Instructor Solutions Manual
for
Physics
by
Halliday, Resnick, and Krane

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Volume 1: Chapters 1-24

A Note To The Instructor...

The solutions here are somewhat brief, as they are designed for the instructor, not for the student. Check with the publishers before electronically posting any part of these solutions; website, ftp, or server access *must* be restricted to your students.

I have been somewhat casual about subscripts whenever it is obvious that a problem is one dimensional, or that the choice of the coordinate system is irrelevant to the *numerical* solution. Although this does not change the validity of the answer, it will sometimes obfuscate the approach if viewed by a novice.

There are some *traditional* formula, such as

$$v_x^2 = v_{0x}^2 + 2a_x x,$$

which are not used in the text. The worked solutions use only material from the text, so there may be times when the solution here seems unnecessarily convoluted and drawn out. Yes, I know an easier approach existed. But if it was not in the text, I did not use it here.

I also tried to avoid reinventing the wheel. There are some exercises and problems in the text which build upon previous exercises and problems. Instead of rederiving expressions, I simply refer you to the previous solution.

I adopt a different approach for rounding of significant figures than previous authors; in particular, I usually round intermediate answers. As such, some of my answers will differ from those in the back of the book.

Exercises and Problems which are enclosed in a box also appear in the Student's Solution Manual with considerably more detail and, when appropriate, include discussion on any physical implications of the answer. These student solutions carefully discuss the steps required for solving problems, point out the relevant equation numbers, or even specify where in the text additional information can be found. When two almost equivalent methods of solution exist, often both are presented. You are encouraged to refer students to the Student's Solution Manual for these exercises and problems. However, the material from the Student's Solution Manual must *not* be copied.

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E1-1 (a) Megaphones; (b) Microphones; (c) Decacards (Deck of Cards); (d) Gigalows (Gigolos); (e) Terabulls (Terribles); (f) Decimates; (g) Centipedes; (h) Nanonanettes (?); (i) Picoboos (Peek-a-Boo); (j) Attoboys ('atta boy); (k) Two Hectowithits (To Heck With It); (l) Two Kilomockingbirds (To Kill A Mockingbird, or Tequila Mockingbird).

E1-2 (a) \$36,000/52 week = \$692/week. (b) \$10,000,000/(20 × 12 month) = \$41,700/month. (c) $30 \times 10^9/8 = 3.75 \times 10^9$.

$\boxed{\textbf{E1-3}}$ Multiply out the factors which make up a century.

$$1 \text{ century} = 100 \text{ years} \left(\frac{365 \text{ days}}{1 \text{ year}}\right)\left(\frac{24 \text{ hours}}{1 \text{ day}}\right)\left(\frac{60 \text{ minutes}}{1 \text{ hour}}\right)$$

This gives 5.256×10^7 minutes in a century, so a microcentury is 52.56 minutes.

The percentage difference from Fermi's approximation is (2.56 min)/(50 min) × 100% or 5.12%.

E1-4 (3000 mi)/(3 hr) = 1000 mi/timezone-hour. There are 24 time-zones, so the circumference is approximately 24 × 1000 mi = 24,000 miles.

E1-5 Actual number of seconds in a year is

$$(365.25 \text{ days})\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 3.1558 \times 10^7 \text{ s}.$$

The percentage error of the approximation is then

$$\frac{3.1416 \times 10^7 \text{ s} - 3.1558 \times 10^7 \text{ s}}{3.1558 \times 10^7 \text{ s}} = -0.45\,\%.$$

E1-6 (a) 10^{-8} seconds per shake means 10^8 shakes per second. There are

$$\left(\frac{365 \text{ days}}{1 \text{ year}}\right)\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 3.1536 \times 10^7 \text{ s/year}.$$

This means there are more shakes in a second.

(b) Humans have existed for a fraction of

$$10^6 \text{ years}/10^{10} \text{ years} = 10^{-4}.$$

That fraction of a day is

$$10^{-4}\,(24 \text{ hr})\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 8.64\,\text{s}.$$

$\boxed{\textbf{E1-7}}$ We'll assume, for convenience only, that the runner with the longer time ran *exactly* one mile. Let the speed of the runner with the shorter time be given by v_1, and call the distance actually ran by this runner d_1. Then $v_1 = d_1/t_1$. Similarly, $v_2 = d_2/t_2$ for the other runner, and $d_2 = 1$ mile.

We want to know when $v_1 > v_2$. Substitute our expressions for speed, and get $d_1/t_1 > d_2/t_2$. Rearrange, and $d_1/d_2 > t_1/t_2$ or $d_1/d_2 > 0.99937$. Then $d_1 > 0.99937$ mile × (5280 feet/1 mile) or $d_1 > 5276.7$ feet is the condition that the first runner was indeed faster. The first track can be no more than 3.3 feet too short to guarantee that the first runner was faster.

E1-8 We will wait until a day's worth of minutes have been gained. That would be

$$(24 \text{ hr})\left(\frac{60 \text{ min}}{1 \text{ hr}}\right) = 1440 \text{ min}.$$

The clock gains one minute per day, so we need to wait 1,440 days, or almost four years. Of course, if it is an older clock with hands that only read 12 hours (instead of 24), then after only 720 days the clock would be correct.

$\boxed{\textbf{E1-9}}$ First find the "logarithmic average" by

$$\begin{aligned} \log t_{\text{av}} &= \frac{1}{2}\left(\log(5\times10^{17}) + \log(6\times10^{-15})\right), \\ &= \frac{1}{2}\log\left(5\times10^{17}\times6\times10^{-15}\right), \\ &= \frac{1}{2}\log 3000 = \log\left(\sqrt{3000}\right). \end{aligned}$$

Solve, and $t_{\text{av}} = 54.8$ seconds.

E1-10 After 20 centuries the day would have increased in length by a total of $20\times0.001\,\text{s} = 0.02\,\text{s}$. The cumulative effect would by the product of the *average* increase and the number of days; that average is half of the maximum, so the cumulative effect is $\frac{1}{2}(2000)(365)(0.02\,\text{s}) = 7300\,\text{s}$. That's about 2 hours.

E1-11 Lunar months are based on the Earth's position, and as the Earth moves around the orbit the Moon has farther to go to complete a phase. In 27.3 days the Moon may have orbited through 360°, but since the Earth moved through $(27.3/365)\times360° = 27°$ the Moon needs to move 27° farther to catch up. That will take $(27°/360°)\times27.3$ days $= 2.05$ days, but in that time the Earth would have moved on yet farther, and the moon will need to catch up again. How much farther? $(2.05/365)\times360° = 2.02°$ which means $(2.02°/360°)\times27.3$ days $= 0.153$ days. The total so far is 2.2 days longer; we could go farther, but at our accuracy level, it isn't worth it.

E1-12 $(1.9\,\text{m})(3.281\text{ ft}/1.000\,\text{m}) = 6.2$ ft, or just under 6 feet, 3 inches.

E1-13 (a) 100 meters = 328.1 feet (Appendix G), or $328.1/3 = 10.9$ yards. This is 28 feet longer than 100 yards, or $(28\text{ ft})(0.3048\text{ m/ft}) = 8.5\,\text{m}$. (b) A metric mile is $(1500\,\text{m})(6.214\times10^{-4}\text{ mi/m}) = 0.932$ mi. I'd rather run the metric mile.

E1-14 There are

$$300,000 \text{ years}\left(\frac{365.25 \text{ days}}{1 \text{ year}}\right)\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 9.5\times10^{12}\text{s}$$

that will elapse before the cesium clock is in error by 1 s. This is almost 1 part in 10^{13}. This kind of accuracy with respect to 2572 miles is

$$10^{-13}(2572 \text{ mi})\left(\frac{1609\,\text{m}}{1 \text{ mi}}\right) = 413 \text{ nm}.$$

$\boxed{\textbf{E1-15}}$ The volume of Antarctica is approximated by the area of the base time the height; the area of the base is the area of a semicircle. Then

$$V = Ah = \left(\frac{1}{2}\pi r^2\right) h.$$

The volume is

$$\begin{aligned} V &= \frac{1}{2}(3.14)(2000 \times 1000 \text{ m})^2(3000 \text{ m}) = 1.88 \times 10^{16} \text{ m}^3 \\ &= 1.88 \times 10^{16} \text{ m}^3 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 1.88 \times 10^{22} \text{ cm}^3. \end{aligned}$$

E1-16 The volume is $(77 \times 10^4\,\text{m}^2)(26\,\text{m}) = 2.00 \times 10^7\,\text{m}^3$. This is equivalent to

$$(2.00 \times 10^7\,\text{m}^3)(10^{-3}\text{ km/m})^3 = 0.02 \text{ km}^3.$$

E1-17 (a) $C = 2\pi r = 2\pi(6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4 \text{ km}$. (b) $A = 4\pi r^2 = 4\pi(6.37 \times 10^3 \text{ km})^2 = 5.10 \times 10^8 \text{ km}$. (c) $V = \frac{4}{3}\pi(6.37 \times 10^3 \text{ km})^3 = 1.08 \times 10^{12} \text{ km}^3$.

E1-18 The conversions: squirrel, 19 km/hr(1000 m/km)/(3600 s/hr) = 5.3 m/s;
rabbit, 30 knots(1.688ft/s/knot)(0.3048 m/ft) = 15 m/s;
snail, 0.030 mi/hr(1609 m/mi)/(3600 s/hr) = 0.013 m/s;
spider, 1.8 ft/s(0.3048 m/ft) = 0.55 m/s;
cheetah, 1.9 km/min(1000 m/km)/(60 s/min) = 32 m/s;
human, 1000 cm/s/(100 cm/m) = 10 m/s;
fox, 1100 m/min/(60 s/min) = 18 m/s;
lion, 1900 km/day(1000 m/km)/(86,400 s/day) = 22 m/s.
The order is snail, spider, squirrel, human, rabbit, fox, lion, cheetah.

$\boxed{\textbf{E1-19}}$ One light-year is the distance traveled by light in one year, or $(3 \times 10^8 \text{ m/s}) \times (1 \text{ year})$. Then

$$19,200\frac{\text{mi}}{\text{hr}}\left(\frac{\text{light-year}}{(3 \times 10^8 \text{ m/s}) \times (1 \text{ year})}\right)\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right)\left(\frac{100 \text{ year}}{1 \text{ century}}\right),$$

which is equal to 0.00286 light-year/century.

E1-20 Start with the British units inverted,

$$\frac{\text{gal}}{30.0 \text{ mi}}\left(\frac{231 \text{ in}^3}{\text{gal}}\right)\left(\frac{1.639 \times 10^{-2} \text{ L}}{\text{in}^3}\right)\left(\frac{\text{mi}}{1.609 \text{ km}}\right) = 7.84 \times 10^{-2} \text{ L/km}.$$

E1-21 (b) A light-year is

$$(3.00 \times 10^5 \text{ km/s})\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right)\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)(365 \text{ days}) = 9.46 \times 10^{12} \text{ km}.$$

A parsec is

$$\frac{1.50 \times 10^8 \text{ km}}{0^\circ\,0'\,1''}\left(\frac{360^\circ}{2\pi \text{ rad}}\right) = \frac{1.50 \times 10^8 \text{ km}}{(1/3600)^\circ}\left(\frac{360^\circ}{2\pi \text{ rad}}\right) = 3.09 \times 10^{13} \text{ km}.$$

(a) $(1.5 \times 10^8 \text{ km})/(3.09 \times 10^{13} \text{ km/pc} = 4.85 \times 10^{-6} \text{ pc}$. $(1.5 \times 10^8 \text{ km})/(9.46 \times 10^{12} \text{ km/ly}) = 1.59 \times 10^{-5} \text{ ly}$.

E1-22 First find the "logarithmic average" by

$$\begin{aligned}\log d_{\text{av}} &= \frac{1}{2}\left(\log(2\times10^{26})+\log(1\times10^{-15})\right),\\ &= \frac{1}{2}\log\left(2\times10^{26}\times1\times10^{-15}\right),\\ &= \frac{1}{2}\log 2\times10^{11}=\log\left(\sqrt{2\times10^{11}}\right).\end{aligned}$$

Solve, and $d_{\text{av}} = 450$ km.

$\boxed{\textbf{E1-23}}$ The number of atoms is given by (1 kg)/(1.00783 × 1.661 × 10^{-27} kg), or 5.974 × 10^{26} atoms.

E1-24 (a) $(2\times1.0+16)\text{u}(1.661\times10^{-27}\text{kg}) = 3.0\times10^{-26}\text{kg}$.
(b) $(1.4\times10^{21}\text{kg})/(3.0\times10^{-26}\text{kg}) = 4.7\times10^{46}$ molecules.

E1-25 The coffee in Paris costs $18.00 per kilogram, or

$$\$18.00\ \text{kg}^{-1}\left(\frac{0.4536\ \text{kg}}{1\ \text{lb}}\right) = \$8.16\ \text{lb}^{-1}.$$

It is cheaper to buy coffee in New York (at least according to the physics textbook, that is.)

E1-26 The room volume is $(21\times13\times12)\text{ft}^3(0.3048\ \text{m/ft})^3 = 92.8\,\text{m}^3$. The mass contained in the room is

$$(92.8\,\text{m}^3)(1.21\,\text{kg/m}^3) = 112\,\text{kg}.$$

$\boxed{\textbf{E1-27}}$ One mole of sugar cubes would have a volume of $N_A\times1.0$ cm^3, where N_A is the Avogadro constant. Since the volume of a cube is equal to the length cubed, $V = l^3$, then $l = \sqrt[3]{N_A}$ cm $= 8.4\times10^7$ cm.

E1-28 The number of seconds in a week is $60\times60\times24\times7 = 6.05\times10^5$. The "weight" loss per second is then

$$(0.23\,\text{kg})/(6.05\times10^5\,\text{s}) = 3.80\times10^{-1}\ \text{mg/s}.$$

$\boxed{\textbf{E1-29}}$ The definition of the meter was wavelengths per meter; the question asks for meters per wavelength, so we want to take the reciprocal. The definition is accurate to 9 figures, so the reciprocal should be written as $1/1,650,763.73 = 6.05780211\times10^{-7}\,\text{m} = 605.780211$ nm.

E1-30 (a) $37.76+0.132 = 37.89$. (b) $16.264-16.26325 = 0.001$.

$\boxed{\textbf{E1-31}}$ The easiest approach is to first solve Darcy's Law for K, and then substitute the known SI units for the other quantities. Then

$$K = \frac{VL}{AHt}\ \text{has units of}\ \frac{(\text{m}^3)\,(\text{m})}{(\text{m}^2)\,(\text{m})\,(\text{s})}$$

which can be simplified to m/s.

E1-32 The Planck length, l_P, is found from

$$\begin{aligned}[l_\mathrm{P}] &= [c^i][G^j][h^k],\\ \mathrm{L} &= (\mathrm{LT}^{-1})^i(\mathrm{L}^3\mathrm{T}^{-2}\mathrm{M}^{-1})^j(\mathrm{ML}^2\mathrm{T}^{-1})^k,\\ &= \mathrm{L}^{i+3j+2k}\mathrm{T}^{-i-2j-k}\mathrm{M}^{-j+k}.\end{aligned}$$

Equate powers on each side,

$$\begin{aligned}\mathrm{L}\colon 1 &= i+3j+2k,\\ \mathrm{T}\colon 0 &= -i-2j-k,\\ \mathrm{M}\colon 0 &= -j+k.\end{aligned}$$

Then $j = k$, and $i = -3k$, and $1 = 2k$; so $k = 1/2$, $j = 1/2$, and $i = -3/2$. Then

$$\begin{aligned}[l_\mathrm{P}] &= [c^{-3/2}][G^{1/2}][h^{1/2}],\\ &= (3.00\times10^8\,\mathrm{m/s})^{-3/2}(6.67\times10^{-11}\,\mathrm{m}^3/\mathrm{s}^2\cdot\mathrm{kg})^{1/2}(6.63\times10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s})^{1/2},\\ &= 4.05\times10^{-35}\,\mathrm{m}.\end{aligned}$$

E1-33 The Planck mass, m_P, is found from

$$\begin{aligned}[m_\mathrm{P}] &= [c^i][G^j][h^k],\\ \mathrm{M} &= (\mathrm{LT}^{-1})^i(\mathrm{L}^3\mathrm{T}^{-2}\mathrm{M}^{-1})^j(\mathrm{ML}^2\mathrm{T}^{-1})^k,\\ &= \mathrm{L}^{i+3j+2k}\mathrm{T}^{-i-2j-k}\mathrm{M}^{-j+k}.\end{aligned}$$

Equate powers on each side,

$$\begin{aligned}\mathrm{L}\colon 0 &= i+3j+2k,\\ \mathrm{T}\colon 0 &= -i-2j-k,\\ \mathrm{M}\colon 1 &= -j+k.\end{aligned}$$

Then $k = j+1$, and $i = -3j-1$, and $0 = -1+2k$; so $k = 1/2$, and $j = -1/2$, and $i = 1/2$. Then

$$\begin{aligned}[m_\mathrm{P}] &= [c^{1/2}][G^{-1/2}][h^{1/2}],\\ &= (3.00\times10^8\,\mathrm{m/s})^{1/2}(6.67\times10^{-11}\,\mathrm{m}^3/\mathrm{s}^2\cdot\mathrm{kg})^{-1/2}(6.63\times10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s})^{1/2},\\ &= 5.46\times10^{-8}\,\mathrm{kg}.\end{aligned}$$

$\boxed{\textbf{P1-1}}$ There are $24\times60 = 1440$ traditional minutes in a day. The conversion plan is then fairly straightforward

$$822.8\text{ dec. min}\left(\frac{1440\text{ trad. min}}{1000\text{ dec. min}}\right) = 1184.8\text{ trad. min.}$$

This is traditional minutes since midnight, the time in traditional hours can be found by dividing by 60 min/hr, the integer part of the quotient is the hours, while the remainder is the minutes. So the time is 19 hours, 45 minutes, which would be 7:45 pm.

P1-2 (a) By similar triangles, the ratio of the distances is the same as the ratio of the diameters—390:1.

(b) Volume is proportional to the radius (diameter) cubed, or $390^3 = 5.93\times10^7$.

(c) $0.52^\circ(2\pi/360^\circ) = 9.1\times10^{-3}$ rad. The diameter is then $(9.1\times10^{-3}\text{ rad})(3.82\times10^5\text{ km}) =$ 3500 km.

P1-3 (a) The circumference of the Earth is approximately 40,000 km; 0.5 seconds of an arc is $0.5/(60 \times 60 \times 360) = 3.9 \times 10^{-7}$ of a circumference, so the north-south error is $\pm(3.9 \times 10^{-7})(4 \times 10^7\,\text{m}) = \pm 15.6\,\text{m}$. This is a range of 31 m.

(b) The east-west range is smaller, because the distance measured along a latitude is smaller than the circumference by a factor of the cosine of the latitude. Then the range is $31 \cos 43.6^\circ = 22\,\text{m}$.

(c) The tanker is in Lake Ontario, some 20 km off the coast of Hamlin?

P1-4 Your position is determined by the time it takes for your longitude to rotate "underneath" the sun (in fact, that's the way longitude was measured originally as in 5 hours west of the Azores...) the rate the sun sweep over at equator is 25,000 miles/86,400 s = 0.29 miles/second. The correction factor because of latitude is the cosine of the latitude, so the sun sweeps overhead near England at approximately 0.19 mi/s. Consequently a 30 mile accuracy requires an error in time of no more than (30 mi)/(0.19 mi/s) = 158 seconds.

Trip takes about 6 months, so clock accuracy needs to be within $(158\,\text{s})/(180\text{ day}) = 1.2$ seconds/day.

(b) Same, except 0.5 miles accuracy requires 2.6 s accuracy, so clock needs to be within 0.007 s/day!

P1-5 Let B be breaths/minute while sleeping. Each breath takes in (1.43 g/L)(0.3 L) = 0.429 g; and lets out (1.96 g/L)(0.3 L) = 0.288 g. The net loss is 0.141 g. Multiply by the number of breaths, (8 hr)(60 min./hr)B(0.141 g) = B(67.68 g). I'll take a short nap, and count my breaths, then finish the problem.

I'm back now, and I found my breaths to be 8/minute. So I lose 541 g/night, or about 1 pound.

P1-6 The mass of the water is $(1000\,\text{kg/m}^3)(5700\,\text{m}^3) = 5.7 \times 10^6\,\text{kg}$. The rate that water leaks drains out is

$$\frac{(5.7 \times 10^6\,\text{kg})}{(12\text{ hr})(3600\text{ s/hr})} = 132\,\text{kg/s}.$$

$\boxed{\textbf{P1-7}}$ Let the radius of the grain be given by r_g. Then the surface area of the grain is $A_g = 4\pi r_g^2$, and the volume is given by $V_g = (4/3)\pi r_g^3$.

If N grains of sand have a total surface area equal to that of a cube 1 m on a edge, then $NA_g = 6\text{ m}^2$. The total volume V_t of this number of grains of sand is NV_g. Eliminate N from these two expressions and get

$$V_t = NV_g = \frac{(6\text{ m}^2)}{A_g}V_g = \frac{(6\text{ m}^2)r_g}{3}.$$

Then $V_t = (2\text{ m}^2)(50 \times 10^{-6}\text{ m}) = 1 \times 10^{-4}\text{ m}^3$.

The mass of a volume V_t is given by

$$1 \times 10^{-4}\text{ m}^3\left(\frac{2600\text{ kg}}{1\text{ m}^3}\right) = 0.26\text{ kg}.$$

P1-8 For a cylinder $V = \pi r^2 h$, and $A = 2\pi r^2 + 2\pi r h$. We want to minimize A with respect to changes in r, so

$$\begin{aligned}\frac{dA}{dr} &= \frac{d}{dr}\left(2\pi r^2 + 2\pi r\frac{V}{\pi r^2}\right),\\ &= 4\pi r - 2\frac{V}{r^2}.\end{aligned}$$

Set this equal to zero; then $V = 2\pi r^3$. Notice that $h = 2r$ in this expression.

P1-9 (a) The volume per particle is

$$(9.27 \times 10^{-26}\text{kg})/(7870\,\text{kg/m}^3) = 1.178 \times 10^{-28}\text{m}^3.$$

The radius of the corresponding sphere is

$$r = \sqrt[3]{\frac{3(1.178 \times 10^{-28}\text{m}^3)}{4\pi}} = 1.41 \times 10^{-10}\text{m}.$$

Double this, and the spacing is 282 pm.

(b) The volume per particle is

$$(3.82 \times 10^{-26}\text{kg})/(1013\,\text{kg/m}^3) = 3.77 \times 10^{-29}\text{m}^3.$$

The radius of the corresponding sphere is

$$r = \sqrt[3]{\frac{3(3.77 \times 10^{-29}\text{m}^3)}{4\pi}} = 2.08 \times 10^{-10}\text{m}.$$

Double this, and the spacing is 416 pm.

P1-10 (a) The area of the plate is (8.43 cm)(5.12 cm) = 43.2 cm^2. (b) (3.14)(3.7 cm)^2 = 43 cm^2.

$\boxed{\textbf{E2-1}}$ Add the vectors as is shown in Fig. 2-4. If $\vec{\mathbf{a}}$ has length $a = 4$ m and $\vec{\mathbf{b}}$ has length $b = 3$ m then the sum is given by $\vec{\mathbf{s}}$. The cosine law can be used to find the magnitude s of $\vec{\mathbf{s}}$,

$$s^2 = a^2 + b^2 - 2ab\cos\theta,$$

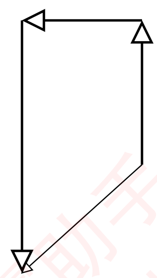
where θ is the angle between sides a and b in the figure.

(a) $(7\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = -1.0$, and $\theta = 180°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing in the same direction.

(b) $(1\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = 1.0$, and $\theta = 0°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing in the opposite direction.

(c) $(5\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = 0$, and $\theta = 90°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing at right angles to each other.

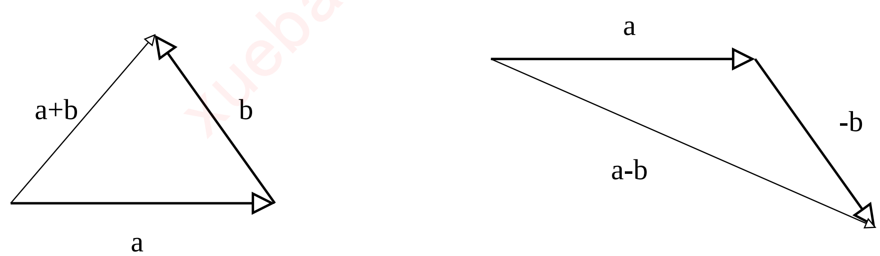
E2-2 (a) Consider the figures below.



(b) Net displacement is 2.4 km west, $(5.2 - 3.1 = 2.1)$ km south. A bird would fly

$$\sqrt{2.4^2 + 2.1^2}\text{ km} = 3.2\text{ km}.$$

E2-3 Consider the figure below.



E2-4 (a) The components are $(7.34)\cos(252°) = -2.27\hat{\mathbf{i}}$ and $(7.34)\sin(252°) = -6.98\hat{\mathbf{j}}$.

(b) The magnitude is $\sqrt{(-25)^2 + (43)^2} = 50$; the direction is $\theta = \tan^{-1}(43/-25) = 120°$. We did need to choose the correct quadrant.

$\boxed{\textbf{E2-5}}$ The components are given by the trigonometry relations

$$O = H\sin\theta = (3.42\text{ km})\sin 35.0° = 1.96\text{ km}$$

and

$$A = H\cos\theta = (3.42\text{ km})\cos 35.0° = 2.80\text{ km}.$$

The stated angle is measured from the east-west axis, counter clockwise from east. So O is measured against the north-south axis, with north being positive; A is measured against east-west with east being positive.

Since her individual steps are displacement vectors which are only north-south or east-west, she must eventually take enough north-south steps to equal 1.96 km, and enough east-west steps to equal 2.80 km. Any individual step can only be along one or the other direction, so the minimum total will be 4.76 km.

E2-6 Let $\vec{\mathbf{r}}_\mathrm{f} = 124\hat{\mathbf{i}}$ km and $\vec{\mathbf{r}}_\mathrm{i} = (72.6\hat{\mathbf{i}} + 31.4\hat{\mathbf{j}})$ km. Then the ship needs to travel

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} = (51.4\hat{\mathbf{i}} + 31.4\hat{\mathbf{j}})\text{ km}.$$

Ship needs to travel $\sqrt{51.4^2 + 31.4^2}$ km $= 60.2$ km in a direction $\theta = \tan^{-1}(31.4/51.4) = 31.4°$ west of north.

$\boxed{\textbf{E2-7}}$ (a) In unit vector notation we need only add the components; $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (5\hat{\mathbf{i}}+3\hat{\mathbf{j}}) + (-3\hat{\mathbf{i}}+2\hat{\mathbf{j}}) = (5-3)\hat{\mathbf{i}} + (3+2)\hat{\mathbf{j}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$.

(b) If we define $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$ and write the magnitude of $\vec{\mathbf{c}}$ as c, then $c = \sqrt{c_x^2 + c_y^2} = \sqrt{2^2 + 5^2} = 5.39$. The direction is given by $\tan\theta = c_y/c_x$ which gives an angle of 68.2°, measured counterclockwise from the positive x-axis.

E2-8 (a) $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (4-1)\hat{\mathbf{i}} + (-3+1)\hat{\mathbf{j}} + (1+4)\hat{\mathbf{k}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$.

(b) $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (4 - -1)\hat{\mathbf{i}} + (-3-1)\hat{\mathbf{j}} + (1-4)\hat{\mathbf{k}} = 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$.

(c) Rearrange, and $\vec{\mathbf{c}} = \vec{\mathbf{b}} - \vec{\mathbf{a}}$, or $\vec{\mathbf{b}} - \vec{\mathbf{a}} = (-1-4)\hat{\mathbf{i}} + (1 - -3)\hat{\mathbf{j}} + (4-1)\hat{\mathbf{k}} = -5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.

E2-9 (a) The magnitude of $\vec{\mathbf{a}}$ is $\sqrt{4.0^2 + (-3.0)^2} = 5.0$; the direction is $\theta = \tan^{-1}(-3.0/4.0) = 323°$.

(b) The magnitude of $\vec{\mathbf{b}}$ is $\sqrt{6.0^2 + 8.0^3} = 10.0$; the direction is $\theta = \tan^{-1}(6.0/8.0) = 36.9°$.

(c) The resultant vector is $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (4.0 + 6.0)\hat{\mathbf{i}} + (-3.0 + 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ is $\sqrt{(10.0)^2 + (5.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(5.0/10.0) = 26.6°$.

(d) The resultant vector is $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (4.0 - 6.0)\hat{\mathbf{i}} + (-3.0 - 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is $\sqrt{(-2.0)^2 + (-11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(-11.0/-2.0) = 260°$.

(e) The resultant vector is $\vec{\mathbf{b}} - \vec{\mathbf{a}} = (6.0 - 4.0)\hat{\mathbf{i}} + (8.0 - -3.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{b}} - \vec{\mathbf{a}}$ is $\sqrt{(2.0)^2 + (11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(11.0/2.0) = 79.7°$.

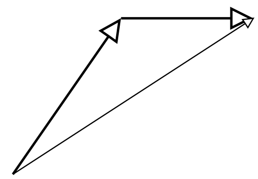
E2-10 (a) Find components of $\vec{\mathbf{a}}$; $a_x = (12.7)\cos(28.2°) = 11.2$, $a_y = (12.7)\sin(28.2°) = 6.00$. Find components of $\vec{\mathbf{b}}$; $b_x = (12.7)\cos(133°) = -8.66$, $b_y = (12.7)\sin(133°) = 9.29$. Then

$$\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} = (11.2 - 8.66)\hat{\mathbf{i}} + (6.00 + 9.29)\hat{\mathbf{j}} = 2.54\hat{\mathbf{i}} + 15.29\hat{\mathbf{j}}.$$

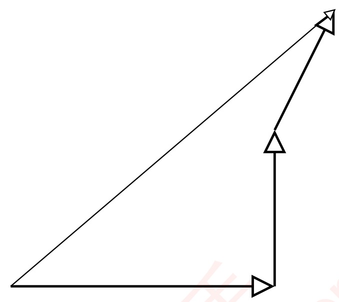
(b) The magnitude of $\vec{\mathbf{r}}$ is $\sqrt{2.54^2 + 15.29^2} = 15.5$.

(c) The angle is $\theta = \tan^{-1}(15.29/2.54) = 80.6°$.

E2-11 Consider the figure below.



E2-12 Consider the figure below.



$\boxed{\textbf{E2-13}}$ Our axes will be chosen so that $\hat{\mathbf{i}}$ points toward 3 O'clock and $\hat{\mathbf{j}}$ points toward 12 O'clock.

(a)

The two relevant positions are $\vec{\mathbf{r}}_\mathrm{i} = (11.3\text{ cm})\hat{\mathbf{i}}$ and $\vec{\mathbf{r}}_\mathrm{f} = (11.3\text{ cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned}\Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} \\ &= (11.3\text{ cm})\hat{\mathbf{j}} - (11.3\text{ cm})\hat{\mathbf{i}}.\end{aligned}$$

(b)

The two relevant positions are now $\vec{\mathbf{r}}_\mathrm{i} = (11.3\text{ cm})\hat{\mathbf{j}}$ and $\vec{\mathbf{r}}_\mathrm{f} = (-11.3\text{ cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned}\Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} \\ &= (11.3\text{ cm})\hat{\mathbf{j}} - (-11.3\text{ cm})\hat{\mathbf{j}} \\ &= (22.6\text{ cm})\hat{\mathbf{j}}.\end{aligned}$$

(c)

The two relevant positions are now $\vec{\mathbf{r}}_\mathrm{i} = (-11.3\text{ cm})\hat{\mathbf{j}}$ and $\vec{\mathbf{r}}_\mathrm{f} = (-11.3\text{ cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned}\Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} \\ &= (-11.3\text{ cm})\hat{\mathbf{j}} - (-11.3\text{ cm})\hat{\mathbf{j}} \\ &= (0\text{ cm})\hat{\mathbf{j}}.\end{aligned}$$

E2-14 (a) The components of $\vec{\mathbf{r}}_1$ are

$$r_{1x} = (4.13\,\mathrm{m})\cos(225^\circ) = -2.92\,\mathrm{m}$$

and

$$r_{1y} = (4.13\,\mathrm{m})\sin(225^\circ) = -2.92\,\mathrm{m}.$$

The components of $\vec{\mathbf{r}}_2$ are

$$r_{1x} = (5.26\,\text{m})\cos(0^\circ) = 5.26\,\text{m}$$

and

$$r_{1y} = (5.26\,\text{m})\sin(0^\circ) = 0\,\text{m}.$$

The components of $\vec{\mathbf{r}}_3$ are

$$r_{1x} = (5.94\,\text{m})\cos(64.0^\circ) = 2.60\,\text{m}$$

and

$$r_{1y} = (5.94\,\text{m})\sin(64.0^\circ) = 5.34\,\text{m}.$$

(b) The resulting displacement is

$$\left[(-2.92 + 5.26 + 2.60)\hat{\mathbf{i}} + (-2.92 + 0 + 5.34)\hat{\mathbf{j}}\right]\text{m} = (4.94\hat{\mathbf{i}} + 2.42\hat{\mathbf{j}})\,\text{m}.$$

(c) The magnitude of the resulting displacement is $\sqrt{4.94^2 + 2.42^2}\,\text{m} = 5.5\,\text{m}$. The direction of the resulting displacement is $\theta = \tan^{-1}(2.42/4.94) = 26.1^\circ$. (d) To bring the particle back to the starting point we need only reverse the answer to (c); the magnitude will be the same, but the angle will be 206°.

E2-15 The components of the initial position are

$$r_{1x} = (12,000\text{ ft})\cos(40^\circ) = 9200\text{ ft}$$

and

$$r_{1y} = (12,000\text{ ft})\sin(40^\circ) = 7700\text{ ft}.$$

The components of the final position are

$$r_{2x} = (25,8000\text{ ft})\cos(163^\circ) = -24,700\text{ ft}$$

and

$$r_{2y} = (25,800\text{ ft})\sin(163^\circ) = 7540\text{ ft}.$$

The displacement is

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \left[(-24,700 - 9,200)\hat{\mathbf{i}} + (7,540 - 9,200)\hat{\mathbf{j}})\right] = (-33900\hat{\mathbf{i}} - 1660\hat{\mathbf{j}})\text{ ft}.$$

E2-16 (a) The displacement vector is $\vec{\mathbf{r}} = (410\hat{\mathbf{i}} - 820\hat{\mathbf{j}})$ mi, where positive x is east and positive y is north. The magnitude of the displacement is $\sqrt{(410)^2 + (-820)^2}$ mi $= 920$ mi. The direction is $\theta = \tan^{-1}(-820/410) = 300^\circ$.

(b) The average velocity is the displacement divided by the *total* time, 2.25 hours. Then

$$\vec{\mathbf{v}}_{\text{av}} = (180\hat{\mathbf{i}} - 360\hat{\mathbf{j}})\text{ mi/hr}.$$

(c) The average speed is total distance over total time, or $(410 + 820)/(2.25)$ mi/hr $= 550$ mi/hr.

$\boxed{\textbf{E2-17}}$ (a) Evaluate $\vec{\mathbf{r}}$ when $t = 2$ s.

$$\begin{aligned}
\vec{\mathbf{r}} &= [(2\text{ m/s}^3)t^3 - (5\text{ m/s})t]\hat{\mathbf{i}} + [(6\text{ m}) - (7\text{ m/s}^4)t^4]\hat{\mathbf{j}} \\
&= [(2\text{ m/s}^3)(2\text{ s})^3 - (5\text{ m/s})(2\text{ s})]\hat{\mathbf{i}} + [(6\text{ m}) - (7\text{ m/s}^4)(2\text{ s})^4]\hat{\mathbf{j}} \\
&= [(16\text{ m}) - (10\text{ m})]\hat{\mathbf{i}} + [(6\text{ m}) - (112\text{ m})]\hat{\mathbf{j}} \\
&= [(6\text{ m})]\hat{\mathbf{i}} + [-(106\text{ m})]\hat{\mathbf{j}}.
\end{aligned}$$

(b) Evaluate:

$$\begin{aligned}
\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} &= [(2\text{ m/s}^3)3t^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(7\text{ m/s}^4)4t^3]\hat{\mathbf{j}} \\
&= [(6\text{ m/s}^3)t^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)t^3]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{v}}(t = 2\text{ s})$ and get

$$\begin{aligned}
\vec{\mathbf{v}} &= [(6\text{ m/s}^3)(2\text{ s})^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)(2\text{ s})^3]\hat{\mathbf{j}} \\
&= [(24\text{ m/s}) - (5\text{ m/s})]\hat{\mathbf{i}} + [-(224\text{ m/s})]\hat{\mathbf{j}} \\
&= [(19\text{ m/s})]\hat{\mathbf{i}} + [-(224\text{ m/s})]\hat{\mathbf{j}},
\end{aligned}$$

for the velocity $\vec{\mathbf{v}}$ when $t = 2$ s.

(c) Evaluate

$$\begin{aligned}
\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} &= [(6\text{ m/s}^3)2t]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)3t^2]\hat{\mathbf{j}} \\
&= [(12\text{ m/s}^3)t]\hat{\mathbf{i}} + [-(84\text{ m/s}^4)t^2]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{a}}(t = 2\text{ s})$ and get

$$\begin{aligned}
\vec{\mathbf{a}} &= [(12\text{ m/s}^3)(2\text{ s})]\hat{\mathbf{i}} + [-(84\text{ m/s}^4)(2\ 2)^2]\hat{\mathbf{j}} \\
&= [(24\text{ m/s}^2)]\hat{\mathbf{i}} + [-(336\text{ m/s}^2)]\hat{\mathbf{j}}.
\end{aligned}$$

E2-18 (a) Let ui point north, $\hat{\mathbf{j}}$ point east, and $\hat{\mathbf{k}}$ point up. The displacement is $(8.7\hat{\mathbf{i}} + 9.7\hat{\mathbf{j}} + 2.9\hat{\mathbf{k}})$ km. The average velocity is found by dividing each term by 3.4 hr; then

$$\vec{\mathbf{v}}_{\text{av}} = (2.6\hat{\mathbf{i}} + 2.9\hat{\mathbf{j}} + 0.85)\text{ km/hr}.$$

The magnitude of the average velocity is $\sqrt{2.6^2 + 2.9^2 + 0.85^2}$ km/hr $= 4.0$ km/hr.

(b) The horizontal velocity has a magnitude of $\sqrt{2.6^2 + 2.9^2}$ km/hr $= 3.9$ km/hr. The angle with the horizontal is given by $\theta = \tan^{-1}(0.85/3.9) = 13°$.

E2-19 (a) The derivative of the velocity is

$$\vec{\mathbf{a}} = [(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t]\hat{\mathbf{i}}$$

so the acceleration at $t = 3\,\text{s}$ is $\vec{\mathbf{a}} = (-18.0\,\text{m/s}^2)\hat{\mathbf{i}}$. (b) The acceleration is zero when $(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t = 0$, or $t = 0.75\,\text{s}$. (c) The velocity is *never* zero; there is no way to "cancel" out the y component. (d) The speed equals $10\,\text{m/s}$ when $10 = \sqrt{v_x^2 + 8^2}$, or $v_x = \pm 6.0\,\text{m/s}$. This happens when $(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t = \pm 6.0\,\text{m/s}$, or when $t = 0\,\text{s}$.

E2-20 If v is constant then so is $v^2 = v_x^2 + v_y^2$. Take the derivative;

$$2v_x \frac{d}{dt} v_x + 2v_y \frac{d}{dt} v_y = 2(v_x a_x + v_y a_y).$$

But if the value is constant the derivative is zero.

$\boxed{\textbf{E2-21}}$ Let the actual flight time, as measured by the passengers, be T. There is some time difference between the two cities, call it ΔT = Namulevu time - Los Angeles time. The ΔT will be positive if Namulevu is east of Los Angeles. The time in Los Angeles can then be found from the time in Namulevu by subtracting ΔT.

The actual time of flight from Los Angeles to Namulevu is then the difference between when the plane lands (LA times) and when the plane takes off (LA time):

$$\begin{aligned} T &= (18{:}50 - \Delta T) - (12{:}50) \\ &= 6{:}00 - \Delta T, \end{aligned}$$

where we have written times in 24 hour format to avoid the AM/PM issue. The return flight time can be found from

$$\begin{aligned} T &= (18{:}50) - (1{:}50 - \Delta T) \\ &= 17{:}00 + \Delta T, \end{aligned}$$

where we have again changed to LA time for the purpose of the calculation.

(b) Now we just need to solve the two equations and two unknowns.

$$\begin{aligned} 17{:}00 + \Delta T &= 6{:}00 - \Delta T \\ 2\Delta T &= 6{:}00 - 17{:}00 \\ \Delta T &= -5{:}30. \end{aligned}$$

Since this is a negative number, Namulevu is located *west* of Los Angeles.

(a) $T = 6{:}00 - \Delta T = 11 : 30$, or eleven and a half hours.

(c) The distance traveled by the plane is given by $d = vt = (520\text{ mi/hr})(11.5\text{ hr}) = 5980$ mi. We'll draw a circle around Los Angeles with a radius of 5980 mi, and then we look for where it intersects with longitudes that would belong to a time zone ΔT away from Los Angeles. Since the Earth rotates once every 24 hours and there are 360 longitude degrees, then each hour corresponds to 15 longitude degrees, and then Namulevu must be located approximately $15^\circ \times 5.5 = 83^\circ$ west of Los Angeles, or at about longitude 160 east. The location on the globe is then latitude 5°, in the vicinity of Vanuatu.

When this exercise was originally typeset the times for the outbound and the inbound flights were inadvertently switched. I suppose that we could blame this on the airlines; nonetheless, when the answers were prepared for the back of the book the reversed numbers put Namulevu *east* of Los Angeles. That would put it in either the North Atlantic or Brazil.

E2-22 There is a three hour time zone difference. So the flight is seven hours long, but it takes 3 hr 51 min for the sun to travel same distance. Look for when the sunset distance has caught up with plane:

$$\begin{aligned} d_{\text{sunset}} &= d_{\text{plane}}, \\ v_{\text{sunset}}(t - 1{:}35) &= v_{\text{plane}} t, \\ (t - 1{:}35)/3{:}51 &= t/7{:}00, \end{aligned}$$

so $t = 3{:}31$ into flight.

E2-23 The distance is

$$d = vt = (112\text{ km/hr})(1\,\text{s})/(3600\text{ s/hr}) = 31\,\text{m}.$$

E2-24 The time taken for the ball to reach the plate is

$$t = \frac{d}{v} = \frac{(18.4\,\text{m})}{(160\text{ km/hr})}(3600\text{ s/hr})/(1000\text{ m/km}) = 0.414\,\text{s}.$$

$\boxed{\textbf{E2-25}}$ Speed is distance traveled divided by time taken; this is equivalent to the inverse of the slope of the line in Fig. 2-32. The line appears to pass through the origin and through the point (1600 km, 80×10^6 y), so the speed is $v = 1600$ km/80×10^6 y$= 2 \times 10^{-5}$ km/y. Converting,

$$v = 2 \times 10^{-5}\text{km/y}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right) = 2\text{ cm/y}$$

E2-26 (a) For Maurice Greene $v_{\text{av}} = (100\,\text{m})/(9.81\,\text{m}) = 10.2\,\text{m/s}$. For Khalid Khannouchi,

$$v_{\text{av}} = \frac{(26.219\text{ mi})}{(2.0950\text{ hr})}\left(\frac{1609\,\text{m}}{1\text{ mi}}\right)\left(\frac{1\text{hr}}{3600\,\text{s}}\right) = 5.594\,\text{m/s}.$$

(b) If Maurice Greene ran the marathon with an average speed equal to his average sprint speed then it would take him

$$t = \frac{(26.219\text{ mi})}{10.2\,\text{m/s}}\left(\frac{1609\,\text{m}}{1\text{ mi}}\right)\left(\frac{1\text{hr}}{3600\,\text{s}}\right) = 1.149\text{ hr},$$

or 1 hour, 9 minutes.

E2-27 The time saved is the difference,

$$\Delta t = \frac{(700\text{ km})}{(88.5\text{ km/hr})} - \frac{(700\text{ km})}{(104.6\text{ km/hr})} = 1.22\text{ hr},$$

which is about 1 hour 13 minutes.

E2-28 The ground elevation will increase by 35 m in a horizontal distance of

$$x = (35.0\,\text{m})/\tan(4.3^\circ) = 465\,\text{m}.$$

The plane will cover that distance in

$$t = \frac{(0.465\text{ km})}{(1300\text{ km/hr})}\left(\frac{3600\,\text{s}}{1\text{hr}}\right) = 1.3\,\text{s}.$$

$\boxed{\textbf{E2-29}}$ Let $v_1 = 40$ km/hr be the speed up the hill, t_1 be the time taken, and d_1 be the distance traveled in that time. We similarly define $v_2 = 60$ km/hr for the down hill trip, as well as t_2 and d_2. Note that $d_2 = d_1$.

$v_1 = d_1/t_1$ and $v_2 = d_2/t_2$. $v_{\rm av} = d/t$, where d total distance and t is the total time. The total distance is $d_1 + d_2 = 2d_1$. The total time t is just the sum of t_1 and t_2, so

$$\begin{aligned} v_{\rm av} &= \frac{d}{t} \\ &= \frac{2d_1}{t_1 + t_2} \\ &= \frac{2d_1}{d_1/v_1 + d_2/v_2} \\ &= \frac{2}{1/v_1 + 1/v_2}, \end{aligned}$$

Take the reciprocal of both sides to get a simpler looking expression

$$\frac{2}{v_{\rm av}} = \frac{1}{v_1} + \frac{1}{v_2}.$$

Then the average speed is 48 km/hr.

E2-30 (a) Average speed is *total* distance divided by *total* time. Then

$$v_{\rm av} = \frac{(240 \text{ ft}) + (240 \text{ ft})}{(240 \text{ ft})/(4.0 \text{ ft/s}) + (240 \text{ ft})/(10 \text{ ft/s})} = 5.7 \text{ ft/s}.$$

(b) Same approach, but different information given, so

$$v_{\rm av} = \frac{(60 \text{ s})(4.0 \text{ ft/s}) + (60 \text{ s})(10 \text{ ft/s})}{(60 \text{ s}) + (60 \text{ s})} = 7.0 \text{ ft/s}.$$

E2-31 The distance traveled is the total area under the curve. The "curve" has four regions: (I) a triangle from 0 to 2 s; (II) a rectangle from 2 to 10 s; (III) a trapezoid from 10 to 12 s; and (IV) a rectangle from 12 to 16 s.

The area underneath the curve is the sum of the areas of the four regions.

$$d = \frac{1}{2}(2\,\text{s})(8\,\text{m/s}) + (8.0\,\text{s})(8\,\text{m/s}) + \frac{1}{2}(2\,\text{s})(8\,\text{m/s} + 4\,\text{m/s}) + (4.0\,\text{s})(4\,\text{m/s}) = 100\,\text{m}.$$

E2-32 The acceleration is the slope of a velocity-time curve,

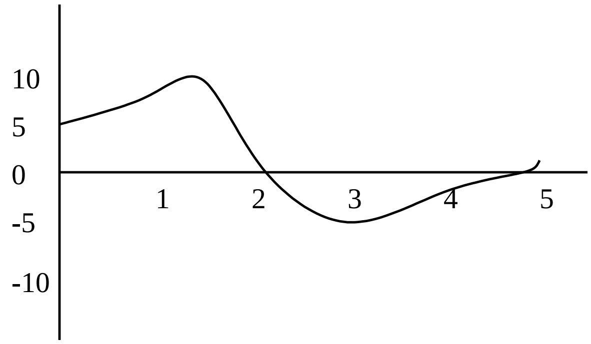
$$a = \frac{(8\,\text{m/s}) - (4\,\text{m/s})}{(10\,\text{s}) - (12\,\text{s})} = -2\,\text{m/s}^2.$$

$\boxed{\textbf{E2-33}}$ The initial velocity is $\vec{\mathbf{v}}_{\rm i} = (18 \text{ m/s})\hat{\mathbf{i}}$, the final velocity is $\vec{\mathbf{v}}_{\rm f} = (-30 \text{ m/s})\hat{\mathbf{i}}$. The average acceleration is then

$$\vec{\mathbf{a}}_{\rm av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_{\rm f} - \vec{\mathbf{v}}_{\rm i}}{\Delta t} = \frac{(-30 \text{ m/s})\hat{\mathbf{i}} - (18 \text{ m/s})\hat{\mathbf{i}}}{2.4 \text{ s}},$$

which gives $\vec{\mathbf{a}}_{\rm av} = (-20.0 \text{ m/s}^2)\hat{\mathbf{i}}$.

E2-34 Consider the figure below.



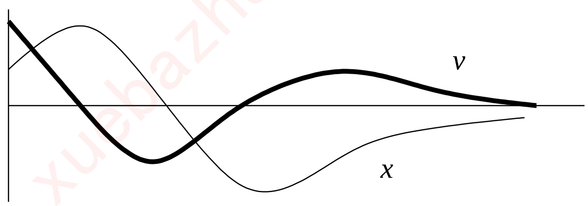
E2-35 (a) Up to A $v_x > 0$ and is constant. From A to B v_x is decreasing, but still positive. From B to C $v_x = 0$. From C to D $v_x < 0$, but $|v_x|$ is decreasing.

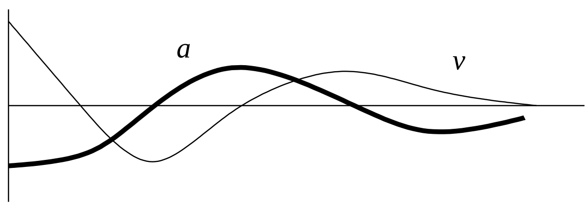
(b) No. Constant acceleration would appear as (part of) a parabola; but it would be challenging to distinguish between a parabola and an almost parabola.

E2-36 (a) Up to A $v_x > 0$ and is decreasing. From A to B $v_x = 0$. From B to C $v_x > 0$ and is increasing. From C to D $v_x > 0$ and is constant.

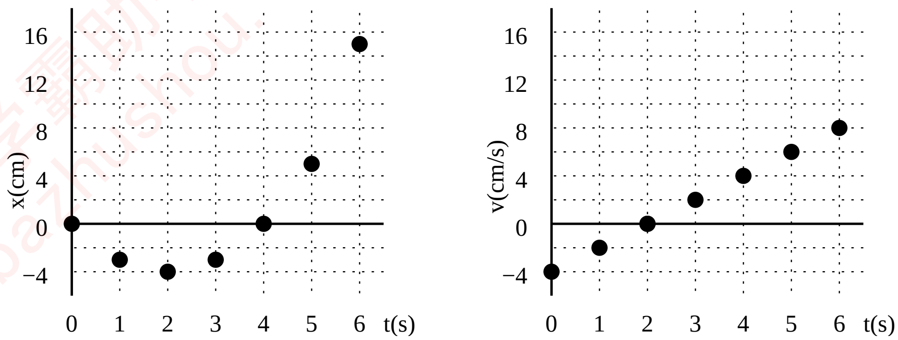
(b) No. Constant acceleration would appear as (part of) a parabola; but it would be challenging to distinguish between a parabola and an almost parabola.

$\boxed{\textbf{E2-37}}$ Consider the figure below.





E2-38 Consider the figure below.



The acceleration is a constant 2 cm/s/s during the entire time interval.

E2-39 (a) A must have units of $\mathrm{m/s^2}$. B must have units of $\mathrm{m/s^3}$.

(b) The maximum positive x position occurs when $v_x = 0$, so

$$v_x = \frac{dx}{dt} = 2At - 3Bt^2$$

implies $v_x = 0$ when either $t = 0$ or $t = 2A/3B = 2(3.0\,\mathrm{m/s^2})/3(1.0\,\mathrm{m/s^3}) = 2.0\,\mathrm{s}$.

(c) Particle starts from rest, then travels in positive direction until $t = 2\,\mathrm{s}$, a distance of

$$x = (3.0\,\mathrm{m/s^2})(2.0\,\mathrm{s})^2 - (1.0\,\mathrm{m/s^3})(2.0\,\mathrm{s})^3 = 4.0\,\mathrm{m}.$$

Then the particle moves back to a final position of

$$x = (3.0\,\mathrm{m/s^2})(4.0\,\mathrm{s})^2 - (1.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^3 = -16.0\,\mathrm{m}.$$

The total path followed was $4.0\,\mathrm{m} + 4.0\,\mathrm{m} + 16.0\,\mathrm{m} = 24.0\,\mathrm{m}$.

(d) The displacement is $-16.0\,\mathrm{m}$ as was found in part (c).

(e) The velocity is $v_x = (6.0\,\mathrm{m/s^2})t - (3.0\,\mathrm{m/s^3})t^2$. When $t = 0$, $v_x = 0.0\,\mathrm{m/s}$. When $t = 1.0\,\mathrm{s}$, $v_x = 3.0\,\mathrm{m/s}$. When $t = 2.0\,\mathrm{s}$, $v_x = 0.0\,\mathrm{m/s}$. When $t = 3.0\,\mathrm{s}$, $v_x = -9.0\,\mathrm{m/s}$. When $t = 4.0\,\mathrm{s}$, $v_x = -24.0\,\mathrm{m/s}$.

(f) The acceleration is the time derivative of the velocity,

$$a_x = \frac{dv_x}{dt} = (6.0\,\mathrm{m/s^2}) - (6.0\,\mathrm{m/s^3})t.$$

When $t = 0\,\mathrm{s}$, $a_x = 6.0\,\mathrm{m/s^2}$. When $t = 1.0\,\mathrm{s}$, $a_x = 0.0\,\mathrm{m/s^2}$. When $t = 2.0\,\mathrm{s}$, $a_x = -6.0\,\mathrm{m/s^2}$. When $t = 3.0\,\mathrm{s}$, $a_x = -12.0\,\mathrm{m/s^2}$. When $t = 4.0\,\mathrm{s}$, $a_x = -18.0\,\mathrm{m/s^2}$.

(g) The distance traveled was found in part (a) to be $-20\,\mathrm{m}$ The average speed during the time interval is then $v_{x,\mathrm{av}} = (-20\,\mathrm{m})/(2.0\,\mathrm{s}) = -10\,\mathrm{m/s}$.

E2-40 $v_{0x} = 0$, $v_x = 360$ km/hr $= 100\,\mathrm{m/s}$. Assuming constant acceleration the average velocity will be

$$v_{x,\mathrm{av}} = \frac{1}{2}(100\,\mathrm{m/s} + 0) = 50\,\mathrm{m/s}.$$

The time to travel the distance of the runway at this average velocity is

$$t = (1800\,\mathrm{m})/(50\,\mathrm{m/s}) = 36\,\mathrm{s}.$$

The acceleration is

$$a_x = 2x/t^2 = 2(1800\,\mathrm{m})/(36.0\,\mathrm{s})^2 = 2.78\,\mathrm{m/s^2}.$$

$\boxed{\textbf{E2-41}}$ (a) Apply Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (3.0 \times 10^7 \text{ m/s}) &= (0) + (9.8 \text{ m/s}^2)t, \\ 3.1 \times 10^6 \text{ s} &= t. \end{aligned}$$

(b) Apply Eq. 2-28 using an initial position of $x_0 = 0$,

$$\begin{aligned} x &= x_0 + v_{0x} + \frac{1}{2}a_x t^2, \\ x &= (0) + (0) + \frac{1}{2}(9.8 \text{ m/s}^2)(3.1 \times 10^6 \text{ s})^2, \\ x &= 4.7 \times 10^{13} \text{ m}. \end{aligned}$$

E2-42 $v_{0x} = 0$ and $v_x = 27.8\,\text{m/s}$. Then

$$t = (v_x - v_{0x})/a = ((27.8\,\text{m/s}) - (0))\,/(50\,\text{m/s}^2) = 0.56\,\text{s}.$$

I want that car.

E2-43 The muon will travel for t seconds before it comes to a rest, where t is given by

$$t = (v_x - v_{0x})/a = \big((0) - (5.20 \times 10^6\text{m/s})\big)\,/(-1.30 \times 10^{14}\text{m/s}^2) = 4 \times 10^{-8}\text{s}.$$

The distance traveled will be

$$x = \frac{1}{2}a_x t^2 + v_{0x}t = \frac{1}{2}(-1.30 \times 10^{14}\text{m/s}^2)(4 \times 10^{-8}\text{s})^2 + (5.20 \times 10^6\text{m/s})(4 \times 10^{-8}\text{s}) = 0.104\,\text{m}.$$

E2-44 The average velocity of the electron was

$$v_{x,\text{av}} = \frac{1}{2}(1.5 \times 10^5 \text{ m/s} + 5.8 \times 10^6 \text{ m/s}) = 3.0 \times 10^6 \text{ m/s}.$$

The time to travel the distance of the runway at this average velocity is

$$t = (0.012\,\text{m})/(3.0 \times 10^6 \text{ m/s}) = 4.0 \times 10^{-9} \text{ s}.$$

The acceleration is

$$a_x = (v_x - v_{0x})/t = ((5.8 \times 10^6 \text{ m/s}) - (1.5 \times 10^5 \text{ m/s}))/(4.0 \times 10^{-9} \text{ s}) = 1.4 \times 10^{15}\text{m/s}^2.$$

$\boxed{\textbf{E2-45}}$ It will be easier to solve the problem if we change the units for the initial velocity,

$$v_{0x} = 1020\frac{\text{km}}{\text{hr}}\left(\frac{1000 \text{ m}}{\text{km}}\right)\left(\frac{\text{hr}}{3600 \text{ s}}\right) = 283\frac{\text{m}}{\text{s}},$$

and then applying Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (0) &= (283 \text{ m/s}) + a_x(1.4 \text{ s}), \\ -202 \text{ m/s}^2 &= a_x. \end{aligned}$$

The problem asks for this in terms of g, so

$$-202 \text{ m/s}^2 \left(\frac{g}{9.8 \text{ m/s}^2}\right) = 21g.$$

E2-46 Change miles to feet and hours to seconds. Then $v_x = 81$ ft/s and $v_{0x} = 125$ ft/s. The time is then

$$t = ((81 \text{ ft/s}) - (125 \text{ ft/s}))/(-17 \text{ ft/s}^2) = 2.6 \text{ s}.$$

E2-47 (a) The time to stop is

$$t = ((0 \text{ m/s}) - (24.6 \text{ m/s}))/(-4.92 \text{ m/s}^2) = 5.00 \text{ s}.$$

(b) The distance traveled is

$$x = \frac{1}{2}a_x t^2 + v_{0x}t = \frac{1}{2}(-4.92 \text{ m/s}^2)(5.00 \text{ s})^2 + (24.6 \text{ m/s})(5.00 \text{ s}) = 62 \text{ m}.$$

E2-48 Answer part (b) first. The average velocity of the arrow while decelerating is

$$v_{y,\text{av}} = \frac{1}{2}((0) + (260 \text{ ft/s})) = 130 \text{ ft/s}.$$

The time for the arrow to travel 9 inches (0.75 feet) is

$$t = (0.75 \text{ ft})/(130 \text{ ft/s}) = 5.8 \times 10^{-3} \text{s}.$$

(a) The acceleration of the arrow is then

$$a_y = (v_y - v_{0y})/t = ((0) - (260 \text{ ft/s}))/(5.8 \times 10^{-3} \text{s}) = -4.5 \times 10^4 \text{ ft/s}^2.$$

$\boxed{\textbf{E2-49}}$ The problem will be somewhat easier if the units are consistent, so we'll write the maximum speed as

$$1000 \frac{\text{ft}}{\text{min}} \left(\frac{\text{min}}{60 \text{ s}}\right) = 16.7 \frac{\text{ft}}{\text{s}}.$$

(a) We can find the time required for the acceleration from Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (16.7 \text{ ft/s}) &= (0) + (4.00 \text{ ft/s}^2)t, \\ 4.18 \text{ s} &= t. \end{aligned}$$

And from this and Eq 2-28 we can find the distance

$$\begin{aligned} x &= x_0 + v_{0x} + \frac{1}{2}a_x t^2, \\ x &= (0) + (0) + \frac{1}{2}(4.00 \text{ ft/s}^2)(4.18 \text{ s})^2, \\ x &= 34.9 \text{ ft}. \end{aligned}$$

(b) The motion of the elevator is divided into three parts: acceleration from rest, constant speed motion, and deceleration to a stop. The total distance is given at 624 ft and in part (a) we found the distance covered during acceleration was 34.9 ft. By symmetry, the distance traveled during deceleration should also be 34.9 ft. The distance traveled at constant speed is then $(624 - 34.9 - 34.9)$ ft $= 554$ ft. The time required for the constant speed portion of the trip is found from Eq. 2-22, rewritten as

$$\Delta t = \frac{\Delta x}{v} = \frac{554 \text{ ft}}{16.7 \text{ ft/s}} = 33.2 \text{ s}.$$

The total time for the trip is the sum of times for the three parts: accelerating (4.18 s), constant speed (33.2 s), and decelerating (4.18 s). The total is 41.6 seconds.

E2-50 (a) The deceleration is found from

$$a_x = \frac{2}{t^2}(x - v_0 t) = \frac{2}{(4.0\,\mathrm{s})^2}((34\,\mathrm{m}) - (16\,\mathrm{m/s})(4.0\,\mathrm{s})) = -3.75\,\mathrm{m/s^2}.$$

(b) The impact speed is

$$v_x = v_{0x} + a_x t = (16\,\mathrm{m/s}) + (-3.75\,\mathrm{m/s^2})(4.0\,\mathrm{s}) = 1.0\,\mathrm{m/s}.$$

E2-51 Assuming the drops fall from rest, the time to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1700\,\mathrm{m})}{(-9.8\,\mathrm{m/s^2})}} = 19\,\mathrm{s}.$$

The velocity of the falling drops would be

$$v_y = a_y t = (-9.8\,\mathrm{m/s^2})(19\,\mathrm{s}) = 190\,\mathrm{m/s},$$

or about 2/3 the speed of sound.

E2-52 Solve the problem out of order.

(b) The time to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-120\,\mathrm{m})}{(-9.8\,\mathrm{m/s^2})}} = 4.9\,\mathrm{s}.$$

(a) The speed at which the elevator hits the ground is

$$v_y = a_y t = (-9.8\,\mathrm{m/s^2})(4.9\,\mathrm{s}) = 48\,\mathrm{m/s}.$$

(d) The time to fall half-way is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-60\,\mathrm{m})}{(-9.8\,\mathrm{m/s^2})}} = 3.5\,\mathrm{s}.$$

(c) The speed at the half-way point is

$$v_y = a_y t = (-9.8\,\mathrm{m/s^2})(3.5\,\mathrm{s}) = 34\,\mathrm{m/s}.$$

$\boxed{\textbf{E2-53}}$ The initial velocity of the "dropped" wrench would be zero. I choose vertical to be along the y axis with up as positive, which is the convention of Eq. 2-29 and Eq. 2-30. It turns out that it is much easier to solve part (b) before solving part (a).

(b) We solve Eq. 2-29 for the time of the fall.

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (-24.0\ \mathrm{m/s}) &= (0) - (9.8\ \mathrm{m/s^2})t, \\ 2.45\ \mathrm{s} &= t. \end{aligned}$$

(a) Now we can use Eq. 2-30 to find the height from which the wrench fell.

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (0) &= y_0 + (0)(2.45\ \mathrm{s}) - \frac{1}{2}(9.8\ \mathrm{m/s^2})(2.45\ \mathrm{s})^2, \\ 0 &= y_0 - 29.4\ \mathrm{m} \end{aligned}$$

We have set $y = 0$ to correspond to the final position of the wrench: on the ground. This results in an initial position of $y_0 = 29.4$ m; it is positive because the wrench was dropped from a point *above* where it landed.

E2-54 (a) It is easier to solve the problem from the point of view of an object which falls from the highest point. The time to fall from the highest point is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-53.7\,\text{m})}{(-9.81\,\text{m/s}^2)}} = 3.31\,\text{s}.$$

The speed at which the object hits the ground is

$$v_y = a_y t = (-9.81\,\text{m/s}^2)(3.31\,\text{s}) = -32.5\,\text{m/s}.$$

But the motion is symmetric, so the object must have been launched up with a velocity of $v_y = 32.5\,\text{m/s}$.

(b) Double the previous answer; the time of flight is 6.62 s.

E2-55 (a) The time to fall the first 50 meters is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-50\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 3.2\,\text{s}.$$

(b) The *total* time to fall 100 meters is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-100\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 4.5\,\text{s}.$$

The time to fall through the second 50 meters is the difference, 1.3 s.

E2-56 The rock returns to the ground with an equal, but opposite, velocity. The acceleration is then

$$a_y = ((-14.6\,\text{m/s}) - (14.6\,\text{m/s}))/(7.72\,\text{s}) = 3.78\,\text{m/s}^2.$$

That would put them on Mercury.

$\boxed{\textbf{E2-57}}$ (a) Solve Eq. 2-30 for the initial velocity. Let the distances be measured from the ground so that $y_0 = 0$.

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (36.8\text{ m}) &= (0) + v_{0y}(2.25\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(2.25\text{ s})^2, \\ 36.8\text{ m} &= v_{0y}(2.25\text{ s}) - 24.8\text{ m}, \\ 27.4\text{ m/s} &= v_{0y}. \end{aligned}$$

(b) Solve Eq. 2-29 for the velocity, using the result from part (a).

$$\begin{aligned} v_y &= v_{0y} - gt, \\ v_y &= (27.4\text{ m/s}) - (9.8\text{ m/s}^2)(2.25\text{ s}), \\ v_y &= 5.4\text{ m/s}. \end{aligned}$$

(c) We need to solve Eq. 2-30 to find the height to which the ball rises, but we don't know how long it takes to get there. So we first solve Eq. 2-29, because we do know the velocity at the highest point ($v_y = 0$).

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= (27.4\text{ m/s}) - (9.8\text{ m/s}^2)t, \\ 2.8\text{ s} &= t. \end{aligned}$$

And then we find the height to which the object rises,

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ y &= (0) + (27.4 \text{ m/s})(2.8 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(2.8 \text{ s})^2, \\ y &= 38.3\text{m}. \end{aligned}$$

This is the height as measured from the ground; so the ball rises $38.3 - 36.8 = 1.5$ m above the point specified in the problem.

E2-58 The time it takes for the ball to fall 2.2 m is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-2.2\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 0.67\,\text{s}.$$

The ball hits the ground with a velocity of

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(0.67\,\text{s}) = -6.6\,\text{m/s}.$$

The ball then bounces up to a height of 1.9 m. It is easier to solve the falling part of the motion, and then apply symmetry. The time is would take to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1.9\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 0.62\,\text{s}.$$

The ball hits the ground with a velocity of

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(0.62\,\text{s}) = -6.1\,\text{m/s}.$$

But we are interested in when the ball moves up, so $v_y = 6.1\,\text{m/s}$.

The acceleration while in contact with the ground is

$$a_y = ((6.1\,\text{m/s}) - (-6.6\,\text{m/s}))/(0.096\,\text{s}) = 130\,\text{m/s}^2.$$

E2-59 The position as a function of time for the first object is

$$y_1 = -\frac{1}{2}gt^2,$$

The position as a function of time for the second object is

$$y_2 = -\frac{1}{2}g(t - 1\,\text{s})^2$$

The difference,

$$\Delta y = y_2 - y_1 = frac12g\left((2\,\text{s})t - 1\right),$$

is the set equal to $10\,\text{m}$, so $t = 1.52\,\text{s}$.

E2-60 Answer part (b) first.

(b) Use the quadratic equation to solve

$$(-81.3\,\text{m}) = \frac{1}{2}(-9.81\,\text{m/s}^2)t^2 + (12.4\,\text{m/s})t$$

for time. Get $t = -3.0\,\text{s}$ and $t = 5.53\,\text{s}$. Keep the positive answer.

(a) Now find final velocity from

$$v_y = (-9.8\,\text{m/s}^2)(5.53\,\text{s}) + (12.4\,\text{m/s}) = -41.8\,\text{m/s}.$$

$\boxed{\textbf{E2-61}}$ The total time the pot is visible is 0.54 s; the pot is visible for 0.27 s on the way down. We'll define the initial position as the highest point and make our measurements from there. Then $y_0 = 0$ and $v_{0y} = 0$. Define t_1 to be the time at which the *falling* pot passes the top of the window y_1, then $t_2 = t_1 + 0.27$ s is the time the pot passes the bottom of the window $y_2 = y_1 - 1.1$ m. We have two equations we can write, both based on Eq. 2-30,

$$\begin{aligned} y_1 &= y_0 + v_{0y}t_1 - \frac{1}{2}gt_1^2, \\ y_1 &= (0) + (0)t_1 - \frac{1}{2}gt_1^2, \end{aligned}$$

and

$$\begin{aligned} y_2 &= y_0 + v_{0y}t_2 - \frac{1}{2}gt_2^2, \\ y_1 - 1.1\text{ m} &= (0) + (0)t_2 - \frac{1}{2}g(t_1 + 0.27\text{ s})^2, \end{aligned}$$

Isolate y_1 in this last equation and then set the two expressions equal to each other so that we can solve for t_1,

$$\begin{aligned} -\frac{1}{2}gt_1^2 &= 1.1\text{ m} - \frac{1}{2}g(t_1 + 0.27\text{ s})^2, \\ -\frac{1}{2}gt_1^2 &= 1.1\text{ m} - \frac{1}{2}g(t_1^2 + [0.54\text{ s}]t_1 + 0.073\text{ s}^2), \\ 0 &= 1.1\text{ m} - \frac{1}{2}g([0.54\text{ s}]t_1 + 0.073\text{ s}^2). \end{aligned}$$

This last line can be directly solved to yield $t_1 = 0.28$ s as the time when the falling pot passes the top of the window. Use this value in the first equation above and we can find $y_1 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.28\text{ s})^2 = -0.38$ m. The negative sign is because the top of the window is beneath the highest point, so the pot must have risen to 0.38 m above the top of the window.

P2-1 (a) The net shift is $\sqrt{(22\,\text{m})^2 + (17\,\text{m})^2)} = 28\,\text{m}$.
(b) The vertical displacement is $(17\,\text{m})\sin(52°) = 13\,\text{m}$.

P2-2 Wheel "rolls" through half of a turn, or $\pi r = 1.41\,\text{m}$. The vertical displacement is $2r = 0.90\,\text{m}$. The net displacement is

$$\sqrt{(1.41\,\text{m})^2 + (0.90\,\text{m})^2} = 1.67\,\text{m}0.$$

The angle is

$$\theta = \tan^{-1}(0.90\,\text{m})/(1.41\,\text{m}) = 33°.$$

$\boxed{\textbf{P2-3}}$ We align the coordinate system so that the origin corresponds to the starting position of the fly and that all positions inside the room are given by positive coordinates.
(a) The displacement vector can just be written,

$$\Delta\vec{\mathbf{r}} = (10\text{ ft})\hat{\mathbf{i}} + (12\text{ ft})\hat{\mathbf{j}} + (14\text{ ft})\hat{\mathbf{k}}.$$

(b) The magnitude of the displacement vector is $|\Delta\vec{\mathbf{r}}| = \sqrt{10^2 + 12^2 + 14^2}$ ft$=$ 21 ft.

(c) The straight line distance between two points is the shortest possible distance, so the length of the path taken by the fly must be greater than or equal to 21 ft.

(d) If the fly walks it will need to cross two faces. The shortest path will be the diagonal across these two faces. If the lengths of sides of the room are l_1, l_2, and l_3, then the diagonal length across two faces will be given by

$$\sqrt{(l_1 + l_2)^2 + l_3^2},$$

where we want to choose the l_i from the set of 10 ft, 12 ft, and 14 ft that will minimize the length. The minimum distance is when $l_1 = 10$ ft, $l_2 = 12$ ft, and $l_3 = 14$. Then the minimal distance the fly would *walk* is 26 ft.

P2-4 Choose vector $\vec{\mathbf{a}}$ to lie on the x axis. Then $\vec{\mathbf{a}} = a\hat{\mathbf{i}}$ and $\vec{\mathbf{b}} = b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}}$ where $b_x = b\cos\theta$ and $b_y = b\sin\theta$. The sum then has components

$$r_x = a + b\cos\theta \text{ and } r_y = b\sin\theta.$$

Then

$$\begin{aligned} r^2 &= (a + b\cos\theta)^2 + (b\sin\theta)^2, \\ &= a^2 + 2ab\cos\theta + b^2. \end{aligned}$$

P2-5 (a) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{(35.0 \text{ mi/hr})(t/2) + (55.0 \text{ mi/hr})(t/2)}{(t/2) + (t/2)} = 45.0 \text{ mi/hr}.$$

(b) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{(d/2) + (d/2)}{(d/2)/(35.0 \text{ mi/hr}) + (d/2)/(55.0 \text{ mi/hr})} = 42.8 \text{ mi/hr}.$$

(c) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{d + d}{(d)/(45.0 \text{ mi/hr}) + (d)/(42.8 \text{ mi/hr})} = 43.9 \text{ mi/hr}$$

P2-6 (a) We'll do just one together. How about $t = 2.0\,\text{s}$?

$$x = (3.0\,\text{m/s})(2.0\,\text{s}) + (-4.0\,\text{m/s}^2)(2.0\,\text{s})^2 + (1.0\,\text{m/s}^3)(2.0\,\text{s})^3 = -2.0\,\text{m}.$$

The rest of the values are, starting from $t = 0$, $x = 0.0$ m, 0.0 m, -2.0 m, 0.0 m, and 12.0 m.

(b) Always final minus initial. The answers are $x_\text{f} - x_\text{i} = -2.0\,\text{m} - 0.0\,\text{m} = -2.0\,\text{m}$ and $x_\text{f} - x_\text{i} = 12.0\,\text{m} - 0.0\,\text{m} = 12.0\,\text{m}$.

(c) Always displacement divided by (change in) time.

$$v_{\text{av}} = \frac{(12.0\,\text{m}) - (-2.0\,\text{m})}{(4.0\,\text{s}) - (2.0\,\text{s})} = 7.0\,\text{m/s},$$

and

$$v_{\text{av}} = \frac{(0.0\,\text{m}) - (0.0\,\text{m})}{(3.0\,\text{s}) - (0.0\,\text{s})} = 0.0\,\text{m/s}.$$

$\boxed{\textbf{P2-7}}$ (a) Assume the bird has no size, the trains have some separation, and the bird is just leaving one of the trains. The bird will be able to fly from one train to the other *before* the two trains collide, regardless of how close together the trains are. After doing so, the bird is now on the other train, the trains are still separated, so once again the bird can fly between the trains before they collide. This process can be repeated every time the bird touches one of the trains, so the bird will make an infinite number of trips between the trains.

(b) The trains collide in the middle; therefore the trains collide after (51 km)/(34 km/hr) = 1.5 hr. The bird was flying with constant speed this entire time, so the distance flown by the bird is (58 km/hr)(1.5 hr) = 87 km.

P2-8 (a) Start with a perfect square:

$$\begin{aligned}
(v_1 - v_2)^2 &> 0,\\
v_1^2 + v_2^2 &> 2v1v2,\\
(v_1^2 + v_2^2)t_1t_2 &> 2v_1v_2t_1t_2,\\
d_1^2 + d_2^2 + (v_1^2 + v_2^2)t_1t_2 &> d_1^2 + d_2^2 + 2v_1v_2t_1t_2,\\
(v_1^2t_1 + v_2^2t_2)(t_1 + t_2) &> (d_1 + d_2)^2,\\
\frac{v_1^2t_1 + v_2^2t_2}{d_1 + d_2} &> \frac{d_1 + d_2}{t_1 + t_2},\\
\frac{v_1d_1 + v_2d_2}{d_1 + d_2} &> \frac{v_1t_1 + v_2t_2}{t_1 + t_2}
\end{aligned}$$

Actually, it only works if $d_1 + d_2 > 0$!

(b) If $v_1 = v_2$.

P2-9 (a) The average velocity during the time interval is $v_{\text{av}} = \Delta x/\Delta t$, or

$$v_{\text{av}} = \frac{(A + B(3\,\text{s})^3) - (A + B(2\,\text{s})^3)}{(3\,\text{s}) - (2\,\text{s})} = (1.50\ \text{cm/s}^3)(19\,\text{s}^3)/(1\,\text{s}) = 28.5\ \text{cm/s}.$$

(b) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2\,\text{s})^2 = 18\ \text{cm/s}.$

(c) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(3\,\text{s})^2 = 40.5\ \text{cm/s}.$

(d) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2.5\,\text{s})^2 = 28.1\ \text{cm/s}.$

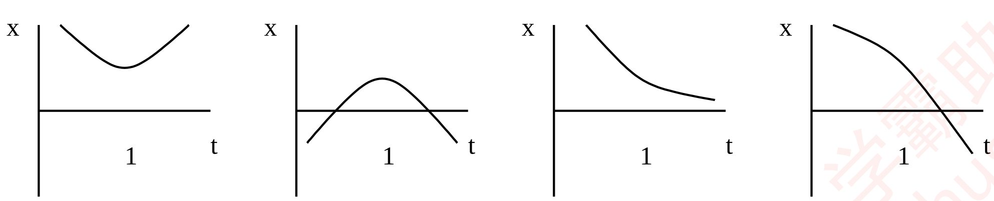
(e) The midway position is $(x_\text{f} + x_\text{i})/2$, or

$$x_{\text{mid}} = A + B[(3\,\text{s})^3 + (2\,\text{s})^3)]/2 = A + (17.5\,\text{s}^3)B.$$

This occurs when $t = \sqrt[3]{(17.5\,\text{s}^3)}$. The instantaneous velocity at this point is

$$v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(\sqrt[3]{(17.5\,\text{s}^3)})^2 = 30.3\ \text{cm/s}.$$

P2-10 Consider the figure below.



(a) (b) (c) (d)

P2-11 (a) The average velocity is displacement divided by change in time,

$$v_{\rm av} = \frac{(2.0\ {\rm m/s}^3)(2.0\ {\rm s})^3 - (2.0\ {\rm m/s}^3)(1.0\ {\rm s})^3}{(2.0\ {\rm s}) - (1.0\ {\rm s})} = \frac{14.0\ {\rm m}}{1.0\ {\rm s}} = 14.0\ {\rm m/s}.$$

The average acceleration is the change in velocity. So we need an expression for the velocity, which is the time derivative of the position,

$$v = \frac{dx}{dt} = \frac{d}{dt}(2.0\ {\rm m/s}^3)t^3 = (6.0\ {\rm m/s}^3)t^2.$$

From this we find average acceleration

$$a_{\rm av} = \frac{(6.0\ {\rm m/s}^3)(2.0\ {\rm s})^2 - (6.0\ {\rm m/s}^3)(1.0\ {\rm s})^2}{(2.0\ {\rm s}) - (1.0\ {\rm s})} = \frac{18.0\ {\rm m/s}}{1.0\ {\rm s}} = 18.0\ {\rm m/s}^2.$$

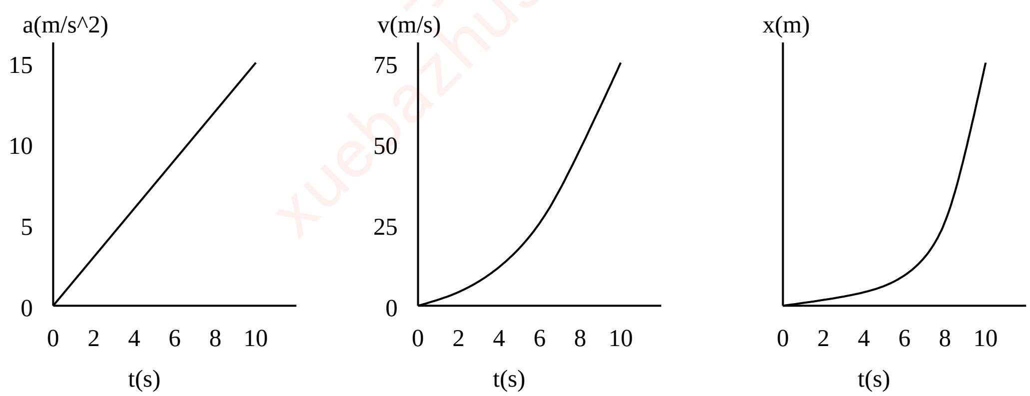
(b) The instantaneous velocities can be found directly from $v = (6.0\ {\rm m/s}^2)t^2$, so $v(2.0\ {\rm s}) = 24.0$ m/s and $v(1.0\ {\rm s}) = 6.0$ m/s. We can get an expression for the instantaneous acceleration by taking the time derivative of the velocity

$$a = \frac{dv}{dt} = \frac{d}{dt}(6.0\ {\rm m/s}^3)t^2 = (12.0\ {\rm m/s}^3)t.$$

Then the instantaneous accelerations are $a(2.0\ {\rm s}) = 24.0\ {\rm m/s}^2$ and $a(1.0\ {\rm s}) = 12.0\ {\rm m/s}^2$

(c) Since the motion is monotonic we expect the average quantities to be somewhere between the instantaneous values at the endpoints of the time interval. Indeed, that is the case.

P2-12 Consider the figure below.



P2-13 Start with $v_{\rm f} = v_{\rm i} + at$, but $v_{\rm f} = 0$, so $v_{\rm i} = -at$, then

$$x = \frac{1}{2}at^2 + v_{\rm i}t = \frac{1}{2}at^2 - at^2 = -\frac{1}{2}at^2,$$

so $t = \sqrt{-2x/a} = \sqrt{-2(19.2\ {\rm ft})/(-32\ {\rm ft/s}^2)} = 1.10\,{\rm s}$. Then $v_{\rm i} = -(-32\ {\rm ft/s}^2)(1.10\,{\rm s}) = 35.2$ ft/s. Converting,

$$35.2\ {\rm ft/s}(1/5280\ {\rm mi/ft})(3600\ {\rm s/h}) = 24\ {\rm mi/h}.$$

P2-14 (b) The average speed during while traveling the 160 m is

$$v_{\rm av} = (33.0\,{\rm m/s} + 54.0\,{\rm m/s})/2 = 43.5\,{\rm m/s}.$$

The time to travel the 160 m is $t = (160\,{\rm m})/(43.5\,{\rm m/s}) = 3.68\,{\rm s}$.

(a) The acceleration is

$$a = \frac{2x}{t^2} - \frac{2v_i}{t} = \frac{2(160\,{\rm m})}{(3.68\,{\rm s})^2} - \frac{2(33.0\,{\rm m/s})}{(3.68\,{\rm s})} = 5.69\,{\rm m/s^2}.$$

(c) The time required to get up to a speed of 33 m/s is

$$t = v/a = (33.0\,{\rm m/s})/(5.69\,{\rm m/s^2}) = 5.80\,{\rm s}.$$

(d) The distance moved from start is

$$d = \frac{1}{2}at^2 = \frac{1}{2}(5.69\,{\rm m/s^2})(5.80\,{\rm s})^2 = 95.7\,{\rm m}.$$

P2-15 (a) The distance traveled during the reaction time happens at constant speed; $t_{\rm reac} = d/v = (15\,{\rm m})/(20\,{\rm m/s}) = 0.75\,{\rm s}$.

(b) The braking distance is proportional to the speed squared (look at the numbers!) and in this case is $d_{\rm brake} = v^2/(20\,{\rm m/s^2})$. Then $d_{\rm brake} = (25\,{\rm m/s})^2/(20\,{\rm m/s^2}) = 31.25\,{\rm m}$. The reaction time distance is $d_{\rm reac} = (25\,{\rm m/s})(0.75\,{\rm s}) = 18.75\,{\rm m}$. The stopping distance is then 50 m.

P2-16 (a) For the car $x_{\rm c} = a_{\rm c}t^2/2$. For the truck $x_{\rm t} = v_{\rm t}t$. Set both x_i to the same value, and then substitute the time from the truck expression:

$$x = a_{\rm c}t^2/2 = a_{\rm c}(x/v_{\rm t})^2/2,$$

or

$$x = 2{v_{\rm t}}^2/a_{\rm c} = 2(9.5\,{\rm m/s})^2/(2.2\,{\rm m/s}) = 82\,{\rm m}.$$

(b) The speed of the car will be given by $v_{\rm c} = a_{\rm c}t$, or

$$v_{\rm c} = a_{\rm c}t = a_{\rm c}x/v_{\rm t} = (2.2\,{\rm m/s})(82\,{\rm m})/(9.5\,{\rm m/s}) = 19\,{\rm m/s}.$$

P2-17 The runner covered a distance d_1 in a time interval t_1 during the acceleration phase and a distance d_2 in a time interval t_2 during the constant speed phase. Since the runner started from rest we know that the constant speed is given by $v = at_1$, where a is the runner's acceleration.

The distance covered during the acceleration phase is given by

$$d_1 = \frac{1}{2}at_1^2.$$

The distance covered during the constant speed phase can also be found from

$$d_2 = vt_2 = at_1t_2.$$

We want to use these two expressions, along with $d_1 + d_2 = 100$ m and $t_2 = (12.2\ {\rm s}) - t_1$, to get

$$\begin{aligned} 100\ {\rm m} &= d_1 + d_2 = \frac{1}{2}at_1^2 + at_1(12.2\ {\rm s} - t_1), \\ &= -\frac{1}{2}at_1^2 + a(12.2\ {\rm s})t_1, \\ &= -(1.40\ {\rm m/s^2})t_1^2 + (34.2\ {\rm m/s})t_1. \end{aligned}$$

This last expression is quadratic in t_1, and is solved to give $t_1 = 3.40\,\mathrm{s}$ or $t_1 = 21.0\,\mathrm{s}$. Since the race only lasted 12.2 s we can ignore the second answer.

(b) The distance traveled during the acceleration phase is then

$$d_1 = \frac{1}{2}at_1^2 = (1.40\ \mathrm{m/s^2})(3.40\,\mathrm{s})^2 = 16.2\,\mathrm{m}.$$

P2-18 (a) The ball will return to the ground with the same speed it was launched. Then the total time of flight is given by

$$t = (v_\mathrm{f} - v_\mathrm{i})/g = (-25\,\mathrm{m/s} - 25\,\mathrm{m/s})/(9.8\,\mathrm{m/s^2}) = 5.1\,\mathrm{s}.$$

(b) For small quantities we can think in terms of derivatives, so

$$\delta t = (\delta v_\mathrm{f} - \delta v_\mathrm{i})/g,$$

or $\tau = 2\epsilon/g$.

P2-19 Use $y = -gt^2/2$, but only keep the absolute value. Then $y_{50} = (9.8\,\mathrm{m/s^2})(0.05\,\mathrm{s})^2/2 =$ 1.2 cm; $y_{100} = (9.8\,\mathrm{m/s^2})(0.10\,\mathrm{s})^2/2 = 4.9$ cm; $y_{150} = (9.8\,\mathrm{m/s^2})(0.15\,\mathrm{s})^2/2 = 11$ cm; $y_{200} = (9.8\,\mathrm{m/s^2})(0.20\,\mathrm{s})^2/2 = 20$ cm; $y_{250} = (9.8\,\mathrm{m/s^2})(0.25\,\mathrm{s})^2/2 = 31$ cm.

P2-20 The truck will move 12 m in $(12\,\mathrm{m})/(55\ \mathrm{km/h}) = 0.785\,\mathrm{s}$. The apple will fall $y = -gt^2/2 = -(9.81\,\mathrm{m/s^2})(0.785\,\mathrm{s})^2/2 = -3.02\,\mathrm{m}$.

$\boxed{\textbf{P2-21}}$ The rocket travels a distance $d_1 = \frac{1}{2}at_1^2 = \frac{1}{2}(20\ \mathrm{m/s^2})(60\ \mathrm{s})^2 = 36,000$ m during the acceleration phase; the rocket velocity at the end of the acceleration phase is $v = at = (20\ \mathrm{m/s^2})(60\ \mathrm{s}) =$ 1200 m/s. The second half of the trajectory can be found from Eqs. 2-29 and 2-30, with $y_0 = 36,000$ m and $v_{0y} = 1200$ m/s.

(a) The highest point of the trajectory occurs when $v_y = 0$, so

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= (1200\ \mathrm{m/s}) - (9.8\ \mathrm{m/s^2})t, \\ 122\ \mathrm{s} &= t. \end{aligned}$$

This time is used to find the height to which the rocket rises,

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ &= (36000\ \mathrm{m}) + (1200\ \mathrm{m/s})(122\mathrm{s}) - \frac{1}{2}(9.8\ \mathrm{m/s^2})(122\ \mathrm{s})^2 = 110000\ \mathrm{m}. \end{aligned}$$

(b) The easiest way to find the total time of flight is to solve Eq. 2-30 for the time when the rocket has returned to the ground. Then

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (0) &= (36000\ \mathrm{m}) + (1200\ \mathrm{m/s})t - \frac{1}{2}(9.8\ \mathrm{m/s^2})t^2. \end{aligned}$$

This quadratic expression has two solutions for t; one is negative so we don't need to worry about it, the other is $t = 270\,\mathrm{s}$. This is the free-fall part of the problem, to find the total time we need to add on the 60 seconds of accelerated motion. The total time is then 330 seconds.

P2-22 (a) The time required for the player to "fall" from the highest point a distance of $y = 15$ cm is $\sqrt{2y/g}$; the total time spent in the top 15 cm is twice this, or $2\sqrt{2y/g} = 2\sqrt{2(0.15\,\text{m})/(9.81\,\text{m/s})} = 0.350\,\text{s}$.

(b) The time required for the player to "fall" from the highest point a distance of 76 cm is $\sqrt{2(0.76\,\text{m})/(9.81\,\text{m/s})} = 0.394\,\text{s}$, the time required for the player to fall from the highest point a distance of $(76 - 15 = 61)$ cm is $\sqrt{2(0.61\,\text{m})/g} = 0.353\,\text{s}$. The time required to fall the bottom 15 cm is the difference, or 0.041 s. The time spent in the bottom 15 cm is twice this, or 0.081 s.

P2-23 (a) The average speed between A and B is $v_{\text{av}} = (v + v/2)/2 = 3v/4$. We can also write $v_{\text{av}} = (3.0\,\text{m})/\Delta t = 3v/4$. Finally, $v/2 = v - g\Delta t$. Rearranging, $v/2 = g\Delta t$. Combining all of the above,

$$\frac{v}{2} = g\left(\frac{4(3.0\,\text{m})}{3v}\right) \text{ or } v^2 = (8.0\,\text{m})g.$$

Then $v = \sqrt{(8.0\,\text{m})(9.8\,\text{m/s}^2)} = 8.85\,\text{m/s}$.

(b) The time to the highest point above B is $v/2 = gt$, so the distance above B is

$$y = -\frac{g}{2}t^2 + \frac{v}{2}t = -\frac{g}{2}\left(\frac{v}{2g}\right)^2 + \frac{v}{2}\left(\frac{v}{2g}\right) = \frac{v^2}{8g}.$$

Then $y = (8.85\,\text{m/s})^2/(8(9.8\,\text{m/s}^2)) = 1.00\,\text{m}$.

P2-24 (a) The time in free fall is $t = \sqrt{-2y/g} = \sqrt{-2(-145\,\text{m})/(9.81\,\text{m/s}^2)} = 5.44\,\text{s}$.

(b) The speed at the bottom is $v = -gt = -(9.81\,\text{m/s}^2)(5.44\,\text{s}) = -53.4\,\text{m/s}$.

(c) The time for deceleration is given by $v = -25gt$, or $t = -(-53.4\,\text{m/s})/(25 \times 9.81\,/\text{s}^2) = 0.218\,\text{s}$. The distance through which deceleration occurred is

$$y = \frac{25g}{2}t^2 + vt = (123\,\text{m/s}^2)(0.218\,\text{s})^2 + (-53.4\,\text{m/s})(0.218\,\text{s}) = -5.80\,\text{m}.$$

$\boxed{\textbf{P2-25}}$ Find the time she fell from Eq. 2-30,

$$(0\text{ ft}) = (144\text{ ft}) + (0)t - \frac{1}{2}(32\text{ ft/s}^2)t^2,$$

which is a simple quadratic with solutions $t = \pm 3.0$ s. Only the positive solution is of interest. Use this time in Eq. 2-29 to find her speed when she hit the ventilator box,

$$v_y = (0) - (32\text{ ft/s}^2)(3.0\text{ s}) = -96\text{ ft/s}.$$

This becomes the initial velocity for the deceleration motion, so her average speed during deceleration is given by Eq. 2-27,

$$v_{\text{av},y} = \frac{1}{2}(v_y + v_{0y}) = \frac{1}{2}((0) + (-96\text{ ft/s})) = -48\text{ ft/s}.$$

This average speed, used with the distance of 18 in (1.5 ft), can be used to find the time of deceleration

$$v_{\text{av},y} = \Delta y/\Delta t,$$

and putting numbers into the expression gives $\Delta t = 0.031$ s. We actually used $\Delta y = -1.5$ ft, where the negative sign indicated that she was still moving downward. Finally, we use this in Eq. 2-26 to find the acceleration,

$$(0) = (-96\text{ ft/s}) + a(0.031\text{ s}),$$

which gives $a = +3100\text{ ft/s}^2$. In terms of g this is $a = 97g$, which can be found by multiplying through by $1 = g/(32\text{ ft/s}^2)$.

P2-26 Let the speed of the disk when it comes into contact with the ground be v_1; then the average speed during the deceleration process is $v_1/2$; so the time taken for the deceleration process is $t_1 = 2d/v_1$, where $d = -2\,\text{mm}$. But d is also give by $d = at_1^2/2 + v_1 t_1$, so

$$d = \frac{100g}{2}\left(\frac{2d}{v_1}\right)^2 + v_1\left(\frac{2d}{v_1}\right) = 200g\frac{d^2}{v_1^2} + 2d,$$

or $v_1^2 = -200gd$. The negative signs *are* necessary!.

The disk was dropped from a height $h = -y$ and it first came into contact with the ground when it had a speed of v_1. Then the average speed is $v_1/2$, and we can repeat most of the above (except $a = -g$ instead of $100g$), and then the time to fall is $t_2 = 2y/v_1$,

$$y = \frac{g}{2}\left(\frac{2y}{v_1}\right)^2 + v_1\left(\frac{2y}{v_1}\right) = 2g\frac{y^2}{v_1^2} + 2y,$$

or $v_1^2 = -2gy$. The negative signs *are* necessary!.

Equating, $y = 100d = 100(-2\,\text{mm}) = -0.2\,\text{m}$, so $h = 0.2\,\text{m}$. Note that although $100g$'s sounds like plenty, you still shouldn't be dropping your hard disk drive!

P2-27 Measure from the feet! Jim is 2.8 cm tall in the photo, so 1 cm on the photo is 60.7 cm in real-life. Then Jim has fallen a distance $y_1 = -3.04\,\text{m}$ while Clare has fallen a distance $y_2 = -5.77\,\text{m}$. Clare jumped first, and the time she has been falling is t_2; Jim jumped seconds, the time he has been falling is $t_1 = t_2 - \Delta t$. Then $y_2 = -gt_2^2/2$ and $y_1 = -gt_1^2/2$, or $t_2 = \sqrt{-2y_2/g} = \sqrt{-2(-5.77\,\text{m})/(9.81\,\text{m/s}^2)} = 1.08\,\text{s}$ and $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(3.04\,\text{m})/(9.81\,\text{m/s}^2)} = 0.79\,\text{s}$. So Jim waited $0.29\,\text{s}$.

P2-28 (a) Assuming she starts from rest and has a speed of v_1 when she opens her chute, then her average speed while falling freely is $v_1/2$, and the time taken to fall $y_1 = -52.0\,\text{m}$ is $t_1 = 2y_1/v_1$. Her speed v_1 is given by $v_1 = -gt_1$, or $v_1^2 = -2gy_1$. Then $v_1 = -\sqrt{-2(9.81\,\text{m/s}^2)(-52.0\,\text{m})} = -31.9\,\text{m/s}$. We must use the *negative* answer, because she fall down! The time in the air is then $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(52.0\,\text{m})/(9.81\,\text{m/s}^2)} = 3.26\,\text{s}$.

Her final speed is $v_2 = -2.90\,\text{m/s}$, so the time for the deceleration is $t_2 = (v_2 - v_1)/a$, where $a = 2.10\,\text{m/s}^2$. Then $t_2 = (-2.90\,\text{m/s} - -31.9\,\text{m/s})/(2.10\,\text{m/s}^2) = 13.8\,\text{s}$.

Finally, the total time of flight is $t = t_1 + t_2 = 3.26\,\text{s} + 13.8\,\text{s} = 17.1\,\text{s}$.

(b) The distance fallen during the deceleration phase is

$$y_2 = -\frac{g}{2}t_2^2 + v_1 t_2 = -\frac{(2.10\,\text{m/s}^2)}{2}(13.8\,\text{s})^2 + (-31.9\,\text{m/s})(13.8\,\text{s}) = -240\,\text{m}.$$

The total distance fallen is $y = y_1 + y_2 = -52.0\,\text{m} - 240\,\text{m} = -292\,\text{m}$. It is negative because she was falling down.

P2-29 Let the speed of the bearing be v_1 at the top of the windows and v_2 at the bottom. These speeds are related by $v_2 = v_1 - gt_{12}$, where $t_{12} = 0.125\,\text{s}$ is the time between when the bearing is at the top of the window and at the bottom of the window. The average speed is $v_{\text{av}} = (v_1 + v_2)/2 = v_1 - gt_{12}/2$. The distance traveled in the time t_{12} is $y_{12} = -1.20\,\text{m}$, so

$$y_{12} = v_{\text{av}} t_{12} = v_1 t_{12} - gt_{12}^2/2,$$

and expression that can be solved for v_1 to yield

$$v_1 = \frac{y_{12} + gt_{12}^2/2}{t_{12}} = \frac{(-1.20\,\text{m}) + (9.81\,\text{m/s}^2)(0.125\,\text{s})^2/2}{(0.125\,\text{s})} = -8.99\,\text{m/s}.$$

Now that we know v_1 we can find the height of the building above the top of the window. The time the object has fallen to get to the top of the window is $t_1 = -v_1/g = -(-8.99\,\text{m/s})/(9.81\,\text{m/s}^2) = 0.916\,\text{m}$.

The total time for falling is then $(0.916\,\text{s}) + (0.125\,\text{s}) + (1.0\,\text{s}) = 2.04\,\text{s}$. Note that we remembered to divide the last time by two! The total distance from the top of the building to the bottom is then

$$y = -gt^2/2 = -(9.81\,\text{m/s}^2)(2.04\,\text{s})^2/2 = 20.4\,\text{m}.$$

P2-30 Each ball falls from a highest point through a distance of 2.0 m in

$$t = \sqrt{-2(2.0\,\text{m})/(9.8\,\text{m/s}^2)} = 0.639\,\text{s}.$$

The time each ball spends in the air is twice this, or 1.28 s. The frequency of tosses per ball is the reciprocal, $f = 1/T = 0.781\,\text{s}^{-1}$. There are five ball, so we multiply this by 5, but there are two hands, so we then divide that by 2. The tosses per hand per second then requires a factor 5/2, and the tosses per hand per minute is 60 times this, or 117.

$\boxed{\textbf{P2-31}}$ Assume each hand can toss n objects per second. Let τ be the amount of time that any one object is in the air. Then $2n\tau$ is the number of objects that are in the air at any time, where the "2" comes from the fact that (most?) jugglers have two hands. We'll estimate n, but τ can be found from Eq. 2-30 for an object which falls a distance h from rest:

$$0 = h + (0)t - \frac{1}{2}gt^2,$$

solving, $t = \sqrt{2h/g}$. But τ is twice this, because the object had to go up before it could come down. So the number of objects that can be juggled is

$$4n\sqrt{2h/g}$$

We estimate $n = 2$ tosses/second. So the maximum number of objects one could juggle to a height h would be

$$3.6\sqrt{h/\text{meters}}.$$

P2-32 (a) We need to look up the height of the leaning tower to solve this! If the height is $h = 56\,\text{m}$, then the time taken to fall a distance $h = -y_1$ is $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(-56\,\text{m})/(9.81\,\text{m/s}^2)} = 3.4\,\text{s}$. The second object, however, has only fallen a a time $t_2 = t_1 - \Delta t = 3.3\,\text{s}$, so the distance the second object falls is $y_2 = -gt_2^2/2 = -(9.81\,\text{m/s}^2)(3.3\,\text{s})^2/2 = 53.4$. The difference is $y_1 - y_2 = 2.9\,\text{m}$.

(b) If the vertical separation is $\Delta y = 0.01\,\text{m}$, then we can approach this problem in terms of differentials,

$$\delta y = at\,\delta t,$$

so $\delta t = (0.01\,\text{m})/[((9.81\,\text{m/s}^2)(3.4\,\text{s})] = 3\times 10^{-4}\,\text{s}$.

P2-33 Use symmetry, and focus on the path from the highest point downward. Then $\Delta t_U = 2t_U$, where t_U is the time from the highest point to the upper level. A similar expression exists for the lower level, but replace U with L. The distance from the highest point to the upper level is $y_U = -gt_U^2/2 = -g(\Delta t_U/2)^2/2$. The distance from the highest point to the lower level is $y_L = -gt_L^2/2 = -g(\Delta t_L/2)^2/2$. Now $H = y_U - y_L = -g\Delta t_U^2/8 - -g\Delta t_L^2/8$, which can be written as

$$g = \frac{8H}{\Delta t_L^2 - \Delta t_U^2}.$$

E3-1 The Earth orbits the sun with a speed of 29.8 km/s. The distance to Pluto is 5900×10^6 km. The time it would take the Earth to reach the orbit of Pluto is

$$t = (5900\times10^6 \text{ km})/(29.8 \text{ km/s}) = 2.0\times10^8 \text{ s},$$

or 6.3 years!

E3-2 (a) $a = F/m = (3.8\,\text{N})/(5.5\,\text{kg}) = 0.69\,\text{m/s}^2$.
(b) $t = v_\text{f}/a = (5.2\,\text{m/s})/(0.69\,\text{m/s}^2) = 7.5\,\text{s}$.
(c) $x = at^2/2 = (0.69\,\text{m/s}^2)(7.5\,\text{s})^2/2 = 20\,\text{m}$.

$\boxed{\textbf{E3-3}}$ Assuming constant acceleration we can find the average speed during the interval from Eq. 2-27

$$v_{\text{av},x} = \frac{1}{2}\left(v_x + v_{0x}\right) = \frac{1}{2}\left((5.8\times10^6 \text{ m/s}) + (0)\right) = 2.9\times10^6 \text{ m/s}.$$

From this we can find the time spent accelerating from Eq. 2-22, since $\Delta x = v_{\text{av},x}\Delta t$. Putting in the numbers $\Delta t = 5.17\times10^{-9}$s. The acceleration is then

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(5.8\times10^6 \text{ m/s}) - (0)}{(5.17\times10^{-9}\text{s})} = 1.1\times10^{15}\text{m/s}^2.$$

The net force on the electron is from Eq. 3-5,

$$\sum F_x = ma_x = (9.11\times10^{-31}\text{kg})(1.1\times10^{15}\text{m/s}^2) = 1.0\times10^{-15}\,\text{N}.$$

E3-4 The average speed while decelerating is $v_\text{av} = 0.7\times10^7\text{m/s}$. The time of deceleration is $t = x/v_\text{av} = (1.0\times10^{-14}\text{m})/(0.7\times10^7\text{m/s}) = 1.4\times10^{-21}\,\text{s}$. The deceleration is $a = \Delta v/t = (-1.4\times10^7\text{m/s})/(1.4\times10^{-21}\,\text{s}) = -1.0\times10^{28}\text{m/s}^2$. The force is $F = ma = (1.67\times10^{-27}\text{kg})(1.0\times10^{28}\text{m/s}^2) = 17\,\text{N}$.

$\boxed{\textbf{E3-5}}$ The *net* force on the sled is 92 N−90 N= 2 N; subtract because the forces are in opposite directions. Then

$$a_x = \frac{\sum F_x}{m} = \frac{(2\,\text{N})}{(25\,\text{kg})} = 8.0\times10^{-2}\text{m/s}^2.$$

E3-6 53 km/hr is 14.7 m/s. The average speed while decelerating is $v_\text{av} = 7.4\,\text{m/s}$. The time of deceleration is $t = x/v_\text{av} = (0.65\,\text{m})/(7.4\,\text{m/s}) = 8.8\times10^{-2}\,\text{s}$. The deceleration is $a = \Delta v/t = (-14.7\,\text{m/s})/(8.8\times10^{-2}\,\text{s}) = -17\times10^2\text{m/s}^2$. The force is $F = ma = (39\text{kg})(1.7\times10^2\text{m/s}^2) = 6600\,\text{N}$.

E3-7 Vertical acceleration is $a = F/m = (4.5\times10^{-15}\text{N})/(9.11\times10^{-31}\text{kg}) = 4.9\times10^{15}\text{m/s}^2$. The electron moves horizontally 33 mm in a time $t = x/v_x = (0.033\,\text{m})/(1.2\times10^7\text{m/s}) = 2.8\times10^{-9}\,\text{s}$. The vertical distance deflected is $y = at^2/2 = (4.9\times10^{15}\text{m/s}^2)(2.8\times10^{-9}\,\text{s})^2/2 = 1.9\times10^{-2}\text{m}$.

E3-8 (a) $a = F/m = (29\,\text{N})/(930\,\text{kg}) = 3.1\times10^{-2}\,\text{m/s}^2$.
(b) $x = at^2/2 = (3.1\times10^{-2}\,\text{m/s}^2)(86400\,\text{s})^2/2 = 1.2\times10^8\,\text{m}$.
(c) $v = at = (3.1\times10^{-2}\,\text{m/s}^2)(86400\,\text{s}) = 2700\,\text{m/s}$.

E3-9 Write the expression for the motion of the first object as $\sum F_x = m_1 a_{1x}$ and that of the second object as $\sum F_x = m_2 a_{2x}$. In both cases there is only one force, F, on the object, so $\sum F_x = F$. We will solve these for the mass as $m_1 = F/a_1$ and $m_2 = F/a_2$. Since $a_1 > a_2$ we can conclude that $m_2 > m_1$

(a) The acceleration of and object with mass $m_2 - m_1$ under the influence of a single force of magnitude F would be

$$a = \frac{F}{m_2 - m_1} = \frac{F}{F/a_2 - F/a_1} = \frac{1}{1/(3.30\,\mathrm{m/s^2}) - 1/(12.0\,\mathrm{m/s^2})},$$

which has a numerical value of $a = 4.55$ m/s^2.

(b) Similarly, the acceleration of an object of mass $m_2 + m_1$ under the influence of a force of magnitude F would be

$$a = \frac{1}{1/a_2 + 1/a_1} = \frac{1}{1/(3.30\,\mathrm{m/s^2}) + 1/(12.0\,\mathrm{m/s^2})},$$

which is the same as part (a) except for the sign change. Then $a = 2.59$ m/s^2.

E3-10 (a) The required acceleration is $a = v/t = 0.1c/t$. The required force is $F = ma = 0.1mc/t$. Then

$$F = 0.1(1200\times10^3\,\mathrm{kg})(3.00\times10^8\mathrm{m/s})/(2.59\times10^5\mathrm{s}) = 1.4\times10^8\mathrm{N},$$

and

$$F = 0.1(1200\times10^3\,\mathrm{kg})(3.00\times10^8\mathrm{m/s})/(5.18\times10^6\mathrm{s}) = 6.9\times10^6\mathrm{N},$$

(b) The distance traveled during the acceleration phase is $x_1 = at_1^2/2$, the time required to travel the remaining distance is $t_2 = x_2/v$ where $x_2 = d - x_1$. d is 5 light-months, or $d = (3.00\times10^8\mathrm{m/s})(1.30\times10^7\mathrm{s}) = 3.90\times10^{15}\mathrm{m}$. Then

$$t = t_1 + t_2 = t_1 + \frac{d - x_1}{v} = t_1 + \frac{2d - at_1^2}{2v} = t_1 + \frac{2d - vt_1}{2v}.$$

If t_1 is 3 days, then

$$t = (2.59\times10^5\mathrm{s}) + \frac{2(3.90\times10^{15}\mathrm{m}) - (3.00\times10^7\mathrm{m/s})(2.59\times10^5\mathrm{s})}{2(3.00\times10^7\mathrm{m/s})} = 1.30\times10^8\mathrm{s} = 4.12\ \mathrm{yr},$$

if t_1 is 2 months, then

$$t = (5.18\times10^6\mathrm{s}) + \frac{2(3.90\times10^{15}\mathrm{m}) - (3.00\times10^7\mathrm{m/s})(5.18\times10^6\mathrm{s})}{2(3.00\times10^7\mathrm{m/s})} = 1.33\times10^8\mathrm{s} = 4.20\ \mathrm{yr},$$

E3-11 (a) The net force on the second block is given by

$$\sum F_x = m_2 a_{2x} = (3.8\,\mathrm{kg})(2.6\,\mathrm{m/s^2}) = 9.9\,\mathrm{N}.$$

There is only one (relevant) force on the block, the force of block 1 on block 2.

(b) There is only one (relevant) force on block 1, the force of block 2 on block 1. By Newton's third law this force has a magnitude of 9.9 N. Then Newton's second law gives $\sum F_x = -9.9$ N$= m_1 a_{1x} = (4.6$ kg$)a_{1x}$. So $a_{1x} = -2.2$ m/s^2 at the instant that $a_{2x} = 2.6$ m/s^2.

E3-12 (a) $W = (5.00$ lb$)(4.448\,\mathrm{N/lb}) = 22.2\,\mathrm{N}$; $m = W/g = (22.2\,\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 2.26\,\mathrm{kg}$.

(b) $W = (240$ lb$)(4.448\,\mathrm{N/lb}) = 1070\,\mathrm{N}$; $m = W/g = (1070\,\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 109\,\mathrm{kg}$.

(c) $W = (3600$ lb$)(4.448\,\mathrm{N/lb}) = 16000\,\mathrm{N}$; $m = W/g = (16000\,\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 1630\,\mathrm{kg}$.

E3-13 (a) $W = (1420.00 \text{ lb})(4.448\,\text{N/lb}) = 6320\,\text{N}$; $m = W/g = (6320\,\text{N})/(9.81\,\text{m/s}^2) = 644\,\text{kg}$.
(b) $m = 412\,\text{kg}$; $W = mg = (412\,\text{kg})(9.81\,\text{m/s}^2) = 4040\,\text{N}$.

E3-14 (a) $W = mg = (75.0\,\text{kg})(9.81\,\text{m/s}^2) = 736\,\text{N}$.
(b) $W = mg = (75.0\,\text{kg})(3.72\,\text{m/s}^2) = 279\,\text{N}$.
(c) $W = mg = (75.0\,\text{kg})(0\,\text{m/s}^2) = 0\,\text{N}$.
(d) The mass is 75.0 kg at all locations.

$\boxed{\textbf{E3-15}}$ If $g = 9.81$ m/s^2, then $m = W/g = (26.0 \text{ N})/(9.81 \text{ m/s}^2) = 2.65$ kg.
(a) Apply $W = mg$ again, but now $g = 4.60$ m/s^2, so at this point $W = (2.65 \text{ kg})(4.60 \text{ m/s}^2) = 12.2$ N.
(b) If there is no gravitational force, there is no weight, because $g = 0$. There is still mass, however, and that mass is still 2.65 kg.

E3-16 Upward force balances weight, so $F = W = mg = (12000\,\text{kg})(9.81\,\text{m/s}^2) = 1.2\times10^5\text{N}$.

E3-17 Mass is $m = W/g$; net force is $F = ma$, or $F = Wa/g$. Then

$$F = (3900 \text{ lb})(13 \text{ ft/s}^2)/(32 \text{ ft/s}^2) = 1600 \text{ lb}.$$

E3-18 $a = \Delta v/\Delta t = (450\,\text{m/s})/(1.82\,\text{s}) = 247\,\text{m/s}^2$. Net force is $F = ma = (523\,\text{kg})(247\,\text{m/s}^2) = 1.29\times10^5\text{N}$.

$\boxed{\textbf{E3-19}}$ $\sum F_x = 2(1.4\times10^5 \text{ N}) = ma_x$. Then $m = 1.22\times10^5$ kg and

$$W = mg = (1.22\times10^5\text{kg})(9.81\,\text{m/s}^2) = 1.20\times10^6\,\text{N}.$$

E3-20 Do part (b) first; there must be a 10 lb force to support the mass. Now do part (a), but cover up the left hand side of both pictures. If you can't tell which picture is which, then they must both be 10 lb!

E3-21 (b) Average speed during deceleration is 40 km/h, or 11 m/s. The time taken to stop the car is then $t = x/v_{\text{av}} = (61\,\text{m})/(11\,\text{m/s}) = 5.6\,\text{s}$.
(a) The deceleration is $a = \Delta v/\Delta t = (22\,\text{m/s})/(5.6\,\text{s}) = 3.9\,\text{m/s}^2$. The braking force is $F = ma = Wa/g = (13,000\,\text{N})(3.9\,\text{m/s}^2)/(9.81\,\text{m/s}^2) = 5200\,\text{N}$.
(d) The deceleration is same; the time to stop the car is then $\Delta t = \Delta v/a = (11\,\text{m/s})/(3.9\,\text{m/s}^2) = 2.8\,s$.
(c) The distance traveled during stopping is $x = v_{\text{av}}t = (5.6\,\text{m/s})(2.8\,\text{s}) = 16\,\text{m}$.

E3-22 Assume acceleration of free fall is $9.81\,\text{m/s}^2$ at the altitude of the meteor. The net force is $F_{\text{net}} = ma = (0.25\,\text{kg})(9.2\,m/\text{s}^2) = 2.30\,\text{N}$. The weight is $W = mg = (0.25\,kg)(9.81\,\text{m/s}^2) = 2.45\,\text{N}$. The retarding force is $F_{\text{net}} - W = (2.3\,\text{N}) - (2.45\,\text{N}) = -0.15\,\text{N}$.

$\boxed{\textbf{E3-23}}$ (a) Find the time during the "jump down" phase from Eq. 2-30.

$$(0 \text{ m}) = (0.48 \text{ m}) + (0)t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2,$$

which is a simple quadratic with solutions $t = \pm0.31$ s. Use this time in Eq. 2-29 to find his speed when he hit ground,

$$v_y = (0) - (9.8 \text{ m/s}^2)(0.31 \text{ s}) = -3.1 \text{ m/s}.$$

This becomes the initial velocity for the deceleration motion, so his average speed during deceleration is given by Eq. 2-27,

$$v_{\mathrm{av},y} = \frac{1}{2}(v_y + v_{0y}) = \frac{1}{2}((0) + (-3.1 \text{ m/s})) = -1.6 \text{ m/s}.$$

This average speed, used with the distance of -2.2 cm (-0.022 m), can be used to find the time of deceleration

$$v_{\mathrm{av},y} = \Delta y/\Delta t,$$

and putting numbers into the expression gives $\Delta t = 0.014$ s. Finally, we use this in Eq. 2-26 to find the acceleration,

$$(0) = (-3.1 \text{ m/s}) + a(0.014 \text{ s}),$$

which gives $a = 220 \text{ m/s}^2$.

(b) The average *net* force on the man is

$$\sum F_y = ma_y = (83\,\mathrm{kg})(220\,\mathrm{m/s^2}) = 1.8\times10^4\mathrm{N}.$$

E3-24 The average speed of the salmon while decelerating is 4.6 ft/s. The time required to stop the salmon is then $t = x/v_{\mathrm{av}} = (0.38 \text{ ft})/(4.6 \text{ ft/s}) = 8.3\times10^{-2}$s. The deceleration of the salmon is $a = \Delta v/\Delta t = (9.2 \text{ ft/s})/(8.2\text{-}2\text{s}) = 110 \text{ ft/s}^2$. The force on the salmon is then $F = Wa/g = (19 \text{ lb})(110 \text{ ft/s}^2)/(32 \text{ ft/s}^2) = 65$ lb.

$\boxed{\textbf{E3-25}}$ From appendix G we find 1 lb = 4.448 N; so the weight is (100 lb)(4.448 N/1 lb) = 445 N; similarly the cord will break if it pulls upward on the object with a force greater than 387 N. The mass of the object is $m = W/g = (445 \text{ N})/(9.8 \text{ m/s}^2) = 45$ kg.

There are two vertical forces on the 45 kg object, an upward force from the cord F_{OC} (which has a maximum value of 387 N) and a downward force from gravity F_{OG}. Then $\sum F_y = F_{OC} - F_{OG} = (387 \text{ N}) - (445 \text{ N}) = -58$ N. Since the net force is negative, the object must be accelerating downward according to

$$a_y = \sum F_y/m = (-58\,\mathrm{N})/(45\,\mathrm{kg}) = -1.3\,\mathrm{m/s^2}.$$

E3-26 (a) Constant speed means no acceleration, hence no net force; this means the weight is balanced by the force from the scale, so the scale reads 65 N.

(b) Net force on mass is $F_{\mathrm{net}} = ma = Wa/g = (65\,\mathrm{N})(-2.4\,\mathrm{m/s^2})/(9.81\,\mathrm{m/s^2}) = -16\,\mathrm{N}$.. Since the weight is 65 N, the scale must be exerting a force of $(-16\,\mathrm{N}) - (-65\,\mathrm{N}) = 49\,\mathrm{N}$.

E3-27 The magnitude of the net force is $W - R = (1600\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) - (3700\,\mathrm{N}) = 12000\,\mathrm{N}$. The acceleration is then $a = F/m = (12000\,\mathrm{N})/(1600\,\mathrm{kg}) = 7.5\,\mathrm{m/s^2}$. The time to fall is

$$t = \sqrt{2y/a} = \sqrt{2(-72\,\mathrm{m})/(-7.5\,\mathrm{m/s^2})} = 4.4\,\mathrm{s}.$$

The final speed is $v = at = (-7.5\,\mathrm{m/s^2})(4.4\,\mathrm{s}) = 33\,\mathrm{m/s}$. Get better brakes, eh?

E3-28 The average speed during the acceleration is 140 ft/s. The time for the plane to travel 300 ft is

$$t = x/v_{\mathrm{av}} = (300 \text{ ft})/(140 \text{ ft/s}) = 2.14\,\mathrm{s}.$$

The acceleration is then

$$a = \Delta v/\Delta t = (280 \text{ ft/s})/(2.14\,\mathrm{s}) = 130 \text{ ft/s}^2.$$

The net force on the plane is $F = ma = Wa/g = (52000 \text{ lb})(130 \text{ ft/s}^2)/(32 \text{ ft/s}^2) = 2.1\times10^5$lb.

The force exerted by the catapult is then $2.1\times10^5\text{lb} - 2.4\times10^4 \text{ lb} = 1.86\times10^5$ lb.

E3-29 (a) The acceleration of a hovering rocket is 0, so the net force is zero; hence the thrust must equal the weight. Then $T = W = mg = (51000\,\text{kg})(9.81\,\text{m/s}^2) = 5.0\times10^5\text{N}$.

(b) If the rocket accelerates upward then the net force is $F = ma = (51000\,\text{kg})(18\,\text{m/s}^2) = 9.2\times10^5\,\text{N}$. Now $F_{\text{net}} = T - W$, so $T = 9.2\times10^5\,\text{N} + 5.0\times10^5\text{N} = 1.42\times10^6\text{N}$.

E3-30 (a) Net force on parachute + person system is $F_{\text{net}} = ma = (77\,\text{kg} + 5.2\,\text{kg})(-2.5\,\text{s}^2) = -210\,\text{N}$. The weight of the system is $W = mg = (77\,\text{kg}+5.2\,\text{kg})(9.81\,\text{s}^2) = 810\,\text{N}$. If P is the upward force of the air on the system (parachute) then $P = F_{\text{net}} + W = (-210\,\text{N}) + (810\,\text{N}) = 600\,\text{N}$.

(b) The net force on the *parachute* is $F_{\text{net}} = ma = (5.2\,\text{kg})(-2.5\,\text{s}^2) = -13\,\text{N}$. The weight of the parachute is $W = mg = (5.2\,\text{kg})(9.81\,\text{m/s}^2) = 51\,\text{N}$. If D is the downward force of the person on the parachute then $D = -F_{\text{net}} - W + P = -(-13\,\text{N}) - (51\,\text{N}) + 600\,\text{N} = 560\,\text{N}$.

$\boxed{\textbf{E3-31}}$ (a) The *total* mass of the helicopter+car system is 19,500 kg; and the only other force acting on the system is the force of gravity, which is

$$W = mg = (19,500\,\text{kg})(9.8\,\text{m/s}^2) = 1.91\times10^5\,\text{N}.$$

The force of gravity is directed down, so the net force on the system is $\sum F_y = F_{BA} - (1.91\times10^5$ N). The net force can also be found from Newton's second law: $\sum F_y = ma_y = (19,500$ kg)(1.4 m/s^2) $= 2.7\times10^4$ N. Equate the two expressions for the net force, $F_{BA} - (1.91\times10^5$ N) $= 2.7\times10^4$ N, and solve; $F_{BA} = 2.2\times10^5$ N.

(b) Repeat the above steps except: (1) the system will consist only of the car, and (2) the upward force on the car comes from the supporting cable only F_{CC}. The weight of the car is $W = mg = (4500$ kg)(9.8 m/s^2) $= 4.4\times10^4$ N. The net force is $\sum F_y = F_{CC} - (4.4\times10^4$ N), it can also be written as $\sum F_y = ma_y = (4500$ kg)(1.4 m/s^2) $= 6300$ N. Equating, $F_{CC} = 50,000$ N.

P3-1 (a) The acceleration is $a = F/m = (2.7\times10^{-5}\text{N})/(280\,\text{kg}) = 9.64\times10^{-8}\text{m/s}^2$. The displacement (from the original trajectory) is

$$y = at^2/2 = (9.64\times10^{-8}\text{m/s}^2)(2.4\,\text{s})^2/2 = 2.8\times10^{-7}\text{m}.$$

(b) The acceleration is $a = F/m = (2.7\times10^{-5}\text{N})/(2.1\,\text{kg}) = 1.3\times10^{-5}\text{m/s}^2$. The displacement (from the original trajectory) is

$$y = at^2/2 = (1.3\times10^{-5}\text{m/s}^2)(2.4\,\text{s})^2/2 = 3.7\times10^{-5}\text{m}.$$

P3-2 (a) The acceleration of the sled is $a = F/m = (5.2\,\text{N})/(8.4\,\text{kg}) = 0.62\,\text{m/s}^2$.

(b) The acceleration of the girl is $a = F/m = (5.2\,\text{N})/(40\,kg) = 0.13\,\text{m/s}^2$.

(c) The distance traveled by girl is $x_1 = a_1t^2/2$; the distance traveled by the sled is $x_2 = a_2t^2/2$. The two meet when $x_1 + x_2 = 15\,\text{m}$. This happens when $(a_1 + a_2)t^2 = 30\,\text{m}$. They then meet when $t = \sqrt{(30\,\text{m})/(0.13\,\text{m/s}^2 + 0.62\,\text{m/s}^2)} = 6.3\,\text{s}$. The girl moves $x_1 = (0.13\,\text{m/s}^2)(6.3\,\text{s})^2/2 = 2.6\,\text{m}$.

$\boxed{\textbf{P3-3}}$ (a) Start with block one. It starts from rest, accelerating through a distance of 16 m in a time of 4.2 s. Applying Eq. 2-28,

$$\begin{aligned} x &= x_0 + v_{0x}t + \frac{1}{2}a_xt^2, \\ -16\ \text{m} &= (0) + (0)(4.2\ \text{s}) + \frac{1}{2}a_x(4.2\ \text{s})^2, \end{aligned}$$

find the acceleration to be $a_x = -1.8$ m/s^2.

Now for the second block. The acceleration of the second block is identical to the first for much the same reason that all objects fall with approximately the same acceleration.

(b) The initial and final velocities are related by a sign, then $v_x = -v_{0x}$ and Eq. 2-26 becomes

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ -v_{0x} &= v_{0x} + a_x t, \\ -2v_{0x} &= (-1.8\ \mathrm{m/s^2})(4.2\ \mathrm{s}). \end{aligned}$$

which gives an initial velocity of $v_{0x} = 3.8$ m/s.

(c) Half of the time is spent coming down from the highest point, so the time to "fall" is 2.1 s. The distance traveled is found from Eq. 2-28,

$$x = (0) + (0)(2.1\,\mathrm{s}) + \frac{1}{2}(-1.8\,\mathrm{m/s^2})(2.1\,\mathrm{s})^2 = -4.0\,\mathrm{m}.$$

P3-4 (a) The weight of the engine is $W = mg = (1400\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1.37\times10^4\,\mathrm{N}$. If each bolt supports 1/3 of this, then the force on a bolt is 4600 N.

(b) If engine accelerates up at $2.60\,\mathrm{m/s^2}$, then net force on the engine is

$$F_{\mathrm{net}} = ma = (1400\,\mathrm{kg})(2.60\,\mathrm{m/s^2}) = 3.64\times10^3\,\mathrm{N}.$$

The upward force from the bolts must then be

$$B = F_{\mathrm{net}} + W = (3.64\times10^3\,\mathrm{N}) + (1.37\times10^4\,\mathrm{N}) = 1.73\times10^4\mathrm{N}.$$

The force per bolt is one third this, or 5800 N.

P3-5 (a) If craft descends with constant speed then net force is zero, so thrust balances weight. The weight is then 3260 N.

(b) If the thrust is 2200 N the net force is $2200\,\mathrm{N} - 3260\,\mathrm{N} = -1060\,\mathrm{N}$. The mass is then $m = F/a = (-1060\,\mathrm{N})/(-0.390\,\mathrm{m/s^2}) = 2720\,\mathrm{kg}$.

(c) The acceleration due to gravity is $g = W/m = (3260\,\mathrm{N})/(2720\,\mathrm{kg}) = 1.20\,\mathrm{m/s^2}$.

P3-6 The weight is originally Mg. The net force is originally $-Ma$. The upward force is then originally $B = Mg - Ma$. The goal is for a net force of $(M-m)a$ and a weight $(M-m)g$. Then

$$(M-m)a = B - (M-m)g = Mg - Ma - Mg + mg = mg - Ma,$$

or $m = 2Ma/(a+g)$.

$\boxed{\textbf{P3-7}}$ (a) Consider all three carts as one system. Then

$$\begin{aligned} \sum F_x &= m_{\mathrm{total}} a_x, \\ 6.5\ \mathrm{N} &= (3.1\,\mathrm{kg} + 2.4\,\mathrm{kg} + 1.2\,\mathrm{kg})a_x, \\ 0.97\ \mathrm{m/s^2} &= a_x. \end{aligned}$$

(b) Now choose your system so that it only contains the third car. Then

$$\sum F_x = F_{23} = m_3 a_x = (1.2\,\mathrm{kg})(0.97\,\mathrm{m/s^2}).$$

The unknown can be solved to give $F_{23} = 1.2\,\mathrm{N}$ directed to the right.

(c) Consider a system involving the second and third carts. Then $\sum F_x = F_{12}$, so Newton's law applied to the system gives

$$F_{12} = (m_2 + m_3)a_x = (2.4\,\mathrm{kg} + 1.2\,\mathrm{kg})(0.97\,\mathrm{m/s^2}) = 3.5\,\mathrm{N}.$$

P3-8 (a) $F = ma = (45.2\,\text{kg} + 22.8\,\text{kg} + 34.3\,\text{kg})(1.32\,\text{m/s}^2) = 135\,\text{N}$.

(b) Consider only m_3. Then $F = ma = (34.3\,\text{kg})(1.32\,\text{m/s}^2) = 45.3\,\text{N}$.

(c) Consider m_2 and m_3. Then $F = ma = (22.8\,\text{kg} + 34.3\,\text{kg})(1.32\,\text{m/s}^2) = 75.4\,\text{N}$.

P3-9 (c) The net force on each link is the same, $F_{\text{net}} = ma = (0.100\,\text{kg})(2.50\,\text{m/s}^2) = 0.250\,\text{N}$.

(a) The weight of each link is $W = mg = (0.100\,\text{kg})(9.81\,\text{m/s}^2) = 0.981\,\text{N}$. On each link (except the top or bottom link) there is a weight, an upward force from the link above, and a downward force from the link below. Then $F_{\text{net}} = U - D - W$. Then $U = F_{\text{net}} + W + D = (0.250\,\text{N}) + (0.981\,\text{N}) + D = 1.231\,\text{N} + D$. For the bottom link $D = 0$. For the bottom link, $U = 1.23\,\text{N}$. For the link above, $U = 1.23\,\text{N} + 1.23\,\text{N} = 2.46\,\text{N}$. For the link above, $U = 1.23\,\text{N} + 2.46\,\text{N} = 3.69\,\text{N}$. For the link above, $U = 1.23\,\text{N} + 3.69\,\text{N} = 4.92\,\text{N}$.

(b) For the top link, the upward force is $U = 1.23\,\text{N} + 4.92\,\text{N} = 6.15\,\text{N}$.

P3-10 (a) The acceleration of the two blocks is $a = F/(m_1 + m_2)$ The net force on block 2 is from the force of contact, and is

$$P = m_2 a = F m_2/(m_1 + m_2) = (3.2\,\text{N})(1.2\,\text{kg})/(2.3\,\text{kg} + 1.2\,\text{kg}) = 1.1\,\text{N}.$$

(b) The acceleration of the two blocks is $a = F/(m_1 + m_2)$ The net force on block 1 is from the force of contact, and is

$$P = m_1 a = F m_1/(m_1 + m_2) = (3.2\,\text{N})(2.3\,\text{kg})/(2.3\,\text{kg} + 1.2\,\text{kg}) = 2.1\,\text{N}.$$

Not a third law pair, eh?

P3-11 (a) Treat the system as including both the block and the rope, so that the mass of the system is $M + m$. There is one (relevant) force which acts on the system, so $\sum F_x = P$. Then Newton's second law would be written as $P = (M+m)a_x$. Solve this for a_x and get $a_x = P/(M+m)$.

(b) Now consider only the block. The horizontal force doesn't act on the block; instead, there is the force of the rope on the block. We'll assume that force has a magnitude R, and this is the *only* (relevant) force on the block, so $\sum F_x = R$ for the net force on the block.. In this case Newton's second law would be written $R = Ma_x$. Yes, a_x is the same in part (a) and (b); the acceleration of the block is the same as the acceleration of the block + rope. Substituting in the results from part (a) we find

$$R = \frac{M}{M+m} P.$$

E4-1 (a) The time to pass between the plates is $t = x/v_x = (2.3\text{ cm})/(9.6{\times}10^8\text{ cm/s}) = 2.4{\times}10^{-9}\text{s}$.

(b) The vertical displacement of the beam is then $y = a_y t^2/2 == (9.4 \times 10^{16}\text{ cm/s}^2)(2.4 \times 10^{-9}\text{s})^2/2 = 0.27\text{ cm}$.

(c) The velocity components are $v_x = 9.6 \times 10^8$ cm/s and $v_y = a_y t = (9.4 \times 10^{16}\text{ cm/s}^2)(2.4 \times 10^{-9}\text{s}) = 2.3 \times 10^8$cm/s.

E4-2 $\vec{\mathbf{a}} = \Delta\vec{\mathbf{v}}/\Delta t = -(6.30\hat{\mathbf{i}} - 8.42\hat{\mathbf{j}})(\text{m/s})/(3\,\text{s}) = (-2.10\hat{\mathbf{i}} + 2.81\hat{\mathbf{j}})(\text{m/s}^2)$.

$\boxed{\textbf{E4-3}}$ (a) The velocity is given by

$$\begin{aligned} \frac{d\vec{\mathbf{r}}}{dt} &= \frac{d}{dt}\left(A\hat{\mathbf{i}}\right) + \frac{d}{dt}\left(Bt^2\hat{\mathbf{j}}\right) + \frac{d}{dt}\left(Ct\hat{\mathbf{k}}\right), \\ \vec{\mathbf{v}} &= (0) + 2Bt\hat{\mathbf{j}} + C\hat{\mathbf{k}}. \end{aligned}$$

(b) The acceleration is given by

$$\begin{aligned} \frac{d\vec{\mathbf{v}}}{dt} &= \frac{d}{dt}\left(2Bt\hat{\mathbf{j}}\right) + \frac{d}{dt}\left(C\hat{\mathbf{k}}\right), \\ \vec{\mathbf{a}} &= (0) + 2B\hat{\mathbf{j}} + (0). \end{aligned}$$

(c) Nothing exciting happens in the x direction, so we will focus on the yz plane. The trajectory in this plane is a parabola.

E4-4 (a) Maximum x is when $v_x = 0$. Since $v_x = a_x t + v_{x,0}$, $v_x = 0$ when $t = -v_{x,0}/a_x = -(3.6\,\text{m/s})/(-1.2\,\text{m/s}^2) = 3.0\,\text{s}$.

(b) Since $v_x = 0$ we have $|\vec{\mathbf{v}}| = |v_y|$. But $v_y = a_y t + v_{y,0} = -(1.4\,\text{m/s})(3.0\,\text{s}) + (0) = -4.2\,\text{m/s}$. Then $|\vec{\mathbf{v}}| = 4.2\,\text{m/s}$.

(c) $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2 + \vec{\mathbf{v}}_0 t$, so

$$\vec{\mathbf{r}} = [-(0.6\,\text{m/s}^2)\hat{\mathbf{i}} - (0.7\,\text{m/s}^2)\hat{\mathbf{j}}](3.0\,\text{s})^2 + [(3.6\,\text{m/s})\hat{\mathbf{i}}](3.0\,\text{s}) = (5.4\,\text{m})\hat{\mathbf{i}} - (6.3\,\text{m})\hat{\mathbf{j}}.$$

E4-5 $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (3.7\,\text{N})\hat{\mathbf{j}} + (4.3\,\text{N})\hat{\mathbf{i}}$. Then $\vec{\mathbf{a}} = \vec{\mathbf{F}}/m = (0.71\,\text{m/s}^2)\hat{\mathbf{j}} + (0.83\,\text{m/s}^2)\hat{\mathbf{i}}$.

E4-6 (a) The acceleration is $\vec{\mathbf{a}} = \vec{\mathbf{F}}/m = (2.2\,\text{m/s}^2)\hat{\mathbf{j}}$. The velocity after 15 seconds is $\vec{\mathbf{v}} = \vec{\mathbf{a}}t + \vec{\mathbf{v}}_{\mathbf{0}}$, or

$$\vec{\mathbf{v}} = [(2.2\,\text{m/s}^2)\hat{\mathbf{j}}](15\,\text{s}) + [(42\,\text{m/s})\hat{\mathbf{i}}] = (42\,\text{m/s})\hat{\mathbf{i}} + (33\,\text{m/s})\hat{\mathbf{j}}.$$

(b) $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2 + \vec{\mathbf{v}}_0 t$, so

$$\vec{\mathbf{r}} = [(1.1\,\text{m/s}^2)\hat{\mathbf{j}}](15\,\text{s})^2 + [(42\,\text{m/s})\hat{\mathbf{i}}](15\,\text{s}) = (630\,\text{m})\hat{\mathbf{i}} + (250\,\text{m})\hat{\mathbf{j}}.$$

$\boxed{\textbf{E4-7}}$ The block has a weight $W = mg = (5.1\text{ kg})(9.8\text{ m/s}^2) = 50$ N.

(a) Initially $P = 12$ N, so $P_y = (12\,\text{N})\sin(25°) = 5.1$ N and $P_x = (12\,\text{N})\cos(25°) = 11$ N. Since the upward component is less than the weight, the block doesn't leave the floor, and a normal force will be present which will make $\sum F_y = 0$. There is only one contribution to the horizontal force, so $\sum F_x = P_x$. Newton's second law then gives $a_x = P_x/m = (11\text{ N})/(5.1\text{ kg}) = 2.2\text{ m/s}^2$.

(b) As P is increased, so is P_y; eventually P_y will be large enough to overcome the weight of the block. This happens just after $P_y = W = 50$ N, which occurs when $P = P_y/\sin\theta = 120$ N.

(c) Repeat part (a), except now $P = 120$ N. Then $P_x = 110$ N, and the acceleration of the block is $a_x = P_x/m = 22\text{ m/s}^2$.

E4-8 (a) The block has weight $W = mg = (96.0\,\text{kg})(9.81\,\text{m/s}^2) = 942\,\text{N}$. $P_x = (450\,\text{N})\cos(38^\circ) = 355\,\text{N}$; $P_y = (450\,\text{N})\sin(38^\circ) = 277\,\text{N}$. Since $P_y < W$ the crate stays on the floor and there is a normal force $N = W - P_y$. The net force in the x direction is $F_x = P_x - (125\,\text{N}) = 230\,\text{N}$. The acceleration is $a_x = F_x/m = (230\,\text{N})/(96.0\,\text{kg}) = 2.40\,\text{m/s}^2$.

(b) The block has mass $m = W/g = (96.0\,\text{N})/(9.81\,\text{m/s}^2) = 9.79\,\text{kg}$. $P_x = (450\,\text{N})\cos(38^\circ) = 355\,\text{N}$; $P_y = (450\,\text{N})\sin(38^\circ) = 277\,\text{N}$. Since $P_y > W$ the crate lifts off of the floor! The net force in the x direction is $F_x = P_x - (125\,\text{N}) = 230\,\text{N}$. The x acceleration is $a_x = F_x/m = (230\,\text{N})/(9.79\,\text{kg}) = 23.5\,\text{m/s}^2$. The net force in the y direction is $F_y = P_y - W = 181\,\text{N}$. The y acceleration is $a_y = F_y/m = (181\,\text{N})/(9.79\,\text{kg}) = 18.5\,\text{m/s}^2$. Wow.

E4-9 Let y be perpendicular and x be parallel to the incline. Then $P = 4600\,\text{N}$;

$$P_x = (4600\,\text{N})\cos(27^\circ) = 4100\,\text{N};$$

$$P_y = (4600\,\text{N})\sin(27^\circ) = 2090\,\text{N}.$$

The weight of the car is $W = mg = (1200\,\text{kg})(9.81\,\text{m/s}^2) = 11800\,\text{N}$;

$$W_x = (11800\,\text{N})\sin(18^\circ) = 3650\,\text{N};$$

$$W_y = (11800\,\text{N})\cos(18^\circ) = 11200\,\text{N}.$$

Since $W_y > P_y$ the car stays on the incline. The net force in the x direction is $F_x = P_x - W_x = 450\,\text{N}$. The acceleration in the x direction is $a_x = F_x/m = (450\,\text{N})/(1200\,\text{kg}) = 0.375\,\text{m/s}^2$. The distance traveled in 7.5 s is $x = a_x t^2/2 = (0.375\,\text{m/s}^2)(7.5\,\text{s})^2/2 = 10.5\,\text{m}$.

E4-10 Constant speed means zero acceleration, so net force is zero. Let y be perpendicular and x be parallel to the incline. The weight is $W = mg = (110\,\text{kg})(9.81\,\text{m/s}^2) = 1080\,\text{N}$; $W_x = W\sin(34^\circ)$; $W_y = W\cos(34^\circ)$. The push F has components $F_x = F\cos(34^\circ)$ and $F_y = -F\sin(34^\circ)$. The y components will balance after a normal force is introduced; the x components will balance if $F_x = W_x$, or $F = W\tan(34^\circ) = (1080\,\text{N})\tan(34^\circ) = 730\,\text{N}$.

$\boxed{\textbf{E4-11}}$ If the x axis is parallel to the river and the y axis is perpendicular, then $\vec{\mathbf{a}} = 0.12\hat{\mathbf{i}}\,\text{m/s}^2$. The net force on the barge is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (9500\,\text{kg})(0.12\hat{\mathbf{i}}\,\text{m/s}^2) = 1100\hat{\mathbf{i}}\,\text{N}.$$

The force exerted on the barge by the horse has components in both the x and y direction. If $P = 7900$ N is the magnitude of the pull and $\theta = 18^\circ$ is the direction, then $\vec{\mathbf{P}} = P\cos\theta\hat{\mathbf{i}} + P\sin\theta\hat{\mathbf{j}} = (7500\hat{\mathbf{i}} + 2400\hat{\mathbf{j}})$ N.

Let the force exerted on the barge by the water be $\vec{\mathbf{F_w}} = F_{w,x}\hat{\mathbf{i}} + F_{w,y}\hat{\mathbf{j}}$. Then $\sum F_x = (7500\text{ N}) + F_{w,x}$ and $\sum F_y = (2400\text{ N}) + F_{w,y}$. But we already found $\sum \vec{\mathbf{F}}$, so

$$\begin{aligned} F_x = 1100\text{ N} &= 7500\text{ N} + F_{w,x}, \\ F_y = 0 &= 2400\text{ N} + F_{w,y}. \end{aligned}$$

Solving, $F_{w,x} = -6400$ N and $F_{w,y} = -2400$ N. The magnitude is found by $F_w = \sqrt{F_{w,x}^2 + F_{w,y}^2} = 6800$ N.

E4-12 (a) Let y be perpendicular and x be parallel to the direction of motion of the plane. Then $W_x = mg\sin\theta$; $W_y = mg\cos\theta$; $m = W/g$. The plane is accelerating in the x direction, so $a_x = 2.62\,\mathrm{m/s^2}$; the net force is in the x direction, where $F_x = ma_x$. But $F_x = T - W_x$, so

$$T = F_x + W_x = W\frac{a_x}{g} + W\sin\theta = (7.93\times10^4\mathrm{N})\left[\frac{(2.62\,\mathrm{m/s^2})}{(9.81\,\mathrm{m/s^2})} + \sin(27^\circ)\right] = 5.72\times10^4\mathrm{N}.$$

(b) There is no motion in the y direction, so

$$L = W_y = (7.93\times10^4\mathrm{N})\cos(27^\circ) = 7.07\times10^4\mathrm{N}.$$

E4-13 (a) The ball rolled off horizontally so $v_{0y} = 0$. Then

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2, \\ (-4.23\text{ ft}) &= (0)t - \frac{1}{2}(32.2\text{ ft/s}^2)t^2, \end{aligned}$$

which can be solved to yield $t = 0.514$ s.

(b) The initial velocity in the x direction can be found from $x = v_{0x}t$; rearranging, $v_{0x} = x/t = (5.11\text{ ft})/(0.514\text{ s}) = 9.94$ ft/s. Since there is no y component to the velocity, then the initial speed is $v_0 = 9.94$ ft/s.

E4-14 The electron travels for a time $t = x/v_x$. The electron "falls" vertically through a distance $y = -gt^2/2$ in that time. Then

$$y = -\frac{g}{2}\left(\frac{x}{v_x}\right)^2 = -\frac{(9.81\,\mathrm{m/s^2})}{2}\left(\frac{(1.0\,\mathrm{m})}{(3.0\times10^7\mathrm{m/s})}\right)^2 = -5.5\times10^{-15}\,\mathrm{m}.$$

E4-15 (a) The dart "falls" vertically through a distance $y = -gt^2/2 = -(9.81\,\mathrm{m/s^2})(0.19\,\mathrm{s})^2/2 = -0.18\,\mathrm{m}$.

(b) The dart travels horizontally $x = v_xt = (10\,\mathrm{m/s})(0.19\,\mathrm{s}) = 1.9\,\mathrm{m}$.

E4-16 The initial velocity components are

$$v_{x,0} = (15\,\mathrm{m/s})\cos(20^\circ) = 14\,\mathrm{m/s}$$

and

$$v_{y,0} = -(15\,\mathrm{m/s})\sin(20^\circ) = -5.1\,\mathrm{m/s}.$$

(a) The horizontal displacement is $x = v_xt = (14\,\mathrm{m/s})(2.3\,\mathrm{s}) = 32\,\mathrm{m}$.

(b) The vertical displacement is

$$y = -gt^2/2 + v_{y,0}t = -(9.81\,\mathrm{m/s^2})(2.3\,\mathrm{s})^2/2 + (-5.1\,\mathrm{m/s})(2.3\,\mathrm{s}) = -38\,\mathrm{m}.$$

E4-17 Find the time in terms of the the initial y component of the velocity:

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= v_{0y} - gt, \\ t &= v_{0y}/g. \end{aligned}$$

Use this time to find the highest point:

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2, \\ y_{\max} &= v_{0y}\left(\frac{v_{0y}}{g}\right) - \frac{1}{2}g\left(\frac{v_{0y}}{g}\right)^2, \\ &= \frac{v_{0y}^2}{2g}. \end{aligned}$$

Finally, we know the initial y component of the velocity from Eq. 2-6, so $y_{\max} = (v_0 \sin\phi_0)^2/2g$.

E4-18 The horizontal displacement is $x = v_x t$. The vertical displacement is $y = -gt^2/2$. Combining, $y = -g(x/v_x)^2/2$. The edge of the nth step is located at $y = -nw$, $x = nw$, where $w = 2/3$ ft. If $|y| > nw$ when $x = nw$ then the ball hasn't hit the step. Solving,

$$\begin{aligned} g(nw/v_x)^2/2 &< nw, \\ gnw/v_x^2 &< 2, \\ n &< 2v_x^2/(gw) = 2(5.0\text{ ft/s})^2/[(32\text{ ft/s}^2)(2/3\text{ ft})] = 2.34. \end{aligned}$$

Then the ball lands on the third step.

E4-19 (a) Start from the observation point 9.1 m above the ground. The ball will reach the highest point when $v_y = 0$, this will happen at a time t after the observation such that $t = v_{y,0}/g = (6.1\,\text{m/s})/(9.81\,\text{m/s}^2) = 0.62\,\text{s}$. The vertical displacement (from the ground) will be

$$y = -gt^2/2 + v_{y,0}t + y_0 = -(9.81\,\text{m/s}^2)(0.62\,\text{s})^2/2 + (6.1\,\text{m/s})(0.62\,\text{s}) + (9.1\,\text{m}) = 11\,\text{m}.$$

(b) The time for the ball to return to the ground from the highest point is $t = \sqrt{2y_{\max}/g} = \sqrt{2(11\,\text{m})/(9.81\,\text{m/s}^2)} = 1.5\,\text{s}$. The total time of flight is twice this, or 3.0 s. The horizontal distance traveled is $x = v_x t = (7.6\,\text{m/s})(3.0\,\text{s}) = 23\,\text{m}$.

(c) The velocity of the ball just prior to hitting the ground is

$$\vec{\mathbf{v}} = \vec{\mathbf{a}}t + \vec{\mathbf{v}}_0 = (-9.81\,\text{m/s}^2)\hat{\mathbf{j}}(1.5\,\text{s}) + (7.6\,\text{m/s})\hat{\mathbf{i}} = 7.6\,\text{m/s}\hat{\mathbf{i}} - 15\,\text{m/s}uj.$$

The magnitude is $\sqrt{7.6^2 + 15^2}(\text{m/s}) = 17\,\text{m/s}$. The direction is

$$\theta = \arctan(-15/7.6) = -63^\circ.$$

E4-20 Focus on the time it takes the ball to get to the plate, assuming it traveled in a straight line. The ball has a "horizontal" velocity of 135 ft/s. Then $t = x/v_x = (60.5\text{ ft})/(135\text{ ft/s}) = 0.448\,\text{s}$. The ball will "fall" a vertical distance of $y = -gt^2/2 = -(32\text{ ft/s}^2)(0.448\,\text{s})^2/2 = -3.2$ ft. That's in the strike zone.

E4-21 Since $R \propto 1/g$ one can write $R_2/R_1 = g_1/g_2$, or

$$\Delta R = R_2 - R_1 = R_1\left(1 - \frac{g_1}{g_2}\right) = (8.09\,\text{m})\left[1 - \frac{(9.7999\,\text{m/s}^2)}{(9.8128\,\text{m/s}^2)}\right] = 1.06\text{ cm}.$$

E4-22 If initial position is $\vec{\mathbf{r}}_0 = 0$, then final position is $\vec{\mathbf{r}} = (13\text{ ft})\hat{\mathbf{i}} + (3\text{ ft})\hat{\mathbf{j}}$. The initial velocity is $\vec{\mathbf{v}}_0 = v\cos\theta\hat{\mathbf{i}} + v\sin\theta\hat{\mathbf{j}}$. The horizontal equation is $(13\text{ ft}) = v\cos\theta t$; the vertical equation is $(3\text{ ft}) = -(g/2)t^2 + v\sin\theta t$. Rearrange the vertical equation and then divide by the horizontal equation to get

$$\frac{3\text{ ft} + (g/2)t^2}{(13\text{ ft})} = \tan\theta,$$

or

$$t^2 = [(13\text{ ft})\tan(55^\circ) - (3\text{ ft})][2/(32\,\text{m/s}^2)] = 0.973\,\text{s}^2,$$

or $t = 0.986\,\text{s}$. Then $v = (13\text{ ft})/(\cos(55^\circ)(0.986\,\text{s})) = 23\text{ ft/s}$.

E4-23 $v_x = x/t = (150\text{ ft})/(4.50\,\text{s}) = 33.3\text{ ft/s}$. The time to the highest point is half the hang time, or 2.25 s. The vertical speed when the ball hits the ground is $v_y = -gt = -(32\text{ ft/s}^2)(2.25\,\text{s}) = 72.0\text{ ft/s}$. Then the initial vertical velocity is 72.0 ft/s. The magnitude of the initial velocity is $\sqrt{72^2 + 33^2}(\text{ft/s}) = 79\text{ ft/s}$. The direction is

$$\theta = \arctan(72/33) = 65^\circ.$$

E4-24 (a) The magnitude of the initial velocity of the projectile is $v = 264\text{ ft/s}$. The projectile was in the air for a time t where

$$t = \frac{x}{v_x} = \frac{x}{v\cos\theta} = \frac{(2300\text{ ft})}{(264\text{ ft/s})\cos(-27^\circ)} = 9.8\,\text{s}.$$

(b) The height of the plane was $-y$ where

$$-y = gt^2/2 - v_{y,0}t = (32\text{ ft/s})(9.8\,\text{s})^2/2 - (264\text{ ft/s})\sin(-27^\circ)(9.8\,\text{s}) = 2700\text{ ft}.$$

$\boxed{\textbf{E4-25}}$ Define the point the ball leaves the racquet as $\vec{\mathbf{r}} = 0$.

(a) The initial conditions are given as $v_{0x} = 23.6$ m/s and $v_{0y} = 0$. The time it takes for the ball to reach the horizontal location of the net is found from

$$\begin{aligned} x &= v_{0x}t, \\ (12\text{ m}) &= (23.6\text{ m/s})t, \\ 0.51\text{ s} &= t, \end{aligned}$$

Find how far the ball has moved horizontally in this time:

$$y = v_{0y}t - \frac{1}{2}gt^2 = (0)(0.51\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(0.51\text{ s})^2 = -1.3\text{ m}.$$

Did the ball clear the net? The ball started 2.37 m above the ground and "fell" through a distance of 1.3 m by the time it arrived at the net. So it is still 1.1 m above the ground and 0.2 m above the net.

(b) The initial conditions are now given by $v_{0x} = (23.6\text{ m/s})(\cos[-5.0^\circ]) = 23.5$ m/s and $v_{0y} = (23.6\text{ m/s})(\sin[-5.0^\circ]) = -2.1$ m/s. Now find the time to reach the net just as done in part (a):

$$t = x/v_{0x} = (12.0\text{ m})/(23.5\text{ m/s}) = 0.51\text{ s}.$$

Find the vertical position of the ball when it arrives at the net:

$$y = v_{0y}t - \frac{1}{2}gt^2 = (-2.1\text{ m/s})(0.51\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(0.51\text{ s})^2 = -2.3\text{ m}.$$

Did the ball clear the net? Not this time; it started 2.37 m above the ground and then passed the net 2.3 m lower, or only 0.07 m above the ground.

E4-26 The initial speed of the ball is given by $v = \sqrt{gR} = \sqrt{(32\text{ ft/s}^2)(350\text{ ft})} = 106$ ft/s. The time of flight from the batter to the wall is

$$t = x/v_x = (320\text{ ft})/[(106\text{ ft/s})\cos(45°)] = 4.3\,\text{s}.$$

The height of the ball at that time is given by $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, or

$$y = (4\text{ ft}) + (106\text{ ft/s})\sin(45°)(4.3\,\text{s}) - (16\text{ ft/s}^2)(4.3\,\text{s})^2 = 31\text{ ft},$$

clearing the fence by 7 feet.

E4-27 The ball lands $x = (358\text{ ft}) + (39\text{ ft})\cos(28°) = 392$ ft from the initial position. The ball lands $y = (39\text{ ft})\sin(28°) - (4.60\text{ ft}) = 14$ ft above the initial position. The horizontal equation is $(392\text{ ft}) = v\cos\theta t$; the vertical equation is $(14\text{ ft}) = -(g/2)t^2 + v\sin\theta t$. Rearrange the vertical equation and then divide by the horizontal equation to get

$$\frac{14\text{ ft} + (g/2)t^2}{(392\text{ ft})} = \tan\theta,$$

or

$$t^2 = [(392\text{ ft})\tan(52°) - (14\text{ ft})][2/(32\,\text{m/s}^2)] = 30.5\,\text{s}^2,$$

or $t = 5.52\,\text{s}$. Then $v = (392\text{ ft})/(\cos(52°)(5.52\,\text{s})) = 115$ ft/s.

E4-28 Since ball is traveling at 45° when it returns to the same level from which it was thrown for maximum range, then we can assume it actually traveled $\approx 1.6\,\text{m}$. farther than it would have had it been launched from the ground level. This won't make a big difference, but that means that $R = 60.0\,\text{m} - 1.6\,\text{m} = 58.4\,\text{m}$. If v_0 is initial speed of ball thrown directly up, the ball rises to the highest point in a time $t = v_0/g$, and that point is $y_{\max} = gt^2/2 = v_0^2/(2g)$ above the launch point. But $v_0^2 = gR$, so $y_{\max} = R/2 = (58.4\,\text{m})/2 = 29.2\,\text{m}$. To this we add the 1.60 m point of release to get 30.8 m.

E4-29 The net force on the pebble is zero, so $\sum F_y = 0$. There are only two forces on the pebble, the force of gravity W and the force of the water on the pebble F_{PW}. These point in opposite directions, so $0 = F_{PW} - W$. But $W = mg = (0.150\text{ kg})(9.81\text{ m/s}^2) = 1.47$ N. Since $F_{PW} = W$ in this problem, the force of the water on the pebble must also be 1.47 N.

E4-30 Terminal speed is when drag force equal weight, or $mg = bv_{\text{T}}{}^2$. Then $v_{\text{T}} = \sqrt{mg/b}$.

E4-31 Eq. 4-22 is

$$v_y(t) = v_{\text{T}}\left(1 - e^{-bt/m}\right),$$

where we have used Eq. 4-24 to substitute for the terminal speed. We want to solve this equation for time when $v_y(t) = v_{\text{T}}/2$, so

$$\begin{aligned}
\tfrac{1}{2}v_{\text{T}} &= v_{\text{T}}\left(1 - e^{-bt/m}\right), \\
\tfrac{1}{2} &= \left(1 - e^{-bt/m}\right), \\
e^{-bt/m} &= \tfrac{1}{2} \\
bt/m &= -\ln(1/2) \\
t &= \tfrac{m}{b}\ln 2
\end{aligned}$$

E4-32 The terminal speed is 7 m/s for a raindrop with $r = 0.15$ cm. The mass of this drop is $m = 4\pi\rho r^3/3$, so

$$b = \frac{mg}{v_\mathrm{T}} = \frac{4\pi(1.0\times10^{-3}\mathrm{kg/cm^3})(0.15\ \mathrm{cm})^3(9.81\,\mathrm{m/s^2})}{3(7\,\mathrm{m/s})} = 2.0\times10^{-5}\mathrm{kg/s}.$$

E4-33 (a) The speed of the train is $v = 9.58\,\mathrm{m/s}$. The drag force on one car is $f = 243(9.58)\,\mathrm{N} = 2330\,\mathrm{N}$. The total drag force is $23(2330\,\mathrm{N}) = 5.36\times10^4\mathrm{N}$. The net force exerted on the cars (treated as a single entity) is $F = ma = 23(48.6\times10^3\mathrm{kg})(0.182\,\mathrm{m/s^2}) = 2.03\times10^5\mathrm{N}$. The pull of the locomotive is then $P = 2.03\times10^5\mathrm{N} + 5.36\times10^4\mathrm{N} = 2.57\times10^5\mathrm{N}$.

(b) If the locomotive is pulling the cars at constant speed up an incline then it must exert a force on the cars equal to the sum of the drag force and the parallel component of the weight. The drag force is the same in each case, so the parallel component of the weight is $W_{||} = W\sin\theta = 2.03\times10^5\mathrm{N} = ma$, where a is the acceleration from part (a). Then

$$\theta = \arcsin(a/g) = \arcsin[(0.182\,\mathrm{m/s^2})/(9.81\,\mathrm{m/s^2})] = 1.06^\circ.$$

E4-34 (a) $a = v^2/r = (2.18\times10^6\mathrm{m/s})^2/(5.29\times10^{-11}\mathrm{m}) = 8.98\times10^{22}\mathrm{m/s^2}$.

(b) $F = ma = (9.11\times10^{-31}\mathrm{kg})(8.98\times10^{22}\mathrm{m/s^2}) = 8.18\times10^{-8}\mathrm{N}$, toward the center.

$\boxed{\textbf{E4-35}}$ (a) $v = \sqrt{ra_\mathrm{c}} = \sqrt{(5.2\ \mathrm{m})(6.8)(9.8\ \mathrm{m/s^2})} = 19\ \mathrm{m/s}$.

(b) Use the fact that one revolution corresponds to a length of $2\pi r$:

$$19\frac{\mathrm{m}}{\mathrm{s}}\left(\frac{1\ \mathrm{rev}}{2\pi(5.2\ \mathrm{m})}\right)\left(\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right) = 35\frac{\mathrm{rev}}{\mathrm{min}}.$$

E4-36 (a) $v = 2\pi r/T = 2\pi(15\,\mathrm{m})/(12\,\mathrm{s}) = 7.85\,\mathrm{m/s}$. Then $a = v^2/r = (7.85\,\mathrm{m/s})^2/(15\,\mathrm{m}) = 4.11\,\mathrm{m/s^2}$, directed toward center, which is down.

(b) Same arithmetic as in (a); direction is still toward center, which is now up.

(c) The magnitude of the net force in both (a) and (b) is $F = ma = (75\,\mathrm{kg})(4.11\,\mathrm{m/s^2}) = 310\,\mathrm{N}$. The weight of the person is the same in both parts: $W = mg = (75\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 740\,\mathrm{N}$. At the top the net force is down, the weight is down, so the Ferris wheel is pushing up with a force of $P = W - F = (740\,\mathrm{N}) - (310\,\mathrm{N}) = 430\,\mathrm{N}$. At the bottom the net force is up, the weight is down, so the Ferris wheel is pushing up with a force of $P = W + F = (740\,\mathrm{N}) + (310\,\mathrm{N}) = 1050\,\mathrm{N}$.

E4-37 (a) $v = 2\pi r/T = 2\pi(20\times10^3\mathrm{m})/(1.0\,\mathrm{s}) = 1.26\times10^5\mathrm{m/s}$.

(b) $a = v^2/r = (1.26\times10^5\mathrm{m/s})^2/(20\times10^3\mathrm{m}) = 7.9\times10^5\mathrm{m/s^2}$.

E4-38 (a) $v = 2\pi r/T = 2\pi(6.37\times10^6\mathrm{m})/(86400\,\mathrm{s}) = 463\,\mathrm{m/s}$. $a = v^2/r = (463\,\mathrm{m/s})^2/(6.37\times10^6\mathrm{m}) = 0.034\,\mathrm{m/s^2}$.

(b) The net force on the object is $F = ma = (25.0\,\mathrm{kg})(0.034\,\mathrm{m/s^2}) = 0.85\,\mathrm{N}$. There are two forces on the object: a force up from the scale (S), and the weight down, $W = mg = (25.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 245\,\mathrm{N}$. Then $S = F + W = 245\,\mathrm{N} + 0.85\,\mathrm{N} = 246\,\mathrm{N}$.

$\boxed{\textbf{E4-39}}$ Let $\Delta x = 15$ m be the length; $t_w = 90$ s, the time to walk the stalled Escalator; $t_s = 60$ s, the time to ride the moving Escalator; and t_m, the time to walk up the moving Escalator.

The walking speed of the person relative to a fixed Escalator is $v_{we} = \Delta x/t_w$; the speed of the Escalator relative to the ground is $v_{eg} = \Delta x/t_s$; and the speed of the walking person relative to the ground on a moving Escalator is $v_{wg} = \Delta x/t_m$. But these three speeds are related by $v_{wg} = v_{we}+v_{eg}$. Combine all the above:

$$\begin{aligned} v_{wg} &= v_{we} + v_{eg}, \\ \frac{\Delta x}{t_m} &= \frac{\Delta x}{t_w} + \frac{\Delta x}{t_s}, \\ \frac{1}{t_m} &= \frac{1}{t_w} + \frac{1}{t_s}. \end{aligned}$$

Putting in the numbers, $t_m = 36$ s.

E4-40 Let v_w be the walking speed, v_s be the sidewalk speed, and $v_\mathrm{m} = v_\mathrm{w} + v_\mathrm{s}$ be Mary's speed while walking on the moving sidewalk. All three cover the same distance x, so $v_i = x/t_i$, where i is one of w, s, or m. Put this into the Mary equation, and

$$1/t_\mathrm{m} = 1/t_\mathrm{w} + 1/t_\mathrm{s} = 1/(150\,\mathrm{s}) + 1/(70\,\mathrm{s}) = 1/48\,\mathrm{s}.$$

E4-41 If it takes longer to fly westward then the speed of the plane (relative to the ground) westward must be less than the speed of the plane eastward. We conclude that the jet-stream must be blowing east. The speed of the plane relative to the ground is $v_\mathrm{e} = v_\mathrm{p} + v_\mathrm{j}$ when going east and $v_\mathrm{w} = v_\mathrm{p} - v_\mathrm{j}$ when going west. In either case the distance is the same, so $x = v_i t_i$, where i is e or w. Since $t_\mathrm{w} - t_\mathrm{e}$ is given, we can write

$$t_\mathrm{w} - t_\mathrm{e} = \frac{x}{v_\mathrm{p} - v_\mathrm{j}} - \frac{x}{v_\mathrm{p} + v_\mathrm{j}} = x\frac{2v_\mathrm{j}}{{v_\mathrm{p}}^2 - {v_\mathrm{j}}^2}.$$

Solve the quadratic if you want, but since $v_\mathrm{j} \ll v_\mathrm{p}$ we can neglect it in the denominator and

$$v_\mathrm{j} = {v_\mathrm{p}}^2(0.83\ \mathrm{h})/(2x) = (600\ \mathrm{mi/h})^2(0.417\ \mathrm{h})/(2700\ \mathrm{mi}) = 56\ \mathrm{mi/hr}.$$

E4-42 The horizontal component of the rain drop's velocity is 28 m/s. Since $v_x = v\sin\theta$, $v = (28\,\mathrm{m/s})/\sin(64^\circ) = 31\,\mathrm{m/s}$.

$\boxed{\textbf{E4-43}}$ (a) The position of the bolt relative to the elevator is y_{be}, the position of the bolt relative to the shaft is y_{bs}, and the position of the elevator relative to the shaft is y_{es}. Zero all three positions at $t = 0$; at this time $v_{0,bs} = v_{0,es} = 8.0$ ft/s.

The three equations describing the positions are

$$\begin{aligned} y_{bs} &= v_{0,bs}t - \frac{1}{2}gt^2, \\ y_{es} &= v_{0,es}t + \frac{1}{2}at^2, \\ y_{be} + r_{es} &= r_{bs}, \end{aligned}$$

where $a = 4.0$ m/s^2 is the upward acceleration of the elevator. Rearrange the last equation and solve for y_{be}; get $y_{be} = -\frac{1}{2}(g + a)t^2$, where advantage was taken of the fact that the initial velocities are the same.

Then

$$t = \sqrt{-2y_{be}/(g+a)} = \sqrt{-2(-9.0\ \mathrm{ft})/(32\ \mathrm{ft/s^2} + 4\ \mathrm{ft/s^2})} = 0.71\,\mathrm{s}$$

(b) Use the expression for y_{bs} to find how the bolt moved relative to the shaft:

$$y_{bs} = v_{0,bs}t - \frac{1}{2}gt^2 = (8.0\ \mathrm{ft})(0.71\ \mathrm{s}) - \frac{1}{2}(32\ \mathrm{ft/s^2})(0.71\ \mathrm{s})^2 = -2.4\ \mathrm{ft}.$$

E4-44 The speed of the plane relative to the ground is $v_{pg} = (810 \text{ km})/(1.9 \text{ h}) = 426$ km/h. The velocity components of the plane relative to the air are $v_N = (480 \text{ km/h})\cos(21^\circ) = 448$ km/h and $v_E = (480 \text{ km/h})\sin(21^\circ) = 172$ km/h. The wind must be blowing with a component of 172 km/h to the west and a component of $448 - 426 = 22$ km/h to the south.

E4-45 (a) Let $\hat{\mathbf{i}}$ point east and $\hat{\mathbf{j}}$ point north. The velocity of the torpedo is $\vec{\mathbf{v}} = (50 \text{ km/h})\hat{\mathbf{i}}\sin\theta + (50 \text{ km/h})\hat{\mathbf{j}}\cos\theta$. The initial coordinates of the battleship are then $\vec{\mathbf{r}}_0 = (4.0 \text{ km})\hat{\mathbf{i}}\sin(20^\circ) + (4.0 \text{ km})\hat{\mathbf{j}}\cos(20^\circ) = (1.37 \text{ km})\hat{\mathbf{i}} + (3.76 \text{ km})\hat{\mathbf{j}}$. The final position of the battleship is $\vec{\mathbf{r}} = (1.37 \text{ km} + 24 \text{ km/h}t)\hat{\mathbf{i}} + (3.76 \text{ km})\hat{\mathbf{j}}$, where t is the time of impact. The final position of the torpedo is the same, so

$$[(50 \text{ km/h})\hat{\mathbf{i}}\sin\theta + (50 \text{ km/h})\hat{\mathbf{j}}\cos\theta]t = (1.37 \text{ km} + 24 \text{ km/h}t)\hat{\mathbf{i}} + (3.76 \text{ km})\hat{\mathbf{j}},$$

or

$$[(50 \text{ km/h})\sin\theta]t - 24 \text{ km/h}t = 1.37 \text{ km}$$

and

$$[(50 \text{ km/h})\cos\theta]t = 3.76 \text{ km}.$$

Dividing the top equation by the bottom and rearranging,

$$50\sin\theta - 24 = 18.2\cos\theta.$$

Use any trick you want to solve this. I used Maple and found $\theta = 46.8^\circ$.

(b) The time to impact is then $t = 3.76 \text{ km}/[(50 \text{ km/h})\cos(46.8^\circ)] = 0.110$ h, or 6.6 minutes.

$\boxed{\textbf{P4-1}}$ Let $\vec{\mathbf{r}}_A$ be the position of particle of particle A, and $\vec{\mathbf{r}}_B$ be the position of particle B. The equations for the motion of the two particles are then

$$\begin{aligned}
\vec{\mathbf{r}}_A &= \vec{\mathbf{r}}_{0,A} + \vec{\mathbf{v}}t, \\
&= d\hat{\mathbf{j}} + vt\hat{\mathbf{i}}; \\
\vec{\mathbf{r}}_B &= \frac{1}{2}\vec{\mathbf{a}}t^2, \\
&= \frac{1}{2}a(\sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}})t^2.
\end{aligned}$$

A collision will occur if there is a time when $\vec{\mathbf{r}}_A = \vec{\mathbf{r}}_B$. Then

$$d\hat{\mathbf{j}} + vt\hat{\mathbf{i}} = \frac{1}{2}a(\sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}})t^2,$$

but this is really *two* equations: $d = \frac{1}{2}at^2\cos\theta$ and $vt = \frac{1}{2}at^2\sin\theta$.

Solve the second one for t and get $t = 2v/(a\sin\theta)$. Substitute that into the first equation, and then rearrange,

$$\begin{aligned}
d &= \frac{1}{2}at^2\cos\theta, \\
d &= \frac{1}{2}a\left(\frac{2v}{a\sin\theta}\right)^2\cos\theta, \\
\sin^2\theta &= \frac{2v}{ad}\cos\theta, \\
1 - \cos^2\theta &= \frac{2v^2}{ad}\cos\theta, \\
0 &= \cos^2\theta + \frac{2v^2}{ad}\cos\theta - 1.
\end{aligned}$$

This last expression is quadratic in $\cos\theta$. It simplifies the solution if we define $b = 2v/(ad) = 2(3.0\ \text{m/s})^2/([0.4\ \text{m/s}^2][30\ \text{m}]) = 1.5$, then

$$\cos\theta = \frac{-b \pm \sqrt{b^2+4}}{2} = -0.75 \pm 1.25.$$

Then $\cos\theta = 0.5$ and $\theta = 60°$.

P4-2 (a) The acceleration of the ball is $\vec{\mathbf{a}} = (1.20\,\text{m/s}^2)\hat{\mathbf{i}} - (9.81\,\text{m/s}^2)\hat{\mathbf{j}}$. Since $\vec{\mathbf{a}}$ is constant the trajectory is given by $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2$, since $\vec{\mathbf{v}}_0 = 0$ and we choose $\vec{\mathbf{r}}_0 = 0$. This is a straight line trajectory, with a direction given by $\vec{\mathbf{a}}$. Then

$$\theta = \arctan(9.81/1.20) = 83.0°.$$

and $R = (39.0\,\text{m})/\tan(83.0°) = 4.79\,\text{m}$. It will be useful to find $H = (39.0\,\text{m})/\sin(83.0°) = 39.3\,\text{m}$.

(b) The magnitude of the acceleration of the ball is $a = \sqrt{9.81^2 + 1.20^2}(\text{m/s}^2) = 9.88\,\text{m/s}^2$. The time for the ball to travel down the hypotenuse of the figure is then $t = \sqrt{2(39.3\,\text{m})/(9.88\,\text{m/s}^2)} = 2.82\,\text{s}$.

(c) The magnitude of the speed of the ball at the bottom will then be

$$v = at = (9.88\,\text{m/s}^2)(2.82\,\text{s}) = 27.9\,\text{m/s}.$$

P4-3 (a) The rocket thrust is $\vec{\mathbf{T}} = (61.2\ \text{kN})\cos(58.0°)\hat{\mathbf{i}} + (61.2\ \text{kN})\sin(58.0°)\hat{\mathbf{i}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 51.9\ \text{kN}\hat{\mathbf{j}}$. The net force on the rocket is the $\vec{\mathbf{F}} = \vec{\mathbf{T}} + \vec{\mathbf{W}}$, or

$$\vec{\mathbf{F}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 51.9\ \text{kN}\hat{\mathbf{j}} - (3030\,\text{kg})(9.81\,\text{m/s}^2)\hat{\mathbf{j}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 22.2\ \text{kN}\hat{\mathbf{j}}.$$

The acceleration (until rocket cut-off) is this net force divided by the mass, or

$$\vec{\mathbf{a}} = 10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}}.$$

The position at rocket cut-off is given by

$$\begin{aligned}\vec{\mathbf{r}} &= \vec{\mathbf{a}}t^2/2 = (10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}})(48.0\,\text{s})^2/2,\\ &= 1.23\times10^4\text{m}\hat{\mathbf{i}} + 8.44\times10^3\text{m}\hat{\mathbf{j}}.\end{aligned}$$

The altitude at rocket cut-off is then 8.44 km.

(b) The velocity at rocket cut-off is

$$\vec{\mathbf{v}} = \vec{\mathbf{a}}t = (10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}})(48.0\,\text{s}) = 514\,\text{m/s}\hat{\mathbf{i}} + 352\,\text{m/s}\hat{\mathbf{j}},$$

this becomes the initial velocity for the "free fall" part of the journey. The rocket will hit the ground after t seconds, where t is the solution to

$$0 = -(9.81\,\text{m/s}^2)t^2/2 + (352\,\text{m/s})t + 8.44\times10^3\,\text{m}.$$

The solution is $t = 90.7\,\text{s}$. The rocket lands a horizontal distance of $x = v_x t = (514\,\text{m/s})(90.7\,\text{s}) = 4.66\times10^4$ m beyond the rocket cut-off; the total horizontal distance covered by the rocket is 46.6 km + 12.3 km = 58.9 km.

P4-4 (a) The horizontal speed of the ball is $v_x = 135$ ft/s. It takes

$$t = x/v_x = (30.0 \text{ ft})/(135 \text{ ft/s}) = 0.222 \,\text{s}$$

to travel the 30 feet horizontally, whether the first 30 feet, the last 30 feet, or 30 feet somewhere in the middle.

(b) The ball "falls" $y = -gt^2/2 = -(32 \text{ ft/s}^2)(0.222\,\text{s})^2/2 = -0.789$ ft while traveling the first 30 feet.

(c) The ball "falls" a total of $y = -gt^2/2 = -(32 \text{ ft/s}^2)(0.444\,\text{s})^2/2 = -3.15$ ft while traveling the first 60 feet, so during the last 30 feet it must have fallen $(-3.15 \text{ ft}) - (-0.789 \text{ ft}) = -2.36$ ft.

(d) The distance fallen because of acceleration is not linear in time; the distance moved horizontally is linear in time.

P4-5 (a) The initial velocity of the ball has components

$$v_{x,0} = (25.3\,\text{m/s})\cos(42.0^\circ) = 18.8\,\text{m/s}$$

and

$$v_{y,0} = (25.3\,\text{m/s})\sin(42.0^\circ) = 16.9\,\text{m/s}.$$

The ball is in the air for $t = x/v_x = (21.8\,\text{m})/(18.8\,\text{m/s}) = 1.16\,\text{s}$ before it hits the wall.

(b) $y = -gt^2/2 + v_{y,0}t = -(4.91\,\text{m/s}^2)(1.16\,\text{s})^2 + (16.9\,\text{m/s})(1.16\,\text{s}) = 13.0\,\text{m}$.

(c) $v_x = v_{x,0} = 18.8\,\text{m/s}$. $v_y = -gt + v_{y,0} = -(9.81\,\text{m/s}^2)(1.16\,\text{s}) + (16.9\,\text{m/s}) = 5.52\,\text{m/s}$.

(d) Since $v_y > 0$ the ball is still heading up.

P4-6 (a) The initial vertical velocity is $v_{y,0} = v_0 \sin\phi_0$. The time to the highest point is $t = v_{y,0}/g$. The highest point is $H = gt^2/2$. Combining,

$$H = g(v_0 \sin\phi_0/g)^2/2 = v_0^2 \sin^2\phi_0/(2g).$$

The range is $R = (v_0^2/g)\sin 2\phi_0 = 2(v_0^2/g)\sin\phi_0\cos\phi_0$. Since $\tan\theta = H/(R/2)$, we have

$$\tan\theta = \frac{2H}{R} = \frac{v_0^2 \sin^2\phi_0/g}{2(v_0^2/g)\sin\phi_0\cos\phi_0} = \frac{1}{2}\tan\phi_0.$$

(b) $\theta = \arctan(0.5\tan 45^\circ) = 26.6^\circ$.

$\boxed{\textbf{P4-7}}$ The components of the initial velocity are given by $v_{0x} = v_0\cos\theta = 56$ ft/s and $v_{0y} = v_0\sin\theta = 106$ ft/s where we used $v_0 = 120$ ft/s and $\theta = 62^\circ$.

(a) To find h we need only find out the vertical position of the stone when $t = 5.5$ s.

$$y = v_{0y}t - \frac{1}{2}gt^2 = (106 \text{ ft/s})(5.5 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(5.5 \text{ s})^2 = 99 \text{ ft}.$$

(b) Look at this as a vector problem:

$$\begin{aligned}
\vec{\mathbf{v}} &= \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t, \\
&= \left(v_{0x}\hat{\mathbf{i}} + v_{0y}\hat{\mathbf{j}}\right) - g\hat{\mathbf{j}}t, \\
&= v_{0x}\hat{\mathbf{i}} + (v_{0y} - gt)\,\hat{\mathbf{j}}, \\
&= (56 \text{ ft/s})\hat{\mathbf{i}} + \Big((106 \text{ ft/s} - (32 \text{ ft/s}^2)(5.5 \text{ s})\Big)\,\hat{\mathbf{j}}, \\
&= (56 \text{ ft/s})\hat{\mathbf{i}} + (-70.0 \text{ ft/s})\hat{\mathbf{j}}.
\end{aligned}$$

The magnitude of this vector gives the speed when $t = 5.5$ s; $v = \sqrt{56^2 + (-70)^2}$ ft/s= 90 ft/s.

(c) Highest point occurs when $v_y = 0$. Solving Eq. 4-9(b) for time; $v_y = 0 = v_{0y} - gt = (106 \text{ ft/s}) - (32 \text{ ft/s}^2)t$; $t = 3.31$ s. Use this time in Eq. 4-10(b),

$$y = v_{0y}t - \frac{1}{2}gt^2 = (106 \text{ ft/s})(3.31 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(3.31 \text{ s})^2 = 176 \text{ ft}.$$

P4-8 (a) Since $R = (v_0^2/g)\sin 2\phi_0$, it is sufficient to prove that $\sin 2\phi_0$ is the same for both $\sin 2(45° + \alpha)$ and $\sin 2(45° - \alpha)$.

$$\sin 2(45° \pm \alpha) = \sin(90° \pm 2\alpha) = \cos(\pm 2\alpha) = \cos(2\alpha).$$

Since the $\pm$ dropped out, the two quantities are equal.

(b) $\phi_0 = (1/2)\arcsin(Rg/v_0^2) = (1/2)\arcsin((20.0\,\text{m})(9.81\,\text{m/s}^2)/(30.0\,\text{m/s})^2) = 6.3°$. The other choice is $90° - 6.3° = 83.7°$.

$\boxed{\textbf{P4-9}}$ To score the ball must pass the horizontal distance of 50 m with an altitude of no less than 3.44 m. The initial velocity components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$ where $v_0 = 25$ m/s, and θ is the unknown.

The time to the goal post is $t = x/v_{0x} = x/(v_0 \cos\theta)$.

The vertical motion is given by

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2, \\ &= x\frac{\sin\theta}{\cos\theta} - \frac{gx^2}{2v_0^2}\frac{1}{\cos^2\theta}. \end{aligned}$$

In this last expression y needs to be greater than 3.44 m. In this last expression use

$$\frac{1}{\cos^2\theta} - 1 + 1 = \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta} + 1 = \frac{1 - \cos^2\theta}{\cos^2\theta} + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1 = \tan^2\theta + 1.$$

This gives for our y expression

$$y = x\tan\theta - \frac{gx^2}{2v_0}\left(\tan^2\theta + 1\right),$$

which can be combined with numbers and constraints to give

$$\begin{aligned} (3.44 \text{ m}) &\leq (50 \text{ m})\tan\theta - \frac{(9.8 \text{ m/s}^2)(50 \text{ m})^2}{2(25 \text{ m/s})^2}\left(\tan^2\theta + 1\right), \\ 3.44 &\leq 50\tan\theta - 20\left(\tan^2\theta + 1\right), \\ 0 &\leq -20\tan^2\theta + 50\tan\theta - 23 \end{aligned}$$

Solve, and $\tan\theta = 1.25 \pm 0.65$, so the allowed kicking angles are between $\theta = 31°$ and $\theta = 62°$.

P4-10 (a) The height of the projectile at the highest point is $H = L\sin\theta$. The amount of time before the projectile hits the ground is $t = \sqrt{2H/g} = \sqrt{2L\sin\theta/g}$. The horizontal distance covered by the projectile in this time is $x = v_x t = v\sqrt{2L\sin\theta/g}$. The horizontal distance to the projectile when it is at the highest point is $x' = L\cos\theta$. The projectile lands at

$$D = x - x' = v\sqrt{2L\sin\theta/g} - L\cos\theta.$$

(b) The projectile will pass overhead if $D > 0$.

P4-11 $v^2 = v_x^2 + v_y^2$. For a projectile v_x is constant, so we need only evaluate $d^2(v_y^2)/dt^2$. The first derivative is $2v_y\,dv_y/dt = -2v_y g$. The derivative of this (the second derivative) is $-2g\,dv_y/dt = 2g^2$.

P4-12 $|\vec{\mathbf{r}}|$ is a maximum when r^2 is a maximum. $r^2 = x^2 + y^2$, or

$$\begin{aligned} r^2 &= (v_{x,0}t)^2 + (-gt^2/2 + v_{y,0}t)^2, \\ &= (v_0 t\cos\phi_0)^2 + \left(v_0 t\sin\phi_0 - gt^2/2\right)^2, \\ &= v_0^2 t^2 - v_0 g t^3 \sin\phi_0 + g^2 t^4/4. \end{aligned}$$

We want to look for the condition which will allow dr^2/dt to vanish. Since

$$dr^2/dt = 2v_0^2 t - 3v_0 g t^2 \sin\phi_0 + g^2 t^3$$

we can focus on the quadratic discriminant, $b^2 - 4ac$, which is

$$9v_0^2 g^2 \sin^2\phi_0 - 8v_0^2 g^2,$$

a quantity which will only be greater than zero if $9\sin^2\phi_0 > 8$. The critical angle is then

$$\phi_c = \arcsin(\sqrt{8/9}) = 70.5^\circ.$$

$\boxed{\textbf{P4-13}}$ There is a downward force on the balloon of 10.8 kN from gravity and an upward force of 10.3 kN from the buoyant force of the air. The resultant of these two forces is 500 N down, but since the balloon is descending at constant speed so the net force on the balloon must be zero. This is possible because there is a drag force on the balloon of $D = bv^2$, this force is directed upward. The magnitude must be 500 N, so the constant b is

$$b = \frac{(500\,\text{N})}{(1.88\,\text{m/s})^2} = 141\,\text{kg/m}.$$

If the crew drops 26.5 kg of ballast they are "lightening" the balloon by

$$(26.5\,\text{kg})(9.81\,\text{m/s}^2) = 260\,\text{N}.$$

This reduced the weight, but not the buoyant force, so the drag force at constant speed will now be $500\,\text{N} - 260\,\text{N} = 240\,\text{N}$.

The new constant downward speed will be

$$v = \sqrt{D/b} = \sqrt{(240\,\text{N})/(141\,\text{kg/m})} = 1.30\,\text{m/s}.$$

P4-14 The constant b is

$$b = (500\,\text{N})/(1.88\,\text{m/s}) = 266\,\text{N}\cdot\text{s/m}.$$

The drag force after "lightening" the load will still be 240 N. The new downward speed will be

$$v = D/b = (240\,\text{N})/(266\,\text{N}\cdot\text{s/m}) = 0.902\,\text{m/s}.$$

P4-15 (a) Initially $v_0 = 0$, so $D = 0$, the only force is the weight, so $a = -g$.

(b) After some time the acceleration is zero, then $W = D$, or $bv_{\text{T}}{}^2 = mg$, or $v_{\text{T}} = \sqrt{mg/b}$.

(c) When $v = v_{\text{T}}/2$ the drag force is $D = bv_{\text{T}}{}^2/4 = mg/4$, so the net force is $F = D - W = -3mg/4$. The acceleration is the $a = -3g/4$.

P4-16 (a) The net force on the barge is $F = -D = -bv$, this results in a differential equation $m\,dv/dt = -bv$, which can be written as

$$\begin{aligned} dv/v &= -(b/m)dt, \\ \int dv/v &= -(b/m)\int dt, \\ \ln(v_\mathrm{f}/v_\mathrm{i}) &= -bt/m. \end{aligned}$$

Then $t = (m/b)\ln(v_\mathrm{i}/v_\mathrm{f})$.

(b) $t = [(970\,\mathrm{kg})/(68\,\mathrm{N}\cdot\mathrm{s/m})]\ln(32/8.3) = 19\,\mathrm{s}$.

$\boxed{\textbf{P4-17}}$ (a) The acceleration is the time derivative of the velocity,

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{mg}{b}\left(1 - e^{-bt/m}\right)\right) = \frac{mg}{b}\frac{b}{m}e^{-bt/m},$$

which can be simplified as $a_y = ge^{-bt/m}$. For large t this expression approaches 0; for small t the exponent can be expanded to give

$$a_y \approx g\left(1 - \frac{bt}{m}\right) = g - v_\mathrm{T}t,$$

where in the last line we made use of Eq. 4-24.

(b) The position is the integral of the velocity,

$$\begin{aligned} \int_0^t v_y\,dt &= \int_0^t \left(\frac{mg}{b}\left(1 - e^{-bt/m}\right)\right)\,dt, \\ \int_0^t \frac{dy}{dt}\,dt &= \frac{mg}{b}\left(t - (-m/b)e^{-bt/m}\right)\Big|_0^t, \\ \int_0^y dy &= v_\mathrm{T}\left(t + \frac{v_\mathrm{T}}{g}\left(e^{-v_\mathrm{T}t/g} - 1\right)\right), \\ y &= v_\mathrm{T}\left(t + \frac{v_\mathrm{T}}{g}\left(e^{-v_\mathrm{T}t/g} - 1\right)\right). \end{aligned}$$

P4-18 (a) We have $v_y = v_\mathrm{T}(1 - e^{-bt/m})$ from Eq. 4-22; this can be substituted into the last line of the solution for P4-17 to give

$$y_{95} = v_\mathrm{T}\left(t - \frac{v_y}{g}\right).$$

We can also rearrange Eq. 4-22 to get $t = -(m/b)\ln(1 - v_y/v_\mathrm{T})$, so

$$y_{95} = {v_\mathrm{T}}^2/g\left(-\ln(1 - v_y/v_\mathrm{T}) - \frac{v_y}{v_\mathrm{T}}\right).$$

But $v_y/v_\mathrm{T} = 0.95$, so

$$y_{95} = {v_\mathrm{T}}^2/g\,(-\ln(0.05) - 0.95) = {v_\mathrm{T}}^2/g\,(\ln 20 - 19/20).$$

(b) $y_{95} = (42\,\mathrm{m/s})^2/(9.81\,\mathrm{m/s^2})(2.05) = 370\,\mathrm{m}$.

P4-19 (a) Convert units first. $v = 86.1\,\mathrm{m/s}$, $a = 0.05(9.81\,\mathrm{m/s^2}) = 0.491\,\mathrm{m/s^2}$. The minimum radius is $r = v^2/a = (86.1\,\mathrm{m/s})/(0.491\,\mathrm{m/s^2}) = 15$ km.

(b) $v = \sqrt{ar} = \sqrt{(0.491\,\mathrm{m/s^2})(940\,\mathrm{m})} = 21.5\,\mathrm{m/s}$. That's 77 km/hr.

P4-20 (a) The position is given by $\vec{\mathbf{r}} = R\sin\omega t\,\hat{\mathbf{i}} + R(1-\cos\omega t)\hat{\mathbf{j}}$, where $\omega = 2\pi/(20\,\mathrm{s}) = 0.314\,\mathrm{s}^{-1}$ and $R = 3.0\,\mathrm{m}$. When $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{r}} = (3.0\,\mathrm{m})\hat{\mathbf{i}} + (3.0\,\mathrm{m})\hat{\mathbf{j}}$; when $t = 7.5\,\mathrm{s}$ $\vec{\mathbf{r}} = (2.1\,\mathrm{m})\hat{\mathbf{i}} + (5.1\,\mathrm{m})\hat{\mathbf{j}}$; when $t = 10\,\mathrm{s}$ $\vec{\mathbf{r}} = (6.0\,\mathrm{m})\hat{\mathbf{j}}$. These vectors have magnitude 4.3 m, 5.5 m and 6.0 m, respectively. The vectors have direction 45°, 68° and 90° respectively.

(b) $\Delta\vec{\mathbf{r}} = (-3.0\,\mathrm{m})\hat{\mathbf{i}} + (3.0\,\mathrm{m})\hat{\mathbf{j}}$, which has magnitude 4.3 m and direction 135°.

(c) $v_{\mathrm{av}} = \Delta r/\Delta t = (4.3\,\mathrm{m})/(5.0\,\mathrm{s}) = 0.86\,\mathrm{m/s}$. The direction is the same as $\Delta\vec{\mathbf{r}}$.

(d) The velocity is given by $\vec{\mathbf{v}} = R\omega\cos\omega t\,\hat{\mathbf{i}} + R\omega\sin\omega t\,\hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{v}} = (0.94\,\mathrm{m/s})\hat{\mathbf{j}}$; at $t = 10\,\mathrm{s}$ $\vec{\mathbf{v}} = (-0.94\,\mathrm{m/s})\hat{\mathbf{i}}$.

(e) The acceleration is given by $\vec{\mathbf{a}} = -R\omega^2\sin\omega t\,\hat{\mathbf{i}} + R\omega^2\cos\omega t\,\hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{a}} = (-0.30\,\mathrm{m/s^2})\hat{\mathbf{i}}$; at $t = 10\,\mathrm{s}$ $\vec{\mathbf{a}} = (-0.30\,\mathrm{m/s^2})\hat{\mathbf{j}}$.

P4-21 Start from where the stone lands; in order to get there the stone fell through a vertical distance of 1.9 m while moving 11 m horizontally. Then

$$y = -\frac{1}{2}gt^2 \text{ which can be written as } t = \sqrt{\frac{-2y}{g}}.$$

Putting in the numbers, $t = 0.62\,\mathrm{s}$ is the time of flight from the moment the string breaks. From this time find the horizontal velocity,

$$v_x = \frac{x}{t} = \frac{(11\,\mathrm{m})}{(0.62\,\mathrm{s})} = 18\,\mathrm{m/s}.$$

Then the centripetal acceleration is

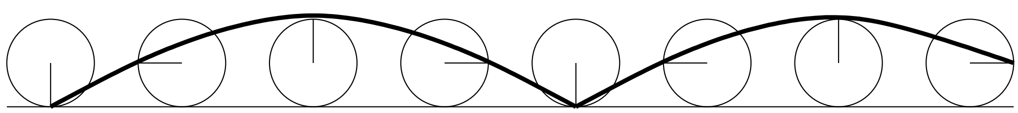
$$a_c = \frac{v^2}{r} = \frac{(18\,\mathrm{m/s})^2}{(1.4\,\mathrm{m})} = 230\,\mathrm{m/s^2}.$$

P4-22 (a) The path traced out by her feet has circumference $c_1 = 2\pi r\cos 50°$, where r is the radius of the earth; the path traced out by her head has circumference $c_2 = 2\pi(r+h)\cos 50°$, where h is her height. The difference is $\Delta c = 2\pi h\cos 50° = 2\pi(1.6\,\mathrm{m})\cos 50° = 6.46\,\mathrm{m}$.

(b) $a = v^2/r = (2\pi r/T)^2/r = 4\pi^2 r/T^2$. Then $\Delta a = 4\pi^2\Delta r/T^2$. Note that $\Delta r = h\cos\theta$! Then

$$\Delta a = 4\pi^2(1.6\,\mathrm{m})\cos 50°/(86400\,\mathrm{s})^2 = 5.44\times 10^{-9}\mathrm{m/s^2}.$$

P4-23 (a) A cycloid looks something like this:



(b) The position of the particle is given by

$$\vec{\mathbf{r}} = (R\sin\omega t + \omega Rt)\hat{\mathbf{i}} + (R\cos\omega t + R)\hat{\mathbf{j}}.$$

The maximum value of y occurs whenever $\cos\omega t = 1$. The minimum value of y occurs whenever $\cos\omega t = -1$. At either of those times $\sin\omega t = 0$.

The velocity is the derivative of the displacement vector,

$$\vec{\mathbf{v}} = (R\omega\cos\omega t + \omega R)\hat{\mathbf{i}} + (-R\omega\sin\omega t)\hat{\mathbf{j}}.$$

When y is a maximum the velocity simplifies to

$$\vec{\mathbf{v}} = (2\omega R)\hat{\mathbf{i}} + (0)\hat{\mathbf{j}}.$$

When y is a minimum the velocity simplifies to

$$\vec{\mathbf{v}} = (0)\hat{\mathbf{i}} + (0)\hat{\mathbf{j}}.$$

The acceleration is the derivative of the velocity vector,

$$\vec{\mathbf{a}} = (-R\omega^2 \sin\omega t)\hat{\mathbf{i}} + (-R\omega^2 \cos\omega t)\hat{\mathbf{j}}.$$

When y is a maximum the acceleration simplifies to

$$\vec{\mathbf{a}} = (0)\hat{\mathbf{i}} + (-R\omega^2)\hat{\mathbf{j}}.$$

When y is a minimum the acceleration simplifies to

$$\vec{\mathbf{a}} = (0)\hat{\mathbf{i}} + (R\omega^2)\hat{\mathbf{j}}.$$

P4-24 (a) The speed of the car is $v_c = 15.3\,\mathrm{m/s}$. The snow appears to fall with an angle $\theta = \arctan(15.3/7.8) = 63°$.

(b) The apparent speed is $\sqrt{(15.3)^2 + (7.8)^2}(\mathrm{m/s}) = 17.2\,\mathrm{m/s}$.

P4-25 (a) The decimal angles are 89.994250° and 89.994278°. The earth moves in the orbit around the sun with a speed of $v = 2.98\times10^4\mathrm{m/s}$ (Appendix C). The speed of light is then between $c = (2.98\times10^4\mathrm{m/s})\tan(89.994250°) = 2.97\times10^8\mathrm{m/s}$ and $c = (2.98\times10^4\mathrm{m/s})\tan(89.994278°) = 2.98\times10^8\mathrm{m/s}$. This method is *highly* sensitive to rounding. Calculating the orbital speed from the radius and period of the Earth's orbit will likely result in different answers!

P4-26 (a) Total distance is $2l$, so $t_0 = 2l/v$.

(b) Assume wind blows east. Time to travel out is $t_1 = l/(v+u)$, time to travel back is $t_2 = l/(v-u)$. Total time is sum, or

$$t_E = \frac{l}{v+u} + \frac{l}{v-u} = \frac{2lv}{v^2-u^2} = \frac{t_0}{1-u^2/v^2}.$$

If wind blows west the times reverse, but the result is otherwise the same.

(c) Assume wind blows north. The airplane will still have a speed of v relative to the wind, but it will need to fly with a heading away from east. The speed of the plane relative to the ground will be $\sqrt{v^2-u^2}$. This will be the speed even when it flies west, so

$$t_N = \frac{2l}{\sqrt{v^2-u^2}} = \frac{t_0}{\sqrt{1-u^2/v^2}}.$$

(d) If $u > v$ the wind sweeps the plane along in one general direction only; it can never fly back. Sort of like a black hole event horizon.

$\boxed{\textbf{P4-27}}$ The velocity of the police car with respect to the ground is $\vec{\mathbf{v}}_{pg} = -76\mathrm{km/h}\hat{\mathbf{i}}$. The velocity of the motorist with respect the ground is $\vec{\mathbf{v}}_{mg} = -62\ \mathrm{km/h}\hat{\mathbf{j}}$.

The velocity of the motorist with respect to the police car is given by solving

$$\vec{\mathbf{v}}_{mg} = \vec{\mathbf{v}}_{mp} + \vec{\mathbf{v}}_{pg},$$

so $\vec{\mathbf{v}}_{mp} = 76\mathrm{km/h}\hat{\mathbf{i}} - 62\ \mathrm{km/h}\hat{\mathbf{j}}$. This velocity has magnitude

$$v_{mp} = \sqrt{(76\mathrm{km/h})^2 + (-62\ \mathrm{km/h})^2} = 98\ \mathrm{km/h}.$$

The direction is

$$\theta = \arctan(-62\text{ km/h})/(76\text{km/h}) = -39^\circ,$$

but that is relative to $\hat{\mathbf{i}}$. We want to know the direction relative to the line of sight. The line of sight is

$$\alpha = \arctan(57\,\text{m})/(41\,\text{m}) = -54^\circ$$

relative to $\hat{\mathbf{i}}$, so the answer must be 15°.

P4-28 (a) The velocity of the plane with respect to the air is $\vec{\mathbf{v}}_{pa}$; the velocity of the air with respect to the ground is $\vec{\mathbf{v}}_{ag}$, the velocity of the plane with respect to the ground is $\vec{\mathbf{v}}_{pg}$. Then $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pa} + \vec{\mathbf{v}}_{ag}$. This can be represented by a triangle; since the sides are given we can find the angle between $\vec{\mathbf{v}}_{ag}$ and $\vec{\mathbf{v}}_{pg}$ (points north) using the cosine law

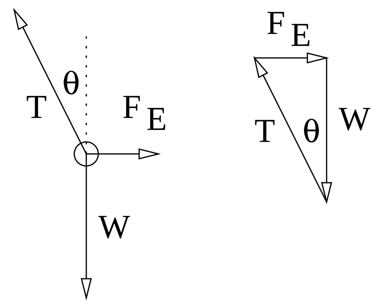
$$\theta = \arccos\left(\frac{(135)^2 - (135)^2 - (70)^2}{-2(135)(70)}\right) = 75^\circ.$$

(b) The direction of $\vec{\mathbf{v}}_{pa}$ can also be found using the cosine law,

$$\theta = \arccos\left(\frac{(70)^2 - (135)^2 - (135)^2}{-2(135)(135)}\right) = 30^\circ.$$

E5-1 There are three forces which act on the charged sphere— an electric force, F_E, the force of gravity, W, and the tension in the string, T. All arranged as shown in the figure on the right below.

(a) Write the vectors so that they geometrically show that the sum is zero, as in the figure on the left below. Now $W = mg = (2.8 \times 10^{-4}\ \text{kg})(9.8\ \text{m/s}^2) = 2.7 \times 10^{-3}$ N. The magnitude of the electric force can be found from the tangent relationship, so $F_E = W \tan\theta = (2.7 \times 10^{-3}$ N$) \tan(33°) = 1.8 \times 10^{-3}$ N.



(a) (b)

(b) The tension can be found from the cosine relation, so

$$T = W/\cos\theta = (2.7 \times 10^{-3}\,\text{N})/\cos(33°) = 3.2 \times 10^{-3}\,\text{N}.$$

E5-2 (a) The net force on the elevator is $F = ma = Wa/g = (6200\ \text{lb})(3.8\ \text{ft/s}^2)/(32\ \text{ft/s}^2) =$ 740 lb. Positive means up. There are two force on the elevator: a weight W down and a tension from the cable T up. Then $F = T - W$ or $T = F + W = (740\ \text{lb}) + (6200\ \text{lb}) = 6940$ lb.

(b) If the elevator acceleration is down then $F = -740$ lb; consequently $T = F + W = (-740\ \text{lb}) + (6200\ \text{lb}) = 5460$ lb.

E5-3 (a) The tension T is up, the weight W is down, and the net force F is in the direction of the acceleration (up). Then $F = T - W$. But $F = ma$ and $W = mg$, so

$$m = T/(a + g) = (89\,\text{N})/[(2.4\,\text{m/s}^2) + (9.8\,\text{m/s}^2)] = 7.3\,\text{kg}.$$

(b) $T = 89\,$N. The direction of velocity is unimportant. In both (a) and (b) the acceleration is up.

E5-4 The average speed of the elevator during deceleration is $v_{\text{av}} = 6.0\,\text{m/s}$. The time to stop the elevator is then $t = (42.0\,\text{m})/(6.0\,\text{m/s}) = 7.0\,\text{s}$. The deceleration is then $a = (12.0\,\text{m/s})/(7.0\,\text{s}) = 1.7\,\text{m/s}^2$. Since the elevator is moving downward but slowing down, then the acceleration is up, which will be positive.

The net force on the elevator is $F = ma$; this is equal to the tension T minus the weight W. Then

$$T = F + W = ma + mg = (1600\,\text{kg})[(1.7\,\text{m/s}^2) + (9.8\,\text{m/s}^2)] = 1.8 \times 10^4\text{N}.$$

E5-5 (a) The magnitude of the man's acceleration is given by

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \frac{(110\ \text{kg}) - (74\ \text{kg})}{(110\ \text{kg}) + (74\ \text{kg})} g = 0.2g,$$

and is directed down. The time which elapses while he falls is found by solving $y = v_{0y}t + \frac{1}{2}a_y t^2$, or, with numbers, $(-12\ \text{m}) = (0)t + \frac{1}{2}(-0.2g)t^2$ which has the solutions $t = \pm 3.5$ s. The velocity with which he hits the ground is then $v = v_{0y} + a_y t = (0) + (-0.2g)(3.5\ \text{s}) = -6.9$ m/s.

(b) Reducing the speed can be accomplished by reducing the acceleration. We can't change Eq. 5-4 without also changing one of the assumptions that went into it. Since the man is hoping to reduce the speed with which he hits the ground, it makes sense that he might want to climb up the rope.

E5-6 (a) Although it might be the monkey which does the work, the upward force to lift him still comes from the tension in the rope. The minimum tension to lift the log is $T = W_\text{l} = m_\text{l} g$. The net force on the monkey is $T - W_\text{m} = m_\text{m} a$. The acceleration of the monkey is then

$$a = (m_\text{l} - m_\text{m})g/m_\text{m} = [(15\,\text{kg}) - (11\,\text{kg})](9.8\,\text{m/s}^2)/(11\,\text{kg}) = 3.6\,\text{m/s}^2.$$

(b) Atwood's machine!

$$a = (m_\text{l} - m_\text{m})g/(m_\text{l} + m_\text{m}) = [(15\,\text{kg}) - (11\,\text{kg})](9.8\,\text{m/s}^2)/[(15\,\text{kg}) + (11\,\text{kg})] = 1.5\,\text{m/s}.$$

(c) Atwood's machine!

$$T = 2m_\text{l} m_\text{m} g/(m_\text{l} + m_\text{m}) = 2(15\,\text{kg})(11\,\text{kg})(9.8\,\text{m/s}^2)/[(15\,\text{kg}) + (11\,\text{kg})] = 120\,\text{N}.$$

E5-7 The weight of each car has two components: a component parallel to the cables $W_{||} = W \sin\theta$ and a component normal to the cables $W_\perp$.. The normal component is "balanced" by the supporting cable. The parallel component acts with the pull cable.

In order to accelerate a car up the incline there must be a net force up with magnitude $F = ma$. Then $F = T_\text{above} - T_\text{below} - W_{||}$, or

$$\Delta T = ma + mg\sin\theta = (2800\,\text{kg})[(0.81\,\text{m/s}^2) + (9.8\,\text{m/s}^2)\sin(35^\circ)] = 1.8\times 10^4\text{N}.$$

E5-8 The tension in the cable is T, the weight of the man + platform system is $W = mg$, and the net force on the man + platform system is $F = ma = Wa/g = T - W$. Then

$$T = Wa/g + W = W(a/g + 1) = (180\text{ lb} + 43\text{ lb})[(1.2\text{ ft/s}^2)/(32\text{ ft/s}^2) + 1] = 231\text{ lb}.$$

$\boxed{\textbf{E5-9}}$ See Sample Problem 5-8. We need only apply the (unlabeled!) equation

$$\mu_s = \tan\theta$$

to find the egg angle. In this case $\theta = \tan^{-1}(0.04) = 2.3^\circ$.

E5-10 (a) The maximum force of friction is $F = \mu_\text{s} N$. If the rear wheels support half of the weight of the automobile then $N = W/2 = mg/2$. The maximum acceleration is then

$$a = F/m = \mu_\text{s} N/m = \mu_\text{s} g/2.$$

(b) $a = (0.56)(9.8\,\text{m/s}^2)/2 = 2.7\,\text{m/s}^2$.

E5-11 The maximum force of friction is $F = \mu_\text{s} N$. Since there is no motion in the y direction the magnitude of the normal force must equal the weight, $N = W = mg$. The maximum acceleration is then

$$a = F/m = \mu_\text{s} N/m = \mu_\text{s} g = (0.95)(9.8\,\text{m/s}^2) = 9.3\,\text{m/s}^2.$$

E5-12 There is no motion in the vertical direction, so $N = W = mg$. Then $\mu_\text{k} = F/N = (470\,\text{N})/[(9.8\,\text{m/s}^2)(79\,\text{kg})] = 0.61$.

E5-13 A 75 kg mass has a weight of $W = (75\,\text{kg})(9.8\,\text{m/s}^2) = 735\,\text{N}$, so the force of friction on each end of the bar must be 368 N. Then

$$F \geq \frac{f_s}{\mu_s} = \frac{(368\,\text{N})}{(0.41)} = 900\,\text{N}.$$

E5-14 (a) There is no motion in the vertical direction, so $N = W = mg$.

To get the box moving you must overcome static friction and push with a force of $P \geq \mu_\text{s} N = (0.41)(240\,\text{N}) = 98\,\text{N}$.

(b) To keep the box moving at constant speed you must push with a force equal to the kinetic friction, $P = \mu_\text{k} N = (0.32)(240\,\text{N}) = 77\,\text{N}$.

(c) If you push with a force of 98 N on a box that experiences a (kinetic) friction of 77 N, then the net force on the box is 21 N. The box will accelerate at

$$a = F/m = Fg/W = (21\,\text{N})(9.8\,\text{m/s}^2)/(240\,\text{N}) = 0.86\,\text{m/s}^2.$$

E5-15 (a) The maximum braking force is $F = \mu_\text{s} N$. There is no motion in the vertical direction, so $N = W = mg$. Then $F = \mu_\text{s} mg = (0.62)(1500\,\text{kg})(9.8\,\text{m/s}^2) = 9100\,\text{N}$.

(b) Although we still use $F = \mu_\text{s} N$, $N \neq W$ on an incline! The weight has two components; one which is parallel to the surface and the other which is perpendicular. Since there is no motion perpendicular to the surface we must have $N = W_\perp = W\cos\theta$. Then

$$F = \mu_\text{s} mg \cos\theta = (0.62)(1500\,\text{kg})(9.8\,\text{m/s}^2)\cos(8.6^\circ) = 9000\,\text{N}.$$

E5-16 $\mu_s = \tan\theta$ is the condition for an object to sit without slipping on an incline. Then $\theta = \arctan(0.55) = 29^\circ$. The angle should be reduced by 13°.

E5-17 (a) The force of static friction is less than $\mu_s N$, where N is the normal force. Since the crate isn't moving up or down, $\sum F_y = 0 = N - W$. So in this case $N = W = mg = (136\text{ kg})(9.81\text{ m/s}^2) = 1330$ N. The force of static friction is less than or equal to $(0.37)(1330\,\text{N}) = 492\,\text{N}$; moving the crate will require a force greater than or equal to 492 N.

(b) The second worker could lift upward with a force L, reducing the normal force, and hence reducing the force of friction. If the first worker can move the block with a 412 N force, then $412 \geq \mu_s N$. Solving for N, the normal force needs to be less than 1110 N. The crate doesn't move off the table, so then $N + L = W$, or $L = W - N = (1330\text{ N}) - (1110\text{ N}) = 220$ N.

(c) Or the second worker can help by adding a push so that the total force of both workers is equal to 492 N. If the first worker pushes with a force of 412 N, the second would need to push with a force of 80 N.

E5-18 The coefficient of static friction is $\mu_\text{s} = \tan(28.0^\circ) = 0.532$. The acceleration is $a = 2(2.53\,\text{m})/(3.92\,\text{s})^2 = .329\,\text{m/s}^2$. We will need to insert a negative sign since this is downward.

The weight has two components: a component parallel to the plane, $W_{||} = mg\sin\theta$; and a component perpendicular to the plane, $W_\perp = mg\cos\theta$. There is no motion perpendicular to the plane, so $N = W_\perp$. The kinetic friction is then $f = \mu_\text{k} N = \mu_\text{k} mg\cos\theta$. The net force parallel to the plane is $F = ma = f - W_{||} = \mu_\text{k} mg\cos\theta - mg\sin\theta$. Solving this for μ_k,

$$\begin{aligned}
\mu_\text{k} &= (a + g\sin\theta)/(g\cos\theta), \\
&= [(-0.329\,\text{m/s}^2) + (9.81\,\text{m/s}^2)\sin(28.0^\circ)]/[((9.81\,\text{m/s}^2)\cos(28.0^\circ)] = 0.494.
\end{aligned}$$

E5-19 The acceleration is $a = -2d/t^2$, where $d = 203\,\text{m}$ is the distance down the slope and t is the time to make the run.

The weight has two components: a component parallel to the incline, $W_{||} = mg\sin\theta$; and a component perpendicular to the incline, $W_\perp = mg\cos\theta$. There is no motion perpendicular tot he plane, so $N = W_\perp$. The kinetic friction is then $f = \mu_\text{k}N = \mu_\text{k}mg\cos\theta$. The net force parallel to the plane is $F = ma = f - W_{||} = \mu_\text{k}mg\cos\theta - mg\sin\theta$. Solving this for μ_k,

$$\begin{aligned}\mu_\text{k} &= (a + g\sin\theta)/(g\cos\theta), \\ &= (g\sin\theta - 2d/t^2)/(g\cos\theta).\end{aligned}$$

If $t = 61\,\text{s}$, then

$$\mu_\text{k} = \frac{(9.81\,\text{m/s}^2)\sin(3.0^\circ) - 2(203\,\text{m})/(61\,\text{s})^2}{(9.81\,\text{m/s}^2)\cos(3.0^\circ)} = 0.041;$$

if $t = 42\,\text{s}$, then

$$\mu_\text{k} = \frac{(9.81\,\text{m/s}^2)\sin(3.0^\circ) - 2(203\,\text{m})/(42\,\text{s})^2}{(9.81\,\text{m/s}^2)\cos(3.0^\circ)} = 0.029.$$

E5-20 (a) If the block slides down with constant velocity then $a = 0$ and $\mu_\text{k} = \tan\theta$. Not only that, but the force of kinetic friction must be equal to the parallel component of the weight, $f = W_{||}$. If the block is projected up the ramp then the net force is now $2W_{||} = 2mg\sin\theta$. The deceleration is $a = 2g\sin\theta$; the block will travel a time $t = v_0/a$ before stopping, and travel a distance

$$d = -at^2/2 + v_0t = -a(v_0/a)^2/2 + v_0(v_0/a) = v_0^2/(2a) = v_0^2/(4g\sin\theta)$$

before stopping.

(b) Since $\mu_\text{k} < \mu_\text{s}$, the incline is *not* steep enough to get the block moving again once it stops.

$\boxed{\textbf{E5-21}}$ Let a_1 be acceleration down frictionless incline of length l, and t_1 the time taken. The a_2 is acceleration down "rough" incline, and $t_2 = 2t_1$ is the time taken. Then

$$l = \frac{1}{2}a_1t_1^2 \text{ and } l = \frac{1}{2}a_2(2t_1)^2.$$

Equate and find $a_1/a_2 = 4$.

There are two force which act on the ice when it sits on the frictionless incline. The normal force acts perpendicular to the surface, so it doesn't contribute any components parallel to the surface. The force of gravity has a component parallel to the surface, given by

$$W_{||} = mg\sin\theta,$$

and a component perpendicular to the surface given by

$$W_\perp = mg\cos\theta.$$

The acceleration down the frictionless ramp is then

$$a_1 = \frac{W_{||}}{m} = g\sin\theta.$$

When friction is present the force of kinetic friction is $f_k = \mu_kN$; since the ice doesn't move perpendicular to the surface we also have $N = W_\perp$; and finally the acceleration down the ramp is

$$a_2 = \frac{W_{||} - f_k}{m} = g(\sin\theta - \mu\cos\theta).$$

Previously we found the ratio of a_1/a_2, so we now have

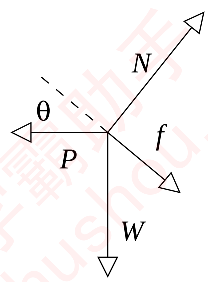
$$\begin{aligned}\sin\theta &= 4\sin\theta - 4\mu\cos\theta,\\ \sin 33^\circ &= 4\sin 33^\circ - 4\mu\cos 33^\circ,\\ \mu &= 0.49.\end{aligned}$$

E5-22 (a) The static friction between A and the table must be equal to the weight of block B to keep A from sliding. This means $m_B g = \mu_s(m_A + m_C)g$, or $m_c = m_B/\mu_s - m_A = (2.6\,\text{kg})/(0.18) - (4.4\,\text{kg}) = 10\,\text{kg}$.

(b) There is no up/down motion for block A, so $f = \mu_k N = \mu_k m_A g$. The net force on the system containing blocks A and B is $F = W_B - f = m_B g - \mu_k m_A g$; the acceleration of this system is then

$$a = g\frac{m_B - \mu_k m_A}{m_A + m_B} = (9.\,\text{m/s}^2)\frac{(2.6\,\text{kg}) - (0.15)(4.4\,\text{kg})}{(2.6\,\text{kg}) + (4.4\,\text{kg})} = 2.7\,\text{m/s}^2.$$

$\boxed{\textbf{E5-23}}$ There are four forces on the block— the force of gravity, $W = mg$; the normal force, N; the horizontal push, P, and the force of friction, f. Choose the coordinate system so that components are either parallel (x-axis) to the plane or perpendicular (y-axis) to it. $\theta = 39^\circ$. Refer to the figure below.



The magnitudes of the x components of the forces are $W_x = W\sin\theta$, $P_x = P\cos\theta$ and f; the magnitudes of the y components of the forces are $W_y = W\cos\theta$, $P_y = P\sin\theta$.

(a) We consider the first the case of the block moving up the ramp; then f is directed down. Newton's second law for each set of components then reads as

$$\begin{aligned}\sum F_x &= P_x - f - W_x = P\cos\theta - f - W\sin\theta = ma_x,\\ \sum F_y &= N - P_y - W_y = N - P\sin\theta - W\cos\theta = ma_y\end{aligned}$$

Then the second equation is easy to solve for N

$$N = P\sin\theta + W\cos\theta = (46\text{ N})\sin(39^\circ) + (4.8\text{ kg})(9.8\text{ m/s}^2)\cos(39^\circ) = 66\text{ N}.$$

The force of friction is found from $f = \mu_k N = (0.33)(66\text{ N}) = 22$ N. This is directed down the incline while the block is moving up. We can now find the acceleration in the x direction.

$$\begin{aligned}ma_x &= P\cos\theta - f - W\sin\theta,\\ &= (46\,\text{N})\cos(39^\circ) - (22\,\text{N}) - (4.8\,\text{kg})(9.8\,\text{m/s}^2)\sin(39^\circ) = -16\,\text{N}.\end{aligned}$$

So the block is slowing down, with an acceleration of magnitude 3.3 m/s^2.

(b) The block has an initial speed of $v_{0x} = 4.3$ m/s; it will rise until it stops; so we can use $v_y = 0 = v_{0y} + a_y t$ to find the time to the highest point. Then $t = (v_y - v_{0y})/a_y = -(-4.3\text{ m/s})/(3.3\text{ m/s}^2 = 1.3$ s. Now that we know the time we can use the other kinematic relation to find the distance

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (4.3\text{ m/s})(1.3\text{ s}) + \frac{1}{2}(-3.3\text{ m/s}^2)(1.3\text{ s})^2 = 2.8\text{ m}$$

(c) When the block gets to the top it *might* slide back down. But in order to do so the frictional force, which is now directed up the ramp, must be sufficiently small so that $f + P_x \leq W_x$. Solving for f we find $f \leq W_x - P_x$ or, using our numbers from above, $f \leq -6$ N. Is this possible? No, so the block will not slide back down the ramp, *even if the ramp were frictionless,* while the horizontal force is applied.

