{20}$$

$$s_g = \sqrt{1 - \frac{w^2}{c^2}} = \frac{2\sqrt{6}}{7} = \frac{t_b}{t_g} \tag{21}$$

(f)

$$\frac{t_{tr}}{t_b} = \frac{\frac{3L}{c}}{\frac{2\sqrt{2}L}{c}} = \frac{3}{2\sqrt{2}} \tag{22}$$

$$s_{tr} = \sqrt{1 - \frac{u^2}{c^2}} = \frac{2\sqrt{2}}{3} = \frac{t_b}{t_{tr}} \tag{23}$$

(g)

$$s = \sqrt{1 - \frac{(\frac{c}{2})^2}{c^2}} = \frac{\sqrt{3}}{2} \tag{24}$$

$$\frac{t_{tr}}{t_g} = \frac{\frac{3L}{c}}{\frac{7\sqrt{3}L}{3c}} = \frac{3\sqrt{3}}{7} \neq s \tag{25}$$

Because these two times are both not proper time.