2019-2020春学期《微分几何》第七周作业

P_{34}

1. **解** 由Frenet公式得

$$\begin{cases} dx &= T \\ dT &= kN \\ dN &= -kT+\tau B \\ dB &= -\tau N \end{cases}$$

其中k,τ为曲线的曲率, 挠率. 从而例1中单参数活动标架场的运动方程为

$$\begin{cases} dx &= e_1 \\ de_1 &= ke_2 \\ de_2 &= -ke_1+\tau e_3 \\ de_3 &= -\tau e_2 \end{cases} \blacksquare$$

2. **证明** (1) 因

$$\begin{aligned} d\bar{e}_i &= \bar{\omega}_i^k \bar{e}_k = \bar{\omega}_i^k A_k^j e_j \\ &= d(A_i^j e_j) = dA_i^j e_j + A_i^k de_k = (dA_i^j + A_i^k \omega_k^j) e_j \end{aligned}$$

比较得

$$\bar{\omega}_i^k A_k^j = dA_i^j + \omega_k^j A_i^k.$$

(2) 由(1)及$\bar{\omega}_i^k = \bar{\Gamma}_{i\alpha}^k du^\alpha$, $\omega_i^k = \Gamma_{i\alpha}^k du^\alpha$得

$$\bar{\Gamma}_{i\alpha}^k du^\alpha A_k^j = \frac{\partial A_i^j}{\partial u^\alpha} du^\alpha + \Gamma_{k\alpha}^j du^\alpha A_i^k$$

从而

$$\bar{\Gamma}_{i\alpha}^k A_k^j = \frac{\partial A_i^j}{\partial u^\alpha} + \Gamma_{k\alpha}^j A_i^k. \blacksquare$$

3. **证明** 由曲面的第一基本形式知

$$|x_u| = \sqrt{E}, \quad |x_v| = \sqrt{G} \quad \text{且} \quad x_u \cdot x_v = 0$$

于是

$$e_1 = \frac{x_u}{|x_u|} = \frac{1}{\sqrt{E}} x_u, \quad e_2 = \frac{x_v}{|x_v|} = \frac{1}{\sqrt{G}} x_v$$

因此

$$dx = x_u du + x_v dv = \sqrt{E} du e_1 + \sqrt{G} dv e_2$$

即

$$\omega^1 = \sqrt{E} du, \quad \omega^2 = \sqrt{G} dv, \quad \omega^3 = 0. \blacksquare$$