E5-24 (a) The horizontal force needs to overcome the maximum static friction, so $P \geq \mu_s N = \mu_s mg = (0.52)(12\,\text{kg})(9.8\,\text{m/s}^2) = 61\,\text{N}$.

(b) If the force acts upward from the horizontal then there are two components: a horizontal component $P_x = P\cos\theta$ and a vertical component $P_y = P\sin\theta$. The normal force is now given by $W = P_y + N$; consequently the maximum force of static friction is now $\mu_s N = \mu_s(mg - P\sin\theta)$. The block will move only if $P\cos\theta \geq \mu_s(mg - P\sin\theta)$, or

$$P \geq \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \frac{(0.52)(12\,\text{kg})(9.8\,\text{m/s}^2)}{\cos(62^\circ) + (0.52)\sin(62^\circ)} = 66\,\text{N}.$$

(c) If the force acts downward from the horizontal then $\theta = -62^\circ$, so

$$P \geq \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \frac{(0.52)(12\,\text{kg})(9.8\,\text{m/s}^2)}{\cos(-62^\circ) + (0.52)\sin(-62^\circ)} = 5900\,\text{N}.$$

E5-25 (a) If the tension acts upward from the horizontal then there are two components: a horizontal component $T_x = T\cos\theta$ and a vertical component $T_y = T\sin\theta$. The normal force is now given by $W = T_y + N$; consequently the maximum force of static friction is now $\mu_s N = \mu_s(W - T\sin\theta)$. The crate will move only if $T\cos\theta \geq \mu_s(W - T\sin\theta)$, or

$$P \geq \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta} = \frac{(0.52)(150\text{ lb})}{\cos(17^\circ) + (0.52)\sin(17^\circ)} = 70\text{ lb}.$$

(b) Once the crate starts to move then the net force on the crate is $F = T_x - f$. The acceleration is then

$$\begin{aligned} a &= \frac{g}{W}[T\cos\theta - \mu_k(W - T\sin\theta)], \\ &= \frac{(32\text{ ft/s}^2)}{(150\text{ lb})}\{(70\text{ lb})\cos(17^\circ) - (0.35)[(150\text{ lb}) - (70\text{ lb})\sin(17^\circ)]\}, \\ &= 4.6\text{ ft/s}^2. \end{aligned}$$

E5-26 If the tension acts upward from the horizontal then there are two components: a horizontal component $T_x = T\cos\theta$ and a vertical component $T_y = T\sin\theta$. The normal force is now given by $W = T_y + N$; consequently the maximum force of static friction is now $\mu_s N = \mu_s(W - T\sin\theta)$. The crate will move only if $T\cos\theta \geq \mu_s(W - T\sin\theta)$, or

$$W \leq T\cos\theta/\mu_s + T\sin\theta.$$

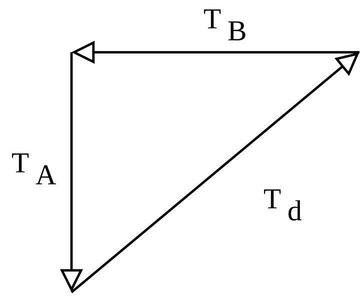
We want the maximum, so we find $dW/d\theta$,

$$dW/d\theta = -(T/\mu_s)\sin\theta + T\cos\theta,$$

which equals zero when $\mu_s = \tan\theta$. For this problem $\theta = \arctan(0.35) = 19^\circ$, so

$$W \leq (1220\,\text{N})\cos(19^\circ)/(0.35) + (1220\,\text{N})\sin(19^\circ) = 3690\,\text{N}.$$

E5-27 The three force on the know above A must add to zero. Construct a vector diagram: $\vec{\mathbf{T}}_A + \vec{\mathbf{T}}_B + \vec{\mathbf{T}}_d = 0$, where $\vec{\mathbf{T}}_d$ refers to the diagonal rope. T_A and T_B must be related by $T_A = T_B \tan\theta$, where $\theta = 41°$.



There is no up/down motion of block B, so $N = W_B$ and $f = \mu_\text{s} W_B$. Since block B is at rest $f = T_B$. Since block A is at rest $W_A = T_A$. Then

$$W_A = W_B(\mu_\text{s} \tan\theta) = (712\,\text{N})(0.25)\tan(41°) = 155\,\text{N}.$$

E5-28 (a) Block 2 doesn't move up/down, so $N = W_2 = m_2 g$ and the force of friction on block 2 is $f = \mu_\text{k} m_2 g$. Block 1 is on a frictionless incline; only the component of the weight parallel to the surface contributes to the motion, and $W_{||} = m_1 g \sin\theta$. There are two relevant forces on the two mass system. The effective net force is the of magnitude $W_{||} - f$, so the acceleration is

$$a = g\frac{m_1 \sin\theta - \mu_\text{k} m_2}{m_1 + m_2} = (9.81\,\text{m/s}^2)\frac{(4.20\,kg)\sin(27°) - (0.47)(2.30\,\text{kg})}{(4.20\,\text{kg}) + (2.30\,\text{kg})} = 1.25\,\text{m/s}^2.$$

The blocks accelerate down the ramp.

(b) The net force on block 2 is $F = m_2 a = T - f$. The tension in the cable is then

$$T = m_2 a + \mu_\text{k} m_2 g = (2.30\,\text{kg})[(1.25\,\text{m/s}^2) + (0.47)(9.81\,\text{m/s}^2)] = 13.5\,\text{N}.$$

$\boxed{\textbf{E5-29}}$ This problem is similar to Sample Problem 5-7, except now there is friction which can act on block B. The relevant equations are now for block B

$$N - m_B g \cos\theta = 0$$

and

$$T - m_B g \sin\theta \pm f = m_B a,$$

where the sign in front of f depends on the direction in which block B is moving. If the block is moving up the ramp then friction is directed down the ramp, and we would use the negative sign. If the block is moving down the ramp then friction will be directed up the ramp, and then we will use the positive sign. Finally, if the block is stationary then friction we be in such a direction as to make $a = 0$.

For block A the relevant equation is

$$m_A g - T = m_A a.$$

Combine the first two equations with $f = \mu N$ to get

$$T - m_B g \sin\theta \pm \mu m_B g \cos\theta = m_B a.$$

Take some care when interpreting friction for the static case, since the static value of μ yields the maximum possible static friction force, which is not necessarily the actual static frictional force.

Combine this last equation with the block A equation,

$$m_A g - m_A a - m_B g \sin\theta \pm \mu m_B g \cos\theta = m_B a,$$

and then rearrange to get

$$a = g\frac{m_A - m_B \sin\theta \pm \mu m_B \cos\theta}{m_A + m_B}.$$

For convenience we will use metric units; then the masses are $m_A = 13.2\,\mathrm{kg}$ and $m_B = 42.6\,\mathrm{kg}$. In addition, $\sin 42^\circ = 0.669$ and $\cos 42^\circ = 0.743$.

(a) If the blocks are originally at rest then

$$m_A - m_B \sin\theta = (13.2\,\mathrm{kg}) - (42.6\,\mathrm{kg})(0.669) = -15.3\,\mathrm{kg}$$

where the negative sign indicates that block B would slide downhill if there were no friction.

If the blocks are originally at rest we need to consider static friction, so the last term can be as large as

$$\mu m_B \cos\theta = (.56)(42.6\,\mathrm{kg})(0.743) = 17.7\,\mathrm{kg}.$$

Since this quantity is larger than the first static friction would be large enough to stop the blocks from accelerating if they are at rest.

(b) If block B is moving up the ramp we use the negative sign, and the acceleration is

$$a = (9.81\,\mathrm{m/s^2})\frac{(13.2\,\mathrm{kg}) - (42.6\,\mathrm{kg})(0.669) - (.25)(42.6\,\mathrm{kg})(0.743)}{(13.2\,\mathrm{kg}) + (42.6\,\mathrm{kg})} = -4.08\,\mathrm{m/s^2}.$$

where the negative sign means down the ramp. The block, originally moving up the ramp, will slow down and stop. Once it stops the static friction takes over and the results of part (a) become relevant.

(c) If block B is moving down the ramp we use the positive sign, and the acceleration is

$$a = (9.81\,\mathrm{m/s^2})\frac{(13.2\,\mathrm{kg}) - (42.6\,\mathrm{kg})(0.669) + (.25)(42.6\,\mathrm{kg})(0.743)}{(13.2\,\mathrm{kg}) + (42.6\,\mathrm{kg})} = -1.30\,\mathrm{m/s^2}.$$

where the negative sign means down the ramp. This means that if the block is moving down the ramp it will continue to move down the ramp, faster and faster.

E5-30 The weight can be resolved into a component parallel to the incline, $W_{||} = W\sin\theta$ and a component that is perpendicular, $W_\perp = W\cos\theta$. There are two normal forces on the crate, one from each side of the trough. By symmetry we expect them to have equal magnitudes; since they both act perpendicular to their respective surfaces we expect them to be perpendicular to each other. They must add to equal the perpendicular component of the weight. Since they are at right angles and equal in magnitude, this yields $N^2 + N^2 = {W_\perp}^2$, or $N = W_\perp/\sqrt{2}$.

Each surface contributes a frictional force $f = \mu_\mathrm{k} N = \mu_\mathrm{k} W_\perp/\sqrt{2}$; the total frictional force is then twice this, or $\sqrt{2}\mu_\mathrm{k} W_\perp$. The net force on the crate is $F = W\sin\theta - \sqrt{2}\mu_\mathrm{k} W\cos\theta$ down the ramp. The acceleration is then

$$a = g(\sin\theta - \sqrt{2}\mu_\mathrm{k}\cos\theta).$$

E5-31 The normal force between the top slab and the bottom slab is $N = W_{\rm t} = m_{\rm t}g$. The force of friction between the top and the bottom slab is $f \le \mu N = \mu m_{\rm t} g$. We don't yet know if the slabs slip relative to each other, so we don't yet know what kind of friction to consider.

The acceleration of the top slab is

$$a_{\rm t} = (110\,{\rm N})/(9.7\,{\rm kg}) - \mu(9.8\,{\rm m/s^2}) = 11.3\,{\rm m/s^2} - \mu(9.8\,{\rm m/s^2}).$$

The acceleration of the bottom slab is

$$a_{\rm b} = \mu(9.8\,{\rm m/s^2})(9.7,{\rm kg})/(42\,{\rm kg}) = \mu(2.3\,{\rm m/s^2}).$$

Can these two be equal? Only if $\mu \ge 0.93$. Since the static coefficient is less than this, the block slide. Then $a_{\rm t} = 7.6\,{\rm m/s^2}$ and $a_{\rm b} = 0.87\,{\rm m/s^2}$.

E5-32 (a) Convert the speed to ft/s: $v = 88$ ft/s. The acceleration is

$$a = v^2/r = (88 \text{ ft/s})^2/(25 \text{ ft}) = 310 \text{ ft/s}^2.$$

(b) $a = 310 \text{ ft/s}^2 g/(32 \text{ ft/s}^2) = 9.7g$.

$\boxed{\textbf{E5-33}}$ (a) The force required to keep the car in the turn is $F = mv^2/r = Wv^2/(rg)$, or

$$F = (10700\,{\rm N})(13.4\,{\rm m/s})^2/[(61.0\,{\rm m})(9.81\,{\rm m/s^2})] = 3210\,{\rm N}.$$

(b) The coefficient of friction required is $\mu_{\rm s} = F/W = (3210\,{\rm N})/(10700\,{\rm N}) = 0.300$.

E5-34 (a) The proper banking angle is given by

$$\theta = \arctan\frac{v^2}{Rg} = \arctan\frac{(16.7\,{\rm m/s})^2}{(150\,{\rm m})(9.8\,{\rm m/s^2})} = 11^\circ.$$

(b) If the road is not banked then the force required to keep the car in the turn is $F = mv^2/r = Wv^2/(Rg)$ and the required coefficient of friction is

$$\mu_{\rm s} = F/W = \frac{v^2}{Rg} = \frac{(16.7\,{\rm m/s})^2}{(150\,{\rm m})(9.8\,{\rm m/s^2})} = 0.19.$$

E5-35 (a) This conical pendulum makes an angle $\theta = \arcsin(0.25/1.4) = 10^\circ$ with the vertical. The pebble has a speed of

$$v = \sqrt{Rg\tan\theta} = \sqrt{(0.25\,{\rm m})(9.8\,{\rm m/s^2})\tan(10^\circ)} = 0.66\,{\rm m/s}.$$

(b) $a = v^2/r = (0.66\,{\rm m/s})^2/(0.25\,{\rm m}) = 1.7\,{\rm m/s^2}$.
(c) $T = mg/cos\theta = (0.053\,{\rm kg})(9.8\,{\rm m/s^2})/\cos(10^\circ) = 0.53\,{\rm N}$.

E5-36 Ignoring air friction (there must be a forward component to the friction!), we have a normal force upward which is equal to the weight: $N = mg = (85\,{\rm kg})(9.8\,{\rm m/s^2}) = 833\,{\rm N}$. There is a sideways component to the friction which is equal tot eh centripetal force, $F = mv^2/r = (85\,{\rm kg})(8.7\,{\rm m/s})^2/(25\,{\rm m}) = 257\,{\rm N}$. The magnitude of the net force of the road on the person is

$$F = \sqrt{(833\,{\rm N})^2 + (257\,{\rm N})^2} = 870\,{\rm N},$$

and the direction is $\theta = \arctan(257/833) = 17^\circ$ off of vertical.

E5-37 (a) The speed is $v = 2\pi r f = 2\pi(5.3\times10^{-11}\,\text{m})(6.6\times10^{15}/\text{s}) = 2.2\times10^{6}\text{m/s}$.

(b) The acceleration is $a = v^2/r = (2.2\times10^{6}\text{m/s})^2/(5.3\times10^{-11}\text{m}) = 9.1\times10^{22}\,\text{m/s}^2$.

(c) The net force is $F = ma = (9.1\times10^{-31}\,\text{kg})(9.1\times10^{22}\,\text{m/s}^2) = 8.3\times10^{-8}\text{N}$.

E5-38 The basket has speed $v = 2\pi r/t$. The basket experiences a frictional force $F = mv^2/r = m(2\pi r/t)^2/r = 4\pi^2 mr/t^2$. The coefficient of static friction is $\mu_\text{s} = F/N = F/W$. Combining,

$$\mu_\text{s} = \frac{4\pi^2 r}{gt^2} = \frac{4\pi^2(4.6\,\text{m})}{(9.8\,\text{m/s}^2)(24\,\text{s})^2} = 0.032.$$

$\boxed{\textbf{E5-39}}$ There are two forces on the hanging cylinder: the force of the cord pulling up T and the force of gravity $W = Mg$. The cylinder is at rest, so these two forces must balance, or $T = W$. There are three forces on the disk, but only the force of the cord on the disk T is relevant here, since there is no friction or vertical motion.

The disk undergoes circular motion, so $T = mv^2/r$. We want to solve this for v and then express the answer in terms of m, M, r, and G.

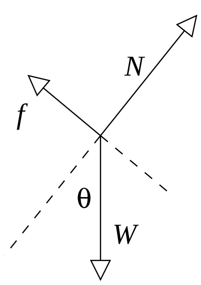
$$v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{Mgr}{m}}.$$

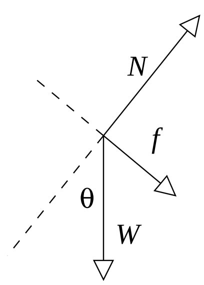
E5-40 (a) The frictional force stopping the car is $F = \mu_\text{s} N = \mu_\text{s} mg$. The deceleration of the car is then $a = \mu_\text{s} g$. If the car is moving at $v = 13.3\,\text{m/s}$ then the average speed while decelerating is half this, or $v_\text{av} = 6.7\,\text{m/s}$. The time required to stop is $t = x/v_\text{av} = (21\,\text{m})/(6.7\,\text{m/s}) = 3.1\,\text{s}$. The deceleration is $a = (13.3\,\text{m/s})/(3.1\,\text{s}) = 4.3\,\text{m/s}^2$. The coefficient of friction is $\mu_\text{s} = a/g = (4.3\,\text{m/s}^2)/(9.8\,\text{m/s}^2) = 0.44$.

(b) The acceleration is the same as in part (a), so $r = v^2/a = (13.3\,\text{m/s})^2/(4.3\,\text{m/s}^2) = 41\,\text{m}$.

$\boxed{\textbf{E5-41}}$ There are three forces to consider: the normal force of the road on the car N; the force of gravity on the car W; and the frictional force on the car f. The acceleration of the car in circular motion is toward the center of the circle; this means the *net* force on the car is horizontal, toward the center. We will arrange our coordinate system so that r is horizontal and z is vertical. Then the components of the normal force are $N_r = N\sin\theta$ and $N_z = N\cos\theta$; the components of the frictional force are $f_r = f\cos\theta$ and $f_z = f\sin\theta$.

The direction of the friction depends on the speed of the car. The figure below shows the two force diagrams.





The turn is designed for 95 km/hr, at this speed a car should require *no* friction to stay on the road. Using Eq. 5-17 we find that the banking angle is given by

$$\tan\theta_b = \frac{v^2}{rg} = \frac{(26\ \text{m/s})^2}{(210\ \text{m})(9.8\ \text{m/s}^2)} = 0.33,$$

for a bank angle of $\theta_b = 18°$.

(a) On the rainy day traffic is moving at 14 m/s. This is slower than the rated speed, so any frictional force must be directed up the incline. Newton's second law is then

$$\begin{aligned} \sum F_r &= N_r - f_r = N\sin\theta - f\cos\theta = \frac{mv^2}{r}, \\ \sum F_z &= N_z + f_z - W = N\cos\theta + f\sin\theta - mg = 0. \end{aligned}$$

We can substitute $f = \mu_s N$ to find the minimum value of μ_s which will keep the cars from slipping. There will then be two equations and two unknowns, μ_s and N. Solving for N,

$$N\left(\sin\theta - \mu_s\cos\theta\right) = \frac{mv^2}{r} \text{ and } N\left(\cos\theta + \mu_s\sin\theta\right) = mg.$$

Combining,

$$\left(\sin\theta - \mu_s\cos\theta\right)mg = \left(\cos\theta + \mu_s\sin\theta\right)\frac{mv^2}{r}$$

Rearrange,

$$\mu_s = \frac{gr\sin\theta - v^2\cos\theta}{gr\cos\theta + v^2\sin\theta}.$$

We know all the numbers. Put them in and we'll find $\mu_s = 0.22$

(b) Now the frictional force will point the other way, so Newton's second law is now

$$\begin{aligned} \sum F_r &= N_r + f_r = N\sin\theta + f\cos\theta = \frac{mv^2}{r}, \\ \sum F_z &= N_z - f_z - W = N\cos\theta - f\sin\theta - mg = 0. \end{aligned}$$

The bottom equation can be rearranged to show that

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}.$$

This can be combined with the top equation to give

$$mg\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{mv^2}{r}.$$

We can solve this final expression for v using all our previous numbers and get $v = 35$ m/s. That's about 130 km/hr.

E5-42 (a) The net force on the person at the top of the Ferris wheel is $mv^2/r = W - N_\mathrm{t}$, pointing down. The net force on the bottom is still mv^2/r, but this quantity now equals $N_\mathrm{b} - W$ and is point up. Then $N_\mathrm{b} = 2W - N_\mathrm{t} = 2(150 \text{ lb}) - (125 \text{ lb}) = 175$ lb.

(b) Doubling the speed would quadruple the net force, so the new scale reading at the top would be $(150 \text{ lb}) - 4[(150 \text{ lb}) - (125 \text{ lb})] = 50$ lb. Wee!

E5-43 The net force on the object when it is *not* sliding is $F = mv^2/r$; the speed of the object is $v = 2\pi r f$ (f is rotational frequency here), so $F = 4\pi^2 mrf^2$. The coefficient of friction is then at least $\mu_\mathrm{s} = F/W = 4\pi^2 rf^2/g$. If the object stays put when the table rotates at $33\frac{1}{3}$ rev/min then

$$\mu_\mathrm{s} \geq 4\pi^2(0.13\,\mathrm{m})(33.3/60\,/\mathrm{s})^2/(9.8\,\mathrm{m/s^2}) = 0.16.$$

If the object slips when the table rotates at 45.0 rev/min then

$$\mu_\mathrm{s} \leq 4\pi^2(0.13\,\mathrm{m})(45.0/60\,/\mathrm{s})^2/(9.8\,\mathrm{m/s^2}) = 0.30.$$

E5-44 This is effectively a banked highway problem if the pilot is flying correctly.

$$r = \frac{v^2}{g\tan\theta} = \frac{(134\,\text{m/s})^2}{(9.8\,\text{m/s}^2)\tan(38.2^\circ)} = 2330\,\text{m}.$$

E5-45 (a) Assume that frigate bird flies as well as a pilot. Then this is a banked highway problem. The speed of the bird is given by $v^2 = gr\tan\theta$. But there is also $vt = 2\pi r$, so $2\pi v^2 = gvt\tan\theta$, or

$$v = \frac{gt\tan\theta}{2\pi} = \frac{(9.8\,\text{m/s}^2)(13\,\text{s})\tan(25^\circ)}{2\pi} = 9.5\,\text{m/s}.$$

(b) $r = vt/(2\pi) = (9.5\,\text{m/s})(13\,\text{s})/(2\pi) = 20\,\text{m}$.

E5-46 (a) The radius of the turn is $r = \sqrt{(33\,\text{m})^2 - (18\,\text{m})^2} = 28\,\text{m}$. The speed of the plane is $v = 2\pi rf = 2\pi(28\,\text{m})(4.4/60\,/\text{s}) = 13\,\text{m/s}$. The acceleration is $a = v^2/r = (13\,\text{m/s})^2/(28\,\text{m}) = 6.0\,m/\text{s}^2$.

(b) The tension has two components: $T_x = T\cos\theta$ and $T_y = T\sin\theta$. In this case $\theta = \arcsin(18/33) = 33^\circ$. All of the centripetal force is provided for by T_x, so

$$T = (0.75\,\text{kg})(6.0\,\text{m/s}^2)/\cos(33^\circ) = 5.4\,\text{N}.$$

(c) The lift is balanced by the weight and T_y. The lift is then

$$T_y + W = (5.4\,\text{N})\sin(33^\circ) + (0.75\,\text{kg})(9.8\,\text{m/s}^2) = 10\,\text{N}.$$

E5-47 (a) The acceleration is $a = v^2/r = 4\pi^2 r/t^2 = 4\pi^2(6.37\times10^6\text{m})/(8.64\times10^4\text{s})^2 = 3.37\times10^{-2}\text{m/s}^2$. The centripetal force on the standard kilogram is $F = ma = (1.00\,\text{kg})(3.37\times10^{-2}\text{m/s}^2) = 0.0337\,\text{N}$.

(b) The tension in the balance would be $T = W - F = (9.80\,\text{N}) - (0.0337\,\text{N}) = 9.77\,\text{N}$.

E5-48 (a) $v = 4(0.179\,\text{m/s}^4)(7.18\,\text{s})^3 - 2(2.08\,\text{m/s}^2)(7.18\,\text{s}) = 235\,\text{m/s}$.

(b) $a = 12(0.179\,\text{m/s}^4)(7.18\,\text{s})^2 - 2(2.08\,\text{m/s}^2) = 107\,\text{m/s}^2$.

(c) $F = ma = (2.17\,\text{kg})(107\,\text{m/s}^2) = 232\,\text{N}$.

$\boxed{\textbf{E5-49}}$ The force only has an x component, so we can use Eq. 5-19 to find the velocity.

$$v_x = v_{0x} + \frac{1}{m}\int_0^t F_x\,dt = v_0 + \frac{F_0}{m}\int_0^t (1 - t/T)\;dt$$

Integrating,

$$v_x = v_0 + a_0\left(t - \frac{1}{2T}t^2\right)$$

Now put this expression into Eq. 5-20 to find the position as a function of time

$$x = x_0 + \int_0^t v_x\,dt = \int_0^t \left(v_{0x} + a_0\left(t - \frac{1}{2T}t^2\right)\right)\;dt$$

Integrating,

$$x = v_0T + a_0\left(\frac{1}{2}T^2 - \frac{1}{6T}T^3\right) = v_0T + a_0\frac{T^2}{3}.$$

Now we can put $t = T$ into the expression for v.

$$v_x = v_0 + a_0\left(T - \frac{1}{2T}T^2\right) = v_0 + a_0T/2.$$

P5-1 (a) There are two forces which accelerate block 1: the tension, T, and the parallel component of the weight, $W_{||,1} = m_1 g \sin\theta_1$. Assuming the block accelerates to the right,

$$m_1 a = m_1 g \sin\theta_1 - T.$$

There are two forces which accelerate block 2: the tension, T, and the parallel component of the weight, $W_{||,2} = m_2 g \sin\theta_2$. Assuming the block 1 accelerates to the right, block 2 must also accelerate to the right, and

$$m_2 a = T - m_2 g \sin\theta_2.$$

Add these two equations,

$$(m_1 + m_2)a = m_1 g \sin\theta_1 - m_2 g \sin\theta_2,$$

and then rearrange:

$$a = \frac{m_1 g \sin\theta_1 - m_2 g \sin\theta_2}{m_1 + m_2}.$$

Or, take the two net force equations, divide each side by the mass, and set them equal to each other:

$$g \sin\theta_1 - T/m_1 = T/m_2 - g \sin\theta_2.$$

Rearrange,

$$T\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = g \sin\theta_1 + g \sin\theta_2,$$

and then rearrange again:

$$T = \frac{m_1 m_2 g}{m_1 + m_2}(\sin\theta_1 + \sin\theta_2).$$

(b) The negative sign we get in the answer means that block 1 accelerates up the ramp.

$$a = \frac{(3.70\,\mathrm{kg})\sin(28^\circ) - (4.86\,\mathrm{kg})\sin(42^\circ)}{(3.70\,\mathrm{kg}) + (4.86\,\mathrm{kg})}(9.81\,\mathrm{m/s^2}) = -1.74\,\mathrm{m/s^2}.$$

$$T = \frac{(3.70\,\mathrm{kg})(4.86\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(3.70\,\mathrm{kg}) + (4.86\,\mathrm{kg})}[\sin(28^\circ) + \sin(42^\circ)] = 23.5\,\mathrm{N}.$$

(c) No acceleration happens when $m_2 = (3.70\,\mathrm{kg})\sin(28^\circ)/\sin(42^\circ) = 2.60\,\mathrm{kg}$. If m_2 is more massive than this m_1 will accelerate up the plane; if m_2 is less massive than this m_1 will accelerate down the plane.

P5-2 (a) Since the pulley is massless, $F = 2T$. The largest value of T that will allow block 2 to remain on the floor is $T \le W_2 = m_2 g$. So $F \le 2(1.9\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 37\,\mathrm{N}$.
(b) $T = F/2 = (110\,\mathrm{N})/2 = 55\,\mathrm{N}$.
(c) The net force on block 1 is $T - W_1 = (55\,\mathrm{N}) - (1.2\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 43\,\mathrm{N}$. This will result in an acceleration of $a = (43\,\mathrm{N})/(1.2\,\mathrm{kg}) = 36\,\mathrm{m/s^2}$.

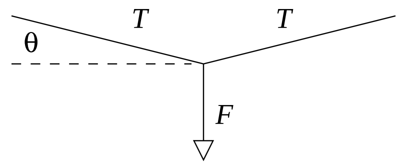
$\boxed{\textbf{P5-3}}$ As the string is pulled the two masses will move together so that the configuration will look like the figure below. The point where the force is applied to the string is massless, so $\sum F = 0$ at that point. We can take advantage of this fact and the figure below to find the tension in the cords, $F/2 = T\cos\theta$. The factor of 1/2 occurs because only 1/2 of F is contained in the right triangle that has T as the hypotenuse. From the figure we can find the x component of the force on one mass to be $T_x = T\sin\theta$. Combining,

$$T_x = \frac{F}{2}\frac{\sin\theta}{\cos\theta} = \frac{F}{2}\tan\theta.$$

But the tangent is equal to

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{x}{\sqrt{L^2 - x^2}}$$

And now we have the answer in the book.



What happens when $x = L$? Well, a_x is infinite according to this expression. Since that could only happen if the tension in the string were infinite, then there must be some other physics that we had previously ignored.

P5-4 (a) The force of static friction can be as large as $f \leq \mu_s N = (0.60)(12\text{ lb}) = 7.2$ lb. That is more than enough to hold the block up.

(b) The force of static friction is actually only large enough to hold up the block: $f = 5.0$ lb. The magnitude of the force of the wall on the block is then $F_{\text{bw}} = \sqrt{(5.0)^2 + (12.0)^2}\text{lb} = 13$ lb.

P5-5 (a) The weight has two components: normal to the incline, $W_\perp = mg\cos\theta$ and parallel to the incline, $W_{||} = mg\sin\theta$. There is no motion perpendicular to the incline, so $N = W_\perp = mg\cos\theta$. The force of friction on the block is then $f = \mu N = \mu mg\cos\theta$, where we use whichever μ is appropriate. The net force on the block is $F - f - W_{||} = F \pm \mu mg\cos\theta - mg\sin\theta$.

To hold the block in place we use μ_s and friction will point *up* the ramp so the $\pm$ is $+$, and

$$F = (7.96\,\text{kg})(9.81\,\text{m/s}^2)[\sin(22.0^\circ) - (0.25)\cos(22.0^\circ)] = 11.2\,\text{N}.$$

(b) To find the minimum force to begin sliding the block up the ramp we still have static friction, but now friction points *down*, so

$$F = (7.96\,\text{kg})(9.81\,\text{m/s}^2)[\sin(22.0^\circ) + (0.25)\cos(22.0^\circ)] = 47.4\,\text{N}.$$

(c) To keep the block sliding up at constant speed we have kinetic friction, so

$$F = (7.96\,\text{kg})(9.81\,\text{m/s}^2)[\sin(22.0^\circ) + (0.15)\cos(22.0^\circ)] = 40.1\,\text{N}.$$

P5-6 The sand will slide if the cone makes an angle greater than θ where $\mu_s = \tan\theta$. But $\tan\theta = h/R$ or $h = R\tan\theta$. The volume of the cone is then

$$Ah/3 = \pi R^2 h/3 = \pi R^3 \tan\theta/3 = \pi\mu_s R^3/3.$$

$\boxed{\textbf{P5-7}}$ There are four forces on the broom: the force of gravity $W = mg$; the normal force of the floor N; the force of friction f; and the applied force from the person P (the book calls it F). Then

$$\begin{aligned}\sum F_x &= P_x - f = P\sin\theta - f = ma_x,\\ \sum F_y &= N - P_y - W = N - P\cos\theta - mg = ma_y = 0\end{aligned}$$

Solve the second equation for N,

$$N = P\cos\theta + mg.$$

(a) If the mop slides at constant speed $f = \mu_k N$. Then

$$P \sin\theta - f = P\sin\theta - \mu_k\left(P\cos\theta + mg\right) = 0.$$

We can solve this for P (which was called F in the book);

$$P = \frac{\mu mg}{\sin\theta - \mu_k \cos\theta}.$$

This is the force required to push the broom at constant speed.

(b) Note that P becomes negative (or infinite) if $\sin\theta \le \mu_k \cos\theta$. This occurs when $\tan\theta_c \le \mu_k$. If this happens the mop stops moving, to get it started again you must overcome the static friction, but this is impossible if $\tan\theta_0 \le \mu_s$

P5-8 (a) The condition to slide is $\mu_s \le \tan\theta$. In this case, $(0.63) > \tan(24^\circ) = 0.445$.

(b) The normal force on the slab is $N = W_\perp = mg\cos\theta$. There are *three* forces parallel to the surface: friction, $f = \mu_s N = \mu_s mg\cos\theta$; the parallel component of the weight, $W_{||} = mg\sin\theta$, and the force F. The block will slide if these don't balance, or

$$F > \mu_s mg\cos\theta - mg\sin\theta = (1.8\times10^7\,\mathrm{kg})(9.8\,\mathrm{m/s^2})[(0.63)\cos(24^\circ) - \sin(24^\circ)] = 3.0\times10^7\,\mathrm{N}.$$

$\boxed{\textbf{P5-9}}$ To hold up the smaller block the frictional force between the larger block and smaller block must be as large as the weight of the smaller block. This can be written as $f = mg$. The normal force of the larger block on the smaller block is N, and the frictional force is given by $f \le \mu_s N$. So the smaller block won't fall if $mg \le \mu_s N$.

There is only one horizontal force on the large block, which is the normal force of the small block on the large block. Newton's third law says this force has a magnitude N, so the acceleration of the large block is $N = Ma$.

There is only one horizontal force on the two block system, the force F. So the acceleration of this system is given by $F = (M+m)a$. The two accelerations are equal, otherwise the blocks won't stick together. Equating, then, gives $N/M = F/(M+m)$.

We can combine this last expression with $mg \le \mu_s N$ and get

$$mg \le \mu_s F\frac{M}{M+m}$$

or

$$F \ge \frac{g(M+m)m}{\mu_s M} = \frac{(9.81\,\mathrm{m/s^2})(88\,\mathrm{kg}+16\,\mathrm{kg})(16\,\mathrm{kg})}{(0.38)(88\,\mathrm{kg})} = 490\,\mathrm{N}$$

P5-10 The normal force on the ith block is $N_i = m_i g\cos\theta$; the force of friction on the ith block is then $f_i = \mu_i m_i g\cos\theta$. The parallel component of the weight on the ith block is $W_{||,i} = m_i g\sin\theta$.

(a) The net force on the system is

$$F = \sum_i m_i g(\sin\theta - \mu_i\cos\theta).$$

Then

$$\begin{aligned} a &= (9.81\,\mathrm{m/s^2})\frac{(1.65\,\mathrm{kg})(\sin 29.5^\circ - 0.226\cos 29.5^\circ) + (3.22\,\mathrm{kg})(\sin 29.5^\circ - 0.127\cos 29.5^\circ)}{(1.65\,\mathrm{kg}) + (3.22\,\mathrm{kg})}, \\ &= 3.46\,\mathrm{m/s^2}. \end{aligned}$$

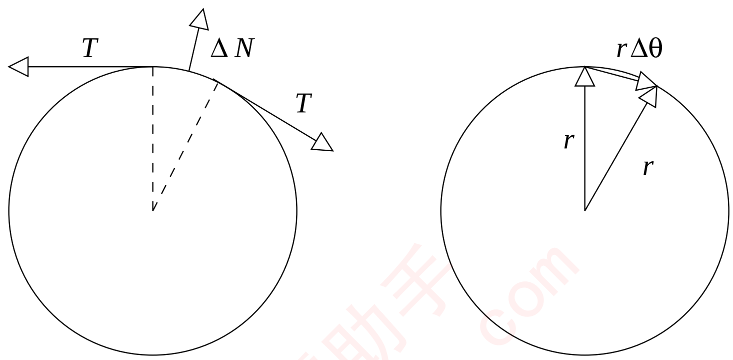
(b) The net force on the lower mass is $m_2 a = W_{||,2} - f_2 - T$, so the tension is

$$T = (9.81\,\mathrm{m/s^2})(3.22\,\mathrm{kg})(\sin 29.5^\circ - 0.127\cos 29.5^\circ) - (3.22\,\mathrm{kg})(3.46\,\mathrm{m/s^2}) = 0.922\,\mathrm{N}.$$

(c) The acceleration will stay the same, since the system is still the same. Reversing the order of the masses can only result in a reversing of the tension: it is still 0.992 N, but is now negative, meaning compression.

$\boxed{\textbf{P5-11}}$ The rope wraps around the dowel and there is a contribution to the frictional force Δf from each small segment of the rope where it touches the dowel. There is also a normal force ΔN at each point where the contact occurs. We can find ΔN much the same way that we solve the circular motion problem.

In the figure on the left below we see that we can form a triangle with long side T and short side ΔN. In the figure on the right below we see a triangle with long side r and short side $r\Delta\theta$. These triangles are similar, so $r\Delta\theta/r = \Delta N/T$.



Now $\Delta f = \mu\Delta N$ and $T(\theta) + \Delta f \approx T(\theta + \Delta\theta)$. Combining, and taking the limit as $\Delta\theta \to 0$, $dT = df$

$$\int \frac{1}{\mu}\frac{dT}{T} = \int d\theta$$

integrating both sides of this expression,

$$\begin{aligned} \int \frac{1}{\mu}\frac{dT}{T} &= \int d\theta, \\ \frac{1}{\mu}\ln T|_{T_1}^{T_2} &= \pi, \\ T_2 &= T_1 e^{\pi\mu}. \end{aligned}$$

In this case T_1 is the weight and T_2 is the downward force.

P5-12 Answer this out of order!

(b) The maximum static friction between the blocks is 12.0 N; the maximum acceleration of the top block is then $a = F/m = (12.0\,\text{N})/(4.40\,\text{kg}) = 2.73\,\text{m/s}^2$.

(a) The net force on a system of two blocks that will accelerate them at $2.73\,\text{m/s}^2$ is $F = (4.40\,\text{kg} + 5.50\,\text{kg})(2.73\,\text{m/s}^2) = 27.0\,\text{N}$.

(c) The coefficient of friction is $\mu_s = F/N = F/mg = (12.0\,\text{N})/[(4.40\,\text{kg})(9.81\,\text{m/s}^2)] = 0.278$.

P5-13 The speed is $v = 23.6\,\text{m/s}$.

(a) The average speed while stopping is half the initial speed, or $v_{av} = 11.8\,\text{m/s}$. The time to stop is $t = (62\,\text{m})/(11.8\,\text{m/s}) = 5.25\,\text{s}$. The rate of deceleration is $a = (23.6\,\text{m/s})/(5.25\,\text{s}) = 4.50\,\text{m/s}^2$. The stopping force is $F = ma$; this is related to the frictional force by $F = \mu_s mg$, so $\mu_s = a/g = (4.50\,\text{m/s}^2)/(9.81\,\text{m/s}^2) = 0.46$.

(b) Turning,

$$a = v^2/r = (23.6\,\text{m/s})^2/(62\,\text{m}) = 8.98\,\text{m/s}^2.$$

Then $\mu_s = a/g = (8.98\,\text{m/s}^2)/(9.81\,\text{m/s}^2) = 0.92$.

P5-14 (a) The net force on car as it travels at the top of a circular hill is $F_{\text{net}} = mv^2/r = W - N$; in this case we are told $N = W/2$, so $F_{\text{net}} = W/2 = (16000\,\text{N})/2 = 8000\,\text{N}$. When the car travels through the bottom valley the net force at the bottom is $F_{\text{net}} = mv^2/r = N - W$. Since the magnitude of v, r, and hence F_{net} is the same in both cases,

$$N = W/2 + W = 3W/2 = 3(16000\,\text{N})/2 = 24000\,\text{N}$$

at the bottom of the valley.

(b) You leave the hill when $N = 0$, or

$$v = \sqrt{rg} = \sqrt{(250\,\text{m})(9.81\,\text{m/s}^2)} = 50\,\text{m/s}.$$

(c) At this speed $F_{\text{net}} = W$, so at the bottom of the valley $N = 2W = 32000\,\text{N}$.

P5-15 (a) $v = 2\pi r/t = 2\pi(0.052\,\text{m})(3/3.3\,\text{s}) = 0.30\,\text{m/s}$.

(b) $a = v^2/r = (0.30\,\text{m/s})^2/(0.052\,\text{m}) = 1.7\,\text{m/s}^2$, toward center.

(c) $F = ma = (0.0017\,\text{kg})(1.7\,\text{m/s}^2) = 2.9\times 10^{-3}\text{N}$.

(d) If the coin can be as far away as r before slipping, then

$$\mu_s = F/mg = (2\pi r/t)^2/(rg) = 4\pi^2 r/(t^2 g) = 4\pi^2(0.12\,\text{m})/[(3/3.3\,\text{s})^2(9.8\,\text{m/s}^2)] = 0.59.$$

P5-16 (a) Whether you assume constant speed or constant energy, the tension in the string must be the greatest at the bottom of the circle, so that's where the string will break.

(b) The net force on the stone at the bottom is $T - W = mv^2/r$. Then

$$v = \sqrt{rg[T/W - 1]} = \sqrt{(2.9\text{ ft})(32\text{ ft/s}^2)[(9.2\text{ lb})/(0.82\text{ lb}) - 1]} = 31\text{ ft/s}.$$

$\boxed{\textbf{P5-17}}$ (a) In order to keep the ball moving in a circle there must be a net centripetal force F_c directed horizontally toward the rod. There are only *three* forces which act on the ball: the force of gravity, $W = mg = (1.34\,\text{kg})(9.81\,\text{m/s}^2) = 13.1\,\text{N}$; the tension in the top string $T_1 = 35.0\,\text{N}$, and the tension in the bottom string, T_2.

The components of the force from the tension in the top string are

$$T_{1,x} = (35.0\,\text{N})\cos 30^\circ = 30.3\,\text{N} \text{ and } T_{1,y} = (35.0\,\text{N})\sin 30^\circ = 17.5\,\text{N}.$$

The vertical components *do* balance, so

$$T_{1,y} + T_{2,y} = W,$$

or $T_{2,y} = (13.1\,\text{N}) - (17.5\,\text{N}) = -4.4\,\text{N}$. From this we can find the tension in the bottom string,

$$T_2 = T_{2,y}/\sin(-30^\circ) = 8.8\,\text{N}.$$

(b) The net force on the object will be the sum of the two horizontal components,

$$F_c = (30.3\,\text{N}) + (8.8\,\text{N})\cos 30^\circ = 37.9\,\text{N}.$$

(c) The speed will be found from

$$\begin{aligned} v &= \sqrt{a_c r} = \sqrt{F_c r/m}, \\ &= \sqrt{(37.9\,\text{m})(1.70\,\text{m})\sin 60^\circ/(1.34\,\text{kg})} = 6.45\,\text{m/s}. \end{aligned}$$

P5-18 The net force on the cube is $F = mv^2/r$. The speed is $2\pi r\omega$. (Note that we are using ω in a non-standard way!) Then $F = 4\pi^2 mr\omega^2$. There are three forces to consider: the normal force of the funnel on the cube N; the force of gravity on the cube W; and the frictional force on the cube f. The acceleration of the cube in circular motion is toward the center of the circle; this means the *net* force on the cube is horizontal, toward the center. We will arrange our coordinate system so that r is horizontal and z is vertical. Then the components of the normal force are $N_r = N\sin\theta$ and $N_z = N\cos\theta$; the components of the frictional force are $f_r = f\cos\theta$ and $f_z = f\sin\theta$.

The direction of the friction depends on the speed of the cube; it will point up if ω is small and down if ω is large.

(a) If ω is small, Newton's second law is

$$\begin{aligned}\sum F_r &= N_r - f_r = N\sin\theta - f\cos\theta = 4\pi^2 mr\omega^2,\\ \sum F_z &= N_z + f_z - W = N\cos\theta + f\sin\theta - mg = 0.\end{aligned}$$

We can substitute $f = \mu_s N$. Solving for N,

$$N(\cos\theta + \mu_s\sin\theta) = mg.$$

Combining,

$$4\pi^2 r\omega^2 = g\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}.$$

Rearrange,

$$\omega = \frac{1}{2\pi}\sqrt{\frac{g}{r}\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}}.$$

This is the minimum value.

(b) Now the frictional force will point the other way, so Newton's second law is now

$$\begin{aligned}\sum F_r &= N_r + f_r = N\sin\theta + f\cos\theta = 4\pi^2 mr\omega^2,\\ \sum F_z &= N_z - f_z - W = N\cos\theta - f\sin\theta - mg = 0.\end{aligned}$$

We've swapped + and - signs, so

$$\omega = \frac{1}{2\pi}\sqrt{\frac{g}{r}\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}$$

is the maximum value.

P5-19 (a) The radial distance from the axis of rotation at a latitude L is $R\cos L$. The speed in the circle is then $v = 2\pi R\cos L/T$. The net force on a hanging object is $F = mv^2/(R\cos L) = 4\pi^2 mR\cos L/T^2$. This net force is *not* directed toward the center of the earth, but is instead directed toward the axis of rotation. It makes an angle L with the Earth's vertical. The tension in the cable must then have two components: one which is vertical (compared to the Earth) and the other which is horizontal. If the cable makes an angle θ with the vertical, then $T_{||} = T\sin\theta$ and $T_\perp = T\cos\theta$. Then $T_{||} = F_{||}$ and $W - T_\perp = F_\perp$. Written with a little more detail,

$$T\sin\theta = 4\pi^2 mR\cos L\,\sin L/T^2 \approx T\theta,$$

and

$$T\cos\theta = 4\pi^2 mR\cos^2 L/T^2 + mg \approx T.$$

But $4\pi^2 R\cos^2 L/T^2 \ll g$, so it can be ignored in the last equation compared to g, and $T \approx mg$. Then from the first equation,

$$\theta = 2\pi^2 R\sin 2L/(gT^2).$$

(b) This is a maximum when $\sin 2L$ is a maximum, which happens when $L = 45°$. Then

$$\theta = 2\pi^2(6.37\times 10^6\mathrm{m})/[(9.8\,\mathrm{m/s^2})(86400\,\mathrm{s})^2] = 1.7\times 10^{-3}\ \mathrm{rad}.$$

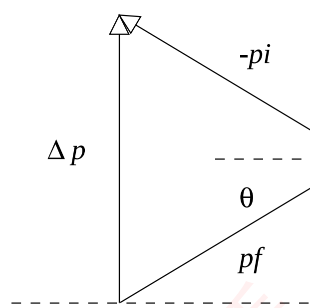
(c) The deflection at both the equator and the poles would be zero.

P5-20 $a = (F_0/m)e^{-t/T}$. Then $v = \int_0^t a\,dt = (F_0T/m)e^{-t/T}$, and $x = \int_0^t v\,dt = (F_0T^2/m)e^{-t/T}$.
(a) When $t = T$ $v = (F_0T/m)e^{-1} = 0.368(F_0T/m)$.
(b) When $t = T$ $x = (F_0T^2/m)e^{-1} = 0.368(F_0T^2/m)$.

E6-1 (a) $v_1 = (m_2/m_1)v_2 = (2650\,\text{kg}/816\,\text{kg})(16.0\text{ km/h}) = 52.0\text{ km/h}$.
(b) $v_1 = (m_2/m_1)v_2 = (9080\,\text{kg}/816\,\text{kg})(16.0\text{ km/h}) = 178\text{ km/h}$.

E6-2 $\vec{\mathbf{p}}_\text{i} = (2000\,\text{kg})(40\text{ km/h})\hat{\mathbf{j}} = 8.00\times10^4\text{kg}\cdot\text{km/h}\hat{\mathbf{j}}$. $\vec{\mathbf{p}}_\text{f} = (2000\,\text{kg})(50\text{ km/h})\hat{\mathbf{i}} = 1.00\times10^5\text{kg}\cdot\text{km/h}\hat{\mathbf{i}}$. $\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_\text{f} - \vec{\mathbf{p}}_\text{i} = 1.00\times10^5\text{kg}\cdot\text{km/h}\hat{\mathbf{i}} - 8.00\times10^4\text{kg}\cdot\text{km/h}\hat{\mathbf{j}}$. $\Delta p = \sqrt{(\Delta p_x)^2 + (\Delta p_y)^2} = 1.28\times10^5\text{kg}\cdot\text{km/h}$. The direction is 38.7° south of east.

E6-3 The figure below shows the initial and final momentum vectors arranged to geometrically show $\vec{\mathbf{p}}_\text{f} - \vec{\mathbf{p}}_\text{i} = \Delta\vec{\mathbf{p}}$. We can use the cosine law to find the length of $\Delta\vec{\mathbf{p}}$.



The angle $\alpha = 42° + 42°$, $p_\text{i} = mv = (4.88\,\text{kg})(31.4\,\text{m/s}) = 153\,\text{kg·m/s}$. Then the magnitude of $\Delta\vec{\mathbf{p}}$ is

$$\Delta p = \sqrt{(153\,\text{kg·m/s})^2 + (153\,\text{kg·m/s})^2 - 2(153\,\text{kg·m/s})^2\cos(84°)} = 205\,\text{kg·m/s},$$

directed up from the plate. By symmetry it must be perpendicular.

E6-4 The change in momentum is $\Delta p = -mv = -(2300\,\text{kg})(15\,\text{m/s}) = -3.5\times10^4\text{kg}\cdot\text{m/s}$. The average force is $F = \Delta p/\Delta t = (-3.5\times10^4\text{kg}\cdot\text{m/s})/(0.54\,\text{s}) = -6.5\times10^4\text{N}$.

E6-5 (a) The change in momentum is $\Delta p = (-mv) - mv$; the average force is $F = \Delta p/\Delta t = -2mv/\Delta t$.
(b) $F = -2(0.14\,\text{kg})(7.8\,\text{m/s})/(3.9\times10^{-3}\text{s}) = 560\,\text{N}$.

E6-6 (a) $J = \Delta p = (0.046\,\text{kg})(52.2\,\text{m/s}) - 0 = 2.4\,\text{N}\cdot\text{s}$.
(b) The impulse imparted to the club is opposite that imparted to the ball.
(c) $F = \Delta p/\Delta t = (2.4\,\text{N}\cdot\text{s})/(1.20\times10^{-3}\text{s}) = 2000\,\text{N}$.

E6-7 Choose the coordinate system so that the ball is only moving along the x axis, with away from the batter as positive. Then $p_{\text{f}x} = mv_{\text{f}x} = (0.150\,\text{kg})(61.5\,\text{m/s}) = 9.23$ kg·m/s and $p_{\text{i}x} = mv_{\text{i}x} = (0.150\,\text{kg})(-41.6\,\text{m/s}) = -6.24$ kg·m/s. The impulse is given by $J_x = p_{\text{f}x} - p_{\text{i}x} = 15.47$ kg·m/s. We can find the average force by application of Eq. 6-7:

$$F_{\text{av},x} = \frac{J_x}{\Delta t} = \frac{(15.47\text{ kg}\cdot\text{m/s})}{(4.7\times10^{-3}\text{ s})} = 3290\,\text{N}.$$

E6-8 The magnitude of the impulse is $J = F\delta t = (-984\,\mathrm{N})(0.0270\,\mathrm{s}) = -26.6\,\mathrm{N}\cdot\mathrm{s}$. Then $p_\mathrm{f} = p_\mathrm{i} + \Delta p$, so

$$v_\mathrm{f} = \frac{(0.420\,\mathrm{kg})(13.8\,\mathrm{m/s}) + (-26.6\,\mathrm{N}\cdot\mathrm{s})}{(0.420\,\mathrm{kg})} = -49.5\,\mathrm{m/s}.$$

The ball moves backward!

E6-9 The change in momentum of the ball is $\Delta p = (mv) - (-mv) = 2mv = 2(0.058\,\mathrm{kg})(32\,\mathrm{m/s}) = 3.7\,\mathrm{kg}\cdot\mathrm{m/s}$. The impulse is the area under a force - time graph; for the trapezoid in the figure this area is $J = F_\mathrm{max}(2\,\mathrm{ms} + 6\,\mathrm{ms})/2 = (4\,\mathrm{ms})F_\mathrm{max}$. Then $F_\mathrm{max} = (3.7\,\mathrm{kg}\cdot\mathrm{m/s})/(4\,\mathrm{ms}) = 930\,\mathrm{N}$.

E6-10 The final speed of each object is given by $v_i = J/m_i$, where i refers to which object (as opposed to "initial"). The object are going in different directions, so the relative speed will be the sum. Then

$$v_\mathrm{rel} = v_1 + v_2 = (300\,\mathrm{N}\cdot\mathrm{s})[1/(1200\,kg) + 1/(1800\,\mathrm{kg})] = 0.42\,\mathrm{m/s}.$$

$\boxed{\textbf{E6-11}}$ Use Simpson's rule. Then the area is given by

$$\begin{aligned} J_x &= \frac{1}{3}h\left(f_0 + 4f_1 + 2f_2 + 4f_3 + ... + 4f_13 + f_14\right), \\ &= \frac{1}{3}(0.2\,\mathrm{ms})\left(200 + 4\cdot 800 + 2\cdot 1200 ...\,\mathrm{N}\right) \end{aligned}$$

which gives $J_x = 4.28$ kg·m/s.

Since the impulse is the change in momentum, and the ball started from rest, $p_{\mathrm{f}x} = J_x + p_{\mathrm{i}x} = 4.28$ kg·m/s. The final velocity is then found from $v_x = p_x/m = 8.6$ m/s.

E6-12 (a) The average speed during the the time the hand is in contact with the board is half of the initial speed, or $v_\mathrm{av} = 4.8\,\mathrm{m/s}$. The time of contact is then $t = y/v_\mathrm{av} = (0.028\,\mathrm{m})/(4.8\,\mathrm{m/s}) = 5.8\,\mathrm{ms}$.

(b) The impulse given to the board is the same as the magnitude in the change in momentum of the hand, or $J = (0.54\,\mathrm{kg})(9.5\,\mathrm{m/s}) = 5.1\,\mathrm{N}\cdot\mathrm{s}$. Then $F_\mathrm{av} = (5.1\,\mathrm{N}\cdot\mathrm{s})/(5.8\,\mathrm{ms}) = 880\,\mathrm{N}$.

E6-13 $\Delta p = J = F\Delta t = (3000\,\mathrm{N})(65.0\,\mathrm{s}) = 1.95\times 10^5\mathrm{N}\cdot\mathrm{s}$. The direction of the thrust relative to the velocity doesn't matter in this exercise.

E6-14 (a) $p = mv = (2.14\times 10^{-3}\mathrm{kg})(483\,\mathrm{m/s}) = 1.03\,\mathrm{kg}\cdot\mathrm{m/s}$.

(b) The impulse imparted to the wall in one second is ten times the above momentum, or $J = 10.3\,\mathrm{kg}\cdot\mathrm{m/s}$. The average force is then $F_\mathrm{av} = (10.3\,\mathrm{kg}\cdot\mathrm{m/s})/(1.0\,\mathrm{s}) = 10.3\,\mathrm{N}$.

(c) The average force for each individual particle is $F_\mathrm{av} = (1.03\,\mathrm{kg}\cdot\mathrm{m/s})/(1.25\times 10^{-3}\,\mathrm{s}) = 830\,\mathrm{N}$.

$\boxed{\textbf{E6-15}}$ A transverse direction means at right angles, so the thrusters have imparted a momentum sufficient to direct the spacecraft 100+3400 = 3500 km to the side of the original path. The spacecraft is half-way through the six-month journey, so it has three months to move the 3500 km to the side. This corresponds to a transverse speed of $v = (3500\times 10^3\mathrm{m})/(90\times 86400\,\mathrm{s}) = 0.45\,\mathrm{m/s}$. The required time for the rocket to fire is $\Delta t = (5400\,\mathrm{kg})(0.45\,\mathrm{m/s})/(1200\,\mathrm{N}) = 2.0\,\mathrm{s}$.

E6-16 Total initial momentum is zero, so

$$v_\mathrm{m} = -\frac{m_\mathrm{s}}{m_\mathrm{m}}v_\mathrm{s} = -\frac{m_\mathrm{s}g}{m_\mathrm{m}g}v_\mathrm{s} = -\frac{(0.158\text{ lb})}{(195\text{ lb})}(12.7\text{ ft/s}) = -1.0\times 10^{-2}\text{ ft/s}.$$

E6-17 Conservation of momentum:

$$\begin{aligned} p_{\rm f,m} + p_{\rm f,c} &= p_{\rm i,m} + p_{\rm i,c}, \\ m_{\rm m}v_{\rm f,m} + m_{\rm c}v_{\rm f,c} &= m_{\rm m}v_{\rm i,m} + m_{\rm c}v_{\rm i,c}, \\ v_{\rm f,c} - v_{\rm i,c} &= \frac{m_{\rm m}v_{\rm i,m} - m_{\rm m}v_{\rm f,m}}{m_{\rm c}}, \\ \Delta v_{\rm c} &= \frac{(75.2\text{ kg})(2.33\text{ m/s}) - (75.2\text{ kg})(0)}{(38.6\text{ kg})}, \\ &= 4.54\,\text{m/s}. \end{aligned}$$

The answer is positive; the cart speed *increases*.

E6-18 Conservation of momentum:

$$\begin{aligned} p_{\rm f,m} + p_{\rm f,c} &= p_{\rm i,m} + p_{\rm i,c}, \\ m_{\rm m}(v_{\rm f,c} - v_{\rm rel}) + m_{\rm c}v_{\rm f,c} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c}, \\ (m_{\rm m} + m_{\rm c})v_{\rm f,c} - m_{\rm m}v_{\rm rel} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c}, \\ \Delta v_{\rm c} &= m_{\rm m}v_{\rm rel}/(m_{\rm m} + m_{\rm c}), \\ &= wv_{\rm rel}/(w + W). \end{aligned}$$

E6-19 Conservation of momentum. Let m refer to motor and c refer to command module:

$$\begin{aligned} p_{\rm f,m} + p_{\rm f,c} &= p_{\rm i,m} + p_{\rm i,c}, \\ m_{\rm m}(v_{\rm f,c} - v_{\rm rel}) + m_{\rm c}v_{\rm f,c} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c}, \\ (m_{\rm m} + m_{\rm c})v_{\rm f,c} - m_{\rm m}v_{\rm rel} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c}, \\ v_{\rm f,c} &= \frac{m_{\rm m}v_{\rm rel} + (m_{\rm m} + m_{\rm c})v_{\rm i,c}}{(m_{\rm m} + m_{\rm c})}, \\ &= \frac{4m_{\rm c}(125\text{ km/h}) + (4m_{\rm c} + m_{\rm c})(3860\text{ km/h})}{(4m_{\rm c} + m_{\rm c})} = 3960\text{ km/h}. \end{aligned}$$

E6-20 Conservation of momentum. The block on the left is 1, the other is 2.

$$\begin{aligned} m_1v_{1,\rm f} + m_2v_{2,\rm f} &= m_1v_{1,\rm i} + m_2v_{2,\rm i}, \\ v_{1,\rm f} &= v_{1,\rm i} + \frac{m_2}{m_1}(v_{2,\rm i} - v_{2,\rm f}), \\ &= (5.5\,\text{m/s}) + \frac{(2.4\,\text{kg})}{(1.6\,\text{kg})}[(2.5\,\text{m/s}) - (4.9\,\text{m/s})], \\ &= 1.9\,\text{m/s}. \end{aligned}$$

E6-21 Conservation of momentum. The block on the left is 1, the other is 2.

$$\begin{aligned} m_1v_{1,\rm f} + m_2v_{2,\rm f} &= m_1v_{1,\rm i} + m_2v_{2,\rm i}, \\ v_{1,\rm f} &= v_{1,\rm i} + \frac{m_2}{m_1}(v_{2,\rm i} - v_{2,\rm f}), \\ &= (5.5\,\text{m/s}) + \frac{(2.4\,\text{kg})}{(1.6\,\text{kg})}[(-2.5\,\text{m/s}) - (4.9\,\text{m/s})], \\ &= -5.6\,\text{m/s}. \end{aligned}$$

E6-22 Assume a completely inelastic collision. Call the Earth 1 and the meteorite 2. Then

$$
\begin{aligned}
m_1 v_{1,\mathrm{f}} + m_2 v_{2,\mathrm{f}} &= m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}, \\
v_{1,\mathrm{f}} &= \frac{m_2 v_{2,\mathrm{i}}}{m_1 + m_2}, \\
&= \frac{(5\times 10^{10}\mathrm{kg})(7200\,\mathrm{m/s})}{(5.98\times 10^{24}\mathrm{kg}) + (5\times 10^{10}\mathrm{kg})} = 7\times 10^{-11}\mathrm{m/s}.
\end{aligned}
$$

That's 2 mm/y!

$\boxed{\textbf{E6-23}}$ Conservation of momentum is used to solve the problem:

$$
\begin{aligned}
P_\mathrm{f} &= P_\mathrm{i}, \\
p_{\mathrm{f,bl}} + p_{\mathrm{f,bu}} &= p_{\mathrm{i,bl}} + p_{\mathrm{i,bu}}, \\
m_\mathrm{bl} v_{\mathrm{f,bl}} + m_\mathrm{bu} v_{\mathrm{f,bu}} &= m_\mathrm{bl} v_{\mathrm{i,bl}} + m_\mathrm{bu} v_{\mathrm{i,bu}}, \\
(715\text{ g}) v_{\mathrm{f,bl}} + (5.18\text{ g})(428\text{ m/s}) &= (715\text{ g})(0) + (5.18\text{ g})(672\text{ m/s}),
\end{aligned}
$$

which has solution $v_{\mathrm{f,bl}} = 1.77\,\mathrm{m/s}$.

E6-24 The y component of the initial momentum is zero; therefore the magnitudes of the y components of the two particles must be equal after the collision. Then

$$
\begin{aligned}
m_\alpha v_\alpha \sin\theta_\alpha &= m_\mathrm{O} v_\mathrm{O} \sin\theta_\mathrm{O}, \\
v_\alpha &= \frac{m_\mathrm{O} v_\mathrm{O} \sin\theta_\mathrm{O}}{m_\alpha v_\alpha \sin\theta_\alpha}, \\
&= \frac{(16\text{ u})(1.20\times 10^5\mathrm{m/s})\sin(51^\circ)}{(4.00\text{ u})\sin(64^\circ)} = 4.15\times 10^5\mathrm{m/s}.
\end{aligned}
$$

E6-25 The total momentum is

$$
\begin{aligned}
\vec{\mathbf{p}} &= (2.0\,\mathrm{kg})[(15\,\mathrm{m/s})\hat{\mathbf{i}} + (30\,\mathrm{m/s})\hat{\mathbf{j}}] + (3.0\,\mathrm{kg})[(-10\,\mathrm{m/s})\hat{\mathbf{i}} + (5\,\mathrm{m/s})\hat{\mathbf{j}}], \\
&= 75\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}.
\end{aligned}
$$

The final velocity of B is

$$
\begin{aligned}
\vec{\mathbf{v}}_{Bf} &= \frac{1}{m_B}(\vec{\mathbf{p}} - m_A \vec{\mathbf{v}}_{Af}), \\
&= \frac{1}{(3.0\,\mathrm{kg})}\{(75\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}) - (2.0\,\mathrm{kg})[(-6.0\,\mathrm{m/s})\hat{\mathbf{i}} + (30\,\mathrm{m/s})\hat{\mathbf{j}}]\}, \\
&= (4.0\,\mathrm{m/s})\hat{\mathbf{i}} + (5.0\,\mathrm{m/s})\hat{\mathbf{j}}.
\end{aligned}
$$

E6-26 Assume electron travels in $+x$ direction while neutrino travels in $+y$ direction. Conservation of momentum requires that

$$
\vec{\mathbf{p}} = -(1.2\times 10^{-22}\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{i}} - (6.4\times 10^{-23}\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{j}}
$$

be the momentum of the nucleus after the decay. This has a magnitude of $p = 1.4\times 10^{-22}\mathrm{kg}\cdot\mathrm{m/s}$ and be directed 152° from the electron.

E6-27 What we know:

$$\begin{aligned}
\vec{\mathbf{p}}_{1,\mathrm{i}} &= (1.50\times10^5\mathrm{kg})(6.20\,\mathrm{m/s})\hat{\mathbf{i}} = 9.30\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}},\\
\vec{\mathbf{p}}_{2,\mathrm{i}} &= (2.78\times10^5\mathrm{kg})(4.30\,\mathrm{m/s})\hat{\mathbf{j}} = 1.20\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}},\\
\vec{\mathbf{p}}_{2,\mathrm{f}} &= (2.78\times10^5\mathrm{kg})(5.10\,\mathrm{m/s})[\sin(18^\circ)\hat{\mathbf{i}} + \cos(18^\circ)\hat{\mathbf{j}}],\\
&= 4.38\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} + 1.35\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}.
\end{aligned}$$

Conservation of momentum then requires

$$\begin{aligned}
\vec{\mathbf{p}}_{1,\mathrm{f}} &= (9.30\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}) - (4.38\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}})\\
&\quad +(1.20\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}) - (1.35\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}),\\
&= 4.92\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} - 1.50\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}.
\end{aligned}$$

This corresponds to a velocity of

$$\vec{\mathbf{v}}_{1,\mathrm{f}} = 3.28\,\mathrm{m/s}\,\hat{\mathbf{i}} - 1.00\,\mathrm{m/s}\,\hat{\mathbf{j}},$$

which has a magnitude of 3.43 m/s directed 17° to the right.

E6-28 $v_\mathrm{f} = -2.1\,\mathrm{m/s}$.

$\boxed{\textbf{E6-29}}$ We want to solve Eq. 6-24 for m_2 given that $v_{1,\mathrm{f}} = 0$ and $v_{1,\mathrm{i}} = -v_{2,\mathrm{i}}$. Making these substitutions

$$\begin{aligned}
(0) &= \frac{m_1 - m_2}{m_1 + m_2}v_{1,\mathrm{i}} + \frac{2m_2}{m_1 + m_2}(-v_{1,\mathrm{i}}),\\
0 &= (m_1 - m_2)v_{1,\mathrm{i}} - (2m_2)v_{1,\mathrm{i}},\\
3m_2 &= m_1
\end{aligned}$$

so $m_2 = 100$ g.

E6-30 (a) Rearrange Eq. 6-27:

$$m_2 = m_1\frac{v_{1\mathrm{i}} - v_{1\mathrm{f}}}{v_{1\mathrm{i}} + v_{1\mathrm{f}}} = (0.342\,\mathrm{kg})\frac{(1.24\,\mathrm{m/s}) - (0.636\,\mathrm{m/s})}{(1.24\,\mathrm{m/s}) + (0.636\,\mathrm{m/s})} = 0.110\,\mathrm{kg}.$$

(b) $v_{2\mathrm{f}} = 2(0.342\,\mathrm{kg})(1.24\,\mathrm{m/s})/(0.342\,\mathrm{kg} + 0.110\,\mathrm{kg}) = 1.88\,\mathrm{m/s}$.

E6-31 Rearrange Eq. 6-27:

$$m_2 = m_1\frac{v_{1\mathrm{i}} - v_{1\mathrm{f}}}{v_{1\mathrm{i}} + v_{1\mathrm{f}}} = (2.0\,\mathrm{kg})\frac{v_{1\mathrm{i}} - v_{1\mathrm{i}}/4}{v_{1\mathrm{i}} + v_{1\mathrm{i}}/4} = 1.2\,\mathrm{kg}.$$

E6-32 I'll multiply all momentum equations by g, then I can use weight directly without converting to mass.

(a) $v_\mathrm{f} = [(31.8\ \mathrm{T})(5.20\ \mathrm{ft/s}) + (24.2\ \mathrm{T})(2.90\ \mathrm{ft/s})]/(31.8\ \mathrm{T} + 24.2\ \mathrm{T}) = 4.21\ \mathrm{ft/s}$.

(b) Evaluate:

$$v_{1\mathrm{f}} = \frac{31.8\ \mathrm{T} - 24.2\ \mathrm{T}}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(5.20\ \mathrm{ft/s}) + \frac{2(24.2\ \mathrm{T})}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(2.90\ \mathrm{ft/s}) = 3.21\ \mathrm{ft/s}.$$

$$v_{2\mathrm{f}} = -\frac{31.8\ \mathrm{T} - 24.2\ \mathrm{T}}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(2.90\ \mathrm{ft/s}) + \frac{2(31.8\ \mathrm{T})}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(5.20\ \mathrm{ft/s}) = 5.51\ \mathrm{ft/s}.$$

$\boxed{\textbf{E6-33}}$ Let the initial momentum of the first object be $\vec{\mathbf{p}}_{1,\mathrm{i}} = m\vec{\mathbf{v}}_{1,\mathrm{i}}$, that of the second object be $\vec{\mathbf{p}}_{2,\mathrm{i}} = m\vec{\mathbf{v}}_{2,\mathrm{i}}$, and that of the combined final object be $\vec{\mathbf{p}}_{\mathrm{f}} = 2m\vec{\mathbf{v}}_{\mathrm{f}}$. Then

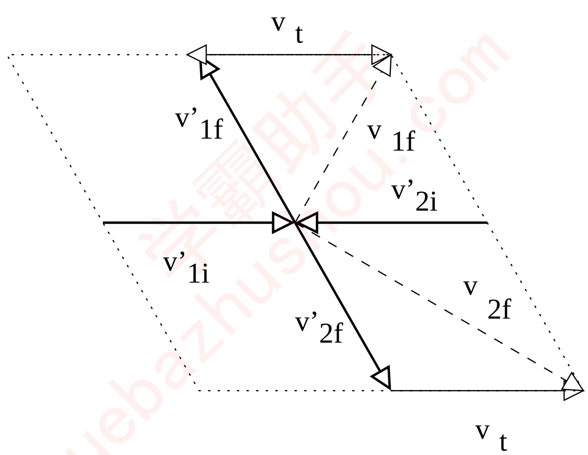
$$\vec{\mathbf{p}}_{1,\mathrm{i}} + \vec{\mathbf{p}}_{2,\mathrm{i}} = \vec{\mathbf{p}}_{\mathrm{f}},$$

implies that we can find a triangle with sides of length $p_{1,\mathrm{i}}$, $p_{2,\mathrm{i}}$, and p_{f}. These lengths are

$$\begin{aligned} p_{1,\mathrm{i}} &= mv_{\mathrm{i}}, \\ p_{2,\mathrm{i}} &= mv_{\mathrm{i}}, \\ p_{\mathrm{f}} &= 2mv_{\mathrm{f}} = 2mv_{\mathrm{i}}/2 = mv_{\mathrm{i}}, \end{aligned}$$

so this is an equilateral triangle. This means the angle between the initial velocities is 120°.

E6-34 We need to change to the center of mass system. Since both particles have the same mass, the conservation of momentum problem is effectively the same as a (vector) conservation of velocity problem. Since one of the particles is originally at rest, the center of mass moves with speed $v_{\mathrm{cm}} = v_{1\mathrm{i}}/2$. In the figure below the center of mass velocities are primed; the transformation velocity is v_t.



Note that since $v_t = v'_{1\mathrm{i}} = v'_{2\mathrm{i}} = v'_{1\mathrm{f}} = v'_{2\mathrm{f}}$ the entire problem can be inscribed in a rhombus. The diagonals of the rhombus are the directions of $v_{1\mathrm{f}}$ and $v_{2\mathrm{f}}$; note that the diagonals of a rhombus are *necessarily* at right angles!

(a) The target proton moves off at 90° to the direction the incident proton moves after the collision, or 26° away from the incident protons original direction.

(b) The y components of the final momenta must be equal, so $v_{2\mathrm{f}}\sin(26°) = v_{1\mathrm{f}}\sin(64°)$, or $v_{2\mathrm{f}} = v_{1\mathrm{f}}\tan(64°)$. The x components must add to the original momentum, so $(514\,\mathrm{m/s}) = v_{2\mathrm{f}}\cos(26°) + v_{1\mathrm{f}}\cos(64°)$, or

$$v_{1\mathrm{f}} = (514\,\mathrm{m/s})/\{\tan(64°)\cos(26°) + \cos(64°)\} = 225\,\mathrm{m/s},$$

and

$$v_{2\mathrm{f}} = (225\,\mathrm{m/s})\tan(64°) = 461\,\mathrm{m/s}.$$

E6-35 $v_{\mathrm{cm}} = \{(3.16\,\mathrm{kg})(15.6\,\mathrm{m/s}) + (2.84\,\mathrm{kg})(-12.2\,\mathrm{m/s})\}/\{(3.16\,\mathrm{kg}) + (2.84\,\mathrm{kg})\} = 2.44\,\mathrm{m/s}$, positive means to the left.

$\boxed{\textbf{P6-1}}$ The force is the change in momentum over change in time; the momentum is the mass time velocity, so

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \Delta v \frac{m}{\Delta t} = 2u\mu,$$

since μ is the mass per unit time.

P6-2 (a) The initial momentum is $\vec{\mathbf{p}}_\mathrm{i} = (1420\,\mathrm{kg})(5.28\,\mathrm{m/s})\hat{\mathbf{j}} = 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}$. After making the right hand turn the final momentum is $\vec{\mathbf{p}}_\mathrm{f} = 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}$. The impulse is $\vec{\mathbf{J}} = 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} - 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}$, which has magnitude $J = 10600\,\mathrm{kg}\cdot\mathrm{m/s}$.

(b) During the collision the impulse is $\vec{\mathbf{J}} = 0 - 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}$. The magnitude is $J = 7500\,\mathrm{kg}\cdot\mathrm{m/s}$.

(c) The average force is $F = J/t = (10600\,\mathrm{kg}\cdot\mathrm{m/s})/(4.60\,\mathrm{s}) = 2300\,\mathrm{N}$.

(d) The average force is $F = J/t = (7500\,\mathrm{kg}\cdot\mathrm{m/s})/(0.350\,\mathrm{s}) = 21400\,\mathrm{N}$.

P6-3 (a) Only the component of the momentum which is perpendicular to the wall changes. Then

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = -2(0.325\,\mathrm{kg})(6.22\,\mathrm{m/s})\sin(33^\circ)\hat{\mathbf{j}} = -2.20\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}.$$

(b) $\vec{\mathbf{F}} = -\vec{\mathbf{J}}/t = -(-2.20\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}})/(0.0104\,\mathrm{s}) = 212\,\mathrm{N}$.

P6-4 The change in momentum of one bullet is $\Delta p = 2mv = 2(0.0030\,\mathrm{kg})(500\,\mathrm{m/s}) = 3.0\,\mathrm{kg}\cdot\mathrm{m/s}$. The average force is the total impulse in one minute divided by one minute, or

$$F_\mathrm{av} = 100(3.0\,\mathrm{kg}\cdot\mathrm{m/s})/(60\,\mathrm{s}) = 5.0\,\mathrm{N}.$$

P6-5 (a) The volume of a hailstone is $V = 4\pi r^3/3 = 4\pi(0.5\ \mathrm{cm})^3/3 = 0.524\ \mathrm{cm}^3$. The mass of a hailstone is $m = \rho V = (9.2\times10^{-4}\mathrm{kg/cm}^3)(0.524\ \mathrm{cm}^3) = 4.8\times10^{-4}\mathrm{kg}$.

(b) The change in momentum of one hailstone when it hits the ground is

$$\Delta p = (4.8\times10^{-4}\mathrm{kg})(25\,\mathrm{m/s}) = 1.2\times10^{-2}\mathrm{kg}\cdot\mathrm{m/s}.$$

The hailstones fall at $25\,\mathrm{m/s}$, which means that in one second the hailstones in a column $25\,\mathrm{m}$ high hit the ground. Over an area of $10\,\mathrm{m}\times20\,\mathrm{m}$ then there would be $(25\,\mathrm{m})(10\,\mathrm{m})(20\,\mathrm{m}) = 500\,\mathrm{m}^3$ worth of hailstones, or 6.00×10^5 hailstones per second striking the surface. Then

$$F_\mathrm{av} = 6.00\times10^5(1.2\times10^{-2}\,\mathrm{kg}\cdot\mathrm{m/s})/(1\,\mathrm{s}) = 7200\,\mathrm{N}.$$

P6-6 Assume the links are *not* connected once the top link is released. Consider the link that starts h above the table; it falls a distance h in a time $t = \sqrt{2h/g}$ and hits the table with a speed $v = gt = \sqrt{2hg}$. When the link hits the table h of the chain is already on the table, and $L-h$ is yet to come. The linear mass density of the chain is M/L, so when this link strikes the table the mass is hitting the table at a rate $dm/dt = (M/L)v = (M/L)\sqrt{2hg}$. The average force required to stop the falling link is then $v\,dm/dt = (M/L)2hg = 2(M/L)hg$. But the weight of the chain that is already on the table is $(M/L)hg$, so the net force on the table is the sum of these two terms, or $F = 3W$.

$\boxed{\textbf{P6-7}}$ The weight of the marbles in the box after a time t is $mgRt$ because Rt is the number of marbles in the box.

The marbles fall a distance h from rest; the time required to fall this distance is $t = \sqrt{2h/g}$, the speed of the marbles when they strike the box is $v = gt = \sqrt{2gh}$. The momentum each marble imparts on the box is then $m\sqrt{2gh}$. If the marbles strike at a rate R then the force required to stop them is $Rm\sqrt{2gh}$.

The reading on the scale is then

$$W = mR(\sqrt{2gh} + gt).$$

This will give a numerical result of

$$(4.60\times10^{-3}\text{kg})(115\,\text{s}^{-1})\left(\sqrt{2(9.81\,\text{m/s}^2)(9.62\,\text{m})} + (9.81\,\text{m/s}^2)(6.50\,\text{s})\right) = 41.0\,\text{N}.$$

P6-8 (a) $v = (108\,\text{kg})(9.74\,\text{m/s})/(108\,\text{kg} + 1930\,\text{kg}) = 0.516\,\text{m/s}$.

(b) Label the person as object 1 and the car as object 2. Then $m_1v_1 + m_2v_2 = (108\,\text{kg})(9.74\,\text{m/s})$ and $v_1 = v_2 + 0.520\,\text{m/s}$. Combining,

$$v_2 = [1050\,\text{kg}\cdot\text{m/s} - (0.520\,\text{m/s})(108\,\text{kg})]/(108\,\text{kg} + 1930\,\text{kg}) = 0.488\,\text{m/s}.$$

P6-9 (a) It takes a time $t_1 = \sqrt{2h/g}$ to fall $h = 6.5$ ft. An object will be moving at a speed $v_1 = gt_1 = \sqrt{2hg}$ after falling this distance. If there is an inelastic collision with the pile then the two will move together with a speed of $v_2 = Mv_1/(M+m)$ after the collision.

If the pile then stops within $d = 1.5$ inches, then the time of stopping is given by $t_2 = d/(v_2/2) = 2d/v_2$.

For inelastic collisions this corresponds to an average force of

$$F_{\text{av}} = \frac{(M+m)v_2}{t_2} = \frac{(M+m)v_2^2}{2d} = \frac{M^2v_1^2}{2(M+m)d} = \frac{(gM)^2}{g(M+m)}\frac{h}{d}.$$

Note that we multiply through by g to get weights. The numerical result is $F_{\text{av}} = 130$ t.

(b) For an elastic collision $v_2 = 2Mv_1/(M+m)$; the time of stopping is still expressed by $t_2 = 2d/v_2$, but we now know F_{av} instead of d. Then

$$F_{\text{av}} = \frac{mv_2}{t_2} = \frac{mv_2^2}{2d} = \frac{4Mmv_1^2}{(M+m)d} = \frac{2(gM)(gm)}{g(M+m)}\frac{h}{d}.$$

or

$$d = \frac{2(gM)(gm)}{g(M+m)}\frac{h}{F_{\text{av}}},$$

which has a numerical result of $d = 0.51$ inches.

But wait! The weight, which just had an elastic collision, "bounced" off of the pile, and then hit it again. This drives the pile deeper into the earth. The weight hits the pile a second time with a speed of $v_3 = (M-m)/(M+m)v_1$; the pile will (in this second elastic collision) then have a speed of $v_4 = 2M(M+m)v_3 = [(M-m)/(M+m)]v_2$. In other words, we have an infinite series of distances traveled by the pile, and if $\alpha = [(M-m)/(M+m)] = 0.71$, the depth driven by the pile is

$$d_{\text{f}} = d(1 + \alpha^2 + \alpha^4 + \alpha^6\cdots) = \frac{d}{1-\alpha^2},$$

or $d = 1.03$.

P6-10 The cat jumps off of sled A; conservation of momentum requires that $Mv_{A,1} + m(v_{A,1} + v_c) = 0$, or

$$v_{A,1} = -mv_c/(m+M) = -(3.63\,\text{kg})(3.05\,\text{m/s})/(22.7\,\text{kg} + 3.63\,\text{kg}) = -0.420\,\text{m/s}.$$

The cat lands on sled B; conservation of momentum requires $v_{B,1} = m(v_{A,1} + v_c)/(m+M)$. The cat jumps off of sled B; conservation of momentum is now

$$Mv_{B,2} + m(v_{B,2} - v_c) = m(v_{A,1} + v_c),$$

or

$$v_{B,2} = 2mv_c/(m+M) = (3.63\,\text{kg})[(-0.420\,\text{m/s}) + 2(3.05\,\text{m/s})]/(22.7\,\text{kg} + 3.63\,\text{kg}) = 0.783\,\text{m/s}.$$

The cat then lands on cart A; conservation of momentum requires that $(M+m)v_{A,2} = -Mv_{B,2}$, or

$$v_{A,2} = -(22.7\,\text{kg})(0.783\,\text{m/s})/(22.7\,\text{kg} + 3.63\,\text{kg}) = -0.675\,\text{m/s}.$$

$\boxed{\textbf{P6-11}}$ We align the coordinate system so that west is $+x$ and south is $+y$. The each car contributes the following to the initial momentum

$$\begin{aligned} A &: (2720\ \text{lb}/g)(38.5\ \text{mi/h})\hat{\mathbf{i}} = 1.05\times10^5\ \text{lb}\cdot\text{mi/h}/g\,\hat{\mathbf{i}}, \\ B &: (3640\ \text{lb}/g)(58.0\ \text{mi/h})\hat{\mathbf{j}} = 2.11\times10^5\ \text{lb}\cdot\text{mi/h}/g\,\hat{\mathbf{j}}. \end{aligned}$$

These become the components of the final momentum. The direction is then

$$\theta = \arctan\frac{2.11\times10^5\ \text{lb}\cdot\text{mi/h}/g}{1.05\times10^5\ \text{lb}\cdot\text{mi/h}/g} = 63.5^\circ,$$

south of west. The magnitude is the square root of the sum of the squares,

$$2.36\times10^5\ \text{lb}\cdot\text{mi/h}/g,$$

and we divide this by the mass ($6360\ \text{lb}/g$) to get the final speed after the collision: 37.1 mi/h.

P6-12 (a) Ball A must carry off a momentum of $\vec{\mathbf{p}} = m_B v\,\hat{\mathbf{i}} - m_B v/2\,\hat{\mathbf{j}}$, which would be in a direction $\theta = \arctan(-0.5/1) = 27^\circ$ from the original direction of B, or 117° from the final direction.

(b) No.

P6-13 (a) We assume all balls have a mass m. The collision imparts a "sideways" momentum to the cue ball of $m(3.50\,\text{m/s})\sin(65^\circ) = m(3.17\,\text{m/s})$. The other ball must have an equal, but opposite "sideways" momentum, so $-m(3.17\,\text{m/s}) = m(6.75\,\text{m/s})\sin\theta$, or $\theta = -28.0^\circ$.

(b) The final "forward" momentum is

$$m(3.50\,\text{m/s})\cos(65^\circ) + m(6.75\,\text{m/s})\cos(-28^\circ) = m(7.44\,\text{m/s}),$$

so the initial speed of the cue ball would have been 7.44 m/s.

P6-14 Assuming $M \gg m$, Eq. 6-25 becomes

$$v_{2\text{f}} = 2v_{1\text{i}} - v_{1\text{i}} = 2(13\ \text{km/s}) - (-12\ \text{km/s}) = 38\ \text{km/s}.$$

$\boxed{\textbf{P6-15}}$ (a) We get

$$v_{2,\text{f}} = \frac{2(220\ \text{g})}{(220\ \text{g}) + (46.0\ \text{g})}(45.0\,\text{m/s}) = 74.4\,\text{m/s}.$$

(b) Doubling the mass of the clubhead we get

$$v_{2,\text{f}} = \frac{2(440\ \text{g})}{(440\ \text{g}) + (46.0\ \text{g})}(45.0\,\text{m/s}) = 81.5\,\text{m/s}.$$

(c) Tripling the mass of the clubhead we get

$$v_{2,\text{f}} = \frac{2(660\ \text{g})}{(660\ \text{g}) + (46.0\ \text{g})}(45.0\,\text{m/s}) = 84.1\,\text{m/s}.$$

Although the heavier club helps some, the maximum speed to get out of the ball will be less than twice the speed of the club.

P6-16 There will always be at least two collisions. The balls are a, b, and c from left to right. After the first collision between a and b one has

$$v_{b,1} = v_0 \text{ and } v_{a,1} = 0.$$

After the first collision between b and c one has

$$v_{c,1} = 2mv_0/(m+M) \text{ and } v_{b,2} = (m-M)v_0/(m+M).$$

(a) If $m \geq M$ then ball b continue to move to the right (or stops) and there are no more collisions.
(b) If $m < M$ then ball b bounces back and strikes ball a which was at rest. Then

$$v_{a,2} = (m-M)v_0/(m+M) \text{ and } v_{b,3} = 0.$$

P6-17 All three balls are identical in mass and radii? Then balls 2 and 3 will move off at 30° to the initial direction of the first ball. By symmetry we expect balls 2 and 3 to have the same speed.

The problem now is to define an elastic three body collision. It is no longer the case that the balls bounce off with the same speed in the center of mass. One can't even treat the problem as two separate collisions, one right after the other. No amount of momentum conservation laws will help solve the problem; we need some additional physics, but at this point in the text we don't have it.

P6-18 The original speed is v_0 in the lab frame. Let α be the angle of deflection in the cm frame and $\vec{\mathbf{v}}_1'$ be the initial velocity in the cm frame. Then the velocity after the collision in the cm frame is $v_1' \cos\alpha\,\hat{\mathbf{i}} + v_1' \sin\alpha\,\hat{\mathbf{j}}$ and the velocity in the lab frame is $(v_1' \cos\alpha + v)\hat{\mathbf{i}} + v_1' \sin\alpha\,\hat{\mathbf{j}}$, where v is the speed of the cm frame. The deflection angle in the lab frame is

$$\theta = \arctan[(v_1' \sin\alpha)/(v_1' \cos\alpha + v)],$$

but $v = m_1 v_0/(m_1 + m_2)$ and $v_1' = v_0 - v$ so $v_1' = m_2 v_0/(m_1 + m_2)$ and

$$\theta = \arctan[(m_2 \sin\alpha)/(m_2 \cos\alpha + m_1)].$$

(c) θ is a maximum when $(\cos\alpha + m_1/m_2)/\sin\alpha$ is a minimum, which happens when $\cos\alpha = -m_1/m_2$ if $m_1 \leq m_2$. Then $[(m_2 \sin\alpha)/(m_2 \cos\alpha + m_1)]$ can have any value between $-\infty$ and ∞, so θ can be between 0 and π.

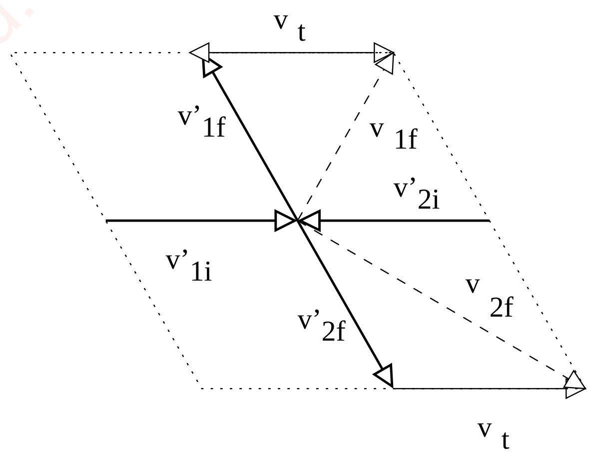
(a) If $m_1 > m_2$ then $(\cos\alpha + m_1/m_2)/\sin\alpha$ is a minimum when $\cos\alpha = -m_2/m_1$, then

$$[(m_2 \sin\alpha)/(m_2 \cos\alpha + m_1)] = m_2/\sqrt{m_1^2 - m_2^2}.$$

If $\tan\theta = m_2/\sqrt{m_1^2 - m_2^2}$ then m_1 is like a hypotenuse and m_2 the opposite side. Then

$$\cos\theta = \sqrt{m_1^2 - m_2^2}/m_1 = \sqrt{1 - (m_2/m_1)^2}.$$

(b) We need to change to the center of mass system. Since both particles have the same mass, the conservation of momentum problem is effectively the same as a (vector) conservation of velocity problem. Since one of the particles is originally at rest, the center of mass moves with speed $v_{\text{cm}} = v_{1\text{i}}/2$. In the figure below the center of mass velocities are primed; the transformation velocity is v_t.



Note that since $v_t = v'_{1\text{i}} = v'_{2\text{i}} = v'_{1\text{f}} = v'_{2\text{f}}$ the entire problem can be inscribed in a rhombus. The diagonals of the rhombus are the directions of $v_{1\text{f}}$ and $v_{2\text{f}}$; note that the diagonals of a rhombus are *necessarily* at right angles!

P6-19 (a) The speed of the bullet after leaving the first block but before entering the second can be determined by momentum conservation.

$$\begin{aligned} P_\text{f} &= P_\text{i}, \\ p_\text{f,bl} + p_\text{f,bu} &= p_\text{i,bl} + p_\text{i,bu}, \\ m_\text{bl}v_\text{f,bl} + m_\text{bu}v_\text{f,bu} &= m_\text{bl}v_\text{i,bl} + m_\text{bu}v_\text{i,bu}, \\ (1.78\text{kg})(1.48\,\text{m/s}) + (3.54\times10^{-3}\text{kg})(1.48\,\text{m/s}) &= (1.78\text{kg})(0) + (3.54\times10^{-3}\text{kg})v_\text{i,bu}, \end{aligned}$$

which has solution $v_\text{i,bl} = 746\,\text{m/s}$.

(b) We do the same steps again, except applied to the first block,

$$\begin{aligned} P_\text{f} &= P_\text{i}, \\ p_\text{f,bl} + p_\text{f,bu} &= p_\text{i,bl} + p_\text{i,bu}, \\ m_\text{bl}v_\text{f,bl} + m_\text{bu}v_\text{f,bu} &= m_\text{bl}v_\text{i,bl} + m_\text{bu}v_\text{i,bu}, \\ (1.22\text{kg})(0.63\,\text{m/s}) + (3.54\times10^{-3}\text{kg})(746\,\text{m/s}) &= (1.22\text{kg})(0) + (3.54\times10^{-3}\text{kg})v_\text{i,bu}, \end{aligned}$$

which has solution $v_\text{i,bl} = 963\,\text{m/s}$.

P6-20 The acceleration of the block down the ramp is $a_1 = g\sin(22°)$. The ramp has a length of $d = h/\sin(22°)$, so it takes a time $t_1 = \sqrt{2d/a_1} = \sqrt{2h/g}/\sin(22°)$ to reach the bottom. The speed when it reaches the bottom is $v_1 = a_1 t_1 = \sqrt{2gh}$. Notice that it is independent of the angle!

The collision is inelastic, so the two stick together and move with an initial speed of $v_2 = m_1 v_1/(m_1 + m_2)$. They slide a distance x before stopping; the average speed while decelerating is $v_\text{av} = v_2/2$, so the stopping time is $t_2 = 2x/v_2$ and the deceleration is $a_2 = v_2/t_2 = v_2^2/(2x)$. If the retarding force is $f = (m_1 + m_2)a_2$, then $f = \mu_\text{k}(m_1 + m_2)g$. Glue it all together and

$$\mu_\text{k} = \frac{m_1^2}{(m_1+m_2)^2}\frac{h}{x} = \frac{(2.0\,\text{kg})^2}{(2.0\,\text{kg} + 3.5\,\text{kg})^2}\frac{(0.65\,\text{m})}{(0.57\,\text{m})} = 0.15.$$

P6-21 (a) For an object with initial speed v and deceleration $-a$ which travels a distance x before stopping, the time t to stop is $t = v/a$, the average speed while stopping is $v/2$, and $d = at^2/2$. Combining, $v = \sqrt{2ax}$. The deceleration in this case is given by $a = \mu_k g$.

Then just after the collision

$$v_A = \sqrt{2(0.130)(9.81\,\mathrm{m/s^2})(8.20\,\mathrm{m})} = 4.57\,\mathrm{m/s},$$

while

$$v_B = \sqrt{2(0.130)(9.81\,\mathrm{m/s^2})(6.10\,\mathrm{m})} = 3.94\,\mathrm{m/s},$$

(b) $v_0 = [(1100\,\mathrm{kg})(4.57\,\mathrm{m/s}) + (1400\mathrm{kg})(3.94\,\mathrm{m/s})]/(1400\,\mathrm{kg}) = 7.53\,\mathrm{m/s}.$

E7-1 $x_{\rm cm} = (7.36\times10^{22}{\rm kg})(3.82\times10^{8}{\rm m})/(7.36\times10^{22}{\rm kg} + 5.98\times10^{34}{\rm kg}) = 4.64\times10^{6}{\rm m}$. This is less than the radius of the Earth.

E7-2 If the particles are l apart then

$$x_1 = m_1 l(m_1 + m_2)$$

is the distance from particle 1 to the center of mass and

$$x_2 = m_2 l(m_1 + m_2)$$

is the distance from particle 2 to the center of mass. Divide the top equation by the bottom and

$$x_1/x_2 = m_1/m_2.$$

$\boxed{\textbf{E7-3}}$ The center of mass velocity is given by Eq. 7-1,

$$\begin{aligned}\vec{\mathbf{v}}_{\rm cm} &= \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1+m_2},\\ &= \frac{(2210\ {\rm kg})(105\ {\rm km/h}) + (2080\ {\rm kg})(43.5\ {\rm km/h})}{(2210\ {\rm kg}) + (2080\ {\rm kg})} = 75.2\ {\rm km/h}.\end{aligned}$$

E7-4 They will meet at the center of mass, so

$$x_{\rm cm} = (65\,{\rm kg})(9.7\,{\rm m})/(65\,{\rm kg} + 42\,{\rm kg}) = 5.9\,{\rm m}.$$

E7-5 (a) No external forces, center of mass doesn't move.

(b) The collide at the center of mass,

$$x_{\rm cm} = (4.29\,{\rm kg})(1.64\,{\rm m})/(4.29\,{\rm kg} + 1.43\,{\rm kg}) = 1.23\,{\rm m}.$$

E7-6 The range of the center of mass is

$$R = v_0^2 \sin 2\theta\,/g = (466\,{\rm m/s})^2 \sin(2\times 57.4^\circ)/(9.81\,{\rm m/s^2}) = 2.01\times10^4\,{\rm m}.$$

Half lands directly underneath the highest point, or 1.00×10^4 m. The other piece must land at x, such that

$$2.01\times10^4\,{\rm m} = (1.00\times10^4\,{\rm m} + x)/2;$$

then $x = 3.02\times10^4$ m.

$\boxed{\textbf{E7-7}}$ The center of mass of the boat + dog doesn't move because there are no external forces on the system. Define the coordinate system so that distances are measured from the shore, so toward the shore is in the negative x direction. The *change* in position of the center of mass is given by

$$\Delta x_{\rm cm} = \frac{m_d\Delta x_d + m_b\Delta x_b}{m_d+m_b} = 0,$$

Both Δx_d and Δx_b are measured with respect to the shore; we are given $\Delta x_{db} = -8.50$ ft, the displacement of the dog with respect to the boat. But

$$\Delta x_d = \Delta x_{db} + \Delta x_b.$$

Since we want to find out about the dog, we'll substitute for the boat's displacement,

$$0 = \frac{m_d \Delta x_d + m_b (\Delta x_d - \Delta x_{db})}{m_d + m_b}.$$

Rearrange and solve for Δx_d. Use $W = mg$ and multiply the top and bottom of the expression by g. Then

$$\Delta x_d = \frac{m_b \Delta x_{db}}{m_d + m_b} \frac{g}{g} = \frac{(46.4 \text{ lb})(-8.50 \text{ ft})}{(10.8 \text{ lb}) + (46.4 \text{ lb})} = -6.90 \text{ ft}.$$

The dog is now $21.4 - 6.9 = 14.5$ feet from shore.

E7-8 Richard has too much time on his hands.

The center of mass of the system is $x_{\rm cm}$ away from the center of the boat. Switching seats is effectively the same thing as rotating the canoe through 180°, so the center of mass of the system has moved through a distance of $2x_{\rm cm} = 0.412\,\text{m}$. Then $x_{\rm cm} = 0.206\,\text{m}$. Then

$$x_{\rm cm} = (Ml - ml)/(M + m + m_c) = 0.206\,\text{m},$$

where $l = 1.47\,\text{m}$, $M = 78.4\,\text{kg}$, $m_c = 31.6\,\text{kg}$, and m is Judy's mass. Rearrange,

$$m = \frac{Ml - (M + m_c)x_{\rm cm}}{l + x_{\rm cm}} = \frac{(78.4\,\text{kg})(1.47\,\text{m}) - (78.4\,\text{kg} + 31.6\,\text{kg})(0.206\,\text{m})}{(1.47\,\text{m}) + (0.206\,\text{m})} = 55.2\,\text{kg}.$$

E7-9 It takes the man $t = (18.2\,\text{m})/(2.08\,\text{m/s}) = 8.75\,\text{s}$ to walk to the front of the boat. During this time the center of mass of the system has moved forward $x = (4.16\,\text{m/s})(8.75\,\text{s}) = 36.4\,\text{m}$. But in walking forward to the front of the boat the man shifted the center of mass by a distance of $(84.4\,\text{kg})(18.2\,\text{m})/(84.4\,\text{kg} + 425\,\text{kg}) = 3.02\,\text{m}$, so the boat only traveled $36.4\,\text{m} - 3.02\,\text{m} = 33.4\,\text{m}$.

E7-10 Do each coordinate separately.

$$x_{\rm cm} = \frac{(3\,\text{kg})(0) + (8\,\text{kg})(1\,\text{m}) + (4\,\text{kg})(2\,\text{m})}{(3\,\text{kg}) + (8\,\text{kg}) + (4\,\text{kg})} = 1.07\,\text{m}$$

and

$$y_{\rm cm} = \frac{(3\,\text{kg})(0) + (8\,\text{kg})(2\,\text{m}) + (4\,\text{kg})(1\,\text{m})}{(3\,\text{kg}) + (8\,\text{kg}) + (4\,\text{kg})} = 1.33\,\text{m}$$

$\boxed{\textbf{E7-11}}$ The center of mass of the three hydrogen atoms will be at the center of the pyramid base. The problem is then reduced to finding the center of mass of the nitrogen atom and the three hydrogen atom triangle. This molecular center of mass must lie on the dotted line in Fig. 7-27.

The location of the plane of the hydrogen atoms can be found from Pythagoras theorem

$$y_{\rm h} = \sqrt{(10.14 \times 10^{-11}\text{m})^2 - (9.40 \times 10^{-11}\text{m})^2} = 3.8 \times 10^{-11}\text{m}.$$

This distance can be used to find the center of mass of the molecule. From Eq. 7-2,

$$y_{\rm cm} = \frac{m_{\rm n} y_{\rm n} + m_{\rm h} y_{\rm h}}{m_{\rm n} + m_{\rm h}} = \frac{(13.9 m_{\rm h})(0) + (3 m_{\rm h})(3.8 \times 10^{-11}\text{m})}{(13.9 m_{\rm h}) + (3 m_{\rm h})} = 6.75 \times 10^{-12}\text{m}.$$

E7-12 The velocity components of the center of mass at $t = 1.42\,\text{s}$ are $v_{{\rm cm},x} = 7.3\,\text{m/s}$ and $v_{{\rm cm},y} = (10.0\,\text{m/s}) - (9.81\,\text{m/s})(1.42\,\text{s}) = -3.93\,\text{m/s}$. Then the velocity components of the "other" piece are

$$v_{2,x} = [(9.6\,\text{kg})(7.3\,\text{m/s}) - (6.5\,\text{kg})(11.4\,\text{m/s})]/(3.1\,\text{kg}) = -1.30\,\text{m/s}.$$

and

$$v_{2,y} = [(9.6\,\text{kg})(-3.9\,\text{m/s}) - (6.5\,\text{kg})(-4.6\,\text{m/s})]/(3.1\,\text{kg}) = -2.4\,\text{m/s}.$$

E7-13 The center of mass should lie on the perpendicular bisector of the rod of mass $3M$. We can view the system as having two parts: the heavy rod of mass $3M$ and the two light rods each of mass M. The two light rods have a center of mass at the center of the square.

Both of these center of masses are located along the vertical line of symmetry for the object. The center of mass of the heavy bar is at $y_{\rm h,cm} = 0$, while the *combined* center of mass of the two light bars is at $y_{\rm l,cm} = L/2$, where down is positive. The center of mass of the system is then at

$$y_{\rm cm} = \frac{2My_{\rm l,cm} + 3My_{\rm h,cm}}{2M + 3M} = \frac{2(L/2)}{5} = L/5.$$

E7-14 The two slabs have the same volume and have mass $m_i = \rho_i V$. The center of mass is located at

$$x_{\rm cm} = \frac{m_1 l - m_2 l}{m_1 + m_2} = l\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} = (5.5 \text{ cm})\frac{(7.85 \text{ g/cm}^3) - (2.70 \text{ g/cm}^3)}{(7.85 \text{ g/cm}^3) + (2.70 \text{ g/cm}^3)} = 2.68 \text{ cm}$$

from the boundary inside the iron; it is centered in the y and z directions.

E7-15 Treat the four sides of the box as one thing of mass $4m$ with a mass located $l/2$ above the base. Then the center of mass is

$$z_{\rm cm} = (l/2)(4m)/(4m + m) = 2l/5 = 2(0.4\,\text{m})/5 = 0.16\,\text{m},$$

$x_{\rm cm} = y_{\rm cm} = 0.2\,\text{m}$.

E7-16 One piece moves off with momentum $m(31.4\,\text{m/s})\hat{\mathbf{i}}$, another moves off with momentum $2m(31.4\,\text{m/s})\hat{\mathbf{j}}$. The third piece must then have momentum $-m(31.4\,\text{m/s})\hat{\mathbf{i}} - 2m(31.4\,\text{m/s})\hat{\mathbf{j}}$ and velocity $-(1/3)(31.4\,\text{m/s})\hat{\mathbf{i}} - 2/3(31.4\,\text{m/s})\hat{\mathbf{j}} = -10.5\,\text{m/s}\,\hat{\mathbf{i}} - 20.9\,\text{m/s}\,\hat{\mathbf{j}}$. The magnitude of v_3 is $23.4\,\text{m/s}$ and direction 63.3° away from the lighter piece.

E7-17 It will take an impulse of $(84.7\,\text{kg})(3.87\,\text{m/s}) = 328\,\text{kg}\cdot\text{m/s}$ to stop the animal. This would come from firing n bullets where $n = (328\,\text{kg}\cdot\text{m/s})/[(0.0126\,\text{kg})(975\,\text{m/s}) = 27$.

E7-18 Conservation of momentum for firing one cannon ball of mass m with muzzle speed v_c forward out of a cannon on a trolley of original total mass M moving forward with original speed v_0 is

$$Mv_0 = (M - m)v_1 + m(v_c + v_1) = Mv_1 + mv_c,$$

where v_1 is the speed of the trolley after the cannonball is fired. Then to stop the trolley we require n cannonballs be fired so that

$$n = (Mv_0)/(mv_c) = [(3500\,\text{kg})(45\,\text{m/s})]/[(65\,\text{kg})(625\,\text{m/s})] = 3.88,$$

so $n = 4$.

E7-19 Label the velocities of the various containers as $\vec{\mathbf{v}}_k$ where k is an integer between one and twelve. The mass of each container is m. The subscript "g" refers to the goo; the subscript k refers to the kth container.

The total momentum before the collision is given by

$$\vec{\mathbf{P}} = \sum_k m\vec{\mathbf{v}}_{k,\rm i} + m_{\rm g}\vec{\mathbf{v}}_{\rm g,i} = 12m\vec{\mathbf{v}}_{\rm cont.,cm} + m_g\vec{\mathbf{v}}_{\rm g,i}.$$

We are told, however, that the initial velocity of the center of mass of the containers is at rest, so the initial momentum simplifies to $\vec{\mathbf{P}} = m_g \vec{\mathbf{v}}_{\mathrm{g,i}}$, and has a magnitude of 4000 kg·m/s.

(a) Then

$$v_{\mathrm{cm}} = \frac{P}{12m + m_{\mathrm{g}}} = \frac{(4000\,\mathrm{kg\cdot m/s})}{12(100.0\,\mathrm{kg}) + (50\,\mathrm{kg})} = 3.2\,\mathrm{m/s}.$$

(b) It doesn't matter if the cord breaks, we'll get the same answer for the motion of the center of mass.

E7-20 (a) $F = (3270\,\mathrm{m/s})(480\,\mathrm{kg/s}) = 1.57\times10^6\mathrm{N}$.

(b) $m = 2.55\times10^5\mathrm{kg} - (480\,\mathrm{kg}/s)(250\,\mathrm{s}) = 1.35\times10^5\mathrm{kg}$.

(c) Eq. 7-32:

$$v_{\mathrm{f}} = (-3270\,\mathrm{m/s})\ln(1.35\times10^5\mathrm{kg}/2.55\times10^5\mathrm{kg}) = 2080\,\mathrm{m/s}.$$

$\boxed{\textbf{E7-21}}$ Use Eq. 7-32. The initial velocity of the rocket is 0. The mass ratio can then be found from a minor rearrangement;

$$\frac{M_{\mathrm{i}}}{M_{\mathrm{f}}} = e^{|v_{\mathrm{f}}/v_{\mathrm{rel}}|}$$

The "flipping" of the left hand side of this expression is possible because the exhaust velocity is *negative* with respect to the rocket. For part (a) $M_{\mathrm{i}}/M_{\mathrm{f}} = e = 2.72$. For part (b) $M_{\mathrm{i}}/M_{\mathrm{f}} = e^2 = 7.39$.

E7-22 Eq. 7-32 rearranged:

$$\frac{M_{\mathrm{f}}}{M_{\mathrm{i}}} = e^{-|\Delta v/v_{\mathrm{rel}}|} = e^{-(22.6\mathrm{m/s})/(1230\mathrm{m/s})} = 0.982.$$

The fraction of the initial mass which is discarded is 0.0182.

E7-23 The loaded rocket has a weight of $(1.11\times10^5\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1.09\times10^6\mathrm{N}$; the thrust must be at least this large to get the rocket off the ground. Then $v \geq (1.09\times10^6\mathrm{N})/(820\,\mathrm{kg/s}) = 1.33\times10^3\mathrm{m/s}$ is the minimum exhaust speed.

E7-24 The acceleration down the incline is $(9.8\,\mathrm{m/s^2})\sin(26^\circ) = 4.3\,\mathrm{m/s^2}$. It will take $t = \sqrt{2(93\,\mathrm{m})/(4.3\,\mathrm{m/s^2})} = 6.6\,\mathrm{s}$. The sand doesn't affect the problem, so long as it only "leaks" out.

$\boxed{\textbf{E7-25}}$ We'll use Eq. 7-4 to solve this problem, but since we are given *weights* instead of *mass* we'll multiply the top and bottom by g like we did in Exercise 7-7. Then

$$\vec{\mathbf{v}}_{\mathrm{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}\frac{g}{g} = \frac{W_1\vec{\mathbf{v}}_1 + W_2\vec{\mathbf{v}}_2}{W_1 + W_2}.$$

Now for the numbers

$$v_{\mathrm{cm}} = \frac{(9.75\ \mathrm{T})(1.36\,\mathrm{m/s}) + (0.50\ \mathrm{T})(0)}{(9.75\ \mathrm{T}) + (0.50\ \mathrm{T})} = 1.29\,\mathrm{m/s}.$$

P7-1 (a) The balloon moves down so that the center of mass is stationary;

$$0 = Mv_{\mathrm{b}} + mv_{\mathrm{m}} = Mv_{\mathrm{b}} + m(v + v_{\mathrm{b}}),$$

or $v_{\mathrm{b}} = -mv/(m + M)$.

(b) When the man stops so does the balloon.

P7-2 (a) The center of mass is midway between them.

(b) Measure from the heavier mass.

$$x_{\text{cm}} = (0.0560\,\text{m})(0.816\,\text{kg})/(1.700\,\text{kg}) = 0.0269\,\text{m},$$

which is 1.12 mm closer to the heavier mass than in part (a).

(c) Think Atwood's machine. The acceleration of the two masses is

$$a = 2\Delta m\, g/(m_1 + m_2) = 2(0.034\,\text{kg})g/(1.700\,\text{kg}) = 0.0400g,$$

the heavier going down while the lighter moves up. The acceleration of the center of mass is

$$a_{\text{cm}} = (am_1 - am_2)/(m_1 + m_2) = (0.0400g)2(0.034\,\text{kg})g/(1.700\,\text{kg}) = 0.00160g.$$

$\boxed{\textbf{P7-3}}$ This is a glorified Atwood's machine problem. The total mass on the right side is the mass per unit length times the length, $m_r = \lambda x$; similarly the mass on the left is given by $m_l = \lambda(L - x)$. Then

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \frac{\lambda x - \lambda(L - x)}{\lambda x + \lambda(L - x)} g = \frac{2x - L}{L} g$$

which solves the problem. The acceleration is in the direction of the side of length x if $x > L/2$.

P7-4 (a) Assume the car is massless. Then moving the cannonballs is moving the center of mass, unless the cannonballs don't move but instead the car does. How far? L.

(b) Once the cannonballs stop moving so does the rail car.

$\boxed{\textbf{P7-5}}$ By symmetry, the center of mass of the empty storage tank should be in the very center, along the axis at a height $y_{\text{t,cm}} = H/2$. We can pretend that the entire mass of the tank, $m_{\text{t}} = M$, is located at this point.

The center of mass of the gasoline is also, by symmetry, located along the axis at half the height of the gasoline, $y_{\text{g,cm}} = x/2$. The mass, if the tank were filled to a height H, is m; assuming a uniform density for the gasoline, the mass present when the level of gas reaches a height x is $m_g = mx/H$.

(a) The center of mass of the entire system is at the center of the cylinder when the tank is full and when the tank is empty. When the tank is half full the center of mass is below the center. So as the tank changes from full to empty the center of mass drops, reaches some lowest point, and then rises back to the center of the tank.

(b) The center of mass of the entire system is found from

$$y_{\text{cm}} = \frac{m_{\text{g}} y_{\text{g,cm}} + m_{\text{t}} y_{\text{t,cm}}}{m_{\text{g}} + m_{\text{t}}} = \frac{(mx/H)(x/2) + (M)(H/2)}{(mx/H) + (M)} = \frac{mx^2 + MH^2}{2mx + 2MH}.$$

Take the derivative:

$$\frac{dy_{\text{cm}}}{dx} = \frac{m\left(mx^2 + 2xMH - MH^2\right)}{(mx + MH)^2}$$

Set this equal to zero to find the minimum; this means we want the numerator to vanish, or $mx^2 + 2xMH - MH^2 = 0$. Then

$$x = \frac{-M + \sqrt{M^2 + mM}}{m} H.$$

P7-6 The center of mass will be located along symmetry axis. Call this the x axis. Then

$$\begin{aligned} x_{\rm cm} &= \frac{1}{M}\int x dm, \\ &= \frac{4}{\pi R^2}\int_0^R \int_0^{\sqrt{R^2-x^2}} x\, dy\, dx, \\ &= \frac{4}{\pi R^2}\int_0^R x\sqrt{R^2-x^2}\, dx, \\ &= \frac{4}{\pi R^2}R^3/3 = \frac{4R}{3\pi}. \end{aligned}$$

P7-7 (a) The components of the shell velocity with respect to the cannon are

$$v'_x = (556\,\mathrm{m/s})\cos(39.0°) = 432\,\mathrm{m/s} \text{ and } v'_y = (556\,\mathrm{m/s})\sin(39.0°) = 350\,\mathrm{m/s}.$$

The vertical component with respect to the ground is the same, $v_y = v'_y$, but the horizontal component is found from conservation of momentum:

$$M(v_x - v'_x) + m(v_x) = 0,$$

so $v_x = (1400\,\mathrm{kg})(432\,\mathrm{m/s})(70.0\,\mathrm{kg} + 1400\,\mathrm{kg}) = 411\,\mathrm{m/s}$. The resulting speed is $v = 540\,\mathrm{m/s}$.
(b) The direction is $\theta = \arctan(350/411) = 40.4°$.

P7-8 $v = (2870\,\mathrm{kg})(252\,\mathrm{m/s})/(2870\,\mathrm{kg} + 917\,\mathrm{kg}) = 191\,\mathrm{m/s}$.

P7-9 It takes $(1.5\,\mathrm{m/s})(20\,\mathrm{kg}) = 30\,\mathrm{N}$ to accelerate the luggage to the speed of the belt. The people when taking the luggage off will (on average) also need to exert a 30 N force to remove it; this force (because of friction) will be exerted on the belt. So the belt requires 60 N of additional force.

P7-10 (a) The thrust must be at least equal to the weight, so

$$dm/dt = (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/(1170\,\mathrm{m/s}) = 49.1\,\mathrm{kg/s}.$$

(b) The net force on the rocket will need to be $F = (5860\,\mathrm{kg})(18.3\,\mathrm{m/s^2}) = 107000\,\mathrm{N}$. Add this to the weight to find the thrust, so

$$dm/dt = [107000\,\mathrm{N} + (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})]/(1170\,\mathrm{m/s}) = 141\,\mathrm{kg/s}$$

$\boxed{\textbf{P7-11}}$ Consider Eq. 7-31. We want the barges to continue at constant speed, so the left hand side of that equation vanishes. Then

$$\sum \vec{\mathbf{F}}_{\rm ext} = -\vec{\mathbf{v}}_{\rm rel}\frac{dM}{dt}.$$

We are told that the frictional force is independent of the weight, since the speed doesn't change the frictional force should be constant and equal in magnitude to the force exerted by the engine *before* the shoveling happens. Then $\sum \vec{\mathbf{F}}_{\rm ext}$ is equal to the additional force required from the engines. We'll call it $\vec{\mathbf{P}}$.

The relative speed of the coal to the faster moving cart has magnitude: $21.2 - 9.65 = 11.6$ km/h= 3.22 m/s. The mass flux is 15.4 kg/s, so $P = (3.22\ \mathrm{m/s})(15.4\mathrm{kg/s}) = 49.6$ N. The faster moving cart will need to *increase* the engine force by 49.6 N. The slower cart won't need to do anything, because the coal left the slower barge with a relative speed of *zero* according to our approximation.

P7-12 (a) Nothing is ejected from the string, so $v_{\rm rel} = 0$. Then Eq. 7-31 reduces to $m\,dv/dt = F_{\rm ext}$.

(b) Since $F_{\rm ext}$ is from the weight of the hanging string, and the fraction that is hanging is y/L, $F_{\rm ext} = mgy/L$. The equation of motion is then $d^2y/dt^2 = gy/L$.

(c) Take first derivative:

$$\frac{dy}{dt} = \frac{y_0}{2}(\sqrt{g/L})\left(e^{\sqrt{g/L}t} - e^{-\sqrt{g/L}t}\right),$$

and then second derivative,

$$\frac{d^2y}{dt^2} = \frac{y_0}{2}(\sqrt{g/L})^2\left(e^{\sqrt{g/L}t} + e^{-\sqrt{g/L}t}\right).$$

Substitute into equation of motion. It works! Note that when $t = 0$ we have $y = y_0$.

$\boxed{\textbf{E8-1}}$ An n-dimensional object can be oriented by stating the position of n different *carefully chosen* points P_i inside the body. Since each point has n coordinates, one might think there are n^2 coordinates required to completely specify the position of the body. But if the body is rigid then the distances between the points are fixed. There is a distance d_{ij} for every pair of points P_i and P_j. For each distance d_{ij} we need one fewer coordinate to specify the position of the body. There are $n(n-1)/2$ ways to connect n objects in pairs, so $n^2 - n(n-1)/2 = n(n+1)/2$ is the number of coordinates required.

E8-2 (1 rev/min)(2π rad/rev)/(60 s/min) = 0.105 rad/s.

E8-3 (a) $\omega = a + 3bt^2 - 4ct^3$.
(b) $\alpha = 6bt - 12t^2$.

E8-4 (a) The radius is $r = (2.3\times10^4\text{ly})(3.0\times10^8\text{m/s}) = 6.9\times10^{12}\text{m}\cdot\text{y/s}$. The time to make one revolution is $t = (2\pi 6.9\times10^{12}\text{m}\cdot\text{y/s})/(250\times10^3\text{m/s}) = 1.7\times10^8\text{y}$.
(b) The Sun has made $4.5\times10^9\text{y}/1.7\times10^8\text{y} = 26$ revolutions.

$\boxed{\textbf{E8-5}}$ (a) Integrate.

$$\omega_z = \omega_0 + \int_0^t \left(4at^3 - 3bt^2\right) dt = \omega_0 + at^4 - bt^3$$

(b) Integrate, again.

$$\Delta\theta = \int_0^t \omega_z dt = \int_0^t \left(\omega_0 + at^4 - bt^3\right) dt = \omega_0 t + \frac{1}{5}at^5 - \frac{1}{4}bt^4$$

E8-6 (a) (1 rev/min)(2π rad/rev)/(60 s/min) = 0.105 rad/s.
(b) (1 rev/h)(2π rad/rev)/(3600 s/h) = 1.75×10^{-3} rad/s.
(c) (1/12 rev/h)(2π rad/rev)/(3600 s/h) = 1.45×10^{-3} rad/s.

E8-7 85 mi/h = 125 ft/s. The ball takes t = (60 ft)/(125 ft/s) = 0.48 s to reach the plate. It makes (30 rev/s)(0.48 s) = 14 revolutions in that time.

E8-8 It takes $t = \sqrt{2(10\,\text{m})/(9.81\,\text{m/s}^2)} = 1.43\,\text{s}$ to fall 10 m. The average angular velocity is then $\omega = (2.5)(2\pi\text{ rad})/(1.43\,\text{s}) = 11$ rad/s.

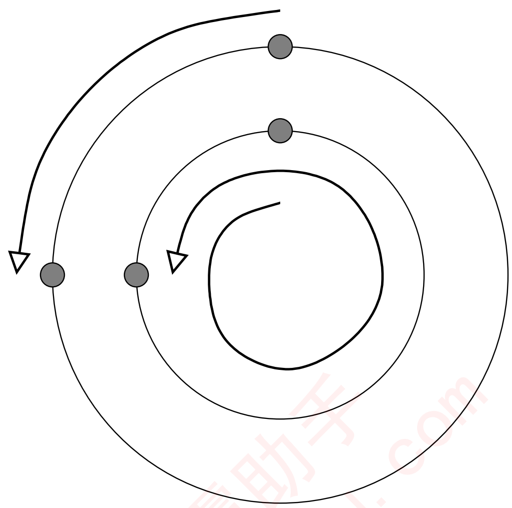
$\boxed{\textbf{E8-9}}$ (a) Since there are eight spokes, this means the wheel can make no more than 1/8 of a revolution while the arrow traverses the plane of the wheel. The wheel rotates at 2.5 rev/s; it makes one revolution every $1/2.5 = 0.4$ s; so the arrow must pass through the wheel in less than $0.4/8 = 0.05$ s.

The arrow is 0.24 m long, and it must move at least one arrow length in 0.05 s. The corresponding minimum speed is (0.24 m)/(0.05 s) = 4.8 m/s.

(b) It does not matter where you aim, because the wheel is rigid. It is the angle through which the spokes have turned, not the distance, which matters here.

E8-10 We look for the times when the Sun, the Earth, and the other planet are collinear in some specified order.

Since the outer planets revolve around the Sun more slowly than Earth, after one year the Earth has returned to the original position, but the outer planet has completed *less* than one revolution. The Earth will then "catch up" with the outer planet *before* the planet has completed a revolution. If θ_E is the angle through which Earth moved and θ_P is the angle through which the planet moved, then $\theta_E = \theta_P + 2\pi$, since the Earth completed one more revolution than the planet.



If ω_P is the angular velocity of the planet, then the angle through which it moves during the time T_S (the time for the planet to line up with the Earth). Then

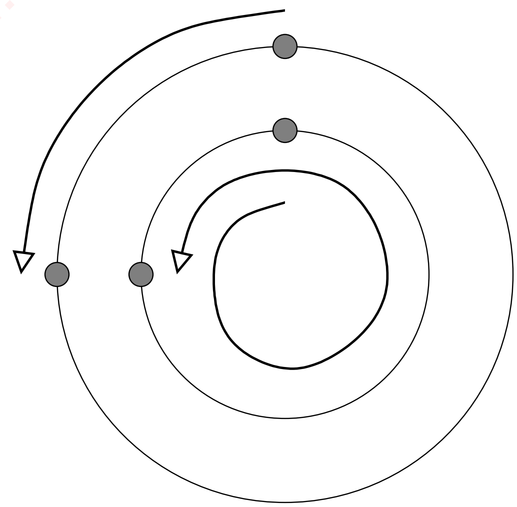
$$\begin{aligned} \theta_E &= \theta_P + 2\pi, \\ \omega_E T_S &= \omega_P T_S + 2\pi, \\ \omega_E &= \omega_P + 2\pi/T_S \end{aligned}$$

The angular velocity of a planet is $\omega = 2\pi/T$, where T is the period of revolution. Substituting this into the last equation above yields

$$1/T_E = 1/T_P + 1/T_S.$$

$\boxed{\textbf{E8-11}}$ We look for the times when the Sun, the Earth, and the other planet are collinear in some specified order.

Since the inner planets revolve around the Sun more quickly than Earth, after one year the Earth has returned to the original position, but the inner planet has completed *more* than one revolution. The inner planet must then have "caught-up" with the Earth *before* the Earth has completed a revolution. If θ_E is the angle through which Earth moved and θ_P is the angle through which the planet moved, then $\theta_P = \theta_E + 2\pi$, since the inner planet completed one more revolution than the Earth.



If ω_P is the angular velocity of the planet, then the angle through which it moves during the time T_S (the time for the planet to line up with the Earth). Then

$$\begin{aligned}\theta_P &= \theta_E + 2\pi,\\ \omega_P T_S &= \omega_E T_S + 2\pi,\\ \omega_P &= \omega_E + 2\pi/T_S\end{aligned}$$

The angular velocity of a planet is $\omega = 2\pi/T$, where T is the period of revolution. Substituting this into the last equation above yields

$$1/T_P = 1/T_E + 1/T_S.$$

E8-12 (a) $\alpha = (-78 \text{ rev/min})/(0.533 \text{ min}) = -150 \text{ rev/min}^2$.

(b) Average angular speed while slowing down is 39 rev/min, so (39 rev/min)(0.533 min) = 21 rev.

E8-13 (a) $\alpha = (2880 \text{ rev/min} - 1170 \text{ rev/min})/(0.210 \text{ min}) = 8140 \text{ rev/min}^2$.

(b) Average angular speed while accelerating is 2030 rev/min, so (2030 rev/min)(0.210 min) = 425 rev.

E8-14 Find area under curve.

$$\frac{1}{2}(5 \text{ min} + 2.5 \text{ min})(3000 \text{ rev/min}) = 1.13\times 10^4 \text{ rev}.$$

E8-15 (a) $\omega_{0z} = 25.2$ rad/s; $\omega_z = 0$; $t = 19.7$ s; and α_z and ϕ are unknown. From Eq. 8-6,

$$\begin{aligned}\omega_z &= \omega_{0z} + \alpha_z t,\\ (0) &= (25.2 \text{ rad/s}) + \alpha_z(19.7 \text{ s}),\\ \alpha_z &= -1.28 \text{ rad/s}^2\end{aligned}$$

(b) We use Eq. 8-7 to find the angle through which the wheel rotates.

$$\phi = \phi_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (0) + (25.2 \text{ rad/s})(19.7 \text{ s}) + \frac{1}{2}(-1.28 \text{ rad/s}^2)(19.7 \text{ s})^2 = 248 \text{ rad}.$$

(c) $\phi = 248 \text{ rad} \frac{1 \text{ rev}}{2\pi \text{ rad}} = 39.5$ rev.

E8-16 (a) $\alpha = (225 \text{ rev/min} - 315 \text{ rev/min})/(1.00 \text{ min}) = -90.0 \text{ rev/min}^2$.
(b) $t = (0 - 225 \text{ rev/min})/(-90.0 \text{ rev/min}^2) = 2.50$ min.
(c) $(-90.0 \text{ rev/min}^2)(2.50 \text{ min})^2/2 + (225 \text{ rev/min})(2.50 \text{ min}) = 281$ rev.

E8-17 (a) The average angular speed was $(90 \text{ rev})/(15\,\text{s}) = 6.0$ rev/s. The angular speed at the beginning of the interval was then $2(6.0 \text{ rev/s}) - (10 \text{ rev/s}) = 2.0$ rev/s.
(b) The angular acceleration was $(10 \text{ rev/s} - 2.0 \text{ rev/s})/(15\,\text{s}) = 0.533 \text{ rev/s}^2$. The time required to get the wheel to 2.0 rev/s was $t = (2.0 \text{ rev/s})/(0.533 \text{ rev/s}^2) = 3.8\,\text{s}$.

E8-18 (a) The wheel will rotate through an angle ϕ where

$$\phi = (563 \text{ cm})/(8.14 \text{ cm}/2) = 138 \text{ rad}.$$

(b) $t = \sqrt{2(138 \text{ rad})/(1.47 \text{ rad/s}^2)} = 13.7\,\text{s}$.

$\boxed{\textbf{E8-19}}$ (a) We are given $\phi = 42.3$ rev$= 266$ rad, $\omega_{0z} = 1.44$ rad/s, and $\omega_z = 0$. Assuming a uniform deceleration, the average angular velocity during the interval is

$$\omega_{\text{av},z} = \frac{1}{2}(\omega_{0z} + \omega_z) = 0.72 \text{ rad/s}.$$

Then the time taken for deceleration is given by $\phi = \omega_{\text{av},z} t$, so $t = 369$ s.
(b) The angular acceleration can be found from Eq. 8-6,

$$\begin{aligned} \omega_z &= \omega_{0z} + \alpha_z t, \\ (0) &= (1.44 \text{ rad/s}) + \alpha_z (369 \text{ s}), \\ \alpha_z &= -3.9 \times 10^{-3} \text{ rad/s}^2. \end{aligned}$$

(c) We'll solve Eq. 8-7 for t,

$$\begin{aligned} \phi &= \phi_0 + \omega_{0z} t + \frac{1}{2}\alpha_z t^2, \\ (133 \text{ rad}) &= (0) + (1.44 \text{ rad/s})t + \frac{1}{2}(-3.9 \times 10^{-3} \text{ rad/s}^2)t^2, \\ 0 &= -133 + (1.44\,\text{s}^{-1})t - (-1.95 \times 10^{-3}\text{s}^{-2})t^2. \end{aligned}$$

Solving this quadratic expression yields two answers: $t = 108$ s and $t = 630$ s.

E8-20 The angular acceleration is $\alpha = (4.96 \text{ rad/s})/(2.33\,\text{s}) = 2.13 \text{ rad/s}^2$. The angle through which the wheel turned while accelerating is $\phi = (2.13 \text{ rad/s}^2)(23.0\,\text{s})^2/2 = 563$ rad. The angular speed at this time is $\omega = (2.13 \text{ rad/s}^2)(23.0\,\text{s}) = 49.0$ rad/s. The wheel spins through an additional angle of $(49.0 \text{ rad/s})(46\,\text{s} - 23\,\text{s}) = 1130$ rad, for a total angle of 1690 rad.

E8-21 $\omega = (14.6\,\text{m/s})/(110\,\text{m}) = 0.133$ rad/s.

E8-22 The linear acceleration is $(25\,\text{m/s} - 12\,\text{m/s})/(6.2\,\text{s}) = 2.1\,\text{m/s}^2$. The angular acceleration is $\alpha = (2.1\,\text{m/s}^2)/(0.75\,\text{m}/2) = 5.6$ rad/s.

$\boxed{\textbf{E8-23}}$ (a) The angular speed is given by $v_T = \omega r$. So $\omega = v_T/r = (28,700 \text{ km/hr})/(3220 \text{ km}) = 8.91$ rad/hr. That's the same thing as 2.48×10^{-3} rad/s.

(b) $a_R = \omega^2 r = (8.91 \text{ rad/h})^2(3220 \text{ km}) = 256000 \text{ km/h}^2$, or

$$a_R = 256000 \text{ km/h}^2(1/3600 \text{ h/s})^2(1000 \text{ m/km}) = 19.8\,\text{m/s}^2.$$

(c) If the speed is constant then the tangential acceleration is zero, regardless of the shape of the trajectory!

E8-24 The bar needs to make

$$(1.50 \text{ cm})(12.0 \text{ turns/cm}) = 18 \text{ turns}.$$

This will happen is $(18 \text{ rev})/(237 \text{ rev/min}) = 4.56\,\text{s}$.

E8-25 (a) The angular speed is $\omega = (2\pi \text{ rad})/(86400\,\text{s}) = 7.27\times10^{-5}$ rad/s.

(b) The distance from the polar axis is $r = (6.37\times10^6\text{m})\cos(40^\circ) = 4.88\times10^6\text{m}$. The linear speed is then $v = (7.27\times10^{-5} \text{ rad/s})(4.88\times10^6\text{m}) = 355\,\text{m/s}$.

(c) The angular speed is the same as part (a). The distance from the polar axis is $r = (6.37\times10^6\text{m})\cos(0^\circ) = 6.37\times10^6\text{m}$. The linear speed is then $v = (7.27\times10^{-5} \text{ rad/s})(6.37\times10^6\text{m}) = 463\,\text{m/s}$.

E8-26 (a) $a_\text{T} = (14.2 \text{ rad/s}^2)(0.0283\,\text{m}) = 0.402\,\text{m/s}^2$.

(b) Full speed is $\omega = 289$ rad/s. $a_\text{R} = (289 \text{ rad/s})^2(0.0283\,\text{m}) = 2360\,\text{m/s}^2$.

(c) It takes

$$t = (289 \text{ rad/s})/(14.2 \text{ rad/s}^2) = 20.4\,\text{s}$$

to get up to full speed. Then $x = (0.402\,\text{m/s}^2)(20.4\,\text{s})^2/2 = 83.6\,\text{m}$ is the distance through which a point on the rim moves.

$\boxed{\textbf{E8-27}}$ (a) The pilot sees the propeller rotate, no more. So the tip of the propeller is moving with a tangential velocity of $v_T = \omega r = (2000 \text{ rev/min})(2\pi \text{ rad/rev})(1.5 \text{ m}) = 18900$ m/min. This is the same thing as 315 m/s.

(b) The observer on the ground sees this tangential motion and sees the forward motion of the plane. These two velocity components are perpendicular, so the magnitude of the sum is $\sqrt{(315\,\text{m/s})^2 + (133\,\text{m/s})^2} = 342\,\text{m/s}$.

E8-28 $a_\text{T} = a_\text{R}$ when $r\alpha = r\omega^2 = r(\alpha t)^2$, or $t = \sqrt{1/(0.236 \text{ rad/s}^2)} = 2.06\,\text{s}$.

E8-29 (a) $a_\text{R} = r\omega^2 = r\alpha^2 t^2$.

(b) $a_\text{T} = r\alpha$.

(c) Since $a_\text{R} = a_\text{T}\tan(57.0^\circ)$, $t = \sqrt{\tan(57.0^\circ)/\alpha}$. Then

$$\phi = \frac{1}{2}\alpha t^2 = \frac{1}{2}\tan(57.0^\circ) = 0.77 \text{ rad} = 44.1^\circ.$$

E8-30 (a) The tangential speed of the edge of the wheel relative axle is $v = 27\,\text{m/s}$. $\omega = (27\,\text{m/s})/(0.38\,\text{m}) = 71$ rad/s.

(b) The average angular speed while slowing is 71 rad/s/2, the time required to stop is then $t = (30\times2\pi \text{ rad})/(71 \text{ rad/s}/2) = 5.3\,\text{s}$. The angular acceleration is then $\alpha = (-71 \text{ rad/s})/(5.3\,\text{s}) = -13$ rad/s.

(c) The car moves forward $(27\,\text{m/s}/2)(5.3\,\text{s}) = 72\,\text{m}$.

E8-31 Yes, the speed would be wrong. The angular velocity of the small wheel would be $\omega = v_\mathrm{t}/r_\mathrm{s}$, but the reported velocity would be $v = \omega r_\mathrm{l} = v_\mathrm{t} r_\mathrm{l}/r_\mathrm{s}$. This would be in error by a fraction

$$\frac{\Delta v}{v_\mathrm{t}} = \frac{(72 \text{ cm})}{(62 \text{ cm})} - 1 = 0.16.$$

E8-32 (a) Square both equations and then add them:

$$x^2 + y^2 = (R\cos\omega t)^2 + (R\sin\omega t)^2 = R^2,$$

which is the equation for a circle of radius R.

(b) $v_x = -R\omega \sin\omega t = -\omega y$; $v_y = R\omega\cos\omega t = \omega x$. Square and add, $v = \omega R$. The direction is tangent to the circle.

(b) $a_x = -R\omega^2 \cos\omega t = -\omega^2 x$; $a_y = -R\omega^2 \sin\omega t = -\omega^2 y$. Square and add, $a = \omega^2 R$. The direction is toward the center.

$\boxed{\textbf{E8-33}}$ (a) The object is "slowing down", so $\vec{\alpha} = (-2.66 \text{ rad/s}^2)\hat{\mathbf{k}}$. We know the direction because it is rotating about the z axis and we are given the direction of $\vec{\omega}$. Then from Eq. 8-19, $\vec{\mathbf{v}} = \vec{\omega} \times \vec{\mathbf{R}} = (14.3 \text{ rad/s})\hat{\mathbf{k}} \times [(1.83 \text{ m})\hat{\mathbf{j}} + (1.26 \text{ m})\hat{\mathbf{k}}]$. But only the cross term $\hat{\mathbf{k}} \times \hat{\mathbf{j}}$ survives, so $\vec{\mathbf{v}} = (-26.2\,\text{m/s})\hat{\mathbf{i}}$.

(b) We find the acceleration from Eq. 8-21,

$$\begin{aligned}
\vec{\mathbf{a}} &= \vec{\alpha} \times \vec{\mathbf{R}} + \vec{\omega} \times \vec{\mathbf{v}}, \\
&= (-2.66 \text{ rad/s}^2)\hat{\mathbf{k}} \times [(1.83\,\text{m})\hat{\mathbf{j}} + (1.26\,\text{m})\hat{\mathbf{k}}] + (14.3 \text{ rad/s})\hat{\mathbf{k}} \times (-26.2\,\text{m/s})\hat{\mathbf{i}}, \\
&= (4.87\,\text{m/s}^2)\hat{\mathbf{i}} + (-375\,\text{m/s}^2)\hat{\mathbf{j}}.
\end{aligned}$$

E8-34 (a) $\vec{\mathbf{F}} = -2m\vec{\omega} \times \vec{\mathbf{v}} = -2m\omega v\cos\theta$, where θ is the latitude. Then

$$F = 2(12\,\text{kg})(2\pi \text{ rad}/86400\,\text{s})(35\,\text{m/s})\cos(45^\circ) = 0.043\,\text{N},$$

and is directed *west*.

(b) Reversing the velocity will reverse the direction, so *east*.

(c) No. The Coriolis force pushes it to the west on the way up and gives it a westerly velocity; on the way down the Coriolis force slows down the westerly motion, but does not push it back east. The object lands to the west of the starting point.

P8-1 (a) $\omega = (4.0 \text{ rad/s}) - (6.0 \text{ rad/s}^2)t + (3.0 \text{ rad/s})t^2$. Then $\omega(2.0\,\text{s}) = 4.0$ rad/s and $\omega(4.0\,\text{s}) = 28.0$ rad/s.

(b) $\alpha_\mathrm{av} = (28.0 \text{ rad/s} - 4.0 \text{ rad/s})/(4.0\,\text{s} - 2.0\,\text{s}) = 12 \text{ rad/s}^2$.

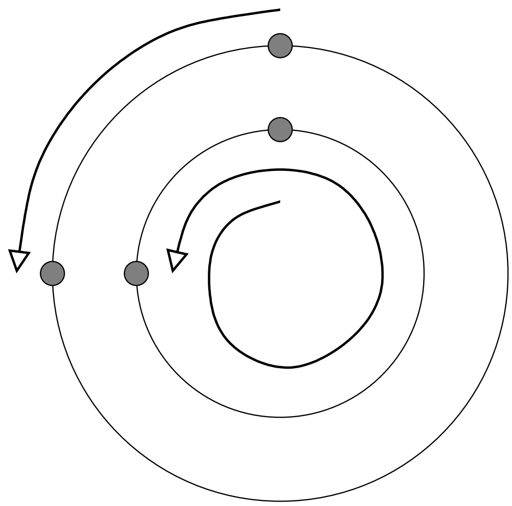
(c) $\alpha = -(6.0 \text{ rad/s}^2) + (6.0 \text{ rad/s})t$. Then $\alpha(2.0\,\text{s}) = 6.0 \text{ rad/s}^2$ and $\alpha(4.0\,\text{s}) = 18.0 \text{ rad/s}^2$.

P8-2 If the wheel really does move counterclockwise at 4.0 rev/min, then it turns through

$$(4.0 \text{ rev/min})/[(60 \text{ s/min})(24 \text{ frames/s})] = 2.78\times10^{-3} \text{ rev/frame}.$$

This means that a spoke has moved 2.78×10^{-3} rev. There are 16 spokes each located 1/16 of a revolution around the wheel. If instead of moving counterclockwise the wheel was instead moving clockwise so that a different spoke had moved $1/16 \text{ rev} - 2.78\times10^{-3} \text{ rev} = 0.0597$ rev, then the same effect would be present. The wheel then would be turning clockwise with a speed of $\omega = (0.0597 \text{ rev})(60 \text{ s/min})(24 \text{ frames/s}) = 86$ rev/min.

P8-3 (a) In the diagram below the Earth is shown at two locations a day apart. The Earth rotates clockwise in this figure.



Note that the Earth rotates through 2π rad in order to be correctly oriented for a complete sidereal day, but because the Earth has moved in the orbit it needs to go farther through an angle θ in order to complete a solar day. By the time the Earth has gone all of the way around the sun the total angle θ will be 2π rad, which means that there was one more sidereal day than solar day.

(b) There are (365.25 d)(24.000 h/d) $= 8.7660\times10^3$ hours in a year with 265.25 solar days. But there are 366.25 sidereal days, so each one has a length of $8.7660\times10^3/366.25 = 23.934$ hours, or 23 hours and 56 minutes and 4 seconds.

P8-4 (a) The period is time per complete rotation, so $\omega = 2\pi/T$.

(b) $\alpha = \Delta\omega/\Delta t$, so

$$\begin{aligned}\alpha &= \left(\frac{2\pi}{T_0+\Delta T} - \frac{2\pi}{T_0}\right)/(\Delta t),\\ &= \frac{2\pi}{\Delta t}\left(\frac{-\Delta T}{T_0(T_0+\Delta T)}\right),\\ &\approx \frac{2\pi}{\Delta t}\frac{-\Delta T}{T_0^2},\\ &= \frac{2\pi}{(3.16\times10^7\text{s})}\frac{-(1.26\times10^{-5}\text{s})}{(0.033\,\text{s})^2} = -2.30\times10^{-9}\text{rad/s}^2.\end{aligned}$$

(c) $t = (2\pi/0.033\,\text{s})/(2.30\times10^{-9}\text{rad/s}^2) = 8.28\times10^{10}\text{s}$, or 2600 years.

(d) $2\pi/T_0 = 2\pi/T - \alpha t$, or

$$T_0 = \Big(1/(0.033\,\text{s}) - (-2.3\times10^{-9}\text{rad/s}^2)(3.0\times10^{10}\,\text{s})/(2\pi)\Big)^{-1} = 0.024\,\text{s}.$$

$\boxed{\textbf{P8-5}}$ The final angular velocity during the acceleration phase is $\omega_z = \alpha_z t$ = (3.0 rad/s)(4.0 s) = 12.0 rad/s. Since both the acceleration and deceleration phases are uniform with endpoints $\omega_z = 0$, the average angular velocity for both phases is the same, and given by half of the maximum: $\omega_{\text{av},z} = 6.0$ rad/s.

The angle through which the wheel turns is then

$$\phi = \omega_{\text{av},z} t = (6.0\ \text{rad/s})(4.1\,\text{s}) = 24.6\ \text{rad}.$$

The time is the total for *both* phases.

(a) The first student sees the wheel rotate through the smallest angle less than one revolution; this student would have no idea that the disk had rotated more than once. Since the disk moved through 3.92 revolutions, the first student will either assume the disk moved forward through 0.92 revolutions or backward through 0.08 revolutions.

(b) According to whom? We've already answered from the perspective of the second student.

P8-6 $\omega = (0.652\ \text{rad/s}^2)t$ and $\alpha = (0.652\ \text{rad/s}^2)$.

(a) $\omega = (0.652\ \text{rad/s}^2)(5.60\,\text{s}) = 3.65\ \text{rad/s}$

(b) $v_\text{T} = \omega r = (3.65\ \text{rad/s})(10.4\,\text{m}) = 38\,\text{m/s}$.

(c) $a_\text{T} = \alpha r = (0.652\ \text{rad/s}^2)(10.4\,\text{m}) = 6.78\,\text{m/s}^2$.

(d) $a_\text{R} = \omega^2 r = (3.65\ \text{rad/s})^2(10.4\,\text{m}) = 139\,\text{m/s}^2$.

P8-7 (a) $\omega = (2\pi\ \text{rad})/(3.16\times10^7\,\text{s}) = 1.99\times10^{-7}\ \text{rad/s}$.

(b) $v_\text{T} = \omega R = (1.99\times10^{-7}\ \text{rad/s})(1.50\times10^{11}\text{m}) = 2.99\times10^4\text{m/s}$.

(c) $a_\text{R} = \omega^2 R = (1.99\times10^{-7}\ \text{rad/s})^2(1.50\times10^{11}\text{m}) = 5.94\times10^{-3}\text{m/s}^2$.

P8-8 (a) $\alpha = (-156\ \text{rev/min})/(2.2\times60\ \text{min}) = -1.18\ \text{rev/min}^2$.

(b) The average angular speed while slowing down is 78 rev/min, so the wheel turns through $(78\ \text{rev/min})(2.2\times60\ \text{min}) = 10300$ revolutions.

(c) $a_\text{T} = (2\pi\ \text{rad/rev})(-1.18\ \text{rev/min}^2)(0.524\,\text{m}) = -3.89\ \text{m/min}^2$. That's the same as $-1.08\times10^{-3}\text{m/s}^2$.

(d) $a_\text{R} = (2\pi\ \text{rad/rev})(72.5\ \text{rev/min})^2(0.524\,\text{m}) = 1.73\times10^4\ \text{m/min}^2$. That's the same as 4.81m/s^2. This is so much larger than the a_T term that the magnitude of the total linear acceleration is simply 4.81m/s^2.

$\boxed{\textbf{P8-9}}$ (a) There are 500 teeth (and 500 spaces between these teeth); so disk rotates $2\pi/500$ rad between the outgoing light pulse and the incoming light pulse. The light traveled 1000 m, so the elapsed time is $t = (1000\,\text{m})/(3\times10^8\ \text{m/s}) = 3.33\times10^{-6}\text{s}$.

Then the angular speed of the disk is $\omega_z = \phi/t = 1.26\times10^{-2}\ \text{rad})/(3.33\times10^{-6}\text{s}) = 3800\ \text{rad/s}$.

(b) The linear speed of a point on the edge of the would be

$$v_T = \omega R = (3800\ \text{rad/s})(0.05\,\text{m}) = 190\,\text{m/s}.$$

P8-10 The linear acceleration of the belt is $a = \alpha_A r_A$. The angular acceleration of C is $\alpha_C = a/r_C = \alpha_A(r_A/r_C)$. The time required for C to get up to speed is

$$t = \frac{(2\pi\ \text{rad/rev})(100\ \text{rev/min})(1/60\ \text{min/s})}{(1.60\ \text{rad/s}^2)(10.0/25.0)} = 16.4\,\text{s}.$$

P8-11 (a) The final angular speed is $\omega_\text{o} = (130\ \text{cm/s})/(5.80\ \text{cm}) = 22.4\ \text{rad/s}$.

(b) The recording area is $\pi({R_\text{o}}^2 - {R_\text{i}}^2)$, the recorded track has a length l and width w, so

$$l = \frac{\pi[(5.80\ \text{cm})^2 - (2.50\ \text{cm}^2)]}{(1.60\times10^{-4}\ \text{cm})} = 5.38\times10^5\ \text{cm}.$$

(c) Playing time is $t = (5.38\times10^5\ \text{cm})/(130\ \text{cm/s}) = 4140\,\text{s}$, or 69 minutes.

P8-12 The angular position is given by $\phi = \arctan(vt/b)$. The derivative (Maple!) is

$$\omega = \frac{vb}{b^2 + v^2t^2},$$

and is directed *up*. Take the derivative again,

$$\alpha = \frac{2bv^3t}{(b^2 + v^2t^2)^2},$$

but is directed down.

$\boxed{\textbf{P8-13}}$ (a) Let the rocket sled move along the line $x = b$. The observer is at the origin and sees the rocket move with a constant angular speed, so the angle made with the x axis increases according to $\theta = \omega t$. The observer, rocket, and starting point form a right triangle; the position y of the rocket is the opposite side of this triangle, so

$$\tan\theta = y/b \text{ implies } y = b/\tan\omega t.$$

We want to take the derivative of this with respect to time and get

$$v(t) = \omega b/\cos^2(\omega t).$$

(b) The speed becomes infinite (which is clearly unphysical) when $t = \pi/2\omega$.

$\boxed{\textbf{E9-1}}$ (a) First, $\vec{\mathbf{F}} = (5.0\text{ N})\hat{\mathbf{i}}$.

$$\begin{aligned}\vec{\tau} &= [yF_z - zF_y]\hat{\mathbf{i}} + [zF_x - xF_z]\hat{\mathbf{j}} + [xF_y - yF_x]\hat{\mathbf{k}},\\ &= [y(0) - (0)(0)]\hat{\mathbf{i}} + [(0)F_x - x(0)]\hat{\mathbf{j}} + [x(0) - yF_x]\hat{\mathbf{k}},\\ &= [-yF_x]\hat{\mathbf{k}} = -(3.0\text{ m})(5.0\text{ N})\hat{\mathbf{k}} = -(15.0\text{ N}\cdot\text{m})\hat{\mathbf{k}}.\end{aligned}$$

(b) Now $\vec{\mathbf{F}} = (5.0\text{ N})\hat{\mathbf{j}}$. Ignoring all zero terms,

$$\vec{\tau} = [xF_y]\hat{\mathbf{k}} = (2.0\text{ m})(5.0\text{ N})\hat{\mathbf{k}} = (10\text{ N}\cdot\text{m})\hat{\mathbf{k}}.$$

(c) Finally, $\vec{\mathbf{F}} = (-5.0\text{ N})\hat{\mathbf{i}}$.

$$\vec{\tau} = [-yF_x]\hat{\mathbf{k}} = -(3.0\text{ m})(-5.0\text{ N})\hat{\mathbf{k}} = (15.0\text{ N}\cdot\text{m})\hat{\mathbf{k}}.$$

E9-2 (a) Everything is in the plane of the page, so the net torque will either be directed normal to the page. Let out be positive, then the net torque is $\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2$.

(b) $\tau = (1.30\,\text{m})(4.20\,\text{N})\sin(75.0^\circ) - (2.15\,\text{m})(4.90\,\text{N})\sin(58.0^\circ) = -3.66\text{N}\cdot\text{m}$.

E9-4 Everything is in the plane of the page, so the net torque will either be directed normal to the page. Let out be positive, then the net torque is

$$\tau = (8.0\,\text{m})(10\,\text{N})\sin(45^\circ) - (4.0\,\text{m})(16\,\text{N})\sin(90^\circ) + (3.0\,\text{m})(19\,\text{N})\sin(20^\circ) = 12\text{N}\cdot\text{m}.$$

$\boxed{\textbf{E9-5}}$ Since $\vec{\mathbf{r}}$ and $\vec{\mathbf{s}}$ lie in the xy plane, then $\vec{\mathbf{t}} = \vec{\mathbf{r}} \times \vec{\mathbf{s}}$ must be perpendicular to that plane, and can only point along the z axis.

The angle between $\vec{\mathbf{r}}$ and $\vec{\mathbf{s}}$ is $320^\circ - 85^\circ = 235^\circ$. So $|\vec{\mathbf{t}}| = rs|\sin\theta| = (4.5)(7.3)|\sin(235^\circ)| = 27$.

Now for the direction of $\vec{\mathbf{t}}$. The smaller rotation to bring $\vec{\mathbf{r}}$ into $\vec{\mathbf{s}}$ is through a counterclockwise rotation; the right hand rule would then show that the cross product points along the *positive* z direction.

E9-6 $\vec{\mathbf{a}} = (3.20)[\cos(63.0^\circ)\hat{\mathbf{j}} + \sin(63.0^\circ)\hat{\mathbf{k}}]$ and $\vec{\mathbf{b}} = (1.40)[\cos(48.0^\circ)\hat{\mathbf{i}} + \sin(48.0^\circ)\hat{\mathbf{k}}]$. Then

$$\begin{aligned}\vec{\mathbf{a}} \times \vec{\mathbf{b}} &= (3.20)\cos(63.0^\circ)(1.40)\sin(48.0^\circ)\hat{\mathbf{i}}\\ &\quad +(3.20)\sin(63.0^\circ)(1.40)\cos(48.0^\circ)\hat{\mathbf{j}}\\ &\quad -(3.20)\cos(63.0^\circ)(1.40)\cos(48.0^\circ)\hat{\mathbf{k}}\\ &= 1.51\hat{\mathbf{i}} + 2.67\hat{\mathbf{j}} - 1.36\hat{\mathbf{k}}.\end{aligned}$$

E9-7 $\vec{\mathbf{b}} \times \vec{\mathbf{a}}$ has magnitude $ab\sin\phi$ and points in the negative z direction. It is then perpendicular to $\vec{\mathbf{a}}$, so $\vec{\mathbf{c}}$ has magnitude $a^2b\sin\phi$. The direction of $\vec{\mathbf{c}}$ is perpendicular to $\vec{\mathbf{a}}$ but lies in the plane containing vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. Then it makes an angle $\pi/2 - \phi$ with $\vec{\mathbf{b}}$.

E9-8 (a) In unit vector notation,

$$\begin{aligned}\vec{\mathbf{c}} &= [(-3)(-3) - (-2)(1)]\hat{\mathbf{i}} + [(1)(4) - (2)(-3)]\hat{\mathbf{j}} + [(2)(-2) - (-3)(4)]\hat{\mathbf{k}},\\ &= 11\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 8\hat{\mathbf{k}}.\end{aligned}$$

(b) Evaluate $\arcsin[|\vec{\mathbf{a}} \times b|/(ab)]$, finding magnitudes with the Pythagoras relationship:

$$\phi = \arcsin(16.8)/[(3.74)(5.39)] = 56^\circ.$$

$\boxed{\textbf{E9-9}}$ This exercise is a three dimensional generalization of Ex. 9-1, except nothing is zero.

$$\begin{aligned}
\vec{\tau} &= [yF_z - zF_y]\hat{\mathbf{i}} + [zF_x - xF_z]\hat{\mathbf{j}} + [xF_y - yF_x]\hat{\mathbf{k}}, \\
&= [(-2.0 \text{ m})(4.3 \text{ N}) - (1.6 \text{ m})(-2.4 \text{ N})]\hat{\mathbf{i}} + [(1.6 \text{ m})(3.5 \text{ N}) - (1.5 \text{ m})(4.3 \text{ N})]\hat{\mathbf{j}} \\
&\quad + [(1.5 \text{ m})(-2.4 \text{ N}) - (-2.0 \text{ m})(3.5 \text{ N})]\hat{\mathbf{k}}, \\
&= [-4.8\,\text{N}\!\cdot\!\text{m}]\hat{\mathbf{i}} + [-0.85\,\text{N}\!\cdot\!\text{m}]\hat{\mathbf{j}} + [3.4\,\text{N}\!\cdot\!\text{m}]\hat{\mathbf{k}}.
\end{aligned}$$

E9-10 (a) $\vec{\mathbf{F}} = (2.6\,\text{N})\hat{\mathbf{i}}$, then $\vec{\tau} = (0.85\,\text{m})(2.6\,\text{N})\hat{\mathbf{j}} - (-0.36\,\text{m})(2.6\,\text{N})\hat{\mathbf{k}} = 2.2\,\text{N}\cdot\text{m}\hat{\mathbf{j}} + 0.94\,\text{N}\cdot\text{m}\hat{\mathbf{k}}$.
(b) $\vec{\mathbf{F}} = (-2.6\,\text{N})\hat{\mathbf{k}}$, then $\vec{\tau} = (-0.36\,\text{m})(-2.6\,\text{N})\hat{\mathbf{i}} - (0.54\,\text{m})(-2.6\,\text{N})\hat{\mathbf{j}} = 0.93\,\text{N}\cdot\text{m}\hat{\mathbf{i}} + 1.4\,\text{N}\cdot\text{m}\hat{\mathbf{j}}$.

E9-11 (a) The rotational inertia about an axis through the origin is

$$I = mr^2 = (0.025\,\text{kg})(0.74\,\text{m})^2 = 1.4\times10^{-2}\text{kg}\cdot\text{m}^2.$$

(b) $\alpha = (0.74\,\text{m})(22\,\text{N})\sin(50°)/(1.4\times10^{-2}\text{kg}\cdot\text{m}^2) = 890$ rad/s.

E9-12 (a) $I_0 = (0.052\,\text{kg})(0.27\,\text{m})^2 + (0.035\,\text{kg})(0.45\,\text{m})^2 + (0.024\,\text{kg})(0.65\,\text{m})^2 = 2.1\times10^{-2}\text{kg}\cdot\text{m}^2$.
(b) The center of mass is located at

$$x_{\text{cm}} = \frac{(0.052\,\text{kg})(0.27\,\text{m}) + (0.035\,\text{kg})(0.45\,\text{m}) + (0.024\,\text{kg})(0.65\,\text{m})}{(0.052\,\text{kg}) + (0.035\,\text{kg}) + (0.024\,\text{kg})} = 0.41\,\text{m}.$$

Applying the parallel axis theorem yields $I_{\text{cm}} = 2.1\times10^{-2}\text{kg}\cdot\text{m}^2 - (0.11\,\text{kg})(0.41\,\text{m})^2 = 2.5\times10^{-3}\text{kg}\cdot\text{m}^2$.

$\boxed{\textbf{E9-13}}$ (a) Rotational inertia is additive so long as we consider the inertia about the same axis. We can use Eq. 9-10:

$$I = \sum m_n r_n^2 = (0.075\,\text{kg})(0.42\,\text{m})^2 + (0.030\,\text{kg})(0.65\,\text{m})^2 = 0.026\,\text{kg}\!\cdot\!\text{m}^2.$$

(b) No change.

E9-14 $\vec{\tau} = [(0.42\,\text{m})(2.5\,\text{N}) - (0.65\,\text{m})(3.6\,\text{N})]\hat{\mathbf{k}} = -1.29\,\text{N}\cdot\text{m}\,\hat{\mathbf{k}}$. Using the result from E9-13, $\vec{\alpha} = (-1.29\,\text{N}\cdot\text{m}\,\hat{\mathbf{k}})/(0.026\,\text{kg}\!\cdot\!\text{m}^2) = 50\ \text{rad/s}^2\hat{\mathbf{k}}$. That's clockwise if viewed from above.

E9-15 (a) $F = m\omega^2 r = (110\,\text{kg})(33.5\ \text{rad/s})^2(3.90\,\text{m}) = 4.81\times10^5\text{N}$.
(b) The angular acceleration is $\alpha = (33.5\ \text{rad/s})/(6.70\,\text{s}) = 5.00\ \text{rad/s}^2$. The rotational inertia about the axis of rotation is $I = (110\,\text{kg})(7.80\,\text{m})^2/3 = 2.23\times10^3\text{kg}\cdot\text{m}^2$. $\tau = I\alpha = (2.23\times10^3\text{kg}\cdot\text{m}^2)(5.00\ \text{rad/s}^2) = 1.12\times10^4\text{N}\cdot\text{m}$.

E9-16 We can add the inertias for the three rods together,

$$I = 3\left(\frac{1}{3}ML^2\right) = (240\,\text{kg})(5.20\,\text{m})^2 = 6.49\times10^3\text{kg}\cdot\text{m}^2.$$

$\boxed{\textbf{E9-17}}$ The diagonal distance from the axis through the center of mass and the axis through the edge is $h = \sqrt{(a/2)^2 + (b/2)^2}$, so

$$I = I_{cm} + Mh^2 = \frac{1}{12}M\left(a^2 + b^2\right) + M\left((a/2)^2 + (b/2)^2\right) = \left(\frac{1}{12} + \frac{1}{4}\right)M\left(a^2 + b^2\right).$$

Simplifying, $I = \frac{1}{3}M\left(a^2 + b^2\right)$.

E9-18 $I = I_{cm} + Mh^2 = (0.56\,\text{kg})(1.0\,\text{m})^2/12 + (0.56)(0.30\,\text{m})^2 = 9.7\times10^{-2}\text{kg}\cdot\text{m}^2$.

$\boxed{\textbf{E9-19}}$ For particle one $I_1 = mr^2 = mL^2$; for particle two $I_2 = mr^2 = m(2L)^2 = 4mL^2$. The rotational inertia of the rod is $I_{\text{rod}} = \frac{1}{3}(2M)(2L)^2 = \frac{8}{3}ML^2$. Add the three inertias:

$$I = \left(5m + \frac{8}{3}M\right)L^2.$$

E9-20 (a) $I = MR^2/2 = M(R/\sqrt{2})^2$.

(b) Let I be the rotational inertia. Assuming that k is the radius of a hoop with an equivalent rotational inertia, then $I = Mk^2$, or $k = \sqrt{I/M}$.

E9-21 Note the mistakes in the equation in the part (b) of the exercise text.

(a) $m_n = M/N$.

(b) Each piece has a thickness $t = L/N$, the distance from the end to the nth piece is $x_n = (n-1/2)t = (n-1/2)L/N$. The axis of rotation is the center, so the distance from the center is $r_n = x_n - L/2 = nL/N - (1+1/2N)L$.

(c) The rotational inertia is

$$\begin{aligned}
I &= \sum_{n=1}^{N} m_n r_n^2, \\
&= \frac{ML^2}{N^3}\sum_{n=1}^{N}(n-1/2-N)^2, \\
&= \frac{ML^2}{N^3}\sum_{n=1}^{N}\left(n^2-(2N+1)n+(N+1/2)^2\right), \\
&= \frac{ML^2}{N^3}\left(\frac{N(N+1)(2N+1)}{6}-(2N+1)\frac{N(N+1)}{2}+(N+1/2)^2N\right), \\
&\approx \frac{ML^2}{N^3}\left(\frac{2N^3}{6}-\frac{2N^3}{2}+N^3\right), \\
&= ML^2/3.
\end{aligned}$$

E9-22 $F = (46\,\text{N})(2.6\text{ cm})/(13\text{ cm}) = 9.2\,\text{N}$.

E9-23 Tower topples when center of gravity is no longer above base. Assuming center of gravity is located at the very center of the tower, then when the tower leans 7.0 m then the tower falls. This is 2.5 m farther than the present.

(b) $\theta = \arcsin(7.0\,\text{m}/55\,\text{m}) = 7.3°$.

E9-24 If the torque from the force is sufficient to lift edge the cube then the cube will tip. The net torque about the edge which stays in contact with the ground will be $\tau = Fd - mgd/2$ if F is sufficiently large. Then $F \geq mg/2$ is the minimum force which will cause the cube to tip.

The minimum force to get the cube to slide is $F \geq \mu_s mg = (0.46)mg$. The cube will slide first.

$\boxed{\textbf{E9-25}}$ The ladder slips if the force of static friction required to keep the ladder up exceeds $\mu_s N$. Equations 9-31 give us the normal force in terms of the masses of the ladder and the firefighter, $N = (m + M)g$, and is independent of the location of the firefighter on the ladder. Also from Eq. 9-31 is the relationship between the force from the wall and the force of friction; the condition at which slipping occurs is $F_w \geq \mu_s(m + M)g$.

Now go straight to Eq. 9-32. The $a/2$ in the second term is the location of the firefighter, who in the example was halfway between the base of the ladder and the top of the ladder. In the exercise we don't know where the firefighter is, so we'll replace $a/2$ with x. Then

$$-F_w h + Mgx + \frac{mga}{3} = 0$$

is an expression for rotational equilibrium. Substitute in the condition of F_w when slipping just starts, and we get

$$-\left(\mu_s(m + M)g\right)h + Mgx + \frac{mga}{3} = 0.$$

Solve this for x,

$$x = \mu_s\left(\frac{m}{M} + 1\right)h - \frac{ma}{3M} = (0.54)\left(\frac{45\text{ kg}}{72\text{ kg}} + 1\right)(9.3\text{ m}) - \frac{(45\text{ kg})(7.6\text{ m})}{3(72\text{ kg})} = 6.6\text{ m}$$

This is the horizontal distance; the fraction of the total length along the ladder is then given by $x/a = (6.6\text{ m})/(7.6\text{ m}) = 0.87$. The firefighter can climb $(0.87)(12\,\text{m}) = 10.4\,\text{m}$ up the ladder.

E9-26 (a) The net torque about the rear axle is $(1360\,\text{kg})(9.8\,\text{m/s}^2)(3.05\,\text{m}-1.78\,\text{m})-F_\text{f}(3.05\,\text{m}) = 0$, which has solution $F_\text{f} = 5.55\times10^3\text{N}$. Each of the front tires support half of this, or $2.77\times10^3\text{N}$.

(b) The net torque about the front axle is $(1360\,\text{kg})(9.8\,\text{m/s}^2)(1.78\,\text{m}) - F_\text{f}(3.05\,\text{m}) = 0$, which has solution $F_\text{f} = 7.78\times10^3\text{N}$. Each of the front tires support half of this, or $3.89\times10^3\text{N}$.

E9-27 The net torque on the bridge about the end closest to the person is

$$(160\text{ lb})L/4 + (600\text{ lb})L/2 - F_\text{f}L = 0,$$

which has a solution for the supporting force on the far end of $F_\text{f} = 340$ lb.

The net force on the bridge is $(160\text{ lb})L/4 + (600\text{ lb})L/2 - (340\text{ lb}) - F_\text{c} = 0$, so the force on the close end of the bridge is $F_\text{c} = 420$ lb.

E9-28 The net torque on the board about the left end is

$$F_\text{r}(1.55\,\text{m}) - (142\,\text{N})(2.24\,\text{m}) - (582\,\text{N})(4.48\,\text{m}) = 0,$$

which has a solution for the supporting force for the right pedestal of $F_\text{r} = 1890\,\text{N}$. The force on the board from the pedestal is up, so the force on the pedestal from the board is down (compression).

The net force on the board is $F_\text{l} + (1890\,\text{N}) - (142\,\text{N}) - (582\,\text{N}) = 0$, so the force from the pedestal on the left is $F_\text{l} = -1170\,\text{N}$. The negative sign means up, so the pedestal is under tension.

$\boxed{\textbf{E9-29}}$ We can assume that both the force $\vec{\mathbf{F}}$ and the force of gravity $\vec{\mathbf{W}}$ act on the center of the wheel. Then the wheel will just start to lift when

$$\vec{\mathbf{W}} \times \vec{\mathbf{r}} + \vec{\mathbf{F}} \times \vec{\mathbf{r}} = 0,$$

or

$$W \sin\theta = F \cos\theta,$$

where θ is the angle between the vertical (pointing down) and the line between the center of the wheel and the point of contact with the step. The use of the sine on the left is a straightforward application of Eq. 9-2. Why the cosine on the right? Because

$$\sin(90^\circ - \theta) = \cos\theta.$$

Then $F = W\tan\theta$. We can express the angle θ in terms of trig functions, h, and r. $r\cos\theta$ is the vertical distance from the center of the wheel to the top of the step, or $r - h$. Then

$$\cos\theta = 1 - \frac{h}{r} \text{ and } \sin\theta = \sqrt{1 - \left(1 - \frac{h}{r}\right)^2}.$$

Finally by combining the above we get

$$F = W\frac{\sqrt{\frac{2h}{r} - \frac{h^2}{r^2}}}{1 - \frac{h}{r}} = W\frac{\sqrt{2hr - h^2}}{r - h}.$$

E9-30 (a) Assume that each of the two support points for the square sign experience the same tension, equal to half of the weight of the sign. The net torque on the rod about an axis through the hinge is

$$(52.3\,\text{kg}/2)(9.81\,\text{m/s}^2)(0.95\,\text{m}) + (52.3\,\text{kg}/2)(9.81\,\text{m/s}^2)(2.88\,\text{m}) - (2.88\,\text{m})T\sin\theta = 0,$$

where T is the tension in the cable and θ is the angle between the cable and the rod. The angle can be found from $\theta = \arctan(4.12\,\text{m}/2.88\,\text{m}) = 55.0^\circ$, so $T = 416\,\text{N}$.

(b) There are two components to the tension, one which is vertical, $(416\,\text{N})\sin(55.0^\circ) = 341\,\text{N}$, and another which is horizontal, $(416\,\text{N})\cos(55.0^\circ) = 239\,\text{N}$. The horizontal force exerted by the wall must then be $239\,\text{N}$. The net vertical force on the rod is $F + (341\,\text{N}) - (52.3\,\text{kg}/2)(9.81\,\text{m/s}^2) = 0$, which has solution $F = 172\,\text{N}$ as the vertical upward force of the wall on the rod.

E9-31 (a) The net torque on the rod about an axis through the hinge is

$$\tau = W(L/2)\cos(54.0^\circ) - TL\sin(153.0^\circ) = 0.$$

or $T = (52.7\text{ lb}/2)(\sin 54.0^\circ/\sin 153.0^\circ) = 47.0$ lb.

(b) The vertical upward force of the wire on the rod is $T_y = T\cos(27.0^\circ)$. The vertical upward force of the wall on the rod is $P_y = W - T\cos(27.0^\circ)$, where W is the weight of the rod. Then

$$P_y = (52.7\text{ lb}) - (47.0\text{ lb})\cos(27.0^\circ) = 10.8\text{ lb}$$

The horizontal force from the wall is balanced by the horizontal force from the wire. Then $P_x = (47.0\text{ lb})\sin(27.0^\circ) = 21.3$ lb.

E9-32 If the ladder is not slipping then the torque about an axis through the point of contact with the ground is

$$\tau = (WL/2)\cos\theta - Nh/\sin\theta = 0,$$

where N is the normal force of the edge on the ladder. Then $N = WL\cos\theta\,\sin\theta\,/(2h)$.

N has two components; one which is vertically up, $N_y = N\cos\theta$, and another which is horizontal, $N_x = N\sin\theta$. The horizontal force must be offset by the static friction.

The normal force on the ladder from the ground is given by

$$N_{\text{g}} = W - N\cos\theta = W[1 - L\cos^2\theta\,\sin\theta\,/(2h)].$$

The force of static friction can be as large as $f = \mu_s N_g$, so

$$\mu_s = \frac{WL\cos\theta\,\sin^2\theta\,/(2h)}{W[1 - L\cos^2\theta\,\sin\theta\,/(2h)]} = \frac{L\cos\theta\,\sin^2\theta}{2h - L\cos^2\theta\,\sin\theta}.$$

Put in the numbers and $\theta = 68.0°$. Then $\mu_s = 0.407$.

E9-33 Let out be positive. The net torque about the axis is then

$$\tau = (0.118\,\mathrm{m})(5.88\,\mathrm{N}) - (0.118\,\mathrm{m})(4.13\,\mathrm{m}) - (0.0493\,\mathrm{m})(2.12\,\mathrm{N}) = 0.102\,\mathrm{N}\cdot\mathrm{m}.$$

The rotational inertia of the disk is $I = (1.92\,\mathrm{kg})(0.118\,\mathrm{m})^2/2 = 1.34\times10^{-2}\mathrm{kg}\cdot\mathrm{m}^2$. Then $\alpha = (0.102\,\mathrm{N}\cdot\mathrm{m})/(1.34\times10^{-2}\mathrm{kg}\cdot\mathrm{m}^2) = 7.61\ \mathrm{rad/s^2}$.

E9-34 (a) $I = \tau/\alpha = (960\,\mathrm{N}\cdot\mathrm{m})/(6.23\ \mathrm{rad/s^2}) = 154\,\mathrm{kg}\cdot\mathrm{m}^2$.
(b) $m = (3/2)I/r^2 = (1.5)(154\,\mathrm{kg}\cdot\mathrm{m}^2)/(1.88\,\mathrm{m})^2 = 65.4\,\mathrm{kg}$.

E9-35 (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{6.20\ \mathrm{rad/s}}{0.22\ \mathrm{s}} = 28.2\ \mathrm{rad/s^2}$$

(b) From Eq. 9-11, $\tau = I\alpha = (12.0\,\mathrm{kg}\cdot\mathrm{m}^2)(28.2\ \mathrm{rad/s^2}) = 338\,\mathrm{N}\cdot\mathrm{m}$.

E9-36 The angular acceleration is $\alpha = 2(\pi/2\ \mathrm{rad})/(30\,\mathrm{s})^2 = 3.5\times10^{-3}\ \mathrm{rad/s^2}$. The required force is then

$$F = \tau/r = I\alpha/r = (8.7\times10^4\mathrm{kg}\cdot\mathrm{m}^2)(3.5\times10^{-3}\ \mathrm{rad/s^2})/(2.4\,\mathrm{m}) = 127\,\mathrm{N}.$$

Don't let the door slam...

E9-37 The torque is $\tau = rF$, the angular acceleration is $\alpha = \tau/I = rF/I$. The angular velocity is

$$\omega = \int_0^t \alpha\,dt = \frac{rAt^2}{2I} + \frac{rBt^3}{3I},$$

so when $t = 3.60\,\mathrm{s}$,

$$\omega = \frac{(9.88\times10^{-2}\mathrm{m})(0.496\,\mathrm{N/s})(3.60\,\mathrm{s})^2}{2(1.14\times10^{-3}\mathrm{kg}\cdot\mathrm{m}^2)} + \frac{(9.88\times10^{-2}\mathrm{m})(0.305\,\mathrm{N/s^2})(3.60\,\mathrm{s})^3}{3(1.14\times10^{-3}\mathrm{kg}\cdot\mathrm{m}^2)} = 690\ \mathrm{rad/s}.$$

E9-38 (a) $\alpha = 2\theta/t^2$.
(b) $a = \alpha R = 2\theta R/t^2$.
(c) T_1 and T_2 are *not* equal. Instead, $(T_1 - T_2)R = I\alpha$. For the hanging block $Mg - T_1 = Ma$. Then

$$T_1 = Mg - 2MR\theta/t^2,$$

and

$$T_2 = Mg - 2MR\theta/t^2 - 2(I/R)\theta/t^2.$$

E9-39 Apply a kinematic equation from chapter 2 to find the acceleration:

$$\begin{aligned} y &= v_{0y}t + \frac{1}{2}a_y t^2, \\ a_y &= \frac{2y}{t^2} = \frac{2(0.765 \text{ m})}{(5.11 \text{ s})^2} = 0.0586 \text{ m/s}^2 \end{aligned}$$

Closely follow the approach in Sample Problem 9-10. For the heavier block, $m_1 = 0.512$ kg, and Newton's second law gives

$$m_1 g - T_1 = m_1 a_y,$$

where a_y is positive and *down*. For the lighter block, $m_2 = 0.463$ kg, and Newton's second law gives

$$T_2 - m_2 g = m_2 a_y,$$

where a_y is positive and *up*. We do know that $T_1 > T_2$; the net force on the pulley creates a torque which results in the pulley rotating toward the heavier mass. That net force is $T_1 - T_2$; so the rotational form of Newton's second law gives

$$(T_1 - T_2)\, R = I\alpha_z = I a_T / R,$$

where $R = 0.049$ m is the radius of the pulley and a_T is the tangential acceleration. But this acceleration is equal to a_y, because everything— both blocks and the pulley— are moving together.

We then have *three* equations and *three* unknowns. We'll add the first two together,

$$\begin{aligned} m_1 g - T_1 + T_2 - m_2 g &= m_1 a_y + m_2 a_y, \\ T_1 - T_2 &= (g - a_y)m_1 - (g + a_y)m_2, \end{aligned}$$

and then combine this with the third equation by substituting for $T_1 - T_2$,

$$\begin{aligned} (g - a_y)m_1 - (g + a_y)m_2 &= I a_y / R^2, \\ \left[\left(\frac{g}{a_y} - 1\right) m_1 - \left(\frac{g}{a_y} + 1\right) m_2\right] R^2 &= I. \end{aligned}$$

Now for the numbers:

$$\begin{aligned} \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} - 1\right)(0.512\,\text{kg}) - \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} + 1\right)(0.463\,\text{kg}) &= 7.23\,\text{kg}, \\ (7.23\,\text{kg})(0.049\,\text{m})^2 &= 0.0174\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

E9-40 The wheel turns with an initial angular speed of $\omega_0 = 88.0$ rad/s. The average speed while decelerating is $\omega_{\text{av}} = \omega_0/2$. The wheel stops turning in a time $t = \phi/\omega_{\text{av}} = 2\phi/\omega_0$. The deceleration is then $\alpha = -\omega_0/t = -\omega_0^2/(2\phi)$.

The rotational inertia is $I = MR^2/2$, so the torque required to stop the disk is $\tau = I\alpha = -MR^2\omega_0^2/(4\phi)$. The force of friction on the disk is $f = \mu N$, so $\tau = Rf$. Then

$$\mu = \frac{MR\omega_0^2}{4N\phi} = \frac{(1.40\,\text{kg})(0.23\,\text{m})(88.0 \text{ rad/s})^2}{4(130\,\text{N})(17.6 \text{ rad})} = 0.272.$$

E9-41 (a) The automobile has an initial speed of $v_0 = 21.8\,\text{m/s}$. The angular speed is then $\omega_0 = (21.8\,\text{m/s})/(0.385\,\text{m}) = 56.6$ rad/s.

(b) The average speed while decelerating is $\omega_{\text{av}} = \omega_0/2$. The wheel stops turning in a time $t = \phi/\omega_{\text{av}} = 2\phi/\omega_0$. The deceleration is then

$$\alpha = -\omega_0/t = -\omega_0^2/(2\phi) = -(56.6 \text{ rad/s})^2/[2(180 \text{ rad})] = -8.90 \text{ rad/s}.$$

(c) The automobile traveled $x = \phi r = (180 \text{ rad})(0.385\,\text{m}) = 69.3\,\text{m}$.

E9-42 (a) The angular acceleration is derived in Sample Problem 9-13,

$$\alpha = \frac{g}{R_0}\frac{1}{1+I/(MR_0^2)} = \frac{(981\ \mathrm{cm/s^2})}{(0.320\ \mathrm{cm})}\frac{1}{1+(0.950\,\mathrm{kg\cdot cm^2})/[(0.120\,\mathrm{kg})(0.320\ \mathrm{cm})^2]} = 39.1\ \mathrm{rad/s^2}.$$

The acceleration is $a = \alpha R_0 = (39.1\ \mathrm{rad/s^2})(0.320\ \mathrm{cm}) = 12.5\ \mathrm{cm/s^2}$.

(b) Starting from rest, $t = \sqrt{2x/a} = \sqrt{2(134\ \mathrm{cm})/(12.5\ \mathrm{cm/s^2})} = 4.63\,\mathrm{s}$.

(c) $\omega = \alpha t = (39.1\ \mathrm{rad/s^2})(4.63\,\mathrm{s}) = 181\ \mathrm{rad/s}$. This is the same as 28.8 rev/s.

(d) The yo-yo accelerates toward the ground according to $y = at^2 + v_0 t$, where *down* is positive. The time required to move to the end of the string is found from

$$t = \frac{-v_0 + \sqrt{v_0^2 + 4ay}}{2a} = \frac{-(1.30\,\mathrm{m/s}) + \sqrt{(1.30\,\mathrm{m/s})^2 + 4(0.125\,\mathrm{m/s^2})(1.34\,\mathrm{m})}}{2(0.125\,\mathrm{m/s^2})} = 0.945\,\mathrm{s}$$

The initial rotational speed was $\omega_0 = (1.30\,\mathrm{m/s})/(3.2\times10^{-3}\mathrm{m}) = 406\ \mathrm{rad/s}$. Then

$$\omega = \omega_0 + \alpha t = (406\ \mathrm{rad/s}) + (39.1\ \mathrm{rad/s^2})(0.945\,\mathrm{s}) = 443\ \mathrm{rad/s},$$

which is the same as 70.5 rev/s.

E9-43 (a) Assuming a perfect hinge at B, the only two vertical forces on the tire will be the normal force from the belt and the force of gravity. Then $N = W = mg$, or $N = (15.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) = 147\ \mathrm{N}$.

While the tire skids we have kinetic friction, so $f = \mu_k N = (0.600)(147\ \mathrm{N}) = 88.2\ \mathrm{N}$. The force of gravity and and the pull from the holding rod AB both act at the axis of rotation, so can't contribute to the net torque. The normal force acts at a point which is parallel to the displacement from the axis of rotation, so it doesn't contribute to the torque either (because the cross product would vanish); so the only contribution to the torque is from the frictional force.

The frictional force is perpendicular to the radial vector, so the magnitude of the torque is just $\tau = rf = (0.300\ \mathrm{m})(88.2\ \mathrm{N}) = 26.5\ \mathrm{N{\cdot}m}$. This means the angular acceleration will be $\alpha = \tau/I = (26.5\ \mathrm{N{\cdot}m})/(0.750\ \mathrm{kg{\cdot}m^2}) = 35.3\ \mathrm{rad/s^2}$.

When $\omega R = v_T = 12.0\ \mathrm{m/s}$ the tire is no longer slipping. We solve for ω and get $\omega = 40\ \mathrm{rad/s}$.

Now we solve $\omega = \omega_0 + \alpha t$ for the time. The wheel started from rest, so $t = 1.13$ s.

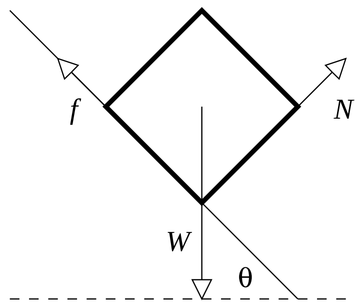
(b) The length of the skid is $x = vt = (12.0\ \mathrm{m/s})(1.13\ \mathrm{s}) = 13.6$ m long.

P9-1 The problem of sliding down the ramp has been solved (see Sample Problem 5-8); the critical angle θ_s is given by $\tan\theta_s = \mu_s$.

The problem of tipping is actually not that much harder: an object tips when the center of gravity is no longer over the base. The important angle for tipping is shown in the figure below; we can find that by trigonometry to be

$$\tan\theta_t = \frac{O}{A} = \frac{(0.56\ \mathrm{m})}{(0.56\ \mathrm{m}) + (0.28\ \mathrm{m})} = 0.67,$$

so $\theta_t = 34°$.



(a) If $\mu_s = 0.60$ then $\theta_s = 31°$ and the crate slides.
(b) If $\mu_s = 0.70$ then $\theta_s = 35°$ and the crate tips before sliding; it tips at $34°$.

P9-2 (a) The total force up on the chain needs to be equal to the total force down; the force down is W. Assuming the tension at the end points is T then $T \sin\theta$ is the upward component, so $T = W/(2\sin\theta)$.

(b) There is a horizontal component to the tension $T\cos\theta$ at the wall; this *must* be the tension at the horizontal point at the bottom of the cable. Then $T_{\text{bottom}} = W/(2\tan\theta)$.

P9-3 (a) The rope exerts a force on the sphere which has horizontal $T\sin\theta$ and vertical $T\cos\theta$ components, where $\theta = \arctan(r/L)$. The weight of the sphere is balanced by the upward force from the rope, so $T\cos\theta = W$. But $\cos\theta = L/\sqrt{r^2 + L^2}$, so $T = W\sqrt{1 + r^2/L^2}$.

(b) The wall pushes outward against the sphere equal to the inward push on the sphere from the rope, or $P = T\sin\theta = W\tan\theta = Wr/L$.

P9-4 Treat the problem as having two forces: the man at one end lifting with force $F = W/3$ and the two men acting together a distance x away from the first man and lifting with a force $2F = 2W/3$. Then the torque about an axis through the end of the beam where the first man is lifting is $\tau = 2xW/3 - WL/2$, where L is the length of the beam. This expression equal zero when $x = 3L/4$.

$\boxed{\textbf{P9-5}}$ (a) We can solve this problem with Eq. 9-32 after a few modifications. We'll assume the center of mass of the ladder is at the center, then the third term of Eq. 9-32 is $mga/2$. The cleaner didn't climb half-way, he climbed $3.10/5.12 = 60.5\%$ of the way, so the second term of Eq. 9-32 becomes $Mga(0.605)$. h, L, and a are related by $L^2 = a^2 + h^2$, so $h = \sqrt{(5.12\text{ m})^2 - (2.45\text{ m})^2} = 4.5$ m. Then, putting the correction into Eq. 9-32,

$$\begin{aligned}
F_w &= \frac{1}{h}\left[Mga(0.605) + \frac{mga}{2}\right], \\
&= \frac{1}{(4.5\text{ m})}\Big[(74.6\text{ kg})(9.81\text{ m/s}^2)(2.45\text{ m})(0.605), \\
&\quad + (10.3\text{ kg})(9.81\text{ m/s}^2)(2.45\text{ m})/2\Big], \\
&= 269\text{ N}
\end{aligned}$$

(b) The vertical component of the force of the ground on the ground is the sum of the weight of the window cleaner and the weight of the ladder, or 833 N.

The horizontal component is equal in magnitude to the force of the ladder on the window. Then the net force of the ground on the ladder has magnitude

$$\sqrt{(269\,\mathrm{N})^2 + (833\,\mathrm{N})^2} = 875\,\mathrm{N}$$

and direction

$$\theta = \arctan(833/269) = 72^\circ \text{ above the horizontal.}$$

P9-6 (a) There are no upward forces other than the normal force on the bottom ball, so the force exerted on the bottom ball by the container is $2W$.

(c) The bottom ball must exert a force on the top ball which has a vertical component equal to the weight of the top ball. Then $W = N \sin\theta$ or the force of contact between the balls is $N = W/\sin\theta$.

(b) The force of contact between the balls has a horizontal component $P = N\cos\theta = W/\tan\theta$, this must also be the force of the walls on the balls.

P9-7 (a) There are three forces on the ball: weight $\vec{\mathbf{W}}$, the normal force from the lower plane $\vec{\mathbf{N}}_1$, and the normal force from the upper plane $\vec{\mathbf{N}}_2$. The force from the lower plane has components $N_{1,x} = -N_1 \sin\theta_1$ and $N_{1,y} = N_1 \cos\theta_1$. The force from the upper plane has components $N_{2,x} = N_2 \sin\theta_2$ and $N_{2,y} = -N_2 \cos\theta_2$. Then $N_1 \sin\theta_1 = N_2 \sin\theta_2$ and $N_1 \cos\theta_1 = W + N_2 \cos\theta_2$.

Solving for N_2 by dividing one expression by the other,

$$\frac{\cos\theta_1}{\sin\theta_1} = \frac{W}{N_2 \sin\theta_2} + \frac{\cos\theta_2}{\sin\theta_2},$$

or

$$\begin{aligned} N_2 &= \frac{W}{\sin\theta_2}\left(\frac{\cos\theta_1}{\sin\theta_1} - \frac{\cos\theta_2}{\sin\theta_2}\right)^{-1}, \\ &= \frac{W}{\sin\theta_2}\frac{\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1}{\sin\theta_1\,\sin\theta_2}, \\ &= \frac{W\sin\theta_1}{\sin(\theta_2 - \theta_1)}. \end{aligned}$$

Then solve for N_1,

$$N_1 = \frac{W\sin\theta_2}{\sin(\theta_2 - \theta_1)}.$$

(b) Friction changes everything.

P9-8 (a) The net torque about a line through A is

$$\tau = Wx - TL\sin\theta = 0,$$

so $T = Wx/(L\sin\theta)$.

(b) The horizontal force on the pin is equal in magnitude to the horizontal component of the tension: $T\cos\theta = Wx/(L\tan\theta)$. The vertical component balances the weight: $W - Wx/L$.

(c) $x = (520\,\mathrm{N})(2.75\,\mathrm{m})\sin(32.0^\circ)/(315\,\mathrm{N}) = 2.41\,\mathrm{m}$.

P9-9 (a) As long as the center of gravity of an object (even if combined) is above the base, then the object will not tip.

Stack the bricks from the top down. The center of gravity of the top brick is $L/2$ from the edge of the top brick. This top brick can be offset nor more than $L/2$ from the one beneath. The center of gravity of the top two bricks is located at

$$x_{\mathrm{cm}} = [(L/2) + (L)]/2 = 3L/4.$$

These top two bricks can be offset no more than $L/4$ from the brick beneath. The center of gravity of the top three bricks is located at

$$x_{\text{cm}} = [(L/2) + 2(L)]/3 = 5L/6.$$

These top three bricks can be offset no more than $L/6$ from the brick beneath. The total offset is then $L/2 + L/4 + L/6 = 11L/12$.

(b) Actually, we never need to know the location of the center of gravity; we now realize that each brick is located with an offset $L/(2n)$ with the brick beneath, where n is the number of the brick counting from the top. The series is then of the form

$$(L/2)[(1/1) + (1/2) + (1/3) + (1/4) + \cdots],$$

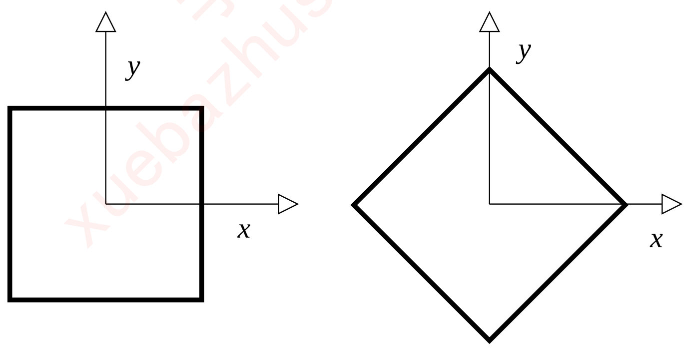
a series (harmonic, for those of you who care) which does not converge.

(c) The center of gravity would be half way between the ends of the two extreme bricks. This would be at NL/n; the pile will topple when this value exceeds L, or when $N = n$.

P9-10 (a) For a planar object which lies in the $x - y$ plane, $I_x = \int x^2 dm$ and $I_y = \int y^2 dm$. Then $I_x + I_y = \int (x^2 + y^2) dm = \int r^2\, dm$. But this is the rotational inertia about the z axis, sent r is the distance from the z axis.

(b) Since the rotational inertia about one diameter (I_x) should be the same as the rotational inertia about any other (I_y) then $I_x = I_y$ and $I_x = I_z/2 = MR^2/4$.

$\boxed{\textbf{P9-11}}$ Problem 9-10 says that $I_x + I_y = I_z$ for any thin, flat object which lies only in the $x - y$ plane.It doesn't matter in which direction the x and y axes are chosen, so long as they are perpendicular. We can then orient our square as in either of the pictures below:



By symmetry $I_x = I_y$ in either picture. Consequently, $I_x = I_y = I_z/2$ for either picture. It is the same square, so I_z is the same for both picture. Then I_x is also the same for both orientations.

P9-12 Let M_0 be the mass of the plate *before* the holes are cut out. Then $M_1 = M_0(a/L)^2$ is the mass of the part cut out of each hole and $M = M_0 - 9M_1$ is the mass of the plate. The rotational inertia (about an axis perpendicular to the plane through the center of the square) for the large uncut square is $M_0L^2/6$ and for each smaller cut out is $M_1a^2/6$.

From the large uncut square's inertia we need to remove $M_1a^2/6$ for the center cut-out, $M_1a^2/6 + M_1(L/3)^2$ for each of the four edge cut-outs, and $M_1a^2/6 + M_1(\sqrt{2}L/3)^2$ for each of the corner sections.

Then

$$I = \frac{M_0L^2}{6} - 9\frac{M_1a^2}{6} - 4\frac{M_1L^2}{9} - 4\frac{2M_1L_2}{9},$$
$$= \frac{M_0L^2}{6} - 3\frac{M_0a^4}{2L^2} - 4\frac{M_0a^2}{3}.$$

P9-13 (a) From Eq. 9-15, $I = \int r^2 dm$ about some axis of rotation when r is measured from that axis. If we consider the x axis as our axis of rotation, then $r = \sqrt{y^2+z^2}$, since the distance to the x axis depends only on the y and z coordinates. We have similar equations for the y and z axes, so

$$I_x = \int (y^2+z^2)\,dm,$$
$$I_y = \int (x^2+z^2)\,dm,$$
$$I_z = \int (x^2+y^2)\,dm.$$

These three equations can be added together to give

$$I_x + I_y + I_z = 2\int (x^2+y^2+z^2)\,dm,$$

so if we *now* define r to be measured from the origin (which is not the definition used above), then we end up with the answer in the text.

(b) The right hand side of the equation is integrated over the entire body, regardless of how the axes are defined. So the integral should be the same, no matter how the coordinate system is rotated.

P9-14 (a) Since the shell is spherically symmetric $I_x = I_y = I_z$, so $I_x = (2/3)\int r^2 dm = (2R^2/3)\int dm = 2MR^2/3$.

(b) Since the solid ball is spherically symmetric $I_x = I_y = I_z$, so

$$I_x = \frac{2}{3}\int r^2 \frac{3Mr^2\,dr}{R^3} = \frac{2}{5}MR^2.$$

P9-15 (a) A simple ratio will suffice:

$$\frac{dm}{2\pi r\,dr} = \frac{M}{\pi R^2} \text{ or } dm = \frac{2Mr}{R^2}dr.$$

(b) $dI = r^2 dm = (2Mr^3/R^2)dr$.

(c) $I = \int_0^R (2Mr^3/R^2)dr = MR^2/2$.

P9-16 (a) Another simple ratio will suffice:

$$\frac{dm}{\pi r^2 dz} = \frac{M}{(4/3)\pi R^3} \text{ or } dm = \frac{3M(R^2-z^2)}{4R^3}dz.$$

(b) $dI = r^2 dm/2 = [3M(R^2-z^2)^2/8R^3]dz$.

(c) There are a few steps to do here:

$$\begin{aligned} I &= \int_{-R}^{R} \frac{3M(R^2-z^2)^2}{8R^3} dz, \\ &= \frac{3M}{4R^3}\int_0^R (R^4 - 2R^2z^2 + z^4)dz, \\ &= \frac{3M}{4R^3}(R^5 - 2R^5/3 + R^5/5) = \frac{2}{5}MR^2. \end{aligned}$$

$\boxed{\textbf{P9-17}}$ The *rotational* acceleration will be given by $\alpha_z = \sum \tau / I$.

The torque about the pivot comes from the force of gravity on each block. This forces will both originally be at right angles to the pivot arm, so the net torque will be $\sum \tau = mgL_2 - mgL_1$, where clockwise as seen on the page is positive.

The rotational inertia about the pivot is given by $I = \sum m_n r_n^2 = m(L_2^2 + L_1^2)$. So we can now find the rotational acceleration,

$$\alpha = \frac{\sum \tau}{I} = \frac{mgL_2 - mgL_1}{m(L_2^2 + L_1^2)} = g\frac{L_2 - L_1}{L_2^2 + L_1^2} = 8.66 \text{ rad/s}^2.$$

The linear acceleration is the tangential acceleration, $a_T = \alpha R$. For the left block, $a_T = 1.73$ m/s^2; for the right block $a_T = 6.93$ m/s^2.

P9-18 (a) The force of friction on the hub is $\mu_k Mg$. The torque is $\tau = \mu_k Mga$. The angular acceleration has magnitude $\alpha = \tau/I = \mu_k ga/k^2$. The time it takes to stop the wheel will be $t = \omega_0/\alpha = \omega_0 k^2/(\mu_k ga)$.

(b) The average rotational speed while slowing is $\omega_0/2$. The angle through which it turns while slowing is $\omega_0 t/2$ radians, or $\omega_0 t/(4\pi) = \omega_0^2 k^2/(4\pi\mu_k ga)$

P9-19 (a) Consider a differential ring of the disk. The torque on the ring because of friction is

$$d\tau = r\,dF = r\frac{\mu_k Mg}{\pi R^2} 2\pi r\,dr = \frac{2\mu_k Mgr^2}{R^2}dr.$$

The net torque is then

$$\tau = \int d\tau = \int_0^R \frac{2\mu_k Mgr^2}{R^2}dr = \frac{2}{3}\mu_k MgR.$$

(b) The rotational acceleration has magnitude $\alpha = \tau/I = \frac{4}{3}\mu_k g/R$. Then it will take a time

$$t = \omega_0/\alpha = \frac{3R\omega_0}{4\mu_k g}$$

to stop.

P9-20 We need only show that the two objects have the same acceleration.

Consider first the hoop. There is a force $W_{||} = W\sin\theta = mg\sin\theta$ pulling it down the ramp and a frictional force f pulling it up the ramp. The frictional force is just large enough to cause a torque that will allow the hoop to roll without slipping. This means $a = \alpha R$; consequently, $fR = \alpha I = aI/R$. In this case $I = mR^2$.

The acceleration down the plane is

$$ma = mg\sin\theta - f = mg\sin\theta - maI/R^2 = mg\sin\theta - ma.$$

Then $a = g\sin\theta/2$. The mass and radius are irrelevant!

For a block sliding with friction there are also two forces: $W_{||} = W\sin\theta = mg\sin\theta$ and $f = \mu_k mg\cos\theta$. Then the acceleration down the plane will be given by

$$a = g\sin\theta - \mu_k g\cos\theta,$$

which will be equal to that of the hoop if

$$\mu_k = \frac{\sin\theta - \sin\theta/2}{\cos\theta} = \frac{1}{2}\tan\theta.$$

P9-21 This problem is equivalent to Sample Problem 9-11, except that we have a sphere instead of a cylinder. We'll have the same two equations for Newton's second law,

$$Mg\sin\theta - f = Ma_{\rm cm} \text{ and } N - Mg\cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{\rm cm}\alpha.$$

The conditions for accelerating without slipping are $a_{\rm cm} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{\rm cm}\alpha}{R} = -\frac{I_{\rm cm}(-a_{\rm cm})}{R^2},$$

and then

$$Mg\sin\theta - \frac{I_{\rm cm}(a_{\rm cm})}{R^2} = Ma_{\rm cm},$$

and solve for $a_{\rm cm}$. For fun, let's write the rotational inertia as $I = \beta MR^2$, where $\beta = 2/5$ for the sphere. Then, upon some mild rearranging, we get

$$a_{\rm cm} = g\frac{\sin\theta}{1+\beta}$$

For the sphere, $a_{\rm cm} = 5/7g\sin\theta$.

(a) If $a_{\rm cm} = 0.133g$, then $\sin\theta = 7/5(0.133) = 0.186$, and $\theta = 10.7°$.

(b) A frictionless block has no rotational properties; in this case $\beta = 0$! Then $a_{\rm cm} = g\sin\theta = 0.186g$.

P9-22 (a) There are three forces on the cylinder: gravity W and the tension from each cable T. The downward acceleration of the cylinder is then given by $ma = W - 2T$.

The ropes unwind according to $\alpha = a/R$, but $\alpha = \tau/I$ and $I = mR^2/2$. Then

$$a = \tau R/I = (2TR)R/(mR^2/2) = 4T/m.$$

Combining the above, $4T = W - 2T$, or $T = W/6$.

(b) $a = 4(mg/6)/m = 2g/3$.

P9-23 The force of friction required to keep the cylinder rolling is given by

$$f = \frac{1}{3}Mg\sin\theta;$$

the normal force is given to be $N = Mg\cos\theta$; so the coefficient of static friction is given by

$$\mu_s \geq \frac{f}{N} = \frac{1}{3}\tan\theta.$$

P9-24 $a = F/M$, since F is the net force on the disk. The torque about the center of mass is FR, so the disk has an angular acceleration of

$$\alpha = \frac{FR}{I} = \frac{FR}{MR^2/2} = \frac{2F}{MR}.$$

P9-25 This problem is equivalent to Sample Problem 9-11, except that we have an unknown rolling object. We'll have the same two equations for Newton's second law,

$$Mg\sin\theta - f = Ma_{\rm cm} \text{ and } N - Mg\cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{\rm cm}\alpha.$$

The conditions for accelerating without slipping are $a_{\rm cm} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{\rm cm}\alpha}{R} = -\frac{I_{\rm cm}(-a_{\rm cm})}{R^2},$$

and then

$$Mg\sin\theta - \frac{I_{\rm cm}(a_{\rm cm})}{R^2} = Ma_{\rm cm},$$

and solve for $a_{\rm cm}$. Write the rotational inertia as $I = \beta MR^2$, where $\beta = 2/5$ for a sphere, $\beta = 1/2$ for a cylinder, and $\beta = 1$ for a hoop. Then, upon some mild rearranging, we get

$$a_{\rm cm} = g\frac{\sin\theta}{1+\beta}$$

Note that a is largest when β is smallest; consequently the cylinder wins. Neither M nor R entered into the final equation.

E10-1 $l = rp = mvr = (13.7\times10^{-3}\text{kg})(380\,\text{m/s})(0.12\,\text{m}) = 0.62\,\text{kg}\cdot\text{m}^2/\text{s}.$

E10-2 (a) $\vec{\mathbf{l}} = m\vec{\mathbf{r}}\times\vec{\mathbf{v}}$, or

$$\vec{\mathbf{l}} = m(yv_z - zv_y)\hat{\mathbf{i}} + m(zv_x - xv_z)\hat{\mathbf{j}} + m(xv_y - yv_x)\hat{\mathbf{k}}.$$

(b) If $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ exist only in the xy plane then $z = v_z = 0$, so only the uk term survives.

$\boxed{\textbf{E10-3}}$ If the angular momentum $\vec{\mathbf{l}}$ is constant in time, then $d\vec{\mathbf{l}}/dt = 0$. Trying this on Eq. 10-1,

$$\begin{aligned}
\frac{d\vec{\mathbf{l}}}{dt} &= \frac{d}{dt}\left(\vec{\mathbf{r}}\times\vec{\mathbf{p}}\right), \\
&= \frac{d}{dt}\left(\vec{\mathbf{r}}\times m\vec{\mathbf{v}}\right), \\
&= m\frac{d\vec{\mathbf{r}}}{dt}\times\vec{\mathbf{v}} + m\vec{\mathbf{r}}\times\frac{d\vec{\mathbf{v}}}{dt}, \\
&= m\vec{\mathbf{v}}\times\vec{\mathbf{v}} + m\vec{\mathbf{r}}\times\vec{\mathbf{a}}.
\end{aligned}$$

Now the cross product of a vector with itself is zero, so the first term vanishes. But in the exercise we are told the particle has constant velocity, so $\vec{\mathbf{a}} = 0$, and consequently the second term vanishes. Hence, $\vec{\mathbf{l}}$ is constant for a single particle if $\vec{\mathbf{v}}$ is constant.

E10-4 (a) $L = \sum l_i$; $l_i = r_i m_i v_i$. Putting the numbers in for each planet and then summing (I won't bore you with the arithmetic details) yields $L = 3.15\times10^{43}\text{kg}\cdot\text{m}^2/\text{s}$.

(b) Jupiter has $l = 1.94\times10^{43}\text{kg}\cdot\text{m}^2/\text{s}$, which is 61.6% of the total.

E10-5 $l = mvr = m(2\pi r/T)r = 2\pi(84.3\,\text{kg})(6.37\times10^{6}\text{m})^2/(86400\,\text{s}) = 2.49\times10^{11}\text{kg}\cdot\text{m}^2/\text{s}.$

E10-6 (a) Substitute and expand:

$$\begin{aligned}
\vec{\mathbf{L}} &= \sum(\vec{\mathbf{r}}_{\text{cm}} + \vec{\mathbf{r}}_i')\times(m_i\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{p}}_i'), \\
&= \sum(m_i\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{p}}_i' + m_i\vec{\mathbf{r}}_i'\times\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i'), \\
&= M\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{r}}_{\text{cm}}\times(\sum\vec{\mathbf{p}}_i') + (\sum m_i\vec{\mathbf{r}}_i')\times\vec{\mathbf{v}}_{\text{cm}} + \sum\vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i'.
\end{aligned}$$

(b) But $\sum\vec{\mathbf{p}}_i' = 0$ and $\sum m_i\vec{\mathbf{r}}_i' = 0$, because these two quantities are *in* the center of momentum and center of mass. Then

$$\vec{\mathbf{L}} = M\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}} + \sum\vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i' = \vec{\mathbf{L}}' + M\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}}.$$

E10-7 (a) Substitute and expand:

$$\vec{\mathbf{p}}_i' = m_i\frac{d\vec{\mathbf{r}}_i'}{dt} = m_i\frac{d\vec{\mathbf{r}}_i}{dt} - m_i\frac{d\vec{\mathbf{r}}_{\text{cm}}}{dt} = \vec{\mathbf{p}}_i - m_i\vec{\mathbf{v}}_{\text{cm}}.$$

(b) Substitute and expand:

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum\frac{d\vec{\mathbf{r}}_i'}{dt}\times\vec{\mathbf{p}}_i' + \sum\vec{\mathbf{r}}_i'\times\frac{d\vec{\mathbf{p}}_i'}{dt} = \sum\vec{\mathbf{r}}_i'\times\frac{d\vec{\mathbf{p}}_i'}{dt}.$$

The first term vanished because $\vec{\mathbf{v}}_i'$ is parallel to $\vec{\mathbf{p}}_i'$.

(c) Substitute and expand:

$$\begin{aligned}
\frac{d\vec{\mathbf{L}}'}{dt} &= \sum \vec{\mathbf{r}}_i' \times \frac{d(\vec{\mathbf{p}}_i - m_i\vec{\mathbf{v}}_{\rm cm})}{dt}, \\
&= \sum \vec{\mathbf{r}}_i' \times (m_i\vec{\mathbf{a}}_i - m_i\vec{\mathbf{a}}_{\rm c}{\rm m}), \\
&= \sum \vec{\mathbf{r}}_i' \times m_i\vec{\mathbf{a}}_i + \left(\sum m_i\vec{\mathbf{r}}_i'\right) \times \vec{\mathbf{a}}_{\rm c}{\rm m})
\end{aligned}$$

The second term vanishes because of the definition of the center of mass. Then

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum \vec{\mathbf{r}}_i' \times \vec{\mathbf{F}}_i,$$

where $\vec{\mathbf{F}}_i$ is the net force on the ith particle. The force $\vec{\mathbf{F}}_i$ may include both internal and external components. If there is an internal component, say between the ith and jth particles, then the torques from these two third law components will cancel out. Consequently,

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum \vec{\tau}_i = \vec{\tau}_{\rm ext}.$$

E10-8 (a) Integrate.

$$\int \vec{\tau}\, dt = \int \frac{d\vec{\mathbf{L}}}{dt} dt = \int d\vec{\mathbf{L}} = \Delta\vec{\mathbf{L}}.$$

(b) If I is fixed, $\Delta L = I\Delta\omega$. Not only that,

$$\int \tau dt = \int Fr\, dt = r\int F\, dt = rF_{\rm av}\Delta t,$$

where we use the definition of average that depends on time.

$\boxed{\textbf{E10-9}}$ (a) $\vec{\tau}\Delta t = \Delta\vec{\mathbf{l}}$. The disk starts from rest, so $\Delta\vec{\mathbf{l}} = \vec{\mathbf{l}} - \vec{\mathbf{l}}_0 = \vec{\mathbf{l}}$. We need only concern ourselves with the magnitudes, so

$$l = \Delta l = \tau\Delta t = (15.8\,{\rm N{\cdot}m})(0.033\,{\rm s}) = 0.521\,{\rm kg{\cdot}m^2/s}.$$

(b) $\omega = l/I = (0.521\,{\rm kg{\cdot}m^2/s})/(1.22\times 10^{-3}{\rm kg{\cdot}m^2}) = 427\ {\rm rad}/s$.

E10-10 (a) Let v_0 be the initial speed; the average speed while slowing to a stop is $v_0/2$; the time required to stop is $t = 2x/v_0$; the acceleration is $a = -v_0/t = -v_0^2/(2x)$. Then

$$a = -(43.3\,{\rm m/s})^2/[2(225\,{\rm m/s})] = -4.17\,{\rm m/s^2}.$$

(b) $\alpha = a/r = (-4.17\,{\rm m/s^2})/(0.247\,{\rm m}) = -16.9\ {\rm rad/s^2}$.
(c) $\tau = I\alpha = (0.155\,{\rm kg\cdot m^2})(-16.9\ {\rm rad/s^2}) = -2.62\,{\rm N\cdot m}$.

E10-11 Let $\vec{\mathbf{r}}_i = \vec{\mathbf{z}} + \vec{\mathbf{r}}_i'$. From the figure, $\vec{\mathbf{p}}_1 = -\vec{\mathbf{p}}_2$ and $\vec{\mathbf{r}}_1' = -\vec{\mathbf{r}}_2'$. Then

$$\begin{aligned}
\vec{\mathbf{L}} &= \vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 = \vec{\mathbf{r}}_1 \times \vec{\mathbf{p}}_1 + \vec{\mathbf{r}}_2 \times \vec{\mathbf{p}}_2, \\
&= (\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \times \vec{\mathbf{p}}_1, \\
&= (\vec{\mathbf{r}}_1' - \vec{\mathbf{r}}_2') \times \vec{\mathbf{p}}_1, \\
&= 2\vec{\mathbf{r}}_1' \times \vec{\mathbf{p}}_1.
\end{aligned}$$

Since $\vec{\mathbf{r}}_1'$ and $\vec{\mathbf{p}}_1$ both lie in the xy plane then $\vec{\mathbf{L}}$ must be along the z axis.

E10-12 Expand:

$$\begin{aligned}\vec{\mathbf{L}} &= \sum \vec{\mathbf{l}}_i = \sum \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i, \\ &= \sum m_i \vec{\mathbf{r}}_i \times \vec{\mathbf{v}}_i = \sum m_i \vec{\mathbf{r}}_i \times (\vec{\omega} \times \vec{\mathbf{r}}_i) \\ &= \sum m_i [(\vec{\mathbf{r}}_i \cdot \vec{\mathbf{r}}_i)\vec{\omega} - (\vec{\mathbf{r}}_i \cdot \vec{\omega})\vec{\mathbf{r}})i], \\ &= \sum m_i [r_i^2 \vec{\omega} - (z_i^2 \omega)\hat{\mathbf{k}} - (z_i x_i \omega)\hat{\mathbf{i}} - (z_i y_i \omega)\hat{\mathbf{j}}],\end{aligned}$$

but if the body is symmetric about the z axis then the last two terms vanish, leaving

$$\vec{\mathbf{L}} = \sum m_i [r_i^2 \vec{\omega} - (z_i^2 \omega)\hat{\mathbf{k}}] = \sum m_i (x_i^2 + y_i^2)\vec{\omega} = I\vec{\omega}.$$

$\boxed{\textbf{E10-13}}$ An impulse of 12.8 N·s will change the linear momentum by 12.8 N·s; the stick starts from rest, so the final momentum *must* be 12.8 N·s. Since $p = mv$, we then can find $v = p/m = (12.8$ N·s$)/(4.42$ kg$) = 2.90$ m/s.

Impulse is a vector, given by $\int \vec{\mathbf{F}} dt$. We can take the cross product of the impulse with the displacement vector $\vec{\mathbf{r}}$ (measured from the axis of rotation to the point where the force is applied) and get

$$\vec{\mathbf{r}} \times \int \vec{\mathbf{F}} dt \approx \int \vec{\mathbf{r}} \times \vec{\mathbf{F}} dt,$$

The two sides of the above expression are only equal if $\vec{\mathbf{r}}$ has a constant magnitude *and* direction. This won't be true, but if the force is of sufficiently short duration then it hopefully won't change much. The right hand side is an integral over a torque, and will equal the change in angular momentum of the stick.

The exercise states that the force is perpendicular to the stick, then $|\vec{\mathbf{r}} \times \vec{\mathbf{F}}| = rF$, and the "torque impulse" is then (0.464 m)(12.8 N·s) = 5.94 kg·m/s. This "torque impulse" is equal to the change in the angular momentum, but the stick started from rest, so the final angular momentum of the stick is 5.94 kg·m/s.

But how fast is it rotating? We can use Fig. 9-15 to find the rotational inertia about the center of the stick: $I = \frac{1}{12}ML^2 = \frac{1}{12}(4.42 \text{ kg})(1.23 \text{ m})^2 = 0.557 \text{ kg}\cdot\text{m}^2$. The angular velocity of the stick is $\omega = l/I = (5.94 \text{ kg}\cdot\text{m/s})/(0.557 \text{ kg}\cdot\text{m}^2) = 10.7$ rad/s.

E10-14 The point of rotation is the point of contact with the plane; the torque about that point is $\tau = rmg\sin\theta$. The angular momentum is $I\omega$, so $\tau = I\alpha$. In this case $I = mr^2/2 + mr^2$, the second term from the parallel axis theorem. Then

$$a = r\alpha = r\tau/I = mr^2 g\sin\theta/(3mr^2/2) = \frac{2}{3}g\sin\theta.$$

E10-15 From Exercise 8 we can immediately write

$$I_1(\omega_1 - \omega_0)/r_1 = I_2(\omega_2 - 0)/r_2,$$

but we also have $r_1\omega_1 = -r_2\omega_2$. Then

$$\omega_2 = -\frac{r_1 r_2 I_1 \omega_0}{r_1^2 I_2 - r_2^2 I_1}.$$

E10-16 (a) $\Delta\omega/\omega = (1/T_1 - 1/T_2)/(1/T_1) = -(T_2 - T_1)/T_2 = -\Delta T/T$, which in this case is $-(6.0\times10^{-3}\text{s})(8.64\times10^4\text{s}) = -6.9\times10^{-8}$.

(b) Assuming conservation of angular momentum, $\Delta I/I = -\Delta\omega/\omega$. Then the fractional change would be 6.9×10^{-8}.

$\boxed{\textbf{E10-17}}$ The rotational inertia of a solid sphere is $I = \frac{2}{5}MR^2$; so as the sun collapses

$$\begin{aligned}
\vec{\mathbf{L}}_\mathrm{i} &= \vec{\mathbf{L}}_\mathrm{f},\\
I_\mathrm{i}\vec{\omega}_\mathrm{i} &= I_\mathrm{f}\vec{\omega}_\mathrm{f},\\
\frac{2}{5}M{R_\mathrm{i}}^2\vec{\omega}_\mathrm{i} &= \frac{2}{5}M{R_\mathrm{f}}^2\vec{\omega}_\mathrm{f},\\
{R_\mathrm{i}}^2\vec{\omega}_i &= {R_\mathrm{f}}^2\vec{\omega}_f.
\end{aligned}$$

The angular frequency is inversely proportional to the period of rotation, so

$$T_\mathrm{f} = T_\mathrm{i}\frac{{R_\mathrm{f}}^2}{{R_\mathrm{i}}^2} = (3.6\times10^4\ \text{min})\left(\frac{(6.37\times10^6\text{m})}{(6.96\times10^8\text{m})}\right)^2 = 3.0\ \text{min}.$$

E10-18 The final angular velocity of the train with respect to the tracks is $\omega_\mathrm{tt} = Rv$. The conservation of angular momentum implies

$$0 = MR^2\omega + mR^2(\omega_\mathrm{tt} + \omega),$$

or

$$\omega = \frac{-mv}{(m+M)R}.$$

E10-19 This is much like a center of mass problem.

$$0 = I_\mathrm{p}\phi_\mathrm{p} + I_\mathrm{m}(\phi_\mathrm{mp} + \phi_\mathrm{p}),$$

or

$$\phi_\mathrm{mp} = -\frac{(I_\mathrm{p} + I_\mathrm{m})\phi_\mathrm{p}}{I_\mathrm{m}} \approx -\frac{(12.6\,\text{kg}\cdot\text{m}^2)(25^\circ)}{(2.47\times10^{-3}\text{kg}\cdot\text{m}^2)} = 1.28\times10^{5\circ}.$$

That's 354 rotations!

E10-20 $\omega_\mathrm{f} = (I_\mathrm{i}/I_\mathrm{f})\omega_\mathrm{i} = [(6.13\,\text{kg}\cdot\text{m}^2)/(1.97\,\text{kg}\cdot\text{m}^2)](1.22\ \text{rev/s}) = 3.80\ \text{rev/s}$.

$\boxed{\textbf{E10-21}}$ We have two disks which are originally not in contact which then come into contact; there are no external torques. We can write

$$\begin{aligned}
\vec{\mathbf{l}}_{1,\mathrm{i}} + \vec{\mathbf{l}}_{2,\mathrm{i}} &= \vec{\mathbf{l}}_{1,\mathrm{f}} + \vec{\mathbf{l}}_{2,\mathrm{f}},\\
I_1\vec{\omega}_{1,\mathrm{i}} + I_2\vec{\omega}_{2,\mathrm{i}} &= I_1\vec{\omega}_{1,\mathrm{f}} + I_2\vec{\omega}_{2,\mathrm{f}}.
\end{aligned}$$

The final angular velocities of the two disks will be equal, so the above equation can be simplified and rearranged to yield

$$\omega_\mathrm{f} = \frac{I_1}{I_1 + I_2}\omega_{1,\mathrm{i}} = \frac{(1.27\ \text{kg}\cdot\text{m}^2)}{(1.27\ \text{kg}\cdot\text{m}^2) + (4.85\ \text{kg}\cdot\text{m}^2)}(824\ \text{rev/min}) = 171\ \text{rev/min}$$

E10-22 $l_\perp = l\cos\theta = mvr\cos\theta = mvh$.

E10-23 (a) $\omega_\mathrm{f} = (I_1/I_2)\omega_\mathrm{i}$, $I_1 = (3.66\,\mathrm{kg})(0.363\,\mathrm{m})^2 = 0.482\,\mathrm{kg}\cdot\mathrm{m}^2$. Then

$$\omega_\mathrm{f} = [(0.482\,\mathrm{kg}\cdot\mathrm{m}^2)/(2.88\,\mathrm{kg}\cdot\mathrm{m}^2)](57.7\ \mathrm{rad/s}) = 9.66\ \mathrm{rad/s},$$

with the same rotational sense as the original wheel.

(b) Same answer, since friction is an internal force internal here.

E10-24 (a) Assume the merry-go-round is a disk. Then conservation of angular momentum yields

$$(\frac{1}{2}m_\mathrm{m}R^2 + m_\mathrm{g}R^2)\omega + (m_\mathrm{r}R^2)(v/R) = 0,$$

or

$$\omega = -\frac{(1.13\,\mathrm{kg})(7.82\,\mathrm{m/s})/(3.72\,\mathrm{m})}{(827\,\mathrm{kg})/2 + (50.6\,\mathrm{kg})} = -5.12\times10^{-3}\mathrm{rad/s}.$$

(b) $v = \omega R = (-5.12\times10^{-3}\mathrm{rad/s})(3.72\,\mathrm{m}) = -1.90\times10^{-2}\mathrm{m/s}$.

