Homework 10

1 Problem 1:

The mass of an electron is $m = 9.1 \times 10^{-31}$kg.
(a)

$$p_1 = \frac{m \cdot 0.01c}{\sqrt{1 - (\frac{0.01c}{c})^2}} \approx 2.73 \times 10^{-24} kg \cdot m/s,$$

(b)

$$p_2 = \frac{m \cdot 0.5c}{\sqrt{1 - (\frac{0.5c}{c})^2}} \approx 1.58 \times 10^{-22} kg \cdot m/s \tag{1}$$

(c)

$$p_3 = \frac{m \cdot 0.9c}{\sqrt{1 - (\frac{0.9c}{c})^2}} \approx 5.64 \times 10^{-22} kg \cdot m/s \tag{2}$$

$$\frac{p' - p}{p'} = \frac{\gamma - 1}{\gamma} = 0.01 \Longrightarrow \sqrt{1 - \frac{v^2}{c^2}} = 0.99 \Longrightarrow v \approx 0.14c \tag{3}$$

2 Problem 2:

(a)

$$E_k = m_0 c^2 (\gamma - 1) = 20.0\text{Gev} = 3.2 \times 10^{-9}\text{J} \Longrightarrow \gamma = 3.907 \times 10^4 \tag{4}$$

(b)

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Longrightarrow v \approx 0.9999999997c \tag{5}$$

(c)

$$l = \frac{l_0}{\gamma} = \frac{3 \times 10^3}{3.907 \times 10^4} \approx 0.077m \tag{6}$$

3 Problem 3:

The mass lost in this reaction is:

$$\Delta m = \frac{\Delta E}{c^2} = \frac{2.86 \times 10^5}{(3 \times 10^8)^2} = 3.18 \times 10^{-12}\text{kg}$$

No.

4 Problem 4:

$$E = Pt = 1.00 \times 10^9 W \times 3 \times 365 \times 24 \times 3600 s \times 80\% = 7.57 \times 10^{16}\text{J} \tag{7}$$

$$E = \Delta m c^2 \Longrightarrow \Delta m = 0.84\text{kg} \tag{8}$$

5 Problem 5:

(i)In the frame K' that moves with speed v along the positive x direction with respect to the rest frame K, the red particle is at rest before the collision.

In the view of the frame K', the velocity of the blue particle(v_2) before the collision :

$$v_2 = \frac{-v - v}{1 - \frac{(-v)\cdot v}{c^2}} = -\frac{2v}{1 + \frac{v^2}{c^2}}$$

(ii)In the view of the frame K', after the collision, the velocities of the blue particle(v_2') and the red particle(v_1') :

$$\begin{cases} v'_{1x} = \frac{0-v}{1-0\cdot v/c^2} = -v, \\ v'_{1y} = \frac{-v}{\gamma(1-0\cdot v/c^2)} = -v\sqrt{1-v^2/c^2} \end{cases} \Longrightarrow v'_1 = v\sqrt{2-v^2/c^2}$$

$$\begin{cases} v'_{2x} = \frac{0-v}{1-0\cdot v/c^2} = -v, \\ v'_{2y} = \frac{v}{\gamma(1-0\cdot v/c^2)} = v\sqrt{1-v^2/c^2} \end{cases} \Longrightarrow v'_2 = v\sqrt{2-v^2/c^2}$$

(iii) With the definition of the relativistic momentum:

$$\boldsymbol{p} = 0 + \gamma m \boldsymbol{v_2} = \frac{m v_2}{\sqrt{1 - {v_2}^2/c^2}} \boldsymbol{i} = -\frac{2mv}{1 - v^2/c^2} \boldsymbol{i}, \tag{9}$$

$$\boldsymbol{p'} = \gamma' m \boldsymbol{v'_1} + \gamma' m \boldsymbol{v'_2} = 2 \cdot \frac{m v'_{1x}}{\sqrt{1 - {v'_1}^2/c^2}} \boldsymbol{i} + 0 \boldsymbol{j} = -\frac{2mv}{1 - v^2/c^2} \boldsymbol{i} = \boldsymbol{p} \tag{10}$$