E10-25 Conservation of angular momentum:

$$(m_\mathrm{m}k^2 + m_\mathrm{g}R^2)\omega = m_\mathrm{g}R^2(v/R),$$

so

$$\omega = \frac{(44.3\,\mathrm{kg})(2.92\,\mathrm{m/s})/(1.22\,\mathrm{m})}{(176\,\mathrm{kg})(0.916\,\mathrm{m})^2 + (44.3\,\mathrm{kg})(1.22\,\mathrm{m})^2} = 0.496\ \mathrm{rad/s}.$$

E10-26 Use Eq. 10-22:

$$\omega_P = \frac{Mgr}{I\omega} = \frac{(0.492\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(3.88\times10^{-2}\mathrm{m})}{(5.12\times10^{-4}\mathrm{kg}\cdot\mathrm{m}^2)(2\pi 28.6\ \mathrm{rad/s})} = 2.04\ \mathrm{rad/s} = 0.324\ \mathrm{rev/s}.$$

$\boxed{\textbf{E10-27}}$ The relevant precession expression is Eq. 10-22.

The rotational inertia will be a sum of the contributions from both the disk and the axle, but the radius of the axle is probably very small compared to the disk, probably as small as 0.5 cm. Since I is proportional to the radius squared, we expect contributions from the axle to be less than $(1/100)^2$ of the value for the disk. For the disk only we use

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(1.14\ \mathrm{kg})(0.487\ \mathrm{m})^2 = 0.135\ \mathrm{kg}\cdot\mathrm{m}^2.$$

Now for ω,

$$\omega = 975\ \mathrm{rev/min}\left(\frac{2\pi\ \mathrm{rad}}{1\ \mathrm{rev}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right) = 102\ \mathrm{rad/s}.$$

Then $L = I\omega = 13.8\ \mathrm{kg{\cdot}m^2/s}$.

Back to Eq. 10-22,

$$\omega_p = \frac{Mgr}{L} = \frac{(1.27\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(0.0610\ \mathrm{m})}{13.8\ \mathrm{kg}\cdot\mathrm{m^2/s}} = 0.0551\ \mathrm{rad/s}.$$

The *time* for one precession is

$$t = \frac{1\mathrm{rev}}{\omega_p} = \frac{2\pi\ \mathrm{rad}}{(0.0551\ \mathrm{rad/s})} = 114\ \mathrm{s}.$$

P10-1 Positive z is out of the page.

(a) $\vec{\mathbf{l}} = rmv \sin\theta \hat{\mathbf{k}} = (2.91\,\mathrm{m})(2.13\,\mathrm{kg})(4.18\,\mathrm{m})\sin(147°)\hat{\mathbf{k}} = 14.1\,\mathrm{kg}\cdot\mathrm{m}^2/s\hat{\mathbf{k}}$.

(b) $\vec{\tau} = rF\sin\theta\hat{\mathbf{k}} = (2.91\,\mathrm{m})(1.88\,\mathrm{N})\sin(26°)\hat{\mathbf{k}} = 2.40\,\mathrm{N}\cdot\mathrm{m}\hat{\mathbf{k}}$.

P10-2 Regardless of where the origin is located one can orient the coordinate system so that the two paths lie in the xy plane and are both parallel to the y axis. The one of the particles travels along the path $x = vt$, $y = a$, $z = b$; the momentum of this particle is $\vec{\mathbf{p}}_1 = mv\hat{\mathbf{i}}$. The other particle will then travel along a path $x = c - vt$, $y = a + d$, $z = b$; the momentum of this particle is $\vec{\mathbf{p}}_2 = -mv\hat{\mathbf{i}}$. The angular momentum of the first particle is

$$\vec{\mathbf{l}}_1 = mvb\hat{\mathbf{j}} - mva\hat{\mathbf{k}},$$

while that of the second is

$$\vec{\mathbf{l}}_2 = -mvb\hat{\mathbf{j}} + mv(a+d)\hat{\mathbf{k}},$$

so the total is $\vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 = mvd\hat{\mathbf{k}}$.

$\boxed{\textbf{P10-3}}$ Assume that the cue stick strikes the ball horizontally with a force of constant magnitude F for a time Δt. Then the magnitude of the change in linear momentum of the ball is given by $F\Delta t = \Delta p = p$, since the initial momentum is zero.

If the force is applied a distance x above the center of the ball, then the magnitude of the torque about a horizontal axis through the center of the ball is $\tau = xF$. The change in angular momentum of the ball is given by $\tau\Delta t = \Delta l = l$, since initially the ball is not rotating.

For the ball to roll without slipping we need $v = \omega R$. We can start with this:

$$\begin{aligned} v &= \omega R, \\ \frac{p}{m} &= \frac{lR}{I}, \\ \frac{F\Delta t}{m} &= \frac{\tau\Delta t R}{I}, \\ \frac{F}{m} &= \frac{xFR}{I}. \end{aligned}$$

Then $x = I/mR$ is the condition for rolling without sliding from the start. For a solid sphere, $I = \frac{2}{5}mR^2$, so $x = \frac{2}{5}R$.

P10-4 The change in momentum of the block is $M(v_2 - v_1)$, this is equal to the magnitude the impulse delivered to the cylinder. According to E10-8 we can write $M(v_2 - v_1)R = I\omega_\mathrm{f}$. But in the end the box isn't slipping, so $\omega_f = v_2/R$. Then

$$Mv_2 - Mv_1 = (I/R^2)v_2,$$

or

$$v_2 = v_1/(1 + I/MR^2).$$

P10-5 Assume that the cue stick strikes the ball horizontally with a force of constant magnitude F for a time Δt. Then the magnitude of the change in linear momentum of the ball is given by $F\Delta t = \Delta p = p$, since the initial momentum is zero. Consequently, $F\Delta t = mv_0$.

If the force is applied a distance h above the center of the ball, then the magnitude of the torque about a horizontal axis through the center of the ball is $\tau = hF$. The change in angular momentum of the ball is given by $\tau\Delta t = \Delta l = l_0$, since initially the ball is not rotating. Consequently, the initial angular momentum of the ball is $l_0 = hmv_0 = I\omega_0$.

The ball originally slips while moving, but eventually it rolls. When it has begun to roll without slipping we have $v = R\omega$. Applying the results from E10-8,

$$m(v - v_0)R + I(\omega - \omega_0) = 0,$$

or

$$m(v - v_0)R + \frac{2}{5}mR^2\frac{v}{R} - hmv_0 = 0,$$

then, if $v = 9v_0/7$,

$$h = \left(\frac{9}{7} - 1\right)R + \frac{2}{5}R\left(\frac{9}{7}\right) = \frac{4}{5}R.$$

P10-6 (a) Refer to the previous answer. We now want $v = \omega = 0$, so

$$m(v - v_0)R + \frac{2}{5}mR^2\frac{v}{R} - hmv_0 = 0,$$

becomes

$$-v_0R - hv_0 = 0,$$

or $h = -R$. That'll scratch the felt.

(b) Assuming only a horizontal force then

$$v = \frac{(h + R)v_0}{R(1 + 2/5)},$$

which can only be negative if $h < -R$, which means hitting below the ball. Can't happen. If instead we allow for a downward component, then we can increase the "reverse English" as much as we want without increasing the initial forward velocity, and as such it would be possible to get the ball to move backwards.

P10-7 We assume the bowling ball is solid, so the rotational inertia will be $I = (2/5)MR^2$ (see Figure 9-15).

The normal force on the bowling ball will be $N = Mg$, where M is the mass of the bowling ball. The kinetic friction on the bowling ball is $F_f = \mu_k N = \mu_k Mg$. The magnitude of the net torque on the bowling ball while skidding is then $\tau = \mu_k MgR$.

Originally the angular momentum of the ball is zero; the final angular momentum will have magnitude $l = I\omega = Iv/R$, where v is the final translational speed of the ball.

(a) The time requires for the ball to stop skidding is the time required to change the angular momentum to l, so

$$\Delta t = \frac{\Delta l}{\tau} = \frac{(2/5)MR^2v/R}{\mu_k MgR} = \frac{2v}{5\mu_k g}.$$

Since we don't know v, we can't solve this for Δt. But the same time through which the angular momentum of the ball is increasing the linear momentum of the ball is decreasing, so we also have

$$\Delta t = \frac{\Delta p}{-F_f} = \frac{Mv - Mv_0}{-\mu_k Mg} = \frac{v_0 - v}{\mu_k g}.$$

Combining,

$$\begin{aligned}\Delta t &= \frac{v_0 - v}{\mu_k g},\\ &= \frac{v_0 - 5\mu_k g\Delta t/2}{\mu_k g},\end{aligned}$$

$$\begin{aligned}
2\mu_k g\Delta t &= 2v_0 - 5\mu_k g\Delta t, \\
\Delta t &= \frac{2v_0}{7\mu_k g}, \\
&= \frac{2(8.50\,\mathrm{m/s})}{7(0.210)(9.81\,\mathrm{m/s^2})} = 1.18\,\mathrm{s}.
\end{aligned}$$

(d) Use the expression for angular momentum and torque,

$$v = 5\mu_k g\Delta t/2 = 5(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})/2 = 6.08\,\mathrm{m/s}.$$

(b) The acceleration of the ball is $F/M = -\mu g$. The distance traveled is then given by

$$\begin{aligned}
x &= \frac{1}{2}at^2 + v_0 t, \\
&= -\frac{1}{2}(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})^2 + (8.50\,\mathrm{m/s})(1.18\,\mathrm{s}) = 8.6\,\mathrm{m},
\end{aligned}$$

(c) The angular acceleration is $\tau/I = 5\mu_k g/(2R)$. Then

$$\begin{aligned}
\theta &= \frac{1}{2}\alpha t^2 + \omega_0 t, \\
&= \frac{5(0.210)(9.81\,\mathrm{m/s^2})}{4(0.11\,\mathrm{m})}(1.18\,\mathrm{s})^2 = 32.6\text{ rad} = 5.19\text{ revolutions}.
\end{aligned}$$

P10-8 (a) $l = I\omega_0 = (1/2)MR^2\omega_0$.

(b) The initial speed is $v_0 = R\omega_0$. The chip decelerates in a time $t = v_0/g$, and during this time the chip travels with an average speed of $v_0/2$ through a distance of

$$y = v_{\mathrm{av}}t = \frac{v_0}{2}\frac{v_0}{g} = \frac{R^2\omega^2}{2g}.$$

(c) Loosing the chip won't change the angular velocity of the wheel.

P10-9 Since $L = I\omega = 2\pi I/T$ and L is constant, then $I \propto T$. But $I \propto R^2$, so $R^2 \propto T$ and

$$\frac{\Delta T}{T} = \frac{2R\Delta R}{R^2} = \frac{2\Delta R}{R}.$$

Then

$$\Delta T = (86400\,\mathrm{s})\frac{2(30\,\mathrm{m})}{(6.37\times10^6\mathrm{m})} \approx 0.8\,\mathrm{s}.$$

P10-10 Originally the rotational inertia was

$$I_{\mathrm{i}} = \frac{2}{5}MR^2 = \frac{8\pi}{15}\rho_0 R^5.$$

The average density can be found from Appendix C. Now the rotational inertia is

$$I_{\mathrm{f}} = \frac{8\pi}{15}(\rho_1 - \rho_2)R_1^5 + \frac{8\pi}{15}\rho_2 R^5,$$

where ρ_1 is the density of the core, R_1 is the radius of the core, and ρ_2 is the density of the mantle. Since the angular momentum is constant we have $\Delta T/T = \Delta I/I$. Then

$$\frac{\Delta T}{T} = \frac{\rho_1 - \rho_2}{\rho_0}\frac{R_1^5}{R^5} + \frac{\rho_2}{\rho_0} - 1 = \frac{10.3 - 4.50}{5.52}\frac{3570^5}{6370^5} + \frac{4.50}{5.52} - 1 = -0.127,$$

so the day is getting longer.

$\boxed{\textbf{P10-11}}$ The cockroach initially has an angular speed of $\omega_{c,i} = -v/r$. The rotational inertia of the cockroach about the axis of the turntable is $I_c = mR^2$. Then conservation of angular momentum gives

$$\begin{aligned} l_{c,i} + l_{s,i} &= l_{c,f} + l_{s,f}, \\ I_c\omega_{c,i} + I_s\omega_{s,i} &= I_c\omega_{c,f} + I_s\omega_{s,f}, \\ -mR^2 v/r + I\omega &= (mR^2 + I)\omega_f, \\ \omega_f &= \frac{I\omega - mvR}{I + mR^2}. \end{aligned}$$

P10-12 (a) The skaters move in a circle of radius $R = (2.92\,\mathrm{m})/2 = 1.46\,\mathrm{m}$ centered midway between the skaters. The angular velocity of the system will be $\omega_i = v/R = (1.38\,\mathrm{m/s})/(1.46\,\mathrm{m}) = 0.945\ \mathrm{rad/s}$.

(b) Moving closer will decrease the rotational inertia, so

$$\omega_f = \frac{2MR_i{}^2}{2MR_f{}^2}\omega_i = \frac{(1.46\,\mathrm{m})^2}{(0.470\,\mathrm{m})^2}(0.945\ \mathrm{rad/s}) = 9.12\ \mathrm{rad/s}.$$

$\boxed{\textbf{E11-1}}$ (a) Apply Eq. 11-2, $W = Fs\cos\phi = (190\,\text{N})(3.3\,\text{m})\cos(22°) = 580\,\text{J}$.

(b) The force of gravity is perpendicular to the displacement of the crate, so there is no work done by the force of gravity.

(c) The normal force is perpendicular to the displacement of the crate, so there is no work done by the normal force.

E11-2 (a) The force required is $F = ma = (106\,\text{kg})(1.97\,\text{m/s}^2) = 209\,\text{N}$. The object moves with an average velocity $v_{\text{av}} = v_0/2$ in a time $t = v_0/a$ through a distance $x = v_{\text{av}}t = v_0^2/(2a)$. So

$$x = (51.3\,\text{m/s})^2/[2(1.97\,\text{m/s}^2)] = 668\,\text{m}.$$

The work done is $W = Fx = (-209\,\text{N})(668\,\text{m}) = 1.40\times10^5\text{J}$.

(b) The force required is

$$F = ma = (106\,\text{kg})(4.82\,\text{m/s}^2) = 511\,\text{N}.$$

$x = (51.3\,\text{m/s})^2/[2(4.82\,\text{m/s}^2)] = 273\,\text{m}$. The work done is $W = Fx = (-511\,\text{N})(273\,\text{m}) = 1.40{\times}10^5\text{J}$.

E11-3 (a) $W = Fx = (120\,\text{N})(3.6\,\text{m}) = 430\,\text{J}$.

(b) $W = Fx\cos\theta = mgx\cos\theta = (25\,\text{kg})(9.8\,\text{m/s}^2)(3.6\,\text{m})\cos(117°) = 400\,\text{N}$.

(c) $W = Fx\cos\theta$, but $\theta = 90°$, so $W = 0$.

E11-4 The worker pushes with a force $\vec{\mathbf{P}}$; this force has components $P_x = P\cos\theta$ and $P_y = P\sin\theta$, where $\theta = -32.0°$. The normal force of the ground on the crate is $N = mg - P_y$, so the force of friction is $f = \mu_\text{k}N = \mu_\text{k}(mg - P_y)$. The crate moves at constant speed, so $P_x = f$. Then

$$\begin{aligned} P\cos\theta &= \mu_\text{k}(mg - P\sin\theta), \\ P &= \frac{\mu_\text{k}mg}{\cos\theta + \mu_\text{k}\sin\theta}. \end{aligned}$$

The work done on the crate is

$$\begin{aligned} W &= \vec{\mathbf{P}}\cdot\vec{\mathbf{x}} = Px\cos\theta = \frac{\mu_\text{k}xmg}{1 + \mu_\text{k}\tan\theta}, \\ &= \frac{(0.21)(31.3\text{ ft})(58.7\text{ lb})}{1 + (0.21)\tan(-32.0°)} = 444\text{ ft}\cdot\text{lb}. \end{aligned}$$

E11-5 The components of the weight are $W_{||} = mg\sin\theta$ and $W_\perp = mg\cos\theta$. The push $\vec{\mathbf{P}}$ has components $P_{||} = P\cos\theta$ and $P_\perp = P\sin\theta$.

The normal force on the trunk is $N = W_\perp + P\perp$ so the force of friction is $f = \mu_\text{k}(mg\cos\theta + P\sin\theta)$. The push required to move the trunk up at constant speed is then found by noting that $P_{||} = W_{||} + f$.

Then

$$P = \frac{mg(\tan\theta + \mu_\text{k})}{1 - \mu_\text{k}\tan\theta}.$$

(a) The work done by the applied force is

$$W = Px\cos\theta = \frac{(52.3\,\text{kg})(9.81\,\text{m/s}^2)[\sin(28.0°) + (0.19)\cos(28.0°)](5.95\,\text{m})}{1 - (0.19)\tan(28.0°)} = 2160\,\text{J}.$$

(b) The work done by the force of gravity is

$$W = mgx\cos(\theta + 90°) = (52.3\,\text{kg})(9.81\,\text{m/s}^2)(5.95\,\text{m})\cos(118°) = -1430\,\text{J}.$$

E11-6 $\theta = \arcsin(0.902\,\text{m}/1.62\,\text{m}) = 33.8°$.

The components of the weight are $W_{||} = mg\sin\theta$ and $W_\perp = mg\cos\theta$.

The normal force on the ice is $N = W_\perp$ so the force of friction is $f = \mu_\text{k} mg\cos\theta$. The push required to allow the ice to slide down at constant speed is then found by noting that $P = W_{||} - f$. Then $P = mg(\sin\theta - \mu_\text{k}\cos\theta)$.

(a) $P = (47.2\,\text{kg})(9.81\,\text{m/s}^2)[\sin(33.8°) - (0.110)\cos(33.8°)] = 215\,\text{N}$.

(b) The work done by the applied force is $W = Px = (215\,\text{N})(-1.62\,\text{m}) = -348\,\text{J}$.

(c) The work done by the force of gravity is

$$W = mgx\cos(90° - \theta) = (47.2\,\text{kg})(9.81\,\text{m/s}^2)(1.62\,\text{m})\cos(56.2°) = 417\,\text{J}.$$

$\boxed{\textbf{E11-7}}$ Equation 11-5 describes how to find the dot product of two vectors from components,

$$\begin{aligned}\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} &= a_x b_x + a_y b_y + a_z b_z,\\ &= (3)(2) + (3)(1) + (3)(3) = 18.\end{aligned}$$

Equation 11-3 can be used to find the angle between the vectors,

$$\begin{aligned}a &= \sqrt{(3)^2 + (3)^2 + (3)^2} = 5.19,\\ b &= \sqrt{(2)^2 + (1)^2 + (3)^2} = 3.74.\end{aligned}$$

Now use Eq. 11-3,

$$\cos\phi = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{ab} = \frac{(18)}{(5.19)(3.74)} = 0.927,$$

and then $\phi = 22.0°$.

E11-8 $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = (12)(5.8)\cos(55°) = 40.$

E11-9 $\vec{\mathbf{r}}\cdot\vec{\mathbf{s}} = (4.5)(7.3)\cos(320° - 85°) = -19.$

E11-10 (a) Add the components individually:

$$\vec{\mathbf{r}} = (5 + 2 + 4)\hat{\mathbf{i}} + (4 - 2 + 3)\hat{\mathbf{j}} + (-6 - 3 + 2)\hat{\mathbf{k}} = 11\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}}.$$

(b) $\theta = \arccos(-7/\sqrt{11^2 + 5^2 + 7^2}) = 120°$.

(c) $\theta = \arccos(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}/\sqrt{ab})$, or

$$\theta = \frac{(5)(-2) + (4)(2) + (-6)(3)}{\sqrt{(5^2 + 4^2 + 6^2)(2^2 + 2^2 + 3^2)}} = 124°.$$

$\boxed{\textbf{E11-11}}$ There are two forces on the woman, the force of gravity directed down and the normal force of the floor directed up. These will be effectively equal, so $N = W = mg$. Consequently, the 57 kg woman must exert a force of $F = (57\text{kg})(9.8\text{m/s}^2) = 560\text{N}$ to propel herself up the stairs.

From the reference frame of the woman the stairs are moving down, and she is exerting a force down, so the work done by the woman is given by

$$W = Fs = (560\text{ N})(4.5\text{ m}) = 2500\text{ J},$$

this work is positive because the force is in the same direction as the displacement.

The average power supplied by the woman is given by Eq. 11-7,

$$P = W/t = (2500\text{ J})/(3.5\text{ s}) = 710\text{ W}.$$

E11-12 $P = W/t = mgy/t = (100 \times 667\,\text{N})(152\,\text{m})/(55.0\,\text{s}) = 1.84\times10^5\text{W}$.

E11-13 $P = Fv = (110\,\text{N})(0.22\,\text{m/s}) = 24\,\text{W}$.

E11-14 $F = P/v$, but the units are awkward.

$$F = \frac{(4800\ \text{hp})}{(77\ \text{knots})}\frac{1\ \text{knot}}{1.688\ \text{ft/s}}\frac{1\ \text{ft/s}}{0.3048\ \text{m/s}}\frac{745.7\,\text{W}}{1\ \text{hp}} = 9.0\times10^4\text{N}.$$

E11-15 $P = Fv = (720\ \text{N})(26\ \text{m/s}) = 19000\ \text{W}$; in horsepower, $P = 19000\,\text{W}(1/745.7\ \text{hp/W}) =$ 25 hp.

E11-16 Change to metric units! Then $P = 4920\,\text{W}$, and the flow rate is $Q = 13.9\ \text{L/s}$. The density of water is approximately 1.00 kg/L , so the mass flow rate is $R = 13.9$ kg/s.

$$y = \frac{P}{gR} = \frac{(4920\,\text{kg})}{(9.81\,\text{m/s}^2)(13.9\ \text{kg/s})} = 36.1\,\text{m},$$

which is the same as approximately 120 feet.

E11-17 (a) Start by converting kilowatt-hours to Joules:

$$1\ \text{kW}\cdot\text{h} = (1000\ \text{W})(3600\ \text{s}) = 3.6\times10^6\ \text{J}.$$

The car gets 30 mi/gal, and one gallon of gas produces 140 MJ of energy. The gas required to produce 3.6×10^6J is

$$3.6\times10^6\text{J}\left(\frac{1\ \text{gal}}{140\times10^6\text{J}}\right) = 0.026\ \text{gal}.$$

The distance traveled on this much gasoline is

$$0.026\ \text{gal}\left(\frac{30\ \text{mi}}{1\ \text{gal}}\right) = 0.78\ \text{mi}.$$

(b) At 55 mi/h, it will take

$$0.78\ \text{mi}\left(\frac{1\ \text{hr}}{55\ \text{mi}}\right) = 0.014\ \text{h} = 51\ \text{s}.$$

The rate of energy expenditure is then $(3.6\times10^6\ \text{J})/(51\ \text{s}) = 71000$ W.

E11-18 The linear speed is $v = 2\pi(0.207\,\text{m})(2.53\ \text{rev/s}) = 3.29\,\text{m/s}$. The frictional force is $f = \mu_\text{k}N = (0.32)(180\,\text{N}) = 57.6\,\text{N}$. The power developed is $P = Fv = (57.6\,\text{N})(3.29\,\text{m}) = 190\,\text{W}$.

E11-19 The net force required will be $(1380\,\text{kg} - 1220\,\text{kg})(9.81\,\text{m/s}^2) = 1570\,\text{N}$. The work is $W = Fy$, the power output is $P = W/t = (1570\,\text{N})(54.5\,\text{m})/(43.0\,\text{s}) = 1990\,\text{W}$, or $P = 2.67$ hp.

E11-20 (a) The momentum change of the ejected material in one second is

$$\Delta p = (70.2\,\text{kg})(497\,\text{m/s} - 184\,\text{m/s}) + (2.92\,\text{kg})(497\,\text{m/s}) = 2.34\times10^4\text{kg}\cdot\text{m/s}.$$

The thrust is then $F = \Delta p/\Delta t = 2.34\times10^4\text{N}$.

(b) The power is $P = Fv = (2.34\times10^4\ \text{N})(184\,\text{m/s}) = 4.31\times10^6\text{W}$. That's 5780 hp.

$\boxed{\textbf{E11-21}}$ The acceleration on the object as a function of position is given by

$$a = \frac{20\ \text{m/s}^2}{8\ \text{m}}x,$$

The work done on the object is given by Eq. 11-14,

$$W = \int_0^8 F_x dx = \int_0^8 (10\ \text{kg})\frac{20\ \text{m/s}^2}{8\ \text{m}}x\ dx = 800\ \text{J}.$$

E11-22 Work is area between the curve and the line $F = 0$. Then

$$W = (10\,\text{N})(2\,\text{s}) + \frac{1}{2}(10\,\text{N})(2\,\text{s}) + \frac{1}{2}(-5\,\text{N})(2\,\text{s}) = 25\,\text{J}.$$

$\boxed{\textbf{E11-23}}$ (a) For a spring, $F = -kx$, and $\Delta F = -k\Delta x$.

$$k = -\frac{\Delta F}{\Delta x} = -\frac{(-240\ \text{N}) - (-110\ \text{N})}{(0.060\ \text{m}) - (0.040\ \text{m})} = 6500\ \text{N/m}.$$

With no force on the spring,

$$\Delta x = -\frac{\Delta F}{k} = -\frac{(0) - (-110\ \text{N})}{(6500\ \text{N/m})} = -0.017\ \text{m}.$$

This is the amount *less* than the 40 mm mark, so the position of the spring with no force on it is 23 mm.

(b) $\Delta x = -10$ mm compared to the 100 N picture, so

$$\Delta F = -k\Delta x = -(6500\,\text{N/m})(-0.010\,\text{m}) = 65\,\text{N}.$$

The weight of the last object is $110\,\text{N} - 65\,\text{N} = 45\,\text{N}$.

E11-24 (a) $W = \frac{1}{2}k({x_\text{f}}^2 - {x_\text{i}}^2) = \frac{1}{2}(1500\,\text{N/m})(7.60\times10^{-3}\text{m})^2 = 4.33\times10^{-2}\text{J}$.

(b) $W = \frac{1}{2}(1500\,\text{N/m})[(1.52\times10^{-2}\text{m})^2 - (7.60\times10^{-3}\text{m})^2 = 1.30\times10^{-1}\text{J}$.

E11-25 Start with Eq. 11-20, and let $F_x = 0$ while $F_y = -mg$:

$$W = \int_i^f (F_x dx + F_y dy) = -mg\int_i^f dy = -mgh.$$

E11-26 (a) $F_0 = mv_0^2/r_0 = (0.675\,\text{kg})(10.0\,\text{m/s})^2/(0.500\,\text{m}) = 135\,\text{N}$.

(b) Angular momentum is conserved, so $v = v_0(r_0/r)$. The force then varies as $F = mv^2/r = mv_0^2r_0^2/r^3 = F_0(r_0/r)^3$. The work done is

$$W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \frac{(-135\,\text{N})(0.500\,\text{m})^3}{-2}\left((0.300\,\text{m})^{-2} - (0.500\,\text{m})^{-2}\right) = 60.0\,\text{J}.$$

$\boxed{\textbf{E11-27}}$ The kinetic energy of the electron is

$$4.2\ \text{eV}\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 6.7\times10^{-19}\ \text{J}.$$

Then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.7\times10^{-19}\text{J})}{(9.1\times10^{-31}\text{kg})}} = 1.2\times10^6\text{m/s}.$$

E11-28 (a) $K = \frac{1}{2}(110\,\text{kg})(8.1\,\text{m/s})^2 = 3600\,\text{J}$.
(b) $K = \frac{1}{2}(4.2\times10^{-3}\text{kg})(950\,\text{m/s})^2 = 1900\,\text{J}$.
(c) $m = 91,400$ tons$(907.2\,\text{kg/ton}) = 8.29\times10^7\text{kg}$;

$$v = 32.0 \text{ knots}(1.688 \text{ ft/s/knot})(0.3048 \text{ m/ft}) = 16.5\,\text{m/s}.$$

$$K = \frac{1}{2}(8.29\times10^7\text{kg})(16.5\,\text{m/s})^2 = 1.13\times10^{10}\text{J}.$$

E11-29 (b) $\Delta K = W = Fx = (1.67\times10^{-27}\text{kg})(3.60\times10^{15}\text{m/s}^2)(0.0350\,\text{m}) = 2.10\times10^{-13}\text{J}$. That's $2.10\times10^{-13}\text{J}/(1.60\times10^{-19}\text{J/eV}) = 1.31\times10^6\text{eV}$.
(a) $K_\text{f} = 2.10\times10^{-13}\text{J} + \frac{1}{2}(1.67\times10^{-27}\text{kg})(2.40\times10^7\text{m/s}^2) = 6.91\times10^{-13}\text{J}$. Then

$$v_\text{f} = \sqrt{2K/m} = \sqrt{2(6.91\times10^{-13}\text{J})/(1.67\times10^{-27}\text{kg})} = 2.88\times10^7\text{m/s}.$$

E11-30 Work is negative if kinetic energy is decreasing. This happens only in region CD. The work is zero in region BC. Otherwise it is positive.

$\boxed{\textbf{E11-31}}$ (a) Find the velocity of the particle by taking the time derivative of the position:

$$v = \frac{dx}{dt} = (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)t + (3.0\,\text{m/s}^3)t^2.$$

Find v at two times: $t = 0$ and $t = 4\,\text{s}$.

$$\begin{aligned} v(0) &= (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)(0) + (3.0\,\text{m/s}^3)(0)^2 = 3.0\,\text{m/s}, \\ v(4) &= (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)(4.0\,\text{s}) + (3.0\,\text{m/s}^3)(4.0\,\text{s})^2 = 19.0\,\text{m/s} \end{aligned}$$

The initial kinetic energy is $K_\text{i} = \frac{1}{2}(2.80\,\text{kg})(3.0\,\text{m/s})^2 = 13\,\text{J}$, while the final kinetic energy is $K_\text{f} = \frac{1}{2}(2.80\,\text{kg})(19.0\,\text{m/s})^2 = 505\,\text{J}$.

The work done by the force is given by Eq. 11-24,

$$W = K_\text{f} - K_\text{i} = 505\,\text{J} - 13\,\text{J} = 492\,\text{J}.$$

(b) This question is asking for the instantaneous power when $t = 3.0\,\text{s}$. $P = Fv$, so first find a;

$$a = \frac{dv}{dt} = -(8.0\,\text{m/s}^2) + (6.0\,\text{m/s}^3)t.$$

Then the power is given by $P = mav$, and when $t = 3\,\text{s}$ this gives

$$P = mav = (2.80\,\text{kg})(10\,\text{m/s}^2)(6\,\text{m/s}) = 168\,\text{W}.$$

E11-32 $W = \Delta K = -K_\text{i}$. Then

$$W = -\frac{1}{2}(5.98\times10^{24}\text{kg})(29.8\times10^3\text{m/s})^2 = 2.66\times10^{33}\text{J}.$$

E11-33 (a) $K = \frac{1}{2}(1600\,\text{kg})(20\,\text{m/s})^2 = 3.2\times10^5\text{J}$.
(b) $P = W/t = (3.2\times10^5\text{J})/(33\,\text{s}) = 9.7\times10^3\text{W}$.
(c) $P = Fv = mav = (1600\,\text{kg})(20\,\text{m/s}/33.0\,\text{s})(20\,\text{m/s}) = 1.9\times10^4\text{W}$.

E11-34 (a) $I = 1.40\times10^4\text{u}\cdot\text{pm}^2(1.66\times10^{-27}\text{kg}/mboxu) = 2.32\times10^{-47}\text{kg}\cdot\text{m}^2$.
(b) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(2.32\times10^{-47}\text{kg}\cdot\text{m}^2)(4.30\times10^{12}\text{ rad/s})^2 = 2.14\times10^{-22}\text{J}$. That's 1.34 meV.

E11-35 The translational kinetic energy is $K_{\rm t} = \frac{1}{2}mv^2$, the rotational kinetic energy is $K_{\rm r} = \frac{1}{2}I\omega^2 = \frac{2}{3}K_{\rm t}$. Then

$$\omega = \sqrt{\frac{2m}{3I}}v = \sqrt{\frac{2(5.30\times10^{-26}\text{kg})}{3(1.94\times10^{-46}\text{kg}\cdot\text{m}^2)}}(500\,\text{m/s}) = 6.75\times10^{12}\text{ rad/s}.$$

E11-36 $K_{\rm r} = \frac{1}{2}I\omega^2 = \frac{1}{4}(512\,\text{kg})(0.976\,\text{m})^2(624\text{ rad/s})^2 = 4.75\times10^7\text{J}$.

(b) $t = W/P = (4.75\times10^7\text{J})/(8130\,\text{W}) = 5840\,\text{s}$, or 97.4 minutes.

$\boxed{\textbf{E11-37}}$ From Eq. 11-29, $K_{\rm i} = \frac{1}{2}I{\omega_{\rm i}}^2$. The object is a hoop, so $I = MR^2$. Then

$$K_{\rm i} = \frac{1}{2}MR^2\omega^2 = \frac{1}{2}(31.4\,\text{kg})(1.21\,\text{m})^2(29.6\,\text{rad/s})^2 = 2.01\times10^4\text{ J}.$$

Finally, the average power required to stop the wheel is

$$P = \frac{W}{t} = \frac{K_{\rm f} - K_{\rm i}}{t} = \frac{(0) - (2.01\times10^4\text{ J})}{(14.8\,\text{s})} = -1360\,\text{W}.$$

E11-38 The wheels are connected by a belt, so $r_A\omega_A = r_B\omega_B$, or $\omega_A = 3\omega_B$.

(a) If $l_A = l_B$ then

$$\frac{I_A}{I_B} = \frac{l_A/\omega_A}{l_B/\omega_B} = \frac{\omega_B}{\omega_A} = \frac{1}{3}.$$

(b) If instead $K_A = K_B$ then

$$\frac{I_A}{I_B} = \frac{2K_A/{\omega_A}^2}{2K_B/{\omega_B}^2} = \frac{{\omega_B}^2}{{\omega_A}^2} = \frac{1}{9}.$$

E11-39 (a) $\omega = 2\pi/T$, so

$$K = \frac{1}{2}I\omega^2 = \frac{4\pi^2}{5}\frac{(5.98\times10^{24}\text{kg})(6.37\times10^6\text{m})^2}{(86,400\,\text{s})^2} = 2.57\times10^{29}\text{J}$$

(b) $t = (2.57\times10^{29}\text{J})/(6.17\times10^{12}\text{W}) = 4.17\times10^{16}\text{s}$, or 1.3 billion years.

E11-40 (a) The velocities relative to the center of mass are $m_1v_1 = m_2v_2$; combine with $v_1 + v_2 = 910.0\,\text{m/s}$ and get

$$(290.0\,\text{kg})v_1 = (150.0\,\text{kg})(910\,\text{m/s} - v_1),$$

or

$$v_1 = (150\,\text{kg})(910\,\text{m/s})/(290\,\text{kg} + 150\,\text{kg}) = 310\,\text{m/s}$$

and $v_2 = 600\,\text{m/s}$. The rocket case was sent back, so $v_{\rm c} = 7600\,\text{m/s} - 310\,\text{m/s} = 7290\,\text{m/s}$. The payload capsule was sent forward, so $v_{\rm p} = 7600\,\text{m/s} + 600\,\text{m/s} = 8200\,\text{m/s}$.

(b) Before,

$$K_{\rm i} = \frac{1}{2}(290\,\text{kg} + 150\,\text{kg})(7600\,\text{m/s})^2 = 1.271\times10^{10}\text{J}.$$

After,

$$K_{\rm f} = \frac{1}{2}(290\,\text{kg})(7290\,\text{m/s})^2 + \frac{1}{2}(150\,\text{kg})(8200\,\text{m/s})^2 = 1.275\times10^{10}\text{J}.$$

The "extra" energy came from the spring.

$\boxed{\textbf{E11-41}}$ Let the mass of the freight car be M and the initial speed be $v_{\rm i}$. Let the mass of the caboose be m and the final speed of the coupled cars be $v_{\rm f}$. The caboose is originally at rest, so the expression of momentum conservation is

$$Mv_{\rm i} = Mv_{\rm f} + mv_{\rm f} = (M+m)v_{\rm f}$$

The decrease in kinetic energy is given by

$$\begin{aligned} K_{\rm i} - K_{\rm f} &= \frac{1}{2}M{v_{\rm i}}^2 - \left(\frac{1}{2}M{v_{\rm f}}^2 + \frac{1}{2}m{v_{\rm f}}^2\right), \\ &= \frac{1}{2}\left(M{v_{\rm i}}^2 - (M+m){v_{\rm f}}^2\right) \end{aligned}$$

What we really want is $(K_{\rm i} - K_{\rm f})/K_{\rm i}$, so

$$\begin{aligned} \frac{K_{\rm i} - K_{\rm f}}{K_{\rm i}} &= \frac{M{v_{\rm i}}^2 - (M+m){v_{\rm f}}^2}{M{v_{\rm i}}^2}, \\ &= 1 - \frac{M+m}{M}\left(\frac{v_{\rm f}}{v_{\rm i}}\right)^2, \\ &= 1 - \frac{M+m}{M}\left(\frac{M}{M+m}\right)^2, \end{aligned}$$

where in the last line we substituted from the momentum conservation expression.

Then

$$\frac{K_{\rm i} - K_{\rm f}}{K_{\rm i}} = 1 - \frac{M}{M+m} = 1 - \frac{Mg}{Mg+mg}.$$

The left hand side is 27%. We want to solve this for mg, the weight of the caboose. Upon rearranging,

$$mg = \frac{Mg}{1-0.27} - Mg = \frac{(35.0\ \text{ton})}{(0.73)} - (35.0\ \text{ton}) = 12.9\ \text{ton}.$$

E11-42 Since the body splits into two parts with equal mass then the velocity gained by one is identical to the velocity "lost" by the other. The initial kinetic energy is

$$K_{\rm i} = \frac{1}{2}(8.0\,\text{kg})(2.0\,\text{m/s})^2 = 16\,\text{J}.$$

The final kinetic energy is 16 J greater than this, so

$$\begin{aligned} K_{\rm f} &= 32\,\text{J} = \frac{1}{2}(4.0\,\text{kg})(2.0\,\text{m/s} + v)^2 + \frac{1}{2}(4.0\,\text{kg})(2.0\,\text{m/s} - v)^2, \\ &= \frac{1}{2}(8.0\,\text{kg})[(2.0\,\text{m/s})^2 + v^2], \end{aligned}$$

so $16.0\,\text{J} = (4.0\,\text{kg})v^2$. Then $v = 2.0\,\text{m/s}$; one chunk comes to a rest while the other moves off at a speed of 4.0 m/s.

E11-43 The initial velocity of the neutron is $v_0\hat{\mathbf{i}}$, the final velocity is $v_1\hat{\mathbf{j}}$. By momentum conservation the final momentum of the deuteron is $m_{\rm n}(v_0\hat{\mathbf{i}} - v_1\hat{\mathbf{j}})$. Then $m_{\rm d}v_2 = m_{\rm n}\sqrt{v_0^2 + v_1^2}$.

There is also conservation of kinetic energy:

$$\frac{1}{2}m_{\rm n}v_0^2 = \frac{1}{2}m_{\rm n}v_1^2 + \frac{1}{2}m_{\rm d}v_2^2.$$

Rounding the numbers slightly we have $m_{\rm d} = 2m_{\rm n}$, then $4v_2^2 = v_0^2 + v_1^2$ is the momentum expression and $v_0^2 = v_1^2 + 2v_2^2$ is the energy expression. Combining,

$$2v_0^2 = 2v_1^2 + (v_0^2 + v_1^2),$$

or $v_1^2 = v_0^2/3$. So the neutron is left with 1/3 of its original kinetic energy.

E11-44 (a) The third particle must have a momentum

$$\begin{aligned}\vec{\mathbf{p}}_3 &= -(16.7\times10^{-27}\text{kg})(6.22\times10^6\text{m/s})\hat{\mathbf{i}} + (8.35\times10^{-27}\text{kg})(7.85\times10^6\text{m/s})\hat{\mathbf{j}} \\ &= (-1.04\hat{\mathbf{i}} + 0.655\hat{\mathbf{j}})\times10^{-19}\text{kg}\cdot\text{m/s}.\end{aligned}$$

(b) The kinetic energy can also be written as $K = \frac{1}{2}mv^2 = \frac{1}{2}m(p/m)^2 = p^2/2m$. Then the kinetic energy appearing in this process is

$$\begin{aligned}K &= \frac{1}{2}(16.7\times10^{-27}\text{kg})(6.22\times10^6\text{m/s})^2 + \frac{1}{2}(8.35\times10^{-27}\text{kg})(7.85\times10^6\text{m/s})^2 \\ &\quad + \frac{1}{2(11.7\times10^{-27}\text{kg})}(1.23\times10^{-19}\text{kg}\cdot\text{m/s})^2 = 1.23\times10^{-12}\text{J}.\end{aligned}$$

This is the same as 7.66 MeV.

P11-1 Change your units! Then

$$F = \frac{W}{s} = \frac{(4.5\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{(3.4\times10^{-9}\text{ m})} = 2.1\times10^{-10}\text{ N}.$$

P11-2 (a) If the acceleration is $-g/4$ the the net force on the block is $-Mg/4$, so the tension in the cord must be $T = 3Mg/4$.

(a) The work done by the cord is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (3Mg/4)(-d) = -(3/4)Mgd$.

(b) The work done by gravity is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (-Mg)(-d) = Mgd$.

P11-3 (a) There are *four* cords which are attached to the bottom load L. Each supports a tension F, so to lift the load $4F = (840\text{ lb}) + (20.0\text{ lb})$, or $F = 215$ lb.

(b) The work done against gravity is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (840\text{ lb})(12.0\text{ ft}) = 10100\text{ ft}\cdot\text{lb}$.

(c) To lift the load 12 feet each segment of the cord must be shortened by 12 ft; there are four segments, so the end of the cord must be pulled through a distance of 48.0 ft.

(d) The work done by the applied force is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (215\text{ lb})(48.0\text{ ft}) = 10300\text{ ft}\cdot\text{lb}$.

P11-4 The incline has a height h where $h = W/mg = (680\,\text{J})/[(75\,\text{kg})(9.81\,\text{m/s}^2)]$. The work required to lift the block is the same regardless of the path, so the length of the incline l is $l = W/F = (680\,\text{J})/(320\,\text{N})$. The angle of the incline is then

$$\theta = \arcsin\frac{h}{l} = \arcsin\frac{F}{mg} = \arcsin\frac{(320\,\text{N})}{(75\,\text{kg})(9.81\,\text{m/s}^2)} = 25.8^\circ.$$

P11-5 (a) In 12 minutes the horse moves $x = (5280\text{ ft/mi})(6.20\text{ mi/h})(0.200\text{ h}) = 6550$ ft. The work done by the horse in that time is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (42.0\text{ lb})(6550\text{ ft})\cos(27.0^\circ) = 2.45\times10^5\text{ ft}\cdot\text{lb}$.

(b) The power output of the horse is

$$P = \frac{(2.45\times10^5\text{ ft}\cdot\text{lb})}{(720\,\text{s})} = 340\text{ ft}\cdot\text{lb/s}\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}} = 0.618\text{ hp}.$$

P11-6 In this problem $\theta = \arctan(28.2/39.4) = 35.5^\circ$.

The weight of the block has components $W_{||} = mg\sin\theta$ and $W_\perp = mg\cos\theta$. The force of friction on the block is $f = \mu_\text{k}N = \mu_\text{k}mg\cos\theta$. The tension in the rope must then be

$$T = mg(\sin\theta + \mu_\text{k}\cos\theta)$$

in order to move the block up the ramp. The power supplied by the winch will be $P = Tv$, so

$$P = (1380\,\text{kg})(9.81\,\text{m/s}^2)[\sin(35.5^\circ) + (0.41)\cos(35.5^\circ)](1.34\,\text{m/s}) = 1.66\times10^4\text{W}.$$

$\boxed{\textbf{P11-7}}$ If the power is constant then the force on the car is given by $F = P/v$. But the force is related to the acceleration by $F = ma$ and to the speed by $F = m\frac{dv}{dt}$ for motion in one dimension. Then

$$\begin{aligned}
F &= \frac{P}{v}, \\
m\frac{dv}{dt} &= \frac{P}{v}, \\
m\frac{dx}{dt}\frac{dv}{dx} &= \frac{P}{v}, \\
mv\frac{dv}{dx} &= \frac{P}{v}, \\
\int_0^v mv^2 dv &= \int_0^x P dx, \\
\frac{1}{3}mv^3 &= Px.
\end{aligned}$$

We can rearrange this final expression to get v as a function of x, $v = (3xP/m)^{1/3}$.

P11-8 (a) If the drag is $D = bv^2$, then the force required to move the plane forward at constant speed is $F = D = bv^2$, so the power required is $P = Fv = bv^3$.

(b) $P \propto v^3$, so if the speed increases to 125% then P increases by a factor of $1.25^3 = 1.953$, or increases by 95.3%.

P11-9 (a) $P = mgh/t$, but m/t is the persons per minute times the average mass, so

$$P = (100\text{ people/min})(75.0\text{kg})(9.81\,\text{m/s}^2)(8.20\,\text{m}) = 1.01\times 10^4\text{W}.$$

(b) In 9.50 s the Escalator has moved $(0.620\,\text{m/s})(9.50\,\text{s}) = 5.89\,\text{m}$; so the Escalator has "lifted" the man through a distance of $(5.89\,\text{m})(8.20\,\text{m}/13.3\,\text{m}) = 3.63\,\text{m}$. The man did the rest himself.

The work done by the Escalator is then $W = (83.5\,\text{kg})(9.81\,\text{m/s}^2)(3.63\,\text{m}) = 2970\,\text{J}$.

(c) Yes, because the point of contact is moving in a direction with at least some non-zero component of the force. The power is

$$P = (83.5\,\text{m/s}^2)(9.81\,\text{m/s}^2)(0.620\,\text{m/s})(8.20\,\text{m}/13.3\,\text{m}) = 313\,\text{W}.$$

(d) If there is a force of contact between the man and the Escalator then the Escalator is doing work on the man.

P11-10 (a) $dP/dv = ab - 3av^2$, so P_{max} occurs when $3v^2 = b$, or $v = \sqrt{b/3}$.

(b) $F = P/v$, so $dF/dv = -2v$, which means F is a maximum when $v = 0$.

(c) No; $P = 0$, but $F = ab$.

$\boxed{\textbf{P11-11}}$ (b) Integrate,

$$W = \int_0^{3x_0} \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}} = \frac{F_0}{x_0}\int_0^{3x_0}(x - x_0)dx = F_0 x_0\left(\frac{9}{2} - 3\right),$$

or $W = 3F_0x_0/2$.

P11-12 (a) Simpson's rule gives

$$W = \frac{1}{3}\left[(10\,\mathrm{N}) + 4(2.4\,\mathrm{N}) + (0.8\,\mathrm{N})\right](1.0\,\mathrm{m}) = 6.8\,\mathrm{J}.$$

(b) $W = \int F\,ds = \int (A/x^2)dx = -A/x$, evaluating this between the limits of integration gives $W = (9\,\mathrm{N}\cdot\mathrm{m}^2)(1/1\,\mathrm{m} - 1/3\,\mathrm{m}) = 6\,\mathrm{J}$.

$\boxed{\textbf{P11-13}}$ The work required to stretch the spring from x_i to x_f is given by

$$W = \int_{x_\mathrm{i}}^{x_\mathrm{f}} kx^3 dx = \frac{k}{4}{x_\mathrm{f}}^4 - \frac{k}{4}{x_\mathrm{i}}^4.$$

The problem gives

$$W_0 = \frac{k}{4}(l)^4 - \frac{k}{4}(0)^4 = \frac{k}{4}l^4.$$

We then want to find the work required to stretch from $x = l$ to $x = 2l$, so

$$\begin{aligned} W_{l\to 2l} &= \frac{k}{4}(2l)^4 - \frac{k}{4}(l)^4, \\ &= 16\frac{k}{4}l^4 - \frac{k}{4}l^4, \\ &= 15\frac{k}{4}l^4 = 15W_0. \end{aligned}$$

P11-14 (a) The spring extension is $\delta l = \sqrt{l_0^2 + x^2} - l_0$. The force from one spring has magnitude $k\delta l$, but only the x component contributes to the problem, so

$$F = 2k\left(\sqrt{l_0^2 + x^2} - l_0\right)\frac{x}{\sqrt{l_0^2 + x^2}}$$

is the force required to move the point.

The work required is the integral, $W = \int_0^x F\,dx$, which is

$$W = kx^2 - 2kl_0\sqrt{l_0^2 + x^2} + 2kl_0^2$$

Note that it does reduce to the expected behavior for $x \gg l_0$.

(b) Binomial expansion of square root gives

$$\sqrt{l_0^2 + x^2} = l_0\left(1 + \frac{1}{2}\frac{x^2}{l_0^2} - \frac{1}{8}\frac{x^4}{l_0^4}\cdots\right),$$

so the first term in the above expansion cancels with the last term in W; the second term cancels with the first term in W, leaving

$$W = \frac{1}{4}k\frac{x^4}{l_0^2}.$$

P11-15 Number the springs clockwise from the top of the picture. Then the four forces *on* each spring are

$$\begin{aligned} F_1 &= k(l_0 - \sqrt{x^2 + (l_0 - y)^2}), \\ F_2 &= k(l_0 - \sqrt{(l_0 - x)^2 + y^2}), \\ F_3 &= k(l_0 - \sqrt{x^2 + (l_0 + y)^2}), \\ F_4 &= k(l_0 - \sqrt{(l_0 + x)^2 + y^2}). \end{aligned}$$

The directions are *much* harder to work out, but for small x and y we can assume that

$$\begin{aligned}
\vec{\mathbf{F_1}} &= k(l_0 - \sqrt{x^2 + (l_0 - y)^2})\hat{\mathbf{j}}, \\
\vec{\mathbf{F_2}} &= k(l_0 - \sqrt{(l_0 - x)^2 + y^2})\hat{\mathbf{i}}, \\
\vec{\mathbf{F_3}} &= k(l_0 - \sqrt{x^2 + (l_0 + y)^2})\hat{\mathbf{j}}, \\
\vec{\mathbf{F_4}} &= k(l_0 - \sqrt{(l_0 + x)^2 + y^2})\hat{\mathbf{i}}.
\end{aligned}$$

Then

$$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \int (F_1 + F_3)dy + \int (F_2 + F_4)dx,$$

Since x and y are small, expand the force(s) in a binomial expansion:

$$F_1(x, y) \approx F_1(0, 0) + \left.\frac{\partial F_1}{\partial x}\right|_{x,y=0} x + \left.\frac{\partial F_1}{\partial y}\right|_{x,y=0} y = ky;$$

there will be similar expression for the other four forces. Then

$$W = \int 2ky\, dy + \int 2kx\, dx = k(x^2 + y^2) = kd^2.$$

P11-16 (a) $K_\mathrm{i} = \frac{1}{2}(1100\,\mathrm{kg})(12.8\,\mathrm{m/s})^2 = 9.0\times10^4\mathrm{J}$. Removing 51 kJ leaves 39 kJ behind, so

$$v_\mathrm{f} = \sqrt{2K_\mathrm{f}/m} = \sqrt{2(3.9\times10^4\mathrm{J})/(1100\,\mathrm{kg})} = 8.4\,\mathrm{m/s},$$

or 30 km/h.

(b) 39 kJ, as was found above.

$\boxed{\textbf{P11-17}}$ Let M be the mass of the man and m be the mass of the boy. Let v_M be the original speed of the man and v_m be the original speed of the boy. Then

$$\frac{1}{2}Mv_M^2 = \frac{1}{2}\left(\frac{1}{2}mv_m^2\right)$$

and

$$\frac{1}{2}M(v_M + 1.0\,\mathrm{m/s})^2 = \frac{1}{2}mv_m^2.$$

Combine these two expressions and solve for v_M,

$$\begin{aligned}
\frac{1}{2}Mv_M^2 &= \frac{1}{2}\left(\frac{1}{2}M(v_M + 1.0\,\mathrm{m/s})^2\right), \\
v_M^2 &= \frac{1}{2}(v_M + 1.0\,\mathrm{m/s})^2, \\
0 &= -v_M^2 + (2.0\,\mathrm{m/s})v_M + (1.0\,\mathrm{m/s})^2.
\end{aligned}$$

The last line can be solved as a quadratic, and $v_M = (1.0\,\mathrm{m/s}) \pm (1.41\,\mathrm{m/s})$. Now we use the very first equation to find the speed of the boy,

$$\begin{aligned}
\frac{1}{2}Mv_M^2 &= \frac{1}{2}\left(\frac{1}{2}mv_m^2\right), \\
v_M^2 &= \frac{1}{4}v_m^2, \\
2v_M &= v_m.
\end{aligned}$$

P11-18 (a) The work done by gravity on the projectile as it is raised up to 140 m is $W = -mgy = -(0.550\,\text{kg})(9.81\,\text{m/s}^2)(140\,\text{m}) = -755\,\text{J}$. Then the kinetic energy at the highest point is $1550\,\text{J} - 755\,\text{J} = 795\,\text{J}$. Since the projectile must be moving horizontally at the highest point, the horizontal velocity is $v_x = \sqrt{2(795\,\text{J})/(0.550\,\text{kg})} = 53.8\,\text{m/s}$.

(b) The magnitude of the velocity at launch is $v = \sqrt{2(1550\,\text{J})/(0.550\,\text{kg})} = 75.1\,\text{m/s}$. Then $v_y = \sqrt{(75.1\,\text{m/s})^2 - (53.8\,\text{m/s})^2} = 52.4\,\text{m/s}$.

(c) The kinetic energy at that point is $\frac{1}{2}(0.550\,\text{kg})[(65.0\,\text{m/s})^2 + (53.8\,\text{m/s})^2] = 1960\,\text{J}$. Since it has extra kinetic energy, it must be below the launch point, and gravity has done 410 J of work on it. That means it is $y = (410\,\text{J})/[(0.550\,\text{kg})(9.81\,\text{m/s}^2)] = 76.0\,\text{m}$ below the launch point.

P11-19 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(8.38\times 10^{11}\text{kg})(3.0\times 10^4\text{m/s})^2 = 3.77\times 10^{20}\text{J}$. In terms of TNT, $K = 9.0\times 10^4$ megatons.

(b) The diameter will be $\sqrt[3]{8.98\times 10^4} = 45$ km.

P11-20 (a) $W_\text{g} = -(0.263\,\text{kg})(9.81\,\text{m/s}^2)(-0.118\,\text{m}) = 0.304\,\text{J}$.

(b) $W_\text{s} = -\frac{1}{2}(252\,\text{N/m})(-0.118\,\text{m})^2 = -1.75\,\text{J}$.

(c) The kinetic energy just before hitting the block would be $1.75\,\text{J} - 0.304\,\text{J} = 1.45\,\text{J}$. The speed is then $v = \sqrt{2(1.45\,\text{J})/(0.263\,\text{kg})} = 3.32\,\text{m/s}$.

(d) Doubling the speed quadruples the initial kinetic energy to 5.78 J. The compression will then be given by

$$-5.78\,\text{J} = -\frac{1}{2}(252\,\text{N/m})y^2 - (0.263\,\text{kg})(9.81\,\text{m/s}^2)y,$$

with solution $y = 0.225\,\text{m}$.

P11-21 (a) We can solve this with a trick of integration.

$$\begin{aligned} W &= \int_0^x F\,dx, \\ &= \int_0^x ma_x \frac{dt}{dt}\,dx = ma_x \int_0^t \frac{dx}{dt}dt \\ &= ma_x \int_0^t v_x\,dt = ma_x \int_0^t at\,dt, \\ &= \frac{1}{2}ma_x^2t^2. \end{aligned}$$

Basically, we changed the variable of integration from x to t, and then used the fact the the acceleration was constant so $v_x = v_{0x} + a_x t$. The object started at rest so $v_{0x} = 0$, and we are given in the problem that $v_\text{f} = at_\text{f}$. Combining,

$$W = \frac{1}{2}ma_x^2t^2 = \frac{1}{2}m\left(\frac{v_\text{f}}{t_\text{f}}\right)^2 t^2.$$

(b) Instantaneous power will be the derivative of this, so

$$P = \frac{dW}{dt} = m\left(\frac{v_\text{f}}{t_\text{f}}\right)^2 t.$$

P11-22 (a) $\alpha = (-39.0 \text{ rev/s})(2\pi \text{ rad/rev})/(32.0\,\text{s}) = -7.66 \text{ rad/s}^2$.

(b) The total rotational inertia of the system about the axis of rotation is

$$I = (6.40\,\text{kg})(1.20\,\text{m})^2/12 + 2(1.06\,\text{kg})(1.20\,\text{m}/2)^2 = 1.53\,\text{kg}\cdot\text{m}^2.$$

The torque is then $\tau = (1.53\,\text{kg}\cdot\text{m}^2)(7.66 \text{ rad/s}^2) = 11.7\,\text{N}\cdot\text{m}$.

(c) $K = \frac{1}{2}(1.53\,\text{kg}\cdot\text{m}^2)(245 \text{ rad/s})^2 = 4.59\times 10^4\text{J}$.

(d) $\theta = \omega_{\text{av}} t = (39.0 \text{ rev/s}/2)(32.0\,\text{s}) = 624$ rev.

(e) Only the loss in kinetic energy is independent of the behavior of the frictional torque.

P11-23 The wheel turn with angular speed $\omega = v/r$, where r is the radius of the wheel and v the speed of the car. The total rotational kinetic energy in the four wheels is then

$$K\text{r} = 4\frac{1}{2}I\omega^2 = 2\left[\frac{1}{2}(11.3\,\text{kg})r^2\right]\left[\frac{v}{r}\right]^2 = (11.3\,\text{kg})v^2.$$

The translational kinetic energy is $K_{\text{t}} = \frac{1}{2}(1040\,\text{kg})v^2$, so the fraction of the total which is due to the rotation of the wheels is

$$\frac{11.3}{520 + 11.3} = 0.0213 \text{ or } 2.13\%.$$

P11-24 (a) Conservation of angular momentum: $\omega_{\text{f}} = (6.13\,\text{kg}\cdot\text{m}^2/1.97\,\text{kg}\cdot\text{m}^2)(1.22 \text{ rev/s}) = 3.80$ rev/s.

(b) $K_{\text{r}} \propto I\omega^2 \propto l^2/I$, so

$$K_{\text{f}}/K_{\text{i}} = I_{\text{i}}/I_{\text{f}} = (6.13\,\text{kg}\cdot\text{m}^2)/(1.97\,\text{kg}\cdot\text{m}^2) = 3.11.$$

P11-25 We did the first part of the solution in Ex. 10-21. The initial kinetic energy is (again, ignoring the shaft),

$$K_{\text{i}} = \frac{1}{2}I_1{\vec{\omega}_{1,\text{i}}}^2,$$

since the second wheel was originally at rest. The final kinetic energy is

$$K_{\text{f}} = \frac{1}{2}(I_1 + I_2){\vec{\omega}_{\text{f}}}^2,$$

since the two wheels moved as one. Then

$$\begin{aligned}
\frac{K_{\text{i}} - K_{\text{f}}}{K_{\text{i}}} &= \frac{\frac{1}{2}I_1{\vec{\omega}_{1,\text{i}}}^2 - \frac{1}{2}(I_1 + I_2){\vec{\omega}_{\text{f}}}^2}{\frac{1}{2}I_1{\vec{\omega}_{1,\text{i}}}^2}, \\
&= 1 - \frac{(I_1 + I_2){\vec{\omega}_{\text{f}}}^2}{I_1{\vec{\omega}_{1,\text{i}}}^2}, \\
&= 1 - \frac{I_1}{I_1 + I_2},
\end{aligned}$$

where in the last line we substituted from the results of Ex. 10-21.

Using the numbers from Ex. 10-21,

$$\frac{K_{\text{i}} - K_{\text{f}}}{K_{\text{i}}} = 1 - \frac{(1.27\,\text{kg}\cdot\text{m}^2)}{(1.27\,\text{kg}\cdot\text{m}^2) + (4.85\,\text{kg}\cdot\text{m}^2)} = 79.2\%.$$

P11-26 See the solution to P10-11.

$$K_{\mathrm{i}} = \frac{I}{2}\omega^2 + \frac{m}{2}v^2$$

while

$$K_{\mathrm{f}} = \frac{1}{2}(I + mR^2){\omega_{\mathrm{f}}}^2$$

according to P10-11,

$$\omega_{\mathrm{f}} = \frac{I\omega - mvR}{I + mR^2}.$$

Then

$$K_{\mathrm{f}} = \frac{1}{2}\frac{(I\omega - mvR)^2}{I + mR^2}.$$

Finally,

$$\begin{aligned}
\Delta K &= \frac{1}{2}\frac{(I\omega - mvR)^2 - (I + mR^2)(I\omega^2 + mv^2)}{I + mR^2}, \\
&= -\frac{1}{2}\frac{ImR^2\omega^2 + 2mvRI\omega + Imv^2}{I + mR^2}, \\
&= -\frac{Im}{2}\frac{(R\omega + v)^2}{I + mR^2}.
\end{aligned}$$

P11-27 See the solution to P10-12.

(a) $K_{\mathrm{i}} = \frac{1}{2}I_{\mathrm{i}}{\omega_{\mathrm{i}}}^2$, so

$$K_{\mathrm{i}} = \frac{1}{2}\left(2(51.2\,\mathrm{kg})(1.46\,\mathrm{m})^2\right)(0.945\ \mathrm{rad/s})^2 = 97.5\,\mathrm{J}.$$

(b) $K_{\mathrm{f}} = \frac{1}{2}\left(2(51.2\,\mathrm{kg})(0.470\,\mathrm{m})^2\right)(9.12\ \mathrm{rad/s})^2 = 941\,\mathrm{J}$. The energy comes from the work they do while pulling themselves closer together.

P11-28 $K = \frac{1}{2}mv^2 = \frac{1}{2m}p^2 = \frac{1}{2m}\vec{\mathbf{p}}\cdot\vec{\mathbf{p}}$. Then

$$\begin{aligned}
K_{\mathrm{f}} &= \frac{1}{2m}(\vec{\mathbf{p}}_{\mathrm{i}} + \Delta\vec{\mathbf{p}})\cdot(\vec{\mathbf{p}}_{\mathrm{i}} + \Delta\vec{\mathbf{p}}), \\
&= \frac{1}{2m}\left({p_{\mathrm{i}}}^2 + 2\vec{\mathbf{p}}_{\mathrm{i}}\cdot\Delta\vec{\mathbf{p}} + (\Delta p)^2\right), \\
\Delta K &= \frac{1}{2m}\left(2\vec{\mathbf{p}}_{\mathrm{i}}\cdot\Delta\vec{\mathbf{p}} + (\Delta p)^2\right).
\end{aligned}$$

In all three cases $\Delta p = (3000\,\mathrm{N})(65.0\,\mathrm{s}) = 1.95\times10^5\mathrm{N}\cdot\mathrm{s}$ and $p_{\mathrm{i}} = (2500\,\mathrm{kg})(300\,\mathrm{m/s}) = 7.50\times10^5\mathrm{kg}\cdot\mathrm{m/s}$.

(a) If the thrust is backward (pushing rocket forward),

$$\Delta K = \frac{+2(7.50\times10^5\mathrm{kg}\cdot\mathrm{m/s})(1.95\times10^5\mathrm{N}\cdot\mathrm{s}) + (1.95\times10^5\mathrm{N}\cdot\mathrm{s})^2}{2(2500\,\mathrm{kg})} = +6.61\times10^7\mathrm{J}.$$

(b) If the thrust is forward,

$$\Delta K = \frac{-2(7.50\times10^5\mathrm{kg}\cdot\mathrm{m/s})(1.95\times10^5\mathrm{N}\cdot\mathrm{s}) + (1.95\times10^5\mathrm{N}\cdot\mathrm{s})^2}{2(2500\,\mathrm{kg})} = -5.09\times10^7\mathrm{J}.$$

(c) If the thrust is sideways the first term vanishes,

$$\Delta K = \frac{+(1.95\times10^5\mathrm{N}\cdot\mathrm{s})^2}{2(2500\,\mathrm{kg})} = 7.61\times10^6\mathrm{J}.$$

$\boxed{\textbf{P11-29}}$ There's nothing to integrate here! Start with the work-energy theorem

$$\begin{aligned} W &= K_\mathrm{f} - K_\mathrm{i} = \frac{1}{2} m {v_\mathrm{f}}^2 - \frac{1}{2} m {v_\mathrm{i}}^2, \\ &= \frac{1}{2} m \left({v_\mathrm{f}}^2 - {v_\mathrm{i}}^2 \right), \\ &= \frac{1}{2} m \left(v_\mathrm{f} - v_\mathrm{i} \right) \left(v_\mathrm{f} + v_\mathrm{i} \right), \end{aligned}$$

where in the last line we factored the difference of two squares. Continuing,

$$\begin{aligned} W &= \frac{1}{2} \left(m v_\mathrm{f} - m v_\mathrm{i} \right) \left(v_\mathrm{f} + v_\mathrm{i} \right), \\ &= \frac{1}{2} \left(\Delta p \right) \left(v_\mathrm{f} + v_\mathrm{i} \right), \end{aligned}$$

but $\Delta p = J$, the impulse. That finishes this problem.

P11-30 Let M be the mass of the helicopter. It will take a force Mg to keep the helicopter airborne. This force comes from pushing the air down at a rate $\Delta m/\Delta t$ with a speed of v; so $Mg = v\Delta m/\Delta t$. The blades sweep out a cylinder of cross sectional area A, so $\Delta m/\Delta t = \rho A v$. The force is then $Mg = \rho A v^2$; the speed that the air must be pushed down is $v = \sqrt{Mg/\rho A}$. The minimum power is then

$$P = Fv = Mg\sqrt{\frac{Mg}{\rho A}} = \sqrt{\frac{(1820\,\mathrm{kg})^3 (9.81\,\mathrm{m/s^2})^3}{(1.23\,\mathrm{kg/m^3})\pi(4.88\,\mathrm{m})^2}} = 2.49\times 10^5\,\mathrm{W}.$$

P11-31 (a) Inelastic collision, so $v_\mathrm{f} = m v_\mathrm{i}/(m+M)$.

(b) $K = \frac{1}{2} m v^2 = p^2/2m$, so

$$\frac{\Delta K}{K_\mathrm{i}} = \frac{1/m - 1/(m+M)}{1/m} = \frac{M}{m+M}.$$

P11-32 Inelastic collision, so

$$v_\mathrm{f} = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s}) + (4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})}{(1.88\,\mathrm{kg}) + (4.92\,\mathrm{kg})} = 5.21\,\mathrm{m/s}.$$

The loss in kinetic energy is

$$\Delta K = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s})^2}{2} + \frac{(4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})^2}{2} - \frac{(1.88\,\mathrm{kg} + 4.92\,\mathrm{kg})(5.21\,\mathrm{m/s})^2}{2} = 33.7\,\mathrm{J}.$$

This change is because of work done on the spring, so

$$x = \sqrt{2(33.7\,\mathrm{J})/(1120\,\mathrm{N/m})} = 0.245\,\mathrm{m}$$

P11-33 $\vec{\mathbf{p}}_{\mathrm{f},B} = \vec{\mathbf{p}}_{\mathrm{i},A} + \vec{\mathbf{p}}_{\mathrm{i},B} - \vec{\mathbf{p}}_{\mathrm{f},A}$, so

$$\begin{aligned} \vec{\mathbf{p}}_{\mathrm{f},B} &= [(2.0\,\mathrm{kg})(15\,\mathrm{m/s}) + (3.0\,\mathrm{kg})(-10\,\mathrm{m/s}) - (2.0\,\mathrm{kg})(-6.0\,\mathrm{m/s})]\hat{\mathbf{i}} \\ &\quad + [(2.0\,\mathrm{kg})(30\,\mathrm{m/s}) + (3.0\,\mathrm{kg})(5.0\,\mathrm{m/s}) - (2.0\,\mathrm{kg})(30\,\mathrm{m/s})]\hat{\mathbf{j}}, \\ &= (12\,\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{i}} + (15\,\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{j}}. \end{aligned}$$

Then $\vec{\mathbf{v}}_{f,B} = (4.0\,\text{m/s})\hat{\mathbf{i}} + (5.0\,\text{m/s})\hat{\mathbf{j}}$. Since $K = \frac{m}{2}(v_x^2 + v_y^2)$, the change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{(2.0\,\text{kg})[(-6.0\,\text{m/s})^2 + (30\,\text{m/s})^2 - (15\,\text{m/s})^2 - (30\,\text{m/s})^2]}{2} \\ &\quad + \frac{(3.0\,\text{kg})[(4.0\,\text{m/s})^2 + (5\,\text{m/s})^2 - (-10\,\text{m/s})^2 - (5.0\,\text{m/s})^2]}{2} \\ &= -315\,\text{J}.\end{aligned}$$

P11-34 For the observer on the train the acceleration of the particle is a, the distance traveled is $x_\text{t} = \frac{1}{2}at^2$, so the work done as measured by the train is $W_\text{t} = max_\text{t} = \frac{1}{2}a^2t^2$. The final speed of the particle as measured by the train is $v_\text{t} = at$, so the kinetic energy as measured by the train is $K = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2$. The particle started from rest, so $\Delta K_\text{t} = W_\text{t}$.

For the observer on the ground the acceleration of the particle is a, the distance traveled is $x_\text{g} = \frac{1}{2}at^2 + ut$, so the work done as measured by the ground is $W_\text{g} = max_\text{g} = \frac{1}{2}a^2t^2 + maut$. The final speed of the particle as measured by the ground is $v_\text{g} = at + u$, so the kinetic energy as measured by the ground is

$$K_\text{g} = \frac{1}{2}mv^2 = \frac{1}{2}m(at+u)^2 = \frac{1}{2}a^2t^2 + maut + \frac{1}{2}mu^2.$$

But the initial kinetic energy as measured by the ground is $\frac{1}{2}mu^2$, so $W_\text{g} = \Delta K_\text{g}$.

P11-35 (a) $K_\text{i} = \frac{1}{2}m_1{v_{1,\text{i}}}^2$.

(b) After collision $v_\text{f} = m_1v_{1,\text{i}}/(m_1+m_2)$, so

$$K_\text{f} = \frac{1}{2}(m_1+m_2)\left(\frac{m_1v_{1,\text{i}}}{m_1+m_2}\right)^2 = \frac{1}{2}m_1{v_{1,\text{i}}}^2\left(\frac{m_1}{m_1+m_2}\right).$$

(c) The fraction lost was

$$1 - \frac{m_1}{m_1+m_2} = \frac{m_2}{m_1+m_2}.$$

(d) Note that $v_\text{cm} = v_\text{f}$. The initial kinetic energy of the system is

$$K_\text{i} = \frac{1}{2}m_1{v'_{1,\text{i}}}^2 + \frac{1}{2}m_2{v'_{2,\text{i}}}^2.$$

The final kinetic energy is zero (they stick together!), so the fraction lost is 1. The *amount* lost, however, is the same.

P11-36 Only consider the first two collisions, the one between m and m', and then the one between m' and M.

Momentum conservation applied to the first collision means the speed of m' will be between $v' = mv_0/(m+m')$ (completely inelastic)and $v' = 2mv_0/(m+m')$ (completely elastic). Momentum conservation applied to the second collision means the speed of M will be between $V = m'v'/(m'+M)$ and $V = 2m'v'/(m'+M)$. The largest kinetic energy for M will occur when it is moving the fastest, so

$$v' = \frac{2mv_0}{m+m'} \text{ and } V = \frac{2m'v'}{m'+M} = \frac{4m'mv_0}{(m+m')(m'+M)}.$$

We want to maximize V as a function of m', so take the derivative:

$$\frac{dV}{dm'} = \frac{4mv_0(mM - m'^2)}{(m'+M)^2(m+m')^2}.$$

This vanishes when $m' = \sqrt{mM}$.

E12-1 (a) Integrate.

$$U(x) = -\int_{\infty}^{x} G\frac{m_1 m_2}{x^2}dx = -G\frac{m_1 m_2}{x}.$$

(b) $W = U(x) - U(x+d)$, so

$$W = Gm_1m_2\left(\frac{1}{x} - \frac{1}{x+d}\right) = Gm_1m_2\frac{d}{x(x+d)}.$$

E12-2 If $d << x$ then $x(x+d) \approx x^2$, so

$$W \approx G\frac{m_1m_2}{x^2}d.$$

$\boxed{\textbf{E12-3}}$ Start with Eq. 12-6.

$$\begin{aligned} U(x) - U(x_0) &= -\int_{x_0}^{x} F_x(x)dx, \\ &= -\int_{x_0}^{x}\left(-\alpha x e^{-\beta x^2}\right)dx, \\ &= \left.\frac{-\alpha}{2\beta}e^{-\beta x^2}\right|_{x_0}^{x}. \end{aligned}$$

Finishing the integration,

$$U(x) = U(x_0) + \frac{\alpha}{2\beta}\left(e^{-\beta x_0^2} - e^{-\beta x^2}\right).$$

If we *choose* $x_0 = \infty$ *and* $U(x_0) = 0$ we would be left with

$$U(x) = -\frac{\alpha}{2\beta}e^{-\beta x^2}.$$

E12-4 $\Delta K = -\Delta U$ so $\Delta K = mg\Delta y$. The power output is then

$$P = (58\%)\frac{(1000\,\text{kg/m}^3)(73,800\,\text{m}^3)}{(60\,\text{s})}(9.81\,\text{m/s}^2)(96.3\,\text{m}) = 6.74\times10^8\text{W}.$$

E12-5 $\Delta U = -\Delta K$, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$. Then

$$k = \frac{mv^2}{x^2} = \frac{(2.38\,\text{kg})(10.0\times10^3\,/\text{s})}{(1.47\,\text{m})^2} = 1.10\times10^8\text{N/m}.$$

Wow.

E12-6 $\Delta U_\text{g} + \Delta U_\text{s} = 0$, since $K = 0$ when the man jumps and when the man stops. Then $\Delta U_\text{s} = -mg\Delta y = (220\text{ lb})(40.4\text{ ft}) = 8900\text{ ft}\cdot\text{lb}$.

$\boxed{\textbf{E12-7}}$ Apply Eq. 12-15,

$$\begin{aligned} K_\text{f} + U_\text{f} &= K_\text{i} + U_\text{i}, \\ \frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} &= \frac{1}{2}m{v_\text{i}}^2 + mgy_\text{i}, \\ \frac{1}{2}{v_\text{f}}^2 + g(-r) &= \frac{1}{2}(0)^2 + g(0). \end{aligned}$$

Rearranging,

$$v_\text{f} = \sqrt{-2g(-r)} = \sqrt{-2(9.81\,\text{m/s}^2)(-0.236\,\text{m})} = 2.15\,\text{m/s}.$$

E12-8 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(2.40\,\text{kg})(150\,\text{m/s})^2 = 2.70\times 10^4\text{J}$.

(b) Assuming that the ground is zero, $U = mgy = (2.40\,\text{kg})(9.81\,\text{m/s}^2)(125\,\text{m}) = 2.94\times 10^3\text{J}$.

(c) $K_\text{f} = K_\text{i} + U_\text{i}$ since $U_\text{f} = 0$. Then

$$v_\text{f} = \sqrt{2\frac{(2.70\times 10^4\text{J}) + (2.94\times 10^3\text{J})}{(2.40\,\text{kg})}} = 158\,\text{m/s}.$$

Only (a) and (b) depend on the mass.

E12-9 (a) Since $\Delta y = 0$, then $\Delta U = 0$ and $\Delta K = 0$. Consequently, at B, $v = v_0$.

(b) At C $K_C = K_A + U_A - U_C$, or

$$\frac{1}{2}m{v_C}^2 = \frac{1}{2}m{v_0}^2 + mgh - mg\frac{h}{2},$$

or

$$v_B = \sqrt{{v_0}^2 + 2g\frac{h}{2}} = \sqrt{{v_0}^2 + gh}.$$

(c) At D $K_D = K_A + U_A - U_D$, or

$$\frac{1}{2}m{v_D}^2 = \frac{1}{2}m{v_0}^2 + mgh - mg(0),$$

or

$$v_B = \sqrt{{v_0}^2 + 2gh}.$$

E12-10 From the slope of the graph, $k = (0.4\,\text{N})/(0.04\,\text{m}) = 10\,\text{N/m}$.

(a) $\Delta K = -\Delta U$, so $\frac{1}{2}m{v_\text{f}}^2 = \frac{1}{2}k{x_\text{i}}^2$, or

$$v_\text{f} = \sqrt{\frac{(10\,\text{N/m})}{(0.00380\,\text{kg})}}(0.0550\,\text{m}) = 2.82\,\text{m/s}.$$

(b) $\Delta K = -\Delta U$, so $\frac{1}{2}m{v_\text{f}}^2 = \frac{1}{2}k({x_\text{i}}^2 - {x_\text{f}}^2)$, or

$$v_\text{f} = \sqrt{\frac{(10\,\text{N/m})}{(0.00380\,\text{kg})}\left[(0.0550\,\text{m})^2 - (0.0150\,\text{m})^2\right]} = 2.71\,\text{m/s}.$$

$\boxed{\textbf{E12-11}}$ (a) The force constant of the spring is

$$k = F/x = mg/x = (7.94\,\text{kg})(9.81\,\text{m/s}^2)/(0.102\,\text{m}) = 764\,\text{N/m}.$$

(b) The potential energy stored in the spring is given by Eq. 12-8,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(764\,\text{N/m})(0.286\,\text{m} + 0.102\,\text{m})^2 = 57.5\,\text{J}.$$

(c) Conservation of energy,

$$\begin{aligned} K_\text{f} + U_\text{f} &= K_\text{i} + U_\text{i}, \\ \frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} + \frac{1}{2}k{x_\text{f}}^2 &= \frac{1}{2}m{v_\text{i}}^2 + mgy_\text{i} + \frac{1}{2}k{x_\text{i}}^2, \\ \frac{1}{2}(0)^2 + mgh + \frac{1}{2}k(0)^2 &= \frac{1}{2}(0)^2 + mg(0) + \frac{1}{2}k{x_\text{i}}^2. \end{aligned}$$

Rearranging,

$$h = \frac{k}{2mg}{x_\text{i}}^2 = \frac{(764\,\text{N/m})}{2(7.94\,\text{kg})(9.81\,\text{m/s}^2)}(0.388\,\text{m})^2 = 0.738\,\text{m}.$$

E12-12 The annual mass of water is $m = (1000\,\mathrm{kg/m^3})(8\times10^{12}\mathrm{m^2})(0.75\,\mathrm{m})$. The change in potential energy each year is then $\Delta U = -mgy$, where $y = -500\,\mathrm{m}$. The power available is then

$$P = \frac{1}{3}(1000\,\mathrm{kg/m^3})(8\times10^{12}\mathrm{m^2})\frac{(0.75\,\mathrm{m})}{(3.15\times10^7\mathrm{m})}(500\,\mathrm{m}) = 3.2\times10^7\mathrm{W}.$$

E12-13 (a) From kinematics, $v = -gt$, so $K = \frac{1}{2}mg^2t^2$ and $U = U_0 - K = mgh - \frac{1}{2}mg^2t^2$.
(b) $U = mgy$ so $K = U_0 - U = mg(h-y)$.

E12-14 The potential energy is the same in both cases. Consequently, $mg_\mathrm{E}\Delta y_\mathrm{E} = mg_\mathrm{M}\Delta y_\mathrm{M}$, and then

$$y_\mathrm{M} = (2.05\,\mathrm{m} - 1.10\,\mathrm{m})(9.81\,\mathrm{m/s^2})/(1.67\,\mathrm{m/s^2}) + 1.10\,\mathrm{m} = 6.68\,\mathrm{m}.$$

$\boxed{\textbf{E12-15}}$ The working is identical to Ex. 12-11,

$$\begin{aligned}
K_\mathrm{f} + U_\mathrm{f} &= K_\mathrm{i} + U_\mathrm{i},\\
\frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f} + \frac{1}{2}k{x_\mathrm{f}}^2 &= \frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i} + \frac{1}{2}k{x_\mathrm{i}}^2,\\
\frac{1}{2}(0)^2 + mgh + \frac{1}{2}k(0)^2 &= \frac{1}{2}(0)^2 + mg(0) + \frac{1}{2}k{x_\mathrm{i}}^2,
\end{aligned}$$

so

$$h = \frac{k}{2mg}{x_\mathrm{i}}^2 = \frac{(2080\,\mathrm{N/m})}{2(1.93\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}(0.187\,\mathrm{m})^2 = 1.92\,\mathrm{m}.$$

The distance up the incline is given by a trig relation,

$$d = h/\sin\theta = (1.92\,\mathrm{m})/\sin(27^\circ) = 4.23\,\mathrm{m}.$$

E12-16 The vertical position of the pendulum is $y = -l\cos\theta$, where θ is measured from the downward vertical and l is the length of the string. The total mechanical energy of the pendulum is

$$E = \frac{1}{2}m{v_\mathrm{b}}^2$$

if we set $U = 0$ at the bottom of the path and v_b is the speed at the bottom. In this case $U = mg(l+y)$.
(a) $K = E - U = \frac{1}{2}m{v_\mathrm{b}}^2 - mgl(1-\cos\theta)$. Then

$$v = \sqrt{(8.12\,\mathrm{m/s})^2 - 2(9.81\,\mathrm{m/s^2})(3.82\,\mathrm{kg})(1-\cos 58.0^\circ)} = 5.54\,\mathrm{m/s}.$$

(b) $U = E - K$, but at highest point $K = 0$. Then

$$\theta = \arccos\left(1 - \frac{1}{2}\frac{(8.12\,\mathrm{m/s})^2}{(9.81\,\mathrm{m/s^2})(3.82\,\mathrm{kg})}\right) = 83.1^\circ.$$

(c) $E = \frac{1}{2}(1.33\,\mathrm{kg})(8.12\,\mathrm{m/s})^2 = 43.8\,\mathrm{J}$.

E12-17 The equilibrium position is when $F = ky = mg$. Then $\Delta U_\mathrm{g} = -mgy$ and $\Delta U_\mathrm{s} = \frac{1}{2}(ky)y = \frac{1}{2}mgy$. So $2\Delta U_\mathrm{s} = -\Delta U_\mathrm{g}$.

E12-18 Let the spring get compressed a distance x. If the object fell from a height $h = 0.436\,\text{m}$, then conservation of energy gives $\frac{1}{2}kx^2 = mg(x+h)$. Solving for x,

$$x = \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + 2\frac{mg}{k}h}$$

only the positive answer is of interest, so

$$x = \frac{(2.14\,\text{kg})(9.81\,\text{m/s}^2)}{(1860\,\text{N/m})} \pm \sqrt{\left(\frac{(2.14\,\text{kg})(9.81\,\text{m/s}^2)}{(1860\,\text{N/m})}\right)^2 + 2\frac{(2.14\,\text{kg})(9.81\,\text{m/s}^2)}{(1860\,\text{N/m})}(0.436\,\text{m})} = 0.111\,\text{m}.$$

$\boxed{\textbf{E12-19}}$ The horizontal distance traveled by the marble is $R = vt_\text{f}$, where t_f is the time of flight and v is the speed of the marble when it leaves the gun. We find *that* speed using energy conservation principles applied to the spring just before it is released and just after the marble leaves the gun.

$$\begin{aligned} K_\text{i} + U_\text{i} &= K_\text{f} + U_\text{f}, \\ 0 + \frac{1}{2}kx^2 &= \frac{1}{2}mv^2 + 0. \end{aligned}$$

$K_\text{i} = 0$ because the marble isn't moving originally, and $U_\text{f} = 0$ because the spring is no longer compressed. Substituting R into this,

$$\frac{1}{2}kx^2 = \frac{1}{2}m\left(\frac{R}{t_\text{f}}\right)^2.$$

We have two values for the compression, x_1 and x_2, and two ranges, R_1 and R_2. We can put both pairs into the above equation and get two expressions; if we divide one expression by the other we get

$$\left(\frac{x_2}{x_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^2.$$

We can easily take the square root of both sides, then

$$\frac{x_2}{x_1} = \frac{R_2}{R_1}.$$

R_1 was Bobby's try, and was equal to $2.20 - 0.27 = 1.93\,\text{m}$. $x_1 = 1.1$ cm was his compression. If Rhoda wants to score, she wants $R_2 = 2.2\,\text{m}$, then

$$x_2 = \frac{2.2\,\text{m}}{1.93\,\text{m}} 1.1 \text{ cm} = 1.25 \text{ cm}.$$

E12-20 Conservation of energy— $U_1 + K_1 = U_2 + K_2$— but $U_1 = mgh$, $K_1 = 0$, and $U_2 = 0$, so $K_2 = \frac{1}{2}mv^2 = mgh$ at the bottom of the swing.

The net force on Tarzan at the bottom of the swing is $F = mv^2/r$, but this net force is equal to the tension T minus the weight $W = mg$. Then $2mgh/r = T - mg$. Rearranging,

$$T = (180 \text{ lb})\left(\frac{2(8.5 \text{ ft})}{(50 \text{ ft})} + 1\right) = 241 \text{ lb}.$$

This isn't enough to break the vine, but it is close.

E12-21 Let point 1 be the start position of the first mass, point 2 be the collision point, and point 3 be the highest point in the swing after the collision. Then $U_1 = K_2$, or $\frac{1}{2}m_1{v_1}^2 = m_1 gd$, where v_1 is the speed of m_1 just before it collides with m_2. Then $v_1 = \sqrt{2gd}$.

After the collision the speed of both objects is, by momentum conservation, $v_2 = m_1v_1/(m_1+m_2)$.

Then, by energy conservation, $U_3 = K_2'$, or $\frac{1}{2}(m_1+m_2){v_2}^2 = (m_1+m_2)gy$, where y is the height to which the combined masses rise.

Combining,

$$y = \frac{{v_2}^2}{2g} = \frac{{m_1}^2{v_1}^2}{2(m_1+m_2)^2 g} = \left(\frac{m_1}{m_1+m_2}\right)^2 d.$$

E12-22 $\Delta K = -\Delta U$, so

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh,$$

where $I = \frac{1}{2}MR^2$ and $\omega = v/R$. Combining,

$$\frac{1}{2}mv^2 + \frac{1}{4}Mv^2 = mgh,$$

so

$$v = \sqrt{\frac{4mgh}{2m+M}} = \sqrt{\frac{4(0.0487\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.540\,\mathrm{m})}{2(0.0487\,\mathrm{kg}) + (0.396\,\mathrm{kg})}} = 1.45\,\mathrm{m/s}.$$

$\boxed{\textbf{E12-23}}$ There are *three* contributions to the kinetic energy: rotational kinetic energy of the shell (K_s), rotational kinetic energy of the pulley (K_p), and translational kinetic energy of the block (K_b). The conservation of energy statement is then

$$\begin{aligned} K_{\mathrm{s,i}} + K_{\mathrm{p,i}} + K_{\mathrm{b,i}} + U_\mathrm{i} &= K_{\mathrm{s,f}} + K_{\mathrm{p,f}} + K_{\mathrm{b,f}} + U_\mathrm{f}, \\ (0) + (0) + (0) + (0) &= \frac{1}{2}I_\mathrm{s}{\omega_\mathrm{s}}^2 + \frac{1}{2}I_\mathrm{p}{\omega_\mathrm{p}}^2 + \frac{1}{2}m{v_\mathrm{b}}^2 + mgy. \end{aligned}$$

Finally, $y = -h$ and

$$\omega_\mathrm{s}R = \omega_\mathrm{p}r = v_\mathrm{b}.$$

Combine all of this together, and our energy conservation statement will look like this:

$$0 = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v_\mathrm{b}}{R}\right)^2 + \frac{1}{2}I_\mathrm{p}\left(\frac{v_\mathrm{b}}{r}\right)^2 + \frac{1}{2}m{v_\mathrm{b}}^2 - mgh$$

which can be fairly easily rearranged into

$${v_\mathrm{b}}^2 = \frac{2mgh}{2M/3 + I_\mathrm{P}/r^2 + m}.$$

E12-24 The angular speed of the flywheel and the speed of the car are related by

$$k = \frac{\omega}{v} = \frac{(1490\ \mathrm{rad/s})}{(24.0\,\mathrm{m/s})} = 62.1\ \mathrm{rad/m}.$$

The height of the slope is $h = (1500\,\mathrm{m})\sin(5.00°) = 131\,\mathrm{m}$. The rotational inertia of the flywheel is

$$I = \frac{1}{2}\frac{(194\,\mathrm{N})}{(9.81\,\mathrm{m/s^2})}(0.54\,\mathrm{m})^2 = 2.88\,\mathrm{kg\cdot m^2}.$$

(a) Energy is conserved as the car moves down the slope: $U_{\rm i} = K_{\rm f}$, or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}Ik^2v^2,$$

or

$$v = \sqrt{\frac{2mgh}{m + Ik^2}} = \sqrt{\frac{2(822\,\text{kg})(9.81\,\text{m/s}^2)(131\,\text{m})}{(822\,\text{kg}) + (2.88\,\text{kg}\cdot\text{m}^2)(62.1\ \text{rad/m})^2}} = 13.3\,\text{m/s},$$

or 47.9 m/s.

(b) The average speed down the slope is $13.3\,\text{m/s}/2 = 6.65\,\text{m/s}$. The time to get to the bottom is $t = (1500\,\text{m})/(6.65\,\text{m/s}) = 226\,\text{s}$. The angular acceleration of the disk is

$$\alpha = \frac{\omega}{t} = \frac{(13.3\,\text{m/s})(62.1\ \text{rad/m})}{(226\,\text{s})} = 3.65\ \text{rad/s}^2.$$

(c) $P = \tau\omega = I\alpha\omega$, so

$$P = (2.88\,\text{kg}\cdot\text{m}^2)(3.65\ \text{rad/s}^2)(13.3\,\text{m/s})(62.1\ \text{rad/m}) = 8680\,\text{W}.$$

E12-25 (a) For the solid sphere $I = \frac{2}{5}mr^2$; if it rolls without slipping $\omega = v/r$; conservation of energy means $K_{\rm i} = U_{\rm f}$. Then

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = mgh.$$

or

$$h = \frac{(5.18\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} + \frac{(5.18\,\text{m/s})^2}{5(9.81\,\text{m/s}^2)} = 1.91\,\text{m}.$$

The distance up the incline is $(1.91\,\text{m})/\sin(34.0^\circ) = 3.42\,\text{m}$.

(b) The sphere will travel a distance of 3.42 m with an average speed of $5.18\,\text{m}/2$, so $t = (3.42\,\text{m})/(2.59\,\text{m/s}) = 1.32\,\text{s}$. But wait, it goes up then comes back down, so double this time to get 2.64 s.

(c) The total distance is 6.84 m, so the number of rotations is $(6.84\,\text{m})/(0.0472\,\text{m})/(2\pi) = 23.1$.

E12-26 Conservation of energy means $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$. But $\omega = v/r$ and we are told $h = 3v^2/4g$, so

$$\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = mg\frac{3v^2}{4g},$$

or

$$I = 2r^2\left(\frac{3}{4}m - \frac{1}{2}m\right) = \frac{1}{2}mr^2,$$

which could be a solid disk or cylinder.

$\boxed{\textbf{E12-27}}$ We assume the cannon ball is solid, so the rotational inertia will be $I = (2/5)MR^2$

The normal force on the cannon ball will be $N = Mg$, where M is the mass of the bowling ball. The kinetic friction on the cannon ball is $F_f = \mu_k N = \mu_k Mg$. The magnitude of the net torque on the bowling ball while skidding is then $\tau = \mu_k MgR$.

Originally the angular momentum of the cannon ball is zero; the final angular momentum will have magnitude $l = I\omega = Iv/R$, where v is the final translational speed of the ball.

The time requires for the cannon ball to stop skidding is the time required to change the angular momentum to l, so

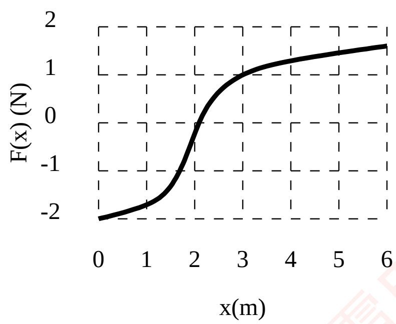
$$\Delta t = \frac{\Delta l}{\tau} = \frac{(2/5)MR^2v/R}{\mu_k MgR} = \frac{2v}{5\mu_k g}.$$

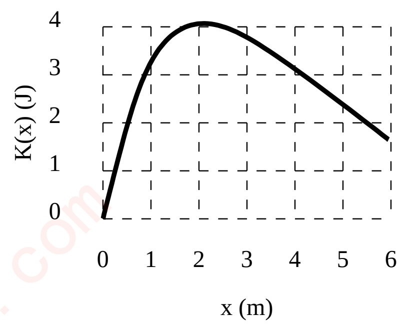
Since we don't know Δt, we can't solve this for v. But the same time through which the angular momentum of the ball is increasing the linear momentum of the ball is decreasing, so we also have

$$\Delta t = \frac{\Delta p}{-F_f} = \frac{Mv - Mv_0}{-\mu_k M g} = \frac{v_0 - v}{\mu_k g}.$$

Combining,

$$\begin{aligned} \frac{2v}{5\mu_k g} &= \frac{v_0 - v}{\mu_k g}, \\ 2v &= 5(v_0 - v), \\ v &= 5v_0/7 \end{aligned}$$





E12-28

E12-29 (a) $F = -\Delta U/\Delta x = -[(-17\,\text{J}) - (-3\,\text{J})]/[(4\,\text{m}) - (1\,\text{m})] = 4.7\,\text{N}$.

(b) The total energy is $\frac{1}{2}(2.0\,\text{kg})(-2.0\,\text{m/s})^2 + (-7\,\text{J})$, or $-3\,\text{J}$. The particle is constrained to move between $x = 1\,\text{m}$ and $x = 14\,\text{m}$.

(c) When $x = 7\,\text{m}$ $K = (-3\,\text{J}) - (-17\,\text{J}) = 14\,\text{J}$. The speed is $v = \sqrt{2(14\,\text{J})/(2.0\,\text{kg})} = 3.7\,\text{m/s}$.

E12-30 Energy is conserved, so

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgy,$$

or

$$v = \sqrt{v_0^2 - 2gy},$$

which depends only on y.

$\boxed{\text{E12-31}}$ (a) We can find F_x and F_y from the appropriate derivatives of the potential,

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} = -kx, \\ F_y &= -\frac{\partial U}{\partial y} = -ky. \end{aligned}$$

The force at point (x, y) is then

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = -kx\hat{\mathbf{i}} - ky\hat{\mathbf{j}}.$$

(b) Since the force vector points directly toward the origin there is *no* angular component, and $F_\theta = 0$. Then $F_r = -kr$ where r is the distance from the origin.

(c) A spring which is attached to a point; the spring is free to rotate, perhaps?

E12-32 (a) By symmetry we expect F_x, F_y, and F_z to all have the same form.

$$F_x = -\frac{\partial U}{\partial x} = \frac{-kx}{(x^2+y^2+z^2)^{3/2}},$$

with similar expressions for F_y and F_z. Then

$$\vec{\mathbf{F}} = \frac{-k}{(x^2+y^2+z^2)^{3/2}}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}).$$

(b) In spherical polar coordinates $r^2 = x^2+y^2+z^2$. Then $U = -k/r$ and

$$F_r = -\frac{\partial U}{\partial r} = -\frac{k}{r^2}.$$

E12-33 We'll just do the paths, showing only non-zero terms.
Path 1: $W = \int_0^b (-k_2 a)\,dy) = -k_2 ab$.
Path 2: $W = \int_0^a (-k_1 b)\,dx) = -k_1 ab$.
Path 3: $W = (\cos\phi\,\sin\phi)\int_0^d (-k_1 - k_2) r\,dr = -(k_1+k_2)ab/2$.
These three are only equal if $k_1 = k_2$.

P12-1 (a) We need to integrate an expression like

$$-\int_\infty^z \frac{k}{(z+l)^2}\,dz = \frac{k}{z+l}.$$

The second half is dealt with in a similar manner, yielding

$$U(z) = \frac{k}{z+l} - \frac{k}{z-l}.$$

(b) If $z \gg l$ then we can expand the denominators, then

$$\begin{aligned} U(z) &= \frac{k}{z+l} - \frac{k}{z-l}, \\ &\approx \left(\frac{k}{z} - \frac{kl}{z^2}\right) - \left(\frac{k}{z} + \frac{kl}{z^2}\right), \\ &= -\frac{2kl}{z^2}. \end{aligned}$$

P12-2 The ball just reaches the top, so $K_2 = 0$. Then $K_1 = U_2 - U_1 = mgL$, so $v_1 = \sqrt{2(mgL)/m} = \sqrt{2gL}$.

P12-3 Measure distances along the incline by x, where $x = 0$ is measured from the maximally compressed spring. The vertical position of the mass is given by $x\sin\theta$. For the spring $k = (268\,\text{N})/(0.0233\,\text{m}) = 1.15\times10^4\,\text{N/m}$. The total energy of the system is

$$\frac{1}{2}(1.15\times10^4\,\text{N/m})(0.0548\,\text{m})^2 = 17.3\,\text{J}.$$

(a) The block needs to have moved a vertical distance $x\sin(32.0^\circ)$, where

$$17.3\,\text{J} = (3.18\,\text{kg})(9.81\,\text{m/s}^2)x\sin(32.0^\circ),$$

or $x = 1.05\,\text{m}$.

(b) When the block hits the top of the spring the gravitational potential energy has changed by

$$\Delta U = -(3.18\,\text{kg})(9.81\,\text{m/s}^2)(1.05\,\text{m} - 0.0548\,\text{m})\sin(32.0^\circ) = 16.5, \text{J};$$

hence the speed is $v = \sqrt{2(16.5\,\text{J})/(3.18\,\text{kg})} = 3.22\,\text{m/s}$.

P12-4 The potential energy associated with the hanging part is

$$U = \int_{-L/4}^{0} \frac{M}{L} g y \, dy = \frac{Mg}{2L} y^2 \Big|_{-L/4}^{0} = -\frac{MgL}{32},$$

so the work required is $W = MgL/32$.

$\boxed{\textbf{P12-5}}$ (a) Considering points P and Q we have

$$\begin{aligned} K_P + U_P &= K_Q + U_Q, \\ (0) + mg(5R) &= \frac{1}{2}mv^2 + mg(R), \\ 4mgR &= \frac{1}{2}mv^2, \\ \sqrt{8gR} &= v. \end{aligned}$$

There are two forces on the block, the normal force from the track,

$$N = \frac{mv^2}{R} = \frac{m(8gR)}{R} = 8mg,$$

and the force of gravity $W = mg$. They are orthogonal so

$$F_{\text{net}} = \sqrt{(8mg)^2 + (mg)^2} = \sqrt{65}\, mg$$

and the angle from the horizontal by

$$\tan\theta = \frac{-mg}{8mg} = -\frac{1}{8},$$

or $\theta = 7.13°$ below the horizontal.

(b) If the block *barely* makes it over the top of the track then the speed at the top of the loop (point S, perhaps?) is just fast enough so that the centripetal force is equal in magnitude to the weight,

$$m{v_{\text{S}}}^2/R = mg.$$

Assume the block was released from point T. The energy conservation problem is then

$$\begin{aligned} K_T + U_T &= K_S + U_S, \\ (0) + mgy_T &= \frac{1}{2}mv_S^2 + mgy_S, \\ y_T &= \frac{1}{2}(R) + m(2R), \\ &= 5R/2. \end{aligned}$$

P12-6 The wedge slides to the left, the block to the right. Conservation of momentum requires $Mv_{\text{w}} + mv_{\text{b},x} = 0$. The block is constrained to move on the surface of the wedge, so

$$\tan\alpha = \frac{v_{\text{b},y}}{v_{\text{b},x} - v_{\text{w}}},$$

or

$$v_{\text{b},y} = v_{\text{b},x} \tan\alpha(1 + m/M).$$

Conservation of energy requires

$$\frac{1}{2}m{v_\mathrm{b}}^2 + \frac{1}{2}M{v_\mathrm{w}}^2 = mgh.$$

Combining,

$$\begin{aligned}
\frac{1}{2}m\left({v_{\mathrm{b},x}}^2 + {v_{\mathrm{b},y}}^2\right) + \frac{1}{2}M\left(\frac{m}{M}v_{\mathrm{b},x}\right)^2 &= mgh,\\
\left(\tan^2\alpha(1+m/M)^2 + 1 + \frac{m}{M}\right){v_{\mathrm{b},x}}^2 &= 2gh,\\
\left(\sin^2\alpha(M+m)^2 + M^2\cos^2\alpha + mM\cos^2\alpha\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha,\\
\left(M^2 + mM + mM\sin^2\alpha + m^2\sin^2\alpha\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha,\\
\left((M+m)(M+m\sin^2\alpha)\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha,
\end{aligned}$$

or

$$v_{\mathrm{b},x} = M\cos\alpha\sqrt{\frac{2gh}{(M+m)(M+m\sin^2\alpha)}}.$$

Then

$$v_\mathrm{w} = -m\cos\alpha\sqrt{\frac{2gh}{(M+m)(M+m\sin^2\alpha)}}.$$

P12-7 $U(x) = -\int F_x\,dx = -Ax^2/2 - Bx^3/3$.

(a) $U = -(-3.00\,\mathrm{N/m})(2.26\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(2.26\,\mathrm{m})^3/3 = 26.9\,\mathrm{J}$.

(b) There are two points to consider:

$$\begin{aligned}
U_1 &= -(-3.00\,\mathrm{N/m})(4.91\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(4.91\,\mathrm{m})^3/3 = 233\,\mathrm{J},\\
U_2 &= -(-3.00\,\mathrm{N/m})(1.77\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(1.77\,\mathrm{m})^3/3 = 13.9\,\mathrm{J},\\
K_1 &= \frac{1}{2}(1.18\mathrm{kg})(4.13\,\mathrm{m/s})^2 = 10.1\,\mathrm{J}.
\end{aligned}$$

Then

$$v_2 = \sqrt{\frac{2(10.1\,\mathrm{J} + 233\,\mathrm{J} - 13.9\,\mathrm{J})}{(1.18\,\mathrm{kg})}} = 19.7\,\mathrm{m/s}.$$

P12-8 Assume that $U_0 = K_0 = 0$. Then conservation of energy requires $K = -U$; consequently, $v = \sqrt{2g(-y)}$.

(a) $v = \sqrt{2(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m})} = 4.85\,\mathrm{m/s}$.

(b) $v = \sqrt{2(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m} - 0.45\,\mathrm{m} - 0.45\,\mathrm{m})} = 2.43\,\mathrm{m/s}$.

$\boxed{\textbf{P12-9}}$ Assume that $U_0 = K_0 = 0$. Then conservation of energy requires $K = -U$; consequently, $v = \sqrt{2g(-y)}$. If the ball *barely* swings around the top of the peg then the speed at the top of the loop is just fast enough so that the centripetal force is equal in magnitude to the weight,

$$mv^2/R = mg.$$

The energy conservation problem is then

$$\begin{aligned}
mv^2 &= 2mg(L - 2(L-d)) = 2mg(2d - L)\\
mg(L-d) &= 2mg(2d-L),\\
d &= 3L/5.
\end{aligned}$$

P12-10 The speed at the top and the speed at the bottom are related by

$$\frac{1}{2}m{v_\mathrm{b}}^2 = \frac{1}{2}m{v_\mathrm{t}}^2 + 2mgR.$$

The magnitude of the net force is $F = mv^2/R$, the tension at the top is

$$T_\mathrm{t} = m{v_\mathrm{t}}^2/R - mg,$$

while tension at the bottom is

$$T_\mathrm{b} = m{v_\mathrm{b}}^2/R + mg,$$

The difference is

$$\Delta T = 2mg + m({v_\mathrm{b}}^2 - {v_\mathrm{t}}^2)/R = 2mg + 4mg = 6mg.$$

P12-11 Let the angle θ be measured from the horizontal to the point on the hemisphere where the boy is located. There are then two components to the force of gravity— a component tangent to the hemisphere, $W_{||} = mg\cos\theta$, and a component directed radially toward the center of the hemisphere, $W_\perp = mg\sin\theta$.

While the boy is in contact with the hemisphere the motion is circular so

$$mv^2/R = W_\perp - N.$$

When the boy leaves the surface we have $mv^2/R = W_\perp$, or $mv^2 = mgR\sin\theta$. Now for energy conservation,

$$\begin{aligned} K + U &= K_0 + U_0, \\ \frac{1}{2}mv^2 + mgy &= \frac{1}{2}m(0)^2 + mgR, \\ \frac{1}{2}gR\sin\theta + mgy &= mgR, \\ \frac{1}{2}y + y &= R, \\ y &= 2R/3. \end{aligned}$$

P12-12 (a) To be in contact at the top requires $m{v_\mathrm{t}}^2/R = mg$. The speed at the bottom would be given by energy conservation

$$\frac{1}{2}m{v_\mathrm{b}}^2 = \frac{1}{2}m{v_\mathrm{t}}^2 + 2mgR,$$

so $v_\mathrm{b} = \sqrt{5gR}$ is the speed at the bottom that will allow the object to make it around the circle without loosing contact.

(b) The particle will lose contact with the track if $mv^2/R \le mg\sin\theta$. Energy conservation gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgR(1 + \sin\theta)$$

for points above the half-way point. Then the condition for "sticking" to the track is

$$\frac{1}{R}v_0^2 - 2g(1 + \sin\theta) \le g\sin\theta,$$

or, if $v_0 = 0.775v_\mathrm{m}$,

$$5(0.775)^2 - 2 \le 3\sin\theta,$$

or $\theta = \arcsin(1/3)$.

P12-13 The rotational inertia is

$$I = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2.$$

Conservation of energy is

$$\frac{1}{2}I\omega^2 = 3Mg(L/2),$$

so $\omega = \sqrt{9g/(4L)}$.

P12-14 The rotational speed of the sphere is $\omega = v/r$; the rotational kinetic energy is $K_\text{r} = \frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$.

(a) For the marble to stay on the track $mv^2/R = mg$ at the top of the track. Then the marble needs to be released from a point

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + 2mgR,$$

or $h = R/2 + R/5 + 2R = 2.7R$.

(b) Energy conservation gives

$$6mgR = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + mgR,$$

or $mv^2/R = 50mg/7$. This corresponds to the horizontal force acting on the marble.

P12-15 $\frac{1}{2}mv_0^2 + mgy = 0$, where y is the distance beneath the rim, or $y = -r\cos\theta_0$. Then

$$v_0 = \sqrt{-2gy} = \sqrt{2gr\cos\theta_0}.$$

P12-16 (a) For E_1 the atoms will eventually move apart completely.

(b) For E_2 the moving atom will bounce back and forth between a closest point and a farthest point.

(c) $U \approx -1.2\times10^{-19}\text{J}$.

(d) $K = E_1 - U \approx 2.2\times10^{-19}\text{J}$.

(e) Find the slope of the curve, so

$$F \approx -\frac{(-1\times10^{-19}\text{J}) - (-2\times10^{-19}\text{J})}{(0.3\times10^{-9}\text{m}) - (0.2\times10^{-9}\text{m})} = -1\times10^{-9}\text{N},$$

which would point toward the larger mass.

$\boxed{\textbf{P12-17}}$ The function needs to fall off at infinity in both directions; an exponential envelope would work, but it will need to have an $-x^2$ term to force the potential to zero on *both* sides. So we propose something of the form

$$U(x) = P(x)e^{-\beta x^2}$$

where $P(x)$ is a polynomial in x and β is a positive constant.

We proposed the *polynomial* because we need a symmetric function which has two zeroes. A quadratic of the form $\alpha x^2 - U_0$ would work, it has two zeroes, a minimum at $x = 0$, and is symmetric.

So our *trial* function is

$$U(x) = \left(\alpha x^2 - U_0\right)e^{-\beta x^2}.$$

This function should have *three* extrema. Take the derivative, and then we'll set it equal to zero,

$$\frac{dU}{dx} = 2\alpha x e^{-\beta x^2} - 2\left(\alpha x^2 - U_0\right)\beta x e^{-\beta x^2}.$$

Setting this equal to zero leaves two possibilities,

$$\begin{aligned} x &= 0, \\ 2\alpha - 2\left(\alpha x^2 - U_0\right)\beta &= 0. \end{aligned}$$

The first equation is trivial, the second is easily rearranged to give

$$x = \pm\sqrt{\frac{\alpha + \beta U_0}{\beta\alpha}}$$

These are the points $\pm x_1$. We can, if we wanted, try to find α and β from the picture, but you might notice we have one equation, $U(x_1) = U_1$ and two unknowns. It really isn't very illuminating to take this problem much farther, but we could.

(b) The force is the derivative of the potential; this expression was found above.

(c) As long as the energy is *less* than the two peaks, then the motion would be oscillatory, trapped in the well.

P12-18 (a) $F = -\partial U/\partial r$, or

$$F = -U_0\left(\frac{r_0}{r^2} + \frac{1}{r}\right)e^{-r/r_0}.$$

(b) Evaluate the force at the four points:

$$\begin{aligned} F(r_0) &= -2(U_0/r_0)e^{-1}, \\ F(2r_0) &= -(3/4)(U_0/r_0)e^{-2}, \\ F(4r_0) &= -(5/16)(U_0/r_0)e^{-4}, \\ F(10r_0) &= -(11/100)(U_0/r_0)e^{-10}. \end{aligned}$$

The ratios are then

$$\begin{aligned} F(2r_0)/F(r_0) &= (3/8)e^{-1} = 0.14, \\ F(4r_0)/F(r_0) &= (5/32)e^{-3} = 7.8\times10^{-3}, \\ F(10r_0)/F(r_0) &= (11/200)e^{-9} = 6.8\times10^{-6}. \end{aligned}$$

E13-1 If the projectile had *not* experienced air drag it would have risen to a height y_2, but because of air drag 68 kJ of mechanical energy was dissipated so it only rose to a height y_1. In either case the initial velocity, and hence initial kinetic energy, was the same; and the velocity at the highest point was zero. Then $W = \Delta U$, so the potential energy would have been 68 kJ greater, and

$$\Delta y = \Delta U/mg = (68\times 10^3\text{J})/(9.4\,\text{kg})(9.81\,\text{m/s}^2) = 740\,\text{m}$$

is how much higher it would have gone without air friction.

E13-2 (a) The road incline is $\theta = \arctan(0.08) = 4.57°$. The frictional forces are the same; the car is now moving with a vertical upward speed of $(15\,\text{m/s})\sin(4.57°) = 1.20\,\text{m/s}$. The additional power required to drive up the hill is then $\Delta P = mgv_y = (1700\,\text{kg})(9.81\,\text{m/s}^2)(1.20\,\text{m/s}) = 20000\,\text{W}$. The total power required is 36000 W.

(b) The car will "coast" if the power generated by rolling downhill is equal to 16000 W, or

$$v_y = (16000\,\text{W})/[(1700\,\text{kg})(9.81\,\text{m/s}^2)] = 0.959\,\text{m/s},$$

down. Then the incline is

$$\theta = \arcsin(0.959\,\text{m/s}/15\,\text{m/s}) = 3.67°.$$

This corresponds to a downward grade of $\tan(3.67°) = 6.4\,\%$.

E13-3 Apply energy conservation:

$$\frac{1}{2}mv^2 + mgy + \frac{1}{2}ky^2 = 0,$$

so

$$v = \sqrt{-2(9.81\,\text{m/s}^2)(-0.084\,\text{m}) - (262\,\text{N/m})(-0.084\,\text{m})^2/(1.25\,\text{kg})} = 0.41\,\text{m/s}.$$

E13-4 The car climbs a vertical distance of $(225\,\text{m})\sin(10°) = 39.1\,\text{m}$ in coming to a stop. The change in energy of the car is then

$$\Delta E = -\frac{1}{2}\frac{(16400\,\text{N})}{(9.81\,\text{m/s}^2)}(31.4\,\text{m/s})^2 + (16400\,\text{N})(39.1\,\text{m}) = -1.83\times 10^5\text{J}.$$

E13-5 (a) Applying conservation of energy to the points where the ball was dropped and where it entered the oil,

$$\begin{aligned}
\frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} &= \frac{1}{2}m{v_\text{i}}^2 + mgy_\text{i},\\
\frac{1}{2}{v_\text{f}}^2 + g(0) &= \frac{1}{2}(0)^2 + gy_\text{i},\\
v_\text{f} &= \sqrt{2gy_\text{i}},\\
&= \sqrt{2(9.81\,\text{m/s}^2)(0.76\,\text{m})} = 3.9\,\text{m/s}.
\end{aligned}$$

(b) The change in internal energy of the ball + oil can be found by considering the points where the ball was released and where the ball reached the bottom of the container.

$$\begin{aligned}
\Delta E &= K_\text{f} + U_\text{f} - K_\text{i} - U_\text{i},\\
&= \frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} - \frac{1}{2}m(0)^2 - mgy_\text{i},\\
&= \frac{1}{2}(12.2\times 10^{-3}\text{kg})(1.48\text{m/s})^2 - (12.2\times 10^{-3}\text{kg})(9.81\text{m/s}^2)(-0.55\text{m} - 0.76\text{m}),\\
&= -0.143\,\text{J}
\end{aligned}$$

E13-6 (a) $U_{\rm i} = (25.3\,{\rm kg})(9.81\,{\rm m/s^2})(12.2\,{\rm m}) = 3030\,{\rm J}$.
(b) $K_{\rm f} = \frac{1}{2}(25.3\,{\rm kg})(5.56\,{\rm m/s})^2 = 391\,{\rm J}$.
(c) $\Delta E_{\rm int} = 3030\,{\rm J} - 391\,{\rm J} = 2640\,{\rm J}$.

E13-7 (a) At atmospheric entry the kinetic energy is

$$K = \frac{1}{2}(7.9\times10^4{\rm kg})(8.0\times10^3{\rm m/s})^2 = 2.5\times10^{12}{\rm J}.$$

The gravitational potential energy is

$$U = (7.9\times10^4{\rm kg})(9.8\,{\rm m/s^2})(1.6\times10^5{\rm m}) = 1.2\times10^{11}{\rm J}.$$

The total energy is 2.6×10^{12}J.
(b) At touch down the kinetic energy is

$$K = \frac{1}{2}(7.9\times10^4{\rm kg})(9.8\times10^1{\rm m/s})^2 = 3.8\times10^8{\rm J}.$$

E13-8 $\Delta E/\Delta t = (68\,{\rm kg})(9.8\,{\rm m/s^2})(59\,{\rm m/s}) = 39000\,{\rm J/s}$.

$\boxed{\textbf{E13-9}}$ Let m be the mass of the water under consideration. Then the percentage of the potential energy “lost” which appears as kinetic energy is

$$\frac{K_{\rm f} - K_{\rm i}}{U_{\rm i} - U_{\rm f}}.$$

Then

$$\begin{aligned}
\frac{K_{\rm f} - K_{\rm i}}{U_{\rm i} - U_{\rm f}} &= \frac{1}{2}m\left({v_{\rm f}}^2 - {v_{\rm i}}^2\right)/\left(mgy_{\rm i} - mgy_{\rm f}\right), \\
&= \frac{{v_{\rm f}}^2 - {v_{\rm i}}^2}{-2g\Delta y}, \\
&= \frac{(13\,{\rm m/s})^2 - (3.2\,{\rm m/s})^2}{-2(9.81\,{\rm m/s^2})(-15\,{\rm m})}, \\
&= 54\,\%.
\end{aligned}$$

The rest of the energy would have been converted to sound and thermal energy.

E13-10 The change in energy is

$$\Delta E = \frac{1}{2}(524\,{\rm kg})(62.6\,{\rm m/s})^2 - (524\,{\rm kg})(9.81\,{\rm m/s^2})(292\,{\rm m}) = 4.74\times10^5{\rm J}.$$

E13-11 $U_{\rm f} = K_{\rm i} - (34.6\,{\rm J})$. Then

$$h = \frac{1}{2}\frac{(7.81\,{\rm m/s})^2}{(9.81\,{\rm m/s^2})} - \frac{(34.6\,{\rm J})}{(4.26\,{\rm kg})(9.81\,{\rm m/s^2})} = 2.28\,{\rm m};$$

which means the distance along the incline is $(2.28\,{\rm m})/\sin(33.0^\circ) = 4.19\,{\rm m}$.

E13-12 (a) $K_f = U_i - U_f$, so

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)[(862\,\text{m}) - (741\,\text{m})]} = 48.7\,\text{m/s}.$$

That's a quick 175 km/h; but the speed at the bottom of the valley is 40% of the speed of sound!

(b) $\Delta E = U_f - U_i$, so

$$\Delta E = (54.4\,\text{kg})(9.81\,\text{m/s}^2)[(862\,\text{m}) - (741\,\text{m})] = -6.46\times10^4\text{J};$$

which means the internal energy of the snow and skis increased by 6.46×10^4J.

$\boxed{\textbf{E13-13}}$ The final potential energy is 15% less than the initial kinetic plus potential energy of the ball, so

$$0.85(K_i + U_i) = U_f.$$

But $U_i = U_f$, so $K_i = 0.15U_f/0.85$, and then

$$v_i = \sqrt{\frac{0.15}{0.85}2gh} = \sqrt{2(0.176)(9.81\,\text{m/s}^2)(12.4\,\text{m})} = 6.54\,\text{m/s}.$$

E13-14 Focus on the potential energy. After the nth bounce the ball will have a potential energy at the top of the bounce of $U_n = 0.9U_{n-1}$. Since $U \propto h$, one can write $h_n = (0.9)^n h_0$. Solving for n,

$$n = \log(h_n/h_0)/\log(0.9) = \log(3\text{ ft}/6\text{ ft})/\log(0.9) = 6.58,$$

which must be rounded up to 7.

E13-15 Let m be the mass of the ball and M be the mass of the block.

The kinetic energy of the ball just before colliding with the block is given by $K_1 = U_0$, so $v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.687\,\text{m})} = 3.67\,\text{m/s}$.

Momentum is conserved, so if v_2 and v_3 are velocities of the ball and block after the collision then $mv_1 = mv_2 + Mv_3$. Kinetic energy is not conserved, instead

$$\frac{1}{2}\left(\frac{1}{2}mv_1^2\right) = \frac{1}{2}mv_2^2 + \frac{1}{2}Mv_3^2.$$

Combine the energy and momentum expressions to eliminate v_3:

$$\begin{aligned} mv_1^2 &= 2mv_2^2 + 2M\left(\frac{m}{M}(v_1 - v_2)\right)^2, \\ Mv_1^2 &= 2Mv_2^2 + 2mv_1^2 - 4mv_1v_2 + 2mv_2^2, \end{aligned}$$

which can be formed into a quadratic. The solution for v_2 is

$$v_2 = \frac{2m \pm \sqrt{2(M^2 - mM)}}{2(M+m)}v_1 = (0.600 \pm 1.95)\,\text{m/s}.$$

The corresponding solutions for v_3 are then found from the momentum expression to be $v_3 = 0.981\,\text{m/s}$ and $v_3 = 0.219$. Since it is unlikely that the ball passed through the block we can toss out the second set of answers.

E13-16 $E_f = K_f + U_f = 3mgh$, or

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)2(0.18\,\text{m})} = 2.66\,\text{m/s}.$$

E13-17 We can find the kinetic energy of the center of mass of the woman when her feet leave the ground by considering energy conservation and her highest point. Then

$$\begin{aligned}
\frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i} &= \frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f},\\
\frac{1}{2}mv_\mathrm{i} &= mg\Delta y,\\
&= (55.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m} - 0.90\,\mathrm{m}) = 162\,\mathrm{J}.
\end{aligned}$$

(a) During the jumping phase her potential energy changed by

$$\Delta U = mg\Delta y = (55.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.50\,\mathrm{m}) = 270\,\mathrm{J}$$

while she was moving up. Then

$$F_\mathrm{ext} = \frac{\Delta K + \Delta U}{\Delta s} = \frac{(162\,\mathrm{J}) + (270\,\mathrm{J})}{(0.5\,\mathrm{m})} = 864\,\mathrm{N}.$$

(b) Her fastest speed was when her feet left the ground,

$$v = \frac{2K}{m} = \frac{2(162\,\mathrm{J})}{(55.0\,\mathrm{kg})} = 2.42\,\mathrm{m/s}.$$

E13-18 (b) The ice skater needs to lose $\frac{1}{2}(116\,\mathrm{kg})(3.24\,\mathrm{m/s})^2 = 609\,\mathrm{J}$ of internal energy.
(a) The average force exerted on the rail is $F = (609\,\mathrm{J})/(0.340\,\mathrm{m}) = 1790\,\mathrm{N}$.

E13-19 12.6 km/h is equal to $3.50\,\mathrm{m/s}$; the initial kinetic energy of the car is

$$\frac{1}{2}(2340\,\mathrm{kg})(3.50\,\mathrm{m/s})^2 = 1.43\times10^4\mathrm{J}.$$

(a) The force exerted on the car is $F = (1.43\times10^4\mathrm{J})/(0.64\,\mathrm{m}) = 2.24\times10^4\mathrm{N}$.
(b) The change increase in internal energy of the car is

$$\Delta E_\mathrm{int} = (2.24\times10^4\mathrm{N})(0.640\,\mathrm{m} - 0.083\,\mathrm{m}) = 1.25\times10^4\mathrm{J}.$$

E13-20 Note that $v_n^2 = {v_n'}^2 - 2\vec{\mathbf{v}}_n' \cdot \vec{\mathbf{v}}_\mathrm{cm} + {v_\mathrm{cm}}^2$. Then

$$\begin{aligned}
K &= \sum_n \frac{1}{2}\left(m_n{v_n'}^2 - 2m_n\vec{\mathbf{v}}_n' \cdot \vec{\mathbf{v}}_\mathrm{cm} + m_n{v_\mathrm{cm}}^2\right),\\
&= \sum_n \frac{1}{2}m_n{v_n'}^2 - \left(\sum_n m_n\vec{\mathbf{v}}_n'\right)\cdot\vec{\mathbf{v}}_\mathrm{cm} + \left(\sum_n \frac{1}{2}m_n\right){v_\mathrm{cm}}^2,\\
&= K_\mathrm{int} - \left(\sum_n m_n\vec{\mathbf{v}}_n'\right)\cdot\vec{\mathbf{v}}_\mathrm{cm} + K_\mathrm{cm}.
\end{aligned}$$

The middle term vanishes because of the definition of velocities relative to the center of mass.

E13-21 Momentum conservation requires $mv_0 = mv + MV$, where the sign indicates the direction. We are assuming one dimensional collisions. Energy conservation requires

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 + E.$$

Combining,

$$\begin{aligned}
\frac{1}{2}mv_0^2 &= \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{m}{M}v_0 - \frac{m}{M}v\right)^2 + E, \\
Mv_0^2 &= Mv^2 + m\left(v_0 - v\right)^2 + 2(M/m)E.
\end{aligned}$$

Arrange this as a quadratic in v,

$$(M+m)\,v^2 - (2mv_0)\,v + \left(2(M/m)E + mv_0^2 - Mv_0^2\right) = 0.$$

This will only have real solutions if the discriminant ($b^2 - 4ac$) is greater than or equal to zero. Then

$$\left(2mv_0\right)^2 \geq 4\,(M+m)\left(2(M/m)E + mv_0^2 - Mv_0^2\right)$$

is the condition for the minimum v_0. Solving the equality condition,

$$4m^2v_0^2 = 4(M+m)\left(2(M/m)E + (m-M)v_0^2\right),$$

or $M^2v_0^2 = 2(M+m)(M/m)E$. One last rearrangement, and $v_0 = \sqrt{2(M+m)E/(mM)}$.

P13-1 (a) The initial kinetic energy will equal the potential energy at the highest point *plus* the amount of energy which is dissipated because of air drag.

$$\begin{aligned}
mgh + fh &= \frac{1}{2}mv_0^2, \\
h &= \frac{v_0^2}{2(g+f/m)} = \frac{v_0^2}{2g(1+f/w)}.
\end{aligned}$$

(b) The final kinetic energy when the stone lands will be equal to the initial kinetic energy *minus* twice the energy dissipated on the way up, so

$$\begin{aligned}
\frac{1}{2}mv^2 &= \frac{1}{2}mv_0^2 - 2fh, \\
&= \frac{1}{2}mv_0^2 - 2f\frac{v_0^2}{2g(1+f/w)}, \\
&= \left(\frac{m}{2} - \frac{f}{g(1+f/w)}\right)v_0^2, \\
v^2 &= \left(1 - \frac{2f}{w+f}\right)v_0^2, \\
v &= \left(\frac{w-f}{w+f}\right)^{1/2} v_0.
\end{aligned}$$

P13-2 The object starts with $U = (0.234\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.05\,\mathrm{m}) = 2.41\,\mathrm{J}$. It will move back and forth across the flat portion $(2.41\,\mathrm{J})/(0.688\,\mathrm{J}) = 3.50$ times, which means it will come to a rest at the center of the flat part while attempting one last right to left journey.

P13-3 (a) The work done on the block block because of friction is

$$(0.210)(2.41\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.83\,\mathrm{m}) = 9.09\,\mathrm{J}.$$

The energy dissipated because of friction is $(9.09\,\mathrm{J})/0.83 = 11.0\,\mathrm{J}$.

The speed of the block just after the bullet comes to a rest is

$$v = \sqrt{2K/m} = \sqrt{2(1.10\,\mathrm{J})/(2.41\,\mathrm{kg})} = 3.02\,\mathrm{m/s}.$$

(b) The initial speed of the bullet is

$$v_0 = \frac{M+m}{m}v = \frac{(2.41\,\mathrm{kg}) + (0.00454\,\mathrm{kg})}{(0.00454\,\mathrm{kg})}(3.02\,\mathrm{m/s}) = 1.60\times 10^3\,\mathrm{m/s}.$$

P13-4 The energy stored in the spring after compression is $\frac{1}{2}(193\,\mathrm{N/m})(0.0416\,\mathrm{m})^2 = 0.167\,\mathrm{J}$. Since 117 mJ was dissipated by friction, the kinetic energy of the block before colliding with the spring was $0.284\,\mathrm{J}$. The speed of the block was then

$$v = \sqrt{2(0.284\,\mathrm{J})/(1.34\,\mathrm{kg})} = 0.651\,\mathrm{m/s}.$$

P13-5 (a) Using Newton's second law, $F = ma$, so for circular motion around the proton

$$\frac{mv^2}{r} = F = k\frac{e^2}{r^2}.$$

The kinetic energy of the electron in an orbit is then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}k\frac{e^2}{r}.$$

The change in kinetic energy is

$$\Delta K = \frac{1}{2}ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

(b) The potential energy difference is

$$\Delta U = -\int_{r_1}^{r_2}\frac{ke^2}{r^2}dr = -ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

(c) The total energy change is

$$\Delta E = \Delta K + \Delta U = -\frac{1}{2}ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

P13-6 (a) The initial energy of the system is $(4000\ \mathrm{lb})(12\mathrm{ft}) = 48,000\ \mathrm{ft\cdot lb}$. The safety device removes $(1000\ \mathrm{lb})(12\mathrm{ft}) = 12,000\ \mathrm{ft\cdot lb}$ before the elevator hits the spring, so the elevator has a kinetic energy of $36,000\ \mathrm{ft\cdot lb}$ when it hits the spring. The speed of the elevator when it hits the spring is

$$v = \sqrt{\frac{2(36,000\ \mathrm{ft\cdot lb})(32.0\ \mathrm{ft/s^2})}{(4000\ \mathrm{lb})}} = 24.0\ \mathrm{ft/s}.$$

(b) Assuming the safety clamp remains in effect while the elevator is in contact with the spring then the distance compressed will be found from

$$36,000\ \mathrm{ft\cdot lb} = \frac{1}{2}(10,000\ \mathrm{lb/ft})y^2 - (4000\ \mathrm{lb})y + (1000\ \mathrm{lb})y.$$

This is a quadratic expression in y which can be simplified to look like

$$5y^2 - 3y - 36 = 0,$$

which has solutions $y = (0.3 \pm 2.7)$ ft. Only $y = 3.00$ ft makes sense here.

(c) The distance through which the elevator will bounce back up is found from

$$33,000 \text{ ft} = (4000 \text{ lb})y - (1000 \text{ lb})y,$$

where y is measured from the most compressed point of the spring. Then $y = 11$ ft, or the elevator bounces back up 8 feet.

(d) The elevator will bounce until it has traveled a total distance so that the safety device dissipates all of the original energy, or 48 ft.

$\boxed{\textbf{P13-7}}$ The net force on the top block while it is being pulled is

$$11.0\,\text{N} - F_f = 11.0\,\text{N} - (0.35)(2.5\,\text{kg})(9.81\,\text{m/s}^2) = 2.42\,\text{N}.$$

This means it is accelerating at $(2.42\,\text{N})/(2.5\,\text{kg}) = 0.968\,\text{m/s}^2$. That acceleration will last a time $t = \sqrt{2(0.30\,\text{m})/(0.968\,\text{m/s}^2)} = 0.787\,\text{s}$. The speed of the top block after the force stops pulling is then $(0.968\,\text{m/s}^2)(0.787\,\text{s}) = 0.762\,\text{m/s}$. The force on the bottom block is F_f, so the acceleration of the bottom block is

$$(0.35)(2.5\,\text{kg})(9.81\,\text{m/s}^2)/(10.0\,\text{kg}) = 0.858\,\text{m/s}^2,$$

and the speed after the force stops pulling on the top block is $(0.858\,\text{m/s}^2)(0.787\,\text{s}) = 0.675\,\text{m/s}$.

(a) $W = Fs = (11.0\,\text{N})(0.30\,\text{m}) = 3.3\,\text{J}$ of energy were delivered to the system, but after the force stops pulling only

$$\frac{1}{2}(2.5\,\text{kg})(0.762\,\text{m/s})^2 + \frac{1}{2}(10.0\,\text{kg})(0.675\,\text{m/s})^2 = 3.004\,\text{J}$$

were present as kinetic energy. So $0.296\,\text{J}$ is "missing" and would be now present as internal energy.

(b) The impulse received by the two block system is then $J = (11.0\,\text{N})(0.787\,\text{s}) = 8.66\,\text{N}{\cdot}\text{s}$. This impulse causes a change in momentum, so the speed of the two block system after the external force stops pulling and both blocks move as one is $(8.66\,\text{N}{\cdot}\text{s})(12.5\,\text{kg}) = 0.693\,\text{m/s}$. The final kinetic energy is

$$\frac{1}{2}(12.5\,\text{kg})(0.693\,\text{m/s})^2 = 3.002\,\text{J};$$

this means that $0.002\,\text{J}$ are dissipated.

P13-8 Hmm.

E14-1 $F_S/F_E = M_S{r_E}^2/M_E{r_S}^2$, since everything else cancels out in the expression. Then

$$\frac{F_S}{F_E} = \frac{(1.99\times10^{30}\text{kg})(3.84\times10^{8}\text{m})^2}{(5.98\times10^{24})(1.50\times10^{11}\text{m})^2} = 2.18$$

E14-2 Consider the force from the Sun and the force from the Earth. $F_S/F_E = M_S{r_E}^2/M_E{r_S}^2$, since everything else cancels out in the expression. We want the ratio to be one; we are also constrained because $r_E + r_S = R$ is the distance from the Sun to the Earth. Then

$$\begin{aligned} M_E\,(R - r_E)^2 &= M_S{r_E}^2, \\ R - r_E &= \sqrt{\frac{M_S}{M_E}}r_E, \\ r_E &= (1.50\times10^{11}\text{m})/\left(1+\sqrt{\frac{(1.99\times10^{30}\text{kg})}{(5.98\times10^{24})}}\right) = 2.6\times10^{8}\text{m}. \end{aligned}$$

E14-3 The masses of each object are $m_1 = 20.0\,\text{kg}$ and $m_2 = 7.0\,\text{kg}$; the distance between the centers of the two objects is $15 + 3 = 18\,\text{m}$.

The magnitude of the force from Newton's law of gravitation is then

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(20.0\,\text{kg})(7.0\,\text{kg})}{(18\,\text{m})^2} = 2.9\times10^{-11}\text{N}.$$

E14-4 (a) The magnitude of the force from Newton's law of gravitation is

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(12.7\,\text{kg})(9.85\times10^{-3}\,\text{kg})}{(0.108\,\text{m})^2} = 7.15\times10^{-10}\text{N}.$$

(b) The torque is $\tau = 2(0.262\,\text{m})(7.15\times10^{-10}\text{N}) = 3.75\times10^{-10}\text{N}\cdot\text{m}$.

E14-5 The force of gravity on an object near the surface of the earth is given by

$$F = \frac{GMm}{(r_e + y)^2},$$

where M is the mass of the Earth, m is the mass of the object, r_e is the radius of the Earth, and y is the height above the surface of the Earth. Expand the expression since $y \ll r_e$. We'll use a Taylor expansion, where $F(r_e + y) \approx F(r_e) + y\partial F/\partial r_e$;

$$F \approx \frac{GMm}{r_e^2} - 2y\frac{GMm}{r_e^3}$$

Since we are interested in the difference between the force at the top and the bottom, we really want

$$\Delta F = 2y\frac{GMm}{r_e^3} = 2\frac{y}{r_e}\frac{GMm}{r_e^2} = 2\frac{y}{r_e}W,$$

where in the last part we substituted for the weight, which is the same as the force of gravity,

$$W = \frac{GMm}{r_e^2}.$$

Finally,

$$\Delta F = 2(411\,\text{m})/(6.37\times10^{6}\text{m})(120\text{ lb}) = 0.015\text{ lb}.$$

E14-6 $g \propto 1/r^2$, so $g_1/g_2 = r_2^2/r_1^2$. Then

$$r_2 = \sqrt{(9.81\,\mathrm{m/s^2})/(7.35\,\mathrm{m/s^2})}(6.37\times10^6\mathrm{m}) = 7.36\times10^6\,\mathrm{m}.$$

That's 990 kilometers above the surface of the Earth.

E14-7 (a) $a = GM/r^2$, or

$$a = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\mathrm{kg})}{(10.0\times10^3\mathrm{m})^2} = 1.33\times10^{12}\mathrm{m/s^2}.$$

(b) $v = \sqrt{2ax} = \sqrt{2(1.33\times10^{12}\mathrm{m/s^2})(1.2\,\mathrm{m/s})} = 1.79\times10^6\mathrm{m/s}.$

E14-8 (a) $g_0 = GM/r^2$, or

$$g_0 = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(7.36\times10^{22}\mathrm{kg})}{(1.74\times10^6\mathrm{m})^2} = 1.62\,\mathrm{m/s^2}.$$

(b) $W_\mathrm{m} = W_\mathrm{e}(g_\mathrm{m}/g_\mathrm{e})$ so

$$W_\mathrm{m} = (100\,\mathrm{N})(1.62\,\mathrm{m/s^2}/9.81\,\mathrm{m/s^2}) = 16.5\,\mathrm{N}.$$

(c) Invert $g = GM/r^2$;

$$r = \sqrt{GM/g} = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})/(1.62\,\mathrm{m/s^2})} = 1.57\times10^7\mathrm{m}.$$

That's 2.46 Earth radii, or 1.46 Earth radii above the surface of the Earth.

$\boxed{\textbf{E14-9}}$ The object fell through $y = -10.0\,\mathrm{m}$; the time required to fall would then be

$$t = \sqrt{-2y/g} = \sqrt{-2(-10.0\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 1.43\,\mathrm{s}.$$

We are interested in the *error*, that means taking the total derivative of $y = -\frac{1}{2}gt^2$. and getting

$$\delta y = -\frac{1}{2}\delta g\, t^2 - gt\,\delta t.$$

$\delta y = 0$ so $-\frac{1}{2}\delta g\, t = g\delta\, t$, which can be rearranged as

$$\delta t = -\frac{\delta g\, t}{2g}$$

The percentage error in t needs to be $\delta t/t = 0.1\,\%/2 = 0.05\,\%$. The absolute error is then $\delta t = (0.05\,\%)(1.43\,\mathrm{s}) = 0.7$ ms.

E14-10 Treat mass which is inside a spherical shell as being located at the center of that shell. Ignore any contributions from shells farther away from the center than the point in question.
(a) $F = G(M_1 + M_2)m/a^2$.
(b) $F = G(M_1)m/b^2$.
(c) $F = 0$.

E14-11 For a sphere of uniform density and radius $R > r$,

$$\frac{M(r)}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3},$$

where M is the total mass.

The force of gravity on the object of mass m is then

$$F = \frac{GMm}{r^2}\frac{r^3}{R^3} = \frac{GMmr}{R^3}.$$

g is the free-fall acceleration of the object, and is the gravitational force divided by the mass, so

$$g = \frac{GMr}{R^3} = \frac{GM}{R^2}\frac{r}{R} = \frac{GM}{R^2}\frac{R-D}{R}.$$

Since R is the distance from the center to the surface, and D is the distance of the object beneath the surface, then $r = R - D$ is the distance from the center to the object. The first fraction is the free-fall acceleration on the surface, so

$$g = \frac{GM}{R^2}\frac{R-D}{R} = g_s\frac{R-D}{R} = g_s\left(1 - \frac{D}{R}\right)$$

E14-12 The work required to move the object is $GM_\mathrm{S}m/r$, where r is the gravitational radius. But if this equals mc^2 we can write

$$\begin{aligned} mc^2 &= GM_\mathrm{S}m/r, \\ r &= GM_\mathrm{S}/c^2. \end{aligned}$$

For the sun, $r = (6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(1.99\times10^{30}\mathrm{kg})/(3.00\times10^8\mathrm{m/s})^2 = 1.47\times10^3\mathrm{m}$. That's $2.1\times10^{-6}R_\mathrm{S}$.

E14-13 The distance from the center is

$$r = (80000)(3.00\times10^8\mathrm{m/s})(3.16\times10^7\mathrm{s}) = 7.6\times10^{20}\mathrm{m}.$$

The mass of the galaxy is

$$M = (1.4\times10^{11})(1.99\times10^{30}\mathrm{kg}) = 2.8\times10^{41}\mathrm{kg}.$$

The escape velocity is

$$v = \sqrt{2GM/r} = \sqrt{2(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(2.8\times10^{41}\mathrm{kg})/(7.6\times10^{20}\mathrm{m})} = 2.2\times10^5\mathrm{m/s}.$$

E14-14 Staying in a circular orbit requires the centripetal force be equal to the gravitational force, so

$$m{v_\mathrm{orb}}^2/r = GMm/r^2,$$

or $m{v_\mathrm{orb}}^2 = GMm/r$. But $-GMm/r$ is the gravitational potential energy; to escape one requires a kinetic energy

$$m{v_\mathrm{esc}}^2/2 = GMm/r = m{v_\mathrm{orb}}^2,$$

which has solution $v_\mathrm{esc} = \sqrt{2}v_\mathrm{orb}$.

E14-15 (a) Near the surface of the Earth the total energy is

$$E = K + U = \frac{1}{2}m\left(2\sqrt{gR_{\rm E}}\right)^2 - \frac{GM_{\rm E}m}{R_{\rm E}}$$

but

$$g = \frac{GM}{{R_{\rm E}}^2},$$

so the total energy is

$$\begin{aligned} E &= 2mgR_{\rm E} - \frac{GM_{\rm E}m}{R_{\rm E}}, \\ &= 2m\left(\frac{GM}{{R_{\rm E}}^2}\right)R_{\rm E} - \frac{GM_{\rm E}m}{R_{\rm E}}, \\ &= \frac{GM_{\rm E}m}{R_{\rm E}} \end{aligned}$$

This is a positive number, so the rocket will escape.

(b) Far from earth there is no gravitational potential energy, so

$$\frac{1}{2}mv^2 = \frac{GM_{\rm E}m}{R_{\rm E}} = \frac{GM_{\rm E}}{{R_{\rm E}}^2}mR_{\rm E} = gmR{\rm E},$$

with solution $v = \sqrt{2gR_{\rm E}}$.

E14-16 The rotational acceleration of the sun is related to the galactic acceleration of free fall by

$$4\pi^2 mr/T^2 = GNm^2/r^2,$$

where N is the number of "sun" sized stars of mass m, r is the size of the galaxy, T is period of revolution of the sun. Then

$$N = \frac{4\pi^2 r^3}{GmT^2} = \frac{4\pi^2(2.2\times10^{20}\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\text{kg})(7.9\times10^{15}\text{s})^2} = 5.1\times10^{10}.$$

E14-17 Energy conservation is $K_{\rm i} + U_{\rm i} = K_{\rm f} + U_{\rm f}$, but at the highest point $K_{\rm f} = 0$, so

$$\begin{aligned} U_{\rm f} &= K_{\rm i} + U_{\rm i}, \\ -\frac{GM_{\rm E}m}{R} &= \frac{1}{2}mv_0^2 - \frac{GM_{\rm E}m}{R_{\rm E}}, \\ \frac{1}{R} &= \frac{1}{R_{\rm E}} - \frac{1}{2GM_{\rm E}}v_0^2, \\ \frac{1}{R} &= \frac{1}{(6.37\times10^6\text{m})} - \frac{(9.42\times10^3\text{m/s})^2)}{2(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})}, \\ R &= 2.19\times10^7\ \text{m}. \end{aligned}$$

The distance above the Earth's surface is $2.19\times10^7\ \text{m} - 6.37\times10^6\text{m} = 1.55\times10^6\ \text{m}$.

E14-18 (a) Free-fall acceleration is $g = GM/r^2$. Escape speed is $v = \sqrt{2GM/r}$. Then $v = \sqrt{2gr} = \sqrt{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})} = 2.02\times10^3\mathrm{m/s}$.

(b) $U_\mathrm{f} = K_\mathrm{i} + U_\mathrm{i}$. But $U/m = -g_0 r_0^2/r$, so

$$\frac{1}{r_\mathrm{f}} = \frac{1}{(1.569\times10^6\mathrm{m})} - \frac{(1.01\times10^3\mathrm{m/s})^2}{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2} = \frac{1}{2.09\times10^6\mathrm{m}}.$$

That's 523 km above the surface.

(c) $K_\mathrm{f} = U_\mathrm{i} - U_\mathrm{f}$. But $U/m = -g_0 r_0^2/r$, so

$$v = \sqrt{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2[1/(1.569\times10^6\mathrm{m}) - 1/(2.569\times10^6\mathrm{m})]} = 1260\,\mathrm{m/s}.$$

(d) $M = gr^2/G$, or

$$M = (1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2/(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2}) = 4.8\times10^{22}\mathrm{kg}.$$

E14-19 (a) Apply $\Delta K = -\Delta U$. Then $mv^2 = Gm^2(1/r_2 - 1/r_1)$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.56\times10^{30}\mathrm{kg})\left(\frac{1}{(4.67\times10^4\mathrm{m})} - \frac{1}{(9.34\times10^4\mathrm{m})}\right)} = 3.34\times10^7\mathrm{m/s}.$$

(b) Apply $\Delta K = -\Delta U$. Then $mv^2 = Gm^2(1/r_2 - 1/r_1)$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.56\times10^{30}\mathrm{kg})\left(\frac{1}{(1.26\times10^4\mathrm{m})} - \frac{1}{(9.34\times10^4\mathrm{m})}\right)} = 5.49\times10^7\mathrm{m/s}.$$

E14-20 Call the particles 1 and 2. Then conservation of momentum requires the particle to have the same momentum of the same magnitude, $p = mv_1 = Mv_2$. The momentum of the particles is given by

$$\begin{aligned}\frac{1}{2m}p^2 + \frac{1}{2M}p^2 &= \frac{GMm}{d},\\ \frac{m+M}{mM}p^2 &= 2GMm/d,\\ p &= mM\sqrt{2G/d(m+M)}.\end{aligned}$$

Then $v_\mathrm{rel} = |v_1| + |v_2|$ is equal to

$$\begin{aligned}v_\mathrm{rel} &= mM\sqrt{2G/d(m+M)}\left(\frac{1}{m} + \frac{1}{M}\right),\\ &= mM\sqrt{2G/d(m+M)}\left(\frac{m+M}{mM}\right),\\ &= \sqrt{2G(m+M)/d}\end{aligned}$$

E14-21 The maximum speed is $mv^2 = Gm^2/d$, or $v = \sqrt{Gm/d}$.

E14-22 $T_1^2/r_1^3 = T_2^2/r_2^3$, or

$$T_2 = T_1(r_2/r_1)^{3/2} = (1.00\ \mathrm{y})(1.52)^{3/2} = 1.87\ \mathrm{y}.$$

$\boxed{\textbf{E14-23}}$ We can use Eq. 14-23 to find the mass of Mars; all we need to do is rearrange to solve for M—

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(9.4\times10^6\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2.75\times10^4\text{s})^2} = 6.5\times10^{23}\text{kg}.$$

E14-24 Use $GM/r^2 = 4\pi^2 r/T^2$, so $M = 4\pi^2 r^3/GT^2$, and

$$M = \frac{4\pi^2(3.82\times10^8\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(27.3\times 86400\,\text{s})^2} = 5.93\times10^{24}\text{kg}.$$

E14-25 $T_1^2/r_1^3 = T_2^2/r_2^3$, or

$$T_2 = T_1(r_2/r_1)^{3/2} = (1.00 \text{ month})(1/2)^{3/2} = 0.354 \text{ month}.$$

E14-26 Geosynchronous orbit was found in Sample Problem 14-8 to be 4.22×10^7m. The latitude is given by

$$\theta = \arccos(6.37\times10^6\text{m}/4.22\times10^7\text{m}) = 81.3°.$$

$\boxed{\textbf{E14-27}}$ (b) Make the assumption that the altitude of the satellite is so low that the radius of the orbit is effectively the radius of the moon. Then

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{GM}\right) r^3, \\ &= \left(\frac{4\pi^2}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\text{kg})}\right)(1.74\times10^6\text{m})^3 = 4.24\times10^7\text{s}^2. \end{aligned}$$

So $T = 6.5\times10^3$s.

(a) The speed of the satellite is the circumference divided by the period, or

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.74\times10^6\text{m})}{(6.5\times10^3\text{s})} = 1.68\times10^3\text{m/s}.$$

E14-28 The total energy is $-GMm/2a$. Then

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a},$$

so

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right).$$

E14-29 $r_\text{a} = a(1+e)$, so from Ex. 14-28,

$$v_\text{a} = \sqrt{GM\left(\frac{2}{a(1+e)} - \frac{1}{a}\right)};$$

$r_\text{p} = a(1-e)$, so from Ex. 14-28,

$$v_\text{p} = \sqrt{GM\left(\frac{2}{a(1-e)} - \frac{1}{a}\right)};$$

Dividing one expression by the other,

$$v_\text{p} = v_\text{a}\sqrt{\frac{2/(1-e)-1}{2/(1+e)-1}} = (3.72\text{ km/s})\sqrt{\frac{2/0.12-1}{2/1.88-1}} = 58.3\text{ km/s}.$$

E14-30 (a) Convert.

$$G = \left(6.67\times10^{-11}\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)\left(\frac{1.99\times10^{30}\text{kg}}{M_\text{S}}\right)\left(\frac{3.156\times10^7\text{s}}{\text{y}}\right)^2\left(\frac{\text{AU}}{1.496\times10^{11}\text{m}}\right)^3,$$

which is $G = 39.49\,\text{AU}^3/{M_\text{S}}^2\cdot\text{y}^2$.

(b) Here is a hint: $4\pi^2 = 39.48$. Kepler's law then looks like

$$T^2 = \left(\frac{{M_\text{S}}^2\cdot\text{y}^2}{\text{AU}^3}\right)\frac{r^3}{M}.$$

E14-31 Kepler's third law states $T^2 \propto r^3$, where r is the mean distance from the *Sun* and T is the period of revolution. Newton was in a position to find the acceleration of the Moon toward the Earth by assuming the Moon moved in a circular orbit, since $a_c = v^2/r = 4\pi^2 r/T^2$. But this means that, because of Kepler's law, $a_c \propto r/T^2 \propto 1/r^2$.

E14-32 (a) The force of attraction between the two bodies is

$$F = \frac{GMm}{(r+R)^2}.$$

The centripetal acceleration for the body of mass m is

$$\begin{aligned} r\omega^2 &= \frac{GM}{(r+R)^2}, \\ \omega^2 &= \frac{GM}{r^3(1+R/r)^2}, \\ T^2 &= \frac{4\pi^2}{GM}r^3(1+R/r)^2. \end{aligned}$$

(b) Note that $r = Md/(m+M)$ and $R = md/(m+M)$. Then $R/r = m/M$, so the correction is $(1 + 5.94\times10^{24}/1.99\times10^{30})^2 = 1.000006$ for the Earth/Sun system and 1.025 for the Earth/Moon system.

E14-33 (a) Use the results of Exercise 14-32. The center of mass is located a distance $r = 2md/(m+2m) = 2d/3$ from the star of mass m and a distance $R = d/3$ from the star of mass $2m$. The period of revolution is then given by

$$T^2 = \frac{4\pi^2}{G(2m)}\left(\frac{2}{3}d\right)^3\left(1+\frac{d/3}{2d/3}\right)^2 = \frac{4\pi^2}{3Gm}d^3.$$

(b) Use $L_m = mr^2\omega$, then

$$\frac{L_m}{L_M} = \frac{mr^2}{MR^2} = \frac{m(2d/3)^2}{(2m)(d/3)^2} = 2.$$

(c) Use $K = I\omega^2/2 = mr^2\omega^2/2$. Then

$$\frac{K_m}{K_M} = \frac{mr^2}{MR^2} = \frac{m(2d/3)^2}{(2m)(d/3)^2} = 2.$$

E14-34 Since we don't know which direction the orbit will be, we will assume that the satellite on the surface of the Earth starts with zero kinetic energy. Then $E_\mathrm{i} = U_\mathrm{i}$.

$\Delta U = U_\mathrm{f} - U_\mathrm{i}$ to get the satellite up to the specified altitude. $\Delta K = K_\mathrm{f} = -U_\mathrm{f}/2$. We want to know if $\Delta U - \Delta K$ is positive (more energy to get it up) or negative (more energy to put it in orbit). Then we are interested in

$$\Delta U - \Delta K = 3U_\mathrm{f}/2 - U_\mathrm{i} = GMm\left(\frac{1}{r_\mathrm{i}} - \frac{3}{2r_\mathrm{f}}\right).$$

The "break-even" point is when $r_\mathrm{f} = 3r_\mathrm{i}/2 = 3(6400\text{ km})/2 = 9600$ km, which is 3200 km above the Earth.

(a) More energy to put it in orbit.
(b) Same energy for both.
(c) More energy to get it up.

$\boxed{\textbf{E14-35}}$ (a) The approximate force of gravity on a 2000 kg pickup truck on Eros will be

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.0\times10^{15}\text{ kg})(2000\,\text{kg})}{(7000\,\text{m})^2} = 13.6\,\text{N}.$$

(b) Use

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.0\times10^{15}\text{ kg})}{(7000\,\text{m})}} = 6.9\,\text{m/s}.$$

E14-36 (a) $U = -GMm/r$. The variation is then

$$\begin{aligned}\Delta U &= (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\mathrm{kg})(5.98\times10^{24}\mathrm{kg})\left(\frac{1}{(1.47\times10^{11}\mathrm{m})} - \frac{1}{(1.52\times10^{11}\mathrm{m})}\right)\\ &= 1.78\times10^{32}\mathrm{J}.\end{aligned}$$

(b) $\Delta K + \Delta U = \Delta E = 0$, so $|\Delta K| = 1.78\times10^{32}\mathrm{J}$.
(c) $\Delta E = 0$.
(d) Since $\Delta l = 0$ and $l = mvr$, we have

$$v_\mathrm{p} - v_\mathrm{a} = v_\mathrm{p}\left(1 - \frac{r_\mathrm{p}}{r_\mathrm{a}}\right) = v_\mathrm{p}\left(1 - \frac{(1.47\times10^{11}\mathrm{m})}{(1.52\times10^{11}\mathrm{m})}\right) = 3.29\times10^{-2}v_\mathrm{p}.$$

But $v_\mathrm{p} \approx v_\mathrm{av} = 2\pi(1.5\times10^{11}\mathrm{m})/(3.16\times10^7\mathrm{s}) = 2.98\times10^4\mathrm{m/s}$. Then $\Delta v = 981\,\mathrm{m/s}$.

E14-37 Draw a triangle. The angle made by Chicago, Earth center, satellite is 47.5°. The distance from Earth center to satellite is 4.22×10^7m. The distance from Earth center to Chicago is 6.37×10^6m. Applying the cosine law we find the distance from Chicago to the satellite is

$$\sqrt{(4.22\times10^7\mathrm{m})^2 + (6.37\times10^6\mathrm{m})^2 - 2(4.22\times10^7\mathrm{m})(6.37\times10^6\mathrm{m})\cos(47.5^\circ)} = 3.82\times10^7\mathrm{m}.$$

Applying the sine law we find the angle made by Earth center, Chicago, satellite to be

$$\arcsin\left(\frac{(4.22\times10^7\mathrm{m})}{(3.82\times10^7\mathrm{m})}\sin(47.5^\circ)\right) = 126^\circ.$$

That's 36° above the horizontal.

E14-38 (a) The new orbit is an ellipse with eccentricity given by $r = a'(1+e)$. Then

$$e = r/a' - 1 = (6.64\times10^6\text{m})/(6.52\times10^6\text{m}) - 1 = 0.0184.$$

The distance at P' is given by $r_{P'} = a'(1-e)$. The potential energy at P' is

$$U_{P'} = U_P\frac{1+e}{1-e} = 2(-9.76\times10^{10}\text{J})\frac{1+0.0184}{1-0.0184} = -2.03\times10^{11}\text{J}.$$

The kinetic energy at P' is then

$$K_{P'} = (-9.94\times10^{10}\text{J}) - (-2.03\times10^{11}\text{J}) = 1.04\times10^{11}\text{J}.$$

That would mean $v = \sqrt{2(1.04\times10^{11}\text{J})/(3250\text{kg})} = 8000\,\text{m/s}$.

(b) The average speed is

$$v = \frac{2\pi(6.52\times10^6\text{m})}{(5240\,\text{s})} = 7820\,\text{m/s}.$$

$\boxed{\textbf{E14-39}}$ (a) The Starshine satellite was approximately 275 km above the surface of the Earth on 1 January 2000. We can find the orbital period from Eq. 14-23,

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{GM}\right)r^3, \\ &= \left(\frac{4\pi^2}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})}\right)(6.65\times10^6\text{m})^3 = 2.91\times10^7\text{s}^2, \end{aligned}$$

so $T = 5.39\times10^3\text{s}$.

(b) Equation 14-25 gives the total energy of the system of a satellite of mass m in a circular orbit of radius r around a stationary body of mass $M \gg m$,

$$E = -\frac{GMm}{2r}.$$

We want the rate of change of this with respect to time, so

$$\frac{dE}{dt} = \frac{GMm}{2r^2}\frac{dr}{dt}$$

We can estimate the value of dr/dt from the diagram. I'll choose February 1 and December 1 as my two reference points.

$$\left.\frac{dr}{dt}\right|_{t=t_0} \approx \frac{\Delta r}{\Delta t} = \frac{(240\text{ km}) - (300\text{ km})}{(62\text{ days})} \approx -1\text{ km/day}$$

The rate of energy loss is then

$$\frac{dE}{dt} = \frac{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})(39\,\text{kg})}{2(6.65\times10^6\text{m})^2}\frac{-1000\,\text{m}}{8.64\times10^4\,\text{s}} = -2.0\,\text{J/s}.$$

$\boxed{\textbf{P14-1}}$ The object on the top experiences a force down from gravity W_1 and a force down from the tension in the rope T. The object on the bottom experiences a force down from gravity W_2 and a force up from the tension in the rope.

In either case, the magnitude of W_i is

$$W_i = \frac{GMm}{r_i^2}$$

where r_i is the distance of the ith object from the center of the Earth. While the objects fall they have the same acceleration, and since they have the same mass we can quickly write

$$\frac{GMm}{r_1^2} + T = \frac{GMm}{r_2^2} - T,$$

or

$$\begin{aligned} T &= \frac{GMm}{2r_2^2} - \frac{GMm}{2r_1^2}, \\ &= \frac{GMm}{2}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right), \\ &= \frac{GMm}{2}\frac{r_2^2 - r_1^2}{r_1^2 r_2^2}. \end{aligned}$$

Now $r_1 \approx r_2 \approx R$ in the denominator, but $r_2 = r_1 + l$, so $r_2^2 - r_1^2 \approx 2Rl$ in the numerator. Then

$$T \approx \frac{GMml}{R^3}.$$

P14-2 For a planet of uniform density, $g = GM/r^2 = G(4\pi\rho r^3/3)/r^2 = 4\pi G\rho r/3$. Then if ρ is doubled while r is halved we find that g will be unchanged.

P14-3 (a) $F = GMm/r^2$, $a = F/m = GM/r$.

(b) The acceleration of the Earth toward the center of mass is $a_{\mathrm{E}} = F/M = Gm/r^2$. The relative acceleration is then $GM/r + Gm/r = G(m + M)/r$. Only if $M \gg m$ can we assume that a is independent of m relative to the Earth.

P14-4 (a) $g = GM/r^2$, $\delta g = -(2GM/r^3)\delta r$. In this case $\delta r = h$ and $M = 4\pi\rho r^3/3$. Then

$$\delta W = m\,\delta g = 8\pi G\rho m h/3.$$

(b) $\Delta W/W = \Delta g/g = 2h/r$. Then an error of one part in a million will occur when h is one part in two million of r, or 3.2 meters.

$\boxed{\textbf{P14-5}}$ (a) The magnitude of the gravitational force from the Moon on a particle at A is

$$F_A = \frac{GMm}{(r-R)^2},$$

where the denominator is the distance from the center of the moon to point A.

(b) At the center of the Earth the gravitational force of the moon on a particle of mass m is $F_C = GMm/r^2$.

(c) Now we want to know the difference between these two expressions:

$$\begin{aligned} F_A - F_C &= \frac{GMm}{(r-R)^2} - \frac{GMm}{r^2}, \\ &= GMm\left(\frac{r^2}{r^2(r-R)^2} - \frac{(r-R)^2}{r^2(r-R)^2}\right), \\ &= GMm\left(\frac{r^2-(r-R)^2}{r^2(r-R)^2}\right), \\ &= GMm\left(\frac{R(2r-R)}{r^2(r-R)^2}\right). \end{aligned}$$

To simplify assume $R \ll r$ and then substitute $(r-R) \approx r$. The force difference simplifies to

$$F_T = GMm\frac{R(2r)}{r^2(r)^2} = \frac{2GMmR}{r^3}$$

(d) Repeat part (c) except we want $r+R$ instead of $r-R$. Then

$$\begin{aligned} F_A - F_C &= \frac{GMm}{(r+R)^2} - \frac{GMm}{r^2}, \\ &= GMm\left(\frac{r^2}{r^2(r+R)^2} - \frac{(r+R)^2}{r^2(r+R)^2}\right), \\ &= GMm\left(\frac{r^2-(r+R)^2}{r^2(r+R)^2}\right), \\ &= GMm\left(\frac{-R(2r+R)}{r^2(r+R)^2}\right). \end{aligned}$$

To simplify assume $R \ll r$ and then substitute $(r+R) \approx r$. The force difference simplifies to

$$F_T = GMm\frac{-R(2r)}{r^2(r)^2} = -\frac{2GMmR}{r^3}$$

The negative sign indicates that this "apparent" force points *away* from the moon, not toward it.

(e) Consider the directions: the water is effectively attracted to the moon when closer, but repelled when farther.

P14-6 $F_{\text{net}} = mr{\omega_{\text{s}}}^2$, where ω_{s} is the rotational speed of the ship. But since the ship is moving relative to the earth with a speed v, we can write $\omega_{\text{s}} = \omega \pm v/r$, where the sign is positive if the ship is sailing east. Then $F_{\text{net}} = mr(\omega \pm v/r)^2$.

The scale measures a force W which is given by $mg - F_{\text{net}}$, or

$$W = mg - mr(\omega \pm v/r)^2.$$

Note that $W_0 = m(g - r\omega^2)$. Then

$$\begin{aligned} W &= W_0\frac{g - r(\omega \pm v/r)^2}{g - r\omega^2}, \\ &\approx W_0\left(1 \pm \frac{2\omega v}{1 - r\omega^2}\right), \\ &\approx W_0(1 \pm 2v\omega/g). \end{aligned}$$

P14-7 (a) $a = GM/r^2 - r\omega^2$. ω is the rotational speed of the Earth. Since Frank observes $a = g/2$ we have

$$\begin{aligned} g/2 &= GM/r^2 - r\omega^2, \\ r^2 &= (2GM - 2r^3\omega^2)/g, \\ r &= \sqrt{2(GM - r^3\omega^2)/g} \end{aligned}$$

Note that

$$GM = (6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg}) = 3.99\times10^{14}\text{m}^3/\text{s}^2$$

while

$$r^3\omega^2 = (6.37\times10^6\text{m})^3(2\pi/86400\,\text{s})^2 = 1.37\times10^{12}\text{m}^3/\text{s}^2.$$

Consequently, $r^3\omega^2$ can be treated as a perturbation of GM bear the Earth. Solving iteratively,

$$\begin{aligned} r_0 &= \sqrt{2[(3.99\times10^{14}\text{m}^3/\text{s}^2) - (6.37\times10^6\text{m})^3(2\pi/86400\,\text{s})^2]/(9.81\,\text{m/s}^2)} = 9.00\times10^6\text{m}, \\ r_1 &= \sqrt{2[(3.99\times10^{14}\text{m}^3/\text{s}^2) - (9.00\times10^6\text{m})^3(2\pi/86400\,\text{s})^2]/(9.81\,\text{m/s}^2)} = 8.98\times10^6\text{m}, \end{aligned}$$

which is close enough for me. Then $h = 8.98\times10^6\text{m} - 6.37\times10^6\text{m} = 2610$ km.

(b) $\Delta E = E_\text{f} - E_\text{i} = U_\text{f}/2 - U_\text{i}$. Then

$$\Delta E = (6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})(100\,\text{kg})\left(\frac{1}{(6.37\times10^6\text{m})} - \frac{1}{2(8.98\times10^6\text{m})}\right) = 4.0\times10^9\text{J}.$$

P14-8 (a) Equate centripetal force with the force of gravity.

$$\begin{aligned} \frac{4\pi^2 mr}{T^2} &= \frac{GMm}{r^2}, \\ \frac{4\pi^2}{T^2} &= \frac{G(4/3)\pi r^3\rho}{r^3}, \\ T &= \sqrt{\frac{3\pi}{G\rho}} \end{aligned}$$

(b) $T = \sqrt{3\pi/(6.7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(3.0\times10^3\text{kg/m}^3)} = 6800\,\text{s}$.

P14-9 (a) One can find δg by pretending the Earth is not there, but the material in the hole is. Concentrate on the vertical component of the resulting force of attraction. Then

$$\delta g = \frac{GM}{r^2}\frac{d}{r},$$

where r is the straight line distance from the prospector to the center of the hole and M is the mass of material that would fill the hole. A few substitutions later,

$$\delta g = \frac{4\pi G\rho R^3 d}{3(\sqrt{d^2+x^2})^3}.$$

(b) Directly above the hole $x = 0$, so a ratio of the two readings gives

$$\frac{(10.0 \text{ milligals})}{(14.0 \text{ milligals})} = \left(\frac{d^2}{d^2 + (150\,\text{m})^2}\right)^{3/2}$$

or

$$(0.800)(d^2 + 2.25\times10^4\text{m}^2) = d^2,$$

which has solution $d = 300\,\text{m}$. Then

$$R^3 = \frac{3(14.0\times10^{-5}\text{m/s}^2)(300\,\text{m})^2}{4\pi(6.7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2800\,\text{kg/m}^3)},$$

so $R = 250\,\text{m}$. The top of the cave is then $300\,\text{m} - 250\,\text{m} = 50\,\text{m}$ beneath the surface.

(b) All of the formulae stay the same *except* replace ρ with the difference between rock and water. d doesn't change, but R will now be given by

$$R^3 = \frac{3(14.0\times10^{-5}\text{m/s}^2)(300\,\text{m})^2}{4\pi(6.7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(1800\,\text{kg/m}^3)},$$

so $R = 292\,\text{m}$, and then the cave is located $300\,\text{m} - 292\,\text{m} = 7\,\text{m}$ beneath the surface.

P14-10 $g = GM/r^2$, where M is the mass enclosed in within the sphere of radius r. Then $dg = (G/r^2)dM - 2(GM/r^3)dr$, so that g is locally constant if $dM/dr = 2M/r$. Expanding,

$$\begin{aligned} 4\pi r^2 \rho_l &= 8\pi r^2 \rho/3, \\ \rho_l &= 2\rho/3. \end{aligned}$$

$\boxed{\textbf{P14-11}}$ The force of gravity on the small sphere of mass m is equal to the force of gravity from a solid lead sphere minus the force which would have been contributed by the smaller lead sphere which would have filled the hole. So we need to know about the size and mass of the lead which was removed to make the hole.

The density of the lead is given by

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

The hole has a radius of $R/2$, so if the density is constant the mass of the hole will be

$$M_h = \rho V = \left(\frac{M}{\frac{4}{3}\pi R^3}\right) \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

The "hole" is closer to the small sphere; the center of the hole is $d - R/2$ away. The force of the whole lead sphere minus the force of the "hole" lead sphere is

$$\frac{GMm}{d^2} - \frac{G(M/8)m}{(d - R/2)^2}$$

P14-12 (a) Use $v = \omega\sqrt{R^2 - r^2}$, where $\omega = \sqrt{GM_E/R^3}$. Then

$$\begin{aligned} T &= \int_0^T dt = \int_R^0 \frac{dr}{dr/dt} = \int_R^0 \frac{dr}{v}, \\ &= \int_R^0 \frac{dr}{\omega\sqrt{R^2 - r^2}}, \\ &= \frac{\pi}{2\omega} \end{aligned}$$

Knowing that

$$\omega = \sqrt{\frac{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})}{(6.37\times10^6\text{m})^3}} = 1.24\times10^{-3}/\text{s},$$

we can find $T = 1260\,\text{s} = 21$ min.

(b) Same time, 21 minutes. To do a complete journey would require four times this, or $2\pi/\omega$. That's 84 minutes!

(c) The answers are the same.

P14-13 (a) $g = GM/r^2$ and $M = 1.93\times10^{24}\text{kg} + 4.01\times10^{24}\text{kg} + 3.94\times10^{22}\text{kg} = 5.98\times10^{24}\text{kg}$ so

$$g = (6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})/(6.37\times10^6\text{m})^2 = 9.83\,\text{m/s}^2.$$

(b) Now $M = 1.93\times10^{24}\text{kg} + 4.01\times10^{24}\text{kg} = 5.94\times10^{24}\text{kg}$ so

$$g = (6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.94\times10^{24}\text{kg})/(6.345\times10^6\text{m})^2 = 9.84\,\text{m/s}^2.$$

(c) For a uniform body, $g = 4\pi G\rho r/3 = GMr/R^3$, so

$$g = (6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})(6.345\times10^6\text{m})/(6.37\times10^6\text{m})^3 = 9.79\,\text{m/s}^2.$$

P14-14 (a) Use $g = GM/r^2$, then

$$g = (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.93\times10^{24}\mathrm{kg})/(3.490\times10^6\mathrm{m})^2 = 106\,\mathrm{m/s^2}.$$

The variation with depth is linear if core has uniform density.

(b) In the mantle we have $g = G(M_\mathrm{c}+M)/r^2$, where M is the amount of the mass of the mantle which is enclosed in the sphere of radius r. The density of the core is

$$\rho_\mathrm{c} = \frac{3(1.93\times10^{24}\mathrm{kg})}{4\pi(3.490\times10^6\mathrm{m})^3} = 1.084\times10^4\mathrm{kg/m^3}.$$

The density of the mantle is harder to find,

$$\rho_\mathrm{c} = \frac{3(4.01\times10^{24}\mathrm{kg})}{4\pi[(6.345\times10^6\mathrm{m})^3 - (3.490\times10^6\mathrm{m})^3} = 4.496\times10^3\mathrm{kg/m^3}.$$

We can pretend that the core is made up of a point mass at the center and the rest has a density equal to that of the mantle. That point mass would be

$$M_\mathrm{p} = \frac{4\pi(3.490\times10^6\mathrm{m})^3(1.084\times10^4\mathrm{kg/m^3} - 4.496\times10^3\mathrm{kg/m^3})}{3} = 1.130\times10^{24}\mathrm{kg}.$$

Then

$$g = GM_\mathrm{p}/r^2 + 4\pi G\rho_\mathrm{m}r/3.$$

Find dg/dr, and set equal to zero. This happens when

$$2M_\mathrm{p}/r^3 = 4\pi\rho_\mathrm{m}/3,$$

or $r = 4.93\times10^6\mathrm{m}$. Then $g = 9.29\,\mathrm{m/s^2}$. Since this is less than the value at the end points it must be a minimum.

$\boxed{\textbf{P14-15}}$ (a) We will use part of the hint, but we will integrate instead of assuming the bit about g_av; doing it this way will become important for later chapters. Consider a small horizontal slice of the column of thickness dr. The weight of the material above the slice exerts a force $F(r)$ on the top of the slice; there is a force of gravity on the slice given by

$$dF = \frac{GM(r)\,dm}{r^2},$$

where $M(r)$ is the mass contained in the sphere of radius r,

$$M(r) = \frac{4}{3}\pi r^3\rho.$$

Lastly, the mass of the slice dm is related to the thickness and cross sectional area by $dm = \rho A\,dr$. Then

$$dF = \frac{4\pi GA\rho^2}{3}r\,dr.$$

Integrate both sides of this expression. On the left the limits are 0 to F_center, on the right the limits are R to 0; we need to throw in an extra negative sign because the force increases as r decreases. Then

$$F = \frac{2}{3}\pi GA\rho^2R^2.$$

Divide both sides by A to get the compressive stress.

(b) Put in the numbers!

$$S = \frac{2}{3}\pi(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(4000\,\mathrm{kg/m}^3)^2(3.0\times10^5\mathrm{m})^2 = 2.0\times10^8\mathrm{N/m}^2.$$

(c) Rearrange, and then put in numbers;

$$R = \sqrt{\frac{3(4.0\times10^7\mathrm{N/m}^2)}{2\pi(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(3000\,\mathrm{kg/m}^3)^2}} = 1.8\times10^5\,\mathrm{m}.$$

P14-16 The two mass increments each exert a vertical and a horizontal force on the particle, but the horizontal components will cancel. The vertical component is proportional to the sine of the angle, so that

$$dF = \frac{2Gm\,dm}{r^2}\frac{y}{r} = \frac{2Gm\lambda\,dx}{r^2}\frac{y}{r},$$

where $r^2 = x^2 + y^2$. We will eventually integrate from 0 to ∞, so

$$\begin{aligned} F &= \int_0^\infty \frac{2Gm\lambda\,dx}{r^2}\frac{y}{r}, \\ &= 2Gm\lambda y\int_0^\infty \frac{dx}{(x^2+y^2)^{3/2}}, \\ &= \frac{2Gm\lambda}{y}. \end{aligned}$$

P14-17 For any arbitrary point P the cross sectional area which is perpendicular to the axis $dA' = r^2 d\Omega$ is not equal to the projection dA onto the surface of the sphere. It depends on the angle that the axis makes with the normal, according to $dA' = \cos\theta dA$. Fortunately, the angle made at point 1 is identical to the angle made at point 2, so we can write

$$\begin{aligned} d\Omega_1 &= d\Omega_2, \\ dA_1/r_1^2 &= dA_2/r_2^2 \end{aligned}$$

But the mass of the shell contained in dA is proportional to dA, so

$$\begin{aligned} r_1^2 dm_1 &= r_2^2 dm_2, \\ Gm\,dm_1/r_1^2 &= Gm\,dm_2/r_2^2. \end{aligned}$$

Consequently, the force on an object at point P is balanced by both cones.

(b) Evaluate $\int d\Omega$ for the top and bottom halves of the sphere. Since every $d\Omega$ on the top is balanced by one on the bottom, the net force is zero.

P14-18

P14-19 Assume that the small sphere is always between the two spheres. Then

$$\begin{aligned} W &= \Delta U_1 + \Delta U_2, \\ &= (6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(0.212\,\mathrm{kg})\,[(7.16\,\mathrm{kg}) - (2.53\,\mathrm{kg})]\left[\frac{1}{(0.420\,\mathrm{m})} - \frac{1}{(1.14\,\mathrm{m})}\right], \\ &= 9.85\times10^{-11}\mathrm{J}. \end{aligned}$$

P14-20 Note that $\frac{1}{2}m{v_{\rm esc}}^2 = -U_0$, where U_0 is the potential energy at the burn-out height. Energy conservation gives

$$\begin{aligned} K &= K_0 + U_0, \\ \frac{1}{2}mv^2 &= \frac{1}{2}mv_0^2 - \frac{1}{2}m{v_{\rm esc}}^2, \\ v &= \sqrt{v_0^2 - {v_{\rm esc}}^2}. \end{aligned}$$

$\boxed{\textbf{P14-21}}$ (a) The force of one star on the other is given by $F = Gm^2/d^2$, where d is the distance between the stars. The stars revolve around the center of mass, which is halfway between the stars so $r = d/2$ is the radius of the orbit of the stars. If a is the centripetal acceleration of the stars, the period of revolution is then

$$T = \sqrt{\frac{4\pi^2 r}{a}} = \sqrt{\frac{4m\pi^2 r}{F}} = \sqrt{\frac{16\pi^2 r^3}{Gm}}.$$

The numerical value is

$$T = \sqrt{\frac{16\pi^2(1.12\times10^{11}\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(3.22\times10^{30}\text{kg})}} = 3.21\times10^7\text{s} = 1.02\text{ y}.$$

(b) The gravitational potential energy per kilogram midway between the stars is

$$-2\frac{Gm}{r} = -2\frac{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(3.22\times10^{30}\text{kg})}{(1.12\times10^{11}\text{m})} = -3.84\times10^9\text{J/kg}.$$

An object of mass M at the center between the stars would need $(3.84\times10^9\text{J/kg})M$ kinetic energy to escape, this corresponds to a speed of

$$v = \sqrt{2K/M} = \sqrt{2(3.84\times10^9\text{J/kg})} = 8.76\times10^4\text{m/s}.$$

P14-22 (a) Each differential mass segment on the ring contributes the same amount to the force on the particle,

$$dF = \frac{Gm\,dm}{r^2}\frac{x}{r},$$

where $r^2 = x^2 + R^2$. Since the differential mass segments are all equal distance, the integration is trivial, and the net force is

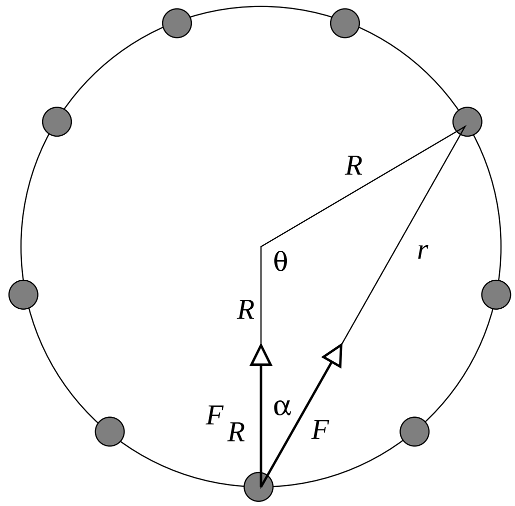
$$F = \frac{GMmx}{(x^2+R^2)^{3/2}}.$$

(b) The potential energy can be found by integrating with respect to x,

$$\Delta U = \int_0^\infty F\,dx = \int_0^\infty \frac{GMmx}{(x^2+R^2)^{3/2}}\,dx = \frac{GMm}{R}.$$

Then the particle of mass m will pass through the center of the ring with a speed $v = \sqrt{2\Delta U/m} = \sqrt{2GM/R}$.

P14-23 (a) Consider the following diagram.



The distance r is given by the cosine law to be

$$r^2 = R^2 + R^2 - 2R^2 \cos\theta = 2R^2(1 - \cos\theta).$$

The force between two particles is then $F = Gm^2/r^2$. Each particle has a symmetric partner, so only the force component directed toward the center contributes. If we call this the R component we have

$$F_R = F\cos\alpha = F\cos(90^\circ - \theta/2) = F\sin(\theta/2).$$

Combining,

$$F_R = \frac{Gm^2}{2R^2}\frac{\sin(\theta/2)}{1-\cos\theta}.$$

But *each* of the other particles contributes to this force, so

$$F_{\text{net}} = \frac{Gm^2}{2R^2}\sum_i \frac{\sin(\theta_i/2)}{1-\cos\theta_i}$$

When there are only 9 particles the angles are in steps of 40°; the θ_i are then 40°, 80°, 120°, 160°, 200°, 240°, 280°, and 320°. With a little patience you will find

$$\sum_i \frac{\sin(\theta_i/2)}{1-\cos\theta_i} = 6.649655,$$

using these angles. Then $F_{\text{net}} = 3.32Gm^2/R^2$.

(b) The rotational period of the ring would need to be

$$T = \sqrt{\frac{4\pi^2 R}{a}} = \sqrt{\frac{4m\pi^2 R}{F}} = \sqrt{\frac{16\pi^2 R^3}{3.32Gm}}.$$

P14-24 The potential energy of the system is $U = -Gm^2/r$. The kinetic energy is mv^2. The total energy is $E = -Gm^2/d$. Then

$$\frac{dr}{dt} = 2\sqrt{Gm(1/r - 1/d)},$$

so the time to come together is

$$T = \int_d^0 \frac{dr}{2\sqrt{Gm(1/r - 1/d)}} = \sqrt{\frac{d^3}{4Gm}} \int_0^1 \sqrt{\frac{x}{1-x}} dx = \frac{\pi}{4}\sqrt{\frac{d^3}{Gm}}.$$

P14-25 (a) $E = U/2$ for each satellite, so the total mechanical energy is $-GMm/r$.

(b) Now there is no K, so the total mechanical energy is simply $U = -2GMm/r$. The factor of 2 is because there are two satellites.

(c) The wreckage falls vertically onto the Earth.

P14-26 Let $r_{\mathrm{a}} = a(1+e)$ and $r_{\mathrm{p}} = a(1-e)$. Then $r_{\mathrm{a}} + r_{\mathrm{e}} = 2a$ and $r_{\mathrm{a}} - r_{\mathrm{p}} = 2ae$. So the answer is

$$2(0.0167)(1.50\times10^{11}\mathrm{m}) = 5.01\times10^9\mathrm{m},$$

or 7.20 solar radii.

P14-27

P14-28 The net force on an orbiting star is

$$F = Gm\left(\frac{M}{r^2} + m4r^2\right).$$

This is the centripetal force, and is equal to $4\pi^2 mr/T^2$. Combining,

$$\frac{4\pi^2}{T^2} = \frac{G}{4r^3}(4M+m),$$

so $T = 4\pi\sqrt{r^3/[G(4M+m)]}$.

P14-29 (a) $v = \sqrt{GM/r}$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times10^{24}\mathrm{kg})/(7.01\times10^6\mathrm{m})} = 7.54\times10^3\mathrm{m/s}.$$

(b) $T = 2\pi(7.01\times10^6\mathrm{m})/(7.54\times10^3\mathrm{m/s}) = 5.84\times10^3\mathrm{s}$.

(c) Originally $E_0 = U/2$, or

$$E = -\frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times10^{24}\mathrm{kg})(220\mathrm{kg})}{2(7.01\times10^6\mathrm{m})} = -6.25\times10^9\mathrm{J}.$$

After 1500 orbits the energy is now $-6.25\times10^9\mathrm{J} - (1500)(1.40\times10^5\mathrm{J}) = -6.46\times10^9\mathrm{J}$. The new distance from the Earth is then

$$r = -\frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times10^{24}\mathrm{kg})(220\mathrm{kg})}{2(-6.46\times10^9\mathrm{J})} = 6.79\times10^6\mathrm{m}.$$

The altitude is now $6790 - 6370 = 420$ km.

(d) $F = (1.40\times10^5\mathrm{J})/(2\pi 7.01\times10^6\mathrm{m}) = 3.2\times10^{-3}\mathrm{N}$.

(e) No.

P14-30 Let the satellite S be directly overhead at some time. The magnitude of the speed is equal to that of a geosynchronous satellite T whose orbit is not inclined, but since there are both parallel and perpendicular components to the motion of S it will appear to move north while "losing ground" compared to T. Eventually, though, it must pass overhead again in 12 hours. When S is as far north as it will go (6 hours) it has a velocity which is parallel to T, but it is located in a region where the required speed to appear fixed is slower. Hence, it will appear to be "gaining ground" against the background stars. Consequently, the motion against the background stars appears to be a figure 8.

P14-31 The net force of gravity on one star because of the other two is

$$F = \frac{2GM^2}{L^2}\cos(30°).$$

The stars orbit about a point $r = L/2\cos(30°)$ from any star. The orbital speed is then found from

$$\frac{Mv^2}{r} = \frac{Mv^2}{L/2\cos(30°)} = \frac{2GM^2}{L^2}\cos(30°),$$

or $v = \sqrt{GM/L}$.

P14-32 A parabolic path will eventually escape; this means that the speed of the comet at any distance is the escape speed for that distance, or $v = \sqrt{2GM/r}$. The angular momentum is constant, and is equal to

$$l = mv_{\rm A}r_{\rm A} = m\sqrt{2GMr_{\rm A}}.$$

For a parabolic path, $r = 2r_{\rm A}/(1+\cos\theta)$. Combining with Eq. 14-21 and the equation before that one we get

$$\frac{d\theta}{dt} = \frac{\sqrt{2GMr_{\rm A}}}{4{r_{\rm A}}^2}(1+\cos\theta)^2.$$

The time required is the integral

$$T = \sqrt{\frac{8{r_{\rm A}}^3}{GM}}\int_0^{\pi/2}\frac{d\theta}{(1+\cos\theta)^2} = \sqrt{\frac{8{r_{\rm A}}^3}{GM}}\left(\frac{2}{3}\right).$$

Note that $\sqrt{{r_{\rm A}}^3/GM}$ is equal to $1/2\pi$ years. Then the time for the comet to move is

$$T = \frac{1}{2\pi}\sqrt{8}\frac{2}{3}\ {\rm y} = 0.300\ {\rm y}.$$

$\boxed{\textbf{P14-33}}$ There are three forces on loose matter (of mass m_0) sitting on the moon: the force of gravity toward the moon, $F_m = Gmm_0/a^2$, the force of gravity toward the planet, $F_M = GMm_0/(r-a)^2$, and the normal force N of the moon pushing the loose matter away from the center of the moon.

The net force on this loose matter is $F_M + N - F_m$, this value is *exactly* equal to the centripetal force necessary to keep the loose matter moving in a uniform circle. The period of revolution of the loose matter is identical to that of the moon,

$$T = 2\pi\sqrt{r^3/GM},$$

but since the loose matter is actually revolving at a radial distance $r-a$ the centripetal force is

$$F_{\rm c} = \frac{4\pi^2 m_0(r-a)}{T^2} = \frac{GMm_0(r-a)}{r^3}.$$

Only if the normal force is zero can the loose matter can lift off, and this will happen when $F_c = F_M - F_m$, or

$$\begin{aligned} \frac{M(r-a)}{r^3} &= \frac{M}{(r-a)^2} - \frac{m}{a^2}, \\ &= \frac{Ma^2 - m(r-a)^2}{a^2(r-a)^2}, \\ Ma^2(r-a)^3 &= Mr^3a^2 - mr^3(r-a)^2, \\ -3r^2a^3 + 3ra^4 - a^4 &= \frac{m}{M}\left(-r^5 + 2r^4a - r^3a^2\right) \end{aligned}$$

Let $r = ax$, then x is dimensionless; let $\beta = m/M$, then β is dimensionless. The expression then simplifies to

$$-3x^2 + 3x - 1 = \beta(-x^5 + 2x^4 - x^3).$$

If we assume than x is very large ($r \gg a$) then only the largest term on each side survives. This means $3x^2 \approx \beta x^5$, or $x = (3/\beta)^{1/3}$. In that case, $r = a(3M/m)^{1/3}$. For the Earth's moon $r_c = 1.1 \times 10^7$m, which is only 4,500 km away from the surface of the Earth. It is somewhat interesting to note that the radius r is actually independent of both a and m if the moon has a uniform density!

$\boxed{\textbf{E15-1}}$ The pressure in the syringe is

$$p = \frac{(42.3\,\mathrm{N})}{\pi(1.12\times10^{-2}\mathrm{m}/s)^2} = 4.29\times10^5\mathrm{Pa}.$$

E15-2 The total mass of fluid is

$$m = (0.5\times10^{-3}\mathrm{m}^3)(2600\,\mathrm{kg/m}^3)+(0.25\times10^{-3}\mathrm{m}^3)(1000\,\mathrm{kg/m}^3)+(0.4\times10^{-3}\mathrm{m}^3)(800\,\mathrm{kg/m}^3) = 1.87\,\mathrm{kg}.$$

The weight is $(18.7\,\mathrm{kg})(9.8\,\mathrm{m/s}^2) = 18\,\mathrm{N}$.

E15-3 $F = A\Delta p$, so

$$F = (3.43\,\mathrm{m})(2.08\,\mathrm{m})(1.00\text{ atm} - 0.962\text{ atm})(1.01\times10^5\mathrm{Pa/\ atm}) = 2.74\times10^4\mathrm{N}.$$

E15-4 $B\Delta V/V = -\Delta p$; $V = L^3$; $\Delta V \approx L^2\Delta L/3$. Then

$$\Delta p = (140\times10^9\mathrm{Pa})\frac{(5\times10^{-3}\mathrm{m})}{3(0.85\,\mathrm{m})} = 2.74\times10^9\mathrm{Pa}.$$

$\boxed{\textbf{E15-5}}$ There is an inward force F_1 pushing the lid closed from the pressure of the air outside the box; there is an outward force F_2 pushing the lid open from the pressure of the air inside the box. To lift the lid we need to exert an additional outward force F_3 to get a net force of zero.

The magnitude of the inward force is $F_1 = P_{\mathrm{out}}A$, where A is the area of the lid and P_{out} is the pressure outside the box. The magnitude of the outward force F_2 is $F_2 = P_{\mathrm{in}}A$. We are told $F_3 = 108$ lb. Combining,

$$\begin{aligned} F_2 &= F_1 - F_3, \\ P_{\mathrm{in}}A &= P_{\mathrm{out}}A - F_3, \\ P_{\mathrm{in}} &= P_{\mathrm{out}} - F_3/A, \end{aligned}$$

so $P_{\mathrm{in}} = (15\text{ lb/in}^2 - (108\text{ lb})/(12\text{ in}^2) = 6.0\text{ lb/in}^2$.

E15-6 $h = \Delta p/\rho g$, so

$$h = \frac{(0.05\text{ atm})(1.01\times10^5\mathrm{Pa/\ atm})}{(1000\,\mathrm{kg/m}^3)(9.8\,\mathrm{m/s}^2)} = 0.52\,\mathrm{m}.$$

E15-7 $\Delta p = (1060\,\mathrm{kg/m}^3)(9.81\,\mathrm{m/s}^2)(1.83\,\mathrm{m}) = 1.90\times10^4\mathrm{Pa}.$

E15-8 $\Delta p = (1024\,\mathrm{kg/m}^3)(9.81\,\mathrm{m/s}^2)(118\,\mathrm{m}) = 1.19\times10^6\mathrm{Pa}$. Add this to p_0; the total pressure is then $1.29\times10^6\mathrm{Pa}$.

$\boxed{\textbf{E15-9}}$ The pressure differential assuming we don't have a sewage pump:

$$p_2 - p_1 = -\rho g\,(y_2 - y_1) = (926\,\mathrm{kg/m}^3)(9.81\,\mathrm{m/s}^2)(8.16\,\mathrm{m} - 2.08\,\mathrm{m}) = 5.52\times10^4\mathrm{Pa}.$$

We need to overcome this pressure difference with the pump.

E15-10 (a) $p = (1.00\text{ atm})e^{-5.00/8.55} = 0.557$ atm.
(b) $h = (8.55\text{ km})\ln(1.00/0.500) = 5.93$ km.

E15-11 The mercury will rise a distance a on one side and fall a distance a on the other so that the difference in mercury height will be $2a$. Since the masses of the "excess" mercury and the water will be proportional, we have $2a\rho_\mathrm{m} = d\rho_\mathrm{w}$, so

$$a = \frac{(0.112\mathrm{m})(1000\,\mathrm{kg/m^3})}{2(13600\,\mathrm{kg/m^3})} = 4.12\times10^{-3}\mathrm{m}.$$

E15-12 (a) The pressure (due to the water alone) at the bottom of the pool is

$$P = (62.45\ \mathrm{lb/ft^3})(8.0\ \mathrm{ft}) = 500\ \mathrm{lb/ft^2}.$$

The force on the bottom is

$$F = (500\ \mathrm{lb/ft^2})(80\ \mathrm{ft})(30\ \mathrm{ft}) = 1.2\times10^6\ \mathrm{lb}.$$

The average pressure on the side is half the pressure on the bottom, so

$$F = (250\ \mathrm{lb/ft^2})(80\ \mathrm{ft})(8.0\ \mathrm{ft}) = 1.6\times10^5\ \mathrm{lb}.$$

The average pressure on the end is half the pressure on the bottom, so

$$F = (250\ \mathrm{lb/ft^2})(30\ \mathrm{ft})(8.0\ \mathrm{ft}) = 6.0\times10^4\ \mathrm{lb}.$$

(b) No, since that additional pressure acts on both sides.

$\boxed{\textbf{E15-13}}$ (a) Equation 15-8 can be used to find the height y_2 of the atmosphere if the density is constant. The pressure at the top of the atmosphere would be $p_2 = 0$, and the height of the bottom y_1 would be zero. Then

$$y_2 = (1.01\times10^5\mathrm{Pa})/\left[(1.21\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})\right] = 8.51\times10^3\mathrm{m}.$$

(b) We have to go back to Eq. 15-7 for an atmosphere which has a density which varies linearly with altitude. Linear variation of density means

$$\rho = \rho_0\left(1 - \frac{y}{y_\mathrm{max}}\right)$$

Substitute this into Eq. 15-7,

$$\begin{aligned} p_2 - p_1 &= -\int_0^{y_\mathrm{max}} \rho g\, dy, \\ &= -\int_0^{y_\mathrm{max}} \rho_0 g\left(1 - \frac{y}{y_\mathrm{max}}\right)\, dy, \\ &= -\rho_0 g\left(y - \frac{y^2}{2y_\mathrm{max}}\right)\Bigg|_0^{y_\mathrm{max}}, \\ &= -\rho g y_\mathrm{max}/2. \end{aligned}$$

In this case we have $y_\mathrm{max} = 2p_1/(\rho g)$, so the answer is twice that in part (a), or 17 km.

E15-14 $\Delta P = (1000\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(112\,\mathrm{m}) = 1.1\times10^6\mathrm{Pa}$. The force required is then $F = (1.1\times10^6\mathrm{Pa})(1.22\,\mathrm{m})(0.590\,\mathrm{m}) = 7.9\times10^5\mathrm{N}$.

E15-15 (a) Choose *any* infinitesimally small spherical region where equal volumes of the two fluids are in contact. The denser fluid will have the larger mass. We can treat the system as being a sphere of uniform mass with a hemisphere of additional mass being superimposed in the region of higher density. The orientation of this hemisphere is the only variable when calculating the potential energy. The center of mass of this hemisphere will be a low as possible only when the surface is horizontal. So all two-fluid interfaces will be horizontal.

(b) If there exists a region where the interface is not horizontal then there will be two different values for $\Delta p = \rho g h$, depending on the path taken. This means that there will be a horizontal pressure gradient, and the fluid will flow along that gradient until the horizontal pressure gradient is equalized.

E15-16 The mass of liquid originally in the first vessel is $m_1 = \rho A h_1$; the center of gravity is at $h_1/2$, so the potential energy of the liquid in the first vessel is originally $U_1 = \rho g A h_1^2/2$. A similar expression exists for the liquid in the second vessel. Since the two vessels have the same cross sectional area the final height in both containers will be $h_\mathrm{f} = (h_1 + h_2)/2$. The final potential energy of the liquid in *each* container will be $U_\mathrm{f} = \rho g A (h_1 + h_2)^2/8$. The work done by gravity is then

$$\begin{aligned} W &= U_1 + U_2 - 2U_\mathrm{f}, \\ &= \frac{\rho g A}{4}\left[2h_1^2 + 2h_2^2 - (h_1^2 + 2h_1h_2 + h_2^2)\right], \\ &= \frac{\rho g A}{4}(h_1 - h_2)^2. \end{aligned}$$

$\boxed{\textbf{E15-17}}$ There are *three* force on the block: gravity ($W = mg$), a buoyant force $B_0 = m_\mathrm{w} g$, and a tension T_0. When the container is at rest all three forces balance, so $B_0 - W - T_0 = 0$. The tension in this case is $T_0 = (m_\mathrm{w} - m)g$.

When the container accelerates upward we now have $B - W - T = ma$. Note that neither the tension *nor* the buoyant force stay the same; the buoyant force increases according to $B = m_\mathrm{w}(g+a)$. The new tension is then

$$T = m_\mathrm{w}(g + a) - mg - ma = (m_\mathrm{w} - m)(g + a) = T_0(1 + a/g).$$

E15-18 (a) $F_1/d_1^2 = F_2/d_2^2$, so

$$F_2 = (18.6\text{ kN})(3.72\text{ cm})^2/(51.3\text{ cm})^2 = 97.8\,\mathrm{N}.$$

(b) $F_2 h_2 = F_1 h_1$, so

$$h_2 = (1.65\,\mathrm{m})(18.6\text{ kN})/(97.8\,\mathrm{N}) = 314\,\mathrm{m}.$$

E15-19 (a) 35.6 kN; the boat doesn't get heavier or lighter just because it is in different water!

(b) Yes.

$$\Delta V = \frac{(35.6\times 10^3\mathrm{N})}{(9.81\,m/\mathrm{s}^2)}\left(\frac{1}{(1024\,\mathrm{kg/m^3})} - \frac{1}{(1000\,\mathrm{kg/m^3})}\right) = -8.51\times 10^{-2}\mathrm{m}^3.$$

E15-20 (a) $\rho_2 = \rho_1(V_1/V_2) = (1000\,\mathrm{kg/m^3})(0.646) = 646\,\mathrm{kg/m^3}$.

(b) $\rho_2 = \rho_1(V_1/V_2) = (646\,\mathrm{kg/m^3})(0.918)^{-1} = 704\,\mathrm{kg/m^3}$.

$\boxed{\textbf{E15-21}}$ The can has a volume of 1200 cm^3, so it can displace that much water. This would provide a buoyant force of

$$B = \rho V g = (998\text{kg/m}^3)(1200\times10^{-6}\text{m}^3)(9.81\,\text{m/s}^2) = 11.7\,\text{N}.$$

This force can then support a total mass of $(11.7\,\text{N})/(9.81\,\text{m/s}^2) = 1.20\,\text{kg}$. If 130 g belong to the can, then the can will be able to carry 1.07 kg of lead.

E15-22 $\rho_2 = \rho_1(V_1/V_2) = (0.98\text{ g/cm}^3)(2/3)^{-1} = 1.47\text{ g/cm}^3$.

E15-23 Let the object have a mass m. The buoyant force of the air on the object is then

$$B_\text{o} = \frac{\rho_\text{a}}{\rho_\text{o}} mg.$$

There is also a buoyant force on the brass, equal to

$$B_\text{b} = \frac{\rho_\text{a}}{\rho_\text{b}} mg.$$

The fractional error in the weighing is then

$$\frac{B_\text{o} - B_\text{b}}{mg} = \frac{(0.0012\text{ g/cm}^3)}{(3.4\text{ g/cm}^3)} - \frac{(0.0012\text{ g/cm}^3)}{(8.0\text{ g/cm}^3)} = 2.0\times10^{-4}$$

E15-24 The volume of iron is

$$V_\text{i} = (6130\,\text{N})/(9.81\,\text{m/s}^2)(7870\,\text{kg/m}^3) = 7.94\times10^{-2}\text{m}^3.$$

The buoyant force of water is $6130\,\text{N} - 3970\,\text{N} = 2160\,\text{N}$. This corresponds to a volume of

$$V_\text{w} = (2160\,\text{N})/(9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3) = 2.20\times10^{-1}\text{m}^3.$$

The volume of air is then $2.20\times10^{-1}\text{m}^3 - 7.94\times10^{-2}\text{m}^3 = 1.41\times10^{-1}\text{m}^3$.

E15-25 (a) The pressure on the top surface is $p = p_0 + \rho g L/2$. The downward force is

$$\begin{aligned} F_\text{t} &= (p_0 + \rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + (944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 3.84\times10^4\text{N}. \end{aligned}$$

(b) The pressure on the bottom surface is $p = p_0 + 3\rho g L/2$. The upward force is

$$\begin{aligned} F_\text{b} &= (p_0 + 3\rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + 3(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 4.05\times10^4\text{N}. \end{aligned}$$

(c) The tension in the wire is given by $T = W + F_\text{t} - F_\text{b}$, or

$$T = (4450\,\text{N}) + (3.84\times10^4\text{N}) - (4.05\times10^4\text{N}) = 2350\,\text{N}.$$

(d) The buoyant force is

$$B = L^3\rho g = (0.608^3)(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2) = 2080\,\text{N}.$$

E15-26 The fish has the (average) density of water if

$$\rho_w = \frac{m_f}{V_c + V_a}$$

or

$$V_a = \frac{m_f}{\rho_w} - V_c.$$

We want the fraction $V_a/(V_c + V_a)$, so

$$\begin{aligned}\frac{V_a}{V_c + V_a} &= 1 - \rho_w \frac{V_c}{m_f}, \\ &= 1 - \rho_w/\rho_c = 1 - (1.024\ \text{g/cm}^3)/(1.08\ \text{g/cm}^3) = 5.19\times 10^{-2}.\end{aligned}$$

$\boxed{\textbf{E15-27}}$ There are *three* force on the dirigible: gravity ($W = m_g g$), a buoyant force $B = m_a g$, and a tension T. Since these forces must balance we have $T = B - W$. The masses are related to the densities, so we can write

$$T = (\rho_a - \rho_g)Vg = (1.21\,\text{kg/m}^3 - 0.796\,\text{kg/m}^3)(1.17\times 10^6\text{m}^3)(9.81\text{m/s}^2) = 4.75\times 10^6\,\text{N}.$$

E15-28 $\Delta m = \Delta\rho V$, so

$$\Delta m = [(0.160\,kg/\text{m}^3) - (0.0810\,\text{kg/m}^3)](5000\,\text{m}^3) = 395\,\text{kg}.$$

E15-29 The volume of one log is $\pi(1.05/2\ \text{ft})^2(5.80\ \text{ft}) = 5.02\ \text{ft}^3$. The weight of the log is $(47.3\ \text{lb/ft}^3)(5.02\ \text{ft}^3) = 237$ lb. Each log if completely submerged will displace a weight of water $(62.4\ \text{lb/ft}^3)(5.02\ \text{ft}^3) = 313$ lb. So each log can support at most 313 lb − 237 lb = 76 lb. The three children have a total weight of 247 lb, so that will require 3.25 logs. Round up to four.

E15-30 (a) The ice will hold up the automobile if

$$\rho_w > \frac{m_a + m_i}{V_i} = \frac{m_a}{At} + \rho_i.$$

Then

$$A = \frac{(1120\,\text{kg})}{(0.305\,\text{m})[(1000\,\text{kg/m}^3) - (917\,\text{kg/m}^3)]} = 44.2\,\text{m}^2.$$

$\boxed{\textbf{E15-31}}$ If there were *no* water vapor pressure above the barometer then the height of the water would be $y_1 = p/(\rho g)$, where $p = p_0$ is the atmospheric pressure. If there is water vapor where there should be a vacuum, then p is the difference, and we would have $y_2 = (p_0 - p_v)/(\rho g)$. The relative error is

$$\begin{aligned}(y_1 - y_2)/y_1 &= [p_0/(\rho g) - (p_0 - p_v)/(\rho g)]\,/\,[p_0/(\rho g)]\,, \\ &= p_v/p_0 = (3169\,\text{Pa})/(1.01\times 10^5\text{Pa}) = 3.14\,\%.\end{aligned}$$

E15-32 $\rho = (1.01\times 10^5\text{Pa})/(9.81\,\text{m/s}^2)(14\,\text{m}) = 740\,\text{kg/m}^3.$

E15-33 $h = (90)(1.01\times 10^5\text{Pa})/(8.60\,\text{m/s}^2)(1.36\times 10^4\text{kg/m}^3) = 78\,\text{m}.$

E15-34 $\Delta U = 2(4.5\times 10^{-2}\text{N/m})4\pi(2.1\times 10^{-2}\text{m})^2 = 5.0\times 10^{-4}\text{J}.$

$\boxed{\textbf{E15-35}}$ The force required is just the surface tension times the circumference of the circular patch. Then

$$F = (0./072\,\mathrm{N/m})2\pi(0.12\,\mathrm{m}) = 5.43\times10^{-2}\mathrm{N}.$$

E15-36 $\Delta U = 2(2.5\times10^{-2}\mathrm{N/m})4\pi(1.4\times10^{-2}\mathrm{m})^2 = 1.23\times10^{-4}\mathrm{J}.$

P15-1 (a) One can replace the two hemispheres with an open flat end with two hemispheres with a closed flat end. Then the area of relevance is the area of the flat end, or πR^2. The net force from the pressure difference is $\Delta pA = \Delta p\pi R^2$; this much force must be applied to pull the hemispheres apart.

(b) $F = \pi(0.9)(1.01\times10^5\mathrm{Pa})(0.305\,\mathrm{m})^2 = 2.6\times10^4\mathrm{N}.$

P15-2 The pressure required is 4×10^9Pa. This will happen at a depth

$$h = \frac{(4\times10^9\mathrm{Pa})}{(9.8\,\mathrm{m/s^2})(3100\,\mathrm{kg/m^3})} = 1.3\times10^5\mathrm{m}.$$

$\boxed{\textbf{P15-3}}$ (a) The resultant force on the wall will be

$$\begin{aligned} F &= \int\int P\,dx\,dy, \\ &= \int(-\rho g y)W\,dy, \\ &= \rho g D^2 W/2. \end{aligned}$$

(b) The torque will is given by $\tau = F(D-y)$ (the distance is from the bottom) so if we generalize,

$$\begin{aligned} \tau &= \int\int Py\,dx\,dy, \\ &= \int(-\rho g(D-y))\,yW\,dy, \\ &= \rho g D^3 W/6. \end{aligned}$$

(c) Dividing to find the location of the equivalent resultant force,

$$d = \tau/F = (\rho g D^3 W/6)/(\rho g D^2 W/2) = D/3,$$

this distance being measured from the bottom.

P15-4 $p = \rho g y = \rho g(3.6\,\mathrm{m})$; the force on the bottom is $F = pA = \rho g(3.6\,\mathrm{m})\pi(0.60\,\mathrm{m})^2 = 1.296\pi\rho g$. The volume of liquid is

$$V = (1.8\,\mathrm{m})\left[\pi(0.60\,\mathrm{m}) + 4.6\times10^{-4}\mathrm{m}^2\right] = 2.037\,\mathrm{m}^3$$

The weight is $W = \rho g(2.037\,\mathrm{m}^3)$. The ratio is 2.000.

P15-5 The pressure at b is $\rho_\mathrm{c}(3.2\times10^4\mathrm{m}) + \rho_\mathrm{m} y$. The pressure at a is $\rho_\mathrm{c}(3.8\times10^4\mathrm{m} + d) + \rho_\mathrm{m}(y-d)$. Set these quantities equal to each other:

$$\begin{aligned} \rho_\mathrm{c}(3.8\times10^4\mathrm{m} + d) + \rho_\mathrm{m}(y-d) &= \rho_\mathrm{c}(3.2\times10^4\mathrm{m}) + \rho_\mathrm{m} y, \\ \rho_\mathrm{c}(6\times10^3\mathrm{m} + d) &= \rho_\mathrm{m} d, \\ d &= \rho_\mathrm{c}(6\times10^3\mathrm{m})/(\rho_\mathrm{m} - \rho_\mathrm{c}), \\ &= (2900\,\mathrm{kg/m^3})(6\times10^3\mathrm{m})/(400\,\mathrm{kg/m^3}) = 4.35\times10^4\mathrm{m}. \end{aligned}$$

P15-6 (a) The pressure (difference) at a depth y is $\Delta p = \rho g y$. Since $\rho = m/V$, then

$$\Delta\rho \approx -\frac{m}{V}\frac{\Delta V}{V} = \rho_{\rm s}\frac{\Delta p}{B}.$$

Then

$$\rho \approx \rho_{\rm s} + \Delta\rho = \rho_{\rm s} + \frac{{\rho_{\rm s}}^2 g y}{B}.$$

(b) $\Delta\rho/\rho_{\rm s} = \rho_{\rm s} g y/B$, so

$$\Delta\rho/\rho \approx (1000\,{\rm kg/m^3})(9.8\,{\rm m/s^2})(4200\,{\rm m})/(2.2\times10^9{\rm Pa}) = 1.9\,\%.$$

$\boxed{\textbf{P15-7}}$ (a) Use Eq. 15-10, $p = (p_0/\rho_0)\rho$, then Eq. 15-13 will look like

$$(p_0/\rho_0)\rho = (p_0/\rho_0)\rho_0 e^{-h/a}.$$

(b) The upward velocity of the rocket as a function of time is given by $v = a_r t$. The height of the rocket above the ground is given by $y = \frac{1}{2}a_r t^2$. Combining,

$$v = a_r\sqrt{\frac{2y}{a_r}} = \sqrt{2ya_r}.$$

Put this into the expression for drag, along with the equation for density variation with altitude;

$$D = CA\rho v^2 = CA\rho_0 e^{-y/a} 2ya_r.$$

Now take the derivative with respect to y,

$$dD/dy = (-1/a)CA\rho_0 e^{-y/a}(2ya_r) + CA\rho_0 e^{-y/a}(2a_r).$$

This will vanish when $y = a$, regardless of the acceleration a_r.

P15-8 (a) Consider a slice of cross section A a depth h beneath the surface. The net force on the fluid above the slice will be

$$F_{\rm net} = ma = \rho h A g,$$

Since the weight of the fluid above the slice is

$$W = mg = \rho h A g,$$

then the upward force on the bottom of the fluid at the slice must be

$$W + F_{\rm net} = \rho h A(g + a),$$

so the pressure is $p = F/A = \rho h(g + a)$.

(b) Change a to $-a$.

(c) The pressure is zero (ignores atmospheric contributions.)

P15-9 (a) Consider a portion of the liquid at the surface. The net force on this portion is $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = ma\hat{\mathbf{i}}$. The force of gravity on this portion is $\vec{\mathbf{W}} = -mg\hat{\mathbf{j}}$. There must then be a buoyant force on the portion with direction $\vec{\mathbf{B}} = \vec{\mathbf{F}} - \vec{\mathbf{W}} = m(a\hat{\mathbf{i}} + g\hat{\mathbf{j}})$. The buoyant force makes an angle $\theta = \arctan(a/g)$ with the vertical. The buoyant force must be perpendicular to the surface of the fluid; there are no pressure-related forces which are parallel to the surface. Consequently, the surface must make an angle $\theta = \arctan(a/g)$ with the *horizontal*.

(b) It will still vary as $\rho g h$; the derivation on page 334 is still valid for vertical displacements.

P15-10 $dp/dr = -\rho g$, but now $g = 4\pi G\rho r/3$. Then

$$\begin{aligned} \int_0^p dp &= -\frac{4}{3}\pi G\rho^2 \int_R^r r\, dr, \\ p &= \frac{2}{3}\pi G\rho^2 \left(R^2 - r^2\right). \end{aligned}$$

$\boxed{\textbf{P15-11}}$ We can start with Eq. 15-11, except that we'll write our distance in terms of r instead if y. Into this we can substitute our expression for g,

$$g = g_0 \frac{R^2}{r^2}.$$

Substituting, then integrating,

$$\begin{aligned} \frac{dp}{p} &= -\frac{g\rho_0}{p_0} dr, \\ \frac{dp}{p} &= -\frac{g_0\rho_0 R^2}{p_0}\frac{dr}{r^2}, \\ \int_{p_0}^p \frac{dp}{p} &= -\int_R^r \frac{g_0\rho_0 R^2}{p_0}\frac{dr}{r^2}, \\ \ln\frac{p}{p_0} &= \frac{g_0\rho_0 R^2}{p_0}\left(\frac{1}{r} - \frac{1}{R}\right) \end{aligned}$$

If $k = g_0\rho_0 R^2/p_0$, then

$$p = p_0 e^{k(1/r - 1/R)}.$$

P15-12 (a) The net force on a small volume of the fluid is $dF = r\omega^2\, dm$ directed toward the center. For radial displacements, then, $dF/dr = -r\omega^2 dm/dr$ or $dp/dr = -r\omega^2\rho$.

(b) Integrating outward,

$$p = p_c + \int_0^r \rho\omega^2 r\, dr = p_c + \frac{1}{2}\rho r^2\omega^2.$$

(c) Do part (d) first.

(d) It will still vary as $\rho g h$; the derivation on page 334 is still valid for vertical displacements.

(c) The pressure anywhere in the liquid is then given by

$$p = p_0 + \frac{1}{2}\rho r^2\omega^2 - \rho g y,$$

where p_0 is the pressure on the surface, y is measured from the bottom of the paraboloid, and r is measured from the center. The surface is defined by $p = p_0$, so

$$\frac{1}{2}\rho r^2\omega^2 - \rho g y = 0,$$

or $y = r^2\omega^2/2g$.

P15-13 The total mass of the shell is $m = \rho_w \pi {d_o}^3/3$, or it wouldn't barely float. The mass of iron in the shell is $m = \rho_i\pi({d_o}^3 - {d_i}^3)/3$, so

$${d_i}^3 = \frac{\rho_i - \rho_w}{\rho_i}{d_o}^3,$$

so

$$d_i = \sqrt[3]{\frac{(7870\,\text{kg/m}^3) - (1000\,\text{kg/m}^3)}{(7870\,\text{kg/m}^3)}}(0.587\,\text{m}) = 0.561\,\text{m}.$$

P15-14 The wood will displace a volume of water equal to $(3.67\,\text{kg})/(594\,\text{kg/m}^3)(0.883) = 5.45\times 10^{-3}\text{m}^3$ in either case. That corresponds to a mass of $(1000\,\text{kg/m}^3)(5.45\times10^{-3}\text{m}^3) = 5.45\,\text{kg}$ that can be supported.

(a) The mass of lead is $5.45\,\text{kg} - 3.67\,\text{kg} = 1.78\,\text{kg}$.

(b) When the lead is submerged beneath the water it displaces water, which affects the "apparent" mass of the lead. The true weight of the lead is mg, the buoyant force is $(\rho_\text{w}/\rho_\text{l})mg$, so the apparent weight is $(1-\rho_\text{w}/\rho_\text{l})mg$. This means the apparent mass of the submerged lead is $(1-\rho_\text{w}/\rho_\text{l})m$. This apparent mass is 1.78 kg, so the true mass is

$$m = \frac{(11400\,\text{kg/m}^3)}{(11400\,\text{kg/m}^3)-(1000\,\text{kg})}(1.78\,\text{kg}) = 1.95\,\text{kg}.$$

$\boxed{\textbf{P15-15}}$ We initially have

$$\frac{1}{4} = \frac{\rho_\text{o}}{\rho_\text{mercury}}.$$

When water is poured over the object the simple relation no longer works.

Once the water is over the object there are two buoyant forces: one from mercury, F_1, and one from the water, F_2. Following a derivation which is similar to Sample Problem 15-3, we have

$$F_1 = \rho_1 V_1 g \text{ and } F_2 = \rho_2 V_2 g$$

where ρ_1 is the density of mercury, V_1 the volume of the object which is in the mercury, ρ_2 is the density of water, and V_2 is the volume of the object which is in the water. We also have

$$F_1 + F_2 = \rho_\text{o} V_\text{o} g \text{ and } V_1 + V_2 = V_\text{o}$$

as expressions for the net force on the object (zero) and the total volume of the object. Combining these four expressions,

$$\rho_1 V_1 + \rho_2 V_2 = \rho_\text{o} V_\text{o},$$

or

$$\begin{aligned}\rho_1 V_1 + \rho_2 (V_\text{o} - V_1) &= \rho_\text{o} V_\text{o},\\ (\rho_1 - \rho_2) V_1 &= (\rho_\text{o} - \rho_2) V_\text{o},\\ \frac{V_1}{V_\text{o}} &= \frac{\rho_\text{o}-\rho_2}{\rho_1-\rho_2}.\end{aligned}$$

The left hand side is the fraction that is submerged in the mercury, so we just need to substitute our result for the density of the material from the beginning to solve the problem. The fraction submerged after adding water is then

$$\begin{aligned}\frac{V_1}{V_\text{o}} &= \frac{\rho_\text{o}-\rho_2}{\rho_1-\rho_2},\\ &= \frac{\rho_1/4-\rho_2}{\rho_1-\rho_2},\\ &= \frac{(13600\,\text{kg/m}^3)/4-(998\,\text{kg/m}^3)}{(13600\,\text{kg/m}^3)-(998\,\text{kg/m}^3)} = 0.191.\end{aligned}$$

P15-16 (a) The car floats if it displaces a mass of water equal to the mass of the car. Then $V = (1820\,\text{kg})/(1000\,\text{kg/m}^3) = 1.82\,\text{m}^3$.

(b) The car has a total volume of $4.87\,\text{m}^3 + 0.750\,\text{m}^3 + 0.810\,\text{m}^3 = 6.43\,\text{m}^3$. It will sink if the total mass inside the car (car + water) is then $(6.43\,\text{m}^3)(1000\,\text{kg/m}^3) = 6430\,\text{kg}$. So the mass of the water in the car is $6430\,\text{kg} - 1820\,\text{kg} = 4610\,\text{kg}$ when it sinks. That's a volume of $(4610\,\text{kg})/(1000\,\text{kg/m}^3) = 4.16\,\text{m}^3$.

P15-17 When the beaker is half filled with water it has a total mass exactly equal to the maximum amount of water it can displace. The total mass of the beaker is the mass of the beaker plus the mass of the water inside the beaker. Then

$$\rho_{\mathrm{w}}(m_{\mathrm{g}}/\rho_{\mathrm{g}} + V_{\mathrm{b}}) = m_{\mathrm{g}} + \rho_{\mathrm{w}} V_{\mathrm{b}}/2,$$

where $m_{\mathrm{g}}/\rho_{\mathrm{g}}$ is the volume of the glass which makes up the beaker. Rearrange,

$$\rho_{\mathrm{g}} = \frac{m_{\mathrm{g}}}{m_{\mathrm{g}}/\rho_{\mathrm{w}} - V_{\mathrm{b}}/2} = \frac{(0.390\,\mathrm{kg})}{(0.390\,\mathrm{kg})/(1000\,\mathrm{kg/m^3}) - (5.00\times10^{-4}\,\mathrm{m^3})/2} = 2790\,\mathrm{kg/m^3}.$$

P15-18 (a) If each atom is a cube then the cube has a side of length

$$l = \sqrt[3]{(6.64\times10^{-27}\,\mathrm{kg})/(145\,\mathrm{kg/m^3})} = 3.58\times10^{-10}\mathrm{m}.$$

Then the atomic surface density is $l^{-2} = (3.58\times10^{-10}\mathrm{m})^{-2} = 7.8\times10^{18}/\mathrm{m}^2$.

(b) The bond surface density is *twice* the atomic surface density. Show this by drawing a square array of atoms and then joining each adjacent pair with a bond. You will need twice as many bonds as there are atoms. Then the energy per bond is

$$\frac{(3.5\times10^{-4}\mathrm{N/m})}{2(7.8\times10^{18}/\mathrm{m}^2)(1.6\times10^{-19}\ \mathrm{J/eV})} = 1.4\times10^{-4}\ \mathrm{eV}.$$

P15-19 Pretend the bubble consists of two hemispheres. The force from surface tension holding the hemispheres together is $F = 2\gamma L = 4\pi r\gamma$. The "extra" factor of two occurs because *each* hemisphere has a circumference which "touches" the boundary that is held together by the surface tension of the liquid. The pressure difference between the inside and outside is $\Delta p = F/A$, where A is the area of the flat side of one of the hemispheres, so $\Delta p = (4\pi r\gamma)/(\pi r^2) = 4\gamma/r$.

P15-20 Use the results of Problem 15-19. To get a numerical answer you need to know the surface tension; try $\gamma = 2.5\times10^{-2}\mathrm{N/m}$. The initial pressure inside the bubble is $p_{\mathrm{i}} = p_0 + 4\gamma/r_{\mathrm{i}}$. The final pressure inside the bell jar is $p = p_{\mathrm{f}} - 4\gamma/r_{\mathrm{f}}$. The initial and final pressure inside the bubble are related by $p_{\mathrm{i}}{r_{\mathrm{i}}}^3 = p_{\mathrm{f}}{r_{\mathrm{f}}}^3$. Now for numbers:

$$p_{\mathrm{i}} = (1.00\times10^5\mathrm{Pa}) + 4(2.5\times10^{-2}\mathrm{N/m})/(1.0\times10^{-3}\mathrm{m}) = 1.001\times10^5\mathrm{Pa}.$$

$$p_{\mathrm{f}} = (1.0\times10^{-3}\mathrm{m}/1.0\times10^{-2}\mathrm{m})^3(1.001\times10^5\mathrm{Pa}) = 1.001\times10^2\mathrm{Pa}.$$

$$p = (1.001\times10^2\mathrm{Pa}) - 4(2.5\times10^{-2}\mathrm{N/m})/(1.0\times10^{-2}\mathrm{m}) = 90.1\mathrm{Pa}.$$

P15-21 The force on the liquid in the space between the rod and the cylinder is $F = \gamma L = 2\pi\gamma(R + r)$. This force can support a mass of water $m = F/g$. This mass has a volume $V = m/\rho$. The cross sectional area is $\pi(R^2 - r^2)$, so the height h to which the water rises is

$$\begin{aligned} h &= \frac{2\pi\gamma(R+r)}{\rho g\pi(R^2 - r^2)} = \frac{2\gamma}{\rho g(R-r)}, \\ &= \frac{2(72.8\times10^{-3}\mathrm{N/m})}{(1000\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(4.0\times10^{-3}\mathrm{m})} = 3.71\times10^{-3}\mathrm{m}. \end{aligned}$$

P15-22 (a) Refer to Problem 15-19. The initial pressure difference is

$$4(2.6\times10^{-2}\text{N/m})/(3.20\times10^{-2}\text{m}) = 3.25\text{Pa}.$$

(b) The final pressure difference is

$$4(2.6\times10^{-2}\text{N/m})/(5.80\times10^{-2}\text{m}) = 1.79\text{Pa}.$$

(c) The work done against the atmosphere is $p\Delta V$, or

$$(1.01\times10^{5}\text{Pa})\frac{4\pi}{3}[(5.80\times10^{-2}\text{m})^3 - (3.20\times10^{-2}\text{m})^3] = 68.7\,\text{J}.$$

(d) The work done in stretching the bubble surface is $\gamma\Delta A$, or

$$(2.60\times10^{-2}\text{N/m})4\pi[(5.80\times10^{-2}\text{m})^2 - (3.20\times10^{-2}\text{m})^2] = 7.65\times10^{-4}\text{J}.$$

E16-1 $R = Av = \pi d^2 v/4$ and $V = Rt$, so

$$t = \frac{4(1600\,\text{m}^3)}{\pi(0.345\,\text{m})^2(2.62\,\text{m})} = 6530\,\text{s}$$

E16-2 $A_1v_1 = A_2v_2$, $A_1 = \pi d_1^2/4$ for the hose, and $A_2 = N\pi d_2^2$ for the sprinkler, where $N = 24$. Then

$$v_2 = \frac{(0.75\text{ in})^2}{(24)(0.050\text{ in})^2}(3.5\text{ ft/s}) = 33\text{ ft/s}.$$

E16-3 We'll assume that each river has a rectangular cross section, despite what the picture implies. The cross section area of the two streams is then

$$A_1 = (8.2\,\text{m})(3.4\,\text{m}) = 28\,\text{m}^2 \text{ and } A_2 = (6.8\,\text{m})(3.2\,\text{m}) = 22\,\text{m}^2.$$

The volume flow rate in the first stream is

$$R_1 = A_1v_1 = (28\,\text{m}^2)(2.3\,\text{m/s}) = 64\,\text{m}^3/\text{s},$$

while the volume flow rate in the second stream is

$$R_2 = A_2v_2 = (22\,\text{m}^2)(2.6\,\text{m/s}) = 57\,\text{m}^3/\text{s}.$$

The amount of fluid in the stream/river system is conserved, so

$$R_3 = R_1 + R_2 = (64\,\text{m}^3/\text{s}) + (57\,\text{m}^3/\text{s}) = 121\,\text{m}^3/\text{s}.$$

where R_3 is the volume flow rate in the river. Then

$$D_3 = R_3/(v_3W_3) = (121\,\text{m}^3/\text{s})/[(10.7\,\text{m})(2.9\,\text{m/s})] = 3.9\,\text{m}.$$

E16-4 The speed of the water is originally zero so both the kinetic and potential energy is zero. When it leaves the pipe at the top it has a kinetic energy of $\frac{1}{2}(5.30\,\text{m/s})^2 = 14.0\,\text{J/kg}$ and a potential energy of $(9.81\,\text{m/s}^2)(2.90\,\text{m}) = 28.4\,\text{J/kg}$. The water is flowing out at a volume rate of $R = (5.30\,\text{m/s})\pi(9.70\times10^{-3}\text{m})^2 = 1.57\times10^{-3}\,\text{m}^3/\text{s}$. The mass rate is $\rho R = (1000\,\text{kg/m}^3)(1.57\times10^{-3}\,\text{m}^3/\text{s}) = 1.57\,\text{kg/s}$.

The power supplied by the pump is $(42.8\,\text{J/kg})(1.57\,\text{kg/s}) = 67.2\text{W}$.

E16-5 There are 8500 km^2 which collects an average of (0.75)(0.48 m/y), where the 0.75 reflects the fact that 1/4 of the water evaporates, so

$$R = \left[8500(10^3\,\text{m})^2\right](0.75)(0.48\,\text{m/y})\left(\frac{1\text{ y}}{365\times24\times60\times60\,\text{s}}\right) = 97\,\text{m}^3/\text{s}.$$

Then the speed of the water in the river is

$$v = R/A = (97\,\text{m}^3/\text{s})/[(21\,\text{m})(4.3\,\text{m})] = 1.1\,\text{m/s}.$$

E16-7 (a) $\Delta p = \rho g(y_1 - y_2) + \rho(v_1^2 - v_2^2)/2$. Then

$$\Delta p = (62.4\text{ lb/ft}^3)(572\text{ ft}) + [(62.4\text{ lb/ft}^3)/(32\text{ ft/s}^2)][(1.33\text{ ft/s})^2 - (31.0\text{ ft/s})^2]/2 = 3.48\times10^4\text{lb/ft}^2.$$

(b) $A_2v_2 = A_1v_1$, so

$$A_2 = (7.60\text{ ft}^2)(1.33\text{ ft/s})/(31.0\text{ ft/s}) = 0.326\text{ ft}^2.$$

E16-8 (a) $A_2 v_2 = A_1 v_1$, so

$$v_2 = (2.76\,\mathrm{m/s})[(0.255\,\mathrm{m})^2 - (0.0480\,\mathrm{m})^2]/(0.255\,\mathrm{m})^2 = 2.66\,\mathrm{m/s}.$$

(b) $\Delta p = \rho(v_1^2 - v_2^2)/2$,

$$\Delta p = (1000\,\mathrm{kg/m^3})[(2.66\,\mathrm{m/s})^2 - (2.76\,\mathrm{m/s})^2]/2 = -271\,\mathrm{Pa}$$

E16-9 (b) We will do part (b) first.

$$R = (100\,\mathrm{m^2})(1.6\,\mathrm{m/y})\left(\frac{1\ \mathrm{y}}{365 \times 24 \times 60 \times 60\,\mathrm{s}}\right) = 5.1\times10^{-6}\,\mathrm{m^3/s}.$$

(b) The speed of the flow R through a hole of cross sectional area a will be $v = R/a$. $p = p_0 + \rho g h$, where $h = 2.0\,\mathrm{m}$ is the depth of the hole. Bernoulli's equation can be applied to find the speed of the water as it travels a horizontal stream line out the hole,

$$p_0 + \frac{1}{2}\rho v^2 = p,$$

where we drop any terms which are either zero or the same on both sides. Then

$$v = \sqrt{2(p - p_0)/\rho} = \sqrt{2gh} = \sqrt{2(9.81\,\mathrm{m/s^2})(2.0\,\mathrm{m})} = 6.3\,\mathrm{m/s}.$$

Finally, $a = (5.1\times10^{-6}\,\mathrm{m^3/s})/(6.3\,\mathrm{m/s}) = 8.1\times10^{-7}\mathrm{m^2}$, or about 0.81 $\mathrm{mm^2}$.

E16-10 (a) $v_2 = (A_1/A_2)v_1 = (4.20\ \mathrm{cm^2})(5.18\,\mathrm{m/s})/(7.60\ \mathrm{cm^2}) = 2.86\,\mathrm{m/s}$.

(b) Use Bernoulli's equation:

$$p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2.$$

Then

$$\begin{aligned} p_2 &= (1.52\times10^5\mathrm{Pa}) + (1000\,\mathrm{kg/m^3})\left[(9.81\,\mathrm{m/s^2})(9.66\,\mathrm{m}) + \frac{1}{2}(5.18\,\mathrm{m/s})^2 - \frac{1}{2}(2.86\,\mathrm{m/s})^2\right], \\ &= 2.56\times10^5\mathrm{Pa}. \end{aligned}$$

E16-11 (a) The wind speed is (110 km/h)(1000 m/km)/(3600 s/h) = 30.6 m/s. The pressure difference is then

$$\Delta p = \frac{1}{2}(1.2\,\mathrm{kg/m^3})(30.6\,\mathrm{m/s})^2 = 562\,\mathrm{Pa}.$$

(b) The lifting force would be $F = (562\,\mathrm{Pa})(93\,\mathrm{m^2}) = 52000\,\mathrm{N}$.

E16-12 The pressure difference is

$$\Delta p = \frac{1}{2}(1.23\,\mathrm{kg/m^3})(28.0\,\mathrm{m/s})^2 = 482\,\mathrm{N}.$$

The net force is then $F = (482\,\mathrm{N})(4.26\,\mathrm{m})(5.26\,\mathrm{m}) = 10800\,\mathrm{N}$.

$\boxed{\textbf{E16-13}}$ The lower pipe has a radius $r_1 = 2.52$ cm, and a cross sectional area of $A_1 = \pi r_1^2$. The speed of the fluid flow at this point is v_1. The higher pipe has a radius of $r_2 = 6.14$ cm, a cross sectional area of $A_2 = \pi r_2^2$, and a fluid speed of v_2. Then

$$A_1 v_1 = A_2 v_2 \text{ or } r_1^2 v_1 = r_2^2 v_2.$$

Set $y_1 = 0$ for the lower pipe. The problem specifies that the pressures in the two pipes are the same, so

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ \frac{1}{2}v_1^2 &= \frac{1}{2}v_2^2 + g y_2, \end{aligned}$$

We can combine the results of the equation of continuity with this and get

$$\begin{aligned} v_1^2 &= v_2^2 + 2 g y_2, \\ v_1^2 &= \left(v_1 r_1^2/r_2^2\right)^2 + 2 g y_2, \\ v_1^2\left(1 - r_1^4/r_2^4\right) &= 2 g y_2, \\ v_1^2 &= 2 g y_2 / \left(1 - r_1^4/r_2^4\right). \end{aligned}$$

Then

$$v_1^2 = 2(9.81\,\text{m/s}^2)(11.5\,\text{m})/\left(1 - (0.0252\,\text{m})^4/(0.0614\,\text{m})^4\right) = 232\,\text{m}^2/\text{s}^2$$

The volume flow rate in the bottom (and top) pipe is

$$R = \pi r_1^2 v_1 = \pi(0.0252\,\text{m})^2(15.2\,\text{m/s}) = 0.0303\,\text{m}^3/\text{s}.$$

E16-14 (a) As instructed,

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_3^2 + \rho g y_3, \\ 0 &= \frac{1}{2}v_3^2 + g(y_3 - y_1), \end{aligned}$$

But $y_3 - y_1 = -h$, so $v_3 = \sqrt{2gh}$.

(b) h above the hole. Just reverse your streamline!

(c) It won't come out as fast and it won't rise as high.

$\boxed{\textbf{E16-15}}$ Sea level will be defined as $y = 0$, and at that point the fluid is assumed to be at rest. Then

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ 0 &= \frac{1}{2}v_2^2 + g y_2, \end{aligned}$$

where $y_2 = -200$ m. Then

$$v_2 = \sqrt{-2 g y_2} = \sqrt{-2(9.81\,\text{m/s}^2)(-200\,\text{m})} = 63\,\text{m/s}.$$

E16-16 Assume streamlined flow, then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ (p_1 - p_2)/\rho + g(y_1 - y_2) &= \frac{1}{2}v_2^2. \end{aligned}$$

Then upon rearranging

$$v_2 = \sqrt{2\left[(2.1)(1.01\times10^5\text{Pa})/(1000\,\text{kg/m}^3) + (9.81\,\text{m/s}^2)(53.0\,\text{m})\right]} = 38.3\,\text{m/s}.$$

E16-17 (a) Points 1 and 3 are both at atmospheric pressure, and both will move at the same speed. But since they are at different heights, Bernoulli's equation will be violated.

(b) The flow isn't steady.

E16-18 The atmospheric pressure difference between the two sides will be $\Delta p = \frac{1}{2}\rho_\text{a} v^2$. The height difference in the U-tube is given by $\Delta p = \rho_\text{w} g h$. Then

$$h = \frac{(1.20\,\text{kg/m}^3)(15.0\,\text{m/s})^2}{2(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} = 1.38\times10^{-2}\text{m}.$$

$\boxed{\textbf{E16-19}}$ (a) There are three forces on the plug. The force from the pressure of the water, $F_1 = P_1 A$, the force from the pressure of the air, $F_2 = P_2 A$, and the force of friction, F_3. These three forces must balance, so $F_3 = F_1 - F_2$, or $F_3 = P_1 A - P_2 A$. But $P_1 - P_2$ is the pressure difference between the surface and the water 6.15 m below the surface, so

$$\begin{aligned} F_3 &= \Delta P A = -\rho g y A, \\ &= -(998\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(-6.15\,\text{m})\pi(0.0215\,\text{m})^2, \\ &= 87.4\,\text{N} \end{aligned}$$

(b) To find the volume of water which flows out in three hours we need to know the volume flow rate, and for that we need both the cross section area of the hole and the speed of the flow. The speed of the flow can be found by an application of Bernoulli's equation. We'll consider the horizontal motion only— a point just inside the hole, and a point just outside the hole. These points are at the same level, so

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combine this with the results of Pascal's principle above, and

$$v_2 = \sqrt{2(p_1 - p_2)/\rho} = \sqrt{-2gy} = \sqrt{-2(9.81\,\text{m/s}^2)(-6.15\,\text{m})} = 11.0\,\text{m/s}.$$

The volume of water which flows out in three hours is

$$V = Rt = (11.0\,\text{m/s})\pi(0.0215\,\text{m})^2(3\times3600\,\text{s}) = 173\,\text{m}^3.$$

E16-20 Apply Eq. 16-12:

$$v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.262\,\text{m})(810\,\text{kg/m}^3)/(1.03\,\text{kg/m}^3)} = 63.6\,\text{m/s}.$$

$\boxed{\textbf{E16-21}}$ We'll assume that the central column of air down the pipe exerts minimal force on the card when it is deflected to the sides. Then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

The resultant upward force on the card is the area of the card times the pressure difference, or

$$F = (p_1 - p_2)A = \frac{1}{2}\rho A v^2.$$

E16-22 If the air blows uniformly over the surface of the plate then there can be no torque about any axis through the center of mass of the plate. Since the weight also doesn't introduce a torque, then the hinge can't exert a force on the plate, because any such force would produce an unbalanced torque. Consequently $mg = \Delta p A$. $\Delta p = \rho v^2/2$, so

$$v = \sqrt{\frac{2mg}{\rho A}} = \sqrt{\frac{2(0.488\,\text{kg})(9.81\,\text{m/s}^2)}{(1.21\,\text{kg/m}^3)(9.10\times10^{-2}\text{m})^2}} = 30.9\,\text{m/s}.$$

E16-23 Consider a streamline which passes above the wing and a streamline which passes beneath the wing. Far from the wing the two streamlines are close together, move with zero relative velocity, and are effectively at the same pressure. So we can pretend they are actually one streamline. Then, since the altitude difference between the two points above and below the wing (on this new, single streamline) is so small we can write

$$\Delta p = \frac{1}{2}\rho({v_\text{t}}^2 - {v_\text{u}}^2)$$

The lift force is then

$$L = \Delta p\, A = \frac{1}{2}\rho A({v_\text{t}}^2 - {v_\text{u}}^2)$$

E16-24 (a) From Exercise 16-23,

$$L = \frac{1}{2}(1.17\,\text{kg/m}^3)(2)(12.5\,\text{m}^2)\left[(49.8\,\text{m/s})^2 - (38.2\,\text{m/s})^2\right] = 1.49\times10^4\text{N}.$$

The mass of the plane must be $m = L/g = (1.49\times10^4\text{N})/(9.81\,\text{m/s}^2) = 1520\,\text{kg}$.

(b) The lift is directed straight up.

(c) The lift is directed 15° off the vertical toward the rear of the plane.

(d) The lift is directed 15° off the vertical toward the front of the plane.

$\boxed{\textbf{E16-25}}$ The larger pipe has a radius $r_1 = 12.7$ cm, and a cross sectional area of $A_1 = \pi r_1^2$. The speed of the fluid flow at this point is v_1. The smaller pipe has a radius of $r_2 = 5.65$ cm, a cross sectional area of $A_2 = \pi r_2^2$, and a fluid speed of v_2. Then

$$A_1 v_1 = A_2 v_2 \text{ or } r_1^2 v_1 = r_2^2 v_2.$$

Now Bernoulli's equation. The two pipes are at the same level, so $y_1 = y_2$. Then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combining this with the results from the equation of continuity,

$$
\begin{aligned}
p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\
v_1^2 &= v_2^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2 &= \left(v_1 \frac{r_1^2}{r_2^2}\right)^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2\left(1 - \frac{r_1^4}{r_2^4}\right) &= \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2 &= \frac{2\left(p_2 - p_1\right)}{\rho\left(1 - r_1^4/r_2^4\right)}.
\end{aligned}
$$

It may look a mess, but we can solve it to find v_1,

$$v_1 = \sqrt{\frac{2(32.6\times10^3\text{Pa} - 57.1\times10^3\text{Pa})}{(998\,\text{kg/m}^3)(1 - (0.127\,\text{m})^4/(0.0565\,\text{m})^4)}} = 1.41\,\text{m/s}.$$

The volume flow rate is then

$$R = Av = \pi(0.127\,\text{m})^2(1.41\,\text{m/s}) = 7.14\times10^{-3}\text{m}^3/\text{s}.$$

That's about 71 liters/second.

E16-26 The lines are parallel and equally spaced, so the velocity is everywhere the same. We can transform to a reference frame where the liquid appears to be at rest, so the Pascal's equation would apply, and $p + \rho g y$ would be a constant. Hence,

$$p_0 = p + \rho g y + \frac{1}{2}\rho v^2$$

is the same for all streamlines.

E16-27 (a) The "particles" of fluid in a whirlpool would obey conservation of angular momentum, meaning a particle off mass m would have $l = mvr$ be constant, so the speed of the fluid as a function of radial distance from the center would be given by $k = vr$, where k is some constant representing the angular momentum per mass. Then $v = k/r$.

(b) Since $v = k/r$ and $v = 2\pi r/T$, the period would be $T \propto r^2$.

(c) Kepler's third law says $T \propto r^{3/2}$.

E16-28 $R_\text{c} = 2000$. Then

$$v < \frac{R_\text{c}\eta}{\rho D} = \frac{(2000)(4.0\times10^{-3}\text{N}\cdot\text{s/m}^2)}{(1060\,\text{kg/m}^3)(3.8\times10^{-3}\text{m})} = 2.0\,\text{m/s}.$$

$\boxed{\textbf{E16-29}}$ (a) The volume flux is given; from that we can find the average speed of the fluid in the pipe.

$$v = \frac{5.35\times10^{-2}\ \text{L/min}}{\pi(1.88\ \text{cm})^2} = 4.81\times10^{-3}\ \text{L/cm}^2\cdot\text{min}.$$

But 1 L is the same as 1000 cm^3 and 1 min is equal to 60 seconds, so $v = 8.03\times10^{-4}$ m/s.

Reynold's number from Eq. 16-22 is then

$$R = \frac{\rho D v}{\eta} = \frac{(13600\,\mathrm{kg/m^3})(0.0376\,\mathrm{m})(8.03\times10^{-4}\,\mathrm{m/s})}{(1.55\times10^{-3}\mathrm{N\cdot s/m^2})} = 265.$$

This is well below the critical value of 2000.

(b) Poiseuille's Law, Eq. 16-20, can be used to find the pressure difference between the ends of the pipe. But first, note that the mass flux dm/dt is equal to the volume rate times the density when the density is constant. Then $\rho\, dV/dt = dm/dt$, and Poiseuille's Law can be written as

$$\delta p = \frac{8\eta L}{\pi R^4}\frac{dV}{dt} = \frac{8(1.55\times10^{-3}\mathrm{N\cdot s/m^2})(1.26\,\mathrm{m})}{\pi(1.88\times10^{-2}\ \mathrm{m})^4}(8.92\times10^{-7}\mathrm{m^3/s}) = 0.0355\,\mathrm{Pa}.$$

P16-1 The volume of water which needs to flow out of the bay is

$$V = (6100\,\mathrm{m})(5200\,\mathrm{m})(3\,\mathrm{m}) = 9.5\times10^7\mathrm{m^3}$$

during a 6.25 hour (22500 s) period. The average speed through the channel must be

$$v = \frac{(9.5\times10^7\mathrm{m^3})}{(22500\,\mathrm{s})(190\,\mathrm{m})(6.5\,\mathrm{m})} = 3.4\,\mathrm{m/s}.$$

P16-2 (a) The speed of the fluid through either hole is $v = \sqrt{2gh}$. The mass flux through a hole is $Q = \rho A v$, so $\rho_1 A_1 = \rho_2 A_2$. Then $\rho_1/\rho_2 = A_2/A_1 = 2$.

(b) $R = Av$, so $R_1/R_2 = A_1/A_2 = 1/2$.

(c) If $R_1 = R_2$ then $A_1\sqrt{2gh_1} = A_2\sqrt{2gh_2}$. Then

$$h_2/h_1 = (A_1/A_2)^2 = (1/2)^2 = 1/4.$$

So $h_2 = h_1/4$.

$\boxed{\textbf{P16-3}}$ (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2gh}$. The speed v is a horizontal velocity, and serves as the initial horizontal velocity of the fluid "projectile" after it leaves the tank. There is no initial vertical velocity.

This fluid "projectile" falls through a vertical distance $H-h$ before splashing against the ground. The equation governing the time t for it to fall is

$$-(H-h) = -\frac{1}{2}gt^2,$$

Solve this for the time, and $t = \sqrt{2(H-h)/g}$. The equation which governs the horizontal distance traveled during the fall is $x = v_x t$, but $v_x = v$ and we just found t, so

$$x = v_x t = \sqrt{2gh}\,\sqrt{2(H-h)/g} = 2\sqrt{h(H-h)}.$$

(b) How many values of h will lead to a distance of x? We need to invert the expression, and we'll start by squaring both sides

$$x^2 = 4h(H-h) = 4hH - 4h^2,$$

and then solving the resulting quadratic expression for h,

$$h = \frac{4H \pm \sqrt{16H^2 - 16x^2}}{8} = \frac{1}{2}\left(H \pm \sqrt{H^2 - x^2}\right).$$

For values of x between 0 and H there are two real solutions, if $x = H$ there is one real solution, and if $x > H$ there are no real solutions.

If h_1 is a solution, then we can write $h_1 = (H + \Delta)/2$, where $\Delta = 2h_1 - H$ could be positive or negative. Then $h_2 = (H + \Delta)/2$ is also a solution, and

$$h_2 = (H + 2h_1 - 2H)/2 = h_1 - H$$

is also a solution.

(c) The farthest distance is $x = H$, and this happens when $h = H/2$, as we can see from the previous section.

P16-4 (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2g(d + h_2)}$, assuming that the liquid remains in contact with the walls of the tube until it exits at the bottom.

(b) The speed of the fluid in the tube is everywhere the same. Then the pressure difference at various points are only functions of height. The fluid exits at C, and assuming that it remains in contact with the walls of the tube the pressure difference is given by $\Delta p = \rho(h_1 + d + h_2)$, so the pressure at B is

$$p = p_0 - \rho(h_1 + d + h_2).$$

(c) The lowest possible pressure at B is zero. Assume the flow rate is *so* slow that Pascal's principle applies. Then the maximum height is given by $0 = p_0 + \rho g h_1$, or

$$h_1 = (1.01 \times 10^5 \text{Pa})/[(9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3)] = 10.3\,\text{m}.$$

P16-5 (a) The momentum per kilogram of the fluid in the smaller pipe is v_1. The momentum per kilogram of the fluid in the larger pipe is v_2. The change in momentum per kilogram is $v_2 - v_1$. There are $\rho a_2 v_2$ kilograms per second flowing past any point, so the change in momentum per second is $\rho a_2 v_2(v_2 - v_1)$. The change in momentum per second is related to the net force according to $F = \Delta p/\Delta t$, so $F = \rho a_2 v_2(v_2 - v_1)$. But $F \approx \Delta p/a_2$, so $p_1 - p_2 \approx \rho v_2(v_2 - v_1)$.

(b) Applying the streamline equation,

$$\begin{aligned} p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2, \\ \frac{1}{2}\rho(v_1^2 - v_2^2) &= p_2 - p_1 \end{aligned}$$

(c) This question asks for the loss of pressure *beyond* that which would occur from a gradually widened pipe. Then we want

$$\begin{aligned} \Delta p &= \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho v_2(v_1 - v_2), \\ &= \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho v_2 v_1 + \rho v_2^2, \\ &= \frac{1}{2}\rho(v_1^2 - 2v_1 v_2 + v_2^2) = \frac{1}{2}\rho(v_1 - v_2)^2. \end{aligned}$$

P16-6 The juice leaves the jug with a speed $v = \sqrt{2gy}$, where y is the height of the juice in the jug. If A is the cross sectional area of the base of the jug and a the cross sectional area of the hole, then the juice flows out the hole with a rate $dV/dt = va = a\sqrt{2gy}$, which means the level of jug varies as $dy/dt = -(a/A)\sqrt{2gy}$. Rearrange and integrate,

$$\int_y^h dy/\sqrt{y} = \int_0^t \sqrt{2g}(a/A)dt,$$

$$
\begin{aligned}
2(\sqrt{h}-\sqrt{y}) &= \sqrt{2g}at/A. \\
\left(\frac{A}{a}\sqrt{\frac{2h}{g}}\right)(\sqrt{h}-\sqrt{y}) &= t
\end{aligned}
$$

When $y = 14h/15$ we have $t = 12.0\,\text{s}$. Then the part in the parenthesis on the left is $3.539\times10^2\text{s}$. The time to empty completely is then 354 seconds, or 5 minutes and 54 seconds. But we want the remaining time, which is 12 seconds less than this.

$\boxed{\textbf{P16-7}}$ The greatest possible value for v will be the value over the wing which results in an air pressure of zero. If the air at the leading edge is stagnant (not moving) and has a pressure of p_0, then Bernoulli's equation gives

$$
p_0 = \frac{1}{2}\rho v^2,
$$

or $v = \sqrt{2p_0/\rho} = \sqrt{2(1.01\times10^5\text{Pa})/(1.2\,\text{kg/m}^3)} = 410\,\text{m/s}$. This value is only slightly larger than the speed of sound; they are related because sound waves involve the movement of air particles which "shove" other air particles out of the way.

P16-8 Bernoulli's equation together with continuity gives

$$
\begin{aligned}
p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\
p_1 - p_2 &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right), \\
&= \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2}v_1^2 - v_1^2\right), \\
&= \frac{v_1^2}{2A_2^2}\rho\left(A_1^2 - A_2^2\right).
\end{aligned}
$$

But $p_1 - p_2 = (\rho' - \rho)gh$. Note that we are *not* assuming ρ is negligible compared to ρ'. Combining,

$$
v_1 = A_2\sqrt{\frac{2(\rho'-\rho)gh}{\rho(A_1^2 - A_2^2)}}.
$$

P16-9 (a) Bernoulli's equation together with continuity gives

$$
\begin{aligned}
p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\
p_1 &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right), \\
&= \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2}v_1^2 - v_1^2\right), \\
&= v_1^2\rho\left[(4.75)^2 - 1\right]/2.
\end{aligned}
$$

Then

$$
v_1 = \sqrt{\frac{2(2.12)(1.01\times10^5\text{Pa})}{(1000\,\text{kg/m}^3)(21.6)}} = 4.45\,\text{m/s},
$$

and then $v_2 = (4.75)(4.45\,\text{m/s}) = 21.2\,\text{m/s}$.

(b) $R = \pi(2.60\times10^{-2}\text{m})^2(4.45\,\text{m/s}) = 9.45\times10^{-3}\,\text{m}^3/\text{s}$.

P16-10 (a) For Fig. 16-13 the velocity is constant, or $\vec{\mathbf{v}} = v\hat{\mathbf{i}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dx = 0,$$

because $\oint dx = 0$.

(b) For Fig. 16-16 the velocity is $\vec{\mathbf{v}} = (k/r)\hat{\mathbf{r}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{r}}dr + \hat{\theta}r\,d\phi$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dr = 0,$$

because $\oint dr = 0$.

P16-11 (a) For an element of the fluid of mass dm the net force as it moves around the circle is $dF = (v^2/r)dm$. $dm/dV = \rho$ and $dV = A\,dr$ and $dF/A = dp$. Then $dp/dr = \rho v^2/r$.

(b) From Bernoulli's equation $p + \rho v^2/2$ is a constant. Then

$$\frac{dp}{dr} + \rho v \frac{dv}{dr} = 0,$$

or $v/r + dv/dr = 0$, or $d(vr) = 0$. Consequently vr is a constant.

(c) The velocity is $\vec{\mathbf{v}} = (k/r)\hat{\mathbf{r}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{r}}dr + \hat{\theta}r\,d\phi$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dr = 0,$$

because $\oint dr = 0$. This means the flow is irrotational.

P16-12 $F/A = \eta v/D$, so

$$F/A = (4.0\times10^{19}\mathrm{N}\cdot\mathrm{s/m^2})(0.048\,\mathrm{m}/3.16\times10^{7}\mathrm{s})/(1.9\times10^{5}\mathrm{m}) = 3.2\times10^{5}\mathrm{Pa}.$$

$\boxed{\textbf{P16-13}}$ A flow will be irrotational if and only if $\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = 0$ for all possible paths. It is fairly easy to construct a rectangular path which is parallel to the flow on the top and bottom sides, but perpendicular on the left and right sides. Then only the top and bottom paths contribute to the integral. $\vec{\mathbf{v}}$ is constant for either path (but not the same), so the magnitude v will come out of the integral sign. Since the lengths of the two paths are the same but v is different the two terms *don't* cancel, so the flow is not irrotational.

P16-14 (a) The area of a cylinder is $A = 2\pi rL$. The velocity gradient if dv/dr. Then the retarding force on the cylinder is $F = -\eta(2\pi rL)dv/dr$.

(b) The force pushing a cylinder through is $F' = A\Delta p = \pi r^2\Delta p$.

(c) Equate, rearrange, and integrate:

$$\begin{aligned} \pi r^2\Delta p &= -\eta(2\pi rL)\frac{dv}{dr}, \\ \Delta p\int_r^R r\,dr &= 2\eta L\int_0^v dv, \\ \Delta p\frac{1}{2}(R^2 - r^2) &= 2\eta Lv. \end{aligned}$$

Then

$$v = \frac{\Delta p}{4\eta L}(R^2 - r^2).$$

$\boxed{\textbf{P16-15}}$ The volume flux (called R_f to distinguish it from the radius R) through an annular ring of radius r and width δr is

$$\delta R_f = \delta A\, v = 2\pi r\, \delta r\, v,$$

where v is a function of r given by Eq. 16-18. The mass flux is the volume flux times the density, so the total mass flux is

$$\begin{aligned}
\frac{dm}{dt} &= \rho \int_0^R \frac{\delta R_f}{\delta r} dr, \\
&= \rho \int_0^R 2\pi r \left(\frac{\Delta p}{4\eta L}(R^2 - r^2) \right) dr, \\
&= \frac{\pi \rho \Delta p}{2\eta L} \int_0^R (rR^2 - r^3) dr, \\
&= \frac{\pi \rho \Delta p}{2\eta L}(R^4/2 - R^4/4), \\
&= \frac{\pi \rho \Delta p R^4}{8\eta L}.
\end{aligned}$$

P16-16 The pressure difference in the tube is $\Delta p = 4\gamma/r$, where r is the (changing) radius of the bubble. The mass flux through the tube is

$$\frac{dm}{dt} = \frac{4\rho\pi R^4 \gamma}{8\eta L r},$$

R is the radius of the tube. $dm = \rho dV$, and $dV = 4\pi r^2 dr$. Then

$$\begin{aligned}
\int_{r_1}^{r_2} r^3\, dr &= \int_0^t \frac{R^4 \gamma}{8\eta L} dt, \\
r_1^4 - r_2^4 &= \frac{\rho R^4 \gamma}{2\eta L} t,
\end{aligned}$$

Then

$$t = \frac{2(1.80\times10^{-5}\mathrm{N\cdot s/m^2})(0.112\,\mathrm{m})}{(0.54\times10^{-3}\mathrm{m})^4(2.50\times10^{-2}\mathrm{N/m})}[(38.2\times10^{-3}\mathrm{m})^4 - (21.6\times10^{-3}\mathrm{m})^4] = 3630\,\mathrm{s}.$$

$\boxed{\textbf{E17-1}}$ For a perfect spring $|F| = k|x|$. $x = 0.157\,\text{m}$ when $3.94\,\text{kg}$ is suspended from it. There would be two forces on the object— the force of gravity, $W = mg$, and the force of the spring, F. These two force must balance, so $mg = kx$ or

$$k = \frac{mg}{x} = \frac{(3.94\,\text{kg})(9.81\,\text{m/s}^2)}{(0.157\,\text{m})} = 0.246\,\text{N/m}.$$

Now that we know k, the spring constant, we can find the period of oscillations from Eq. 17-8,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(0.520\,\text{kg})}{(0.246\,\text{N/m})}} = 0.289\,\text{s}.$$

E17-2 (a) $T = 0.484\,\text{s}$.
(b) $f = 1/T = 1/(0.484\,\text{s}) = 2.07\,\text{s}^{-1}$.
(c) $\omega = 2\pi f = 13.0$ rad/s.
(d) $k = m\omega^2 = (0.512\,\text{kg})(13.0\text{ rad/s})^2 = 86.5\,\text{N/m}$.
(e) $v_\text{m} = \omega x_\text{m} = (13.0\text{ rad/s})(0.347\,\text{m}) = 4.51\,\text{m/s}$.
(f) $F_\text{m} = ma_\text{m} = (0.512\,\text{kg})(13.0\text{ rad/s})^2(0.347\,\text{m}) = 30.0\,\text{N}$.

E17-3 $a_\text{m} = (2\pi f)^2 x_\text{m}$. Then

$$f = \sqrt{(9.81\,\text{m/s}^2)/(1.20\times10^{-6}\text{m})}/(2\pi) = 455\text{ Hz}.$$

E17-4 (a) $\omega = (2\pi)/(0.645\,\text{s}) = 9.74$ rad/s. $k = m\omega^2 = (5.22\,\text{kg})(9.74\text{ rad/s})^2 = 495\,\text{N/m}$.
(b) $x_\text{m} = v_\text{m}/\omega = (0.153\,\text{m/s})/(9.74\text{ rad/s}) = 1.57\times10^{-2}\text{m}$.
(c) $f = 1/(0.645\,\text{s}) = 1.55$ Hz.

$\boxed{\textbf{E17-5}}$ (a) The amplitude is half of the distance between the extremes of the motion, so $A = (2.00\text{ mm})/2 = 1.00$ mm.
(b) The maximum blade speed is given by $v_\text{m} = \omega x_\text{m}$. The blade oscillates with a frequency of 120 Hz, so $\omega = 2\pi f = 2\pi(120\,\text{s}^{-1}) = 754$ rad/s, and then $v_\text{m} = (754\text{ rad/s})(0.001\,\text{m}) = 0.754\,\text{m/s}$.
(c) Similarly, $a_\text{m} = \omega^2 x_\text{m}$, $a_\text{m} = (754\text{ rad/s})^2(0.001\,\text{m}) = 568\,\text{m/s}^2$.

E17-6 (a) $k = m\omega^2 = (1460\,\text{kg}/4)(2\pi 2.95/\text{s})^2 = 1.25\times10^5\text{N/m}$
(b) $f = \sqrt{k/m}/2\pi = \sqrt{(1.25\times10^5\text{N/m})/(1830\,\text{kg}/4)}/2\pi = 2.63/\text{s}$.

E17-7 (a) $x = (6.12\,\text{m})\cos[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = 3.27\,\text{m}$.
(b) $v = -(6.12\,\text{m})(8.38/\text{s})\sin[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = 43.4\,\text{m/s}$.
(c) $a = -(6.12\,\text{m})(8.38/\text{s})^2\cos[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = -229\,\text{m/s}^2$.
(d) $f = (8.38\text{ rad/s})/2\pi = 1.33/\text{s}$.
(e) $T = 1/f = 0.750\,\text{s}$.

E17-8 $k = (50.0\text{ lb})/(4.00\text{ in}) = 12.5$ lb/in.

$$mg = \frac{(32\text{ ft/s}^2)(12\text{ in/ft})(12.5\text{ lb/in})}{[2\pi(2.00/\text{s})]^2} = 30.4\text{ lb}.$$

$\boxed{\textbf{E17-9}}$ If the drive wheel rotates at 193 rev/min then

$$\omega = (193\text{ rev/min})(2\pi\text{ rad/rev})(1/60\text{ s/min}) = 20.2\text{ rad/s},$$

then $v_\text{m} = \omega x_\text{m} = (20.2\text{ rad/s})(0.3825\,\text{m}) = 7.73\,\text{m/s}$.

E17-10 $k = (0.325\,\text{kg})(9.81\,\text{m/s}^2)/(1.80\times10^{-2}\,\text{m}) = 177\,\text{N/m}.$.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(2.14\,\text{kg})}{(177\,\text{N/m})}} = 0.691\,\text{s}.$$

E17-11 For the tides $\omega = 2\pi/(12.5\text{ h})$. Half the maximum occurs when $\cos\omega t = 1/2$, or $\omega t = \pi/3$. Then $t = (12.5\text{ h})/6 = 2.08$ h.

E17-12 The two will separate if the (maximum) acceleration exceeds g.

(a) Since $\omega = 2\pi/T = 2\pi/(1.18\,\text{s}) = 5.32$ rad/s the maximum amplitude is

$$x_\text{m} = (9.81\,\text{m/s}^2)/(5.32\text{ rad/s})^2 = 0.347\,\text{m}.$$

(b) In this case $\omega = \sqrt{(9.81\,\text{m/s}^2)/(0.0512\,\text{m})} = 13.8$ rad/s. Then $f = (13.8\text{ rad/s})/2\pi = 2.20/\text{s}$.

E17-13 (a) $a_x/x = -\omega^2$. Then

$$\omega = \sqrt{-(-123\,\text{m/s})/(0.112\,m)} = 33.1\text{ rad/s},$$

so $f = (33.1\text{ rad/s})/2\pi = 5.27/\text{s}$.

(b) $m = k/\omega^2 = (456\,\text{N/m})/(33.1\text{ rad/s})^2 = 0.416\,\text{kg}$.

(c) $x = x_\text{m}\cos\omega t$; $v = -x_\text{m}\omega\sin\omega t$. Combining,

$$x^2 + (v/\omega)^2 = {x_\text{m}}^2\cos^2\omega t + {x_\text{m}}^2\sin^2\omega t = {x_\text{m}}^2.$$

Consequently,

$$x_\text{m} = \sqrt{(0.112\,\text{m})^2 + (-13.6\,\text{m/s})^2/(33.1\text{ rad/s})^2} = 0.426\,\text{m}.$$

E17-14 $x_1 = x_\text{m}\cos\omega t$, $x_2 = x_\text{m}\cos(\omega t + \phi)$. The crossing happens when $x_1 = x_\text{m}/2$, or when $\omega t = \pi/3$ (and other values!). The same constraint happens for x_2, except that it is moving in the other direction. The closest value is $\omega t + \phi = 2\pi/3$, or $\phi = \pi/3$.

E17-15 (a) The net force on the three cars is zero before the cable breaks. There are three forces on the cars: the weight, W, a normal force, N, and the upward force from the cable, F. Then

$$F = W\sin\theta = 3mg\sin\theta.$$

This force is from the elastic properties of the cable, so

$$k = \frac{F}{x} = \frac{3mg\sin\theta}{x}$$

The frequency of oscillation of the remaining two cars after the bottom car is released is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} = \frac{1}{2\pi}\sqrt{\frac{3mg\sin\theta}{2mx}} = \frac{1}{2\pi}\sqrt{\frac{3g\sin\theta}{2x}}.$$

Numerically, the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{3g\sin\theta}{2x}} = \frac{1}{2\pi}\sqrt{\frac{3(9.81\,\text{m/s}^2)\sin(26^\circ)}{2(0.142\,\text{m})}} = 1.07\text{ Hz}.$$

(b) Each car contributes equally to the stretching of the cable, so one car causes the cable to stretch $14.2/3 = 4.73$ cm. The amplitude is then 4.73 cm.

E17-16 Let the height of one side over the equilibrium position be x. The net restoring force on the liquid is $2\rho Axg$, where A is the cross sectional area of the tube and g is the acceleration of free-fall. This corresponds to a spring constant of $k = 2\rho Ag$. The mass of the fluid is $m = \rho AL$. The period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{2L}{g}}.$$

$\boxed{\textbf{E17-17}}$ (a) There are two forces on the log. The weight, $W = mg$, and the buoyant force B. We'll assume the log is cylindrical. If x is the length of the log beneath the surface and A the cross sectional area of the log, then $V = Ax$ is the volume of the displaced water. Furthermore, $m_\mathrm{w} = \rho_\mathrm{w} V$ is the mass of the displaced water and $B = m_\mathrm{w} g$ is then the buoyant force on the log. Combining,

$$B = \rho_\mathrm{w} Agx,$$

where ρ_w is the density of water. This certainly looks similar to an elastic spring force law, with $k = \rho_\mathrm{w} Ag$. We would then expect the motion to be simple harmonic.

(b) The period of the oscillation would be

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\rho_\mathrm{w} Ag}},$$

where m is the total mass of the log and lead. We are told the log is in equilibrium when $x = L = 2.56$ m. This would give us the *weight* of the log, since $W = B$ is the condition for the log to float. Then

$$m = \frac{B}{g} = \frac{\rho_\mathrm{w} AgL}{g} = \rho AL.$$

From this we can write the period of the motion as

$$T = 2\pi\sqrt{\frac{\rho AL}{\rho_\mathrm{w} Ag}} = 2\pi\sqrt{L/g} = 2\pi\sqrt{\frac{(2.56\,\mathrm{m})}{(9.81\,\mathrm{m/s^2})}} = 3.21\,\mathrm{s}.$$

E17-18 (a) $k = 2(1.18\,\mathrm{J})/(0.0984\,\mathrm{m})^2 = 244\,\mathrm{N/m}$.

(b) $m = 2(1.18\,\mathrm{J})/(1.22\,\mathrm{m/s})^2 = 1.59\,\mathrm{kg}$.

(c) $f = [(1.22\,\mathrm{m/s})/(0.0984\,\mathrm{m})]/(2\pi) = 1.97/\mathrm{s}$.

E17-19 (a) Equate the kinetic energy of the object just after it leaves the slingshot with the potential energy of the stretched slingshot.

$$k = \frac{mv^2}{x^2} = \frac{(0.130\,\mathrm{kg})(11.2\times 10^3\mathrm{m/s})^2}{(1.53\,\mathrm{m})^2} = 6.97\times 10^6\mathrm{N/m}.$$

(b) $N = (6.97\times 10^6\,\mathrm{N/m})(1.53\,\mathrm{m})/(220\,\mathrm{N}) = 4.85\times 10^4$ people.

E17-20 (a) $E = k{x_\mathrm{m}}^2/2$, $U = kx^2/2 = k(x_\mathrm{m}/2)^2/2 = E/4$. $K = E - U = 3E/4$. The the energy is 25% potential and 75% kinetic.

(b) If $U = E/2$ then $kx^2/2 = k{x_\mathrm{m}}^2/4$, or $x = x_\mathrm{m}/\sqrt{2}$.

E17-21 (a) $a_m = \omega^2 x_m$ so

$$\omega = \sqrt{\frac{a_m}{x_m}} = \sqrt{\frac{(7.93 \times 10^3\,\mathrm{m/s^2})}{(1.86 \times 10^{-3}\,\mathrm{m})}} = 2.06 \times 10^3 \text{ rad/s}$$

The period of the motion is then

$$T = \frac{2\pi}{\omega} = 3.05 \times 10^{-3}\mathrm{s}.$$

(b) The maximum speed of the particle is found by

$$v_m = \omega x_m = (2.06 \times 10^3 \text{ rad/s})(1.86 \times 10^{-3}\,\mathrm{m}) = 3.83\,\mathrm{m/s}.$$

(c) The mechanical energy is given by Eq. 17-15, except that we will focus on when $v_x = v_m$, because then $x = 0$ and

$$E = \frac{1}{2} m {v_m}^2 = \frac{1}{2}(12.3\,\mathrm{kg})(3.83\,\mathrm{m/s})^2 = 90.2\,\mathrm{J}.$$

E17-22 (a) $f = \sqrt{k/m}/2\pi = \sqrt{(988\,\mathrm{N/m})/(5.13\,\mathrm{kg})}/2\pi = 2.21/\mathrm{s}$.
(b) $U_i = (988\,\mathrm{N/m})(0.535\,\mathrm{m})^2/2 = 141\,\mathrm{J}$.
(c) $K_i = (5.13\,\mathrm{kg})(11.2\,\mathrm{m/s})^2/2 = 322\,\mathrm{J}$.
(d) $x_m = \sqrt{2E/k} = \sqrt{2(322\,\mathrm{J} + 141\,\mathrm{J})/(988\,\mathrm{N/m})} = 0.968\,\mathrm{m}$.

E17-23 (a) $\omega = \sqrt{(538\,\mathrm{N/m})/(1.26\,\mathrm{kg})} = 20.7$ rad/s.

$$x_m = \sqrt{(0.263\,\mathrm{m})^2 + (3.72\,\mathrm{m/s})^2/(20.7 \text{ rad/s})^2} = 0.319\,\mathrm{m}.$$

(b) $\phi = \arctan\{-(-3.72\,\mathrm{m/s})/[(20.7 \text{ rad/s})(0.263\,\mathrm{m})]\} = 34.3°$.

E17-24 Before doing anything else apply conservation of momentum. If v_0 is the speed of the bullet just before hitting the block and v_1 is the speed of the bullet/block system just after the two begin moving as one, then $v_1 = mv_0/(m + M)$, where m is the mass of the bullet and M is the mass of the block.

For this system $\omega = \sqrt{k/(m + M)}$.

(a) The total energy of the oscillation is $\frac{1}{2}(m + M)v_1^2$, so the amplitude is

$$x_m = \sqrt{\frac{m + M}{k}} v_1 = \sqrt{\frac{m + M}{k}} \frac{mv_0}{m + M} = mv_0 \sqrt{\frac{1}{k(m + M)}}.$$

The numerical value is

$$x_m = (0.050\,\mathrm{kg})(150\,\mathrm{m/s}) \sqrt{\frac{1}{(500\,\mathrm{N/m})(0.050\,\mathrm{kg} + 4.00\,\mathrm{kg})}} = 0.167\,\mathrm{m}.$$

(b) The fraction of the energy is

$$\frac{(m + M)v_1^2}{mv_0^2} = \frac{m + M}{m} \left(\frac{m}{m + M}\right)^2 = \frac{m}{m + M} = \frac{(0.050\,\mathrm{kg})}{(0.050\,\mathrm{kg} + 4.00\,\mathrm{kg})} = 1.23 \times 10^{-2}.$$

E17-25 $L = (9.82\,\mathrm{m/s^2})(1.00\,\mathrm{s}/2\pi)^2 = 0.249\,\mathrm{m}$.

E17-26 $T = (180\,\text{s})/(72.0)$. Then

$$g = \left(\frac{2\pi(72.0)}{(180\,\text{s})}\right)^2 (1.53\,\text{m}) = 9.66\,\text{m/s}.$$

$\boxed{\textbf{E17-27}}$ We are interested in the value of θ_m which will make the second term 2% of the first term. We want to solve

$$0.02 = \frac{1}{2^2}\sin^2\frac{\theta_\text{m}}{2},$$

which has solution

$$\sin\frac{\theta_\text{m}}{2} = \sqrt{0.08}$$

or $\theta_m = 33°$.

(b) How large is the third term at this angle?

$$\frac{3^2}{2^2 4^2}\sin^4\frac{\theta_m}{2} = \frac{3^2}{2^2}\left(\frac{1}{2^2}\sin^2\frac{\theta_m}{2}\right)^2 = \frac{9}{4}(0.02)^2$$

or 0.0009, which is very small.

E17-28 Since $T \propto \sqrt{1/g}$ we have

$$T_\text{p} = T_\text{e}\sqrt{g_\text{e}/g_\text{p}} = (1.00\,\text{s})\sqrt{\frac{(9.78\,\text{m/s}^2)}{(9.834\,\text{m/s}^2)}} = 0.997\,\text{s}.$$

$\boxed{\textbf{E17-29}}$ Let the period of the clock in Paris be T_1. In a day of length D_1 = 24 hours it will undergo $n = D/T_1$ oscillations. In Cayenne the period is T_2. n oscillations should occur in 24 hours, but since the clock runs slow, D_2 is 24 hours + 2.5 minutes elapse. So

$$T_2 = D_2/n = (D_2/D_1)T_1 = [(1442.5\text{ min})/(1440.0\text{ min})]T_1 = 1.0017T_1.$$

Since the ratio of the periods is $(T_2/T_1) = \sqrt{(g_1/g_2)}$, the g_2 in Cayenne is

$$g_2 = g_1(T_1/T_2)^2 = (9.81\,\text{m/s}^2)/(1.0017)^2 = 9.78\,\text{m/s}^2.$$

E17-30 (a) Take the differential of

$$g = \left(\frac{2\pi(100)}{T}\right)^2 (10\,\text{m}) = \frac{4\pi^2\times 10^5\,\text{m}}{T^2},$$

so $\delta g = (-8\pi^2\times 10^5\,\text{m}/T^3)\delta T$. Note that T is not the period here, it is the time for 100 oscillations! The relative error is then

$$\frac{\delta g}{g} = -2\frac{\delta T}{T}.$$

If $\delta g/g = 0.1\%$ then $\delta T/T = 0.05\%$.

(b) For $g \approx 10\,\text{m/s}^2$ we have

$$T \approx 2\pi(100)\sqrt{(10\,\text{m})/(10\,\text{m/s}^2)} = 628\,\text{s}.$$

Then $\delta T \approx (0.0005)(987\,\text{s}) \approx 300$ ms.

E17-31 $T = 2\pi\sqrt{(17.3\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 8.34\,\mathrm{s}$.

E17-32 The spring will extend until the force from the spring balances the weight, or when $Mg = kh$. The frequency of this system is then

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{Mg/h}{M}} = \frac{1}{2\pi}\sqrt{\frac{g}{h}},$$

which is the frequency of a pendulum of length h. The mass of the bob is irrelevant.

E17-33 The frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}},$$

where d is the distance from the pivot about which the hoop oscillates and the center of mass of the hoop.

The rotational inertia I is about an axis through the pivot, so we apply the parallel axis theorem. Then

$$I = Md^2 + I_{\mathrm{cm}} = Md^2 + Mr^2.$$

But d is r, since the pivot point is on the rim of the hoop. So $I = 2Md^2$, and the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{2Md^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{2d}} = \frac{1}{2\pi}\sqrt{\frac{(9.81\,\mathrm{m/s^2})}{2(0.653\,\mathrm{m})}} = 0.436\ \mathrm{Hz}.$$

(b) Note the above expression looks like the simple pendulum equation if we replace $2d$ with l. Then the equivalent length of the simple pendulum is $2(0.653\,\mathrm{m}) = 1.31\,\mathrm{m}$.

E17-34 Apply Eq. 17-21:

$$I = \frac{T^2\kappa}{4\pi^2} = \frac{(48.7\,\mathrm{s}/20.0)^2(0.513\,\mathrm{N\cdot m})}{4\pi^2} = 7.70\times10^{-2}\mathrm{kg\cdot m^2}.$$

E17-35 $\kappa = (0.192\,\mathrm{N\cdot m})/(0.850\ \mathrm{rad}) = 0.226\,\mathrm{N\cdot m}$. $I = \frac{2}{5}(95.2\,\mathrm{kg})(0.148\,\mathrm{m})^2 = 0.834\,\mathrm{kg\cdot m^2}$. Then

$$T = 2\pi\sqrt{I/\kappa} = 2\pi\sqrt{(0.834\,\mathrm{kg\cdot m^2})/(0.226\,\mathrm{N\cdot m})} = 12.1\,\mathrm{s}.$$

E17-36 x is d in Eq. 17-29. Since the hole is drilled off center we apply the principle axis theorem to find the rotational inertia:

$$I = \frac{1}{12}ML^2 + Mx^2.$$

Then

$$\begin{aligned}
\frac{1}{12}ML^2 + Mx^2 &= \frac{T^2Mgx}{4\pi^2},\\
\frac{1}{12}(1.00\,\mathrm{m})^2 + x^2 &= \frac{(2.50\,\mathrm{s})^2(9.81\,\mathrm{m/s^2})}{4\pi^2}x,\\
(8.33\times10^{-2}\mathrm{m^2}) - (1.55\,\mathrm{m})x + x^2 &= 0.
\end{aligned}$$

This has solutions $x = 1.49\,\mathrm{m}$ and $x = 0.0557\,\mathrm{m}$. Use the latter.

$\boxed{\textbf{E17-37}}$ For a stick of length L which can pivot about the end, $I = \frac{1}{3}ML^2$. The center of mass of such a stick is located $d = L/2$ away from the end.

The frequency of oscillation of such a stick is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}}, \\ f &= \frac{1}{2\pi}\sqrt{\frac{Mg\,(L/2)}{\frac{1}{3}ML^2}}, \\ f &= \frac{1}{2\pi}\sqrt{\frac{3g}{2L}}. \end{aligned}$$

This means that f is proportional to $\sqrt{1/L}$, regardless of the mass or density of the stick. The ratio of the frequency of two such sticks is then $f_2/f_1 = \sqrt{L_1/L_2}$, which in our case gives

$$f_2 = f_1\sqrt{L_2/L_1} = f_1\sqrt{(L_1)/(2L_1/3)} = 1.22 f_1.$$

E17-38 The rotational inertia of the pipe section about the cylindrical axis is

$$I_{\rm cm} = \frac{M}{2}\left[r_1^2 + r_2^2\right] = \frac{M}{2}\left[(0.102\,{\rm m})^2 + (0.1084\,{\rm m})^2\right] = (1.11\times 10^{-2}{\rm m}^2)M$$

(a) The total rotational inertia about the pivot axis is

$$I = 2I_{\rm cm} + M(0.102\,{\rm m})^2 + M(0.3188\,{\rm m})^2 = (0.134\,{\rm m}^2)M.$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{(0.134\,{\rm m}^2)M}{M(9.81\,{\rm m/s}^2)(0.2104\,{\rm m})}} = 1.60\,{\rm s}$$

(b) The rotational inertia of the pipe section about a diameter is

$$I_{\rm cm} = \frac{M}{4}\left[r_1^2 + r_2^2\right] = \frac{M}{4}\left[(0.102\,{\rm m})^2 + (0.1084\,{\rm m})^2\right] = (5.54\times 10^{-3}{\rm m}^2)M$$

The total rotational inertia about the pivot axis is now

$$I = M(1.11\times 10^{-2}{\rm m}^2) + M(0.102\,{\rm m})^2 + M(5.54\times 10^{-3}{\rm m}^2) + M(0.3188\,{\rm m})^2 = (0.129\,{\rm m}^2)M$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{(0.129\,{\rm m}^2)M}{M(9.81\,{\rm m/s}^2)(0.2104\,{\rm m})}} = 1.57\,{\rm s}.$$

The percentage difference with part (a) is $(0.03\,{\rm s})/(1.60\,{\rm s}) = 1.9\%$.

E17-39

E17-40

E17-41 (a) Since effectively $x = y$, the path is a diagonal line.

(b) The path will be an ellipse which is symmetric about the line $x = y$.

(c) Since $\cos(\omega t + 90°) = -\sin(\omega t)$, the path is a circle.

E17-42 (a)

(b) Take two time derivatives and multiply by m,

$$\vec{\mathbf{F}} = -mA\omega^2\left(\hat{\mathbf{i}}\cos\omega t + 9\hat{\mathbf{j}}\cos 3\omega t\right).$$

(c) $U = -\int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$, so

$$U = \frac{1}{2}mA^2\omega^2\left(\cos^2\omega t + 9\cos^2 3\omega t\right).$$

(d) $K = \frac{1}{2}mv^2$, so

$$K = \frac{1}{2}mA^2\omega^2\left(\sin^2\omega t + 9\sin^2 3\omega t\right);$$

And then $E = K + U = 5mA^2\omega^2$.

(e) Yes; the period is $2\pi/\omega$.

$\boxed{\textbf{E17-43}}$ The ω which describes the angular velocity in uniform circular motion is effectively the same ω which describes the angular frequency of the corresponding simple harmonic motion. Since $\omega = \sqrt{k/m}$, we can find the effective force constant k from knowledge of the Moon's mass and the period of revolution.

The moon orbits with a period of T, so

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(27.3\times 24\times 3600\,\text{s})} = 2.66\times 10^{-6}\text{rad/s}.$$

This can be used to find the value of the effective force constant k from

$$k = m\omega^2 = (7.36\times 10^{22}\text{kg})(2.66\times 10^{-6}\text{rad/s})^2 = 5.21\times 10^{11}\text{N/m}.$$

E17-44 (a) We want to know when $e^{-bt/2m} = 1/3$, or

$$t = \frac{2m}{b}\ln 3 = \frac{2(1.52\,\text{kg})}{(0.227\,\text{kg/s})}\ln 3 = 14.7\,\text{s}$$

(b) The (angular) frequency is

$$\omega' = \sqrt{\left(\frac{(8.13\,\text{N/m})}{(1.52\,\text{kg})}\right) - \left(\frac{(0.227\,\text{kg/s})}{2(1.52\,\text{kg})}\right)^2} = 2.31\text{ rad/s}.$$

The number of oscillations is then

$$(14.7\,\text{s})(2.31\text{ rad/s})/2\pi = 5.40$$

$\boxed{\textbf{E17-45}}$ The first derivative of Eq. 17-39 is

$$\begin{aligned}\frac{dx}{dt} &= x_\text{m}(-b/2m)e^{-bt/2m}\cos(\omega' t+\phi) + x_\text{m}e^{-bt/2m}(-\omega')\sin(\omega' t+\phi),\\ &= -x_\text{m}e^{-bt/2m}\left((b/2m)\cos(\omega' t+\phi) + \omega'\sin(\omega' t+\phi)\right)\end{aligned}$$

The second derivative is quite a bit messier;

$$\begin{aligned}\frac{d^2}{dx^2} &= -x_\text{m}(-b/2m)e^{-bt/2m}\left((b/2m)\cos(\omega' t+\phi) + \omega'\sin(\omega' t+\phi)\right)\\ &\quad -x_\text{m}e^{-bt/2m}\left((b/2m)(-\omega')\sin(\omega' t+\phi) + (\omega')^2\cos(\omega' t+\phi)\right),\\ &= x_\text{m}e^{-bt/2m}\left((\omega' b/m)\sin(\omega' t+\phi) + (b^2/4m^2 - \omega'^2)\cos(\omega' t+\phi)\right).\end{aligned}$$

Substitute these three expressions into Eq. 17-38. There are, however, some fairly obvious simplifications. Every one of the terms above has a factor of $x_{\rm m}$, and every term above has a factor of $e^{-bt/2m}$, so simultaneously with the substitution we will cancel out those factors. Then Eq. 17-38 becomes

$$m\left[(\omega' b/m)\sin(\omega' t+\phi)+(b^2/4m^2-\omega'^2)\cos(\omega' t+\phi)\right]$$
$$-b\left[(b/2m)\cos(\omega' t+\phi)+\omega'\sin(\omega' t+\phi)\right]+k\cos(\omega' t+\phi)=0$$

Now we collect terms with cosine and terms with sine,

$$(\omega' b-\omega' b)\sin(\omega' t+\phi)+\left(mb^2/4m^2-\omega'^2-b^2/2m+k\right)\cos(\omega' t+\phi)=0.$$

The coefficient for the sine term is identically zero; furthermore, because the cosine term must then vanish regardless of the value of t, the coefficient for the sine term must also vanish. Then

$$\left(mb^2/4m^2-m\omega'^2-b^2/2m+k\right)=0,$$

or

$$\omega'^2=\frac{k}{m}-\frac{b^2}{4m^2}.$$

If this condition is met, then Eq. 17-39 is indeed a solution of Eq. 17-38.

E17-46 (a) Four complete cycles requires a time $t_4 = 8\pi/\omega'$. The amplitude decays to 3/4 the original value in this time, so $0.75 = e^{-bt_4/2m}$, or

$$\ln(4/3)=\frac{8\pi b}{2m\omega'}.$$

It is probably reasonable at this time to assume that $b/2m$ is small compared to ω so that $\omega' \approx \omega$. We'll do it the hard way anyway. Then

$$\begin{aligned}
\omega'^2 &= \left(\frac{8\pi}{\ln(4/3)}\right)^2\left(\frac{b}{2m}\right)^2,\\
\frac{k}{m}-\left(\frac{b}{2m}\right)^2 &= \left(\frac{8\pi}{\ln(4/3)}\right)^2\left(\frac{b}{2m}\right)^2,\\
\frac{k}{m} &= (7630)\left(\frac{b}{2m}\right)^2
\end{aligned}$$

Numerically, then,

$$b=\sqrt{\frac{4(1.91\,\text{kg})(12.6\,\text{N/m})}{(7630)}}=0.112\,\text{kg/s}.$$

(b)

E17-47 (a) Use Eqs. 17-43 and 17-44. At resonance $\omega''=\omega$, so

$$G=\sqrt{b^2\omega^2}=b\omega,$$

and then $x_{\rm m}=F_{\rm m}/b\omega$.

(b) $v_{\rm m}=\omega x_{\rm m}=F_{\rm m}/b$.

E17-48 We need the first two derivatives of

$$x = \frac{F_{\mathrm{m}}}{G}\cos(\omega'' t - \beta)$$

The derivatives are easy enough to find,

$$\frac{dx}{dt} = \frac{F_{\mathrm{m}}}{G}(-\omega'')\sin(\omega'' t - \beta),$$

and

$$\frac{d^2x}{dt^2} = -\frac{F_{\mathrm{m}}}{G}(\omega'')^2\cos(\omega'' t - \beta),$$

We'll substitute this into Eq. 17-42,

$$\begin{aligned} &m\left(-\frac{F_{\mathrm{m}}}{G}(\omega'')^2\cos(\omega'' t - \beta)\right) \\ &\quad +b\left(\frac{F_{\mathrm{m}}}{G}(-\omega'')\sin(\omega'' t - \beta)\right) + k\frac{F_{\mathrm{m}}}{G}\cos(\omega'' t - \beta) = F_{\mathrm{m}}\cos\omega'' t. \end{aligned}$$

Then we'll cancel out as much as we can and collect the sine and cosine terms,

$$\left(k - m(\omega'')^2\right)\cos(\omega'' t - \beta) - (b\omega'')\sin(\omega'' t - \beta) = G\cos\omega'' t.$$

We can write the left hand side of this equation in the form

$$A\cos\alpha_1\cos\alpha_2 - A\sin\alpha_1\sin\alpha_2,$$

if we let $\alpha_2 = \omega'' t - \beta$ and choose A and α_1 correctly. The best choice is

$$\begin{aligned} A\cos\alpha_1 &= k - m(\omega'')^2, \\ A\sin\alpha_1 &= b\omega'', \end{aligned}$$

and then taking advantage of the fact that $\sin^2 + \cos^2 = 1$,

$$A^2 = \left(k - m(\omega'')^2\right)^2 + (b\omega'')^2,$$

which looks like Eq. 17-44! But then we can apply the cosine angle addition formula, and

$$A\cos(\alpha_1 + \omega'' t - \beta) = G\cos\omega'' t.$$

This expression needs to be true for all time. This means that $A = G$ and $\alpha_1 = \beta$.

$\boxed{\textbf{E17-49}}$ The derivatives are easy enough to find,

$$\frac{dx}{dt} = \frac{F_{\mathrm{m}}}{G}(-\omega''')\sin(\omega''' t - \beta),$$

and

$$\frac{d^2x}{dt^2} = -\frac{F_{\mathrm{m}}}{G}(\omega''')^2\cos(\omega''' t - \beta),$$

We'll substitute this into Eq. 17-42,

$$\begin{aligned} &m\left(-\frac{F_{\mathrm{m}}}{G}(\omega''')^2\cos(\omega''' t - \beta)\right) \\ &\quad +b\left(\frac{F_{\mathrm{m}}}{G}(-\omega''')\sin(\omega''' t - \beta)\right) + k\frac{F_{\mathrm{m}}}{G}\cos(\omega''' t - \beta) = F_{\mathrm{m}}\cos\omega'' t. \end{aligned}$$

Then we'll cancel out as much as we can and collect the sine and cosine terms,

$$\left(k - m(\omega''')^2\right)\cos(\omega'''t - \beta) - (b\omega''')\sin(\omega'''t - \beta) = G\cos\omega''t.$$

We can write the left hand side of this equation in the form

$$A\cos\alpha_1\cos\alpha_2 - A\sin\alpha_1\sin\alpha_2,$$

if we let $\alpha_2 = \omega'''t - \beta$ and choose A and α_1 correctly. The best choice is

$$\begin{aligned} A\cos\alpha_1 &= k - m(\omega''')^2, \\ A\sin\alpha_1 &= b\omega''', \end{aligned}$$

and then taking advantage of the fact that $\sin^2 + \cos^2 = 1$,

$$A^2 = \left(k - m(\omega''')^2\right)^2 + (b\omega''')^2,$$

which looks like Eq. 17-44! But then we can apply the cosine angle addition formula, and

$$A\cos(\alpha_1 + \omega'''t - \beta) = G\cos\omega''t.$$

This expression needs to be true for all time. This means that $A = G$ and $\alpha_1 + \omega'''t - \beta = \omega''t$ and $\alpha_1 = \beta$ and $\omega''' = \omega''$.

E17-50 Actually, Eq. 17-39 is *not* a solution to Eq. 17-42 by itself, this is a wording mistake in the exercise. Instead, Eq. 17-39 can be added to *any* solution of Eq. 17-42 and the result will still be a solution.

Let x_n be *any* solution to Eq. 17-42 (such as Eq. 17-43.) Let x_h be given by Eq. 17-39. Then

$$x = x_\text{n} + x_\text{h}.$$

Take the first two time derivatives of this expression.

$$\begin{aligned} \frac{dx}{dt} &= \frac{dx_\text{n}}{dt} + \frac{dx_\text{h}}{dt}, \\ \frac{d^2x}{dt^2} &= \frac{d^2x_\text{n}}{dt^2} + \frac{d^2x_\text{h}}{dt^2} \end{aligned}$$

Substitute these three expressions into Eq. 17-42.

$$m\left(\frac{d^2x_\text{n}}{dt^2} + \frac{d^2x_\text{h}}{dt^2}\right) + b\left(\frac{dx_\text{n}}{dt} + \frac{dx_\text{h}}{dt}\right) + k\left(x_\text{n} + x_\text{h}\right) = F_\text{m}\cos\omega''t.$$

Rearrange and regroup.

$$\left(m\frac{d^2x_\text{n}}{dt^2} + b\frac{dx_\text{n}}{dt} + kx_\text{n}\right) + \left(m\frac{d^2x_\text{h}}{dt^2} + b\frac{dx_\text{h}}{dt} + kx_\text{h}\right) = F_\text{m}\cos\omega''t.$$

Consider the second term on the left. The parenthetical expression is just Eq. 17-38, the damped harmonic oscillator equation. It is given in the text (and proved in Ex. 17-45) the x_h is a solution, so this term is identically zero. What remains is Eq. 17-42; and we took as a given that x_n was a solution.

(b) The "add-on" solution of x_h represents the transient motion that will die away with time.

E17-51 The time between "bumps" is the solution to

$$\begin{aligned} vt &= x, \\ t &= \frac{(13\text{ ft})}{(10\text{ mi/hr})}\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = 0.886\,\text{s} \end{aligned}$$

The angular frequency is

$$\omega = \frac{2\pi}{T} = 7.09\text{ rad/s}$$

This is the driving frequency, and the problem states that at this frequency the up-down bounce oscillation is at a maximum. This occurs when the driving frequency is approximately equal to the natural frequency of oscillation. The force constant for the car is k, and this is related to the natural angular frequency by

$$k = m\omega^2 = \frac{W}{g}\omega^2,$$

where $W = (2200 + 4 \times 180)$ lb= 2920 lb is the weight of the car and occupants. Then

$$k = \frac{(2920\text{ lb})}{(32\text{ ft/s}^2)}(7.09\text{ rad/s})^2 = 4590\text{ lb/ft}$$

When the four people get out of the car there is less downward force on the car springs. The important relationship is

$$\Delta F = k\Delta x.$$

In this case $\Delta F = 720$ lb, the weight of the four people who got out of the car. Δx is the distance the car will rise when the people get out. So

$$\Delta x = \frac{\Delta F}{k} = \frac{(720\text{ lb})}{4590\text{ lb/ft}} = 0.157\text{ ft} \approx 2\text{ in.}$$

E17-52 The derivative is easy enough to find,

$$\frac{dx}{dt} = \frac{F_\text{m}}{G}(-\omega'')\sin(\omega'' t - \beta),$$

The velocity amplitude is

$$\begin{aligned} v_\text{m} &= \frac{F_\text{m}}{G}\omega'', \\ &= \frac{F_\text{m}}{\frac{1}{\omega''}\sqrt{m^2(\omega''^2 - \omega^2)^2 + b^2\omega''^2}}, \\ &= \frac{F_\text{m}}{\sqrt{(m\omega'' - k/\omega'')^2 + b^2}}. \end{aligned}$$

Note that this *is exactly* a maximum when $\omega'' = \omega$.

E17-53 The reduced mass is

$$m = (1.13\,\text{kg})(3.24\,\text{kg})/(1.12\,\text{kg} + 3.24\,\text{kg}) = 0.840\,\text{kg}.$$

The period of oscillation is

$$T = 2\pi\sqrt{(0.840\,\text{kg})/(252\,\text{N/m})} = 0.363\,\text{s}$$

E17-54

E17-55 Start by multiplying the kinetic energy expression by $(m_1 + m_2)/(m_1 + m_2)$.

$$\begin{aligned} K &= \frac{(m_1+m_2)}{2(m_1+m_2)}\left(m_1 v_1^2 + m_2 v_2^2\right), \\ &= \frac{1}{2(m_1+m_2)}\left(m_1^2 v_1^2 + m_1 m_2 (v_1^2 + v_2^2) + m_2^2 v_2^2\right), \end{aligned}$$

and then add $2m_1 m_2 v_1 v_2 - 2m_1 m_2 v_1 v_2$,

$$\begin{aligned} K &= \frac{1}{2(m_1+m_2)}\left(m_1^2 v_1^2 + 2m_1 m_2 v_1 v_2 + m_2^2 v_2^2 + m_1 m_2 (v_1^2 + v_2^2 - 2v_1 v_2)\right), \\ &= \frac{1}{2(m_1+m_2)}\left((m_1 v_1 + m_2 v_2)^2 + m_1 m_2 (v_1 - v_2)^2\right). \end{aligned}$$

But $m_1 v_2 + m_2 v_2 = 0$ by conservation of momentum, so

$$\begin{aligned} K &= \frac{(m_1 m_2)}{2(m_1+m_2)}\left(v_1 - v_2\right)^2, \\ &= \frac{m}{2}(v_1 - v_2)^2. \end{aligned}$$

P17-1 The mass of one silver atom is $(0.108\,\mathrm{kg})/(6.02\times10^{23}) = 1.79\times10^{-25}\mathrm{kg}$. The effective spring constant is

$$k = (1.79\times10^{-25}\mathrm{kg})4\pi^2(10.0\times10^{12}/\mathrm{s})^2 = 7.07\times10^{2}\mathrm{N/m}.$$

P17-2 (a) Rearrange Eq. 17-8 except replace m with the total mass, or $m+M$. Then $(M+m)/k = T^2/(4\pi^2)$, or

$$M = (k/4\pi^2)T^2 - m.$$

(b) When $M = 0$ we have

$$m = [(605.6\,\mathrm{N/m})/(4\pi^2)](0.90149\,\mathrm{s})^2 = 12.467\,\mathrm{kg}.$$

(c) $M = [(605.6\,\mathrm{N/m})/(4\pi^2)](2.08832\,\mathrm{s})^2 - (12.467\,\mathrm{kg}) = 54.432\,\mathrm{kg}.$

$\boxed{\textbf{P17-3}}$ The maximum static friction is $F_f \le \mu_s N$. Then

$$F_f = \mu_s N = \mu_s W = \mu_s m g$$

is the maximum available force to accelerate the upper block. So the maximum acceleration is

$$a_\mathrm{m} = \frac{F_f}{m} = \mu_s g$$

The maximum possible amplitude of the oscillation is then given by

$$x_\mathrm{m} = \frac{a_\mathrm{m}}{\omega^2} = \frac{\mu_s g}{k/(m+M)},$$

where in the last part we substituted the total mass of the two blocks because both blocks are oscillating. Now we put in numbers, and find

$$x_\mathrm{m} = \frac{(0.42)(1.22\,\mathrm{kg} + 8.73\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(344\,\mathrm{N/m})} = 0.119\,\mathrm{m}.$$

P17-4 (a) Equilibrium occurs when $F = 0$, or $b/r^3 = a/r^2$. This happens when $r = b/a$.
(b) $dF/dr = 2a/r^3 - 3b/r^4$. At $r = b/a$ this becomes

$$dF/dr = 2a^4/b^3 - 3a^4/b^3 = -a^4/b^3,$$

which corresponds to a force constant of a^4/b^3.
(c) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{mb^3}/a^2$, where m is the reduced mass.

P17-5 Each spring helps to restore the block. The net force on the block is then of magnitude $F_1 + F_2 = k_1 x + k_2 x = (k_1 + k_2)x = kx$. We can then write the frequency as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}.$$

With a little algebra,

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}, \\ &= \sqrt{\frac{1}{4\pi^2}\frac{k_1}{m} + \frac{1}{4\pi^2}\frac{k_2}{m}}, \\ &= \sqrt{f_1^2 + f_2^2}. \end{aligned}$$

P17-6 The tension in the two spring is the same, so $k_1 x_1 = k_2 x_2$, where x_i is the extension of the ith spring. The total extension is $x_1 + x_2$, so the effective spring constant of the combination is

$$\frac{F}{x} = \frac{F}{x_1 + x_2} = \frac{1}{x_1/F + x_2/F} = \frac{1}{1/k_1 + 1/k_2} = \frac{k_1 k_2}{k_1 + k_2}.$$

The period is then

$$T = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

With a little algebra,

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{1}{m/k_1 + m/k_2}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{1}{1/\omega_1^2 + 1/\omega_2^2}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{\omega_1^2\omega_2^2}{\omega_1^2 + \omega_2^2}}, \\ &= \frac{f_1 f_2}{\sqrt{f_1^2 + f_2^2}}. \end{aligned}$$

$\boxed{\textbf{P17-7}}$ (a) When a spring is stretched the tension is the same everywhere in the spring. The stretching, however, is distributed over the entire length of the spring, so that the relative amount of stretch is proportional to the length of the spring under consideration. Half a spring, half the extension. But $k = -F/x$, so half the extension means twice the spring constant.

In short, cutting the spring in half will create two stiffer springs with twice the spring constant, so $k = 7.20$ N/cm for each spring.

(b) The two spring halves now support a mass M. We can view this as each spring is holding one-half of the total mass, so in effect

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M/2}}$$

or, solving for M,

$$M = \frac{2k}{4\pi^2 f^2} = \frac{2(720\,\mathrm{N/m})}{4\pi^2(2.87\,\mathrm{s}^{-1})^2} = 4.43\,\mathrm{kg}.$$

P17-8 Treat the spring a being composed of N massless springlets each with a point mass m_s/N at the end. The spring constant for each springlet will be kN. An expression for the conservation of energy is then

$$\frac{m}{2}v^2 + \frac{m_\mathrm{s}}{2N}\sum_1^N {v_n}^2 + \frac{Nk}{2}\sum_1^N {x_n}^2 = E.$$

Since the spring stretches proportionally along the length then we conclude that each springlet compresses the same amount, and then $x_n = A/N \sin\omega t$ could describe the *change* in length of each springlet. The energy conservation expression becomes

$$\frac{m}{2}v^2 + \frac{m_\mathrm{s}}{2N}\sum_1^N {v_n}^2 + \frac{k}{2}A^2\sin^2\omega t = E.$$

$v = A\omega\cos\omega t$. The hard part to sort out is the v_n, since the displacements for all springlets to one side of the nth must be added to get the net displacement. Then

$$v_n = n\frac{A}{N}\omega\cos\omega t,$$

and the energy expression becomes

$$\left(\frac{m}{2} + \frac{m_\mathrm{s}}{2N^3}\sum_1^N n^2\right)A^2\omega^2\cos^2\omega t + \frac{k}{2}A^2\sin^2\omega t = E.$$

Replace the sum with an integral, then

$$\frac{1}{N^3}\int_0^N n^2\,dn = \frac{1}{3},$$

and the energy expression becomes

$$\frac{1}{2}\left(m + \frac{m_\mathrm{s}}{3}\right)A^2\omega^2\cos^2\omega t + \frac{k}{2}A^2\sin^2\omega t = E.$$

This will only be constant if

$$\omega^2 = \left(m + \frac{m_\mathrm{s}}{3}\right)/k,$$

or $T = 2\pi\sqrt{(m + m_\mathrm{s}/3)/k}$.

P17-9 (a) Apply conservation of energy. When $x = x_\mathrm{m}$ $v = 0$, so

$$\begin{aligned}
\frac{1}{2}k{x_\mathrm{m}}^2 &= \frac{1}{2}m[v(0)]^2 + \frac{1}{2}k[x(0)]^2,\\
{x_\mathrm{m}}^2 &= \frac{m}{k}[v(0)]^2 + [x(0)]^2,\\
x_\mathrm{m} &= \sqrt{[v(0)/\omega]^2 + [x(0)]^2}.
\end{aligned}$$

(b) When $t = 0$ $x(0) = x_\mathrm{m}\cos\phi$ and $v(0) = -\omega x_\mathrm{m}\sin\phi$, so

$$\frac{v(0)}{\omega x(0)} = -\frac{\sin\phi}{\cos\phi} = \tan\phi.$$

P17-10

$\boxed{\textbf{P17-11}}$ Conservation of momentum for the bullet block collision gives $mv = (m+M)v_\mathrm{f}$ or

$$v_\mathrm{f} = \frac{m}{m+M}v.$$

This v_f will be equal to the maximum oscillation speed v_m. The angular frequency for the oscillation is given by

$$\omega = \sqrt{\frac{k}{m+M}}.$$

Then the amplitude for the oscillation is

$$x_\mathrm{m} = \frac{v_\mathrm{m}}{\omega} = v\frac{m}{m+M}\sqrt{\frac{m+M}{k}} = \frac{mv}{\sqrt{k(m+M)}}.$$

P17-12 (a) $W = F_\mathrm{s}$, or $mg = kx$, so $x = mg/k$.

(b) $F = ma$, but $F = W - F_\mathrm{s} = mg - kx$, and since $ma = m\,d^2x/dt^2$,

$$m\frac{d^2x}{dt^2} + kx = mg.$$

The solution can be verified by direct substitution.

(c) Just look at the answer!

(d) dE/dt is

$$\begin{aligned}
mv\frac{dv}{dt} + kx\frac{dx}{dt} - mg\frac{dx}{dt} &= 0,\\
m\frac{dv}{dt} + kx &= mg.
\end{aligned}$$

P17-13 The initial energy stored in the spring is $k{x_\mathrm{m}}^2/2$. When the cylinder passes through the equilibrium point it has a translational velocity v_m and a rotational velocity $\omega_\mathrm{r} = v_\mathrm{m}/R$, where R is the radius of the cylinder. The total kinetic energy at the equilibrium point is

$$\frac{1}{2}m{v_\mathrm{m}}^2 + \frac{1}{2}I{\omega_\mathrm{r}}^2 = \frac{1}{2}\left(m + \frac{1}{2}m\right){v_\mathrm{m}}^2.$$

Then the kinetic energy is 2/3 translational and 1/3 rotational. The total energy of the system is

$$E = \frac{1}{2}(294\,\mathrm{N/\,m})(0.239\,\mathrm{m})^2 = 8.40\,\mathrm{J}.$$

(a) $K_t = (2/3)(8.40\,\mathrm{J}) = 5.60\,\mathrm{J}$.
(b) $K_r = (1/3)(8.40\,\mathrm{J}) = 2.80\,\mathrm{J}$.
(c) The energy expression is

$$\frac{1}{2}\left(\frac{3m}{2}\right)v^2 + \frac{1}{2}kx^2 = E,$$

which leads to a standard expression for the period with $3M/2$ replacing m. Then $T = 2\pi\sqrt{3M/2k}$.

P17-14 (a) Integrate the potential energy expression over one complete period and then divide by the time for one period:

$$\begin{aligned}
\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1}{2}kx^2\,dt &= \frac{k\omega}{4\pi}\int_0^{2\pi/\omega}{x_\mathrm{m}}^2\cos^2\omega t\,dt,\\
&= \frac{k\omega}{4\pi}{x_\mathrm{m}}^2\frac{\pi}{\omega},\\
&= \frac{1}{4}k{x_\mathrm{m}}^2.
\end{aligned}$$

This is half the total energy; since the average total energy is E, then the average kinetic energy must be the other half of the average total energy, or $(1/4)k{x_\mathrm{m}}^2$.

(b) Integrate over half a cycle and divide by twice the amplitude.

$$\begin{aligned}
\frac{1}{2x_\mathrm{m}}\int_{-x_\mathrm{m}}^{x_\mathrm{m}}\frac{1}{2}kx^2\,dx &= \frac{1}{2x_\mathrm{m}}\frac{1}{3}k{x_\mathrm{m}}^3,\\
&= \frac{1}{6}k{x_\mathrm{m}}^2.
\end{aligned}$$

This is one-third the total energy. The average kinetic energy must be two-thirds the total energy, or $(1/3)k{x_\mathrm{m}}^2$.

P17-15 The rotational inertia is

$$I = \frac{1}{2}MR^2 + Md^2 = M\left(\frac{1}{2}(0.144\,\mathrm{m})^2 + (0.102\,\mathrm{m})^2\right) = (2.08\times10^{-2}\mathrm{m}^2)M.$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{(2.08\times10^{-2}\mathrm{m}^2)M}{M(9.81\,\mathrm{m/s^2})(0.102\,\mathrm{m})}} = 0.906\,\mathrm{s}.$$

P17-16 (a) The rotational inertia of the pendulum about the pivot is

$$(0.488\,\mathrm{kg})\left(\frac{1}{2}(0.103\,\mathrm{m})^2 + (0.103\,\mathrm{m} + 0.524\,\mathrm{m})^2\right) + \frac{1}{3}(0.272\,\mathrm{kg})(0.524\,\mathrm{m})^2 = 0.219\,\mathrm{kg}\cdot\mathrm{m}^2.$$

(b) The center of mass location is

$$d = \frac{(0.524\,\mathrm{m})(0.272\,\mathrm{kg})/2 + (0.103\,\mathrm{m} + 0.524\,\mathrm{m})(0.488\,\mathrm{kg})}{(0.272\,\mathrm{kg}) + (0.488\,\mathrm{kg})} = 0.496\,\mathrm{m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{(0.291\,\mathrm{kg}\cdot\mathrm{m}^2)/(0.272\,\mathrm{kg} + 0.488\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.496\,\mathrm{m})} = 1.76\,\mathrm{s}.$$

$\boxed{\textbf{P17-17}}$ (a) The rotational inertia of a stick about an axis through a point which is a distance d from the center of mass is given by the parallel axis theorem,

$$I = I_{\rm cm} + md^2 = \frac{1}{12}mL^2 + md^2.$$

The period of oscillation is given by Eq. 17-28,

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}$$

(b) We want to find the minimum period, so we need to take the derivative of T with respect to d. It'll look weird, but

$$\frac{dT}{dd} = \pi\frac{12d^2 - L^2}{\sqrt{12gd^3(L^2+12d^2)}}.$$

This will vanish when $12d^2 = L^2$, or when $d = L/\sqrt{12}$.

P17-18 The energy stored in the spring is given by $kx^2/2$, the kinetic energy of the rotating wheel is

$$\frac{1}{2}(MR^2)\left(\frac{v}{r}\right)^2,$$

where v is the tangential velocity of the point of attachment of the spring to the wheel. If $x = x_{\rm m}\sin\omega t$, then $v = x_{\rm m}\omega\cos\omega t$, and the energy will only be constant if

$$\omega^2 = \frac{k}{M}\frac{r^2}{R^2}.$$

P17-19 The method of solution is identical to the approach for the simple pendulum on page 381 *except* replace the tension with the normal force of the bowl on the particle. The effective pendulum with have a length R.

P17-20 Let x be the distance from the center of mass to the first pivot point. Then the period is given by

$$T = 2\pi\sqrt{\frac{I + Mx^2}{Mgx}}.$$

Solve this for x by expressing the above equation as a quadratic:

$$\left(\frac{MgT^2}{4\pi^2}\right)x = I + Mx^2,$$

or

$$Mx^2 - \left(\frac{MgT^2}{4\pi^2}\right)x + I = 0$$

There are *two* solutions. One corresponds to the first location, the other the second location. Adding the two solutions together will yield L; in this case the discriminant of the quadratic will drop out, leaving

$$L = x_1 + x_2 = \frac{MgT^2}{M4\pi^2} = \frac{gT^2}{4\pi^2}.$$

Then $g = 4\pi^2 L/T^2$.

P17-21 In this problem

$$I = (2.50\,\text{kg})\left(\frac{(0.210\,\text{m})^2}{2} + (0.760\,\text{m} + 0.210\,\text{m})^2\right) = 2.41\,\text{kg}/\cdot\text{m}^2.$$

The center of mass is at the center of the disk.

(a) $T = 2\pi\sqrt{(2.41\,\text{kg}/\cdot\text{m}^2)/(2.50\,\text{kg})(9.81\,\text{m/s}^2)(0.760\,\text{m} + 0.210\,\text{m})} = 2.00\,\text{s}.$

(b) Replace Mgd with $Mgd + \kappa$ and $2.00\,\text{s}$ with $1.50\,\text{s}$. Then

$$\kappa = \frac{4\pi^2(2.41\,\text{kg}/\cdot\text{m}^2)}{(1.50\,\text{s})^2} - (2.50\,\text{kg})(9.81\,\text{m/s}^2)(0.760\,\text{m} + 0.210\,\text{m}) = 18.5\,\text{N}\cdot\text{m/rad}.$$

P17-22 The net force on the bob is toward the center of the circle, and has magnitude $F_{\text{net}} = mv^2/R$. This net force comes from the horizontal component of the tension. There is also a vertical component of the tension of magnitude mg. The tension then has magnitude

$$T = \sqrt{(mg)^2 + (mv^2/R)^2} = m\sqrt{g^2 + v^4/R^2}.$$

It is this tension which is important in finding the restoring force in Eq. 17-22; in effect we want to replace g with $\sqrt{g^2 + v^4/R^2}$ in Eq. 17-24. The frequency will then be

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{\sqrt{g^2 + v^4/R^2}}}.$$

$\boxed{\textbf{P17-23}}$ (a) Consider an object of mass m at a point P on the axis of the ring. It experiences a gravitational force of attraction to all points on the ring; by symmetry, however, the net force is not directed toward the circumference of the ring, but instead along the axis of the ring. There is then a factor of $\cos\theta$ which will be thrown in to the mix.

The distance from P to *any* point on the ring is $r = \sqrt{R^2 + z^2}$, and θ is the angle between the axis on the line which connects P and *any* point on the circumference. Consequently,

$$\cos\theta = z/r,$$

and then the net force on the star of mass m at P is

$$F = \frac{GMm}{r^2}\cos\theta = \frac{GMmz}{r^3} = \frac{GMmz}{(R^2 + z^2)^{3/2}}.$$

(b) If $z \ll R$ we can apply the binomial expansion to the denominator, and

$$(R^2 + z^2)^{-3/2} = R^{-3}\left(1 + \left(\frac{z}{R}\right)^2\right)^{-3/2} \approx R^{-3}\left(1 - \frac{3}{2}\left(\frac{z}{R}\right)^2\right).$$

Keeping terms only linear in z we have

$$F = \frac{GMm}{R^3}z,$$

which corresponds to a spring constant $k = GMm/R^3$. The frequency of oscillation is then

$$f = \sqrt{k/m}/(2\pi) = \sqrt{GM/R^3}/(2\pi).$$

(c) Using some numbers from the Milky Way galaxy,

$$f = \sqrt{(7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2\times10^{43}\text{kg})/(6\times10^{19}\text{m})^3}/(2\pi) = 1\times10^{-14}\ \text{Hz}.$$

P17-24 (a) The acceleration of the center of mass (point C) is $a = F/M$. The torque about an axis through the center of mass is $tau = FR/2$, since O is $R/2$ away from the center of mass. The angular acceleration of the disk is then

$$\alpha = \tau/I = (FR/2)/(MR^2/2) = F/(MR).$$

Note that the angular acceleration will tend to rotate the disk anti-clockwise. The tangential component to the angular acceleration at P is $a_{\mathrm{T}} = -\alpha R = -F/M$; this is exactly the opposite of the linear acceleration, so P will not (initially) accelerate.

(b) There is no net force at P.

P17-25 The value for k is closest to

$$k \approx (2000\,\mathrm{kg}/4)(9.81\,\mathrm{m/s^2})/(0.10\,\mathrm{m}) = 4.9\times10^4\mathrm{N/m}.$$

One complete oscillation requires a time $t_1 = 2\pi/\omega'$. The amplitude decays to 1/2 the original value in this time, so $0.5 = e^{-bt_1/2m}$, or

$$\ln(2) = \frac{2\pi b}{2m\omega'}.$$

It is *not* reasonable at this time to assume that $b/2m$ is small compared to ω so that $\omega' \approx \omega$. Then

$$\begin{aligned}
\omega'^2 &= \left(\frac{2\pi}{\ln(2)}\right)^2 \left(\frac{b}{2m}\right)^2, \\
\frac{k}{m} - \left(\frac{b}{2m}\right)^2 &= \left(\frac{2\pi}{\ln(2)}\right)^2 \left(\frac{b}{2m}\right)^2, \\
\frac{k}{m} &= (81.2)\left(\frac{b}{2m}\right)^2
\end{aligned}$$

Then the value for b is

$$b = \sqrt{\frac{4(2000\,\mathrm{kg}/4)(4.9\times10^4\mathrm{N/m})}{(81.2)}} = 1100\,\mathrm{kg/s}.$$

P17-26 $a = d^2x/dt^2 = -A\omega^2 \cos\omega t$. Substituting into the non-linear equation,

$$-mA\omega^2 \cos\omega t + kA^3 \cos^3\omega t = F\cos\omega_{\mathrm{d}} t.$$

Now let $\omega_{\mathrm{d}} = 3\omega$. Then

$$kA^3 \cos^3\omega t - mA\omega^2 \cos\omega t = F\cos 3\omega t$$

Expand the right hand side as $\cos 3\omega t = 4\cos^3\omega t - 3\cos\omega t$, then

$$kA^3 \cos^3\omega t - mA\omega^2 \cos\omega t = F(4\cos^3\omega t - 3\cos\omega t)$$

This will only work if $4F = kA^3$ and $3F = mA\omega^2$. Dividing one condition by the other means $4mA\omega^2 = kA^3$, so $A \propto \omega$ and then $F \propto \omega^3 \propto \omega_{\mathrm{d}}{}^3$.

P17-27 (a) Divide the top and the bottom by m_2. Then

$$\frac{m_1 m_2}{m_1 + m_2} = \frac{m_1}{(m_1/m_2) + 1},$$

and in the limit as $m_2 \to \infty$ the value of $(m_1/m_2) \to 0$, so

$$\lim_{m_2 \to \infty} \frac{m_1 m_2}{m_1 + m_2} = \lim_{m_2 \to \infty} \frac{m_1}{(m_1/m_2) + 1} = m_1.$$

(b) m is called the reduced mass because it is always less than either m_1 or m_2. Think about it in terms of

$$m = \frac{m_1}{(m_1/m_2) + 1} = \frac{m_2}{(m_2/m_1) + 1}.$$

Since mass is always positive, the denominator is always greater than or equal to 1. Equality only occurs if one of the masses is infinite. Now $\omega = \sqrt{k/m}$, and since m is always less than m_1, so the existence of a finite wall will cause ω to be larger, and the period to be smaller.

(c) If the bodies have equal mass then $m = m_1/2$. This corresponds to a value of $\omega = \sqrt{2k/m_1}$. In effect, the spring constant is doubled, which is what happens if a spring is cut in half.

E18-1 (a) $f = v/\lambda = (243\,\text{m/s})/(0.0327\,\text{m}) = 7.43\times 10^3$ Hz.
(b) $T = 1/f = 1.35\times 10^{-4}$s.

E18-2 (a) $f = (12)/(30\,\text{s}) = 0.40$ Hz.
(b) $v = (15\,\text{m})/(5.0\,\text{s}) = 3.0\,\text{m/s}$.
(c) $\lambda = v/f = (3.0\,\text{m/s})/(0.40\text{ Hz}) = 7.5\,\text{m}$.

E18-3 (a) The time for a particular point to move from maximum displacement to zero displacement is one-quarter of a period; the point must then go to maximum negative displacement, zero displacement, and finally maximum positive displacement to complete a cycle. So the period is 4(178 ms) = 712 ms.
(b) The frequency is $f = 1/T = 1/(712\times 10^{-3}\text{s}) = 1.40$ Hz.
(c) The wave-speed is $v = f\lambda = (1.40\text{ Hz})(1.38\,\text{m}) = 1.93\,\text{m/s}$.

E18-4 Use Eq. 18-9, except let $f = 1/T$:

$$y = (0.0213\,\text{m})\sin 2\pi\left(\frac{x}{(0.114\,\text{m})} - (385\text{ Hz})t\right) = (0.0213\,\text{m})\sin\left[(55.1\text{ rad/m})x - (2420\text{ rad/s})t\right].$$

E18-5 The dimensions for tension are $[\mathrm{F}] = [\mathrm{M}][\mathrm{L}]/[\mathrm{T}]^2$ where M stands for mass, L for length, T for time, and F stands for force. The dimensions for linear mass density are [M]/[L]. The dimensions for velocity are [L]/[T].

Inserting this into the expression $v = F^a/\mu^b$,

$$\begin{aligned}\frac{[\mathrm{L}]}{[\mathrm{T}]} &= \left(\frac{[\mathrm{M}][\mathrm{L}]}{[\mathrm{T}]^2}\right)^{\mathrm{a}} \Big/ \left(\frac{[\mathrm{M}]}{[\mathrm{L}]}\right)^{\mathrm{b}},\\ \frac{[\mathrm{L}]}{[\mathrm{T}]} &= \frac{[\mathrm{M}]^{\mathrm{a}}[\mathrm{L}]^{\mathrm{a}}}{[\mathrm{T}]^{2\mathrm{a}}}\frac{[\mathrm{L}]^{\mathrm{b}}}{[\mathrm{M}]^{\mathrm{b}}},\\ \frac{[\mathrm{L}]}{[\mathrm{T}]} &= \frac{[\mathrm{M}]^{\mathrm{a-b}}[\mathrm{L}]^{\mathrm{a+b}}}{[\mathrm{T}]^{2\mathrm{a}}}\end{aligned}$$

There are three equations here. One for time, $-1 = -2a$; one for length, $1 = a + b$; and one for mass, $0 = a - b$. We need to satisfy all three equations. The first is fairly quick; $a = 1/2$. Either of the other equations can be used to show that $b = 1/2$.

E18-6 (a) $y_{\mathrm{m}} = 2.30\,\text{mm}$.
(b) $f = (588\text{ rad/s})/(2\pi\text{ rad}) = 93.6$ Hz.
(c) $v = (588\text{ rad/s})/(1822\text{ rad/m}) = 0.323\,\text{m/s}$.
(d) $\lambda = (2\pi\text{ rad})/(1822\text{ rad/m}) = 3.45\,\text{mm}$.
(e) $u_y = y_{\mathrm{m}}\omega = (2.30\,\text{mm})(588\text{ rad/s}) = 1.35\,\text{m/s}$.

E18-7 (a) $y_{\mathrm{m}} = 0.060\,\text{m}$.
(b) $\lambda = (2\pi\text{ rad})/(2.0\pi\text{ rad/m}) = 1.0\,\text{m}$.
(c) $f = (4.0\pi\text{ rad/s})/(2\pi\text{ rad}) = 2.0$ Hz.
(d) $v = (4.0\pi\text{ rad/s})/(2.0\pi\text{ rad/m}) = 2.0\,\text{m/s}$.
(e) Since the second term is positive the wave is moving in the $-x$ direction.
(f) $u_y = y_{\mathrm{m}}\omega = (0.060\,\text{m})(4.0\pi\text{ rad/s}) = 0.75\,\text{m/s}$.

E18-8 $v = \sqrt{F/\mu} = \sqrt{(487\,\text{N})/[(0.0625\,\text{kg})/(2.15\,\text{m})]} = 129\,\text{m/s}$.

$\boxed{\textbf{E18-9}}$ We'll first find the linear mass density by rearranging Eq. 18-19,

$$\mu = \frac{F}{v^2}$$

Since this is the same string, we expect that changing the tension will not significantly change the linear mass density. Then for the two different instances,

$$\frac{F_1}{v_1^2} = \frac{F_2}{v_2^2}$$

We want to know the new tension, so

$$F_2 = F_1\frac{v_2^2}{v_1^2} = (123\,\mathrm{N})\frac{(180\,\mathrm{m/s})^2}{(172\,\mathrm{m/s})^2} = 135\,\mathrm{N}$$

E18-10 First $v = (317\ \mathrm{rad/s})/(23.8\ \mathrm{rad/m}) = 13.32\,\mathrm{m/s}$. Then

$$\mu = F/v^2 = (16.3\,\mathrm{N})/(13.32\,\mathrm{m/s})^2 = 0.0919\,\mathrm{kg/m}.$$

E18-11 (a) $y_m = 0.05\,\mathrm{m}$.

(b) $\lambda = (0.55\,\mathrm{m}) - (0.15\,\mathrm{m}) = 0.40\,\mathrm{m}$.

(c) $v = \sqrt{F/\mu} = \sqrt{(3.6\,\mathrm{N})/(0.025\,\mathrm{kg/m})} = 12\,\mathrm{m/s}$.

(d) $T = 1/f = \lambda/v = (0.40\,\mathrm{m})/(12\,\mathrm{m/s}) = 3.33\times10^{-2}\mathrm{s}$.

(e) $u_y = y_\mathrm{m}\omega = 2\pi y_\mathrm{m}/T = 2\pi(0.05\,\mathrm{m})/(3.33\times10^{-2}\mathrm{s}) = 9.4\,\mathrm{m/s}$.

(f) $k = (2\pi\ \mathrm{rad})/(0.40\,\mathrm{m}) = 5.0\pi\ \mathrm{rad/m}$; $\omega = kv = (5.0\pi\ \mathrm{rad/m})(12\,\mathrm{m/s}) = 60\pi\ \mathrm{rad/s}$. The phase angle can be found from

$$(0.04\,\mathrm{m}) = (0.05\,\mathrm{m})\sin(\phi),$$

or $\phi = 0.93$ rad. Then

$$y = (0.05\,\mathrm{m})\sin[(5.0\pi\ \mathrm{rad/m})x + (60\pi\ \mathrm{rad/s})t + (0.93\ \mathrm{rad})].$$

E18-12 (a) The tensions in the two strings are equal, so $F = (0.511\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/2 = 2.506\,\mathrm{N}$. The wave speed in string 1 is

$$v = \sqrt{F/\mu} = \sqrt{(2.506\,\mathrm{N})/(3.31\times10^{-3}\mathrm{kg/m})} = 27.5\,\mathrm{m/s},$$

while the wave speed in string 2 is

$$v = \sqrt{F/\mu} = \sqrt{(2.506\,\mathrm{N})/(4.87\times10^{-3}\mathrm{kg/m})} = 22.7\,\mathrm{m/s}.$$

(b) We have $\sqrt{F_1/\mu_1} = \sqrt{F_2/\mu_2}$, or $F_1/\mu_1 = F_2/\mu_2$. But $F_i = M_i g$, so $M_1/\mu_1 = M_2/\mu_2$. Using $M = M_1 + M_2$,

$$\begin{aligned}
\frac{M_1}{\mu_1} &= \frac{M - M_1}{\mu_2},\\
\frac{M_1}{\mu_1} + \frac{M_1}{\mu_2} &= \frac{M}{\mu_2},\\
M_1 &= \frac{M}{\mu_2}\bigg/\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right),\\
&= \frac{(0.511\,\mathrm{kg})}{(4.87\times10^{-3}\mathrm{kg/m})}\bigg/\left(\frac{1}{(3.31\times10^{-3}\mathrm{kg/m})} + \frac{1}{(4.87\times10^{-3}\mathrm{kg/m})}\right),\\
&= 0.207\,\mathrm{kg}
\end{aligned}$$

and $M_2 = (0.511\,\mathrm{kg}) - (0.207\,\mathrm{kg}) = 0.304\,\mathrm{kg}$.

E18-13 We need to know the wave speed before we do anything else. This is found from Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{(248\,\text{N})}{(0.0978\,\text{kg})/(10.3\,\text{m})}} = 162\,\text{m/s}.$$

The two pulses travel in opposite directions on the wire; one travels as distance x_1 in a time t, the other travels a distance x_2 in a time $t + 29.6$ ms, and since the pulses meet, we have $x_1 + x_2 = 10.3$ m.

Our equations are then $x_1 = vt = (162\,\text{m/s})t$, and $x_2 = v(t+29.6\text{ ms}) = (162\,\text{m/s})(t+29.6\text{ ms}) = (162\,\text{m/s})t + 4.80\,\text{m}$. We can add these two expressions together to solve for the time t at which the pulses meet,

$$10.3\,\text{m} = x_1 + x_2 = (162\,\text{m/s})t + (162\,\text{m/s})t + 4.80\,\text{m} = (324\,\text{m/s})t + 4.80\,\text{m}.$$

which has solution $t = 0.0170\,\text{s}$. The two pulses meet at $x_1 = (162\,\text{m/s})(0.0170\,\text{s}) = 2.75\,\text{m}$, or $x_2 = 7.55\,\text{m}$.

E18-14 (a) $\partial y/\partial r = (A/r)k\cos(kr - \omega t) - (A/r^2)\sin(kr - \omega t)$. Multiply this by r^2, and then find

$$\frac{\partial}{\partial r} r^2 \frac{\partial y}{\partial r} = Ak\cos(kr - \omega t) - Ak^2 r\sin(kr - \omega t) - Ak\cos(kr - \omega t).$$

Simplify, and then divide by r^2 to get

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial y}{\partial r} = -(Ak^2/r)\sin(kr - \omega t).$$

Now find $\partial^2 y/\partial t^2 = -A\omega^2\sin(kr - \omega t)$. But since $1/v^2 = k^2/\omega^2$, the two sides are equal.

(b) [length]^2.

E18-15 The liner mass density is $\mu = (0.263\,\text{kg})/(2.72\,\text{m}) = 9.669\times10^{-2}\text{kg/m}$. The wave speed is $v = \sqrt{(36.1\,\text{N})/(9.669\times10^{-2}\text{kg/m})} = 19.32\,\text{m/s}$.

$P_{\text{av}} = \frac{1}{2}\mu\omega^2 {y_{\text{m}}}^2 v$, so

$$\omega = \sqrt{\frac{2(85.5\,\text{W})}{(9.669\times10^{-2}\text{kg/m})(7.70\times10^{-3}\text{m})^2(19.32\,\text{m/s})}} = 1243\text{ rad/s}.$$

Then $f = (1243\text{ rad/s})/2\pi = 199$ Hz.

E18-16 (a) If the medium absorbs no energy then the power flow through any closed surface which contains the source must be constant. Since for a cylindrical surface the area grows as r, then intensity must fall of as $1/r$.

(b) Intensity is proportional to the amplitude squared, so the amplitude must fall off as $1/\sqrt{r}$.

E18-17 The intensity is the average power per unit area (Eq. 18-33); as you get farther from the source the intensity falls off because the perpendicular area increases. At some distance r from the source the total possible area is the area of a spherical shell of radius r, so intensity as a function of distance from the source would be

$$I = \frac{P_{\text{av}}}{4\pi r^2}$$

We are given two intensities: $I_1 = 1.13\,\mathrm{W/m^2}$ at a distance r_1; $I_2 = 2.41\,\mathrm{W/m^2}$ at a distance $r_2 = r_1 - 5.30\,\mathrm{m}$. Since the average power of the source is the same in both cases we can equate these two values as

$$\begin{aligned} 4\pi r_1^2 I_1 &= 4\pi r_2^2 I_2, \\ 4\pi r_1^2 I_1 &= 4\pi (r_1 - d)^2 I_2, \end{aligned}$$

where $d = 5.30\,\mathrm{m}$, and then solve for r_1. Doing this we find a quadratic expression which is

$$\begin{aligned} r_1^2 I_1 &= (r_1^2 - 2dr_1 + d^2) I_2, \\ 0 &= \left(1 - \frac{I_1}{I_2}\right) r_1^2 - 2dr_1 + d^2, \\ 0 &= \left(1 - \frac{(1.13\,\mathrm{W/m^2})}{(2.41\,\mathrm{W/m^2})}\right) r_1^2 - 2(5.30\,\mathrm{m})r_1 + (5.30\,\mathrm{m})^2, \\ 0 &= (0.531)r_1^2 - (10.6\,\mathrm{m})r_1 + (28.1\,\mathrm{m^2}). \end{aligned}$$

The solutions to this are $r_1 = 16.8\,\mathrm{m}$ and $r_1 = 3.15\,\mathrm{m}$; but since the person walked 5.3 m toward the lamp we will assume they started at least that far away. Then the power output from the light is

$$P = 4\pi r_1^2 I_1 = 4\pi (16.8\,\mathrm{m})^2 (1.13\,\mathrm{W/m^2}) = 4.01 \times 10^3\mathrm{W}.$$

E18-18 Energy density is energy per volume, or $u = U/V$. A wave front of cross sectional area A sweeps out a volume of $V = Al$ when it travels a distance l. The wave front travels that distance l in a time $t = l/v$. The energy flow per time is the power, or $P = U/t$. Combine this with the definition of intensity, $I = P/A$, and

$$I = \frac{P}{A} = \frac{U}{At} = \frac{uV}{At} = \frac{uAl}{At} = uv.$$

E18-19 Refer to Eq. 18-40, where the amplitude of the combined wave is

$$2y_\mathrm{m}\cos(\Delta\phi/2),$$

where y_m is the amplitude of the combining waves. Then

$$\cos(\Delta\phi/2) = (1.65 y_\mathrm{m})/(2y_\mathrm{m}) = 0.825,$$

which has solution $\Delta\phi = 68.8°$.

E18-20 Consider only the point $x = 0$. The displacement y at that point is given by

$$y = y_{\mathrm{m}1}\sin(\omega t) + y_{\mathrm{m}2}\sin(\omega t + \pi/2) = y_{\mathrm{m}1}\sin(\omega t) + y_{\mathrm{m}2}\cos(\omega t).$$

This can be written as

$$y = y_\mathrm{m}(A_1 \sin\omega t + A_2\cos\omega t),$$

where $A_i = y_{\mathrm{m}i}/y_\mathrm{m}$. But if y_m is judiciously chosen, $A_1 = \cos\beta$ and $A_2 = \sin\beta$, so that

$$y = y_\mathrm{m}\sin(\omega t + \beta).$$

Since we then require $A_1^2 + A_2^2 = 1$, we must have

$$y_\mathrm{m} = \sqrt{(3.20\text{ cm})^2 + (4.19\text{ cm})^2} = 5.27\text{ cm}.$$

E18-21 The easiest approach is to use a phasor representation of the waves.

Write the phasor components as

$$\begin{aligned} x_1 &= ym1 \cos\phi_1, \\ y_1 &= ym1 \sin\phi_1, \\ x_2 &= ym2 \cos\phi_2, \\ y_2 &= ym2 \sin\phi_2, \end{aligned}$$

and then use the cosine law to find the magnitude of the resultant.

The phase angle can be found from the arcsine of the opposite over the hypotenuse.

E18-22 (a) The diagrams for all times except $t = 15\,\text{ms}$ should show two distinct pulses, first moving closer together, then moving farther apart. The pulses don not flip over when passing each other. The $t = 15\,\text{ms}$ diagram, however, should simply be a flat line.

E18-23 Use a program such as Maple or Mathematica to plot this.

E18-24 Use a program such as Maple or Mathematica to plot this.

$\boxed{\textbf{E18-25}}$ (a) The wave speed can be found from Eq. 18-19; we need to know the linear mass density, which is $\mu = m/L = (0.122\,\text{kg})/(8.36\,\text{m}) = 0.0146\,\text{kg/m}$. The wave speed is then given by

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{(96.7\,\text{N})}{(0.0146\,\text{kg/m})}} = 81.4\,\text{m/s}.$$

(b) The longest possible standing wave will be twice the length of the string; so $\lambda = 2L = 16.7\,\text{m}$.

(c) The frequency of the wave is found from Eq. 18-13, $v = f\lambda$.

$$f = \frac{v}{\lambda} = \frac{(81.4\,\text{m/s})}{(16.7\,\text{m})} = 4.87\ \text{Hz}$$

E18-26 (a) $v = \sqrt{(152\,\text{N})/(7.16\times10^{-3}\text{kg/m})} = 146\,\text{m/s}$.

(b) $\lambda = (2/3)(0.894\,\text{m}) = 0.596\,\text{m}$.

(c) $f = v/\lambda = (146\,\text{m/s})/(0.596\,\text{m}) = 245$ Hz.

E18-27 (a) $y = -3.9$ cm.

(b) $y = (0.15\,\text{m})\sin[(0.79\ \text{rad/m})x + (13\ \text{rad/s})t]$.

(c) $y = 2(0.15\,\text{m})\sin[(0.79\ \text{rad/m})(2.3\,\text{m})]\cos[(13\ \text{rad/s})(0.16\,\text{s})] = -0.14\,\text{m}$.

E18-28 (a) The amplitude is half of 0.520 cm, or 2.60 mm. The speed is

$$v = (137\ \text{rad/s})/(1.14\ \text{rad/cm}) = 1.20\,\text{m/s}.$$

(b) The nodes are $(\pi\text{rad})/(1.14\ \text{rad/cm}) = 2.76$ cm apart.

(c) The velocity of a particle on the string at position x and time t is the derivative of the wave equation with respect to time, or

$$u_y = -(0.520\ \text{cm})(137\ \text{rad/s})\sin[(1.14\ \text{rad/cm})(1.47\ \text{cm})]\sin[(137\ \text{rad/s})(1.36\,\text{s})] = -0.582\,\text{m/s}.$$

$\boxed{\textbf{E18-29}}$ (a) We are given the wave frequency and the wave-speed, the wavelength is found from Eq. 18-13,

$$\lambda = \frac{v}{f} = \frac{(388\,\mathrm{m/s})}{(622\ \mathrm{Hz})} = 0.624\,\mathrm{m}$$

The standing wave has four loops, so from Eq. 18-45

$$L = n\frac{\lambda}{2} = (4)\frac{(0.624\,\mathrm{m})}{2} = 1.25\,\mathrm{m}$$

is the length of the string.

(b) We can just write it down,

$$y = (1.90\ \mathrm{mm})\sin[(2\pi/0.624\,\mathrm{m})x]\cos[(2\pi 622\,\mathrm{s}^{-1})t].$$

E18-30 (a) $f_n = nv/2L = (1)(250\,\mathrm{m/s})/2(0.150\,\mathrm{m}) = 833$ Hz.

(b) $\lambda = v/f = (348\,\mathrm{m/s})/(833\ \mathrm{Hz}) = 0.418\,\mathrm{m}$.

E18-31 $v = \sqrt{F/\mu} = \sqrt{FL/m}$. Then $f_n = nv/2L = n\sqrt{F/4mL}$, so

$$f_1 = (1)\sqrt{(236\,\mathrm{N})/4(0.107\,\mathrm{kg})(9.88\,\mathrm{m})}7.47\ \mathrm{Hz},$$

and $f_2 = 2f_1 = 14.9$ Hz while $f_3 = 3f_1 = 22.4$ Hz.

E18-32 (a) $v = \sqrt{F/\mu} = \sqrt{FL/m} = \sqrt{(122\,\mathrm{N})(1.48\,\mathrm{m})/(8.62\times10^{-3}\mathrm{kg})} = 145\,\mathrm{m/s}$.

(b) $\lambda_1 = 2(1.48\,\mathrm{m}) = 2.96\,\mathrm{m}$; $\lambda_2 = 1.48\,\mathrm{m}$.

(c) $f_1 = (145\,\mathrm{m/s})/(2.96\,\mathrm{m}) = 49.0$ Hz; $f_2 = (145\,\mathrm{m/s})/(1.48\,\mathrm{m}) = 98.0$ Hz.

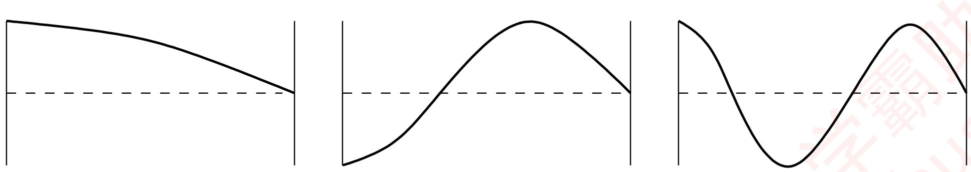
$\boxed{\textbf{E18-33}}$ Although the tied end of the string forces it to be a node, the fact that the other end is loose means that it should be an anti-node. The discussion of Section 18-10 indicated that the spacing between nodes is always $\lambda/2$. Since anti-nodes occur between nodes, we can expect that the distance between a node and the nearest anti-node is $\lambda/4$.

The longest possible wavelength will have one node at the tied end, an anti-node at the loose end, and no other nodes or anti-nodes. In this case $\lambda/4 = 120$ cm, or $\lambda = 480$ cm.

The next longest wavelength will have a node somewhere in the middle region of the string. But this means that there must be an anti-node between this new node and the node at the tied end of the string. Moving from left to right, we then have an anti-node at the loose end, a node, and anti-node, and finally a node at the tied end. There are four points, each separated by $\lambda/4$, so the wavelength would be given by $3\lambda/4 = 120$ cm, or $\lambda = 160$ cm.

To progress to the next wavelength we will add another node, and another anti-node. This will add another two lengths of $\lambda/4$ that need to be fit onto the string; hence $5\lambda/4 = 120$ cm, or $\lambda = 100$ cm.

In the figure below we have sketched the first three standing waves.



E18-34 (a) Note that $f_n = nf_1$. Then $f_{n+1} - f_n = f_1$. Since there is no resonant frequency between these two then they must differ by 1, and consequently $f_1 = (420\text{ Hz}) - (315\text{ Hz}) = 105\text{ Hz}$.

(b) $v = f\lambda = (105\text{ Hz})[2(0.756\,\text{m})] = 159\,\text{m/s}$.

P18-1 (a) $\lambda = v/f$ and $k = 360°/\lambda$. Then

$$x = (55°)\lambda/(360°) = 55(353\,\text{m/s})/360(493\text{ Hz}) = 0.109\,\text{m}.$$

(b) $\omega = 360° f$, so

$$\phi = \omega t = (360°)(493\text{ Hz})(1.12\times10^{-3}\text{s}) = 199°.$$

P18-2 $\omega = (2\pi\text{ rad})(548\text{ Hz}) = 3440\text{ rad/s}$; $\lambda = v/f$ and then

$$k = (2\pi\text{ rad})/[(326\,\text{m/s})/(548\text{ Hz})] = 10.6\text{ rad/m}.$$

Finally, $y = (1.12\times10^{-2}\text{m})\sin[(10.6\text{ rad/m})x + (3440\text{ rad/s})t]$.

$\boxed{\textbf{P18-3}}$ (a) This problem really isn't as bad as it might look. The tensile stress S is tension per unit cross sectional area, so

$$S = \frac{F}{A}\text{ or }F = SA.$$

We already know that linear mass density is $\mu = m/L$, where L is the length of the wire. Substituting into Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{SA}{m/L}}\sqrt{\frac{S}{m/(AL)}}.$$

But AL is the volume of the wire, so the denominator is just the mass density ρ.

(b) The maximum speed of the transverse wave will be

$$v = \sqrt{\frac{S}{\rho}} = \sqrt{\frac{(720\times10^6\text{ Pa})}{(7800\text{ kg/m}^3)}} = 300\,\text{m/s}.$$

P18-4 (a) $f = \omega/2\pi = (4.08\text{ rad/s})/(2\pi\text{ rad}) = 0.649\text{ Hz}$.

(b) $\lambda = v/f = (0.826\,\text{m/s})/(0.649\text{ Hz}) = 1.27\,\text{m}$.

(c) $k = (2\pi\text{ rad})/(1.27\,\text{m}) = 4.95\text{ rad/m}$, so

$$y = (5.12\text{ cm})\sin[(4.95\text{ rad/m}x - (4.08\text{ rad/s})t + \phi],$$

where ϕ is determined by $(4.95\text{ rad/m})(9.60\times10^{-2}\text{m}) + \phi = (1.16\text{ rad})$, or $\phi = 0.685\text{ rad}$.

(d) $F = \mu v^2 = (0.386\,\text{kg/m})(0.826\,\text{m/s})^2 = 0.263\,\text{m/s}$.

P18-5 We want to show that $dy/dx = u_y/v$. The easy way, although not mathematically rigorous:

$$\frac{dy}{dx} = \frac{dy}{dx}\frac{dt}{dt} = \frac{dy}{dt}\frac{dt}{dx} = u_y\frac{1}{v} = \frac{u_y}{x}.$$

P18-6 The maximum value for u_y occurs when the cosine function in Eq. 18-14 returns unity. Consequently, $u_\text{m}/y_\text{m} = \omega$.

P18-7 (a) The linear mass density changes as the rubber band is stretched! In this case,

$$\mu = \frac{m}{L + \Delta L}.$$

The tension in the rubber band is given by $F = k\Delta L$. Substituting this into Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta L(L + \Delta L)}{m}}.$$

(b) We want to know the time it will take to travel the length of the rubber band, so

$$v = \frac{L + \Delta L}{t} \text{ or } t = \frac{L + \Delta L}{v}.$$

Into this we will substitute our expression for wave speed

$$t = (L + \Delta L)\sqrt{\frac{m}{k\Delta L(L + \Delta L)}} = \sqrt{\frac{m(L + \Delta L)}{k\Delta L}}$$

We have to possibilities to consider: either $\Delta L \ll L$ or $\Delta L \gg L$. In either case we are only interested in the part of the expression with $L + \Delta L$; whichever term is much larger than the other will be the only significant part.

Then if $\Delta L \ll L$ we get $L + \Delta L \approx L$ and

$$t = \sqrt{\frac{m(L + \Delta L)}{k\Delta L}} \approx \sqrt{\frac{mL}{k\Delta L}},$$

so that t is proportional to $1/\sqrt{\Delta L}$.

But if $\Delta L \gg L$ we get $L + \Delta L \approx \Delta L$ and

$$t = \sqrt{\frac{m(L + \Delta L)}{k\Delta L}} \approx \sqrt{\frac{m\Delta L}{k\Delta L}} = \sqrt{\frac{m}{k}},$$

so that t is constant.

P18-8 (a) The tension in the rope at some point is a function of the weight of the cable beneath it. If the bottom of the rope is $y = 0$, then the weight beneath some point y is $W = y(m/L)g$. The speed of the wave at that point is $v = \sqrt{T/(m/L)} = \sqrt{y(M/L)g/(m/L)} = \sqrt{gy}$.

(b) $dy/dt = \sqrt{gy}$, so

$$\begin{aligned} dt &= \frac{dy}{\sqrt{gy}}, \\ t &= \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{L/g}. \end{aligned}$$

(c) No.

P18-9 (a) $M = \int \mu\, dx$, so

$$M = \int_0^L kx\, dx = \frac{1}{2}kL^2.$$

Then $k = 2M/L^2$.

(b) $v = \sqrt{F/\mu} = \sqrt{F/kx}$, then

$$\begin{aligned} dt &= \sqrt{kx/F}\, dx, \\ t &= \int_0^L \sqrt{2M/FL^2}\sqrt{x}\, dx = \frac{2}{3}\sqrt{2M/FL^2}L^{3/2} = \sqrt{8ML/9F}. \end{aligned}$$

P18-10 Take a cue from pressure and surface tension. In the rotating *non-inertial* reference frame for which the hoop appears to be at rest there is an effective force per unit length acting to push on each part of the loop directly away from the center. This force per unit length has magnitude

$$\frac{\Delta F}{\Delta L} = (\Delta m/\Delta L)\frac{v^2}{r} = \mu\frac{v^2}{r}.$$

There must be a tension T in the string to hold the loop together. Imagine the loop to be replaced with two semicircular loops. Each semicircular loop has a diameter part; the force tending to pull off the diameter section is $(\Delta F/\Delta L)2r = 2\mu v^2$. There are two connections to the diameter section, so the tension in the string must be half the force on the diameter section, or $T = \mu v^2$.

The wave speed is $v_{\mathrm{w}} = \sqrt{T/\mu} = v$.

Note that the wave on the string can travel in either direction relative to an inertial observer. One wave will appear to be fixed in space; the other will move around the string with twice the speed of the string.

$\boxed{\textbf{P18-11}}$ If we assume that Handel wanted his violins to play in tune with the other instruments then all we need to do is find an instrument from Handel's time that will accurately keep pitch over a period of several hundred years. Most instruments won't keep pitch for even a few days because of temperature and humidity changes; some (like the piccolo?) can't even play in tune for more than a few notes! But if someone found a tuning fork...

Since the length of the string doesn't change, and we are using a string with the same mass density, the only choice is to change the tension. But $f \propto v \propto \sqrt{T}$, so the percentage change in the tension of the string is

$$\frac{T_{\mathrm{f}} - T_{\mathrm{i}}}{T_{\mathrm{i}}} = \frac{{f_{\mathrm{f}}}^2 - {f_{\mathrm{i}}}^2}{{f_{\mathrm{i}}}^2} = \frac{(440\ \mathrm{Hz})^2 - (422.5\ \mathrm{Hz})^2}{(422.5\ \mathrm{Hz})^2} = 8.46\,\%.$$

P18-12

P18-13 (a) The point sources emit spherical waves; the solution to the appropriate wave equation is found in Ex. 18-14:

$$y_i = \frac{A}{r_i}\sin(kr_i - \omega t).$$

If r_i is sufficiently large compared to A, and $r_1 \approx r_2$, then let $r_1 = r - \delta r$ and $r_2 = r - \delta r$;

$$\frac{A}{r_1} + \frac{A}{r_2} \approx \frac{2A}{r},$$

with an error of order $(\delta r/r)^2$. So ignore it.

Then

$$\begin{aligned} y_1 + y_2 &\approx \frac{A}{r}\left[\sin(kr_1 - \omega t) + \sin(kr_2 - \omega t)\right], \\ &= \frac{2A}{r}\sin(kr - \omega t)\cos\frac{k}{2}(r_1 - r_2), \\ y_m &= \frac{2A}{r}\cos\frac{k}{2}(r_1 - r_2). \end{aligned}$$

(b) A maximum (minimum) occurs when the operand of the cosine, $k(r_1 - r_2)/2$ is an integer multiple of π (a half odd-integer multiple of π)

P18-14 The direct wave travels a distance d from S to D. The wave which reflects off the original layer travels a distance $\sqrt{d^2 + 4H^2}$ between S and D. The wave which reflects off the layer after it has risen a distance h travels a distance $\sqrt{d^2 + 4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2 + 4H^2} - d = n\lambda,$$

and later we have destructive interference so

$$\sqrt{d^2 + 4(H+h)^2} - d = (n + 1/2)\lambda.$$

We don't know n, but we can subtract the top equation from the bottom and get

$$\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} = \lambda/2$$

$\boxed{\textbf{P18-15}}$ The wavelength is

$$\lambda = v/f = (3.00\times10^8\text{m/s})/(13.0\times10^6\text{Hz}) = 23.1\,\text{m}.$$

The direct wave travels a distance d from S to D. The wave which reflects off the original layer travels a distance $\sqrt{d^2 + 4H^2}$ between S and D. The wave which reflects off the layer one minute later travels a distance $\sqrt{d^2 + 4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2 + 4H^2} - d = n_1\lambda,$$

and then one minute later we have

$$\sqrt{d^2 + 4(H+h)^2} - d = n_2\lambda.$$

We don't know either n_1 or n_2, but we do know the difference is 6, so we can subtract the top equation from the bottom and get

$$\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} = 6\lambda$$

We could use that expression as written, do some really obnoxious algebra, and then get the answer. But we don't want to; we want to take advantage of the fact that h is small compared to d and H. Then the first term can be written as

$$\begin{aligned}
\sqrt{d^2 + 4(H+h)^2} &= \sqrt{d^2 + 4H^2 + 8Hh + 4h^2},\\
&\approx \sqrt{d^2 + 4H^2 + 8Hh},\\
&\approx \sqrt{d^2 + 4H^2}\sqrt{1 + \frac{8H}{d^2 + 4H^2}h},\\
&\approx \sqrt{d^2 + 4H^2}\left(1 + \frac{1}{2}\frac{8H}{d^2 + 4H^2}h\right).
\end{aligned}$$

Between the second and the third lines we factored out $d^2 + 4H^2$; that last line is from the binomial expansion theorem. We put this into the previous expression, and

$$\begin{aligned}
\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} &= 6\lambda,\\
\sqrt{d^2 + 4H^2}\left(1 + \frac{4H}{d^2 + 4H^2}h\right) - \sqrt{d^2 + 4H^2} &= 6\lambda,\\
\frac{4H}{\sqrt{d^2 + 4H^2}}h &= 6\lambda.
\end{aligned}$$

Now what were we doing? We were trying to find the speed at which the layer is moving. We know H, d, and λ; we can then find h,

$$h = \frac{6(23.1\,\mathrm{m})}{4(510\times10^3\mathrm{m})}\sqrt{(230\times10^3\mathrm{m})^2 + 4(510\times10^3\mathrm{m})^2} = 71.0\,\mathrm{m}.$$

The layer is then moving at $v = (71.0\,\mathrm{m})/(60\,\mathrm{s}) = 1.18\,\mathrm{m/s}$.

P18-16 The equation of the standing wave is

$$y = 2y_\mathrm{m}\sin kx\cos\omega t.$$

The transverse speed of a point on the string is the derivative of this, or

$$u_y = -2y_\mathrm{m}\omega\sin kx\sin\omega t,$$

this has a maximum value when $\omega t - \pi/2$ is a integer multiple of π. The maximum value is

$$u_\mathrm{m} = 2y_\mathrm{m}\omega\sin kx.$$

Each mass element on the string dm then has a maximum kinetic energy

$$dK_\mathrm{m} = (dm/2){u_\mathrm{m}}^2 = {y_\mathrm{m}}^2\omega^2\sin^2 kx\;dm.$$

Using $dm = \mu dx$, and integrating over one loop from $kx = 0$ to $kx = \pi$, we get

$$K_\mathrm{m} = {y_\mathrm{m}}^2\omega^2\mu/2k = 2\pi^2{y_\mathrm{m}}^2 f\mu v.$$

P18-17 (a) For 100% reflection the amplitudes of the incident and reflected wave are equal, or $A_i = A_r$, which puts a zero in the denominator of the equation for SWR. If there is no reflection, $A_r = 0$ leaving the expression for SWR to reduce to $A_i/A_i = 1$.

(b) $P_r/P_i = A_r^2/A_i^2$. Do the algebra:

$$\begin{aligned}
\frac{A_i + A_r}{A_i - A_r} &= \mathrm{SWR},\\
A_i + A_r &= \mathrm{SWR}(A_i - A_r),\\
A_r(\mathrm{SWR}+1) &= A_i(\mathrm{SWR}-1),\\
A_r/A_i &= (\mathrm{SWR}-1)/(\mathrm{SWR}+1).
\end{aligned}$$

Square this, and multiply by 100.

P18-18 Measure with a ruler; I get $2A_\mathrm{max} = 1.1$ cm and $2A_\mathrm{min} = 0.5$ cm.

(a) $\mathrm{SWR} = (1.1/0.5) = 2.2$

(b) $(2.2-1)^2/(2.2+1)^2 = 0.14\,\%$.

$\boxed{\textbf{P18-19}}$ (a) Call the three waves

$$\begin{aligned}
y_\mathrm{i} &= A\sin k_1(x - v_1 t),\\
y_\mathrm{t} &= B\sin k_2(x - v_2 t),\\
y_\mathrm{r} &= C\sin k_1(x + v_1 t),
\end{aligned}$$

where the subscripts i, t, and r refer to the incident, transmitted, and reflected waves respectively.

Apply the principle of superposition. Just to the left of the knot the wave has amplitude $y_\mathrm{i} + y_\mathrm{r}$ while just to the right of the knot the wave has amplitude y_t. These two amplitudes must line up at the knot for all times t, or the knot will come undone. Remember the knot is at $x = 0$, so

$$\begin{aligned} y_\mathrm{i} + y_\mathrm{r} &= y_\mathrm{t}, \\ A\sin k_1(-v_1 t) + C\sin k_1(+v_1 t) &= B\sin k_2(-v_2 t), \\ -A\sin k_1 v_1 t + C\sin k_1 v_1 t &= -B\sin k_2 v_2 t \end{aligned}$$

We know that $k_1 v_1 = k_2 v_2 = \omega$, so the three sin functions are all equivalent, and can be canceled. This leaves $A = B + C$.

(b) We need to match more than the displacement, we need to match the slope just on either side of the knot. In that case we need to take the derivative of

$$y_\mathrm{i} + y_\mathrm{r} = y_\mathrm{t}$$

with respect to x, and then set $x = 0$. First we take the derivative,

$$\begin{aligned} \frac{d}{dx}\left(y_\mathrm{i} + y_\mathrm{r}\right) &= \frac{d}{dx}\left(y_\mathrm{t}\right), \\ k_1 A\cos k_1(x - v_1 t) + k_1 C\cos k_1(x + v_1 t) &= k_2 B\cos k_2(x - v_2 t), \end{aligned}$$

and then we set $x = 0$ and simplify,

$$\begin{aligned} k_1 A\cos k_1(-v_1 t) + k_1 C\cos k_1(+v_1 t) &= k_2 B\cos k_2(-v_2 t), \\ k_1 A\cos k_1 v_1 t + k_1 C\cos k_1 v_1 t &= k_2 B\cos k_2 v_2 t. \end{aligned}$$

This last expression simplifies like the one in part (a) to give

$$k_1(A + C) = k_2 B$$

We can combine this with $A = B + C$ to solve for C,

$$\begin{aligned} k_1(A + C) &= k_2(A - C), \\ C(k_1 + k_2) &= A(k_2 - k_1), \\ C &= A\frac{k_2 - k_1}{k_1 + k_2}. \end{aligned}$$

If $k_2 < k_1$ C will be negative; this means the reflected wave will be inverted.

P18-20

P18-21 Find the wavelength from

$$\lambda = 2(0.924\,\mathrm{m})/4 = 0.462\,\mathrm{m}.$$

Find the wavespeed from

$$v = f\lambda = (60.0\ \mathrm{Hz})(0.462\,\mathrm{m}) = 27.7\,\mathrm{m/s}.$$

Find the tension from

$$F = \mu v^2 = (0.0442\,\mathrm{kg})(27.7\,\mathrm{m/s})^2/(0.924\,\mathrm{m}) = 36.7\,\mathrm{N}.$$

P18-22 (a) The frequency of vibration f is the same for both the aluminum and steel wires; they don not, however, need to vibrate in the same mode. The speed of waves in the aluminum is v_1, that in the steel is v_2. The aluminum vibrates in a mode given by $n_1 = 2L_1 f/v_1$, the steel vibrates in a mode given by $n_2 = 2L_2 f/v_2$. Both n_1 and n_2 need be integers, so the ratio must be a rational fraction. Note that the ratio is independent of f, so that L_1 and L_2 must be chosen correctly for this problem to work at all!

This ratio is

$$\frac{n_2}{n_1} = \frac{L_2}{L_1}\sqrt{\frac{\mu_2}{\mu_1}} = \frac{(0.866\,\mathrm{m})}{(0.600\,\mathrm{m})}\sqrt{\frac{(7800\,\mathrm{kg/m^3})}{(2600\,\mathrm{kg/m^3})}} = 2.50 \approx \frac{5}{2}$$

Note that since the wires have the same tension and the same cross sectional area it is acceptable to use the volume density instead of the linear density in the problem.

The smallest integer solution is then $n_1 = 2$ and $n_2 = 5$. The frequency of vibration is then

$$f = \frac{n_1 v}{2L_1} = \frac{n_1}{2L_1}\sqrt{\frac{T}{\rho_1 A}} = \frac{(2)}{2(0.600\,\mathrm{m})}\sqrt{\frac{(10.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(2600\,\mathrm{kg/m^3})(1.00\times 10^{-6}\mathrm{m^2})}} = 323\ \mathrm{Hz}.$$

(b) There are three nodes in the aluminum and six in the steel. But one of those nodes is shared, and two are on the ends of the wire. The answer is then six.

E19-1 (a) $v = f\lambda = (25\text{ Hz})(0.24\,\text{m}) = 6.0\,\text{m/s}$.

(b) $k = (2\pi\text{ rad})/(0.24\,\text{m}) = 26\text{ rad/m}$; $\omega = (2\pi\text{ rad})(25\text{ Hz}) = 160\text{ rad/s}$. The wave equation is

$$s = (3.0\times10^{-3}\text{m})\sin[(26\text{ rad/m})x + (160\text{ rad/s})t]$$

E19-2 (a) $[\Delta P]_\text{m} = 1.48\,\text{Pa}$.

(b) $f = (334\pi\text{ rad/s})/(2\pi\text{ rad}) = 167\text{ Hz}$.

(c) $\lambda = (2\pi\text{ rad})/(1.07\pi\text{ rad/m}) = 1.87\,\text{m}$.

(d) $v = (167\text{ Hz})(1.87\,\text{m}) = 312\,\text{m/s}$.

$\boxed{\textbf{E19-3}}$ (a) The wavelength is given by $\lambda = v/f = (343\,\text{m/s})/(4.50\times10^{6}\text{Hz} = 7.62\times10^{-5}\text{m}$.

(b) The wavelength is given by $\lambda = v/f = (1500\,\text{m/s})/(4.50\times10^{6}\text{Hz} = 3.33\times10^{-4}\text{m}$.

E19-4 Note: There is a typo; the mean free path should have been measured in "μm" instead of "pm".

$\lambda_\text{min} = 1.0\times10^{-6}\text{m}$; $f_\text{max} = (343\,\text{m/s})/(1.0\times10^{-6}\text{m}) = 3.4\times10^{8}\text{Hz}$.

E19-5 (a) $\lambda = (240\,\text{m/s})/(4.2\times10^{9}\text{Hz}) = 5.7\times10^{-8}\text{m}$.

E19-6 (a) The speed of sound is

$$v = (331\,\text{m/s})(6.21\times10^{-4}\text{ mi/m}) = 0.206\text{ mi/s}.$$

In five seconds the sound travels $(0.206\text{ mi/s})(5.0\,\text{s}) = 1.03$ mi, which is 3% too large.

(b) Count seconds and divide by 3.

$\boxed{\textbf{E19-7}}$ Marching at 120 paces per minute means that you move a foot every half a second. The soldiers in the back are moving the wrong foot, which means they are moving the correct foot half a second later than they should. If the speed of sound is 343 m/s, then the column of soldiers must be $(343\,\text{m/s})(0.5\,\text{s}) = 172\,\text{m}$ long.

E19-8 It takes $(300\,\text{m})/(343\,\text{m/s}) = 0.87\,\text{s}$ for the concert goer to hear the music after it has passed the microphone. It takes $(5.0\times10^{6}\text{m})/(3.0\times10^{8}\text{m/s}) = 0.017\,\text{s}$ for the radio listener to hear the music after it has passed the microphone. The radio listener hears the music first, 0.85 s before the concert goer.

E19-9 $x/v_\text{P} = t_\text{P}$ and $x/v_\text{S} = t_\text{S}$; subtracting and rearranging,

$$x = \Delta t/[1/v_\text{S} - 1/v_\text{P}] = (180\,\text{s})/[1/(4.5\text{ km/s}) - 1/(8.2\text{ km/s})] = 1800\text{ km}.$$

E19-10 Use Eq. 19-8, $s_\text{m} = [\Delta p]_\text{m}/kB$, and Eq. 19-14, $v = \sqrt{B/\rho_0}$. Then

$$[\Delta p]_\text{m} = kBs_\text{m} = kv^2\rho_0 s_\text{m} = 2\pi f v\rho_0 s_\text{m}.$$

Insert into Eq. 19-18, and

$$I = 2\pi^2\rho v f^2 {s_\text{m}}^2.$$

$\boxed{\textbf{E19-11}}$ If the source emits equally in all directions the intensity at a distance r is given by the average power divided by the surface area of a sphere of radius r centered on the source.

The power output of the source can then be found from

$$P = IA = I(4\pi r^2) = (197\times10^{-6}\text{W/m}^2)4\pi(42.5\,\text{m})^2 = 4.47\,\text{W}.$$

E19-12 Use the results of Exercise 19-10.

$$s_{\rm m} = \sqrt{\frac{(1.13\times10^{-6}{\rm W/m^2})}{2\pi^2(1.21\,{\rm kg/m^3})(343\,{\rm m/s})(313\ {\rm Hz})^2}} = 3.75\times10^{-8}{\rm m}.$$

E19-13 $U = IAt = (1.60\times10^{-2}{\rm W/m^2})(4.70\times10^{-4}{\rm m^2})(3600\,{\rm s}) = 2.71\times10^{-2}{\rm W}.$

E19-14 Invert Eq. 19-21:

$$I_1/I_2 = 10^{(1.00{\rm dB})/10} = 1.26.$$

$\boxed{\textbf{E19-15}}$ (a) Relative sound level is given by Eq. 19-21,

$$SL_1 - SL_2 = 10\log\frac{I_1}{I_2} \text{ or } \frac{I_1}{I_2} = 10^{(SL_1-SL_2)/10},$$

so if $\Delta SL = 30$ then $I_1/I_2 = 10^{30/10} = 1000$.

(b) Intensity is proportional to pressure amplitude squared according to Eq. 19-19; so

$$\Delta p_{\rm m,1}/\Delta p_{\rm m,2} = \sqrt{I_1/I_2} = \sqrt{1000} = 32.$$

E19-16 We know where her ears hurt, so we know the intensity at that point. The power output is then

$$P = 4\pi(1.3\,{\rm m})^2(1.0\,{\rm W/m^2}) = 21\,{\rm W}.$$

This is less than the advertised power.

E19-17 Use the results of Exercise 18-18, $I = uv$. The intensity is

$$I = (5200\,{\rm W})/4\pi(4820\,{\rm m})^2 = 1.78\times10^{-5}{\rm W/m^2},$$

so the energy density is

$$u = I/v = (1.78\times10^{-5}{\rm W/m^2})/(343\,{\rm m/s}) = 5.19\times10^{-8}{\rm J/m^3}.$$

E19-18 $I_2 = 2I_1$, since $I \propto 1/r^2$ then $r_1^2 = 2r_2^2$. Then

$$\begin{aligned} D &= \sqrt{2}(D - 51.4\,{\rm m}), \\ D(\sqrt{2}-1) &= \sqrt{2}(51.4\,{\rm m}), \\ D &= 176\,{\rm m}. \end{aligned}$$

$\boxed{\textbf{E19-19}}$ The sound level is given by Eq. 19-20,

$$SL = 10\log\frac{I}{I_0}$$

where I_0 is the threshold intensity of 10^{-12} W/m^2. Intensity is given by Eq. 19-19,

$$I = \frac{(\Delta p_{\rm m})^2}{2\rho v}$$

If we assume the maximum possible pressure amplitude is equal to one atmosphere, then

$$I = \frac{(\Delta p_{\rm m})^2}{2\rho v} = \frac{(1.01\times10^5{\rm Pa})^2}{2(1.21\,{\rm kg/m^3})(343\,{\rm m/s})} = 1.22\times10^7{\rm W/m^2}.$$

The sound level would then be

$$SL = 10\log\frac{I}{I_0} = 10\log\frac{1.22\times10^7\mathrm{W/m^2}}{(10^{-12}\ \mathrm{W/m^2})} = 191\ \mathrm{dB}$$

E19-20 Let one person speak with an intensity I_1. N people would have an intensity NI_1. The ratio is N, so by inverting Eq. 19-21,

$$N = 10^{(15\mathrm{dB})/10} = 31.6,$$

so 32 people would be required.

E19-21 Let one leaf rustle with an intensity I_1. N leaves would have an intensity NI_1. The ratio is N, so by Eq. 19-21,

$$SL_N = (8.4\ \mathrm{dB}) + 10\log(2.71\times10^5) = 63\ \mathrm{dB}.$$

E19-22 Ignoring the finite time means that we can assume the sound waves travels vertically, which considerably simplifies the algebra.

The intensity ratio can be found by inverting Eq. 19-21,

$$I_1/I_2 = 10^{(30\mathrm{dB})/10} = 1000.$$

But intensity is proportional to the inverse distance squared, so $I_1/I_2 = (r_2/r_1)^2$, or

$$r_2 = (115\,\mathrm{m})\sqrt{(1000)} = 3640\,\mathrm{m}.$$

$\boxed{\textbf{E19-23}}$ A minimum will be heard at the detector if the path length difference between the straight path and the path through the curved tube is half of a wavelength. Both paths involve a straight section from the source to the start of the curved tube, and then from the end of the curved tube to the detector. Since it is the path difference that matters, we'll only focus on the part of the path between the start of the curved tube and the end of the curved tube. The length of the straight path is one diameter, or $2r$. The length of the curved tube is half a circumference, or πr. The difference is $(\pi-2)r$. This difference is equal to half a wavelength, so

$$\begin{aligned}(\pi-2)r &= \lambda/2,\\ r &= \frac{\lambda}{2\pi-4} = \frac{(42.0\ \mathrm{cm})}{2\pi-4} = 18.4\ \mathrm{cm}.\end{aligned}$$

E19-24 The path length difference here is

$$\sqrt{(3.75\,\mathrm{m})^2+(2.12\,\mathrm{m})^2} - (3.75\,\mathrm{m}) = 0.5578\,\mathrm{m}.$$

(a) A minimum will occur if this is equal to a half integer number of wavelengths, or $(n-1/2)\lambda = 0.5578\,\mathrm{m}$. This will occur when

$$f = (n-1/2)\frac{(343\,\mathrm{m/s})}{(0.5578\,\mathrm{m})} = (n-1/2)(615\ \mathrm{Hz}).$$

(b) A maximum will occur if this is equal to an integer number of wavelengths, or $n\lambda = 0.5578\,\mathrm{m}$. This will occur when

$$f = n\frac{(343\,\mathrm{m/s})}{(0.5578\,\mathrm{m})} = n(615\ \mathrm{Hz}).$$

E19-25 The path length difference here is

$$\sqrt{(24.4\,\mathrm{m} + 6.10\,\mathrm{m})^2 + (15.2\,\mathrm{m})^2} - \sqrt{(24.4\,\mathrm{m})^2 + (15.2\,\mathrm{m})^2} = 5.33\,\mathrm{m}.$$

A maximum will occur if this is equal to an integer number of wavelengths, or $n\lambda = 5.33\,\mathrm{m}$. This will occur when

$$f = n(343\,\mathrm{m/s})/(5.33\,\mathrm{m}) = n(64.4\ \mathrm{Hz})$$

The two lowest frequencies are then 64.4 Hz and 129 Hz.

E19-26 The wavelength is $\lambda = (343\,\mathrm{m/s})/(300\ \mathrm{Hz}) = 1.143\,\mathrm{m}$. This means that the sound maxima will be half of this, or $0.572\,\mathrm{m}$ apart. Directly in the center the path length difference is zero, but since the waves are out of phase, this will be a minimum. The maxima should be located on either side of this, a distance $(0.572\,\mathrm{m})/2 = 0.286\,\mathrm{m}$ from the center. There will then be maxima located each $0.572\,\mathrm{m}$ farther along.

E19-27 (a) $f_1 = v/2L$ and $f_2 = v/2(L - \Delta L)$. Then

$$\frac{1}{r} = \frac{f_1}{f_2} = \frac{L - \Delta L}{L} = 1 - \frac{\Delta L}{L},$$

or $\Delta L = L(1 - 1/r)$.

(b) The answers are $\Delta L = (0.80\,\mathrm{m})(1 - 5/6) = 0.133\,\mathrm{m}$; $\Delta L = (0.80\,\mathrm{m})(1 - 4/5) = 0.160\,\mathrm{m}$; $\Delta L = (0.80\,\mathrm{m})(1 - 3/4) = 0.200\,\mathrm{m}$; and $\Delta L = (0.80\,\mathrm{m})(1 - 2/3) = 0.267\,\mathrm{m}$.

E19-28 The wavelength is twice the distance between the nodes in this case, so $\lambda = 7.68$ cm. The frequency is

$$f = (1520\,\mathrm{m/s})/(7.68\times 10^{-2}\mathrm{m}) = 1.98\times 10^{4}\mathrm{Hz}.$$

$\boxed{\textbf{E19-29}}$ The well is a tube open at one end and closed at the other; Eq. 19-28 describes the allowed frequencies of the resonant modes. The lowest frequency is when $n = 1$, so $f_1 = v/4L$. We know f_1; to find the depth of the well, L, we need to know the speed of sound.

We should use the information provided, instead of looking up the speed of sound, because maybe the well is filled with some kind of strange gas.

Then, from Eq. 19-14,

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{(1.41 \times 10^5\ \mathrm{Pa})}{(1.21\,\mathrm{kg/m^3})}} = 341\,\mathrm{m/s}.$$

The depth of the well is then

$$L = v/(4f_1) = (341\,\mathrm{m/s})/[4(7.20\ \mathrm{Hz})] = 11.8\,\mathrm{m}.$$

E19-30 (a) The resonant frequencies of the pipe are given by $f_n = nv/2L$, or

$$f_n = n(343\,\mathrm{m/s})/2(0.457\,\mathrm{m}) = n(375\ \mathrm{Hz}).$$

The lowest frequency in the specified range is $f_3 = 1130$ Hz; the other allowed frequencies in the specified range are $f_4 = 1500$ Hz, and $f_5 = 1880$ Hz.

$\boxed{\textbf{E19-31}}$ The maximum reflected frequencies will be the ones that undergo constructive interference, which means the path length difference will be an integer multiple of a wavelength. A wavefront will strike a terrace wall and part will reflect, the other part will travel on to the next terrace, and then reflect. Since part of the wave had to travel to the next terrace and back, the path length difference will be $2 \times 0.914\,\text{m} = 1.83\,\text{m}$.

If the speed of sound is $v = 343\,\text{m/s}$, the lowest frequency wave which undergoes constructive interference will be

$$f = \frac{v}{\lambda} = \frac{(343\,\text{m/s})}{(1.83\,\text{m})} = 187 \text{ Hz}$$

Any integer multiple of this frequency will also undergo constructive interference, and will also be heard. The ear and brain, however, will most likely interpret the complex mix of frequencies as a single tone of frequency 187 Hz.

E19-32 Assume there is no frequency between these two that is amplified. Then one of these frequencies is $f_n = nv/2L$, and the other is $f_{n+1} = (n+1)v/2L$. Subtracting the larger from the smaller, $\Delta f = v/2L$, or

$$L = v/2\Delta f = (343\,\text{m/s})/2(138 \text{ Hz} - 135 \text{ Hz}) = 57.2\,\text{m}.$$

E19-33 (a) $v = 2Lf = 2(0.22\,\text{m})(920 \text{ Hz}) = 405\,\text{m/s}$.
(b) $F = v^2\mu = (405\,\text{m/s})^2(820\times10^{-6}\text{kg})/(0.220\,\text{m}) = 611\,\text{N}$.

E19-34 $f \propto v$, and $v \propto \sqrt{F}$, so $f \propto \sqrt{F}$. Doubling f requires F increase by a factor of 4.

$\boxed{\textbf{E19-35}}$ The speed of a wave on the string is the same, regardless of where you put your finger, so $f\lambda$ is a constant. The string will vibrate (mostly) in the lowest harmonic, so that $\lambda = 2L$, where L is the length of the part of the string that is allowed to vibrate. Then

$$\begin{aligned} f_2\lambda_2 &= f_1\lambda_1, \\ 2f_2L_2 &= 2f_1L_1, \\ L_2 &= L_1\frac{f_1}{f_2} = (30 \text{ cm})\frac{(440 \text{ Hz})}{(528 \text{ Hz})} = 25 \text{ cm}. \end{aligned}$$

So you need to place your finger 5 cm from the end.

E19-36 The open organ pipe has a length

$$L_\text{o} = v/2f_1 = (343\,\text{m/s})/2(291 \text{ Hz}) = 0.589\,\text{m}.$$

The second harmonic of the open pipe has frequency $2f_1$; this is the first overtone of the closed pipe, so the closed pipe has a length

$$L_\text{c} = (3)v/4(2f_1) = (3)(343\,\text{m/s})/4(2)(291 \text{ Hz}) = 0.442\,\text{m}.$$

$\boxed{\textbf{E19-37}}$ The unknown frequency is either 3 Hz higher or lower than the standard fork. A small piece of wax placed on the fork of this unknown frequency tuning fork will result in a lower frequency because $f \propto \sqrt{k/m}$. If the beat frequency decreases then the two tuning forks are getting *closer* in frequency, so the frequency of the first tuning fork must be above the frequency of the standard fork. Hence, 387 Hz.

E19-38 If the string is too taut then the frequency is too high, or $f = (440 + 4)$Hz. Then $T = 1/f = 1/(444 \text{ Hz}) = 2.25\times10^{-3}$s.

E19-39 One of the tuning forks need to have a frequency 1 Hz different from another. Assume then one is at 501 Hz. The next fork can be played against the first or the second, so it could have a frequency of 503 Hz to pick up the 2 and 3 Hz beats. The next one needs to pick up the 5, 7, and 8 Hz beats, and 508 Hz will do the trick. There are other choices.

E19-40 $f = v/\lambda = (5.5\,\text{m/s})/(2.3\,\text{m}) = 2.39$ Hz. Then

$$f' = f(v + v_O)/v = (2.39 \text{ Hz})(5.5\,\text{m/s} + 3.3\,\text{m/s})/(5.5\,\text{m/s}) = 3.8 \text{ Hz}.$$

$\boxed{\textbf{E19-41}}$ We'll use Eq. 19-44, since both the observer and the source are in motion. Then

$$f' = f\frac{v \pm v_O}{v \mp v_S} = (15.8 \text{ kHz})\frac{(343\,\text{m/s}) + (246\,\text{m/s})}{(343\,\text{m/s}) + (193\,\text{m/s})} = 17.4 \text{ kHz}r$$

E19-42 Solve Eq. 19-44 for v_S;

$$v_S = (v + v_O)f/f' - v = (343\,\text{m/s} + 2.63\,\text{m/s})(1602 \text{ Hz})/(1590 \text{ Hz}) - (343\,\text{m/s}) = 5.24\,\text{m/s}.$$

E19-43 $v_S = (14.7 \text{ Rad/s})(0.712\,\text{m}) = 10.5\,\text{m/s}$.

(a) The low frequency heard is

$$f' = (538 \text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} + 10.5\,\text{m/s}) = 522 \text{ Hz}.$$

(a) The high frequency heard is

$$f' = (538 \text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} - 10.5\,\text{m/s}) = 555 \text{ Hz}.$$

E19-44 Solve Eq. 19-44 for v_S;

$$v_S = v - vf/f' = (343\,\text{m/s}) - (343\,\text{m/s})(440 \text{ Hz})/(444 \text{ Hz}) = 3.1\,\text{m/s}.$$

E19-45

E19-46 The approaching car "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (148 \text{ Hz})\frac{(343\,\text{m/s}) + (44.7\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 167 \text{ Hz}$$

This sound is reflected back at the same frequency, so the police car "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (167 \text{ Hz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) - (44.7\,\text{m/s})} = 192 \text{ Hz}$$

E19-47 The departing intruder "hears"

$$f' = f\frac{v - v_O}{v + v_S} = (28.3 \text{ kHz})\frac{(343\,\text{m/s}) - (0.95\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 28.22 \text{ kHz}$$

This sound is reflected back at the same frequency, so the alarm "hears"

$$f' = f\frac{v - v_O}{v + v_S} = (28.22 \text{ kHz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) + (0.95\,\text{m/s})} = 28.14 \text{ kHz}$$

The beat frequency is 28.3 kHz − 28.14 kHz = 160 Hz.

E19-48 (a) $f' = (1000 \text{ Hz})(330\,\text{m/s})(330\,\text{m/s} + 10.0\,\text{m/s}) = 971$ Hz.
(b) $f' = (1000 \text{ Hz})(330\,\text{m/s})(330\,\text{m/s} - 10.0\,\text{m/s}) = 1030$ Hz.
(c) $1030 \text{ Hz} - 971 \text{ Hz} = 59$ Hz.

E19-49 (a) The frequency "heard" by the wall is

$$f' = f\frac{v + v_O}{v - v_S} = (438 \text{ Hz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) - (19.3\,\text{m/s})} = 464 \text{ Hz}$$

(b) The wall then reflects a frequency of 464 Hz back to the trumpet player. Sticking with Eq. 19-44, the source is now at rest while the observer moving,

$$f' = f\frac{v + v_O}{v - v_S} = (464 \text{ Hz})\frac{(343\,\text{m/s}) + (19.3\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 490 \text{ Hz}$$

E19-50 The body part "hears"

$$f' = f\frac{v + v_b}{v}.$$

This sound is reflected back to the detector which then "hears"

$$f'' = f'\frac{v}{v - v_b} = f\frac{v + v_b}{v - v_b}.$$

Rearranging,

$$v_b/v = \frac{f'' - f}{f'' + f} \approx \frac{1}{2}\frac{\Delta f}{f},$$

so $v \approx 2(1\times10^{-3}\text{m/s})/(1.3\times10^{-6}) \approx 1500\,\text{m/s}$.

E19-51 The wall "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (39.2 \text{ kHz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) - (8.58\,\text{m/s})} = 40.21 \text{ kHz}$$

This sound is reflected back at the same frequency, so the bat "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (40.21 \text{ kHz})\frac{(343\,\text{m/s}) + (8.58\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 41.2 \text{ kHz}.$$

P19-1 (a) $t_{\text{air}} = L/v_{\text{air}}$ and $t_{\text{m}} = L/v$, so the difference is

$$\Delta t = L(1/v_{\text{air}} - 1/v)$$

(b) Rearrange the above result, and

$$L = (0.120\,\text{s})/[1/(343\,\text{m/s}) - 1/(6420\,\text{m/s})] = 43.5\,\text{m}.$$

P19-2 The stone falls for a time t_1 where $y = gt_1^2/2$ is the depth of the well. Note y is positive in this equation. The sound travels back in a time t_2 where $v = y/t_2$ is the speed of sound in the well. $t_1 + t_2 = 3.00\,\text{s}$, so

$$2y = g(3.00\,\text{s} - t_2)^2 = g[(9.00\,\text{s}^2) - (6.00\,\text{s})y/v + y^2/v^2],$$

or, using $g = 9.81\,\text{m/s}^2$ and $v = 343\,\text{m/s}$,

$$y^2 - (2.555\times10^5\,\text{m})y + (1.039\times10^7\,\text{m}^2) = 0,$$

which has a positive solution $y = 40.7\,\text{m}$.

$\boxed{\textbf{P19-3}}$ (a) The intensity at 28.5 m is found from the $1/r^2$ dependence;

$$I_2 = I_1(r_1/r_2)^2 = (962\,\mu\text{W/m}^2)(6.11\,\text{m}/28.5\,\text{m})^2 = 44.2\,\mu\text{W/m}^2.$$

(c) We'll do this part first. The pressure amplitude is found from Eq. 19-19,

$$\Delta p_\text{m} = \sqrt{2\rho v I} = \sqrt{2(1.21\,\text{kg/m}^3)(343\,\text{m/s})(962\times10^{-6}\text{W/m}^2)} = 0.894\,\text{Pa}.$$

(b) The displacement amplitude is found from Eq. 19-8,

$$s_\text{m} = \Delta p_\text{m}/(kB),$$

where $k = 2\pi f/v$ is the wave number. From Eq. 19-14 w know that $B = \rho v^2$, so

$$s_\text{m} = \frac{\Delta p_\text{m}}{2\pi f \rho v} = \frac{(0.894\,\text{Pa})}{2\pi(2090\text{ Hz})(1.21\,\text{kg/m}^3)(343\,\text{m/s})} = 1.64\times10^{-7}\text{m}.$$

P19-4 (a) If the intensities are equal, then $\Delta p_\text{m} \propto \sqrt{\rho v}$, so

$$\frac{[\Delta p_\text{m}]_\text{water}}{[\Delta p_\text{m}]_\text{air}} = \sqrt{\frac{(998\,\text{kg/m}^3)(1482\,\text{m/s})}{(1.2\,\text{kg/m}^3)(343\,\text{m/s})}} = 59.9.$$

(b) If the pressure amplitudes are equal, then $I \propto\propto 1/\rho v$, so

$$\frac{I_\text{water}}{I_\text{air}} = \frac{(1.2\,\text{kg/m}^3)(343\,\text{m/s})}{(998\,\text{kg/m}^3)(1482\,\text{m/s})} = 2.78\times10^{-4}.$$

P19-5 The energy is dissipated on a cylindrical surface which grows in area as r, so the intensity is proportional to $1/r$. The amplitude is proportional to the square root of the intensity, so $s_\text{m} \propto 1/\sqrt{r}$.

P19-6 (a) The first position corresponds to maximum destructive interference, so the waves are half a wavelength out of phase; the second position corresponds to maximum constructive interference, so the waves are in phase. Shifting the tube has in effect added half a wavelength to the path through B. But each segment is added, so

$$\lambda = (2)(2)(1.65\text{ cm}) = 6.60\text{ cm},$$

and $f = (343\,\text{m/s})/(6.60\text{ cm}) = 5200$ Hz.

(b) $I_\text{min} \propto (s_1 - s_2)^2$, $I_\text{max} \propto (s_1 + s_2)^2$, then dividing one expression by the other and rearranging we find

$$\frac{s_1}{s_2} = \frac{\sqrt{I_\text{max}} + \sqrt{I_\text{min}}}{\sqrt{I_\text{max}} - \sqrt{I_\text{min}}} = \frac{\sqrt{90} + \sqrt{10}}{\sqrt{90} - \sqrt{10}} = 2$$

P19-7 (a) $I = P/4\pi r^2 = (31.6\,\text{W})/4\pi(194\,\text{m})^2 = 6.68\times10^{-5}\text{W/m}^2$.

(b) $P = IA = (6.68\times10^{-5}\text{W/m}^2)(75.2\times10^{-6}\text{m}^2) = 5.02\times10^{-9}\text{W}$.

(c) $U = Pt = (5.02\times10^{-9}\text{W})(25.0\text{ min})(60.0\text{ s/min}) = 7.53\,\mu\text{J}$.

P19-8 Note that the reverberation time is logarithmically related to the intensity, but linearly related to the sound level. As such, the reverberation time is the amount of time for the sound level to decrease by

$$\Delta SL = 10\log(10^{-6}) = 60\text{ dB}.$$

Then

$$t = (87\text{ dB})(2.6\,s)/(60\text{ dB}) = 3.8\,\text{s}$$

P19-9 What the device is doing is taking all of the energy which strikes a large surface area and concentrating it into a small surface area. It doesn't succeed; only 12% of the energy is concentrated. We can think, however, in terms of power: 12% of the average power which strikes the parabolic reflector is transmitted into the tube.

If the sound intensity on the reflector is I_1, then the average power is $P_1 = I_1 A_1 = I_1 \pi r_1^2$, where r_1 is the radius of the reflector. The average power in the tube will be $P_2 = 0.12 P_1$, so the intensity in the tube will be

$$I_2 = \frac{P_2}{A_2} = \frac{0.12 I_1 \pi r_1^2}{\pi r_2^2} = 0.12 I_1 \frac{r_1^2}{r_2^2}$$

Since the lowest audible sound has an intensity of $I_0 = 10^{-12}$ W/m^2, we can set $I_2 = I_0$ as the condition for "hearing" the whisperer through the apparatus. The minimum sound intensity at the parabolic reflector is

$$I_1 = \frac{I_0}{0.12} \frac{r_2^2}{r_1^2}.$$

Now for the whisperers. Intensity falls off as $1/d^2$, where d is the distance from the source. We are told that when $d = 1.0$ m the sound level is 20 dB; this sound level has an intensity of

$$I = I_0 10^{20/10} = 100 I_0$$

Then at a distance d from the source the intensity must be

$$I_1 = 100 I_0 \frac{(1\,\text{m})^2}{d^2}.$$

This would be the intensity "picked-up" by the parabolic reflector. Combining this with the condition for being able to hear the whisperers through the apparatus, we have

$$\frac{I_0}{0.12} \frac{r_2^2}{r_1^2} = 100 I_0 \frac{(1\,\text{m})^2}{d^2}$$

or, upon some rearranging,

$$d = (\sqrt{12}\,\text{m}) \frac{r_1}{r_2} = (\sqrt{12}\,\text{m}) \frac{(0.50\,\text{m})}{(0.005\,\text{m})} = 346\,\text{m}.$$

P19-10 (a) A displacement node; at the center the particles have nowhere to go.

(b) This systems acts like a pipe which is closed at one end.

(c) $v\sqrt{B/\rho}$, so

$$T = 4(0.009)(6.96\times10^8\text{m})\sqrt{(1.0\times10^{10}\text{kg/m}^3)/(1.33\times10^{22}\text{Pa})} = 22\,\text{s}.$$

P19-11 The cork filing collect at pressure antinodes when standing waves are present, and the antinodes are each half a wavelength apart. Then $v = f\lambda = f(2d)$.

P19-12 (a) $f = v/4L = (343\,\text{m/s})/4(1.18\,\text{m}) = 72.7$ Hz.

(b) $F = \mu v^2 = \mu f^2 \lambda^2$, or

$$F = (9.57\times10^{-3}\text{kg}/0.332\,\text{m})(72.7\text{ Hz})^2[2(0.332\,\text{m})]^2 = 67.1\,N.$$

$\boxed{\textbf{P19-13}}$ In this problem the string is observed to resonate at 880 Hz and then again at 1320 Hz, so the two corresponding values of n must differ by 1. We can then write two equations

$$(880\text{ Hz}) = \frac{nv}{2L} \text{ and } (1320\text{ Hz}) = \frac{(n+1)v}{2L}$$

and solve these for v. It is somewhat easier to first solve for n. Rearranging both equations, we get

$$\frac{(880\text{ Hz})}{n} = \frac{v}{2L} \text{ and } \frac{(1320\text{ Hz})}{n+1} = \frac{v}{2L}.$$

Combining these two equations we get

$$\begin{aligned} \frac{(880\text{ Hz})}{n} &= \frac{(1320\text{ Hz})}{n+1}, \\ (n+1)(880\text{ Hz}) &= n(1320\text{ Hz}), \\ n &= \frac{(880\text{ Hz})}{(1320\text{ Hz}) - (880\text{ Hz})} = 2. \end{aligned}$$

Now that we know n we can find v,

$$v = 2(0.300\,\text{m})\frac{(880\text{ Hz})}{2} = 264\,\text{m/s}$$

And, finally, we are in a position to find the tension, since

$$F = \mu v^2 = (0.652\times 10^{-3}\text{kg/m})(264\,\text{m/s})^2 = 45.4\,\text{N}.$$

P19-14 (a) There are five choices for the first fork, and four for the second. That gives 20 pairs. But order doesn't matter, so we need divide that by two to get a maximum of 10 possible beat frequencies.

(b) If the forks are ordered to have equal differences (say, 400 Hz, 410 Hz, 420 Hz, 430 Hz, and 440 Hz) then there will actually be only 4 beat frequencies.

P19-15 $v = (2.25\times 10^8\text{m/s})/\sin(58.0°) = 2.65\times 10^8\text{m/s}.$

P19-16 (a) $f_1 = (442\text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} - 31.3\,\text{m/s}) = 486\text{ Hz}$, while

$$f_2 = (442\text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} + 31.3\,\text{m/s}) = 405\text{ Hz},$$

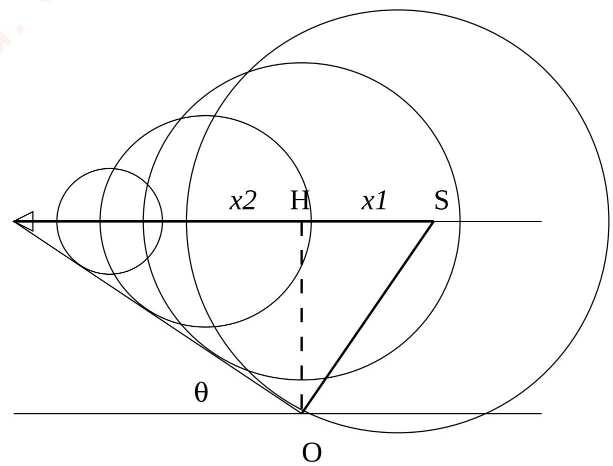
so $\Delta f = 81$ Hz.

(b) $f_1 = (442\text{ Hz})(343\,\text{m/s} - 31.3\,\text{m/s})/(343\,\text{m/s}) = 402\text{ Hz}$, while

$$f_2 = (442\text{ Hz})(343\,\text{m/s} + 31.3\,\text{m/s})/(343\,\text{m/s}) = 482\text{ Hz},$$

so $\Delta f = 80$ Hz.

$\boxed{\textbf{P19-17}}$ The sonic boom that you hear is not from the sound given off by the plane when it is overhead, it is from the sound given off *before* the plane was overhead. So this problem isn't as simple as distance equals velocity $\times$ time. It is *very* useful to sketch a picture.



We can find the angle θ from the figure, we'll get Eq. 19-45, so

$$\sin\theta = \frac{v}{v_s} = \frac{(330\,\text{m/s})}{(396\,\text{m/s})} = 0.833 \text{ or } \theta = 56.4^\circ$$

Note that v_s is the speed of the source, not the speed of sound!

Unfortunately $t = 12\,\text{s}$ is *not* the time between when the sonic boom leaves the plane and when it arrives at the observer. It is the time between when the plane is overhead and when the sonic boom arrives at the observer. That's why there are so many marks and variables on the figure. x_1 is the distance from where the sonic boom which is heard by the observer is emitted to the point directly overhead; x_2 is the distance from the point which is directly overhead to the point where the plane is when the sonic boom is heard by the observer. We do have $x_2 = v_s(12.0\,\text{s})$. This length forms one side of a right triangle HSO, the opposite side of this triangle is the side HO, which is the height of the plane above the ground, so

$$h = x_2 \tan\theta = (343\,\text{m/s})(12.0\,\text{s})\tan(56.4^\circ) = 7150\,\text{m}.$$

P19-18 (a) The target "hears"

$$f' = f_s \frac{v+V}{v}.$$

This sound is reflected back to the detector which then "hears"

$$f_r = f' \frac{v}{v-V} = f_s \frac{v+V}{v-V}.$$

(b) Rearranging,

$$V/v = \frac{f_r - f_s}{f_r + f_s} \approx \frac{1}{2}\frac{f_r - f_s}{f_s},$$

where we have assumed that the source frequency and the reflected frequency are almost identical, so that when added $f_r + f_s \approx 2f_s$.

$\boxed{\textbf{P19-19}}$ (a) We apply Eq. 19-44

$$f' = f\frac{v+v_O}{v-v_S} = (1030\text{ Hz})\frac{(5470\text{ km/h}) + (94.6\text{ km/h})}{(5470\text{ km/h}) - (20.2\text{ km/h})} = 1050\text{ Hz}$$

(b) The reflected signal has a frequency equal to that of the signal received by the second sub originally. Applying Eq. 19-44 again,

$$f' = f\frac{v + v_O}{v - v_S} = (1050\text{ Hz})\frac{(5470\text{ km/h}) + (20.2\text{ km/h})}{(5470\text{ km/h}) - (94.6\text{ km/h})} = 1070\text{ Hz}$$

P19-20 In this case $v_S = 75.2\text{ km/h} - 30.5\text{ km/h} = 12.4\,\text{m/s}$. Then

$$f' = (989\text{ Hz})(1482\,\text{m/s})(1482\,\text{m/s} - 12.4\,\text{m/s}) = 997\text{ Hz}.$$

P19-21 There is no relative motion between the source and observer, so there is no frequency shift regardless of the wind direction.

P19-22 (a) $v_S = 34.2\,\text{m/s}$ and $v_O = 34.2\,\text{m/s}$, so

$$f' = (525\text{ Hz})(343\,\text{m/s} + 34.2\,\text{m/s})/(343\,\text{m/s} - 34.2\,\text{m/s}) = 641\text{ Hz}.$$

(b) $v_S = 34.2\,\text{m/s} + 15.3\,\text{m/s} = 49.5\,\text{m/s}$ and $v_O = 34.2\,\text{m/s} - 15.3\,\text{m/s} = 18.9\,\text{m/s}$, so

$$f' = (525\text{ Hz})(343\,\text{m/s} + 18.9\,\text{m/s})/(343\,\text{m/s} - 49.5\,\text{m/s}) = 647\text{ Hz}.$$

(c) $v_S = 34.2\,\text{m/s} - 15.3\,\text{m/s} = 18.9\,\text{m/s}$ and $v_O = 34.2\,\text{m/s} + 15.3\,\text{m/s} = 49.5\,\text{m/s}$, so

$$f' = (525\text{ Hz})(343\,\text{m/s} + 49.5\,\text{m/s})/(343\,\text{m/s} - 18.9\,\text{m/s}) = 636\text{ Hz}.$$

E20-1 (a) $t = x/v = (0.20\,\text{m})/(0.941)(3.00\times10^8\text{m/s}) = 7.1\times10^{-10}\text{s}$.
(b) $y = -gt^2/2 = -(9.81\,\text{m/s}^2)(7.1\times10^{-10}\text{s})^2/2 = 2.5\times10^{-18}\text{m}$.

E20-2 $L = L_0\sqrt{1-u^2/c^2} = (2.86\,\text{m})\sqrt{1-(0.999987)^2} = 1.46$ cm.

$\boxed{\textbf{E20-3}}$ $L = L_0\sqrt{1-u^2/c^2} = (1.68\,\text{m})\sqrt{1-(0.632)^2} = 1.30\,\text{m}$.

E20-4 Solve $\Delta t = \Delta t_0/\sqrt{1-u^2/c^2}$ for u:

$$u = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = (3.00\times10^8\text{m/s})\sqrt{1-\left(\frac{(2.20\,\mu\text{s})}{(16.0\,\mu\text{s})}\right)^2} = 2.97\times10^8\text{m/s}.$$

$\boxed{\textbf{E20-5}}$ We can apply $\Delta x = v\Delta t$ to find the time the particle existed before it decayed. Then

$$\Delta t = \frac{x}{v}\frac{(1.05\times10^{-3}\,\text{m})}{(0.992)(3.00\times10^8\,\text{m/s})} = 3.53\times10^{-12}\,\text{s}.$$

The *proper lifetime* of the particle is

$$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (3.53\times10^{-12}\,\text{s})\sqrt{1-(0.992)^2} = 4.46\times10^{-13}\,\text{s}.$$

E20-6 Apply Eq. 20-12:

$$v = \frac{(0.43c)+(0.587c)}{1+(0.43c)(0.587c)/c^2} = 0.812c.$$

E20-7 (a) $L = L_0\sqrt{1-u^2/c^2} = (130\,\text{m})\sqrt{1-(0.740)^2} = 87.4\,\text{m}$
(b) $\Delta t = L/v = (87.4\,\text{m})/(0.740)(3.00\times10^8\text{m/s}) = 3.94\times10^{-7}\text{s}$.

E20-8 $\Delta t = \Delta t_0/\sqrt{1-u^2/c^2} = (26\text{ ns})\sqrt{1-(0.99)^2} = 184$ ns. Then

$$L = v\Delta t = (0.99)(3.00\times10^8\text{m/s})(184\times10^{-9}\text{s}) = 55\,\text{m}.$$

E20-9 (a) $v_\text{g} = 2v = (7.91+7.91)$ km/s $= 15.82$ km/s.
(b) A relativistic treatment yields $v_\text{r} = 2v/(1+v^2/c^2)$. The fractional error is

$$\frac{v_\text{g}}{v_\text{r}} - 1 = \left(1+\frac{v^2}{c^2}\right) - 1 = \frac{v^2}{c^2} = \frac{(7.91\times10^3\text{m/s})^2}{(3.00\times10^8\text{m/s})^2} = 6.95\times10^{-10}.$$

E20-10 Invert Eq. 20-15 to get $\beta = \sqrt{1-1/\gamma^2}$.
(a) $\beta = \sqrt{1-1/(1.01)^2} = 0.140$.
(b) $\beta = \sqrt{1-1/(10.0)^2} = 0.995$.
(c) $\beta = \sqrt{1-1/(100)^2} = 0.99995$.
(d) $\beta = \sqrt{1-1/(1000)^2} = 0.9999995$.

$\boxed{\textbf{E20-11}}$ The distance traveled by the particle is (6.0 y)c; the time required for the particle to travel this distance is 8.0 years. Then the speed of the particle is

$$v = \frac{\Delta x}{\Delta t} = \frac{(6.0 \text{ y})c}{(8.0 \text{ y})} = \frac{3}{4}c.$$

The speed parameter β is given by

$$\beta = \frac{v}{c} = \frac{\frac{3}{4}c}{c} = \frac{3}{4}.$$

E20-12 $\gamma = 1/\sqrt{1-(0.950)^2} = 3.20$. Then

$$\begin{aligned} x' &= (3.20)[(1.00\times10^5\text{m}) - (0.950)(3.00\times10^8\text{m/s})(2.00\times10^{-4}\text{s})] = 1.38\times10^5\text{m}, \\ t' &= (3.20)[(2.00\times10^{-4}\text{s}) - (1.00\times10^5\text{m})(0.950)/(3.00\times10^8\text{m/s})] = -3.73\times10^{-4}\text{s}. \end{aligned}$$

E20-13 (a) $\gamma = 1/\sqrt{1-(0.380)^2} = 1.081$. Then

$$\begin{aligned} x' &= (1.081)[(3.20\times10^8\text{m}) - (0.380)(3.00\times10^8\text{m/s})(2.50\,\text{s})] = 3.78\times10^7\text{m}, \\ t' &= (1.081)[(2.50\,\text{s}) - (3.20\times10^8\text{m})(0.380)/(3.00\times10^8\text{m/s})] = 2.26\,\text{s}. \end{aligned}$$

(b) $\gamma = 1/\sqrt{1-(0.380)^2} = 1.081$. Then

$$\begin{aligned} x' &= (1.081)[(3.20\times10^8\text{m}) - (-0.380)(3.00\times10^8\text{m/s})(2.50\,\text{s})] = 6.54\times10^8\text{m}, \\ t' &= (1.081)[(2.50\,\text{s}) - (3.20\times10^8\text{m})(-0.380)/(3.00\times10^8\text{m/s})] = 3.14\,\text{s}. \end{aligned}$$

E20-14

E20-15 (a) $v_x' = (-u)/(1-0)$ and $v_y' = c\sqrt{1-u^2/c^2}$.
(b) $(v_x')^2 + (v_y')^2 = u^2 + c^2 - u^2 = c^2$.

E20-16 $v' = (0.787c + 0.612c)/[1 + (0.787)(0.612)] = 0.944c$.

$\boxed{\textbf{E20-17}}$ (a) The first part is easy; we appear to be moving away from A at the same speed as A appears to be moving away from us: $0.347c$.

(b) Using the velocity transformation formula, Eq. 20-18,

$$v_x' = \frac{v_x - u}{1 - uv_x/c^2} = \frac{(0.347c) - (-0.347c)}{1 - (-0.347c)(0.347c)/c^2} = 0.619c.$$

The negative sign reflects the fact that these two velocities are in *opposite* directions.

E20-18 $v' = (0.788c - 0.413c)/[1 + (0.788)(-0.413)] = 0.556c$.

E20-19 (a) $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

$$\begin{aligned} v_x' &= \frac{v_x}{\gamma(1 - uv_y/c^2)} = \frac{3(0.8c)}{5[1-(0)]} = \frac{12}{25}c, \\ v_y' &= \frac{v_y - u}{1 - uv_y/c^2} = \frac{(0) - (0.8c)}{1 - (0)} = -\frac{4}{5}c. \end{aligned}$$

Then $v' = c\sqrt{(-4/5)^2 + (12/25)^2} = 0.933c$ directed $\theta = \arctan(-12/20) = 31°$ East of South.

(b) $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

$$\begin{aligned} v'_x &= \frac{v_x - u}{1 - uv_x/c^2} = \frac{(0)-(-0.8c)}{1-(0)} = +\frac{4}{5}c, \\ v'_y &= \frac{v_y}{\gamma(1-uv_x/c^2)} = \frac{3(0.8c)}{5[1-(0)]} = \frac{12}{25}c. \end{aligned}$$

Then $v' = c\sqrt{(4/5)^2 + (12/25)^2} = 0.933c$ directed $\theta = \arctan(20/12) = 59°$ West of North.

E20-20 This exercise should occur in Section 20-9.
(a) $v = 2\pi(6.37\times10^6\text{m})c/(1\,\text{s})(3.00\times10^8\text{m/s}) = 0.133c$.
(b) $K = (\gamma-1)mc^2 = (1/\sqrt{1-(0.133)^2} - 1)(511\text{ keV}) = 4.58\text{ keV}$.
(c) $K_c = mv^2/2 = mc^2(v^2/c^2)/2 = (511\text{ keV})(0.133)^2/2 = 4.52\text{ keV}$. The percent error is

$$(4.52 - 4.58)/(4.58) = -1.31\%.$$

E20-21 $\Delta L = L' - L_0$ so

$$\Delta L = 2(6.370\times10^6\text{m})(1 - \sqrt{1-(29.8\times10^3\text{m/s})^2/(3.00\times10^8\text{m/s})^2}) = 6.29\times10^{-2}\text{m}.$$

E20-22 (a) $\Delta L/L_0 = 1 - L'/L_0$ so

$$\Delta L = (1 - \sqrt{1-(522\,\text{m/s})^2/(3.00\times10^8\text{m/s})^2}) = 1.51\times10^{-12}.$$

(b) We want to solve $\Delta t - \Delta t' = 1\,\mu\text{s}$, or

$$1\,\mu\text{s} = \Delta t(1 - 1/\sqrt{1-(522\,\text{m/s})^2/(3.00\times10^8\text{m/s})^2}),$$

which has solution $\Delta t = 6.61\times10^5\text{s}$. That's 7.64 days.

E20-23 The length of the ship as measured in the "certain" reference frame is

$$L = L_0\sqrt{1-v^2/c^2} = (358\,\text{m})\sqrt{1-(0.728)^2} = 245\,\text{m}.$$

In a time Δt the ship will move a distance $x_1 = v_1\Delta t$ while the micrometeorite will move a distance $x_2 = v_2\Delta t$; since they are moving toward each other then the micrometeorite will pass then ship when $x_1 + x_2 = L$. Then

$$\Delta t = L/(v_1+v_2) = (245\,\text{m})/[(0.728+0.817)(3.00\times10^8\text{m/s})] = 5.29\times10^{-7}\,\text{s}.$$

This answer is the time measured in the "certain" reference frame. We can use Eq. 20-21 to find the time as measured on the ship,

$$\Delta t = \frac{\Delta t' + u\Delta x'/c^2}{\sqrt{1-u^2/c^2}} = \frac{(5.29\times10^{-7}\,\text{s}) + (0.728c)(116\,\text{m})/c^2}{\sqrt{1-(0.728)^2}} = 1.23\times10^{-6}\text{s}.$$

E20-24 (a) $\gamma = 1/\sqrt{1-(0.622)^2} = 1.28$.
(b) $\Delta t = (183\,\text{m})/(0.622)(3.00\times10^8\text{m/s}) = 9.81\times10^{-7}\text{s}$. On the clock, however,

$$\Delta t' = \Delta t/\gamma = (9.81\times10^{-7}\text{s})/(1.28) = 7.66\times10^{-7}\text{s}.$$

E20-25 (a) $\Delta t = (26.0 \text{ ly})/(0.988)(1.00 \text{ ly/y}) = 26.3$ y.
(b) The signal takes 26 years to return, so $26 + 26.3 = 52.3$ years.
(c) $\Delta t' = (26.3 \text{ y})\sqrt{1-(0.988)^2} = 4.06$ y.

E20-26 (a) $\gamma = (1000 \text{ y})(1 \text{ y}) = 1000$;

$$v = c\sqrt{1-1/\gamma^2} \approx c(1-1/2\gamma^2) = 0.9999995c$$

(b) No.

E20-27 $(5.61\times10^{29} \text{ MeV}/c^2)c/(3.00\times10^8\text{m/s}) = 1.87\times10^{21} \text{ MeV}/c$.

E20-28 $p^2 = m^2c^2 = m^2v^2/(1-v^2/c^2)$, so $2v^2/c^2 = 1$, or $v = \sqrt{2}c$.

E20-29 The magnitude of the momentum of a relativistic particle in terms of the magnitude of the velocity is given by Eq. 20-23,

$$p = \frac{mv}{\sqrt{1-v^2/c^2}}.$$

The speed parameter, β, is what we are looking for, so we need to rearrange the above expression for the quantity v/c.

$$\begin{aligned}
p/c &= \frac{mv/c}{\sqrt{1-v^2/c^2}},\\
\frac{p}{c} &= \frac{m\beta}{\sqrt{1-\beta^2}},\\
\frac{mc}{p} &= \frac{\sqrt{1-\beta^2}}{\beta},\\
\frac{mc}{p} &= \sqrt{1/\beta^2-1}.
\end{aligned}$$

Rearranging,

$$\begin{aligned}
\frac{mc^2}{pc} &= \sqrt{1/\beta^2-1},\\
\left(\frac{mc^2}{pc}\right)^2 &= \frac{1}{\beta^2}-1,\\
\sqrt{\left(\frac{mc^2}{pc}\right)^2+1} &= \frac{1}{\beta},\\
\frac{pc}{\sqrt{m^2c^4+p^2c^2}} &= \beta
\end{aligned}$$

(a) For the electron,

$$\beta = \frac{(12.5 \text{ MeV}/c)c}{\sqrt{(0.511 \text{ MeV}/c^2)^2c^4 + (12.5 \text{ MeV}/c)^2c^2}} = 0.999.$$

(b) For the proton,

$$\beta = \frac{(12.5 \text{ MeV}/c)c}{\sqrt{(938 \text{ MeV}/c^2)^2c^4 + (12.5 \text{ MeV}/c)^2c^2}} = 0.0133.$$

E20-30 $K = mc^2(\gamma - 1)$, so $\gamma = 1 + K/mc^2$. $\beta = \sqrt{1 - 1/\gamma^2}$.
(a) $\gamma = 1 + (1.0\text{ keV})/(511\text{ keV}) = 1.00196$. $\beta = 0.0625c$.
(b) $\gamma = 1 + (1.0\text{ MeV})/(0.511\text{ MeV}) = 2.96$. $\beta = 0.941c$.
(c) $\gamma = 1 + (1.0\text{ GeV})/(0.511\text{ MeV}) = 1960$. $\beta = 0.99999987c$.

$\boxed{\textbf{E20-31}}$ The kinetic energy is given by Eq. 20-27,

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2.$$

We rearrange this to solve for $\beta = v/c$,

$$\beta = \sqrt{1 - \left(\frac{mc^2}{K + mc^2}\right)^2}.$$

It is actually *much* easier to find γ, since

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

so $K = \gamma mc^2 - mc^2$ implies

$$\gamma = \frac{K + mc^2}{mc^2}$$

(a) For the electron,

$$\beta = \sqrt{1 - \left(\frac{(0.511\text{ MeV}/c^2)c^2}{(10\text{ MeV}) + (0.511\text{ MeV}/c^2)c^2}\right)^2} = 0.9988,$$

and

$$\gamma = \frac{(10\text{ MeV}) + (0.511\text{ MeV}/c^2)c^2}{(0.511\text{ MeV}/c^2)c^2} = 20.6.$$

(b) For the proton,

$$\beta = \sqrt{1 - \left(\frac{(938\text{ MeV}/c^2)c^2}{(10\text{ MeV}) + (938\text{ MeV}/c^2)c^2}\right)^2} = 0.0145,$$

and

$$\gamma = \frac{(10\text{ MeV}) + (938\text{ MeV}/c^2)c^2}{(938\text{ MeV}/c^2)c^2} = 1.01.$$

(b) For the alpha particle,

$$\beta = \sqrt{1 - \left(\frac{4(938\text{ MeV}/c^2)c^2}{(10\text{ MeV}) + 4(938\text{ MeV}/c^2)c^2}\right)^2} = 0.73,$$

and

$$\gamma = \frac{(10\text{ MeV}) + 4(938\text{ MeV}/c^2)c^2}{4(938\text{ MeV}/c^2)c^2} = 1.0027.$$

E20-32 $\gamma = 1/\sqrt{1 - (0.99)^2} = 7.089$.
(a) $E = \gamma mc^2 = (7.089)(938.3\text{ MeV}) = 6650\text{ MeV}$. $K = E - mc^2 = 5710\text{ MeV}$. $p = mv\gamma = (938.3\text{ MeV}/c^2)(0.99c)(7.089) = 6580\text{ MeV}/c$.
(b) $E = \gamma mc^2 = (7.089)(0.511\text{ MeV}) = 3.62\text{ MeV}$. $K = E - mc^2 = 3.11\text{ MeV}$. $p = mv\gamma = (0.511\text{ MeV}/c^2)(0.99c)(7.089) = 3.59\text{ MeV}/c$.

E20-33 $\Delta m/\Delta t = (1.2\times10^{41}\text{W})/(3.0\times10^{8}\text{m/s})^2 = 1.33\times10^{24}\text{kg/s}$, which is

$$\frac{\Delta m}{\Delta t} = \frac{(1.33\times10^{24}\text{kg/s})(3.16\times10^{7}\text{s/y})}{(1.99\times10^{30}\text{kg/sun})} = 21.1$$

E20-34 (a) If $K = E - mc^2 = 2mc^2$, then $E = 3mc^2$, so $\gamma = 3$, and

$$v = c\sqrt{1-1/\gamma^2} = c\sqrt{1-1/(3)^2} = 0.943c.$$

(b) If $E = 2mc^2$, then $\gamma = 2$, and

$$v = c\sqrt{1-1/\gamma^2} = c\sqrt{1-1/(2)^2} = 0.866c.$$

E20-35 (a) The kinetic energy is given by Eq. 20-27,

$$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2\left(\left(1-\beta^2\right)^{-1/2} - 1\right).$$

We want to expand the $1-\beta^2$ part for small β,

$$\left(1-\beta^2\right)^{-1/2} = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots$$

Inserting this into the kinetic energy expression,

$$K = \frac{1}{2}mc^2\beta^2 + \frac{3}{8}mc^2\beta^4 + \cdots$$

But $\beta = v/c$, so

$$K = \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$$

(b) We want to know when the error because of neglecting the second (and higher) terms is 1%; or

$$0.01 = (\frac{3}{8}m\frac{v^4}{c^2})/(\frac{1}{2}mv^2) = \frac{3}{4}\left(\frac{v}{c}\right)^2.$$

This will happen when $v/c = \sqrt{(0.01)4/3)} = 0.115$.

E20-36 $K_c = (1000\,\text{kg})(20\,\text{m/s})^2/2 = 2.0\times10^5\text{J}$. The relativistic calculation is slightly harder:

$$\begin{aligned}
K_r &= (1000\,\text{kg})(3\times10^8\text{m/s})^2(1/\sqrt{1-(20\,\text{m/s})^2/(3\times10^8\text{m/s})^2}-1),\\
&\approx (1000\,\text{kg})\left[\frac{1}{2}(20\,\text{m/s})^2 + \frac{3}{8}(20\,\text{m/s})^4/(3\times10^8\text{m/s})^2 + \ldots\right],\\
&= 2.0\times10^5\text{J} + 6.7\times10^{-10}\text{J}.
\end{aligned}$$

E20-37 Start with Eq. 20-34 in the form

$$E^2 = (pc)^2 + (mc^2)^2$$

The rest energy is mc^2, and if the total energy is three times this then $E = 3mc^2$, so

$$\begin{aligned}
(3mc^2)^2 &= (pc)^2 + (mc^2)^2,\\
8(mc^2)^2 &= (pc)^2,\\
\sqrt{8}\,mc &= p.
\end{aligned}$$

E20-38 The initial kinetic energy is

$$K_{\rm i} = \frac{1}{2}m\left(\frac{2v}{1+v^2/c^2}\right) = \frac{2mv^2}{(1+v^2/c^2)^2}.$$

The final kinetic energy is

$$K_{\rm f} = 2\frac{1}{2}m\left(v\sqrt{2-v^2/c^2}\right)^2 = mv^2(2-v^2/c^2).$$

E20-39 This exercise is much more involved than the previous one!

The initial kinetic energy is

$$\begin{aligned} K_{\rm i} &= \frac{mc^2}{\sqrt{1-\left(\frac{2v}{1+v^2/c^2}\right)^2/c^2}} - mc^2, \\ &= \frac{mc^2(1+v^2/c^2)}{\sqrt{(1+v^2/c^2)^2-4v^2/c^2}} - mc^2, \\ &= \frac{m(c^2+v^2)}{1-v^2/c^2} - \frac{m(c^2-v^2)}{1-v^2/c^2}, \\ &= \frac{2mv^2}{1-v^2/c^2}. \end{aligned}$$

The final kinetic energy is

$$\begin{aligned} K_{\rm f} &= 2\frac{mc^2}{\sqrt{1-\left(v\sqrt{2-v^2/c^2}\right)^2/c^2}} - 2mc^2, \\ &= 2\frac{mc^2}{\sqrt{1-(v^2/c^2)(2-v^2/c^2)}} - 2mc^2, \\ &= 2\frac{mc^2}{1-v^2/c^2} - 2mc^2, \\ &= 2\frac{mc^2}{1-v^2/c^2} - 2\frac{m(c^2-v^2)}{1-v^2/c^2}, \\ &= \frac{2mv^2}{1-v^2/c^2}. \end{aligned}$$

E20-40 For a particle with mass, $\gamma = K/mc^2 + 1$. For the electron, $\gamma = (0.40)/(0.511) + 1 = 1.78$. For the proton, $\gamma = (10)/(938) + 1 = 1.066$.

For the photon, $pc = E$. For a particle with mass, $pc = \sqrt{(K+mc^2)^2 - m^2c^4}$. For the electron,

$$pc = \sqrt{[(0.40\text{ MeV}) + (0.511\text{ MeV})]^2 - (0.511\text{ MeV})^2} = 0.754\text{ MeV}.$$

For the proton,

$$pc = \sqrt{[(10\text{ MeV}) + (938\text{ MeV})]^2 - (938\text{ MeV})^2} = 137\text{ MeV}.$$

(a) Only photons move at the speed of light, so it is moving the fastest.
(b) The proton, since it has smallest value for γ.
(c) The proton has the greatest momentum.
(d) The photon has the least.

E20-41 Work is change in energy, so

$$W = mc^2/\sqrt{1-(v_\mathrm{f}/c)^2} - mc^2/\sqrt{1-(v_\mathrm{i}/c)^2}.$$

(a) Plug in the numbers,

$$W = (0.511 \text{ MeV})(1/\sqrt{1-(0.19)^2} - 1/\sqrt{1-(0.18)^2}) = 0.996 \text{ keV}.$$

(b) Plug in the numbers,

$$W = (0.511 \text{ MeV})(1/\sqrt{1-(0.99)^2} - 1/\sqrt{1-(0.98)^2}) = 1.05 \text{ MeV}.$$

E20-42 $E = 2\gamma m_0 c^2 = mc^2$, so

$$m = 2\gamma m_0 = 2(1.30 \text{ mg})/\sqrt{1-(0.580)^2} = 3.19 \text{ mg}.$$

E20-43 (a) Energy conservation requires $E_\mathrm{k} = 2E_\pi$, or $m_\mathrm{k}c^2 = 2\gamma m_\pi c^2$. Then

$$\gamma = (498 \text{ MeV})/2(140 \text{ MeV}) = 1.78$$

This corresponds to a speed of $v = c\sqrt{1-1/(1.78)^2} = 0.827c$.

(b) $\gamma = (498 \text{ MeV} + 325 \text{ MeV})/(498 \text{ MeV}) = 1.65$, so $v = c\sqrt{1-1/(1.65)^2} = 0.795c$.

(c) The lab frame velocities are then

$$v_1' = \frac{(0.795)+(-0.827)}{1+(0.795)(-0.827)}c = -0.0934c,$$

and

$$v_2' = \frac{(0.795)+(0.827)}{1+(0.795)(0.827)}c = 0.979c,$$

The corresponding kinetic energies are

$$K_1 = (140 \text{ MeV})(1/\sqrt{1-(-0.0934)^2} - 1) = 0.614 \text{ MeV}$$

and

$$K_1 = (140 \text{ MeV})(1/\sqrt{1-(0.979)^2} - 1) = 548 \text{ MeV}$$

E20-44

P20-1 (a) $\gamma = 2$, so $v = \sqrt{1-1/(2)^2} = 0.866c$.

(b) $\gamma = 2$.

P20-2 (a) Classically, $v' = (0.620c) + (0.470c) = 1.09c$. Relativistically,

$$v' = \frac{(0.620c)+(0.470c)}{1+(0.620)(0.470)} = 0.844c.$$

(b) Classically, $v' = (0.620c) + (-0.470c) = 0.150c$. Relativistically,

$$v' = \frac{(0.620c)+(-0.470c)}{1+(0.620)(-0.470)} = 0.211c.$$

P20-3 (a) $\gamma = 1/\sqrt{1-(0.247)^2} = 1.032$. Use the equations from Table 20-2.

$$\Delta t = (1.032)[(0) - (0.247)(30.4\times10^3\text{m})/(3.00\times10^8\text{m/s})] = -2.58\times10^{-5}\text{s}.$$

(b) The red flash appears to go first.

P20-4 Once again, the "pico" should have been a μ.
$\gamma = 1/\sqrt{1-(0.60)^2} = 1.25$. Use the equations from Table 20-2.

$$\Delta t = (1.25)[(4.0\times10^{-6}\text{s}) - (0.60)(3.0\times10^3\text{m})/(3.00\times10^8\text{m/s})] = -2.5\times10^{-6}\text{s}.$$

$\boxed{\textbf{P20-5}}$ We can choose our coordinate system so that u is directed along the x axis without any loss of generality. Then, according to Table 20-2,

$$\begin{aligned}
\Delta x' &= \gamma(\Delta x - u\Delta t),\\
\Delta y' &= \Delta y,\\
\Delta z' &= \Delta z,\\
c\Delta t' &= \gamma(c\Delta t - u\Delta x/c).
\end{aligned}$$

Square these expressions,

$$\begin{aligned}
(\Delta x')^2 &= \gamma^2(\Delta x - u\Delta t)^2 = \gamma^2\left((\Delta x)^2 - 2u(\Delta x)(\Delta t) + (\Delta t)^2\right),\\
(\Delta y')^2 &= (\Delta y)^2,\\
(\Delta z')^2 &= (\Delta z)^2,\\
c^2(\Delta t')^2 &= \gamma^2(c\Delta t - u\Delta x/c)^2 = \gamma^2\left(c^2(\Delta t)^2 - 2u(\Delta t)(\Delta x) + u^2(\Delta x)^2/c^2\right).
\end{aligned}$$

We'll add the first three equations and then subtract the fourth. The left hand side is the equal to

$$(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - c^2(\Delta t')^2,$$

while the right hand side will equal

$$\gamma^2\left((\Delta x)^2 + u^2(\Delta t)^2 - c^2(\Delta t)^2 - u^2/c^2(\Delta x)^2\right) + (\Delta y)^2 + (\Delta z)^2,$$

which can be rearranged as

$$\begin{gathered}
\gamma^2\left(1-u^2/c^2\right)(\Delta x)^2 + \gamma^2\left(u^2-c^2\right)(\Delta t)^2 + (\Delta y)^2 + (\Delta z)^2,\\
\gamma^2\left(1-u^2/c^2\right)(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2\gamma^2\left(1-u^2/c^2\right)(\Delta t)^2.
\end{gathered}$$

But

$$\gamma^2 = \frac{1}{1-u^2/c^2},$$

so the previous expression will simplify to

$$(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2.$$

P20-6 (a) $v_x = [(0.780c) + (0.240c)]/[1 + (0.240)(0.780)] = 0.859c$.
(b) $v_x = [(0) + (0.240c)]/[1 + (0)] = 0.240c$, while

$$v_y = (0.780c)\sqrt{1-(0.240)^2}/[1+(0)] = 0.757c.$$

Then $v = \sqrt{(0.240c)^2 + (0.757c)^2} = 0.794c$.
(b) $v'_x = [(0) - (0.240c)]/[1 + (0)] = -0.240c$, while

$$v'_y = (0.780c)\sqrt{1-(0.240)^2}/[1+(0)] = 0.757c.$$

Then $v' = \sqrt{(-0.240c)^2 + (0.757c)^2} = 0.794c$.

$\boxed{\textbf{P20-7}}$ If we look back at the boost equation we might notice that it looks *very* similar to the rule for the tangent of the sum of two angles. It is exactly the same as the rule for the *hyperbolic* tangent,

$$\tanh(\alpha_1 + \alpha_2) = \frac{\tanh\alpha_1 + \tanh\alpha_2}{1 + \tanh\alpha_1\,\tanh\alpha_2}.$$

This means that each boost of $\beta = 0.5$ is the same as a "hyperbolic" rotation of α_r where $\tanh\alpha_r = 0.5$. We need only add these rotations together until we get to α_f, where $\tanh\alpha_\mathrm{f} = 0.999$.

$\alpha_\mathrm{f} = 3.800$, and $\alpha_R = 0.5493$. We can fit $(3.800)/(0.5493) = 6.92$ boosts, but we need an integral number, so there are seven boosts required. The final speed after these seven boosts will be $0.9991c$.

P20-8 (a) If $\Delta x' = 0$, then $\Delta x = u\Delta t$, or

$$u = (730\,\mathrm{m})/(4.96\times10^{-6}\mathrm{s}) = 1.472\times10^{8}\mathrm{m/s} = 0.491c.$$

(b) $\gamma = 1/\sqrt{1-(0.491)^2} = 1.148$,

$$\Delta t' = (1.148)[(4.96\times10^{-6}\mathrm{s}) - (0.491)(730\,\mathrm{m})/(3\times10^{8})] = 4.32\times10^{-6}\mathrm{s}.$$

P20-9 Since the maximum value for u is c, then the minimum Δt is

$$\Delta t \geq (730\,\mathrm{m})/(3.00\times10^{8}\mathrm{m/s}) = 2.43\times10^{-6}\mathrm{s}.$$

P20-10 (a) Yes.

(b) The speed will be very close to the speed of light, consequently $\gamma \approx (23,000)/(30) = 766.7$. Then

$$v = \sqrt{1 - 1/\gamma^2} \approx 1 - 1/2\gamma^2 = 1 - 1/2(766.7)^2 = 0.99999915c.$$

P20-11 (a) $\Delta t' = (5.00\,\mu\mathrm{s})\sqrt{1-(0.6)^2} = 4.00\,\mu\mathrm{s}$.

(b) Note: it takes time for the reading on the S' clock to be seen by the S clock. In this case, $\Delta t_1 + \Delta t_2 = 5.00\,\mu\mathrm{s}$, where $\Delta t_1 = x/u$ and $\Delta t_2 = x/c$. Solving for Δt_1,

$$\Delta t_1 = \frac{(5.00\,\mu\mathrm{s})/(0.6c)}{1/(0.6c) + 1/c} = 3.125\,\mathrm{s},$$

and

$$\Delta t_1' = (3.125\,\mu\mathrm{s})\sqrt{1-(0.6)^2} = 2.50\,\mu\mathrm{s}.$$

P20-12 The only change in the components of Δr occur parallel to the boost. Then we can choose the boost to be parallel to Δr and then

$$\Delta r' = \gamma[\Delta r - u(0)] = \gamma\Delta r \geq \Delta r,$$

since $\gamma \geq 1$.

$\boxed{\textbf{P20-13}}$ (a) Start with Eq. 20-34,

$$E^2 = (pc)^2 + (mc^2)^2,$$

and substitute into this $E = K + mc^2$,

$$K^2 + 2Kmc^2 + (mc^2)^2 = (pc)^2 + (mc^2)^2.$$

We can rearrange this, and then

$$\begin{aligned} K^2 + 2Kmc^2 &= (pc)^2, \\ m &= \frac{(pc)^2 - K^2}{2Kc^2} \end{aligned}$$

(b) As $v/c \to 0$ we have $K \to \frac{1}{2}mv^2$ and $p \to mv$, the classical limits. Then the above expression becomes

$$\begin{aligned} m &= \frac{m^2v^2c^2 - \frac{1}{4}m^2v^4}{mv^2c^2}, \\ &= m\frac{v^2c^2 - \frac{1}{4}v^4}{v^2c^2}, \\ &= m\left(1 - \frac{1}{4}\frac{v^2}{c^2}\right) \end{aligned}$$

But $v/c \to 0$, so this expression reduces to $m = m$ in the classical limit, which is a *good thing.*

(c) We get

$$m = \frac{(121 \text{ MeV})^2 - (55.0 \text{ MeV})^2}{2(55.0 \text{ MeV})c^2} = 1.06 \text{ MeV}/c^2,$$

which is $(1.06 \text{ MeV}/c^2)/(0.511 \text{ MeV}/c^2) = 207m_e$. A muon.

P20-14 Since $E \gg mc^2$ the particle is ultra-relativistic and $v \approx c$. $\gamma = (135)/(0.1396) = 967$. Then the particle has a lab-life of $\Delta t' = (967)(35.0\times10^{-9}\text{s}) = 3.385\times10^{-5}\text{s}$. The distance traveled is

$$x = (3.00\times10^8\text{m/s})(3.385\times10^{-5}\text{s}) = 1.016\times10^4\text{m},$$

so the pion decays 110 km above the Earth.

P20-15 (a) A completely inelastic collision means the two particles, each of mass m_1, stick together after the collision, in effect becoming a new particle of mass m_2. We'll use the subscript 1 for moving particle of mass m_1, the subscript 0 for the particle which is originally at rest, and the subscript 2 for the new particle after the collision. We need to conserve momentum,

$$\begin{aligned} p_1 + p_0 &= p_2, \\ \gamma_1 m_1 u_1 + (0) &= \gamma_2 m_2 u_2, \end{aligned}$$

and we need to conserve total energy,

$$\begin{aligned} E_1 + E_0 &= E_2, \\ \gamma_1 m_1 c^2 + m_1 c^2 &= \gamma_2 m_2 c^2, \end{aligned}$$

Divide the momentum equation by the energy equation and then

$$\frac{\gamma_1 u_1}{\gamma_1 + 1} = u_2.$$

But $u_1 = c\sqrt{1 - 1/\gamma_1^2}$, so

$$\begin{aligned} u_2 &= c\frac{\gamma_1\sqrt{1 - 1/\gamma_1^2}}{\gamma_1 + 1}, \\ &= c\frac{\sqrt{\gamma_1^2 - 1}}{\gamma_1 + 1}, \end{aligned}$$

$$\begin{aligned} &= c\frac{\sqrt{(\gamma_1+1)(\gamma_1-1)}}{\gamma_1+1}, \\ &= c\sqrt{\frac{\gamma_1-1}{\gamma_1+1}}. \end{aligned}$$

(b) Using the momentum equation,

$$\begin{aligned} m_2 &= m_1\frac{\gamma_1 u_1}{\gamma_2 u_2}, \\ &= m_1\frac{c\gamma_1\sqrt{1-1/\gamma_1^2}}{u_2/\sqrt{1-(u_2/c)^2}}, \\ &= m_1\frac{\sqrt{\gamma_1^2-1}}{1/\sqrt{(c/u_2)^2-1}}, \\ &= m_1\frac{\sqrt{\gamma_1^2-1}}{1/\sqrt{(\gamma_1+1)/(\gamma_1-1)-1}}, \\ &= m_1\frac{\sqrt{(\gamma_1+1)(\gamma_1-1)}}{\sqrt{(\gamma_1-1)/2}}, \\ &= m_1\sqrt{2(\gamma_1+1)}. \end{aligned}$$

P20-16 (a) $K = W = \int F\,dx = \int(dp/dt)dx = \int(dx/dt)dp = \int v\,dp$.

(b) $dp = m\gamma\,dv + mv(d\gamma/dv)dv$. Now use Maple or Mathematica to save time, and get

$$dp = \frac{m\,dv}{(1-v^2/c^2)^{1/2}} + \frac{mv^2\,dv}{c^2(1-v^2/c^2)^{3/2}}.$$

Now integrate:

$$\begin{aligned} K &= \int v\left(\frac{m}{(1-v^2/c^2)^{1/2}} + \frac{mv^2}{c^2(1-v^2/c^2)^{3/2}}\right)dv, \\ &= \frac{mv^2}{\sqrt{1-v^2/c^2}}. \end{aligned}$$

P20-17 (a) Since $E = K + mc^2$, then

$$E_{\text{new}} = 2E = 2mc^2 + 2K = 2mc^2(1 + K/mc^2).$$

(b) $E_{\text{new}} = 2(0.938\text{ GeV}) + 2(100\text{ GeV}) = 202\text{ GeV}$.

(c) $K = (100\text{ GeV})/2 - (0.938\text{ GeV}) = 49.1\text{ GeV}$.

P20-18 (a) Assume only one particle is formed. That particle can later decay, but it sets the standard on energy and momentum conservation. The momentum of this one particle must equal that of the incident proton, or

$$p^2c^2 = [(mc^2+K)^2 - m^2c^4].$$

The initial energy was $K + 2mc^2$, so the mass of the "one" particle is given by

$$M^2c^4 = [(K+2mc^2)^2 - p^2c^2] = 2Kmc^2 + 4m^2c^4.$$

This is a measure of the available energy; the remaining energy is required to conserve momentum. Then

$$E_{\text{new}} = \sqrt{M^2c^4} = 2mc^2\sqrt{1 + K/2mc^2}.$$

P20-19 The initial momentum is $m\gamma_{\rm i}v_{\rm i}$. The final momentum is $(M-m)\gamma_{\rm f}v_{\rm f}$. Manipulating the momentum conservation equation,

$$\begin{aligned}
m\gamma_{\rm i}v_{\rm i} &= (M-m)\gamma_{\rm f}v_{\rm f},\\
\frac{1}{m\gamma_{\rm i}\beta_{\rm i}} &= \frac{\sqrt{1-{\beta_{\rm f}}^2}}{(M-m)\beta_{\rm f}},\\
\frac{M-m}{m\gamma_{\rm i}\beta_{\rm i}} &= \left(\frac{1}{{\beta_{\rm f}}^2}-1\right),\\
\frac{M-m}{m\gamma_{\rm i}\beta_{\rm i}}+1 &= \frac{1}{{\beta_{\rm f}}^2},
\end{aligned}$$

$\boxed{\textbf{E21-1}}$ (a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then $T_S = mT_C + b$, where m and b are constants to be determined, T_S is the temperature measurement in the "new" scale, and T_C is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

$$\begin{aligned} T_S &= mT_C + b, \\ (0) &= m(-273.15^\circ\text{C}) + b. \end{aligned}$$

We have two other points, the melting and boiling points for water,

$$\begin{aligned} (T_S)_{\text{bp}} &= m(100^\circ\text{C}) + b, \\ (T_S)_{\text{mp}} &= m(0^\circ\text{C}) + b; \end{aligned}$$

we can subtract the top equation from the bottom equation to get

$$(T_S)_{\text{bp}} - (T_s)S_{\text{mp}} = 100\ \text{C}^\circ m.$$

We are told this is 180 S$^\circ$, so $m = 1.8\ \text{S}^\circ/\text{C}^\circ$. Put this into the first equation and then find b, $b = 273.15^\circ\text{C}m = 491.67^\circ\text{S}$. The conversion is then

$$T_S = (1.8\ \text{S}^\circ/\text{C}^\circ)T_C + (491.67^\circ\text{S}).$$

(b) The melting point for water is 491.67°S; the boiling point for water is 180 S° above this, or 671.67°S.

E21-2 $T_\text{F} = 9(-273.15\,\text{deg C})/5 + 32^\circ\text{F} = -459.67^\circ\text{F}$.

E21-3 (a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then $T_S = mT_C + b$, where m and b are constants to be determined, T_S is the temperature measurement in the "new" scale, and T_C is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

$$\begin{aligned} T_S &= mT_C + b, \\ (0) &= m(-273.15^\circ\text{C}) + b. \end{aligned}$$

We have two other points, the melting and boiling points for water,

$$\begin{aligned} (T_S)_{\text{bp}} &= m(100^\circ\text{C}) + b, \\ (T_S)_{\text{mp}} &= m(0^\circ\text{C}) + b; \end{aligned}$$

we can subtract the top equation from the bottom equation to get

$$(T_S)_{\text{bp}} - (T_s)S_{\text{mp}} = 100\ \text{C}^\circ m.$$

We are told this is 100 Q$^\circ$, so $m = 1.0\ \text{Q}^\circ/\text{C}^\circ$. Put this into the first equation and then find b, $b = 273.15^\circ\text{C} = 273.15^\circ\text{Q}$. The conversion is then

$$T_S = T_C + (273.15^\circ\text{S}).$$

(b) The melting point for water is 273.15°Q; the boiling point for water is 100 Q° above this, or 373.15°Q.

(c) Kelvin Scale.

E21-4 (a) $T = (9/5)(6000\,\mathrm{K} - 273.15) + 32 = 10000°\mathrm{F}$.

(b) $T = (5/9)(98.6°\mathrm{F} - 32) = 37.0°\mathrm{C}$.

(c) $T = (5/9)(-70°\mathrm{F} - 32) = -57°\mathrm{C}$.

(d) $T = (9/5)(-183°\mathrm{C}) + 32 = -297°\mathrm{F}$.

(e) It depends on what you think is hot. My mom thinks 79°F is too warm; that's $T = (5/9)(79°\mathrm{F} - 32) = 26°\mathrm{C}$.

E21-5 $T = (9/5)(310\,\mathrm{K} - 273.15) + 32 = 98.3°\mathrm{F}$, which is fine.

E21-6 (a) $T = 2(5/9)(T - 32)$, so $-T/10 = -32$, or $T = 320°\mathrm{F}$.

(b) $2T = (5/9)(T - 32)$, so $13T/5 = -32$, or $T = -12.3°\mathrm{F}$.

E21-7 If the temperature (in Kelvin) is directly proportional to the resistance then $T = kR$, where k is a constant of proportionality. We are given one point, $T = 273.16$ K when $R = 90.35\ \Omega$, but that is okay; we only have one unknown, k. Then $(273.16\ \mathrm{K}) = k(90.35\ \Omega)$ or $k = 3.023\ \mathrm{K}/\Omega$.

If the resistance is measured to be $R = 96.28\ \Omega$, we have a temperature of

$$T = kR = (3.023\ \mathrm{K}/\Omega)(96.28\ \Omega) = 291.1\ \mathrm{K}.$$

E21-8 $T = (510°\mathrm{C})/(0.028\,\mathrm{V})V$, so $T = (1.82\times10^{4}°\mathrm{C/V})(0.0102\,\mathrm{V}) = 186°\mathrm{C}$.

E21-9 We must first find the equation which relates gain to temperature, and then find the gain at the specified temperature. If we let G be the gain we can write this linear relationship as

$$G = mT + b,$$

where m and b are constants to be determined. We have two known points:

$$\begin{aligned}(30.0) &= m(20.0°\ \mathrm{C}) + b,\\ (35.2) &= m(55.0°\ \mathrm{C}) + b.\end{aligned}$$

If we subtract the top equation from the bottom we get $5.2 = m(35.0°\ \mathrm{C})$, or $m = 1.49\,\mathrm{C}^{-1}$. Put this into either of the first two equations and

$$(30.0) = (0.149\,\mathrm{C}^{-1})(20.0°\ \mathrm{C}) + b,$$

which has a solution $b = 27.0$

Now to find the gain when $T = 28.0\,°\mathrm{C}$:

$$G = mT + b = (0.149\,\mathrm{C}^{-1})(28.0°\ \mathrm{C}) + (27.0) = 31.2$$

E21-10 $p/p_{\mathrm{tr}} = (373.15\,\mathrm{K})/(273.16\,\mathrm{K}) = 1.366$.

E21-11 100 cm Hg is 1000 torr. $P_{\mathrm{He}} = (100\ \mathrm{cm\ Hg})(373\,\mathrm{K})/(273.16\,\mathrm{K}) = 136.550$ cm Hg. Nitrogen records a temperature which is 0.2 K higher, so $P_{\mathrm{N}} = (100\ \mathrm{cm\ Hg})(373.2\,\mathrm{K})/(273.16\,\mathrm{K}) = 136.623$ cm Hg. The difference is 0.073 cm Hg.

E21-12 $\Delta L = (23\times10^{-6}/\mathrm{C}°)(33\,\mathrm{m})(15\mathrm{C}°) = 1.1\times10^{-2}\mathrm{m}$.

E21-13 $\Delta L = (3.2\times10^{-6}/\mathrm{C}°)(200\ \mathrm{in})(60\mathrm{C}°) = 3.8\times10^{-2}\mathrm{in}$.

E21-14 $L' = (2.725\text{cm})[1 + (23\times10^{-6}/\text{C}^\circ)(128\text{C}^\circ)] = 2.733$ cm.

$\boxed{\textbf{E21-15}}$ We want to focus on the temperature change, not the absolute temperature. In this case, $\Delta T = T_\text{f} - T_\text{i} = (42°\text{C}) - (-5.0°\text{C}) = 47\,\text{C}°$.

Then

$$\Delta L = (11 \times 10^{-6}\,\text{C}^{-1})(12.0\,\text{m})(47\,\text{C}°) = 6.2 \times 10^{-3}\,\text{m}.$$

E21-16 $\Delta A = 2\alpha A \Delta T$, so

$$\Delta A = 2(9\times10^{-6}/\text{C}^\circ)(2.0\,\text{m})(3.0\,\text{m})(30\text{C}^\circ) = 3.2\times10^{-3}\text{m}^2.$$

$\boxed{\textbf{E21-17}}$ (a) We'll apply Eq. 21-10. The surface area of a cube is six times the area of one face, which is the edge length squared. So $A = 6(0.332\,\text{m})^2 = 0.661\,\text{m}^2$. The temperature change is $\Delta T = (75.0°\text{C}) - (20.0°\text{C}) = 55.0\,\text{C}°$. Then the increase in surface area is

$$\Delta A = 2\alpha A \Delta T = 2(19 \times 10^{-6}\,\text{C}^{-1})(0.661\,\text{m}^2)(55.0\,\text{C}°) = 1.38 \times 10^{-3}\,\text{m}^2$$

(b) We'll now apply Eq. 21-11. The volume of the cube is the edge length cubed, so

$$V = (0.332\,\text{m})^3 = 0.0366\,\text{m}^3.$$

and then from Eq. 21-11,

$$\Delta V = 2\alpha V \Delta T = 3(19 \times 10^{-6}\,\text{C}^{-1})(0.0366\,\text{m}^3)(55.0\,\text{C}°) = 1.15 \times 10^{-4}\,\text{m}^3,$$

is the change in volume of the cube.

E21-18 $V' = V(1 + 3\alpha\Delta T)$, so

$$V' = (530\ \text{cm}^3)[1 + 3(29\times10^{-6}/\text{C}^\circ)(-172\,\text{C}^\circ)] = 522\ \text{cm}^3.$$

E21-19 (a) The slope is approximately $1.6\times10^{-4}/\text{C}^\circ$.

(b) The slope is zero.

E21-20 $\Delta r = (\beta/3) r \Delta T$, so

$$\Delta r = [(3.2\times10^{-5}/\text{K})/3](6.37\times10^6\text{m})(2700\,\text{K}) = 1.8\times10^5\text{m}.$$

$\boxed{\textbf{E21-21}}$ We'll assume that the steel ruler measures length correctly at room temperature. Then the 20.05 cm measurement of the rod is correct. But both the rod and the ruler will expand in the oven, so the 20.11 cm measurement of the rod is *not* the actual length of the rod in the oven. What is the actual length of the rod in the oven? We can only answer that after figuring out how the 20.11 cm mark on the ruler moves when the ruler expands.

Let $L = 20.11$ cm correspond to the ruler mark at room temperature. Then

$$\Delta L = \alpha_\text{steel} L \Delta T = (11 \times 10^{-6}\,\text{C}^{-1})(20.11\ \text{cm})(250\,\text{C}°) = 5.5 \times 10^{-2}\ \text{cm}$$

is the shift in position of the mark as the ruler is raised to the higher temperature. Then the change in length of the rod is *not* (20.11 cm) − (20.05 cm) = 0.06 cm, because the 20.11 cm mark is shifted out. We need to add 0.055 cm to this; the rod changed length by 0.115 cm.

The coefficient of thermal expansion for the rod is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{(0.115\ \text{cm})}{(20.05\ \text{cm})(250\,\text{C}°)} = 23 \times 10^{-6}\,\text{C}^{-1}.$$

E21-22 $A = ab$, $A' = (a + \Delta a)(b + \Delta b) = ab + a\Delta b + b\Delta a + \Delta a \Delta b$, so

$$\begin{aligned}\Delta A &= a\Delta b + b\Delta a + \Delta a\Delta b,\\ &= A(\Delta b/b + \Delta a/a + \Delta a\Delta b/ab),\\ &\approx A(\alpha\Delta T + \alpha\Delta T),\\ &= 2\alpha A\Delta T.\end{aligned}$$

$\boxed{\textbf{E21-23}}$ Solve this problem by assuming the solid is in the form of a cube.

If the length of one side of a cube is originally L_0, then the volume is originally $V_0 = L_0^3$. After heating, the volume of the cube will be $V = L^3$, where $L = L_0 + \Delta L$.

Then

$$\begin{aligned}V &= L^3,\\ &= (L_0 + \Delta L)^3,\\ &= (L_0 + \alpha L_0 \Delta T)^3,\\ &= L_0^3(1 + \alpha\Delta T)^3.\end{aligned}$$

As long as the quantity $\alpha\Delta T$ is much less than one we can expand the last line in a binomial expansion as

$$V \approx V_0(1 + 3\alpha\Delta T + \cdots),$$

so the change in volume is $\Delta V \approx 3\alpha V_0 \Delta T$.

E21-24 (a) $\Delta A/A = 2(0.18\%) = (0.36\%)$.

(b) $\Delta L/L = 0.18\%$.

(c) $\Delta V/V = 3(0.18\%) = (0.54\%)$.

(d) Zero.

(e) $\alpha = (0.0018)/(100\,\mathrm{C}^\circ) = 1.8\times 10^{-5}/\mathrm{C}^\circ$.

E21-25 $\rho' - \rho = m/V' - m/V = m/(V + \Delta V) - m/V \approx -m\Delta V/V^2$. Then

$$\Delta\rho = -(m/V)(\Delta V/V) = -\rho\beta\Delta T.$$

E21-26 Use the results of Exercise 21-25.

(a) $\Delta V/V = 3\Delta L/L = 3(0.092\%) = 0.276\%$. The change in density is

$$\Delta\rho/\rho = -\Delta V/V = -(0.276\%) = -0.28$$

(b) $\alpha = \beta/3 = (0.28\%)/3(40\,\mathrm{C}^\circ) = 2.3\times 10^{-5}/\mathrm{C}^\circ$. Must be aluminum.

$\boxed{\textbf{E21-27}}$ The diameter of the rod as a function of temperature is

$$d_\mathrm{s} = d_{\mathrm{s},0}(1 + \alpha_\mathrm{s}\Delta T),$$

The diameter of the ring as a function of temperature is

$$d_\mathrm{b} = d_{\mathrm{b},0}(1 + \alpha_\mathrm{b}\Delta T).$$

We are interested in the temperature when the diameters are equal,

$$\begin{aligned}
d_{\mathrm{s},0}(1+\alpha_{\mathrm{s}}\Delta T) &= d_{\mathrm{b},0}(1+\alpha_{\mathrm{b}}\Delta T),\\
\alpha_{\mathrm{s}}d_{\mathrm{s},0}\Delta T-\alpha_{\mathrm{b}}d_{\mathrm{b},0}\Delta T &= d_{\mathrm{b},0}-d_{\mathrm{s},0},\\
\Delta T &= \frac{d_{\mathrm{b},0}-d_{\mathrm{s},0}}{\alpha_{\mathrm{s}}d_{\mathrm{s},0}-\alpha_{\mathrm{b}}d_{\mathrm{b},0}},\\
\Delta T &= \frac{(2.992\text{ cm})-(3.000\text{ cm})}{(11\times10^{-6}/\mathrm{C}^\circ)(3.000\text{ cm})-(19\times10^{-6}/\mathrm{C}^\circ)(2.992\text{ cm})},\\
&= 335\,\mathrm{C}^\circ.
\end{aligned}$$

The final temperature is then $T_{\mathrm{f}} = (25^\circ) + 335\,\mathrm{C}^\circ = 360^\circ$.

E21-28 (a) $\Delta L = \Delta L_1 + \Delta L_2 = (L_1\alpha_1 + L_2\alpha_2)\Delta T$. The effective value for α is then

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L}.$$

(b) Since $L_2 = L - L_1$ we can write

$$\begin{aligned}
\alpha_1 L_1 + \alpha_2(L-L_1) &= \alpha L,\\
L_1 &= L\frac{\alpha-\alpha_2}{\alpha_1-\alpha_2},\\
&= (0.524\,\mathrm{m})\frac{(13\times10^{-6})-(11\times10^{-6})}{(19\times10^{-6})-(11\times10^{-6})} = 0.131\,\mathrm{m}.
\end{aligned}$$

The brass length is then 13.1 cm and the steel is 39.3 cm.

E21-29 At 100°C the glass and mercury each have a volume V_0. After cooling, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_{\mathrm{g}} - \beta_{\mathrm{m}})\Delta T.$$

Since $m = \rho V$, the mass of mercury that needs to be added can be found by multiplying though by the density of mercury. Then

$$\Delta m = (0.891\,\mathrm{kg})[3(9.0\times10^{-6}/\mathrm{C}^\circ) - (1.8\times10^{-4}/\mathrm{C}^\circ)](-135\mathrm{C}^\circ) = 0.0184\,\mathrm{kg}.$$

This is the additional amount required, so the total is now 909 g.

E21-30 (a) The rotational inertia is given by $I = \int r^2 dm$; changing the temperature requires $r \to r' = r + \Delta r = r(1+\alpha\Delta T)$. Then

$$I' = \int (1+\alpha\Delta T)^2 r^2 dm \approx (1+2\alpha\Delta T)\int r^2 dm,$$

so $\Delta I = 2\alpha I\Delta T$.

(b) Since $L = I\omega$, then $0 = \omega\Delta I + I\Delta\omega$. Rearranging, $\Delta\omega/\omega = -\Delta I/I = -2\alpha\Delta T$. Then

$$\Delta\omega = -2(19\times10^{-6}/\mathrm{C}^\circ)(230\text{ rev/s})(170\,\mathrm{C}^\circ) = -1.5\text{ rev/s}.$$

$\boxed{\textbf{E21-31}}$ This problem is related to objects which expand when heated, but we never actually need to calculate any temperature changes. We will, however, be interested in the change in rotational inertia. Rotational inertia is directly proportional to the square of the (appropriate) linear dimension, so

$$I_f/I_i = (r_f/r_i)^2.$$

(a) If the bearings are frictionless then there are no external torques, so the angular momentum is constant.

(b) If the angular momentum is constant, then

$$\begin{aligned} L_i &= L_f, \\ I_i\omega_i &= I_f\omega_f. \end{aligned}$$

We are interested in the percent change in the angular velocity, which is

$$\frac{\omega_f - \omega_i}{\omega_i} = \frac{\omega_f}{\omega_i} - 1 = \frac{I_i}{I_f} - 1 = \left(\frac{r_i}{r_f}\right)^2 - 1 = \left(\frac{1}{1.0018}\right)^2 - 1 = -0.36\%.$$

(c) The rotational kinetic energy is proportional to $I\omega^2 = (I\omega)\omega = L\omega$, but L is constant, so

$$\frac{K_f - K_i}{K_i} = \frac{\omega_f - \omega_i}{\omega_i} = -0.36\%.$$

E21-32 (a) The period of a physical pendulum is given by Eq. 17-28. There are two variables in the equation that depend on length. I, which is proportional to a length squared, and d, which is proportional to a length. This means that the period have an overall dependence on length proportional to $\sqrt{r}$. Taking the derivative,

$$\Delta P \approx dP = \frac{1}{2}\frac{P}{r}dr \approx \frac{1}{2}P\alpha\Delta T.$$

(b) $\Delta P/P = (0.7\times10^{-6}\text{C}^\circ)(10\text{C}^\circ)/2 = 3.5\times10^{-6}$. After 30 days the clock will be slow by

$$\Delta t = (30 \times 24 \times 60 \times 60\,\text{s})(3.5\times10^{-6}) = 9.07\,\text{s}.$$

E21-33 Refer to the Exercise 21-32.

$$\Delta P = (3600\,\text{s})(19\times10^{-6}\text{C}^\circ)(-20\text{C}^\circ)/2 = 0.68\,\text{s}.$$

E21-34 At 22°C the aluminum cup and glycerin each have a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_a - \beta_g)\Delta T.$$

The amount that spills out is then

$$\Delta V = (110\text{ cm}^3)[3(23\times10^{-6}/\text{C}^\circ) - (5.1\times10^{-4}/\text{C}^\circ)](6\text{C}^\circ) = -0.29\text{ cm}^3.$$

E21-35 At 20.0°C the glass tube is filled with liquid to a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_g - \beta_l)\Delta T.$$

The cross sectional area of the tube changes according to

$$\Delta A = A_0 2\alpha_g\Delta T.$$

Consequently, the height of the liquid changes according to

$$\begin{aligned}\Delta V &= (h_0 + \Delta h)(A_0 + \Delta A) - h_0 A, \\ &\approx h_0 \Delta A + A_0 \Delta h, \\ \Delta V / V_0 &= \Delta A / A_0 + \Delta h / h_0.\end{aligned}$$

Then

$$\Delta h = (1.28\,\mathrm{m}/2)[(1.1\times10^{-5}/\mathrm{C}^\circ) - (4.2\times10^{-5}/\mathrm{C}^\circ)](13\,\mathrm{C}^\circ) = 2.6\times10^{-4}\mathrm{m}.$$

E21-36 (a) $\beta = (dV/dT)/V$. If $pV = nRT$, then $p\,dV = nR\,dT$, so

$$\beta = (nR/p)/V = nR/pV = 1/T.$$

(b) Kelvins.
(c) $\beta \approx 1/(300/\mathrm{K}) = 3.3\times10^{-3}/\mathrm{K}$.

E21-37 (a) $V = (1\text{ mol})(8.31\,\mathrm{J/mol\cdot K})(273\,\mathrm{K})/(1.01\times10^5\mathrm{Pa}) = 2.25\times10^{-2}\mathrm{m}^3$.
(b) $(6.02\times10^{23}\mathrm{mol}^{-1})/(2.25\times10^4/\mathrm{cm}^3) = 2.68\times10^{19}$.

E21-38 $n/V = p/kT$, so

$$n/V = (1.01\times10^{-13}\mathrm{Pa})/(1.38\times10^{-23}\mathrm{J/K})(295\,\mathrm{K}) = 25\text{ part/cm}^3.$$

$\boxed{\textbf{E21-39}}$ (a) Using Eq. 21-17,

$$n = \frac{pV}{RT} = \frac{(108\times10^3\mathrm{Pa})(2.47\,\mathrm{m}^3)}{(8.31\,\mathrm{J/mol\cdot K})([12+273]\,\mathrm{K})} = 113\text{ mol}.$$

(b) Use the same expression again,

$$V = \frac{nRT}{p} = \frac{(113\text{ mol})(8.31\,\mathrm{J/mol\cdot K})([31+273]\,\mathrm{K})}{(316\times10^3\mathrm{Pa})} = 0.903\,\mathrm{m}^3.$$

E21-40 (a) $n = pV/RT = (1.01\times10^5\mathrm{Pa})(1.13\times10^{-3}\mathrm{m}^3)/(8.31\,\mathrm{J/mol\cdot K})(315\,\mathrm{K}) = 4.36\times10^{-2}\mathrm{mol}$.
(b) $T_\mathrm{f} = T_\mathrm{i} p_\mathrm{f} V_\mathrm{f} / p_\mathrm{i} V_\mathrm{i}$, so

$$T_\mathrm{f} = \frac{(315\,\mathrm{K})(1.06\times10^5\mathrm{Pa})(1.530\times10^{-3}\mathrm{m}^3)}{(1.01\times10^5\mathrm{Pa})(1.130\times10^{-3}\mathrm{m}^3)} = 448\,\mathrm{K}.$$

E21-41 $p_\mathrm{i} = (14.7 + 24.2)\text{ lb/in}^2 = 38.9\text{ lb/in}^2$. $p_\mathrm{f} = p_\mathrm{i} T_\mathrm{f} V_\mathrm{i} / T_\mathrm{i} V_\mathrm{f}$, so

$$p_\mathrm{f} = \frac{(38.9\text{ lb/in}^2)(299\mathrm{K})(988\text{ in}^3)}{(270\,\mathrm{K})(1020\text{ in}^3)} = 41.7\text{ lb/in}^2.$$

The gauge pressure is then $(41.7 - 14.7)\text{ lb/in}^2 = 27.0\text{ lb/in}^2$.

E21-42 Since $p = F/A$ and $F = mg$, a reasonable estimate for the mass of the atmosphere is

$$m = pA/g = (1.01\times10^5\mathrm{Pa})4\pi(6.37\times10^6\mathrm{m})^2/(9.81\,\mathrm{m/s}^2) = 5.25\times10^{18}\mathrm{kg}.$$

E21-43 $p = p_0 + \rho g h$, where h is the depth. Then $P_\text{f} = 1.01\times10^5\text{Pa}$ and

$$p_\text{i} = (1.01\times10^5\text{Pa}) + (998\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(41.5\,\text{m}) = 5.07\times10^5\text{Pa}.$$

$V_\text{f} = V_\text{i}p_\text{i}T_\text{f}/p_\text{f}T_\text{i}$, so

$$V_\text{f} = \frac{(19.4\ \text{cm}^3)(5.07\times10^5\text{Pa.})(296\,\text{K})}{(1.01\times10^5\text{Pa})(277\,\text{K})} = 104\ \text{cm}^3.$$

E21-44 The new pressure in the pipe is

$$p_\text{f} = p_\text{i}V_\text{i}/V_\text{f} = (1.01\times10^5\text{Pa})(2) = 2.02\times10^5\text{Pa}.$$

The water pressure at some depth y is given by $p = p_0 + \rho g y$, so

$$y = \frac{(2.02\times10^5\text{Pa}) - (1.01\times10^5\text{Pa})}{(998\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} = 10.3\,\text{m}.$$

Then the water/air interface inside the tube is at a depth of 10.3 m; so $h = (10.3\,\text{m}) + (25.0\,\text{m})/2 = 22.8\,\text{m}$.

$\boxed{\textbf{P21-1}}$ (a) The dimensions of A must be $[\text{time}]^{-1}$, as can be seen with a quick inspection of the equation. We would expect that A would depend on the surface area at the very least; however, that means that it must also depend on some other factor to fix the dimensionality of A.

(b) Rearrange and integrate,

$$\begin{aligned} \int_{\Delta T_0}^{T} \frac{d\Delta T}{\Delta T} &= -\int_0^t A\,dt, \\ \ln(\Delta T/\Delta T_0) &= -At, \\ \Delta T &= \Delta T_0 e^{-At}. \end{aligned}$$

P21-2 First find A.

$$A = \frac{\ln(\Delta T_0/\Delta T)}{t} = \frac{\ln[(29\,\text{C}^\circ)/(25\,\text{C}^\circ)]}{(45\ \text{min})} = 3.30\times10^{-3}/\text{min}.$$

Then find time to new temperature difference.

$$t = \frac{\ln(\Delta T_0/\Delta T)}{t} = \frac{\ln[(29\,\text{C}^\circ)/(21\,\text{C}^\circ)]}{(3.30\times10^{-3}/\text{min})} = 97.8\text{min}$$

This happens $97.8 - 45 = 53$ minutes later.

P21-3 If we neglect the expansion of the tube then we can assume the cross sectional area of the tube is constant. Since $V = Ah$, we can assume that $\Delta V = A\Delta h$. Then since $\Delta V = \beta V_0 \Delta T$, we can write $\Delta h = \beta h_0 \Delta T$.

P21-4 For either container we can write $p_i V_i = n_i R T_i$. We are told that V_i and n_i are constants. Then $\Delta p = AT_1 - BT_2$, where A and B are constants. When $T_1 = T_2$ $\Delta p = 0$, so $A = B$. When $T_1 = T_\text{tr}$ and $T_2 = T_\text{b}$ we have

$$(120\ \text{mm Hg}) = A(373\,\text{K} - 273.16\,\text{K}),$$

so $A = 1.202$ mm Hg/K. Then

$$T = \frac{(90\ \text{mm Hg}) + (1.202\ \text{mm Hg/K})(273.16\,\text{K})}{(1.202\ \text{mm Hg/K})} = 348\,\text{K}.$$

Actually, we could have assumed A was negative, and then the answer would be 198 K.

P21-5 Start with a differential form for Eq. 21-8, $dL/dT = \alpha L_0$, rearrange, and integrate:

$$\begin{aligned}\int_{L_0}^{L} dL &= \int_{T_0}^{T} \alpha L_0 \, dT, \\ L - L_0 &= L_0 \int_{T_0}^{T} \alpha \, dT, \\ L &= L_0 \left(1 + \int_{T_0}^{T} \alpha \, dT\right).\end{aligned}$$

P21-6 $\Delta L = \alpha L \Delta T$, so

$$\frac{\Delta T}{\Delta t} = \frac{1}{\alpha L}\frac{\Delta L}{\Delta t} = \frac{(96\times10^{-9}\text{m/s})}{(23\times10^{-6}/\text{C}^\circ)(1.8\times10^{-2}\text{m})} = 0.23^\circ\text{C/s}.$$

P21-7 (a) Consider the work that was done for Ex. 21-27. The length of rod a is

$$L_a = L_{a,0}(1 + \alpha_a \Delta T),$$

while the length of rod b is

$$L_b = L_{b,0}(1 + \alpha_b \Delta T).$$

The difference is

$$\begin{aligned}L_a - L_b &= L_{a,0}(1 + \alpha_a \Delta T) - L_{b,0}(1 + \alpha_b \Delta T), \\ &= L_{a,0} - L_{b,0} + (L_{a,0}\alpha_a - L_{b,0}\alpha_b)\Delta T,\end{aligned}$$

which will be a constant is $L_{a,0}\alpha_a = L_{b,0}\alpha_b$ or

$$L_{i,0} \propto 1/\alpha_i.$$

(b) We want $L_{a,0} - L_{b,0} = 0.30\,\text{m}$ so

$$k/\alpha_a - k/\alpha_b = 0.30\,\text{m},$$

where k is a constant of proportionality;

$$k = (0.30\,\text{m})/\left(1/(11\times10^{-6}/\text{C}^\circ) - 1/(19\times10^{-6}/\text{C}^\circ)\right) = 7.84\times10^{-6}\text{m/C}^\circ.$$

The two lengths are

$$L_a = (7.84\times10^{-6}\text{m/C}^\circ)/(11\times10^{-6}/\text{C}^\circ) = 0.713\,\text{m}$$

for steel and

$$L_b = (7.84\times10^{-6}\text{m/C}^\circ)/(19\times10^{-6}/\text{C}^\circ) = 0.413\,\text{m}$$

for brass.

P21-8 The fractional increase in length of the bar is $\Delta L/L_0 = \alpha\Delta T$. The right triangle on the left has base $L_0/2$, height x, and hypotenuse $(L_0 + \Delta L)/2$. Then

$$x = \frac{1}{2}\sqrt{(L_0 + \Delta L)^2 - L_0^2} = \frac{L_0}{2}\sqrt{2\frac{\Delta L}{L_0}}.$$

With numbers,

$$x = \frac{(3.77\,\text{m})}{2}\sqrt{2(25\times10^{-6}/\text{C}^\circ)(32\,\text{C}^\circ)} = 7.54\times10^{-2}\text{m}.$$

P21-9 We want to evaluate $V = V_0(1 + \int \beta\, dT)$; the integral is the area under the graph; the graph looks like a triangle, so the result is

$$V = V_0[1 + (16\,\mathrm{C}^\circ)(0.0002/\mathrm{C}^\circ)/2] = (1.0016)V_0.$$

The density is then

$$\rho = \rho_0(V_0/V) = (1000\,\mathrm{kg/m^3})/(1.0016) = 0.9984\,\mathrm{kg/m^3}.$$

P21-10 At 0.00°C the glass bulb is filled with mercury to a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(\beta - 3\alpha)\Delta T.$$

Since $T_0 = 0.0°\mathrm{C}$, then $\Delta T = T$, if it is measured in °C. The amount of mercury in the capillary is ΔV, and since the cross sectional area is fixed at A, then the length is $L = \Delta V/A$, or

$$L = \frac{V}{A}(\beta - 3\alpha)\Delta T.$$

P21-11 Let a, b, and c correspond to aluminum, steel, and invar, respectively. Then

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

We can replace a with $a_0(1 + \alpha_a \Delta T)$, and write similar expressions for b and c. Since $a_0 = b_0 = c_0$, this can be simplified to

$$\cos C = \frac{(1 + \alpha_a\Delta T)^2 + (1 + \alpha_b\Delta T)^2 - (1 + \alpha_c\Delta T)^2}{2(1 + \alpha_a\Delta T)(1 + \alpha_b\Delta T)}.$$

Expand this as a Taylor series in terms of ΔT, and we find

$$\cos C \approx \frac{1}{2} + \frac{1}{2}\left(\alpha_a + \alpha_b - 2\alpha_c\right)\Delta T.$$

Now solve:

$$\Delta T = \frac{2\cos(59.95^\circ) - 1}{(23\times10^{-6}/\mathrm{C}^\circ) + (11\times10^{-6}/\mathrm{C}^\circ) - 2(0.7\times10^{-6}/\mathrm{C}^\circ)} = 46.4^\circ\mathrm{C}.$$

The final temperature is then 66.4°C.

P21-12 The bottom of the iron bar moves downward according to $\Delta L = \alpha L \Delta T$. The center of mass of the iron bar is located in the center; it moves downward half the distance. The mercury expands in the glass upwards; subtracting off the distance the iron moves we get

$$\Delta h = \beta h \Delta T - \Delta L = (\beta h - \alpha L)\Delta T.$$

The center of mass in the mercury is located in the center. If the center of mass of the system is to remain constant we require

$$m_\mathrm{i}\Delta L/2 = m_\mathrm{m}(\Delta h - \Delta L)/2;$$

or, since $\rho = mV = mAy$,

$$\rho_\mathrm{i}\alpha L = \rho_\mathrm{m}(\beta h - 2\alpha L).$$

Solving for h,

$$h = \frac{(12\times10^{-6}/\mathrm{C}^\circ)(1.00\,\mathrm{m})[(7.87\times10^3\mathrm{kg/m^3}) + 2(13.6\times10^3\mathrm{kg/m^3})]}{(13.6\times10^3\mathrm{kg/m^3})(18\times10^{-5}/\mathrm{C}^\circ)} = 0.17\,\mathrm{m}.$$

$\boxed{\textbf{P21-13}}$ The volume of the block which is beneath the surface of the mercury displaces a mass of mercury equal to the mass of the block. The mass of the block is independent of the temperature but the volume of the displaced mercury changes according to

$$V_{\rm m} = V_{\rm m,0}(1 + \beta_{\rm m}\Delta T).$$

This volume is equal to the depth which the block sinks times the cross sectional area of the block (which *does* change with temperature). Then

$$h_{\rm s}{h_{\rm b}}^2 = h_{\rm s,0}{h_{\rm b,0}}^2(1 + \beta_{\rm m}\Delta T),$$

where $h_{\rm s}$ is the depth to which the block sinks and $h_{\rm b,0} = 20$ cm is the length of the side of the block. But

$$h_{\rm b} = h_{\rm b,0}(1 + \alpha_{\rm b}\Delta T),$$

so

$$h_{\rm s} = h_{\rm s,0}\frac{1 + \beta_{\rm m}\Delta T}{(1 + \alpha_{\rm b}\Delta T)^2}.$$

Since the changes are small we can expand the right hand side using the binomial expansion; keeping terms only in ΔT we get

$$h_{\rm s} \approx h_{\rm s,0}(1 + (\beta_{\rm m} - 2\alpha_{\rm b})\Delta T),$$

which means the block will sink a distance $h_{\rm s} - h_{\rm s,0}$ given by

$$h_{\rm s,0}(\beta_{\rm m} - 2\alpha_{\rm b})\Delta T = h_{\rm s,0}\left[(1.8\times10^{-4}/{\rm C}^\circ) - 2(23\times10^{-6}/{\rm C}^\circ)\right](50\,{\rm C}^\circ) = (6.7\times10^{-3})h_{\rm s,0}.$$

In order to finish we need to know how much of the block was submerged in the first place. Since the fraction submerged is equal to the ratio of the densities, we have

$$h_{\rm s,0}/h_{\rm b,0} = \rho_{\rm b}/\rho_{\rm m} = (2.7\times10^3{\rm kg/m^3})/(1.36\times10^4{\rm kg/m^3}),$$

so $h_{\rm s,0} = 3.97$ cm, and the change in depth is 0.27 mm.

P21-14 The area of glass expands according to $\Delta A_{\rm g} = 2\alpha_{\rm g}A_{\rm g}\Delta T$. The are of Dumet wire expands according to

$$\Delta A_{\rm c} + \Delta A{\rm i} = 2(\alpha_{\rm c}A_{\rm c} + \alpha_{\rm i}A_{\rm i})\Delta T.$$

We need these to be equal, so

$$\begin{aligned}
\alpha_{\rm g}A_{\rm g} &= \alpha_{\rm c}A_{\rm c} + \alpha_{\rm i}A_{\rm i},\\
\alpha_{\rm g}{r_{\rm g}}^2 &= \alpha_{\rm c}({r_{\rm c}}^2 - {r_{\rm i}}^2) + \alpha_{\rm i}{r_{\rm i}}^2,\\
\alpha_{\rm g}({r_{\rm c}}^2 + {r_{\rm i}}^2) &= \alpha_{\rm c}({r_{\rm c}}^2 - {r_{\rm i}}^2) + \alpha_{\rm i}{r_{\rm i}}^2,\\
\frac{{r_{\rm i}}^2}{{r_{\rm c}}^2} &= \frac{\alpha_{\rm c} - \alpha_{\rm g}}{\alpha_{\rm c} - \alpha_{\rm i}}.
\end{aligned}$$

P21-15

P21-16 $V_2 = V_1(p_1/p_2)(T_1/T_2)$, so

$$V_2 = (3.47\,{\rm m^3})[(76\text{ cm Hg})/(36\text{ cm Hg})][(225\,{\rm K})/(295\,{\rm K})] = 5.59\,{\rm m^3}.$$

P21-17 Call the containers one and two so that $V_1 = 1.22$ L and $V_2 = 3.18$ L. Then the initial number of moles in the two containers are

$$n_{1,\mathrm{i}} = \frac{p_\mathrm{i} V_1}{RT_\mathrm{i}} \text{ and } n_{2,\mathrm{i}} = \frac{p_\mathrm{i} V_2}{RT_\mathrm{i}}.$$

The total is

$$n = p_\mathrm{i}(V_1 + V_2)/(RT_\mathrm{i}).$$

Later the temperatures are changed and then the number of moles of gas in each container is

$$n_{1,\mathrm{f}} = \frac{p_\mathrm{f} V_1}{RT_{1,\mathrm{f}}} \text{ and } n_{2,\mathrm{f}} = \frac{p_\mathrm{f} V_2}{RT_{2,\mathrm{f}}}.$$

The total is still n, so

$$\frac{p_\mathrm{f}}{R}\left(\frac{V_1}{T_{1,\mathrm{f}}} + \frac{V_2}{T_{2,\mathrm{f}}}\right) = \frac{p_\mathrm{i}(V_1 + V_2)}{RT_\mathrm{i}}.$$

We can solve this for the final pressure, so long as we remember to convert all temperatures to Kelvins,

$$p_\mathrm{f} = \frac{p_\mathrm{i}(V_1 + V_2)}{T_\mathrm{i}}\left(\frac{V_1}{T_{1,\mathrm{f}}} + \frac{V_2}{T_{2,\mathrm{f}}}\right)^{-1},$$

or

$$p_\mathrm{f} = \frac{(1.44\text{ atm})(1.22\text{L} + 3.18\text{ L})}{(289\,\text{K})}\left(\frac{(1.22\text{ L})}{(289\,\text{K})} + \frac{(3.18\text{ L})}{(381\,\text{K})}\right)^{-1} = 1.74\text{ atm}.$$

P21-18 Originally $n_A = p_A V_A/RT_A$ and $n_B = p_B V_B/RT_B$; $V_B = 4V_A$. Label the final state of A as C and the final state of B as D. After mixing, $n_C = p_C V_A/RT_A$ and $n_D = p_D V_B/RT_B$, but $P_C = P_D$ and $n_A + n_B = n_C + n_D$. Then

$$p_A/T_A + 4p_B/T_B = p_C(1/T_A + 4/T_B),$$

or

$$p_C = \frac{(5\times10^5\text{Pa})/(300\,\text{K}) + 4(1\times10^5\text{Pa})/(400\,\text{K})}{1/(300\,\text{K}) + 4/(400\,K)} = 2.00\times10^5\text{Pa}.$$

P21-19 If the temperature is uniform then all that is necessary is to substitute $p_0 = nRT/V$ and $p = nRT/V$; cancel RT from both sides, and then equate n/V with n_V.

P21-20 Use the results of Problem 15-19. The initial pressure inside the bubble is $p_\mathrm{i} = p_0 + 4\gamma/r_\mathrm{i}$. The final pressure inside the bell jar is zero, so$p_\mathrm{f} = 4\gamma/r_\mathrm{f}$. The initial and final pressure inside the bubble are related by $p_\mathrm{i} {r_\mathrm{i}}^3 = p_\mathrm{f} {r_\mathrm{f}}^3 = 4\gamma {r_\mathrm{f}}^2$. Now for numbers:

$$p_\mathrm{i} = (1.01\times10^5\text{Pa}) + 4(2.5\times10^{-2}\text{N/m})/(2.0\times10^{-3}\text{m}) = 1.0105\times10^5\text{Pa}.$$

and

$$r_\mathrm{f} = \sqrt{\frac{(1.0105\times10^5\text{Pa})(2.0\times10^{-3}\text{m})^3}{4(2.5\times10^{-2}\text{N/m})}} = 8.99\times10^{-2}\text{m}.$$

P21-21

P21-22

E22-1 (a) $n = (2.56 \text{ g})/(197 \text{ g/mol}) = 1.30 \times 10^{-2}\text{mol}$.

(b) $N = (6.02 \times 10^{23}\text{mol}^{-1})(1.30 \times 10^{-2}\text{mol}) = 7.83 \times 10^{21}$.

E22-2 (a) $N = pV/kT = (1.01 \times 10^5\text{Pa})(1.00\,\text{m}^3)/(1.38 \times 10^{-23}\text{J/K})(293\,\text{K}) = 2.50 \times 10^{25}$.

(b) $n = (2.50 \times 10^{25})/(6.02 \times 10^{23}\text{mol}^{-1}) = 41.5$ mol. Then

$$m = (41.5 \text{ mol})[75\%(28 \text{ g/mol}) + 25\%(32 \text{ g/mol})] = 1.20\,\text{kg}.$$

$\boxed{\textbf{E22-3}}$ (a) We first need to calculate the molar mass of ammonia. This is

$$M = M(\text{N}) + 3M(\text{H}) = (14.0 \text{ g/mol}) + 3(1.01 \text{ g/mol}) = 17.0 \text{ g/mol}$$

The number of moles of nitrogen present is

$$n = m/M_\text{r} = (315 \text{ g})/(17.0 \text{ g/mol}) = 18.5 \text{ mol}.$$

The volume of the tank is

$$V = nRT/p = (18.5 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(350\,\text{K})/(1.35 \times 10^6\text{Pa}) = 3.99 \times 10^{-2}\text{m}^3.$$

(b) After the tank is checked the number of moles of gas in the tank is

$$n = pV/(RT) = (8.68 \times 10^5\text{Pa})(3.99 \times 10^{-2}\text{m}^3)/[(8.31\,\text{J/mol}\cdot\text{K})(295\,\text{K})] = 14.1 \text{ mol}.$$

In that case, 4.4 mol must have escaped; that corresponds to a mass of

$$m = nM_\text{r} = (4.4 \text{ mol})(17.0 \text{ g/mol}) = 74.8 \text{ g}.$$

E22-4 (a) The volume per particle is $V/N = kT/P$, so

$$V/N = (1.38 \times 10^{-23}\text{J/K})(285\,\text{K})/(1.01 \times 10^5\text{Pa}) = 3.89 \times 10^{-26}\text{m}^3.$$

The edge length is the cube root of this, or 3.39×10^{-9}m. The ratio is 11.3.

(b) The volume per particle is V/N_A, so

$$V/N_A = (18 \times 10^{-6}\text{m}^3)/(6.02 \times 10^{23}) = 2.99 \times 10^{-29}\text{m}^3.$$

The edge length is the cube root of this, or 3.10×10^{-10}m. The ratio is 1.03.

E22-5 The volume per particle is $V/N = kT/P$, so

$$V/N = (1.38 \times 10^{-23}\text{J/K})(308\,\text{K})/(1.22)(1.01 \times 10^5\text{Pa}) = 3.45 \times 10^{-26}\text{m}^3.$$

The fraction actually occupied by the particle is

$$\frac{4\pi(0.710 \times 10^{-10}\text{m})^3/3)}{(3.45 \times 10^{-26}\text{m}^3)} = 4.34 \times 10^{-5}.$$

E22-6 The component of the momentum normal to the wall is

$$p_y = (3.3 \times 10^{-27}\text{kg})(1.0 \times 10^3\text{m/s})\cos(55°) = 1.89 \times 10^{-24}\text{kg}\cdot\text{m/s}.$$

The pressure exerted on the wall is

$$p = \frac{F}{A} = \frac{(1.6 \times 10^{23}/\text{s})2(1.89 \times 10^{-24}\text{kg}\cdot\text{m/s})}{(2.0 \times 10^{-4}\text{m}^2)} = 3.0 \times 10^3\text{Pa}.$$

E22-7 (a) From Eq. 22-9,

$$v_{\rm rms} = \sqrt{\frac{3p}{\rho}}.$$

Then

$$p = 1.23\times 10^{-3}\ \text{atm}\left(\frac{1.01\times 10^5\,\text{Pa}}{1\ \text{atm}}\right) = 124\,\text{Pa}$$

and

$$\rho = 1.32\times 10^{-5}\ \text{g/cm}^3\left(\frac{1\text{kg}}{1000\ \text{g}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 1.32\times 10^{-2}\ \text{kg/m}^3.$$

Finally,

$$v_{\rm rms} = \sqrt{\frac{3(1240\,\text{Pa})}{(1.32\times 10^{-2}\ \text{kg/m}^3)}} = 531\,\text{m/s}.$$

(b) The molar density of the gas is just n/V; but this can be found quickly from the ideal gas law as

$$\frac{n}{V} = \frac{p}{RT} = \frac{(1240\,\text{Pa})}{(8.31\,\text{J/mol}\cdot\text{K})(317\,\text{K})} = 4.71\times 10^{-1}\ \text{mol/m}^3.$$

(c) We were given the density, which is mass per volume, so we could find the molar mass from

$$\frac{\rho}{n/V} = \frac{(1.32\times 10^{-2}\ \text{kg/m}^3)}{(4.71\times 10^{-1}\ \text{mol/m}^3)} = 28.0\ \text{g/mol}.$$

But what gas is it? It could contain any atom lighter than silicon; trial and error is the way to go. Some of my guesses include C_2H_4 (ethene), CO (carbon monoxide), and N_2. There's no way to tell which is correct at this point, in fact, the gas could be a mixture of all three.

E22-8 The density is $\rho = m/V = nM_r/V$, or

$$\rho = (0.350\ \text{mol})(0.0280\ \text{kg/mol})/\pi(0.125\,\text{m}/2)^2(0.560\,\text{m}) = 1.43\,\text{kg/m}^3.$$

The rms speed is

$$v_{\rm rms} = \sqrt{\frac{3(2.05)(1.01\times 10^5\text{Pa})}{(1.43\,\text{kg/m}^3)}} = 659\,\text{m/s}.$$

E22-9 (a) $N/V = p/kT = (1.01\times 10^5\text{Pa})/(1.38\times 10^{-23}\text{J/K})(273\,\text{K}) = 2.68\times 10^{25}/\text{m}^3$.

(b) Note that Eq. 22-11 is *wrong*; for the explanation read the last two paragraphs in the first column on page 502. We need an extra factor of $\sqrt{2}$, so $\pi d^2 = V/\sqrt{2}N\lambda$, so

$$d = \sqrt{1/\sqrt{2}\pi(2.68\times 10^{25}/\text{m}^3)(285\times 10^{-9}\text{m})} = 1.72\times 10^{-10}\text{m}.$$

E22-10 (a) $\lambda = V/\sqrt{2}N\pi d^2$, so

$$\lambda = \frac{1}{\sqrt{2}(1.0\times 10^6/\text{m}^3)\pi(2.0\times 10^{-10}\text{m})^2} = 5.6\times 10^{12}\text{m}.$$

(b) Particles effectively follow ballistic trajectories.

$\boxed{\textbf{E22-11}}$ We have $v = f\lambda$, where λ is the wavelength (which we will set equal to the mean free path), and v is the speed of sound. The mean free path is, from Eq. 22-13,

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p}$$

so

$$f = \frac{\sqrt{2}\pi d^2 pv}{kT} = \frac{\sqrt{2}\pi(315\times10^{-12}\mathrm{m})^2(1.02\times1.01\times10^5\mathrm{Pa})(343\,\mathrm{m/s})}{(1.38\times10^{-23}\mathrm{J/K})(291\,\mathrm{K})} = 3.88\times10^9\mathrm{Hz}.$$

E22-12 (a) $p = (1.10\times10^{-6}$ mm Hg$)(133\,\mathrm{Pa/mm\ Hg}) = 1.46\times10^{-4}\mathrm{Pa}$. The particle density is

$$N/V = (1.46\times10^{-4}\mathrm{Pa})/(1.38\times10^{-23}\mathrm{J/K})(295\,\mathrm{K}) = 3.59\times10^{16}/\mathrm{m}^3.$$

(b) The mean free path is

$$\lambda = 1/\sqrt{(}2)(3.59\times10^{16}/\mathrm{m}^3)\pi(2.20\times10^{-10}\mathrm{m})^2 = 130\,\mathrm{m}.$$

E22-13 Note that $v_{\mathrm{av}} \propto \sqrt{T}$, while $\lambda \propto T$. Then the collision rate is proportional to $1/\sqrt{T}$. Then

$$T = (300\,\mathrm{K})\frac{(5.1\times10^9/\mathrm{s})^2}{(6.0\times10^9/\mathrm{s})^2} = 216\,\mathrm{K}.$$

E22-14 (a) $v_{\mathrm{av}} = (65\ \mathrm{km/s})/(10) = 6.5\ \mathrm{km/s}$.
(b) $v_{\mathrm{rms}} = \sqrt{(505\ \mathrm{km/s})/(10)} = 7.1\ \mathrm{km/s}$.

$\boxed{\textbf{E22-15}}$ (a) The average is

$$\frac{4(200\,\mathrm{s}) + 2(500\,\mathrm{m/s}) + 4(600\,\mathrm{m/s})}{4+2+4} = 420\,\mathrm{m/s}.$$

The mean-square value is

$$\frac{4(200\,\mathrm{s})^2 + 2(500\,\mathrm{m/s})^2 + 4(600\,\mathrm{m/s})^2}{4+2+4} = 2.1\times10^5\ \mathrm{m^2/s^2}.$$

The root-mean-square value is the square root of this, or 458 m/s.

(b) I'll be lazy. Nine particles are not moving, and the tenth has a speed of 10 m/s. Then the average speed is 1 m/s, and the root-mean-square speed is 3.16 m/s. Look, v_{rms} is larger than v_{av}!

(c) Can $v_{\mathrm{rms}}{=}v_{\mathrm{av}}$? Assume that the speeds are *not* all the same. Transform to a frame of reference where $v_{\mathrm{av}} = 0$, then some of the individual speeds must be greater than zero, and some will be less than zero. Squaring these speeds will result in positive, non-zero, numbers; the mean square will necessarily be greater than zero, so $v_{\mathrm{rms}} > 0$.

Only if *all* of the particles have the same speed will $v_{\mathrm{rms}}{=}v_{\mathrm{av}}$.

E22-16 Use Eq. 22-20:

$$v_{\mathrm{rms}} = \sqrt{\frac{3(1.38\times10^{-23}\mathrm{J/K})(329\,\mathrm{K})}{(2.33\times10^{-26}\mathrm{kg} + 3\times1.67\times10^{-27}\mathrm{kg})}} = 694\,\mathrm{m/s}.$$

E22-17 Use Eq. 22-20:

$$v_{\mathrm{rms}} = \sqrt{\frac{3(1.38\times10^{-23}\mathrm{J/K})(2.7\,\mathrm{K})}{(2\times1.67\times10^{-27}\mathrm{kg})}} = 180\,\mathrm{m/s}.$$

E22-18 Eq. 22-14 is in the form $N = Av^2 e^{-Bv^2}$. Taking the derivative,

$$\frac{dN}{dv} = 2Ave^{-Bv^2} - 2ABv^3 e^{-Bv^2},$$

and setting this equal to zero,

$$v^2 = 1/B = 2kT/m.$$

$\boxed{\textbf{E22-19}}$ We want to integrate

$$\begin{aligned} v_{\text{av}} &= \frac{1}{N}\int_0^\infty N(v) v\,dv, \\ &= \frac{1}{N}\int_0^\infty 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} v\,dv, \\ &= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^2 e^{-mv^2/2kT} v\,dv. \end{aligned}$$

The easiest way to attack this is first with a change of variables— let $x = mv^2/2kT$, then $kT\,dx = mv\,dv$. The limits of integration don't change, since $\sqrt{\infty} = \infty$. Then

$$\begin{aligned} v_{\text{av}} &= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty \frac{2kT}{m} xe^{-x} \frac{kT}{m} dx, \\ &= 2\left(\frac{2kT}{\pi m}\right)^{1/2} \int_0^\infty xe^{-\alpha x} dx \end{aligned}$$

The factor of α that was introduced in the last line is a Feynman trick; we'll set it equal to one when we are finished, so it won't change the result.

Feynman's trick looks like

$$\frac{d}{d\alpha}\int e^{-\alpha x} dx = \int \frac{\partial}{\partial \alpha} e^{-\alpha x} dx = \int (-x) e^{-\alpha x} dx.$$

Applying this to our original problem,

$$\begin{aligned} v_{\text{av}} &= 2\left(\frac{2kT}{\pi m}\right)^{1/2} \int_0^\infty xe^{-\alpha x} dx, \\ &= -\frac{d}{d\alpha} 2\left(\frac{2kT}{\pi m}\right)^{1/2} \int_0^\infty e^{-\alpha x} dx, \\ &= -2\left(\frac{2kT}{\pi m}\right)^{1/2} \frac{d}{d\alpha}\left(\left.\frac{-1}{\alpha} e^{-\alpha x}\right|_0^\infty\right), \\ &= -2\left(\frac{2kT}{\pi m}\right)^{1/2} \frac{d}{d\alpha}\left(\frac{1}{\alpha}\right), \\ &= -2\left(\frac{2kT}{\pi m}\right)^{1/2} \frac{-1}{\alpha^2}. \end{aligned}$$

We promised, however, that we would set $\alpha = 1$ in the end, so this last line is

$$\begin{aligned} v_{\text{av}} &= 2\left(\frac{2kT}{\pi m}\right)^{1/2}, \\ &= \sqrt{\frac{8kT}{\pi m}}. \end{aligned}$$

E22-20 We want to integrate

$$
\begin{aligned}
(v^2)_{\rm av} &= \frac{1}{N}\int_0^\infty N(v)v^2\,dv,\\
&= \frac{1}{N}\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}v^2\,dv,\\
&= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT}v^2\,dv.
\end{aligned}
$$

The easiest way to attack this is first with a change of variables— let $x^2 = mv^2/2kT$, then $\sqrt{2kT/m}dx = dv$. The limits of integration don't change. Then

$$
\begin{aligned}
(v^2)_{\rm av} &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty \left(\frac{2kT}{m}\right)^{5/2} x^4 e^{-x^2}\,dx,\\
&= \frac{8kT}{\sqrt{\pi}\,m}\int_0^\infty x^4 e^{-x^2}\,dx
\end{aligned}
$$

Look up the integral; although you can solve it by first applying a Feynman trick (see solution to Exercise 22-21) and then squaring the integral and changing to polar coordinates. I looked it up. I found $3\sqrt{\pi}/8$, so

$$(v^2)_{\rm av} = \frac{8kT}{\sqrt{\pi}\,m}3\sqrt{\pi}/8 = 3kT/m.$$

E22-21 Apply Eq. 22-20:

$$v_{\rm rms} = \sqrt{3(1.38\times10^{-23}{\rm J/K})(287\,{\rm K})/(5.2\times10^{-17}{\rm kg})} = 1.5\times10^{-2}{\rm m/s}.$$

E22-22 Since $v_{\rm rms} \propto \sqrt{T/m}$, we have

$$T = (299\,{\rm K})(4/2) = 598\,{\rm K},$$

or 325°C.

E22-23 (a) The escape speed is found on page 310; $v = 11.2\times10^3$m/s. For hydrogen,

$$T = (2)(1.67\times10^{-27}{\rm kg})(11.2\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 1.0\times10^4{\rm K}.$$

For oxygen,

$$T = (32)(1.67\times10^{-27}{\rm kg})(11.2\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 1.6\times10^5{\rm K}.$$

(b) The escape speed is found on page 310; $v = 2.38\times10^3$m/s. For hydrogen,

$$T = (2)(1.67\times10^{-27}{\rm kg})(2.38\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 460{\rm K}.$$

For oxygen,

$$T = (32)(1.67\times10^{-27}{\rm kg})(2.38\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 7300{\rm K}.$$

(c) There should be more oxygen than hydrogen.

E22-24 (a) $v_{\rm av} = (70\ {\rm km/s})/(22) = 3.18\ {\rm km/s}$.

(b) $v_{\rm rms} = \sqrt{(250\ {\rm km}^2/{\rm s}^2)/(22)} = 3.37\ {\rm km/s}$.

(c) 3.0 km/s.

$\boxed{\textbf{E22-25}}$ According to the equation directly beneath Fig. 22-8,

$$\omega = v\phi/L = (212\,\text{m/s})(0.0841\text{ rad})/(0.204\,\text{m}) = 87.3\text{ rad/s}.$$

E22-26 If $v_\text{p} = v_\text{rms}$ then $2T_2 = 3T_1$, or $T_2/T_1 = 3/2$.

E22-27 Read the last paragraph on the first column of page 505. The distribution of speeds is proportional to

$$v^3 e^{-mv^2/2kT} = v^3 e^{-Bv^2},$$

taking the derivative dN/dv and setting equal to zero yields

$$\frac{dN}{dv} = 3v^2 e^{-Bv^2} - 2Bv^4 e^{-Bv^2},$$

and setting this equal to zero,

$$v^2 = 3/2B = 3kT/m.$$

E22-28 (a) $v = \sqrt{3(8.31\,\text{J/mol}\cdot\text{K})(4220\,\text{K})/(0.07261\,\text{kg/mol})} = 1200\,\text{m/s}$.

(b) Half of the sum of the diameters, or 273 pm.

(c) The mean free path of the germanium in the argon is

$$\lambda = 1/\sqrt{2}(4.13\times10^{25}/\text{m}^3)\pi(273\times10^{-12}\text{m})^2 = 7.31\times10^{-8}\text{m}.$$

The collision rate is

$$(1200\,\text{m/s})/(7.31\times10^{-8}\text{m}) = 1.64\times10^{10}/\text{s}.$$

$\boxed{\textbf{E22-29}}$ The fraction of particles that interests us is

$$\frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\int_{0.01kT}^{0.03kT} E^{1/2} e^{-E/kT}\,dE.$$

Change variables according to $E/kT = x$, so that $dE = kT\,dx$. The integral is then

$$\frac{2}{\sqrt{\pi}}\int_{0.01}^{0.03} x^{1/2} e^{-x}\,dx.$$

Since the value of x is so small compared to 1 throughout the range of integration, we can expand according to

$$e^{-x} \approx 1 - x \text{ for } x \ll 1.$$

The integral then simplifies to

$$\frac{2}{\sqrt{\pi}}\int_{0.01}^{0.03} x^{1/2}(1-x)\,dx = \frac{2}{\sqrt{\pi}}\left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_{0.01}^{0.03} = 3.09\times10^{-3}.$$

E22-30 Write $N(E) = N(E_\text{p} + \epsilon)$. Then

$$N(E_\text{p} + \epsilon) \approx N(E_\text{p}) + \epsilon\,\frac{dN(E)}{dE}\bigg|_{E_\text{p}} + ...$$

But the very definition of E_p implies that the first derivative is zero. Then the fraction of [particles with energies in the range $E_\text{p} \pm 0.02kT$ is

$$\frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}(kT/2)^{1/2}e^{-1/2}(0.02kT),$$

or $0.04\sqrt{1/2e\pi} = 9.68\times10^{-3}$.

$\boxed{\textbf{E22-31}}$ The volume correction is on page 508; we need first to find d. If we assume that the particles in water are arranged in a cubic lattice (a bad guess, but we'll use it anyway), then 18 grams of water has a volume of $18\times10^{-6}\text{m}^3$, and

$$d^3 = \frac{(18\times10^{-6}\,\text{m}^3)}{(6.02\times10^{23})} = 3.0\times10^{-29}\text{m}^3$$

is the volume allocated to each water molecule. In this case $d = 3.1\times10^{-10}$m. Then

$$b = \frac{1}{2}(6.02\times10^{23})(\frac{4}{3}\pi(3.1\times10^{-10}\text{m})^3) = 3.8\,\text{m}^3/\text{mol}.$$

E22-32 $d^3 = 3b/2\pi N_A$, or

$$d = \sqrt[3]{\frac{3(32\times10^{-6}\text{m}^3/\text{mol})}{2\pi(6.02\times10^{23}/\text{mol})}} = 2.9\times10^{-10}\text{m}.$$

E22-33 a has units of energy volume per square mole, which is the same as energy per mole times volume per mole.

P22-1 Solve $(1-x)(1.429) + x(1.250) = 1.293$ for x. The result is $x = 0.7598$.

P22-2

$\boxed{\textbf{P22-3}}$ The only thing that matters is the total number of moles of gas (2.5) and the number of moles of the second gas (0.5). Since 1/5 of the total number of moles of gas is associated with the second gas, then 1/5 of the total pressure is associated with the second gas.

P22-4 Use Eq. 22-11 with the appropriate $\sqrt{2}$ inserted.

$$\lambda = \frac{(1.0\times10^{-3}\text{m}^3)}{\sqrt{2}(35)\pi(1.0\times10^{-2}\text{m})^2} = 6.4\times10^{-2}\text{m}.$$

P22-5 (a) Since $\lambda \propto 1/d^2$, we have

$$\frac{d_\text{a}}{d_\text{n}} = \sqrt{\frac{\lambda_\text{n}}{\lambda_\text{a}}} = \sqrt{\frac{(27.5\times10^{-8}\text{m})}{(9.90\times10^{-8}\text{m})}} = 1.67.$$

(b) Since $\lambda \propto 1/p$, we have

$$\lambda_2 = \lambda_1\frac{p_1}{p_2} = (9.90\times10^{-8}\text{m})\frac{(75.0\text{ cm Hg})}{(15.0\text{ cm Hg})} = 49.5\times10^{-8}\text{m}.$$

(c) Since $\lambda \propto T$, we have

$$\lambda_2 = \lambda_1\frac{T_2}{T_1} = (9.90\times10^{-8}\text{m})\frac{(233\,\text{K})}{(293\,\text{K})} = 7.87\times10^{-8}\,\text{m}.$$

P22-6 We can assume the molecule will collide with something. Then

$$1 = \int_0^\infty A e^{-cr} dr = A/c,$$

so $A = c$. If the molecule has a mean free path of λ, then

$$\lambda = \int_0^\infty r c e^{-cr} dr = 1/c,$$

so $A = c = 1/\lambda$.

$\boxed{\textbf{P22-7}}$ What is important here is the temperature; since the temperatures are the same then the average kinetic energies per particle are the same. Then

$$\frac{1}{2} m_1 (v_{\mathrm{rms},1})^2 = \frac{1}{2} m_2 (v_{\mathrm{rms},2})^2.$$

We are given in the problem that $v_{\mathrm{av},2} = 2 v_{\mathrm{rms},1}$. According to Eqs. 22-18 and 22-20 we have

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\pi}{8}} \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{3\pi}{8}} v_{\mathrm{av}}.$$

Combining this with the kinetic energy expression above,

$$\frac{m_1}{m_2} = \left(\frac{v_{\mathrm{rms},2}}{v_{\mathrm{rms},1}} \right)^2 = \left(2 \sqrt{\frac{3\pi}{8}} \right)^2 = 4.71.$$

P22-8 (a) Assume that the speeds are *not* all the same. Transform to a frame of reference where $v_{\mathrm{av}} = 0$, then some of the individual speeds must be greater than zero, and some will be less than zero. Squaring these speeds will result in positive, non-zero, numbers; the mean square will necessarily be greater than zero, so $v_{\mathrm{rms}} > 0$.

(b) Only if *all* of the particles have the same speed will $v_{\mathrm{rms}} = v_{\mathrm{av}}$.

$\boxed{\textbf{P22-9}}$ (a) We need to first find the number of particles by integrating

$$\begin{aligned} N &= \int_0^\infty N(v)\, dv, \\ &= \int_0^{v_0} C v^2\, dv + \int_{v_0}^\infty (0)\, dv = C \int_0^{v_0} v^2\, dv = \frac{C}{3} v_0^3. \end{aligned}$$

Invert, then $C = 3N/v_0^3$.

(b) The average velocity is found from

$$v_{\mathrm{av}} = \frac{1}{N} \int_0^\infty N(v) v\, dv.$$

Using our result from above,

$$\begin{aligned} v_{\mathrm{av}} &= \frac{1}{N} \int_0^{v_0} \left(\frac{3N}{v_0^3} v^2 \right) v\, dv, \\ &= \frac{3}{v_0^3} \int_0^{v_0} v^3\, dv = \frac{3}{v_0^3} \frac{v_0^4}{4} = \frac{3}{4} v_0. \end{aligned}$$

As expected, the average speed is less than the maximum speed. We can make a prediction about the root mean square speed; it will be larger than the average speed (see Exercise 22-15 above) but smaller than the maximum speed.

(c) The root-mean-square velocity is found from

$$v_{\rm rms}^2 = \frac{1}{N}\int_0^\infty N(v)v^2\,dv.$$

Using our results from above,

$$\begin{aligned} v_{\rm rms}^2 &= \frac{1}{N}\int_0^{v_0}\left(\frac{3N}{v_0^3}v^2\right)v^2\,dv, \\ &= \frac{3}{v_0^3}\int_0^{v_0} v^4\,dv = \frac{3}{v_0^3}\frac{v_0^5}{5} = \frac{3}{5}v_0^2. \end{aligned}$$

Then, taking the square root,

$$v_{\rm rms}^2 = \sqrt{\frac{3}{5}}\,v_0$$

Is $\sqrt{3/5} > 3/4$? It had better be.

P22-10

P22-11

P22-12

P22-13

P22-14

$\boxed{\textbf{P22-15}}$ The mass of air displaced by 2180 m^3 is $m = (1.22\,\text{kg/m}^3)(2180\,\text{m}^3) = 2660\,\text{kg}$. The mass of the balloon and basket is 249 kg and we want to lift 272 kg; this leaves a remainder of 2140 kg for the mass of the air inside the balloon. This corresponds to $(2140\,\text{kg})/(0.0289\,\text{kg/mol}) = 7.4{\times}10^4\text{mol}$.

The temperature of the gas inside the balloon is then

$$T = (pV)/(nR) = [(1.01\times10^5\text{Pa})(2180\,\text{m}^3)]/[(7.4\times10^4\text{mol})(8.31\,\text{J/mol}\cdot\text{K}) = 358\,\text{K}.$$

That's 85°C.

P22-16

P22-17

$\boxed{\textbf{E23-1}}$ We apply Eq. 23-1,

$$H = kA\frac{\Delta T}{\Delta x}$$

The rate at which heat flows out is given as a power per area (mW/m^2), so the quantity given is really H/A. Then the temperature difference is

$$\Delta T = \frac{H}{A}\frac{\Delta x}{k} = (0.054\,\text{W/m}^2)\frac{(33,000\,\text{m})}{(2.5\,\text{W/m}\cdot\text{K})} = 710\,\text{K}$$

The heat flow is out, so that the temperature is higher at the base of the crust. The temperature there is then

$$710 + 10 = 720\,^\circ\text{C}.$$

E23-2 We apply Eq. 23-1,

$$H = kA\frac{\Delta T}{\Delta x} = (0.74\,\text{W/m}\cdot\text{K})(6.2\,\text{m})(3.8\,\text{m})\frac{(44\,\text{C}^\circ)}{(0.32\,\text{m})} = 2400\,\text{W}.$$

E23-3 (a) $\Delta T/\Delta x = (136\,\text{C}^\circ)/(0.249\,\text{m}) = 546\,\text{C}^\circ/\text{m}$.

(b) $H = kA\Delta T/\Delta x = (401\,\text{W/m}\cdot\text{K})(1.80\,\text{m}^2)(546\,\text{C}^\circ/\text{m}) = 3.94\times10^5\text{W}$.

(c) $T_\text{H} = (-12^\circ\text{C} + 136\,\text{C}^\circ) = 124^\circ\text{C}$. Then

$$T = (124^\circ\text{C}) - (546\,\text{C}^\circ/\text{m})(0.11\,\text{m}) = 63.9^\circ\text{C}.$$

E23-4 (a) $H = (0.040\,\text{W/m}\cdot\text{K})(1.8\,\text{m}^2)(32\,\text{C}^\circ)/(0.012\,\text{m}) = 190\,\text{W}$.

(b) Since k has increased by a factor of $(0.60)/(0.04) = 15$ then H should also increase by a factor of 15.

$\boxed{\textbf{E23-5}}$ There are three possible arrangements: a sheet of type 1 with a sheet of type 1; a sheet of type 2 with a sheet of type 2; and a sheet of type 1 with a sheet of type 2. We can look back on Sample Problem 23-1 to see how to start the problem; the heat flow will be

$$H_{12} = \frac{A\Delta T}{(L/k_1) + (L/k_2)}$$

for substances of different types; and

$$H_{11} = \frac{A\Delta T/L}{(L/k_1) + (L/k_1)} = \frac{1}{2}\frac{A\Delta T k_1}{L}$$

for a double layer if substance 1. There is a similar expression for a double layer of substance 2.

For configuration (a) we then have

$$H_{11} + H_{22} = \frac{1}{2}\frac{A\Delta T k_1}{L} + \frac{1}{2}\frac{A\Delta T k_2}{L} = \frac{A\Delta T}{2L}(k_1 + k_2),$$

while for configuration (b) we have

$$H_{12} + H_{21} = 2\frac{A\Delta T}{(L/k_1) + (L/k_2)} = \frac{2A\Delta T}{L}\left((1/k_1) + (1/k_2)\right)^{-1}.$$

We want to compare these, so expanding the relevant part of the second configuration

$$\left((1/k_1) + (1/k_2)\right)^{-1} = \left((k_1 + k_2)/(k_2 k_2)\right)^{-1} = \frac{k_1 k_2}{k_1 + k_2}.$$

Then which is larger

$$(k_1 + k_2)/2 \text{ or } \frac{2k_1k_2}{k_1 + k_2}\,?$$

If $k_1 \gg k_2$ then the expression become

$$k_1/2 \text{ and } 2k_2,$$

so the first expression is larger, and therefore configuration (b) has the lower heat flow. Notice that we get the same result if $k_1 \ll k_2$!

E23-6 There's a typo in the exercise.

$H = A\Delta T/R$; since the heat flows through one slab and then through the other, we can write $(T_1 - T_x)/R_1 = (T_x - T_2)/R_2$. Rearranging,

$$T_x = (T_1R_2 + T_2R_1)/(R_1 + R_2).$$

E23-7 Use the results of Exercise 23-6. At the interface between ice and water $T_x = 0°\text{C}$. Then $R_1T_2 + R_2T_1 = 0$, or $k_1T_1/L_1 + k_2T_2/L_2 = 0$. Not only that, $L_1 + L_2 = L$, so

$$k_1T_1L_2 + (L - L_2)k_2T_2 = 0,$$

so

$$L_2 = \frac{(1.42\,\text{m})(1.67\,\text{W/m}\cdot\text{K})(-5.20°\text{C})}{(1.67\,\text{W/m}\cdot\text{K})(-5.20°\text{C}) - (0.502\,\text{W/m}\cdot\text{K})(3.98°\text{C})} = 1.15\,\text{m}.$$

E23-8 ΔT is the same in both cases. So is k. The top configuration has $H_\text{t} = kA\Delta T/(2L)$. The bottom configuration has $H_\text{b} = k(2A)\Delta T/L$. The ratio of $H_\text{b}/H_\text{t} = 4$, so heat flows through the bottom configuration at 4 times the rate of the top. For the top configuration $H_\text{t} = (10\,\text{J})/(2\text{ min}) = 5\,\text{J/min}$. Then $H_\text{b} = 20\,\text{J/min}$. It will take

$$t = (30\,\text{J})/(20\,\text{J/min}) = 1.5\text{ min}.$$

$\boxed{\textbf{E23-9}}$ (a) This exercise has a distraction: it asks about the heat flow through the window, but what you need to find first is the heat flow through the air near the window. We are given the temperature gradient both inside and outside the window. Inside,

$$\frac{\Delta T}{\Delta x} = \frac{(20°\text{C}) - (5°\text{C})}{(0.08\,\text{m})} = 190\,\text{C}°/\text{m};$$

a similar expression exists for outside.

From Eq. 23-1 we find the heat flow *through the air*;

$$H = kA\frac{\Delta T}{\Delta x} = (0.026\,\text{W/m}\cdot\text{K})(0.6\,\text{m})^2(190\,\text{C}°/\text{m}) = 1.8\,\text{W}.$$

The value that we arrived at is the rate that heat flows through the air across an area the size of the window on either side of the window. This heat flow had to occur through the window as well, so

$$H = 1.8\,\text{W}$$

answers the window question.

(b) Now that we know the rate that heat flows through the window, we are in a position to find the temperature difference across the window. Rearranging Eq. 32-1,

$$\Delta T = \frac{H\Delta x}{kA} = \frac{(1.8\,\text{W})(0.005\,\text{m})}{(1.0\,\text{W/m}\cdot\text{K})(0.6\,\text{m})^2} = 0.025\,\text{C}°,$$

so we were well justified in our approximation that the temperature drop across the glass is very small.

E23-10 (a) $W = +214\,\mathrm{J}$, done on means positive.
(b) $Q = -293\,\mathrm{J}$, extracted from means negative.
(c) $\Delta E_{\mathrm{int}} = Q + W = (-293\,\mathrm{J}) + (+214\,\mathrm{J}) = -79.0\,\mathrm{J}$.

E23-11 (a) ΔE_{int} along *any* path between these two points is

$$\Delta E_{\mathrm{int}} = Q + W = (50\,\mathrm{J}) + (-20\,\mathrm{J}) = 30\,\mathrm{J}.$$

Then along ibf $W = (30\,\mathrm{J}) - (36\,\mathrm{J}) = -6\,\mathrm{J}$.
(b) $Q = (-30\,\mathrm{J}) - (+13\,\mathrm{J}) = -43\,\mathrm{J}$.
(c) $E_{\mathrm{int},f} = E_{\mathrm{int},i} + \Delta E_{\mathrm{int}} = (10\,\mathrm{J}) + (30\,\mathrm{J}) = 40\,\mathrm{J}$.
(d) $\Delta E_{\mathrm{int}\,ib} = (22\,\mathrm{J}) - (10\,\mathrm{J}) = 12\,\mathrm{J}$; while $\Delta E_{\mathrm{int}\,bf} = (40\,\mathrm{J}) - (22\,\mathrm{J}) = 18\,\mathrm{J}$. There is no work done on the path bf, so

$$Q_{bf} = \Delta E_{\mathrm{int}\,bf} - W_{bf} = (18\,\mathrm{J}) - (0) = 18\,\mathrm{J},$$

and $Q_{ib} = Q_{ibf} - Q_{bf} = (36\,\mathrm{J}) - (18\,\mathrm{J}) = 18\,\mathrm{J}$.

E23-12 $Q = mL = (0.10)(2.1\times10^{8}\,\mathrm{kg})(333\times10^{3}\,\mathrm{J/kg}) = 7.0\times10^{12}\,\mathrm{J}$.

$\boxed{\textbf{E23-13}}$ We don't need to know the outside temperature because the amount of heat energy required is explicitly stated: 5.22 GJ. We just need to know how much water is required to transfer this amount of heat energy. Use Eq. 23-11, and then

$$m = \frac{Q}{c\Delta T} = \frac{(5.22\times10^{9}\,\mathrm{J})}{(4190\,\mathrm{J/kg\cdot K})(50.0^\circ\mathrm{C} - 22.0^\circ\mathrm{C})} = 4.45\times10^{4}\,\mathrm{kg}.$$

This is the mass of the water, we want to know the volume, so we'll use the density, and then

$$V = \frac{m}{\rho} = \frac{(4.45\times10^{4}\,\mathrm{kg})}{(998\,\mathrm{kg/m^3})} = 44.5\,\mathrm{m}^3.$$

E23-14 The heat energy required is $Q = mc\Delta T$. The time required is $t = Q/P$. Then

$$t = \frac{(0.136\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot K})(100^\circ\mathrm{C} - 23.5^\circ\mathrm{C})}{(220\,\mathrm{W})} = 198\,\mathrm{s}.$$

E23-15 $Q = mL$, so $m = (50.4\times10^{3}\mathrm{J})/(333\times10^{3}\mathrm{J/kg}) = 0.151\,\mathrm{kg}$ is the mount which freezes. Then $(0.258\,\mathrm{kg}) - (0.151\,\mathrm{kg}) = 0.107\,\mathrm{kg}$ is the amount which does not freeze.

E23-16 (a) $W = mg\Delta y$; if $|Q| = |W|$, then

$$\Delta T = \frac{mg\Delta y}{mc} = \frac{(9.81\,\mathrm{m/s^2})(49.4\,\mathrm{m})}{(4190\,\mathrm{J/kg\cdot K})} = 0.112\,\mathrm{C}^\circ.$$

$\boxed{\textbf{E23-17}}$ There are three "things" in this problem: the copper bowl (b), the water (w), and the copper cylinder (c). The total internal energy changes must add up to zero, so

$$\Delta E_{\mathrm{int,b}} + \Delta E_{\mathrm{int,w}} + \Delta E_{\mathrm{int,c}} = 0.$$

As in Sample Problem 23-3, no work is done on any object, so

$$Q_{\mathrm{b}} + Q_{\mathrm{w}} + Q_{\mathrm{c}} = 0.$$

The heat transfers for these three objects are

$$\begin{aligned}
Q_\mathrm{b} &= m_\mathrm{b}c_\mathrm{b}(T_\mathrm{f,b} - T_\mathrm{i,b}),\\
Q_\mathrm{w} &= m_\mathrm{w}c_\mathrm{w}(T_\mathrm{f,w} - T_\mathrm{i,w}) + L_\mathrm{v}m_2,\\
Q_\mathrm{c} &= m_\mathrm{c}c_\mathrm{c}(T_\mathrm{f,c} - T_\mathrm{i,c}).
\end{aligned}$$

For the most part, this looks exactly like the presentation in Sample Problem 23-3; but there is an extra term in the second line. This term reflects the extra heat required to vaporize $m_2 = 4.70$ g of water at 100°C into steam 100°C.

Some of the initial temperatures are specified in the exercise: $T_\mathrm{i,b} = T_\mathrm{i,w} = 21.0°\mathrm{C}$ and $T_\mathrm{f,b} = T_\mathrm{f,w} = T_\mathrm{f,c} = 100°\mathrm{C}$.

(a) The heat transferred to the water, then, is

$$\begin{aligned}
Q_\mathrm{w} &= (0.223\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot K})\left((100°\mathrm{C}) - (21.0°\mathrm{C})\right),\\
&\quad +(2.26\times10^6\mathrm{J/kg})(4.70\times10^{-3}\mathrm{kg}),\\
&= 8.44\times10^4\,\mathrm{J}.
\end{aligned}$$

This answer differs from the back of the book. I think that they (or was it me) used the latent heat of fusion when they should have used the latent heat of vaporization!

(b) The heat transfered to the bowl, then, is

$$Q_\mathrm{w} = (0.146\,\mathrm{kg})(387\,\mathrm{J/kg\cdot K})\left((100°\mathrm{C}) - (21.0°\mathrm{C})\right) = 4.46\times10^3\,\mathrm{J}.$$

(c) The heat transfered from the cylinder was transfered into the water and bowl, so

$$Q_\mathrm{c} = -Q_\mathrm{b} - Q_\mathrm{w} = -(4.46\times10^3\,\mathrm{J}) - (8.44\times10^4\,\mathrm{J}) = -8.89\times10^4\,\mathrm{J}.$$

The initial temperature of the cylinder is then given by

$$T_\mathrm{i,c} = T_\mathrm{f,c} - \frac{Q_\mathrm{c}}{m_\mathrm{c}c_\mathrm{c}} = (100°\mathrm{C}) - \frac{(-8.89\times10^4\,\mathrm{J})}{(0.314\,\mathrm{kg})(387\,\mathrm{J/kg\cdot K})} = 832°\mathrm{C}.$$

E23-18 The temperature of the silver must be raised to the melting point and then the heated silver needs to be melted. The heat required is

$$Q = mL + mc\Delta T = (0.130\,\mathrm{kg})[(105\times10^3\mathrm{J/kg}) + (236\,\mathrm{J/kg\cdot K})(1235\,\mathrm{K} - 289\,\mathrm{K})] = 4.27\times10^4\mathrm{J}.$$

E23-19 (a) Use $Q = mc\Delta T$, $m = \rho V$, and $t = Q/P$. Then

$$\begin{aligned}
t &= \frac{[m_\mathrm{a}c_\mathrm{a} + \rho_\mathrm{w}V_\mathrm{w}c_\mathrm{w}]\Delta T}{P},\\
&= \frac{[(0.56\,\mathrm{kg})(900\,\mathrm{J/kg\cdot K}) + (998\,\mathrm{kg/m^3})(0.64\times10^{-3}\mathrm{m^3})(4190\,\mathrm{J/kg\cdot K})](100°\mathrm{C} - 12°\mathrm{C})}{(2400\,\mathrm{W})} = 117\,\mathrm{s}.
\end{aligned}$$

(b) Use $Q = mL$, $m = \rho V$, and $t = Q/P$. Then

$$t = \frac{\rho_\mathrm{w}V_\mathrm{w}L_\mathrm{w}}{P} = \frac{(998\,\mathrm{kg/m^3})(0.640\times10^{-3}\mathrm{m^3})(2256\times10^3\,\mathrm{J/kg})}{(2400\,\mathrm{W})} = 600\,\mathrm{s}$$

is the *additional* time required.

E23-20 The heat given off by the steam will be

$$Q_\mathrm{s} = m_\mathrm{s} L_\mathrm{v} + m_\mathrm{s} c_\mathrm{w} (50\,\mathrm{C}^\circ).$$

The hear taken in by the ice will be

$$Q_\mathrm{i} = m_\mathrm{i} L_\mathrm{f} + m_\mathrm{i} c_\mathrm{w} (50\,\mathrm{C}^\circ).$$

Equating,

$$\begin{aligned} m_\mathrm{s} &= mrm_i \frac{L_\mathrm{f} + c_\mathrm{w}(50\,\mathrm{C}^\circ)}{L_\mathrm{v} + c_\mathrm{w}(50\,\mathrm{C}^\circ)}, \\ &= (0.150\,\mathrm{kg}) \frac{(333\times 10^3\,\mathrm{J/kg}) + (4190\,\mathrm{J/kg\cdot K})(50\,\mathrm{C}^\circ)}{(2256\times 10^3\,\mathrm{J/kg}) + (4190\,\mathrm{J/kg\cdot K})(50\,\mathrm{C}^\circ)} = 0.033\,\mathrm{kg}. \end{aligned}$$

E23-21 The linear dimensions of the ring and sphere change with the temperature change according to

$$\begin{aligned} \Delta d_\mathrm{r} &= \alpha_\mathrm{r} d_\mathrm{r} (T_\mathrm{f,r} - T_\mathrm{i,r}), \\ \Delta d_\mathrm{s} &= \alpha_\mathrm{s} d_\mathrm{s} (T_\mathrm{f,s} - T_\mathrm{i,s}). \end{aligned}$$

When the ring and sphere are at the same (final) temperature the ring and the sphere have the same diameter. This means that

$$d_\mathrm{r} + \Delta d_\mathrm{r} = d_\mathrm{s} + \Delta d_\mathrm{s}$$

when $T_\mathrm{f,s} = T_\mathrm{f,r}$. We'll solve these expansion equations first, and then go back to the heat equations.

$$\begin{aligned} d_\mathrm{r} + \Delta d_\mathrm{r} &= d_\mathrm{s} + \Delta d_\mathrm{s}, \\ d_\mathrm{r}\left(1 + \alpha_\mathrm{r}(T_\mathrm{f,r} - T_\mathrm{i,r})\right) &= d_\mathrm{s}\left(1 + \alpha_\mathrm{s}(T_\mathrm{f,s} - T_\mathrm{i,s})\right), \end{aligned}$$

which can be rearranged to give

$$\alpha_\mathrm{r} d_\mathrm{r} T_\mathrm{f,r} - \alpha_\mathrm{s} d_\mathrm{s} T_\mathrm{f,s} = d_\mathrm{s}\left(1 - \alpha_\mathrm{s} T_\mathrm{i,s}\right) - d_\mathrm{r}\left(1 - \alpha_\mathrm{r} T_\mathrm{i,r}\right),$$

but since the final temperatures are the same,

$$T_\mathrm{f} = \frac{d_\mathrm{s}\left(1 - \alpha_\mathrm{s} T_\mathrm{i,s}\right) - d_\mathrm{r}\left(1 - \alpha_\mathrm{r} T_\mathrm{i,r}\right)}{\alpha_\mathrm{r} d_\mathrm{r} - \alpha_\mathrm{s} d_\mathrm{s}}$$

Putting in the numbers,

$$\begin{aligned} T_\mathrm{f} &= \\ &\frac{(2.54533\mathrm{cm})[1-(23\times 10^{-6}/\mathrm{C}^\circ)(100^\circ\mathrm{C})]-(2.54000\mathrm{cm})[1-(17\times 10^{-6}/\mathrm{C}^\circ)(0^\circ\mathrm{C})]}{(2.54000\mathrm{cm})(17\times 10^{-6}/\mathrm{C}^\circ)-(2.54533\mathrm{cm})(23\times 10^{-6}/\mathrm{C}^\circ)}, \\ &= 34.1^\circ\mathrm{C}. \end{aligned}$$

No work is done, so we only have the issue of heat flow, then

$$Q_\mathrm{r} + Q_\mathrm{s} = 0.$$

Where "r" refers to the copper ring and "s" refers to the aluminum sphere. The heat equations are

$$\begin{aligned} Q_\mathrm{r} &= m_\mathrm{r} c_\mathrm{r} (T_\mathrm{f} - T_\mathrm{i,r}), \\ Q_\mathrm{s} &= m_\mathrm{s} c_\mathrm{s} (T_\mathrm{f} - T_\mathrm{i,s}). \end{aligned}$$

Equating and rearranging,

$$m_\mathrm{s} = \frac{m_\mathrm{r} c_\mathrm{r} (T_\mathrm{i,r} - T_\mathrm{f})}{c_\mathrm{s} (T_\mathrm{f} - T_\mathrm{i,s})}$$

or

$$m_\mathrm{s} = \frac{(21.6\ \mathrm{g})(387\,\mathrm{J/kg\cdot K})(0^\circ\mathrm{C} - 34.1^\circ\mathrm{C})}{(900\,\mathrm{J/kg\cdot K})(34.1^\circ\mathrm{C} - 100^\circ\mathrm{C})} = 4.81\ \mathrm{g}.$$

E23-22 The problem is compounded because we don't know if the final state is only water, only ice, or a mixture of the two.

Consider first the water. Cooling it to 0°C would require the removal of

$$Q_{\rm w} = (0.200\,{\rm kg})(4190\,{\rm J/kg\cdot K})(0^\circ{\rm C} - 25^\circ{\rm C}) = -2.095\times10^4{\rm J}.$$

Consider now the ice. Warming the ice to would require the addition of

$$Q_{\rm i} = (0.100\,{\rm kg})(2220\,{\rm J/kg\cdot K})(0^\circ{\rm C} + 15^\circ{\rm C}) = 3.33\times10^3{\rm J}.$$

The heat absorbed by the warming ice isn't enough to cool the water to freezing. However, the ice can melt; and if it does it will require the addition of

$$Q_{\rm im} = (0.100\,{\rm kg})(333\times10^3{\rm J/kg}) = 3.33\times10^4{\rm J}.$$

This is far more than will be liberated by the cooling water, so the final temperature is 0°C, and consists of a mixture of ice and water.

(b) Consider now the ice. Warming the ice to would require the addition of

$$Q_{\rm i} = (0.050\,{\rm kg})(2220\,{\rm J/kg\cdot K})(0^\circ{\rm C} + 15^\circ{\rm C}) = 1.665\times10^3{\rm J}.$$

The heat absorbed by the warming ice isn't enough to cool the water to freezing. However, the ice can melt; and if it does it will require the addition of

$$Q_{\rm im} = (0.050\,{\rm kg})(333\times10^3{\rm J/kg}) = 1.665\times10^4{\rm J}.$$

This is still not enough to cool the water to freezing. Hence, we need to solve

$$Q_{\rm i} + Q_{\rm im} + m_{\rm i}c_{\rm w}(T - 0^\circ{\rm C}) + m_{\rm w}c_{\rm w}(T - 25^\circ{\rm C}) = 0,$$

which has solution

$$T = \frac{(4190\,{\rm J/kg\cdot K})(0.200\,{\rm kg})(25^\circ{\rm C}) - (1.665\times10^3{\rm J}) + (1.665\times10^4{\rm J})}{(4190\,{\rm J/kg\cdot K})(0.250\,{\rm kg})} = 2.5^\circ{\rm C}.$$

E23-23 (a) $c = (320\,{\rm J})/(0.0371\,{\rm kg})(42.0^\circ{\rm C} - 26.1^\circ{\rm C}) = 542\,{\rm J/kg\cdot K}$.

(b) $n = m/M = (37.1\ {\rm g})/(51.4\ {\rm g/mol}) = 0.722$ mol.

(c) $c = (542\,{\rm J/kg\cdot K})(51.4\times10^{-3}\ {\rm kg/mol}) = 27.9\,{\rm J/mol\cdot K}$.

E23-24 (1) $W = -p\Delta V = (15\,{\rm Pa})(4\,{\rm m}^3) = -60\,{\rm J}$ for the horizontal path; no work is done during the vertical path; the net work done on the gas is $-60\,{\rm J}$.

(2) It is easiest to consider work as the (negative of) the area under the curve; then $W = -(15\,{\rm Pa} + 5\,{\rm Pa})(4\,{\rm m}^3)/2 = -40\,{\rm J}$.

(3) No work is done during the vertical path; $W = -p\Delta V = (5\,{\rm Pa})(4\,{\rm m}^3) = -20\,{\rm J}$ for the horizontal path; the net work done on the gas is $-20\,{\rm J}$.

$\boxed{\textbf{E23-25}}$ Net work done on the gas is given by Eq. 23-15,

$$W = -\int p\,dV.$$

But integrals are just the area under the curve; and that's the easy way to solve this problem. In the case of closed paths, it becomes the area inside the curve, with a clockwise sense giving a positive value for the integral.

The magnitude of the area is the same for either path, since it is a rectangle divided in half by a square. The area of the rectangle is

$$(15\times10^3{\rm Pa})(6\,{\rm m}^3) = 90\times10^3{\rm J},$$

so the area of path 1 (counterclockwise) is -45 kJ; this means the work done on the gas is -(-45 kJ) or 45 kJ. The work done on the gas for path 2 is the negative of this because the path is clockwise.

E23-26 During the isothermal expansion,

$$W_1 = -nRT \ln \frac{V_2}{V_1} = -p_1 V_1 \ln \frac{p_1}{p_2}.$$

During cooling at constant pressure,

$$W_2 = -p_2 \Delta V = -p_2(V_1 - V_2) = -p_2 V_1 (1 - p_1/p_2) = V_1(p_1 - p_2).$$

The work done is the sum, or

$$-(204\times10^3\text{Pa})(0.142\,\text{m}^3) \ln \frac{(204\times10^3\text{Pa})}{(101\times10^3\text{Pa})} + (0.142\,\text{m}^3)(103\,\text{Pa}) = -5.74\times10^3\,\text{J}.$$

E23-27 During the isothermal expansion,

$$W = -nRT \ln \frac{V_2}{V_1} = -p_1 V_1 \ln \frac{V_2}{V_1},$$

so

$$W = -(1.32)(1.01\times10^5\text{Pa})(0.0224\,\text{m}^3) \ln \frac{(0.0153\,\text{m}^3)}{(0.0224\,\text{m}^3)} = 1.14\times10^3\,\text{J}.$$

E23-28 (a) pV^γ is a constant, so

$$p_2 = p_1 (V_1/V_2)^\gamma = (1.00 \text{ atm})[(1 \text{ l})/(0.5 \text{ l})]^{1.32} = 2.50 \text{ atm};$$

$T_2 = T_1(p_2/p_1)(V_2/V_1)$, so

$$T_2 = (273\,\text{K}) \frac{(2.50 \text{ atm})}{(1.00 \text{ atm})} \frac{(0.5 \text{ l})}{(1 \text{ l})} = 341\,\text{K}.$$

(b) $V_3 = V_2(p_2/p_1)(T_3/T_2)$, so

$$V_3 = (0.5 \text{ l}) \frac{(273\,\text{K})}{(341\,\text{K})} = 0.40 \text{ l}.$$

(c) During the adiabatic process,

$$W_{12} = \frac{(1.01\times10^5\text{Pa/atm})(1\times10^{-3}\text{m}^3/\text{l})}{(1.32) - 1}[(2.5 \text{ atm})(0.5 \text{ l}) - (1.0 \text{ atm})(1 \text{ l})] = 78.9\,\text{J}.$$

During the cooling process,

$$W_{23} = -p\Delta V = -(1.01\times10^5\text{Pa/atm})(2.50 \text{ atm})(1\times10^{-3}\text{m}^3/\text{l})[(0.4 \text{ l}) - (0.5 \text{ l})] = 25.2\,\text{J}.$$

The net work done is $W_{123} = 78.9\,\text{J} + 25.2\,\text{J} = 104.1\,\text{J}$.

E23-29 (a) According to Eq. 23-20,

$$p_\text{f} = \frac{p_\text{i} V_\text{i}{}^\gamma}{V_\text{f}{}^\gamma} = \frac{(1.17 \text{ atm})(4.33 \text{ L})^{(1.40)}}{(1.06 \text{ L})^{(1.40)}} = 8.39 \text{ atm}.$$

(b) The final temperature can be found from the ideal gas law,

$$T_\text{f} = T_\text{i} \frac{p_\text{f} V_\text{f}}{p_\text{i} V_\text{i}} = (310\,\text{K}) \frac{(8.39 \text{ atm})(1.06 \text{ L})}{(1.17 \text{ atm})(4.33 \text{ L})} = 544\,\text{K}.$$

(c) The work done (for an adiabatic process) is given by Eq. 23-22,

$$\begin{aligned} W &= \frac{1}{(1.40) - 1} \left[(8.39 \times 1.01\times10^5\text{Pa})(1.06\times10^{-3}\text{m}^3)\right. \\ &\quad \left. -(1.17 \times 1.01\times10^5\text{Pa})(4.33\times10^{-3}\text{m}^3)\right], \\ &= 966\,\text{J}. \end{aligned}$$

E23-30 Air is mostly diatomic (N_2 and O_2), so use $\gamma = 1.4$.

(a) pV^γ is a constant, so

$$V_2 = V_1 \sqrt[\gamma]{p_1/p_2} = V_1 \sqrt[1.4]{(1.0 \text{ atm})/(2.3 \text{ atm})} = 0.552 V_1.$$

$T_2 = T_1(p_2/p_1)(V_2/V_1)$, so

$$T_2 = (291\,\text{K})\frac{(2.3 \text{ atm})}{(1.0 \text{ atm})}\frac{(0.552V_1)}{V_1} = 369\,\text{K},$$

or 96°C.

(b) The work required for delivering 1 liter of compressed air is

$$W_{12} = \frac{(1.01\times10^5\text{Pa/atm})(1\times10^{-3}\text{m}^3/\text{l})}{(1.40)-1}[(2.3 \text{ atm})(1.0 \text{ l}) - (1.0 \text{ atm})(1.0 \text{ l}/0.552)] = 123\,\text{J}.$$

The number of liters per second that can be delivered is then

$$\Delta V/\Delta t = (230\,\text{W})/(123\,\text{J/l}) = 1.87 \text{ l}.$$

E23-31 $E_{\text{int,rot}} = nRT = (1 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(298\,\text{K}) = 2480\,\text{J}.$

E23-32 $E_{\text{int,rot}} = \frac{3}{2}nRT = (1.5)(1 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(523\,\text{K}) = 6520\,\text{J}.$

$\boxed{\textbf{E23-33}}$ (a) Invert Eq. 32-20,

$$\gamma = \frac{\ln(p_1/p_2)}{\ln(V_2/V_1)} = \frac{\ln(122 \text{ kPa}/1450 \text{ kPa})}{\ln(1.36\,\text{m}^3/10.7\,\text{m}^3)} = 1.20.$$

(b) The final temperature is found from the ideal gas law,

$$T_\text{f} = T_\text{i}\frac{p_\text{f}V_\text{f}}{p_\text{i}V_\text{i}} = (250\,\text{K})\frac{(1450\times10^3\text{Pa})(1.36\,\text{m}^3)}{(122\times10^3\text{Pa})(10.7\,\text{m}^3)} = 378\,\text{K},$$

which is the same as 105°C.

(c) Ideal gas law, again:

$$n = [pV]/[RT] = [(1450\times10^3\text{Pa})(1.36\,\text{m}^3)]/[(8.31\,\text{J/mol}\cdot\text{K})(378\,\text{K})] = 628 \text{ mol}.$$

(d) From Eq. 23-24,

$$E_\text{int} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(250\,\text{K}) = 1.96\times10^6\,\text{J}$$

before the compression and

$$E_\text{int} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(378\,\text{K}) = 2.96\times10^6\,\text{J}$$

after the compression.

(e) The ratio of the rms speeds will be proportional to the square root of the ratio of the internal energies,

$$\sqrt{(1.96\times10^6\text{J})/(2.96\times10^6\text{J})} = 0.813;$$

we can do this because the number of particles is the same before and after, hence the ratio of the energies per particle is the same as the ratio of the total energies.

E23-34 We can assume neon is an ideal gas. Then $\Delta T = 2\Delta E_{\rm int}/3nR$, or

$$\Delta T = \frac{2(1.34\times10^{12}\text{eV})(1.6\times10^{-19}\text{J/eV})}{3(0.120\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})} = 1.43\times10^{-7}\text{J}.$$

E23-35 At constant pressure, doubling the volume is the same as doubling the temperature. Then

$$Q = nC_p\Delta T = (1.35\text{ mol})\frac{7}{2}(8.31\,\text{J/mol}\cdot\text{K})(568\,\text{K} - 284\,\text{K}) = 1.12\times10^{4}\text{J}.$$

E23-36 (a) $n = m/M = (12\text{ g})/(28\text{ g/mol}) = 0.429\text{ mol}.$

(b) This is a constant volume process, so

$$Q = nC_V\Delta T = (0.429\text{ mol})\frac{5}{2}(8.31\,\text{J/mol}\cdot\text{K})(125^\circ\text{C} - 25^\circ\text{C}) = 891\text{J}.$$

E23-37 (a) From Eq. 23-37,

$$Q = nc_{\rm p}\Delta T = (4.34\text{ mol})(29.1\,\text{J/mol}\cdot\text{K})(62.4\,\text{K}) = 7880\,\text{J}.$$

(b) From Eq. 23-28,

$$E_{\rm int} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.34\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(62.4\,\text{K}) = 5630\,\text{J}.$$

(c) From Eq. 23-23,

$$K_{\rm trans} = \frac{3}{2}nR\Delta T = \frac{5}{2}(4.34\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(62.4\,\text{K}) = 3380\,\text{J}.$$

E23-38 $c_V = \frac{3}{2}(8.31\,\text{J/mol}\cdot\text{K})/(4.00\text{ g/mol}) = 3120\,\text{J/kg}\cdot\text{K}.$

E23-39 Each species will experience the same temperature change, so

$$\begin{aligned} Q &= Q_1 + Q_2 + Q_3, \\ &= n_1C_1\Delta T + n_2C_2\Delta T + n_3C_3\Delta T, \end{aligned}$$

Dividing this by $n = n_1 + n_2 + n_3$ and ΔT will return the specific heat capacity of the mixture, so

$$C = \frac{n_1C_1 + n_2C_2 + n_3C_3}{n_1 + n_2 + n_3}.$$

E23-40 $W_{AB} = 0$, since it is a constant volume process, consequently, $W = W_{AB} + W_{ABC} = -15\,\text{J}$. But around a closed path $Q = -W$, so $Q = 15\,\text{J}$. Then

$$Q_{CA} = Q - Q_{AB} - Q_{BC} = (15\,\text{J}) - (20\,\text{J}) - (0\,\text{J}) = -5\,\text{J}.$$

Note that this heat is *removed* from the system!

$\boxed{\textbf{E23-41}}$ According to Eq. 23-25 (which is specific to ideal gases),

$$\Delta E_{\text{int}} = \frac{3}{2}nR\Delta T,$$

and for an isothermal process $\Delta T = 0$, so for an ideal gas $\Delta E_{\text{int}} = 0$. Consequently, $Q + W = 0$ for an ideal gas which undergoes an isothermal process.

But we know W for an isotherm, Eq. 23-18 shows

$$W = -nRT \ln \frac{V_{\text{f}}}{V_{\text{i}}}$$

Then finally

$$Q = -W = nRT \ln \frac{V_{\text{f}}}{V_{\text{i}}}$$

E23-42 Q is greatest for constant pressure processes and least for adiabatic. W is greatest (in magnitude, it is negative for increasing volume processes) for constant pressure processes and least for adiabatic. ΔE_{int} is greatest for constant pressure (for which it is positive), and least for adiabatic (for which is is negative).

E23-43 (a) For a monatomic gas, $\gamma = 1.667$. Fast process are often adiabatic, so

$$T_2 = T_1(V_1/V_2)^{\gamma-1} = (292\,\text{K})[(1)(1/10)]^{1.667-1} = 1360\,\text{K}.$$

(b) For a diatomic gas, $\gamma = 1.4$. Fast process are often adiabatic, so

$$T_2 = T_1(V_1/V_2)^{\gamma-1} = (292\,\text{K})[(1)(1/10)]^{1.4-1} = 733\,\text{K}.$$

E23-44 This problem cannot be solved without making some assumptions about the type of process occurring on the two curved portions.

$\boxed{\textbf{E23-45}}$ If the pressure and volume are both doubled along a straight line then the process can be described by

$$p = \frac{p_1}{V_1}V$$

The final point involves the doubling of both the pressure and the volume, so according to the ideal gas law, $pV = nRT$, the final temperature T_2 will be *four* times the initial temperature T_1.

Now for the exercises.

(a) The work done on the gas is

$$W = -\int_1^2 p\,dV = -\int_1^2 \frac{p_1}{V_1}V\,dV = -\frac{p_1}{V_1}\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)$$

We want to express our answer in terms of T_1. First we take advantage of the fact that $V_2 = 2V_1$, then

$$W = -\frac{p_1}{V_1}\left(\frac{4V_1^2}{2} - \frac{V_1^2}{2}\right) = -\frac{3}{2}p_1V_1 = -\frac{3}{2}nRT_1$$

(b) The nice thing about ΔE_{int} is that it is path independent, we care only of the initial and final points. From Eq. 23-25,

$$\Delta E_{\text{int}} = \frac{3}{2}nR\Delta T = \frac{3}{2}nR\,(T_2 - T_1) = \frac{9}{2}nRT_1$$

(c) Finally we are in a position to find Q by applying the first law,

$$Q = \Delta E_{\text{int}} - W = \frac{9}{2}nRT_1 + \frac{3}{2}nRT_1 = 6nRT_1.$$

(d) If we define specific heat as heat divided by temperature change, then

$$c = \frac{Q}{n\Delta T} = \frac{6RT_1}{4T_1 - T_1} = 2R.$$

E23-46 The work done is the area enclosed by the path. If the pressure is measured in units of 10MPa, then the shape is a semi-circle, and the area is

$$W = (\pi/2)(1.5)^2(10\text{MPa})(1\times10^{-3}\text{m}^3) = 3.53\times10^4\text{J}.$$

The heat is given by $Q = -W = -3.53\times10^4$ J.

E23-47 (a) Internal energy changes according to $\Delta E_{\text{int}} = Q + W$, so

$$\Delta E_{\text{int}} = (20.9\,\text{J}) - (1.01\times10^5\text{Pa})(113\times10^{-6}\text{m}^3 - 63\times10^{-6}\text{m}^3) = 15.9\,\text{J}.$$

(b) $T_1 = p_1V_1/nR$ and $T_2 = p_2V_2/nR$, but p is constant, so $\Delta T = p\Delta V/nR$. Then

$$C_P = \frac{Q}{n\Delta T} = \frac{QR}{p\Delta V} = \frac{(20.9\,\text{J})(8.31\,\text{J/mol}\cdot\text{K})}{(1.01\times10^5\text{Pa})(113\times10^{-6}\text{m}^3 - 63\times10^{-6}\text{m}^3)} = 34.4\,\text{J/mol}\cdot\text{K}.$$

(c) $C_V = C_P - R = (34.4\,\text{J/mol}\cdot\text{K}) - (8.31\,\text{J/mol}\cdot\text{K}) = 26.1\,\text{J/mol}\cdot\text{K}$.

E23-48 **Constant Volume**

(a) $Q = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.
(b) $W = 0$.
(c) $\Delta Erm_{int} = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.

Constant Pressure

(a) $Q = 4(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 5450\,\text{J}$.
(b) $W = -p\Delta V = -nR\Delta T = -(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = -1360\,\text{J}$.
(c) $\Delta Erm_{int} = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.

Adiabatic

(a) $Q = 0$.
(b) $W = (p_{\text{f}}V_{\text{f}} - p_{\text{i}}V_{\text{i}})/(\gamma - 1) = nR\Delta T/(\gamma - 1) = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.
(c) $\Delta Erm_{int} = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.

P23-1 (a) The temperature difference is

$$(5\,\text{C}^\circ/9\ \text{F}^\circ)(72^\circ\text{F} - -20^\circ\text{F}) = 51.1\,\text{C}^\circ.$$

The rate of heat loss is

$$H = (1.0\,\text{W/m}\cdot\text{K})(1.4\,\text{m}^2)(51.1\,\text{C}^\circ)/(3.0\times10^{-3}\text{m}) = 2.4\times10^4\text{W}.$$

(b) Start by finding the R values.

$$R_{\text{g}} = (3.0\times10^{-3}\text{m})/(1.0\,\text{W/m}\cdot\text{K}) = 3.0\times10^{-3}\text{m}^2\cdot\text{K/W},$$

$$R_{\text{a}} = (7.5\times10^{-2}\text{m})/(0.026\text{W/m}\cdot\text{K}) = 2.88\text{m}^2\cdot\text{K/W}.$$

Then use Eq. 23-5,

$$H = \frac{(1.4\,\text{m}^2)(51.1\,\text{C}^\circ)}{2(3.0\times10^{-3}\text{m}^2\cdot\text{K/W}) + (2.88\text{m}^2\cdot\text{K/W})} = 25\,\text{W}.$$

Get double pane windows!

P23-2 (a) $H = (428\,\text{W/m}\cdot\text{K})(4.76\times10^{-4}\text{m}^2)(100\,\text{C}^\circ)/(1.17\,\text{m}) = 17.4\,\text{W}$.
(b) $\Delta m/\Delta t = H/L = (17.4\,\text{W})/(333\times10^3\text{J/kg}) = 5.23\times10^{-5}\text{kg/s}$, which is the same as 188 g/h.

$\boxed{\textbf{P23-3}}$ Follow the example in Sample Problem 23-2. We start with Eq. 23-1:

$$\begin{aligned}
H &= kA\frac{dT}{dr},\\
H &= k(4\pi r^2)\frac{dT}{dr},\\
\int_{r_1}^{r_2} H\frac{dr}{4\pi r^2} &= \int_{T_1}^{T_2} k dT,\\
\frac{H}{4pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) &= k(T_1-T_2),\\
H\left(\frac{r_2-r_1}{r_1r_2}\right) &= 4\pi k(T_1-T_2),\\
H &= \frac{4\pi k(T_1-T_2)r_1r_2}{r_2-r_1}.
\end{aligned}$$

P23-4 (a) $H = (54\times10^{-3}\text{W/m}^2)4\pi(6.37\times10^6\text{m})^2 = 2.8\times10^{13}\text{W}$.
(b) Using the results of Problem 23-3,

$$\Delta T = \frac{(2.8\times10^{13}\text{W})(6.37\times10^6\text{m} - 3.47\times10^6\text{m})}{4\pi(4.2\,\text{W/m}\cdot\text{K})(6.37\times10^6\text{m})(3.47\times10^6\text{m})} = 7.0\times10^4\text{C}^\circ.$$

Since $T_2 = 0^\circ\text{C}$, we expect $T_1 = 7.0\times10^4\text{C}^\circ$.

P23-5 Since $H = -kA\,dT/dx$, then $H\,dx = -aT\,dT$. H is a constant, so integrate both side according to

$$\begin{aligned}
\int H\,dx &= -\int aT\,dT,\\
HL &= -a\frac{1}{2}(T_2^2-T_1^2),\\
H &= \frac{aA}{2L}(T_1^2-T_2^2).
\end{aligned}$$

P23-6 Assume the water is all at 0°C. The heat flow through the ice is then $H = kA\Delta T/x$; the rate of ice formation is $\Delta m/\Delta t = H/L$. But $\Delta m = \rho A\Delta x$, so

$$\frac{\Delta x}{\Delta t} = \frac{H}{\rho AL} = \frac{k\Delta T}{\rho Lx},$$

$$\frac{(1.7\,\text{W/m}\cdot\text{K})(10\,\text{C}^\circ)}{(920\,\text{kg/m}^3)(333\times10^3\text{J/kg})(0.05\,\text{m})} = 1.11\times10^{-6}\text{m/s}.$$

That's the same as 0.40 cm/h.

$\boxed{\textbf{P23-7}}$ (a) Start with the heat equation:

$$Q_\text{t} + Q_\text{i} + Q_\text{w} = 0,$$

where $Q_{\rm t}$ is the heat from the tea, $Q_{\rm i}$ is the heat from the ice when it melts, and $Q_{\rm w}$ is the heat from the water (which used to be ice). Then

$$m_{\rm t}c_{\rm t}(T_{\rm f}-T_{\rm t,i})+m_{\rm i}L_{\rm f}+m_{\rm w}c_{\rm w}(T_{\rm f}-T_{\rm w,i})=0,$$

which, since we have assumed all of the ice melts and the masses are all equal, can be solved for T_f as

$$\begin{aligned} T_f &= \frac{c_{\rm t}T_{\rm t,i}+c_{\rm w}T_{\rm w,i}-L_{\rm f}}{c_{\rm t}+c_{\rm w}}, \\ &= \frac{(4190\,{\rm J/kg\cdot K})(90^\circ{\rm C})+(4190\,{\rm J/kg\cdot K})(0^\circ{\rm C})-(333\times10^3\,{\rm J/kg})}{(4190\,{\rm J/kg\cdot K})+(4190\,{\rm J/kg\cdot K})}, \\ &= 5.3^\circ{\rm C}. \end{aligned}$$

(b) Once again, assume all of the ice melted. Then we can do the same steps, and we get

$$\begin{aligned} T_f &= \frac{c_{\rm t}T_{\rm t,i}+c_{\rm w}T_{\rm w,i}-L_{\rm f}}{c_{\rm t}+c_{\rm w}}, \\ &= \frac{(4190\,{\rm J/kg\cdot K})(70^\circ{\rm C})+(4190\,{\rm J/kg\cdot K})(0^\circ{\rm C})-(333\times10^3\,{\rm J/kg})}{(4190\,{\rm J/kg\cdot K})+(4190\,{\rm J/kg\cdot K})}, \\ &= -4.7^\circ{\rm C}. \end{aligned}$$

So we must have guessed wrong when we assumed that all of the ice melted. The heat equation then simplifies to

$$m_{\rm t}c_{\rm t}(T_{\rm f}-T_{\rm t,i})+m_{\rm i}L_{\rm f}=0,$$

and then

$$\begin{aligned} m_{\rm i} &= \frac{m_{\rm t}c_{\rm t}(T_{\rm t,i}-T_{\rm f})}{L_{\rm f}}, \\ &= \frac{(0.520\,{\rm kg})(4190\,{\rm J/kg\cdot K})(90^\circ{\rm C}-0^\circ)}{(333\times10^3\,{\rm J/kg})}, \\ &= 0.458\,{\rm kg}. \end{aligned}$$

P23-8 $c=Q/m\Delta T=H/(\Delta m/\Delta t)\Delta T$. But $\Delta m/\Delta t=\rho\Delta V/\Delta t$. Combining,

$$c=\frac{H}{(\Delta V/\Delta t)\rho\Delta T}=\frac{(250\,{\rm W})}{(8.2\times10^{-6}{\rm m^3/s})(0.85\times10^3{\rm kg/m^3})(15\,{\rm C^\circ})}=2.4\times10^3\,{\rm J/kg\cdot K}.$$

P23-9 (a) $n=N_A/M$, so

$$\epsilon=\frac{(2256\times10^3\,{\rm J/kg})}{(6.02\times10^{23}/{\rm mol})/(0.018\ {\rm kg/mol})}=6.75\times10^{-20}\,{\rm J}.$$

(b) $E_{\rm av}=\frac{3}{2}kT$, so

$$\frac{\epsilon}{E_{\rm av}}=\frac{2(6.75\times10^{-20}\,{\rm J})}{3(1.38\times10^{-23}\,{\rm J/K})(305\,{\rm K})}=10.7.$$

P23-10 $Q_{\rm w}+Q_{\rm t}=0$, so

$$C_{\rm t}\Delta T_{\rm t}+m_{\rm w}c_{\rm w}(T_{\rm f}-T_{\rm i})=0,$$

or

$$T_{\rm i}=\frac{(0.3\,{\rm kg})(4190\,{\rm J/kg\cdot m})(44.4^\circ{\rm C})+(46.1\,{\rm J/K})(44.4^\circ{\rm C}-15.0^\circ{\rm C})}{(0.3\,{\rm kg})(4190\,{\rm J/kg\cdot m})}=45.5^\circ{\rm C}.$$

$\boxed{\textbf{P23-11}}$ We can use Eq. 23-10, but we will need to approximate c first. If we assume that the line is straight then we use $c = mT + b$. I approximate m from

$$m = \frac{(14\,\mathrm{J/mol\cdot K}) - (3\,\mathrm{J/mol\cdot K})}{(500\,\mathrm{K}) - (200\,\mathrm{K})} = 3.67\times10^{-2}\mathrm{J/mol}.$$

Then I find b from those same data points,

$$b = (3\,\mathrm{J/mol\cdot K}) - (3.67\times10^{-2}\mathrm{J/mol})(200\,\mathrm{K}) = -4.34\,\mathrm{J/mol\cdot K}.$$

Then from Eq. 23-10,

$$\begin{aligned}
Q &= n\int_{T_\mathrm{i}}^{T_\mathrm{f}} c\,dT,\\
&= n\int_{T_\mathrm{i}}^{T_\mathrm{f}} (mT+b)\,dT,\\
&= n\left[\frac{m}{2}T^2 + bT\right]_{T_\mathrm{i}}^{T_\mathrm{f}},\\
&= n\left(\frac{m}{2}({T_\mathrm{f}}^2 - {T_\mathrm{i}}^2) + b(T_\mathrm{f} - T_\mathrm{i})\right),\\
&= (0.45\mathrm{mol})\left(\frac{(3.67\times10^{-2}\mathrm{J/mol})}{2}((500\,\mathrm{K})^2 - (200\,\mathrm{K})^2)\right.\\
&\quad \left. + (-4.34\,\mathrm{J/mol\cdot K})(500\,\mathrm{K} - 200\,\mathrm{K})\right),\\
&= 1.15\times10^3\mathrm{J}.
\end{aligned}$$

P23-12 $\delta Q = nC\delta T$, so

$$\begin{aligned}
Q &= n\int C\,dT,\\
&= n\left[(0.318\ \mathrm{J/mol\cdot K^2})T^2/2 - (0.00109\ \mathrm{J/mol\cdot K^3})T^3/3 - (0.628\ \mathrm{J/mol\cdot K})T\right]_{50\,\mathrm{K}}^{90\,\mathrm{K}},\\
&= n(645.8\ \mathrm{J/mol}).
\end{aligned}$$

Finally,

$$Q = (645.8\ \mathrm{J/mol})(316\ \mathrm{g})/(107.87\ \mathrm{g/mol}) = 189\,\mathrm{J}.$$

P23-13 $TV^{\gamma-1}$ is a constant, so

$$T_2 = (292\,\mathrm{K})(1/1.28)^{(1.40)-1} = 265\,\mathrm{K}$$

P23-14 $W = -\int p\,dV$, so

$$\begin{aligned}
W &= -\int\left[\frac{nRT}{V-nb} - \frac{an^2}{V^2}\right]dV,\\
&= -\left. nRT\ln(V-nb) - \frac{an^2}{V}\right|_i^f,\\
&= -nRT\ln\frac{V_\mathrm{f}-nb}{V_\mathrm{i}-nb} - an^2\left(\frac{1}{V_\mathrm{f}} - \frac{1}{V_\mathrm{i}}\right).
\end{aligned}$$

P23-15 When the tube is horizontal there are two regions filled with gas, one at $p_{1,\mathrm{i}}$, $V_{1,\mathrm{i}}$; the other at $p_{2,\mathrm{i}}$, $V_{2,\mathrm{i}}$. Originally $p_{1,\mathrm{i}} = p_{2,\mathrm{i}} = 1.01\times10^5\mathrm{Pa}$ and $V_{1,\mathrm{i}} = V_{2,\mathrm{i}} = (0.45\,\mathrm{m})A$, where A is the cross sectional area of the tube.

When the tube is held so that region 1 is on top then the mercury has three forces on it: the force of gravity, mg; the force from the gas above pushing down $p_{2,\mathrm{f}}A$; and the force from the gas below pushing up $p_{1,\mathrm{f}}A$. The balanced force expression is

$$p_{1,\mathrm{f}}A = p_{2,\mathrm{f}}A + mg.$$

If we write $m = \rho l_{\mathrm{m}} A$ where $l_{\mathrm{m}} = 0.10\,\mathrm{m}$, then

$$p_{1,\mathrm{f}} = p_{2,\mathrm{f}} + \rho g l_{\mathrm{m}}.$$

Finally, since the tube has uniform cross section, we can write $V = Al$ everywhere.

(a) For an isothermal process $p_{\mathrm{i}}l_{\mathrm{i}} = p_{\mathrm{f}}l_{\mathrm{f}}$, where we have used $V = Al$, and then

$$p_{1,\mathrm{i}}\frac{l_{1,\mathrm{i}}}{l_{1,\mathrm{f}}} - p_{2,\mathrm{i}}\frac{l_{2,\mathrm{i}}}{l_{2,\mathrm{f}}} = \rho g l_{\mathrm{m}}.$$

But we can factor out $p_{1,\mathrm{i}} = p_{2,\mathrm{i}}$ and $l_{1,\mathrm{i}} = l_{2,\mathrm{i}}$, and we can apply $l_{1,\mathrm{f}} + l_{2,\mathrm{f}} = 0.90\,\mathrm{m}$. Then

$$\frac{1}{l_{1,\mathrm{f}}} - \frac{1}{0.90\,\mathrm{m} - l_{1,\mathrm{f}}} = \frac{\rho g l_{\mathrm{m}}}{p_{\mathrm{i}}l_{\mathrm{i}}}.$$

Put in some numbers and rearrange,

$$0.90\,\mathrm{m} - 2l_{1,\mathrm{f}} = (0.294\,\mathrm{m}^{-1})l_{1,\mathrm{f}}(0.90\,\mathrm{m} - l_{1,\mathrm{f}}),$$

which can be written as an ordinary quadratic,

$$(0.294\,\mathrm{m}^{-1}){l_{1,\mathrm{f}}}^2 - (2.265)l_{1,\mathrm{f}} + (0.90\,\mathrm{m}) = 0$$

The solutions are $l_{1,\mathrm{f}} = 7.284\,\mathrm{m}$ and $0.421\,\mathrm{m}$. Only one of these solutions is reasonable, so the mercury shifted down $0.450 - 0.421 = 0.029\,\mathrm{m}$.

(b) The math is a wee bit uglier here, but we can start with $p_{\mathrm{i}}{l_{\mathrm{i}}}^\gamma = p_{\mathrm{f}}{l_{\mathrm{f}}}^\gamma$, and this means that everywhere we had a $l_{1,\mathrm{f}}$ in the previous derivation we need to replace it with ${l_{1,\mathrm{f}}}^\gamma$. Then we have

$$\frac{1}{{l_{1,\mathrm{f}}}^\gamma} - \frac{1}{(0.90\,\mathrm{m} - l_{1,\mathrm{f}})^\gamma} = \frac{\rho g l_{\mathrm{m}}}{p_{\mathrm{i}}{l_{\mathrm{i}}}^\gamma}.$$

This can be written as

$$(0.90\,\mathrm{m} - l_{1,\mathrm{f}})^\gamma - {l_{1,\mathrm{f}}}^\gamma - = (0.404\,\mathrm{m}^{-\gamma}){l_{1,\mathrm{f}}}^\gamma(0.90\,\mathrm{m} - l(0._{1,\mathrm{f}})^\gamma,$$

which looks nasty to me! I'll use Maple to get the answer, and find $l_{1,\mathrm{f}} = 0.429$, so the mercury shifted down $0.450 - 0.429 = 0.021\,\mathrm{m}$.

Which is more likely? Turn the tube fast, and the adiabatic approximation works. Eventually the system will return to room temperature, and then the isothermal approximation is valid.

P23-16 Internal energy for an ideal diatomic gas can be written as

$$E_{\mathrm{int}} = \frac{5}{2}nRT = \frac{5}{2}pV,$$

simply by applying the ideal gas law. The room, however, has a fixed pressure and volume, so the internal energy is independent of the temperature. As such, any energy supplied by the furnace leaves the room, either as heat or as expanding gas doing work on the outside.

P23-17 The speed of sound in the iodine gas is

$$v = f\lambda = (1000\text{ Hz})(2 \times 0.0677\,\text{m}) = 135\,\text{m/s}.$$

Then

$$\gamma = \frac{Mv^2}{RT} = \frac{n(0.127\,\text{kg/mol})(135\,\text{m/s})^2}{(8.31\,\text{J/mol}\cdot\text{K})(400\,\text{K})} = n(0.696).$$

Since γ is greater than one, $n \geq 2$. If $n = 2$ then $\gamma = 1.39$, which is consistent; if $n = 3$ then $\gamma = 2.08$, which is not consistent.

Consequently, iodine gas is diatomic.

P23-18 $W = -Q = mL = (333\times10^3\text{J/kg})(0.122\,\text{kg}) = 4.06\times10^4\text{J}.$

P23-19 (a)

Process AB

$Q = \frac{3}{2}(1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = 3740\,\text{J}.$

$W = 0.$

$\Delta E_{\text{int}} = Q + W = 3740\,\text{J}.$

Process BC

$Q = 0.$

$W = (p_{\text{f}}V_{\text{f}} - p_{\text{i}}V_{\text{i}})/(\gamma - 1) = nR\Delta T/(\gamma - 1) = (1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(-145\,\text{K})/(1.67 - 1) = -1800\,\text{J}$

$\Delta E_{\text{int}} = Q + W = -1800\,\text{J}.$

Process AB

$Q = \frac{5}{2}(1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(-155\,\text{K}) = -3220\,\text{J}.$

$W = -p\Delta V = -nR\Delta T = -(1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(-155\,\text{K}) = 1290\,\text{J}.$

$\Delta E_{\text{int}} = Q + W = 1930\,\text{J}.$

Cycle

$Q = 520\,\text{J}$; $W = -510\,\text{J}$ (rounding error!); $\Delta E_{\text{int}} = 10\,\text{J}$ (rounding error!)

P23-20

P23-21 $p_{\text{f}} = (16.0\text{ atm})(50/250)^{1.40} = 1.68\text{ atm}$. The work done by the gas during the expansion is

$$\begin{aligned} W &= [p_{\text{i}}V_{\text{i}} - p_{\text{f}}V_{\text{f}}]/(\gamma - 1), \\ &= \frac{(16.0\text{ atm})(50\times10^{-6}\text{m}^3) - (1.68\text{ atm})(250\times10^{-6}\text{m}^3)}{(1.40) - 1}(1.01\times10^5\text{Pa/atm}), \\ &= 96.0\,\text{J}. \end{aligned}$$

This process happens 4000 times per minute, but the actual time to complete the process is half of the cycle, or 1/8000 of a minute. Then $P = (96\,\text{J})(8000)/(60\,\text{s}) = 12.8\times10^3\text{W}.$

$\boxed{\textbf{E24-1}}$ For isothermal processes the entropy expression is almost trivial, $\Delta S = Q/T$, where if Q is positive (heat flow into system) the entropy increases.

Then $Q = T\Delta S = (405\,\text{K})(46.2\,\text{J/K}) = 1.87\times 10^4\,\text{J}$.

E24-2 Entropy is a state variable and is path independent, so

(a) $\Delta S_{ab,2} = \Delta S_{ab,1} = +0.60\,\text{J/K}$,

(b) $\Delta S_{ba,2} = -\Delta S_{ab,2} = -0.60\,\text{J/K}$,

E24-3 (a) Heat only enters along the top path, so

$$Q_{\text{in}} = T\Delta S = (400\,\text{K})(0.6\,\text{J/K} - 0.1\,\text{J/K}) = 200\,\text{J}.$$

(b) Heat leaves only bottom path, so

$$Q_{\text{out}} = T\Delta S = (250\,\text{K})(0.1\,\text{J/K} - 0.6\,\text{J/K}) = -125\,\text{J}.$$

Since $Q + W = 0$ for a cyclic path,

$$W = -Q = -[(200\,\text{J}) + (-125\,\text{J})] = -75\,\text{J}.$$

E24-4 (a) The work done for isothermal expansion is given by Eq. 23-18,

$$W = -(4.00\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(410\,\text{K})\ln\frac{3.45V_1}{V_1} = -1.69\times 10^4\,\text{J}.$$

(b) For isothermal process, $Q = -W$, then

$$\Delta S = Q/T = (1.69\times 10^4\,\text{J})/(410\,\text{K}) = 41.2\,\text{J/K}.$$

(c) Entropy change is zero for reversible adiabatic processes.

$\boxed{\textbf{E24-5}}$ (a) We want to find the heat absorbed, so

$$Q = mc\Delta T = (1.22\,\text{kg})(387\,\text{J/mol}\cdot\text{K})\,((105^\circ\text{C}) - (25.0^\circ\text{C})) = 3.77\times 10^4\,\text{J}.$$

(b) We want to find the entropy change, so, according to Eq. 24-1,

$$\begin{aligned} \Delta S &= \int_{T_\text{i}}^{T_\text{f}} \frac{dQ}{T}, \\ &= \int_{T_\text{i}}^{T_\text{f}} \frac{mc\,dT}{T}, \\ &= mc\ln\frac{T_\text{f}}{T_\text{i}}. \end{aligned}$$

The entropy change of the copper block is then

$$\Delta S = mc\ln\frac{T_\text{f}}{T_\text{i}} = (1.22\,\text{kg})(387\,\text{J/mol}\cdot\text{K})\ln\frac{(378\,\text{K})}{(298\,\text{K})} = 112\,\text{J/K}.$$

E24-6 $\Delta S = Q/T = mL/T$, so

$$\Delta S = (0.001\,\text{kg})(-333\times 10^3\,\text{J/kg})/(263\,\text{K}) = -1.27\text{J/K}.$$

E24-7 Use the first equation on page 551.

$$n = \frac{\Delta S}{R \ln(V_\text{f}/V_\text{i})} = \frac{(24\,\text{J/K})}{(8.31\,\text{J/mol}\cdot\text{K})\ln(3.4/1.3)} = 3.00 \text{ mol}.$$

E24-8 $\Delta S = Q/T_\text{c} - Q/T_\text{h}$.

(a) $\Delta S = (260\,\text{J})(1/100\,\text{K} - 1/400\,\text{K}) = 1.95\,\text{J/K}$.
(b) $\Delta S = (260\,\text{J})(1/200\,\text{K} - 1/400\,\text{K}) = 0.65\,\text{J/K}$.
(c) $\Delta S = (260\,\text{J})(1/300\,\text{K} - 1/400\,\text{K}) = 0.217\,\text{J/K}$.
(d) $\Delta S = (260\,\text{J})(1/360\,\text{K} - 1/400\,\text{K}) = 0.0722\,\text{J/K}$.

$\boxed{\textbf{E24-9}}$ (a) If the rod is in a steady state we wouldn't expect the entropy of the rod to change. Heat energy is flowing out of the hot reservoir into the rod, but this process happens at a fixed temperature, so the entropy change in the hot reservoir is

$$\Delta S_\text{H} = \frac{Q_\text{H}}{T_\text{H}} = \frac{(-1200\,\text{J})}{(403\,\text{K})} = -2.98\,\text{J/K}.$$

The heat energy flows into the cold reservoir, so

$$\Delta S_\text{C} = \frac{Q_\text{H}}{T_\text{H}} = \frac{(1200\,\text{J})}{(297\,\text{K})} = 4.04\,\text{J/K}.$$

The total change in entropy of the system is the sum of these two terms

$$\Delta S = \Delta S_\text{H} + \Delta S_\text{C} = 1.06\,\text{J/K}.$$

(b) Since the rod is in a steady state, nothing is changing, not even the entropy.

E24-10 (a) $Q_\text{c} + Q_\text{l} = 0$, so

$$m_\text{c}c_\text{c}(T - T_\text{c}) + m_\text{l}c_\text{l}(T - T_\text{l}) = 0,$$

which can be solved for T to give

$$T = \frac{(0.05\,\text{kg})(387\,\text{J/kg}\cdot\text{K})(400\,\text{K}) + (0.10\,\text{kg})(129\,\text{J/kg}\cdot\text{K})(200\,\text{K})}{(0.05\,\text{kg})(387\,\text{J/kg}\cdot\text{K}) + (0.10\,\text{kg})(129\,\text{J/kg}\cdot\text{K})} = 320\,\text{K}.$$

(b) Zero.
(c) $\Delta S = mc \ln T_\text{f}/T_\text{i}$, so

$$\Delta S = (0.05\,\text{kg})(387\,\text{J/kg}\cdot\text{K}) \ln\frac{(320\,\text{K})}{(400\,\text{K})} + (0.10\,\text{kg})(129\,\text{J/kg}\cdot\text{K}) \ln\frac{(320\,\text{K})}{(200\,\text{K})} = 1.75\,\text{J/K}.$$

$\boxed{\textbf{E24-11}}$ The total mass of ice and water is 2.04 kg. If eventually the ice and water have the same mass, then the final state will have 1.02 kg of each. This means that $1.78\,\text{kg} - 1.02\,\text{kg} = 0.76\,\text{kg}$ of water changed into ice.

(a) The change of water at 0°C to ice at 0°C is isothermal, so the entropy change is

$$\Delta S = \frac{Q}{T} = \frac{-mL}{T} = \frac{(0.76\,\text{kg})(333\times10^3\,\text{J/kg})}{(273\,\text{K})} = -927\,\text{J/K}.$$

(b) The entropy change is now $+927\,\text{J/K}$.

E24-12 (a) $Q_a + Q_w = 0$, so

$$m_a c_a (T - T_a) + m_w c_w (T - T_w) = 0,$$

which can be solved for T to give

$$T = \frac{(0.196\,\text{kg})(900\,\text{J/kg}\cdot\text{K})(380\,\text{K}) + (0.0523\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(292\,\text{K})}{(0.196\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) + (0.0523\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})} = 331\,\text{K}.$$

That's the same as 58°C.
(b) $\Delta S = mc \ln T_f/T_i$, so

$$\Delta S_a = (0.196\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) \ln \frac{(331\,\text{K})}{(380\,\text{K})} = -24.4\,\text{J/K}.$$

(c) For the water,

$$(0.0523\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}) \ln \frac{(331\,\text{K})}{(292\,\text{K})} = 27.5\,\text{J/K}.$$

(d) $\Delta S = (27.5\,\text{J/K}) + (-24.4\,\text{J/K}) = 3.1\,\text{J/K}$.

E24-13 (a) $e = 1 - (36.2\,\text{J}/52.4\,\text{J}) = 0.309$.
(b) $W = Q_h - Q_c = (52.4\,\text{J}) - (36.2\,\text{J}) = 16.2\,\text{J}$.

E24-14 (a) $Q_h = (8.18\text{ kJ})/(0.25) = 32.7\text{ kJ}$, $Q_c = Q_h - W = (32.7\text{ kJ}) - (8.18\text{ kJ}) = 24.5\text{ kJ}$.
(b) $Q_h = (8.18\text{ kJ})/(0.31) = 26.4\text{ kJ}$, $Q_c = Q_h - W = (26.4\text{ kJ}) - (8.18\text{ kJ}) = 18.2\text{ kJ}$.

E24-15 One hour's worth of coal, when burned, will provide energy equal to

$$(382\times10^3\text{kg})(28.0\times10^6\text{J/kg}) = 1.07\times10^{13}\text{J}.$$

In this hour, however, the plant only generates

$$(755\times10^6\text{ W})(3600\,s) = 2.72\times10^{12}\text{J}.$$

The efficiency is then

$$e = (2.72\times10^{12}\text{J})/(1.07\times10^{13}\text{J}) = 25.4\%.$$

E24-16 We use the convention that all quantities are positive, regardless of direction. $W_A = 5W_B$; $Q_{i,A} = 3Q_{i,B}$; and $Q_{o,A} = 2Q_{o,B}$. But $W_A = Q_{i,A} - Q_{o,A}$, so

$$5W_B = 3Q_{i,B} - 2Q_{o,B},$$

or, applying $W_B = Q_{i,B} - Q_{o,B}$,

$$\begin{aligned} 5W_B &= 3Q_{i,B} - 2(Q_{i,B} - W_B), \\ 3W_B &= Q_{i,B}, \\ W_B/Q_{i,B} &= 1/3 = e_B. \end{aligned}$$

Then

$$e_A = \frac{W_A}{Q_{i,A}} = \frac{5W_B}{3Q_{i,B}} = \frac{5}{3}\frac{1}{3} = \frac{5}{9}.$$

E24-17 (a) During an isothermal process $W = -Q = -2090\,\mathrm{J}$. The negative indicates that the gas did work on the environment.

(b) The efficiency is $e = 1 - (297\,\mathrm{K})/(412\,\mathrm{K}) = 0.279$. Then

$$Q_\mathrm{o} = Q_\mathrm{i}(1-e) = (2090\,\mathrm{J})[1-(0.279)] = 1510\,\mathrm{J}.$$

Since this is rejected heat it should actually be negative.

(c) During an isothermal process $W = -Q = 1510\,\mathrm{J}$. Positive indicates that the gas did work on the environment.

E24-18 $1 - e = T_\mathrm{c}/T_\mathrm{h}$, or $T_\mathrm{c} = T_\mathrm{h}(1-e)$. The difference is

$$\Delta T = T_\mathrm{h} - T_\mathrm{c} = T_\mathrm{h} e,$$

so $T_\mathrm{h} = (75\,\mathrm{C}^\circ)/(0.22) = 341\,\mathrm{K}$, and

$$T_\mathrm{c} = (341\,\mathrm{K})[(1-(0.22)] = 266\,\mathrm{K}.$$

$\boxed{\textbf{E24-19}}$ The BC and DA processes are both adiabatic; so if we could find an expression for work done during an adiabatic process we might be almost done. But what is an adiabatic process? It is a process for which $Q = 0$, so according to the first law

$$\Delta E_\mathrm{int} = W.$$

But for an ideal gas

$$\Delta E_\mathrm{int} = nC_V\Delta T,$$

as was pointed out in Table 23-5. So we have

$$|W| = nC_V|\Delta T|$$

and since the adiabatic paths BC and DA operate between the same two isotherms, we can conclude that the magnitude of the work is the same for both paths.

E24-20 (a) To save typing, assume that all quantities are positive. Then

$$e_1 = 1 - T_2/T_1,$$

$W_1 = e_1 Q_1$, and $Q_2 = Q_1 - W_1$. Not only that, but

$$e_2 = 1 - T_3/T_2,$$

and $W_2 = e_2 Q_2$. Combining,

$$e = \frac{W_1 + W_2}{Q_1} = \frac{e_1 Q_1 + e_2(Q_1 - W_1)}{Q_1} = e_1 + e_2(1-e_1) = e_1 + e_2 - e_1 e_2,$$

or

$$e = 1 - \frac{T_2}{T_1} + 1 - \frac{T_3}{T_2} - 1 + \frac{T_2}{T_1} + \frac{T_3}{T_2} - \frac{T_3}{T_1} = 1 - -\frac{T_3}{T_1}.$$

(b) $e = 1 - (311\,\mathrm{K})/(742\,\mathrm{K}) = 0.581$.

E24-21 (a) $p_2 = (16.0\ \mathrm{atm})(1/5.6)^{(1.33)} = 1.62\ \mathrm{atm}$.

(b) $T_2 = T_1(1/5.6)^{(1.33)-1} = (0.567)T_1$, so

$$e = 1 - (0.567) = 0.433.$$

E24-22 (a) The area of the cycle is $\Delta V \Delta p = p_0 V_0$, so the work done by the gas is

$$W = (1.01\times10^5\text{Pa})(0.0225\,\text{m}^3) = 2270\,\text{J}.$$

(b) Let the temperature at a be T_a. Then

$$T_b = T_a(V_b/V_a)(p_b/p_a) = 2T_a.$$

Let the temperature at c be T_c. Then

$$T_c = T_a(V_c/V_a)(p_c/p_a) = 4T_a.$$

Consequently, $\Delta T_{ab} = T_a$ and $\Delta T_{bc} = 2T_a$. Putting this information into the constant volume and constant pressure heat expressions,

$$Q_{ab} = \frac{3}{2}nR\Delta T_{ab} = \frac{3}{2}nRT_a = \frac{3}{2}p_a V_a,$$

and

$$Q_{bc} = \frac{5}{2}nR\Delta T_{bc} = \frac{5}{2}nR2T_a = 5p_a V_a,$$

so that $Q_{ac} = \frac{13}{2}p_0V_0$, or

$$Q_{ac} = \frac{13}{2}(1.01\times10^5\text{Pa})(0.0225\,\text{m}^3) = 1.48\times10^4\text{J}.$$

(c) $e = (2270\,\text{J})/(14800\,\text{J}) = 0.153$.
(d) $e_c = 1 - (T_a/4T_a) = 0.75$.

$\boxed{\textbf{E24-23}}$ According to Eq. 24-15,

$$K = \frac{T_L}{T_H - T_L} = \frac{(261\,\text{K})}{(299\,\text{K}) - (261\,\text{K})} = 6.87$$

Now we solve the question out of order.
(b) The work required to run the freezer is

$$|W| = |Q_L|/K = (185\text{ kJ})/(5.70) = 32.5\text{ kJ}.$$

(a) The freezer will discharge heat into the room equal to

$$|Q_L| + |W| = (185\text{ kJ}) + (32.5\text{ kJ}) = 218\text{ kJ}.$$

E24-24 (a) $K = |Q_L|/|W| = (568\,\text{J})/(153\,\text{J}) = 3.71$.
(b) $|Q_H| = |Q_L| + |W| = (568\,\text{J}) + (153\,\text{J}) = 721\,\text{J}$.

E24-25 $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$.
(a) $|W| = (10.0\,\text{J})(300\,\text{K}/280\text{K} - 1) = 0.714\,\text{J}$.
(b) $|W| = (10.0\,\text{J})(300\,\text{K}/200\text{K} - 1) = 5.00\,\text{J}$.
(c) $|W| = (10.0\,\text{J})(300\,\text{K}/100\text{K} - 1) = 20.0\,\text{J}$.
(d) $|W| = (10.0\,\text{J})(300\,\text{K}/50\text{K} - 1) = 50.0\,\text{J}$.

E24-26 $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$. Then

$$|Q_H| = |Q_L| + |W| = |Q_L|(T_H/T_L) = (0.150\,\text{J})(296\,\text{K}/4.0\,\text{K}) = 11\,\text{J}.$$

$\boxed{\textbf{E24-27}}$ We will start with the assumption that the air conditioner is a Carnot refrigerator. $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$. For fun, I'll convert temperature to the absolute Rankine scale! Then

$$|Q_L| = (1.0\,\text{J})/(555°\text{R}/530°\text{R} - 1) = 21\,\text{J}.$$

E24-28 The best coefficient of performance is

$$K_c = (276\,\text{K})/(308\,\text{K} - 276\,\text{K}) = 8.62.$$

The inventor claims they have a machine with

$$K = (20\text{ kW} - 1.9\text{kW})/(1.9\text{ kW}) = 9.53.$$

Can't be done.

E24-29 (a) $e = 1 - (258\,\text{K}/322\,\text{K}) = 0.199$. $|W| = (568\,\text{J})(0.199) = 113\,\text{J}$.
(b) $K = (258\,\text{K})/(322\,\text{K} - 258\,\text{K}) = 4.03$. $|W| = (1230\,\text{J})/(4.03) = 305\,\text{J}$.

E24-30 The temperatures are distractors!

$$|W| = |Q_H| - |Q_L| = |Q_H| - K|W|,$$

so

$$|W| = |Q_H|/(1 + K) = (7.6\text{ MJ})/(1 + 3.8) = 1.58\text{ MJ}.$$

Then $P = (1.58\text{ MJ})/(3600\,\text{s}) = 440\,\text{W}$.

E24-31 $K = (260\,\text{K})/(298\,\text{K} - 260\,\text{K}) = 6.8$.

E24-32 $K = (0.85)(270\text{K})/(299\text{K}–270\text{K}) = 7.91$. In 15 minutes the motor can do $(210\,\text{W})(900\,\text{s}) = 1.89\times10^5\text{J}$ of work. Then

$$|Q_L| = K|W| = (7.91)(1.89\times10^5\text{J}) = 1.50\times10^6\text{J}.$$

$\boxed{\textbf{E24-33}}$ The Carnot engine has an efficiency

$$\epsilon = 1 - \frac{T_2}{T_1} = \frac{|W|}{|Q_1|}.$$

The Carnot refrigerator has a coefficient of performance

$$K = \frac{T_4}{T_3 - T_4} = \frac{|Q_4|}{|W|}.$$

Lastly, $|Q_4| = |Q_3| - |W|$. We just need to combine these three expressions into one. Starting with the first, and solving for $|W|$,

$$|W| = |Q_1|\frac{T_1 - T_2}{T_1}.$$

Then we combine the last two expressions, and

$$\frac{T_4}{T_3 - T_4} = \frac{|Q_3| - |W|}{|W|} = \frac{|Q_3|}{|W|} - 1.$$

Finally, combine them all,

$$\frac{T_4}{T_3 - T_4} = \frac{|Q_3|}{|Q_1|}\frac{T_1}{T_1 - T_2} - 1.$$

Now, we rearrange,

$$\begin{aligned}
\frac{|Q_3|}{|Q_1|} &= \left(\frac{T_4}{T_3 - T_4} + 1\right)\frac{T_1 - T_2}{T_1}, \\
&= \left(\frac{T_3}{T_3 - T_4}\right)\frac{T_1 - T_2}{T_1}, \\
&= (1 - T_2/T_1)/(1 - T_4/T_3).
\end{aligned}$$

E24-34 (a) Integrate:

$$\ln N! \approx \int_1^N \ln x\, dx = N\ln N - N + 1 \approx N\ln N - N.$$

(b) 91, 752, and about 615,000. You will need to use the Stirling approximation extended to a double inequality to do the last two:

$$\sqrt{2\pi}n^{n+1/2}e^{-n+1/(12n+1)} < n! < \sqrt{2\pi}n^{n+1/2}e^{-n+1/(12n)}.$$

$\boxed{\textbf{E24-35}}$ (a) For this problem we don't care how the particles are arranged inside a section, we only care how they are divided up between the two sides.

Consequently, there is only one way to arrange the particles: you put them all on one side, and you have no other choices. So the multiplicity in this case is one, or $w_1 = 1$.

(b) Once the particles are allowed to mix we have more work in computing the multiplicity. Using Eq. 24-19, we have

$$w_2 = \frac{N!}{(N/2)!(N/2)!} = \frac{N!}{((N/2)!)^2}$$

(c) The entropy of a state of multiplicity w is given by Eq. 24-20,

$$S = k\ln w$$

For part (a), with a multiplicity of 1, $S_1 = 0$. Now for part (b),

$$S_2 = k\ln\left(\frac{N!}{((N/2)!)^2}\right) = k\ln N! - 2k\ln(N/2)!$$

and we need to expand each of those terms with Stirling's approximation.

Combining,

$$\begin{aligned}
S_2 &= = k\left(N\ln N - N\right) - 2k\left((N/2)\ln(N/2) - (N/2)\right), \\
&= kNlnN - kN - kN\ln N + kN\ln 2 + kN, \\
&= kN\ln 2
\end{aligned}$$

Finally, $\Delta S = S_2 - S_1 = kN\ln 2$.

(d) The answer should be the same; it is a free expansion problem in both cases!

P24-1 We want to evaluate

$$
\begin{aligned}
\Delta S &= \int_{T_\mathrm{i}}^{T_\mathrm{f}} \frac{nC_\mathrm{V}\,dT}{T}, \\
&= \int_{T_\mathrm{i}}^{T_\mathrm{f}} \frac{nAT^3\,dT}{T}, \\
&= \int_{T_\mathrm{i}}^{T_\mathrm{f}} nAT^2\,dT, \\
&= \frac{nA}{3}\left({T_\mathrm{f}}^3 - {T_\mathrm{i}}^3\right).
\end{aligned}
$$

Into this last expression, which is true for many substances at sufficiently low temperatures, we substitute the given numbers.

$$\Delta S = \frac{(4.8\text{ mol})(3.15\times10^{-5}\text{J/mol}\cdot\text{K}^4)}{3}\left((10\,\text{K})^3 - (5.0\,\text{K})^3\right) = 4.41\times10^{-2}\text{J/K}.$$

P24-2

P24-3 (a) Work is only done along path ab, where $W_{ab} = -p\Delta V = -3p_0\Delta V_0$. So $W_{abc} = -3p_0V_0$.

(b) $\Delta E_{\int bc} = \frac{3}{2}nR\Delta T_{bc}$, with a little algebra,

$$\Delta E_{\mathrm{int}\,bc} = \frac{3}{2}(nRT_c - nRT_b) = \frac{3}{2}(p_cV_c - p_bV_b) = \frac{3}{2}(8-4)p_0V_0 = 6p_0V_0.$$

$\Delta S_{bc} = \frac{3}{2}nR\ln(T_c/T_b)$, with a little algebra,

$$\Delta S_{bc} = \frac{3}{2}nR\ln(p_c/p_b) = \frac{3}{2}nR\ln 2.$$

(c) Both are zero for a cyclic process.

P24-4 (a) For an isothermal process,

$$p_2 = p_1(V_1/V_2) = p_1/3.$$

For an adiabatic process,

$$p_3 = p_1(V_1/V_2)^\gamma = p_1(1/3)^{1.4} = 0.215p_1.$$

For a constant volume process,

$$T_3 = T_2(p_3/p_2) = T_1(0.215/0.333) = 0.646T_1.$$

(b) The easiest ones first: $\Delta E_\mathrm{i}\mathrm{nt}_{12} = 0$, $W_{23} = 0$, $Q_{31} = 0$, $\Delta S_{31} = 0$. The next easier ones:

$$\Delta E_\mathrm{i}\mathrm{nt}_{23} = \frac{5}{2}nR\Delta T_{23} = \frac{5}{2}nR(0.646T_1 - T_1) = -0.885p_1V_1,$$

$$Q_{23} = \Delta E_{\mathrm{int}\,23} - W_{23} = -0.885p_1V_1,$$

$$\Delta E_{\mathrm{int}\,31} = -\Delta E_{\mathrm{int}\,23} - \Delta E_\mathrm{i}\mathrm{nt}_{12} = 0.885p_1V_1,$$

$$W_{31} = \Delta E_{\mathrm{int}\,31} - Q_{31} = 0.885p_1V_1.$$

Finally, some harder ones:

$$W_{12} = -nRT_1 \ln(V_2/V_1) = -p_1V_1 \ln(3) = -1.10 p_1V_1,$$

$$Q_{12} = \Delta E_{\text{int}\,12} - W_{12} = 1.10p_1V_1.$$

And now, the hardest:

$$\Delta S_{12} = Q_{12}/T_1 = 1.10nR,$$

$$\Delta S_{23} = -\Delta S_{12} - \Delta S_{31} = -1.10nR.$$

P24-5 Note that $T_A = T_B = T_C/4 = T_D$.

Process I: *ABC*

(a) $Q_{AB} = -W_{AB} = nRT_0 \ln(V_B/V_A) = p_0V_0 \ln 2$. $Q_{BC} = \frac{3}{2}nR(T_C - T_B) = \frac{3}{2}(p_CV_C - p_BV_B) = \frac{3}{2}(4p_0V_0 - p_0V_0) = 4.5p_0V_0$.

(b) $W_{AB} = -nRT_0 \ln(V_B/V_A) = -p_0V_0 \ln 2$. $W_{BC} = 0$.

(c) $E_{\text{int}} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(p_CV_C - p_AV_A) = \frac{3}{2}(4p_0V_0 - p_0V_0) = 4.5p_0V_0$.

(d) $\Delta S_{AB} = nR\ln(V_B/V_A) = nR\ln 2$; $\Delta S_{BC} = \frac{3}{2}nR\ln(T_C/T_B) = \frac{3}{2}nR\ln 4 = 3nR\ln 2$. Then $\Delta S_{AC} = 4nR$.

Process II: *ADC*

(a) $Q_{AD} = -W_{AD} = nRT_0 \ln(V_D/V_A) = -p_0V_0 \ln 2$. $Q_{DC} = \frac{5}{2}nR(T_C - T_D) = \frac{5}{2}(p_CV_C - p_DV_D) = \frac{5}{2}(4p_0V_0 - p_0V_0) = 10p_0V_0$.

(b) $W_{AB} = -nRT_0 \ln(V_D/V_A) = p_0V_0 \ln 2$. $W_{DC} = -p\Delta V = -p_0(2V_0 - V_0/2) = -\frac{3}{2}p_0V_0$.

(c) $E_{\text{int}} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(p_CV_C - p_AV_A) = \frac{3}{2}(4p_0V_0 - p_0V_0) = 4.5p_0V_0$.

(d) $\Delta S_{AD} = nR\ln(V_D/V_A) = -nR\ln 2$; $\Delta S_{DC} = \frac{5}{2}nR\ln(T_C/T_D) = \frac{5}{2}nR\ln 4 = 5nR\ln 2$. Then $\Delta S_{AC} = 4nR$.

P24-6 The heat required to melt the ice is

$$\begin{aligned} Q &= m(c_\text{w}\Delta T_{23} + L + c_\text{i}\Delta T_{12}), \\ &= (0.0126\,\text{kg})[(4190\,\text{J/kg}\cdot\text{K})(15\,\text{C}^\circ) + (333\times 10^3\text{J/kg}) + (2220\,\text{J/kg}\cdot\text{K})(10\,\text{C}^\circ)], \\ &= 5270\,\text{J}. \end{aligned}$$

The change in entropy of the ice is

$$\begin{aligned} \Delta S_\text{i} &= m[c_\text{w}\ln(T_3/T_2) + L/T_2 + c_\text{i}\ln(T_2/T_1)], \\ &= (0.0126\text{kg})[(4190\text{J/kg}\cdot\text{K})\ln(288/273) + (333\times 10^3\text{J/kg})/(273\text{K}), \\ &\quad +(2220\text{J/kg}\cdot\text{K})\ln(273/263)], \\ &= 19.24\,\text{J/K} \end{aligned}$$

The change in entropy of the lake is $\Delta S_\text{l} = (-5270\,\text{J})/(288\,\text{K}) = 18.29\,\text{J}/K$. The change in entropy of the system is $0.95\,\text{J/kg}$.

$\boxed{\textbf{P24-7}}$ (a) This is a problem where the total internal energy of the two objects doesn't change, but since no work is done during the process, we can start with the simpler expression $Q_1 + Q_2 = 0$. The heat transfers by the two objects are

$$\begin{aligned} Q_1 &= m_1c_1(T_1 - T_{1,\text{i}}), \\ Q_2 &= m_2c_2(T_2 - T_{2,\text{i}}). \end{aligned}$$

Note that we don't call the final temperature T_f here, because we *are not* assuming that the two objects are at equilibrium.

We combine these three equations,

$$\begin{aligned}
m_2c_2(T_2 - T_{2,\mathrm{i}}) &= -m_1c_1(T_1 - T_{1,\mathrm{i}}),\\
m_2c_2T_2 &= m_2c_2T_{2,\mathrm{i}} + m_1c_1(T_{1,\mathrm{i}} - T_1),\\
T_2 &= T_{2,\mathrm{i}} + \frac{m_1c_1}{m_2c_2}(T_{1,\mathrm{i}} - T_1)
\end{aligned}$$

As object 1 "cools down", object 2 "heats up", as expected.

(b) The entropy change of *one* object is given by

$$\Delta S = \int_{T_\mathrm{i}}^{T_\mathrm{f}} \frac{mc\,dT}{T} = mc\ln\frac{T_\mathrm{f}}{T_\mathrm{i}},$$

and the total entropy change for the system will be the sum of the changes for each object, so

$$\Delta S = m_1c_1\ln\frac{T_1}{T_{\mathrm{i},1}} + m_2c_2\ln\frac{T_2}{T_{\mathrm{i},2}}.$$

Into the this last equation we need to substitute the expression for T_2 in as a function of T_1. There's no new physics in doing this, just a mess of algebra.

(c) We want to evaluate $d(\Delta S)/dT_1$. To save on algebra we will work with the last expression, remembering that T_2 is a function, not a variable. Then

$$\frac{d(\Delta S)}{dT_1} = \frac{m_1c_1}{T_1} + \frac{m_2c_2}{T_2}\frac{dT_2}{dT_1}.$$

We've saved on algebra, but now we need to evaluate dT_2/dT_1. Starting with the results from part (a),

$$\begin{aligned}
\frac{dT_2}{dT_1} &= \frac{d}{dT_1}\left(T_{2,\mathrm{i}} + \frac{m_1c_1}{m_2c_2}(T_{1,\mathrm{i}} - T_1)\right),\\
&= -\frac{m_1c_1}{m_2c_2}.
\end{aligned}$$

Now we collect the two results and write

$$\begin{aligned}
\frac{d(\Delta S)}{dT_1} &= \frac{m_1c_1}{T_1} + \frac{m_2c_2}{T_2}\left(-\frac{m_1c_1}{m_2c_2}\right),\\
&= m_1c_1\left(\frac{1}{T_1} - \frac{1}{T_2}\right).
\end{aligned}$$

We could consider writing T_2 out in all of its glory, but what would it gain us? Nothing. There is actually considerably more physics in the expression as written, because...

(d) ...we get a maximum for ΔS when $d(\Delta S)/dT_1 = 0$, and this can only occur when $T_1 = T_2$ according to the expression.

P24-8 $T_b = (10.4 \times 1.01\times 10^5\mathrm{Pa})(1.22\,\mathrm{m}^3)/(2\text{ mol})(8.31\,\mathrm{J/mol\cdot K}) = 7.71\times 10^4\mathrm{K}$. Maybe not so realistic? T_a can be found after finding

$$p_c = p_b(V_b/V_c)^\gamma = (10.4\times 1.01\times 10^5\mathrm{Pa})(1.22/9.13)^{1.67} = 3.64\times 10^4\mathrm{Pa},$$

Then

$$T_a = T_b(p_a/p_b) = (7.71\times 10^4\mathrm{K})(3.64\times 10^4/1.05\times 10^6) = 2.67\times 10^3\mathrm{K}.$$

Similarly,

$$T_c = T_a(V_c/V_a) = (2.67\times10^3\text{K})(9.13/1.22) = 2.00\times10^4\text{K}.$$

(a) Heat is added during process ab only;

$$Q_{ab} = \frac{3}{2}nR(T_b - T_a) = \frac{3}{2}(2\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(7.71\times10^4\text{K} - 2.67\times10^3\text{K}) = 1.85\times10^6\text{J}.$$

(b) Heat is removed during process ca only;

$$Q_{ca} = \frac{5}{2}nR(T_a - T_c) = \frac{5}{2}(2\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(2.67\times10^3\text{K} - 2.00\times10^4\text{K}) = -0.721\times10^6\text{J}.$$

(c) $W = |Q_{ab}| - |Q_{ca}| = (1.85\times10^6\text{J}) - (0.721\times10^6\text{J}) = 1.13\times10^6\text{J}$.
(d) $e = W/Q_{ab} = (1.13\times10^6)/(1.85\times10^6) = 0.611$.

P24-9 The pV diagram for this process is Figure 23-21, except the cycle goes clockwise.

(a) Heat is input during the constant volume heating and the isothermal expansion. During heating,

$$Q_1 = \frac{3}{2}nR\Delta T = \frac{3}{2}(1\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 3740\,\text{J};$$

During isothermal expansion,

$$Q_2 = -W_2 = nRT\ln(V_\text{f}/V_\text{i}) = (1\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K})\ln(2) = 3460\,\text{J};$$

so $Q_\text{in} = 7200\,\text{J}$.

(b) Work is only done during the second and third processes; we've already solved the second, $W_2 = -3460\,\text{J}$;

$$W_3 = -p\Delta V = p_aV_c - p_aV_a = nR(T_c - T_a) = (1\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 2490\,\text{J};$$

So $W = -970\,\text{J}$.

(c) $e = |W|/|Q_\text{in}| = (970\,\text{J})/(7200\,\text{J}) = 0.13$.

P24-10 (a) $T_b = T_a(p_b/p_a) = 3T_a$;

$$T_c = T_b(V_b/V_c)^{\gamma-1} = 3T_a(1/4)^{0.4} = 1.72T_a;$$

$$p_c = p_b(V_b/V_c)^{\gamma} = 3p_a(1/4)^{1.4} = 0.430p_a;$$

$$T_d = T_a(V_a/V_d)^{\gamma-1} = T_a(1/4)^{0.4} = 0.574T_a;$$

$$p_d = p_a(V_a/V_d)^{\gamma} = p_a(1/4)^{1.4} = 0.144p_a.$$

(b) Heat in occurs during process ab, so $Q_\text{i} = \frac{5}{2}nR\Delta T_{ab} = 5nRT_a$; Heat out occurs during process cd, so $Q_\text{o} = \frac{5}{2}nR\Delta T_{cd} = 2.87nRT_a$. Then

$$e = 1 - (2.87nRT_a/5nRT_a) = 0.426.$$

P24-11 (c) $(V_B/V_A) = (p_A/p_B) = (0/0.5) = 2$. The work done on the gas during the isothermal compression is

$$W = -nRT\ln(V_B/V_A) = -(1\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K})\ln(2) = -1730\,\text{J}.$$

Since $\Delta E_\text{int} = 0$ along an isotherm $Q_\text{h} = 1730\,\text{J}$.

The cycle has an efficiency of $e = 1 - (100/300) = 2/3$. Then for the cycle,

$$W = eQ_\text{h} = (2/3)(1730\,\text{J}) = 1150\,\text{J}.$$

Instructor Solutions Manual
for
Physics
by
Halliday, Resnick, and Krane

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Volume 2

A Note To The Instructor...

The solutions here are somewhat brief, as they are designed for the instructor, not for the student. Check with the publishers before electronically posting any part of these solutions; website, ftp, or server access *must* be restricted to your students.

I have been somewhat casual about subscripts whenever it is obvious that a problem is one dimensional, or that the choice of the coordinate system is irrelevant to the *numerical* solution. Although this does not change the validity of the answer, it will sometimes obfuscate the approach if viewed by a novice.

There are some *traditional* formula, such as

$$v_x^2 = v_{0x}^2 + 2a_x x,$$

which are not used in the text. The worked solutions use only material from the text, so there may be times when the solution here seems unnecessarily convoluted and drawn out. Yes, I know an easier approach existed. But if it was not in the text, I did not use it here.

I also tried to avoid reinventing the wheel. There are some exercises and problems in the text which build upon previous exercises and problems. Instead of rederiving expressions, I simply refer you to the previous solution.

I adopt a different approach for rounding of significant figures than previous authors; in particular, I usually round intermediate answers. As such, some of my answers will differ from those in the back of the book.

Exercises and Problems which are enclosed in a box also appear in the Student's Solution Manual with considerably more detail and, when appropriate, include discussion on any physical implications of the answer. These student solutions carefully discuss the steps required for solving problems, point out the relevant equation numbers, or even specify where in the text additional information can be found. When two almost equivalent methods of solution exist, often both are presented. You are encouraged to refer students to the Student's Solution Manual for these exercises and problems. However, the material from the Student's Solution Manual must *not* be copied.

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E25-1 The charge transferred is

$$Q = (2.5 \times 10^4\,\mathrm{C/s})(20 \times 10^{-6}\,\mathrm{s}) = 5.0 \times 10^{-1}\,\mathrm{C}.$$

E25-2 Use Eq. 25-4:

$$r = \sqrt{\frac{(8.99\times10^9\mathrm{N\cdot m^2/C^2})(26.3\times10^{-6}\mathrm{C})(47.1\times10^{-6}\mathrm{C})}{(5.66\,\mathrm{N})}} = 1.40\,\mathrm{m}$$

E25-3 Use Eq. 25-4:

$$F = \frac{(8.99\times10^9\mathrm{N\cdot m^2/C^2})(3.12\times10^{-6}\mathrm{C})(1.48\times10^{-6}\mathrm{C})}{(0.123\,\mathrm{m})^2} = 2.74\,\mathrm{N}.$$

E25-4 (a) The forces are equal, so $m_1a_1 = m_2a_2$, or

$$m_2 = (6.31\times10^{-7}\mathrm{kg})(7.22\,\mathrm{m/s^2})/(9.16\,\mathrm{m/s^2}) = 4.97\times10^{-7}\mathrm{kg}.$$

(b) Use Eq. 25-4:

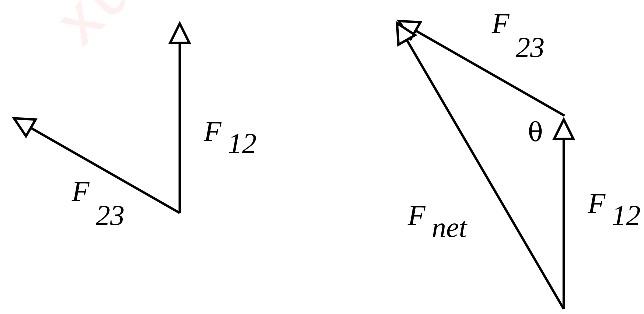
$$q = \sqrt{\frac{(6.31\times10^{-7}\mathrm{kg})(7.22\,\mathrm{m/s^2})(3.20\times10^{-3}\mathrm{m})^2}{(8.99\times10^9\mathrm{N\cdot m^2/C^2})}} = 7.20\times10^{-11}\mathrm{C}$$

E25-5 (a) Use Eq. 25-4,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2} = \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(21.3\,\mu\mathrm{C})(21.3\,\mu\mathrm{C})}{(1.52\,\mathrm{m})^2} = 1.77\,\mathrm{N}$$

(b) In part (a) we found F_{12}; to solve part (b) we need to first find F_{13}. Since $q_3 = q_2$ and $r_{13} = r_{12}$, we can immediately conclude that $F_{13} = F_{12}$.

We must assess the direction of the force of q_3 on q_1; it will be directed along the line which connects the two charges, and will be directed away from q_3. The diagram below shows the directions.



From this diagram we want to find the magnitude of the *net* force on q_1. The cosine law is appropriate here:

$$\begin{aligned}
{F_{\mathrm{net}}}^2 &= F_{12}^2 + F_{13}^2 - 2F_{12}F_{13}\cos\theta, \\
&= (1.77\,\mathrm{N})^2 + (1.77\,\mathrm{N})^2 - 2(1.77\,\mathrm{N})(1.77\,\mathrm{N})\cos(120^\circ), \\
&= 9.40\,\mathrm{N}^2, \\
F_{\mathrm{net}} &= 3.07\,\mathrm{N}.
\end{aligned}$$

E25-6 Originally $F_0 = CQ_0^2 = 0.088\,\mathrm{N}$, where C is a constant. When sphere 3 touches 1 the charge on both becomes $Q_0/2$. When sphere 3 the touches sphere 2 the charge on each becomes $(Q_0 + Q_0/2)/2 = 3Q_0/4$. The force between sphere 1 and 2 is then

$$F = C(Q_0/2)(3Q_0/4) = (3/8)CQ_0^2 = (3/8)F_0 = 0.033\,\mathrm{N}.$$

$\boxed{\textbf{E25-7}}$ The forces on q_3 are $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$. These forces are given by the vector form of Coulomb's Law, Eq. 25-5,

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{r_{31}^2}\hat{\mathbf{r}}_{31} = \frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{(2d)^2}\hat{\mathbf{r}}_{31},\\
\vec{\mathbf{F}}_{32} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{r_{32}^2}\hat{\mathbf{r}}_{32} = \frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{(d)^2}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

These two forces are the only forces which act on q_3, so in order to have q_3 in equilibrium the forces must be equal in magnitude, but opposite in direction. In short,

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= -\vec{\mathbf{F}}_{32},\\
\frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{(2d)^2}\hat{\mathbf{r}}_{31} &= -\frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{(d)^2}\hat{\mathbf{r}}_{32},\\
\frac{q_1}{4}\hat{\mathbf{r}}_{31} &= -\frac{q_2}{1}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

Note that $\hat{\mathbf{r}}_{31}$ and $\hat{\mathbf{r}}_{32}$ both point in the same direction and are both of unit length. We then get

$$q_1 = -4q_2.$$

E25-8 The horizontal and vertical contributions from the upper left charge and lower right charge are straightforward to find. The contributions from the upper left charge require slightly more work. The diagonal distance is $\sqrt{2}a$; the components will be weighted by $\cos 45° = \sqrt{2}/2$. The diagonal charge will contribute

$$\begin{aligned}
F_x &= \frac{1}{4\pi\epsilon_0}\frac{(q)(2q)}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}\hat{\mathbf{i}} = \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}},\\
F_y &= \frac{1}{4\pi\epsilon_0}\frac{(q)(2q)}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}\hat{\mathbf{j}} = \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}}.
\end{aligned}$$

(a) The horizontal component of the net force is then

$$\begin{aligned}
F_x &= \frac{1}{4\pi\epsilon_0}\frac{(2q)(2q)}{a^2}\hat{\mathbf{i}} + \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}},\\
&= \frac{4+\sqrt{2}/2}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}},\\
&= (4.707)(8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.13\times10^{-6}\mathrm{C})^2/(0.152\,m)^2\hat{\mathbf{i}} = 2.34\,\mathrm{N}\,\hat{\mathbf{i}}.
\end{aligned}$$

(b) The vertical component of the net force is then

$$\begin{aligned}
F_y &= -\frac{1}{4\pi\epsilon_0}\frac{(q)(2q)}{a^2}\hat{\mathbf{j}} + \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}},\\
&= \frac{-2+\sqrt{2}/2}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}},\\
&= (-1.293)(8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.13\times10^{-6}\mathrm{C})^2/(0.152\,m)^2\hat{\mathbf{j}} = -0.642\,\mathrm{N}\,\hat{\mathbf{j}}.
\end{aligned}$$

E25-9 The magnitude of the force on the negative charge from each positive charge is

$$F = (8.99\times 10^9 \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(4.18\times 10^{-6}\mathrm{C})(6.36\times 10^{-6}\mathrm{C})/(0.13\,\mathrm{m})^2 = 14.1\,\mathrm{N}.$$

The force from each positive charge is directed along the side of the triangle; but from symmetry only the component along the bisector is of interest. This means that we need to weight the above answer by a factor of $2\cos(30^\circ) = 1.73$. The net force is then 24.5 N.

E25-10 Let the charge on one sphere be q, then the charge on the other sphere is $Q = (52.6\times 10^{-6}\mathrm{C}) - q$. Then

$$\begin{aligned}
\frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2} &= F, \\
(8.99\times 10^9 \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)q(52.6\times 10^{-6}\mathrm{C} - q) &= (1.19\,\mathrm{N})(1.94\,\mathrm{m})^2.
\end{aligned}$$

Solve this quadratic expression for q and get answers $q_1 = 4.02\times 10^{-5}\mathrm{C}$ and $q_2 = 1.24\times 10^{-6}\mathrm{N}$.

$\boxed{\textbf{E25-11}}$ This problem is similar to Ex. 25-7. There are some additional issues, however. It is easy enough to write expressions for the forces on the third charge

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{r_{31}^2}\hat{\mathbf{r}}_{31}, \\
\vec{\mathbf{F}}_{32} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{r_{32}^2}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

Then

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= -\vec{\mathbf{F}}_{32}, \\
\frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{r_{31}^2}\hat{\mathbf{r}}_{31} &= -\frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{r_{32}^2}\hat{\mathbf{r}}_{32}, \\
\frac{q_1}{r_{31}^2}\hat{\mathbf{r}}_{31} &= -\frac{q_2}{r_{32}^2}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

The only way to satisfy the *vector* nature of the above expression is to have $\hat{\mathbf{r}}_{31} = \pm\hat{\mathbf{r}}_{32}$; this means that q_3 must be collinear with q_1 and q_2. q_3 could be between q_1 and q_2, or it could be on either side. Let's resolve this issue now by putting the values for q_1 and q_2 into the expression:

$$\begin{aligned}
\frac{(1.07\,\mu\mathrm{C})}{r_{31}^2}\hat{\mathbf{r}}_{31} &= -\frac{(-3.28\,\mu\mathrm{C})}{r_{32}^2}\hat{\mathbf{r}}_{32}, \\
r_{32}^2\hat{\mathbf{r}}_{31} &= (3.07)r_{31}^2\hat{\mathbf{r}}_{32}.
\end{aligned}$$

Since squared quantities are positive, we can only get this to work if $\hat{\mathbf{r}}_{31} = \hat{\mathbf{r}}_{32}$, so q_3 is *not* between q_1 and q_2. We are then left with

$$r_{32}^2 = (3.07)r_{31}^2,$$

so that q_3 is closer to q_1 than it is to q_2. Then $r_{32} = r_{31} + r_{12} = r_{31} + 0.618\,\mathrm{m}$, and if we take the square root of both sides of the above expression,

$$\begin{aligned}
r_{31} + (0.618\,\mathrm{m}) &= \sqrt{(3.07)}r_{31}, \\
(0.618\,\mathrm{m}) &= \sqrt{(3.07)}r_{31} - r_{31}, \\
(0.618\,\mathrm{m}) &= 0.752r_{31}, \\
0.822\,\mathrm{m} &= r_{31}
\end{aligned}$$

E25-12 The magnitude of the magnetic force between any two charges is kq^2/a^2, where $a = 0.153\,\mathrm{m}$. The force between each charge is directed along the side of the triangle; but from symmetry only the component along the bisector is of interest. This means that we need to weight the above answer by a factor of $2\cos(30°) = 1.73$. The net force on any charge is then $1.73kq^2/a^2$.

The length of the angle bisector, d, is given by $d = a\cos(30°)$.

The distance from any charge to the center of the equilateral triangle is x, given by $x^2 = (a/2)^2 + (d-x)^2$. Then

$$x = a^2/8d + d/2 = 0.644a.$$

The angle between the strings and the plane of the charges is θ, given by

$$\sin\theta = x/(1.17\,\mathrm{m}) = (0.644)(0.153\,\mathrm{m})/(1.17\,\mathrm{m}) = 0.0842,$$

or $\theta = 4.83°$.

The force of gravity on each ball is directed vertically and the electric force is directed horizontally. The two must then be related by

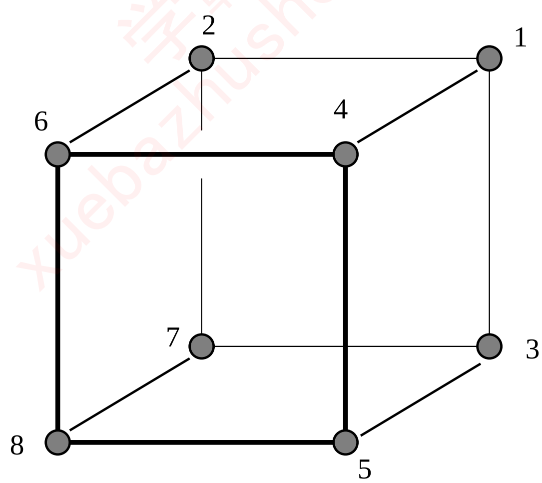
$$\tan\theta = F_\mathrm{E}/F_\mathrm{G},$$

so

$$1.73(8.99\times10^9\mathrm{N\cdot m^2/C^2})q^2/(0.153\,\mathrm{m})^2 = (0.0133\,\mathrm{kg})(9.81\,\mathrm{m/s^2})\tan(4.83°),$$

or $q = 1.29\times10^{-7}\mathrm{C}$.

$\boxed{\textbf{E25-13}}$ On any corner charge there are seven forces; one from each of the other seven charges. The net force will be the sum. Since all eight charges are the same all of the forces will be repulsive. We need to sketch a diagram to show how the charges are labeled.



The magnitude of the force of charge 2 on charge 1 is

$$F_{12} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r_{12}^2},$$

where $r_{12} = a$, the length of a side. Since both charges are the same we wrote q^2. By symmetry we expect that the magnitudes of F_{12}, F_{13}, and F_{14} will all be the same and they will all be at right angles to each other directed along the edges of the cube. Written in terms of vectors the forces would be

$$\vec{\mathbf{F}}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}},$$
$$\vec{\mathbf{F}}_{13} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}},$$
$$\vec{\mathbf{F}}_{14} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{k}}.$$

The force from charge 5 is

$$F_{15} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r_{15}^2},$$

and is directed along the side diagonal away from charge 5. The distance r_{15} is also the side diagonal distance, and can be found from

$$r_{15}^2 = a^2 + a^2 = 2a^2,$$

then

$$F_{15} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}.$$

By symmetry we expect that the magnitudes of F_{15}, F_{16}, and F_{17} will all be the same and they will all be directed along the diagonals of the faces of the cube. In terms of components we would have

$$\vec{\mathbf{F}}_{15} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}\left(\hat{\mathbf{j}}/\sqrt{2} + \hat{\mathbf{k}}/\sqrt{2}\right),$$
$$\vec{\mathbf{F}}_{16} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}\left(\hat{\mathbf{i}}/\sqrt{2} + \hat{\mathbf{k}}/\sqrt{2}\right),$$
$$\vec{\mathbf{F}}_{17} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}\left(\hat{\mathbf{i}}/\sqrt{2} + \hat{\mathbf{j}}/\sqrt{2}\right).$$

The last force is the force from charge 8 on charge 1, and is given by

$$F_{18} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r_{18}^2},$$

and is directed along the cube diagonal away from charge 8. The distance r_{18} is also the cube diagonal distance, and can be found from

$$r_{18}^2 = a^2 + a^2 + a^2 = 3a^2,$$

then in term of components

$$\vec{\mathbf{F}}_{18} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{3a^2}\left(\hat{\mathbf{i}}/\sqrt{3} + \hat{\mathbf{j}}/\sqrt{3} + \hat{\mathbf{k}}/\sqrt{3}\right).$$

We can add the components together. By symmetry we expect the same answer for each components, so we'll just do one. How about $\hat{\mathbf{i}}$. This component has contributions from charge 2, 6, 7, and 8:

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left(\frac{1}{1} + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right),$$

or

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}(1.90)$$

The three components add according to Pythagoras to pick up a final factor of $\sqrt{3}$, so

$$F_{\text{net}} = (0.262)\frac{q^2}{\epsilon_0 a^2}.$$

E25-14 (a) Yes. Changing the sign of y will change the sign of F_y; since this is equivalent to putting the charge q_0 on the "other" side, we would expect the force to also push in the "other" direction.

(b) The equation should look Eq. 25-15, except all y's should be replaced by x's. Then

$$F_x = \frac{1}{4\pi\epsilon_0}\frac{q_0\,q}{x\sqrt{x^2+L^2/4}}.$$

(c) Setting the particle a distance d away should give a force with the same magnitude as

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_0\,q}{d\sqrt{d^2+L^2/4}}.$$

This force is directed along the 45° line, so $F_x = F\cos 45°$ and $F_y = F\sin 45°$.

(d) Let the distance be $d=\sqrt{x^2+y^2}$, and then use the fact that $F_x/F = \cos\theta = x/d$. Then

$$F_x = F\frac{x}{d} = \frac{1}{4\pi\epsilon_0}\frac{x\,q_0\,q}{(x^2+y^2+L^2/4)^{3/2}}.$$

and

$$F_y = F\frac{y}{d} = \frac{1}{4\pi\epsilon_0}\frac{y\,q_0\,q}{(x^2+y^2+L^2/4)^{3/2}}.$$

E25-15 (a) The equation *is* valid for both positive and negative z, so in vector form it would read

$$\vec{\mathbf{F}} = F_z\hat{\mathbf{k}} = \frac{1}{4\pi\epsilon_0}\frac{q_0\,q\,z}{(z^2+R^2)^{3/2}}\hat{\mathbf{k}}.$$

(b) The equation *is not* valid for both positive and negative z. Reversing the sign of z should reverse the sign of F_z, and one way to fix this is to write $1 = z/\sqrt{z^2}$. Then

$$\vec{\mathbf{F}} = F_z\hat{\mathbf{k}} = \frac{1}{4\pi\epsilon_0}\frac{2q_0\,qz}{R^2}\left(\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2}}\right)\hat{\mathbf{k}}.$$

E25-16 Divide the rod into small differential lengths dr, each with charge $dQ = (Q/L)dr$. Each differential length contributes a differential force

$$dF = \frac{1}{4\pi\epsilon_0}\frac{q\,dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2L}dr.$$

Integrate:

$$\begin{aligned} F &= \int dF = \int_x^{x+L}\frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2L}dr, \\ &= \frac{1}{4\pi\epsilon_0}\frac{qQ}{L}\left(\frac{1}{x} - \frac{1}{x+L}\right) \end{aligned}$$

E25-17 You must solve Ex. 16 before solving this problem! q_0 refers to the charge that had been called q in that problem. In either case the distance from q_0 will be the same regardless of the sign of q; if $q = Q$ then q will be on the right, while if $q = -Q$ then q will be on the left.

Setting the forces equal to each other one gets

$$\frac{1}{4\pi\epsilon_0}\frac{qQ}{L}\left(\frac{1}{x} - \frac{1}{x+L}\right) = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2},$$

or

$$r = \sqrt{x(x+L)}.$$

E25-18 You must solve Ex. 16 and Ex. 17 before solving this problem.

If all charges are positive then moving q_0 off axis will result in a net force away from the axis. That's unstable.

If $q = -Q$ then both q and Q are on the same side of q_0. Moving q_0 closer to q will result in the attractive force growing faster than the repulsive force, so q_0 will move away from equilibrium.

$\boxed{\textbf{E25-19}}$ We can start with the work that was done for us on Page 577, except since we are concerned with $\sin\theta = z/r$ we would have

$$dF_x = dF\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{q_0\lambda\,dz}{(y^2+z^2)}\frac{z}{\sqrt{y^2+z^2}}.$$

We will need to take into consideration that λ changes sign for the two halves of the rod. Then

$$\begin{aligned}
F_x &= \frac{q_0\lambda}{4\pi\epsilon_0}\left(\int_{-L/2}^{0}\frac{-z\,dz}{(y^2+z^2)^{3/2}}+\int_0^{L/2}\frac{+z\,dz}{(y^2+z^2)^{3/2}}\right),\\
&= \frac{q_0\lambda}{2\pi\epsilon_0}\int_0^{L/2}\frac{z\,dz}{(y^2+z^2)^{3/2}},\\
&= \frac{q_0\lambda}{2\pi\epsilon_0}\left.\frac{-1}{\sqrt{y^2+z^2}}\right|_0^{L/2},\\
&= \frac{q_0\lambda}{2\pi\epsilon_0}\left(\frac{1}{y}-\frac{1}{\sqrt{y^2+(L/2)^2}}\right).
\end{aligned}$$

E25-20 Use Eq. 25-15 to find the magnitude of the force from any one rod, but write it as

$$F = \frac{1}{4\pi\epsilon_0}\frac{q\,Q}{r\sqrt{r^2+L^2/4}},$$

where $r^2 = z^2 + L^2/4$. The component of this along the z axis is $F_z = Fz/r$. Since there are 4 rods, we have

$$F = \frac{1}{\pi\epsilon_0}\frac{q\,Q\,z}{r^2\sqrt{r^2+L^2/4}}, = \frac{1}{\pi\epsilon_0}\frac{q\,Q\,z}{(z^2+L^2/4)\sqrt{z^2+L^2/2}},$$

Equating the electric force with the force of gravity and solving for Q,

$$Q = \frac{\pi\epsilon_0 mg}{qz}(z^2+L^2/4)\sqrt{z^2+L^2/2};$$

putting in the numbers,

$$\frac{\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(3.46\times10^{-7}\mathrm{kg})(9.8\mathrm{m/s}^2)}{(2.45\times10^{-12}\mathrm{C})(0.214\,\mathrm{m})}((0.214\mathrm{m})^2+(0.25\mathrm{m})^2/4)\sqrt{(0.214\mathrm{m})^2+(0.25\mathrm{m})^2/2}$$

so $Q = 3.07\times10^{-6}$C.

$\boxed{\textbf{E25-21}}$ In each case we conserve charge by making sure that the total number of protons is the same on both sides of the expression. We also need to conserve the number of neutrons.

(a) Hydrogen has one proton, Beryllium has four, so X must have five protons. Then X must be Boron, B.

(b) Carbon has six protons, Hydrogen has one, so X must have seven. Then X is Nitrogen, N.

(c) Nitrogen has seven protons, Hydrogen has one, but Helium has two, so X has $7 + 1 - 2 = 6$ protons. This means X is Carbon, C.

E25-22 (a) Use Eq. 25-4:

$$F = \frac{(8.99\times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(2)(90)(1.60\times 10^{-19}\text{C})^2}{(12\times 10^{-15}\text{m})^2} = 290\,\text{N}.$$

(b) $a = (290\,\text{N})/(4)(1.66\times 10^{-27}\text{kg}) = 4.4\times 10^{28}\text{m/s}^2$.

E25-23 Use Eq. 25-4:

$$F = \frac{(8.99\times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\text{C})^2}{(282\times 10^{-12}\text{m})^2} = 2.89\times 10^{-9}\text{N}.$$

E25-24 (a) Use Eq. 25-4:

$$q = \sqrt{\frac{(3.7\times 10^{-9}\text{N})(5.0\times 10^{-10}\text{m})^2}{(8.99\times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)}} = 3.20\times 10^{-19}\text{C}.$$

(b) $N = (3.20\times 10^{-19}\text{C})/(1.60\times 10^{-19}\text{C}) = 2$.

$\boxed{\textbf{E25-25}}$ Use Eq. 25-4,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2} = \frac{(\frac{1}{3}1.6\times 10^{-19}\,\text{C})(\frac{1}{3}1.6\times 10^{-19}\,\text{C})}{4\pi(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.6\times 10^{-15}\,\text{m})^2} = 3.8\,\text{N}.$$

E25-26 (a) $N = (1.15\times 10^{-7}\text{C})/(1.60\times 10^{-19}\text{C}) = 7.19\times 10^{11}$.

(b) The penny has enough electrons to make a total charge of $-1.37\times 10^5\text{C}$. The fraction is then

$$(1.15\times 10^{-7}\text{C})/(1.37\times 10^5\text{C}) = 8.40\times 10^{-13}.$$

E25-27 Equate the magnitudes of the forces:

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = mg,$$

so

$$r = \sqrt{\frac{(8.99\times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\text{C})^2}{(9.11\times 10^{-31}\text{kg})(9.81\,\text{m/s}^2)}} = 5.07\,\text{m}$$

E25-28 $Q = (75.0\,\text{kg})(-1.60\times 10^{-19}\text{C})/(9.11\times 10^{-31}\text{kg}) = -1.3\times 10^{13}\text{C}$.

E25-29 The mass of water is $(250\ \text{cm}^3)(1.00\ \text{g/cm}^3) = 250$ g. The number of moles of water is $(250\ \text{g})/(18.0\ \text{g/mol}) = 13.9$ mol. The number of water molecules is $(13.9\ \text{mol})(6.02\times 10^{23}\text{mol}^{-1}) = 8.37\times 10^{24}$. Each molecule has ten protons, so the total positive charge is

$$Q = (8.37\times 10^{24})(10)(1.60\times 10^{-19}\text{C}) = 1.34\times 10^7\text{C}.$$

E25-30 The total positive charge in 0.250 kg of water is 1.34×10^7C. Mary's imbalance is then

$$q_1 = (52.0)(4)(1.34\times10^7\mathrm{C})(0.0001) = 2.79\times10^5\mathrm{C},$$

while John's imbalance is

$$q_2 = (90.7)(4)(1.34\times10^7\mathrm{C})(0.0001) = 4.86\times10^5\mathrm{C},$$

The electrostatic force of attraction is then

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = (8.99\times10^9\mathrm{N\cdot m^2/C^2})\frac{(2.79\times10^5)(4.86\times10^5)}{(28.0\,\mathrm{m})^2} = 1.6\times10^{18}\mathrm{N}.$$

$\boxed{\textbf{E25-31}}$ (a) The gravitational force of attraction between the Moon and the Earth is

$$F_\mathrm{G} = \frac{GM_\mathrm{E}M_\mathrm{M}}{R^2},$$

where R is the distance between them. If both the Earth and the moon are provided a charge q, then the electrostatic repulsion would be

$$F_\mathrm{E} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{R^2}.$$

Setting these two expression equal to each other,

$$\frac{q^2}{4\pi\epsilon_0} = GM_\mathrm{E}M_\mathrm{M},$$

which has solution

$$\begin{aligned} q &= \sqrt{4\pi\epsilon_0 GM_\mathrm{E}M_\mathrm{M}}, \\ &= \sqrt{4\pi(8.85\times10^{-12}\mathrm{C^2/Nm^2})(6.67\times10^{-11}\mathrm{Nm^2/kg^2})(5.98\times10^{24}\mathrm{kg})(7.36\times10^{22}\mathrm{kg})}, \\ &= 5.71\times10^{13}\,\mathrm{C}. \end{aligned}$$

(b) We need

$$(5.71\times10^{13}\,\mathrm{C})/(1.60\times10^{-19}\,\mathrm{C}) = 3.57\times10^{32}$$

protons on each body. The mass of protons needed is then

$$(3.57\times10^{32})(1.67\times10^{-27}\,\mathrm{kg}) = 5.97\times10^{65}\,\mathrm{kg}.$$

Ignoring the mass of the electron (why not?) we can assume that hydrogen is all protons, so we need that much hydrogen.

$\boxed{\textbf{P25-1}}$ Assume that the spheres initially have charges q_1 and q_2. The force of attraction between them is

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2} = -0.108\,\mathrm{N},$$

where $r_{12} = 0.500\,\mathrm{m}$. The net charge is $q_1 + q_2$, and after the conducting wire is connected each sphere will get *half* of the total. The spheres will have the same charge, and repel with a force of

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{\frac{1}{2}(q_1+q_2)\frac{1}{2}(q_1+q_2)}{r_{12}^2} = 0.0360\,\mathrm{N}.$$

Since we know the separation of the spheres we can find $q_1 + q_2$ quickly,

$$q_1 + q_2 = 2\sqrt{4\pi\epsilon_0 r_{12}^2 (0.0360\,\mathrm{N})} = 2.00\,\mu\mathrm{C}$$

We'll put this back into the first expression and solve for q_2.

$$\begin{aligned} -0.108\,\mathrm{N} &= \frac{1}{4\pi\epsilon_0}\frac{(2.00\,\mu\mathrm{C} - q_2)q_2}{r_{12}^2}, \\ -3.00\times10^{-12}\,\mathrm{C}^2 &= (2.00\,\mu\mathrm{C} - q_2)q_2, \\ 0 &= -q_2^2 + (2.00\,\mu\mathrm{C})q_2 + (1.73\,\mu\mathrm{C})^2. \end{aligned}$$

The solution is $q_2 = 3.0\,\mu\mathrm{C}$ or $q_2 = -1.0\,\mu\mathrm{C}$. Then $q_1 = -1.0\,\mu\mathrm{C}$ or $q_1 = 3.0\,\mu\mathrm{C}$.

P25-2 The electrostatic force on Q from each q has magnitude $qQ/4\pi\epsilon_0 a^2$, where a is the length of the side of the square. The magnitude of the vertical (horizontal) component of the force of Q on Q is $\sqrt{2}Q^2/16\pi\epsilon_0 a^2$.

(a) In order to have a zero net force on Q the magnitudes of the two contributions must balance, so

$$\frac{\sqrt{2}Q^2}{16\pi\epsilon_0 a^2} = \frac{qQ}{4\pi\epsilon_0 a^2},$$

or $q = \sqrt{2}Q/4$. The charges must actually have opposite charge.

(b) No.

$\boxed{\textbf{P25-3}}$ (a) The third charge, q_3, will be between the first two. The net force on the third charge will be zero if

$$\frac{1}{4\pi\epsilon_0}\frac{q\,q_3}{r_{31}^2} = \frac{1}{4\pi\epsilon_0}\frac{4q\,q_3}{r_{32}^2},$$

which will occur if

$$\frac{1}{r_{31}} = \frac{2}{r_{32}}$$

The total distance is L, so $r_{31} + r32 = L$, or $r_{31} = L/3$ and $r_{32} = 2L/3$.

Now that we have found the position of the third charge we need to find the magnitude. The second and third charges both exert a force on the first charge; we want this net force on the first charge to be zero, so

$$\frac{1}{4\pi\epsilon_0}\frac{q\,q_3}{r_{13}^2} = \frac{1}{4\pi\epsilon_0}\frac{q\,4q}{r_{12}^2},$$

or

$$\frac{q_3}{(L/3)^2} = \frac{4q}{L^2},$$

which has solution $q_3 = -4q/9$. The negative sign is because the force between the first and second charge must be in the opposite direction to the force between the first and third charge.

(b) Consider what happens to the net force on the middle charge if is is displaced a small distance z. If the charge 3 is moved toward charge 1 then the force of attraction with charge 1 will increase. But moving charge 3 closer to charge 1 means moving charge 3 away from charge 2, so the force of attraction between charge 3 and charge 2 will decrease. So charge 3 experiences more attraction to ward the charge that it moves toward, and less attraction to the charge it moves away from. Sounds unstable to me.

P25-4 (a) The electrostatic force on the charge on the right has magnitude

$$F = \frac{q^2}{4\pi\epsilon_0 x^2},$$

The weight of the ball is $W = mg$, and the two forces are related by

$$F/W = \tan\theta \approx \sin\theta = x/2L.$$

Combining, $2Lq^2 = 4\pi\epsilon_0 mgx^3$, so

$$x = \left(\frac{q^2 L}{2\pi\epsilon_0}\right)^{1/3}.$$

(b) Rearrange and solve for q,

$$q = \sqrt{\frac{2\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(0.0112\,\mathrm{kg})(9.81\,\mathrm{m/s}^2)(4.70\times10^{-2}\mathrm{m})^3}{(1.22\,\mathrm{m})}} = 2.28\times10^{-8}\mathrm{C}.$$

P25-5 (a) Originally the balls would not repel, so they would move together and touch; after touching the balls would "split" the charge ending up with $q/2$ each. They would then repel again.

(b) The new equilibrium separation is

$$x' = \left(\frac{(q/2)^2 L}{2\pi\epsilon_0 mg}\right)^{1/3} = \left(\frac{1}{4}\right)^{1/3} x = 2.96\ \mathrm{cm}.$$

P25-6 Take the time derivative of the expression in Problem 25-4. Then

$$\frac{dx}{dt} = \frac{2}{3}\frac{x}{q}\frac{dq}{dt} = \frac{2}{3}\frac{(4.70\times10^{-2}\mathrm{m})}{(2.28\times10^{-8}\mathrm{C})}(-1.20\times10^{-9}\mathrm{C/s}) = 1.65\times10^{-3}\mathrm{m/s}.$$

$\boxed{\textbf{P25-7}}$ The force between the two charges is

$$F = \frac{1}{4\pi\epsilon_0}\frac{(Q-q)q}{r_{12}^2}.$$

We want to maximize this force with respect to variation in q, this means finding dF/dq and setting it equal to 0. Then

$$\frac{dF}{dq} = \frac{d}{dq}\left(\frac{1}{4\pi\epsilon_0}\frac{(Q-q)q}{r_{12}^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q-2q}{r_{12}^2}.$$

This will vanish if $Q - 2q = 0$, or $q = \frac{1}{2}Q$.

P25-8 Displace the charge q a distance y. The net restoring force on q will be approximately

$$F \approx 2\frac{qQ}{4\pi\epsilon_0}\frac{1}{(d/2)^2}\frac{y}{(d/2)} = \frac{qQ}{4\pi\epsilon_0}\frac{16}{d^3}y.$$

Since F/y is effectively a force constant, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = \left(\frac{\epsilon_0 m\pi^3 d^3}{qQ}\right)^{1/2}.$$

P25-9 Displace the charge q a distance x toward one of the positive charges Q. The net restoring force on q will be

$$\begin{aligned} F &= \frac{qQ}{4\pi\epsilon_0}\left(\frac{1}{(d/2-x)^2} - \frac{1}{(d/2+x)^2}\right), \\ &\approx \frac{qQ}{4\pi\epsilon_0}\frac{32}{d^3}x. \end{aligned}$$

Since F/x is effectively a force constant, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = \left(\frac{\epsilon_0 m\pi^3 d^3}{2qQ}\right)^{1/2}.$$

P25-10 (a) Zero, by symmetry.

(b) Removing a positive Cesium ion is equivalent to adding a singly charged negative ion at that same location. The net force is then

$$F = e^2/4\pi\epsilon_0 r^2,$$

where r is the distance between the Chloride ion and the newly placed negative ion, or

$$r = \sqrt{3(0.20\times10^{-9}\mathrm{m})^2}$$

The force is then

$$F = \frac{(1.6\times10^{-19}\mathrm{C})^2}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)3(0.20\times10^{-9}\mathrm{m})^2} = 1.92\times10^{-9}\mathrm{N}.$$

P25-11 We can pretend that this problem is in a single plane containing all three charges. The magnitude of the force on the test charge q_0 from the charge q on the left is

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q\,q_0}{(a^2+R^2)}.$$

A force of identical magnitude exists from the charge on the right. we need to add these two forces as vectors. Only the components along R will survive, and each force will contribute an amount

$$F_1\sin\theta = F_1\frac{R}{\sqrt{R^2+a^2}},$$

so the net force on the test particle will be

$$\frac{2}{4\pi\epsilon_0}\frac{q\,q_0}{(a^2+R^2)}\frac{R}{\sqrt{R^2+a^2}}.$$

We want to find the maximum value as a function of R. This means take the derivative, and set it equal to zero. The derivative is

$$\frac{2q\,q_0}{4\pi\epsilon_0}\left(\frac{1}{(a^2+R^2)^{3/2}} - \frac{3R^2}{(a^2+R^2)^{5/2}}\right),$$

which will vanish when

$$a^2+R^2 = 3R^2,$$

a *simple* quadratic equation with solutions $R = \pm a/\sqrt{2}$.

E26-1 $E = F/q = ma/q$. Then

$$E = (9.11\times10^{-31}\text{kg})(1.84\times10^{9}\text{m/s}^2)/(1.60\times10^{-19}\text{C}) = 1.05\times10^{-2}\text{N/C}.$$

E26-2 The answers to (a) and (b) are the same!
$F = Eq = (3.0\times10^{6}\text{N/C})(1.60\times10^{-19}\text{C}) = 4.8\times10^{-13}\text{N}.$

$\boxed{\textbf{E26-3}}$ $F = W$, or $Eq = mg$, so

$$E = \frac{mg}{q} = \frac{(6.64 \times 10^{-27}\,\text{kg})(9.81\,\text{m/s}^2)}{2(1.60 \times 10^{-19}\,\text{C})} = 2.03 \times 10^{-7}\,\text{N/C}.$$

The alpha particle has a positive charge, this means that it will experience an electric force which is in the same direction as the electric field. Since the gravitational force is down, the electric force, and consequently the electric field, must be directed up.

E26-4 (a) $E = F/q = (3.0\times10^{-6}\text{N})/(2.0\times10^{-9}\text{C}) = 1.5\times10^{3}\text{N/C}.$
(b) $F = Eq = (1.5\times10^{3}\text{N/C})(1.60\times10^{-19}\text{C}) = 2.4\times10^{-16}\text{N}.$
(c) $F = mg = (1.67\times10^{-27}\text{kg})(9.81\,\text{m/s}^2) = 1.6\times10^{-26}\text{N}.$
(d) $(2.4\times10^{-16}\text{N})/(1.6\times10^{-26}\text{N}) = 1.5\times10^{10}.$

E26-5 Rearrange $E = q/4\pi\epsilon_0 r^2$,

$$q = 4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.750\,\text{m})^2(2.30\,\text{N/C}) = 1.44\times10^{-10}\text{C}.$$

E26-6 $p = qd = (1.60\times10^{-19}\text{C})(4.30\times10^{-9}) = 6.88\times10^{-28}\text{C}\cdot\text{m}.$

$\boxed{\textbf{E26-7}}$ Use Eq. 26-12 for points along the perpendicular bisector. Then

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{x^3} = (8.99 \times 10^{9}\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.56 \times 10^{-29}\,\text{C}\cdot\text{m})}{(25.4 \times 10^{-9}\,\text{m})^3} = 1.95 \times 10^{4}\text{N/C}.$$

E26-8 If the charges on the line $x = a$ where $+q$ and $-q$ instead of $+2q$ and $-2q$ then at the center of the square $E = 0$ by symmetry. This simplifies the problem into finding E for a charge $+q$ at $(a, 0)$ and $-q$ at (a, a). This is a dipole, and the field is given by Eq. 26-11. For this exercise we have $x = a/2$ and $d = a$, so

$$E = \frac{1}{4\pi\epsilon_0}\frac{qa}{[2(a/2)^2]^{3/2}},$$

or, putting in the numbers, $E = 1.11\times10^{5}\text{N/C}.$

E26-9 The charges at 1 and 7 are opposite and can be effectively replaced with a single charge of $-6q$ at 7. The same is true for 2 and 8, 3 and 9, on up to 6 and 12. By symmetry we expect the field to point along a line so that three charges are above and three below. That would mean 9:30.

E26-10 If both charges are positive then Eq. 26-10 would read $E = 2E_+ \sin\theta$, and Eq. 26-11 would look like

$$\begin{aligned} E &= 2\frac{1}{4\pi\epsilon_0}\frac{q}{x^2 + (d/2)^2}\frac{x}{\sqrt{x^2 + (d/2)^2}}, \\ &\approx 2\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\frac{x}{\sqrt{x^2}} \end{aligned}$$

when $x \gg d$. This can be simplified to $E = 2q/4\pi\epsilon_0 x^2$.

$\boxed{\textbf{E26-11}}$ Treat the two charges on the left as one dipole and treat the two charges on the right as a second dipole. Point P is on the perpendicular bisector of both dipoles, so we can use Eq. 26-12 to find the two fields.

For the dipole on the left $p = 2aq$ and the electric field due to this dipole at P has magnitude

$$E_\mathrm{l} = \frac{1}{4\pi\epsilon_0}\frac{2aq}{(x+a)^3}$$

and is directed *up*.

For the dipole on the right $p = 2aq$ and the electric field due to this dipole at P has magnitude

$$E_\mathrm{r} = \frac{1}{4\pi\epsilon_0}\frac{2aq}{(x-a)^3}$$

and is directed *down*.

The net electric field at P is the sum of these two fields, but since the two component fields point in opposite directions we must actually subtract these values,

$$\begin{aligned} E &= E_\mathrm{r} - E_\mathrm{l}, \\ &= \frac{2aq}{4\pi\epsilon_0}\left(\frac{1}{(x-a)^3} - \frac{1}{(x+a)^3}\right), \\ &= \frac{aq}{2\pi\epsilon_0}\frac{1}{x^3}\left(\frac{1}{(1-a/x)^3} - \frac{1}{(1+a/x)^3}\right). \end{aligned}$$

We can use the binomial expansion on the terms containing $1 \pm a/x$,

$$\begin{aligned} E &\approx \frac{aq}{2\pi\epsilon_0}\frac{1}{x^3}\left((1+3a/x) - (1-3a/x)\right), \\ &= \frac{aq}{2\pi\epsilon_0}\frac{1}{x^3}\left(6a/x\right), \\ &= \frac{3(2qa^2)}{2\pi\epsilon_0 x^4}. \end{aligned}$$

E26-12 Do a series expansion on the part in the parentheses

$$1 - \frac{1}{\sqrt{1+R^2/z^2}} \approx 1 - \left(1 - \frac{1}{2}\frac{R^2}{z^2}\right) = \frac{R^2}{2z^2}.$$

Substitute this in,

$$E_z \approx \frac{\sigma}{2\epsilon_0}\frac{R^2}{2z^2}\frac{\pi}{\pi} = \frac{Q}{4\pi\epsilon_0 z^2}.$$

E26-13 At the surface $z = 0$ and $E_z = \sigma/2\epsilon_0$. Half of this value occurs when z is given by

$$\frac{1}{2} = 1 - \frac{z}{\sqrt{z^2+R^2}},$$

which can be written as $z^2 + R^2 = (2z)^2$. Solve this, and $z = R/\sqrt{3}$.

E26-14 Look at Eq. 26-18. The electric field will be a maximum when $z/(z^2+R^2)^{3/2}$ is a maximum. Take the derivative of this with respect to z, and get

$$\frac{1}{(z^2+R^2)^{3/2}} - \frac{3}{2}\frac{2z^2}{(z^2+R^2)^{5/2}} = \frac{z^2+R^2-3z^2}{(z^2+R^2)^{5/2}}.$$

This will vanish when the numerator vanishes, or when $z = R/\sqrt{2}$.

$\boxed{\textbf{E26-15}}$ (a) The electric field strength just above the center surface of a charged disk is given by Eq. 26-19, but with $z = 0$,

$$E = \frac{\sigma}{2\epsilon_0}$$

The surface charge density is $\sigma = q/A = q/(\pi R^2)$. Combining,

$$q = 2\epsilon_0 \pi R^2 E = 2(8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})\pi(2.5 \times 10^{-2}\mathrm{m})^2(3 \times 10^6\,\mathrm{N/C}) = 1.04 \times 10^{-7}\mathrm{C}.$$

Notice we used an electric field strength of $E = 3 \times 10^6\,\mathrm{N/C}$, which is the field at air breaks down and sparks happen.

(b) We want to find out how many atoms are on the surface; if a is the cross sectional area of one atom, and N the number of atoms, then $A = Na$ is the surface area of the disk. The number of atoms is

$$N = \frac{A}{a} = \frac{\pi(0.0250\,\mathrm{m})^2}{(0.015 \times 10^{-18}\,\mathrm{m^2})} = 1.31 \times 10^{17}$$

(c) The total charge on the disk is $1.04 \times 10^{-7}\mathrm{C}$, this corresponds to

$$(1.04 \times 10^{-7}\mathrm{C})/(1.6 \times 10^{-19}\mathrm{C}) = 6.5 \times 10^{11}$$

electrons. (We are ignoring the sign of the charge here.) If each surface atom can have at most one excess electron, then the fraction of atoms which are charged is

$$(6.5 \times 10^{11})/(1.31 \times 10^{17}) = 4.96 \times 10^{-6},$$

which isn't very many.

E26-16 Imagine switching the positive and negative charges. The electric field would also need to switch directions. By symmetry, then, the electric field can only point vertically down. Keeping only that component,

$$\begin{aligned} E &= 2\int_0^{\pi/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda d\theta}{r^2}\sin\theta, \\ &= \frac{2}{4\pi\epsilon_0}\frac{\lambda}{r^2}. \end{aligned}$$

But $\lambda = q/(\pi/2)$, so $E = q/\pi^2\epsilon_0 r^2$.

$\boxed{\textbf{E26-17}}$ We want to fit the data to Eq. 26-19,

$$E_z = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right).$$

There are only two variables, R and q, with $q = \sigma\pi R^2$.

We can find σ *very* easily if we assume that the measurements have no error because then at the surface (where $z = 0$), the expression for the electric field simplifies to

$$E = \frac{\sigma}{2\epsilon_0}.$$

Then $\sigma = 2\epsilon_0 E = 2(8.854 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})(2.043 \times 10^7\,\mathrm{N/C}) = 3.618 \times 10^{-4}\,\mathrm{C/m^2}$.

Finding the radius will take a little more work. We can choose one point, and make that the reference point, and then solve for R. Starting with

$$E_z = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right),$$

and then rearranging,

$$\begin{aligned}
\frac{2\epsilon_0 E_z}{\sigma} &= 1 - \frac{z}{\sqrt{z^2+R^2}}, \\
\frac{2\epsilon_0 E_z}{\sigma} &= 1 - \frac{1}{\sqrt{1+(R/z)^2}}, \\
\frac{1}{\sqrt{1+(R/z)^2}} &= 1 - \frac{2\epsilon_0 E_z}{\sigma}, \\
1+(R/z)^2 &= \frac{1}{\left(1-2\epsilon_0 E_z/\sigma\right)^2}, \\
\frac{R}{z} &= \sqrt{\frac{1}{\left(1-2\epsilon_0 E_z/\sigma\right)^2}-1}.
\end{aligned}$$

Using $z = 0.03\,\mathrm{m}$ and $E_z = 1.187 \times 10^7\,\mathrm{N/C}$, along with our value of $\sigma = 3.618 \times 10^{-4}\,\mathrm{C/m^2}$, we find

$$\begin{aligned}
\frac{R}{z} &= \sqrt{\frac{1}{\left(1-2(8.854\times10^{-12}\mathrm{C^2/Nm^2})(1.187\times10^{7}\mathrm{N/C})/(3.618\times10^{-4}\mathrm{C/m^2})\right)^2}-1}, \\
R &= 2.167(0.03\,\mathrm{m}) = 0.065\,\mathrm{m}.
\end{aligned}$$

(b) And now find the charge from the charge density and the radius,

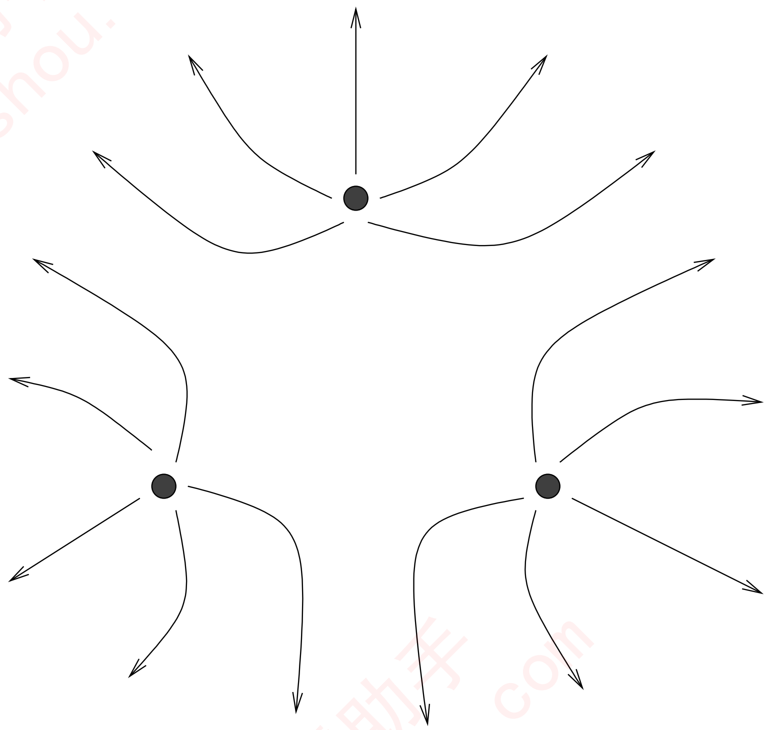
$$q = \pi R^2 \sigma = \pi(0.065\,\mathrm{m})^2(3.618\times10^{-4}\,\mathrm{C/m^2}) = 4.80\,\mu\mathrm{C}.$$

E26-18 (a) $\lambda = -q/L$.
(b) Integrate:

$$\begin{aligned}
E &= \int_a^{L+a} \frac{1}{4\pi\epsilon_0} \lambda\, dx x^2, \\
&= \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{L+a}\right), \\
&= \frac{q}{4\pi\epsilon_0}\frac{1}{a(L+a)},
\end{aligned}$$

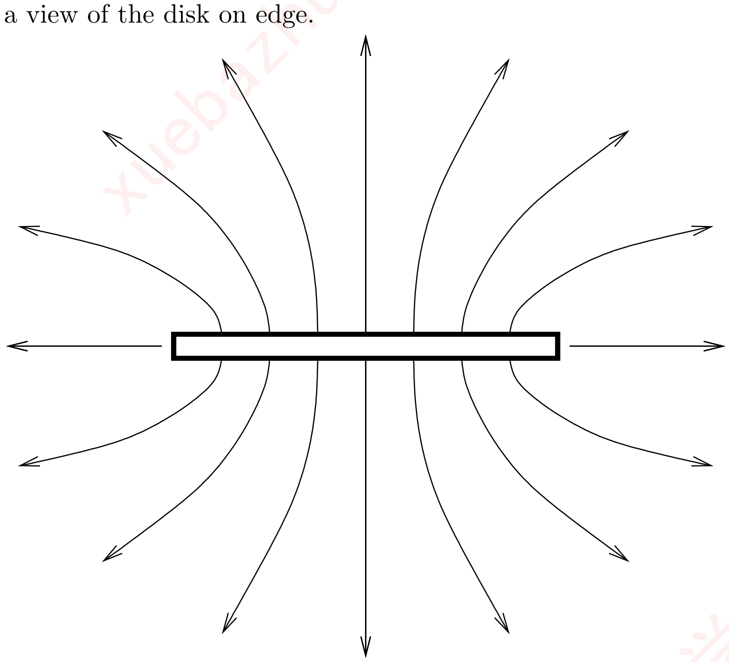
since $\lambda = q/L$.
(c) If $a \gg L$ then L can be replaced with 0 in the above expression.

E26-19 A sketch of the field looks like this.

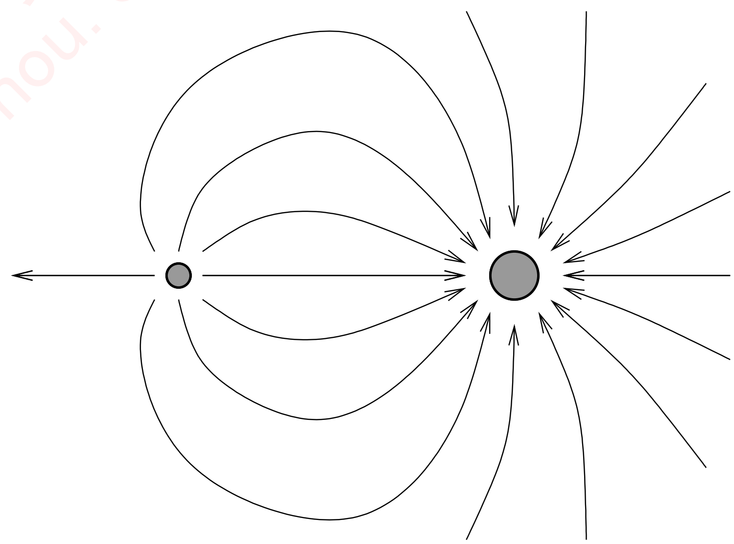


E26-20 (a) $F = Eq = (40\,\mathrm{N/C})(1.60\times10^{-19}\mathrm{C}) = 6.4\times10^{-18}\mathrm{N}$
(b) Lines are twice as far apart, so the field is half as large, or $E = 20\mathrm{N/C}$.

E26-21 Consider a view of the disk on edge.



E26-22 A sketch of the field looks like this.



E26-23 To the right.

E26-24 (a) The electric field is zero nearer to the smaller charge; since the charges have opposite signs it must be to the right of the $+2q$ charge. Equating the magnitudes of the two fields,

$$\frac{2q}{4\pi\epsilon_0 x^2} = \frac{5q}{4\pi\epsilon_0 (x+a)^2},$$

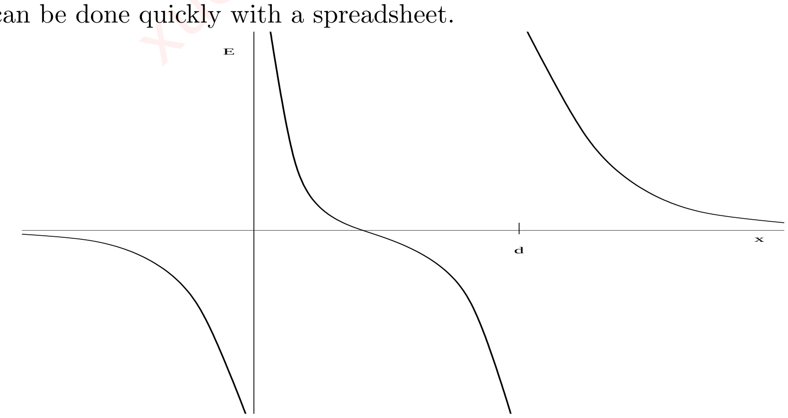
or

$$\sqrt{5}x = \sqrt{2}(x+a),$$

which has solution

$$x = \frac{\sqrt{2}\,a}{\sqrt{5}-\sqrt{2}} = 2.72a.$$

E26-25 This can be done quickly with a spreadsheet.



E26-26 (a) At point A,

$$E = \frac{1}{4\pi\epsilon_0}\left(-\frac{q}{d^2} - \frac{-2q}{(2d)^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{-q}{2d^2},$$

where the negative sign indicates that $\vec{\mathbf{E}}$ is directed to the left.

At point B,

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(d/2)^2} - \frac{-2q}{(d/2)^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{6q}{d^2},$$

where the positive sign indicates that $\vec{\mathbf{E}}$ is directed to the right.

At point C,

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(2d)^2} + \frac{-2q}{d^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{-7q}{4d^2},$$

where the negative sign indicates that $\vec{\mathbf{E}}$ is directed to the left.

E26-27 (a) The electric field does (negative) work on the electron. The magnitude of this work is $W = Fd$, where $F = Eq$ is the magnitude of the electric force on the electron and d is the distance through which the electron moves. Combining,

$$W = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = q\vec{\mathbf{E}}\cdot\vec{\mathbf{d}},$$

which gives the work done by the electric field on the electron. The electron originally possessed a kinetic energy of $K = \frac{1}{2}mv^2$, since we want to bring the electron to a rest, the work done must be negative. The charge q of the electron is negative, so $\vec{\mathbf{E}}$ and $\vec{\mathbf{d}}$ are pointing in the same direction, and $\vec{\mathbf{E}}\cdot\vec{\mathbf{d}} = Ed$.

By the work energy theorem,

$$W = \Delta K = 0 - \frac{1}{2}mv^2.$$

We put all of this together and find d,

$$d = \frac{W}{qE} = \frac{-mv^2}{2qE} = \frac{-(9.11\times10^{-31}\text{kg})(4.86\times10^6\,\text{m/s})^2}{2(-1.60\times10^{-19}\text{C})(1030\,\text{N/C})} = 0.0653\,\text{m}.$$

(b) $Eq = ma$ gives the magnitude of the acceleration, and $v_\text{f} = v_\text{i} + at$ gives the time. But $v_\text{f} = 0$. Combining these expressions,

$$t = -\frac{mv_\text{i}}{Eq} = -\frac{(9.11\times10^{-31}\text{kg})(4.86\times10^6\,\text{m/s})}{(1030\,\text{N/C})(-1.60\times10^{-19}\text{C})} = 2.69\times10^{-8}\,\text{s}.$$

(c) We will apply the work energy theorem again, except now we don't assume the final kinetic energy is zero. Instead,

$$W = \Delta K = K_\text{f} - K_\text{i},$$

and dividing through by the initial kinetic energy to get the fraction lost,

$$\frac{W}{K_\text{i}} = \frac{K_\text{f} - K_\text{i}}{K_\text{i}} = \text{ fractional change of kinetic energy.}$$

But $K_\text{i} = \frac{1}{2}mv^2$, and $W = qEd$, so the fractional change is

$$\frac{W}{K_\text{i}} = \frac{qEd}{\frac{1}{2}mv^2} = \frac{(-1.60\times10^{-19}\text{C})(1030\,\text{N/C})(7.88\times10^{-3}\text{m})}{\frac{1}{2}(9.11\times10^{-31}\text{kg})(4.86\times10^6\,\text{m/s})^2} = -12.1\%.$$

E26-28 (a) $a = Eq/m = (2.16\times10^4\text{N/C})(1.60\times10^{-19}\text{C})/(1.67\times10^{-27}\text{kg}) = 2.07\times10^{12}\text{m/s}^2$.

(b) $v = \sqrt{2ax} = \sqrt{2(2.07\times10^{12}\text{m/s}^2)(1.22\times10^{-2}\text{m})} = 2.25\times10^5\text{m/s}$.

E26-29 (a) $E = 2q/4\pi\epsilon_0 r^2$, or

$$E = \frac{(1.88\times10^{-7}\mathrm{C})}{2\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(0.152\,\mathrm{m}/2)^2} = 5.85\times10^5\mathrm{N/C}.$$

(b) $F = Eq = (5.85\times10^5\mathrm{N/C})(1.60\times10^{-19}\mathrm{C}) = 9.36\times10^{-14}\mathrm{N}.$

E26-30 (a) The average speed between the plates is $(1.95\times10^{-2}\mathrm{m})/(14.7\times10^{-9}\mathrm{s}) = 1.33\times10^6\mathrm{m/s}$. The speed with which the electron hits the plate is twice this, or $2.65\times10^6\mathrm{m/s}$.

(b) The acceleration is $a = (2.65\times10^6\mathrm{m/s})/(14.7\times10^{-9}\mathrm{s}) = 1.80\times10^{14}\mathrm{m/s}^2$. The electric field then has magnitude $E = ma/q$, or

$$E = (9.11\times10^{-31}\mathrm{kg})(1.80\times10^{14}\mathrm{m/s}^2)/(1.60\times10^{-19}\mathrm{C}) = 1.03\times10^3\mathrm{N/C}.$$

$\boxed{\textbf{E26-31}}$ The drop is balanced if the electric force is equal to the force of gravity, or $Eq = mg$. The mass of the drop is given in terms of the density by

$$m = \rho V = \rho\frac{4}{3}\pi r^3.$$

Combining,

$$q = \frac{mg}{E} = \frac{4\pi\rho r^3 g}{3E} = \frac{4\pi(851\,\mathrm{kg/m}^3)(1.64\times10^{-6}\mathrm{m})^3(9.81\,\mathrm{m/s}^2)}{3(1.92\times10^5\mathrm{N/C})} = 8.11\times10^{-19}\mathrm{C}.$$

We want the charge in terms of e, so we divide, and get

$$\frac{q}{e} = \frac{(8.11\times10^{-19}\mathrm{C})}{(1.60\times10^{-19}\mathrm{C})} = 5.07 \approx 5.$$

E26-32 (b) $F = (8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(2.16\times10^{-6}\mathrm{C})(85.3\times10^{-9}\mathrm{C})/(0.117\mathrm{m})^2 = 0.121\,\mathrm{N}.$

(a) $E_2 = F/q_1 = (0.121\,\mathrm{N})/(2.16\times10^{-6}\mathrm{C}) = 5.60\times10^4\mathrm{N/C}.$

$E_1 = F/q_2 = (0.121\,\mathrm{N})/(85.3\times10^{-9}\mathrm{C}) = 1.42\times10^6\mathrm{N/C}.$

$\boxed{\textbf{E26-33}}$ If each value of q measured by Millikan was a multiple of e, then the difference between any two values of q must also be a multiple of q. The smallest difference would be the smallest multiple, and this multiple might be unity. The differences are 1.641, 1.63, 1.60, 1.63, 3.30, 3.35, 3.18, 3.24, all times 10^{-19} C. This is a pretty clear indication that the fundamental charge is on the order of 1.6×10^{-19} C. If so, the likely number of fundamental charges on each of the drops is shown below in a table arranged like the one in the book:

4	8	12
5	10	14
7	11	16

The total number of charges is 87, while the total charge is 142.69×10^{-19} C, so the average charge per quanta is 1.64×10^{-19} C.

E26-34 Because of the electric field the acceleration toward the ground of a charged particle is not g, but $g \pm Eq/m$, where the sign depends on the direction of the electric field.

(a) If the lower plate is positively charged then $a = g - Eq/m$. Replace g in the pendulum period expression by this, and then

$$T = 2\pi\sqrt{\frac{L}{g - Eq/m}}.$$

(b) If the lower plate is negatively charged then $a = g + Eq/m$. Replace g in the pendulum period expression by this, and then

$$T = 2\pi\sqrt{\frac{L}{g + Eq/m}}.$$

E26-35 The ink drop travels an additional time $t' = d/v_x$, where d is the additional horizontal distance between the plates and the paper. During this time it travels an additional vertical distance $y' = v_y t'$, where $v_y = at = 2y/t = 2yv_x/L$. Combining,

$$y' = \frac{2yv_x t'}{L} = \frac{2yd}{L} = \frac{2(6.4\times10^{-4}\text{m})(6.8\times10^{-3}\text{m})}{(1.6\times10^{-2}\text{m})} = 5.44\times10^{-4}\text{m},$$

so the total deflection is $y + y' = 1.18\times10^{-3}$m.

E26-36 (a) $p = (1.48\times10^{-9}\text{C})(6.23\times10^{-6}\text{m}) = 9.22\times10^{-15}\text{C}\cdot\text{m}$.

(b) $\Delta U = 2pE = 2(9.22\times10^{-15}\text{C}\cdot\text{m})(1100\,\text{N/C}) = 2.03\times10^{-11}\text{J}$.

$\boxed{\textbf{E26-37}}$ Use $\tau = pE\sin\theta$, where θ is the angle between $\vec{\mathbf{p}}$ and $\vec{\mathbf{E}}$. For this dipole $p = qd = 2ed$ or $p = 2(1.6\times10^{-19}\,\text{C})(0.78\times10^{-9}\,\text{m}) = 2.5\times10^{-28}\,\text{C}\cdot\text{m}$. For all three cases

$$pE = (2.5\times10^{-28}\,\text{C}\cdot\text{m})(3.4\times10^{6}\text{N/C}) = 8.5\times10^{-22}\,\text{N}\cdot\text{m}.$$

The only thing we care about is the angle.

(a) For the parallel case $\theta = 0$, so $\sin\theta = 0$, and $\tau = 0$.

(b) For the perpendicular case $\theta = 90°$, so $\sin\theta = 1$, and $\tau = 8.5\times10^{-22}\,\text{N}\cdot\text{m}$..

(c) For the anti-parallel case $\theta = 180°$, so $\sin\theta = 0$, and $\tau = 0$.

E26-38 (c) Equal and opposite, or 5.22×10^{-16}N.

(d) Use Eq. 26-12 and $F = Eq$. Then

$$\begin{aligned}
p &= \frac{4\pi\epsilon_0 x^3 F}{q}, \\
&= \frac{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.285\text{m})^3(5.22\times10^{-16}\text{N})}{(3.16\times10^{-6}\text{C})}, \\
&= 4.25\times10^{-22}\text{C}\cdot\text{m}.
\end{aligned}$$

$\boxed{\textbf{E26-39}}$ The point-like nucleus contributes an electric field

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{Ze}{r^2},$$

while the uniform sphere of negatively charged electron cloud of radius R contributes an electric field given by Eq. 26-24,

$$E_- = \frac{1}{4\pi\epsilon_0}\frac{-Zer}{R^3}.$$

The net electric field is just the sum,

$$E = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

E26-40 The shell theorem first described for gravitation in chapter 14 is applicable here since both electric forces and gravitational forces fall off as $1/r^2$. The net positive charge inside the sphere of radius $d/2$ is given by $Q = 2e(d/2)^3/R^3 = ed^3/4R^3$.

The net force on either electron will be zero when

$$\frac{e^2}{d^2} = \frac{eQ}{(d/2)^2} = \frac{4e^2}{d^2}\frac{d^3}{4R^3} = \frac{e^2 d}{R^3},$$

which has solution $d = R$.

$\boxed{\textbf{P26-1}}$ (a) Let the positive charge be located *closer* to the point in question, then the electric field from the positive charge is

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{(x - d/2)^2}$$

and is directed *away from* the dipole.

The negative charge is located farther from the point in question, so

$$E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{(x + d/2)^2}$$

and is directed *toward* the dipole.

The net electric field is the sum of these two fields, but since the two component fields point in opposite direction we must actually subtract these values,

$$\begin{aligned} E &= E_+ - E_-, \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{(z - d/2)^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{(z + d/2)^2}, \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\left(\frac{1}{(1 - d/2z)^2} - \frac{1}{(1 + d/2z)^2}\right) \end{aligned}$$

We can use the binomial expansion on the terms containing $1 \pm d/2z$,

$$\begin{aligned} E &\approx \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\left((1 + d/z) - (1 - d/z)\right), \\ &= \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3} \end{aligned}$$

(b) The electric field is directed away from the positive charge when you are closer to the positive charge; the electric field is directed toward the negative charge when you are closer to the negative charge. In short, along the axis the electric field is directed in the same direction as the dipole moment.

P26-2 The key to this problem will be the expansion of

$$\frac{1}{(x^2 + (z \pm d/2)^2)^{3/2}} \approx \frac{1}{(x^2 + z^2)^{3/2}}\left(1 \mp \frac{3}{2}\frac{zd}{x^2 + z^2}\right).$$

for $d \ll \sqrt{x^2+z^2}$. Far from the charges the electric field of the positive charge has magnitude

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+(z-d/2)^2},$$

the components of this are

$$\begin{aligned} E_{x,+} &= \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+z^2}\frac{x}{\sqrt{x^2+(z-d/2)^2}}, \\ E_{z,+} &= \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+z^2}\frac{(z-d/2)}{\sqrt{x^2+(z-d/2)^2}}. \end{aligned}$$

Expand both according to the first sentence, then

$$\begin{aligned} E_{x,+} &\approx \frac{1}{4\pi\epsilon_0}\frac{xq}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right), \\ E_{z,+} &= \frac{1}{4\pi\epsilon_0}\frac{(z-d/2)q}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right). \end{aligned}$$

Similar expression exist for the negative charge, except we must replace q with $-q$ and the $+$ in the parentheses with a $-$, and $z-d/2$ with $z+d/2$ in the E_z expression. All that is left is to add the expressions. Then

$$\begin{aligned} E_x &= \frac{1}{4\pi\epsilon_0}\frac{xq}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right)+\frac{1}{4\pi\epsilon_0}\frac{-xq}{(x^2+z^2)^{3/2}}\left(1-\frac{3}{2}\frac{zd}{x^2+z^2}\right), \\ &= \frac{1}{4\pi\epsilon_0}\frac{3xqzd}{(x^2+z^2)^{5/2}}, \\ E_z &= \frac{1}{4\pi\epsilon_0}\frac{(z-d/2)q}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right)+\frac{1}{4\pi\epsilon_0}\frac{-(z+d/2)q}{(x^2+z^2)^{3/2}}\left(1-\frac{3}{2}\frac{zd}{x^2+z^2}\right), \\ &= \frac{1}{4\pi\epsilon_0}\frac{3z^2dq}{(x^2+z^2)^{5/2}}-\frac{1}{4\pi\epsilon_0}\frac{dq}{(x^2+z^2)^{3/2}}, \\ &= \frac{1}{4\pi\epsilon_0}\frac{(2z^2-x^2)dq}{(x^2+z^2)^{5/2}}. \end{aligned}$$

P26-3 (a) Each point on the ring is a distance $\sqrt{z^2+R^2}$ from the point on the axis in question. Since all points are equal distant and subtend the same angle from the axis then the top half of the ring contributes

$$E_{1z} = \frac{q_1}{4\pi\epsilon_0(x^2+R^2)}\frac{z}{\sqrt{z^2+R^2}},$$

while the bottom half contributes a similar expression. Add, and

$$E_z = \frac{q_1+q_2}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}} = \frac{q}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}},$$

which is identical to Eq. 26-18.

(b) The perpendicular component would be zero if $q_1 = q_2$. It isn't, so it must be the difference $q_1 - q_2$ which is of interest. Assume this charge difference is evenly distributed on the *top* half of the ring. If it is a positive difference, then $E_\perp$ must point down. We are only interested then in the vertical component as we integrate around the top half of the ring. Then

$$\begin{aligned} E_\perp &= \int_0^\pi \frac{1}{4\pi\epsilon_0}\frac{(q_1-q_2)/\pi}{z^2+R^2}\cos\theta\,d\theta, \\ &= \frac{q_1-q_2}{2\pi^2\epsilon_0}\frac{1}{z^2+R^2}. \end{aligned}$$

P26-4 Use the approximation $1/(z \pm d)^2 \approx (1/z^2)(1 \mp 2d/z + 3d^2/z^2)$.

Add the contributions:

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(z+d)^2} - \frac{2q}{z^2} + \frac{q}{(z-d)^2}\right), \\ &\approx \frac{q}{4\pi\epsilon_0 z^2}\left(1 - \frac{2d}{z} + \frac{3d^2}{z^2} - 2 + 1 + \frac{2d}{z} + \frac{3d^2}{z^2}\right), \\ &= \frac{q}{4\pi\epsilon_0 z^2}\frac{6d^2}{z^2} = \frac{3Q}{4\pi\epsilon_0 z^4}, \end{aligned}$$

where $Q = 2qd^2$.

$\boxed{\textbf{P26-5}}$ A monopole field falls off as $1/r^2$. A dipole field falls off as $1/r^3$, and consists of two oppositely charge monopoles close together. A quadrupole field (see Exercise 11 above or read Problem 4) falls off as $1/r^4$ and (can) consist of two otherwise identical dipoles arranged with anti-parallel dipole moments. Just taking a leap of faith it seems as if we can construct a $1/r^6$ field behavior by extending the reasoning.

First we need an *octopole* which is constructed from a quadrupole. We want to keep things as simple as possible, so the construction steps are

1. The monopole is a charge $+q$ at $x = 0$.
2. The dipole is a charge $+q$ at $x = 0$ and a charge $-q$ at $x = a$. We'll call this a dipole at $x = a/2$
3. The quadrupole is the dipole at $x = a/2$, and a second dipole pointing the other way at $x = -a/2$. The charges are then $-q$ at $x = -a$, $+2q$ at $x = 0$, and $-q$ at $x = a$.
4. The octopole will be two stacked, offset quadrupoles. There will be $-q$ at $x = -a$, $+3q$ at $x = 0$, $-3q$ at $x = a$, and $+q$ at $x = 2a$.
5. Finally, our distribution with a far field behavior of $1/r^6$. There will be $+q$ at $x = 2a$, $-4q$ at $x = -a$, $+6q$ at $x = 0$, $-4q$ at $x = a$, and $+q$ at $x = 2a$.

P26-6 The vertical component of $\vec{\mathbf{E}}$ is simply half of Eq. 26-17. The horizontal component is given by a variation of the work required to derive Eq. 26-16,

$$dE_z = dE \sin\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dz}{y^2 + z^2}\frac{z}{\sqrt{y^2 + z^2}},$$

which integrates to zero if the limits are $-\infty$ to $+\infty$, but in this case,

$$E_z = \int_0^\infty dE_z = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{z}.$$

Since the vertical and horizontal components are equal then $\vec{\mathbf{E}}$ makes an angle of 45°.

P26-7 (a) Swap all positive and negative charges in the problem and the electric field must reverse direction. But this is the same as flipping the problem over; consequently, the electric field must point parallel to the rod. This only holds true at point P, because point P doesn't move when you flip the rod.

(b) We are only interested in the vertical component of the field as contributed from each point on the rod. We can integrate only half of the rod and double the answer, so we want to evaluate

$$\begin{aligned} E_z &= 2\int_0^{L/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dz}{y^2+z^2}\frac{z}{\sqrt{y^2+z^2}}, \\ &= \frac{2\lambda}{4\pi\epsilon_0}\frac{\sqrt{(L/2)^2+y^2}-y}{y\sqrt{(L/2)^2+y^2}}. \end{aligned}$$

(c) The previous expression is exact. If $y \gg L$, then the expression simplifies with a Taylor expansion to

$$E_z = \frac{\lambda}{4\pi\epsilon_0}\frac{L^2}{y^3},$$

which looks similar to a dipole.

P26-8 Evaluate

$$E = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{z\,dq}{(z^2+r^2)^{3/2}},$$

where r is the radius of the ring, z the distance to the plane of the ring, and dq the differential charge on the ring. But $r^2+z^2=R^2$, and $dq=\sigma(2\pi r\,dr)$, where $\sigma=q/2\pi R^2$. Then

$$\begin{aligned} E &= \int_0^R \frac{q}{4\pi\epsilon_0}\frac{\sqrt{R^2-r^2}\,r\,dr}{R^5}, \\ &= \frac{q}{4\pi\epsilon_0}\frac{1}{3R^2}. \end{aligned}$$

$\boxed{\textbf{P26-9}}$ The key statement is the second italicized paragraph on page 595; the number of field lines through a unit cross-sectional area is proportional to the electric field strength. If the exponent is n, then the electric field strength a distance r from a point charge is

$$E = \frac{kq}{r^n},$$

and the *total* cross sectional area at a distance r is the area of a spherical shell, $4\pi r^2$. Then the number of field lines through the shell is proportional to

$$EA = \frac{kq}{r^n}4\pi r^2 = 4\pi kq r^{2-n}.$$

Note that the number of field lines varies with r if $n \neq 2$. This means that as we go farther from the point charge we need more and more field lines (or fewer and fewer). But the field lines can only start on charges, and we don't have any except for the point charge. We have a problem; we really do need $n=2$ if we want workable field lines.

P26-10 The distance traveled by the electron will be $d_1 = a_1t^2/2$; the distance traveled by the proton will be $d_2 = a_2t^2/2$. a_1 and a_2 are related by $m_1a_1 = m_2a_2$, since the electric force is the same (same charge magnitude). Then $d_1+d_2 = (a_1+a_2)t^2/2$ is the 5.00 cm distance. Divide by the proton distance, and then

$$\frac{d_1+d_2}{d_2} = \frac{a_1+a_2}{a_2} = \frac{m_2}{m_1}+1.$$

Then

$$d_2 = (5.00\times10^{-2}\mathrm{m})/(1.67\times10^{-27}/9.11\times10^{-31}+1) = 2.73\times10^{-5}\mathrm{m}.$$

P26-11 This is merely a fancy projectile motion problem. $v_x = v_0 \cos\theta$ while $v_{y,0} = v_0 \sin\theta$. The x and y positions are $x = v_x t$ and

$$y = \frac{1}{2}at^2 + v_{y,0}t = \frac{ax^2}{2v_0^2\cos^2\theta} + x\tan\theta.$$

The acceleration of the electron is vertically down and has a magnitude of

$$a = \frac{F}{m} = \frac{Eq}{m} = \frac{(1870\,\mathrm{N/C})(1.6\times10^{-19}\mathrm{C})}{(9.11\times10^{-31}\mathrm{kg})} = 3.284\times10^{14}\mathrm{m/s^2}.$$

We want to find out how the vertical velocity of the electron at the location of the top plate. If we get an imaginary answer, then the electron doesn't get as high as the top plate.

$$\begin{aligned} v_y &= \sqrt{{v_{y,0}}^2 + 2a\Delta y}, \\ &= \sqrt{(5.83\times10^6\mathrm{m/s})^2\sin(39^\circ)^2 + 2(-3.284\times10^{14}\mathrm{m/s^2})(1.97\times10^{-2}\mathrm{m})}, \\ &= 7.226\times10^5\mathrm{m/s}. \end{aligned}$$

This is a real answer, so this means the electron either hits the top plate, or it misses both plates. The time taken to reach the height of the top plate is

$$t = \frac{\Delta v_y}{a} = \frac{(7.226\times10^5\mathrm{m/s}) - (5.83\times10^6\mathrm{m/s})\sin(39^\circ)}{(-3.284\times10^{14}\mathrm{m/s^2})} = 8.972\times10^{-9}\mathrm{s}.$$

In this time the electron has moved a horizontal distance of

$$x = (5.83\times10^6\mathrm{m/s})\cos(39^\circ)(8.972\times10^{-9}\mathrm{s}) = 4.065\times10^{-2}\mathrm{m}.$$

This is clearly on the upper plate.

P26-12 Near the center of the ring $z \ll R$, so a Taylor expansion yields

$$E = \frac{\lambda}{2\epsilon_0}\frac{z}{R^2}.$$

The force on the electron is $F = Ee$, so the effective "spring" constant is $k = e\lambda/2\epsilon_0 R^2$. This means

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{e\lambda}{2\epsilon_0 m R^2}} = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}}.$$

P26-13 $U = -pE\cos\theta$, so the work required to flip the dipole is

$$W = -pE\left[\cos(\theta_0 + \pi) - \cos\theta_0\right] = 2pE\cos\theta_0.$$

P26-14 If the torque on a system is given by $|\tau| = \kappa\theta$, where κ is a constant, then the frequency of oscillation of the system is $f = \sqrt{\kappa/I}/2\pi$. In this case $\tau = pE\sin\theta \approx pE\theta$, so

$$f = \sqrt{pE/I}/2\pi.$$

P26-15 Use the a variation of the *exact* result from Problem 26-1. The two charge are positive, but since we will eventually focus on the area between the charges we must *subtract* the two field contributions, since they point in opposite directions. Then

$$E_z = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(z-a/2)^2} - \frac{1}{(z+a/2)^2}\right)$$

and then take the derivative,

$$\frac{dE_z}{dz} = -\frac{q}{2\pi\epsilon_0}\left(\frac{1}{(z-a/2)^3} - \frac{1}{(z+a/2)^3}\right).$$

Applying the binomial expansion for points $z \ll a$,

$$\begin{aligned}
\frac{dE_z}{dz} &= -\frac{8q}{2\pi\epsilon_0}\frac{1}{a^3}\left(\frac{1}{(2z/a-1)^3} - \frac{1}{(2z/a+1)^3}\right), \\
&\approx -\frac{8q}{2\pi\epsilon_0}\frac{1}{a^3}\left(-(1+6z/a) - (1-6z/a)\right), \\
&= \frac{8q}{\pi\epsilon_0}\frac{1}{a^3}.
\end{aligned}$$

There were some fancy sign flips in the second line, so review those steps carefully!

(b) The electrostatic force on a dipole is the difference in the magnitudes of the electrostatic forces on the two charges that make up the dipole. Near the center of the above charge arrangement the electric field behaves as

$$E_z \approx E_z(0) + \left.\frac{dE_z}{dz}\right|_{z=0} z + \text{ higher ordered terms.}$$

The net force on a dipole is

$$F_+ - F_- = q(E_+ - E_-) = q\left(E_z(0) + \left.\frac{dE_z}{dz}\right|_{z=0} z_+ - E_z(0) - \left.\frac{dE_z}{dz}\right|_{z=0} z_-\right)$$

where the "+" and "-" subscripts refer to the locations of the positive and negative charges. This last line can be simplified to yield

$$q\left.\frac{dE_z}{dz}\right|_{z=0}(z_+ - z_-) = qd\left.\frac{dE_z}{dz}\right|_{z=0}.$$

E27-1 $\Phi_E = (1800\,\mathrm{N/C})(3.2\times10^{-3}\mathrm{m})^2\cos(145°) = -7.8\times10^{-3}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}$.

E27-2 The right face has an area element given by $\vec{\mathbf{A}} = (1.4\,\mathrm{m})^2\hat{\mathbf{j}}$.

(a) $\Phi_E = \vec{\mathbf{A}}\cdot\vec{\mathbf{E}} = (2.0\,\mathrm{m}^2)\hat{\mathbf{j}}\cdot(6\,\mathrm{N/C})\hat{\mathbf{i}} = 0$.

(b) $\Phi_E = (2.0\,\mathrm{m}^2)\hat{\mathbf{j}}\cdot(-2\,\mathrm{N/C})\hat{\mathbf{j}} = -4\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}$.

(c) $\Phi_E = (2.0\,\mathrm{m}^2)\hat{\mathbf{j}}\cdot[(-3\,\mathrm{N/C})\hat{\mathbf{i}} + (4\,\mathrm{N/C})\hat{\mathbf{k}}] = 0$.

(d) In each case the field is uniform so we can simply evaluate $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}}$, where $\vec{\mathbf{A}}$ has six parts, one for every face. The faces, however, have the same size but are organized in pairs with opposite directions. These will cancel, so the total flux is zero in all three cases.

$\boxed{\textbf{E27-3}}$ (a) The flat base is easy enough, since according to Eq. 27-7,

$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}.$$

There are two important facts to consider in order to integrate this expression. $\vec{\mathbf{E}}$ is parallel to the axis of the hemisphere, $\vec{\mathbf{E}}$ points inward while $\mathbf{d}\vec{\mathbf{A}}$ points outward on the flat base. $\vec{\mathbf{E}}$ is uniform, so it is everywhere the same on the flat base. Since $\vec{\mathbf{E}}$ is anti-parallel to $d\vec{\mathbf{A}}$, $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = -E\,dA$, then

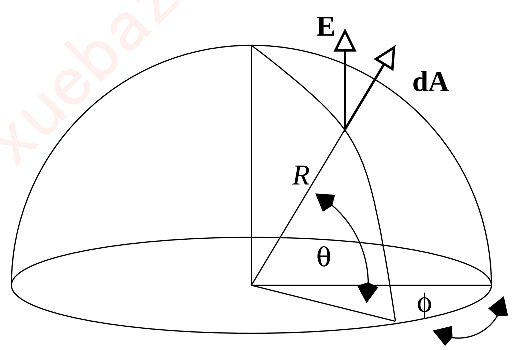
$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = -\int E\,dA.$$

Since $\vec{\mathbf{E}}$ is uniform we can simplify this as

$$\Phi_E = -\int E\,dA = -E\int dA = -EA = -\pi R^2 E.$$

The last steps are just substituting the area of a circle for the flat side of the hemisphere.

(b) We must first sort out the dot product



We can simplify the vector part of the problem with $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \cos\theta E\,dA$, so

$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int \cos\theta E\,dA$$

Once again, $\vec{\mathbf{E}}$ is uniform, so we can take it out of the integral,

$$\Phi_E = \int \cos\theta E\,dA = E\int \cos\theta\,dA$$

Finally, $dA = (R\,d\theta)(R\sin\theta\,d\phi)$ on the surface of a sphere centered on $R = 0$.

We'll integrate ϕ around the axis, from 0 to 2π. We'll integrate θ from the axis to the equator, from 0 to $\pi/2$. Then

$$\Phi_E = E\int \cos\theta\, dA = E\int_0^{2\pi}\int_0^{\pi/2} R^2\cos\theta\sin\theta\, d\theta\, d\phi.$$

Pulling out the constants, doing the ϕ integration, and then writing $2\cos\theta\sin\theta$ as $\sin(2\theta)$,

$$\Phi_E = 2\pi R^2 E\int_0^{\pi/2}\cos\theta\sin\theta\, d\theta = \pi R^2 E\int_0^{\pi/2}\sin(2\theta)\, d\theta,$$

Change variables and let $\beta = 2\theta$, then we have

$$\Phi_E = \pi R^2 E\int_0^{\pi}\sin\beta\frac{1}{2}d\beta = \pi R^2 E.$$

E27-4 Through S_1, $\Phi_E = q/\epsilon_0$. Through S_2, $\Phi_E = -q/\epsilon_0$. Through S_3, $\Phi_E = q/\epsilon_0$. Through S_4, $\Phi_E = 0$. Through S_5, $\Phi_E = q/\epsilon_0$.

E27-5 By Eq. 27-8,

$$\Phi_E = \frac{q}{\epsilon_0} = \frac{(1.84\,\mu\mathrm{C})}{(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)} = 2.08\times10^5\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}.$$

E27-6 The total flux through the sphere is

$$\Phi_E = (-1+2-3+4-5+6)(\times10^3\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}) = 3\times10^3\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}.$$

The charge inside the die is $(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(3\times10^3\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}) = 2.66\times10^{-8}\mathrm{C}$.

E27-7 The total flux through a cube would be q/ϵ_0. Since the charge is in the center of the cube we expect that the flux through any side would be the same, or 1/6 of the total flux. Hence the flux through the square surface is $q/6\epsilon_0$.

E27-8 If the electric field is uniform then there are no free charges near (or inside) the net. The flux through the netting must be equal to, but opposite in sign, from the flux through the opening. The flux through the opening is $E\pi a^2$, so the flux through the netting is $-E\pi a^2$.

E27-9 There is no flux through the sides of the cube. The flux through the top of the cube is $(-58\,\mathrm{N/C})(100\,\mathrm{m})^2 = -5.8\times10^5\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}$. The flux through the bottom of the cube is

$$(110\,\mathrm{N/C})(100\,\mathrm{m})^2 = 1.1\times10^6\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}.$$

The total flux is the sum, so the charge contained in the cube is

$$q = (8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(5.2\times10^5\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}) = 4.60\times10^{-6}\mathrm{C}.$$

E27-10 (a) There is only a flux through the right and left faces. Through the right face

$$\Phi_\mathrm{R} = (2.0\,\mathrm{m}^2)\hat{\mathbf{j}}\cdot(3\,\mathrm{N/C}\cdot\mathrm{m})(1.4\,\mathrm{m})\hat{\mathbf{j}} = 8.4\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}.$$

The flux through the left face is zero because $y = 0$.

$\boxed{\textbf{E27-11}}$ There are *eight* cubes which can be "wrapped" around the charge. Each cube has three external faces that are indistinguishable for a total of twenty-four faces, each with the same flux Φ_E. The total flux is q/ϵ_0, so the flux through one face is $\Phi_E = q/24\epsilon_0$. Note that this is the flux through faces opposite the charge; for faces which touch the charge the electric field is parallel to the surface, so the flux would be zero.

E27-12 Use Eq. 27-11,

$$\lambda = 2\pi\epsilon_0 rE = 2\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.96\,\mathrm{m})(4.52\times10^4\mathrm{N/C}) = 4.93\times10^{-6}\mathrm{C/m}.$$

E27-13 (a) $q = \sigma A = (2.0\times10^{-6}\mathrm{C/m}^2)\pi(0.12\,\mathrm{m})(0.42\,\mathrm{m}) = 3.17\times10^{-7}\mathrm{C}$.

(b) The charge density will be the same! $q = \sigma A = (2.0\times10^{-6}\mathrm{C/m}^2)\pi(0.08\,\mathrm{m})(0.28\,\mathrm{m}) = 1.41\times10^{-7}\mathrm{C}$.

E27-14 The electric field from the sheet on the left is of magnitude $E_l = \sigma/2\epsilon_0$, and points directly away from the sheet. The magnitude of the electric field from the sheet on the right is the same, but it points directly away from the sheet on the right.

(a) To the left of the sheets the two fields add since they point in the same direction. This means that the electric field is $\vec{\mathbf{E}} = -(\sigma/\epsilon_0)\hat{\mathbf{i}}$.

(b) Between the sheets the two electric fields cancel, so $\vec{\mathbf{E}} = 0$.

(c) To the right of the sheets the two fields add since they point in the same direction. This means that the electric field is $\vec{\mathbf{E}} = (\sigma/\epsilon_0)\hat{\mathbf{i}}$.

E27-15 The electric field from the plate on the left is of magnitude $E_l = \sigma/2\epsilon_0$, and points directly toward the plate. The magnitude of the electric field from the plate on the right is the same, but it points directly away from the plate on the right.

(a) To the left of the plates the two fields cancel since they point in the opposite directions. This means that the electric field is $\vec{\mathbf{E}} = 0$.

(b) Between the plates the two electric fields add since they point in the same direction. This means that the electric field is $\vec{\mathbf{E}} = -(\sigma/\epsilon_0)\hat{\mathbf{i}}$.

(c) To the right of the plates the two fields cancel since they point in the opposite directions. This means that the electric field is $\vec{\mathbf{E}} = 0$.

E27-16 The magnitude of the electric field is $E = mg/q$. The surface charge density on the plates is $\sigma = \epsilon_0 E = \epsilon_0 mg/q$, or

$$\sigma = \frac{(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(9.11\times10^{-31}\mathrm{kg})(9.81\,\mathrm{m/s}^2)}{(1.60\times10^{-19}\mathrm{C})} = 4.94\times10^{-22}\mathrm{C/m}^2.$$

$\boxed{\textbf{E27-17}}$ We don't really need to write an integral, we just need the charge per unit length in the cylinder to be equal to zero. This means that the positive charge in cylinder must be $+3.60n\mathrm{C/m}$. This positive charge is uniformly distributed in a circle of radius $R = 1.50$ cm, so

$$\rho = \frac{3.60n\mathrm{C/m}}{\pi R^2} = \frac{3.60n\mathrm{C/m}}{\pi(0.0150\,\mathrm{m})^2} = 5.09\mu\mathrm{C/m}^3.$$

E27-18 The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\rm enc}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = 4\pi r^2 E.$$

(a) For point P_1 the charge enclosed is $q_{\rm enc} = 1.26\times10^{-7}$C, so

$$E = \frac{(1.26\times10^{-7}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.83\times10^{-2}\mathrm{m})^2} = 3.38\times10^6\mathrm{N/C}.$$

(b) Inside a conductor $E = 0$.

$\boxed{\textbf{E27-19}}$ The proton orbits with a speed v, so the centripetal force on the proton is $F_C = mv^2/r$. This centripetal force is from the electrostatic attraction with the sphere; so long as the proton is outside the sphere the electric field is equivalent to that of a point charge Q (Eq. 27-15),

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}.$$

If q is the charge on the proton we can write $F = Eq$, or

$$\frac{mv^2}{r} = q\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

Solving for Q,

$$\begin{aligned} Q &= \frac{4\pi\epsilon_0 mv^2 r}{q}, \\ &= \frac{4\pi(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.67\times10^{-27}\mathrm{kg})(294\times10^3\mathrm{m/s})^2(0.0113\,\mathrm{m})}{(1.60\times10^{-19}\mathrm{C})}, \\ &= -1.13\times10^{-9}\mathrm{C}. \end{aligned}$$

E27-20 The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\rm enc}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = 4\pi r^2 E.$$

(a) At $r = 0.120\,\mathrm{m}$ $q_{\rm enc} = 4.06\times10^{-8}$C. Then

$$E = \frac{(4.06\times10^{-8}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.20\times10^{-1}\mathrm{m})^2} = 2.54\times10^4\mathrm{N/C}.$$

(b) At $r = 0.220\,\mathrm{m}$ $q_{\rm enc} = 5.99\times10^{-8}$C. Then

$$E = \frac{(5.99\times10^{-8}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(2.20\times10^{-1}\mathrm{m})^2} = 1.11\times10^4\mathrm{N/C}.$$

(c) At $r = 0.0818\,\mathrm{m}$ $q_{\rm enc} = 0\,\mathrm{C}$. Then $E = 0$.

E27-21 The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\text{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E\,dA = E\int dA = 2\pi rLE,$$

where L is the length of the cylinder. Note that $\sigma = q/2\pi rL$ represents a surface charge density.

(a) $r = 0.0410\,\text{m}$ is between the two cylinders. Then

$$E = \frac{(24.1\times10^{-6}\text{C/m}^2)(0.0322\,\text{m})}{(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.0410\,\text{m})} = 2.14\times10^6\text{N/C}.$$

It points outward.

(b) $r = 0.0820\,\text{m}$ is outside the two cylinders. Then

$$E = \frac{(24.1\times10^{-6}\text{C/m}^2)(0.0322\,\text{m}) + (-18.0\times10^{-6}\text{C/m}^2)(0.0618\,\text{m})}{(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.0820\,\text{m})} = -4.64\times10^5\text{N/C}.$$

The negative sign is because it is pointing inward.

E27-22 The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\text{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E\,dA = E\int dA = 2\pi rLE,$$

where L is the length of the cylinder. The charge enclosed is

$$q_{\text{enc}} = \int \rho dV = \rho\pi L\left(r^2 - R^2\right)$$

The electric field is given by

$$E = \frac{\rho\pi L\left(r^2 - R^2\right)}{2\pi\epsilon_0 rL} = \frac{\rho\left(r^2 - R^2\right)}{2\epsilon_0 r}.$$

At the surface,

$$E_{\text{s}} = \frac{\rho\left((2R)^2 - R^2\right)}{2\epsilon_0 2R} = \frac{3\rho R}{4\epsilon_0}.$$

Solve for r when E is half of this:

$$\begin{aligned}\frac{3R}{8} &= \frac{r^2 - R^2}{2r},\\ 3rR &= 4r^2 - 4R^2,\\ 0 &= 4r^2 - 3rR - 4R^2.\end{aligned}$$

The solution is $r = 1.443R$. That's $(2R - 1.443R) = 0.557R$ beneath the surface.

E27-23 The electric field must do work on the electron to stop it. The electric field is given by $E = \sigma/2\epsilon_0$. The work done is $W = Fd = Eqd$. d is the distance in question, so

$$d = \frac{2\epsilon_0 K}{\sigma q} = \frac{2(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(1.15\times10^5\ \text{eV})}{(2.08\times10^{-6}\text{C/m}^2)e} = 0.979\,\text{m}$$

E27-24 Let the spherical Gaussian surface have a radius of R and be centered on the origin. Choose the orientation of the axis so that the infinite line of charge is along the z axis. The electric field is then directed radially outward from the z axis with magnitude $E = \lambda/2\pi\epsilon_0\rho$, where ρ is the perpendicular distance from the z axis. Now we want to evaluate

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}},$$

over the surface of the sphere. In spherical coordinates, $dA = R^2 \sin\theta \, d\theta \, d\phi$, $\rho = R\sin\theta$, and $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA\sin\theta$. Then

$$\Phi_E = \oint \frac{\lambda}{2\pi\epsilon_0} \sin\theta R \, d\theta \, d\phi = \frac{2\lambda R}{\epsilon_0}.$$

$\boxed{\textbf{E27-25}}$ (a) The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\text{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E \, dA = E \int dA = 2\pi r L E,$$

where L is the length of the cylinder. Now for the q_{enc} part. If the (uniform) volume charge density is ρ, then the charge enclosed in the Gaussian cylinder is

$$q_{\text{enc}} = \int \rho dV = \rho \int dV = \rho V = \pi r^2 L \rho.$$

Combining, $\pi r^2 L\rho/\epsilon_0 = E2\pi rL$ or $E = \rho r/2\epsilon_0$.

(b) Outside the charged cylinder the charge enclosed in the Gaussian surface is just the charge in the cylinder. Then

$$q_{\text{enc}} = \int \rho dV = \rho \int dV = \rho V = \pi R^2 L \rho.$$

and

$$\pi R^2 L\rho/\epsilon_0 = E2\pi rL,$$

and then finally

$$E = \frac{R^2 \rho}{2\epsilon_0 r}.$$

E27-26 (a) $q = 4\pi(1.22\,\text{m})^2(8.13\times10^{-6}\text{C/m}^2) = 1.52\times10^{-4}\text{C}$.

(b) $\Phi_E = q/\epsilon_0 = (1.52\times10^{-4}\text{C})/(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2) = 1.72\times10^{7}\text{N}\cdot\text{m}^2/\text{C}$.

(c) $E = \sigma/\epsilon_0 = (8.13\times10^{-6}\text{C/m}^2)/(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2) = 9.19\times10^{5}\text{N/C}$

E27-27 (a) $\sigma = (2.4\times10^{-6}\text{C})/4\pi(0.65\,\text{m})^2 = 4.52\times10^{-7}\text{C/m}^2$.

(b) $E = \sigma/\epsilon_0 = (4.52\times10^{-7}\text{C/m}^2)/(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2) = 5.11\times10^{4}\text{N/C}$.

E27-28 $E = \sigma/\epsilon_0 = q/4\pi r^2\epsilon_0$.

$\boxed{\textbf{E27-29}}$ (a) The near field is given by Eq. 27-12, $E = \sigma/2\epsilon_0$, so

$$E \approx \frac{(6.0\times10^{-6}\text{C})/(8.0\times10^{-2}\,\text{m})^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = 5.3\times10^{7}\text{N/C}.$$

(b) Very far from *any* object a point charge approximation is valid. Then

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\frac{(6.0\times10^{-6}\text{C})}{(30\,\text{m})^2} = 60\text{N/C}.$$

P27-1 For a spherically symmetric mass distribution choose a spherical Gaussian shell. Then

$$\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = \oint g\, dA = g \oint dA = 4\pi r^2 g.$$

Then

$$\frac{\Phi_g}{4\pi G} = \frac{gr^2}{G} = -m,$$

or

$$g = -\frac{Gm}{r^2}.$$

The negative sign indicates the direction; $\vec{\mathbf{g}}$ point toward the mass center.

P27-2 (a) The flux through all surfaces *except* the right and left faces will be zero. Through the left face,

$$\Phi_\mathrm{l} = -E_y A = -b\sqrt{a}a^2.$$

Through the right face,

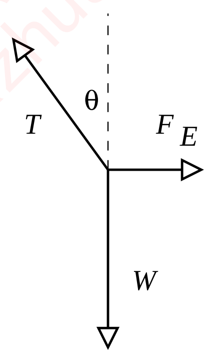
$$\Phi_\mathrm{r} = E_y A = b\sqrt{2a}a^2.$$

The net flux is then

$$\Phi = ba^{5/2}(\sqrt{2}-1) = (8830\,\mathrm{N/C\cdot m^{1/2}})(0.130\,\mathrm{m})^{5/2}(\sqrt{2}-1) = 22.3\,\mathrm{N\cdot m^2/C}.$$

(b) The charge enclosed is $q = (8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(22.3\,\mathrm{N\cdot m^2/C}) = 1.97\times10^{-10}\mathrm{C}$.

$\boxed{\textbf{P27-3}}$ The net force on the small sphere is zero; this force is the vector sum of the force of gravity W, the electric force F_E, and the tension T.



These forces are related by $Eq = mg\tan\theta$. We also have $E = \sigma/2\epsilon_0$, so

$$\begin{aligned}
\sigma &= \frac{2\epsilon_0 mg\tan\theta}{q}, \\
&= \frac{2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.12\times10^{-6}\mathrm{kg})(9.81\,\mathrm{m/s^2})\tan(27.4°)}{(19.7\times10^{-9}\mathrm{C})}, \\
&= 5.11\times10^{-9}\mathrm{C/m^2}.
\end{aligned}$$

P27-4 The materials are conducting, so *all* charge will reside on the surfaces. The electric field inside any conductor is zero. The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\mathrm{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = 4\pi r^2 E.$$

Consequently, $E = q_{\mathrm{enc}}/4\pi\epsilon_0 r^2$.

(a) Within the sphere $E = 0$.

(b) Between the sphere and the shell $q_{\mathrm{enc}} = q$. Then $E = q/4\pi\epsilon_0 r^2$.

(c) Within the shell $E = 0$.

(d) Outside the shell $q_{\mathrm{enc}} = +q - q = 0$. Then $E = 0$.

(e) Since $E = 0$ inside the shell, $q_{\mathrm{enc}} = 0$, this requires that $-q$ reside on the inside surface. This is no charge on the outside surface.

P27-5 The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\mathrm{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E\, dA = E \int dA = 2\pi r L E,$$

where L is the length of the cylinder. Consequently, $E = q_{\mathrm{enc}}/2\pi\epsilon_0 r L$.

(a) Outside the conducting shell $q_{\mathrm{enc}} = +q - 2q = -q$. Then $E = -q/2\pi\epsilon_0 r L$. The negative sign indicates that the field is pointing inward toward the axis of the cylinder.

(b) Since $E = 0$ inside the conducting shell, $q_{\mathrm{enc}} = 0$, which means a charge of $-q$ is on the inside surface of the shell. The remaining $-q$ must reside on the outside surface of the shell.

(c) In the region between the cylinders $q_{\mathrm{enc}} = +q$. Then $E = +q/2\pi\epsilon_0 r L$. The positive sign indicates that the field is pointing outward from the axis of the cylinder.

P27-6 Subtract Eq. 26-19 from Eq. 26-20. Then

$$E = \frac{\sigma}{2\epsilon_0} \frac{z}{\sqrt{z^2 + R^2}}.$$

$\boxed{\textbf{P27-7}}$ This problem is closely related to Ex. 27-25, except for the part concerning q_{enc}. We'll set up the problem the same way: the Gaussian surface will be a (imaginary) cylinder centered on the axis of the physical cylinder. For Gaussian surfaces of radius $r < R$, there is *no* charge enclosed while for Gaussian surfaces of radius $r > R$, $q_{\mathrm{enc}} = \lambda l$.

We've already worked out the integral

$$\int_{\mathrm{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi r l E,$$

for the cylinder, and then from Gauss' law,

$$q_{\mathrm{enc}} = \epsilon_0 \int_{\mathrm{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi\epsilon_0 r l E.$$

(a) When $r < R$ there is no enclosed charge, so the left hand vanishes and consequently $E = 0$ inside the physical cylinder.

(b) When $r > R$ there is a charge λl enclosed, so

$$E = \frac{\lambda}{2\pi\epsilon_0 r}.$$

P27-8 This problem is closely related to Ex. 27-25, except for the part concerning q_{enc}. We'll set up the problem the same way: the Gaussian surface will be a (imaginary) cylinder centered on the axis of the physical cylinders. For Gaussian surfaces of radius $r < a$, there is *no* charge enclosed while for Gaussian surfaces of radius $b > r > a$, $q_{\text{enc}} = \lambda l$.

We've already worked out the integral

$$\int_{\text{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi r l E,$$

for the cylinder, and then from Gauss' law,

$$q_{\text{enc}} = \epsilon_0 \int_{\text{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi\epsilon_0 r l E.$$

(a) When $r < a$ there is no enclosed charge, so the left hand vanishes and consequently $E = 0$ inside the inner cylinder.

(b) When $b > r > a$ there is a charge λl enclosed, so

$$E = \frac{\lambda}{2\pi\epsilon_0 r}.$$

P27-9 Uniform circular orbits require a constant net force towards the center, so $F = Eq = \lambda q/2\pi\epsilon_0 r$. The speed of the positron is given by $F = mv^2/r$; the kinetic energy is $K = mv^2/2 = Fr/2$. Combining,

$$\begin{aligned} K &= \frac{\lambda q}{4\pi\epsilon_0}, \\ &= \frac{(30\times10^{-9}\text{C/m})(1.6\times10^{-19}\text{C})}{4\pi((8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}, \\ &= 4.31\times10^{-17}\,\text{J} = 270\text{ eV}. \end{aligned}$$

P27-10 $\lambda = 2\pi\epsilon_0 r E$, so

$$q = 2\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.014\,\text{m})(0.16\,\text{m})(2.9\times10^4\text{N/C}) = 3.6\times10^{-9}\text{C}.$$

$\boxed{\textbf{P27-11}}$ (a) Put a spherical Gaussian surface inside the shell centered on the point charge. Gauss' law states

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\epsilon_0}.$$

Since there is spherical symmetry the electric field is normal to the spherical Gaussian surface, and it is everywhere the same on this surface. The dot product simplifies to $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\,dA$, while since E is a constant on the surface we can pull it out of the integral, and we end up with

$$E \oint dA = \frac{q}{\epsilon_0},$$

where q is the point charge in the center. Now $\oint dA = 4\pi r^2$, where r is the radius of the Gaussian surface, so

$$E = \frac{q}{4\pi\epsilon_0 r^2}.$$

(b) Repeat the above steps, except put the Gaussian surface outside the conducting shell. Keep it centered on the charge. Two things are different from the above derivation: (1) r is bigger, and

(2) there is an uncharged spherical conducting shell inside the Gaussian surface. Neither change will affect the surface integral or q_{enc}, so the electric field outside the shell is still

$$E = \frac{q}{4\pi\epsilon_0 r^2},$$

(c) This is a subtle question. With all the symmetry here it appears as if the shell has no effect; the field just looks like a point charge field. If, however, the charge were moved off center the field inside the shell would become distorted, and we wouldn't be able to use Gauss' law to find it. So the shell does make a difference.

Outside the shell, however, we can't tell what is going on inside the shell. So the electric field outside the shell looks like a point charge field originating from the center of the shell *regardless of where inside the shell the point charge is placed!*

(d) Yes, q induces surface charges on the shell. There will be a charge $-q$ on the inside surface and a charge q on the outside surface.

(e) Yes, as there is an electric field from the shell, isn't there?

(f) No, as the electric field from the outside charge won't make it through a conducting shell. The conductor acts as a shield.

(g) No, this is not a contradiction, because the outside charge never experienced any electrostatic attraction or repulsion from the inside charge. The force is between the shell and the outside charge.

P27-12 The repulsive electrostatic forces must exactly balance the attractive gravitational forces. Then

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = G\frac{m^2}{r^2},$$

or $m = q/\sqrt{4\pi\epsilon_0 G}$. Numerically,

$$m = \frac{(1.60\times10^{-19}\text{C})}{\sqrt{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)}} = 1.86\times10^{-9}\text{kg}.$$

P27-13 The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\text{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \oint E\,dA = E\oint dA = 4\pi r^2 E.$$

Consequently, $E = q_{\text{enc}}/4\pi\epsilon_0 r^2$.

$q_{\text{enc}} = q + 4\pi\int \rho\, r^2 dr$, or

$$q_{\text{enc}} = q + 4\pi\int_a^r Ar\,dr = q + 2\pi A(r^2 - a^2).$$

The electric field will be constant if q_{enc} behaves as r^2, which requires $q = 2\pi A a^2$, or $A = q/2\pi a^2$.

P27-14 (a) The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\text{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \oint E\,dA = E\oint dA = 4\pi r^2 E.$$

Consequently, $E = q_{\text{enc}}/4\pi\epsilon_0 r^2$.

$q_{\text{enc}} = 4\pi\int \rho\, r^2 dr = 4\pi\rho r^3/3$, so

$$E = \rho r/3\epsilon_0$$

and is directed radially out from the center. Then $\vec{\mathbf{E}} = \rho\vec{\mathbf{r}}/3\epsilon_0$.

(b) The electric field in the hole is given by $\vec{\mathbf{E}}_h = \vec{\mathbf{E}} - \vec{\mathbf{E}}_b$, where $\vec{\mathbf{E}}$ is the field from part (a) and $\vec{\mathbf{E}}_b$ is the field that would be produced by the matter that would have been in the hole had the hole not been there. Then

$$\vec{\mathbf{E}}_b = \rho\vec{\mathbf{b}}/3\epsilon_0,$$

where $\vec{\mathbf{b}}$ is a vector pointing from the center of the hole. Then

$$\vec{\mathbf{E}}_h = \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} - \frac{\rho\vec{\mathbf{b}}}{3\epsilon_0} = \frac{\rho}{3\epsilon_0}(\vec{\mathbf{r}} - \vec{\mathbf{b}}).$$

But $\vec{\mathbf{r}} - \vec{\mathbf{b}} = \vec{\mathbf{a}}$, so $\vec{\mathbf{E}}_h = \rho\vec{\mathbf{a}}/3\epsilon_0$.

$\boxed{\textbf{P27-15}}$ If a point is an equilibrium point then the electric field at that point should be zero. If it is a stable point then moving the test charge (assumed positive) a small distance from the equilibrium point should result in a restoring force directed back toward the equilibrium point. In other words, there will be a point where the electric field is zero, and around this point there will be an electric field pointing inward. Applying Gauss' law to a small surface surrounding our point P, we have a net inward flux, so there must be a negative charge *inside* the surface. But there should be nothing inside the surface except an empty point P, so we have a contradiction.

P27-16 (a) Follow the example on Page 618. By symmetry $E = 0$ along the median plane. The charge enclosed between the median plane and a surface a distance x from the plane is $q = \rho A x$. Then

$$E = \rho A x/\epsilon_0 A = \rho A/\epsilon_0.$$

(b) Outside the slab the charge enclosed between the median plane and a surface a distance x from the plane is is $q = \rho A d/2$, regardless of x. The

$$E = \rho A d/2/\epsilon_0 A = \rho d/2\epsilon_0.$$

$\boxed{\textbf{P27-17}}$ (a) The total charge is the volume integral over the whole sphere,

$$Q = \int \rho \, dV.$$

This is actually a three dimensional integral, and $dV = A\,dr$, where $A = 4\pi r^2$. Then

$$\begin{aligned} Q &= \int \rho\, dV, \\ &= \int_0^R \left(\frac{\rho_S r}{R}\right) 4\pi r^2\, dr, \\ &= \frac{4\pi\rho_S}{R}\frac{1}{4}R^4, \\ &= \pi\rho_S R^3. \end{aligned}$$

(b) Put a spherical Gaussian surface inside the sphere centered on the center. We can use Gauss' law here because there is spherical symmetry in the entire problem, both inside and outside the Gaussian surface. Gauss' law states

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\epsilon_0}.$$

Since there is spherical symmetry the electric field is normal to the spherical Gaussian surface, and it is everywhere the same on this surface. The dot product simplifies to $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\,dA$, while since E is a constant on the surface we can pull it out of the integral, and we end up with

$$E \oint dA = \frac{q_{\text{enc}}}{\epsilon_0},$$

Now $\oint dA = 4\pi r^2$, where r is the radius of the Gaussian surface, so

$$E = \frac{q_{\text{enc}}}{4\pi\epsilon_0 r^2}.$$

We aren't done yet, because the charge enclosed depends on the radius of the Gaussian surface. We need to do part (a) again, except this time we don't want to do the whole volume of the sphere, we only want to go out as far as the Gaussian surface. Then

$$\begin{aligned} q_{\text{enc}} &= \int \rho\, dV, \\ &= \int_0^r \left(\frac{\rho_S r}{R}\right) 4\pi r^2\, dr, \\ &= \frac{4\pi\rho_S}{R}\frac{1}{4}r^4, \\ &= \pi\rho_S \frac{r^4}{R}. \end{aligned}$$

Combine these last two results and

$$\begin{aligned} E &= \frac{\pi\rho_S}{4\pi\epsilon_0 r^2}\frac{r^4}{R}, \\ &= \frac{\pi\rho_S}{4\pi\epsilon_0}\frac{r^2}{R}, \\ &= \frac{Q}{4\pi\epsilon_0}\frac{r^2}{R^4}. \end{aligned}$$

In the last line we used the results of part (a) to eliminate ρ_S from the expression.

P27-18 (a) Inside the conductor $E = 0$, so a Gaussian surface which is embedded in the conductor but containing the hole must have a net enclosed charge of zero. The cavity wall must then have a charge of $-3.0\,\mu$C.

(b) The net charge on the conductor is $+10.0\,\mu$C; the charge on the outer surface must then be $+13.0\,\mu$C.

P27-19 (a) Inside the shell $E = 0$, so the net charge inside a Gaussian surface embedded in the shell must be zero, so the inside surface has a charge $-Q$.

(b) Still $-Q$; the outside has nothing to do with the inside.

(c) $-(Q + q)$; see reason (a).

(d) Yes.

Throughout this chapter we will use the convention that $V(\infty) = 0$ unless explicitly stated otherwise. Then the potential in the vicinity of a point charge will be given by Eq. 28-18, $V = q/4\pi\epsilon_0 r$.

E28-1 (a) Let U_{12} be the potential energy of the interaction between the two "up" quarks. Then

$$U_{12} = (8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)\frac{(2/3)^2 e(1.60\times10^{-19}\mathrm{C})}{(1.32\times10^{-15}\mathrm{m})} = 4.84\times10^5\mathrm{eV}.$$

(b) Let U_{13} be the potential energy of the interaction between an "up" quark and a "down" quark. Then

$$U_{13} = (8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)\frac{(-1/3)(2/3)e(1.60\times10^{-19}\mathrm{C})}{(1.32\times10^{-15}\mathrm{m})} = -2.42\times10^5\mathrm{eV}$$

Note that $U_{13} = U_{23}$. The total electric potential energy is the sum of these three terms, or zero.

E28-2 There are six interaction terms, one for every charge pair. Number the charges clockwise from the upper left hand corner. Then

$$\begin{aligned}
U_{12} &= -q^2/4\pi\epsilon_0 a,\\
U_{23} &= -q^2/4\pi\epsilon_0 a,\\
U_{34} &= -q^2/4\pi\epsilon_0 a,\\
U_{41} &= -q^2/4\pi\epsilon_0 a,\\
U_{13} &= (-q)^2/4\pi\epsilon_0(\sqrt{2}a),\\
U_{24} &= q^2/4\pi\epsilon_0(\sqrt{2}a).
\end{aligned}$$

Add these terms and get

$$U = \left(\frac{2}{\sqrt{2}} - 4\right)\frac{q^2}{4\pi\epsilon_0 a} = -0.206\frac{q^2}{\epsilon_0 a}$$

The amount of work required is $W = U$.

$\boxed{\textbf{E28-3}}$ (a) We build the electron one part at a time; each part has a charge $q = e/3$. Moving the first part from infinity to the location where we want to construct the electron is easy and takes no work at all. Moving the second part in requires work to change the potential energy to

$$U_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r},$$

which is basically Eq. 28-7. The separation $r = 2.82 \times 10^{-15}\,\mathrm{m}$.

Bringing in the third part requires work against the force of repulsion between the third charge and both of the other two charges. Potential energy then exists in the form U_{13} and U_{23}, where all three charges are the same, and all three separations are the same. Then $U_{12} = U_{13} = U_{12}$, so the total potential energy of the system is

$$U = 3\frac{1}{4\pi\epsilon_0}\frac{(e/3)^2}{r} = \frac{3}{4\pi(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)}\frac{(1.60\times10^{-19}\,\mathrm{C}/3)^2}{(2.82\times10^{-15}\,\mathrm{m})} = 2.72\times10^{-14}\,\mathrm{J}$$

(b) Dividing our answer by the speed of light squared to find the mass,

$$m = \frac{2.72\times10^{-14}\,\mathrm{J}}{(3.00\times10^8\,\mathrm{m/s})^2} = 3.02\times10^{-31}\,\mathrm{kg}.$$

E28-4 There are three interaction terms, one for every charge pair. Number the charges from the left; let $a = 0.146\,\text{m}$. Then

$$\begin{aligned}
U_{12} &= \frac{(25.5\times10^{-9}\text{C})(17.2\times10^{-9}\text{C})}{4\pi\epsilon_0 a},\\
U_{13} &= \frac{(25.5\times10^{-9}\text{C})(-19.2\times10^{-9}\text{C})}{4\pi\epsilon_0 (a+x)},\\
U_{23} &= \frac{(17.2\times10^{-9}\text{C})(-19.2\times10^{-9}\text{C})}{4\pi\epsilon_0 x}.
\end{aligned}$$

Add these and set it equal to zero. Then

$$\frac{(25.5)(17.2)}{a} = \frac{(25.5)(19.2)}{a+x} + \frac{(17.2)(19.2)}{x},$$

which has solution $x = 1.405a = 0.205\,\text{m}$.

E28-5 The volume of the nuclear material is $4\pi a^3/3$, where $a = 8.0\times10^{-15}\text{m}$. Upon dividing in half each part will have a radius r where $4\pi r^3/3 = 4\pi a^3/6$. Consequently, $r = a/\sqrt[3]{2} = 6.35\times10^{-15}\text{m}$. Each fragment will have a charge of $+46e$.

(a) The force of repulsion is

$$F = \frac{(46)^2(1.60\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)[2(6.35\times10^{-15}\text{m})]^2} = 3000\,\text{N}$$

(b) The potential energy is

$$U = \frac{(46)^2 e(1.60\times10^{-19}\text{C})}{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)2(6.35\times10^{-15}\text{m})} = 2.4\times10^8\text{eV}$$

E28-6 This is a work/kinetic energy problem: $\frac{1}{2}mv_0^2 = q\Delta V$. Then

$$v_0 = \sqrt{\frac{2(1.60\times10^{-19}\text{C})(10.3\times10^3\text{V})}{(9.11\times10^{-31}\text{kg})}} = 6.0\times10^7\text{m/s}.$$

$\boxed{\textbf{E28-7}}$ (a) The energy released is equal to the charges times the potential through which the charge was moved. Then

$$\Delta U = q\Delta V = (30\,\text{C})(1.0\times10^9\,\text{V}) = 3.0\times10^{10}\,\text{J}.$$

(b) Although the problem mentions acceleration, we want to focus on energy. The energy will change the kinetic energy of the car from 0 to $K_\text{f} = 3.0\times10^{10}\,\text{J}$. The speed of the car is then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.0\times10^{10}\,\text{J})}{(1200\,\text{kg})}} = 7100\,\text{m/s}.$$

(c) The energy required to melt ice is given by $Q = mL$, where L is the latent heat of fusion. Then

$$m = \frac{Q}{L} = \frac{(3.0\times10^{10}\,\text{J})}{(3.33\times10^5\text{J/kg})} = 90,100\text{kg}.$$

E28-8 (a) $\Delta U = (1.60\times10^{-19}\text{C})(1.23\times10^{9}\text{V}) = 1.97\times10^{-10}\text{J}$.
(b) $\Delta U = e(1.23\times10^{9}\text{V}) = 1.23\times10^{9}\text{eV}$.

E28-9 This is an energy conservation problem: $\frac{1}{2}mv^2 = q\Delta V$; $\Delta V = q/4\pi\epsilon_0(1/r_1 - 1/r_2)$. Combining,

$$\begin{aligned}
v &= \sqrt{\frac{q^2}{2\pi\epsilon_0 m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}, \\
&= \sqrt{\frac{(3.1\times10^{-6}\text{C})^2}{2\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(18\times10^{-6}\text{kg})}\left(\frac{1}{(0.90\times10^{-3}\text{m})}-\frac{1}{(2.5\times10^{-3}\text{m})}\right)}, \\
&= 2600\,\text{m/s}.
\end{aligned}$$

E28-10 This is an energy conservation problem:

$$\frac{1}{2}m(2v)^2 - \frac{q^2}{4\pi\epsilon_0 r} = \frac{1}{2}mv^2.$$

Rearrange,

$$\begin{aligned}
r &= \frac{q^2}{6\pi\epsilon_0 mv^2}, \\
&= \frac{(1.60\times10^{-19}\text{C})^2}{6\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(9.11\times10^{-31}\text{kg})(3.44\times10^{5}\text{m/s})^2)} = 1.42\times10^{-9}\text{m}.
\end{aligned}$$

E28-11 (a) $V = (1.60\times10^{-19}\text{C})/4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(5.29\times10^{-11}\text{m}) = 27.2\,\text{V}$.
(b) $U = qV = (-e)(27.2\,\text{V}) = -27.2$ eV.
(c) For uniform circular orbits $F = mv^2/r$; the force is electrical, or $F = e^2/4\pi\epsilon_0 r^2$. Kinetic energy is $K = mv^2/2 = Fr/2$, so

$$K = \frac{e^2}{8\pi\epsilon_0 r} = \frac{(1.60\times10^{-19}\text{C})}{8\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(5.29\times10^{-11}\text{m})} = 13.6\text{ eV}.$$

(d) The ionization energy is $-(K+U)$, or

$$E_{\text{ion}} = -[(13.6\text{ eV}) + (-27.2\text{ eV})] = 13.6\text{ eV}.$$

E28-12 (a) The electric potential at A is

$$V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right) = (8.99\times10^{9}\text{N}\cdot\text{m}^2/\text{C})\left(\frac{(-5.0\times10^{-6}\text{C})}{(0.15\,\text{m})}+\frac{(2.0\times10^{-6}\text{C})}{(0.05\,\text{m})}\right) = 6.0\times10^{4}\text{V}.$$

The electric potential at B is

$$V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_2}+\frac{q_2}{r_1}\right) = (8.99\times10^{9}\text{N}\cdot\text{m}^2/\text{C})\left(\frac{(-5.0\times10^{-6}\text{C})}{(0.05\,\text{m})}+\frac{(2.0\times10^{-6}\text{C})}{(0.15\,\text{m})}\right) = -7.8\times10^{5}\text{V}.$$

(b) $W = q\Delta V = (3.0\times10^{-6}\text{C})(6.0\times10^{4}\text{V} - -7.8\times10^{5}\text{V}) = 2.5\,\text{J}$.
(c) Since work is positive then external work is converted to electrostatic potential energy.

E28-13 (a) The magnitude of the electric field would be found from

$$E = \frac{F}{q} = \frac{(3.90 \times 10^{-15}\,\mathrm{N})}{(1.60 \times 10^{-19}\,\mathrm{C})} = 2.44 \times 10^4\,\mathrm{N/C}.$$

(b) The potential difference between the plates is found by evaluating Eq. 28-15,

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}.$$

The electric field between two parallel plates is uniform and perpendicular to the plates. Then $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E\,ds$ along this path, and since E is uniform,

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_a^b E\,ds = -E\int_a^b ds = E\Delta x,$$

where Δx is the separation between the plates. Finally, $\Delta V = (2.44 \times 10^4\,\mathrm{N/C})(0.120\,\mathrm{m}) = 2930\,\mathrm{V}$.

E28-14 $\Delta V = E\Delta x$, so

$$\Delta x = \frac{2\epsilon_0}{\sigma}\Delta V = \frac{2(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)}{(0.12\times10^{-6}\mathrm{C/m}^2)}(48\,\mathrm{V}) = 7.1\times10^{-3}\mathrm{m}$$

E28-15 The electric field around an infinitely long straight wire is given by $E = \lambda/2\pi\epsilon_0 r$. The potential difference between the inner wire and the outer cylinder is given by

$$\Delta V = -\int_a^b (\lambda/2\pi\epsilon_0 r)\,dr = (\lambda/2\pi\epsilon_0)\ln(a/b).$$

The electric field near the surface of the wire is then given by

$$E = \frac{\lambda}{2\pi\epsilon_0 a} = \frac{\Delta V}{a\ln(a/b)} = \frac{(-855\,\mathrm{V})}{(6.70\times10^{-7}\mathrm{m})\ln(6.70\times10^{-7}\,\mathrm{m}/1.05\times10^{-2}\,\mathrm{m})} = 1.32\times10^8\mathrm{V/m}.$$

The electric field near the surface of the cylinder is then given by

$$E = \frac{\lambda}{2\pi\epsilon_0 a} = \frac{\Delta V}{a\ln(a/b)} = \frac{(-855\,\mathrm{V})}{(1.05\times10^{-2}\mathrm{m})\ln(6.70\times10^{-7}\,\mathrm{m}/1.05\times10^{-2}\,\mathrm{m})} = 8.43\times10^3\mathrm{V/m}.$$

E28-16 $\Delta V = E\Delta x = (1.92\times10^5\mathrm{N/C})(1.50\times10^{-2}\mathrm{m}) = 2.88\times10^3\mathrm{V}$.

E28-17 (a) This is an energy conservation problem:

$$K = \frac{1}{4\pi\epsilon_0}\frac{(2)(79)e^2}{r} = (8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C})\frac{(2)(79)e(1.60\times10^{-19}\mathrm{C})}{(7.0\times10^{-15}\mathrm{m})} = 3.2\times10^7\ \mathrm{eV}$$

(b) The alpha particles used by Rutherford never came close to hitting the gold nuclei.

E28-18 This is an energy conservation problem: $mv^2/2 = eq/4\pi\epsilon_0 r$, or

$$v = \sqrt{\frac{(1.60\times10^{-19}\mathrm{C})(1.76\times10^{-15}\mathrm{C})}{2\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.22\times10^{-2}\mathrm{m})(9.11\times10^{-31}\mathrm{kg})}} = 2.13\times10^4\mathrm{m/s}$$

E28-19 (a) We evaluate V_A and V_B individually, and then find the difference.

$$V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(1.16\mu\mathrm{C})}{(2.06\,\mathrm{m})} = 5060\,\mathrm{V},$$

and

$$V_B = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(1.16\mu\mathrm{C})}{(1.17\,\mathrm{m})} = 8910\,\mathrm{V},$$

The difference is then $V_A - V_B = -3850\,\mathrm{V}$.

(b) The answer is the same, since when concerning ourselves with electric potential we only care about distances, and not directions.

E28-20 The number of "excess" electrons on each grain is

$$n = \frac{4\pi\epsilon_0 rV}{e} = \frac{4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m})(1.0\times10^{-6}\mathrm{m})(-400\,\mathrm{V})}{(-1.60\times10^{-19}\mathrm{C})} = 2.8\times10^5$$

E28-21 The excess charge on the shuttle is

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m})(10\,\mathrm{m})(-1.0\,\mathrm{V}) = -1.1\times10^{-9}\mathrm{C}$$

E28-22 $q = 1.37\times10^5\mathrm{C}$, so

$$V = (8.99\times10^9\mathrm{N\cdot m^2/C^2})\frac{(1.37\times10^5\mathrm{C})}{(6.37\times10^6\mathrm{m})} = 1.93\times10^8\mathrm{V}.$$

E28-23 The ratio of the electric potential to the electric field strength is

$$\frac{V}{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right) / \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right) = r.$$

In this problem r is the radius of the Earth, so at the surface of the Earth the potential is

$$V = Er = (100\,\mathrm{V/m})(6.38\times10^6\mathrm{m}) = 6.38\times10^8\,\mathrm{V}.$$

E28-24 Use Eq. 28-22:

$$V = (8.99\times10^9\mathrm{N\cdot m^2/C^2})\frac{(1.47)(3.34\times10^{-30}\mathrm{C\cdot m})}{(52.0\times10^{-9}\mathrm{m})^2} = 1.63\times10^{-5}\mathrm{V}.$$

E28-25 (a) When finding V_A we need to consider the contribution from both the positive and the negative charge, so

$$V_A = \frac{1}{4\pi\epsilon_0}\left(qa + \frac{-q}{a+d}\right)$$

There will be a similar expression for V_B,

$$V_B = \frac{1}{4\pi\epsilon_0}\left(-qa + \frac{q}{a+d}\right).$$

Now to evaluate the difference.

$$\begin{aligned}
V_A - V_B &= \frac{1}{4\pi\epsilon_0}\left(qa + \frac{-q}{a+d}\right) - \frac{1}{4\pi\epsilon_0}\left(-qa + \frac{q}{a+d}\right), \\
&= \frac{q}{2\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{a+d}\right), \\
&= \frac{q}{2\pi\epsilon_0}\left(\frac{a+d}{a(a+d)} - \frac{a}{a(a+d)}\right), \\
&= \frac{q}{2\pi\epsilon_0}\frac{d}{a(a+d)}.
\end{aligned}$$

(b) Does it do what we expect when $d = 0$? I expect it the difference to go to zero as the two points A and B get closer together. The numerator will go to zero as d gets smaller. The denominator, however, stays finite, which is a good thing. So yes, $V_a - V_B \to 0$ as $d \to 0$.

E28-26 (a) Since both charges are positive the electric potential from both charges will be positive. There will be no *finite* points where $V = 0$, since two positives can't add to zero.

(b) Between the charges the electric field from each charge points toward the other, so $\vec{\mathbf{E}}$ will vanish when $q/x^2 = 2q/(d-x)^2$. This happens when $d - x = \sqrt{2}x$, or $x = d/(1+\sqrt{2})$.

E28-27 The distance from C to either charge is $\sqrt{2}d/2 = 1.39\times 10^{-2}\text{m}$.

(a) V at C is

$$V = (8.99\times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2(2.13\times 10^{-6}\text{C})}{(1.39\times 10^{-2}\text{m})} = 2.76\times 10^6\text{V}$$

(b) $W = q\delta V = (1.91\times 10^{-6}\text{C})(2.76\times 10^6\text{V}) = 5.27\,\text{J}$.

(c) Don't forget about the potential energy of the original two charges!

$$U_0 = (8.99\times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.13\times 10^{-6}\text{C})^2}{(1.96\times 10^{-2}\text{m})} = 2.08\,\text{J}$$

Add this to the answer from part (b) to get 7.35 J.

E28-28 The potential is given by Eq. 28-32; at the surface $V_s = \sigma R/2\epsilon_0$, half of this occurs when

$$\begin{aligned}
\sqrt{R^2+z^2} - z &= R/2, \\
R^2 + z^2 &= R^2/4 + Rz + z^2, \\
3R/4 &= z.
\end{aligned}$$

$\boxed{\textbf{E28-29}}$ We can find the linear charge density by dividing the charge by the circumference,

$$\lambda = \frac{Q}{2\pi R},$$

where Q refers to the charge on the ring. The work done to move a charge q from a point x to the origin will be given by

$$\begin{aligned}
W &= q\Delta V, \\
W &= q\left(V(0) - V(x)\right), \\
&= q\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2}} - \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+x^2}}\right), \\
&= \frac{qQ}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+x^2}}\right).
\end{aligned}$$

Putting in the numbers,

$$\frac{(-5.93\times10^{-12}\mathrm{C})(-9.12\times10^{-9}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)}\left(\frac{1}{1.48\mathrm{m}}-\frac{1}{\sqrt{(1.48\mathrm{m})^2+(3.07\mathrm{m})^2}}\right)=1.86\times10^{-10}\mathrm{J}.$$

E28-30 (a) The electric field strength is greatest where the gradient of V is greatest. That is between d and e.

(b) The least absolute value occurs where the gradient is zero, which is between b and c and again between e and f.

E28-31 The potential on the positive plate is $2(5.52\,\mathrm{V}) = 11.0\,\mathrm{V}$; the electric field between the plates is $E = (11.0\,\mathrm{V})/(1.48\times10^{-2}\mathrm{m}) = 743\,\mathrm{V/m}$.

E28-32 Take the derivative: $E = -\partial V/\partial z$.

$\boxed{\textbf{E28-33}}$ The radial potential gradient is just the magnitude of the radial component of the electric field,

$$E_r = -\frac{\partial V}{\partial r}$$

Then

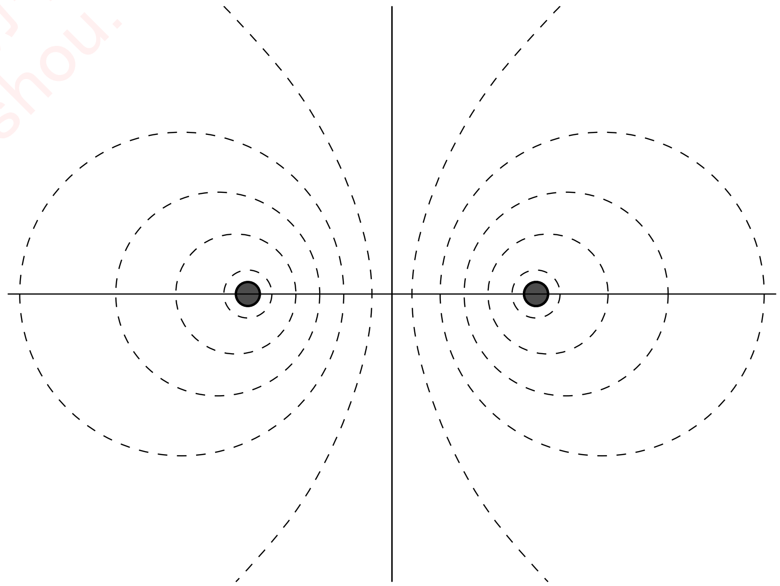
$$\begin{aligned}\frac{\partial V}{\partial r} &= -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2},\\ &= \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)}\frac{79(1.60\times10^{-19}\mathrm{C})}{(7.0\times10^{-15}\mathrm{m})^2},\\ &= -2.32\times10^{21}\,\mathrm{V/m}.\end{aligned}$$

E28-34 Evaluate $\partial V/\partial r$, and

$$E = -\frac{Ze}{4\pi\epsilon_0}\left(\frac{-1}{r^2}+2\frac{r}{2R^3}\right).$$

E28-35 $E_x = -\partial V/\partial x = -2(1530\,\mathrm{V/m}^2)x$. At the point in question, $E = -2(1530\,\mathrm{V/m}^2)(1.28\times10^{-2}\mathrm{m}) = 39.2\,\mathrm{V/m}$.

E28-36 Draw the wires so that they are perpendicular to the plane of the page; they will then “come out of” the page. The equipotential surfaces are then lines where they intersect the page, and they look like



$\boxed{\textbf{E28-37}}$ (a) $|V_B - V_A| = |W/q| = |(3.94 \times 10^{-19}\,\mathrm{J})/(1.60 \times 10^{-19}\,\mathrm{C})| = 2.46\,\mathrm{V}$. The electric field did work on the electron, so the electron was moving from a region of low potential to a region of high potential; or $V_B > V_A$. Consequently, $V_B - V_A = 2.46\,\mathrm{V}$.

(b) V_C is at the same potential as V_B (both points are on the same equipotential line), so $V_C - V_A = V_B - V_A = 2.46\,\mathrm{V}$.

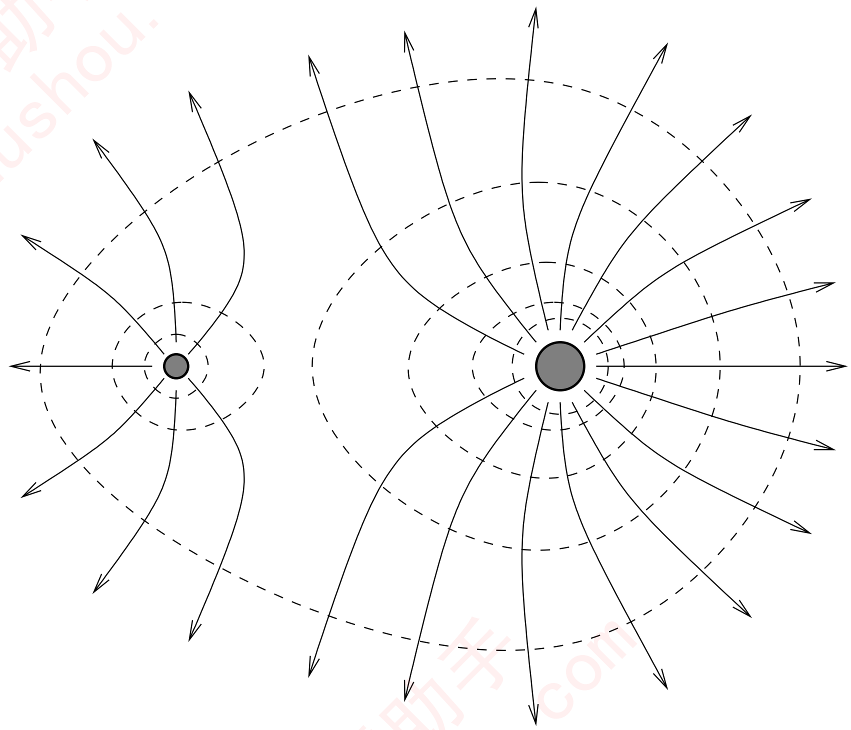
(c) V_C is at the same potential as V_B (both points are on the same equipotential line), so $V_C - V_B = 0\,\mathrm{V}$.

E28-38 (a) For point charges $r = q/4\pi\epsilon_0 V$, so

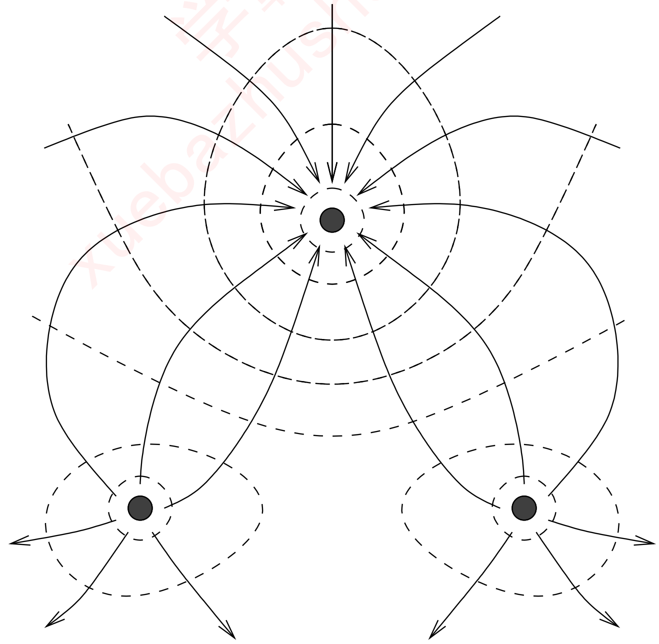
$$r = (8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.5\times10^{-8}\mathrm{C})/(30\,\mathrm{V}) = 4.5\,\mathrm{m}$$

(b) No, since $V \propto 1/r$.

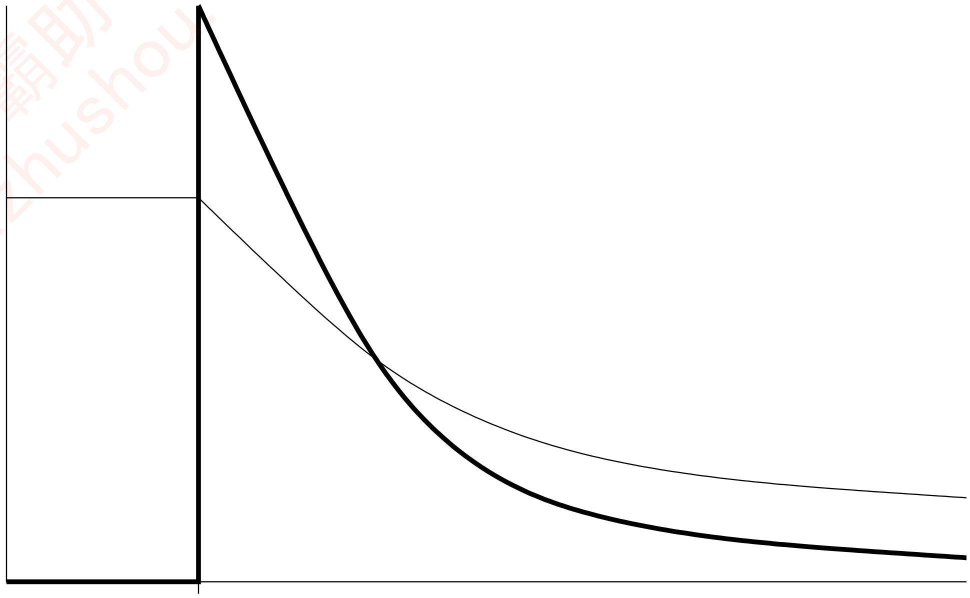
E28-39 The dotted lines are equipotential lines, the solid arrows are electric field lines. Note that there are twice as many electric field lines from the larger charge!



E28-40 The dotted lines are equipotential lines, the solid arrows are electric field lines.



E28-41 This can easily be done with a spreadsheet. The following is a *sketch*; the electric field is the bold curve, the potential is the thin curve.



E28-42 Originally $V = q/4\pi\epsilon_0 r$, where r is the radius of the smaller sphere.

(a) Connecting the spheres will bring them to the same potential, or $V_1 = V_2$.

(b) $q_1 + q_2 = q$; $V_1 = q_1/4\pi\epsilon_0 r$ and $V_2 = q_2/4\pi\epsilon_0 2r$; combining all of the above $q_2 = 2q_1$ and $q_1 = q/3$ and $q_2 = 2q/3$.

E28-43 (a) $q = 4\pi R^2\sigma$, so $V = q/4\pi\epsilon_0 R = \sigma R/\epsilon_0$, or

$$V = (-1.60\times10^{-19}\mathrm{C/m^2})(6.37\times10^6\mathrm{m})/(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2}) = 0.115\,\mathrm{V}$$

(b) Pretend the Earth is a conductor, then $E = \sigma/epsilon_0$, so

$$E = (-1.60\times10^{-19}\mathrm{C/m^2})/(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2}) = 1.81\times10^{-8}\mathrm{V/m}.$$

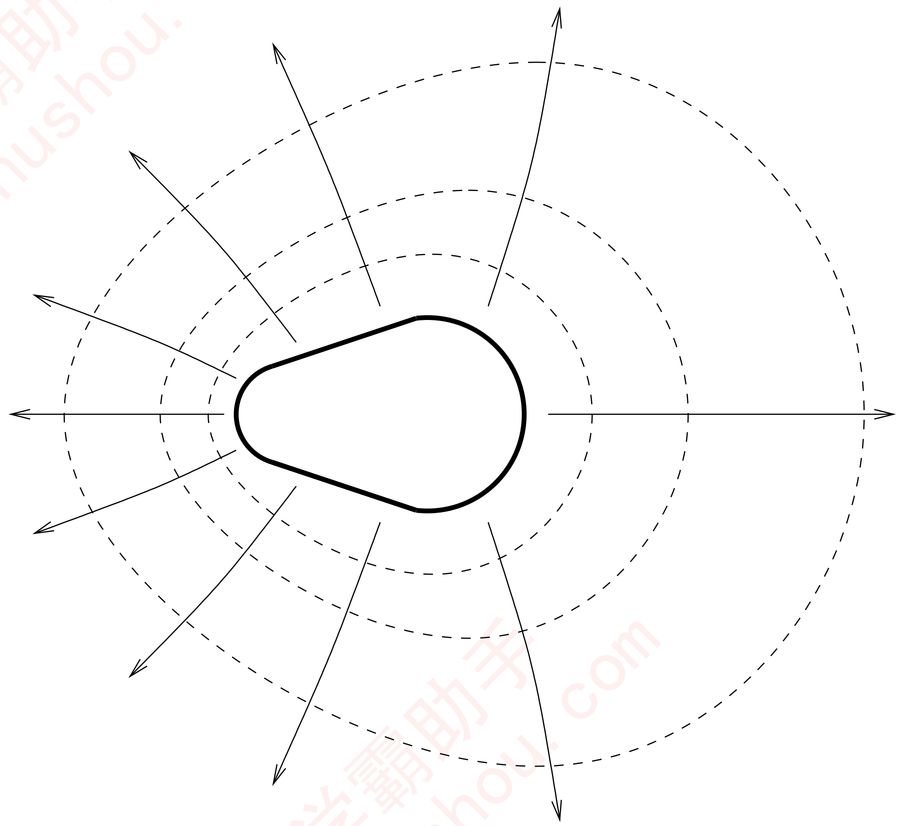
E28-44 $V = q/4\pi\epsilon_0 R$, so

$$V = (8.99\times10^9\mathrm{N\cdot m^2/C^2})(15\times10^{-9}\mathrm{C})/(0.16\,\mathrm{m}) = 850\,\mathrm{V}.$$

E28-45 (a) $q = 4\pi\epsilon_0 RV = 4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.152\,\mathrm{m})(215\,\mathrm{V}) = 3.63\times10^{-9}\mathrm{C}$

(b) $\sigma = q/4\pi R^2 = (3.63\times10^{-9}\mathrm{C})/4\pi(0.152\,\mathrm{m})^2 = 1.25\times10^{-8}\mathrm{C/m^2}$.

E28-46 The dotted lines are equipotential lines, the solid arrows are electric field lines.



$\boxed{\textbf{E28-47}}$ (a) The total charge ($Q = 57.2$nC) will be divided up between the two spheres so that they are at the same potential. If q_1 is the charge on one sphere, then $q_2 = Q - q_1$ is the charge on the other. Consequently

$$\begin{aligned} V_1 &= V_2, \\ \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} &= \frac{1}{4\pi\epsilon_0}\frac{Q-q_1}{r_2}, \\ q_1 r_2 &= (Q-q_1)r_1, \\ q_1 &= \frac{Qr_2}{r_2+r_1}. \end{aligned}$$

Putting in the numbers, we find

$$q_1 = \frac{Qr_1}{r_2+r_1} = \frac{(57.2\text{ nC})(12.2\text{ cm})}{(5.88\text{ cm}) + (12.2\text{ cm})} = 38.6\text{ nC},$$

and $q_2 = Q - q_1 = (57.2\text{ nC}) - (38.6\text{ nC}) = 18.6\text{ nC}$.

(b) The potential on each sphere should be the same, so we only need to solve one. Then

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\frac{(38.6\text{ nC})}{(12.2\text{ cm})} = 2850\,\text{V}.$$

E28-48 (a) $V = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(31.5\times10^{-9}\text{C})/(0.162\,\text{m}) = 1.75\times10^3\text{V}$.

(b) $V = q/4\pi\epsilon_0 r$, so $r = q/4\pi\epsilon_0 V$, and then

$$r = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(31.5\times10^{-9}\text{C})/(1.20\times10^3\text{V}) = 0.236\,\text{m}.$$

That is $(0.236\,\text{m}) - (0.162\,\text{m}) = 0.074\,\text{m}$ above the surface.

E28-49 (a) Apply the point charge formula, but solve for the charge. Then

$$\begin{aligned} \frac{1}{4\pi\epsilon_0}\frac{q}{r} &= V, \\ q &= 4\pi\epsilon_0 rV, \\ q &= 4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(1\,\text{m})(10^6\,\text{V}) = 0.11\,\text{mC}. \end{aligned}$$

Now that's a fairly small charge. But if the radius were decreased by a factor of 100, so would the charge ($1.10\,\mu$C). Consequently, smaller metal balls can be raised to higher potentials with less charge.

(b) The electric field near the surface of the ball is a function of the surface charge density, $E = \sigma/\epsilon_0$. But surface charge density depends on the area, and varies as r^{-2}. For a given potential, the electric field near the surface would then be given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 r^2} = \frac{V}{r}.$$

Note that the electric field grows as the ball gets smaller. This means that the break down field is more likely to be exceeded with a low voltage small ball; you'll get sparking.

E28-50 A "Volt" is a Joule per Coulomb. The power required by the drive belt is the product $(3.41\times10^6\text{V})(2.83\times10^{-3}\text{C/s}) = 9650\,\text{W}$.

P28-1 (a) According to Newtonian mechanics we want $K = \frac{1}{2}mv^2$ to be equal to $W = q\Delta V$ which means

$$\Delta V = \frac{mv^2}{2q} = \frac{(0.511\text{ MeV})}{2e} = 256\text{ kV}.$$

mc^2 is the rest mass energy of an electron.

(b) Let's do some rearranging first.

$$\begin{aligned} K &= mc^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right], \\ \frac{K}{mc^2} &= \frac{1}{\sqrt{1-\beta^2}} - 1, \\ \frac{K}{mc^2} + 1 &= \frac{1}{\sqrt{1-\beta^2}}, \\ \frac{1}{\frac{K}{mc^2}+1} &= \sqrt{1-\beta^2}, \\ \frac{1}{\left(\frac{K}{mc^2}+1\right)^2} &= 1-\beta^2, \end{aligned}$$

and finally,

$$\beta = \sqrt{1 - \frac{1}{\left(\frac{K}{mc^2}+1\right)^2}}$$

Putting in the numbers,

$$\sqrt{1 - \frac{1}{\left(\frac{(256\text{ keV})}{(511\text{ keV})}+1\right)^2}} = 0.746,$$

so $v = 0.746c$.

P28-2 (a) The potential of the hollow sphere is $V = q/4\pi\epsilon_0 r$. The work required to increase the charge by an amount dq is $dW = V/, dq$. Integrating,

$$W = \int_0^e \frac{q}{4\pi\epsilon_0 r} dq = \frac{e^2}{8\pi\epsilon_0 r}.$$

This corresponds to an electric potential energy of

$$W = \frac{e(1.60\times 10^{-19}\mathrm{C})}{8\pi(8.85\times 10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(2.82\times 10^{-15}\mathrm{m})} = 2.55\times 10^5 \text{ eV} = 4.08\times 10^{-14}\mathrm{J}.$$

(b) This would be a mass of $m = (4.08\times 10^{-14}\mathrm{J})/(3.00\times 10^8\mathrm{m/s})^2 = 4.53\times 10^{-31}\mathrm{kg}$.

P28-3 The negative charge is held in orbit by electrostatic attraction, or

$$\frac{mv^2}{r} = \frac{qQ}{4\pi\epsilon_0 r^2}.$$

The kinetic energy of the charge is

$$K = \frac{1}{2}mv^2 = \frac{qQ}{8\pi\epsilon_0 r}.$$

The electrostatic potential energy is

$$U = -\frac{qQ}{4\pi\epsilon_0 r},$$

so the total energy is

$$E = -\frac{qQ}{8\pi\epsilon_0 r}.$$

The work required to change orbit is then

$$W = \frac{qQ}{8\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

P28-4 (a) $V = -\int E\,dr$, so

$$V = -\int_0^r \frac{qr}{4\pi\epsilon_0 R^3} dr = -\frac{qr^2}{8\pi\epsilon_0 R^3}.$$

(b) $\Delta V = q/8\pi\epsilon_0 R$.

(c) If instead of $V = 0$ at $r = 0$ as was done in part (a) we take $V = 0$ at $r = \infty$, then $V = q/4\pi\epsilon_0 R$ on the surface of the sphere. The new expression for the potential inside the sphere will look like $V = V' + V_s$, where V' is the answer from part (a) and V_s is a constant so that the surface potential is correct. Then

$$V_s = \frac{q}{4\pi\epsilon_0 R} + \frac{qR^2}{8\pi\epsilon_0 R^3} = \frac{3qR^2}{8\pi\epsilon_0 R^3},$$

and then

$$V = -\frac{qr^2}{8\pi\epsilon_0 R^3} + \frac{3qR^2}{8\pi\epsilon_0 R^3} = \frac{q(3R^2 - r^2)}{8\pi\epsilon_0 R^3}.$$

P28-5 The total electric potential energy of the system is the sum of the three interaction pairs. One of these pairs does not change during the process, so it can be ignored when finding the change in potential energy. The change in electrical potential energy is then

$$\Delta U = 2\frac{q^2}{4\pi\epsilon_0 r_\mathrm{f}} - 2\frac{q^2}{4\pi\epsilon_0 r_\mathrm{i}} = \frac{q^2}{2\pi\epsilon_0}\left(\frac{1}{r_\mathrm{f}} - \frac{1}{r_\mathrm{i}}\right).$$

In this case $r_\mathrm{i} = 1.72\,\mathrm{m}$, while $r_\mathrm{f} = 0.86\,\mathrm{m}$. The change in potential energy is then

$$\Delta U = 2(8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(0.122\,\mathrm{C})^2\left(\frac{1}{(0.86\,\mathrm{m})} - \frac{1}{(1.72\,\mathrm{m})}\right) = 1.56\times10^8\mathrm{J}$$

The time required is

$$t = (1.56\times10^8)/(831\,\mathrm{W}) = 1.87\times10^5\mathrm{s} = 2.17\ \mathrm{days}.$$

P28-6 (a) Apply conservation of energy:

$$K = \frac{qQ}{4\pi\epsilon_0 d},\ \text{or}\ d = \frac{qQ}{4\pi\epsilon_0 K},$$

where d is the distance of closest approach.

(b) Apply conservation of energy:

$$K = \frac{qQ}{4\pi\epsilon_0(2d)} + \frac{1}{2}mv^2,$$

so, combining with the results in part (a), $v = \sqrt{K/m}$.

$\boxed{\textbf{P28-7}}$ (a) First apply Eq. 28-18, but solve for r. Then

$$r = \frac{q}{4\pi\epsilon_0 V} = \frac{(32.0\times10^{-12}\,\mathrm{C})}{4\pi(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(512\,\mathrm{V})} = 562\,\mu\mathrm{m}.$$

(b) If two such drops join together the charge doubles, and the volume of water doubles, but the radius of the new drop only increases by a factor of $\sqrt[3]{2} = 1.26$ because volume is proportional to the radius cubed.

The potential on the surface of the new drop will be

$$\begin{aligned} V_\mathrm{new} &= \frac{1}{4\pi\epsilon_0}\frac{q_\mathrm{new}}{r_\mathrm{new}}, \\ &= \frac{1}{4\pi\epsilon_0}\frac{2q_\mathrm{old}}{\sqrt[3]{2}\,r_\mathrm{old}}, \\ &= (2)^{2/3}\frac{1}{4\pi\epsilon_0}\frac{q_\mathrm{old}}{r_\mathrm{old}} = (2)^{2/3}V_\mathrm{old}. \end{aligned}$$

The new potential is 813 V.

P28-8 (a) The work done is $W = -Fz = -Eqz = -q\sigma z/2\epsilon_0$.

(b) Since $W = q\Delta V$, $\Delta V = -\sigma z/2\epsilon_0$, so

$$V = V_0 - (\sigma/2\epsilon_0)z.$$

$\boxed{\textbf{P28-9}}$ (a) The potential at any point will be the sum of the contribution from each charge,

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2},$$

where r_1 is the distance the point in question from q_1 and r_2 is the distance the point in question from q_2. Pick a point, call it (x, y). Since q_1 is at the origin,

$$r_1 = \sqrt{x^2 + y^2}.$$

Since q_2 is at $(d, 0)$, where $d = 9.60$ nm,

$$r_2 = \sqrt{(x-d)^2 + y^2}.$$

Define the "Stanley Number" as $S = 4\pi\epsilon_0 V$. Equipotential surfaces are also equi-Stanley surfaces. In particular, when $V = 0$, so does S. We can then write the potential expression in a sightly simplified form

$$S = \frac{q_1}{r_1} + \frac{q_2}{r_2}.$$

If $S = 0$ we can rearrange and square this expression.

$$\begin{aligned} \frac{q_1}{r_1} &= -\frac{q_2}{r_2}, \\ \frac{r_1^2}{q_1^2} &= \frac{r_2^2}{q_2^2}, \\ \frac{x^2+y^2}{q_1^2} &= \frac{(x-d)^2+y^2}{q_2^2}, \end{aligned}$$

Let $\alpha = q_2/q_1$, then we can write

$$\begin{aligned} \alpha^2\left(x^2+y^2\right) &= (x-d)^2 + y^2, \\ \alpha^2 x^2 + \alpha^2 y^2 &= x^2 - 2xd + d^2 + y^2, \\ (\alpha^2-1)x^2 + 2xd + (\alpha^2-1)y^2 &= d^2. \end{aligned}$$

We complete the square for the $(\alpha^2-1)x^2 + 2xd$ term by adding $d^2/(\alpha^2-1)$ to both sides of the equation. Then

$$(\alpha^2-1)\left[\left(x + \frac{d}{\alpha^2-1}\right)^2 + y^2\right] = d^2\left(1 + \frac{1}{\alpha^2-1}\right).$$

The center of the circle is at

$$-\frac{d}{\alpha^2-1} = \frac{(9.60\text{ nm})}{(-10/6)^2-1} = -5.4\text{ nm}.$$

(b) The radius of the circle is

$$\sqrt{d^2\frac{\left(1+\frac{1}{\alpha^2-1}\right)}{\alpha^2-1}},$$

which can be simplified to

$$d\frac{\alpha}{\alpha^2-1} = (9.6\text{ nm})\frac{|(-10/6)|}{(-10/6)^2-1} = 9.00\text{ nm}.$$

(c) No.

P28-10 An annulus is composed of differential rings of varying radii r and width dr; the charge on any ring is the product of the area of the ring, $dA = 2\pi r\,dr$, and the surface charge density, or

$$dq = \sigma\,dA = \frac{k}{r^3}2\pi r\,dr = \frac{2\pi k}{r^2}dr.$$

The potential at the center can be found by adding up the contributions from each ring. Since we are at the center, the contributions will each be $dV = dq/4\pi\epsilon_0 r$. Then

$$V = \int_a^b \frac{k}{2\epsilon_0}\frac{dr}{r^3} = \frac{k}{4\epsilon_0}\left(\frac{1}{a^2} - \frac{1}{b^2}\right). = \frac{k}{4\epsilon_0}\frac{b^2-a^2}{b^2a^2}.$$

The total charge on the annulus is

$$Q = \int_a^b \frac{2\pi k}{r^2}dr = 2\pi k\left(\frac{1}{a} - \frac{1}{b}\right) = 2\pi k\frac{b-a}{ba}.$$

Combining,

$$V = \frac{Q}{8\pi\epsilon_0}\frac{a+b}{ab}.$$

P28-11 Add the three contributions, and then do a series expansion for $d \ll r$.

$$\begin{aligned}
V &= \frac{q}{4\pi\epsilon_0}\left(\frac{-1}{r+d} + \frac{1}{r} + \frac{1}{r-d}\right),\\
&= \frac{q}{4\pi\epsilon_0 r}\left(\frac{-1}{1+d/r} + 1 + \frac{1}{1-d/r}\right),\\
&\approx \frac{q}{4\pi\epsilon_0 r}\left(-1 + \frac{d}{r} + 1 + 1 + \frac{d}{r}\right),\\
&\approx \frac{q}{4\pi\epsilon_0 r}\left(1 + \frac{2d}{r}\right).
\end{aligned}$$

P28-12 (a) Add the contributions from each differential charge: $dq = \lambda\,dy$. Then

$$V = \int_y^{y+L}\frac{\lambda}{4\pi\epsilon_0 y}dy = \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{y+L}{y}\right).$$

(b) Take the derivative:

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\lambda}{4\pi\epsilon_0}\frac{y}{y+L}\frac{-L}{y^2} = \frac{\lambda}{4\pi\epsilon_0}\frac{L}{y(y+L)}.$$

(c) By symmetry it must be zero, since the system is invariant under rotations about the axis of the rod. Note that we can't determine $E_\perp$ from derivatives because we don't have the functional form of V for points off-axis!

$\boxed{\textbf{P28-13}}$ (a) We follow the work done in Section 28-6 for a uniform line of charge, starting with Eq. 28-26,

$$\begin{aligned}
dV &= \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dx}{\sqrt{x^2+y^2}},\\
dV &= \frac{1}{4\pi\epsilon_0}\int_0^L\frac{kx\,dx}{\sqrt{x^2+y^2}},
\end{aligned}$$

$$
\begin{aligned}
&= \frac{k}{4\pi\epsilon_0} \left. \sqrt{x^2+y^2} \right|_0^L, \\
&= \frac{k}{4\pi\epsilon_0} \left(\sqrt{L^2+y^2} - y \right).
\end{aligned}
$$

(b) The y component of the electric field can be found from

$$E_y = -\frac{\partial V}{\partial y},$$

which (using a computer-aided math program) is

$$E_y = \frac{k}{4\pi\epsilon_0} \left(1 - \frac{y}{\sqrt{L^2+y^2}} \right).$$

(c) We could find E_x if we knew the x variation of V. But we don't; we only found the values of V along a fixed value of x.

(d) We want to find y such that the ratio

$$\left[\frac{k}{4\pi\epsilon_0} \left(\sqrt{L^2+y^2} - y \right) \right] / \left[\frac{k}{4\pi\epsilon_0} (L) \right]$$

is one-half. Simplifying, $\sqrt{L^2+y^2} - y = L/2$, which can be written as

$$L^2 + y^2 = L^2/4 + Ly + y^2,$$

or $3L^2/4 = Ly$, with solution $y = 3L/4$.

P28-14 The spheres are small compared to the separation distance. Assuming only *one* sphere at a potential of 1500 V, the charge would be

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N\cdot m})(0.150\,\mathrm{m})(1500\,\mathrm{V}) = 2.50\times10^{-8}\mathrm{C}.$$

The potential from the sphere at a distance of 10.0 m would be

$$V = (1500\,\mathrm{V})\frac{(0.150\,\mathrm{m})}{(10.0\,\mathrm{m})} = 22.5\,\mathrm{V}.$$

This is small compared to 1500 V, so we will treat it as a perturbation. This means that we can assume that the spheres have charges of

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N\cdot m})(0.150\,\mathrm{m})(1500\,\mathrm{V} + 22.5\,\mathrm{V}) = 2.54\times10^{-8}\mathrm{C}.$$

$\boxed{\textbf{P28-15}}$ Calculating the fraction of excess electrons is the same as calculating the fraction of excess charge, so we'll skip counting the electrons. This problem is effectively the same as Exercise 28-47; we have a total charge that is divided between two unequal size spheres which are at the same potential on the surface. Using the result from that exercise we have

$$q_1 = \frac{Qr_1}{r_2 + r_1},$$

where $Q = -6.2$ nC is the total charge available, and q_1 is the charge left on the sphere. r_1 is the radius of the small ball, r_2 is the radius of Earth. Since the fraction of charge remaining is q_1/Q, we can write

$$\frac{q_1}{Q} = \frac{r_1}{r_2 + r_1} \approx \frac{r_1}{r_2} = 2.0 \times 10^{-8}.$$

P28-16 The positive charge on the sphere would be

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(1.08\times10^{-2}\text{m})(1000\,\text{V}) = 1.20\times10^{-9}\text{C}.$$

The number of decays required to build up this charge is

$$n = 2(1.20\times10^{-9}\text{C})/(1.60\times10^{-19}\text{C}) = 1.50\times10^{10}.$$

The extra factor of two is because only half of the decays result in an increase in charge. The time required is

$$t = (1.50\times10^{10})/(3.70\times10^{8}\text{s}^{-1}) = 40.6\,\text{s}.$$

P28-17 (a) None.
(b) None.
(c) None.
(d) None.
(e) No.

P28-18 (a) Outside of an isolated charged spherical object $E = q/4\pi\epsilon_0 r^2$ and $V = q/4\pi\epsilon_0 r$. Then $E = V/r$. Consequently, the sphere must have a radius larger than $r = (9.15\times10^6\text{V})/(100\times10^6\text{V/m}) = 9.15\times10^{-2}\text{m}$.
(b) The power required is $(320\times10^{-6}\text{C/s})(9.15\times10^6\text{V}) = 2930\,\text{W}$.
(c) $\sigma w v = (320\times10^{-6}\text{C/s})$, so

$$\sigma = \frac{(320\times10^{-6}\text{C/s})}{(0.485\,\text{m})(33.0\,\text{m/s})} = 2.00\times10^{-5}\text{C/m}^2.$$

$\boxed{\textbf{E29-1}}$ (a) The charge which flows through a cross sectional surface area in a time t is given by $q = it$, where i is the current. For this exercise we have

$$q = (4.82\,\mathrm{A})(4.60 \times 60\,\mathrm{s}) = 1330\,\mathrm{C}$$

as the charge which passes through a cross section of this resistor.

(b) The number of electrons is given by $(1330\,\mathrm{C})/(1.60 \times 10^{-19}\,\mathrm{C}) = 8.31 \times 10^{21}$ electrons.

E29-2 $Q/t = (200\times10^{-6}\mathrm{A/s})(60\mathrm{s/min})/(1.60\times10^{-19}\mathrm{C}) = 7.5\times10^{16}$ electrons per minute.

E29-3 (a) $j = nqv = (2.10\times10^{14}/\mathrm{m}^3)2(1.60\times10^{-19}\mathrm{C})(1.40\times10^5\mathrm{m/s}) = 9.41\,\mathrm{A/m}^2$. Since the ions have positive charge then the current density is in the same direction as the velocity.

(b) We need an area to calculate the current.

E29-4 (a) $j = i/A = (123\times10^{-12}\mathrm{A})/\pi(1.23\times10^{-3}\mathrm{m})^2 = 2.59\times10^{-5}\mathrm{A/m}^2$.

(b) $v_\mathrm{d} = j/ne = (2.59\times10^{-5}\mathrm{A/m}^2)/(8.49\times10^{28}/\mathrm{m}^3)(1.60\times10^{-19}\mathrm{C}) = 1.91\times10^{-15}\mathrm{m/s}$.

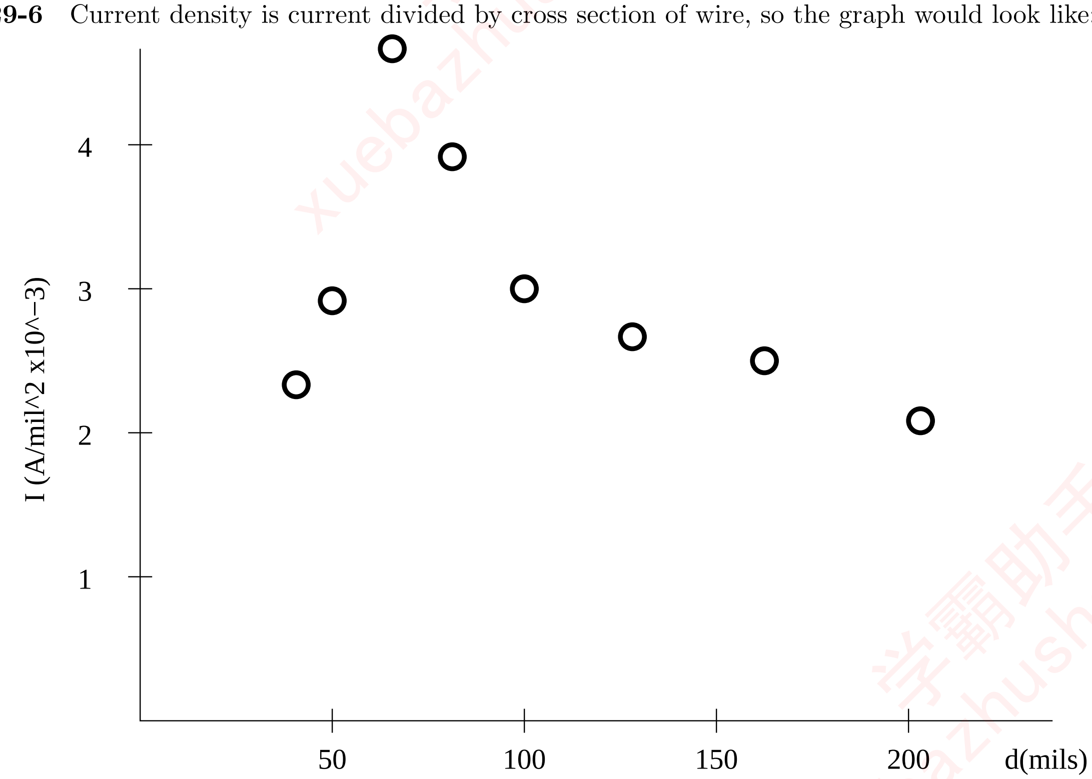
$\boxed{\textbf{E29-5}}$ The current rating of a fuse of cross sectional area A would be

$$i_\mathrm{max} = (440\,\mathrm{A/cm}^2)A,$$

and if the fuse wire is cylindrical $A = \pi d^2/4$. Then

$$d = \sqrt{\frac{4}{\pi}\frac{(0.552\,\mathrm{A})}{(440\,\mathrm{A/m}^2)}} = 4.00\times10^{-2}\ \mathrm{cm}.$$

E29-6 Current density is current divided by cross section of wire, so the graph would look like:



E29-7 The current is in the direction of the motion of the positive charges. The magnitude of the current is

$$i = (3.1\times10^{18}/\text{s} + 1.1\times10^{18}/\text{s})(1.60\times10^{-19}\text{C}) = 0.672\,\text{A}.$$

E29-8 (a) The total current is

$$i = (3.50\times10^{15}/\text{s} + 2.25\times10^{15}/\text{s})(1.60\times10^{-19}\text{C}) = 9.20\times10^{-4}\text{A}.$$

(b) The current density is

$$j = (9.20\times10^{-4}\text{A})/\pi(0.165\times10^{-3}\text{m})^2 = 1.08\times10^4\text{A/m}^2.$$

E29-9 (a) $j = (8.70\times10^6/\text{m}^3)(1.60\times10^{-19}\text{C})(470\times10^3\text{m/s}) = 6.54\times10^{-7}\text{A/m}^2$.
(b) $i = (6.54\times10^{-7}\text{A/m}^2)\pi(6.37\times10^6\text{m})^2 = 8.34\times10^7\text{A}$.

E29-10 $i = \sigma w v$, so

$$\sigma = (95.0\times10^{-6}\text{A})/(0.520\,\text{m})(28.0\,\text{m/s}) = 6.52\times10^{-6}\text{C/m}^2.$$

$\boxed{\textbf{E29-11}}$ The drift velocity is given by Eq. 29-6,

$$v_\text{d} = \frac{j}{ne} = \frac{i}{Ane} = \frac{(115\,\text{A})}{(31.2\times10^{-6}\text{m}^2)(8.49\times10^{28}/m^3)(1.60\times10^{-19}\text{C})} = 2.71\times10^{-4}\text{m/s}.$$

The time it takes for the electrons to get to the starter motor is

$$t = \frac{x}{v} = \frac{(0.855\,\text{m})}{(2.71\times10^{-4}\text{m/s})} = 3.26\times10^3\text{s}.$$

That's about 54 minutes.

E29-12 $\Delta V = iR = (50\times10^{-3}\text{A})(1800\,\Omega) = 90\,\text{V}$.

$\boxed{\textbf{E29-13}}$ The resistance of an object with constant cross section is given by Eq. 29-13,

$$R = \rho\frac{L}{A} = (3.0\times10^{-7}\,\Omega\cdot\text{m})\frac{(11,000\,\text{m})}{(0.0056\,\text{m}^2)} = 0.59\,\Omega.$$

E29-14 The slope is approximately $[(8.2-1.7)/1000]\mu\Omega\cdot\text{cm}/^\circ\text{C}$, so

$$\alpha = \frac{1}{1.7\mu\Omega\cdot\text{cm}}6.5\times10^{-3}\mu\Omega\cdot\text{cm}/^\circ\text{C} \approx 4\times10^{-3}/\text{C}^\circ$$

E29-15 (a) $i = \Delta V/R = (23\,\text{V})/(15\times10^{-3}\Omega) = 1500\,\text{A}$.
(b) $j = i/A = (1500\,\text{A})/\pi(3.0\times10^{-3}\text{m})^2 = 5.3\times10^7\text{A/m}^2$.
(c) $\rho = RA/L = (15\times10^{-3}\Omega)\pi(3.0\times10^{-3}\text{m})^2/(4.0\,\text{m}) = 1.1\times10^{-7}\Omega\cdot\text{m}$. The material is possibly platinum.

E29-16 Use the equation from Exercise 29-17. $\Delta R = 8\,\Omega$; then

$$\Delta T = (8\,\Omega)/(50\,\Omega)(4.3\times10^{-3}/\text{C}^\circ) = 37\,\text{C}^\circ.$$

The final temperature is then 57°C.

$\boxed{\textbf{E29-17}}$ Start with Eq. 29-16,

$$\rho - \rho_0 = \rho_0 \alpha_{\rm av}(T - T_0),$$

and multiply through by L/A,

$$\frac{L}{A}(\rho - \rho_0) = \frac{L}{A}\rho_0 \alpha_{\rm av}(T - T_0),$$

to get

$$R - R_0 = R_0 \alpha_{\rm av}(T - T_0).$$

E29-18 The wire has a length $L = (250)2\pi(0.122\,\text{m}) = 192\,\text{m}$. The diameter is 0.129 inches; the cross sectional area is then

$$A = \pi(0.129 \times 0.0254\,\text{m})^2/4 = 8.43\times10^{-6}\text{m}^2.$$

The resistance is

$$R = \rho L/A = (1.69\times10^{-8}\Omega\cdot\text{m})(192\,\text{m})/(8.43\times10^{-6}\text{m}^2) = 0.385\,\Omega.$$

$\boxed{\textbf{E29-19}}$ If the length of each conductor is L and has resistivity ρ, then

$$R_A = \rho\frac{L}{\pi D^2/4} = \rho\frac{4L}{\pi D^2}$$

and

$$R_B = \rho\frac{L}{(\pi 4D^2/4 - \pi D^2/4)} = \rho\frac{4L}{3\pi D^2}.$$

The ratio of the resistances is then

$$\frac{R_A}{R_B} = 3.$$

E29-20 $R = R$, so $\rho_1 L_1/\pi(d_1/2)^2 = \rho_2 L_2/\pi(d_2/2)^2$. Simplifying, $\rho_1/d_1^2 = \rho_2/d_2^2$. Then

$$d_2 = (1.19\times10^{-3}\text{m})\sqrt{(9.68\times10^{-8}\Omega\cdot\text{m})/(1.69\times10^{-8}\Omega\cdot\text{m})} = 2.85\times10^{-3}\text{m}.$$

E29-21 (a) $(750\times10^{-3}\text{A})/(125) = 6.00\times10^{-3}\text{A}$.
(b) $\Delta V = iR = (6.00\times10^{-3}\text{A})(2.65\times10^{-6}\Omega) = 1.59\times10^{-8}\text{V}$.
(c) $R = \Delta V/i = (1.59\times10^{-8}\text{V})/(750\times10^{-3}\text{A}) = 2.12\times10^{-8}\Omega$.

E29-22 Since $\Delta V = iR$, then if ΔV and i are the same, then R must be the same.
(a) Since $R = R$, $\rho_1 L_1/\pi r_1^2 = \rho_2 L_2/\pi r_2^2$, or $\rho_1/r_1^2 = \rho_2/r_2^2$. Then

$$r_{\rm iron}/r_{\rm copper} = \sqrt{(9.68\times10^{-8}\Omega\cdot\text{m})(1.69\times10^{-8}\Omega\cdot\text{m})} = 2.39.$$

(b) Start with the definition of current density:

$$j = \frac{i}{A} = \frac{\Delta V}{RA} = \frac{\Delta V}{\rho L}.$$

Since ΔV and L is the same, but ρ is different, then the current densities *will* be different.

E29-23 Conductivity is given by Eq. 29-8, $\vec{\mathbf{j}} = \sigma\vec{\mathbf{E}}$. If the wire is long and thin, then the magnitude of the electric field in the wire will be given by

$$E \approx \Delta V/L = (115\,\mathrm{V})/(9.66\,\mathrm{m}) = 11.9\,\mathrm{V/m}.$$

We can now find the conductivity,

$$\sigma = \frac{j}{E} = \frac{(1.42\times 10^4\,\mathrm{A/m^2})}{(11.9\,\mathrm{V/m})} = 1.19\times 10^3(\Omega\cdot\mathrm{m})^{-1}.$$

E29-24 (a) $v_\mathrm{d} = j/en = \sigma E/en$. Then

$$v_\mathrm{d} = (2.70\times 10^{-14}/\Omega\cdot\mathrm{m})(120\,\mathrm{V/m})/(1.60\times 10^{-19}\mathrm{C})(620\times 10^6/\mathrm{m}^3 + 550\times 10^6/\mathrm{m}^3) = 1.73\times 10^{-2}\mathrm{m/s}.$$

(b) $j = \sigma E = (2.70\times 10^{-14}/\Omega\cdot\mathrm{m})(120\,\mathrm{V/m}) = 3.24\times 10^{-14}\mathrm{A/m^2}$.

E29-25 (a) $R/L = \rho/A$, so $j = i/A = (R/L)i/\rho$. For copper,

$$j = (0.152\times 10^{-3}\Omega/\mathrm{m})(62.3\,\mathrm{A})/(1.69\times 10^{-8}\Omega\cdot\mathrm{m}) = 5.60\times 10^5\mathrm{A/m^2};$$

for aluminum,

$$j = (0.152\times 10^{-3}\Omega/\mathrm{m})(62.3\,\mathrm{A})/(2.75\times 10^{-8}\Omega\cdot\mathrm{m}) = 3.44\times 10^5\mathrm{A/m^2}.$$

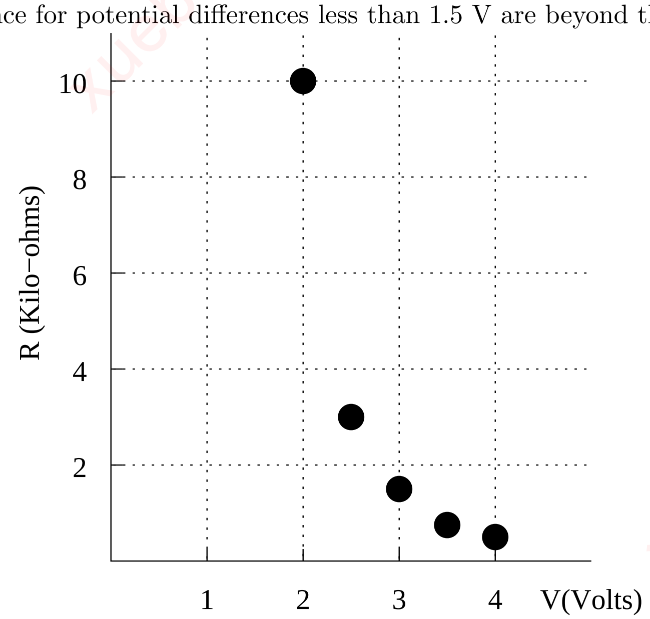
(b) $A = \rho L/R$; if δ is density, then $m = \delta l A = l\delta\rho/(R/L)$. For copper,

$$m = (1.0\,\mathrm{m})(8960\,\mathrm{kg/m^3})(1.69\times 10^{-8}\Omega\cdot\mathrm{m})/(0.152\times 10^{-3}\Omega/\mathrm{m}) = 0.996\,\mathrm{kg};$$

for aluminum,

$$m = (1.0\,\mathrm{m})(2700\,\mathrm{kg/m^3})(2.75\times 10^{-8}\Omega\cdot\mathrm{m})/(0.152\times 10^{-3}\Omega/\mathrm{m}) = 0.488\,\mathrm{kg}.$$

E29-26 The resistance for potential differences less than 1.5 V are beyond the scale.



$\boxed{\textbf{E29-27}}$ (a) The resistance is defined as

$$R = \frac{\Delta V}{i} = \frac{(3.55\times 10^6\,\mathrm{V/A^2})i^2}{i} = (3.55\times 10^6\,\mathrm{V/A^2})i.$$

When $i = 2.40$ mA the resistance would be

$$R = (3.55\times 10^6\,\mathrm{V/A^2})(2.40\times 10^{-3}\mathrm{A}) = 8.52\,\mathrm{k\Omega}.$$

(b) Invert the above expression, and

$$i = R/(3.55\times 10^6\,\mathrm{V/A^2}) = (16.0\,\Omega)/(3.55\times 10^6\,\mathrm{V/A^2}) = 4.51\,\mu\mathrm{A}.$$

E29-28 First, $n = 3(6.02\times10^{23})(2700\,\mathrm{kg/m^3})(27.0\times10^{-3}\mathrm{kg}) = 1.81\times10^{29}/\mathrm{m^3}$. Then

$$\tau = \frac{m}{ne^2\rho} = \frac{(9.11\times10^{-31}\mathrm{kg})}{(1.81\times10^{29}/\mathrm{m^3})(1.60\times10^{-19}\mathrm{C})^2(2.75\times10^{-8}\Omega\cdot\mathrm{m})} = 7.15\times10^{-15}\mathrm{s}.$$

E29-29 (a) $E = E_0/\kappa_\mathrm{e} = q/4\pi\epsilon_0\kappa_\mathrm{e}R^2$, so

$$E = \frac{(1.00\times10^{-6}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(4.7)(0.10\,\mathrm{m})^2} =$$

(b) $E = E_0 = q/4\pi\epsilon_0 R^2$, so

$$E = \frac{(1.00\times10^{-6}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.10\,\mathrm{m})^2} =$$

(c) $\sigma_\mathrm{ind} = \epsilon_0(E_0 - E) = q(1 - 1/\kappa_\mathrm{e})/4\pi R^2$. Then

$$\sigma_\mathrm{ind} = \frac{(1.00\times10^{-6}\mathrm{C})}{4\pi(0.10\,\mathrm{m})^2}\left(1 - \frac{1}{(4.7)}\right) = 6.23\times10^{-6}\mathrm{C/m^2}.$$

E29-30 Midway between the charges $E = q/\pi\epsilon_0 d$, so

$$q = \pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.10\,\mathrm{m})(3\times10^6\mathrm{V/m}) = 8.3\times10^{-6}\mathrm{C}.$$

$\boxed{\textbf{E29-31}}$ (a) At the surface of a conductor of radius R with charge Q the magnitude of the electric field is given by

$$E = \frac{1}{4\pi\epsilon_0}QR^2,$$

while the potential (assuming $V = 0$ at infinity) is given by

$$V = \frac{1}{4\pi\epsilon_0}QR.$$

The ratio is $V/E = R$.

The potential on the sphere that would result in "sparking" is

$$V = ER = (3\times10^6\mathrm{N/C})R.$$

(b) It is "easier" to get a spark off of a sphere with a smaller radius, because any potential on the sphere will result in a larger electric field.

(c) The points of a lighting rod are like small hemispheres; the electric field will be large near these points so that this will be the likely place for sparks to form and lightning bolts to strike.

$\boxed{\textbf{P29-1}}$ If there is more current flowing into the sphere than is flowing out then there must be a change in the net charge on the sphere. The net current is the difference, or $2\,\mu$A. The potential on the surface of the sphere will be given by the point-charge expression,

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r},$$

and the charge will be related to the current by $q = it$. Combining,

$$V = \frac{1}{4\pi\epsilon_0}\frac{it}{r},$$

or

$$t = \frac{4\pi\epsilon_0 Vr}{i} = \frac{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(980\,\mathrm{V})(0.13\,\mathrm{m})}{(2\,\mu\mathrm{A})} = 7.1\,\mathrm{ms}.$$

P29-2 The net current density is in the direction of the positive charges, which is to the east. There are two electrons for every alpha particle, and each alpha particle has a charge equal in magnitude to two electrons. The current density is then

$$\begin{aligned} j &= q_\mathrm{e}n_\mathrm{e}v_\mathrm{e} + q_\alpha + n_\alpha v_\alpha, \\ &= (-1.6\times10^{-19}\mathrm{C})(5.6\times10^{21}/\mathrm{m^3})(-88\,\mathrm{m/s}) + (3.2\times10^{-19}\mathrm{C})(2.8\times10^{21}/\mathrm{m^3})(25\,\mathrm{m/s}), \\ &= 1.0\times10^{5}\mathrm{C/m^2}. \end{aligned}$$

P29-3 (a) The resistance of the segment of the wire is

$$R = \rho L/A = (1.69\times10^{-8}\Omega\cdot\mathrm{m})(4.0\times10^{-2}\mathrm{m})/\pi(2.6\times10^{-3}\mathrm{m})^2 = 3.18\times10^{-5}\Omega.$$

The potential difference across the segment is

$$\Delta V = iR = (12\,\mathrm{A})(3.18\times10^{-5}\Omega) = 3.8\times10^{-4}\mathrm{V}.$$

(b) The tail is negative.
(c) The drift speed is $v = j/en = i/Aen$, so

$$v = (12\,\mathrm{A})/\pi(2.6\times10^{-3}\mathrm{m})^2(1.6\times10^{-19}\mathrm{C})(8.49\times10^{28}/\mathrm{m^3}) = 4.16\times10^{-5}\mathrm{m/s}.$$

The electrons will move 1 cm in $(1.0\times10^{-2}\mathrm{m})/(4.16\times10^{-5}\mathrm{m/s}) = 240\,\mathrm{s}$.

P29-4 (a) $N = it/q = (250\times10^{-9}\mathrm{A})(2.9\,\mathrm{s})/(3.2\times10^{-19}\mathrm{C}) = 2.27\times10^{12}$.
(b) The speed of the particles in the beam is given by $v = \sqrt{2K/m}$, so

$$v = \sqrt{2(22.4\text{ MeV})/4(932\text{ MeV}/c^2)} = 0.110c.$$

It takes $(0.180\,\mathrm{m})/(0.110)(3.00\times10^{8}\mathrm{m/s}) = 5.45\times10^{-9}\mathrm{s}$ for the beam to travel 18.0 cm. The number of charges is then

$$N = it/q = (250\times10^{-9}\mathrm{A})(5.45\times10^{-9}\mathrm{s})/(3.2\times10^{-19}\mathrm{C}) = 4260.$$

(c) $W = q\Delta V$, so $\Delta V = (22.4\text{ MeV})/2e = 11.2$ MV.

P29-5 (a) The time it takes to complete one turn is $t = (250\,\mathrm{m})/c$. The total charge is

$$q = it = (30.0\,\mathrm{A})(950\,\mathrm{m})/(3.00\times 10^8\mathrm{m/s}) = 9.50\times 10^{-5}\mathrm{C}.$$

(b) The number of charges is $N = q/e$, the total energy absorbed by the block is then

$$\Delta U = (28.0\times 10^9\ \mathrm{eV})(9.50\times 10^{-5}\mathrm{C})/e = 2.66\times 10^6\mathrm{J}.$$

This will raise the temperature of the block by

$$\Delta T = \Delta U/mC = (2.66\times 10^6\mathrm{J})/(43.5\,\mathrm{kg})(385\mathrm{J/kgC^\circ}) = 159\,\mathrm{C^\circ}.$$

P29-6 (a) $i = \int j\,dA = 2\pi \int jr\,dr$;

$$i = 2\pi \int -0^R j_0(1 - r/R)r\,dr = 2\pi j_0(R^2/2 - R^3/3R) = \pi j_0 R^2/6.$$

(b) Integrate, again:

$$i = 2\pi \int -0^R j_0(r/R)r\,dr = 2\pi j_0(R^3/3R) = \pi j_0 R^2/3.$$

P29-7 (a) Solve $2\rho_0 = \rho_0[1 + \alpha(T - 20^\circ\mathrm{C})]$, or

$$T = 20^\circ\mathrm{C} + 1/(4.3\times 10^{-3}/\mathrm{C^\circ}) = 250^\circ\mathrm{C}.$$

(b) Yes, ignoring changes in the physical dimensions of the resistor.

P29-8 The resistance when on is $(2.90\,\mathrm{V})/(0.310\,\mathrm{A}) = 9.35\,\Omega$. The temperature is given by

$$T = 20^\circ\mathrm{C} + (9.35\,\Omega - 1.12\,\Omega)/(1.12\,\Omega)(4.5\times 10^{-3}/^\circ\mathrm{C}) = 1650^\circ\mathrm{C}.$$

$\boxed{\textbf{P29-9}}$ Originally we have a resistance R_1 made out of a wire of length l_1 and cross sectional area A_1. The volume of this wire is $V_1 = A_1 l_1$. When the wire is drawn out to the new length we have $l_2 = 3l_1$, but the volume of the wire should be constant so

$$\begin{aligned} A_2 l_2 &= A_1 l_1, \\ A_2(3l_1) &= A_1 l_1, \\ A_2 &= A_1/3. \end{aligned}$$

The original resistance is

$$R_1 = \rho\frac{l_1}{A_1}.$$

The new resistance is

$$R_2 = \rho\frac{l_2}{A_2} = \rho\frac{3l_1}{A_1/3} = 9R_1,$$

or $R_2 = 54\,\Omega$.

P29-10 (a) $i = (35.8\,\mathrm{V})/(935\,\Omega) = 3.83\times 10^{-2}\mathrm{A}$.
(b) $j = i/A = (3.83\times 10^{-2}\mathrm{A})/(3.50\times 10^{-4}\mathrm{m}^2) = 109\,\mathrm{A/m^2}$.
(c) $v = (109\,\mathrm{A/m^2})/(1.6\times 10^{-19}\mathrm{C})(5.33\times 10^{22}/\mathrm{m}^3) = 1.28\times 10^{-2}\mathrm{m/s}$.
(d) $E = (35.8\,\mathrm{V})/(0.158\,\mathrm{m}) = 227\,\mathrm{V/m}$.

P29-11 (a) $\rho = (1.09\times10^{-3}\Omega)\pi(5.5\times10^{-3}\text{m})^2/4(1.6\,\text{m}) = 1.62\times10^{-8}\Omega\cdot\text{m}$. This is possibly silver.
(b) $R = (1.62\times10^{-8}\Omega\cdot\text{m})(1.35\times10^{-3}\text{m})4/\pi(2.14\times10^{-2}\text{m})^2 = 6.08\times10^{-8}\Omega$.

P29-12 (a) $\Delta L/L = 1.7\times10^{-5}$ for a temperature change of $1.0\,\text{C}^\circ$. Area changes are twice this, or $\Delta A/A = 3.4\times10^{-5}$.

Take the differential of $RA = \rho L$: $R\,dA + A\,dR = \rho\,dL + L\,d\rho$, or $dR = \rho\,dL/A + L\,d\rho/A - R\,dA/A$. For finite changes this can be written as

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + \frac{\Delta\rho}{\rho} - \frac{\Delta A}{A}.$$

$\Delta\rho/\rho = 4.3\times10^{-3}$. Since this term is so much larger than the other two it is the only significant effect.

$\boxed{\textbf{P29-13}}$ We will use the results of Exercise 29-17,

$$R - R_0 = R_0\alpha_{\text{av}}(T - T_0).$$

To save on subscripts we will drop the "av" notation, and just specify whether it is carbon "c" or iron "i".

The disks will be effectively in series, so we will add the resistances to get the total. Looking only at *one* disk pair, we have

$$\begin{aligned} R_{\text{c}} + R_{\text{i}} &= R_{0,\text{c}}\left(\alpha_{\text{c}}(T - T_0) + 1\right) + R_{0,\text{i}}\left(\alpha_{\text{i}}(T - T_0) + 1\right), \\ &= R_{0,\text{c}} + R_{0,\text{i}} + \left(R_{0,\text{c}}\alpha_{\text{c}} + R_{0,\text{i}}\alpha_{\text{i}}\right)(T - T_0). \end{aligned}$$

This last equation will only be constant if the coefficient for the term $(T - T_0)$ vanishes. Then

$$R_{0,\text{c}}\alpha_{\text{c}} + R_{0,\text{i}}\alpha_{\text{i}} = 0,$$

but $R = \rho L/A$, and the disks have the same cross sectional area, so

$$L_{\text{c}}\rho_{\text{c}}\alpha_{\text{c}} + L_{\text{i}}\rho_{\text{i}}\alpha_{\text{i}} = 0,$$

or

$$\frac{L_{\text{c}}}{L_{\text{i}}} = -\frac{\rho_{\text{i}}\alpha_{\text{i}}}{\rho_{\text{c}}\alpha_{\text{c}}} = -\frac{(9.68\times10^{-8}\Omega\cdot\text{m})(6.5\times10^{-3}/\text{C}^\circ)}{(3500\times10^{-8}\Omega\cdot\text{m})(-0.50\times10^{-3}/\text{C}^\circ)} = 0.036.$$

P29-14 The current entering the cone is i. The current density as a function of distance x from the left end is then

$$j = \frac{i}{\pi[a + x(b-a)/L]^2}.$$

The electric field is given by $E = \rho j$. The potential difference between the ends is then

$$\Delta V = \int_0^L E\,dx = \int_0^L \frac{i\rho}{\pi[a + x(b-a)/L]^2}dx = \frac{i\rho L}{\pi ab}$$

The resistance is $R = \Delta V/i = \rho L/\pi ab$.

$\boxed{\textbf{P29-15}}$ The current is found from Eq. 29-5,

$$i = \int \vec{\mathbf{j}} \cdot d\vec{\mathbf{A}},$$

where the region of integration is over a spherical shell concentric with the two conducting shells but between them. The current density is given by Eq. 29-10,

$$\vec{\mathbf{j}} = \vec{\mathbf{E}}/\rho,$$

and we will have an electric field which is perpendicular to the spherical shell. Consequently,

$$i = \frac{1}{\rho}\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\rho}\int E\,dA$$

By symmetry we expect the electric field to have the same magnitude anywhere on a spherical shell which is concentric with the two conducting shells, so we can bring it out of the integral sign, and then

$$i = \frac{1}{\rho}E\int dA = \frac{4\pi r^2 E}{\rho},$$

where E is the magnitude of the electric field on the shell, which has radius r such that $b > r > a$.

The above expression can be inverted to give the electric field as a function of radial distance, since the current is a constant in the above expression. Then $E = i\rho/4\pi r^2$ The potential is given by

$$\Delta V = -\int_b^a \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}},$$

we will integrate along a radial line, which is parallel to the electric field, so

$$\begin{aligned}
\Delta V &= -\int_b^a E\,dr, \\
&= -\int_b^a \frac{i\rho}{4\pi r^2}\,dr, \\
&= -\frac{i\rho}{4\pi}\int_b^a \frac{dr}{r}, \\
&= \frac{i\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right).
\end{aligned}$$

We divide this expression by the current to get the resistance. Then

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

P29-16 Since $\tau = \lambda/v_{\mathrm{d}}$, $\rho \propto v_{\mathrm{d}}$. For an ideal gas the kinetic energy is proportional to the temperature, so $\rho \propto \sqrt{K} \propto \sqrt{T}$.

$\boxed{\textbf{E30-1}}$ We apply Eq. 30-1,

$$q = C\Delta V = (50 \times 10^{-12}\,\text{F})(0.15\,\text{V}) = 7.5 \times 10^{-12}\,\text{C};$$

E30-2 (a) $C = \Delta V/q = (73.0\times10^{-12}\,C)/(19.2\,\text{V}) = 3.80\times10^{-12}\text{F}$.
(b) The capacitance doesn't change!
(c) $\Delta V = q/C = (210\times10^{-12}\text{C})/(3.80\times10^{-12}\text{F}) = 55.3\,\text{V}$.

E30-3 $q = C\Delta V = (26.0\times10^{-6}\text{F})(125\,\text{V}) = 3.25\times10^{-3}\text{C}$.

E30-4 (a) $C = \epsilon_0 A/d = (8.85\times10^{-12}\text{F/m})\pi(8.22\times10^{-2}\text{m})^2/(1.31\times10^{-3}\text{m}) = 1.43\times10^{-10}\text{F}$.
(b) $q = C\Delta V = (1.43\times10^{-10}\text{F})(116\,\text{V}) = 1.66\times10^{-8}\text{C}$.

$\boxed{\textbf{E30-5}}$ Eq. 30-11 gives the capacitance of a cylinder,

$$C = 2\pi\epsilon_0\frac{L}{\ln(b/a)} = 2\pi(8.85\times10^{-12}\,\text{F/m})\frac{(0.0238\,\text{m})}{\ln((9.15\text{mm})/(0.81\text{mm}))} = 5.46\times10^{-13}\text{F}.$$

E30-6 (a) $A = Cd/\epsilon_0 = (9.70\times10^{-12}\text{F})(1.20\times10^{-3}\text{m})/(8.85\times10^{-12}\text{F/m}) = 1.32\times10^{-3}\text{m}^2$.
(b) $C = C_0 d_0/d = (9.70\times10^{-12}\text{F})(1.20\times10^{-3}\text{m})/(1.10\times10^{-3}\text{m}) = 1.06\times10^{-11}\text{F}$.
(c) $\Delta V = q_0/C = [\Delta V]_0 C_0/C = [\Delta V]_0 d/d_0$. Using this formula, the new potential difference would be $[\Delta V]_0 = (13.0\,\text{V})(1.10\times10^{-3}\text{m})/(1.20\times10^{-3}\text{m}) = 11.9\,\text{V}$. The potential energy has *changed* by $(11.9\,\text{V}) - (30.0\,\text{V}) = -1.1\,\text{V}$.

E30-7 (a) From Eq. 30-8,

$$C = 4\pi(8.85\times10^{-12}\text{F/m})\frac{(0.040\,\text{m})(0.038\,\text{m})}{(0.040\,\text{m}) - (0.038\,\text{m})} = 8.45\times10^{-11}\text{F}.$$

(b) $A = Cd/\epsilon_0 = (8.45\times10^{-11}\text{F})(2.00\times10^{-3}\text{m})/(8.85\times10^{-12}\text{F/m}) = 1.91\times10^{-2}\text{m}^2$.

E30-8 Let $a = b + d$, where d is the *small* separation between the shells. Then

$$\begin{aligned} C &= 4\pi\epsilon_0\frac{ab}{a-b} = 4\pi\epsilon_0\frac{(b+d)b}{d}, \\ &\approx 4\pi\epsilon_0\frac{b^2}{d} = \epsilon_0 A/d. \end{aligned}$$

$\boxed{\textbf{E30-9}}$ The potential difference across each capacitor in parallel is the same; it is equal to 110 V. The charge on each of the capacitors is then

$$q = C\Delta V = (1.00 \times 10^{-6}\,\text{F})(110\,\text{V}) = 1.10 \times 10^{-4}\,\text{C}.$$

If there are N capacitors, then the total charge will be Nq, and we want this total charge to be 1.00 C. Then

$$N = \frac{(1.00\,\text{C})}{q} = \frac{(1.00\,\text{C})}{(1.10 \times 10^{-4}\,\text{C})} = 9090.$$

E30-10 First find the equivalent capacitance of the parallel part:

$$C_{\text{eq}} = C_1 + C_2 = (10.3\times10^{-6}\text{F}) + (4.80\times10^{-6}\text{F}) = 15.1\times10^{-6}\text{F}.$$

Then find the equivalent capacitance of the series part:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{(15.1\times10^{-6}\text{F})} + \frac{1}{(3.90\times10^{-6}\text{F})} = 3.23\times10^{5}\text{F}^{-1}.$$

Then the equivalent capacitance of the entire arrangement is 3.10×10^{-6}F.

E30-11 First find the equivalent capacitance of the series part:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{(10.3\times10^{-6}\text{F})} + \frac{1}{(4.80\times10^{-6}\text{F})} = 3.05\times10^{5}\text{F}^{-1}.$$

The equivalent capacitance is 3.28×10^{-6}F. Then find the equivalent capacitance of the parallel part:

$$C_{\text{eq}} = C_1 + C_2 = (3.28\times10^{-6}\text{F}) + (3.90\times10^{-6}\text{F}) = 7.18\times10^{-6}\text{F}.$$

This is the equivalent capacitance for the entire arrangement.

E30-12 For one capacitor $q = C\Delta V = (25.0\times10^{-6}\text{F})(4200\,\text{V}) = 0.105\,\text{C}$. There are three capacitors, so the total charge to pass through the ammeter is 0.315 C.

E30-13 (a) The equivalent capacitance is given by Eq. 30-21,

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{(4.0\mu\text{F})} + \frac{1}{(6.0\mu\text{F})} = \frac{5}{(12.0\mu\text{F})}$$

or $C_{\text{eq}} = 2.40\mu$F.

(b) The charge on the equivalent capacitor is $q = C\Delta V = (2.40\mu\text{F})(200\,\text{V}) = 0.480\,\text{mC}$. For series capacitors, the charge on the equivalent capacitor is the same as the charge on each of the capacitors. *This statement is wrong in the Student Solutions!*

(c) The potential difference across the equivalent capacitor is *not* the same as the potential difference across each of the individual capacitors. We need to apply $q = C\Delta V$ to each capacitor using the charge from part (b). Then for the 4.0μF capacitor,

$$\Delta V = \frac{q}{C} = \frac{(0.480\,\text{mC})}{(4.0\mu\text{F})} = 120\,\text{V};$$

and for the 6.0μF capacitor,

$$\Delta V = \frac{q}{C} = \frac{(0.480\,\text{mC})}{(6.0\mu\text{F})} = 80\,\text{V}.$$

Note that the sum of the potential differences across each of the capacitors is equal to the potential difference across the equivalent capacitor.

E30-14 (a) The equivalent capacitance is

$$C_{\text{eq}} = C_1 + C_2 = (4.0\mu\text{F}) + (6.0\mu\text{F}) = (10.0\mu\text{F}).$$

(c) For parallel capacitors, the potential difference across the equivalent capacitor is the same as the potential difference across either of the capacitors.

(b) For the 4.0μF capacitor,

$$q = C\Delta V = (4.0\mu\text{F})(200\,\text{V}) = 8.0\times10^{-4}\,\text{C};$$

and for the 6.0μF capacitor,

$$q = C\Delta V = (6.0\mu\text{F})(200\,\text{V}) = 12.0\times10^{-4}\,\text{C}.$$

E30-15 (a) $C_{\text{eq}} = C + C + C = 3C$;

$$d_{\text{eq}} = \frac{\epsilon_0 A}{C_{\text{eq}}} = \frac{\epsilon_0 A}{3C} = \frac{d}{3}.$$

(b) $1/C_{\text{eq}} = 1/C + 1/C + 1/C = 3/C$;

$$d_{\text{eq}} = \frac{\epsilon_0 A}{C_{\text{eq}}} = \frac{\epsilon_0 A}{C/3} = 3d.$$

E30-16 (a) The maximum potential across any individual capacitor is 200 V; so there must be at least $(1000\,\text{V})/(200\,\text{V}) = 5$ series capacitors in any parallel branch. This branch would have an equivalent capacitance of $C_{\text{eq}} = C/5 = (2.0\times10^{-6}\text{F})/5 = 0.40\times10^{-6}\text{F}$.

(b) For parallel branches we add, which means we need $(1.2\times10^{-6}\text{F})/(0.40\times10^{-6}\text{F}) = 3$ parallel branches of the combination found in part (a).

E30-17 Look back at the solution to Ex. 30-10. If C_3 breaks down electrically then the circuit is effectively two capacitors in parallel.

(b) $\Delta V = 115\,\text{V}$ after the breakdown.

(a) $q_1 = (10.3\times10^{-6}\text{F})(115\,\text{V}) = 1.18\times10^{-3}\text{C}$.

E30-18 The 108μF capacitor originally has a charge of $q = (108\times10^{-6}\text{F})(52.4\,\text{V}) = 5.66\times10^{-3}\text{C}$. After it is connected to the second capacitor the 108μF capacitor has a charge of $q = (108\times10^{-6}\text{F})(35.8\,\text{V}) = 3.87\times10^{-3}\text{C}$. The difference in charge must reside on the second capacitor, so the capacitance is $C = (1.79\times10^{-3}\text{C})/(35.8\,\text{V}) = 5.00\times10^{-5}\text{F}$.

$\boxed{\textbf{E30-19}}$ Consider any junction other than A or B. Call this junction point 0; label the four nearest junctions to this as points 1, 2, 3, and 4. The charge on the capacitor that links point 0 to point 1 is $q_1 = C\Delta V_{01}$, where ΔV_{01} is the potential difference across the capacitor, so $\Delta V_{01} = V_0 - V_1$, where V_0 is the potential at the junction 0, and V_1 is the potential at the junction 1. Similar expressions exist for the other three capacitors.

For the junction 0 the net charge must be zero; there is no way for charge to cross the plates of the capacitors. Then $q_1 + q_2 + q_3 + q_4 = 0$, and this means

$$C\Delta V_{01} + C\Delta V_{02} + C\Delta V_{03} + C\Delta V_{04} = 0$$

or

$$\Delta V_{01} + \Delta V_{02} + \Delta V_{03} + \Delta V_{04} = 0.$$

Let $\Delta V_{0i} = V_0 - V_i$, and then rearrange,

$$4V_0 = V_1 + V_2 + V_3 + V_4,$$

or

$$V_0 = \frac{1}{4}\left(V_1 + V_2 + V_3 + V_4\right).$$

E30-20 $U = uV = \epsilon_0 E^2 V/2$, where V is the volume. Then

$$U = \frac{1}{2}(8.85\times10^{-12}\text{F/m})(150\,\text{V/m})^2(2.0\,\text{m}^3) = 1.99\times10^{-7}\text{J}.$$

E30-21 The total capacitance is $(2100)(5.0\times10^{-6}\text{F}) = 1.05\times10^{-2}\text{F}$. The total energy stored is

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(1.05\times10^{-2}\text{F})(55\times10^3\text{V})^2 = 1.59\times10^7\,\text{J}.$$

The cost is

$$(1.59\times10^7\text{J})\left(\frac{\$0.03}{3600\times10^3\text{J}}\right) = \$0.133.$$

E30-22 (a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(0.061\,\text{F})(1.0\times10^4\text{V})^2 = 3.05\times10^6\text{J}$.
(b) $(3.05\times10^6\text{J})/(3600\times10^3\text{J/kW}\cdot\text{h}) = 0.847\text{kW}\cdot\text{h}$.

$\boxed{\textbf{E30-23}}$ (a) The capacitance of an air filled parallel-plate capacitor is given by Eq. 30-5,

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\text{F/m})(42.0\times10^{-4}\text{m}^2)}{(1.30\times10^{-3}\text{m})} = 2.86\times10^{-11}\,\text{F}.$$

(b) The magnitude of the charge on each plate is given by

$$q = C\Delta V = (2.86\times10^{-11}\,\text{F})(625\,\text{V}) = 1.79\times10^{-8}\,\text{C}.$$

(c) The stored energy in a capacitor is given by Eq. 30-25, regardless of the type or shape of the capacitor, so

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(2.86\times10^{-11}\,\text{F})(625\,\text{V})^2 = 5.59\,\mu\text{J}.$$

(d) Assuming a parallel plate arrangement with *no* fringing effects, the magnitude of the electric field between the plates is given by $Ed = \Delta V$, where d is the separation between the plates. Then

$$E = \Delta V/d = (625\,\text{V})/(0.00130\,\text{m}) = 4.81\times10^5\,\text{V/m}.$$

(e) The energy density is Eq. 30-28,

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}((8.85\times10^{-12}\text{F/m}))(4.81\times10^5\,\text{V/m})^2 = 1.02\,\text{J/m}^3.$$

E30-24 The equivalent capacitance is given by

$$1/C_{\text{eq}} = 1/(2.12\times10^{-6}\text{F}) + 1/(3.88\times10^{-6}\text{F}) = 1/(1.37\times10^{-6}\text{F}).$$

The energy stored is $U = \frac{1}{2}(1.37\times10^{-6}\text{F})(328\,\text{V})^2 = 7.37\times10^{-2}\text{J}$.

E30-25 $V/r = q/4\pi\epsilon_0 r^2 = E$, so that if V is the potential of the sphere then $E = V/r$ is the electric field on the surface. Then the energy density of the electric field near the surface is

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{(8.85\times10^{-12}\text{F/m})}{2}\left(\frac{(8150\,\text{V})}{(0.063\,\text{m})}\right)^2 = 7.41\times10^{-2}\text{J/m}^3.$$

E30-26 The charge on C_3 can be found from considering the equivalent capacitance. $q_3 = (3.10\times10^{-6}\text{F})(112\,\text{V}) = 3.47\times10^{-4}\text{C}$. The potential across C_3 is given by $[\Delta V]_3 = (3.47\times10^{-4}\text{C})/(3.90\times10^{-6}\text{F}) = 89.0\,\text{V}$.

The potential across the parallel segment is then $(112\,\text{V}) - (89.0\,\text{V}) = 23.0\,\text{V}$. So $[\Delta V]_1 = [\Delta V]_2 = 23.0\,\text{V}$.

Then $q_1 = (10.3\times10^{-6}\text{F})(23.0\,\text{V}) = 2.37\times10^{-4}\text{C}$ and $q_2 = (4.80\times10^{-6}\text{F})(23.0\,\text{V}) = 1.10\times10^{-4}\text{C}$..

$\boxed{\textbf{E30-27}}$ There is enough work on this problem without deriving once again the electric field between charged cylinders. I will instead refer you back to Section 26-4, and state

$$E = \frac{1}{2\pi\epsilon_0}\frac{q}{Lr},$$

where q is the magnitude of the charge on a cylinder and L is the length of the cylinders.

The energy density as a function of radial distance is found from Eq. 30-28,

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{8\pi^2\epsilon_0}\frac{q^2}{L^2r^2}$$

The total energy stored in the electric field is given by Eq. 30-24,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{q^2}{2}\frac{\ln(b/a)}{2\pi\epsilon_0 L},$$

where we substituted into the last part Eq. 30-11, the capacitance of a cylindrical capacitor.

We want to show that integrating a volume integral from $r = a$ to $r = \sqrt{ab}$ over the energy density function will yield $U/2$. Since we want to do this problem the hard way, we will pretend we don't know the answer, and integrate from $r = a$ to $r = c$, and then find out what c is.

Then

$$\begin{aligned}
\frac{1}{2}U &= \int u\,dV, \\
&= \int_a^c \int_0^{2\pi} \int_0^L \left(\frac{1}{8\pi^2\epsilon_0}\frac{q^2}{L^2r^2}\right) r\,dr\,d\phi\,dz, \\
&= \frac{q^2}{8\pi^2\epsilon_0 L^2}\int_a^c \int_0^{2\pi} \int_0^L \frac{dr}{r} d\phi\,dz, \\
&= \frac{q^2}{4\pi\epsilon_0 L}\int_a^c \frac{dr}{r}, \\
&= \frac{q^2}{4\pi\epsilon_0 L}\ln\frac{c}{a}.
\end{aligned}$$

Now we equate this to the value for U that we found above, and we solve for c.

$$\begin{aligned}
\frac{1}{2}\frac{q^2}{2}\frac{\ln(b/a)}{2\pi\epsilon_0 L} &= \frac{q^2}{4\pi\epsilon_0 L}\ln\frac{c}{a}, \\
\ln(b/a) &= 2\ln(c/a), \\
(b/a) &= (c/a)^2, \\
\sqrt{ab} &= c.
\end{aligned}$$

E30-28 (a) $d = \epsilon_0 A/C$, or

$$d = (8.85\times10^{-12}\text{F/m})(0.350\,\text{m}^2)/(51.3\times10^{-12}\text{F}) = 6.04\times10^{-3}\text{m}.$$

(b) $C = (5.60)(51.3\times10^{-12}\,\text{F}) = 2.87\times10^{-10}\text{F}.$

E30-29 Originally, $C_1 = \epsilon_0 A/d_1$. After the changes, $C_2 = \kappa\epsilon_0 A/d_2$. Dividing C_2 by C_1 yields $C_2/C_1 = \kappa d_1/d_2$, so

$$\kappa = d_2C_2/d_1C_1 = (2)(2.57\times10^{-12}\text{F})/(1.32\times10^{-12}\text{F}) = 3.89.$$

E30-30 The required capacitance is found from $U = \frac{1}{2}C(\Delta V)^2$, or

$$C = 2(6.61\times10^{-6}\text{J})/(630\,\text{V})^2 = 3.33\times10^{-11}\text{F}.$$

The dielectric constant required is $\kappa = (3.33\times10^{-11}\text{F})/(7.40\times10^{-12}\text{F}) = 4.50$. Try transformer oil.

$\boxed{\textbf{E30-31}}$ Capacitance with dielectric media is given by Eq. 30-31,

$$C = \frac{\kappa_\text{e}\epsilon_0 A}{d}.$$

The various sheets have different dielectric constants and different thicknesses, and we want to maximize C, which means maximizing κ_e/d. For mica this ratio is 54 mm^{-1}, for glass this ratio is 35 mm^{-1}, and for paraffin this ratio is 0.20 mm^{-1}. Mica wins.

E30-32 The minimum plate separation is given by

$$d = (4.13\times10^3\text{V})/(18.2\times10^6\text{V/m}) = 2.27\times10^{-4}\text{m}.$$

The minimum plate area is then

$$A = \frac{dC}{\kappa\epsilon_0} = \frac{(2.27\times10^{-4}\text{m})(68.4\times10^{-9}\text{F})}{(2.80)(8.85\times10^{-12}\text{F/m})} = 0.627\,\text{m}^2.$$

E30-33 The capacitance of a cylindrical capacitor is given by Eq. 30-11,

$$C = 2\pi(8.85\times10^{-12}\text{F/m})(2.6)\frac{1.0\times10^3\text{m}}{\ln(0.588/0.11)} = 8.63\times10^{-8}\text{F}.$$

E30-34 (a) $U = C'(\Delta V)^2/2$, $C' = \kappa_\text{e}\epsilon_0 A/d$, and $\Delta V/d$ is less than or equal to the dielectric strength (which we will call S). Then $\Delta V = Sd$ and

$$U = \frac{1}{2}\kappa_\text{e}\epsilon_0 AdS^2,$$

so the volume is given by

$$V = 2U/\kappa_\text{e}\epsilon_0 S^2.$$

This quantity is a minimum for mica, so

$$V = 2(250\times10^3\text{J})/(5.4)(8.85\times10^{-12}\text{F/m})(160\times10^6\text{V/m})^2 = 0.41\,\text{m}^3.$$

(b) $\kappa_\text{e} = 2U/V\epsilon_0 S^2$, so

$$\kappa_\text{e} = 2(250\times10^3\text{J})/(0.087\text{m}^3)(8.85\times10^{-12}\text{F/m})(160\times10^6\text{V/m})^2 = 25.$$

$\boxed{\textbf{E30-35}}$ (a) The capacitance of a cylindrical capacitor is given by Eq. 30-11,

$$C = 2\pi\epsilon_0\kappa_\text{e}\frac{L}{\ln(b/a)}.$$

The factor of κ_e is introduced because there is now a dielectric (the Pyrex drinking glass) between the plates. We can look back to Table 29-2 to get the dielectric properties of Pyrex. The capacitance of our "glass" is then

$$C = 2\pi(8.85\times10^{-12}\text{F/m})(4.7)\frac{(0.15\,\text{m})}{\ln((3.8\text{ cm})/(3.6\text{ cm}))} = 7.3\times10^{-10}\,\text{F}.$$

(b) The breakdown potential is (14 kV/mm)(2 mm) = 28 kV.

E30-36 (a) $C' = \kappa_e C = (6.5)(13.5\times10^{-12}\text{F}) = 8.8\times10^{-11}\text{F}$.

(b) $Q = C'\Delta V = (8.8\times10^{-11}\text{F})(12.5\,\text{V}) = 1.1\times10^{-9}\text{C}$.

(c) $E = \Delta V/d$, but we don't know d.

(d) $E' = E/\kappa_e$, but we couldn't find E.

$\boxed{\textbf{E30-37}}$ (a) Insert the slab so that it is a distance a above the lower plate. Then the distance between the slab and the upper plate is $d-a-b$. Inserting the slab has the same effect as having two capacitors wired in series; the separation of the bottom capacitor is a, while that of the top capacitor is $d-a-b$.

The bottom capacitor has a capacitance of $C_1 = \epsilon_0 A/a$, while the top capacitor has a capacitance of $C_2 = \epsilon_0 A/(d-a-b)$. Adding these in series,

$$\begin{aligned}\frac{1}{C_{\text{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2},\\ &= \frac{a}{\epsilon_0 A} + \frac{d-a-b}{\epsilon_0 A},\\ &= \frac{d-b}{\epsilon_0 A}.\end{aligned}$$

So the capacitance of the system after putting the copper slab in is $C = \epsilon_0 A/(d-b)$.

(b) The energy stored in the system before the slab is inserted is

$$U_{\text{i}} = \frac{q^2}{2C_{\text{i}}} = \frac{q^2}{2}\frac{d}{\epsilon_0 A}$$

while the energy stored after the slab is inserted is

$$U_{\text{f}} = \frac{q^2}{2C_{\text{f}}} = \frac{q^2}{2}\frac{d-b}{\epsilon_0 A}$$

The ratio is $U_{\text{i}}/U_{\text{f}} = d/(d-b)$.

(c) Since there was more energy *before* the slab was inserted, then the slab must have gone in willingly, *it was pulled in!*. To get the slab back out we will need to do work on the slab equal to the energy difference.

$$U_{\text{i}} - U_{\text{f}} = \frac{q^2}{2}\frac{d}{\epsilon_0 A} - \frac{q^2}{2}\frac{d-b}{\epsilon_0 A} = \frac{q^2}{2}\frac{b}{\epsilon_0 A}.$$

E30-38 (a) Insert the slab so that it is a distance a above the lower plate. Then the distance between the slab and the upper plate is $d-a-b$. Inserting the slab has the same effect as having two capacitors wired in series; the separation of the bottom capacitor is a, while that of the top capacitor is $d-a-b$.

The bottom capacitor has a capacitance of $C_1 = \epsilon_0 A/a$, while the top capacitor has a capacitance of $C_2 = \epsilon_0 A/(d-a-b)$. Adding these in series,

$$\begin{aligned}\frac{1}{C_{\text{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2},\\ &= \frac{a}{\epsilon_0 A} + \frac{d-a-b}{\epsilon_0 A},\\ &= \frac{d-b}{\epsilon_0 A}.\end{aligned}$$

So the capacitance of the system after putting the copper slab in is $C = \epsilon_0 A/(d-b)$.

(b) The energy stored in the system before the slab is inserted is

$$U_{\rm i} = \frac{C_{\rm i}(\Delta V)^2}{2} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d}$$

while the energy stored after the slab is inserted is

$$U_{\rm f} = \frac{C_{\rm f}(\Delta V)^2}{2} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d-b}$$

The ratio is $U_{\rm i}/U_{\rm f} = (d-b)/d$.

(c) Since there was more energy *after* the slab was inserted, then the slab must not have gone in willingly, *it was being repelled!*. To get the slab in we will need to do work on the slab equal to the energy difference.

$$U_{\rm f} - U_{\rm i} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d-b} - \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A b}{d(d-b)}.$$

E30-39 $C = \kappa_{\rm e}\epsilon_0 A/d$, so $d = \kappa_{\rm e}\epsilon_0 A/C$.

(a) $E = \Delta V/d = C\Delta V/\kappa_{\rm e}\epsilon_0 A$, or

$$E = \frac{(112\times10^{-12}\mathrm{F})(55.0\,\mathrm{V})}{(5.4)(8.85\times10^{-12}\mathrm{F/m})(96.5\times10^{-4}\mathrm{m}^2)} = 13400\,\mathrm{V/m}.$$

(b) $Q = C\Delta V = (112\times10^{-12}\mathrm{F})(55.0\,\mathrm{V}) = 6.16\times10^{-9}\mathrm{C}$..

(c) $Q' = Q(1-1/\kappa_{\rm e}) = (6.16\times10^{-9}\mathrm{C})(1-1/(5.4)) = 5.02\times10^{-9}\mathrm{C}$.

E30-40 (a) $E = q/\kappa_{\rm e}\epsilon_0 A$, so

$$\kappa_{\rm e} = \frac{(890\times10^{-9}\mathrm{C})}{(1.40\times10^{6}\mathrm{V/m})(8.85\times10^{-12}\mathrm{F/m})(110\times10^{-4}\mathrm{m}^2)} = 6.53$$

(b) $q' = q(1-1/\kappa_{\rm e}) = (890\times10^{-9}\mathrm{C})(1-1/(6.53)) = 7.54\times10^{-7}\mathrm{C}$.

P30-1 The capacitance of the cylindrical capacitor is from Eq. 30-11,

$$C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}.$$

If the cylinders are very close together we can write $b = a + d$, where d, the separation between the cylinders, is a small number, so

$$C = \frac{2\pi\epsilon_0 L}{\ln((a+d)/a)} = \frac{2\pi\epsilon_0 L}{\ln(1+d/a)}.$$

Expanding according to the hint,

$$C \approx \frac{2\pi\epsilon_0 L}{d/a} = \frac{2\pi a\epsilon_0 L}{d}$$

Now $2\pi a$ is the circumference of the cylinder, and L is the length, so $2\pi aL$ is the area of a cylindrical plate. Hence, for small separation between the cylinders we have

$$C \approx \frac{\epsilon_0 A}{d},$$

which is the expression for the parallel plates.

P30-2 (a) $C = \epsilon_0 A/x$; take the derivative and

$$\begin{aligned}\frac{dC}{dT} &= \frac{\epsilon_0}{x}\frac{dA}{dT} - \frac{\epsilon_0 A}{x^2}\frac{dx}{dT},\\ &= C\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right).\end{aligned}$$

(b) Since $(1/A)dA/dT = 2\alpha_\mathrm{a}$ and $(1/x)dx/dT = \alpha_\mathrm{s}$, we need

$$\alpha_\mathrm{s} = 2\alpha_\mathrm{a} = 2(23\times10^{-6}/\mathrm{C}^\circ) = 46\times10^{-6}/\mathrm{C}^\circ.$$

$\boxed{\textbf{P30-3}}$ Insert the slab so that it is a distance d above the lower plate. Then the distance between the slab and the upper plate is $a-b-d$. Inserting the slab has the same effect as having two capacitors wired in series; the separation of the bottom capacitor is d, while that of the top capacitor is $a-b-d$.

The bottom capacitor has a capacitance of $C_1 = \epsilon_0 A/d$, while the top capacitor has a capacitance of $C_2 = \epsilon_0 A/(a-b-d)$. Adding these in series,

$$\begin{aligned}\frac{1}{C_\mathrm{eq}} &= \frac{1}{C_1} + \frac{1}{C_2},\\ &= \frac{d}{\epsilon_0 A} + \frac{a-b-d}{\epsilon_0 A},\\ &= \frac{a-b}{\epsilon_0 A}.\end{aligned}$$

So the capacitance of the system after putting the slab in is $C = \epsilon_0 A/(a-b)$.

P30-4 The potential difference between any two adjacent plates is ΔV. Each interior plate has a charge q on *each* surface; the exterior plate (one pink, one gray) has a charge of q on the interior surface only.

The capacitance of one pink/gray plate pair is $C = \epsilon_0 A/d$. There are n plates, but only $n-1$ plate pairs, so the total charge is $(n-1)q$. This means the total capacitance is $C = \epsilon_0(n-1)A/d$.

$\boxed{\textbf{P30-5}}$ Let $\Delta V_0 = 96.6\,\mathrm{V}$.

As far as point e is concerned point a looks like it is originally positively charged, and point d is originally negatively charged. It is then convenient to define the charges on the capacitors in terms of the charges on the top sides, so the original charge on C_1 is $q_{1,\mathrm{i}} = C_1\Delta V_0$ while the original charge on C_2 is $q_{2,\mathrm{i}} = -C_2\Delta V_0$. Note the negative sign reflecting the opposite polarity of C_2.

(a) Conservation of charge requires

$$q_{1,\mathrm{i}} + q_{2,\mathrm{i}} = q_{1,\mathrm{f}} + q_{2,\mathrm{f}},$$

but since $q = C\Delta V$ and the two capacitors will be at the same potential after the switches are closed we can write

$$\begin{aligned}C_1\Delta V_0 - C_2\Delta V_0 &= C_1\Delta V + C_2\Delta V,\\ (C_1 - C_2)\,\Delta V_0 &= (C_1 + C_2)\,\Delta V,\\ \frac{C_1 - C_2}{C_1 + C_2}\Delta V_0 &= \Delta V.\end{aligned}$$

With numbers,

$$\Delta V = (96.6\,\mathrm{V})\frac{(1.16\,\mu\mathrm{F}) - (3.22\,\mu\mathrm{F})}{(1.16\,\mu\mathrm{F}) + (3.22\,\mu\mathrm{F})} = -45.4\,\mathrm{V}.$$

The negative sign means that the top sides of *both* capacitor will be negatively charged after the switches are closed.

(b) The charge on C_1 is $C_1\Delta V = (1.16\,\mu\text{F})(45.4\,\text{V}) = 52.7\mu\text{C}$.

(c) The charge on C_2 is $C_2\Delta V = (3.22\,\mu\text{F})(45.4\,\text{V}) = 146\mu\text{C}$.

P30-6 C_2 and C_3 form an effective capacitor with equivalent capacitance $C_a = C_2C_3/(C_2 + C_3)$. The charge on C_1 is originally $q_0 = C_1\Delta V_0$. After throwing the switch the potential across C_1 is given by $q_1 = C_1\Delta V_1$. The same potential is across C_a; $q_2 = q_3$, so $q_2 = C_a\Delta V_1$. Charge is conserved, so $q_1 + q_2 = q_0$. Combining some of the above,

$$\Delta V_1 = \frac{q_0}{C_1 + C_a} = \frac{C_1}{C_1 + C_a}\Delta V_0,$$

and then

$$q_1 = \frac{C_1^2}{C_1 + C_a}\Delta V_0 = \frac{C_1^2(C_2 + C_3)}{C_1C_2 + C_1C_3 + C_2C_3}\Delta V_0.$$

Similarly,

$$q_2 = \frac{C_aC_1}{C_1 + C_a}\Delta V_0 = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}\Delta V_0.$$

$q_3 = q_2$ because they are in series.

$\boxed{\textbf{P30-7}}$ (a) If terminal a is more positive than terminal b then current can flow that will charge the capacitor on the left, the current can flow through the diode on the top, and the current can charge the capacitor on the right. Current will not flow through the diode on the left. The capacitors are effectively in series.

Since the capacitors are identical and series capacitors have the same charge, we expect the capacitors to have the same potential difference across them. But the total potential difference across both capacitors is equal to 100 V, so the potential difference across either capacitor is 50 V.

The output pins are connected to the capacitor on the right, so the potential difference across the output is 50 V.

(b) If terminal b is more positive than terminal a the current can flow through the diode on the left. If we assume the diode is resistanceless in this configuration then the potential difference across it will be zero. The net result is that the potential difference across the output pins is 0 V.

In real life the potential difference across the diode would not be zero, even if forward biased. It will be somewhere around 0.5 Volts.

P30-8 Divide the strip of width a into N segments, each of width $\Delta x = a/N$. The capacitance of each strip is $\Delta C = \epsilon_0 a\Delta x/y$. If θ is small then

$$\frac{1}{y} = \frac{1}{d + x\sin\theta} \approx \frac{1}{d + x\theta} \approx \frac{d}{(}1 - x\theta/d).$$

Since parallel capacitances *add*,

$$C = \sum \Delta C = \frac{\epsilon_0 a}{d}\int_0^a (1 - x\theta/d)dx = \frac{\epsilon_0 a^2}{d}\left(1 - \frac{a\theta}{2d}\right).$$

P30-9 (a) When S_2 is open the circuit acts as two parallel capacitors. The branch on the left has an effective capacitance given by

$$\frac{1}{C_l} = \frac{1}{(1.0\times10^{-6}\text{F})} + \frac{1}{(3.0\times10^{-6}\text{F})} = \frac{1}{7.5\times10^{-7}\text{F}},$$

while the branch on the right has an effective capacitance given by

$$\frac{1}{C_l} = \frac{1}{(2.0\times10^{-6}\text{F})} + \frac{1}{(4.0\times10^{-6}\text{F})} = \frac{1}{1.33\times10^{-6}\text{F}}.$$

The charge on *either* capacitor in the branch on the left is

$$q = (7.5\times10^{-7}\text{F})(12\,\text{V}) = 9.0\times10^{-6}\text{C},$$

while the charge on *either* capacitor in the branch on the right is

$$q = (1.33\times10^{-6}\text{F})(12\,\text{V}) = 1.6\times10^{-5}\text{C}.$$

(b) After closing S_2 the circuit is effectively two capacitors in series. The top part has an effective capacitance of

$$C_\text{t} = (1.0\times10^{-6}\text{F}) + (2.0\times10^{-6}\text{F}) = (3.0\times10^{-6}\text{F}),$$

while the effective capacitance of the bottom part is

$$C_\text{b} = (3.0\times10^{-6}\text{F}) + (4.0\times10^{-6}\text{F}) = (7.0\times10^{-6}\text{F}).$$

The effective capacitance of the series combination is given by

$$\frac{1}{C_\text{eq}} = \frac{1}{(3.0\times10^{-6}\text{F})} + \frac{1}{(7.0\times10^{-6}\text{F})} = \frac{1}{2.1\times10^{-6}\text{F}}.$$

The charge on each part is $q = (2.1\times10^{-6}\text{F})(12\,\text{V}) = 2.52\times10^{-5}\text{C}$. The potential difference across the top part is

$$\Delta V_\text{t} = (2.52\times10^{-5}\text{C})/(3.0\times10^{-6}\text{F}) = 8.4\,\text{V},$$

and then the charge on the top two capacitors is $q_1 = (1.0\times10^{-6}\text{F})(8.4\,\text{V}) = 8.4\times10^{-6}\text{C}$ and $q_2 = (2.0\times10^{-6}\text{F})(8.4\,\text{V}) = 1.68\times10^{-5}\text{C}$. The potential difference across the bottom part is

$$\Delta V_\text{t} = (2.52\times10^{-5}\text{C})/(7.0\times10^{-6}\text{F}) = 3.6\,\text{V},$$

and then the charge on the top two capacitors is $q_1 = (3.0\times10^{-6}\text{F})(3.6\,\text{V}) = 1.08\times10^{-5}\text{C}$ and $q_2 = (4.0\times10^{-6}\text{F})(3.6\,\text{V}) = 1.44\times10^{-5}\text{C}$.

P30-10 Let $\Delta V = \Delta V_{xy}$. By symmetry $\Delta V_2 = 0$ and $\Delta V_1 = \Delta V_4 = \Delta V_5 = \Delta V_3 = \Delta V/2$. Suddenly the problem is *very* easy. The charges on each capacitor is q_1, except for $q_2 = 0$. Then the equivalent capacitance of the circuit is

$$C_\text{eq} = \frac{q}{\Delta V} = \frac{q_1 + q_4}{2\Delta V_1} = C_1 = 4.0\times10^{-6}\text{F}.$$

$\boxed{\textbf{P30-11}}$ (a) The charge on the capacitor with stored energy $U_0 = 4.0\,\text{J}$ is q_0, where

$$U_0 = \frac{q_0^2}{2C}.$$

When this capacitor is connected to an identical uncharged capacitor the charge is shared equally, so that the charge on either capacitor is now $q = q_0/2$. The stored energy in *one* capacitor is then

$$U = \frac{q^2}{2C} = \frac{q_0^2/4}{2C} = \frac{1}{4}U_0.$$

But there are two capacitors, so the total energy stored is $2U = U_0/2 = 2.0\,\text{J}$.

(b) Good question. Current had to flow through the connecting wires to get the charge from one capacitor to the other. Originally the second capacitor was uncharged, so the potential difference across that capacitor would have been zero, which means the potential difference across the connecting wires would have been equal to that of the first capacitor, and there would then have been energy dissipation in the wires according to

$$P = i^2 R.$$

That's where the missing energy went.

P30-12 $R = \rho L/A$ and $C = \epsilon_0 A/L$. Combining, $R = \rho\epsilon_0/C$, or

$$R = (9.40\,\Omega\cdot\text{m})(8.85\times10^{-12}\text{F/m})/(110\times10^{-12}\text{F}) = 0.756\,\Omega.$$

P30-13 (a) $u = \frac{1}{2}\epsilon_0 E^2 = e^2/32\pi^2\epsilon_0 r^4$.

(b) $U = \int u\,dV$ where $dV = 4\pi r^2 dr$. Then

$$U = 4pi\int_R^\infty \frac{e^2}{32\pi^2\epsilon_0 r^4} r^2\,dr = \frac{e^2}{8\pi\epsilon_0}\frac{1}{R}.$$

(c) $R = e^2/8\pi\epsilon_0 mc^2$, or

$$R = \frac{(1.60\times10^{-19}\text{C})^2}{8\pi(8.85\times10^{-12}\text{F/m})(9.11\times10^{-31}\text{kg})(3.00\times10^{8}\text{m/s})^2} = 1.40\times10^{-15}\text{m}.$$

P30-14 $U = \frac{1}{2}q^2/C = q^2x/2A\epsilon_0$. $F = dU/dx = q^2/2A\epsilon_0$.

$\boxed{\textbf{P30-15}}$ According to Problem 14, the force on a plate of a parallel plate capacitor is

$$F = \frac{q^2}{2\epsilon_0 A}.$$

The force per unit area is then

$$\frac{F}{A} = \frac{q^2}{2\epsilon_0 A^2} = \frac{\sigma^2}{2\epsilon_0},$$

where $\sigma = q/A$ is the surface charge density. But we know that the electric field near the surface of a conductor is given by $E = \sigma/\epsilon_0$, so

$$\frac{F}{A} = \frac{1}{2}\epsilon_0 E^2.$$

P30-16 A small surface area element dA carries a charge $dq = q\,dA/4\pi R^2$. There are three forces on the elements which balance, so

$$p(V_0/V)dA + q\,dq/4\pi\epsilon_0 R^2 = p\,dA,$$

or

$$pR_0^3 + q^2/16\pi^2\epsilon_0 R = pR^3.$$

This can be rearranged as

$$q^2 = 16\pi^2\epsilon_0 pR(R^3 - R_0^3).$$

P30-17 The magnitude of the electric field in the cylindrical region is given by $E = \lambda/2\pi\epsilon_0 r$, where λ is the linear charge density on the anode. The potential difference is given by $\Delta V = (\lambda/2\pi\epsilon_0)\ln(b/a)$, where a is the radius of the anode b the radius of the cathode. Combining, $E = \Delta V/r\ln(b/a)$, this will be a maximum when $r = a$, so

$$\Delta V = (0.180\times10^{-3}\mathrm{m})\ln[(11.0\times10^{-3}\mathrm{m})/(0.180\times10^{-3}\mathrm{m})](2.20\times10^{6}\mathrm{V/m}) = 1630\,\mathrm{V}.$$

P30-18 This is effectively two capacitors in parallel, each with an area of $A/2$. Then

$$C_{\mathrm{eq}} = \kappa_{\mathrm{e1}}\frac{\epsilon_0 A/2}{d} + \kappa_{\mathrm{e2}}\frac{\epsilon_0 A/2}{d} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_{\mathrm{e1}} + \kappa_{\mathrm{e2}}}{2}\right).$$

$\boxed{\textbf{P30-19}}$ We will treat the system as two capacitors in series by pretending there is an infinitesimally thin conductor between them. The slabs are (I assume) the same thickness. The capacitance of one of the slabs is then given by Eq. 30-31,

$$C_1 = \frac{\kappa_{\mathrm{e1}}\epsilon_0 A}{d/2},$$

where $d/2$ is the thickness of the slab. There would be a similar expression for the other slab. The equivalent series capacitance would be given by Eq. 30-21,

$$\begin{aligned}
\frac{1}{C_{\mathrm{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2},\\
&= \frac{d/2}{\kappa_{\mathrm{e1}}\epsilon_0 A} + \frac{d/2}{\kappa_{\mathrm{e2}}\epsilon_0 A},\\
&= \frac{d}{2\epsilon_0 A}\frac{\kappa_{\mathrm{e2}} + \kappa_{\mathrm{e1}}}{\kappa_{\mathrm{e1}}\kappa_{\mathrm{e2}}},\\
C_{\mathrm{eq}} &= \frac{2\epsilon_0 A}{d}\frac{\kappa_{\mathrm{e1}}\kappa_{\mathrm{e2}}}{\kappa_{\mathrm{e2}} + \kappa_{\mathrm{e1}}}.
\end{aligned}$$

P30-20 Treat this as three capacitors. Find the equivalent capacitance of the series combination on the right, and then add on the parallel part on the left. The right hand side is

$$\frac{1}{C_{\mathrm{eq}}} = \frac{d}{\kappa_{\mathrm{e2}}\epsilon_0 A/2} + \frac{d}{\kappa_{\mathrm{e3}}\epsilon_0 A/2} = \frac{2d}{\epsilon_0 A}\left(\frac{\kappa_{\mathrm{e2}} + \kappa_{\mathrm{e3}}}{\kappa_{\mathrm{e2}}\kappa_{\mathrm{e3}}}\right).$$

Add this to the left hand side, and

$$\begin{aligned}
C_{\mathrm{eq}} &= \frac{\kappa_{\mathrm{e1}}\epsilon_0 A/2}{2d} + \frac{\epsilon_0 A}{2d}\left(\frac{\kappa_{\mathrm{e2}}\kappa_{\mathrm{e3}}}{\kappa_{\mathrm{e2}} + \kappa_{\mathrm{e3}}}\right),\\
&= \frac{\epsilon_0 A}{2d}\left(\frac{\kappa_{\mathrm{e1}}}{2} + \frac{\kappa_{\mathrm{e2}}\kappa_{\mathrm{e3}}}{\kappa_{\mathrm{e2}} + \kappa_{\mathrm{e3}}}\right).
\end{aligned}$$

P30-21 (a) q doesn't change, but $C' = C/2$. Then $\Delta V' = q/C = 2\Delta V$.
(b) $U = C(\Delta V)^2/2 = \epsilon_0 A(\Delta V)^2/2d$. $U' = C'(\Delta V')^2/2 = \epsilon_0 A(2\Delta V)^2/4d = 2U$.
(c) $W = U' - U = 2U - U = U = \epsilon_0 A(\Delta V)^2/2d$.

P30-22 The total energy is $U = q\delta V/2 = (7.02\times 10^{-10}\mathrm{C})(52.3\,\mathrm{V})/2 = 1.84\times 10^{-8}\mathrm{J}$.
(a) In the air gap we have

$$U_{\mathrm{a}} = \frac{\epsilon_0 E_0^2 V}{2} = \frac{(8.85\times 10^{-12}\mathrm{F/m})(6.9\times 10^3\mathrm{V/m})^2(1.15\times 10^{-2}\mathrm{m}^2)(4.6\times 10^{-3}\mathrm{m})}{2} = 1.11\times 10^{-8}\mathrm{J}.$$

That is $(1.11/1.85) = 60\%$ of the total.
(b) The remaining 40% is in the slab.

P30-23 (a) $C = \epsilon_0 A/d = (8.85\times 10^{-12}\mathrm{F/m})(0.118\,\mathrm{m}^2)/(1.22\times 10^{-2}\mathrm{m}) = 8.56\times 10^{-11}\mathrm{F}$.
(b) Use the results of Problem 30-24.

$$C' = \frac{(4.8)(8.85\times 10^{-12}\mathrm{F/m})(0.118\,\mathrm{m}^2)}{(4.8)(1.22\times 10^{-2}\mathrm{m}) - (4.3\times 10^{-3}\mathrm{m})(4.8-1)} = 1.19\times 10^{-10}\mathrm{F}$$

(c) $q = C\Delta V = (8.56\times 10^{-11}\mathrm{F})(120\,\mathrm{V}) = 1.03\times 10^{-8}\mathrm{C}$; since the battery is disconnected $q' = q$.
(d) $E = q/\epsilon_0 A = (1.03\times 10^{-8}\mathrm{C})/(8.85\times 10^{-12}\mathrm{F/m})(0.118\,\mathrm{m}^2) = 9860\,\mathrm{V/m}$ in the space between the plates.
(e) $E' = E/\kappa_{\mathrm{e}} = (9860\,\mathrm{V/m})/(4.8) = 2050\,\mathrm{V/m}$ in the dielectric.
(f) $\Delta V' = q/C' = (1.03\times 10^{-8}\mathrm{C})/(1.19\times 10^{-10}\mathrm{F}) = 86.6\,\mathrm{V}$.
(g) $W = U' - U = q^2(1/C - 1/C')/2$, or

$$W = \frac{(1.03\times 10^{-8}\mathrm{C})^2}{2}[1/(8.56\times 10^{-11}\mathrm{F}) - 1/(1.19\times 10^{-10}\mathrm{F})] = 1.73\times 10^{-7}\mathrm{J}.$$

P30-24 The result is effectively three capacitors in series. Two are air filled with thicknesses of x and $d-b-x$, the third is dielectric filled with thickness b. All have an area A. The effective capacitance is given by

$$\begin{aligned}
\frac{1}{C} &= \frac{x}{\epsilon_0 A} + \frac{d-b-x}{\epsilon_0 A} + \frac{b}{\kappa_{\mathrm{e}}\epsilon_0 A}, \\
&= \frac{1}{\epsilon_0 A}\left((d-b) + \frac{b}{\kappa_{\mathrm{e}}}\right), \\
C &= \frac{\epsilon_0 A}{d-b+b/\kappa_{\mathrm{e}}}, \\
&= \frac{\kappa_{\mathrm{e}}\epsilon_0 A}{\kappa_{\mathrm{e}} - b(\kappa_{\mathrm{e}}-1)}.
\end{aligned}$$

E31-1 $(5.12\,\mathrm{A})(6.00\,\mathrm{V})(5.75\text{ min})(60\text{ s/min}) = 1.06\times10^4\,\mathrm{J}$.

E31-2 (a) $(12.0\,\mathrm{V})(1.60\times10^{-19}\mathrm{C}) = 1.92\times10^{-18}\mathrm{J}$.
(b) $(1.92\times10^{-18}\mathrm{J})(3.40\times10^{18}/\mathrm{s}) = 6.53\,\mathrm{W}$.

$\boxed{\textbf{E31-3}}$ If the energy is delivered at a rate of 110 W, then the current through the battery is

$$i = \frac{P}{\Delta V} = \frac{(110\,\mathrm{W})}{(12\,\mathrm{V})} = 9.17\,\mathrm{A}.$$

Current is the flow of charge in some period of time, so

$$\Delta t = \frac{\Delta q}{i} = \frac{(125\,\mathrm{A}\cdot\mathrm{h})}{(9.2\,\mathrm{A})} = 13.6\text{ h},$$

which is the same as 13 hours and 36 minutes.

E31-4 $(100\,\mathrm{W})(8\text{ h}) = 800\,\mathrm{W}\cdot\mathrm{h}$.
(a) $(800\,\mathrm{W}\cdot\mathrm{h})/(2.0\,\mathrm{W}\cdot\mathrm{h}) = 400$ batteries, at a cost of $(400)(\$0.80) = \$320$.
(b) $(800\,\mathrm{W}\cdot\mathrm{h})(\$0.12\times10^{-3}\,\mathrm{W}\cdot\mathrm{h}) = \$0.096$.

$\boxed{\textbf{E31-5}}$ Go all of the way around the circuit. It is a simple one loop circuit, and although it does not matter which way we go around, we will follow the direction of the larger emf. Then

$$(150\,\mathrm{V}) - i(2.0\,\Omega) - (50\,\mathrm{V}) - i(3.0\,\Omega) = 0,$$

where i is positive if it is counterclockwise. Rearranging,

$$100\,\mathrm{V} = i(5.0\,\Omega),$$

or $i = 20\,\mathrm{A}$.

Assuming the potential at P is $V_P = 100\,\mathrm{V}$, then the potential at Q will be given by

$$V_Q = V_P - (50\,\mathrm{V}) - i(3.0\,\Omega) = (100\,\mathrm{V}) - (50\,\mathrm{V}) - (20\,\mathrm{A})(3.0\,\Omega) = -10\,\mathrm{V}.$$

E31-6 (a) $R_{\mathrm{eq}} = (10\,\Omega) + (140\,\Omega) = 150\,\Omega$. $i = (12.0\,\mathrm{V})/(150\,\Omega) = 0.080\,\mathrm{A}$.
(b) $R_{\mathrm{eq}} = (10\,\Omega) + (80\,\Omega) = 90\,\Omega$. $i = (12.0\,\mathrm{V})/(90\,\Omega) = 0.133\,\mathrm{A}$.
(c) $R_{\mathrm{eq}} = (10\,\Omega) + (20\,\Omega) = 30\,\Omega$. $i = (12.0\,\mathrm{V})/(30\,\Omega) = 0.400\,\mathrm{A}$.

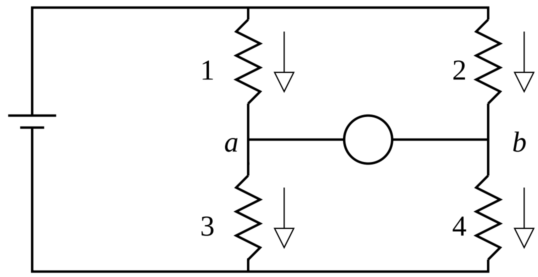
E31-7 (a) $R_{\mathrm{eq}} = (3.0\,\mathrm{V} - 2.0\,\mathrm{V})/(0.050\,\mathrm{A}) = 20\,\Omega$. Then $R = (20\,\Omega) - (3.0\,\Omega) - (3.0\,\Omega) = 14\,\Omega$.
(b) $P = i\Delta V = i^2R = (0.050\,\mathrm{A})^2(14\,\Omega) = 3.5\times10^{-2}\mathrm{W}$.

E31-8 $(5.0\,\mathrm{A})R_1 = \Delta V$. $(4.0\,\mathrm{A})(R_1 + 2.0\,\Omega) = \Delta V$. Combining, $5R_1 = 4R_1 + 8.0\,\Omega$, or $R_1 = 8.0\,\Omega$.

E31-9 (a) $(53.0\,\mathrm{W})/(1.20\,\mathrm{A}) = 44.2\,\mathrm{V}$.
(b) $(1.20\,\mathrm{A})(19.0\,\Omega) = 22.8\,\mathrm{V}$ is the potential difference across R. Then an additional potential difference of $(44.2\,\mathrm{V}) - (22.8\,\mathrm{V}) = 21.4\,\mathrm{V}$ must exist across C.
(c) The left side is positive; it is a reverse emf.

E31-10 (a) The current in the resistor is $\sqrt{(9.88\,\mathrm{W})/(0.108\,\Omega)} = 9.56\,\mathrm{A}$. The total resistance of the circuit is $(1.50\,\mathrm{V})/(9.56\,\mathrm{A}) = 0.157\,\Omega$. The internal resistance of the battery is then $(0.157\,\Omega) - (0.108\,\Omega) = 0.049\,\Omega$.
(b) $(9.88\,\mathrm{W})/(9.56\,\mathrm{A}) = 1.03\,\mathrm{V}$.

E31-11 We assign directions to the currents through the four resistors as shown in the figure.



Since the ammeter has no resistance the potential at a is the same as the potential at b. Consequently the potential difference ($\Delta V_{\rm b}$) across both of the bottom resistors is the same, and the potential difference ($\Delta V_{\rm t}$) across the two top resistors is also the same (but different from the bottom). We then have the following relationships:

$$\begin{aligned}\Delta V_{\rm t} + \Delta V_{\rm b} &= \mathcal{E},\\ i_1 + i_2 &= i_3 + i_4,\\ \Delta V_j &= i_j R_j,\end{aligned}$$

where the j subscript in the last line refers to resistor 1, 2, 3, or 4.

For the top resistors,

$$\Delta V_1 = \Delta V_2 \text{ implies } 2i_1 = i_2;$$

while for the bottom resistors,

$$\Delta V_3 = \Delta V_4 \text{ implies } i_3 = i_4.$$

Then the junction rule requires $i_4 = 3i_1/2$, and the loop rule requires

$$(i_1)(2R) + (3i_1/2)(R) = \mathcal{E} \text{ or } i_1 = 2\mathcal{E}/(7R).$$

The current that flows through the ammeter is the difference between i_2 and i_4, or $4\mathcal{E}/(7R) - 3\mathcal{E}/(7R) = \mathcal{E}/(7R)$.

E31-12 (a) Define the current i_1 as moving to the left through r_1 and the current i_2 as moving to the left through r_2. $i_3 = i_1 + i_2$ is moving to the right through R. Then there are two loop equations:

$$\begin{aligned}\mathcal{E}_1 &= i_1 r_1 + i_3 R,\\ \mathcal{E}_2 &= (i_3 - i_1) r_2 + i_3 R.\end{aligned}$$

Multiply the top equation by r_2 and the bottom by r_1 and then add:

$$r_2\mathcal{E}_1 + r_1\mathcal{E}_2 = i_3 r_1 r_2 + i_3 R(r_1 + r_2),$$

which can be rearranged as

$$i_3 = \frac{r_2\mathcal{E}_1 + r_1\mathcal{E}_2}{r_1 r_2 + R r_1 + R r_2}.$$

(b) There is only one current, so

$$\mathcal{E}_1 + \mathcal{E}_2 = i(r_1 + r_2 + R),$$

or

$$i = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R}.$$

$\boxed{\textbf{E31-13}}$ (a) Assume that the current flows through each source of emf in the same direction as the emf. The the loop rule will give us three equations

$$\begin{aligned}
\mathcal{E}_1 - i_1R_1 + i_2R_2 - \mathcal{E}_2 - i_1R_1 &= 0,\\
\mathcal{E}_2 - i_2R_2 + i_3R_1 - \mathcal{E}_3 + i_3R_1 &= 0,\\
\mathcal{E}_1 - i_1R_1 + i_3R_1 - \mathcal{E}_3 + i_3R_1 - i_1R_1 &= 0.
\end{aligned}$$

The junction rule (looks at point a) gives us $i_1 + i_2 + i_3 = 0$. Use this to eliminate i_2 from the second loop equation,

$$\mathcal{E}_2 + i_1R_2 + i_3R_2 + 2i_3R_1 - \mathcal{E}_3 = 0,$$

and then combine this with the the third equation to eliminate i_3,

$$\mathcal{E}_1R_2 - \mathcal{E}_3R_2 + 2i_3R_1R_2 + 2\mathcal{E}_2R_1 + 2i_3R_1R_2 + 4i_3R_1^2 - 2\mathcal{E}_3R_1 = 0,$$

or

$$i_3 = \frac{2\mathcal{E}_3R_1 + \mathcal{E}_3R_2 - \mathcal{E}_1R_2 - 2\mathcal{E}_2R_1}{4R_1R_2 + 4R_1^2} = 0.582\,\mathrm{A}.$$

Then we can find i_1 from

$$i_1 = \frac{\mathcal{E}_3 - \mathcal{E}_2 - i_3R_2 - 2i_3R_1}{R_2} = -0.668\,\mathrm{A},$$

where the negative sign indicates the current is *down*.

Finally, we can find $i_2 = -(i_1 + i_3) = 0.0854\,\mathrm{A}$.

(b) Start at a and go to b (final minus initial!),

$$+i_2R_2 - \mathcal{E}_2 = -3.60\,\mathrm{V}.$$

E31-14 (a) The current through the circuit is $i = \mathcal{E}/(r + R)$. The power delivered to R is then $P = i\Delta V = i^2R = \mathcal{E}^2R/(r + R)^2$. Evaluate dP/dR and set it equal to zero to find the maximum. Then

$$0 = \frac{dP}{dR} = \mathcal{E}^2R\frac{r - R}{(r + R)^3},$$

which has the solution $r = R$.

(b) When $r = R$ the power is

$$P = \mathcal{E}^2R\frac{1}{(R + R)^2} = \frac{\mathcal{E}^2}{4r}.$$

$\boxed{\textbf{E31-15}}$ (a) We first use $P = Fv$ to find the power output by the electric motor. Then $P = (2.0\,\mathrm{N})(0.50\,\mathrm{m/s}) = 1.0\,\mathrm{W}$.

The potential difference across the motor is $\Delta V_\mathrm{m} = \mathcal{E} - ir$. The power output from the motor is the rate of energy dissipation, so $P_\mathrm{m} = \Delta V_\mathrm{m}i$. Combining these two expressions,

$$\begin{aligned}
P_\mathrm{m} &= (\mathcal{E} - ir)\,i,\\
&= \mathcal{E}i - i^2r,\\
0 &= -i^2r + \mathcal{E}i - P_\mathrm{m},\\
0 &= (0.50\,\Omega)i^2 - (2.0\,\mathrm{V})i + (1.0\,\mathrm{W}).
\end{aligned}$$

Rearrange and solve for i,

$$i = \frac{(2.0\,\mathrm{V}) \pm \sqrt{(2.0\,\mathrm{V})^2 - 4(0.50\,\Omega)(1.0\,\mathrm{W})}}{2(0.50\,\Omega)},$$

which has solutions $i = 3.4\,\text{A}$ and $i = 0.59\,\text{A}$.

(b) The potential difference across the terminals of the motor is $\Delta V_{\text{m}} = \mathcal{E} - ir$ which if $i = 3.4\,\text{A}$ yields $\Delta V_{\text{m}} = 0.3\,\text{V}$, but if $i = 0.59\,\text{A}$ yields $\Delta V_{\text{m}} = 1.7\,\text{V}$. The battery provides an emf of 2.0 V; it isn't possible for the potential difference across the motor to be larger than this, but both solutions seem to satisfy this constraint, so we will move to the next part and see what happens.

(c) So what is the significance of the two possible solutions? It is a consequence of the fact that power is related to the current squared, and with any quadratics we expect two solutions. Both are possible, but it might be that only one is stable, or even that neither is stable, and a small perturbation to the friction involved in turning the motor will cause the system to break down. We will learn in a later chapter that the effective resistance of an electric motor depends on the speed at which it is spinning, and although that won't affect the problem here as worded, it will affect the physical problem that provided the numbers in this problem!

E31-16 $r_{\text{eq}} = 4r = 4(18\,\Omega) = 72\,\Omega$. The current is $i = (27\,\text{V})/(72\,\Omega) = 0.375\,\text{A}$.

$\boxed{\textbf{E31-17}}$ In parallel connections of two resistors the effective resistance is less than the smaller resistance but larger than half the smaller resistance. In series connections of two resistors the effective resistance is greater than the larger resistance but less than twice the larger resistance.

Since the effective resistance of the parallel combination is less than either single resistance and the effective resistance of the series combinations is larger than either single resistance we can conclude that $3.0\,\Omega$ must have been the parallel combination and $16\,\Omega$ must have been the series combination.

The resistors are then $4.0\,\Omega$ and $12\,\Omega$ resistors.

E31-18 Points B and C are effectively the same point!

(a) The three resistors are in parallel. Then $r_{\text{eq}} = R/3$.

(b) See (a).

(c) 0, since there is no resistance between B and C.

E31-19 Focus on the loop through the battery, the $3.0\,\Omega$, and the $5.0\,\Omega$ resistors. The loop rule yields

$$(12.0\,\text{V}) = i[(3.0\,\Omega) + (5.0\,\Omega)] = i(8.0\,\Omega).$$

The potential difference across the $5.0\,\Omega$ resistor is then

$$\Delta V = i(5.0\,\Omega) = (5.0\,\Omega)(12.0\,\text{V})/(8.0\,\Omega) = 7.5\,\text{V}.$$

E31-20 Each lamp draws a current of $(500\,\text{W})/(120\,\text{V}) = 4.17\,\text{A}$. Furthermore, the fuse can support $(15\,\text{A})/(4.17\,\text{A}) = 3.60$ lamps. That is a maximum of 3.

E31-21 The current in the series combination is $i_{\text{s}} = \mathcal{E}/(R_1 + R_2)$. The power dissipated is $P_{\text{s}} = i\mathcal{E} = \mathcal{E}^2/(R_1 + R_2)$.

In a parallel arrangement R_1 dissipates $P_1 = i_1\mathcal{E} = \mathcal{E}^2/R_1$. A similar expression exists for R_2, so the total power dissipated is $P_{\text{p}} = \mathcal{E}^2(1/R_1 + 1/R_2)$.

The ratio is 5, so $5 = P_{\text{p}}/P_{\text{s}} = (1/R_1 + 1/R_2)(R_1 + R_2)$, or $5R_1R_2 = (R_1 + R_2)^2$. Solving for R_2 yields $2.618R_1$ or $0.382R_1$. Then $R_2 = 262\,\Omega$ or $R_2 = 38.2\,\Omega$.

E31-22 Combining n identical resistors in series results in an equivalent resistance of $r_{\text{eq}} = nR$. Combining n identical resistors in parallel results in an equivalent resistance of $r_{\text{eq}} = R/n$. If the resistors are arranged in a square array consisting of n parallel branches of n series resistors, then the effective resistance is R. Each will dissipate a power P, together they will dissipate n^2P.

So we want nine resistors, since four would be too small.

E31-23 (a) Work through the circuit one step at a time. We first "add" R_2, R_3, and R_4 in parallel:

$$\frac{1}{R_{\text{eff}}} = \frac{1}{42.0\,\Omega} + \frac{1}{61.6\,\Omega} + \frac{1}{75.0\,\Omega} = \frac{1}{18.7\,\Omega}$$

We then "add" this resistance in series with R_1,

$$R_{\text{eff}} = (112\,\Omega) + (18.7\,\Omega) = 131\,\Omega.$$

(b) The current through the battery is $i = \mathcal{E}/R = (6.22\,\text{V})/(131\,\Omega) = 47.5$ mA. This is also the current through R_1, since all the current through the battery must also go through R_1.

The potential difference across R_1 is $\Delta V_1 = (47.5 \text{ mA})(112\,\Omega) = 5.32\,\text{V}$. The potential difference across each of the three remaining resistors is $6.22\,\text{V} - 5.32\,\text{V} = 0.90\,\text{V}$.

The current through each resistor is then

$$\begin{aligned} i_2 &= (0.90\,\text{V})/(42.0\,\Omega) = 21.4 \text{ mA}, \\ i_3 &= (0.90\,\text{V})/(61.6\,\Omega) = 14.6 \text{ mA}, \\ i_4 &= (0.90\,\text{V})/(75.0\,\Omega) = 12.0 \text{ mA}. \end{aligned}$$

E31-24 The equivalent resistance of the parallel part is $r' = R_2R/(R_2 + R)$. The equivalent resistance for the circuit is $r = R_1 + R_2R/(R_2 + R)$. The current through the circuit is $i' = \mathcal{E}/r$. The potential difference across R is $\Delta V = \mathcal{E} - i'R_1$, or

$$\begin{aligned} \Delta V &= \mathcal{E}(1 - R_1/r), \\ &= \mathcal{E}\left(1 - R_1\frac{R_2 + R}{R_1R_2 + R_1R + RR_2}\right), \\ &= \mathcal{E}\frac{RR_2}{R_1R_2 + R_1R + RR_2}. \end{aligned}$$

Since $P = i\Delta V = (\Delta V)^2/R$,

$$P = \mathcal{E}^2\frac{RR_2^2}{(R_1R_2 + R_1R + RR_2)^2}.$$

Set $dP/dR = 0$, the solution is $R = R_1R_2/(R_1 + R_2)$.

E31-25 (a) First "add" the left two resistors in series; the effective resistance of that branch is $2R$. Then "add" the right two resistors in series; the effective resistance of that branch is also $2R$.

Now we combine the three parallel branches and find the effective resistance to be

$$\frac{1}{R_{\text{eff}}} = \frac{1}{2R} + \frac{1}{R} + \frac{1}{2R} = \frac{4}{2R},$$

or $R_{\text{eff}} = R/2$.

(b) First we "add" the right two resistors in series; the effective resistance of that branch is $2R$. We then combine this branch with the resistor which connects points F and H. This is a parallel connection, so the effective resistance is

$$\frac{1}{R_{\text{eff}}} = \frac{1}{2R} + \frac{1}{R} = \frac{3}{2R},$$

or $2R/3$.

This value is effectively in series with the resistor which connects G and H, so the "total" is $5R/3$.

Finally, we can combine this value in parallel with the resistor that directly connects F and G according to

$$\frac{1}{R_{\text{eff}}} = \frac{1}{R} + \frac{3}{5R} = \frac{8}{5R},$$

or $R_{\text{eff}} = 5R/8$.

E31-26 The resistance of the second resistor is $r_2 = (2.4\,\text{V})/(0.001\,\text{A}) = 2400\,\Omega$. The potential difference across the first resistor is $(12\,\text{V}) - (2.4\,\text{V}) = 9.6\,\text{V}$. The resistance of the first resistor is $(9.6\,\text{V})/(0.001\,\text{A}) = 9600\,\Omega$.

E31-27 See Exercise 31-26. The resistance ratio is

$$\frac{r_1}{r_1 + r_2} = \frac{(0.95 \pm 0.1\,\text{V})}{(1.50\,\text{V})},$$

or

$$\frac{r_2}{r_1} = \frac{(1.50\,\text{V})}{(0.95 \pm 0.1\,\text{V})} - 1.$$

The allowed range for the ratio r_2/r_1 is between 0.5625 and 0.5957.

We can choose any standard resistors we want, and we could use any tolerance, but then we will need to check our results. 22Ω and 39Ω would work; as would 27Ω and 47Ω. There are other choices.

$\boxed{\textbf{E31-28}}$ Consider any junction other than A or B. Call this junction point 0; label the four nearest junctions to this as points 1, 2, 3, and 4. The current through the resistor that links point 0 to point 1 is $i_1 = \Delta V_{01}/R$, where ΔV_{01} is the potential difference across the resistor, so $\Delta V_{01} = V_0 - V_1$, where V_0 is the potential at the junction 0, and V_1 is the potential at the junction 1. Similar expressions exist for the other three resistor.

For the junction 0 the net current must be zero; there is no way for charge to accumulate on the junction. Then $i_1 + i_2 + i_3 + i_4 = 0$, and this means

$$\Delta V_{01}/R + \Delta V_{02}/R + \Delta V_{03}/R + \Delta V_{04}/R = 0$$

or

$$\Delta V_{01} + \Delta V_{02} + \Delta V_{03} + \Delta V_{04} = 0.$$

Let $\Delta V_{0i} = V_0 - V_i$, and then rearrange,

$$4V_0 = V_1 + V_2 + V_3 + V_4,$$

or

$$V_0 = \frac{1}{4}\left(V_1 + V_2 + V_3 + V_4\right).$$

$\boxed{\textbf{E31-29}}$ The current through the radio is $i = P/\Delta V = (7.5\,\text{W})/(9.0\,\text{V}) = 0.83\,\text{A}$. The radio was left one for 6 hours, or $2.16\times10^4\,\text{s}$. The total charge to flow through the radio in that time is $(0.83\,\text{A})(2.16\times10^4\,\text{s}) = 1.8\times10^4\,\text{C}$.

E31-30 The power dissipated by the headlights is $(9.7\,\text{A})(12.0\,\text{V}) = 116\,\text{W}$. The power required by the engine is $(116\,\text{W})/(0.82) = 142\,\text{W}$, which is equivalent to 0.190 hp.

E31-31 (a) $P = (120\,\mathrm{V})(120\,\mathrm{V})/(14.0\,\Omega) = 1030\,\mathrm{W}$.

(b) $W = (1030\,\mathrm{W})(6.42\ \mathrm{h}) = 6.61\ \mathrm{kW\cdot h}$. The cost is \$0.345.

E31-32

$\boxed{\textbf{E31-33}}$ We want to apply either Eq. 31-21,

$$P_R = i^2 R,$$

or Eq. 31-22,

$$P_R = (\Delta V_R)^2/R,$$

depending on whether we are in series (the current is the same through each bulb), or in parallel (the potential difference across each bulb is the same. The brightness of a bulb will be measured by P, even though P is not necessarily a measure of the rate radiant energy is emitted from the bulb.

(b) If the bulbs are in parallel then $P_R = (\Delta V_R)^2/R$ is how we want to compare the brightness. The potential difference across each bulb is the same, so the bulb with the smaller resistance is brighter.

(b) If the bulbs are in series then $P_R = i^2 R$ is how we want to compare the brightness. Both bulbs have the same current, so the larger value of R results in the brighter bulb.

One direct consequence of this can be tried at home. Wire up a 60 W, 120 V bulb and a 100 W, 120 V bulb in series. Which is brighter? You should observe that the 60 W bulb will be brighter.

E31-34 (a) $j = i/A = (25\,\mathrm{A})/\pi(0.05\ \mathrm{in}) = 3180\ \mathrm{A/in^2} = 4.93\times10^6\mathrm{A/m^2}$.

(b) $E = \rho j = (1.69\times10^{-8}\Omega\cdot\mathrm{m})(4.93\times10^6\mathrm{A/m^2}) = 8.33\times10^{-2}\mathrm{V/m}$.

(c) $\Delta V = Ed = (8.33\times10^{-2}\mathrm{V/m})(305\,\mathrm{m}) = 25\,\mathrm{V}$.

(d) $P = i\Delta V = (25\,\mathrm{A})(25\,\mathrm{V}) = 625\,\mathrm{W}$.

E31-35 (a) The bulb is on for 744 hours. The energy consumed is $(100\,\mathrm{W})(744\ \mathrm{h}) = 74.4\ \mathrm{kW\cdot h}$, at a cost of $(74.4)(0.06) = \$4.46$.

(b) $r = V^2/P = (120\,\mathrm{V})^2/(100\,\mathrm{W}) = 144\,\Omega$.

(c) $i = P/V = (100\,\mathrm{W})/(120\,\mathrm{V}) = 0.83\,\mathrm{A}$.

E31-36 $P = (\Delta V)^2/r$ and $r = r_0(1+\alpha\Delta T)$. Then

$$P = \frac{P_0}{1+\alpha\Delta T} = \frac{(500\,\mathrm{W})}{1+(4.0\times10^{-4}/\mathrm{C^\circ})(-600\mathrm{C^\circ})} = 660\,\mathrm{W}$$

E31-37 (a) $n = q/e = it/e$, so

$$n = (485\times10^{-3}\mathrm{A})(95\times10^{-9}\mathrm{s})/(1.6\times10^{-19}\mathrm{C}) = 2.88\times10^{11}.$$

(b) $i_{\mathrm{av}} = (520/\mathrm{s})(485\times10^{-3}\mathrm{A})(95\times10^{-9}\mathrm{s}) = 2.4\times10^{-5}\mathrm{A}$.

(c) $P_{\mathrm{p}} = i_{\mathrm{p}}\Delta V = (485\times10^{-3}\mathrm{A})(47.7\times10^6\mathrm{V}) = 2.3\times10^6\mathrm{W}$; while $P_{\mathrm{a}} = i_{\mathrm{a}}\Delta V = (2.4\times10^{-5}\mathrm{A})(47.7\times10^6\mathrm{V}) = 1.14\times10^3\mathrm{W}$.

E31-38 $r = \rho L/A = (3.5\times10^{-5}\Omega\cdot\mathrm{m})(1.96\times10^{-2}\mathrm{m})/\pi(5.12\times10^{-3}\mathrm{m})^2 = 8.33\times10^{-3}\Omega$.

(a) $i = \sqrt{P/r} = \sqrt{(1.55\,\mathrm{W})/(8.33\times10^{-3}\Omega)} = 13.6\,\mathrm{A}$, so

$$j = i/A = (13.6\,\mathrm{A})/\pi(5.12\times10^{-3}\mathrm{m})^2 = 1.66\times10^5\mathrm{A/m^2}.$$

(b) $\Delta V = \sqrt{Pr} = \sqrt{(1.55\,\mathrm{W})(8.33\times10^{-3}\Omega)} = 0.114\,\mathrm{V}$.

$\boxed{\textbf{E31-39}}$ (a) The current through the wire is

$$i = P/\Delta V = (4800\,\mathrm{W})/(75\,\mathrm{V}) = 64\,\mathrm{A},$$

The resistance of the wire is

$$R = \Delta V/i = (75\,\mathrm{V})/(64\,\mathrm{A}) = 1.17\,\Omega.$$

The length of the wire is then found from

$$L = \frac{RA}{\rho} = \frac{(1.17\,\Omega)(2.6\times10^{-6}\,\mathrm{m}^2)}{(5.0\times10^{-7}\,\Omega\mathrm{m})} = 6.1\,\mathrm{m}.$$

One could easily wind this much nichrome to make a toaster oven. Of course allowing 64 Amps to be drawn through household wiring will likely blow a fuse.

(b) We want to combine the above calculations into one formula, so

$$L = \frac{RA}{\rho} = \frac{A\Delta V/i}{\rho} = \frac{A(\Delta V)^2}{P\rho},$$

then

$$L = \frac{(2.6\times10^{-6}\,\mathrm{m}^2)(110\,\mathrm{V})^2}{(4800\,\mathrm{W})(5.0\times10^{-7}\,\Omega\mathrm{m})} = 13\,\mathrm{m}.$$

Hmm. We need more wire if the potential difference is increased? Does this make sense? Yes, it does. We need more wire because we need more resistance to *decrease* the current so that the same power output occurs.

E31-40 (a) The energy required to bring the water to boiling is $Q = mC\Delta T$. The time required is

$$t = \frac{Q}{0.77P} = \frac{(2.1\,\mathrm{kg})(4200\,\mathrm{J/kg})(100^\circ\mathrm{C} - 18.5^\circ\mathrm{C})}{0.77(420\,\mathrm{W})} = 2.22\times10^3\mathrm{s}$$

(b) The additional time required to boil half of the water away is

$$t = \frac{mL/2}{0.77P} = \frac{(2.1\,\mathrm{kg})(2.26\times10^6\mathrm{J/kg})/2}{0.77(420\,\mathrm{W})} = 7340\,\mathrm{s}.$$

$\boxed{\textbf{E31-41}}$ (a) Integrate both sides of Eq. 31-26;

$$\begin{aligned}
\int_0^q \frac{dq}{q - \mathcal{E}C} &= -\int_0^t \frac{dt}{RC}, \\
\ln(q - \mathcal{E}C)\Big|_0^q &= -\left.\frac{t}{RC}\right|_0^t, \\
\ln\left(\frac{q - \mathcal{E}C}{-\mathcal{E}C}\right) &= -\frac{t}{RC}, \\
\frac{q - \mathcal{E}C}{-\mathcal{E}C} &= e^{-t/RC}, \\
q &= \mathcal{E}C\left(1 - e^{-t/RC}\right).
\end{aligned}$$

That wasn't so bad, was it?

(b) Rearrange Eq. 31-26 in order to get q terms on the left and t terms on the right, then integrate;

$$\begin{aligned}
\int_{q_0}^{q} \frac{dq}{q} &= -\int_0^t \frac{dt}{RC}, \\
\ln q|_{q_0}^{q} &= -\left.\frac{t}{RC}\right|_0^t, \\
\ln\left(\frac{q}{q_0}\right) &= -\frac{t}{RC}, \\
\frac{q}{q_0} &= e^{-t/RC}, \\
q &= q_0 e^{-t/RC}.
\end{aligned}$$

That wasn't so bad either, was it?

E31-42 (a) $\tau_C = RC = (1.42\times10^6\Omega)(1.80\times10^{-6}\text{F}) = 2.56\,\text{s}$.
(b) $q_0 = C\Delta V = (1.80\times10^{-6}\text{F})(11.0\,\text{V}) = 1.98\times10^{-5}\text{C}$.
(c) $t = -\tau_C \ln(1 - q/q_0)$, so

$$t = -(2.56\,\text{s})\ln(1 - 15.5\times10^{-6}\text{C}/1.98\times10^{-5}\text{C}) = 3.91\,\text{s}.$$

E31-43 Solve $n = t/\tau_C = -\ln(1 - 0.99) = 4.61$.

E31-44 (a) $\Delta V = \mathcal{E}(1 - e^{-t/\tau_C})$, so

$$\tau_C = -(1.28\times10^{-6}\text{s})/\ln(1 - 5.00\,\text{V}/13.0\,\text{V}) = 2.64\times10^{-6}\text{s}$$

(b) $C = \tau_C/R = (2.64\times10^{-6}\text{s})/(15.2\times10^3\Omega) = 1.73\times10^{-10}\text{F}$

E31-45 (a) $\Delta V = \mathcal{E}e^{-t/\tau_C}$, so

$$\tau_C = -(10.0\,\text{s})/\ln(1.06\,\text{V}/100\,\text{V}) = 2.20\,\text{s}$$

(b) $\Delta V = (100\,\text{V})e^{-17\,\text{S}/2.20\,\text{S}} = 4.4\times10^{-2}\text{V}$.

E31-46 $\Delta V = \mathcal{E}e^{-t/\tau_C}$ and $\tau_C = RC$, so

$$R = -\frac{t}{C\ln(\Delta V/\Delta V_0)} = -\frac{t}{(220\times10^{-9}\text{F})\ln(0.8\,\text{V}/5\,\text{V})} = \frac{t}{4.03\times10^{-7}\text{F}}.$$

If t is between $10.0\,\mu\text{s}$ and $6.0\,\text{ms}$, then R is between

$$R = (10\times10^{-6}\text{s})/(4.03\times10^{-7}\text{F}) = 24.8\Omega,$$

and

$$R = (6\times10^{-3}\text{s})/(4.03\times10^{-7}\text{F}) = 14.9\times10^3\Omega.$$

$\boxed{\textbf{E31-47}}$ The charge on the capacitor needs to build up to a point where the potential across the capacitor is $V_L = 72\,\text{V}$, and this needs to happen within 0.5 seconds. This means that we want to solve

$$C\Delta V_L = C\mathcal{E}\left(1 - e^{T/RC}\right)$$

for R knowing that $T = 0.5\,\text{s}$. This expression can be written as

$$R = -\frac{T}{C\ln(1 - V_L/\mathcal{E})} = -\frac{(0.5\,\text{s})}{(0.15\,\mu\text{C})\ln(1 - (72\,\text{V})/(95\,\text{V}))} = 2.35\times10^6\Omega.$$

E31-48 (a) $q_0 = \sqrt{2UC} = \sqrt{2(0.50\,\mathrm{J})(1.0\times10^{-6}\mathrm{F})} = 1\times10^{-3}\mathrm{C}$.

(b) $i_0 = \Delta V_0/R = q_0/RC = (1\times10^{-3}\mathrm{C})/(1.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F}) = 1\times10^{-3}\mathrm{A}$.

(c) $\Delta V_C = \Delta V_0 e^{-t/\tau_C}$, so

$$\Delta V_C = \frac{(1\times10^{-3}\mathrm{C})}{(1.0\times10^{-6}\mathrm{F})}e^{-t/(1.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})} = (1000\,\mathrm{V})e^{-t/(1.0\,\mathrm{S})}$$

Note that $\Delta V_R = \Delta V_C$.

(d) $P_R = (\Delta V_R)^2/R$, so

$$P_R = (1000\,\mathrm{V})^2 e^{-2t/(1.0\,\mathrm{S})}/(1\times10^{6}\omega) = (1\,\mathrm{W})e^{-2t/(1.0\,\mathrm{S})}.$$

E31-49 (a) $i = dq/dt = \mathcal{E}e^{-t/\tau_C}/R$, so

$$i = \frac{(4.0\,\mathrm{V})}{(3.0\times10^{6}\Omega)}e^{-(1.0\,\mathrm{S})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})} = 9.55\times10^{-7}\mathrm{A}.$$

(b) $P_C = i\Delta V = (\mathcal{E}^2/R)e^{-t/\tau_C}(1-e^{-t/\tau_C})$, so

$$P_C = \frac{(4.0\,\mathrm{V})^2}{(3.0\times10^{6}\Omega)}e^{-(1.0\,\mathrm{S})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})}\left(1-e^{-(1.0\,\mathrm{S})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})}\right) = 1.08\times10^{-6}\mathrm{W}.$$

(c) $P_R = i^2R = (\mathcal{E}^2/R)e^{-2t/\tau_C}$, so

$$P_R = \frac{(4.0\,\mathrm{V})^2}{(3.0\times10^{6}\Omega)}e^{-2(1.0\,\mathrm{S})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})} = 2.74\times10^{-6}\mathrm{W}.$$

(d) $P = P_R + P_C$, or

$$P = 2.74\times10^{-6}\mathrm{W} + 1.08\times10^{-6}\mathrm{W} = 3.82\times10^{-6}\mathrm{W}$$

E31-50 The rate of energy dissipation in the resistor is

$$P_R = i^2R = (\mathcal{E}^2/R)e^{-2t/\tau_C}.$$

Evaluating

$$\int_0^\infty P_R dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/RC}dt = \frac{\mathcal{E}^2}{2}C,$$

but that is the original energy stored in the capacitor.

$\boxed{\textbf{P31-1}}$ The terminal voltage of the battery is given by $V = \mathcal{E} - ir$, so the internal resistance is

$$r = \frac{\mathcal{E}-V}{i} = \frac{(12.0\,\mathrm{V})-(11.4\,\mathrm{V})}{(50\,\mathrm{A})} = 0.012\,\Omega,$$

so the battery appears within specs.

The resistance of the wire is given by

$$R = \frac{\Delta V}{i} = \frac{(3.0\,\mathrm{V})}{(50\,\mathrm{A})} = 0.06\,\Omega,$$

so the cable appears to be bad.

What about the motor? Trying it,

$$R = \frac{\Delta V}{i} = \frac{(11.4\,\mathrm{V})-(3.0\,\mathrm{V})}{(50\,\mathrm{A})} = 0.168\,\Omega,$$

so it appears to be within spec.

P31-2 Traversing the circuit we have

$$\mathcal{E} - ir_1 + \mathcal{E} - ir_2 - iR = 0,$$

so $i = 2\mathcal{E}/(r_1 + r_2 + R)$. The potential difference across the first battery is then

$$\Delta V_1 = \mathcal{E} - ir_1 = \mathcal{E}\left(1 - \frac{2r_1}{r_1 + r_2 + R}\right) = \mathcal{E}\frac{r_2 - r_1 + R}{r_1 + r_2 + R}$$

This quantity will only vanish if $r_2 - r_1 + R = 0$, or $r_1 = R + r_2$. Since $r_1 > r_2$ this is actually possible; $R = r_1 - r_2$.

P31-3 $\Delta V = \mathcal{E} - ir_\mathrm{i}$ and $i = \mathcal{E}/(r_\mathrm{i} + R)$, so

$$\Delta V = \mathcal{E}\frac{R}{r_\mathrm{i} + R},$$

There are then two simultaneous equations:

$$(0.10\,\mathrm{V})(500\,\Omega) + (0.10\,\mathrm{V})r_\mathrm{i} = \mathcal{E}(500\,\Omega)$$

and

$$(0.16\,\mathrm{V})(1000\,\Omega) + (0.16\,\mathrm{V})r_\mathrm{i} = \mathcal{E}(1000\,\Omega),$$

with solution

(a) $r_\mathrm{i} = 1.5\times10^3\Omega$ and

(b) $\mathcal{E} = 0.400\,\mathrm{V}$.

(c) The cell receives energy from the sun at a rate $(2.0\ \mathrm{mW/cm^2})(5.0\ \mathrm{cm^2}) = 0.010\,\mathrm{W}$. The cell converts energy at a rate of $V^2/R = (0.16\,\mathrm{V})^2/(1000\,\Omega) = 0.26\,\%$

P31-4 (a) The emf of the battery can be found from

$$\mathcal{E} = ir_\mathrm{i} + \Delta V_1 = (10\,\mathrm{A})(0.05\,\Omega) + (12\,\mathrm{V}) = 12.5\,\mathrm{V}$$

(b) Assume that resistance is *not* a function of temperature. The resistance of the headlights is then

$$r_1 = (12.0\,\mathrm{V})/(10.0\,\mathrm{A}) = 1.2\,\Omega.$$

The potential difference across the lights when the starter motor is on is

$$\Delta V_1 = (8.0\,\mathrm{A})(1.2\,\Omega) = 9.6\,\mathrm{V},$$

and this is also the potential difference across the terminals of the battery. The current through the battery is then

$$i = \frac{\mathcal{E} - \Delta V}{r_\mathrm{i}} = \frac{(12.5\,\mathrm{V}) - (9.6\,\mathrm{V})}{(0.05\,\Omega)} = 58\,\mathrm{A},$$

so the current through the motor is 50 Amps.

P31-5 (a) The resistivities are

$$\rho_A = r_A A/L = (76.2\times10^{-6}\Omega)(91.0\times10^{-4}\mathrm{m^2})/(42.6\,\mathrm{m}) = 1.63\times10^{-8}\Omega\cdot\mathrm{m},$$

and

$$\rho_B = r_B A/L = (35.0\times10^{-6}\Omega)(91.0\times10^{-4}\mathrm{m^2})/(42.6\,\mathrm{m}) = 7.48\times10^{-9}\Omega\cdot\mathrm{m}.$$

(b) The current is $i = \Delta V/(r_A + r_B) = (630\,\mathrm{V})/(111.2\,\mu\Omega) = 5.67\times10^6\mathrm{A}$. The current density is then

$$j = (5.67\times10^6\mathrm{A})/(91.0\times10^{-4}\mathrm{m^2}) = 6.23\times10^8\mathrm{A/m^2}.$$

(c) $E_A = \rho_A j = (1.63\times10^{-8}\Omega\cdot\mathrm{m})(6.23\times10^8\mathrm{A/m^2}) = 10.2\,\mathrm{V/m}$ and $E_B = \rho_B j = (7.48\times10^{-9}\Omega\cdot\mathrm{m})(6.23\times10^8\mathrm{A/m^2}) = 4.66\,\mathrm{V/m}$.

(d) $\Delta V_A = E_A L = (10.2\,\mathrm{V/m})(42.6\,\mathrm{m}) = 435\,\mathrm{V}$ and $\Delta V_B = E_B L = (4.66\,\mathrm{V/m})(42.6\,\mathrm{m}) = 198\,\mathrm{V}$.

P31-6 Set up the problem with the traditional presentation of the Wheatstone bridge problem. Then the symmetry of the problem (flip it over on the line between x and y) implies that there is *no* current through r. As such, the problem is equivalent to two identical parallel branches each with two identical series resistances.

Each branch has resistance $R + R = 2R$, so the overall circuit has resistance

$$\frac{1}{R_{\text{eq}}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R},$$

so $R_{\text{eq}} = R$.

P31-7

P31-8 (a) The loop through R_1 is trivial: $i_1 = \mathcal{E}_2/R_1 = (5.0\,\text{V})/(100\,\Omega) = 0.05\,\text{A}$. The loop through R_2 is only slightly harder: $i_2 = (\mathcal{E}_2 + \mathcal{E}_3 - \mathcal{E}_1)/R_2 = 0.06\,\text{A}$.

(b) $\Delta V_{ab} = \mathcal{E}_3 + \mathcal{E}_2 = (5.0\,\text{V}) + (4.0\,\text{V}) = 9.0\,\text{V}$.

$\boxed{\textbf{P31-9}}$ (a) The three way light-bulb has two filaments (or so we are told in the question). There are four ways for these two filaments to be wired: either one alone, both in series, or both in parallel. Wiring the filaments in series will have the largest total resistance, and since $P = V^2/R$ this arrangement would result in the dimmest light. But we are told the light still operates at the lowest setting, and if a filament burned out in a series arrangement the light would go out.

We then conclude that the lowest setting is one filament, the middle setting is another filament, and the brightest setting is both filaments in parallel.

(b) The beauty of parallel settings is that then power is additive (it is also addictive, but that's a different field.) One filament dissipates 100 W at 120 V; the other filament (the one that burns out) dissipates 200 W at 120 V, and both together dissipate 300 W at 120 V.

The resistance of one filament is then

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\,\text{V})^2}{(100\,\text{W})} = 144\,\Omega.$$

The resistance of the other filament is

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\,\text{V})^2}{(200\,\text{W})} = 72\,\Omega.$$

P31-10 We can assume that R "contains" all of the resistance of the resistor, the battery and the ammeter, then

$$R = (1.50\,\text{V})/(1.0\,\text{m/A}) = 1500\,\Omega.$$

For each of the following parts we apply $R + r = \Delta V/i$, so

(a) $r = (1.5\,\text{V})/(0.1\,\text{mA}) - (1500\,\Omega) = 1.35\times10^4\Omega$,

(b) $r = (1.5\,\text{V})/(0.5\,\text{mA}) - (1500\,\Omega) = 1.5\times10^3\Omega$,

(c) $r = (1.5\,\text{V})/(0.9\,\text{mA}) - (1500\,\Omega) = 167\Omega$.

(d) $R = (1500\,\Omega) - (18.5\,\Omega) = 1482\,\Omega$

P31-11 (a) The effective resistance of the parallel branches on the middle and the right is

$$\frac{R_2 R_3}{R_2 + R_3}.$$

The effective resistance of the circuit as seen by the battery is then

$$R_1 + \frac{R_2R_3}{R_2+R_3} = \frac{R_1R_2+R_1R_3+R_2R_3}{R_2+R_3},$$

The current through the battery is

$$i = \mathcal{E}\frac{R_2+R_3}{R_1R_2+R_1R_3+R_2R_3},$$

The potential difference across R_1 is then

$$\Delta V_1 = \mathcal{E}\frac{R_2+R_3}{R_1R_2+R_1R_3+R_2R_3}R_1,$$

while $\Delta V_3 = \mathcal{E} - \Delta V_1$, or

$$\Delta V_3 = \mathcal{E}\frac{R_2R_3}{R_1R_2+R_1R_3+R_2R_3},$$

so the current through the ammeter is

$$i_3 = \frac{\Delta V_3}{R_3} = \mathcal{E}\frac{R_2}{R_1R_2+R_1R_3+R_2R_3},$$

or

$$i_3 = (5.0\,\mathrm{V})\frac{(4\,\Omega)}{(2\,\Omega)(4\,\Omega)+(2\,\Omega)(6\,\Omega)+(4\,\Omega)(6\,\Omega)} = 0.45\,\mathrm{A}.$$

(b) Changing the locations of the battery and the ammeter is equivalent to swapping R_1 and R_3. But since the expression for the current doesn't change, then the current is the same.

P31-12 $\Delta V_1 + \Delta V_2 = \Delta V_S + \Delta V_X$; if $V_a = V_b$, then $\Delta V_1 = \Delta V_S$. Using the first expression,

$$i_a(R_1+R_2) = i_b(R_S+R_X),$$

using the second,

$$i_aR_1 = i_bR_2.$$

Dividing the first by the second,

$$1 + R_2/R_1 = 1 + R_X/R_S,$$

or $R_X = R_S(R_2/R_1)$.

P31-13

P31-14 $L_v = \Delta Q/\Delta m$ and $\Delta Q/\Delta t = P = i\Delta V$, so

$$L_v = \frac{i\Delta V}{\Delta m/\Delta t} = \frac{(5.2\,\mathrm{A})(12\,\mathrm{V})}{(21\times10^{-6}\mathrm{kg/s})} = 2.97\times10^{6}\mathrm{J/kg}.$$

P31-15 $P = i^2R$. $W = p\Delta V$, where V is volume. $p = mg/A$ and $V = Ay$, where y is the height of the piston. Then $P = dW/dt = mgv$. Combining all of this,

$$v = \frac{i^2R}{mg} = \frac{(0.240\,\mathrm{A})^2(550\,\Omega)}{(11.8\,\mathrm{kg})(9.8\,\mathrm{m/s^2})} = 0.274\,\mathrm{m/s}.$$

P31-16 (a) Since $q = CV$, then

$$q = (32\times10^{-6}\text{F})\left[(6\,\text{V}) + (4\,\text{V/s})(0.5\,\text{s}) - (2\,\text{V/s}^2)(0.5\,\text{s})^2\right] = 2.4\times10^{-4}\text{C}.$$

(b) Since $i = dq/dt = C\,dV/dt$, then

$$i = (32\times10^{-6}\text{F})\left[(4\,\text{V/s}) - 2(2\,\text{V/s}^2)(0.5\,\text{s})\right] = 6.4\times10^{-5}\text{A}.$$

(c) Since $P = iV$,

$$P = [\left[(4\,\text{V/s}) - 2(2\,\text{V/s}^2)(0.5\,\text{s})\right]\left[(6\,\text{V}) + (4\,\text{V/s})(0.5\,\text{s}) - (2\,\text{V/s}^2)(0.5\,\text{s})^2\right] = 4.8\times10^{-4}\text{W}.$$

$\boxed{\textbf{P31-17}}$ (a) We have $P = 30P_0$ and $i = 4i_0$. Then

$$R = \frac{P}{i^2} = \frac{30P_0}{(4i_0)^2} = \frac{30}{16}R_0.$$

We don't really care what happened with the potential difference, since knowing the change in resistance of the wire should give all the information we need.

The volume of the wire is a constant, even upon drawing the wire out, so $LA = L_0A_0$; the product of the length and the cross sectional area must be a constant.

Resistance is given by $R = \rho L/A$, but $A = L_0A_0/L$, so the length of the wire is

$$L = \sqrt{\frac{A_0L_0R}{\rho}} = \sqrt{\frac{30}{16}\frac{A_0L_0R_0}{\rho}} = 1.37L_0.$$

(b) We know that $A = L_0A_0/L$, so

$$A = \frac{L}{L_0}A_0 = \frac{A_0}{1.37} = 0.73A_0.$$

P31-18 (a) The capacitor charge as a function of time is given by Eq. 31-27,

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right),$$

while the current through the circuit (and the resistor) is given by Eq. 31-28,

$$i = \frac{\mathcal{E}}{R}e^{-t/RC}.$$

The energy supplied by the emf is

$$U = \int \mathcal{E}i\,dt = \mathcal{E}\int dq = \mathcal{E}q;$$

but the energy in the capacitor is $U_C = q\Delta V/2 = \mathcal{E}q/2$.

(b) Integrating,

$$U_R = \int i^2 R dt = \frac{\mathcal{E}^2}{R}\int e^{-2t/RC}dt = \frac{\mathcal{E}^2}{2C} = \frac{\mathcal{E}q}{2}.$$

P31-19 The capacitor charge as a function of time is given by Eq. 31-27,

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right),$$

while the current through the circuit (and the resistor) is given by Eq. 31-28,

$$i = \frac{\mathcal{E}}{R} e^{-t/RC}.$$

The energy stored in the capacitor is given by

$$U = \frac{q^2}{2C},$$

so the rate that energy is being stored in the capacitor is

$$P_C = \frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} = \frac{q}{C} i.$$

The rate of energy dissipation in the resistor is

$$P_R = i^2 R,$$

so the time at which the rate of energy dissipation in the resistor is equal to the rate of energy storage in the capacitor can be found by solving

$$\begin{aligned}
P_C &= P_R, \\
i^2 R &= \frac{q}{C} i, \\
iRC &= q, \\
\mathcal{E} C e^{-t/RC} &= C\mathcal{E}\left(1 - e^{-t/RC}\right), \\
e^{-t/RC} &= 1/2, \\
t &= RC \ln 2.
\end{aligned}$$

$\boxed{\textbf{E32-1}}$ Apply Eq. 32-3, $\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$.

All of the paths which involve left hand turns are positive particles (path 1); those paths which involve right hand turns are negative particle (path 2 and path 4); and those paths which don't turn involve neutral particles (path 3).

E32-2 (a) The greatest magnitude of force is $F = qvB = (1.6\times10^{-19}\text{C})(7.2\times10^{6}\text{m/s})(83\times10^{-3}\text{T}) = 9.6\times10^{-14}\text{N}$. The least magnitude of force is 0.

(b) The force on the electron is $F = ma$; the angle between the velocity and the magnetic field is θ, given by $ma = qvB\sin\theta$. Then

$$\theta = \arcsin\left(\frac{(9.1\times10^{-31}\text{kg})(4.9\times10^{16}\text{m/s}^2)}{(1.6\times10^{-19}\text{C})(7.2\times10^{6}\text{m/s})(83\times10^{-3}\text{T})}\right) = 28^\circ.$$

E32-3 (a) $v = E/B = (1.5\times10^3\text{V/m})/(0.44\,\text{T}) = 3.4\times10^3\text{m/s}$.

E32-4 (a) $v = F/qB\sin\theta = (6.48\times10^{-17}\text{N}/(1.60\times10^{-19}\text{C})(2.63\times10^{-3}\text{T})\sin(23.0^\circ) = 3.94\times10^5\text{m/s}$.

(b) $K = mv^2/2 = (938\text{ MeV}/c^2)(3.94\times10^5\text{m/s})^2/2 = 809\text{ eV}$.

$\boxed{\textbf{E32-5}}$ The magnetic force on the proton is

$$F_B = qvB = (1.6\times10^{-19}\,\text{C})(2.8\times10^{7}\,\text{m/s})(30eex{-}6\,\text{T}) = 1.3\times10^{-16}\text{N}.$$

The gravitational force on the proton is

$$mg = (1.7\times10^{-27}\text{kg})(9.8\,\text{m/s}^2) = 1.7\times10^{-26}\,\text{N}.$$

The ratio is then 7.6×10^9. If, however, you carry the number of significant digits for the intermediate answers farther you will get the answer which is in the back of the book.

E32-6 The speed of the electron is given by $v = \sqrt{2q\Delta V/m}$, or

$$v = \sqrt{2(1000\text{ eV})/(5.1\times10^5\text{ eV}/c^2)} = 0.063c.$$

The electric field between the plates is $E = (100\,\text{V})/(0.020\,\text{m}) = 5000\,\text{V/m}$. The required magnetic field is then

$$B = E/v = (5000\,\text{V/m})/(0.063c) = 2.6\times10^{-4}\text{T}.$$

E32-7 Both have the same velocity. Then $K_\text{p}/K_\text{e} = m_\text{p}v^2/m_\text{e}v^2 = m_\text{p}/m_\text{e} =$.

E32-8 The speed of the ion is given by $v = \sqrt{2q\Delta V/m}$, or

$$v = \sqrt{2(10.8\text{ keV})/(6.01)(932\text{ MeV}/c^2)} = 1.96\times10^{-3}c.$$

The required electric field is $E = vB = (1.96\times10^{-3}c)(1.22\,\text{T}) = 7.17\times10^5\text{V/m}$.

$\boxed{\textbf{E32-9}}$ (a) For a charged particle moving in a circle in a magnetic field we apply Eq. 32-10;

$$r = \frac{mv}{|q|B} = \frac{(9.11\times10^{-31}\text{kg})(0.1)(3.00\times10^{8}\,\text{m/s})}{(1.6\times10^{-19}\,\text{C})(0.50\,\text{T})} = 3.4\times10^{-4}\,\text{m}.$$

(b) The (non-relativistic) kinetic energy of the electron is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.511\text{ MeV})(0.10c)^2 = 2.6\times10^{-3}\text{ MeV}.$$

E32-10 (a) $v = \sqrt{2K/m} = \sqrt{2(1.22\text{ keV})/(511\text{ keV}/c^2)} = 0.0691c$.
(b) $B = mv/qr = (9.11\times10^{-31}\text{kg})(0.0691c)/(1.60\times10^{-19}\text{C})(0.247\,\text{m}) = 4.78\times10^{-4}\text{T}$.
(c) $f = qB/2\pi m = (1.60\times10^{-19}\text{C})(4.78\times10^{-4}\text{T})/2\pi(9.11\times10^{-31}\text{kg}) = 1.33\times10^{7}$ Hz.
(d) $T = 1/f = 1/(1.33\times10^{7}\text{ Hz}) = 7.48\times10^{-8}\text{s}$.

E32-11 (a) $v = \sqrt{2K/m} = \sqrt{2(350\text{ eV})/(511\text{ keV}/c^2)} = 0.037c$.
(b) $r = mv/qB = (9.11\times10^{-31}\text{kg})(0.037c)/(1.60\times10^{-19}\text{C})(0.20\text{T}) = 3.16\times10^{-4}\text{m}$.

E32-12 The frequency is $f = (7.00)/(1.29\times10^{-3}\text{s}) = 5.43\times10^{3}$ Hz. The mass is given by $m = qB/2\pi f$, or

$$m = \frac{(1.60\times10^{-19}\text{C})(45.0\times10^{-3}\text{T})}{2\pi(5.43\times10^{3}\text{ Hz})} = 2.11\times10^{-25}\text{kg} = 127\text{ u}.$$

$\boxed{\textbf{E32-13}}$ (a) Apply Eq. 32-10, but rearrange it as

$$v = \frac{|q|rB}{m} = \frac{2(1.6\times10^{-19}\,\text{C})(0.045\,\text{m})(1.2\,\text{T})}{4.0(1.66\times10^{-27}\text{kg})} = 2.6\times10^{6}\,\text{m/s}.$$

(b) The speed is equal to the circumference divided by the period, so

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B} = \frac{2\pi 4.0(1.66\times10^{-27}\text{kg})}{2(1.6\times10^{-19}\,\text{C})(1.2\,\text{T})} = 1.1\times10^{-7}\,\text{s}.$$

(c) The (non-relativistic) kinetic energy is

$$K = \frac{|q|^2r^2B}{2m} = \frac{(2\times1.6\times10^{-19}\,\text{C})^2(0.045\,\text{m})^2(1.2\,\text{T})^2}{2(4.0\times1.66\times10^{-27}\text{kg}))} = 2.24\times10^{-14}\,\text{J}.$$

To change to electron volts we need merely divide this answer by the charge on one electron, so

$$K = \frac{(2.24\times10^{-14}\,\text{J})}{(1.6\times10^{-19}\,\text{C})} = 140\text{ keV}.$$

(d) $\Delta V = \frac{K}{q} = (140\text{ keV})/(2e) = 70\,\text{V}$.

E32-14 (a) $R = mv/qB = (938\text{ MeV}/c^2)(0.100c)/e(1.40\,\text{T}) = 0.223\,\text{m}$.
(b) $f = qB/2\pi m = e(1.40\,\text{T})/2\pi(938\text{ MeV}/c^2) = 2.13\times10^{7}$ Hz.

E32-15 (a) $K_\alpha/K_\text{p} = (q_\alpha^2/m_\alpha)/({q_\text{p}}^2/m_\text{p}) = 2^2/4 = 1$.
(b) $K_\text{d}/K_\text{p} = ({q_\text{d}}^2/m_\text{d})/({q_\text{p}}^2/m_\text{p}) = 1^2/2 = 1/2$.

E32-16 (a) $K = q\Delta V$. Then $K_\text{p} = e\Delta V$, $K_\text{d} = e\Delta V$, and $K_\alpha = 2e\Delta V$.
(b) $r = sqrt2mK/qB$. Then $r_\text{d}/r_\text{p} = \sqrt{(2/1)(1/1)}/(1/1) = \sqrt{2}$.
(c) $r = sqrt2mK/qB$. Then $r_\alpha/r_\text{p} = \sqrt{(4/1)(2/1)}/(2/1) = \sqrt{2}$.

$\boxed{\textbf{E32-17}}$ $r = \sqrt{2mK}/|q|B = (\sqrt{m}/|q|)(\sqrt{2K}/B)$. All three particles are traveling with the same kinetic energy in the same magnetic field. The relevant factors are in front; we just need to compare the mass and charge of each of the three particles.
(a) The radius of the deuteron path is $\frac{\sqrt{2}}{1}r_\text{p}$.
(b) The radius of the alpha particle path is $\frac{\sqrt{4}}{2}r_\text{p} = r_\text{p}$.

E32-18 The neutron, being neutral, is unaffected by the magnetic field and moves off in a line tangent to the original path. The proton moves at the same original speed as the deuteron and has the same charge, but since it has half the mass it moves in a circle with half the radius.

E32-19 (a) The proton momentum would be $pc = qcBR = e(3.0{\times}10^8\text{m/s})(41{\times}10^{-6}\text{T})(6.4{\times}10^6\text{m}) = 7.9{\times}10^4$ MeV. Since 79000 MeV is much, much greater than 938 MeV the proton is ultra-relativistic. Then $E \approx pc$, and since $\gamma = E/mc^2$ we have $\gamma = p/mc$. Inverting,

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{m^2c^2}{p^2}} \approx 1 - \frac{m^2c^2}{2p^2} \approx 0.99993.$$

E32-20 (a) Classically, $R = \sqrt{2mK}/qB$, or

$$R = \sqrt{2(0.511\ \text{MeV}/c^2)(10.0\ \text{MeV})}/e(2.20\,\text{T}) = 4.84\times10^{-3}\text{m}.$$

(b) This would be an ultra-relativistic electron, so $K \approx E \approx pc$, then $R = p/qB = K/qBc$, or

$$R = (10.0\ \text{MeV})/e(2.2\,\text{T})(3.00\times10^8\text{m/s}) = 1.52\times10^{-2}\text{m}.$$

(c) The electron is effectively traveling at the speed of light, so $T = 2\pi R/c$, or

$$T = 2\pi(1.52\times10^{-2}\text{m})/(3.00\times10^8\ \text{m/s}) = 3.18\times10^{-10}\text{s}.$$

This result *does* depend on the speed!

$\boxed{\textbf{E32-21}}$ Use Eq. 32-10, except we rearrange for the mass,

$$m = \frac{|q|rB}{v} = \frac{2(1.60\times10^{-19}\,\text{C})(4.72\,\text{m})(1.33\,\text{T})}{0.710(3.00\times10^8\ \text{m/s})} = 9.43\times10^{-27}\,\text{kg}$$

However, if it is moving at this velocity then the "mass" which we have here is not the true mass, but a relativistic correction. For a particle moving at $0.710c$ we have

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (0.710)^2}} = 1.42,$$

so the true mass of the particle is $(9.43\times10^{-27}\,\text{kg})/(1.42) = 6.64\times10^{-27}$kg. The number of nucleons present in this particle is then $(6.64\times10^{-27}\text{kg})/(1.67\times10^{-27}\,\text{kg}) = 3.97 \approx 4$. The charge was $+2$, which implies two protons, the other two nucleons would be neutrons, so this must be an alpha particle.

E32-22 (a) Since 950 GeV is much, much greater than 938 MeV the proton is ultra-relativistic. $\gamma = E/mc^2$, so

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{m^2c^4}{E^2}} \approx 1 - \frac{m^2c^4}{2E^2} \approx 0.9999995.$$

(b) Ultra-relativistic motion requires $pc \approx E$, so

$$B = pc/qRc = (950\ \text{GeV})/e(750\,\text{m})(3.00\times10^8\text{m/s}) = 4.44\,\text{T}.$$

E32-23 First use $2\pi f = qB/m$. The use $K = q^2B^2R^2/2m = mR^2(2\pi f)^2/2$. The number of turns is $n = K/2q\Delta V$, on average the particle is located at a distance $R/\sqrt{2}$ from the center, so the distance traveled is $x = n2\pi R/\sqrt{2} = n\sqrt{2}\pi R$. Combining,

$$x = \frac{\sqrt{2}\pi^3 R^3 m f^2}{q\Delta V} = \frac{\sqrt{2}\pi^3(0.53\,\mathrm{m})^3(2\times 932\times 10^3\ \mathrm{keV}/c^2)(12\times 10^6/\mathrm{s})^2}{e(80\ \mathrm{kV})} = 240\,\mathrm{m}.$$

E32-24 The particle moves in a circle. $x = R\sin\omega t$ and $y = R\cos\omega t$.

E32-25 We will use Eq. 32-20, $E_\mathrm{H} = v_\mathrm{d}B$, except we will not take the derivation through to Eq. 32-21. Instead, we will set the drift velocity equal to the speed of the strip. We will, however, set $E_\mathrm{H} = \Delta V_\mathrm{H}/w$. Then

$$v = \frac{E_\mathrm{H}}{B} = \frac{\Delta V_\mathrm{H}/w}{B} = \frac{(3.9\times 10^{-6}\mathrm{V})/(0.88\times 10^{-2}\mathrm{m})}{(1.2\times 10^{-3}\mathrm{T})} = 3.7\times 10^{-1}\mathrm{m/s}.$$

E32-26 (a) $v = E/B = (40\times 10^{-6}\mathrm{V})/(1.2\times 10^{-2}\mathrm{m})/(1.4\,\mathrm{T}) = 2.4\times 10^{-3}\mathrm{m/s}$.
(b) $n = (3.2\,\mathrm{A})(1.4\,\mathrm{T})/(1.6\times 10^{-19}\mathrm{C})(9.5\times 10^{-6}\mathrm{m})(40\times 10^{-6}\mathrm{V}) = 7.4\times 10^{28}/\mathrm{m}^3$.; Silver.

E32-27 $E_\mathrm{H} = v_\mathrm{d}B$ and $v_\mathrm{d} = j/ne$. Combine and rearrange.

E32-28 (a) Use the result of the previous exercise and $E_\mathrm{c} = \rho j$.
(b) $(0.65\,\mathrm{T})/(8.49\times 10^{28}/\mathrm{m}^3)(1.60\times 10^{-19}\mathrm{C})(1.69\times 10^{-8}\Omega\cdot\mathrm{m}) = 0.0028$.

E32-29 Since $\vec{\mathbf{L}}$ is perpendicular to $\vec{\mathbf{B}}$ can use

$$F_B = iLB.$$

Equating the two forces,

$$\begin{aligned} iLB &= mg, \\ i &= \frac{mg}{LB} = \frac{(0.0130\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(0.620\,\mathrm{m})(0.440\,\mathrm{T})} = 0.467\,\mathrm{A}. \end{aligned}$$

Use of an appropriate right hand rule will indicate that the current must be directed to the right in order to have a magnetic force directed upward.

E32-30 $F = iLB\sin\theta = (5.12\times 10^3\mathrm{A})(100\,\mathrm{m})(58\times 10^{-6}\mathrm{T})\sin(70^\circ) = 27.9\,\mathrm{N}$. The direction is horizontally west.

E32-31 (a) We use Eq. 32-26 again, and since the (horizontal) axle is perpendicular to the vertical component of the magnetic field,

$$i = \frac{F}{BL} = \frac{(10,000\,\mathrm{N})}{(10\,\mu\mathrm{T})(3.0\,\mathrm{m})} = 3.3\times 10^8\mathrm{A}.$$

(b) The power lost per ohm of resistance in the rails is given by

$$P/r = i^2 = (3.3\times 10^8\mathrm{A})^2 = 1.1\times 10^{17}\,\mathrm{W}.$$

(c) If such a train were to be developed the rails would melt well before the train left the station.

E32-32 $F = idB$, so $a = F/m = idB/m$. Since a is constant, $v = at = idBt/m$. The direction is to the left.

E32-33 Only the $\hat{\mathbf{j}}$ component of $\vec{\mathbf{B}}$ is of interest. Then $F = \int dF = i \int B_y\, dx$, or

$$F = (5.0\,\mathrm{A})(8\times 10^{-3}\mathrm{T/m^2}) \int_{1.2}^{3.2} x^2\, dx = 0.414\,\mathrm{N}.$$

The direction is $-\hat{\mathbf{k}}$.

E32-34 The magnetic force will have two components: one will lift vertically ($F_y = F \sin\alpha$), the other push horizontally ($F_x = F\cos\alpha$). The rod will move when $F_x > \mu(W - F_y)$. We are interested in the minimum value for F as a function of α. This occurs when

$$\frac{dF}{d\alpha} = \frac{d}{d\alpha}\left(\frac{\mu W}{\cos\alpha + \mu\sin\alpha}\right) = 0.$$

This happens when $\mu = \tan\alpha$. Then $\alpha = \arctan(0.58) = 30°$, and

$$F = \frac{(0.58)(1.15\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{\cos(30°) + (0.58)\sin(30°)} = 5.66\,\mathrm{N}$$

is the minimum force. Then $B = (5.66\,\mathrm{N})/(53.2\,\mathrm{A})(0.95\,\mathrm{m}) = 0.112\,\mathrm{T}$.

E32-35 We choose that the field points from the shorter side to the longer side.

(a) The magnetic field is parallel to the 130 cm side so there is no magnetic force on that side.

The magnetic force on the 50 cm side has magnitude

$$F_B = iLB\sin\theta,$$

where θ is the angle between the 50 cm side and the magnetic field. This angle is larger than 90°, but the sine can be found directly from the triangle,

$$\sin\theta = \frac{(120\ \mathrm{cm})}{(130\ \mathrm{cm})} = 0.923,$$

and then the force on the 50 cm side can be found by

$$F_B = (4.00\,\mathrm{A})(0.50\,\mathrm{m})(75.0\times 10^{-3}\,\mathrm{T})\frac{(120\ \mathrm{cm})}{(130\ \mathrm{cm})} = 0.138\,\mathrm{N},$$

and is directed out of the plane of the triangle.

The magnetic force on the 120 cm side has magnitude

$$F_B = iLB\sin\theta,$$

where θ is the angle between the 1200 cm side and the magnetic field. This angle is larger than 180°, but the sine can be found directly from the triangle,

$$\sin\theta = \frac{(-50\ \mathrm{cm})}{(130\ \mathrm{cm})} = -0.385,$$

and then the force on the 50 cm side can be found by

$$F_B = (4.00\,\mathrm{A})(1.20\,\mathrm{m})(75.0\times 10^{-3}\,\mathrm{T})\frac{(-50\ \mathrm{cm})}{(130\ \mathrm{cm})} = -0.138\,\mathrm{N},$$

and is directed into the plane of the triangle.

(b) Look at the three numbers above.

E32-36 $\tau = NiAB\sin\theta$, so

$$\tau = (20)(0.1\,\mathrm{A})(0.12\,\mathrm{m})(0.05\,\mathrm{m})(0.5\,\mathrm{T})\sin(90^\circ - 33^\circ) = 5.0\times10^{-3}\mathrm{N}\cdot\mathrm{m}.$$

E32-37 The external magnetic field must be in the plane of the clock/wire loop. The clockwise current produces a magnetic dipole moment directed into the plane of the clock.

(a) Since the magnetic field points along the 1 pm line and the torque is perpendicular to both the external field and the dipole, then the torque must point along either the 4 pm or the 10 pm line. Applying Eq. 32-35, the direction is along the 4 pm line. It will take the minute hand 20 minutes to get there.

(b) $\tau = (6)(2.0\,\mathrm{A})\pi(0.15\,\mathrm{m})^2(0.07\,\mathrm{T}) = 0.059\,\mathrm{N}\cdot\mathrm{m}$.

P32-1 Since $\vec{\mathbf{F}}$ must be perpendicular to $\vec{\mathbf{B}}$ then $\vec{\mathbf{B}}$ must be along $\hat{\mathbf{k}}$. The magnitude of v is $\sqrt{(40)^2 + (35)^2}$ km/s $= 53.1$ km/s; the magnitude of F is $\sqrt{(-4.2)^2 + (4.8)^2}$fN $= 6.38$ fN. Then

$$B = F/qv = (6.38\times10^{-15}\mathrm{N})/(1.6\times10^{-19}\mathrm{C})(53.1\times10^3\mathrm{m/s}) = 0.75\,\mathrm{T}.$$

or $\vec{\mathbf{B}} = 0.75\,\mathrm{T}\,\hat{\mathbf{k}}$.

P32-2 $\vec{\mathbf{a}} = (q/m)(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}})$. For the initial velocity given,

$$\vec{\mathbf{v}}\times\vec{\mathbf{B}} = (15.0\times10^3\mathrm{m/s})(400\times10^{-6}\mathrm{T})\hat{\mathbf{j}} - (12.0\times10^3\mathrm{m/s})(400\times10^{-6}\mathrm{T})\hat{\mathbf{k}}.$$

But since there is no acceleration in the $\hat{\mathbf{j}}$ or $\hat{\mathbf{k}}$ direction this must be offset by the electric field. Consequently, two of the electric field components are $E_y = -6.00\,\mathrm{V/m}$ and $E_z = 4.80\,\mathrm{V/m}$. The third component of the electric field is the source of the acceleration, so

$$E_x = ma_x/q = (9.11\times10^{-31}\mathrm{kg})(2.00\times10^{12}\mathrm{m/s^2})/(-1.60\times10^{-19}\mathrm{C}) = -11.4\,\mathrm{V/m}.$$

P32-3 (a) Consider first the cross product, $\vec{\mathbf{v}}\times\vec{\mathbf{B}}$. The electron moves horizontally, there is a component of the $\vec{\mathbf{B}}$ which is down, so the cross product results in a vector which points to the left of the electron's path.

But the force on the electron is given by $\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$, and since the electron has a negative charge the force on the electron would be directed to the *right* of the electron's path.

(b) The kinetic energy of the electrons is much less than the rest mass energy, so this is non-relativistic motion. The speed of the electron is then $v = \sqrt{2K/m}$, and the magnetic force on the electron is $F_B = qvB$, where we are assuming $\sin\theta = 1$ because the electron moves horizontally through a magnetic field with a vertical component. We can ignore the effect of the magnetic field's horizontal component because the electron is moving parallel to this component.

The acceleration of the electron because of the magnetic force is then

$$\begin{aligned} a &= \frac{qvB}{m} = \frac{qB}{m}\sqrt{\frac{2K}{m}}, \\ &= \frac{(1.60\times10^{-19}\mathrm{C})(55.0\times10^{-6}\mathrm{T})}{(9.11\times10^{-31}\mathrm{kg})}\sqrt{\frac{2(1.92\times10^{-15}\mathrm{J})}{(9.11\times10^{-31}\mathrm{kg})}} = 6.27\times10^{14}\,\mathrm{m/s^2}. \end{aligned}$$

(c) The electron travels a horizontal distance of 20.0 cm in a time of

$$t = \frac{(20.0\text{ cm})}{\sqrt{2K/m}} = \frac{(20.0\text{ cm})}{\sqrt{2(1.92\times10^{-15}\mathrm{J})/(9.11\times10^{-31}\mathrm{kg})}} = 3.08\times10^{-9}\,\mathrm{s}.$$

In this time the electron is accelerated to the side through a distance of

$$d = \frac{1}{2}at^2 = \frac{1}{2}(6.27\times10^{14}\,\mathrm{m/s^2})(3.08\times10^{-9}\,\mathrm{s})^2 = 2.98\text{ mm}.$$

P32-4 (a) d needs to be larger than the turn radius, so $R \le d$; but $2mK/q^2B^2 = R^2 \le d^2$, or $B \ge \sqrt{2mK/q^2d^2}$.

(b) Out of the page.

P32-5 Only undeflected ions emerge from the velocity selector, so $v = E/B$. The ions are then deflected by B' with a radius of curvature of $r = mv/qB$; combining and rearranging, $q/m = E/rBB'$.

P32-6 The ions are given a kinetic energy $K = q\Delta V$; they are then deflected with a radius of curvature given by $R^2 = 2mK/q^2B^2$. But $x = 2R$. Combine all of the above, and $m = B^2qx^2/8\Delta V$.

$\boxed{\textbf{P32-7}}$ (a) Start with the equation in Problem 6, and take the square root of both sides to get

$$\sqrt{m} = \left(\frac{B^2q}{8\Delta V}\right)^{\frac{1}{2}} x,$$

and then take the derivative of x with respect to m,

$$\frac{1}{2}\frac{dm}{\sqrt{m}} = \left(\frac{B^2q}{8\Delta V}\right)^{\frac{1}{2}} dx,$$

and then consider finite differences instead of differential quantities,

$$\Delta m = \left(\frac{mB^2q}{2\Delta V}\right)^{\frac{1}{2}} \Delta x,$$

(b) Invert the above expression,

$$\Delta x = \left(\frac{2\Delta V}{mB^2q}\right)^{\frac{1}{2}} \Delta m,$$

and then put in the given values,

$$\begin{aligned}\Delta x &= \left(\frac{2(7.33\times10^3\text{V})}{(35.0)(1.66\times10^{-27}\text{kg})(0.520\,\text{T})^2(1.60\times10^{-19}\text{C})}\right)^{\frac{1}{2}}(2.0)(1.66\times10^{-27}\text{kg}),\\ &= 8.02\text{ mm}.\end{aligned}$$

Note that we used 35.0 u for the mass; if we had used 37.0 u the result would have been closer to the answer in the back of the book.

P32-8 (a) $B = \sqrt{2\Delta Vm/qr^2} = \sqrt{2(0.105\text{ MV})(238)(932\text{ MeV}/c^2)/2e(0.973\,\text{m})^2} = 5.23\times10^{-7}\text{T}$.

(b) The number of atoms in a gram is $6.02\times10^{23}/238 = 2.53\times10^{21}$. The current is then

$$(0.090)(2.53\times10^{21})(2)(1.6\times10^{-19}\text{C})/(3600\,\text{s}) = 20.2\text{ mA}.$$

P32-9 (a) $-q$.

(b) Regardless of speed, the orbital period is $T = 2\pi m/qB$. But they collide halfway around a complete orbit, so $t = \pi m/qB$.

P32-10

$\boxed{\textbf{P32-11}}$ (a) The period of motion can be found from the reciprocal of Eq. 32-12,

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi(9.11\times10^{-31}\text{kg})}{(1.60\times10^{-19}\text{C})(455\times10^{-6}\text{T})} = 7.86\times10^{-8}\text{s}.$$

(b) We need to find the velocity of the electron from the kinetic energy,

$$v = \sqrt{2K/m} = \sqrt{2(22.5\text{ eV})(1.60\times10^{-19}\text{ J/eV})/(9.11\times10^{-31}\text{kg})} = 2.81\times10^{6}\text{ m/s}.$$

The velocity can written in terms of components which are parallel and perpendicular to the magnetic field. Then

$$v_{||} = v\cos\theta \text{ and } v_{\perp} = v\sin\theta.$$

The pitch is the parallel distance traveled by the electron in one revolution, so

$$p = v_{||}T = (2.81\times10^{6}\text{m/s})\cos(65.5^\circ)(7.86\times10^{-8}\text{s}) = 9.16\text{ cm}.$$

(c) The radius of the helical path is given by Eq. 32-10, except that we use the perpendicular velocity component, so

$$R = \frac{mv_{\perp}}{|q|B} = \frac{(9.11\times10^{-31}\text{kg})(2.81\times10^{6}\text{m/s})\sin(65.5^\circ)}{(1.60\times10^{-19}\text{C})(455\times10^{-6}\text{T})} = 3.20\text{ cm}$$

P32-12 $\vec{\mathbf{F}} = i\int_a^b d\vec{\mathbf{l}}\times\vec{\mathbf{B}}$. $d\vec{\mathbf{l}}$ has two components, those parallel to the path, say $d\vec{\mathbf{x}}$ and those perpendicular, say $d\vec{\mathbf{y}}$. Then the integral can be written as

$$\vec{\mathbf{F}} = \int_a^b d\vec{\mathbf{x}}\times\vec{\mathbf{B}} + \int_a^b d\vec{\mathbf{y}}\times\vec{\mathbf{B}}.$$

But $\vec{\mathbf{B}}$ is constant, and can be removed from the integral. $\int_a^b d\vec{\mathbf{x}} = \vec{\mathbf{l}}$, a vector that points from a to b. $\int_a^b d\vec{\mathbf{y}} = 0$, because there is no net motion perpendicular to $\vec{\mathbf{l}}$.

P32-13 $qv_yB = F_x = m\,dv_x/dt$; $-qv_xB = F_y = m\,dv_y/dt$. Taking the time derivative of the second expression and inserting into the first we get

$$qv_yB = m\left(-\frac{m}{qB}\right)\frac{d^2v_y}{dt^2},$$

which has solution $v_y = -v\sin(mt/qB)$, where v is a constant. Using the second equation we find that there is a similar solution for v_x, except that it is out of phase, and so $v_x = v\cos(mt/qB)$.

Integrating,

$$x = \int v_x dt = v\int\cos(mt/qB) = \frac{qBv}{m}\sin(mt/qB).$$

Similarly,

$$y = \int v_y dt = -v\int\sin(mt/qB) = \frac{qBv}{m}\cos(mt/qB).$$

This is the equation of a circle.

P32-14 $d\vec{\mathbf{L}} = \hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy + \hat{\mathbf{k}}dz$. $\vec{\mathbf{B}}$ is uniform, so that the integral can be written as

$$\vec{\mathbf{F}} = i\oint(\hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy + \hat{\mathbf{k}}dz)\times\vec{\mathbf{B}} = i\hat{\mathbf{i}}\times\vec{\mathbf{B}}\oint dx + i\hat{\mathbf{j}}\times\vec{\mathbf{B}}\oint dy + i\hat{\mathbf{k}}\times\vec{\mathbf{B}}\oint dz,$$

but since $\oint dx = \oint dy = \oint dz = 0$, the entire expression vanishes.

P32-15 The current pulse provides an impulse which is equal to

$$\int F\,dt = \int BiL\,dt = BL\int i\,dt = BLq.$$

This gives an initial velocity of $v_0 = BLq/m$, which will cause the rod to hop to a height of

$$h = v_0^2/2g = B^2L^2q^2/2m^2g.$$

Solving for q,

$$q = \frac{m}{BL}\sqrt{2gh} = \frac{(0.013\,\mathrm{kg})}{(0.12\,\mathrm{T})(0.20\,\mathrm{m})}\sqrt{2(9.8\,\mathrm{m/s^2})(3.1\,\mathrm{m})} = 4.2\,\mathrm{C}.$$

P32-16

$\boxed{\textbf{P32-17}}$ The torque on a current carrying loop depends on the orientation of the loop; the maximum torque occurs when the plane of the loop is parallel to the magnetic field. In this case the magnitude of the torque is from Eq. 32-34 with $\sin\theta = 1$—

$$\tau = NiAB.$$

The area of a circular loop is $A = \pi r^2$ where r is the radius, but since the circumference is $C = 2\pi r$, we can write

$$A = \frac{C^2}{4\pi}.$$

The circumference is *not* the length of the wire, because there may be more than one turn. Instead, $C = L/N$, where N is the number of turns.

Finally, we can write the torque as

$$\tau = Ni\frac{L^2}{4\pi N^2}B = \frac{iL^2B}{4\pi N},$$

which is a maximum when N is a minimum, or $N = 1$.

P32-18 $d\vec{\mathbf{F}} = i\,d\vec{\mathbf{L}} \times \vec{\mathbf{B}}$; the direction of $d\vec{\mathbf{F}}$ will be upward and somewhat toward the center. $\vec{\mathbf{L}}$ and $\vec{\mathbf{B}}$ are a right angles, but only the upward component of $d\vec{\mathbf{F}}$ will survive the integration as the central components will cancel out by symmetry. Hence

$$F = iB\sin\theta\int dL = 2\pi riB\sin\theta.$$

P32-19 The torque on the cylinder from gravity is

$$\tau_{\mathrm{g}} = mgr\sin\theta,$$

where r is the radius of the cylinder. The torque from magnetism needs to balance this, so

$$mgr\sin\theta = NiAB\sin\theta = Ni2rLB\sin\theta,$$

or

$$i = \frac{mg}{2NLB} = \frac{(0.262\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2(13)(0.127\,\mathrm{m})(0.477\,\mathrm{T})} = 1.63\,\mathrm{A}.$$

$\boxed{\textbf{E33-1}}$ (a) The magnetic field from a moving charge is given by Eq. 33-5. If the protons are moving side by side then the angle is $\phi = \pi/2$, so

$$B = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

and we are interested is a distance $r = d$. The electric field at that distance is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2},$$

where in both of the above expressions q is the charge of the source proton.

On the receiving end is the other proton, and the force on that proton is given by

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}).$$

The velocity is the same as that of the first proton (otherwise they wouldn't be moving side by side.) This velocity is then perpendicular to the magnetic field, and the resulting direction for the cross product will be opposite to the direction of $\vec{\mathbf{E}}$. Then for balance,

$$\begin{aligned} E &= vB, \\ \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} &= v\frac{\mu_0}{4\pi}\frac{qv}{r^2}, \\ \frac{1}{\epsilon_0\mu_0} &= v^2. \end{aligned}$$

We can solve this easily enough, and we find $v \approx 3 \times 10^8\,\mathrm{m/s}$.

(b) This is clearly a relativistic speed!

E33-2 $B = \mu_0 i/2\pi d = (4\pi\times10^{-7}\mathrm{T\cdot m/A})(120\,\mathrm{A})/2\pi(6.3\,\mathrm{m}) = 3.8\times10^{-6}\mathrm{T}$. This will deflect the compass needle by as much as one degree. However, there is unlikely to be a place on the Earth's surface where the magnetic field is 210 μT. This was likely a typo, and should probably have been 21.0 μT. The deflection would then be some ten degrees, and that *is* significant.

E33-3 $B = \mu_0 i/2\pi d = (4\pi\times10^{-7}\mathrm{T\cdot m/A})(50\,\mathrm{A})/2\pi(1.3\times10^{-3}\,\mathrm{m}) = 37.7\times10^{-3}\mathrm{T}$.

E33-4 (a) $i = 2\pi dB/\mu_0 = 2\pi(8.13\times10^{-2}\mathrm{m})(39.0\times10^{-6}\mathrm{T})/(4\pi\times10^{-7}\mathrm{T\cdot m/A}) = 15.9\,\mathrm{A}$.

(b) Due East.

$\boxed{\textbf{E33-5}}$ Use

$$B = \frac{\mu_0 i}{2\pi d} = \frac{(4\pi\times10^{-7}\mathrm{N/A^2})(1.6\times10^{-19}\,\mathrm{C})(5.6\times10^{14}\,\mathrm{s^{-1}})}{2\pi(0.0015\,\mathrm{m})} = 1.2\times10^{-8}\mathrm{T}.$$

E33-6 Zero, by symmetry. Any contributions from the top wire are exactly canceled by contributions from the bottom wire.

E33-7 $B = \mu_0 i/2\pi d = (4\pi\times10^{-7}\mathrm{T\cdot m/A})(48.8\,\mathrm{A})/2\pi(5.2\times10^{-2}\,\mathrm{m}) = 1.88\times10^{-4}\mathrm{T}$.

$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. All cases are either parallel or perpendicular, so either $F = 0$ or $F = qvB$.

(a) $F = qvB = (1.60\times10^{-19}\mathrm{C})(1.08\times10^7\mathrm{m/s})(1.88\times10^{-4}\mathrm{T}) = 3.24\times10^{-16}\mathrm{N}$. The direction of $\vec{\mathbf{F}}$ is parallel to the current.

(b) $F = qvB = (1.60\times10^{-19}\mathrm{C})(1.08\times10^7\mathrm{m/s})(1.88\times10^{-4}\mathrm{T}) = 3.24\times10^{-16}\mathrm{N}$. The direction of $\vec{\mathbf{F}}$ is radially outward from the current.

(c) $F = 0$.

E33-8 We want $B_1 = B_2$, but with opposite directions. Then $i_1/d_1 = i_2/d_2$, since all constants cancel out. Then $i_2 = (6.6\,\mathrm{A})(1.5\ \mathrm{cm})/(2.25\ \mathrm{cm}) = 4.4\,\mathrm{A}$, directed out of the page.

$\boxed{\textbf{E33-9}}$ For a single long straight wire, $B = \mu_0 i/2\pi d$ but we need a factor of "2" since there are two wires, then $i = \pi d B/\mu_0$. Finally

$$i = \frac{\pi d B}{\mu_0} = \frac{\pi(0.0405\,\mathrm{m})(296, \mu\mathrm{T})}{(4\pi\times 10^{-7}\mathrm{N/A^2})} = 30\,\mathrm{A}$$

E33-10 (a) The semi-circle part contributes half of Eq. 33-21, or $\mu_0 i/4R$. Each long straight wire contributes half of Eq. 33-13, or $\mu_0 i/4\pi R$. Add the three contributions and get

$$B_a = \frac{\mu_0 i}{4R}\left(\frac{2}{\pi}+1\right) = \frac{(4\pi\times 10^{-7}\mathrm{N/A^2})(11.5\,\mathrm{A})}{4(5.20\times 10^{-3}\mathrm{m})}\left(\frac{2}{\pi}+1\right) = 1.14\times 10^{-3}\mathrm{T}.$$

The direction is out of the page.

(b) Each long straight wire contributes Eq. 33-13, or $\mu_0 i/2\pi R$. Add the two contributions and get

$$B_a = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times 10^{-7}\mathrm{N/A^2})(11.5\,\mathrm{A})}{\pi(5.20\times 10^{-3}\mathrm{m})} = 8.85\times 10^{-4}\mathrm{T}.$$

The direction is out of the page.

E33-11 $z^3 = \mu_0 i R^2/2B = (4\pi\times 10^{-7}\mathrm{N/A^2})(320)(4.20\,\mathrm{A})(2.40\times 10^{-2}\mathrm{m})^2/2(5.0\times 10^{-6}\mathrm{T}) = 9.73\times 10^{-2}\mathrm{m}^3$. Then $z = 0.46\,\mathrm{m}$.

E33-12 The circular part contributes a fraction of Eq. 33-21, or $\mu_0 i\theta/4\pi R$. Each long straight wire contributes half of Eq. 33-13, or $\mu_0 i/4\pi R$. Add the three contributions and get

$$B = \frac{\mu_0 i}{4\pi R}(\theta - 2).$$

The goal is to get $B = 0$ that will happen if $\theta = 2$ radians.

$\boxed{\textbf{E33-13}}$ There are four current segments that could contribute to the magnetic field. The straight segments, however, contribute nothing because the straight segments carry currents either directly toward or directly away from the point P.

That leaves the two rounded segments. Each contribution to $\vec{\mathbf{B}}$ can be found by starting with Eq. 33-21, or $\mu_0 i\theta/4\pi b$. The direction is out of the page.

There is also a contribution from the top arc; the calculations are almost identical except that this is pointing into the page and $r = a$, so $\mu_0 i\theta/4\pi a$. The net magnetic field at P is then

$$B = B_1 + B_2 = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right).$$

E33-14 For each straight wire segment use Eq. 33-12. When the length of wire is L, the distance to the center is $W/2$; when the length of wire is W the distance to the center is $L/2$. There are four terms, but they are equal in pairs, so

$$\begin{aligned} B &= \frac{\mu_0 i}{4\pi}\left(\frac{4L}{W\sqrt{L^2/4+W^2/4}} + \frac{4W}{L\sqrt{L^2/4+W^2/4}}\right), \\ &= \frac{2\mu_0 i}{\pi\sqrt{L^2+W^2}}\left(\frac{L^2}{WL}+\frac{W^2}{WL}\right) = \frac{2\mu_0 i}{\pi}\frac{\sqrt{L^2+W^2}}{WL}. \end{aligned}$$

$\boxed{\textbf{E33-15}}$ We imagine the ribbon conductor to be a collection of thin wires, each of thickness dx and carrying a current di. di and dx are related by $di/dx = i/w$. The contribution of one of these thin wires to the magnetic field at P is $dB = \mu_0\, di/2\pi x$, where x is the distance from this thin wire to the point P. We want to change variables to x and integrate, so

$$B = \int dB = \int \frac{\mu_0 i\, dx}{2\pi w x} = \frac{\mu_0 i}{2\pi w}\int \frac{dx}{x}.$$

The limits of integration are from d to $d+w$, so

$$B = \frac{\mu_0 i}{2\pi w}\ln\left(\frac{d+w}{d}\right).$$

E33-16 The fields from each wire are perpendicular at P. Each contributes an amount $B' = \mu_0 i/2\pi d$, but since they are perpendicular there is a net field of magnitude $B = \sqrt{2B'^2} = \sqrt{2}\mu_0 i/2\pi d$. Note that $a = \sqrt{2}d$, so $B = \mu_0 i/\pi a$.

(a) $B = (4\pi\times10^{-7}\mathrm{T\cdot m/A})(115\,\mathrm{A})/\pi(0.122\,\mathrm{m}) = 3.77\times10^{-4}\mathrm{T}$. The direction is to the left.

(b) Same numerical result, except the direction is up.

E33-17 Follow along with Sample Problem 33-4.

Reversing the direction of the second wire (so that now both currents are directed out of the page) will also reverse the direction of B_2. Then

$$\begin{aligned} B &= B_1 - B_2 = \frac{\mu_0 i}{2\pi}\left(\frac{1}{b+x} - \frac{1}{b-x}\right), \\ &= \frac{\mu_0 i}{2\pi}\left(\frac{(b-x)-(b+x)}{b^2-x^2}\right), \\ &= \frac{\mu_0 i}{\pi}\left(\frac{x}{x^2-b^2}\right). \end{aligned}$$

E33-18 (b) By symmetry, only the horizontal component of $\vec{\mathbf{B}}$ survives, and must point to the right.

(a) The horizontal component of the field contributed by the top wire is given by

$$B = \frac{\mu_0 i}{2\pi h}\sin\theta = \frac{\mu_0 i}{2\pi h}\frac{b/2}{h} = \frac{\mu_0 i b}{\pi(4R^2+b^2)},$$

since h is the hypotenuse, or $h = \sqrt{R^2 + b^2/4}$. But there are two such components, one from the top wire, and an identical component from the bottom wire.

$\boxed{\textbf{E33-19}}$ (a) We can use Eq. 33-21 to find the magnetic field strength at the center of the large loop,

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi\times10^{-7}\mathrm{T\cdot m/A})(13\,\mathrm{A})}{2(0.12\,\mathrm{m})} = 6.8\times10^{-5}\mathrm{T}.$$

(b) The torque on the smaller loop in the center is given by Eq. 32-34,

$$\vec{\tau} = Ni\vec{\mathbf{A}}\times\vec{\mathbf{B}},$$

but since the magnetic field from the large loop is perpendicular to the plane of the large loop, and the plane of the small loop is also perpendicular to the plane of the large loop, the magnetic field is in the plane of the small loop. This means that $|\vec{\mathbf{A}}\times\vec{\mathbf{B}}| = AB$. Consequently, the magnitude of the torque on the small loop is

$$\tau = NiAB = (50)(1.3\,\mathrm{A})(\pi)(8.2\times10^{-3}\mathrm{m})^2(6.8\times10^{-5}\mathrm{T}) = 9.3\times10^{-7}\mathrm{N\cdot m}.$$

E33-20 (a) There are two contributions to the field. One is from the circular loop, and is given by $\mu_0 i/2R$. The other is from the long straight wire, and is given by $\mu_0 i/2\pi R$. The two fields are out of the page and parallel, so

$$B = \frac{\mu_0 i}{2R}(1 + 1/\pi).$$

(b) The two components are now at right angles, so

$$B = \frac{\mu_0 i}{2R}\sqrt{1 + 1/\pi^2}.$$

The direction is given by $\tan\theta = 1/\pi$ or $\theta = 18°$.

E33-21 The force per meter for any pair of parallel currents is given by Eq. 33-25, $F/L = \mu_0 i^2/2\pi d$, where d is the separation. The direction of the force is along the line connecting the intersection of the currents with the perpendicular plane. Each current experiences three forces; two are at right angles and equal in magnitude, so $|\vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{14}|/L = \sqrt{F_{12}^2 + F_{14}^2}/L = \sqrt{2}\mu_0 i^2/2\pi a$. The third force points parallel to this sum, but $d = \sqrt{a}$, so the resultant force is

$$\frac{F}{L} = \frac{\sqrt{2}\mu_0 i^2}{2\pi a} + \frac{\mu_0 i^2}{2\pi\sqrt{2}a} = \frac{4\pi \times \times 10^{-7}\mathrm{N/A^2}(18.7\,\mathrm{A})^2}{2\pi(0.245\,\mathrm{m})}(\sqrt{2} + 1/\sqrt{2}) = 6.06\times 10^{-4}\mathrm{N/m}.$$

It is directed toward the center of the square.

E33-22 By symmetry we expect the middle wire to have a net force of zero; the two on the outside will each be attracted toward the center, but the answers will be symmetrically distributed.

For the wire which is the farthest left,

$$\frac{F}{L} = \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} + \frac{1}{4a}\right) = \frac{4\pi \times \times 10^{-7}\mathrm{N/A^2}(3.22\,\mathrm{A})^2}{2\pi(0.083\,\mathrm{m})}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 5.21\times 10^{-5}\mathrm{N/m}.$$

For the second wire over, the contributions from the two adjacent wires should cancel. This leaves

$$\frac{F}{L} = \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{2a} + \frac{1}{3a} +\right) = \frac{4\pi \times \times 10^{-7}\mathrm{N/A^2}(3.22\,\mathrm{A})^2}{2\pi(0.083\,\mathrm{m})}\left(\frac{1}{2} + \frac{1}{3}\right) = 2.08\times 10^{-5}\mathrm{N/m}.$$

$\boxed{\textbf{E33-23}}$ (a) The force on the projectile is given by the integral of

$$d\vec{\mathbf{F}} = i\,d\vec{\mathbf{l}} \times \vec{\mathbf{B}}$$

over the length of the projectile (which is w). The magnetic field strength can be found from adding together the contributions from each rail. If the rails are circular and the distance between them is small compared to the length of the wire we can use Eq. 33-13,

$$B = \frac{\mu_0 i}{2\pi x},$$

where x is the distance from the center of the rail. There is one problem, however, because these are not wires of infinite length. Since the current *stops* traveling along the rail when it reaches the projectile we have a rod that is only half of an infinite rod, so we need to multiply by a factor of 1/2. But there are two rails, and each will contribute to the field, so the net magnetic field strength between the rails is

$$B = \frac{\mu_0 i}{4\pi x} + \frac{\mu_0 i}{4\pi(2r + w - x)}.$$

In that last term we have an expression that is a measure of the distance from the center of the lower rail in terms of the distance x from the center of the upper rail.

The magnitude of the force on the projectile is then

$$\begin{aligned} F &= i\int_r^{r+w} B\,dx, \\ &= \frac{\mu_0 i^2}{4\pi}\int_r^{r+w}\left(\frac{1}{x}+\frac{1}{2r+w-x}\right)dx, \\ &= \frac{\mu_0 i^2}{4\pi}2\ln\left(\frac{r+w}{r}\right) \end{aligned}$$

The current through the projectile is down the page; the magnetic field through the projectile is into the page; so the force on the projectile, according to $\vec{\mathbf{F}} = i\vec{\mathbf{l}}\times\vec{\mathbf{B}}$, is to the right.

(b) Numerically the magnitude of the force on the rail is

$$F = \frac{(450\times 10^3\,\mathrm{A})^2(4\pi\times 10^{-7}\mathrm{N/A^2})}{2\pi}\ln\left(\frac{(0.067\,\mathrm{m})+(0.012\,\mathrm{m})}{(0.067\,\mathrm{m})}\right) = 6.65\times 10^3\,\mathrm{N}$$

The speed of the rail can be found from either energy conservation so we first find the work done on the projectile,

$$W = Fd = (6.65\times 10^3\,\mathrm{N})(4.0\,\mathrm{m}) = 2.66\times 10^4\,\mathrm{J}.$$

This work results in a change in the kinetic energy, so the final speed is

$$v = \sqrt{2K/m} = \sqrt{2(2.66\times 10^4\,\mathrm{J})/(0.010\,\mathrm{kg})} = 2.31\times 10^3\,\mathrm{m/s}.$$

E33-24 The contributions from the left end and the right end of the square cancel out. This leaves the top and the bottom. The net force is the difference, or

$$\begin{aligned} F &= \frac{(4\pi\times 10^{-7}\mathrm{N/A^2})(28.6\,\mathrm{A})(21.8\,\mathrm{A})(0.323\,\mathrm{m})}{2\pi}\left(\frac{1}{(1.10\times 10^{-2}\mathrm{m})}-\frac{1}{(10.30\times 10^{-2}\mathrm{m})}\right), \\ &= 3.27\times 10^{-3}\mathrm{N}. \end{aligned}$$

E33-25 The magnetic force on the upper wire near the point d is

$$F_B = \frac{\mu_0 i_a i_b L}{2\pi(d+x)} \approx \frac{\mu_0 i_a i_b L}{2\pi d} - \frac{\mu_0 i_a i_b L}{2\pi d^2}x,$$

where x is the distance from the equilibrium point d. The equilibrium magnetic force is equal to the force of gravity mg, so near the equilibrium point we can write

$$F_B = mg - mg\frac{x}{d}.$$

There is then a restoring force against small perturbations of magnitude mgx/d which corresponds to a spring constant of $k = mg/d$. This would give a frequency of oscillation of

$$f = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{g/d},$$

which is identical to the pendulum.

E33-26 $B = (4\pi\times 10^{-7}\mathrm{N/A^2})(3.58\,\mathrm{A})(1230)/(0.956\mathrm{m}) = 5.79\times 10^{-3}\mathrm{T}.$

E33-27 The magnetic field inside an ideal solenoid is given by Eq. 33-28 $B = \mu_0 i n$, where n is the turns per unit length. Solving for n,

$$n = \frac{B}{\mu_0 i} = \frac{(0.0224\,\mathrm{T})}{(4\pi\times10^{-7}\mathrm{N/A^2})(17.8\,\mathrm{A})} = 1.00\times10^{3}/\mathrm{m}^{-1}.$$

The solenoid has a length of 1.33 m, so the total number of turns is

$$N = nL = (1.00\times10^{3}/\mathrm{m}^{-1})(1.33\,\mathrm{m}) = 1330,$$

and since each turn has a length of one circumference, then the total length of the wire which makes up the solenoid is $(1330)\pi(0.026\,\mathrm{m}) = 109\,\mathrm{m}$.

E33-28 From the solenoid we have

$$B_\mathrm{s} = \mu_0 n i_\mathrm{s} = \mu_0(11500/\mathrm{m})(1.94\,\mathrm{mA}) = \mu_0(22.3\mathrm{A/m}).$$

From the wire we have

$$B_\mathrm{w} = \frac{\mu_0 i_\mathrm{w}}{2\pi r} = \frac{\mu_0(6.3\,\mathrm{A})}{2\pi r} = (1.002\,\mathrm{A})\frac{\mu_0}{r}$$

These fields are at right angles, so we are interested in when $\tan(40^\circ) = B_\mathrm{w}/B_\mathrm{s}$, or

$$r = \frac{(1.002\,\mathrm{A})}{\tan(40^\circ)(22.3\,\mathrm{A/m})} = 5.35\times10^{-2}\mathrm{m}.$$

E33-29 Let $u = z - d$. Then

$$\begin{aligned}
B &= \frac{\mu_0 n i R^2}{2}\int_{d-L/2}^{d+L/2}\frac{du}{[R^2+u^2]^{3/2}},\\
&= \frac{\mu_0 n i R^2}{2}\left.\frac{u}{R^2\sqrt{R^2+u^2}}\right|_{d-L/2}^{d+L/2},\\
&= \frac{\mu_0 n i}{2}\left(\frac{d+L/2}{\sqrt{R^2+(d+L/2)^2}} - \frac{d-L/2}{\sqrt{R^2+(d-L/2)^2}}\right).
\end{aligned}$$

If L is much, much greater than R and d then $|L/2 \pm d| >> R$, and R can be ignored in the denominator of the above expressions, which then simplify to

$$\begin{aligned}
B &= \frac{\mu_0 n i}{2}\left(\frac{d+L/2}{\sqrt{R^2+(d+L/2)^2}} - \frac{d-L/2}{\sqrt{R^2+(d-L/2)^2}}\right).\\
&= \frac{\mu_0 n i}{2}\left(\frac{d+L/2}{\sqrt{(d+L/2)^2}} - \frac{d-L/2}{\sqrt{(d-L/2)^2}}\right).\\
&= \mu_0 i n.
\end{aligned}$$

It *is* important that we consider the relative size of $L/2$ and d!

E33-30 The net current in the loop is $1i_0 + 3i_0 + 7i_0 - 6i_0 = 5i_0$. Then $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = 5\mu_0 i_0$.

E33-31 (a) The path is clockwise, so a positive current is *into* page. The net current is 2.0 A out, so $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = -\mu_0 i_0 = -2.5\times10^{-6}\mathrm{T\cdot m}$.

(b) The net current is zero, so $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = 0$.

E33-32 Let R_0 be the radius of the wire. On the surface of the wire $B_0 = \mu_0 i/2\pi R_0$.

Outside the wire we have $B = \mu_0 i/2\pi R$, this is half B_0 when $R = 2R_0$.

Inside the wire we have $B = \mu_0 iR/2\pi R_0^2$, this is half B_0 when $R = R_0/2$.

$\boxed{\textbf{E33-33}}$ (a) We don't want to reinvent the wheel. The answer is found from Eq. 33-34, except it looks like

$$B = \frac{\mu_0 ir}{2\pi c^2}.$$

(b) In the region between the wires the magnetic field looks like Eq. 33-13,

$$B = \frac{\mu_0 i}{2\pi r}.$$

This is derived on the right hand side of page 761.

(c) Ampere's law (Eq. 33-29) is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 i$, where i is the current enclosed. Our Amperian loop will still be a circle centered on the axis of the problem, so the left hand side of the above equation will reduce to $2\pi rB$, just like in Eq. 33-32. The right hand side, however, depends on the *net* current enclosed which is the current i in the center wire minus the fraction of the current enclosed in the outer conductor. The cross sectional area of the outer conductor is $\pi(a^2 - b^2)$, so the fraction of the outer current enclosed in the Amperian loop is

$$i\frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = i\frac{r^2 - b^2}{a^2 - b^2}.$$

The net current in the loop is then

$$i - i\frac{r^2 - b^2}{a^2 - b^2} = i\frac{a^2 - r^2}{a^2 - b^2},$$

so the magnetic field in this region is

$$B = \frac{\mu_0 i}{2\pi r}\frac{a^2 - r^2}{a^2 - b^2}.$$

(d) This part is easy since the net current is zero; consequently $B = 0$.

E33-34 (a) Ampere's law (Eq. 33-29) is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 i$, where i is the current enclosed. Our Amperian loop will still be a circle centered on the axis of the problem, so the left hand side of the above equation will reduce to $2\pi rB$, just like in Eq. 33-32. The right hand side, however, depends on the *net* current enclosed which is the fraction of the current enclosed in the conductor. The cross sectional area of the conductor is $\pi(a^2 - b^2)$, so the fraction of the current enclosed in the Amperian loop is

$$i\frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = i\frac{r^2 - b^2}{a^2 - b^2}.$$

The magnetic field in this region is

$$B = \frac{\mu_0 i}{2\pi r}\frac{r^2 - b^2}{a^2 - b^2}.$$

(b) If $r = a$, then

$$B = \frac{\mu_0 i}{2\pi a}\frac{a^2 - b^2}{a^2 - b^2} = \frac{\mu_0 i}{2\pi a},$$

which is what we expect.

If $r = b$, then

$$B = \frac{\mu_0 i}{2\pi b}\frac{b^2 - b^2}{a^2 - b^2} = 0,$$

which is what we expect.

If $b = 0$, then

$$B = \frac{\mu_0 i}{2\pi r}\frac{r^2 - 0^2}{a^2 - 0^2} = \frac{\mu_0 i r}{2\pi a^2}$$

which is what I expected.

E33-35 The magnitude of the magnetic field due to the cylinder will be *zero* at the center of the cylinder and $\mu_0 i_0/2\pi(2R)$ at point P. The magnitude of the magnetic field field due to the wire will be $\mu_0 i/2\pi(3R)$ at the center of the cylinder but $\mu_0 i/2\pi R$ at P. In order for the net field to have different directions in the two locations the currents in the wire and pipe must be in different direction. The net field at the center of the pipe is $\mu_0 i/2\pi(3R)$, while that at P is then $\mu_0 i_0/2\pi(2R) - \mu_0 i/2\pi R$. Set these equal and solve for i;

$$i/3 = i_0/2 - i,$$

or $i = 3i_0/8$.

E33-36 (a) $B = (4\pi\times10^{-7}\mathrm{N/A^2})(0.813\,\mathrm{A})(535)/2\pi(0.162\,\mathrm{m}) = 5.37\times10^{-4}\mathrm{T}$.

(b) $B = (4\pi\times10^{-7}\mathrm{N/A^2})(0.813\,\mathrm{A})(535)/2\pi(0.162\,\mathrm{m} + 0.052\,\mathrm{m}) = 4.07\times10^{-4}\mathrm{T}$.

E33-37 (a) A positive particle would experience a magnetic force directed to the right for a magnetic field out of the page. This particle is going the other way, so it must be negative.

(b) The magnetic field of a toroid is given by Eq. 33-36,

$$B = \frac{\mu_0 i N}{2\pi r},$$

while the radius of curvature of a charged particle in a magnetic field is given by Eq. 32-10

$$R = \frac{mv}{|q|B}.$$

We use the R to distinguish it from r. Combining,

$$R = \frac{2\pi m v}{\mu_0 i N|q|}r,$$

so the two radii are directly proportional. This means

$$R/(11\text{ cm}) = (110\text{ cm})/(125\text{ cm}),$$

so $R = 9.7$ cm.

P33-1 The field from one coil is given by Eq. 33-19

$$B = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

There are N turns in the coil, so we need a factor of N. There are two coils and we are interested in the magnetic field at P, a distance $R/2$ from each coil. The magnetic field strength will be twice the above expression but with $z = R/2$, so

$$B = \frac{2\mu_0 N i R^2}{2(R^2 + (R/2)^2)^{3/2}} = \frac{8\mu_0 N i}{(5)^{3/2}R}.$$

P33-2 (a) Change the limits of integration that lead to Eq. 33-12:

$$\begin{aligned} B &= \frac{\mu_0 i d}{4\pi}\int_0^L \frac{dz}{(z^2+d^2)^{3/2}}, \\ &= \frac{\mu_0 i d}{4\pi}\frac{z}{(z^2+d^2)^{1/2}}\bigg|_0^L, \\ &= \frac{\mu_0 i d}{4\pi}\frac{L}{(L^2+d^2)^{1/2}}. \end{aligned}$$

(b) The angle ϕ in Eq. 33-11 would always be 0, so $\sin\phi = 0$, and therefore $B = 0$.

P33-3 This problem is the all important derivation of the Helmholtz coil properties.
(a) The magnetic field from one coil is

$$B_1 = \frac{\mu_0 N i R^2}{2(R^2+z^2)^{3/2}}.$$

The magnetic field from the other coil, located a distance s away, but for points measured from the first coil, is

$$B_2 = \frac{\mu_0 N i R^2}{2(R^2+(z-s)^2)^{3/2}}.$$

The magnetic field on the axis between the coils is the sum,

$$B = \frac{\mu_0 N i R^2}{2(R^2+z^2)^{3/2}} + \frac{\mu_0 N i R^2}{2(R^2+(z-s)^2)^{3/2}}.$$

Take the derivative with respect to z and get

$$\frac{dB}{dz} = -\frac{3\mu_0 N i R^2}{2(R^2+z^2)^{5/2}}z - \frac{3\mu_0 N i R^2}{2(R^2+(z-s)^2)^{5/2}}(z-s).$$

At $z = s/2$ this expression vanishes! We expect this by symmetry, because the magnetic field will be strongest in the plane of either coil, so the mid-point should be a local minimum.

(b) Take the derivative again and

$$\begin{aligned} \frac{d^2B}{dz^2} = & -\frac{3\mu_0 N i R^2}{2(R^2+z^2)^{5/2}} + \frac{15\mu_0 N i R^2}{2(R^2+z^2)^{5/2}}z^2 \\ & -\frac{3\mu_0 N i R^2}{2(R^2+(z-s)^2)^{5/2}} + \frac{15\mu_0 N i R^2}{2(R^2+(z-s)^2)^{5/2}}(z-s)^2. \end{aligned}$$

We could try and simplify this, but we don't really want to; we instead want to set it equal to zero, then let $z = s/2$, and then solve for s. The second derivative will equal zero when

$$-3(R^2+z^2) + 15z^2 - 3(R^2+(z-s)^2) + 15(z-s)^2 = 0,$$

and is $z = s/2$ this expression will simplify to

$$\begin{aligned} 30(s/2)^2 &= 6(R^2+(s/2)^2), \\ 4(s/2)^2 &= R^2, \\ s &= R. \end{aligned}$$

P33-4 (a) Each of the side of the square is a straight wire segment of length a which contributes a field strength of

$$B = \frac{\mu_0 i}{4\pi r}\frac{a}{\sqrt{a^2/4 + r^2}},$$

where r is the distance to the point on the axis of the loop, so

$$r = \sqrt{a^2/4 + z^2}.$$

This field is *not* parallel to the z axis; the z component is $B_z = B(a/2)/r$. There are four of these contributions. The off axis components cancel. Consequently, the field for the square is

$$\begin{aligned}
B &= 4\frac{\mu_0 i}{4\pi r}\frac{a}{\sqrt{a^2/4 + r^2}}\frac{a/2}{r},\\
&= \frac{\mu_0 i}{2\pi r^2}\frac{a^2}{\sqrt{a^2/4 + r^2}},\\
&= \frac{\mu_0 i}{2\pi(a^2/4 + z^2)}\frac{a^2}{\sqrt{a^2/2 + z^2}},\\
&= \frac{4\mu_0 i}{\pi(a^2 + 4z^2)}\frac{a^2}{\sqrt{2a^2 + 4z^2}}.
\end{aligned}$$

(b) When $z = 0$ this reduces to

$$B = \frac{4\mu_0 i}{\pi(a^2)}\frac{a^2}{\sqrt{2a^2}} = \frac{4\mu_0 i}{\sqrt{2}\,\pi a}.$$

P33-5 (a) The polygon has n sides. A perpendicular bisector of each side can be drawn to the center and has length x where $x/a = \cos(\pi/n)$. Each side has a length $L = 2a\sin(\pi/n)$. Each of the side of the polygon is a straight wire segment which contributes a field strength of

$$B = \frac{\mu_0 i}{4\pi x}\frac{L}{\sqrt{L^2/4 + x^2}},$$

This field *is* parallel to the z axis. There are n of these contributions. The off axis components cancel. Consequently, the field for the polygon

$$\begin{aligned}
B &= n\frac{\mu_0 i}{4\pi x}\frac{L}{\sqrt{L^2/4 + x^2}},\\
&= n\frac{\mu_0 i}{4\pi}\frac{2}{\sqrt{L^2/4 + x^2}}\tan(\pi/n),\\
&= n\frac{\mu_0 i}{2\pi}\frac{1}{a}\tan(\pi/n),
\end{aligned}$$

since $(L/2)^2 + x^2 = a^2$.

(b) Evaluate:

$$\lim_{n\to\infty} n\tan(\pi/n) = \lim_{n\to\infty} n\sin(\pi/n) \approx n\pi/n = \pi.$$

Then the answer to part (a) simplifies to

$$B = \frac{\mu_0 i}{2a}.$$

P33-6 For a square loop of wire we have four finite length segments each contributing a term which looks like Eq. 33-12, except that L is replaced by $L/4$ and d is replaced by $L/8$. Then at the center,

$$B = 4\frac{\mu_0 i}{4\pi L/8}\frac{L/4}{\sqrt{L^2/64 + L^2/64}} = \frac{16\mu_0 i}{\sqrt{2}\pi L}.$$

For a circular loop $R = L/2\pi$ so

$$B = \frac{\mu_0 i}{2R} = \frac{\pi\mu_0}{L}.$$

Since $16/\sqrt{2}\,\pi > \pi$, the square wins. But only by some 7%!

$\boxed{\textbf{P33-7}}$ We want to use the differential expression in Eq. 33-11, except that the limits of integration are going to be different. We have four wire segments. From the top segment,

$$\begin{aligned} B_1 &= \frac{\mu_0 i}{4\pi}\frac{d}{\sqrt{z^2+d^2}}\bigg|_{-L/4}^{3L/4}, \\ &= \frac{\mu_0 i}{4\pi d}\left(\frac{3L/4}{\sqrt{(3L/4)^2+d^2}} - \frac{-L/4}{\sqrt{(-L/4)^2+d^2}}\right). \end{aligned}$$

For the top segment $d = L/4$, so this simplifies even further to

$$B_1 = \frac{\mu_0 i}{10\pi L}\left(\sqrt{2}(3\sqrt{5}+5)\right).$$

The bottom segment has the same integral, but $d = 3L/4$, so

$$B_3 = \frac{\mu_0 i}{30\pi L}\left(\sqrt{2}(\sqrt{5}+5)\right).$$

By symmetry, the contribution from the right hand side is the same as the bottom, so $B_2 = B_3$, and the contribution from the left hand side is the same as that from the top, so $B_4 = B_1$. Adding all four terms,

$$\begin{aligned} B &= \frac{2\mu_0 i}{30\pi L}\left(3\sqrt{2}(3\sqrt{5}+5) + \sqrt{2}(\sqrt{5}+5)\right), \\ &= \frac{2\mu_0 i}{3\pi L}(2\sqrt{2}+\sqrt{10}). \end{aligned}$$

P33-8 Assume a current ring has a radius r and a width dr, the charge on the ring is $dq = 2\pi\sigma r\,dr$, where $\sigma = q/\pi R^2$. The current in the ring is $di = \omega\,dq/2\pi = \omega\sigma r\,dr$. The ring contributes a field $dB = \mu_0\,di/2r$. Integrate over all the rings:

$$B = \int_0^R \mu_0\omega\sigma r\,dr/2r = \mu_0\omega R/2 = \mu\omega q/2\pi R.$$

P33-9 $B = \mu_0 in$ and $mv = qBr$. Combine, and

$$i = \frac{mv}{\mu_0 qrn} = \frac{(5.11\times10^5\mathrm{eV}/c^2)(0.046c)}{(4\pi\times10^{-7}\mathrm{N/A^2})e(0.023\,\mathrm{m})(10000/\mathrm{m})} = 0.271\,\mathrm{A}.$$

P33-10 This shape is a triangle with area $A = (4d)(3d)/2 = 6d^2$. The enclosed current is then

$$i = jA = (15\,\mathrm{A/m^2})6(0.23\,\mathrm{m})^2 = 4.76\,\mathrm{A}$$

The line integral is then

$$\mu_0 i = 6.0\times10^{-6}\mathrm{T\cdot m}.$$

P33-11 Assume that B does vary as the picture implies. Then the line integral along the path shown *must* be nonzero, since $\vec{\mathbf{B}}\cdot\vec{\mathbf{l}}$ on the right is not zero, while it is along the three other sides. Hence $\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{l}}$ is non zero, implying some current passes through the dotted path. But it doesn't, so $\vec{\mathbf{B}}$ cannot have an abrupt change.

P33-12 (a) Sketch an Amperian loop which is a rectangle which enclosed N wires, has a vertical sides with height h, and horizontal sides with length L. Then $\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = \mu_0 Ni$. Evaluate the integral along the four sides. The vertical side contribute nothing, since $\vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{h}}$, and then $\vec{\mathbf{B}}\cdot\vec{\mathbf{h}} = 0$. If the integral is performed in a counterclockwise direction (it must, since the sense of integration was determined by assuming the current is positive), we get BL for each horizontal section. Then

$$B = \frac{\mu_0 iN}{2L} = \frac{1}{2}\mu_0 in.$$

(b) As $a\to\infty$ then $\tan^{-1}(a/2R)\to\pi/2$. Then $B\to\mu_0 i/2a$. If we assume that i is made up of several wires, each with current i_0, then $i/a = i_0 n$.

$\boxed{\textbf{P33-13}}$ Apply Ampere's law with an Amperian loop that is a circle centered on the center of the wire. Then

$$\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \oint B\,ds = B\oint ds = 2\pi rB,$$

because $\vec{\mathbf{B}}$ is tangent to the path and B is uniform along the path by symmetry. The current enclosed is

$$i_{\mathrm{enc}} = \int j\,dA.$$

This integral is best done in polar coordinates, so $dA = (dr)(r\,d\theta)$, and then

$$\begin{aligned} i_{\mathrm{enc}} &= \int_0^r\int_0^{2\pi}(j_0 r/a)\,r dr\,d\theta, \\ &= 2\pi j_0/a\int_0^r r^2 dr, \\ &= \frac{2\pi j_0}{3a}r^3. \end{aligned}$$

When $r = a$ the current enclosed is i, so

$$i = \frac{2\pi j_0 a^2}{3}\text{ or }j_0 = \frac{3i}{2\pi a^2}.$$

The magnetic field strength inside the wire is found by gluing together the two parts of Ampere's law,

$$\begin{aligned} 2\pi rB &= \mu_0\frac{2\pi j_0}{3a}r^3, \\ B &= \frac{\mu_0 j_0 r^2}{3a}, \\ &= \frac{\mu_0 i r^2}{2\pi a^3}. \end{aligned}$$

P33-14 (a) According to Eq. 33-34, the magnetic field inside the wire *without a hole* has magnitude $B = \mu_0 ir/2\pi R^2 = \mu_0 jr/2$ and is directed radially. If we superimpose a second current to create the hole, the additional field at the center of the hole is zero, so $B = \mu_0 jb/2$. But the current in the remaining wire is

$$i = jA = j\pi(R^2 - a^2),$$

so

$$B = \frac{\mu_0 ib}{2\pi(R^2 - a^2)}.$$

E34-1 $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = (42\times 10^{-6}\mathrm{T})(2.5\,\mathrm{m}^2)\cos(57^\circ) = 5.7\times 10^{-5}\mathrm{Wb}$.

E34-2 $|\mathcal{E}| = |d\Phi_B/dt| = A\,dB/dt = (\pi/4)(0.112\,\mathrm{m})^2(0.157\,\mathrm{T/s}) = 1.55\,\mathrm{mV}$.

E34-3 (a) The magnitude of the emf induced in a loop is given by Eq. 34-4,

$$\begin{aligned} |\mathcal{E}| &= N\left|\frac{d\Phi_B}{dt}\right|, \\ &= N\left|(12\,\mathrm{mWb/s^2})t + (7\,\mathrm{mWb/s})\right| \end{aligned}$$

There is only one loop, and we want to evaluate this expression for $t = 2.0\,\mathrm{s}$, so

$$|\mathcal{E}| = (1)\left|(12\,\mathrm{mWb/s^2})(2.0\,\mathrm{s}) + (7\,\mathrm{mWb/s})\right| = 31\,\mathrm{mV}.$$

(b) This part isn't harder. The magnetic flux through the loop is increasing when $t = 2.0\,\mathrm{s}$. The induced current needs to flow in such a direction to create a *second* magnetic field to oppose this increase. The original magnetic field is out of the page and we oppose the increase by pointing the other way, so the *second* field will point into the page (inside the loop).

By the right hand rule this means the induced current is clockwise through the loop, or to the left through the resistor.

E34-4 $\mathcal{E} = -d\Phi_B/dt = -A\,dB/dt$.
(a) $\mathcal{E} = -\pi(0.16\,\mathrm{m})^2(0.5\,\mathrm{T})/(2\,\mathrm{s}) = -2.0\times 10^{-2}\mathrm{V}$.
(b) $\mathcal{E} = -\pi(0.16\,\mathrm{m})^2(0.0\,\mathrm{T})/(2\,\mathrm{s}) = 0.0\times 10^{-2}\mathrm{V}$.
(c) $\mathcal{E} = -\pi(0.16\,\mathrm{m})^2(-0.5\,\mathrm{T})/(4\,\mathrm{s}) = 1.0\times 10^{-2}\mathrm{V}$.

E34-5 (a) $R = \rho L/A = (1.69\times 10^{-8}\Omega\cdot\mathrm{m})[(\pi)(0.104\,\mathrm{m})]/[(\pi/4)(2.50\times 10^{-3}\mathrm{m})^2] = 1.12\times 10^{-3}\Omega$.
(b) $\mathcal{E} = iR = (9.66\,\mathrm{A})(1.12\times 10^{-3}\Omega) = 1.08\times 10^{-2}\mathrm{V}$. The required dB/dt is then given by

$$\frac{dB}{dt} = \frac{\mathcal{E}}{A} = (1.08\times 10^{-2}\,\mathrm{V})/(\pi/4)(0.104\,\mathrm{m})^2 = 1.27\,\mathrm{T/s}.$$

E34-6 $\mathcal{E} = -A\,\Delta B/\Delta t = AB/\Delta t$. The power is $P = i\mathcal{E} = \mathcal{E}^2/R$. The energy dissipated is

$$E = P\Delta t = \frac{\mathcal{E}^2\Delta t}{R} = \frac{A^2B^2}{R\Delta t}.$$

E34-7 (a) We could re-derive the steps in the sample problem, or we could start with the end result. We'll start with the result,

$$\mathcal{E} = NA\mu_0 n\left|\frac{di}{dt}\right|,$$

except that we have gone ahead and used the derivative instead of the Δ.

The rate of change in the current is

$$\frac{di}{dt} = (3.0\,\mathrm{A/s}) + (1.0\,\mathrm{A/s^2})t,$$

so the induced emf is

$$\begin{aligned} \mathcal{E} &= (130)(3.46\times 10^{-4}\mathrm{m}^2)(4\pi\times 10^{-7}\mathrm{Tm/A})(2.2\times 10^4/\mathrm{m})\left((3.0\mathrm{A/s}) + (2.0\mathrm{A/s^2})t\right), \\ &= (3.73\times 10^{-3}\,\mathrm{V}) + (2.48\times 10^{-3}\,\mathrm{V/s})t. \end{aligned}$$

(b) When $t = 2.0\,\mathrm{s}$ the induced emf is $8.69\times 10^{-3}\,\mathrm{V}$, so the induced current is

$$i = (8.69\times 10^{-3}\,\mathrm{V})/(0.15\,\Omega) = 5.8\times 10^{-2}\,\mathrm{A}.$$

E34-8 (a) $i = \mathcal{E}/R = NA\,dB/dt$. Note that A refers to the area enclosed by the outer solenoid where B is non-zero. This A is then the cross sectional area of the inner solenoid! Then

$$i = \frac{1}{R}NA\mu_0 n\frac{di}{dt} = \frac{(120)(\pi/4)(0.032\,\text{m})^2(4\pi\times10^{-7}\text{N/A}^2)(220\times10^2/\text{m})}{(5.3\,\Omega)}\frac{(1.5\,\text{A})}{(0.16\,\text{s})} = 4.7\times10^{-3}\text{A}.$$

E34-9 $P = \mathcal{E}i = \mathcal{E}^2/R = (A\,dB/dt)^2/(\rho L/a)$, where A is the area of the loop and a is the cross sectional area of the wire. But $a = \pi d^2/4$ and $A = L^2/4\pi$, so

$$P = \frac{L^3 d^2}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = \frac{(0.525\,\text{m})^3(1.1\times10^{-3}\text{m})^2}{64\pi(1.69\times10^{-8}\Omega\cdot\text{m})}(9.82\times10^{-3}\text{T/s})^2 = 4.97\times10^{-6}\text{W}.$$

E34-10 $\Phi_B = BA = B(2.3\,\text{m})^2/2$. $\mathcal{E}_B = -d\Phi_B/dt = -AdB/dt$, or

$$\mathcal{E}_B = -\frac{(2.3\,\text{m})^2}{2}[-(0.87\,\text{T/s})] = 2.30\,\text{V},$$

so $\mathcal{E} = (2.0\,\text{V}) + (2.3\,\text{V}) = 4.3\,\text{V}$.

$\boxed{\textbf{E34-11}}$ (a) The induced emf, as a function of time, is given by Eq. 34-5, $\mathcal{E}(t) = -d\Phi_B(t)/dt$ This emf drives a current through the loop which obeys $\mathcal{E}(t) = i(t)R$ Combining,

$$i(t) = -\frac{1}{R}\frac{d\Phi_B(t)}{dt}.$$

Since the current is defined by $i = dq/dt$ we can write

$$\frac{dq(t)}{dt} = -\frac{1}{R}\frac{d\Phi_B(t)}{dt}.$$

Factor out the dt from both sides, and then integrate:

$$\begin{aligned} dq(t) &= -\frac{1}{R}\,d\Phi_B(t), \\ \int dq(t) &= -\int \frac{1}{R}\,d\Phi_B(t), \\ q(t) - q(0) &= \frac{1}{R}\left(\Phi_B(0) - \Phi_B(t)\right) \end{aligned}$$

(b) No. The induced current could have increased from zero to some positive value, then decreased to zero and became negative, so that the net charge to flow through the resistor was zero. This would be like sloshing the charge back and forth through the loop.

E34-12 $\Delta Phi_B = 2\Phi_B = 2NBA$. Then the charge to flow through is

$$q = 2(125)(1.57\,\text{T})(12.2\times10^{-4}\text{m}^2)/(13.3\,\Omega) = 3.60\times10^{-2}\text{C}.$$

E34-13 The part above the long straight wire (a distance $b-a$ above it) cancels out contributions below the wire (a distance $b-a$ beneath it). The flux through the loop is then

$$\Phi_B = \int_{2a-b}^{a}\frac{\mu_0 i}{2\pi r}b\,dr = \frac{\mu_0 ib}{2\pi}\ln\left(\frac{a}{2a-b}\right).$$

The emf in the loop is then

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{2a-b}\right) [2(4.5\,\mathrm{A/s^2})t - (10\,\mathrm{A/s})].$$

Evaluating,

$$\mathcal{E} = \frac{4\pi\times10^{-7}\mathrm{N/A^2}(0.16\,\mathrm{m})}{2\pi} \ln\left(\frac{(0.12\,\mathrm{m})}{2(0.12\,\mathrm{m})-(0.16\,\mathrm{m})}\right) [2(4.5\,\mathrm{A/s^2})(3.0\,\mathrm{s})-(10\,\mathrm{A/s})] = 2.20\times10^{-7}\mathrm{V}.$$

E34-14 Use Eq. 34-6: $\mathcal{E} = BDv = (55\times10^{-6}\mathrm{T})(1.10\,\mathrm{m})(25\,\mathrm{m/s}) = 1.5\times10^{-3}\mathrm{V}$.

E34-15 If the angle doesn't vary then the flux, given by

$$\Phi = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$$

is constant, so there is no emf.

E34-16 (a) Use Eq. 34-6: $\mathcal{E} = BDv = (1.18\mathrm{T})(0.108\,\mathrm{m})(4.86\,\mathrm{m/s}) = 0.619\,\mathrm{V}$.
(b) $i = (0.619\,\mathrm{V})/(0.415\,\Omega) = 1.49\,\mathrm{A}$.
(c) $P = (0.619\,\mathrm{V})(1.49\,\mathrm{A}) = 0.922\,\mathrm{W}$.
(d) $F = iLB = (1.49\,\mathrm{A})(0.108\,\mathrm{m})(1.18\mathrm{T}) = 0.190\,\mathrm{N}$.
(e) $P = Fv = (0.190\,\mathrm{V})(4.86\,\mathrm{m/s}) = 0.923\,\mathrm{W}$.

$\boxed{\textbf{E34-17}}$ The magnetic field is out of the page, and the current through the rod is down. Then Eq. 32-26 $\vec{\mathbf{F}} = i\vec{\mathbf{L}}\times\vec{\mathbf{B}}$ shows that the direction of the magnetic force is to the right; furthermore, since everything is perpendicular to everything else, we can get rid of the vector nature of the problem and write $F = iLB$. Newton's second law gives $F = ma$, and the acceleration of an object from rest results in a velocity given by $v = at$. Combining,

$$v(t) = \frac{iLB}{m}t.$$

E34-18 (b) The rod will accelerate as long as there is a net force on it. This net force comes from $F = iLB$. The current is given by $iR = \mathcal{E} - BLv$, so as v increases i decreases. When $i = 0$ the rod stops accelerating and assumes a terminal velocity.
(a) $\mathcal{E} = BLv$ will give the terminal velocity. In this case, $v = \mathcal{E}/BL$.

E34-19

E34-20 The acceleration is $a = R\omega^2$; since $\mathcal{E} = B\omega R^2/2$, we can find

$$a = 4\mathcal{E}^2/B^2R^3 = 4(1.4\,\mathrm{V})^2/(1.2\,\mathrm{T})^2(5.3\times10^{-2}\mathrm{m})^3 = 3.7\times10^4\mathrm{m/s^2}.$$

$\boxed{\textbf{E34-21}}$ We will use the results of Exercise 11 that were worked out above. All we need to do is find the initial flux; flipping the coil up-side-down will simply change the sign of the flux.
So

$$\Phi_B(0) = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = (59\,\mu\mathrm{T})(\pi)(0.13\,\mathrm{m})^2\sin(20^\circ) = 1.1\times10^{-6}\,\mathrm{Wb}.$$

Then using the results of Exercise 11 we have

$$\begin{aligned} q &= \frac{N}{R}(\Phi_B(0) - \Phi_B(t)), \\ &= \frac{950}{85\,\Omega}((1.1\times10^{-6}\,\text{Wb}) - (-1.1\times10^{-6}\,\text{Wb})), \\ &= 2.5\times10^{-5}\,\text{C}. \end{aligned}$$

E34-22 (a) The flux through the loop is

$$\Phi_B = \int_0^{vt} dx \int_a^{a+L} dr \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i v t}{2\pi} \ln \frac{a+L}{a}.$$

The emf is then

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 i v}{2\pi} \ln \frac{a+L}{a}.$$

Putting in the numbers,

$$\mathcal{E} = \frac{(4\pi\times10^{-7}\text{N/A}^2)(110\,\text{A})(4.86\,\text{m/s})}{2\pi} \ln \frac{(0.0102\,\text{m}) + (0.0983\,\text{m})}{(0.0102\,\text{m})} = 2.53\times10^{-4}\text{V}.$$

(b) $i = \mathcal{E}/R = (2.53\times10^{-4}\text{V})/(0.415\,\Omega) = 6.10\times10^{-4}\text{A}$.
(c) $P = i^2 R = (6.10\times10^{-4}\text{A})^2(0.415\,\Omega) = 1.54\times10^{-7}\text{W}$.
(d) $F = \int B i_l\, dl$, or

$$F = i_l \int_a^{a+L} dr \frac{\mu_0 i}{2\pi r} = i_l \frac{\mu_0 i}{2\pi} \ln \frac{a+L}{a}.$$

Putting in the numbers,

$$F = (6.10\times10^{-4}\text{A}) \frac{(4\pi\times10^{-7}\text{N/A}^2)(110\,\text{A})}{2\pi} \ln \frac{(0.0102\,\text{m}) + (0.0983\,\text{m})}{(0.0102\,\text{m})} = 3.17\times10^{-8}\text{N}.$$

(e) $P = Fv = (3.17\times10^{-8}\text{N})(4.86\,\text{m/s}) = 1.54\times10^{-7}\text{W}$.

$\boxed{\textbf{E34-23}}$ (a) Starting from the beginning, Eq. 33-13 gives

$$B = \frac{\mu_0 i}{2\pi y}.$$

The flux through the loop is given by

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}},$$

but since the magnetic field from the long straight wire goes through the loop perpendicular to the plane of the loop this expression simplifies to a scalar integral. The loop is a rectangular, so use $dA = dx\,dy$, and let x be parallel to the long straight wire.

Combining the above,

$$\begin{aligned} \Phi_B &= \int_D^{D+b} \int_0^a \left(\frac{\mu_0 i}{2\pi y}\right) dx\,dy, \\ &= \frac{\mu_0 i}{2\pi} a \int_D^{D+b} \frac{dy}{y}, \\ &= \frac{\mu_0 i}{2\pi} a \ln\left(\frac{D+b}{D}\right) \end{aligned}$$

(b) The flux through the loop is a function of the distance D from the wire. If the loop moves away from the wire at a constant speed v, then the distance D varies as vt. The induced emf is then

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt}, \\ &= \frac{\mu_0 i}{2\pi} a \frac{b}{t(vt+b)}. \end{aligned}$$

The current will be this emf divided by the resistance R. The "back-of-the-book" answer is somewhat different; the answer is expressed in terms of D instead if t. The two answers are otherwise identical.

E34-24 (a) The area of the triangle is $A = x^2 \tan\theta/2$. In this case $x = vt$, so

$$\Phi_B = B(vt)^2 \tan\theta/2,$$

and then

$$\mathcal{E} = 2Bv^2 t \tan\theta/2,$$

(b) $t = \mathcal{E}/2Bv^2 \tan\theta/2$, so

$$t = \frac{(56.8\,\mathrm{V})}{2(0.352\,\mathrm{T})(5.21\,\mathrm{m/s})^2 \tan(55^\circ)} = 2.08\,\mathrm{s}.$$

E34-25 $\mathcal{E} = NBA\omega$, so

$$\omega = \frac{(24\,\mathrm{V})}{(97)(0.33\,\mathrm{T})(0.0190\,\mathrm{m}^2)} = 39.4\,\mathrm{rad/s}.$$

That's 6.3 rev/second.

E34-26 (a) The frequency of the emf is the same as the frequency of rotation, f.

(b) The flux changes by $BA = B\pi a^2$ during a half a revolution. This is a sinusoidal change, so the amplitude of the sinusoidal variation in the emf is $\mathcal{E} = \Phi_B \omega/2$. Then $\mathcal{E} = B\pi^2 a^2 f$.

$\boxed{\textbf{E34-27}}$ We can use Eq. 34-10; the emf is $\mathcal{E} = BA\omega \sin\omega t$, This will be a maximum when $\sin\omega t = 1$. The angular frequency, ω is equal to $\omega = (1000)(2\pi)/(60)$ rad/s $= 105$ rad/s The maximum emf is then

$$\mathcal{E} = (3.5\,\mathrm{T})\left[(100)(0.5\,\mathrm{m})(0.3\,\mathrm{m})\right](105\ \mathrm{rad/s}) = 5.5\ \mathrm{kV}.$$

E34-28 (a) The amplitude of the emf is $\mathcal{E} = BA\omega$, so

$$A = \mathcal{E}/2\pi f B = (150\,\mathrm{V})/2\pi(60/\mathrm{s})(0.50\,\mathrm{T}) = 0.798\mathrm{m}^2.$$

(b) Divide the previous result by 100. $A = 79.8\ \mathrm{cm}^2$.

E34-29 $d\Phi_B/dt = A\,dB/dt = A(-8.50\,\mathrm{mT/s})$.

(a) For this path

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -d\Phi_B/dt = --\pi(0.212\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) = -1.20\,\mathrm{mV}.$$

(b) For this path

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -d\Phi_B/dt = --\pi(0.323\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) = -2.79\,\mathrm{mV}.$$

(c) For this path

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -d\Phi_B/dt = - - \pi(0.323\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) - \pi(0.323\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) = 1.59\,\mathrm{mV}.$$

E34-30 $d\Phi_B/dt = A\,dB/dt = A(-6.51\,\mathrm{mT/s})$, while $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 2\pi r E$.

(a) The path of integration is *inside* the solenoid, so

$$E = \frac{-\pi r^2(-6.51\,\mathrm{mT/s})}{2\pi r} = \frac{(0.022\,\mathrm{m})(-6.51\,\mathrm{mT/s})}{2} = 7.16\times10^{-5}\mathrm{V/m}.$$

(b) The path of integration is *outside* the solenoid, so

$$E = \frac{-\pi r^2(-6.51\,\mathrm{mT/s})}{2\pi R} = \frac{(0.063\,\mathrm{m})^2(-6.51\,\mathrm{mT/s})}{2(0.082\,\mathrm{m})} = 1.58\times10^{-4}\mathrm{V/m}$$

E34-31 The induced electric field can be found from applying Eq. 34-13,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}.$$

We start with the left hand side of this expression. The problem has cylindrical symmetry, so the induced electric field lines should be circles centered on the axis of the cylindrical volume. If we choose the path of integration to lie along an electric field line, then the electric field $\vec{\mathbf{E}}$ will be parallel to $d\vec{\mathbf{s}}$, and E will be uniform along this path, so

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = \oint E\,ds = E\oint ds = 2\pi r E,$$

where r is the radius of the circular path.

Now for the right hand side. The flux is contained in the path of integration, so $\Phi_B = B\pi r^2$. All of the time dependence of the flux is contained in B, so we can immediately write

$$2\pi r E = -\pi r^2 \frac{dB}{dt} \text{ or } E = -\frac{r}{2}\frac{dB}{dt}.$$

What does the negative sign mean? The path of integration is chosen so that if our right hand fingers curl around the path our thumb gives the direction of the magnetic field which cuts through the path. Since the field points into the page a positive electric field would have a clockwise orientation. Since B is decreasing the derivative is negative, but we get another negative from the equation above, so the electric field has a positive direction.

Now for the magnitude.

$$E = (4.82\times10^{-2}\,\mathrm{m})(10.7\times10^{-3}\,T/\mathrm{s})/2 = 2.58\times10^{-4}\,\mathrm{N/C}.$$

The acceleration of the electron at either a or c then has magnitude

$$a = Eq/m = (2.58\times10^{-4}\,\mathrm{N/C})(1.60\times10^{-19}\,\mathrm{C})/(9.11\times10^{-31}\,\mathrm{kg}) = 4.53\times10^{7}\mathrm{m/s^2}.$$

P34-1 The induced current is given by $i = \mathcal{E}/R$. The resistance of the loop is given by $R = \rho L/A$, where A is the cross sectional area. Combining, and writing in terms of the radius of the wire, we have

$$i = \frac{\pi r^2 \mathcal{E}}{\rho L}.$$

The length of the wire is related to the radius of the wire because we have a fixed mass. The total volume of the wire is $\pi r^2 L$, and this is related to the mass and density by $m = \delta \pi r^2 L$. Eliminating r we have

$$i = \frac{m\mathcal{E}}{\rho \delta L^2}.$$

The length of the wire loop is the same as the circumference, which is related to the radius R of the loop by $L = 2\pi R$. The emf is related to the changing flux by $\mathcal{E} = -d\Phi_B/dt$, but if the shape of the loop is fixed this becomes $\mathcal{E} = -A\,dB/dt$. Combining all of this,

$$i = \frac{mA}{\rho\delta(2\pi R)^2}\frac{dB}{dt}.$$

We dropped the negative sign because we are only interested in absolute values here.

Now $A = \pi R^2$, so this expression can also be written as

$$i = \frac{m\pi R^2}{\rho\delta(2\pi R)^2}\frac{dB}{dt} = \frac{m}{4\pi\rho\delta}\frac{dB}{dt}.$$

P34-2 For the lower surface $\vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = (76\times10^{-3}\mathrm{T})(\pi/2)(0.037\,\mathrm{m})^2\cos(62°) = 7.67\times10^{-5}\mathrm{Wb}$. For the upper surface $\vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = (76\times10^{-3}\mathrm{T})(\pi/2)(0.037\,\mathrm{m})^2\cos(28°) = 1.44\times10^{-4}\mathrm{Wb}$.. The induced emf is then

$$\mathcal{E} = (7.67\times10^{-5}\mathrm{Wb} + 1.44\times10^{-4}\mathrm{Wb})/(4.5\times10^{-3}\mathrm{s}) = 4.9\times10^{-2}\mathrm{V}.$$

P34-3 (a) We are only interested in the portion of the ring in the yz plane. Then $\mathcal{E} = (3.32\times10^{-3}\mathrm{T/s})(\pi/4)(0.104\,\mathrm{m})^2 = 2.82\times10^{-5}\mathrm{V}$.

(b) From c to b. Point your right thumb along $-x$ to oppose the increasing $\vec{\mathbf{B}}$ field. Your right fingers will curl from c to b.

P34-4 $\mathcal{E} \propto NA$, but $A = \pi r^2$ and $N 2\pi r = L$, so $\mathcal{E} \propto 1/N$. This means use only one loop to maximize the emf.

$\boxed{\textbf{P34-5}}$ This is a integral best performed in rectangular coordinates, then $dA = (dx)(dy)$. The magnetic field is perpendicular to the surface area, so $\vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = B\,dA$. The flux is then

$$\begin{aligned}
\Phi_B &= \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA, \\
&= \int_0^a \int_0^a (4\,\mathrm{T/m\cdot s^2})t^2 y\,dy\,dx, \\
&= (4\,\mathrm{T/m\cdot s^2})t^2 \left(\frac{1}{2}a^2\right) a, \\
&= (2\,\mathrm{T/m\cdot s^2})a^3 t^2.
\end{aligned}$$

But $a = 2.0$ cm, so this becomes

$$\Phi_B = (2\,\mathrm{T/m\cdot s^2})(0.02\,\mathrm{m})^3 t^2 = (1.6\times10^{-5}\mathrm{Wb/s^2})t^2.$$

The emf around the square is given by

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -(3.2\times10^{-5}\mathrm{Wb/s^2})t,$$

and at $t = 2.5\,\mathrm{s}$ this is $-8.0\times10^{-5}\mathrm{V}$. Since the magnetic field is directed out of the page, a positive emf would be counterclockwise (hold your right thumb in the direction of the magnetic field and your fingers will give a counter clockwise sense around the loop). But the answer was negative, so the emf must be clockwise.

P34-6 (a) Far from the plane of the large loop we can approximate the large loop as a dipole, and then

$$B = \frac{\mu_0 i \pi R^2}{2x^3}.$$

The flux through the small loop is then

$$\Phi_B = \pi r^2 B = \frac{\mu_0 i \pi^2 r^2 R^2}{2x^3}.$$

(b) $\mathcal{E} = -d\Phi_B/dt$, so

$$\mathcal{E} = \frac{3\mu_0 i \pi^2 r^2 R^2}{2x^4} v.$$

(c) Anti-clockwise when viewed from above.

$\boxed{\textbf{P34-7}}$ The magnetic field is perpendicular to the surface area, so $\vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = B\,dA$. The flux is then

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = BA,$$

since the magnetic field is uniform. The area is $A = \pi r^2$, where r is the radius of the loop. The induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -2\pi r B \frac{dr}{dt}.$$

It is given that $B = 0.785\,\mathrm{T}$, $r = 1.23\,\mathrm{m}$, and $dr/dt = -7.50\times10^{-2}\mathrm{m/s}$. The negative sign indicate a decreasing radius. Then

$$\mathcal{E} = -2\pi(1.23\,\mathrm{m})(0.785\,\mathrm{T})(-7.50\times10^{-2}\mathrm{m/s}) = 0.455\,\mathrm{V}.$$

P34-8 (a) $d\Phi_B/dt = B\,dA/dt$, but dA/dt is $\Delta A/\Delta t$, where ΔA is the area swept out during one rotation and $\Delta t = 1/f$. But the area swept out is πR^2, so

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi f B R^2.$$

(b) If the output current is i then the power is $P = \mathcal{E} i$. But $P = \tau\omega = \tau 2\pi f$, so

$$\tau = \frac{P}{2\pi f} = iBR^2/2.$$

P34-9 (a) $\mathcal{E} = -d\Phi_B/dt$, and $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$,so

$$\mathcal{E} = BLv\cos\theta.$$

The component of the force of gravity on the rod which pulls it down the incline is $F_G = mg\sin\theta$. The component of the magnetic force on the rod which pulls it up the incline is $F_B = BiL\cos\theta$. Equating,

$$BiL\cos\theta = mg\sin\theta,$$

and since $\mathcal{E} = iR$,

$$v = \frac{\mathcal{E}}{BL\cos\theta} = \frac{mgR\sin\theta}{B^2L^2\cos^2\theta}.$$

(b) $P = i\mathcal{E} = \mathcal{E}^2/R = B^2L^2v^2\cos^2\theta/R = mgv\sin\theta$. This is identical to the rate of change of gravitational potential energy.

P34-10 Let the cross section of the wire be a.

(a) $R = \rho L/a = \rho(r\theta + 2r)/a$; with numbers,

$$R = (3.4\times 10^{-3}\Omega)(2+\theta).$$

(b) $\Phi_B = B\theta r^2/2$; with numbers,

$$\Phi_B = (4.32\times 10^{-3}\text{Wb})\theta.$$

(c) $i = \mathcal{E}/R = B\omega r^2/2R = Ba\omega r/2\rho(\theta+2)$, or

$$i = \frac{Ba\alpha tr}{\rho(\alpha t^2+4)}.$$

Take the derivative and set it equal to zero,

$$0 = \frac{4 - at^2}{(\alpha t^2+4)^2},$$

so $at^2 = 4$, or $\theta = \frac{1}{2}at^2 = 2$ rad.

(d) $\omega = \sqrt{2\alpha\theta}$, so

$$i = \frac{(0.15\,\text{T})(1.2\times 10^{-6}\text{m}^2)\sqrt{2(12\text{ rad/s}^2)(2\text{ rad})}(0.24\,\text{m})}{(1.7\times 10^{-8}\Omega\cdot\text{m})(6\text{ rad})} = 2.2\,\text{A}.$$

P34-11 It does say approximate, so we will be making some rather bold assumptions here. First we will find an expression for the emf. Since B is constant, the emf must be caused by a change in the area; in this case a shift in position. The small square where $B \neq 0$ has a width a and sweeps around the disk with a speed $r\omega$. An approximation for the emf is then $\mathcal{E} = Bar\omega$. This emf causes a current. We don't know exactly where the current flows, but we can reasonably assume that it occurs near the location of the magnetic field. Let us assume that it is constrained to that region of the disk. The resistance of this portion of the disk is the approximately

$$R = \frac{1}{\sigma}\frac{L}{A} = \frac{1}{\sigma}\frac{a}{at} = \frac{1}{\sigma t},$$

where we have assumed that the current is flowing radially when defining the cross sectional area of the "resistor". The induced current is then (on the order of)

$$\frac{\mathcal{E}}{R} = \frac{Bar\omega}{1/(\sigma t)} = Bar\omega\sigma t.$$

This current experiences a breaking force according to $F = BIl$, so

$$F = B^2a^2r\omega\sigma t,$$

where l is the length through which the current flows, which is a.

Finally we can find the torque from $\tau = rF$, and

$$\tau = B^2a^2r^2\omega\sigma t.$$

P34-12 The induced electric field in the ring is given by Eq. 34-11: $2\pi RE = |d\Phi_B/dt|$. This electric field will result in a force on the free charge carries (electrons?), given by $F = Ee$. The acceleration of the electrons is then $a = Ee/m_\text{e}$. Then

$$a = \frac{e}{2\pi Rm_\text{e}}\frac{d\Phi_B}{dt}.$$

Integrate both sides with respect to time to find the speed of the electrons.

$$\begin{aligned}\int a\,dt &= \int \frac{e}{2\pi Rm_\text{e}}\frac{d\Phi_B}{dt}dt,\\ v &= \frac{e}{2\pi Rm_\text{e}}\int \frac{d\Phi_B}{,}\\ &= \frac{e}{2\pi Rm_\text{e}}\Delta\Phi_B.\end{aligned}$$

The current density is given by $j = nev$, and the current by $iA = i\pi a^2$. Combining,

$$i = \frac{ne^2a^2}{2Rm_\text{e}}\Delta Phi_B.$$

Actually, it should be pointed out that ΔPhi_B refers to the change in flux from *external* sources. The current induced in the wire will produce a flux which will exactly offset ΔPhi_B so that the *net* flux through the superconducting ring is fixed at the value present when the ring became superconducting.

P34-13 Assume that E does vary as the picture implies. Then the line integral along the path shown *must* be nonzero, since $\vec{\mathbf{E}}\cdot\vec{\mathbf{l}}$ on the right is not zero, while it is along the three other sides. Hence $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$ is non zero, implying a change in the magnetic flux through the dotted path. But it doesn't, so $\vec{\mathbf{E}}$ cannot have an abrupt change.

P34-14 The electric field a distance r from the center is given by

$$E = \frac{\pi r^2 dB/dT}{2\pi r} = \frac{r}{2}\frac{dB}{dt}.$$

This field is directed perpendicular to the radial lines.

Define h to be the distance from the center of the circle to the center of the rod, and evaluate $\mathcal{E} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$,

$$\begin{aligned}\mathcal{E} &= \frac{dB}{dt}\int \frac{r}{2}\frac{h}{r}dx,\\ &= \frac{dB}{dt}\frac{L}{2}h.\end{aligned}$$

But $h^2 = R^2 - (L/2)^2$, so

$$\mathcal{E} = \frac{dB}{dt}\frac{L}{2}\sqrt{R^2 - (L/2)^2}.$$

P34-15 (a) $\Phi_B = \pi r^2 B_\text{av}$, so

$$E = \frac{\mathcal{E}}{2\pi r} = \frac{(0.32\,\text{m})}{2}2(0.28\,\text{T})(120\,\pi) = 34\,\text{V/m}.$$

(b) $a = F/m = Eq/m = (33.8\,\text{V/m})(1.6\times10^{-19}\text{C})/(9.1\times10^{-31}\text{kg}) = 6.0\times10^{12}\text{m/s}^2$.

$\boxed{\textbf{E35-1}}$ If the Earth's magnetic dipole moment were produced by a single current around the core, then that current would be

$$i = \frac{\mu}{A} = \frac{(8.0 \times 10^{22}\,\mathrm{J/T})}{\pi(3.5 \times 10^{6}\mathrm{m})^2} = 2.1\times 10^{9}\,\mathrm{A}$$

E35-2 (a) $i = \mu/A = (2.33\,\mathrm{A}\cdot\mathrm{m}^2)/(160)\pi(0.0193\,\mathrm{m})^2 = 12.4\,\mathrm{A}$.
(b) $\tau = \mu B = (2.33\,\mathrm{A}\cdot\mathrm{m}^2)(0.0346\,\mathrm{T}) = 8.06\times 10^{-2}\mathrm{N}\cdot\mathrm{m}$.

E35-3 (a) Using the right hand rule a clockwise current would generate a magnetic moment which would be into the page. Both currents are clockwise, so add the moments:

$$\mu = (7.00\,\mathrm{A})\pi(0.20\,\mathrm{m})^2 + (7.00\,\mathrm{A})\pi(0.30\,\mathrm{m})^2 = 2.86\,\mathrm{A}\cdot\mathrm{m}^2.$$

(b) Reversing the current reverses the moment, so

$$\mu = (7.00\,\mathrm{A})\pi(0.20\,\mathrm{m})^2 - (7.00\,\mathrm{A})\pi(0.30\,\mathrm{m})^2 = -1.10\,\mathrm{A}\cdot\mathrm{m}^2.$$

E35-4 (a) $\mu = iA = (2.58\,\mathrm{A})\pi(0.16\,\mathrm{m})^2 = 0.207\,\mathrm{A}\cdot\mathrm{m}^2$.
(b) $\tau = \mu B \sin\theta = (0.207\,\mathrm{A}\cdot\mathrm{m}^2)(1.20\,\mathrm{T})\sin(41^\circ) = 0.163\,\mathrm{N}\cdot\mathrm{m}$.

$\boxed{\textbf{E35-5}}$ (a) The result from Problem 33-4 for a square loop of wire was

$$B(z) = \frac{4\mu_0 i a^2}{\pi(4z^2 + a^2)(4z^2 + 2a^2)^{1/2}}.$$

For z much, much larger than a we can ignore any a terms which are added to or subtracted from z terms. This means that

$$4z^2 + a^2 \to 4z^2 \text{ and } (4z^2 + 2a^2)^{1/2} \to 2z,$$

but we can't ignore the a^2 in the numerator.

The expression for B then simplifies to

$$B(z) = \frac{\mu_0 i a^2}{2\pi z^3},$$

which certainly looks like Eq. 35-4.

(b) We can rearrange this expression and get

$$B(z) = \frac{\mu_0}{2\pi z^3} i a^2,$$

where it is rather evident that ia^2 must correspond to $\vec{\mu}$, the dipole moment, in Eq. 35-4. So that must be the answer.

E35-6 $\mu = iA = (0.2\,\mathrm{A})\pi(0.08\,\mathrm{m})^2 = 4.02\times 10^{-3}\mathrm{A}\cdot\mathrm{m}^2$; $\vec{\mu} = \mu\hat{\mathbf{n}}$.

(a) For the torque,

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}} = (-9.65\times 10^{-4}\mathrm{N}\cdot\mathrm{m})\hat{\mathbf{i}} + (-7.24\times 10^{-4}\mathrm{N}\cdot\mathrm{m})\hat{\mathbf{j}} + (8.08\times 10^{-4}\mathrm{N}\cdot\mathrm{m})\hat{\mathbf{k}}.$$

(b) For the magnetic potential energy,

$$U = \vec{\mu}\cdot\vec{\mathbf{B}} = \mu[(0.60)(0.25\,\mathrm{T})] = 0.603\times 10^{-3}\mathrm{J}.$$

E35-7 $\mu = iA = i\pi(a^2 + b^2/2) = i\pi(a^2 + b^2)/2$.

E35-8 If the distance to P is very large compared to a or b we can write the Law of Biot and Savart as

$$\vec{\mathbf{B}} = \frac{\mu_0 i}{4\pi} \frac{\vec{\mathbf{s}} \times \vec{\mathbf{r}}}{r^3}.$$

$\vec{\mathbf{s}}$ is perpendicular to $\vec{\mathbf{r}}$ for the left and right sides, so the left side contributes

$$B_1 = \frac{\mu_0 i}{4\pi} \frac{b}{(x + a/2)^2},$$

and the right side contributes

$$B_3 = -\frac{\mu_0 i}{4\pi} \frac{b}{(x - a/2)^2}.$$

The top and bottom sides each contribute an equal amount

$$B_2 = B_4 = \frac{\mu_0 i}{4\pi} \frac{a \sin\theta}{x^2 + b^2/4} \approx \frac{\mu_0 i}{4\pi} \frac{a(b/2)}{x^3}.$$

Add the four terms, expand the denominators, and keep only terms in x^3,

$$B = -\frac{\mu_0 i}{4\pi} \frac{ab}{x^3} = -\frac{\mu_0}{4\pi} \frac{\mu}{x^3}.$$

The negative sign indicates that it is into the page.

$\boxed{\textbf{E35-9}}$ (a) The electric field at this distance from the proton is

$$E = \frac{1}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)} \frac{(1.60\times10^{-19}\mathrm{C})}{(5.29\times10^{-11}\mathrm{m})^2} = 5.14\times10^{11}\mathrm{N/C}.$$

(b) The magnetic field at this from the proton is given by the dipole approximation,

$$\begin{aligned} B(z) &= \frac{\mu_0 \mu}{2\pi z^3}, \\ &= \frac{(4\pi\times10^{-7}\mathrm{N/A}^2)(1.41\times10^{-26}\mathrm{A/m}^2)}{2\pi(5.29\times10^{-11}\mathrm{m})^3}, \\ &= 1.90\times10^{-2}\,\mathrm{T} \end{aligned}$$

E35-10 1.50 g of water has $(2)(6.02\times10^{23})(1.5)/(18) = 1.00\times10^{23}$ hydrogen nuclei. If all are aligned the net magnetic moment would be $\mu = (1.00\times10^{23})(1.41\times10^{-26}\mathrm{J/T}) = 1.41\times10^{-3}\mathrm{J/T}$. The field strength is then

$$B = \frac{\mu_0}{4\pi} \frac{\mu}{x^3} = (1.00\times10^{-7}\mathrm{N/A}^2) \frac{(1.41\times10^{-3}\mathrm{J/T})}{(5.33\,\mathrm{m})^3} = 9.3\times10^{-13}\mathrm{T}.$$

E35-11 (a) There is effectively a current of $i = fq = q\omega/2\pi$. The dipole moment is then $\mu = iA = (q\omega/2\pi)(\pi r^2) = \frac{1}{2}q\omega r^2$.

(b) The rotational inertia of the ring is mr^2 so $L = I\omega = mr^2\omega$. Then

$$\frac{\mu}{L} = \frac{(1/2)q\omega r^2}{mr^2\omega} = \frac{q}{2m}.$$

E35-12 The mass of the bar is

$$m = \rho V = (7.87\ \text{g/cm}^3)(4.86\ \text{cm})(1.31\ \text{cm}^2) = 50.1\ \text{g}.$$

The number of atoms in the bar is

$$N = (6.02\times 10^{23})(50.1\ \text{g})/(55.8\ \text{g}) = 5.41\times 10^{23}.$$

The dipole moment of the bar is then

$$\mu = (5.41\times 10^{23})(2.22)(9.27\times 10^{-24}\,\text{J/T}) = 11.6\,\text{J/T}.$$

(b) The torque on the magnet is $\tau = (11.6\,\text{J/T})(1.53\,\text{T}) = 17.7\,\text{N}\cdot\text{m}$.

$\boxed{\textbf{E35-13}}$ The magnetic dipole moment is given by $\mu = MV$, Eq. 35-13. Then

$$\mu = (5,300\,\text{A/m})(0.048\,\text{m})\pi(0.0055\,\text{m})^2 = 0.024\,\text{A}\cdot\text{m}^2.$$

E35-14 (a) The original field is $B_0 = \mu_0 in$. The field will increase to $B = \kappa_\text{m} B_0$, so the increase is

$$\Delta B = (\kappa_1 - 1)\mu_0 in = (3.3\times 10^{-4})(4\pi\times 10^{-7}\text{N/A}^2)(1.3\,\text{A})(1600/\text{m}) = 8.6\times 10^{-7}\text{T}.$$

(b) $M = (\kappa_1 - 1)B_0/\mu_0 = (\kappa_1 - 1)in = (3.3\times 10^{-4})(1.3\,\text{A})(1600/\text{m}) = 0.69\,\text{A/m}$.

E35-15 The energy to flip the dipoles is given by $U = 2\mu B$. The temperature is then

$$T = \frac{2\mu B}{3k/2} = \frac{4(1.2\times 10^{-23}\text{J/T})(0.5\,\text{T})}{3(1.38\times 10^{-23}\text{J/K})} = 0.58\,\text{K}.$$

E35-16 The Curie temperature of iron is 770°C, which is 750°C higher than the surface temperature. This occurs at a depth of (750°C)/(30 C°/km) = 25 km.

$\boxed{\textbf{E35-17}}$ (a) Look at the figure. At 50% (which is 0.5 on the vertical axis), the curve is at $B_0/T \approx 0.55\,\text{T/K}$. Since $T = 300\,\text{K}$, we have $B_0 \approx 165\,\text{T}$.

(b) Same figure, but now look at the 90% mark. $B_0/T \approx 1.60\,\text{T/K}$, so $B_0 \approx 480\,\text{T}$.

(c) Good question. I think both fields are far beyond our current abilities.

E35-18 (a) Look at the figure. At 50% (which is 0.5 on the vertical axis), the curve is at $B_0/T \approx 0.55\,\text{T/K}$. Since $B_0 = 1.8\,\text{T}$, we have $T \approx (1.8\,\text{T})/(0.55\,\text{T/K}) = 3.3\,\text{K}$.

(b) Same figure, but now look at the 90% mark. $B_0/T \approx 1.60\,\text{T/K}$, so $T \approx (1.8\,\text{T})/(1.60\,\text{T/K}) = 1.1\,\text{K}$.

E35-19 Since $(0.5\,\text{T})/(10\,\text{K}) = 0.05\,\text{T/K}$, and all higher temperatures have lower values of the ratio, and this puts all points in the region near where Curie's Law (the straight line) is valid, then the answer is yes.

E35-20 Using Eq. 35-19,

$$\mu_n = \frac{VM}{N} = \frac{M_r M}{A\rho} = \frac{(108\text{g/mol})(511\times 10^3\,\text{A/m})}{(10490\,\text{kg/m}^3)(6.02\times 10^{23}/\text{mol})} = 8.74\times 10^{-21}\text{A/m}^2$$

E35-21 (a) $B = \mu_0\mu/2z^3$, so

$$B = \frac{(4\pi\times 10^{-7}\text{N/A}^2)(1.5\times 10^{-23}\text{J/T})}{2(10\times 10^{-9}\text{m})^3} = 9.4\times 10^{-6}\text{T}.$$

(b) $U = 2\mu B = 2(1.5\times 10^{-23}\text{J/T})(9.4\times 10^{-6}\text{T}) = 2.82\times 10^{-28}\text{J}.$

E35-22 $\Phi_B = (43\times 10^{-6}\text{T})(295,000\times 10^6\text{m}^2) = 1.3\times 10^7\text{Wb}.$

E35-23 (a) We'll assume, however, that all of the iron atoms are perfectly aligned. Then the dipole moment of the earth will be related to the dipole moment of one atom by

$$\mu_{\text{Earth}} = N\mu_{\text{Fe}},$$

where N is the number of iron atoms in the magnetized sphere. If m_A is the relative atomic mass of iron, then the total mass is

$$m = \frac{Nm_A}{A} = \frac{m_A}{A}\frac{\mu_{\text{Earth}}}{\mu_{\text{Fe}}},$$

where A is Avogadro's number. Next, the volume of a sphere of mass m is

$$V = \frac{m}{\rho} = \frac{m_A}{\rho A}\frac{\mu_{\text{Earth}}}{\mu_{\text{Fe}}},$$

where ρ is the density.

And finally, the radius of a sphere with this volume would be

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3\mu_{\text{Earth}}m_A}{4\pi\rho\mu_{\text{Fe}}A}\right)^{1/3}.$$

Now we find the radius by substituting in the known values,

$$r = \left(\frac{3(8.0\times 10^{22}\text{J/T})(56\text{ g/mol})}{4\pi(14\times 10^6\text{g/m}^3)(2.1\times 10^{-23}\text{J/T})(6.0\times 10^{23}/\text{mol})}\right)^{1/3} = 1.8\times 10^5\text{m}.$$

(b) The fractional volume is the cube of the fractional radius, so the answer is

$$(1.8\times 10^5\text{ m}/6.4\times 10^6)^3 = 2.2\times 10^{-5}.$$

E35-24 (a) At magnetic equator $L_{\text{m}} = 0$, so

$$B = \frac{\mu_0\mu}{4\pi r^3} = \frac{(1.00\times 10^{-7}\text{N/A}^2)(8.0\times 10^{22}\text{J/T})}{(6.37\times 10^6\text{m})^3} = 31\mu\text{T}.$$

There is no vertical component, so the inclination is zero.

(b) Here $L_{\text{m}} = 60°$, so

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2 L_{\text{m}}} = \frac{(1.00\times 10^{-7}\text{N/A}^2)(8.0\times 10^{22}\text{J/T})}{(6.37\times 10^6\text{m})^3}\sqrt{1+3\sin^2(60°)} = 56\mu\text{T}.$$

The inclination is given by

$$\arctan(B_{\text{v}}/B_{\text{h}}) = \arctan(2\tan L_{\text{m}}) = 74°.$$

(c) At magnetic north pole $L_{\text{m}} = 90°$, so

$$B = \frac{\mu_0\mu}{2\pi r^3} = \frac{2(1.00\times 10^{-7}\text{N/A}^2)(8.0\times 10^{22}\text{J/T})}{(6.37\times 10^6\text{m})^3} = 62\mu\text{T}.$$

There is no horizontal component, so the inclination is 90°.

E35-25 This problem is effectively solving $1/r^3 = 1/2$ for r measured in Earth radii. Then $r = 1.26 r_{\rm E}$, and the altitude above the Earth is $(0.26)(6.37\times10^6{\rm m}) = 1.66\times10^6{\rm m}$.

E35-26 The radial distance from the center is $r = (6.37\times10^6{\rm m}) - (2900\times10^3{\rm m}) = 3.47\times10^6{\rm m}$. The field strength is

$$B = \frac{\mu_0\mu}{2\pi r^3} = \frac{2(1.00\times10^{-7}{\rm N/A^2})(8.0\times10^{22}{\rm J/T})}{(3.47\times10^6{\rm m})^3} = 380\mu{\rm T}.$$

E35-27 Here $L_{\rm m} = 90° - 11.5° = 78.5°$, so

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2 L_{\rm m}} = \frac{(1.00\times10^{-7}{\rm N/A^2})(8.0\times10^{22}{\rm J/T})}{(6.37\times10^6{\rm m})^3}\sqrt{1+3\sin^2(78.5°)} = 61\mu{\rm T}.$$

The inclination is given by

$$\arctan(B_{\rm v}/B_{\rm h}) = \arctan(2\tan L_{\rm m}) = 84°.$$

E35-28 The flux out the "other" end is $(1.6\times10^{-3}{\rm T})\pi(0.13\,{\rm m})^2 = 85\mu{\rm Wb}$. The net flux through the surface is zero, so the flux through the curved surface is $0 - (85\mu{\rm Wb}) - (-25\mu{\rm Wb}) = -60\mu{\rm Wb}$.. The negative indicates inward.

$\boxed{\textbf{E35-29}}$ The total magnetic flux through a closed surface is zero. There is inward flux on faces one, three, and five for a total of -9 Wb. There is outward flux on faces two and four for a total of +6 Wb. The difference is +3 Wb; consequently the outward flux on the sixth face must be +3 Wb.

E35-30 The stable arrangements are (a) and (c). The torque in each case is zero.

E35-31 The field on the x axis between the wires is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{2r+x} + \frac{1}{2r-x}\right).$$

Since $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$, we can assume the flux through the curved surface is equal to the flux through the xz plane within the cylinder. This flux is

$$\begin{aligned}\Phi_B &= L\int_{-r}^{r}\left[\frac{\mu_0 i}{2\pi}\left(\frac{1}{2r+x}+\frac{1}{2r-x}\right)\right]dx,\\ &= L\frac{\mu_0 i}{2\pi}\left(\ln\frac{3r}{r} - \ln\frac{r}{3r}\right),\\ &= L\frac{\mu_0 i}{\pi}\ln 3.\end{aligned}$$

$\boxed{\textbf{P35-1}}$ We can imagine the rotating disk as being composed of a number of rotating rings of radius r, width dr, and circumference $2\pi r$. The surface charge density on the disk is $\sigma = q/\pi R^2$, and consequently the (differential) charge on any ring is

$$dq = \sigma(2\pi r)(dr) = \frac{2qr}{R^2}dr$$

The rings "rotate" with angular frequency ω, or period $T = 2\pi/\omega$. The effective (differential) current for each ring is then

$$di = \frac{dq}{T} = \frac{qr\omega}{\pi R^2}dr.$$

Each ring contributes to the magnetic moment, and we can glue all of this together as

$$\begin{aligned}\mu &= \int d\mu,\\ &= \int \pi r^2\, di,\\ &= \int_0^R \frac{qr^3\omega}{R^2} dr,\\ &= \frac{qR^2\omega}{4}.\end{aligned}$$

P35-2 (a) The sphere can be sliced into disks. The disks can be sliced into rings. Each ring has some charge q_i, radius r_i, and mass m_i; the period of rotation for a ring is $T = 2\pi/\omega$, so the current in the ring is $q_i/T = \omega q_i/2\pi$. The magnetic moment is

$$\mu_i = (\omega q_i/2\pi)\pi r_i^2 = \omega q_i r_i^2/2.$$

Note that this is *closely* related to the expression for angular momentum of a ring: $l_i = \omega m_i r_i^2$. Equating,

$$\mu_i = q_i l_i/2m_i.$$

If both mass density and charge density are uniform then we can write $q_i/m_i = q/m$,

$$\mu = \int d\mu = (q/2m)\int dl = qL/2m$$

For a solid sphere $L = \omega I = 2\omega mR^2/5$, so

$$\mu = q\omega R^2/5.$$

(b) See part (a)

P35-3 (a) The orbital speed is given by $K = mv^2/2$. The orbital radius is given by $mv = qBr$, or $r = mv/qB$. The frequency of revolution is $f = v/2\pi r$. The effective current is $i = qf$. Combining all of the above to find the dipole moment,

$$\mu = iA = q\frac{v}{2\pi r}\pi r^2 = q\frac{vr}{2} = q\frac{mv^2}{2qB} = \frac{K}{B}.$$

(b) Since q and m cancel out of the above expression the answer is the same!
(c) Work it out:

$$M = \frac{\mu}{V} = \frac{(5.28\times10^{21}/\mathrm{m}^3)(6.21\times10^{-20}\,\mathrm{J})}{(1.18\,\mathrm{T})} + \frac{(5.28\times10^{21}/\mathrm{m}^3)(7.58\times10^{-21}\,\mathrm{J})}{(1.18\,\mathrm{T})} = 312\,\mathrm{A/m}.$$

P35-4 (b) Point the thumb or your right hand to the right. Your fingers curl in the direction of the current in the wire loop.

(c) In the vicinity of the wire of the loop $\vec{\mathbf{B}}$ has a component which is directed radially outward. Then $\vec{\mathbf{B}} \times d\vec{s}$ has a component directed to the left. Hence, the net force is directed to the left.

P35-5 (b) Point the thumb or your right hand to the left. Your fingers curl in the direction of the current in the wire loop.

(c) In the vicinity of the wire of the loop $\vec{\mathbf{B}}$ has a component which is directed radially outward. Then $\vec{\mathbf{B}} \times d\vec{s}$ has a component directed to the right. Hence, the net force is directed to the right.

P35-6 (a) Let $x = \mu B/kT$. Adopt the convention that N_+ refers to the atoms which have parallel alignment and N_- those which are anti-parallel. Then $N_+ + N_- = N$, so

$$N_+ = Ne^x/(e^x + e^{-x}),$$

and

$$N_- = Ne^{-x}/(e^x + e^{-x}),$$

Note that the denominators are necessary so that $N_+ + N_- = N$. Finally,

$$M = \mu(N_+ - N_-) = \mu N \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

(b) If $\mu B \ll kT$ then x is very small and $e^{\pm x} \approx 1 \pm x$. The above expression reduces to

$$M = \mu N \frac{(1+x) - (1-x)}{(1+x) + (1-x)} = \mu N x = \frac{\mu^2 B}{kT}.$$

(c) If $\mu B \gg kT$ then x is very large and $e^{\pm x} \to \infty$ while $e^{-x} \to 0$. The above expression reduces to

$$N = \mu N.$$

$\boxed{\textbf{P35-7}}$ (a) Centripetal acceleration is given by $a = r\omega^2$. Then

$$\begin{aligned} a - a_0 &= r(\omega_0 + \Delta\omega)^2 - r\omega_0^2, \\ &= 2r\omega_0\Delta\omega + r(\Delta\omega_0)^2, \\ &\approx 2r\omega_0\Delta\omega. \end{aligned}$$

(b) The change in centripetal acceleration is caused by the additional magnetic force, which has magnitude $F_B = qvB = er\omega B$. Then

$$\Delta\omega = \frac{a - a_0}{2r\omega_0} = \frac{eB}{2m}.$$

Note that we boldly canceled ω against ω_0 in this last expression; we are assuming that $\Delta\omega$ is small, and for these problems it is.

P35-8 (a) $i = \mu/A = (8.0\times10^{22}\mathrm{J/T})/\pi(6.37\times10^6\mathrm{m})^2 = 6.3\times10^8\mathrm{A}$.

(b) Far enough away both fields act like perfect dipoles, and can then cancel.

(c) Close enough neither field acts like a perfect dipole and the fields will not cancel.

P35-9 (a) $B = \sqrt{{B_\mathrm{h}}^2 + {B_\mathrm{v}}^2}$, so

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2 L_\mathrm{m} + 4\sin^2 L_\mathrm{m}} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 + 3\sin^2 L_\mathrm{m}}.$$

(b) $\tan\phi_\mathrm{i} = B_\mathrm{v}/B_\mathrm{h} = 2\sin L_\mathrm{m}/\cos L_\mathrm{m} = 2\tan L_\mathrm{m}$.

$\boxed{\textbf{E36-1}}$ The important relationship is Eq. 36-4, written as

$$\Phi_B = \frac{iL}{N} = \frac{(5.0\,\text{mA})(8.0\,\text{mH})}{(400)} = 1.0\times10^{-7}\text{Wb}$$

E36-2 (a) $\Phi = (34)(2.62\times10^{-3}\text{T})\pi(0.103\,\text{m})^2 = 2.97\times10^{-3}\text{Wb}$.
(b) $L = \Phi/i = (2.97\times10^{-3}\text{Wb})/(3.77\,\text{A}) = 7.88\times10^{-4}\text{H}$.

E36-3 $n = 1/d$, where d is the diameter of the wire. Then

$$\frac{L}{l} = \mu_0 n^2 A = \frac{\mu_0 A}{d^2} = \frac{(4\pi\times10^{-7}\text{H/m})(\pi/4)(4.10\times10^{-2}\text{m})^2}{(2.52\times10^{-3}\text{m})^2} = 2.61\times10^{-4}\text{H/m}.$$

E36-4 (a) The emf supports the current, so the current must be decreasing.
(b) $L = \mathcal{E}/(di/dt) = (17\,\text{V})/(25\times10^3\text{A/s}) = 6.8\times10^{-4}\text{H}$.

$\boxed{\textbf{E36-5}}$ (a) Eq. 36-1 can be used to find the inductance of the coil.

$$L = \frac{\mathcal{E}_L}{di/dt} = \frac{(3.0\,\text{mV})}{(5.0\,\text{A/s})} = 6.0\times10^{-4}\text{H}.$$

(b) Eq. 36-4 can then be used to find the number of turns in the coil.

$$N = \frac{iL}{\Phi_B} = \frac{(8.0\,\text{A})(6.0\times10^{-4}\text{H})}{(40\mu\text{Wb})} = 120$$

E36-6 Use the equation between Eqs. 36-9 and 36-10.

$$\begin{aligned}\Phi_B &= \frac{(4\pi\times10^{-7}\text{H/m})(0.81\,\text{A})(536)(5.2\times10^{-2}\text{m})}{2\pi}\ln\frac{(5.2\times10^{-2}\text{m})+(15.3\times10^{-2}\text{m})}{(15.3\times10^{-2}\text{m})},\\ &= 1.32\times10^{-6}\text{Wb}.\end{aligned}$$

E36-7 $L = \kappa_\text{m}\mu_0 n^2 Al = \kappa_\text{m}\mu_0 N^2 A/l$, or

$$L = (968)(4\pi\times10^{-7}\text{H/m})(1870)^2(\pi/4)(5.45\times10^{-2}\text{m})^2/(1.26\,\text{m}) = 7.88\,\text{H}.$$

E36-8 In each case apply $\mathcal{E} = L\Delta i/\Delta t$.
(a) $\mathcal{E} = (4.6\,\text{H})(7\,\text{A})/(2\times10^{-3}\text{s}) = 1.6\times10^4\text{V}$.
(b) $\mathcal{E} = (4.6\,\text{H})(2\,\text{A})/(3\times10^{-3}\text{s}) = 3.1\times10^3\text{V}$.
(c) $\mathcal{E} = (4.6\,\text{H})(5\,\text{A})/(1\times10^{-3}\text{s}) = 2.3\times10^4\text{V}$.

$\boxed{\textbf{E36-9}}$ (a) If two inductors are connected in parallel then the current through each inductor will add to the total current through the circuit, $i = i_1 + i_2$, Take the derivative of the current with respect to time and then $di/dt = di_1/dt + di_2/dt$,

The potential difference across each inductor is the same, so if we divide by $\mathcal{E}$ and apply we get

$$\frac{di/dt}{\mathcal{E}} = \frac{di_1/dt}{\mathcal{E}} + \frac{di_2/dt}{\mathcal{E}},$$

But

$$\frac{di/dt}{\mathcal{E}} = \frac{1}{L},$$

so the previous expression can also be written as

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

(b) If the inductors are close enough together then the magnetic field from one coil will induce currents in the other coil. Then we will need to consider mutual induction effects, but that is a topic not covered in this text.

E36-10 (a) If two inductors are connected in series then the emf across each inductor will add to the total emf across both, $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$,

Then the current through each inductor is the same, so if we divide by di/dt and apply we get

$$\frac{\mathcal{E}}{di/dt} = \frac{\mathcal{E}_1}{di/dt} + \frac{\mathcal{E}_2}{di/dt},$$

But

$$\frac{\mathcal{E}}{di/dt} = L,$$

so the previous expression can also be written as

$$L_{\text{eq}} = L_1 + L_2.$$

(b) If the inductors are close enough together then the magnetic field from one coil will induce currents in the other coil. Then we will need to consider mutual induction effects, but that is a topic not covered in this text.

E36-11 Use Eq. 36-17, but rearrange:

$$\tau_L = \frac{t}{\ln[i_0/i]} = \frac{(1.50\,\text{s})}{\ln[(1.16\,\text{A})/(10.2\times10^{-3}\text{A})]} = 0.317\,\text{s}.$$

Then $R = L/\tau_L = (9.44\,\text{H})/(0.317\,\text{s}) = 29.8\,\Omega$.

E36-12 (a) There is no current through the resistor, so $\mathcal{E}_R = 0$ and then $\mathcal{E}_L = \mathcal{E}$.

(b) $\mathcal{E}_L = \mathcal{E}e^{-2} = (0.135)\mathcal{E}$.

(c) $n = -\ln(\mathcal{E}_L/\mathcal{E}) = -ln(1/2) = 0.693$.

$\boxed{\textbf{E36-13}}$ (a) From Eq. 36-4 we find the inductance to be

$$L = \frac{N\Phi_B}{i} = \frac{(26.2\times10^{-3}\text{Wb})}{(5.48\,\text{A})} = 4.78\times10^{-3}\text{H}.$$

Note that Φ_B is the *flux*, while the quantity $N\Phi_B$ is the *number of flux linkages.*

(b) We can find the time constant from Eq. 36-14,

$$\tau_L = L/R = (4.78\times10^{-3}\text{H})/(0.745\,\Omega) = 6.42\times10^{-3}\,\text{s}.$$

The we can invert Eq. 36-13 to get

$$\begin{aligned} t &= -\tau_L \ln\left(1 - \frac{Ri(t)}{\mathcal{E}}\right), \\ &= -(6.42\times10^{-3}\,\text{s})\ln\left(1 - \frac{(0.745\,\text{A})(2.53\,\text{A})}{(6.00\,\text{V})}\right) = 2.42\times10^{-3}\,\text{s}. \end{aligned}$$

E36-14 (a) Rearrange:

$$\begin{aligned}
\mathcal{E} &= iR + L\frac{di}{dt}, \\
\frac{\mathcal{E}}{R} - i &= \frac{L}{R}\frac{di}{dt}, \\
\frac{R}{L}dt &= \frac{di}{\mathcal{E}/R - i}.
\end{aligned}$$

(b) Integrate:

$$\begin{aligned}
-\int_0^t \frac{R}{L}dt &= \int_0^i \frac{di}{i - \mathcal{E}/R}, \\
-\frac{R}{L}t &= \ln\frac{i + \mathcal{E}/R}{\mathcal{E}/R}, \\
\frac{\mathcal{E}}{R}e^{-t/\tau_L} &= i + \mathcal{E}/R, \\
\frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) &= i.
\end{aligned}$$

E36-15 $di/dt = (5.0\,\mathrm{A/s})$. Then

$$\mathcal{E} = iR + L\frac{di}{dt} = (3.0\,\mathrm{A})(4.0\,\Omega) + (5.0\,\mathrm{A/s})t(4.0\,\Omega) + (6.0\,\mathrm{H})(5.0\,\mathrm{A/s}) = 42\,\mathrm{V} + (20\,\mathrm{V/s})t.$$

E36-16 $(1/3) = (1 - e^{-t/\tau_L})$, so

$$\tau_L = -\frac{t}{\ln(2/3)} = -\frac{(5.22\,\mathrm{s})}{\ln(2/3)} = 12.9\,\mathrm{s}.$$

E36-17 We want to take the derivative of the current in Eq. 36-13 with respect to time,

$$\frac{di}{dt} = \frac{\mathcal{E}}{R}\frac{1}{\tau_L}e^{-t/\tau_L} = \frac{\mathcal{E}}{L}e^{-t/\tau_L}.$$

Then $\tau_L = (5.0{\times}10^{-2}\mathrm{H})/(180\,\Omega) = 2.78{\times}10^{-4}\mathrm{s}$. Using this we find the rate of change in the current when $t = 1.2$ ms to be

$$\frac{di}{dt} = \frac{(45\,\mathrm{V})}{((5.0\times10^{-2}\mathrm{H})}e^{-(1.2{\times}10^{-3}\mathrm{s})/(2.78{\times}10^{-4}\mathrm{s})} = 12\,\mathrm{A/s}.$$

E36-18 (b) Consider some time t_i:

$$\mathcal{E}_L(t_i) = \mathcal{E}e^{-t_i/\tau_L}.$$

Taking a ratio for two different times,

$$\frac{\mathcal{E}_L(t_1)}{\mathcal{E}_L(t_2)} = e^{(t_2 - t_1)/\tau_L},$$

or

$$\tau_L = \frac{t_2 - t_1}{\ln[\mathcal{E}_L(t_1)/\mathcal{E}_L(t_2)]} = \frac{(2\,\mathrm{ms}) - (1\,\mathrm{ms})}{\ln[(18.24\,\mathrm{V})/(13.8\,\mathrm{V})]} = 3.58\,\mathrm{ms}$$

(a) Choose any time, and

$$\mathcal{E} = \mathcal{E}_L e^{t/\tau_L} = (18.24\,\mathrm{V})e^{(1\,\mathrm{ms})/(3.58\,\mathrm{ms})} = 24\,\mathrm{V}.$$

E36-19 (a) When the switch is just closed there is *no* current through the inductor. So $i_1 = i_2$ is given by

$$i_1 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{(100\,\mathrm{V})}{(10\,\Omega) + (20\,\Omega)} = 3.33\,\mathrm{A}.$$

(b) A long time later there is current through the inductor, but it is as if the inductor has no effect on the circuit. Then the effective resistance of the circuit is found by first finding the equivalent resistance of the parallel part

$$1/(30\,\Omega) + 1/(20\,\Omega) = 1/(12\,\Omega),$$

and then finding the equivalent resistance of the circuit

$$(10\,\Omega) + (12\,\Omega) = 22\,\Omega.$$

Finally, $i_1 = (100\,\mathrm{V})/(22\,\Omega) = 4.55\,\mathrm{A}$ and

$$\Delta V_2 = (100\,\mathrm{V}) - (4.55\,\mathrm{A})(10\,\Omega) = 54.5\,\mathrm{V};$$

consequently, $i_2 = (54.5\,\mathrm{V})/(20\,\Omega) = 2.73\,\mathrm{A}$. It didn't ask, but $i_2 = (4.55\,\mathrm{A}) - (2.73\,\mathrm{A}) = 1.82\,\mathrm{A}$.

(c) After the switch is just opened the current through the battery stops, while that through the inductor continues on. Then $i_2 = i_3 = 1.82\,\mathrm{A}$.

(d) All go to zero.

E36-20 (a) For toroids $L = \mu_0 N^2 h \ln(b/a)/2\pi$. The number of turns is limited by the inner radius: $Nd = 2\pi a$. In this case,

$$N = 2\pi(0.10\,\mathrm{m})/(0.00096\,\mathrm{m}) = 654.$$

The inductance is then

$$L = \frac{(4\pi\times10^{-7}\mathrm{H/m})(654)^2(0.02\,\mathrm{m})}{2\pi}\ln\frac{(0.12\,\mathrm{m})}{(0.10\,\mathrm{m})} = 3.1\times10^{-4}\mathrm{H}.$$

(b) Each turn has a length of $4(0.02\,\mathrm{m}) = 0.08\,\mathrm{m}$. The resistance is then

$$R = N(0.08\,\mathrm{m})(0.021\,\Omega/\mathrm{m}) = 1.10\,\Omega$$

The time constant is

$$\tau_L = L/R = (3.1\times10^{-4}\mathrm{H})/(1.10\,\Omega) = 2.8\times10^{-4}\mathrm{s}.$$

E36-21 (I) When the switch is just closed there is *no* current through the inductor or R_2, so the potential difference across the inductor must be 10 V. The potential difference across R_1 is always 10 V when the switch is closed, regardless of the amount of time elapsed since closing.

(a) $i_1 = (10\,\mathrm{V})/(5.0\,\Omega) = 2.0\,\mathrm{A}$.

(b) Zero; read the above paragraph.

(c) The current through the switch is the sum of the above two currents, or 2.0 A.

(d) Zero, since the current through R_2 is zero.

(e) 10 V, since the potential across R_2 is zero.

(f) Look at the results of Exercise 36-17. When $t = 0$ the rate of change of the current is $di/dt = \mathcal{E}/L$. Then

$$di/dt = (10\,\mathrm{V})/(5.0\,\mathrm{H}) = 2.0\,\mathrm{A/s}.$$

(II) After the switch has been closed for a long period of time the currents are stable and the inductor no longer has an effect on the circuit. Then the circuit is a simple two resistor parallel network, each resistor has a potential difference of 10 V across it.

(a) Still 2.0 A; nothing has changed.
(b) $i_2 = (10\,\mathrm{V})/(10\,\Omega) = 1.0\,\mathrm{A}$.
(c) Add the two currents and the current through the switch will be 3.0 A.
(d) 10 V; see the above discussion.
(e) Zero, since the current is no longer changing.
(f) Zero, since the current is no longer changing.

E36-22 $U = (71\,\mathrm{J/m^3})(0.022\,\mathrm{m^3}) = 1.56\,\mathrm{J}$. Then using $U = i^2L/2$ we get

$$i = \sqrt{2U/L} = \sqrt{2(1.56\,\mathrm{J})/(0.092\,\mathrm{H})} = 5.8\,\mathrm{A}.$$

E36-23 (a) $L = 2U/i^2 = 2(0.0253\,\mathrm{J})/(0.062\,\mathrm{A})^2 = 13.2\,\mathrm{H}$.

(b) Since the current is squared in the energy expression, doubling the current would quadruple the energy. Then $i' = 2i_0 = 2(0.062\,\mathrm{A}) = 0.124\,\mathrm{A}$.

E36-24 (a) $B = \mu_0 in$ and $u = B^2/2\mu_0$, or

$$u = \mu_0 i^2 n^2/2 = (4\pi\times10^{-7}\mathrm{N/A^2})(6.57\,\mathrm{A})^2(950/0.853\,\mathrm{m})^2/2 = 33.6\,\mathrm{J/m^3}.$$

(b) $U = uAL = (33.6\,\mathrm{J/m^3})(17.2\times10^{-4}\mathrm{m^2})(0.853\,\mathrm{m}) = 4.93\times10^{-2}\mathrm{J}$.

E36-25 $u_B = B^2/2\mu_0$, and from Sample Problem 33-2 we know B, hence

$$u_B = \frac{(12.6\,\mathrm{T})^2}{2(4\pi\times10^{-7}\mathrm{N/A^2})} = 6.32\times10^7\mathrm{J/m^3}.$$

E36-26 (a) $u_B = B^2/2\mu_0$, so

$$u_B = \frac{(100\times10^{-12}\mathrm{T})^2}{2(4\pi\times10^{-7}\mathrm{N/A^2})}\frac{1}{(1.6\times10^{-19}\mathrm{J/eV})} = 2.5\times10^{-2}\ \mathrm{eV/cm^3}.$$

(b) $x = (10)(9.46\times10^{15}\mathrm{m}) = 9.46\times10^{16}\mathrm{m}$. Using the results from part (a) expressed in $\mathrm{J/m^3}$ we find the energy contained is

$$U = (3.98\times10^{-15}\mathrm{J/m^3})(9.46\times10^{16}\mathrm{m})^3 = 3.4\times10^{36}\mathrm{J}$$

E36-27 The energy density of an electric field is given by Eq. 36-23; that of a magnetic field is given by Eq. 36-22. Equating,

$$\begin{aligned}\frac{\epsilon_0}{2}E^2 &= \frac{1}{2\mu_0}B^2,\\ E &= \frac{B}{\sqrt{\epsilon_0\mu_0}}.\end{aligned}$$

The answer is then

$$E = (0.50\,\mathrm{T})/\sqrt{(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(4\pi\times10^{-7}\mathrm{N/A^2})} = 1.5\times10^8\,\mathrm{V/m}.$$

E36-28 The rate of internal energy increase in the resistor is given by $P = i\Delta V_R$. The rate of energy storage in the inductor is $dU/dt = Li\,di/dt = i\Delta V_L$. Since the current is the same through both we want to find the time when $\Delta V_R = \Delta V_L$. Using Eq. 36-15 we find

$$\begin{aligned} 1 - e^{-t/\tau_L} &= e^{-t/\tau_L}, \\ \ln 2 &= t/\tau_L, \end{aligned}$$

so $t = (37.5\,\text{ms})\ln 2 = 26.0\,\text{ms}$.

E36-29 (a) Start with Eq. 36-13:

$$\begin{aligned} i &= \mathcal{E}(1 - e^{-t/\tau_L})/R, \\ 1 - \frac{iR}{\mathcal{E}} &= e^{-t/\tau_L}, \\ \tau_L &= \frac{-t}{\ln(1 - iR/\mathcal{E})}, \\ &= \frac{-(5.20\times 10^{-3}\text{s})}{\ln[1 - (1.96\times 10^{-3}\text{A})(10.4\times 10^{3}\Omega)/(55.0\,\text{V})]}, \\ &= 1.12\times 10^{-2}\text{s}. \end{aligned}$$

Then $L = \tau_L R = (1.12\times 10^{-2}\text{s})(10.4\times 10^{3}\Omega) = 116\,\text{H}$.
(b) $U = (1/2)(116\,\text{H})(1.96\times 10^{-3}\text{A})^2 = 2.23\times 10^{-4}\text{J}$.

E36-30 (a) $U = \mathcal{E}\Delta q$; $q = \int i dt$.

$$\begin{aligned} U &= \mathcal{E}\int \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) dt, \\ &= \frac{\mathcal{E}^2}{R}\left(t + \tau_L e^{-t/\tau_L}\right)_0^2, \\ &= \frac{\mathcal{E}^2}{R}t + \frac{\mathcal{E}^2 L}{R^2}(e^{-t/\tau_L} - 1). \end{aligned}$$

Using the numbers provided,

$$\tau_L = (5.48\,\text{H})/(7.34\,\Omega) = 0.7466\,\text{s}.$$

Then

$$U = \frac{(12.2\,\text{V})^2}{(7.34\,\Omega)}\left[(2\,\text{s}) + (0.7466\,\text{s})(e^{-(2\,\text{S})/0.7466\,\text{S})} - 1)\right] = 26.4\,\text{J}$$

(b) The energy stored in the inductor is $U_L = Li^2/2$, or

$$\begin{aligned} U_L &= \frac{L\mathcal{E}^2}{2R^2}\int \left(1 - e^{-t/\tau_L}\right)^2 dt, \\ &= 6.57\,\text{J}. \end{aligned}$$

(c) $U_R = U - U_L = 19.8\,\text{J}$.

$\boxed{\textbf{E36-31}}$ This shell has a volume of

$$V = \frac{4\pi}{3}\left((R_\text{E} + a)^3 - {R_\text{E}}^3\right).$$

Since $a << R_{\rm E}$ we can expand the polynomials but keep only the terms which are linear in a. Then

$$V \approx 4\pi {R_{\rm E}}^2 a = 4\pi(6.37\times10^6{\rm m})^2(1.6\times10^4{\rm m}) = 8.2\times10^{18}{\rm m}^3.$$

The magnetic energy density is found from Eq. 36-22,

$$u_B = \frac{1}{2\mu_0}B^2 = \frac{(60\times10^{-6}\,{\rm T})^2}{2(4\pi\times10^{-7}{\rm N/A}^2)} = 1.43\times10^{-3}{\rm J/m}^3.$$

The total energy is then $(1.43\times10^{-3}{\rm J/m}^3)(8.2eex18{\rm m}^3) = 1.2\times10^{16}{\rm J}$.

E36-32 (a) $B = \mu_0 i/2\pi r$ and $u_B = B^2/2\mu_0 = \mu_0 i^2/8\pi^2 r^2$, or

$$u_B = (4\pi\times10^{-7}{\rm H/m})(10\,{\rm A})^2/8\pi^2(1.25\times10^{-3}{\rm m})^2 = 1.0\,{\rm J/m}^3.$$

(b) $E = \Delta V/l = iR/l$ and $u_E = \epsilon_0 E^2/2 = \epsilon_0 i^2(R/l)^2/2$. Then

$$u_E = (8.85\times10^{-12}{\rm F/m})(10\,{\rm A})^2(3.3\times10^{-3}\Omega/{\rm m})^2/2 = 4.8\times10^{-15}{\rm J/m}^3.$$

E36-33 $i = \sqrt{2U/L} = \sqrt{2(11.2\times10^{-6}{\rm J})/(1.48\times10^{-3}{\rm H})} = 0.123\,{\rm A}$.

E36-34 $C = q^2/2U = (1.63\times10^{-6}{\rm C})^2/2(142\times10^{-6}{\rm J}) = 9.36\times10^{-9}{\rm F}$.

E36-35 $1/2\pi f = \sqrt{LC}$ so $L = 1/4\pi^2 f^2 C$, or

$$L = 1/4\pi^2(10\times10^3{\rm Hz})^2(6.7\times10^{-6}{\rm F}) = 3.8\times10^{-5}{\rm H}.$$

E36-36 ${q_{\rm max}}^2/2C = L{i_{\rm max}}^2/2$, or

$$i_{\rm max} = q_{\rm max}/\sqrt{LC} = (2.94\times10^{-6}{\rm C})/\sqrt{(1.13\times10^{-3}{\rm H})(3.88\times10^{-6}{\rm F})} = 4.44\times10^{-2}{\rm A}.$$

$\boxed{\textbf{E36-37}}$ Closing a switch has the effect of “shorting” out the relevant circuit element, which effectively removes it from the circuit. If S_1 is closed we have $\tau_C = RC$ or $C = \tau_C/R$, if instead S_2 is closed we have $\tau_L = L/R$ or $L = R\tau_L$, but if instead S_3 is closed we have a LC circuit which will oscillate with period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}.$$

Substituting from the expressions above,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\tau_L\tau_C}.$$

E36-38 The capacitors can be used individually, or in series, or in parallel. The four possible capacitances are then $2.00\mu{\rm F}$, $5.00\mu{\rm F}$, $2.00\mu_F + 5.00\mu{\rm F} = 7.00\mu{\rm F}$, and $(2.00\mu_F)(5.00\mu{\rm F})(2.00\mu_F + 5.00\mu{\rm F}) = 1.43\mu{\rm F}$. The possible resonant frequencies are then

$$\begin{aligned} \frac{1}{2\pi}\sqrt{\frac{1}{LC}} &= f, \\ \frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,{\rm mH})(1.43\mu_F)}} &= 1330\ {\rm Hz}, \end{aligned}$$

$$\frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,\mathrm{mH})(2.00\mu_F)}} = 1130 \text{ Hz},$$
$$\frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,\mathrm{mH})(5.00\mu_F)}} = 712 \text{ Hz},$$
$$\frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,\mathrm{mH})(7.00\mu_F)}} = 602 \text{ Hz}.$$

E36-39 (a) $k = (8.13\,\mathrm{N})/(0.0021\,\mathrm{m}) = 3.87\times10^3\mathrm{N/m}$. $\omega = \sqrt{k/m} = \sqrt{(3870\,\mathrm{N/m})/(0.485\,\mathrm{kg})} = 89.3$ rad/s.

(b) $T = 2\pi/\omega = 2\pi/(89.3 \text{ rad/s}) = 7.03\times10^{-2}$s.

(c) $LC = 1/\omega^2$, so

$$C = 1/(89.3 \text{ rad/s})^2(5.20\,\mathrm{H}) = 2.41\times10^{-5}\mathrm{F}.$$

E36-40 The period of oscillation is $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(52.2\mathrm{mH})(4.21\mu\mathrm{F})} = 2.95\,\mathrm{ms}$. It requires one-quarter period for the capacitor to charge, or 0.736 ms.

$\boxed{\textbf{E36-41}}$ (a) An LC circuit oscillates so that the energy is converted from all magnetic to all electrical *twice* each cycle. It occurs twice because once the energy is magnetic with the current flowing in one direction through the inductor, and later the energy is magnetic with the current flowing the other direction through the inductor.

The period is then *four* times 1.52μs, or 6.08μs.

(b) The frequency is the reciprocal of the period, or 164000 Hz.

(c) Since it occurs twice during each oscillation it is equal to half a period, or 3.04μs.

E36-42 (a) $q = C\Delta V = (1.13\times10^{-9}\mathrm{F})(2.87\,\mathrm{V}) = 3.24\times10^{-9}\mathrm{C}$.

(c) $U = q^2/2C = (3.24\times10^{-9}\mathrm{C})^2/2(1.13\times10^{-9}\mathrm{F}) = 4.64\times10^{-9}\mathrm{J}$.

(b) $i = \sqrt{2U/L} = \sqrt{2(4.64\times10^{-9}\mathrm{J})/(3.17\times10^{-3}\mathrm{H})} = 1.71\times10^{-3}\mathrm{A}$.

E36-43 (a) $i_\mathrm{m} = q_\mathrm{m}\omega$ and $q_\mathrm{m} = CV_\mathrm{m}$. Multiplying the second expression by L we get $Lq_\mathrm{m} = V_\mathrm{m}/\omega^2$. Combining, $Li_\mathrm{m}\omega = V_\mathrm{m}$. Then

$$f = \frac{\omega}{2\pi} = \frac{(50\,\mathrm{V})}{2\pi(0.042\,\mathrm{H})(0.031\,\mathrm{A})} = 6.1\times10^3/\mathrm{s}.$$

(b) See (a) above.

(c) $C = 1/\omega^2 L = 1/(2\pi 6.1\times10^3/\mathrm{s})^2(0.042\,\mathrm{H}) = 1.6\times10^{-8}\mathrm{F}$.

E36-44 (a) $f = 1/2\pi\sqrt{LC} = 1/2\pi\sqrt{(6.2\times10^{-6}\mathrm{F})(54\times10^{-3}\mathrm{H})} = 275$ Hz.

(b) Note that from Eq. 36-32 we can deduce $i_\mathrm{max} = \omega q_\mathrm{max}$. The capacitor starts with a charge $q = C\Delta V = (6.2\times10^{-6}\mathrm{F})(34\,\mathrm{V}) = 2.11\times10^{-4}\mathrm{C}$. Then the current amplitude is

$$i_\mathrm{max} = q_\mathrm{max}/\sqrt{LC} = (2.11\times10^{-4}\mathrm{C})/\sqrt{(6.2\times10^{-6}\mathrm{F})(54\times10^{-3}\mathrm{H})} = 0.365\,\mathrm{A}.$$

E36-45 (a) $\omega = 1/\sqrt{LC} = 1/\sqrt{(10\times10^{-6}\mathrm{F})(3.0\times10^{-3}\mathrm{H})} = 5800$ rad/s.

(b) $T = 2\pi/\omega = 2\pi/(5800 \text{ rad/s}) = 1.1\times10^{-3}$s.

E36-46 $f = (2\times10^5\text{Hz})(1+\theta/180°)$. $C = 4\pi^2/f^2L$, so

$$C = \frac{4\pi^2}{(2\times10^5\text{Hz})^2(1+\theta/180°)^2(1\,\text{mH})} = \frac{(9.9\times10^{-7}\text{F})}{(1+\theta/180°)^2}.$$

E36-47 (a) $U_E = U_B/2$ and $U_E + U_B = U$, so $3U_E = U$, or $3(q^2/2C) = q_{\max}^2/2C$, so $q = q_{\max}/\sqrt{3}$.
(b) Solve $q = q_{\max}\cos\omega t$, or

$$t = \frac{T}{2\pi}\arccos 1/\sqrt{3} = 0.152T.$$

E36-48 (a) Add the contribution from the inductor and the capacitor,

$$U = \frac{(24.8\times10^{-3}\text{H})(9.16\times10^{-3}\text{A})^2}{2} + \frac{(3.83\times10^{-6}\text{C})^2}{2(7.73\times10^{-6}\text{F})} = 1.99\times10^{-6}\text{J}.$$

(b) $q_\text{m} = \sqrt{2(7.73\times10^{-6}\text{F})(1.99\times10^{-6}\text{J})} = 5.55\times10^{-6}\text{C}$.
(c) $i_\text{m} = \sqrt{2(1.99\times10^{-6}\text{J})/(24.8\times10^{-3}\text{H})} = 1.27\times10^{-2}\text{A}$.

$\boxed{\textbf{E36-49}}$ (a) The frequency of such a system is given by Eq. 36-26, $f = 1/2\pi\sqrt{LC}$. Note that maximum frequency occurs with minimum capacitance. Then

$$\frac{f_1}{f_2} = \sqrt{\frac{C_2}{C_1}} = \sqrt{\frac{(365\text{ pF})}{(10\text{ pF})}} = 6.04.$$

(b) The desired ratio is $1.60/0.54 = 2.96$ Adding a capacitor in parallel will result in an effective capacitance given by

$$C_{1,\text{eff}} = C_1 + C_\text{add},$$

with a similar expression for C_2. We want to choose C_add so that

$$\frac{f_1}{f_2} = \sqrt{\frac{C_{2,\text{eff}}}{C_{1,\text{eff}}}} = 2.96.$$

Solving,

$$\begin{aligned} C_{2,\text{eff}} &= C_{1,\text{eff}}(2.96)^2, \\ C_2 + C_\text{add} &= (C_1 + C_\text{add})8.76, \\ C_\text{add} &= \frac{C_2 - 8.76C_1}{7.76}, \\ &= \frac{(365\text{ pF}) - 8.76(10\text{ pF})}{7.76} = 36\text{ pF}. \end{aligned}$$

The necessary inductance is then

$$L = \frac{1}{4\pi^2f^2C} = \frac{1}{4\pi^2(0.54\times10^6\text{Hz})^2(401\times10^{-12}\text{F})} = 2.2\times10^{-4}\text{H}.$$

E36-50 The key here is that $U_E = C(\Delta V)^2/2$. We want to charge a capacitor with one-ninth the capacitance to have three times the potential difference. Since $3^2 = 9$, it is reasonable to assume that we want to take *all* of the energy from the first capacitor and give it to the second. Closing S_1 and S_2 will not work, because the energy will be shared. Instead, close S_2 until the capacitor has completely discharged into the inductor, then simultaneously open S_2 while closing S_1. The inductor will then discharge into the second capacitor. Open S_1 when it is "full".

E36-51 (a) $\omega = 1/\sqrt{LC}$.

$$q_{\mathrm{m}} = \frac{i_{\mathrm{m}}}{\omega} = (2.0\,\mathrm{A})\sqrt{(3.0\times10^{-3}\mathrm{H})(2.7\times10^{-6}\mathrm{F})} = 1.80\times10^{-4}\mathrm{C}$$

(b) $dU_C/dt = qi/C$. Since $q = q_{\mathrm{m}} \sin\omega t$ and $i = i_{\mathrm{m}} \cos\omega t$ then dU_C/dt is a maximum when $\sin\omega t \cos\omega t$ is a maximum, or when $\sin 2\omega t$ is a maximum. This happens when $2\omega t = \pi/2$, or $t = T/8$.

(c) $dU_C/dt = q_{\mathrm{m}} i_{\mathrm{m}}/2C$, or

$$dU_C/dt = (1.80\times10^{-4}\mathrm{C})(2.0\,\mathrm{A})/2(2.7\times10^{-6}\mathrm{F}) = 67\,\mathrm{W}.$$

E36-52 After only complete cycles $q = q_{\max} e^{-Rt/2L}$. Not only that, but $t = N\tau$, where $\tau = 2\pi/\omega'$. Finally, $\omega' = \sqrt{(1/LC) - (R/2L)^2}$. Since the first term under the square root is so much larger than the second, we can ignore the effect of damping on the frequency, and simply use $\omega' \approx \omega = 1/\sqrt{LC}$. Then

$$q = q_{\max} e^{-NR\tau/2L} = q_{\max} e^{-N\pi R\sqrt{LC}/L} = q_{\max} e^{-N\pi R\sqrt{C/L}}.$$

Finally, $\pi R\sqrt{C/L} = \pi(7.22\,\Omega)\sqrt{(3.18\,\mu\mathrm{F})/(12.3\,\mathrm{H})} = 1.15\times10^{-2}$. Then

$$\begin{aligned} N = 5 &\;:\; q = (6.31\mu\mathrm{C})e^{-5(0.0115)} = 5.96\mu\mathrm{C}, \\ N = 5 &\;:\; q = (6.31\mu\mathrm{C})e^{-10(0.0115)} = 5.62\mu\mathrm{C}, \\ N = 5 &\;:\; q = (6.31\mu\mathrm{C})e^{-100(0.0115)} = 1.99\mu\mathrm{C}. \end{aligned}$$

E36-53 Use Eq. 36-40, and since $U \propto q^2$, we want to solve $e^{-Rt/L} = 1/2$, then

$$t = \frac{L}{R}\ln 2.$$

E36-54 Start by assuming that the presence of the resistance does not significantly change the frequency. Then $\omega = 1/\sqrt{LC}$, $q = q_{\max} e^{-Rt/2L}$, $t = N\tau$, and $\tau = 2\pi/\omega$. Combining,

$$q = q_{\max} e^{-NR\tau/2L} = q_{\max} e^{-N\pi R\sqrt{LC}/L} = q_{\max} e^{-N\pi R\sqrt{C/L}}.$$

Then

$$R = -\frac{\sqrt{L/C}}{N\pi}\ln(q/q_{\max}) = -\frac{\sqrt{(220\mathrm{mH})/(12\mu\mathrm{F})}}{(50)\pi}\ln(0.99) = 8700\,\Omega.$$

It remains to be verified that $1/LC \gg (R/2L)^2$.

E36-55 The damped (angular) frequency is given by Eq. 36-41; the fractional change would then be

$$\frac{\omega - \omega'}{\omega} = 1 - \sqrt{1 - (R/2L\omega)^2} = 1 - \sqrt{1 - (R^2C/4L)}.$$

Setting this equal to 0.01% and then solving for R,

$$R = \sqrt{\frac{4L}{C}\left(1 - (1 - 0.0001)^2\right)} = \sqrt{\frac{4(12.6\times10^{-3}\mathrm{H})}{(1.15\times10^{-6}\mathrm{F})}(1.9999\times10^{-4})} = 2.96\,\Omega.$$

$\boxed{\textbf{P36-1}}$ The inductance of a toroid is

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}.$$

If the toroid is very large and thin then we can write $b = a + \delta$, where $\delta << a$. The natural log then can be approximated as

$$\ln \frac{b}{a} = \ln \left(1 + \frac{\delta}{a}\right) \approx \frac{\delta}{a}.$$

The product of δ and h is the cross sectional area of the toroid, while $2\pi a$ is the circumference, which we will call l. The inductance of the toroid then reduces to

$$L \approx \frac{\mu_0 N^2}{2\pi} \frac{\delta}{a} = \frac{\mu_0 N^2 A}{l}.$$

But N is the number of turns, which can also be written as $N = nl$, where n is the turns per unit length. Substitute this in and we arrive at Eq. 36-7.

P36-2 (a) Since ni is the net current per unit length and is this case i/W, we can simply write $B = \mu_0 i/W$.

(b) There is only one loop of wire, so

$$L = \phi_B/i = BA/i = \mu_0 i \pi R^2/Wi = \mu_0 \pi R^2/W.$$

P36-3 Choose the y axis so that it is parallel to the wires and directly between them. The field in the plane between the wires is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d/2 + x} + \frac{1}{d/2 - x}\right).$$

The flux per length l of the wires is

$$\begin{aligned}
\Phi_B &= l\int_{-d/2+a}^{d/2-a} B\,dx = l\frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2 + x} + \frac{1}{d/2 - x}\right) dx, \\
&= 2l\frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2 + x}\right) dx, \\
&= 2l\frac{\mu_0 i}{2\pi}\ln\frac{d-a}{a}.
\end{aligned}$$

The inductance is then

$$L = \frac{\phi_B}{i} = \frac{\mu_0 l}{\pi}\ln\frac{d-a}{a}.$$

P36-4 (a) Choose the y axis so that it is parallel to the wires and directly between them. The field in the plane between the wires is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d/2 + x} + \frac{1}{d/2 - x}\right).$$

The flux per length l between the wires is

$$\begin{aligned}
\Phi_1 &= \int_{-d/2+a}^{d/2-a} B\,dx = \frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2 + x} + \frac{1}{d/2 - x}\right) dx, \\
&= 2\frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2 + x}\right) dx, \\
&= 2\frac{\mu_0 i}{2\pi}\ln\frac{d-a}{a}.
\end{aligned}$$

The field in the plane *inside* one of the wires, but still between the centers is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d/2+x} + \frac{d/2-x}{a^2}\right).$$

The additional flux is then

$$\begin{aligned}\Phi_2 &= 2\int_{d/2-a}^{d/2} B\,dx = 2\frac{\mu_0 i}{2\pi}\int_{d/2-a}^{d/2}\left(\frac{1}{d/2+x} + \frac{d/2-x}{a^2}\right)dx,\\ &= 2\frac{\mu_0 i}{2\pi}\left(\ln\frac{d}{d-a} + \frac{1}{2}\right).\end{aligned}$$

The flux per meter between the axes of the wire is the sum, or

$$\begin{aligned}\Phi_B &= \frac{\mu_0 i}{\pi}\left(\ln\frac{d}{a} + \frac{1}{2}\right),\\ &= \frac{(4\pi\times10^{-7}\mathrm{H/m})(11.3\,\mathrm{A})}{\pi}\left(\ln\frac{(21.8,\mathrm{mm})}{(1.3\,\mathrm{mm})} + \frac{1}{2}\right),\\ &= 1.5\times10^{-5}\mathrm{Wb/m}.\end{aligned}$$

(b) The fraction f inside the wires is

$$\begin{aligned}f &= \left(\ln\frac{d}{d-a} + \frac{1}{2}\right)/\left(\ln\frac{d}{a} + \frac{1}{2}\right),\\ &= \left(\frac{(21.8,\mathrm{mm})}{(21.8,\mathrm{mm})-(1.3\,\mathrm{mm})} + \frac{1}{2}\right)/\left(\frac{(21.8,\mathrm{mm})}{(1.3\,\mathrm{mm})} + \frac{1}{2}\right),\\ &= 0.09.\end{aligned}$$

(c) The net flux is zero for parallel currents.

$\boxed{\textbf{P36-5}}$ The magnetic field in the region between the conductors of a coaxial cable is given by

$$B = \frac{\mu_0 i}{2\pi r},$$

so the flux through an area of length l, width $b-a$, and perpendicular to $\vec{\mathbf{B}}$ is

$$\begin{aligned}\Phi_B &= \int \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = \int B\,dA,\\ &= \int_a^b\int_0^l \frac{\mu_0 i}{2\pi r}dz\,dr,\\ &= \frac{\mu_0 i l}{2\pi}\ln\frac{b}{a}.\end{aligned}$$

We evaluated this integral is cylindrical coordinates: $dA = (dr)(dz)$. As you have been warned so many times before, learn these differentials!

The inductance is then

$$L = \frac{\Phi_B}{i} = \frac{\mu_0 l}{2\pi}\ln\frac{b}{a}.$$

P36-6 (a) So long as the fuse is not blown there is effectively no resistance in the circuit. Then the equation for the current is $\mathcal{E} = L\,di/dt$, but since $\mathcal{E}$ is constant, this has a solution $i = \mathcal{E}t/L$. The fuse blows when $t = i_{\max}L/\mathcal{E} = (3.0\,\mathrm{A})(5.0\,\mathrm{H})/(10\,\mathrm{V}) = 1.5\,\mathrm{s}$.

(b) Note that once the fuse blows the maximum steady state current is $2/3\,\mathrm{A}$, so there must be an exponential approach to that current.

P36-7 The initial rate of increase is $di/dt = \mathcal{E}/L$. Since the steady state current is $\mathcal{E}/R$, the current will reach the steady state value in a time given by $\mathcal{E}/R = i = \mathcal{E}t/L$, or $t = L/R$. But that's τ_L.

P36-8 (a) $U = \frac{1}{2}Li^2 = (152\,\mathrm{H})(32\,\mathrm{A})^2/2 = 7.8\times10^4\,\mathrm{J}$.

(b) If the coil develops at finite resistance then all of the energy in the field will be dissipated as heat. The mass of Helium that will boil off is

$$m = Q/L_v = (7.8\times10^4\mathrm{J})/(85\,\mathrm{J/mol})/(4.00\mathrm{g/mol}) = 3.7\,\mathrm{kg}.$$

P36-9 (a) $B = (\mu_0 N i)/(2\pi r)$, so

$$u = \frac{B^2}{2\mu_0} = \frac{\mu_0 N^2 i^2}{8\pi^2 r^2}.$$

(b) $U = \int u\,dV = \int u r dr\,d\theta\,dz$. The field inside the toroid is uniform in z and θ, so

$$\begin{aligned} U &= 2\pi h \int_a^b \frac{\mu_0 N^2 i^2}{8\pi^2 r^2} r\,dr, \\ &= \frac{h\mu_0 N^2 i^2}{4\pi} \ln\frac{b}{a}. \end{aligned}$$

(c) The answers are the same!

P36-10 The energy in the inductor is originally $U = Li_0^2/2$. The internal energy in the resistor increases at a rate $P = i^2 R$. Then

$$\int_0^\infty P\,dt = R\int_0^\infty i_0^2 e^{-2t/\tau_L}\,dt = \frac{Ri_0^2\tau_L}{2} = \frac{Li_0^2}{2}.$$

$\boxed{\textbf{P36-11}}$ (a) In Chapter 33 we found the magnetic field inside a wire carrying a uniform current density is

$$B = \frac{\mu_0 i r}{2\pi R^2}.$$

The magnetic energy density in this wire is

$$u_B = \frac{1}{2\mu_0}B^2 = \frac{\mu_0 i^2 r^2}{8\pi^2 R^4}.$$

We want to integrate in cylindrical coordinates over the volume of the wire. Then the volume element is $dV = (dr)(r\,d\theta)(dz)$, so

$$\begin{aligned} U_B &= \int u_B dV, \\ &= \int_0^R \int_0^l \int_0^{2\pi} \frac{\mu_0 i^2 r^2}{8\pi^2 R^4} d\theta\,dz\,r dr, \\ &= \frac{\mu_0 i^2 l}{4\pi R^4}\int_0^R r^3\,dr, \\ &= \frac{\mu_0 i^2 l}{16\pi}. \end{aligned}$$

(b) Solve

$$U_B = \frac{L}{2}i^2$$

for L, and

$$L = \frac{2U_B}{i^2} = \frac{\mu_0 l}{8\pi}.$$

P36-12 $1/C = 1/C_1 + 1/C_2$ and $L = L_1 + L_2$. Then

$$\frac{1}{\omega} = \sqrt{LC} = \sqrt{(L_1+L_2)\frac{C_1C_2}{C_1+C_2}} = \sqrt{\frac{C_2/\omega_0^2 + C_1/\omega_0^2}{C_1+C_2}} = \frac{1}{\omega_0}.$$

P36-13 (a) There is no current in the middle inductor; the loop equation becomes

$$L\frac{d^2q}{dt^2} + \frac{q}{C} + L\frac{d^2q}{dt^2} + \frac{q}{C} = 0.$$

Try $q = q_\mathrm{m}\cos\omega t$ as a solution:

$$-L\omega^2 + \frac{1}{C} - L\omega^2 + \frac{1}{C} = 0;$$

which requires $\omega = 1/\sqrt{LC}$.

(b) Look at only the left hand loop; the loop equation becomes

$$L\frac{d^2q}{dt^2} + \frac{q}{C} + 2L\frac{d^2q}{dt^2} = 0.$$

Try $q = q_\mathrm{m}\cos\omega t$ as a solution:

$$-L\omega^2 + \frac{1}{C} - 2L\omega^2 = 0;$$

which requires $\omega = 1/\sqrt{3LC}$.

P36-14 (b) $(\omega' - \omega)/\omega$ is the fractional shift; this can also be written as

$$\begin{aligned}
\frac{\omega'}{\omega - 1} &= \sqrt{1 - (LC)(R/2L)^2} - 1, \\
&= \sqrt{1 - R^2C/4L} - 1, \\
&= \sqrt{1 - \frac{(100\,\Omega)^2(7.3\times 10^{-6}\mathrm{F})}{4(4.4\,\mathrm{H})}} - 1 = -2.1\times 10^{-3}.
\end{aligned}$$

$\boxed{\textbf{P36-15}}$ We start by focusing on the charge on the capacitor, given by Eq. 36-40 as

$$q = q_\mathrm{m}e^{-Rt/2L}\cos(\omega' t + \phi).$$

After one oscillation the cosine term has returned to the original value but the exponential term has attenuated the charge on the capacitor according to

$$q = q_\mathrm{m}e^{-RT/2L},$$

where T is the period. The fractional energy loss on the capacitor is then

$$\frac{U_0 - U}{U_0} = 1 - \frac{q^2}{q_m^2} = 1 - e^{-RT/L}.$$

For small enough damping we can expand the exponent. Not only that, but $T = 2\pi/\omega$, so

$$\frac{\Delta U}{U} \approx 2\pi R/\omega L.$$

P36-16 We are given $1/2 = e^{-t/2\tau_L}$ when $t = 2\pi n/\omega'$. Then

$$\omega' = \frac{2\pi n}{t} = \frac{2\pi n}{2(L/R)\ln 2} = \frac{\pi n R}{L \ln 2}.$$

From Eq. 36-41,

$$\begin{aligned}
\omega^2 - \omega'^2 &= (R/2L)^2, \\
(\omega - \omega')(\omega + \omega') &= (R/2L)^2, \\
(\omega - \omega')2\omega' &\approx (R/2L)^2, \\
\frac{\omega - \omega'}{\omega} &= \approx \frac{(R/2L)^2}{2\omega'^2}, \\
&= \frac{(\ln 2)^2}{8\pi^2 n^2}, \\
&= \frac{0.0061}{n^2}.
\end{aligned}$$

$\boxed{\textbf{E37-1}}$ The frequency, f, is related to the angular frequency ω by

$$\omega = 2\pi f = 2\pi(60\ \text{Hz}) = 377\ \text{rad/s}$$

The current is alternating because that is what the generator is designed to produce. It does this through the configuration of the magnets and coils of wire. One complete turn of the generator will (could?) produce one "cycle"; hence, the generator is turning 60 times per second. Not only does this set the frequency, it also sets the emf, since the emf is proportional to the speed at which the coils move through the magnetic field.

E37-2 (a) $X_L = \omega L$, so

$$f = X_L/2\pi L = (1.28\times10^3\Omega)/2\pi(0.0452\,\text{H}) = 4.51\times10^3/\text{s}.$$

(b) $X_C = 1/\omega C$, so

$$C = 1/2\pi f X_C = 1/2\pi(4.51\times10^3/\text{s})(1.28\times10^3\Omega) = 2.76\times10^{-8}\text{F}.$$

(c) The inductive reactance doubles while the capacitive reactance is cut in half.

E37-3 (a) $X_L = X_C$ implies $\omega L = 1/\omega C$ or $\omega = 1/\sqrt{LC}$, so

$$\omega = 1/\sqrt{(6.23\times10^{-3}\text{H})(11.4\times10^{-6}\text{F})} = 3750\ \text{rad/s}.$$

(b) $X_L = \omega L = (3750\ \text{rad/s})(6.23\times10^{-3}\text{H}) = 23.4\,\Omega$
(c) See (a) above.

E37-4 (a) $i_\text{m} = \mathcal{E}/X_L = \mathcal{E}/\omega L$, so

$$i_\text{m} = (25.0\,\text{V})/(377\ \text{rad/s})(12.7\,\text{H}) = 5.22\times10^{-3}\text{A}.$$

(b) The current and emf are 90° out of phase. When the current is a maximum, $\mathcal{E} = 0$.
(c) $\omega t = \arcsin[\mathcal{E}(t)/\mathcal{E}_\text{m}]$, so

$$\omega t = \arcsin\frac{(-13.8\,\text{V})}{(25.0\,\text{V})} = 0.585\ \text{rad}.$$

and

$$i = (5.22\times10^{-3}\text{A})\cos(0.585) = 4.35\times10^{-3}\text{A}.$$

(d) Taking energy.

$\boxed{\textbf{E37-5}}$ (a) The reactance of the capacitor is from Eq. 37-11, $X_C = 1/\omega C$. The AC generator from Exercise 4 has $\mathcal{E} = (25.0\,\text{V})\sin(377\,\text{rad/s})t$. So the reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\,\text{rad/s})(4.1\mu\text{F})} = 647\,\Omega.$$

The maximum value of the current is found from Eq. 37-13,

$$i_\text{m} = \frac{(\Delta V_C)_\text{max}}{X_C} = \frac{(25.0\,\text{V})}{(647\,\Omega)} = 3.86\times10^{-2}\text{A}.$$

(b) The generator emf is 90° out of phase with the current, so when the current is a maximum the emf is zero.

(c) The emf is -13.8 V when

$$\omega t = \arcsin\frac{(-13.8\,\mathrm{V})}{(25.0\,\mathrm{V})} = 0.585\text{ rad}.$$

The current leads the voltage by $90° = \pi/2$, so

$$i = i_\mathrm{m}\sin(\omega t - \phi) = (3.86\times10^{-2}\mathrm{A})\sin(0.585 - \pi/2) = -3.22\times10^{-2}\mathrm{A}.$$

(d) Since both the current and the emf are negative the product is positive and the generator is supplying energy to the circuit.

E37-6 $R = (\omega L - 1/omegaC)/\tan\phi$ and $\omega = 2\pi f = 2\pi(941/\mathrm{s}) = 5910$ rad/s , so

$$R = \frac{(5910\text{ rad/s})(88.3\times10^{-3}\mathrm{H}) - 1/(5910\text{ rad/s})(937\times10^{-9}\mathrm{F})}{\tan(75°)} = 91.5\,\Omega.$$

E37-7

E37-8 (a) X_L doesn't change, so $X_L = 87\,\Omega$.
(b) $X_C = 1/\omega C = 1/2\pi(60/\mathrm{s})(70\times10^{-6}\mathrm{F}) = 37.9\Omega$.
(c) $Z = \sqrt{(160\,\Omega)^2 + (87\,\Omega - 37.9\,\Omega)^2} = 167\,\Omega$.
(d) $i_\mathrm{m} = (36\,\mathrm{V})/(167\,\Omega) = 0.216\,\mathrm{A}$.
(e) $\tan\phi = (87\,\Omega - 37.9\,\Omega)/(160\,\Omega) = 0.3069$, so

$$\phi = \arctan(0.3069) = 17°.$$

$\boxed{\textbf{E37-9}}$ A circuit is considered inductive if $X_L > X_C$, this happens when i_m lags $\mathcal{E}_\mathrm{m}$. If, on the other hand, $X_L < X_C$, and i_m leads $\mathcal{E}_\mathrm{m}$, we refer to the circuit as capacitive. This is discussed on page 850, although it is slightly hidden in the text of column one.

(a) At resonance, $X_L = X_C$. Since $X_L = \omega L$ and $X_C = 1/\omega C$ we expect that X_L grows with increasing frequency, while X_C decreases with increasing frequency.

Consequently, at frequencies above the resonant frequency $X_L > X_C$ and the circuit is predominantly inductive. But what does this really mean? It means that the inductor plays a major role in the current through the circuit while the capacitor plays a minor role. The more inductive a circuit is, the less significant any capacitance is on the behavior of the circuit.

For frequencies below the resonant frequency the reverse is true.

(b) Right at the resonant frequency the inductive effects are exactly canceled by the capacitive effects. The impedance is equal to the resistance, and it is (almost) as if neither the capacitor or inductor are even in the circuit.

E37-10 The net y component is $X_C - X_L$. The net x component is R. The magnitude of the resultant is

$$Z = \sqrt{R^2 + (X_C - X_L)^2},$$

while the phase angle is

$$\tan\phi = \frac{-(X_C - X_L)}{R}.$$

E37-11 Yes.

At resonance $\omega = 1/\sqrt{(1.2\,\mathrm{H})(1.3\times10^{-6}\mathrm{F})} = 800$ rad/s and $Z = R$. Then $i_\mathrm{m} = \mathcal{E}/Z = (10\,\mathrm{V})/(9.6\,\Omega) = 1.04\,\mathrm{A}$, so

$$[\Delta V_L]_\mathrm{m} = i_\mathrm{m} X_L = (1.08\,\mathrm{A})(800\text{ rad/s})(1.2\,\mathrm{H}) = 1000\,\mathrm{V}.$$

E37-12 (a) Let $O = X_L - X_C$ and $A = R$, then $H^2 = A^2 + O^2 = Z^2$, so

$$\sin\phi = (X_L - X_C)/Z$$

and

$$\cos\phi = R/Z.$$

$\boxed{\textbf{E37-13}}$ (a) The voltage across the generator is the generator emf, so when it is a maximum from Sample Problem 37-3, it is 36 V. This corresponds to $\omega t = \pi/2$.

(b) The current through the circuit is given by $i = i_{\rm m}\sin(\omega t - \phi)$. We found in Sample Problem 37-3 that $i_{\rm m} = 0.196\,{\rm A}$ and $\phi = -29.4° = 0.513$ rad.

For a resistive load we apply Eq. 37-3,

$$\Delta V_R = i_{\rm m} R \sin(\omega t - \phi) = (0.196\,{\rm A})(160\Omega)\sin((\pi/2) - (-0.513)) = 27.3\,{\rm V}.$$

(c) For a capacitive load we apply Eq. 37-12,

$$\Delta V_C = i_{\rm m} X_C \sin(\omega t - \phi - \pi/2) = (0.196\,{\rm A})(177\Omega)\sin(-(-0.513)) = 17.0\,{\rm V}.$$

(d) For an inductive load we apply Eq. 37-7,

$$\Delta V_L = i_{\rm m} X_L \sin(\omega t - \phi + \pi/2) = (0.196\,{\rm A})(87\Omega)\sin(\pi - (-0.513)) = -8.4\,{\rm V}.$$

(e) $(27.3\,{\rm V}) + (17.0\,{\rm V}) + (-8.4\,{\rm V}) = 35.9\,{\rm V}$.

E37-14 If circuit 1 and 2 have the same resonant frequency then $L_1C_1 = L_2C_2$. The series combination for the inductors is

$$L = L_1 + L_2,$$

The series combination for the capacitors is

$$1/C = 1/C_1 + 1/C_2,$$

so

$$LC = (L_1 + L_2)\frac{C_1C_2}{C_1 + C_2} = \frac{L_1C_1C_2 + L_2C_2C_1}{C_1 + C_2} = L_1C_1,$$

which is the same as both circuit 1 and 2.

E37-15 (a) $Z = (125\,{\rm V})/(3.20\,{\rm A}) = 39.1\,\Omega$.

(b) Let $O = X_L - X_C$ and $A = R$, then $H^2 = A^2 + O^2 = Z^2$, so

$$\cos\phi = R/Z.$$

Using this relation,

$$R = (39.1\,\Omega)\cos(56.3°) = 21.7\,\Omega.$$

(c) If the current leads the emf then the circuit is capacitive.

E37-16 (a) Integrating over a single cycle,

$$\begin{aligned} \frac{1}{T}\int_0^T \sin^2\omega t\,dt &= \frac{1}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t)\,dt, \\ &= \frac{1}{2T}T = \frac{1}{2}. \end{aligned}$$

(b) Integrating over a single cycle,

$$\begin{aligned} \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\,dt &= \frac{1}{T}\int_0^T \frac{1}{2}\sin 2\omega t dt, \\ &= 0. \end{aligned}$$

$\boxed{\textbf{E37-17}}$ The resistance would be given by Eq. 37-32,

$$R = \frac{P_{\rm av}}{{i_{\rm rms}}^2} = \frac{(0.10)(746\,\mathrm{W})}{(0.650\,\mathrm{A})^2} = 177\,\Omega.$$

This would not be the same as the direct current resistance of the coils of a stopped motor, because there would be no inductive effects.

E37-18 Since $i_{\rm rms} = \mathcal{E}_{\rm rms}/Z$, then

$$P_{\rm av} = i^2{}_{\rm rms}R = \frac{\mathcal{E}^2{}_{\rm rms}R}{Z^2}.$$

E37-19 (a) $Z = \sqrt{(160\,\Omega)^2 + (177\,\Omega)^2} = 239\,\Omega$; then

$$P_{\rm av} = \frac{1}{2}\frac{(36\,\mathrm{V})^2(160\,\Omega)}{(239\,\Omega)^2} = 1.82\,\mathrm{W}.$$

(b) $Z = \sqrt{(160\,\Omega)^2 + (87\,\Omega)^2} = 182\,\Omega$; then

$$P_{\rm av} = \frac{1}{2}\frac{(36\,\mathrm{V})^2(160\,\Omega)}{(182\,\Omega)^2} = 3.13\,\mathrm{W}.$$

E37-20 (a) $Z = \sqrt{(12.2\,\Omega)^2 + (2.30\,\Omega)^2} = 12.4\,\Omega$
(b) $P_{\rm av} = (120\,\mathrm{V})^2(12.2\,\Omega)/(12.4\,\Omega)^2 = 1140\,\mathrm{W}$.
(c) $i_{\rm rms} = (120\,\mathrm{V})/(12.4\,\Omega) = 9.67\,\mathrm{A}$.

$\boxed{\textbf{E37-21}}$ The rms value of any sinusoidal quantity is related to the maximum value by $\sqrt{2}\,v_{\rm rms} = v_{\rm max}$. Since this factor of $\sqrt{2}$ appears in all of the expressions, we can conclude that if the rms values are equal then so are the maximum values. This means that

$$(\Delta V_R)_{\rm max} = (\Delta V_C)_{\rm max} = (\Delta V_L)_{\rm max}$$

or $i_m R = i_m X_C = i_m X_L$ or, with one last simplification, $R = X_L = X_C$. Focus on the right hand side of the last equality. If $X_C = X_L$ then we have a resonance condition, and the impedance (see Eq. 37-20) is a minimum, and is equal to R. Then, according to Eq. 37-21,

$$i_m = \frac{\mathcal{E}_{\rm m}}{R},$$

which has the immediate consequence that the rms voltage across the resistor is the same as the rms voltage across the generator. So everything is 100 V.

E37-22 (a) The antenna is "in-tune" when the impedance is a minimum, or $\omega = 1/\sqrt{LC}$. So

$$f = \omega/2\pi = 1/2\pi\sqrt{(8.22\times10^{-6}\mathrm{H})(0.270\times10^{-12}\mathrm{F})} = 1.07\times10^{8}\mathrm{Hz}.$$

(b) $i_{\rm rms} = (9.13\,\mu\mathrm{V})/(74.7\,\Omega) = 1.22\times10^{-7}\mathrm{A}$.
(c) $X_C = 1/2\pi fC$, so

$$V_C = iX_C = (1.22\times10^{-7}\mathrm{A})/2\pi(1.07\times10^{8}\mathrm{Hz})(0.270\times10^{-12}\mathrm{F}) = 6.72\times10^{-4}\mathrm{V}.$$

E37-23 Assuming no inductors or capacitors in the circuit, then the circuit effectively behaves as a DC circuit. The current through the circuit is $i = \mathcal{E}/(r + R)$. The power delivered to R is then $P = i\Delta V = i^2 R = \mathcal{E}^2 R/(r + R)^2$. Evaluate dP/dR and set it equal to zero to find the maximum. Then

$$0 = \frac{dP}{dR} = \mathcal{E}^2 R \frac{r - R}{(r + R)^3},$$

which has the solution $r = R$.

E37-24 (a) Since $P_{\text{av}} = {i_{\text{m}}}^2 R/2 = {\mathcal{E}_{\text{m}}}^2 R/2Z^2$, then P_{av} is a maximum when Z is a minimum, and vise-versa. Z is a minimum at resonance, when $Z = R$ and $f = 1/2\pi\sqrt{LC}$. When Z is a minimum

$$C = 1/4\pi^2 f^2 L = 1/4\pi^2(60\text{ Hz})^2(60\,\text{mH}) = 1.2\times10^{-7}\text{F}.$$

(b) Z is a maximum when X_C is a maximum, which occurs when C is very small, like zero.

(c) When X_C is a maximum $P = 0$. When P is a maximum $Z = R$ so

$$P = (30\,\text{V})^2/2(5.0\,\Omega) = 90\,\text{W}.$$

(d) The phase angle is zero for resonance; it is 90° for infinite X_C or X_L.

(e) The power factor is zero for a system which has no power. The power factor is one for a system in resonance.

$\boxed{\textbf{E37-25}}$ (a) The resistance is $R = 15.0\,\Omega$. The inductive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(550\,\text{s}^{-1})(4.72\mu\text{F})} = 61.3\,\Omega.$$

The inductive reactance is given by

$$X_L = \omega L = 2\pi(550\,\text{s}^{-1})(25.3\,\text{mH}) = 87.4\,\Omega.$$

The impedance is then

$$Z = \sqrt{(15.0\,\Omega)^2 + ((87.4\,\Omega) - (61.3\,\Omega))^2} = 30.1\,\Omega.$$

Finally, the rms current is

$$i_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{(75.0\,\text{V})}{(30.1\,\Omega)} = 2.49\,\text{A}.$$

(b) The rms voltages between any two points is given by

$$(\Delta V)_{\text{rms}} = i_{\text{rms}} Z,$$

where Z is *not* the impedance of the circuit but instead the impedance between the two points in question. When only one device is between the two points the impedance is equal to the reactance (or resistance) of that device.

We're not going to show *all* of the work here, but we will put together a nice table for you

Points	Impedance Expression	Impedance Value	$(\Delta V)_{\text{rms}}$,
ab	$Z = R$	$Z = 15.0\,\Omega$	37.4 V,
bc	$Z = X_C$	$Z = 61.3\,\Omega$	153 V,
cd	$Z = X_L$	$Z = 87.4\,\Omega$	218 V,
bd	$Z = \lvert X_L - X_C \rvert$	$Z = 26.1\,\Omega$	65 V,
ac	$Z = \sqrt{R^2 + X_C^2}$	$Z = 63.1\,\Omega$	157 V,

Note that this last one was ΔV_{ac}, and not ΔV_{ad}, because it is more entertaining. You probably should use ΔV_{ad} for your homework.

(c) The average power dissipated from a capacitor or inductor is zero; that of the resistor is

$$P_R = [(\Delta V_R)_{\text{rms}}]^2/R = (37.4\,\text{V})^2/(15.0\Omega) = 93.3\,\text{W}.$$

E37-26 (a) The energy stored in the capacitor is a function of the charge on the capacitor; although the charge does vary with time it varies periodically and at the end of the cycle has returned to the original values. As such, the energy stored in the capacitor doesn't change from one period to the next.

(b) The energy stored in the inductor is a function of the current in the inductor; although the current does vary with time it varies periodically and at the end of the cycle has returned to the original values. As such, the energy stored in the inductor doesn't change from one period to the next.

(c) $P = \mathcal{E}i = \mathcal{E}_{\rm m}i_{\rm m}\sin(\omega t)\sin(\omega t - \phi)$, so the energy generated in one cycle is

$$\begin{aligned} U &= \int_0^T P\,dt = \mathcal{E}_{\rm m}i_{\rm m}\int_0^T \sin(\omega t)\sin(\omega t-\phi)dt, \\ &= \mathcal{E}_{\rm m}i_{\rm m}\int_0^T \sin(\omega t)\sin(\omega t-\phi)dt, \\ &= \frac{T}{2}\mathcal{E}_{\rm m}i_{\rm m}\cos\phi. \end{aligned}$$

(d) $P = {i_{\rm m}}^2R\sin^2(\omega t-\phi)$, so the energy dissipated in one cycle is

$$\begin{aligned} U &= \int_0^T P\,dt = {i_{\rm m}}^2R\int_0^T \sin^2(\omega t-\phi)dt, \\ &= {i_{\rm m}}^2R\int_0^T \sin^2(\omega t-\phi)dt, \\ &= \frac{T}{2}{i_{\rm m}}^2R. \end{aligned}$$

(e) Since $\cos\phi = R/Z$ and $\mathcal{E}_{\rm m}/Z = i_{\rm m}$ we can equate the answers for (c) and (d).

$\boxed{\textbf{E37-27}}$ Apply Eq. 37-41,

$$\Delta V_{\rm s} = \Delta V_{\rm p}\frac{N_{\rm s}}{N_{\rm p}} = (150\,{\rm V})\frac{(780)}{(65)} = 1.8\times10^3\,{\rm V}.$$

E37-28 (a) Apply Eq. 37-41,

$$\Delta V_{\rm s} = \Delta V_{\rm p}\frac{N_{\rm s}}{N_{\rm p}} = (120\,{\rm V})\frac{(10)}{(500)} = 2.4\,{\rm V}.$$

(b) $i_{\rm s} = (2.4\,{\rm V})/(15\,\Omega) = 0.16\,{\rm A}$;

$$i_{\rm p} = i_{\rm s}\frac{N_{\rm s}}{N_{\rm p}} = (0.16\,{\rm A})\frac{(10)}{(500)} = 3.2\times10^{-3}{\rm A}.$$

$\boxed{\textbf{E37-29}}$ The autotransformer could have a primary connected between taps T_1 and T_2 (200 turns), T_1 and T_3 (1000 turns), and T_2 and T_3 (800 turns).

The same possibilities are true for the secondary connections. Ignoring the one-to-one connections there are 6 choices— three are step up, and three are step down. The step up ratios are $1000/200 = 5$, $800/200 = 4$, and $1000/800 = 1.25$. The step down ratios are the reciprocals of these three values.

E37-30 $\rho = (1.69\times10^{-8}\Omega\cdot\mathrm{m})[1-(4.3\times10^{-3}/\mathrm{C}^\circ)(14.6^\circ\mathrm{C})] = 1.58\times10^{-8}\Omega\cdot\mathrm{m}$. The resistance of the two wires is

$$R = \frac{\rho L}{A} = \frac{(1.58\times10^{-8}\Omega\cdot\mathrm{m})2(1.2\times10^{3}\mathrm{m})}{\pi(0.9\times10^{-3}\mathrm{m})^2} = 14.9\,\Omega.$$

$P = i^2R = (3.8\,\mathrm{A})^2(14.9\,\Omega) = 220\,\mathrm{W}$.

E37-31 The supply current is

$$i_\mathrm{p} = (0.270\,\mathrm{A})(74\times10^{3}\mathrm{V}/\sqrt{2})/(220\,\mathrm{V}) = 64.2\,\mathrm{A}.$$

The potential drop across the supply lines is

$$\Delta V = (64.2\,\mathrm{A})(0.62\,\Omega) = 40\,\mathrm{V}.$$

This is the amount by which the supply voltage must be increased.

E37-32 Use Eq. 37-46:

$$N_\mathrm{p}/N_\mathrm{s} = \sqrt{(1000\,\Omega)/(10\,\Omega)} = 10.$$

P37-1 (a) The emf is a maximum when $\omega t - \pi/4 = \pi/2$, so $t = 3\pi/4\omega = 3\pi/4(350\ \mathrm{rad/s}) = 6.73\times10^{-3}\mathrm{s}$.

(b) The current is a maximum when $\omega t - 3\pi/4 = \pi/2$, so $t = 5\pi/4\omega = 5\pi/4(350\ \mathrm{rad/s}) = 1.12\times10^{-2}\mathrm{s}$.

(c) The current lags the emf, so the circuit contains an inductor.

(d) $X_L = \mathcal{E}_\mathrm{m}/i_\mathrm{m}$ and $X_L = \omega L$, so

$$L = \frac{\mathcal{E}_\mathrm{m}}{i_\mathrm{m}\omega} = \frac{(31.4\,\mathrm{V})}{(0.622\,\mathrm{A})(350\ \mathrm{rad/s})} = 0.144\,\mathrm{H}.$$

P37-2 (a) The emf is a maximum when $\omega t - \pi/4 = \pi/2$, so $t = 3\pi/4\omega = 3\pi/4(350\ \mathrm{rad/s}) = 6.73\times10^{-3}\mathrm{s}$.

(b) The current is a maximum when $\omega t+\pi/4 = \pi/2$, so $t = \pi/4\omega = \pi/4(350\ \mathrm{rad/s}) = 2.24\times10^{-3}\mathrm{s}$.

(c) The current leads the emf, so the circuit contains an capacitor.

(d) $X_C = \mathcal{E}_\mathrm{m}/i_\mathrm{m}$ and $X_C = 1/\omega C$, so

$$C = \frac{i_\mathrm{m}}{\mathcal{E}_\mathrm{m}\omega} = \frac{(0.622\,\mathrm{A})}{(31.4\,\mathrm{V})(350\ \mathrm{rad/s})} = 5.66\times10^{-5}\mathrm{F}.$$

$\boxed{\textbf{P37-3}}$ (a) Since the maximum values for the voltages across the individual devices are proportional to the reactances (or resistances) for devices in series (the constant of proportionality is the maximum current), we have $X_L = 2R$ and $X_C = R$.

From Eq. 37-18,

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2R - R}{R} = 1,$$

or $\phi = 45^\circ$.

(b) The impedance of the circuit, in terms of the resistive element, is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2R-R)^2} = \sqrt{2}\,R.$$

But $\mathcal{E}_\mathrm{m} = i_\mathrm{m}Z$, so $Z = (34.4\,\mathrm{V})/(0.320\,\mathrm{A}) = 108\Omega$. Then we can use our previous work to finds that $R = 76\Omega$.

P37-4 When the switch is open the circuit is an LRC circuit. In position 1 the circuit is an RLC circuit, but the capacitance is equal to the two capacitors of C in parallel, or $2C$. In position 2 the circuit is a simple LC circuit with no resistance.

The impedance when the switch is in position 2 is $Z_2 = |\omega L - 1/\omega C|$. But

$$Z_2 = (170\,\mathrm{V})/(2.82\,\mathrm{A}) = 60.3\,\Omega.$$

The phase angle when the switch is open is $\phi_0 = 20°$. But

$$\tan\phi_0 = \frac{\omega L - 1/\omega C}{R} = \frac{Z_2}{R},$$

so $R = (60.3\,\Omega)/\tan(20°) = 166\,\Omega$.

The phase angle when the switch is in position 1 is

$$\tan\phi_1 = \frac{\omega L - 1/\omega 2C}{R},$$

so $\omega L - 1/\omega 2C = (166\,\Omega)\tan(10°) = 29.2\,\Omega$. Equating the ωL part,

$$\begin{aligned}(29.2\,\Omega) + 1/\omega 2C &= (-60.3\,\Omega) + 1/\omega C,\\ C &= 1/2(377\text{ rad/s})[(60.3\,\Omega) + (29.2\,\Omega)] = 1.48\times 10^{-5}\mathrm{F}.\end{aligned}$$

Finally,

$$L = \frac{(-60.3\Omega) + 1/(377\text{ rad/s})(1.48\times 10^{-5}\mathrm{F})}{(377\text{ rad/s})} = 0.315\,\mathrm{H}.$$

P37-5 All three wires have emfs which vary sinusoidally in time; if we choose *any* two wires the phase difference will have an absolute value of 120°. We can then choose any two wires and expect (by symmetry) to get the same result. We choose 1 and 2. The potential difference is then

$$V_1 - V_2 = V_\mathrm{m}\left(\sin\omega t - \sin(\omega t - 120°)\right).$$

We need to add these two sine functions to get just one. We use

$$\sin\alpha - \sin\beta = 2\sin\frac{1}{2}(\alpha - \beta)\cos\frac{1}{2}(\alpha + \beta).$$

Then

$$\begin{aligned}V_1 - V_2 &= 2V_\mathrm{m}\sin\frac{1}{2}(120°)\cos\frac{1}{2}(2\omega t - 120°),\\ &= 2V_\mathrm{m}(\frac{\sqrt{3}}{2})\cos(\omega t - 60°),\\ &= \sqrt{3}V_\mathrm{m}\sin(\omega t + 30°).\end{aligned}$$

P37-6 (a) $\cos\phi = \cos(-42°) = 0.74$.

(b) The current leads.

(c) The circuit is capacitive.

(d) No. Resonance circuits have a power factor of one.

(e) There must be at least a capacitor and a resistor.

(f) $P = (75\,\mathrm{V})(1.2\,\mathrm{A})(0.74)/2 = 33\,\mathrm{W}$.

P37-7 (a) $\omega = 1/\sqrt{LC} = 1/\sqrt{(0.988\,\text{H})(19.3\times10^{-6}\text{F})} = 229$ rad/s.

(b) $i_m = (31.3\,\text{V})/(5.12\,\Omega) = 6.11\,\text{A}$.

(c) The current amplitude will be halved when the impedance is doubled, or when $Z = 2R$. This occurs when $3R^2 = (\omega L - 1/\omega C)^2$, or

$$3R^2\omega^2 = \omega^4 L^2 - 2\omega^2 L/C + 1/C^2.$$

The solution to this quadratic is

$$\omega^2 = \frac{2L + 3CR^2 \pm \sqrt{9C^2R^4 + 12CR^2L}}{2L^2C},$$

so $\omega_1 = 224.6$ rad/s and $\omega_2 = 233.5$ rad/s.

(d) $\Delta\omega/\omega = (8.9\text{ rad/s})/(229\text{ rad/s}) = 0.039$.

P37-8 (a) The current amplitude will be halved when the impedance is doubled, or when $Z = 2R$. This occurs when $3R^2 = (\omega L - 1/\omega C)^2$, or

$$3R^2\omega^2 = \omega^4 L^2 - 2\omega^2 L/C + 1/C^2.$$

The solution to this quadratic is

$$\omega^2 = \frac{2L + 3CR^2 \pm \sqrt{9C^2R^4 + 12CR^2L}}{2L^2C},$$

Note that $\Delta\omega = \omega_+ - \omega_-$; with a wee bit of algebra,

$$\Delta\omega(\omega_+ + \omega_-) = \omega_+^2 - \omega_-^2.$$

Also, $\omega_+ + \omega_- \approx 2\omega$. Hence,

$$\begin{aligned}
\omega\Delta\omega &\approx \frac{\sqrt{9C^2R^4 + 12CR^2L}}{2L^2C}, \\
\omega\Delta\omega &\approx \frac{\omega^2 R\sqrt{9C^2R^2 + 12LC}}{2L}, \\
\omega\Delta\omega &\approx \frac{\omega R\sqrt{9\omega^2C^2R^2 + 12}}{2L}, \\
\frac{\Delta\omega}{\omega} &\approx \frac{R\sqrt{9CR^2/L + 12}}{2L\omega}, \\
&\approx \frac{\sqrt{3}R}{\omega L},
\end{aligned}$$

assuming that $CR^2 \ll 4L/3$.

P37-9

P37-10 Use Eq. 37-46.

P37-11 (a) The resistance of this bulb is

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\,\text{V})^2}{(1000\,\text{W})} = 14.4\,\Omega.$$

The power is directly related to the brightness; if the bulb is to be varied in brightness by a factor of 5 then it would have a minimum power of 200 W. The rms current through the bulb at this power would be

$$i_{\rm rms} = \sqrt{P/R} = \sqrt{(200\,{\rm W})/(14.4\,\Omega)} = 3.73\,{\rm A}.$$

The impedance of the circuit must have been

$$Z = \frac{\mathcal{E}_{\rm rms}}{i_{\rm rms}} = \frac{(120\,{\rm V})}{(3.73\,{\rm A})} = 32.2\,\Omega.$$

The inductive reactance would then be

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(32.2\,\Omega)^2 - (14.4\,\Omega)^2} = 28.8\,\Omega.$$

Finally, the inductance would be

$$L = X_L/\omega = (28.8\,\Omega)/(2\pi(60.0\,{\rm s}^{-1})) = 7.64\,{\rm H}.$$

(b) One could use a variable resistor, and since it would be in series with the lamp a value of

$$32.2\,\Omega - 14.4\Omega = 17.8\,\Omega$$

would work. But the resistor would get hot, while on average there is no power radiated from a pure inductor.

E38-1 The maximum value occurs where $r = R$; there $B_{\max} = \frac{1}{2}\mu_0\epsilon_0 R\, dE/dt$. For $r < R$ B is half of $B_{\max}$ when $r = R/2$. For $r > R$ B is half of $B_{\max}$ when $r = 2R$. Then the two values of r are 2.5 cm and 10.0 cm.

E38-2 For a parallel plate capacitor $E = \sigma/\epsilon_0$ and the flux is then $\Phi_E = \sigma A/\epsilon_0 = q/\epsilon_0$. Then

$$i_\mathrm{d} = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt} = \frac{d}{dt}CV = C\frac{dV}{dt}.$$

$\boxed{\textbf{E38-3}}$ Use the results of Exercise 2, and change the potential difference across the plates of the capacitor at a rate

$$\frac{dV}{dt} = \frac{i_\mathrm{d}}{C} = \frac{(1.0\,\mathrm{mA})}{(1.0\mu\mathrm{F})} = 1.0\ \mathrm{kV/s}.$$

Provide a constant current to the capacitor

$$i = \frac{dQ}{dt} = \frac{d}{dt}CV = C\frac{dV}{dt} = i_\mathrm{d}.$$

E38-4 Since E is uniform between the plates $\Phi_E = EA$, regardless of the size of the region of interest. Since $j_\mathrm{d} = i_\mathrm{d}/A$,

$$j_\mathrm{d} = \frac{i_\mathrm{d}}{A} = \frac{1}{A}\epsilon_0\frac{d\Phi_E}{dt} = \epsilon_0\frac{dE}{dt}.$$

E38-5 (a) In this case $i_\mathrm{d} = i = 1.84\,\mathrm{A}$.

(b) Since $E = q/\epsilon_0 A$, $dE/dt = i/\epsilon_0 A$, or

$$dE/dt = (1.84\,\mathrm{A})/(8.85\times10^{-12}\mathrm{F/m})(1.22\,\mathrm{m})^2 = 1.40\times10^{11}\mathrm{V/m}.$$

(c) $i_\mathrm{d} = \epsilon_0 d\Phi_E/dt = \epsilon_0 a dE/dt$. a here refers to the area of the smaller square. Combine this with the results of part (b) and

$$i_\mathrm{d} = ia/A = (1.84\,\mathrm{A})(0.61\,\mathrm{m}/1.22\,\mathrm{m})^2 = 0.46\,\mathrm{A}.$$

(d) $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 i_\mathrm{d} = (4\pi\times10^{-7}\mathrm{H/m})(0.46\,\mathrm{A}) = 5.78\times10^{-7}\mathrm{T\cdot m}.$

E38-6 Substitute Eq. 38-8 into the results obtained in Sample Problem 38-1. Outside the capacitor $\Phi_E = \pi R^2 E$, so

$$B = \frac{\mu_0}{2\pi r}\frac{\epsilon_0 \pi R^2 dE}{dt} = \frac{\mu_0}{2\pi r}i_\mathrm{d}.$$

Inside the capacitor the enclosed flux is $\Phi_E = \pi r^2 E$; but we want instead to define i_d in terms of the total Φ_E inside the capacitor as was done above. Consequently, inside the conductor

$$B = \frac{\mu_0 r}{2\pi R^2}\frac{\epsilon_0 \pi R^2 dE}{dt} = \frac{\mu_0 r}{2\pi R^2}i_\mathrm{d}.$$

$\boxed{\textbf{E38-7}}$ Since the electric field is uniform in the area and perpendicular to the surface area we have

$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int E\,dA = E\int dA = EA.$$

The displacement current is then

$$i_\mathrm{d} = \epsilon_0 A\frac{dE}{dt} = (8.85\times10^{-12}\mathrm{F/m})(1.9\,\mathrm{m}^2)\frac{dE}{dt}.$$

(a) In the first region the electric field decreases by 0.2 MV/m in 4μs, so

$$i_{\rm d} = (8.85\times10^{-12}{\rm F/m})(1.9\,{\rm m}^2)\frac{(-0.2\times10^6{\rm V/m})}{(4\times10^{-6}{\rm s})} = -0.84\,{\rm A}.$$

(b) The electric field is constant so there is no change in the electric flux, and hence there is no displacement current.

(c) In the last region the electric field decreases by 0.4 MV/m in 5μs, so

$$i_{\rm d} = (8.85\times10^{-12}{\rm F/m})(1.9\,{\rm m}^2)\frac{(-0.4\times10^6{\rm V/m})}{(5\times10^{-6}{\rm s})} = -1.3\,{\rm A}.$$

E38-8 (a) Because of the circular symmetry $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = 2\pi rB$, where r is the distance from the center of the circular plates. Not only that, but $i_{\rm d} = j_{\rm d}A = \pi r^2 j_{\rm d}$. Equate these two expressions, and

$$B = \mu_0 r j_{\rm d}/2 = (4\pi\times10^{-7}{\rm H/m})(0.053\,{\rm m})(1.87\times10^1{\rm A/m})/2 = 6.23\times10^{-7}{\rm T}.$$

(b) $dE/dt = i_{\rm d}/\epsilon_0 A = j_{\rm d}/\epsilon_0 = (1.87\times10^1{\rm A/m})/(8.85\times10^{-12}{\rm F/m}) = 2.11\times10^{-12}{\rm V/m}.$

E38-9 The magnitude of E is given by

$$E = \frac{(162\,{\rm V})}{(4.8\times10^{-3}{\rm m})}\sin 2\pi(60/{\rm s})t;$$

Using the results from Sample Problem 38-1,

$$\begin{aligned} B_{\rm m} &= \frac{\mu_0\epsilon_0 R}{2}\frac{dE}{dt}\bigg|_{t=0}, \\ &= \frac{(4\pi\times10^{-7}{\rm H/m})(8.85\times10^{-12}{\rm F/m})(0.0321\,{\rm m})}{2}2\pi(60/{\rm s})\frac{(162\,{\rm V})}{(4.8\times10^{-3}{\rm m})}, \\ &= 2.27\times10^{-12}{\rm T}. \end{aligned}$$

E38-10 (a) Eq. 33-13 from page 764 and Eq. 33-34 from page 762.

(b) Eq. 27-11 from page 618 and the equation from Ex. 27-25 on page 630.

(c) The equations from Ex. 38-6 on page 876.

(d) Eqs. 34-16 and 34-17 from page 785.

$\boxed{\textbf{E38-11}}$ (a) Consider the path $abefa$. The closed line integral consists of *two* parts: $b\to e$ and $e\to f\to a\to b$. Then

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d\Phi}{dt}$$

can be written as

$$\int_{b\to e}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} + \int_{e\to f\to a\to b}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\Phi_{abef}.$$

Now consider the path $bcdeb$. The closed line integral consists of *two* parts: $b\to c\to d\to e$ and $e\to b$. Then

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d\Phi}{dt}$$

can be written as

$$\int_{b\to c\to d\to e}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} + \int_{e\to b}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\Phi_{bcde}.$$

These two expressions can be added together, and since

$$\int_{e\to b} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{b\to e} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

we get

$$\int_{e\to f\to a\to b} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} + \int_{b\to c\to d\to e} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\left(\Phi_{abef} + \Phi_{bcde}\right).$$

The left hand side of this is just the line integral over the closed path $efadcde$; the right hand side is the net change in flux through the two surfaces. Then we can simplify this expression as

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi}{dt}.$$

(b) Do everything above again, except substitute B for E.

(c) If the equations were not self consistent we would arrive at different values of E and B depending on how we defined our surfaces. This multi-valued result would be quite unphysical.

E38-12 (a) Consider the part on the left. It has a shared surface s, and the other surfaces l. Applying Eq. I,

$$q_l/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_l \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

Note that $d\vec{\mathbf{A}}$ is directed to the right on the shared surface.

Consider the part on the right. It has a shared surface s, and the other surfaces r. Applying Eq. I,

$$q_r/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_r \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

Note that $d\vec{\mathbf{A}}$ is directed to the left on the shared surface.

Adding these two expressions will result in a canceling out of the part

$$\int_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

since one is oriented opposite the other. We are left with

$$\frac{q_r + q_l}{\epsilon_0} = \int_r \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_l \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

E38-13

E38-14 (a) Electric dipole is because the charges are separating like an electric dipole. Magnetic dipole because the current loop acts like a magnetic dipole.

$\boxed{\textbf{E38-15}}$ A series LC circuit will oscillate naturally at a frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

We will need to combine this with $v = f\lambda$, where $v = c$ is the speed of EM waves.

We want to know the inductance required to produce an EM wave of wavelength $\lambda = 550\times10^{-9}$m, so

$$L = \frac{\lambda^2}{4\pi^2c^2C} = \frac{(550\times10^{-9}\text{m})^2}{4\pi^2(3.00\times10^8\text{m/s})^2(17\times10^{-12}\,\text{F})} = 5.01\times10^{-21}\,\text{H}.$$

This is a small inductance!

E38-16 (a) $B = E/c$, and B must be pointing in the negative y direction in order that the wave be propagating in the positive x direction. Then $B_x = B_z = 0$, and

$$B_y = -E_z/c = -(2.34\times10^{-4}\mathrm{V/m})/(3.00\times10^{8}\mathrm{m/s}) = (-7.80\times10^{-13}\mathrm{T})\sin k(x-ct).$$

(b) $\lambda = 2\pi/k = 2\pi/(9.72\times10^{6}/\mathrm{m}) = 6.46\times10^{-7}\mathrm{m}$.

$\boxed{\textbf{E38-17}}$ The electric and magnetic field of an electromagnetic wave are related by Eqs. 38-15 and 38-16,

$$B = \frac{E}{c} = \frac{(321\mu\mathrm{V/m})}{(3.00\times10^{8}\mathrm{m/s})} = 1.07\ \mathrm{pT}.$$

E38-18 Take the partial of Eq. 38-14 with respect to x,

$$\begin{aligned}\frac{\partial}{\partial x}\frac{\partial E}{\partial x} &= -\frac{\partial}{\partial x}\frac{\partial B}{\partial t},\\ \frac{\partial^2 E}{\partial x^2} &= -\frac{\partial^2 B}{\partial x\partial t}.\end{aligned}$$

Take the partial of Eq. 38-17 with respect to t,

$$\begin{aligned}-\frac{\partial}{\partial t}\frac{\partial B}{\partial x} &= \mu_0\epsilon_0\frac{\partial}{\partial t}\frac{\partial E}{\partial t},\\ -\frac{\partial^2 B}{\partial t\partial x} &= \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}.\end{aligned}$$

Equate, and let $\mu_0\epsilon_0 = 1/c^2$, then

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}.$$

Repeat, except now take the partial of Eq. 38-14 with respect to t, and then take the partial of Eq. 38-17 with respect to x.

E38-19 (a) Since $\sin(kx-\omega t)$ is of the form $f(kx\pm\omega t)$, then we only need do part (b).

(b) The constant E_m drops out of the wave equation, so we need only concern ourselves with $f(kx\pm\omega t)$. Letting $g = kx\pm\omega t$,

$$\begin{aligned}\frac{\partial^2 f}{\partial t^2} &= c^2\frac{\partial^2 f}{\partial x^2},\\ \frac{\partial^2 f}{\partial g^2}\left(\frac{\partial g}{\partial t}\right)^2 &= c^2\frac{\partial^2 f}{\partial g^2}\left(\frac{\partial g}{\partial x}\right)^2,\\ \frac{\partial g}{\partial t} &= c\frac{\partial g}{\partial x},\\ \omega &= ck.\end{aligned}$$

E38-20 Use the right hand rule.

E38-21 $U = Pt = (100\times10^{12}\mathrm{W})(1.0\times10^{-9}\mathrm{s}) = 1.0\times10^{5}\mathrm{J}$.

E38-22 $E = Bc = (28\times10^{-9}\mathrm{T})(3.0\times10^{8}\mathrm{m/s}) = 8.4\,\mathrm{V/m}$. It is in the positive x direction.

$\boxed{\textbf{E38-23}}$ Intensity is given by Eq. 38-28, which is simply an expression of power divided by surface area. To find the intensity of the TV signal at α-Centauri we need to find the distance in meters;

$$r = (4.30 \text{ light-years})(3.00\times10^8 \text{ m/s})(3.15\times10^7 \text{ s/year}) = 4.06\times10^{16} \text{ m}.$$

The intensity of the signal when it has arrived at out nearest neighbor is then

$$I = \frac{P}{4\pi r^2} = \frac{(960 \text{ kW})}{4\pi(4.06\times10^{16} \text{ m})^2} = 4.63\times10^{-29} \text{ W/m}^2$$

E38-24 (a) From Eq. 38-22, $S = cB^2/\mu_0$. $B = B_\mathrm{m}\sin\omega t$. The time average is defined as

$$\frac{1}{T}\int_0^T S\,dt = \frac{c{B_\mathrm{m}}^2}{\mu_0 T}\int_0^T \cos^2\omega t\,dt = \frac{c{B_\mathrm{m}}^2}{2\mu_0}.$$

(b) $S_\mathrm{av} = (3.0\times10^8\mathrm{m/s})(1.0\times10^{-4}\mathrm{T})^2/2(4\pi\times10^{-7}\mathrm{H/m}) = 1.2\times10^6\mathrm{W/m^2}$.

E38-25 $I = P/4\pi r^2$, so

$$r = \sqrt{P/4\pi I} = \sqrt{(1.0\times10^3\mathrm{W})/4\pi(130\,\mathrm{W/m^2})} = 0.78\,\mathrm{m}.$$

E38-26 $u_E = \epsilon_0 E^2/2 = \epsilon_0(cB)^2/2 = B^2/2\mu_0 = u_B$.

$\boxed{\textbf{E38-27}}$ (a) Intensity is related to distance by Eq. 38-28. If r_1 is the original distance from the street lamp and I_1 the intensity at that distance, then

$$I_1 = \frac{P}{4\pi r_1^2}.$$

There is a similar expression for the closer distance $r_2 = r_1 - 162\,\mathrm{m}$ and the intensity at that distance $I_2 = 1.50 I_1$. We can combine the two expression for intensity,

$$\begin{aligned} I_2 &= 1.50 I_1, \\ \frac{P}{4\pi r_2^2} &= 1.50\frac{P}{4\pi r_1^2}, \\ r_1^2 &= 1.50 r_2^2, \\ r_1 &= \sqrt{1.50}\,(r_1 - 162\,\mathrm{m}). \end{aligned}$$

The last line is easy enough to solve and we find $r_1 = 883\,\mathrm{m}$.

(b) No, we can't find the power output from the lamp, because we were never provided with an absolute intensity reference.

E38-28 (a) $E_\mathrm{m} = \sqrt{2\mu_0 c I}$, so

$$E_\mathrm{m} = \sqrt{2(4\pi\times10^{-7}\mathrm{H/m})(3.00\times10^8\mathrm{m/s})(1.38\times10^3\mathrm{W/m^2})} = 1.02\times10^3\mathrm{V/m}.$$

(b) $B_\mathrm{m} = E_\mathrm{m}/c = (1.02\times10^3\mathrm{V/m})/(3.00\times10^8\mathrm{m/s}) = 3.40\times10^{-6}\mathrm{T}$.

E38-29 (a) $B_\mathrm{m} = E_\mathrm{m}/c = (1.96\,\mathrm{V/m})/(3.00\times10^8\mathrm{m/s}) = 6.53\times10^{-9}\mathrm{T}$.
(b) $I = {E_\mathrm{m}}^2/2\mu_0 c = (1.96\,\mathrm{V})^2/2(4\pi\times10^{-7}\mathrm{H/m})(3.00\times10^8\mathrm{m/s}) = 5.10\times10^{-3}\mathrm{W/m^2}$.
(c) $P = 4\pi r^2 I = 4\pi(11.2\,\mathrm{m})^2(5.10\times10^{-3}\mathrm{W/m^2}) = 8.04\,\mathrm{W}$.

E38-30 (a) The intensity is

$$I = \frac{P}{A} = \frac{(1\times10^{-12}\mathrm{W})}{4\pi(6.37\times10^{6}\mathrm{m})^2} = 1.96\times10^{-27}\mathrm{W/m^2}.$$

The power received by the Arecibo antenna is

$$P = IA = (1.96\times10^{-27}\mathrm{W/m^2})\pi(305\,\mathrm{m})^2/4 = 1.4\times10^{-22}\mathrm{W}.$$

(b) The power of the transmitter at the center of the galaxy would be

$$P = IA = (1.96\times10^{-27}\mathrm{W})\pi(2.3\times10^{4}\mathrm{ly})^2(9.46\times10^{15}\mathrm{m/ly})^2 = 2.9\times10^{14}\mathrm{W}.$$

$\boxed{\textbf{E38-31}}$ (a) The electric field amplitude is related to the intensity by Eq. 38-26,

$$I = \frac{E_{\mathrm{m}}^2}{2\mu_0 c},$$

or

$$E_{\mathrm{m}} = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\mathrm{H/m})(3.00\times10^{8}\mathrm{m/s})(7.83\mu\mathrm{W/m^2})} = 7.68\times10^{-2}\,\mathrm{V/m}.$$

(b) The magnetic field amplitude is given by

$$B_{\mathrm{m}} = \frac{E_{\mathrm{m}}}{c} = \frac{(7.68\times10^{-2}\,\mathrm{V/m})}{(3.00\times10^{8}\mathrm{m/s})} = 2.56\times10^{-10}\,\mathrm{T}$$

(c) The power radiated by the transmitter can be found from Eq. 38-28,

$$P = 4\pi r^2 I = 4\pi(11.3\ \mathrm{km})^2(7.83\mu\mathrm{W/m^2}) = 12.6\ \mathrm{kW}.$$

E38-32 (a) The power incident on (and then reflected by) the target craft is $P_1 = I_1 A = P_0 A/2\pi r^2$. The intensity of the reflected beam is $I_2 = P_1/2\pi r^2 = P_0 A/4\pi^2 r^4$. Then

$$I_2 = (183\times10^{3}\mathrm{W})(0.222\,\mathrm{m^2})/4\pi^2(88.2\times10^{3}\mathrm{m})^4 = 1.70\times10^{-17}\mathrm{W/m^2}.$$

(b) Use Eq. 38-26:

$$E_{\mathrm{m}} = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\mathrm{H/m})(3.00\times10^{8}\mathrm{m/s})(1.70\times10^{-17}\mathrm{W/m^2})} = 1.13\times10^{-7}\,\mathrm{V/m}.$$

(c) $B_{\mathrm{rms}} = E_{\mathrm{m}}/\sqrt{2}c = (1.13\times10^{-7}\,\mathrm{V/m})/\sqrt{2}(3.00\times10^{8}\mathrm{m/s}) = 2.66\times10^{-16}\mathrm{T}.$

$\boxed{\textbf{E38-33}}$ Radiation pressure for absorption is given by Eq. 38-34, but we need to find the energy absorbed before we can apply that. We are given an intensity, a surface area, and a time, so

$$\Delta U = (1.1\times10^{3}\mathrm{W/m^2})(1.3\,\mathrm{m^2})(9.0\times10^{3}\mathrm{s}) = 1.3\times10^{7}\mathrm{J}.$$

The momentum delivered is

$$p = (\Delta U)/c = (1.3\times10^{7}\mathrm{J})/(3.00\times10^{8}\mathrm{m/s}) = 4.3\times10^{-2}\mathrm{kg\cdot m/s}.$$

E38-34 (a) $F/A = I/c = (1.38\times10^{3}\mathrm{W/m^2})/(3.00\times10^{8}\mathrm{m/s}) = 4.60\times10^{-6}\mathrm{Pa}.$
(b) $(4.60\times10^{-6}\mathrm{Pa})/(101\times10^{5}\mathrm{Pa}) = 4.55\times10^{-11}.$

E38-35 $F/A = 2P/Ac = 2(1.5\times10^{9}\mathrm{W})/(1.3\times10^{-6}\mathrm{m^2})(3.0\times10^{8}\mathrm{m/s}) = 7.7\times10^{6}\mathrm{Pa}.$

E38-36 $F/A = P/4\pi r^2 c$, so

$$F/A = (500\,\mathrm{W})/4\pi(1.50\,\mathrm{m})^2(3.00\times10^8\mathrm{m/s}) = 5.89\times10^{-8}\mathrm{Pa}.$$

E38-37 (a) $F = IA/c$, so

$$F = \frac{(1.38\times10^3\mathrm{W/m^2})\pi(6.37\times10^6\mathrm{m})^2}{(3.00\times10^8\mathrm{m/s})} = 5.86\times10^8\mathrm{N}.$$

E38-38 (a) Assuming MKSA, the units are

$$\frac{\mathrm{m}}{\mathrm{s}}\frac{\mathrm{F}}{\mathrm{m}}\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{N}}{\mathrm{Am}} = \frac{\mathrm{m}}{\mathrm{s}}\frac{\mathrm{C}}{\mathrm{Vm}}\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{sN}}{\mathrm{Cm}} = \frac{\mathrm{Ns}}{\mathrm{m^2 s}}.$$

(b) Assuming MKSA, the units are

$$\frac{\mathrm{A}^2}{\mathrm{N}}\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{N}}{\mathrm{Am}} = \frac{\mathrm{A}^2}{\mathrm{N}}\frac{\mathrm{J}}{\mathrm{Cm}}\frac{\mathrm{N}}{\mathrm{Am}} = \frac{1}{\mathrm{sm}}\frac{\mathrm{J}}{\mathrm{m}} = \frac{\mathrm{J}}{\mathrm{m^2 s}}.$$

$\boxed{\textbf{E38-39}}$ We can treat the object as having two surfaces, one completely reflecting and the other completely absorbing. If the entire surface has an area A then the absorbing part has an area fA while the reflecting part has area $(1-f)A$. The average force is then the sum of the force on each part,

$$F_{\mathrm{av}} = \frac{I}{c}fA + \frac{2I}{c}(1-f)A,$$

which can be written in terms of pressure as

$$\frac{F_{\mathrm{av}}}{A} = \frac{I}{c}(2-f).$$

E38-40 We can treat the object as having two surfaces, one completely reflecting and the other completely absorbing. If the entire surface has an area A then the absorbing part has an area fA while the reflecting part has area $(1-f)A$. The average force is then the sum of the force on each part,

$$F_{\mathrm{av}} = \frac{I}{c}fA + \frac{2I}{c}(1-f)A,$$

which can be written in terms of pressure as

$$\frac{F_{\mathrm{av}}}{A} = \frac{I}{c}(2-f).$$

The intensity I is that of the incident beam; the reflected beam will have an intensity $(1-f)I$. Each beam will contribute to the energy density— I/c and $(1-f)I/c$, respectively. Add these two energy densities to get the net energy density outside the surface. The result is $(2-f)I/c$, which is the left hand side of the pressure relation above.

E38-41 The bullet density is $\rho = Nm/V$. Let $V = Ah$; the kinetic energy density is $K/V = \frac{1}{2}Nmv^2/Ah$. h/v, however, is the time taken for N balls to strike the surface, so that

$$P = \frac{F}{A} = \frac{Nmv}{At} = \frac{Nmv^2}{Ah} = \frac{2K}{V}.$$

E38-42 $F = IA/c$; $P = IA$; $a = F/m$; and $v = at$. Combine:

$$v = Pt/mc = (10\times10^3\mathrm{W})(86400\,\mathrm{s})/(1500\,\mathrm{kg})(3\times10^8\mathrm{m/s}) = 1.9\times10^{-3}\mathrm{m/s}.$$

E38-43 The force of radiation on the bottom of the cylinder is $F = 2IA/c$. The force of gravity on the cylinder is

$$W = mg = \rho HAg.$$

Equating, $2I/c = \rho Hg$. The intensity of the beam is given by $I = 4P/\pi d^2$. Solving for H,

$$H = \frac{8P}{\pi c\rho g d^2} = \frac{8(4.6\,\mathrm{W})}{\pi(3.0\times10^8\mathrm{m/s})(1200\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(2.6\times10^{-3}\mathrm{m})^2} = 4.9\times10^{-7}\mathrm{m}.$$

E38-44 $F = 2IA/c$. The value for I is in Ex. 38-37, among other places. Then

$$F = (1.38\times10^3\mathrm{W/m^2})(3.1\times10^6\mathrm{m^2})/(3.00\times10^8\mathrm{m/s}) = 29\,\mathrm{N}.$$

P38-1 For the two outer circles use Eq. 33-13. For the inner circle use $E = V/d$, $Q = CV$, $C = \epsilon_0 A/d$, and $i = dQ/dt$. Then

$$i = \frac{dQ}{dt} = \frac{\epsilon_0 A}{d}\frac{dV}{dt} = \epsilon_0 A\frac{dE}{dt}.$$

The change in flux is $d\Phi_E/dt = A\,dE/dt$. Then

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = \mu_0\epsilon_0\frac{d\Phi_E}{dt} = \mu_0 i,$$

so $B = \mu_0 i/2\pi r$.

P38-2 (a) $i_\mathrm{d} = i$. Assuming $\Delta V = (174\times10^3\mathrm{V})\sin\omega t$, then $q = C\Delta V$ and $i = dq/dt = Cd(\Delta V)/dt$. Combine, and use $\omega = 2\pi(50.0/\mathrm{s})$,

$$i_\mathrm{d} = (100\times10^{-12}\mathrm{F})(174\times10^3\mathrm{V})2\pi(50.0/\mathrm{s}) = 5.47\times10^{-3}\mathrm{A}.$$

P38-3 (a) $i = i_\mathrm{d} = 7.63\mu\mathrm{A}$.

(b) $d\Phi_E/dt = i_\mathrm{d}/\epsilon_0 = (7.63\mu\mathrm{A})/(8.85\times10^{-12}\mathrm{F/m}) = 8.62\times10^5\mathrm{V/m}$.

(c) $i = dq/dt = Cd(\Delta V)/dt$; $C = \epsilon_0 A/d$; $[d(\Delta V)/dt]_\mathrm{m} = \mathcal{E}_\mathrm{m}\omega$. Combine, and

$$d = \frac{\epsilon_0 A}{C} = \frac{\epsilon_0 A\mathcal{E}_\mathrm{m}\omega}{i} = \frac{(8.85\times10^{-12}\mathrm{F/m})\pi(0.182\,\mathrm{m})^2(225\,\mathrm{V})(128\ \mathrm{rad/s})}{(7.63\mu\mathrm{A})} = 3.48\times10^{-3}\mathrm{m}.$$

P38-4 (a) $q = \int i\,dt = \alpha\int t\,dt = \alpha t^2/2$.

(b) $E = \sigma/\epsilon_0 = q/\epsilon_0 A = \alpha t^2/2\pi R^2\epsilon_0$.

(d) $2\pi rB = \mu_0\epsilon_0\pi r^2 dE/dt$, so

$$B = \mu_0 r(dE/dt)/2 = \mu_0\alpha rt/2\pi R^2.$$

(e) Check Exercise 38-10!

P38-5 (a) $\vec{\mathbf{E}} = E\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = B\hat{\mathbf{k}}$. Then $\vec{\mathbf{S}} = \vec{\mathbf{E}}\times\vec{\mathbf{B}}/\mu_0$, or

$$\vec{\mathbf{S}} = -EB/mu_0\,\hat{\mathbf{i}}.$$

Energy only passes through the yz faces; it goes in one face and out the other. The rate is $P = SA = EBa^2/mu_0$.

(b) The net change is zero.

P38-6 (a) For a sinusoidal time dependence $|dE/dt|_{\rm m} = \omega E_{\rm m} = 2\pi f E_{\rm m}$. Then

$$|dE/dt|_{\rm m} = 2\pi(2.4\times10^{9}/{\rm s})(13\times10^{3}{\rm V/m}) = 1.96\times10^{14}{\rm V/m\cdot s}.$$

(b) Using the result of part (b) of Sample Problem 38-1,

$$B = \frac{1}{2}(4\pi\times10^{-7}{\rm H/m})(8.9\times10^{-12}{\rm F/m})(2.4\times10^{-2}{\rm m})\frac{1}{2}(1.96\times10^{14}{\rm V/m\cdot s}) = 1.3\times10^{-5}{\rm T}.$$

$\boxed{\textbf{P38-7}}$ Look back to Chapter 14 for a discussion on the elliptic orbit. On page 312 it is pointed out that the closest distance to the sun is $R_{\rm p} = a(1-e)$ while the farthest distance is $R_{\rm a} = a(1+e)$, where a is the semi-major axis and e the eccentricity.

The fractional variation in intensity is

$$\begin{aligned}\frac{\Delta I}{I} &\approx \frac{I_{\rm p} - I_{\rm a}}{I_{\rm a}},\\ &= \frac{I_{\rm p}}{I_{\rm a}} - 1,\\ &= \frac{{R_{\rm a}}^2}{{R_{\rm p}}^2} - 1,\\ &= \frac{(1+e)^2}{(1-e)^2} - 1.\end{aligned}$$

We need to expand this expression for small e using $(1+e)^2 \approx 1+2e$, and $(1-e)^{-2} \approx 1+2e$, and finally $(1+2e)^2 \approx 1+4e$. Combining,

$$\frac{\Delta I}{I} \approx (1+2e)^2 - 1 \approx 4e.$$

P38-8 The beam radius grows as $r = (0.440\,\mu{\rm rad})R$, where R is the distance from the origin. The beam intensity is

$$I = \frac{P}{\pi r^2} = \frac{(3850\,{\rm W})}{\pi(0.440\,\mu{\rm rad})^2(3.82\times10^{8}{\rm m})^2} = 4.3\times10^{-2}{\rm W}.$$

$\boxed{\textbf{P38-9}}$ Eq. 38-14 requires

$$\begin{aligned}\frac{\partial E}{\partial x} &= -\frac{\partial B}{\partial t},\\ E_{\rm m}k\cos kx\sin\omega t &= B_{\rm m}\omega\cos kx\sin\omega t,\\ E_{\rm m}k &= B_{\rm m}\omega.\end{aligned}$$

Eq. 38-17 requires

$$\begin{aligned}\mu_0\epsilon_0\frac{\partial E}{\partial t} &= -\frac{\partial B}{\partial x},\\ \mu_0\epsilon_0 E_{\rm m}\omega\sin kx\cos\omega t &= B_{\rm m}k\sin kx\cos\omega t,\\ \mu_0\epsilon_0 E_{\rm m}\omega &= B_{\rm m}k.\end{aligned}$$

Dividing one expression by the other,

$$\mu_0\epsilon_0 k^2 = \omega^2,$$

or

$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

.

Not only that, but $E_{\rm m} = cB_{\rm m}$. You've seen an expression similar to this before, and you'll see expressions similar to it again.

(b) We'll assume that Eq. 38-21 is applicable here. Then

$$\begin{aligned} S &= \frac{1}{\mu_0} = \frac{E_{\rm m}B_{\rm m}}{\mu_0}\sin kx \sin \omega t \cos kx \cos \omega t, \\ &= \frac{E_{\rm m}^2}{4\mu_0 c}\sin 2kx \sin 2\omega t \end{aligned}$$

is the magnitude of the instantaneous Poynting vector.

(c) The time averaged power flow across any surface is the value of

$$\frac{1}{T}\int_0^T \int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}}\, dt,$$

where T is the period of the oscillation. We'll just gloss over any concerns about direction, and assume that the $\vec{\mathbf{S}}$ will be constant in direction so that we will, at most, need to concern ourselves about a constant factor $\cos\theta$. We can then deal with a scalar, instead of vector, integral, and we can integrate it in any order we want. We want to do the t integration first, because an integral over $\sin \omega t$ for a period $T = 2\pi/\omega$ is zero. Then we are done!

(d) There is no energy flow; the energy remains inside the container.

P38-10 (a) The electric field is parallel to the wire and given by

$$E = V/d = iR/d = (25.0\,\text{A})(1.00\,\Omega/300\,\text{m}) = 8.33\times10^{-2}\text{V/m}$$

(b) The magnetic field is in rings around the wire. Using Eq. 33-13,

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times10^{-7}\text{H/m})(25\,\text{A})}{2\pi(1.24\times10^{-3}\text{m})} = 4.03\times10^{-3}\text{T}.$$

(c) $S = EB/\mu_0$, so

$$S = (8.33\times10^{-2}\text{V/m})(4.03\times10^{-3}\text{T})/(4\pi\times10^{-7}\text{H/m}) = 267\,\text{W/m}^2.$$

$\boxed{\textbf{P38-11}}$ (a) We've already calculated B previously. It is

$$B = \frac{\mu_0 i}{2\pi r} \text{ where } i = \frac{\mathcal{E}}{R}.$$

The electric field of a long straight wire has the form $E = k/r$, where k is some constant. But

$$\Delta V = -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_a^b E\, dr = -k\ln(b/a).$$

In this problem the inner conductor is at the higher potential, so

$$k = \frac{-\Delta V}{\ln(b/a)} = \frac{\mathcal{E}}{\ln(b/a)},$$

and then the electric field is

$$E = \frac{\mathcal{E}}{r\ln(b/a)}.$$

This is also a vector field, and if $\mathcal{E}$ is positive the electric field points radially out from the central conductor.

(b) The Poynting vector is

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}};$$

$\vec{\mathbf{E}}$ is radial while $\vec{\mathbf{B}}$ is circular, so they are perpendicular. Assuming that $\mathcal{E}$ is positive the direction of $\vec{\mathbf{S}}$ is away from the battery. Switching the sign of $\mathcal{E}$ (connecting the battery in reverse) will flip the direction of both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, so $\vec{\mathbf{S}}$ will pick up *two* negative signs and therefore *still* point away from the battery.

The magnitude is

$$S = \frac{EB}{\mu_0} = \frac{\mathcal{E}^2}{2\pi R\ln(b/a)r^2}$$

(c) We want to evaluate a surface integral in polar coordinates and so $dA = (dr)(rd\theta)$. We have already established that $\vec{\mathbf{S}}$ is pointing away from the battery parallel to the central axis. Then we can integrate

$$\begin{aligned} P &= \int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = \int S\,dA, \\ &= \int_a^b \int_0^{2\pi} \frac{\mathcal{E}^2}{2\pi R\ln(b/a)r^2} d\theta\, r\, dr, \\ &= \int_a^b \frac{\mathcal{E}^2}{R\ln(b/a)r} dr, \\ &= \frac{\mathcal{E}^2}{R}. \end{aligned}$$

(d) Read part (b) above.

P38-12 (a) $\vec{\mathbf{B}}$ is oriented as rings around the cylinder. If the thumb is in the direction of current then the fingers of the right hand grip ion the direction of the magnetic field lines. $\vec{\mathbf{E}}$ is directed parallel to the wire in the direction of the current. $\vec{\mathbf{S}}$ is found from the cross product of these two, and must be pointing radially inward.

(b) The magnetic field on the surface is given by Eq. 33-13:

$$B = \mu_0 i/2\pi a.$$

The electric field on the surface is given by

$$E = V/l = iR/l$$

Then S has magnitude

$$S = EB/\mu_0 = \frac{i}{2\pi a}\frac{iR}{l} = \frac{i^2 R}{2\pi a l}.$$

$\int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}}$ is only evaluated on the surface of the cylinder, not the end caps. $\vec{\mathbf{S}}$ is everywhere parallel to $d\vec{\mathbf{A}}$, so the dot product reduces to $S\,dA$; S is uniform, so it can be brought out of the integral; $\int dA = 2\pi a l$ on the surface.

Hence,

$$\int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = i^2 R,$$

as it should.

P38-13 (a) $f = vlambda = (3.00\times10^8\mathrm{m/s})/(3.18\,\mathrm{m}) = 9.43\times10^7$ Hz.

(b) $\vec{\mathbf{B}}$must be directed along the z axis. The magnitude is

$$B = E/c = (288\,\mathrm{V/m})/(3.00\times10^8\mathrm{m/s}) = 9.6\times10^{-7}\mathrm{T}.$$

(c) $k = 2\pi/\lambda = 2\pi/(3.18\,\mathrm{m}) = 1.98/\mathrm{m}$ while $\omega = 2\pi f$, so

$$\omega = 2\pi(9.43\times10^7\mathrm{Hz}) = 5.93\times10^8 \text{ rad/s}.$$

(d) $I = E_\mathrm{m}B_\mathrm{m}/2\mu_0$, so

$$I = \frac{(288\,\mathrm{V})(9.6\times10^{-7}\mathrm{T})}{2(4\pi\times10^{-7}\mathrm{H/m})} = 110\,\mathrm{W}.$$

(e) $P = I/c = (110\,\mathrm{W})/(3.00\times10^8\mathrm{m/s}) = 3.67\times10^{-7}\mathrm{Pa}.$

P38-14 (a) $\vec{\mathbf{B}}$ is oriented as rings around the cylinder. If the thumb is in the direction of current then the fingers of the right hand grip ion the direction of the magnetic field lines. $\vec{\mathbf{E}}$ is directed parallel to the wire in the direction of the current. $\vec{\mathbf{S}}$ is found from the cross product of these two, and must be pointing radially inward.

(b) The magnitude of the electric field is

$$E = \frac{V}{d} = \frac{Q}{Cd} = \frac{Q}{\epsilon_0 A} = \frac{it}{\epsilon_0 A}.$$

The magnitude of the magnetic field on the outside of the plates is given by Sample Problem 38-1,

$$B = \frac{\mu_0\epsilon_0 R}{2}\frac{dE}{dt} = \frac{\mu_0\epsilon_0 iR}{2\epsilon_0 A} = \frac{\mu_0\epsilon_0 R}{2t}E.$$

$\vec{\mathbf{S}}$ has magnitude

$$S = \frac{EB}{\mu_0} = \frac{\epsilon_0 R}{2t}E^2.$$

Integrating,

$$\int \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = \frac{\epsilon_0 R}{2t}E^2 2\pi Rd = Ad\frac{\epsilon_0 E^2}{t}.$$

But E is linear in t, so $d(E^2)/dt = 2E^2/t$; and then

$$\int \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = Ad\frac{d}{dt}\left(\frac{1}{2}\epsilon_0 E^2\right).$$

P38-15 (a) $I = P/A = (5.00\times10^{-3}\mathrm{W})/\pi(1.05)^2(633\times10^{-9}\mathrm{m})^2 = 3.6\times10^9\mathrm{W/m^2}.$

(b) $p = I/c = (3.6\times10^9\mathrm{W/m^2})/(3.00\times10^8\mathrm{m/s}) = 12\,\mathrm{Pa}$

(c) $F = pA = P/c = (5.00\times10^{-3}\mathrm{W})/(3.00\times10^8\mathrm{m/s}) = 1.67\times10^{-11}\mathrm{N}.$

(d) $a = F/m = F/\rho V$, so

$$a = \frac{(1.67\times10^{-11}\mathrm{N})}{4(4880\,\mathrm{kg/m^3})(1.05)^3(633\times10^{-9})^3/3} = 2.9\times10^3\mathrm{m/s^2}.$$

P38-16 The force from the sun is $F = GMm/r^2$. The force from radiation pressure is

$$F = \frac{2IA}{c} = \frac{2PA}{4\pi r^2 c}.$$

Equating,

$$A = \frac{4\pi GMm}{2P/c},$$

so

$$A = \frac{4\pi(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\mathrm{kg})(1650\,\mathrm{kg})}{2(3.9\times10^{26}\mathrm{W})/(3.0\times10^{8}\mathrm{m/s})} = 1.06\times10^{6}\mathrm{m}^2.$$

That's about one square kilometer.

E39-1 Both scales are logarithmic; choose any data point from the right hand side such as

$$c = f\lambda \approx (1 \text{ Hz})(3\times 10^8\text{m}) = 3\times 10^8\text{m/s},$$

and another from the left hand side such as

$$c = f\lambda \approx (1\times 10^{21} \text{ Hz})(3\times 10^{-13}\text{m}) = 3\times 10^8\text{m/s}.$$

E39-2 (a) $f = v/\lambda = (3.0\times 10^8\text{m/s})/(1.0\times 10^4)(6.37\times 10^6\text{m}) = 4.7\times 10^{-3}$ Hz. If we assume that this is the data transmission rate in bits per second (a generous assumption), then it would take 140 days to download a web-page which would take only 1 second on a 56K modem!

(b) $T = 1/f = 212\,\text{s} = 3.5$ min.

$\boxed{\textbf{E39-3}}$ (a) Apply $v = f\lambda$. Then

$$f = (3.0\times 10^8\text{m/s})/(0.067\times 10^{-15}\text{m}) = 4.5\times 10^{24} \text{ Hz}.$$

(b) $\lambda = (3.0\times 10^8\text{m/s})/(30 \text{ Hz}) = 1.0\times 10^7\text{m}$.

E39-4 Don't simply take reciprocal of linewidth! $f = c/\lambda$, so $\delta f = (-c/\lambda^2)\delta\lambda$. Ignore the negative, and

$$\delta f = (3.00\times 10^8\text{m/s})(0.010\times 10^{-9}\text{m})/(632.8\times 10^{-9}\text{m})^2 = 7.5\times 10^9 \text{ Hz}.$$

$\boxed{\textbf{E39-5}}$ (a) We refer to Fig. 39-6 to answer this question. The limits are approximately 520 nm and 620 nm.

(b) The wavelength for which the eye is most sensitive is 550 nm. This corresponds to to a frequency of

$$f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(550 \times 10^{-9}\text{m}) = 5.45 \times 10^{14} \text{ Hz}.$$

This frequency corresponds to a period of $T = 1/f = 1.83 \times 10^{-15}$s.

E39-6 $f = c/\lambda$. The number of complete pulses is ft, or

$$ft = ct/\lambda = (3.00\times 10^8\text{m/s})(430\times 10^{-12}\text{s})/(520\times 10^{-9}\text{m}) = 2.48\times 10^5.$$

E39-7 (a) $2(4.34 \text{ y}) = 8.68$ y.

(b) $2(2.2\times 10^6 \text{ y}) = 4.4\times 10^6$ y.

E39-8 (a) $t = (150\times 10^3\text{m})/(3\times 10^8\text{m/s}) = 5\times 10^{-4}$s.

(b) The distance traveled by the light is $(1.5\times 10^{11}\text{m}) + 2(3.8\times 10^8\text{m})$, so

$$t = (1.51\times 10^{11}\text{m})/(3\times 10^8\text{m/s}) = 503\,\text{s}.$$

(c) $t = 2(1.3\times 10^{12}\text{m})/(3\times 10^8\text{m/s}) = 8670\,\text{s}$.

(d) $1054 - 6500 \approx 5400$ BC.

$\boxed{\textbf{E39-9}}$ This is a question of how much time it takes light to travel 4 cm, because the light traveled from the Earth to the moon, bounced off of the reflector, and then traveled back. The time to travel 4 cm is $\Delta t = (0.04\,\text{m})/(3 \times 10^8 \text{ m/s}) = 0.13$ ns. Note that I interpreted the question differently than the answer in the back of the book.

E39-10 Consider any incoming ray. The path of the ray can be projected onto the xy plane, the xz plane, or the yz plane. If the projected rays is exactly reflected in all three cases then the three dimensional incoming ray will be reflected exactly reversed. But the problem is symmetric, so it is sufficient to show that any plane works.

Now the problem has been reduced to Sample Problem 39-2, so we are done.

E39-11 We will choose the mirror to lie in the xy plane at $z = 0$. There is no loss of generality in doing so; we had to define our coordinate system somehow. The choice is convenient in that any normal is then parallel to the z axis. Furthermore, we can arbitrarily define the incident ray to originate at $(0, 0, z_1)$. Lastly, we can rotate the coordinate system about the z axis so that the reflected ray passes through the point $(0, y_3, z_3)$.

The point of reflection for this ray is somewhere on the surface of the mirror, say $(x_2, y_2, 0)$. This distance traveled from the point 1 to the reflection point 2 is

$$d_{12} = \sqrt{(0 - x_2)^2 + (0 - y_2)^2 + (z_1 - 0)^2} = \sqrt{x_2^2 + y_2^2 + z_1^2}$$

and the distance traveled from the reflection point 2 to the final point 3 is

$$d_{23} = \sqrt{(x_2 - 0)^2 + (y_2 - y_3)^2 + (0 - z_3)^2} = \sqrt{x_2^2 + (y_2 - y_3)^2 + z_3^2}.$$

The only point which is free to move is the reflection point, $(x_2, y_2, 0)$, and that point can only move in the xy plane. Fermat's principle states that the reflection point will be such to minimize the total distance,

$$d_{12} + d_{23} = \sqrt{x_2^2 + y_2^2 + z_1^2} + \sqrt{x_2^2 + (y_2 - y_3)^2 + z_3^2}.$$

We do this minimization by taking the partial derivative with respect to both x_2 and y_2. But we can do part by inspection alone. Any non-zero value of x_2 can only *add* to the total distance, regardless of the value of any of the other quantities. Consequently, $x_2 = 0$ is one of the conditions for minimization.

We are done! Although you are invited to finish the minimization process, once we know that $x_2 = 0$ we have that point 1, point 2, and point 3 all lie in the yz plane. The normal is parallel to the z axis, so it also lies in the yz plane. Everything is then in the yz plane.

E39-12 Refer to Page 442 of Volume 1.

E39-13 (a) $\theta_1 = 38°$.

(b) $(1.58)\sin(38°) = (1.22)\sin\theta_2$. Then $\theta_2 = \arcsin(0.797) = 52.9°$.

E39-14 $n_\mathrm{g} = n_\mathrm{v} \sin\theta_1/\sin\theta_2 = (1.00)\sin(32.5°)/\sin(21.0°) = 1.50$.

E39-15 $n = c/v = (3.00\times10^8\mathrm{m/s})/(1.92\times10^8\mathrm{m/s}) = 1.56$.

E39-16 $v = c/n = (3.00\times10^8\mathrm{m/s})/(1.46) = 2.05\times10^8\mathrm{m/s}$.

E39-17 The speed of light in a substance with index of refraction n is given by $v = c/n$. An electron will then emit Cerenkov radiation in this particular liquid if the speed exceeds

$$v = c/n = (3.00 \times 10^8\,\mathrm{m/s})/(1.54) = 1.95\times10^8\,\mathrm{m/s}.$$

E39-18 Since $t = d/v = nd/c$, $\Delta t = \Delta n\, d/c$. Then

$$\Delta t = (1.00029 - 1.00000)(1.61\times 10^3\text{m})/(3.00\times 10^8\text{m/s}) = 1.56\times 10^{-9}\text{s}.$$

$\boxed{\textbf{E39-19}}$ The angle of the refracted ray is $\theta_2 = 90°$, the angle of the incident ray can be found by trigonometry,

$$\tan\theta_1 = \frac{(1.14\,\text{m})}{(0.85\,\text{m})} = 1.34,$$

or $\theta_1 = 53.3°$.

We can use these two angles, along with the index of refraction of air, to find that the index of refraction of the liquid from Eq. 39-4,

$$n_1 = n_2\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\frac{(\sin 90°)}{(\sin 53.3°)} = 1.25.$$

There are no units attached to this quantity.

E39-20 For an equilateral prism $\phi = 60°$. Then

$$n = \frac{\sin[\psi+\phi]/2}{\sin[\phi/2]} = \frac{\sin[(37°)+(60°)]/2}{\sin[(60°)/2]} = 1.5.$$

E39-21

E39-22 $t = d/v$; but $L/d = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}$ and $v = c/n$. Combining,

$$t = \frac{nL}{c\sqrt{1-\sin^2\theta_2}} = \frac{n^2L}{c\sqrt{n^2-\sin^2\theta_1}} = \frac{(1.63)^2(0.547\,\text{m})}{(3\times 10^8\text{m/s})\sqrt{(1.63^2)-\sin^2(24°)}} = 3.07\times 10^{-9}\text{s}.$$

E39-23 The ray of light from the top of the smokestack to the life ring is θ_1, where $\tan\theta_1 = L/h$ with h the height and L the distance of the smokestack.

Snell's law gives $n_1\sin\theta_1 = n_2\sin\theta_2$, so

$$\theta_1 = \arcsin[(1.33)\sin(27°)/(1.00)] = 37.1°.$$

Then $L = h\tan\theta_1 = (98\,\text{m})\tan(37.1°) = 74\,\text{m}$.

E39-24 The length of the shadow on the surface of the water is

$$x_1 = (0.64\,\text{m})/\tan(55°) = 0.448\,\text{m}.$$

The ray of light which forms the "end" of the shadow has an angle of incidence of 35°, so the ray travels into the water at an angle of

$$\theta_2 = \arcsin\left(\frac{(1.00)}{(1.33)}\sin(35°)\right) = 25.5°.$$

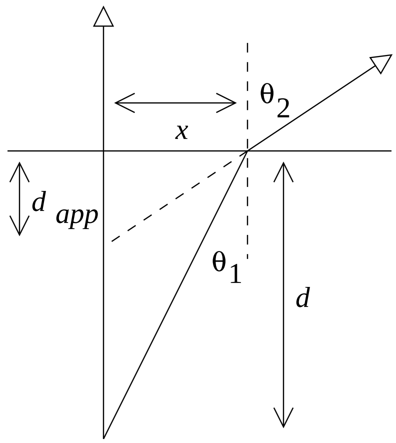
The ray travels an additional distance

$$x_2 = (2.00\,\text{m} - 0.64\,\text{m})/\tan(90° - 25.5°) = 0.649\,\text{m}$$

The total length of the shadow is

$$(0.448\,\text{m}) + (0.649\,\text{m}) = 1.10\,\text{m}.$$

$\boxed{\textbf{E39-25}}$ We'll rely heavily on the figure for our arguments. Let x be the distance between the points on the surface where the vertical ray crosses and the bent ray crosses.



In this exercise we will take advantage of the fact that, for small angles θ, $\sin\theta \approx \tan\theta \approx \theta$ In this approximation Snell's law takes on the particularly simple form $n_1\theta_1 = n_2\theta_2$ The two angles here are conveniently found from the figure,

$$\theta_1 \approx \tan\theta_1 = \frac{x}{d},$$

and

$$\theta_2 \approx \tan\theta_2 = \frac{x}{d_{\text{app}}}.$$

Inserting these two angles into the simplified Snell's law, as well as substituting $n_1 = n$ and $n_2 = 1.0$,

$$\begin{aligned} n_1\theta_1 &= n_2\theta_2, \\ n\frac{x}{d} &= \frac{x}{d_{\text{app}}}, \\ d_{\text{app}} &= \frac{d}{n}. \end{aligned}$$

E39-26 (a) You need to address the issue of total internal reflection to answer this question.

(b) Rearrange

$$n = \frac{\sin[\psi+\phi]/2}{/}\sin[\phi/2]$$

and $\theta = (\psi+\phi)/2$ to get

$$\theta = \arcsin\left(n\sin[\phi/2]\right) = \arcsin\left((1.60)\sin[(60^\circ)/2]\right) = 53.1^\circ.$$

E39-27 Use the results of Ex. 39-35. The apparent thickness of the carbon tetrachloride layer, as viewed by an observer in the water, is

$$d_{\text{c,w}} = n_{\text{w}}d_{\text{c}}/n_{\text{c}} = (1.33)(41\,\text{mm})/(1.46) = 37.5\,\text{mm}.$$

The total "thickness" from the water perspective is then $(37.5\,\text{mm}) + (20\,\text{mm}) = 57.5\,\text{mm}$. The apparent thickness of the entire system as view from the air is then

$$d_{\text{app}} = (57.5\,\text{mm})/(1.33) = 43.2\,\text{mm}.$$

E39-28 (a) Use the results of Ex. 39-35. $d_{\rm app} = (2.16\,{\rm m})/(1.33) = 1.62\,{\rm m}$.

(b) Need a diagram here!

E39-29 (a) $\lambda_n = \lambda/n = (612\text{ nm})/(1.51) = 405\text{ nm}$.

(b) $L = nL_n = (1.51)(1.57\text{ pm}) = 2.37\text{ pm}$. There is actually a typo: the "p" in "pm" was supposed to be a μ. This makes a huge difference for part (c)!

E39-30 (a) $f = c/\lambda = (3.00\times10^8\text{m/s})/(589\text{ nm}) = 5.09\times10^{14}\text{Hz}$.

(b) $\lambda_n = \lambda/n = (589\text{ nm})/(1.53) = 385\text{ nm}$.

(c) $v = f\lambda = (5.09\times10^{14}\text{Hz})(385\text{ nm}) = 1.96\times10^8\text{m/s}$.

E39-31 (a) The second derivative of

$$L = \sqrt{a^2+x^2} + \sqrt{b^2+(d-x)^2}$$

is

$$\frac{a^2(b^2+(d-2)^2)^{3/2} + b^2(a^2+x^2)^{3/2}}{(b^2+(d-2)^2)^{3/2}(a^2+x^2)^{3/2}}.$$

This is *always* a positive number, so $dL/dx = 0$ is a minimum.

(a) The second derivative of

$$L = n_1\sqrt{a^2+x^2} + n_2\sqrt{b^2+(d-x)^2}$$

is

$$\frac{n_1a^2(b^2+(d-2)^2)^{3/2} + n_2b^2(a^2+x^2)^{3/2}}{(b^2+(d-2)^2)^{3/2}(a^2+x^2)^{3/2}}.$$

This is *always* a positive number, so $dL/dx = 0$ is a minimum.

E39-32 (a) The angle of incidence on the face ac will be $90^\circ - \phi$. Total internal reflection occurs when $\sin(90^\circ - \phi) > 1/n$, or

$$\phi < 90^\circ - \arcsin[1/(1.52)] = 48.9^\circ.$$

(b) Total internal reflection occurs when $\sin(90^\circ - \phi) > n_{\rm w}/n$, or

$$\phi < 90^\circ - \arcsin[(1.33)/(1.52)] = 29.0^\circ.$$

E39-33 (a) The critical angle is given by Eq. 39-17,

$$\theta_{\rm c} = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{(1.586)}{(1.667)} = 72.07^\circ.$$

(b) Critical angles only exist when "attempting" to travel from a medium of higher index of refraction to a medium of lower index of refraction; in this case from A to B.

E39-34 If the fire is at the water's edge then the light travels along the surface, entering the water near the fish with an angle of incidence of effectively 90°. Then the angle of refraction in the water is numerically equivalent to a critical angle, so the fish needs to look up at an angle of $\theta = \arcsin(1/1.33) = 49^\circ$ with the vertical. That's the same as 41° with the horizontal.

E39-35 Light can only emerge from the water if it has an angle of incidence less than the critical angle, or

$$\theta < \theta_c = \arcsin 1/n = \arcsin 1/(1.33) = 48.8^\circ.$$

The radius of the circle of light is given by $r/d = \tan\theta_c$, where d is the depth. The diameter is twice this radius, or

$$2(0.82\,\text{m})\tan(48.8^\circ) = 1.87\,\text{m}.$$

E39-36 The refracted angle is given by $n\sin\theta_1 = \sin(39^\circ)$. This ray strikes the left surface with an angle of incidence of $90^\circ - \theta_1$. Total internal reflection occurs when

$$\sin(90^\circ - \theta_1) = 1/n;$$

but $\sin(90^\circ - \theta_1) = \cos\theta_1$, so we can combine and get $\tan\theta = \sin(39^\circ)$ with solution $\theta_1 = 32.2^\circ$. The index of refraction of the glass is then

$$n = \sin(39^\circ)/\sin(32.2) = 1.18.$$

$\boxed{\textbf{E39-37}}$ The light strikes the quartz-air interface from the inside; it is originally "white", so if the reflected ray is to appear "bluish" (reddish) then the refracted ray should have been "reddish" (bluish). Since part of the light undergoes total internal reflection while the other part does not, then the angle of incidence must be approximately equal to the critical angle.

(a) Look at Fig. 39-11, the index of refraction of fused quartz is given as a function of the wavelength. As the wavelength increases the index of refraction decreases. The critical angle is a function of the index of refraction; for a substance in air the critical angle is given by $\sin\theta_c = 1/n$. As n decreases $1/n$ increases so θ_c increases. For fused quartz, then, as wavelength increases θ_c also increases.

In short, red light has a larger critical angle than blue light. If the angle of incidence is midway between the critical angle of red and the critical angle of blue, then the blue component of the light will experience total internal reflection while the red component will pass through as a refracted ray.

So yes, the light can be made to appear bluish.

(b) No, the light can't be made to appear reddish. See above.

(c) Choose an angle of incidence between the two critical angles as described in part (a). Using a value of $n = 1.46$ from Fig. 39-11,

$$\theta_c = \sin^{-1}(1/1.46) = 43.2^\circ.$$

Getting the effect to work will require considerable sensitivity.

E39-38 (a) There needs to be an opaque spot in the center of each face so that no refracted ray emerges. The radius of the spot will be large enough to cover rays which meet the surface at less than the critical angle. This means $\tan\theta_c = r/d$, where d is the distance from the surface to the spot, or 6.3 mm. Since

$$\theta_c = \arcsin 1/(1.52) = 41.1^\circ,$$

then $r = (6.3\,\text{mm})\tan(41.1^\circ) = 5.50\,\text{mm}$.

(b) The circles have an area of $a = \pi(5.50\,\text{mm})^2 = 95.0\,\text{mm}^2$. Each side has an area of $(12.6\,\text{mm})^2$; the fraction covered is then $(95.0\,\text{mm}^2)/(12.6\,\text{mm})^2 = 0.598$.

E39-39 For $u \ll c$ the relativistic Doppler shift simplifies to

$$\Delta f = -f_0 u/c = -u/\lambda_0,$$

so

$$u = \lambda_0 \Delta f = (0.211\,\text{m})\Delta f.$$

E39-40 $c = f\lambda$, so $0 = f\Delta\lambda + \lambda\Delta f$. Then $\Delta\lambda/\lambda = -\Delta f/f$. Furthermore, $f_0 - f$, from Eq. 39-21, is $f_0 u/c$ for small enough u. Then

$$\frac{\Delta\lambda}{\lambda} = -\frac{f - f_0}{f_0} = \frac{u}{c}.$$

$\boxed{\textbf{E39-41}}$ The Doppler theory for light gives

$$f = f_0 \frac{1 - u/c}{\sqrt{1 - u^2/c^2}} = f_0 \frac{1 - (0.2)}{\sqrt{1 - (0.2)^2}} = 0.82\, f_0.$$

The frequency is shifted down to about 80%, which means the wavelength is shifted up by an additional 25%. Blue light (480 nm) would appear yellow/orange (585 nm).

E39-42 Use Eq. 39-20:

$$f = f_0 \frac{1 - u/c}{\sqrt{1 - u^2/c^2}} = (100 \text{ Mhz}) \frac{1 - (0.892)}{\sqrt{1 - (0.892)^2}} = 23.9 \text{ MHz}.$$

E39-43 (a) If the wavelength is three times longer then the frequency is one-third, so for the classical Doppler shift

$$f_0/3 = f_0(1 - u/c),$$

or $u = 2c$.

(b) For the relativistic shift,

$$\begin{aligned} f_0/3 &= f_0 \frac{1 - u/c}{\sqrt{1 - u^2/c^2}}, \\ \sqrt{1 - u^2/c^2} &= 3(1 - u/c), \\ c^2 - u^2 &= 9(c - u)^2, \\ 0 &= 10u^2 - 18uc + 8c^2. \end{aligned}$$

The solution is $u = 4c/5$.

E39-44 (a) $f_0/f = \lambda/\lambda_0$. This shift is *small*, so we apply the approximation:

$$u = c\left(\frac{\lambda_0}{\lambda} - 1\right) = (3\times10^8 \text{m/s})\left(\frac{(462 \text{ nm})}{(434 \text{ nm})} - 1\right) = 1.9\times10^7 \text{m/s}.$$

(b) A red shift corresponds to objects moving away from us.

$\boxed{\textbf{E39-45}}$ The sun rotates once every 26 days at the equator, while the radius is 7.0×10^8m. The speed of a point on the equator is then

$$v = \frac{2\pi R}{T} = \frac{2\pi(7.0\times10^8 \text{m})}{(2.2\times10^6 \text{s})} = 2.0\times10^3 \text{ m/s}.$$

This corresponds to a velocity parameter of

$$\beta = u/c = (2.0\times10^3 \text{ m/s})/(3.0\times10^8 \text{ m/s}) = 6.7\times10^{-6}.$$

This is a case of small numbers, so we'll use the formula that you derived in Exercise 39-40:

$$\Delta\lambda = \lambda\beta = (553 \text{ nm})(6.7\times10^{-6}) = 3.7\times10^{-3} \text{ nm}.$$

E39-46 Use Eq. 39-23 written as

$$(1 - u/c)\lambda^2 = \lambda_0^2(1 + u/c),$$

which can be rearranged as

$$u/c = \frac{\lambda^2 - \lambda_0^2}{\lambda^2 + \lambda_0^2} = \frac{(540\text{ nm})^2 - (620\text{ nm})^2}{(540\text{ nm})^2 + (620\text{ nm})^2} = -0.137.$$

The negative sign means that you should be going toward the red light.

E39-47 (a) $f_1 = cf/(c + v)$ and $f_2 = cf/(c - v)$.

$$\Delta f = (f_2 - f) - (f - f_1) = f_2 + f_1 - 2f,$$

so

$$\begin{aligned}
\frac{\Delta f}{f} &= \frac{c}{c+v} + \frac{c}{c-v} - 2, \\
&= \frac{2v^2}{c^2 - v^2}, \\
&= \frac{2(8.65\times 10^5\text{m/s})^2}{(3.00\times 10^8\text{m/s})^2 - (8.65\times 10^5\text{m/s})^2}, \\
&= 1.66\times 10^{-5}.
\end{aligned}$$

(b) $f_1 = f(c - u)/sqrtc^2 - u^2$ and $f_2 = f(c + u)/\sqrt{c^2 - u^2}$.

$$\Delta f = (f_2 - f) - (f - f_1) = f_2 + f_1 - 2f,$$

so

$$\begin{aligned}
\frac{\Delta f}{f} &= \frac{2c}{\sqrt{c^2 - u^2}} - 2, \\
&= \frac{2(3.00\times 10^8\text{m/s})}{\sqrt{(3.00\times 10^8\text{m/s})^2 - (8.65\times 10^5\text{m/s})^2}} - 2, \\
&= 8.3\times 10^{-6}.
\end{aligned}$$

E39-48 (a) No relative motion, so every 6 minutes.

(b) The Doppler effect at this speed is

$$\frac{1 - u/c}{\sqrt{1 - u^2/c^2}} = \frac{1 - (0.6)}{\sqrt{1 - (0.6)^2}} = 0.5;$$

this means the frequency is one half, so the period is doubled to 12 minutes.

(c) If C send the signal at the instant the signal from A passes, then the two signals travel together to C, so C would get B's signals at the same rate that it gets A's signals: every six minutes.

E39-49

E39-50 The transverse Doppler effect is $\lambda = \lambda_0/\sqrt{1 - u^2/c^2}$. Then

$$\lambda = (589.00\text{ nm})/\sqrt{1 - (0.122)^2} = 593.43\text{ nm}.$$

The shift is $(593.43\text{ nm}) - (589.00\text{ nm}) = 4.43\text{ nm}$.

E39-51 The frequency observed by the detector from the first source is (Eq. 39-31)

$$f = f_1\sqrt{1-(0.717)^2} = 0.697 f_1.$$

The frequency observed by the detector from the second source is (Eq. 39-30)

$$f = f_2\frac{\sqrt{1-(0.717)^2}}{1+(0.717)\cos\theta} = \frac{0.697 f_2}{1+(0.717)\cos\theta}.$$

We need to equate these and solve for θ. Then

$$\begin{aligned}
0.697 f_1 &= \frac{0.697 f_2}{1+0.717\cos\theta},\\
1+0.717\cos\theta &= f_2/f_1,\\
\cos\theta &= (f_2/f_1-1)/0.717,\\
\theta &= 101.1^\circ.
\end{aligned}$$

Subtract from 180° to find the angle with the line of sight.

E39-52

P39-1 Consider the triangle in Fig. 39-45. The true position corresponds to the speed of light, the opposite side corresponds to the velocity of earth in the orbit. Then

$$\theta = \arctan(29.8\times10^3\mathrm{m/s})/(3.00\times10^8\mathrm{m/s}) = 20.5''.$$

P39-2 The distance to Jupiter from point x is $d_x = r_j - r_e$. The distance to Jupiter from point y is

$$d_2 = \sqrt{r_e^2 + r_j^2}.$$

The difference in distance is related to the time according to

$$(d_2 - d_1)/t = c,$$

so

$$\frac{\sqrt{(778\times10^9\mathrm{m})^2 + (150\times10^9\mathrm{m})^2} - (778\times10^9\mathrm{m}) + (150\times10^9\mathrm{m})}{(600\,\mathrm{s})} = 2.7\times10^8\mathrm{m/s}.$$

P39-3 $\sin(30^\circ)/(4.0\,\mathrm{m}/s) = \sin\theta/(3.0\,\mathrm{m/s})$. Then $\theta = 22^\circ$. Water waves travel more slowly in shallower water, which means they always bend toward the normal as they approach land.

P39-4 (a) If the ray is normal to the water's surface then it passes into the water undeflected. Once in the water the problem is identical to Sample Problem 39-2. The reflected ray in the water is parallel to the incident ray in the water, so it also strikes the water normal, and is transmitted normal.

(b) Assume the ray strikes the water at an angle θ_1. It then passes into the water at an angle θ_2, where

$$n_w \sin\theta_2 = n_a \sin\theta_1.$$

Once the ray is in the water then the problem is identical to Sample Problem 39-2. The reflected ray in the water is parallel to the incident ray in the water, so it also strikes the water at an angle θ_2. When the ray travels back into the air it travels with an angle θ_3, where

$$n_w \sin\theta_2 = n_a \sin\theta_3.$$

Comparing the two equations yields $\theta_1 = \theta_3$, so the outgoing ray in the air is parallel to the incoming ray.

$\boxed{\textbf{P39-5}}$ (a) As was done in Ex. 39-25 above we use the small angle approximation of

$$\sin\theta \approx \theta \approx \tan\theta$$

The incident angle is θ; if the light were to go in a straight line we would expect it to strike a distance y_1 beneath the normal on the right hand side. The various distances are related to the angle by

$$\theta \approx \tan\theta \approx y_1/t.$$

The light, however, does *not* go in a straight line, it is refracted according to (the small angle approximation to) Snell's law, $n_1\theta_1 = n_2\theta_2$, which we will simplify further by letting $\theta_1 = \theta$, $n_2 = n$, and $n_1 = 1$, $\theta = n\theta_2$. The point where the refracted ray *does* strike is related to the angle by $\theta_2 \approx \tan\theta_2 = y_2/t$. Combining the three expressions,

$$y_1 = ny_2.$$

The difference, $y_1 - y_2$ is the vertical distance between the displaced ray and the original ray as measured on the plate glass. A little algebra yields

$$\begin{aligned} y_1 - y_2 &= y_1 - y_1/n, \\ &= y_1\left(1 - 1/n\right), \\ &= t\theta\frac{n-1}{n}. \end{aligned}$$

The perpendicular distance x is related to this difference by

$$\cos\theta = x/(y_1 - y_2).$$

In the small angle approximation $\cos\theta \approx 1 - \theta^2/2$. If θ is sufficiently small we can ignore the square term, and $x \approx y_2 - y_1$.

(b) Remember to use *radians* and not degrees whenever the small angle approximation is applied. Then

$$x = (1.0 \text{ cm})(0.175 \text{ rad})\frac{(1.52) - 1}{(1.52)} = 0.060 \text{ cm}.$$

P39-6 (a) At the top layer,

$$n_1 \sin\theta_1 = \sin\theta;$$

at the next layer,

$$n_2 \sin\theta_2 = n_1 \sin\theta_1;$$

at the next layer,

$$n_3 \sin\theta_3 = n_2 \sin\theta_2.$$

Combining all three expressions,

$$n_3 \sin\theta_3 = \sin\theta.$$

(b) $\theta_3 = \arcsin[sin(50^\circ)/(1.00029)] = 49.98^\circ$. Then shift is $(50^\circ) - (49.98^\circ) = 0.02^\circ$.

$\boxed{\textbf{P39-7}}$ The "big idea" of Problem 6 is that when light travels through layers the angle that it makes in any layer depends only on the incident angle, the index of refraction where that incident angle occurs, and the index of refraction at the current point.

That means that light which leaves the surface of the runway at 90° to the normal will make an angle

$$n_0 \sin 90^\circ = n_0(1 + ay)\sin\theta$$

at some height y above the runway. It is mildly entertaining to note that the value of n_0 is unimportant, only the value of a!

The expression

$$\sin\theta = \frac{1}{1+ay} \approx 1 - ay$$

can be used to find the angle made by the curved path against the normal as a function of y. The slope of the curve at any point is given by

$$\frac{dy}{dx} = \tan(90^\circ - \theta) = \cot\theta = \frac{\cos\theta}{\sin\theta}.$$

Now we need to know $\cos\theta$. It is

$$\cos\theta = \sqrt{1 - \sin^2\theta} \approx \sqrt{2ay}.$$

Combining

$$\frac{dy}{dx} \approx \frac{\sqrt{2ay}}{1 - ay},$$

and now we integrate. We will ignore the ay term in the denominator because it will always be small compared to 1. Then

$$\begin{aligned} \int_0^d dx &= \int_0^h \frac{dy}{\sqrt{2ay}}, \\ d &= \sqrt{\frac{2h}{a}} = \sqrt{\frac{2(1.7\,\mathrm{m})}{(1.5\times 10^{-6}\mathrm{m}^{-1})}} = 1500\,\mathrm{m}. \end{aligned}$$

P39-8 The energy of a particle is given by $E^2 = p^2c^2 + m^2c^4$. This energy is related to the mass by $E = \gamma mc^2$. γ is related to the speed by $\gamma = 1/\sqrt{1 - u^2/c^2}$. Rearranging,

$$\begin{aligned} \frac{u}{c} &= \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{m^2c^2}{p^2 + m^2c^2}}, \\ &= \sqrt{\frac{p^2}{p^2 + m^2c^2}}. \end{aligned}$$

Since $n = c/u$ we can write this as

$$n = \sqrt{1 + \frac{m^2c^2}{p^2}} = \sqrt{1 + \left(\frac{mc^2}{pc}\right)^2}.$$

For the pion,

$$n = \sqrt{1 + \left(\frac{(135\text{ MeV})}{(145\text{ MeV})}\right)^2} = 1.37.$$

For the muon,

$$n = \sqrt{1 + \left(\frac{(106\text{ MeV})}{(145\text{ MeV})}\right)^2} = 1.24.$$

P39-9 (a) Before adding the drop of liquid project the light ray along the angle θ so that $\theta = 0$. Increase θ slowly until total internal reflection occurs at angle θ_1. Then

$$n_{\rm g} \sin\theta_1 = 1$$

is the equation which can be solved to find $n_{\rm g}$.

Now put the liquid on the glass and repeat the above process until total internal reflection occurs at angle θ_2. Then

$$n_{\rm g} \sin\theta_2 = n_{\rm l}.$$

Note that $n_{\rm g} < n_{\rm g}$ for this method to work.

(b) This is not terribly practical.

P39-10 Let the internal angle at Q be θ_Q. Then $n\sin\theta_Q = 1$, because it is a critical angle. Let the internal angle at P be θ_P. Then $\theta_P + \theta_Q = 90^\circ$. Combine this with the other formula and

$$1 = n\sin(90 - \theta_P) = n\cos\theta_Q = n\sqrt{1-\sin^2\theta_P}.$$

Not only that, but $\sin\theta_1 = n\sin\theta_P$, or

$$1 = n\sqrt{1-(\sin\theta_1)^2/n^2},$$

which can be solved for n to yield

$$n = \sqrt{1+\sin^2\theta_1}.$$

(b) The largest value of the sine function is one, so $n_{\rm max} = \sqrt{2}$.

P39-11 (a) The fraction of light energy which escapes from the water is dependent on the critical angle. Light radiates in all directions from the source, but only that which strikes the surface at an angle less than the critical angle will escape. This critical angle is

$$\sin\theta_{\rm c} = 1/n.$$

We want to find the solid angle of the light which escapes; this is found by integrating

$$\Omega = \int_0^{2\pi}\int_0^{\theta_{\rm c}} \sin\theta\, d\theta\, d\phi.$$

This is *not* a hard integral to do. The result is

$$\Omega = 2\pi(1-\cos\theta_{\rm c}).$$

There are 4π steradians in a spherical surface, so the fraction which escapes is

$$f = \frac{1}{2}(1-\cos\theta_{\rm c}) = \frac{1}{2}(1-\sqrt{1-\sin^2\theta_{\rm c}}).$$

The last substitution is easy enough. We never needed to know the depth h.

(b) $f = \frac{1}{2}(1-\sqrt{1-(1/(1.3))^2}) = 0.18$.

P39-12 (a) The beam of light strikes the face of the fiber at an angle θ and is refracted according to

$$n_1 \sin\theta_1 = \sin\theta.$$

The beam then travels inside the fiber until it hits the cladding interface; it does so at an angle of $90^\circ - \theta_1$ to the normal. It will be reflected if it exceeds the critical angle of

$$n_1 \sin\theta_c = n_2,$$

or if

$$\sin(90^\circ - \theta_1) \geq n_2/n_1,$$

which can be written as

$$\cos\theta_1 \geq n_2/n_1.$$

but if this is the cosine, then we can use $\sin^2 + \cos^2 = 1$ to find the sine, and

$$\sin\theta_1 \leq \sqrt{1 - n_2^2/n_1^2}.$$

Combine this with the first equation and

$$\theta \leq \arcsin\sqrt{n_1^2 - n_2^2}.$$

(b) $\theta = \arcsin\sqrt{(1.58)^2 - (1.53)^2} = 23.2^\circ$.

$\boxed{\textbf{P39-13}}$ Consider the two possible extremes: a ray of light can propagate in a straight line directly down the axis of the fiber, or it can reflect off of the sides with the minimum possible angle of incidence. Start with the harder option.

The minimum angle of incidence that will still involve reflection is the critical angle, so

$$\sin\theta_c = \frac{n_2}{n_1}.$$

This light ray has farther to travel than the ray down the fiber axis because it is traveling at an angle. The distance traveled by this ray is

$$L' = L/\sin\theta_c = L\frac{n_1}{n_2},$$

The time taken for this bouncing ray to travel a length L down the fiber is then

$$t' = \frac{L'}{v} = \frac{L' n_1}{c} = \frac{L}{c}\frac{n_1^2}{n_2}.$$

Now for the easier ray. It travels straight down the fiber in a time

$$t = \frac{L}{c} n_1.$$

The difference is

$$t' - t = \Delta t = \frac{L}{c}\left(\frac{n_1^2}{n_2} - n_1\right) = \frac{L n_1}{c n_2}(n_1 - n_2).$$

(b) For the numbers in Problem 12 we have

$$\Delta t = \frac{(350\times 10^3\,\mathrm{m})(1.58)}{(3.00\times 10^8\mathrm{m/s})(1.53)}((1.58) - (1.53)) = 6.02\times 10^{-5}\mathrm{s}.$$

P39-14

P39-15 We can assume the airplane speed is small compared to the speed of light, and use Eq. 39-21. $\Delta f = 990$ Hz; so

$$|\Delta f| = f_0 u/c = u/\lambda_0,$$

hence $u = (990/\mathrm{s})(0.12\,\mathrm{m}) = 119\,\mathrm{m/s}$. The actual answer for the speed of the airplane is *half* this because there were two Doppler shifts: once when the microwaves struck the plane, and one when the reflected beam was received by the station. Hence, the plane approaches with a speed of $59.4\,\mathrm{m/s}$.

E40-1 (b) Since $i = -o$, $v_i = di/dt = -do/dt = -v_o$.

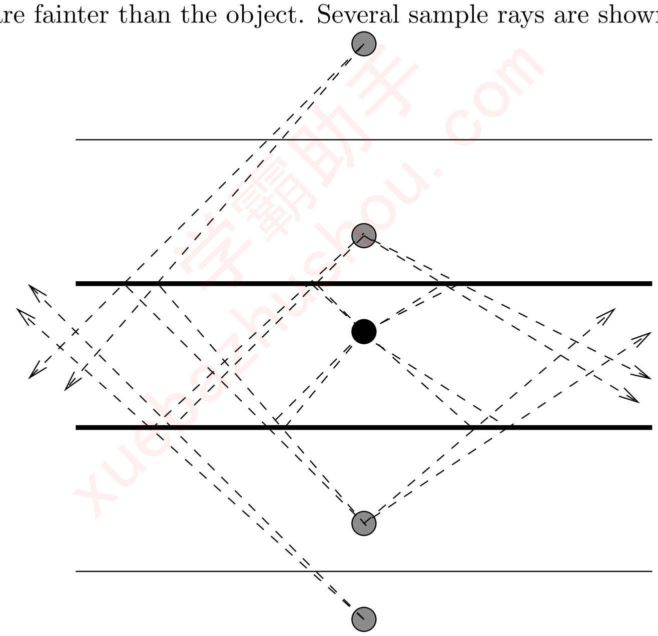
(a) In order to change from the frame of reference of the mirror to your own frame of reference you need to subtract v_o from all velocities. Then your velocity is $v_o - v_0 = 0$, the mirror is moving with velocity $0 - v_o = -v_o$ and your image is moving with velocity $-v_o - v_o = -2v_o$.

E40-2 You are 30 cm from the mirror, the image is 10 cm behind the mirror. You need to focus 40 cm away.

$\boxed{\textbf{E40-3}}$ If the mirror rotates through an angle α then the angle of incidence will increase by an angle α, and so will the angle of reflection. But that means that the angle between the incident angle and the reflected angle has increased by α twice.

E40-4 Sketch a line from Sarah through the right edge of the mirror and then beyond. Sarah can see any image which is located between that line and the mirror. By similar triangles, the image of Bernie will be $d/2 = (3.0\,\text{m})/2 = 1/5\,\text{m}$ from the mirror when it becomes visible. Since $i = -o$, Bernie will also be $1.5\,\text{m}$ from the mirror.

E40-5 The images are fainter than the object. Several sample rays are shown.



E40-6 The image is displaced. The eye would need to look up to see it.

$\boxed{\textbf{E40-7}}$ The apparent depth of the swimming pool is given by the work done for Exercise 39-25, $d_{\text{app}} = d/n$ The water then "appears" to be only 186 cm/1.33 = 140 cm deep. The apparent distance between the light and the mirror is then 250 cm + 140 cm = 390 cm; consequently the image of the light is 390 cm beneath the surface of the mirror.

E40-8 Three. There is a single direct image in each mirror and one more image of an image in one of the mirrors.

$\boxed{\textbf{E40-9}}$ We want to know over what surface area of the mirror are rays of light reflected from the object into the eye. By similar triangles the diameter of the pupil and the diameter of the part of the mirror (d) which reflects light into the eye are related by

$$\frac{d}{(10\text{ cm})} = \frac{(5.0\text{ mm})}{(24\text{ cm}) + (10\text{ cm})},$$

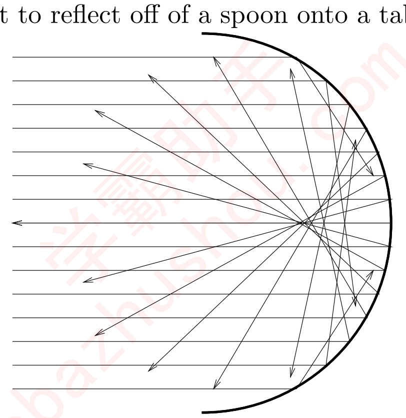
which has solution $d = 1.47$ mm The area of the circle on the mirror is

$$A = \pi(1.47\text{ mm})^2/4 = 1.7\text{ mm}^2.$$

E40-10 (a) Seven; (b) Five; and (c) Two. This is a problem of symmetry.

E40-11 Seven. Three images are the ones from Exercise 8. But each image has an image in the ceiling mirror. That would make a total of six, except that you also have an image in the ceiling mirror (look up, eh?). So the total is seven!

E40-12 A point focus is not formed. The envelope of rays is called the caustic. You can see a similar effect when you allow light to reflect off of a spoon onto a table.



$\boxed{\textbf{E40-13}}$ The image is magnified by a factor of 2.7, so the image distance is 2.7 times farther from the mirror than the object. An important question to ask is whether or not the image is real or virtual. If it is a virtual image it is behind the mirror and someone looking at the mirror could see it. If it were a real image it would be in front of the mirror, and the man, who serves as the object and is therefore closer to the mirror than the image, would not be able to see it.

So we shall assume that the image is virtual. The image distance is then a negative number. The focal length is half of the radius of curvature, so we want to solve Eq. 40-6, with $f = 17.5$ cm and $i = -2.7o$

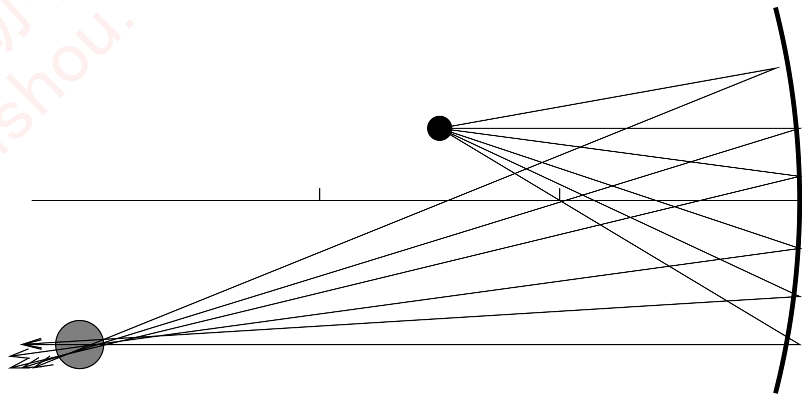
$$\frac{1}{(17.5\text{ cm})} = \frac{1}{o} + \frac{1}{-2.7o} = \frac{0.63}{o},$$

which has solution $o = 11$ cm.

E40-14 The image will be located at a point given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(10\text{ cm})} - \frac{1}{(15\text{ cm})} = \frac{1}{(30\text{ cm})}.$$

The vertical scale is three times the horizontal scale in the figure below.



E40-15 This problem requires repeated application of $1/f = 1/o + 1/i$, $r = 2f$, $m = -i/o$, or the properties of plane, convex, or concave mirrors. All dimensioned variables below (f, r, i, o) are measured in centimeters.

(a) Concave mirrors have positive focal lengths, so $f = +20$; $r = 2f = +40$;

$$1/i = 1/f - 1/o = 1/(20) - 1/(10) = 1/(-20);$$

$m = -i/o = -(-20)/(10) = 2$; the image is virtual and upright.

(b) $m = +1$ for plane mirrors only; $r = \infty$ for flat surface; $f = \infty/2 = \infty$; $i = -o = -10$; the image is virtual and upright.

(c) If f is positive the mirror is concave; $r = 2f = +40$;

$$1/i = 1/f - 1/o = 1/(20) - 1/(30) = 1/(60);$$

$m = -i/o = -(60)/(30) = -2$; the image is real and inverted.

(d) If m is negative then the image is real and inverted; only Concave mirrors produce real images (from real objects); $i = -mo = -(-0.5)(60) = 30$;

$$1/f = 1/o + 1/i = 1/(30) + 1/(60) = 1/(20);$$

$r = 2f = +40$.

(e) If r is negative the mirror is convex; $f = r/2 = (-40)/2 = -20$;

$$1/o = 1/f - 1/i = 1/(-20) - 1/(-10) = 1/(20);$$

$m = -(-10)/(20) = 0.5$; the image is virtual and upright.

(f) If m is positive the image is virtual and upright; if m is less than one the image is reduced, but only convex mirrors produce reduced virtual images (from real objects); $f = -20$ for convex mirrors; $r = 2f = -40$; let $i = -mo = -o/10$, then

$$1/f = 1/o + 1/i = 1/o - 10/o = -9/o,$$

so $o = -9f = -9(-20) = 180$; $i = -o/10 = -(180)/10 = -18$.

(g) r is negative for convex mirrors, so $r = -40$; $f = r/2 = -20$; convex mirrors produce only virtual upright images (from real objects); so i is negative; and

$$1/o = 1/f - 1/i = 1/(-20) - 1/(-4) = 1/(5);$$

$m = -i/o = -(-4)/(5) = 0.8$.

(h) Inverted images are real; only concave mirrors produce real images (from real objects); inverted images have negative m; $i = -mo = -(-0.5)(24) = 12$;

$$1/f = 1/o + 1/i = 1/(24) + 1/(12) = 1/(8);$$

$r = 2f = 16$.

E40-16 Use the angle definitions provided by Eq. 40-8. From triangle OaI we have

$$\alpha + \gamma = 2\theta,$$

while from triangle IaC we have

$$\beta + \theta = \gamma.$$

Combining to eliminate θ we get

$$\alpha - \gamma = -2\beta.$$

Substitute Eq. 40-8 and eliminate s,

$$\frac{1}{o} - \frac{1}{i} = -\frac{2}{r},$$

or

$$\frac{1}{o} + \frac{1}{-i} = \frac{2}{-r},$$

which is the same as Eq. 40-4 if $i \to -i$ and $r \to -r$.

$\boxed{\textbf{E40-17}}$ (a) Consider the point A. Light from this point travels along the line ABC and will be parallel to the horizontal center line from the center of the cylinder. Since the tangent to a circle defines the outer limit of the intersection with a line, this line must describe the apparent size.

(b) The angle of incidence of ray AB is given by

$$\sin\theta_1 = r/R.$$

The angle of refraction of ray BC is given by

$$\sin\theta_2 = r^*/R.$$

Snell's law, and a little algebra, yields

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2, \\ n_1 \frac{r}{R} &= n_2 \frac{r^*}{R}, \\ nr &= r^*. \end{aligned}$$

In the last line we used the fact that $n_2 = 1$, because it is in the air, and $n_1 = n$, the index of refraction of the glass.

E40-18 This problem requires repeated application of $(n_2 - n_1)/r = n_1/o + n_2/i$. All dimensioned variables below (r, i, o) are measured in centimeters.

(a)

$$\frac{(1.5) - (1.0)}{(30)} - \frac{(1.0)}{(10)} = -0.08333,$$

so $i = (1.5)/(-0.08333) = -18$, and the image is virtual.

(b)

$$\frac{(1.0)}{(10)} + \frac{(1.5)}{(-13)} = -0.015385,$$

so $r = (1.5 - 1.0)/(-0.015385) = -32.5$, and the image is virtual.

(c)

$$\frac{(1.5) - (1.0)}{(30)} - \frac{(1.5)}{(600)} = 0.014167,$$

so $o = (1.0)/(0.014167) = 71$. The image was real since $i > 0$.

(d) Rearrange the formula to solve for n_2, then

$$n_2 \left(\frac{1}{r} - 1i \right) = n_1 \left(\frac{1}{r} + \frac{1}{o} \right).$$

Substituting the numbers,

$$n_2 \left(\frac{1}{(-20)} - \frac{1}{(-20)} \right) = (1.0) \left(\frac{1}{(-20)} + \frac{1}{(20)} \right),$$

which has *any* solution for n_2! Since $i < 0$ the image is virtual.

(e)

$$\frac{(1.5)}{(10)} + \frac{(1.0)}{(-6)} = -0.016667,$$

so $r = (1.0 - 1.5)/(-0.016667) = 30$, and the image is virtual.

(f)

$$\frac{(1.0) - (1.5)}{(-30)} - \frac{(1.0)}{(-7.5)} = 0.15,$$

so $o = (1.5)/(0.15) = 10$. The image was virtual since $i < 0$.

(g)

$$\frac{(1.0) - (1.5)}{(30)} - \frac{(1.5)}{(70)} = -3.81 \times 10^{-2},$$

so $i = (1.0)/(-3.81 \times 10^{-2}) = -26$, and the image is virtual.

(h) Solving Eq. 40-10 for n_2 yields

$$n_2 = n_1 \frac{1/o + 1/r}{1/r - 1/i},$$

so

$$n_2 = (1.5) \frac{1/(100) + 1/(-30)}{1/(-30) - 1/(600)} = 1.0$$

and the image is real.

E40-19 (b) If the beam is small we can use Eq. 40-10. Parallel incoming rays correspond to an object at infinity. Solving for n_2 yields

$$n_2 = n_1 \frac{1/o + 1/r}{1/r - 1/i},$$

so if $o \to \infty$ and $i = 2r$, then

$$n_2 = (1.0) \frac{1/\infty + 1/r}{1/r - 1/2r} = 2.0$$

(c) There is no solution if $i = r$!

E40-20 The image will be located at a point given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(10 \text{ cm})} - \frac{1}{(6 \text{ cm})} = \frac{1}{(-15 \text{ cm})}.$$

E40-21 The image location can be found from Eq. 40-15,

$$\frac{1}{i}=\frac{1}{f}-\frac{1}{o}=\frac{1}{(-30\text{ cm})}-\frac{1}{(20\text{ cm})}=\frac{1}{-12\text{ cm}},$$

so the image is located 12 cm from the thin lens, *on the same side as the object.*

E40-22 For a double convex lens $r_1>0$ and $r_2<0$ (see Fig. 40-21 and the accompanying text). Then the problem states that $r_2=-r_1/2$. The lens maker's equation can be applied to get

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{3(n-1)}{r_1},$$

so $r_1=3(n-1)f=3(1.5-1)(60\,\text{mm})=90\,\text{mm}$, and $r_2=-45\,\text{mm}$.

E40-23 The object distance is essentially $o=\infty$, so $1/f=1/o+1/i$ implies $f=i$, and the image forms at the focal point. In reality, however, the object distance is not infinite, so the magnification is given by $m=-i/o\approx -f/o$, where o is the Earth/Sun distance. The size of the image is then

$$h_i=h_of/o=2(6.96\times10^8\text{m})(0.27\,\text{m})/(1.50\times10^{11}\text{m})=2.5\,\text{mm}.$$

The factor of two is because the sun's radius is given, and we need the diameter!

E40-24 (a) The flat side has $r_2=\infty$, so $1/f=(n-1)/r$, where r is the curved side. Then $f=(0.20\,\text{m})/(1.5-1)=0.40\,\text{m}$.

(b) $1/i=1/f-1/o=1/(0.40\,\text{m})-1/(0.40\,\text{m})=0$. Then i is ∞.

E40-25 (a) $1/f=(1.5-1)[1/(0.4\,\text{m})-1/(-0.4\,\text{m})]=1/(0.40\,\text{m})$.

(b) $1/f=(1.5-1)[1/(\infty)-1/(-0.4\,\text{m})]=1/(0.80\,\text{m})$.

(c) $1/f=(1.5-1)[1/(0.4\,\text{m})-1/(0.6\,\text{m})]=1/(2.40\,\text{m})$.

(d) $1/f=(1.5-1)[1/(-0.4\,\text{m})-1/(0.4\,\text{m})]=1/(-0.40\,\text{m})$.

(e) $1/f=(1.5-1)[1/(\infty)-1/(0.8\,\text{m})]=1/(-0.80\,\text{m})$.

(f) $1/f=(1.5-1)[1/(0.6\,\text{m})-1/(0.4\,\text{m})]=1/(-2.40\,\text{m})$.

E40-26 (a) $1/f=(n-1)[1/(-r)-1/r]$, so $1/f=2(1-n)/r$. $1/i=1/f-1/o$ so if $o=r$, then

$$1/i=2(1-n)/r-1/r=(1-2n)/r,$$

so $i=r/(1-2n)$. For $n>0.5$ the image is virtual.

(b) For $n>0.5$ the image is virtual; the magnification is

$$m=-i/o=-r/(1-2n)/r=1/(2n-1).$$

E40-27 According to the definitions, $o=f+x$ and $i=f+x'$. Starting with Eq. 40-15,

$$\begin{aligned}\frac{1}{o}+\frac{1}{i}&=\frac{1}{f},\\ \frac{i+o}{oi}&=\frac{1}{f},\\ \frac{2f+x+x'}{(f+x)(f+x')}&=\frac{1}{f},\\ 2f^2+fx+fx'&=f^2+fx+fx'+xx',\\ f^2&=xx'.\end{aligned}$$

E40-28 (a) You can't determine r_1, r_2, or n. i is found from

$$\frac{1}{i} = \frac{1}{+10} - \frac{1}{+20} = \frac{1}{+20},$$

the image is real and inverted. $m = -(20)/(20) = -1$.

(b) You can't determine r_1, r_2, or n. The lens is converging since f is positive. i is found from

$$\frac{1}{i} = \frac{1}{+10} - \frac{1}{+5} = \frac{1}{-10},$$

the image is virtual and upright. $m = -(-10)/(+5) = 2$.

(c) You can't determine r_1, r_2, or n. Since m is positive and greater than one the lens is converging. Then f is positive. i is found from

$$\frac{1}{i} = \frac{1}{+10} - \frac{1}{+5} = \frac{1}{-10},$$

the image is virtual and upright. $m = -(-10)/(+5) = 2$.

(d) You can't determine r_1, r_2, or n. Since m is positive and less than one the lens is diverging. Then f is negative. i is found from

$$\frac{1}{i} = \frac{1}{-10} - \frac{1}{+5} = \frac{1}{-3.3},$$

the image is virtual and upright. $m = -(-3.3)/(+5) = 0.66$.

(e) f is found from

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{+30} - \frac{1}{-30}\right) = \frac{1}{+30}.$$

The lens is converging. i is found from

$$\frac{1}{i} = \frac{1}{+30} - \frac{1}{+10} = \frac{1}{-15},$$

the image is virtual and upright. $m = -(-15)/(+10) = 1.5$.

(f) f is found from

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-30} - \frac{1}{+30}\right) = \frac{1}{-30}.$$

The lens is diverging. i is found from

$$\frac{1}{i} = \frac{1}{-30} - \frac{1}{+10} = \frac{1}{-7.5},$$

the image is virtual and upright. $m = -(-7.5)/(+10) = 0.75$.

(g) f is found from

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-30} - \frac{1}{-60}\right) = \frac{1}{-120}.$$

The lens is diverging. i is found from

$$\frac{1}{i} = \frac{1}{-120} - \frac{1}{+10} = \frac{1}{-9.2},$$

the image is virtual and upright. $m = -(-9.2)/(+10) = 0.92$.

(h) You can't determine r_1, r_2, or n. Upright images have positive magnification. i is found from

$$i = -(0.5)(10) = -5;$$

f is found from

$$\frac{1}{f} = \frac{1}{+10} + \frac{1}{-5} = \frac{1}{-10},$$

so the lens is diverging.

(h) You can't determine r_1, r_2, or n. Real images have negative magnification. i is found from

$$i = -(-0.5)(10) = 5;$$

f is found from

$$\frac{1}{f} = \frac{1}{+10} + \frac{1}{5} = \frac{1}{+3.33},$$

so the lens is converging.

E40-29 $o + i = 0.44\,\text{m} = L$, so

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} = \frac{1}{o} + \frac{1}{L-o} = \frac{L}{o(L-o)},$$

which can also be written as $o^2 - oL + fL = 0$. This has solution

$$o = \frac{L \pm \sqrt{L^2 - 4fL}}{2} = \frac{(0.44\,\text{m}) \pm \sqrt{(0.44\,\text{m}) - 4(0.11\,\text{m})(0.44\,\text{m})}}{2} = 0.22\,\text{m}.$$

There is only one solution to this problem, but sometimes there are two, and other times there are none!

E40-30 (a) Real images (from real objects) are only produced by converging lenses.

(b) Since $h_i = -h_0/2$, then $i = o/2$. But $d = i + o = o + o/2 = 3o/2$, so $o = 2(0.40\,\text{m})/3 = 0.267\,\text{m}$, and $i = 0.133\,\text{m}$.

(c) $1/f = 1/o + 1/i = 1/(0.267\,\text{m}) + 1/(0.133\,\text{m}) = 1/(0.0889\,\text{m})$.

$\boxed{\textbf{E40-31}}$ Step through the exercise one lens at a time. The object is 40 cm to the left of a converging lens with a focal length of +20 cm. The image from this first lens will be located by solving

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(20\text{ cm})} - \frac{1}{(40\text{ cm})} = \frac{1}{40\text{ cm}},$$

so $i = 40$ cm. Since i is positive it is a real image, and it is located to the right of the converging lens. This image becomes the object for the diverging lens.

The image from the converging lens is located 40 cm - 10 cm from the diverging lens, but it is located on the wrong side: the diverging lens is "in the way" so the rays which would form the image hit the diverging lens before they have a chance to form the image. That means that the real image from the converging lens is a *virtual* object in the diverging lens, so that the object distance for the diverging lens is $o = -30$ cm.

The image formed by the diverging lens is located by solving

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(-15\text{ cm})} - \frac{1}{(-30\text{ cm})} = \frac{1}{-30\text{ cm}},$$

or $i = -30$ cm. This would mean the image formed by the diverging lens would be a virtual image, and would be located to the left of the diverging lens.

The image is virtual, so it is upright. The magnification from the first lens is

$$m_1 = -i/o = -(40\text{ cm})/(40\text{ cm})) = -1;$$

the magnification from the second lens is

$$m_2 = -i/o = -(-30 \text{ cm})/(-30 \text{ cm})) = -1;$$

which implies an overall magnification of $m_1 m_2 = 1$.

E40-32 (a) The parallel rays of light which strike the lens of focal length f will converge on the focal point. This point will act like an object for the second lens. If the second lens is located a distance L from the first then the object distance for the second lens will be $L - f$. Note that this will be a negative value for $L < f$, which means the object is virtual. The image will form at a point

$$1/i = 1/(-f) - 1/(L - f) = L/f(f - L).$$

Note that i will be positive if $L < f$, so the rays really do converge on a point.

(b) The same equation applies, except switch the sign of f. Then

$$1/i = 1/(f) - 1/(L - f) = L/f(L - f).$$

This is *negative* for $L < f$, so there is no real image, and no converging of the light rays.

(c) If $L = 0$ then $i = \infty$, which means the rays coming from the second lens are parallel.

E40-33 The image from the converging lens is found from

$$\frac{1}{i_1} = \frac{1}{(0.58\,\text{m})} - \frac{1}{(1.12\,\text{m})} = \frac{1}{1.20\,\text{m}}$$

so $i_1 = 1.20\,\text{m}$, and the image is real and inverted.

This real image is $1.97\,\text{m} - 1.20\,\text{m} = 0.77\,\text{m}$ in front of the plane mirror. It acts as an object for the mirror. The mirror produces a virtual image $0.77\,\text{m}$ behind the plane mirror. This image is upright relative to the object which formed it, which was inverted relative to the original object.

This second image is $1.97\,\text{m} + 0.77\,\text{m} = 2.74\,\text{m}$ away from the lens. This second image acts as an object for the lens, the image of which is found from

$$\frac{1}{i_3} = \frac{1}{(0.58\,\text{m})} - \frac{1}{(2.74\,\text{m})} = \frac{1}{0.736\,\text{m}}$$

so $i_3 = 0.736\,\text{m}$, and the image is real and inverted relative to the object which formed it, which was inverted relative to the original object. So this image is actually upright.

E40-34 (a) The first lens forms a real image at a location given by

$$1/i = 1/f - 1/o = 1/(0.1\,\text{m}) - 1/(0.2\,\text{m}) = 1/(0.2\,\text{m}).$$

The image and object distance are the same, so the image has a magnification of 1. This image is $0.3\,\text{m} - 0.2\,\text{m} = 0.1\,\text{m}$ from the second lens. The second lens forms an image at a location given by

$$1/i = 1/f - 1/o = 1/(0.125\,\text{m}) - 1/(0.1\,\text{m}) = 1/(-0.5\,\text{m}).$$

Note that this puts the final image at the location of the original object! The image is magnified by a factor of $(0.5\,\text{m})/(0.1\,\text{m}) = 5$.

(c) The image is virtual, but inverted.

E40-35 If the two lenses "pass" the same amount of light then the solid angle subtended by each lens as seen from the respective focal points must be the same. If we assume the lenses have the same round shape then we can write this as $d_o/f_o = d_e/f_e$. Then

$$\frac{d_e}{d_o} = \frac{f_o}{f_e} = m_\theta,$$

or $d_e = (72\,\text{mm})/36 = 2\,\text{mm}$.

E40-36 (a) $f = (0.25\,\text{m})/(200) \approx 1.3\,\text{mm}$. Then $1/f = (n-1)(2/r)$ can be used to find r; $r = 2(n-1)f = 2(1.5-1)(1.3\,\text{mm}) = 1.3\,\text{mm}$.

(b) The diameter would be twice the radius. In effect, these were tiny glass balls.

E40-37 (a) In Fig. 40-46(a) the image is at the focal point. This means that in Fig. 40-46(b) $i = f = 2.5$ cm, even though $f' \neq f$. Solving,

$$\frac{1}{f} = \frac{1}{(36\text{ cm})} + \frac{1}{(2.5\text{ cm})} = \frac{1}{2.34\text{ cm}}.$$

(b) The effective radii of curvature must have decreased.

E40-38 (a) $s = (25\text{ cm}) - (4.2\text{ cm}) - (7.7\text{ cm}) = 13.1$ cm.

(b) $i = (25\text{ cm}) - (7.7\text{ cm}) = 17.3$ cm. Then

$$\frac{1}{o} = \frac{1}{(4.2\text{ cm})} - \frac{1}{(17.3\text{ cm})} = \frac{1}{5.54\text{ cm.}}$$

The object should be placed $5.5 - 4.2 = 1.34$ cm beyond F_1.

(c) $m = -(17.3)/(5.5) = -3.1$.

(d) $m_\theta = (25\text{ cm})/(7.7\text{ cm}) = 3.2$.

(e) $M = mm_\theta = -10$.

E40-39 Microscope magnification is given by Eq. 40-33. We need to first find the focal length of the objective lens before we can use this formula. We are told in the text, however, that the microscope is constructed so the at the object is placed just beyond the focal point of the objective lens, then $f_{ob} \approx 12.0$ mm. Similarly, the intermediate image is formed at the focal point of the eyepiece, so $f_{ey} \approx 48.0$ mm. The magnification is then

$$m = \frac{-s(250\text{ mm})}{f_{ob}f_{ey}} = -\frac{(285\text{ mm})(250\text{ mm})}{(12.0\text{ mm})(48.0\text{ mm})} = 124.$$

A more accurate answer can be found by calculating the *real* focal length of the objective lens, which is 11.4 mm, but since there is a *huge* uncertainty in the near point of the eye, I see no point in trying to be more accurate than this.

P40-1 The old intensity is $I_o = P/4\pi d^2$, where P is the power of the point source. With the mirror in place there is an additional amount of light which needs to travel a total distance of $3d$ in order to get to the screen, so it contributes an additional $P/4\pi(3d)^2$ to the intensity. The new intensity is then

$$I_n = P/4\pi d^2 + P/4\pi(3d)^2 = (10/9)P/4\pi d^2 = (10/9)I_o.$$

P40-2 (a) $v_i = di/dt$; but $i = fo/(o-f)$ and $f = r/2$ so

$$v_i = \frac{d}{dt}\left(\frac{ro}{2o-r}\right) = -\left(\frac{r}{2o-r}\right)^2 \frac{do}{dt} = -\left(\frac{r}{2o-r}\right)^2 v_o.$$

(b) Put in the numbers!

$$v_i = -\left(\frac{(15\text{ cm})}{2(75\text{ cm}) - (15\text{ cm})}\right)^2 (5.0\text{ cm/s}) = -6.2\times10^{-2}\text{ cm/s}.$$

(c) Put in the numbers!

$$v_i = -\left(\frac{(15\text{ cm})}{2(7.7\text{ cm}) - (15\text{ cm})}\right)^2 (5.0\text{ cm/s}) = -70\,\text{m/s}$$

(d) Put in the numbers!

$$v_i = -\left(\frac{(15\text{ cm})}{2(0.15\text{ cm}) - (15\text{ cm})}\right)^2 (5.0\text{ cm/s}) = -5.2\text{ cm/s}.$$

$\boxed{\textbf{P40-3}}$ (b) There are two ends to the object of length L, one of these ends is a distance o_1 from the mirror, and the other is a distance o_2 from the mirror. The images of the two ends will be located at i_1 and i_2.

Since we are told that the object has a short length L we will assume that a differential approach to the problem is in order. Then

$$L = \Delta o = o_1 - o_2 \text{ and } L' = \Delta i = i_1 - i_2,$$

Finding the ratio of L'/L is then reduced to

$$\frac{L'}{L} = \frac{\Delta i}{\Delta o} \approx \frac{di}{do}.$$

We can take the derivative of Eq. 40-15 with respect to changes in o and i,

$$\frac{di}{i^2} + \frac{do}{o^2} = 0,$$

or

$$\frac{L'}{L} \approx \frac{di}{do} = -\frac{i^2}{o^2} = -m^2,$$

where m is the lateral magnification.

(a) Since i is given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{o-f}{of},$$

the fraction i/o can also be written

$$\frac{i}{o} = \frac{of}{o(o-f)} = \frac{f}{o-f}.$$

Then

$$L \approx -\frac{i^2}{o^2} = -\left(\frac{f}{o-f}\right)^2$$

P40-4 The left surface produces an image which is found from $n/i = (n-1)/R - 1/o$, but since the incoming rays are parallel we take $o = \infty$ and the expression simplifies to $i = nR/(n-1)$. This image is located a distance $o = 2R - i = (n-2)R/(n-1)$ from the right surface, and the image produced by this surface can be found from

$$1/i = (1-n)/(-R) - n/o = (n-1)/R - n(n-1)/(n-2)R = 2(1-n)/(n-2)R.$$

Then $i = (n-2)R/2(n-1)$.

P40-5 The "1" in Eq. 40-18 is actually n_{air}; the assumption is that the thin lens is in the air. If that isn't so, then we need to replace "1" with n', so Eq. 40-18 becomes

$$\frac{n'}{o} - \frac{n}{|i'|} = \frac{n-n'}{r_1}.$$

A similar correction happens to Eq. 40-21:

$$\frac{n}{|i'|} + \frac{n'}{i} = -\frac{n-n'}{r_2}.$$

Adding these two equations,

$$\frac{n'}{o} + \frac{n'}{i} = (n-n')\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

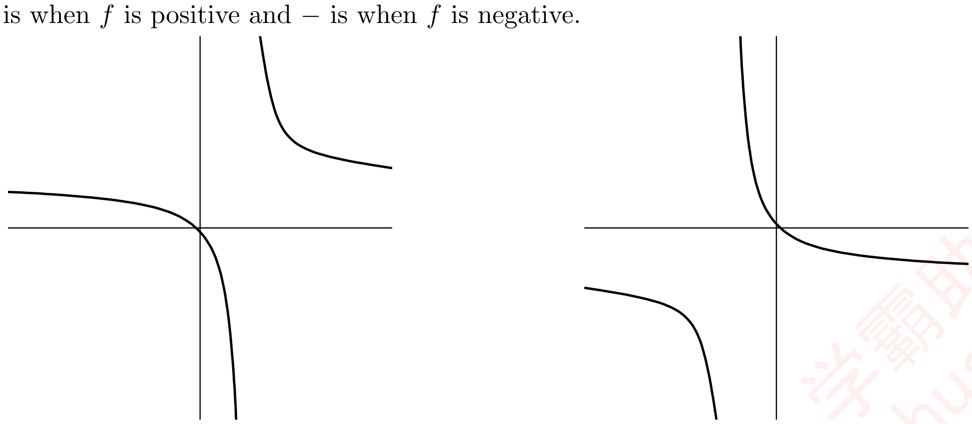
This yields a focal length given by

$$\frac{1}{f} = \frac{n-n'}{n}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

P40-6 Start with Eq. 40-4

$$\begin{aligned}
\frac{1}{o} + \frac{1}{i} &= \frac{1}{f}, \\
\frac{|f|}{o} + \frac{|f|}{i} &= \frac{|f|}{f}, \\
\frac{1}{y} + \frac{1}{y'} &= \pm 1,
\end{aligned}$$

where $+$ is when f is positive and $-$ is when f is negative.



The plot on the right is for $+$, that on the left for $-$.
Real image and objects occur when y or y' is positive.

$\boxed{\textbf{P40-7}}$ (a) The image (which will appear on the screen) and object are a distance $D = o + i$ apart. We can use this information to eliminate one variable from Eq. 40-15,

$$\begin{aligned}\frac{1}{o} + \frac{1}{i} &= \frac{1}{f},\\ \frac{1}{o} + \frac{1}{D-o} &= \frac{1}{f},\\ \frac{D}{o(D-o)} &= \frac{1}{f},\\ o^2 - oD + fD &= 0.\end{aligned}$$

This last expression is a quadratic, and we would expect to get two solutions for o. These solutions will be of the form "something" plus/minus "something else"; the distance between the two locations for o will evidently be twice the "something else", which is then

$$d = o_+ - o_- = \sqrt{(-D)^2 - 4(fD)} = \sqrt{D(D-4f)}.$$

(b) The ratio of the image sizes is m_+/m_-, or i_+o_-/i_-o_+. Now it seems we must find the actual values of o_+ and o_-. From the quadratic in part (a) we have

$$o_\pm = \frac{D \pm \sqrt{D(D-4f)}}{2} = \frac{D \pm d}{2},$$

so the ratio is

$$\frac{o_-}{o_+} = \left(\frac{D-d}{D+d}\right).$$

But $i_- = o_+$, and vice-versa, so the ratio of the image sizes is this quantity squared.

P40-8 $1/i = 1/f - 1/o$ implies $i = fo/(o-f)$. i is only real if $o \geq f$. The distance between the image and object is

$$y = i + o = \frac{of}{o-f} + o = \frac{o^2}{o-f}.$$

This quantity is a minimum when $dy/do = 0$, which occurs when $o = 2f$. Then $i = 2f$, and $y = 4f$.

P40-9 (a) The angular size of each lens is the same when viewed from the *shared* focal point. This means $W_1/f_1 = W_2/f_2$, or

$$W_2 = (f_2/f_1)W_1.$$

(b) Pass the light through the diverging lens first; choose the separation of the lenses so that the focal point of the converging lens is at the same location as the focal point of the diverging lens which is on the opposite side of the diverging lens.

(c) Since $I \propto 1/A$, where A is the area of the beam, we have $I \propto 1/W^2$. Consequently,

$$I_2/I_1 = (W_1/W_2)^2 = (f_1/f_2)^2$$

P40-10 The location of the image in the mirror is given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{a+b}.$$

The location of the image in the plate is given by $i' = -a$, which is located at $b - a$ relative to the mirror. Equating,

$$\begin{aligned}
\frac{1}{b-a} + \frac{1}{b+a} &= \frac{1}{f}, \\
\frac{2b}{b^2 - a^2} &= \frac{1}{f}, \\
b^2 - a^2 &= 2bf, \\
a &= \sqrt{b^2 - 2bf}, \\
&= \sqrt{(7.5 \text{ cm})^2 - 2(7.5 \text{ cm})(-28.2 \text{ cm})} = 21.9 \text{ cm}.
\end{aligned}$$

$\boxed{\textbf{P40-11}}$ We'll solve the problem by finding out what happens if you put an object in front of the combination of lenses.

Let the object distance be o_1. The first lens will create an image at i_1, where

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{o_1}$$

This image will act as an object for the second lens.

If the first image is real (i_1 positive) then the image will be on the "wrong" side of the second lens, and as such the real image will act like a virtual object. In short, $o_2 = -i_1$ will give the correct sign to the object distance when the image from the first lens acts like an object for the second lens. The image formed by the second lens will then be at

$$\begin{aligned}
\frac{1}{i_2} &= \frac{1}{f_2} - \frac{1}{o_2}, \\
&= \frac{1}{f_2} + \frac{1}{i_2}, \\
&= \frac{1}{f_2} + \frac{1}{f_1} - \frac{1}{o_1}.
\end{aligned}$$

In this case it appears as if the combination

$$\frac{1}{f_2} + \frac{1}{f_1}$$

is equivalent to the reciprocal of a focal length. We will go ahead and make this connection, and

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1} = \frac{f_1 + f_2}{f_1 f_2}.$$

The rest is straightforward enough.

P40-12 (a) The image formed by the first lens can be found from

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{2f_1} = \frac{1}{2f_1}.$$

This is a distance $o_2 = 2(f_1 + f_2) = 2f_2$ from the mirror. The image formed by the mirror is at an image distance given by

$$\frac{1}{i_2} = \frac{1}{f_2} - \frac{1}{2f_2} = \frac{1}{2f_2}.$$

Which is at the same point as i_1!. This means it will act as an object o_3 in the lens, and, reversing the first step, produce a final image at O, the location of the original object. There are then three images formed; each is real, same size, and inverted. Three inversions nets an inverted image. The final image at O is therefore inverted.

P40-13 (a) Place an object at o. The image will be at a point i' given by

$$\frac{1}{i'} = \frac{1}{f} - \frac{1}{o},$$

or $i' = fo/(o - f)$.

(b) The lens must be shifted a distance $i' - i$, or

$$i' - i = \frac{fo}{o - f} - 1.$$

(c) The range of motion is

$$\Delta i = \frac{(0.05\,\text{m})(1.2\,\text{m})}{(1.2\,\text{m}) - (0.05\,\text{m})} - 1 = -5.2 \text{ cm}.$$

P40-14 (a) Because magnification is proportional to $1/f$.

(b) Using the results of Problem 40-11,

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1},$$

so $P = P_1 + P_2$.

$\boxed{\textbf{P40-15}}$ We want the maximum linear motion of the train to move no more than 0.75 mm on the film; this means we want to find the size of an object on the train that will form a 0.75 mm image. The object distance is much larger than the focal length, so the image distance is approximately equal to the focal length. The magnification is then $m = -i/o = (3.6 \text{ cm})/(44.5\,\text{m}) = -0.00081$.

The size of an object on the train that would produce a 0.75 mm image on the film is then $0.75 \text{ mm}/0.00081 = 0.93$ m.

How much time does it take the train to move that far?

$$t = \frac{(0.93 \text{ m})}{(135 \text{ km/hr})(1/3600 \text{ hr/s})} = 25 \text{ ms}.$$

P40-16 (a) The derivation leading to Eq. 40-34 depends only on the fact that two converging optical devices are used. Replacing the objective lens with an objective mirror doesn't change anything except the ray diagram.

(b) The image will be located very close to the focal point, so $|m| \approx f/o$, and

$$h_\text{i} = (1.0\,\text{m})\frac{(16.8\,\text{m})}{(2000\,\text{m})} = 8.4\times 10^{-3}\text{m}$$

(c) $f_\text{e} = (5\,\text{m})/(200) = 0.025\,\text{m}$. Note that we were given the radius of curvature, not the focal length, of the mirror!

$\boxed{\textbf{E41-1}}$ In this problem we look for the location of the third-order bright fringe, so

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(3)(554\times10^{-9}\text{m})}{(7.7\times10^{-6}\text{m})} = 12.5^\circ = 0.22 \text{ rad}.$$

E41-2 $d_1 \sin\theta = \lambda$ gives the first maximum; $d_2\sin\theta = 2\lambda$ puts the second maximum at the location of the first. Divide the second expression by the first and $d_2 = 2d_1$. This is a 100% increase in d.

E41-3 $\Delta y = \lambda D/d = (512\times10^{-9}\text{m})(5.4\,\text{m})/(1.2\times10^{-3}\text{m}) = 2.3\times10^{-3}\text{m}$.

E41-4 $d = \lambda/\sin\theta = (592\times10^{-9}\text{m})/\sin(1.00^\circ) = 3.39\times10^{-5}\text{m}$.

$\boxed{\textbf{E41-5}}$ Since the angles are *very* small, we can assume $\sin\theta\approx\theta$ for angles measured in radians.

If the interference fringes are 0.23° apart, then the angular position of the first bright fringe is 0.23° away from the central maximum. Eq. 41-1, written with the small angle approximation in mind, is $d\theta = \lambda$ for this first ($m = 1$) bright fringe. The goal is to find the wavelength which increases θ by 10%. To do this we must increase the right hand side of the equation by 10%, which means increasing λ by 10%. The new wavelength will be $\lambda' = 1.1\lambda = 1.1(589 \text{ nm}) = 650$ nm

E41-6 Immersing the apparatus in water will shorten the wavelengths to λ/n. Start with $d\sin\theta_0 = \lambda$; and then find θ from $d\sin\theta = \lambda/n$. Combining the two expressions,

$$\theta = \arcsin[\sin\theta_0/n] = \arcsin[\sin(0.20^\circ)/(1.33)] = 0.15^\circ.$$

E41-7 The third-order fringe for a wavelength λ will be located at $y = 3\lambda D/d$, where y is measured from the central maximum. Then Δy is

$$y_1 - y_2 = 3(\lambda_1-\lambda_2)D/d = 3(612\times10^{-9}\text{m} - 480\times10^{-9}\text{m})(1.36\,\text{m})/(5.22\times10^{-3}\text{m}) = 1.03\times10^{-4}\text{m}.$$

E41-8 $\theta = \arctan(y/D)$;

$$\lambda = d\sin\theta = (0.120\,\text{m})\sin[\arctan(0.180\,\text{m}/2.0\,\text{m})] = 1.08\times10^{-2}\text{m}.$$

Then $f = v/\lambda = (0.25\,\text{m/s})/(1.08\times10^{-2}\text{m}) = 23$ Hz.

$\boxed{\textbf{E41-9}}$ A variation of Eq. 41-3 is in order:

$$y_m = \left(m+\frac{1}{2}\right)\frac{\lambda D}{d}$$

We are given the distance (on the screen) between the first minima ($m = 0$) and the tenth minima ($m = 9$). Then

$$18\text{ mm} = y_9 - y_0 = 9\frac{\lambda(50\text{ cm})}{(0.15\text{ mm})},$$

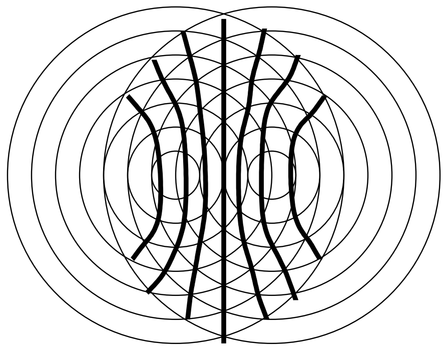
or $\lambda = 6\times10^{-4}$ mm $= 600$ nm.

E41-10 The "maximum" maxima is given by the integer part of

$$m = d\sin(90^\circ)/\lambda = (2.0\,\text{m})/(0.50\,\text{m}) = 4.$$

Since there is no integer part, the "maximum" maxima occurs at 90°. These are point sources radiating in both directions, so there are two central maxima, and four maxima each with $m = 1$, $m = 2$, and $m = 3$. But the $m = 4$ values overlap at 90°, so there are only two. The total is 16.

$\boxed{\textbf{E41-11}}$ This figure should explain it well enough.



E41-12 $\Delta y = \lambda D/d = (589\times10^{-9}\text{m})(1.13\,\text{m})/(0.18\times10^{-3}\text{m}) = 3.70\times10^{-3}\text{m}$.

E41-13 Consider Fig. 41-5, and solve it *exactly* for the information given. For the tenth bright fringe $r_1 = 10\lambda + r_2$. There are two important triangles:

$$r_2^2 = D^2 + (y - d/2)^2$$

and

$$r_1^2 = D^2 + (y + d/2)^2$$

Solving to eliminate r_2,

$$\sqrt{D^2 + (y + d/2)^2} = \sqrt{D^2 + (y - d/2)^2} + 10\lambda.$$

This has solution

$$y = 5\lambda\sqrt{\frac{4D^2 + d^2 - 100\lambda^2}{d^2 - 100\lambda^2}}.$$

The solution predicted by Eq. 41-1 is

$$y' = \frac{10\lambda}{d}\sqrt{D^2 + y'^2},$$

or

$$y' = 5\lambda\sqrt{\frac{4D^2}{d^2 - 100\lambda^2}}.$$

The fractional error is $y'/y - 1$, or

$$\sqrt{\frac{4D^2}{4D^2 + d^2 - 100\lambda^2}} - 1,$$

or

$$\sqrt{\frac{4(40\,\text{mm})^2}{4(40\,\text{mm})^2 + (2\,\text{mm})^2 - 100(589\times10^{-6}\text{mm})^2}} - 1 = -3.1\times10^{-4}.$$

E41-14 (a) $\Delta x = c/\Delta t = (3.00\times10^8\text{m/s})/(1\times10^{-8}\text{s}) = 3\,\text{m}$.
(b) No.

$\boxed{\textbf{E41-15}}$ Leading by 90° is the same as leading by a quarter wavelength, since there are 360° in a circle. The distance from A to the detector is 100 m longer than the distance from B to the detector. Since the wavelength is 400 m, 100 m corresponds to a quarter wavelength.

So a wave peak starts out from source A and travels to the detector. When it has traveled a quarter wavelength a wave peak leaves source B. But when the wave peak from A has traveled a quarter wavelength it is now located at the same distance from the detector as source B, which means the two wave peaks arrive at the detector at the same time.

They are in phase.

E41-16 The first dark fringe involves waves π radians out of phase. Each dark fringe after that involves an additional 2π radians of phase difference. So the mth dark fringe has a phase difference of $(2m+1)\pi$ radians.

E41-17 $I = 4I_0 \cos^2\left(\frac{2\pi d}{\lambda}\sin\theta\right)$, so for this problem we want to plot

$$I/I_0 = \cos^2\left(\frac{2\pi(0.60\,\mathrm{mm})}{(600\times 10^{-9}\mathrm{m})}\sin\theta\right) = \cos^2\left(6280\sin\theta\right).$$

E41-18 The resultant quantity will be of the form $A\sin(\omega t+\beta)$. Solve the problem by looking at $t=0$; then $y_1 = 0$, but $x_1 = 10$, and $y_2 = 8\sin 30° = 4$ and $x_2 = 8\cos 30 = 6.93$. Then the resultant is of length

$$A = \sqrt{(4)^2 + (10+6.93)^2} = 17.4,$$

and has an angle β given by

$$\beta = \arctan(4/16.93) = 13.3°.$$

$\boxed{\textbf{E41-19}}$ (a) We want to know the path length difference of the two sources to the detector. Assume the detector is at x and the second source is at $y = d$. The distance S_1D is x; the distance S_2D is $\sqrt{x^2+d^2}$. The difference is $\sqrt{x^2+d^2} - x$. If this difference is an integral number of wavelengths then we have a maximum; if instead it is a half integral number of wavelengths we have a minimum. For part (a) we are looking for the maxima, so we set the path length difference equal to $m\lambda$ and solve for x_m.

$$\begin{aligned}
\sqrt{x_m^2+d^2} - x_m &= m\lambda,\\
x_m^2 + d^2 &= (m\lambda + x_m)^2,\\
x_m^2 + d^2 &= m^2\lambda^2 + 2m\lambda x_m + x_m^2,\\
x_m &= \frac{d^2 - m^2\lambda^2}{2m\lambda}
\end{aligned}$$

The first question we need to ask is what happens when $m = 0$. The right hand side becomes indeterminate, so we need to go back to the first line in the above derivation. If $m = 0$ then $d^2 = 0$; since this is *not* true in this problem, there is no $m = 0$ solution.

In fact, we may have even more troubles. x_m needs to be a positive value, so the maximum allowed value for m will be given by

$$\begin{aligned}
m^2\lambda^2 &< d^2,\\
m &< d/\lambda = (4.17\,\mathrm{m})/(1.06\,\mathrm{m}) = 3.93;
\end{aligned}$$

but since m is an integer, $m = 3$ is the maximum value.

The first three maxima occur at $m = 3$, $m = 2$, and $m = 1$. These maxima are located at

$$\begin{aligned} x_3 &= \frac{(4.17\,\mathrm{m})^2 - (3)^2(1.06\,\mathrm{m})^2}{2(3)(1.06\,\mathrm{m})} = 1.14 \text{ m}, \\ x_2 &= \frac{(4.17\,\mathrm{m})^2 - (2)^2(1.06\,\mathrm{m})^2}{2(2)(1.06\,\mathrm{m})} = 3.04 \text{ m}, \\ x_1 &= \frac{(4.17\,\mathrm{m})^2 - (1)^2(1.06\,\mathrm{m})^2}{2(1)(1.06\,\mathrm{m})} = 7.67 \text{ m}. \end{aligned}$$

Interestingly enough, as m decreases the maxima get farther away!

(b) The closest maxima to the origin occurs at $x = \pm 6.94$ cm. What then is $x = 0$? It is a local minimum, but the intensity isn't zero. It corresponds to a point where the path length difference is 3.93 wavelengths. It should be half an integer to be a complete minimum.

E41-20 The resultant can be written in the form $A\sin(\omega t + \beta)$. Consider $t = 0$. The three components can be written as

$$\begin{aligned} y_1 &= 10\sin 0^\circ = 0, \\ y_2 &= 14\sin 26^\circ = 6.14, \\ y_3 &= 4.7\sin(-41^\circ) = -3.08, \\ y &= 0 + 6.14 - 3.08 = 3.06. \end{aligned}$$

and

$$\begin{aligned} x_1 &= 10\cos 0^\circ = 10, \\ x_2 &= 14\cos 26^\circ = 12.6, \\ x_3 &= 4.7\cos(-41^\circ) = 3.55, \\ x &= 10 + 12.6 + 3.55 = 26.2. \end{aligned}$$

Then $A = \sqrt{(3.06)^2 + (26.2)^2} = 26.4$ and $\beta = \arctan(3.06/26.2) = 6.66^\circ$.

E41-21 The order of the indices of refraction is the same as in Sample Problem 41-4, so

$$d = \lambda/4n = (620 \text{ nm})/4(1.25) = 124 \text{ nm}.$$

E41-22 Follow the example in Sample Problem 41-3.

$$\lambda = \frac{2dn}{m - 1/2} = \frac{2(410 \text{ nm})(1.50)}{m - 1/2} = \frac{1230 \text{ nm}}{m - 1/2}.$$

The result is only in the visible range when $m = 3$, so $\lambda = 492$ nm.

$\boxed{\textbf{E41-23}}$ (a) Light from above the oil slick can be reflected back up from the top of the oil layer or from the bottom of the oil layer. For both reflections the light is reflecting off a substance with a higher index of refraction so both reflected rays pick up a phase change of π. Since both waves have this phase the equation for a maxima is

$$2d + \frac{1}{2}\lambda_n + \frac{1}{2}\lambda_n = m\lambda_n.$$

Remember that $\lambda_n = \lambda/n$, where n is the index of refraction of the thin film. Then $2nd = (m-1)\lambda$ is the condition for a maxima. We know $n = 1.20$ and $d = 460$ nm. We don't know m or λ. It might seem as if there isn't enough information to solve the problem, but we can. We need to find the wavelength in the visible range (400 nm to 700 nm) which has an integer m. Trial and error might work. If $\lambda = 700$ nm, then m is

$$m = \frac{2nd}{\lambda} + 1 = \frac{2(1.20)(460 \text{ nm})}{(700 \text{ nm})} + 1 = 2.58$$

But m needs to be an integer. If we increase m to 3, then

$$\lambda = \frac{2(1.20)(460 \text{ nm})}{(3-1)} = 552 \text{ nm}$$

which is in the visible range. So the oil slick will appear green.

(b) One of the most profound aspects of thin film interference is that wavelengths which are maximally reflected are minimally transmitted, and vice versa. Finding the maximally transmitted wavelengths is the same as finding the minimally reflected wavelengths, or looking for values of m that are half integer.

The most obvious choice is $m = 3.5$, and then

$$\lambda = \frac{2(1.20)(460 \text{ nm})}{(3.5-1)} = 442 \text{ nm}.$$

E41-24 The condition for constructive interference is $2nd = (m - 1/2)\lambda$. Assuming a minimum value of $m = 1$ one finds

$$d = \lambda/4n = (560 \text{ nm})/4(2.0) = 70 \text{ nm}.$$

E41-25 The top surface contributes a phase difference of π, so the phase difference because of the thickness is 2π, or one complete wavelength. Then $2d = \lambda/n$, or $d = (572 \text{ nm})/2(1.33) = 215$ nm.

E41-26 The wave reflected from the first surface picks up a phase shift of π. The wave which is reflected off of the second surface travels an additional path difference of $2d$. The interference will be bright if $2d + \lambda_n/2 = m\lambda_n$ results in m being an integer.

$$m = 2nd/\lambda + 1/2 = 2(1.33)(1.21\times10^{-6}\text{m})/(585\times10^{-9}\text{m}) + 1/2 = 6.00,$$

so the interference is bright.

$\boxed{\textbf{E41-27}}$ As with the oil on the water in Ex. 41-23, both the light which reflects off of the acetone and the light which reflects off of the glass undergoes a phase shift of π. Then the maxima for reflection are given by $2nd = (m-1)\lambda$. We don't know m, but at some integer value of m we have $\lambda = 700$ nm. If m is increased by exactly $\frac{1}{2}$ then we are at a minimum of $\lambda = 600$ nm. Consequently,

$$2(1.25)d = (m-1)(700 \text{ nm}) \text{ and } 2(1.25)d = (m-1/2)(600 \text{ nm}),$$

we can set these two expressions equal to each other to find m,

$$(m-1)(700 \text{ nm}) = (m-1/2)(600 \text{ nm}),$$

so $m = 4$. Then we can find the thickness,

$$d = (4-1)(700 \text{ nm})/2(1.25) = 840 \text{ nm}.$$

E41-28 The wave reflected from the first surface picks up a phase shift of π. The wave which is reflected off of the second surface travels an additional path difference of $2d$. The interference will be bright if $2d + \lambda_n/2 = m\lambda_n$ results in m being an integer. Then $2nd = (m - 1/2)\lambda_1$ is bright, and $2nd = m\lambda_2$ is dark. Divide one by the other and $(m - 1/2)\lambda_1 = m\lambda_2$, so

$$m = \lambda_1/2(\lambda_1 - \lambda_2) = (600\text{ nm})/2(600\text{ nm} - 450\text{ nm}) = 2,$$

then $d = m\lambda_2/2n = (2)(450\text{ nm})/2(1.33) = 338$ nm.

E41-29 Constructive interference happens when $2d = (m - 1/2)\lambda$. The minimum value for m is $m = 1$; the maximum value is the integer portion of $2d/\lambda + 1/2 = 2(4.8\times10^{-5}\text{m})/(680\times10^{-9}\text{m}) + 1/2 = 141.67$, so $m_{\text{max}} = 141$. There are then 141 bright bands.

E41-30 (a) A half wavelength phase shift occurs for both the air/water interface and the water/oil interface, so if $d = 0$ the two reflected waves are in phase. It will be bright!

(b) $2nd = 3\lambda$, or $d = 3(475\text{ nm})/2(1.20) = 594$ nm.

E41-31 There is a phase shift on one surface only, so the bright bands are given by $2nd = (m - 1/2)\lambda$. Let the first band be given by $2nd_1 = (m_1 - 1/2)\lambda$. The last bright band is then given by $2nd_2 = (m_1 + 9 - 1/2)\lambda$. Subtract the two equations to get the change in thickness:

$$\Delta d = 9\lambda/2n = 9(630\text{ nm})/2(1.50) = 1.89\,\mu\text{m}.$$

E41-32 Apply Eq. 41-21: $2nd = m\lambda$. In one case we have

$$2n_{\text{air}} = (4001)\lambda,$$

in the other,

$$2n_{\text{vac}} = (4000)\lambda.$$

Equating, $n_{\text{air}} = (4001)/(4000) = 1.00025$.

$\boxed{\textbf{E41-33}}$ (a) We can start with the last equation from Sample Problem 41-5,

$$r = \sqrt{(m - \frac{1}{2})\lambda R},$$

and solve for m,

$$m = \frac{r^2}{\lambda R} + \frac{1}{2}$$

In this exercise $R = 5.0\,\text{m}$, $r = 0.01\,\text{m}$, and $\lambda = 589$ nm. Then

$$m = \frac{(0.01\,\text{m})^2}{(589\text{ nm})(5.0\,\text{m})} = 34$$

is the number of rings observed.

(b) Putting the apparatus in water effectively changes the wavelength to

$$(589\text{ nm})/(1.33) = 443\text{ nm},$$

so the number of rings will now be

$$m = \frac{(0.01\,\text{m})^2}{(443\text{ nm})(5.0\,\text{m})} = 45.$$

E41-34 $(1.42 \text{ cm}) = \sqrt{(10 - \frac{1}{2})R\lambda}$, while $(1.27 \text{ cm}) = \sqrt{(10 - \frac{1}{2})R\lambda/n}$. Divide one expression by the other, and $(1.42 \text{ cm})/(1.27 \text{ cm}) = \sqrt{n}$, or $n = 1.25$.

E41-35 $(0.162 \text{ cm}) = \sqrt{(n - \frac{1}{2})R\lambda}$, while $(0.368 \text{ cm}) = \sqrt{(n + 20 - \frac{1}{2})R\lambda}$. Square both expressions, the divide one by the other, and find

$$(n + 19.5)/(n - 0.5) = (0.368 \text{ cm}/0.162 \text{ cm})^2 = 5.16$$

which can be rearranged to yield

$$n = \frac{19.5 + 5.16 \times 0.5}{5.16 - 1} = 5.308.$$

Oops! That should be an integer, shouldn't it? The above work is correct, which means that there really aren't bright bands at the specified locations. I'm just going to gloss over that fact and solve for R using the value of $m = 5.308$. Then

$$R = r^2/(m - 1/2)\lambda = (0.162 \text{ cm})^2/(5.308 - 0.5)(546 \text{ nm}) = 1.00\,\text{m}.$$

Well, at least we got the answer which is in the back of the book...

E41-36 Pretend the ship is a two point source emitter, one h above the water, and one h below the water. The one below the water is out of phase by half a wavelength. Then $d \sin\theta = \lambda$, where $d = 2h$, gives the angle for theta for the first minimum.

$$\lambda/2h = (3.43\,\text{m})/2(23\,\text{m}) = 7.46\times10^{-2} = \sin\theta \approx H/D,$$

so $D = (160\,\text{m})/(7.46\times10^{-2}) = 2.14$ km.

E41-37 The phase difference is $2\pi/\lambda_n$ times the path difference which is $2d$, so

$$\phi = 4\pi d/\lambda_n = 4\pi n d/\lambda.$$

We are given that $d = 100\times10^{-9}$m and $n = 1.38$.

(a) $\phi = 4\pi(1.38)(100\times10^{-9}\text{m})/(450\times10^{-9}\text{m}) = 3.85$. Then

$$\frac{I}{I_0} = \cos^2\frac{(3.85)}{2} = 0.12.$$

The reflected ray is diminished by $1 - 0.12 = 88\%$.

(b) $\phi = 4\pi(1.38)(100\times10^{-9}\text{m})/(650\times10^{-9}\text{m}) = 2.67$. Then

$$\frac{I}{I_0} = \cos^2\frac{(2.67)}{2} = 0.055.$$

The reflected ray is diminished by $1 - 0.055 = 95\%$.

E41-38 The change in the optical path length is $2(d - d/n)$, so $7\lambda/n = 2d(1 - 1/n)$, or

$$d = \frac{7(589\times10^{-9}\text{m})}{2(1.42) - 2} = 4.9\times10^{-6}\text{m}.$$

$\boxed{\textbf{E41-39}}$ When M_2 moves through a distance of $\lambda/2$ a fringe has will be produced, destroyed, and then produced again. This is because the light travels twice through any change in distance. The wavelength of light is then

$$\lambda = \frac{2(0.233 \text{ mm})}{792} = 588 \text{ nm}.$$

E41-40 The change in the optical path length is $2(d - d/n)$, so $60\lambda = 2d(1 - 1/n)$, or

$$n = \frac{1}{1 - 60\lambda/2d} = \frac{1}{1 - 60(500\times10^{-9}\text{m})/2(5\times10^{-2}\text{m})} = 1.00030.$$

$\boxed{\textbf{P41-1}}$ (a) This is a small angle problem, so we use Eq. 41-4. The distance to the screen is $2 \times 20\,\text{m}$, because the light travels to the mirror and back again. Then

$$d = \frac{\lambda D}{\Delta y} = \frac{(632.8 \text{ nm})(40.0\,\text{m})}{(0.1\,\text{m})} = 0.253 \text{ mm}.$$

(b) Placing the cellophane over one slit will cause the interference pattern to shift to the left or right, but not disappear or change size. How does it shift? Since we are picking up 2.5 waves then we are, in effect, swapping bright fringes for dark fringes.

P41-2 The change in the optical path length is $d - d/n$, so $7\lambda/n = d(1 - 1/n)$, or

$$d = \frac{7(550\times10^{-9}\text{m})}{(1.58) - 1} = 6.64\times10^{-6}\text{m}.$$

P41-3 The distance from S_1 to P is $r_1 = \sqrt{(x + d/2)^2 + y^2}$. The distance from S_2 to P is $r_2 = \sqrt{(x - d/2)^2 + y^2}$. The difference in distances is fixed at some value, say c, so that

$$\begin{aligned}
r_1 - r_2 &= c, \\
r_1^2 - 2r_1r_2 + r_2^2 &= c^2, \\
(r_1^2 + r_2^2 - c^2)^2 &= 4r_1^2r_2^2, \\
(r_1^2 - r_2^2)^2 - 2c^2(r_1^2 + r_2^2) + c^4 &= 0, \\
(2xd)^2 - 2c^2(2x^2 + d^2/2 + 2y^2) + c^4 &= 0, \\
4x^2d^2 - 4c^2x^2 - c^2d^2 - 4c^2y^2 + c^4 &= 0, \\
4(d^2 - c^2)x^2 - 4c^2y^2 &= c^2(d^2 - c^2).
\end{aligned}$$

Yes, that is the equation of a hyperbola.

P41-4 The change in the optical path length for each slit is $nt - t$, where n is the corresponding index of refraction. The net change in the path difference is then $n_2t - n_1t$. Consequently, $m\lambda = t(n_2 - n_1)$, so

$$t = \frac{(5)(480\times10^{-9}\text{m})}{(1.7) - (1.4)} = 8.0\times10^{-6}\text{m}.$$

$\boxed{\textbf{P41-5}}$ The intensity is given by Eq. 41-17, which, in the small angle approximation, can be written as

$$I_\theta = 4I_0 \cos^2\left(\frac{\pi d\theta}{\lambda}\right).$$

The intensity will be half of the maximum when

$$\frac{1}{2} = \cos^2\left(\frac{\pi d\Delta\theta/2}{\lambda}\right)$$

or

$$\frac{\pi}{4} = \frac{\pi d\Delta\theta}{2\lambda},$$

which will happen if $\Delta\theta = \lambda/2d$.

P41-6 Follow the construction in Fig. 41-10, except that one of the electric field amplitudes is twice the other. The resultant field will have a length given by

$$\begin{aligned} E' &= \sqrt{(2E_0 + E_0\cos\phi)^2 + (E_0\sin\phi)^2}, \\ &= E_0\sqrt{5+4\cos\phi}, \end{aligned}$$

so squaring this yields

$$\begin{aligned} I &= I_0\left(5 + 4\cos\frac{2\pi d\sin\theta}{\lambda}\right), \\ &= I_0\left(1 + 8\cos^2\frac{\pi d\sin\theta}{\lambda}\right), \\ &= \frac{I_{\mathrm{m}}}{9}\left(1 + 8\cos^2\frac{\pi d\sin\theta}{\lambda}\right). \end{aligned}$$

P41-7 We actually did this problem in Exercise 41-27, although slightly differently. One maximum is

$$2(1.32)d = (m - 1/2)(679 \text{ nm}),$$

the other is

$$2(1.32)d = (m + 1/2)(485 \text{ nm}).$$

Set these equations equal to each other,

$$(m - 1/2)(679 \text{ nm}) = (m + 1/2)(485 \text{ nm}),$$

and find $m = 3$. Then the thickness is

$$d = (3 - 1/2)(679 \text{ nm})/2(1.32) = 643 \text{ nm}.$$

P41-8 (a) Since we are concerned with transmission there is a phase shift for two rays, so

$$2d = m\lambda_n$$

The minimum thickness occurs when $m = 1$; solving for d yields

$$d = \frac{\lambda}{2n} = \frac{(525\times10^{-9}\text{m})}{2(1.55)} = 169\times10^{-9}\text{m}.$$

(b) The wavelengths are different, so the other parts have differing phase differences.
(c) The nearest destructive interference wavelength occurs when $m = 1.5$, or

$$\lambda = 2nd = 2(1.55)1.5(169\times10^{-9}\text{m}) = 393\times10^{-9}\text{m}.$$

This is blue-violet.

P41-9 It doesn't matter if we are looking at bright are dark bands. It doesn't even matter if we concern ourselves with phase shifts. All that cancels out. Consider $2\delta d = \delta m\lambda$; then

$$\delta d = (10)(480 \text{ nm})/2 = 2.4\,\mu\text{m}.$$

P41-10 (a) Apply $2d = m\lambda$. Then

$$d = (7)(600\times10^{-9}\text{m})/2 = 2100\times10^{-9}\text{m}.$$

(b) When water seeps in it introduces an extra phase shift. Point A becomes then a bright fringe, and the equation for the number of bright fringes is $2nd = m\lambda$. Solving for m,

$$m = 2(1.33)(2100\times10^{-9}\text{m})/(600\times10^{-9}\text{m}) = 9.3;$$

this means that point B is almost, but not quite, a dark fringe, and there are *nine* of them.

$\boxed{\textbf{P41-11}}$ (a) Look back at the work for Sample Problem 41-5 where it was found

$$r_m = \sqrt{(m - \frac{1}{2})\lambda R},$$

We can write this as

$$r_m = \sqrt{\left(1 - \frac{1}{2m}\right) m\lambda R}$$

and expand the part in parentheses in a binomial expansion,

$$r_m \approx \left(1 - \frac{1}{2}\frac{1}{2m}\right)\sqrt{m\lambda R}.$$

We will do the same with

$$r_{m+1} = \sqrt{(m + 1 - \frac{1}{2})\lambda R},$$

expanding

$$r_{m+1} = \sqrt{\left(1 + \frac{1}{2m}\right) m\lambda R}$$

to get

$$r_{m+1} \approx \left(1 + \frac{1}{2}\frac{1}{2m}\right)\sqrt{m\lambda R}.$$

Then

$$\Delta r \approx \frac{1}{2m}\sqrt{m\lambda R},$$

or

$$\Delta r \approx \frac{1}{2}\sqrt{\lambda R/m}.$$

(b) The area between adjacent rings is found from the difference,

$$A = \pi\left(r_{m+1}^2 - r_m^2\right),$$

and into this expression we will substitute the *exact* values for r_m and r_{m+1},

$$\begin{aligned} A &= \pi\left((m + 1 - \frac{1}{2})\lambda R - (m - \frac{1}{2})\lambda R\right), \\ &= \pi\lambda R. \end{aligned}$$

Unlike part (a), we did not need to assume $m \gg 1$ in order to arrive at this expression; it is exact for all m.

P41-12 The path length shift that occurs when moving the mirror as distance x is $2x$. This means $\phi = 2\pi 2x/\lambda = 4\pi x/\lambda$. The intensity is then

$$I = 4I_0 \cos^2 \frac{2\pi x}{\lambda}$$

E42-1 $\lambda = a\sin\theta = (0.022\,\mathrm{mm})\sin(1.8°) = 6.91\times10^{-7}\mathrm{m}$.

E42-2 $a = \lambda/\sin\theta = (0.10\times10^{-9}\mathrm{m})/\sin(0.12\times10^{-3}\ \mathrm{rad}/2) = 1.7\,\mu\mathrm{m}$.

$\boxed{\textbf{E42-3}}$ (a) This is a valid small angle approximation problem: the distance between the points on the screen is much less than the distance to the screen. Then

$$\theta \approx \frac{(0.0162\,\mathrm{m})}{(2.16\,\mathrm{m})} = 7.5\times10^{-3}\ \mathrm{rad}.$$

(b) The diffraction minima are described by Eq. 42-3,

$$\begin{aligned} a\sin\theta &= m\lambda, \\ a\sin(7.5\times10^{-3}\ \mathrm{rad}) &= (2)(441\times10^{-9}\,\mathrm{m}), \\ a &= 1.18\times10^{-4}\,\mathrm{m}. \end{aligned}$$

E42-4 $a = \lambda/\sin\theta = (633\times10^{-9}\mathrm{m})/\sin(1.97°/2) = 36.8\,\mu\mathrm{m}$.

$\boxed{\textbf{E42-5}}$ (a) We again use Eq. 42-3, but we will need to throw in a few extra subscripts to distinguish between which wavelength we are dealing with. If the angles match, then so will the sine of the angles. We then have $\sin\theta_{a,1} = \sin\theta_{b,2}$ or, using Eq. 42-3,

$$\frac{(1)\lambda_a}{a} = \frac{(2)\lambda_b}{a},$$

from which we can deduce $\lambda_a = 2\lambda_b$.

(b) Will any other minima coincide? We want to solve for the values of m_a and m_b that will be integers and have the same angle. Using Eq. 42-3 one more time,

$$\frac{m_a\lambda_a}{a} = \frac{m_b\lambda_b}{a},$$

and then substituting into this the relationship between the wavelengths, $m_a = m_b/2$. whenever m_b is an even integer m_a is an integer. Then *all* of the diffraction minima from λ_a are overlapped by a minima from λ_b.

E42-6 The angle is given by $\sin\theta = 2\lambda/a$. This is a small angle, so we can use the small angle approximation of $\sin\theta = y/D$. Then

$$y = 2D\lambda/a = 2(0.714\,\mathrm{m})(593\times10^{-9}\mathrm{m})/(420\times10^{-6}\mathrm{m}) = 2.02\,\mathrm{mm}.$$

E42-7 Small angles, so $y/D = \sin\theta = \lambda/a$. Then

$$a = D\lambda/y = (0.823\,\mathrm{m})(546\times10^{-9}\mathrm{m})/(5.20\times10^{-3}\mathrm{m}/2) = 1.73\times10^{-4}\mathrm{m}.$$

E42-8 (b) Small angles, so $\Delta y/D = \Delta m\lambda/a$. Then

$$a = \Delta m D\lambda/\Delta y = (5-1)(0.413\,\mathrm{m})(546\times10^{-9}\mathrm{m})/(0.350\times10^{-3}\mathrm{m}) = 2.58\,\mathrm{mm}.$$

(a) $\theta = \arcsin(\lambda/a) = \arcsin[(546\times10^{-9}\mathrm{m})/(2.58\,\mathrm{mm})] = 1.21\times10^{-2\circ}$.

E42-9 Small angles, so $\Delta y/D = \Delta m\lambda/a$. Then

$$\Delta y = \Delta m D\lambda/a = (2-1)(2.94\,\mathrm{m})(589\times10^{-9}\mathrm{m})/(1.16\times10^{-3}\mathrm{m}) = 1.49\times10^{-3}\mathrm{m}.$$

E42-10 Doubling the width of the slit results in a narrowing of the diffraction pattern. Since the width of the central maximum is effectively cut in half, then there is twice the energy in half the space, producing four times the intensity.

$\boxed{\textbf{E42-11}}$ (a) This is a small angle approximation problem, so

$$\theta = (1.13 \text{ cm})/(3.48\,\text{m}) = 3.25 \times 10^{-3} \text{ rad}.$$

(b) A convenient measure of the phase difference, α is related to θ through Eq. 42-7,

$$\alpha = \frac{\pi a}{\lambda}\sin\theta = \frac{\pi(25.2 \times 10^{-6}\text{m})}{(538 \times 10^{-9}\text{m})}\sin(3.25 \times 10^{-3} \text{ rad}) = 0.478 \text{ rad}$$

(c) The intensity at a point is related to the intensity at the central maximum by Eq. 42-8,

$$\frac{I_\theta}{I_\text{m}} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin(0.478 \text{ rad})}{(0.478 \text{ rad})}\right)^2 = 0.926$$

E42-12 Consider Fig. 42-11; the angle with the vertical is given by $(\pi - \phi)/2$. For Fig. 42-10(d) the circle has wrapped once around onto itself so the angle with the vertical is $(3\pi - \phi)/2$. Substitute α into this expression and the angel against the vertical is $3\pi/2 - \alpha$.

Use the result from Problem 42-3 that $\tan\alpha = \alpha$ for the maxima. The lowest non-zero solution is $\alpha = 4.49341$ rad. The angle against the vertical is then 0.21898 rad, or 12.5°.

E42-13 Drawing heavily from Sample Problem 42-4,

$$\theta_x = \arcsin\left(\frac{\alpha_x\lambda}{\pi a}\right) = \arcsin\left(\frac{1.39}{10\pi}\right) = 2.54°.$$

Finally, $\Delta\theta = 2\theta_x = 5.1°$.

E42-14 (a) Rayleigh's criterion for resolving images (Eq. 42-11) requires that two objects have an angular separation of at least

$$\theta_\text{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(540 \times 10^{-9})}{(4.90 \times 10^{-3}\text{m})}\right) = 1.34 \times 10^{-4} \text{ rad}$$

(b) The linear separation is $y = \theta D = (1.34 \times 10^{-4} \text{ rad})(163\times 10^3\text{m}) = 21.9\,\text{m}$.

$\boxed{\textbf{E42-15}}$ (a) Rayleigh's criterion for resolving images (Eq. 42-11) requires that two objects have an angular separation of at least

$$\theta_\text{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(562 \times 10^{-9})}{(5.00 \times 10^{-3}\text{m})}\right) = 1.37 \times 10^{-4} \text{ rad}.$$

(b) Once again, this is a small angle, so we can use the small angle approximation to find the distance to the car. In that case $\theta_\text{R} = y/D$, where y is the headlight separation and D the distance to the car. Solving,

$$D = y/\theta_R = (1.42\,\text{m})/(1.37 \times 10^{-4} \text{ rad}) = 1.04 \times 10^4\text{m},$$

or about six or seven miles.

E42-16 $y/D = 1.22\lambda/a$; or

$$D = (5.20\times10^{-3}\text{m})(4.60\times10^{-3}/m)/1.22(542\times10^{-9}\text{m}) = 36.2\,\text{m}.$$

$\boxed{\textbf{E42-17}}$ The smallest resolvable angular separation will be given by Eq. 42-11,

$$\theta_\text{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(565\times10^{-9}\text{m})}{(5.08\,\text{m})}\right) = 1.36\times10^{-7}\text{ rad},$$

The smallest objects resolvable on the Moon's surface by this telescope have a size y where

$$y = D\theta_\text{R} = (3.84\times10^{8}\,\text{m})(1.36\times10^{-7}\text{ rad}) = 52.2\,\text{m}$$

E42-18 $y/D = 1.22\lambda/a$; or

$$y = 1.22(1.57\times10^{-2}\text{m})(6.25\times10^{3}\text{m})/(2.33\,\text{m}) = 51.4\,\text{m}$$

E42-19 $y/D = 1.22\lambda/a$; or

$$D = (4.8\times10^{-2}\text{m})(4.3\times10^{-3}/m)/1.22(0.12\times10^{-9}\text{m}) = 1.4\times10^{6}\,\text{m}.$$

E42-20 $y/D = 1.22\lambda/a$; or

$$d = 1.22(550\times10^{-9}\text{m})(160\times10^{3}\text{m})/(0.30\,\text{m}) = 0.36\,\text{m}.$$

$\boxed{\textbf{E42-21}}$ Using Eq. 42-11, we find the minimum resolvable angular separation is given by

$$\theta_\text{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(475\times10^{-9}\text{m})}{(4.4\times10^{-3}\text{m})}\right) = 1.32\times10^{-4}\text{ rad}$$

The dots are 2 mm apart, so we want to stand a distance D away such that

$$D > y/\theta_\text{R} = (2\times10^{-3}\text{m})/(1.32\times10^{-4}\text{ rad}) = 15\,\text{m}.$$

E42-22 $y/D = 1.22\lambda/a$; or

$$y = 1.22(500\times10^{-9}\text{m})(354\times10^{3}\text{m})/(9.14\,\text{m}/2) = 4.73\times10^{-2}\,\text{m}.$$

E42-23 (a) $\lambda = v/f$. Now use Eq. 42-11:

$$\theta = \arcsin\left(1.22\frac{(1450\,\text{m/s})}{(25\times10^{3}\text{Hz})(0.60\,\text{m})}\right) = 6.77^\circ.$$

(b) Following the same approach,

$$\theta = \arcsin\left(1.22\frac{(1450\,\text{m/s})}{(1\times10^{3}\text{Hz})(0.60\,\text{m})}\right)$$

has no real solution, so there is no minimum.

E42-24 (a) $\lambda = v/f$. Now use Eq. 42-11:

$$\theta = \arcsin\left(1.22\frac{(3\times10^8\,\mathrm{m/s})}{(220\times10^9\mathrm{Hz})(0.55\,\mathrm{m})}\right) = 0.173^\circ.$$

This is the angle from the central maximum; the angular width is twice this, or 0.35°.

(b) use Eq. 42-11:

$$\theta = \arcsin\left(1.22\frac{(0.0157\,\mathrm{m})}{(2.33\,\mathrm{m})}\right) = 0.471^\circ.$$

This is the angle from the central maximum; the angular width is twice this, or 0.94°.

$\boxed{\textbf{E42-25}}$ The linear separation of the fringes is given by

$$\frac{\Delta y}{D} = \Delta\theta = \frac{\lambda}{d} \text{ or } \Delta y = \frac{\lambda D}{d}$$

for sufficiently small d compared to λ.

E42-26 (a) $d\sin\theta = 4\lambda$ gives the location of the fourth interference maximum, while $a\sin\theta = \lambda$ gives the location of the first diffraction minimum. Hence, if $d = 4a$ there will be no fourth interference maximum!

(b) Since $d\sin\theta_{m_i} = m_i\lambda$ gives the interference maxima and $a\sin\theta_{m_d} = m_d\lambda$ gives the diffraction minima, and $d = 4a$, then whenever $m_i = 4m_d$ there will be a missing maximum.

E42-27 (a) The central diffraction envelope is contained in the range

$$\theta = \arcsin\frac{\lambda}{a}$$

This angle corresponds to the mth maxima of the interference pattern, where

$$\sin\theta = m\lambda/d = m\lambda/2a.$$

Equating, $m = 2$, so there are three interference bands, since the $m = 2$ band is "washed out" by the diffraction minimum.

(b) If $d = a$ then $\beta = \alpha$ and the expression reduces to

$$\begin{aligned} I\theta &= I_\mathrm{m} cos^2\alpha\frac{\sin^2\alpha}{\alpha^2}, \\ &= I_\mathrm{m}\frac{\sin^2(2\alpha)}{2\alpha^2}, \\ &= 2I_\mathrm{m}\left(\frac{\sin\alpha'}{\alpha'}\right)^2, \end{aligned}$$

where $\alpha = 2\alpha'$, which is the same as replacing a by $2a$.

E42-28 Remember that the central peak has an envelope width twice that of any other peak. Ignoring the central maximum there are $(11-1)/2 = 5$ fringes in any other peak envelope.

$\boxed{\textbf{E42-29}}$ (a) The first diffraction minimum is given at an angle θ such that $a\sin\theta = \lambda$; the order of the interference maximum at that point is given by $d\sin\theta = m\lambda$. Dividing one expression by the other we get $d/a = m$, with solution $m = (0.150)/(0.030) = 5$. The fact that the answer is *exactly* 5 implies that the fifth interference maximum is squelched by the diffraction minimum. Then there are only four complete fringes on either side of the central maximum. Add this to the central maximum and we get nine as the answer.

(b) For the third fringe $m = 3$, so $d\sin\theta = 3\lambda$. Then β is Eq. 42-14 is 3π, while α in Eq. 42-16 is

$$\alpha = \frac{\pi a}{\lambda}\frac{3\lambda}{d} = 3\pi\frac{a}{d},$$

so the relative intensity of the third fringe is, from Eq. 42-17,

$$(\cos 3\pi)^2\left(\frac{\sin(3\pi a/d)}{(3\pi a/d)}\right)^2 = 0.255.$$

P42-1 $y = m\lambda D/a$. Then

$$y = (10)(632.8\times10^{-9}\mathrm{m})(2.65\,\mathrm{m})/(1.37\times10^{-3}\mathrm{m}) = 1.224\times10^{-2}\mathrm{m}.$$

The separation is twice this, or 2.45 cm.

P42-2 If $a \gg \lambda$ then the diffraction pattern is extremely tight, and there is effectively no light at P. In the event that either shape produces an interference pattern at P then the other shape must produce an equal but opposite electric field vector at that point so that when both patterns from both shapes are superimposed the field cancel.

But the intensity is the field vector squared; hence the two patterns look identical.

$\boxed{\textbf{P42-3}}$ (a) We want to take the derivative of Eq. 42-8 with respect to α, so

$$\begin{aligned}
\frac{dI_\theta}{d\alpha} &= \frac{d}{d\alpha}I_\mathrm{m}\left(\frac{\sin\alpha}{\alpha}\right)^2,\\
&= I_\mathrm{m}2\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\cos\alpha}{\alpha}-\frac{\sin\alpha}{\alpha^2}\right),\\
&= I_\mathrm{m}2\frac{\sin\alpha}{\alpha^3}\left(\alpha\cos\alpha-\sin\alpha\right).
\end{aligned}$$

This equals zero whenever $\sin\alpha = 0$ or $\alpha\cos\alpha = \sin\alpha$; the former is the case for a minima while the latter is the case for the maxima. The maxima case can also be written as

$$\tan\alpha = \alpha.$$

(b) Note that as the order of the maxima increases the solutions get closer and closer to odd integers times $\pi/2$. The solutions are

$$\alpha = 0,\ 1.43\pi,\ 2.46\pi,\ \text{etc.}$$

(c) The m values are $m = \alpha/\pi - 1/2$, and correspond to

$$m = 0.5,\ 0.93,\ 1.96,\ \text{etc.}$$

These values will get closer and closer to integers as the values are increased.

P42-4 The outgoing beam strikes the moon with a circular spot of radius

$$r = 1.22\lambda D/a = 1.22(0.69\times10^{-6}\text{m})(3.82\times10^{8}\text{m})/(2\times1.3\,\text{m}) = 123\,\text{m}.$$

The light is *not* evenly distributed over this circle.

If P_0 is the power in the light, then

$$P_0 = \int I_\theta r\,dr\,d\phi = 2\pi\int_0^R I_\theta r\,dr,$$

where R is the radius of the central peak and I_θ is the angular intensity. For $a \gg \lambda$ we can write $\alpha \approx \pi a r/\lambda D$, then

$$P_0 = 2\pi I_m\left(\frac{\lambda D}{\pi a}\right)^2\int_0^{\pi/2}\frac{\sin^2\alpha}{\alpha}d\alpha \approx 2\pi I_m\left(\frac{\lambda D}{\pi a}\right)^2(0.82).$$

Then the intensity at the center falls off with distance D as

$$I_m = 1.9\,(a/\lambda D)^2\,P_0$$

The fraction of light collected by the mirror on the moon is then

$$P_1/P_0 = 1.9\left(\frac{(2\times1.3\,\text{m})}{(0.69\times10^{-6}\text{m})(3.82\times10^{8}\text{m})}\right)^2\pi(0.10\,\text{m})^2 = 5.6\times10^{-6}.$$

The fraction of light collected by the mirror on the Earth is then

$$P_2/P_1 = 1.9\left(\frac{(2\times0.10\,\text{m})}{(0.69\times10^{-6}\text{m})(3.82\times10^{8}\text{m})}\right)^2\pi(1.3\,\text{m})^2 = 5.6\times10^{-6}.$$

Finally, $P_2/P_0 = 3\times10^{-11}$.

P42-5 (a) The ring is reddish because it occurs at the blue minimum.

(b) Apply Eq. 42-11 for *blue* light:

$$d = 1.22\lambda/\sin\theta = 1.22(400\text{ nm})/\sin(0.375^\circ) = 70\,\mu\text{m}.$$

(c) Apply Eq. 42-11 for *red* light:

$$\theta = \arcsin\left(1.22(700\text{ nm})/(70\,\mu\text{m})\right) \approx 0.7^\circ,$$

which occurs 3 lunar radii from the moon.

P42-6 The diffraction pattern is a property of the speaker, not the interference between the speakers. The diffraction pattern should be unaffected by the phase shift. The interference pattern, however, should shift up or down as the phase of the second speaker is varied.

$\boxed{\textbf{P42-7}}$ (a) The missing fringe at $\theta = 5^\circ$ is a good hint as to what is going on. There should be some sort of *interference* fringe, unless the *diffraction* pattern has a minimum at that point. This would be the first minimum, so

$$a\sin(5^\circ) = (440\times10^{-9}\text{m})$$

would be a good measure of the width of each slit. Then $a = 5.05\times10^{-6}$m.

(b) If the diffraction pattern envelope were not present we could expect that the fourth interference maxima beyond the central maximum would occur at this point, and then

$$d\sin(5^\circ) = 4(440 \times 10^{-9}\text{m})$$

yielding

$$d = 2.02 \times 10^{-5}\text{m}.$$

(c) Apply Eq. 42-17, where $\beta = m\pi$ and

$$\alpha = \frac{\pi a}{\lambda}\sin\theta = \frac{\pi a}{\lambda}\frac{m\lambda}{d} = m\frac{\pi a}{d} = m\pi/4.$$

Then for $m = 1$ we have

$$I_1 = (7)\left(\frac{\sin(\pi/4)}{(\pi/4)}\right)^2 = 5.7;$$

while for $m = 2$ we have

$$I_2 = (7)\left(\frac{\sin(2\pi/4)}{(2\pi/4)}\right)^2 = 2.8.$$

These are in good agreement with the figure.

E43-1 (a) $d = (21.5\times10^{-3}\text{m})/(6140) = 3.50\times10^{-6}\text{m}$.

(b) There are a number of angles allowed:

$$\begin{aligned}
\theta &= \arcsin[(1)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 9.7^\circ,\\
\theta &= \arcsin[(2)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 19.5^\circ,\\
\theta &= \arcsin[(3)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 30.3^\circ,\\
\theta &= \arcsin[(4)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 42.3^\circ,\\
\theta &= \arcsin[(5)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 57.3^\circ.
\end{aligned}$$

E43-2 The distance between adjacent rulings is

$$d = \frac{(2)(612\times10^{-9}\text{m})}{\sin(33.2^\circ)} = 2.235\times10^{-6}\text{m}.$$

The number of lines is then

$$N = D/d = (2.86\times10^{-2}\text{m})/(2.235\times10^{-6}\text{m}) = 12,800.$$

E43-3 We want to find a relationship between the angle and the order number which is linear. We'll plot the data in this representation, and then use a least squares fit to find the wavelength.

The data to be plotted is

m	θ	$\sin\theta$	m	θ	$\sin\theta$
1	17.6°	0.302	-1	-17.6°	-0.302
2	37.3°	0.606	-2	-37.1°	-0.603
3	65.2°	0.908	-3	-65.0°	-0.906

On my calculator I get the best straight line fit as

$$0.302m + 8.33\times10^{-4} = \sin\theta_m,$$

which means that

$$\lambda = (0.302)(1.73\,\mu\text{m}) = 522\text{ nm}.$$

E43-4 Although an approach like the solution to Exercise 3 should be used, we'll assume that each measurement is perfect and error free. Then randomly choosing the third maximum,

$$\lambda = \frac{d\sin\theta}{m} = \frac{(5040\times10^{-9}\text{m})\sin(20.33^\circ)}{(3)} = 586\times10^{-9}\text{m}.$$

E43-5 (a) The principle maxima occur at points given by Eq. 43-1,

$$\sin\theta_m = m\frac{\lambda}{d}.$$

The difference of the sine of the angle between any two adjacent orders is

$$\sin\theta_{m+1} - \sin\theta_m = (m+1)\frac{\lambda}{d} - m\frac{\lambda}{d} = \frac{\lambda}{d}.$$

Using the information provided we can find d from

$$d = \frac{\lambda}{\sin\theta_{m+1} - \sin\theta_m} = \frac{(600\times10^{-9})}{(0.30)-(0.20)} = 6\,\mu\text{m}.$$

It doesn't take much imagination to recognize that the second and third order maxima were given.

(b) If the fourth order maxima is missing it must be because the diffraction pattern envelope has a minimum at that point. Any fourth order maxima should have occurred at $\sin\theta_4 = 0.4$. If it is a diffraction minima then

$$a \sin\theta_m = m\lambda \text{ where } \sin\theta_m = 0.4$$

We can solve this expression and find

$$a = m\frac{\lambda}{\sin\theta_m} = m\frac{(600 \times 10^{-9}\text{m})}{(0.4)} = m1.5\,\mu\text{m}.$$

The minimum width is when $m = 1$, or $a = 1.5\,\mu$m.

(c) The visible orders would be integer values of m *except* for when m is a multiple of four.

E43-6 (a) Find the maximum integer value of $m = d/\lambda = (930\text{ nm})/(615\text{ nm}) = 1.5$, hence $m = -1, 0, +1$; there are three diffraction maxima.

(b) The first order maximum occurs at

$$\theta = \arcsin(615\text{ nm})/(930\text{ nm}) = 41.4^\circ.$$

The width of the maximum is

$$\delta\theta = \frac{(615\text{ nm})}{(1120)(930\text{ nm})\cos(41.4^\circ)} = 7.87\times 10^{-4}\text{ rad},$$

or 0.0451°.

E43-7 The fifth order maxima will be visible if $d/\lambda \geq 5$; this means

$$\lambda \leq \frac{d}{5} = \frac{(1\times 10^{-3}\text{m})}{(315\text{ rulings})(5)} = 635\times 10^{-9}\text{m}.$$

E43-8 (a) The maximum could be the first, and then

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1\times 10^{-3}\text{m})\sin(28^\circ)}{(200)(1)} = 2367\times 10^{-9}\text{m}.$$

That's not visible. The first visible wavelength is at $m = 4$, then

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1\times 10^{-3}\text{m})\sin(28^\circ)}{(200)(4)} = 589\times 10^{-9}\text{m}.$$

The next is at $m = 5$, then

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1\times 10^{-3}\text{m})\sin(28^\circ)}{(200)(5)} = 469\times 10^{-9}\text{m}.$$

Trying $m = 6$ results in an ultraviolet wavelength.

(b) Yellow-orange and blue.

E43-9 A grating with 400 rulings/mm has a slit separation of

$$d = \frac{1}{400\ \text{mm}^{-1}} = 2.5 \times 10^{-3}\ \text{mm}.$$

To find the number of orders of the entire visible spectrum that will be present we need only consider the wavelength which will be on the outside of the maxima. That will be the longer wavelengths, so we only need to look at the 700 nm behavior. Using Eq. 43-1,

$$d \sin\theta = m\lambda,$$

and using the maximum angle 90°, we find

$$m < \frac{d}{\lambda} = \frac{(2.5 \times 10^{-6}\text{m})}{(700 \times 10^{-9}\text{m})} = 3.57,$$

so there can be at most three orders of the entire spectrum.

E43-10 In this case $d = 2a$. Since interference maxima are given by $\sin\theta = m\lambda/d$ while diffraction minima are given at $\sin\theta = m'\lambda/a = 2m'\lambda/d$ then diffraction minima overlap with interference maxima whenever $m = 2m'$. Consequently, all even m are at diffraction minima and therefore vanish.

E43-11 If the second-order spectra overlaps the third-order, it is because the 700 nm second-order line is at a larger angle than the 400 nm third-order line.

Start with the wavelengths multiplied by the appropriate order parameter, then divide both side by d, and finally apply Eq. 43-1.

$$\begin{aligned} 2(700\ \text{nm}) &> 3(400\ \text{nm}), \\ \frac{2(700\ \text{nm})}{d} &> \frac{3(400\ \text{nm})}{d}, \\ \sin\theta_{2,\lambda=700} &> \sin\theta_{3,\lambda=400}, \end{aligned}$$

regardless of the value of d.

E43-12 Fig. 32-2 shows the path length difference for the right hand side of the grating as $d \sin\theta$. If the beam strikes the grating at ang angle ψ then there will be an additional path length difference of $d \sin\psi$ on the right hand side of the figure. The diffraction pattern then has two contributions to the path length difference, these add to give

$$d(\sin\theta + \sin psi) = m\lambda.$$

E43-13

E43-14 Let $d \sin\theta_i = \lambda_i$ and $\theta_1 + 20° = \theta_2$. Then

$$\sin\theta_2 = \sin\theta_1 \cos(20°) + \cos\theta_1 \sin(20°).$$

Rearranging,

$$\sin\theta_2 = \sin\theta_1 \cos(20°) + \sqrt{1 - \sin^2\theta_1}\,\sin(20°).$$

Substituting the equations together yields a rather nasty expression,

$$\frac{\lambda_2}{d} = \frac{\lambda_1}{d}\cos(20°) + \sqrt{1 - (\lambda_1/d)^2}\,\sin(20°).$$

Rearranging,

$$(\lambda_2 - \lambda_1\cos(20°))^2 = \left(d^2 - \lambda_1^2\right)\sin^2(20°).$$

Use $\lambda_1 = 430$ nm and $\lambda_2 = 680$ nm, then solve for d to find $d = 914$ nm. This corresponds to 1090 rulings/mm.

E43-15 The shortest wavelength passes through at an angle of

$$\theta_1 = \arctan(50\,\text{mm})/(300\,\text{mm}) = 9.46°.$$

This corresponds to a wavelength of

$$\lambda_1 = \frac{(1\times10^{-3}\text{m})\sin(9.46°)}{(350)} = 470\times10^{-9}\text{m}.$$

The longest wavelength passes through at an angle of

$$\theta_2 = \arctan(60\,\text{mm})/(300\,\text{mm}) = 11.3°.$$

This corresponds to a wavelength of

$$\lambda_2 = \frac{(1\times10^{-3}\text{m})\sin(11.3°)}{(350)} = 560\times10^{-9}\text{m}.$$

E43-16 (a) $\Delta\lambda = \lambda/R = \lambda/Nm$, so

$$\Delta\lambda = (481\text{ nm})/(620\text{ rulings/mm})(5.05\,\text{mm})(3) = 0.0512\text{ nm}.$$

(b) m_m is the largest integer smaller than d/λ, or

$$m_\text{m} \le 1/(481\times10^{-9}\text{m})(620\text{ rulings/mm}) = 3.35,$$

so $m = 3$ is highest order seen.

$\boxed{\textbf{E43-17}}$ The required resolving power of the grating is given by Eq. 43-10

$$R = \frac{\lambda}{\Delta\lambda} = \frac{(589.0\text{ nm})}{(589.6\text{ nm}) - (589.0\text{ nm})} = 982.$$

Our resolving power is then $R = 1000$.

Using Eq. 43-11 we can find the number of grating lines required. We are looking at the second-order maxima, so

$$N = \frac{R}{m} = \frac{(1000)}{(2)} = 500.$$

E43-18 (a) $N = R/m = \lambda/m\Delta\lambda$, so

$$N = \frac{(415.5\text{ nm})}{(2)(415.496\text{ nm} - 415.487\text{ nm})} = 23100.$$

(b) $d = w/N$, where w is the width of the grating. Then

$$\theta = \arcsin\frac{m\lambda}{d} = \arcsin\frac{(23100)(2)(415.5\times10^{-9}\text{m})}{(4.15\times10^{-2}\text{m})} = 27.6°.$$

E43-19 $N = R/m = \lambda/m\Delta\lambda$, so

$$N = \frac{(656.3\text{ nm})}{(1)(0.180\text{ nm})} = 3650$$

E43-20 Start with Eq. 43-9:

$$D = \frac{m}{d\cos\theta} = \frac{d\sin\theta/\lambda}{d\cos\theta} = \frac{\tan\theta}{\lambda}.$$

$\boxed{\textbf{E43-21}}$ (a) We find the ruling spacing by Eq. 43-1,

$$d = \frac{m\lambda}{\sin\theta_m} = \frac{(3)(589\text{ nm})}{\sin(10.2^\circ)} = 9.98\,\mu\text{m}.$$

(b) The resolving power of the grating needs to be at least $R = 1000$ for the third-order line; see the work for Ex. 43-17 above. The number of lines required is given by Eq. 43-11,

$$N = \frac{R}{m} = \frac{(1000)}{(3)} = 333,$$

so the width of the grating (or at least the part that is being used) is $333(9.98\,\mu\text{m}) = 3.3$ mm.

E43-22 (a) Condition (1) is satisfied if

$$d \geq 2(600\text{ nm})/\sin(30^\circ) = 2400\text{ nm}.$$

The dispersion is maximal for the smallest d, so $d = 2400$ nm.

(b) To remove the third order requires $d = 3a$, or $a = 800$ nm.

E43-23 (a) The angles of the first three orders are

$$\begin{aligned}
\theta_1 &= \arcsin\frac{(1)(589\times10^{-9}\text{m})(40000)}{(76\times10^{-3}\text{m})} = 18.1^\circ,\\
\theta_2 &= \arcsin\frac{(2)(589\times10^{-9}\text{m})(40000)}{(76\times10^{-3}\text{m})} = 38.3^\circ,\\
\theta_3 &= \arcsin\frac{(3)(589\times10^{-9}\text{m})(40000)}{(76\times10^{-3}\text{m})} = 68.4^\circ.
\end{aligned}$$

The dispersion for each order is

$$\begin{aligned}
D_1 &= \frac{(1)(40000)}{(76\times10^{6}\text{nm})\cos(18.1^\circ)}\frac{360^\circ}{2\pi} = 3.2\times10^{-2\circ}/\text{nm},\\
D_2 &= \frac{(2)(40000)}{(76\times10^{6}\text{nm})\cos(38.3^\circ)}\frac{360^\circ}{2\pi} = 7.7\times10^{-2\circ}/\text{nm},\\
D_3 &= \frac{(3)(40000)}{(76\times10^{6}\text{nm})\cos(68.4^\circ)}\frac{360^\circ}{2\pi} = 2.5\times10^{-1\circ}/\text{nm}.
\end{aligned}$$

(b) $R = Nm$, so

$$\begin{aligned}
R_1 &= (40000)(1) = 40000,\\
R_2 &= (40000)(2) = 80000,\\
R_3 &= (40000)(3) = 120000.
\end{aligned}$$

E43-24 $d = m\lambda/2\sin\theta$, so

$$d = \frac{(2)(0.122\text{ nm})}{2\sin(28.1^\circ)} = 0.259\text{ nm}.$$

E43-25 Bragg reflection is given by Eq. 43-12

$$2d\sin\theta = m\lambda,$$

where the angles are measured not against the normal, but against the plane. The value of d depends on the family of planes under consideration, but it is at never larger than a_0, the unit cell dimension.

We are looking for the smallest angle; this will correspond to the largest d and the smallest m. That means $m = 1$ and $d = 0.313$ nm. Then the minimum angle is

$$\theta = \sin^{-1}\frac{(1)(29.3\times10^{-12}\,\text{m})}{2(0.313\times10^{-9}\,\text{m})} = 2.68^\circ.$$

E43-26 $2d/\lambda = \sin\theta_1$ and $2d/2\lambda = \sin\theta_2$. Then

$$\theta_2 = \arcsin[2\sin(3.40^\circ)] = 6.81^\circ.$$

E43-27 We apply Eq. 43-12 to each of the peaks and find the product

$$m\lambda = 2d\sin\theta.$$

The four values are 26 pm, 39 pm, 52 pm, and 78 pm. The last two values are twice the first two, so the wavelengths are 26 pm and 39 pm.

E43-28 (a) $2d\sin\theta = m\lambda$, so

$$d = \frac{(3)(96.7\text{ pm})}{2\sin(58.0^\circ)} = 171\text{ pm}.$$

(b) $\lambda = 2(171\text{ pm})\sin(23.2^\circ)/(1) = 135$ pm.

E43-29 The angle against the face of the crystal is $90^\circ - 51.3^\circ = 38.7^\circ$. The wavelength is

$$\lambda = 2(39.8\text{ pm})\sin(38.7^\circ)/(1) = 49.8\text{ pm}.$$

E43-30 If $\lambda > 2d$ then $\lambda/2d > 1$. But

$$\lambda/2d = \sin\theta/m.$$

This means that $\sin\theta > m$, but the sine function can never be greater than one.

E43-31 There are too many unknowns. It is only possible to determine the ratio d/λ.

E43-32 A wavelength will be diffracted if $m\lambda = 2d\sin\theta$. The possible solutions are

$$\begin{aligned}\lambda_3 &= 2(275\text{ pm})\sin(47.8)/(3) = 136\text{ pm},\\ \lambda_4 &= 2(275\text{ pm})\sin(47.8)/(4) = 102\text{ pm}.\end{aligned}$$

$\boxed{\textbf{E43-33}}$ We use Eq. 43-12 to first find d;

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.261\times10^{-9}\,\mathrm{m})}{2\sin(63.8^\circ)} = 1.45\times10^{-10}\,\mathrm{m}.$$

d is the spacing between the planes in Fig. 43-28; it correspond to half of the diagonal distance between two cell centers. Then

$$(2d)^2 = a_0^2 + a_0^2,$$

or

$$a_0 = \sqrt{2}d = \sqrt{2}(1.45\times10^{-10}\,\mathrm{m}) = 0.205\ \mathrm{nm}.$$

E43-34 Diffraction occurs when $2d\sin\theta = m\lambda$. The angles in this case are then given by

$$\sin\theta = m\frac{(0.125\times10^{-9}\mathrm{m})}{2(0.252\times10^{-9}\mathrm{m})} = (0.248)m.$$

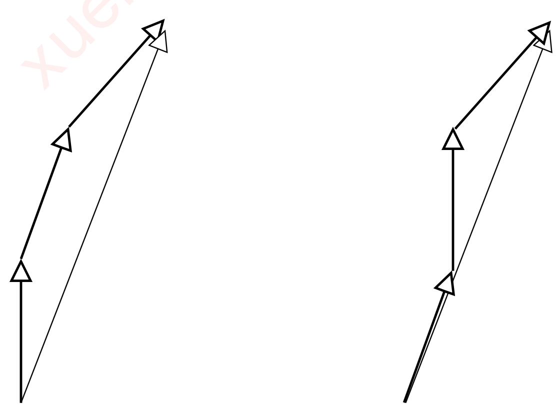
There are four solutions to this equation. They are 14.4°, 29.7°, 48.1°, and 82.7°. They involve rotating the crystal from the original orientation ($90^\circ - 42.4^\circ = 47.6^\circ$) by amounts

$$\begin{aligned}47.6^\circ - 14.4^\circ &= 33.2^\circ,\\ 47.6^\circ - 29.7^\circ &= 17.9^\circ,\\ 47.6^\circ - 48.1^\circ &= -0.5^\circ,\\ 47.6^\circ - 82.7^\circ &= -35.1^\circ.\end{aligned}$$

$\boxed{\textbf{P43-1}}$ Since the slits are *so* narrow we only need to consider interference effects, not diffraction effects. There are three waves which contribute at any point. The phase angle between adjacent waves is

$$\phi = 2\pi d\sin\theta/\lambda.$$

We can add the electric field vectors as was done in the previous chapters, or we can do it in a different order as is shown in the figure below.



Then the vectors sum to

$$E(1+2\cos\phi).$$

We need to square this quantity, and then normalize it so that the central maximum is the maximum. Then

$$I = I_\mathrm{m}\frac{(1+4\cos\phi+4\cos^2\phi)}{9}.$$

P43-2 (a) Solve ϕ for $I = I_\mathrm{m}/2$, this occurs when

$$\frac{3}{\sqrt{2}} = 1 + 2\cos\phi,$$

or $\phi = 0.976$ rad. The corresponding angle θ_x is

$$\theta_x \approx \frac{\lambda\phi}{2\pi d} = \frac{\lambda(0.976)}{2\pi d} = \frac{\lambda}{6.44d}.$$

But $\Delta\theta = 2\theta_x$, so

$$\Delta\theta \approx \frac{\lambda}{3.2d}.$$

(b) For the two slit pattern the half width was found to be $\Delta\theta = \lambda/2d$. The half width in the three slit case *is* smaller.

$\boxed{\textbf{P43-3}}$ (a) and (b) A plot of the intensity quickly reveals that there is an alternation of large maximum, then a smaller maximum, etc. The large maxima are at $\phi = 2n\pi$, the smaller maxima are half way between those values.

(c) The intensity at these secondary maxima is then

$$I = I_\mathrm{m}\frac{(1 + 4\cos\pi + 4\cos^2\pi)}{9} = \frac{I_\mathrm{m}}{9}.$$

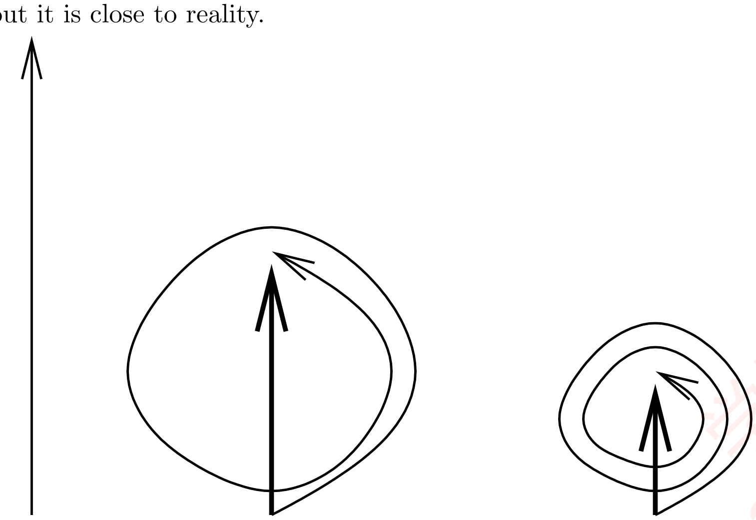
Note that the minima are *not* located half-way between the maxima!

P43-4 Covering up the middle slit will result in a two slit apparatus with a slit separation of $2d$. The half width, as found in Problem 41-5, is then

$$\Delta\theta = \lambda/2(2d), = \lambda/4d,$$

which is *narrower* than before covering up the middle slit by a factor of $3.2/4 = 0.8$.

P43-5 (a) If N is large we can treat the phasors as summing to form a flexible "line" of length $N\delta E$. We then assume (incorrectly) that the secondary maxima occur when the loop wraps around on itself as shown in the figures below. Note that the resultant phasor always points straight up. This isn't right, but it is close to reality.



The length of the resultant depends on how many loops there are. For $k = 0$ there are none. For $k = 1$ there are one and a half loops. The circumference of the resulting circle is $2N\delta E/3$, the diameter is $N\delta E/3\pi$. For $k = 2$ there are two and a half loops. The circumference of the resulting circle is $2N\delta E/5$, the diameter is $N\delta E/5\pi$. The pattern for higher k is similar: the circumference is $2N\delta E/(2k+1)$, the diameter is $N\delta E/(k+1/2)\pi$.

The intensity at this "approximate" maxima is proportional to the resultant squared, or

$$I_k \propto \frac{(N\delta E)^2}{(k+1/2)^2\pi^2}.$$

but I_m is proportional to $(N\delta E)^2$, so

$$I_k = I_\mathrm{m}\frac{1}{(k+1/2)^2\pi^2}.$$

(b) Near the middle the vectors simply fold back on one another, leaving a resultant of δE. Then

$$I_k \propto (\delta E)^2 = \frac{(N\delta E)^2}{N^2},$$

so

$$I_k = \frac{I_\mathrm{m}}{N^2},$$

(c) Let α have the values which result in $\sin\alpha = 1$, and then the two expressions are identical!

P43-6 (a) $v = f\lambda$, so $\delta v = f\delta\lambda + \lambda\delta f$. Assuming $\delta v = 0$, we have $\delta f/f = -\delta\lambda/\lambda$. Ignore the negative sign (we don't need it here). Then

$$R = \frac{\lambda}{\Delta\lambda} = \frac{f}{\Delta f} = \frac{c}{\lambda\Delta f},$$

and then

$$\Delta f = \frac{c}{R\lambda} = \frac{c}{Nm\lambda}.$$

(b) The ray on the top gets there first, the ray on the bottom must travel an additional distance of $Nd\sin\theta$. It takes a time

$$\Delta t = Nd\sin\theta/c$$

to do this.

(c) Since $m\lambda = d\sin\theta$, the two resulting expression can be multiplied together to yield

$$(\Delta f)(\Delta t) = \frac{c}{Nm\lambda}\frac{Nd\sin\theta}{c} = 1.$$

This is almost, but not quite, one of Heisenberg's uncertainty relations!

P43-7 (b) We sketch parallel lines which connect centers to form almost any right triangle similar to the one shown in the Fig. 43-18. The triangle will have two sides which have integer multiple lengths of the lattice spacing a_0. The hypotenuse of the triangle will then have length $\sqrt{h^2+k^2}a_0$, where h and k are the integers. In Fig. 43-18 $h = 2$ while $k = 1$. The number of planes which cut the diagonal is equal to $h^2 + k^2$ if, and only if, h and k are relatively prime. The inter-planar spacing is then

$$d = \frac{\sqrt{h^2+k^2}a_0}{h^2+k^2} = \frac{a_0}{\sqrt{h^2+k^2}}.$$

(a) The next five spacings are then

$$\begin{aligned}
h=1, \quad k=1, \quad d &= a_0/\sqrt{2}, \\
h=1, \quad k=2, \quad d &= a_0/\sqrt{5}, \\
h=1, \quad k=3, \quad d &= a_0/\sqrt{10}, \\
h=2, \quad k=3, \quad d &= a_0/\sqrt{13}, \\
h=1, \quad k=4, \quad d &= a_0/\sqrt{17}.
\end{aligned}$$

P43-8 The middle layer cells will also diffract a beam, but this beam will be exactly π out of phase with the top layer. The two beams will then cancel out exactly because of destructive interference.

$\boxed{\textbf{E44-1}}$ (a) The direction of propagation is determined by considering the argument of the sine function. As t increases y must decrease to keep the sine function "looking" the same, so the wave is propagating in the negative y direction.

(b) The electric field is orthogonal (perpendicular) to the magnetic field (so $E_x = 0$) and the direction of motion (so $E_y = 0$); Consequently, the only non-zero term is E_z. The magnitude of E will be equal to the magnitude of B times c. Since $\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{B}}/\mu_0$, when $\vec{\mathbf{B}}$ points in the positive x direction then $\vec{\mathbf{E}}$ must point in the negative z direction in order that $\vec{\mathbf{S}}$ point in the negative y direction. Then

$$E_z = -cB\sin(ky + \omega t).$$

(c) The polarization is given by the direction of the electric field, so the wave is linearly polarized in the z direction.

E44-2 Let one wave be polarized in the x direction and the other in the y direction. Then the net electric field is given by $E^2 = E_x^2 + E_y^2$, or

$$E^2 = E_0^2\left(\sin^2(kz - \omega t) + \sin^2(kz - \omega t + \beta)\right),$$

where β is the phase difference. We can consider any point in space, including $z = 0$, and then average the result over a full cycle. Since β merely shifts the integration limits, then the result is independent of β. Consequently, there are no interference effects.

E44-3 (a) The transmitted intensity is $I_0/2 = 6.1\times10^{-3}W/\mathrm{m}^2$. The maximum value of the electric field is

$$E_\mathrm{m} = \sqrt{2\mu_0 cI} = \sqrt{2(1.26\times10^{-6}\mathrm{H/m})(3.00\times10^{8}\mathrm{m/s})(6.1\times10^{-3}\mathrm{W/m^2})} = 2.15\,\mathrm{V/m}.$$

(b) The radiation pressure is caused by the absorbed half of the incident light, so

$$p = I/c = (6.1\times10^{-3}\mathrm{W/m^2})/(3.00\times10^{8}\mathrm{m/s}) = 2.03\times10^{-11}\mathrm{Pa}.$$

E44-4 The first sheet transmits half the original intensity, the second transmits an amount proportional to $\cos^2\theta$. Then $I = (I_0/2)\cos^2\theta$, or

$$\theta = \arccos\sqrt{2I/I_0} = \arccos\sqrt{2(I_0/3)/I_0}35.3^\circ.$$

$\boxed{\textbf{E44-5}}$ The first sheet polarizes the un-polarized light, half of the intensity is transmitted, so $I_1 = \frac{1}{2}I_0$.

The second sheet transmits according to Eq. 44-1,

$$I_2 = I_1\cos^2\theta = \frac{1}{2}I_0\cos^2(45^\circ) = \frac{1}{4}I_0,$$

and the transmitted light is polarized in the direction of the second sheet.

The third sheet is 45° to the second sheet, so the intensity of the light which is transmitted through the third sheet is

$$I_3 = I_2\cos^2\theta = \frac{1}{4}I_0\cos^2(45^\circ) = \frac{1}{8}I_0.$$

E44-6 The transmitted intensity through the first sheet is proportional to $\cos^2\theta$, the transmitted intensity through the second sheet is proportional to $\cos^2(90^\circ - \theta) = \sin^2\theta$. Then

$$I = I_0 \cos^2\theta \sin^2\theta = (I_0/4)\sin^2 2\theta,$$

or

$$\theta = \frac{1}{2}\arcsin\sqrt{4I/I_0} = \frac{1}{2}\arcsin\sqrt{4(0.100 I_0)/I_0} = 19.6^\circ.$$

Note that 70.4° is also a valid solution!

E44-7 The first sheet transmits half of the original intensity; each of the remaining sheets transmits an amount proportional to $\cos^2\theta$, where $\theta = 30^\circ$. Then

$$\frac{I}{I_0} = \frac{1}{2}\left(\cos^2\theta\right)^3 = \frac{1}{2}\left(\cos(30^\circ)\right)^6 = 0.211$$

E44-8 The first sheet transmits an amount proportional to $\cos^2\theta$, where $\theta = 58.8^\circ$. The second sheet transmits an amount proportional to $\cos^2(90^\circ - \theta) = \sin^2\theta$. Then

$$I = I_0\cos^2\theta\sin^2\theta = (43.3\,\mathrm{W/m^2})\cos^2(58.8^\circ)\sin^2(58.8^\circ) = 8.50\,\mathrm{W/m^2}.$$

$\boxed{\textbf{E44-9}}$ Since the incident beam is unpolarized the first sheet transmits 1/2 of the original intensity. The transmitted beam then has a polarization set by the first sheet: 58.8° to the vertical. The second sheet is horizontal, which puts it 31.2° to the first sheet. Then the second sheet transmits $\cos^2(31.2^\circ)$ of the intensity incident on the second sheet. The final intensity transmitted by the second sheet can be found from the product of these terms,

$$I = (43.3\,\mathrm{W/m^2})\left(\frac{1}{2}\right)\left(\cos^2(31.2^\circ)\right) = 15.8\,\mathrm{W/m^2}.$$

E44-10 $\theta_\mathrm{p} = \arctan(1.53/1.33) = 49.0^\circ$.

$\boxed{\textbf{E44-11}}$ (a) The angle for complete polarization of the reflected ray is Brewster's angle, and is given by Eq. 44-3 (since the first medium is air)

$$\theta_\mathrm{p} = \tan^{-1} n = \tan^{-1}(1.33) = 53.1^\circ.$$

(b) Since the index of refraction depends (slightly) on frequency, then so does Brewster's angle.

E44-12 (b) Since $\theta_\mathrm{r} + \theta_\mathrm{p} = 90^\circ$, $\theta_\mathrm{p} = 90^\circ - (31.8^\circ) = 58.2^\circ$.
(a) $n = \tan\theta_\mathrm{p} = \tan(58.2^\circ) = 1.61$.

E44-13 The angles are between

$$\theta_\mathrm{p} = \tan^{-1} n = \tan^{-1}(1.472) = 55.81^\circ.$$

and

$$\theta_\mathrm{p} = \tan^{-1} n = \tan^{-1}(1.456) = 55.52^\circ.$$

E44-14 The smallest possible thickness t will allow for one half a wavelength phase difference for the o and e waves. Then $\Delta n t = \lambda/2$, or

$$t = (525\times10^{-9}\mathrm{m})/2(0.022) = 1.2\times10^{-5}\mathrm{m}.$$

$\boxed{\textbf{E44-15}}$ (a) The incident wave is at 45° to the optical axis. This means that there are two components; assume they originally point in the $+y$ and $+z$ direction. When they travel through the half wave plate they are now out of phase by 180°; this means that when one component is in the $+y$ direction the other is in the $-z$ direction. In effect the polarization has been rotated by 90°.

(b) Since the half wave plate will delay one component so that it emerges 180° "later" than it should, it will in effect reverse the handedness of the circular polarization.

(c) Pretend that an unpolarized beam can be broken into two orthogonal linearly polarized components. Both are then rotated through 90°; but when recombined it looks like the original beam. As such, there is no apparent change.

E44-16 The quarter wave plate has a thickness of $x = \lambda/4\Delta n$, so the number of plates that can be cut is given by

$$N = (0.250\times10^{-3}\text{m})4(0.181)/(488\times10^{-9}\text{m}) = 371.$$

$\boxed{\textbf{P44-1}}$ Intensity is proportional to the electric field *squared*, so the original intensity reaching the eye is I_0, with components $I_\text{h} = (2.3)^2 I_\text{v}$, and then

$$I_0 = I_\text{h} + I_\text{v} = 6.3 I_\text{v} \text{ or } I_\text{v} = 0.16 I_0.$$

Similarly, $I_\text{h} = (2.3)^2 I_\text{v} = 0.84 I_0$.

(a) When the sun-bather is standing only the vertical component passes, while

(b) when the sun-bather is lying down only the horizontal component passes.

P44-2 The intensity of the transmitted light which was originally unpolarized is reduced to $I_\text{u}/2$, regardless of the orientation of the polarizing sheet. The intensity of the transmitted light which was originally polarized is between 0 and I_p, depending on the orientation of the polarizing sheet. Then the maximum transmitted intensity is $I_\text{u}/2 + I_\text{p}$, while the minimum transmitted intensity is $I_\text{u}/2$. The ratio is 5, so

$$5 = \frac{I_\text{u}/2 + I_\text{p}}{I_\text{u}/2} = 1 + 2\frac{I_\text{p}}{I_\text{u}},$$

or $I_\text{p}/I_\text{u} = 2$. Then the beam is 1/3 unpolarized and 2/3 polarized.

$\boxed{\textbf{P44-3}}$ Each sheet transmits a fraction

$$\cos^2\alpha = \cos^2\left(\frac{\theta}{N}\right).$$

There are N sheets, so the fraction transmitted through the stack is

$$\left(\cos^2\left(\frac{\theta}{N}\right)\right)^N.$$

We want to evaluate this in the limit as $N \to \infty$.

As N gets larger we can use a small angle approximation to the cosine function,

$$\cos x \approx 1 - \frac{1}{2}x^2 \text{ for } x \ll 1$$

The the transmitted intensity is

$$\left(1 - \frac{1}{2}\frac{\theta^2}{N^2}\right)^{2N}.$$

This expression can also be expanded in a binomial expansion to get

$$1 - 2N\frac{1}{2}\frac{\theta^2}{N^2},$$

which in the limit as $N \to \infty$ approaches 1.

The stack then transmits *all* of the light which makes it past the first filter. Assuming the light is originally unpolarized, then the stack transmits half the original intensity.

P44-4 (a) Stack several polarizing sheets so that the angle between any two sheets is sufficiently small, but the total angle is 90°.

(b) The transmitted intensity fraction needs to be 0.95. Each sheet will transmit a fraction $\cos^2\theta$, where $\theta = 90°/N$, with N the number of sheets. Then we want to solve

$$0.95 = \left(\cos^2(90°/N)\right)^N$$

for N. For large enough N, θ will be small, so we can expand the cosine function as

$$\cos^2\theta = 1 - \sin^2\theta \approx 1 - \theta^2,$$

so

$$0.95 \approx \left(1 - (\pi/2N)^2\right)^N \approx 1 - N(\pi/2N)^2,$$

which has solution $N = \pi^2/4(0.05) = 49$.

$\boxed{\textbf{P44-5}}$ Since passing through a quarter wave plate twice can rotate the polarization of a linearly polarized wave by 90°, then if the light passes through a polarizer, through the plate, reflects off the coin, then through the plate, and through the polarizer, it would be possible that when it passes through the polarizer the second time it is 90° to the polarizer and no light will pass. You won't see the coin.

On the other hand if the light passes first through the plate, then through the polarizer, then is reflected, the passes again through the polarizer, *all* the reflected light will pass through he polarizer and eventually work its way out through the plate. So the coin will be visible.

Hence, side A must be the polarizing sheet, and that sheet must be at 45° to the optical axis.

P44-6 (a) The displacement of a ray is given by

$$\tan\theta_k = y_k/t,$$

so the shift is

$$\Delta y = t(\tan\theta_e - \tan\theta_o).$$

Solving for each angle,

$$\begin{aligned}\theta_e &= \arcsin\left(\frac{1}{(1.486)}\sin(38.8°)\right) = 24.94°,\\ \theta_o &= \arcsin\left(\frac{1}{(1.658)}\sin(38.8°)\right) = 22.21°.\end{aligned}$$

The shift is then

$$\Delta y = (1.12\times10^{-2}\text{m})\,(\tan(24.94) - \tan(22.21)) = 6.35\times10^{-4}\text{m}.$$

(b) The e-ray bends less than the o-ray.

(c) The rays have polarizations which are perpendicular to each other; the o-wave being polarized along the direction of the optic axis.

(d) One ray, then the other, would disappear.

P44-7 The method is outline in Sample Problem 44-24; use a polarizing sheet to pick out the o-ray or the e-ray.

$\boxed{\textbf{E45-1}}$ (a) The energy of a photon is given by Eq. 45-1, $E = hf$, so

$$E = hf = \frac{hc}{\lambda}.$$

Putting in "best" numbers

$$hc = \frac{(6.62606876\times10^{-34}\text{J}\cdot\text{s})}{(1.602176462\times10^{-19}\text{ C})}(2.99792458\times10^{8}\text{m/s}) = 1.23984\times10^{-6}\text{ eV}\cdot\text{m}.$$

This means that $hc = 1240$ eV $\cdot$ nm is accurate to almost one part in 8000!

(b) $E = (1240 \text{ eV}\cdot\text{nm})/(589 \text{ nm}) = 2.11$ eV.

E45-2 Using the results of Exercise 45-1,

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{(0.60 \text{ eV})} = 2100 \text{ nm},$$

which is in the infrared.

E45-3 Using the results of Exercise 45-1,

$$E_1 = \frac{(1240 \text{ eV}\cdot\text{nm})}{(375 \text{ nm})} = 3.31 \text{ eV},$$

and

$$E_2 = \frac{(1240 \text{ eV}\cdot\text{nm})}{(580 \text{ nm})} = 2.14 \text{ eV},$$

The difference is $\Delta E = (3.307 \text{ eV}) - (2.138 \text{ eV}) = 1.17$ eV.

E45-4 $P = E/t$, so, using the result of Exercise 45-1,

$$P = (100/\text{s})\frac{(1240 \text{ eV}\cdot\text{nm})}{(540 \text{ nm})} = 230 \text{ eV/s}.$$

That's a small 3.68×10^{-17}W.

$\boxed{\textbf{E45-5}}$ When talking about the regions in the sun's spectrum it is more common to refer to wavelengths than frequencies. So we will use the results of Exercise 45-1(a), and solve

$$\lambda = hc/E = (1240 \text{ eV}\cdot\text{nm})/E.$$

The energies are between $E = (1.0\times10^{18}\text{J})/(1.6\times10^{-19}\text{C}) = 6.25$ eV and $E = (1.0\times10^{16}\text{J})/(1.6\times10^{-19}\text{C}) = 625$ eV. These energies correspond to wavelengths between 198 nm and 1.98 nm; this is the ultraviolet range.

E45-6 The energy per photon is $E = hf = hc/\lambda$. The intensity is power per area, which is energy per time per area, so

$$I = \frac{P}{A} = \frac{E}{At} = \frac{nhc}{\lambda At} = \frac{hc}{\lambda A}\frac{n}{t}.$$

But $R = n/t$ is the rate of photons per unit time. Since h and c are constants and I and A are equal for the two beams, we have $R_1/\lambda_1 = R_2/\lambda_2$, or

$$R_1/R_2 = \lambda_1/\lambda_2.$$

$\boxed{\textbf{E45-7}}$ (a) Since the power is the same, the bulb with the larger energy per photon will emits *fewer* photons per second. Since longer wavelengths have lower energies, the bulb emitting 700 nm must be giving off more photons per second.

(b) How many more photons per second? If E_1 is the energy per photon for one of the bulbs, then $N_1 = P/E_1$ is the number of photons per second emitted. The difference is then

$$N_1 - N_2 = \frac{P}{E_1} - \frac{P}{E_2} = \frac{P}{hc}(\lambda_1 - \lambda_2),$$

or

$$N_1 - N_2 = \frac{(130\,\mathrm{W})}{(6.63\times10^{-34}\mathrm{J\cdot s})(3.00\times10^{8}\mathrm{m/s})}((700\times10^{-9}\mathrm{m}) - (400\times10^{-9}\mathrm{m})) = 1.96\times10^{20}.$$

E45-8 Using the results of Exercise 45-1, the energy of one photon is

$$E = \frac{(1240\ \mathrm{eV\cdot nm})}{(630\ \mathrm{nm})} = 1.968\ \mathrm{eV},$$

The total *light* energy given off by the bulb is

$$E_\mathrm{t} = Pt = (0.932)(70\,\mathrm{W})(730\ \mathrm{hr})(3600\ \mathrm{s/hr}) = 1.71\times10^{8}\mathrm{J}.$$

The number of photons is

$$n = \frac{E_\mathrm{t}}{E_0} = \frac{(1.71\times10^{8}\mathrm{J})}{(1.968\ \mathrm{eV})(1.6\times10^{-19}\mathrm{J/eV})} = 5.43\times10^{26}.$$

E45-9 Apply Wien's law, Eq. 45-4, $\lambda_\mathrm{max}T = 2898\,\mu\mathrm{m\cdot K}$; so

$$T = \frac{(2898\,\mu\mathrm{m\cdot K})}{(32\times10^{-12}\mathrm{m})} = 91\times10^{6}\mathrm{K}.$$

Actually, the wavelength was supposed to be $32\,\mu$m. Then the temperature would be 91 K.

E45-10 Apply Wien's law, Eq. 45-4, $\lambda_\mathrm{max}T = 2898\,\mu\mathrm{m\cdot K}$; so

$$\lambda = \frac{(2898\,\mu\mathrm{m\cdot K})}{(0.0020\,\mathrm{K})} = 1.45\,\mathrm{m}.$$

This is in the radio region, near 207 on the FM dial.

E45-11 The wavelength of the maximum spectral radiancy is given by Wien's law, Eq. 45-4,

$$\lambda_\mathrm{max}T = 2898\,\mu\mathrm{m\cdot K}.$$

Applying to each temperature in turn,

(a) $\lambda = 1.06\times10^{-3}$m, which is in the microwave;
(b) $\lambda = 9.4\times10^{-6}$m, which is in the infrared;
(c) $\lambda = 1.6\times10^{-6}$m, which is in the infrared;
(d) $\lambda = 5.0\times10^{-7}$m, which is in the visible;
(e) $\lambda = 2.9\times10^{-10}$m, which is in the x-ray;
(f) $\lambda = 2.9\times10^{-41}$m, which is in a hard gamma ray.

E45-12 (a) Apply Wien's law, Eq. 45-4, $\lambda_{\max}T = 2898\,\mu\text{m}\cdot\text{K}$.; so

$$\lambda = \frac{(2898\,\mu\text{m}\cdot\text{K})}{(5800\,\text{K})} = 5.00\times10^{-7}\text{m}.$$

That's blue-green.

(b) Apply Wien's law, Eq. 45-4, $\lambda_{\max}T = 2898\,\mu\text{m}\cdot\text{K}$.; so

$$T = \frac{(2898\,\mu\text{m}\cdot\text{K})}{(550\times10^{-9}\text{m})} = 5270\,\text{K}.$$

E45-13 $I = \sigma T^4$ and $P = IA$. Then

$$P = (5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)(1900\,\text{K})^4\pi(0.5\times10^{-3}\text{m})^2 = 0.58\,\text{W}.$$

E45-14 Since $I \propto T^4$, doubling T results in a $2^4 = 16$ times increase in I. Then the new power level is

$$(16)(12.0\,\text{mW}) = 192\,\text{mW}.$$

$\boxed{\textbf{E45-15}}$ (a) We want to apply Eq. 45-6,

$$R(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}.$$

We know the ratio of the spectral radiancies at two different wavelengths. Dividing the above equation at the first wavelength by the same equation at the second wavelength,

$$3.5 = \frac{\lambda_1^5\left(e^{hc/\lambda_1 kT}-1\right)}{\lambda_2^5\left(e^{hc/\lambda_2 kT}-1\right)},$$

where $\lambda_1 = 200$ nm and $\lambda_2 = 400$ nm. We can considerably simplify this expression if we let

$$x = e^{hc/\lambda_2 kT},$$

because since $\lambda_2 = 2\lambda_1$ we would have

$$e^{hc/\lambda_1 kT} = e^{2hc/\lambda_2 kT} = x^2.$$

Then we get

$$3.5 = \left(\frac{1}{2}\right)^5\frac{x^2-1}{x-1} = \frac{1}{32}(x+1).$$

We will use the results of Exercise 45-1 for the exponents and then rearrange to get

$$T = \frac{hc}{\lambda_1 k\ln(111)} = \frac{(3.10\text{ eV})}{(8.62\times10^{-5}\text{ eV/K})\ln(111)} = 7640\,\text{K}.$$

(b) The method is the same, except that instead of 3.5 we have 1/3.5; this means the equation for x is

$$\frac{1}{3.5} = \frac{1}{32}(x+1),$$

with solution $x = 8.14$, so then

$$T = \frac{hc}{\lambda_1 k\ln(8.14)} = \frac{(3.10\text{ eV})}{(8.62\times10^{-5}\text{ eV/K})\ln(8.14)} = 17200\,\text{K}.$$

E45-16 $hf = \phi$, so

$$f = \frac{\phi}{h} = \frac{(5.32 \text{ eV})}{(4.14\times 10^{-15}\text{eV}\cdot\text{s})} = 1.28\times 10^{15}\text{Hz}.$$

E45-17 We'll use the results of Exercise 45-1. Visible red light has an energy of

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(650 \text{ nm})} = 1.9 \text{ eV}.$$

The substance must have a work function *less* than this to work with red light. This means that only cesium will work with red light. Visible blue light has an energy of

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(450 \text{ nm})} = 2.75 \text{ eV}.$$

This means that barium, lithium, and cesium will work with blue light.

E45-18 Since $K_\text{m} = hf - \phi$,

$$K_\text{m} = (4.14\times 10^{-15}\text{eV}\cdot\text{s})(3.19\times 10^{15}\text{Hz}) - (2.33 \text{ eV}) = 10.9 \text{ eV}.$$

$\boxed{\textbf{E45-19}}$ (a) Use the results of Exercise 45-1 to find the energy of the corresponding photon,

$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(678 \text{ nm})} = 1.83 \text{ eV}.$$

Since this energy is less than than the minimum energy required to remove an electron then the photo-electric effect will not occur.

(b) The cut-off wavelength is the longest possible wavelength of a photon that will still result in the photo-electric effect occurring. That wavelength is

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{E} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(2.28 \text{ eV})} = 544 \text{ nm}.$$

This would be visible as green.

E45-20 (a) Since $K_\text{m} = hc/\lambda - \phi$,

$$K_\text{m} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(200 \text{ nm})}(4.2 \text{ eV}) = 2.0 \text{ eV}.$$

(b) The minimum kinetic energy is zero; the electron just barely makes it off the surface.

(c) $V_\text{s} = K_\text{m}/q = 2.0\,\text{V}$.

(d) The cut-off wavelength is the longest possible wavelength of a photon that will still result in the photo-electric effect occurring. That wavelength is

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{E} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(4.2 \text{ eV})} = 295 \text{ nm}.$$

E45-21 $K_\text{m} = qV_\text{s} = 4.92$ eV. But $K_\text{m} = hc/\lambda - \phi$, so

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{(4.92 \text{ eV} + 2.28 \text{ eV})} = 172 \text{ nm}.$$

E45-22 (a) $K_\mathrm{m} = qV_\mathrm{s}$ and $K_\mathrm{m} = hc/\lambda - \phi$. We have two different values for qV_s and λ, so subtracting this equation from itself yields

$$q(V_{\mathrm{s},1} - V_{\mathrm{s},2}) = hc/\lambda_1 - hc/\lambda_2.$$

Solving for λ_2,

$$\begin{aligned}\lambda_2 &= \frac{hc}{hc/\lambda_1 - q(V_{\mathrm{s},1} - V_{\mathrm{s},2})},\\ &= \frac{(1240 \text{ eV} \cdot \text{nm})}{(1240 \text{ eV} \cdot \text{nm})/(491 \text{ nm}) - (0.710 \text{ eV}) + (1.43 \text{ eV})},\\ &= 382 \text{ nm}.\end{aligned}$$

(b) $K_\mathrm{m} = qV_\mathrm{s}$ and $K_\mathrm{m} = hc/\lambda - \phi$, so

$$\phi = (1240 \text{ eV} \cdot \text{nm})/(491 \text{ nm}) - (0.710 \text{ eV}) = 1.82 \text{ eV}.$$

$\boxed{\textbf{E45-23}}$ (a) The stopping potential is given by Eq. 45-11,

$$V_0 = \frac{h}{e}f - \frac{\phi}{e},$$

so

$$V_0 = \frac{(1240 \text{ eV} \cdot \text{nm})}{e(410 \text{ nm}} - \frac{(1.85 \text{ eV}}{e} = 1.17\,\text{V}.$$

(b) These are *not* relativistic electrons, so

$$v = \sqrt{2K/m} = c\sqrt{2K/mc^2} = c\sqrt{2(1.17 \text{ eV})/(0.511\times10^6 \text{ eV})} = 2.14\times10^{-3}c,$$

or $v = 64200\,\text{m/s}$.

E45-24 It will have become the stopping potential, or

$$V_0 = \frac{h}{e}f - \frac{\phi}{e},$$

so

$$V_0 = \frac{(4.14\times10^{-15}\text{eV} \cdot \text{m})}{(1.0e)}(6.33\times10^{14}/\text{s}) - \frac{(2.49 \text{ eV})}{(1.0e)} = 0.131\,\text{V}.$$

E45-25

E45-26 (a) Using the results of Exercise 45-1,

$$\lambda = \frac{(1240 \text{ eV} \cdot \text{nm})}{(20\times10^3 \text{ eV})} = 62 \text{ pm}.$$

(b) This is in the x-ray region.

E45-27 (a) Using the results of Exercise 45-1,

$$E = \frac{(1240 \text{ eV} \cdot \text{nm})}{(41.6\times10^{-3} \text{ nm})} = 29,800 \text{ eV}.$$

(b) $f = c/\lambda = (3\times10^8\text{m/s})/(41.6 \text{ pm}) = 7.21\times10^{18}/\text{s}$.
(c) $p = E/c = 29,800 \text{ eV}/c = 2.98\times10^4 \text{ eV}/c$.

E45-28 (a) $E = hf$, so

$$f = \frac{(0.511\times10^6\text{eV})}{(4.14\times10^{-15}\text{eV}\cdot\text{s})} = 1.23\times10^{20}/\text{s}.$$

(b) $\lambda = c/f = (3\times10^8\text{m/s})/(1.23\times10^{20}/\text{s}) = 2.43$ pm.
(c) $p = E/c = (0.511\times10^6\text{eV})/c$.

$\boxed{\textbf{E45-29}}$ The initial momentum of the system is the momentum of the photon, $p = h/\lambda$. This momentum is imparted to the sodium atom, so the final speed of the sodium is $v = p/m$, where m is the mass of the sodium. Then

$$v = \frac{h}{\lambda m} = \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})}{(589\times10^{-9}\text{m})(23)(1.7\times10^{-27}\text{kg})} = 2.9 \text{ cm/s}.$$

E45-30 (a) $\lambda_\text{C} = h/mc = hc/mc^2$, so

$$\lambda_\text{C} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(0.511\times10^6\text{eV})} = 2.43 \text{ pm}.$$

(c) Since $E_\lambda = hf = hc/\lambda$, and $\lambda = h/mc = hc/mc^2$, then

$$E_\lambda = hc/\lambda = mc^2.$$

(b) See part (c).

E45-31 The change in the wavelength of a photon during Compton scattering is given by Eq. 45-17,

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi).$$

We'll use the results of Exercise 45-30 to save some time, and let $h/mc = \lambda_\text{C}$, which is 2.43 pm.

(a) For $\phi = 35°$,

$$\lambda' = (2.17 \text{ pm}) + (2.43 \text{ pm})(1-\cos 35°) = 2.61 \text{ pm}.$$

(b) For $\phi = 115°$,

$$\lambda' = (2.17 \text{ pm}) + (2.43 \text{ pm})(1-\cos 115°) = 5.63 \text{ pm}.$$

E45-32 (a) We'll use the results of Exercise 45-1:

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{(0.511\times10^6\text{eV})} = 2.43 \text{ pm}.$$

(b) The change in the wavelength of a photon during Compton scattering is given by Eq. 45-17,

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi).$$

We'll use the results of Exercise 45-30 to save some time, and let $h/mc = \lambda_\text{C}$, which is 2.43 pm.

$$\lambda' = (2.43 \text{ pm}) + (2.43 \text{ pm})(1-\cos 72°) = 4.11 \text{ pm}.$$

(c) We'll use the results of Exercise 45-1:

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(4.11 \text{ pm})} = 302 \text{ keV}.$$

$\boxed{\textbf{E45-33}}$ The change in the wavelength of a photon during Compton scattering is given by Eq. 45-17,

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi).$$

We are *not* using the expression with the form $\Delta\lambda$ because $\Delta\lambda$ and ΔE are *not* simply related.

The wavelength is related to frequency by $c = f\lambda$, while the frequency is related to the energy by Eq. 45-1, $E = hf$. Then

$$\begin{aligned}
\Delta E &= E - E' = hf - hf', \\
&= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right), \\
&= hc\frac{\lambda' - \lambda}{\lambda\,\lambda'}.
\end{aligned}$$

Into this last expression we substitute the Compton formula. Then

$$\Delta E = \frac{h^2}{m}\frac{(1 - \cos\phi)}{\lambda\,\lambda'}.$$

Now $E = hf = hc/\lambda$, and we can divide this on both sides of the above equation. Also, $\lambda' = c/f'$, and we can substitute this into the right hand side of the above equation. Both of these steps result in

$$\frac{\Delta E}{E} = \frac{hf'}{mc^2}(1 - \cos\phi).$$

Note that mc^2 is the rest energy of the scattering particle (usually an electron), while hf' is the energy of the scattered photon.

E45-34 The wavelength is related to frequency by $c = f\lambda$, while the frequency is related to the energy by Eq. 45-1, $E = hf$. Then

$$\begin{aligned}
\Delta E &= E - E' = hf - hf', \\
&= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right), \\
&= hc\frac{\lambda' - \lambda}{\lambda\,\lambda'}, \\
\frac{\Delta E}{E} &= \frac{\Delta\lambda}{\lambda + \Delta\lambda},
\end{aligned}$$

But $\Delta E/E = 3/4$, so

$$3\lambda + 3\Delta\lambda = 4\Delta\lambda,$$

or $\Delta\lambda = 3\lambda$.

E45-35 The maximum shift occurs when $\phi = 180°$, so

$$\Delta\lambda_{\mathrm{m}} = 2\frac{h}{mc} = 2\frac{(1240\ \mathrm{eV}\cdot\mathrm{nm})}{938\ \mathrm{MeV})} = 2.64\times10^{-15}\mathrm{m}.$$

E45-36 Since $E = hf$ frequency shifts are identical to energy shifts. Then we can use the results of Exercise 45-33 to get

$$(0.0001) = \frac{(0.9999)(6.2\ \mathrm{keV})}{(511\ \mathrm{keV})}(1 - \cos\phi),$$

which has solution $\phi = 7.4°$.

(b) $(0.0001)(6.2\ \mathrm{keV}) = 0.62\ \mathrm{eV}$.

E45-37 (a) The change in wavelength is independent of the wavelength and is given by Eq. 45-17,

$$\Delta\lambda = \frac{hc}{mc^2}(1-\cos\phi) = 2\frac{(1240\text{ eV}\cdot\text{nm})}{(0.511\times10^6\text{ eV})} = 4.85\times10^{-3}\text{ nm}.$$

(b) The change in energy is given by

$$\begin{aligned}\Delta E &= \frac{hc}{\lambda_\text{f}} - \frac{hc}{\lambda_\text{i}},\\ &= hc\left(\frac{1}{\lambda_\text{i}+\Delta\lambda} - \frac{1}{\lambda_\text{i}}\right),\\ &= (1240\text{ eV}\cdot\text{nm})\left(\frac{1}{(9.77\text{ pm})+(4.85\text{ pm})} - \frac{1}{(9.77\text{ pm})}\right) = -42.1\text{ keV}\end{aligned}$$

(c) This energy went to the electron, so the final kinetic energy of the electron is 42.1 keV.

E45-38 For $\phi = 90°$ $\Delta\lambda = h/mc$. Then

$$\begin{aligned}\frac{\Delta E}{E} &= 1-\frac{hf'}{hf} = 1-\frac{\lambda}{\lambda+\Delta\lambda},\\ &= \frac{h/mc}{\lambda+h/mc}.\end{aligned}$$

But $h/mc = 2.43$ pm for the electron (see Exercise 45-30).

(a) $\Delta E/E = (2.43\text{ pm})/(3.00\text{ cm}+2.43\text{ pm}) = 8.1\times10^{-11}$.
(b) $\Delta E/E = (2.43\text{ pm})/(500\text{ nm}+2.43\text{ pm}) = 4.86\times10^{-6}$.
(c) $\Delta E/E = (2.43\text{ pm})/(0.100\text{ nm}+2.43\text{ pm}) = 0.0237$.
(d) $\Delta E/E = (2.43\text{ pm})/(1.30\text{ pm}+2.43\text{ pm}) = 0.651$.

E45-39 We can use the results of Exercise 45-33 to get

$$(0.10) = \frac{(0.90)(215\text{ keV})}{(511\text{ keV})}(1-\cos\phi),$$

which has solution $\phi = 42/6°$.

E45-40 (a) A crude estimate is that the photons can't arrive more frequently than once every $10-8$s. That would provide an emission rate of 10^8/s.

(b) The power output would be

$$P = (10^8)\frac{(1240\text{ eV}\cdot\text{nm})}{(550\text{ nm})} = 2.3\times10^8\text{eV/s},$$

which is 3.6×10^{-11} W!

E45-41 We can follow the example of Sample Problem 45-6, and apply

$$\lambda = \lambda_0(1-v/c).$$

(a) Solving for λ_0,

$$\lambda_0 = \frac{(588.995\text{ nm})}{(1-(-300\,\text{m/s})(3\times10^8\text{m/s})} = 588.9944\text{ nm}.$$

(b) Applying Eq. 45-18,

$$\Delta v = -\frac{h}{m\lambda} = -\frac{(6.6\times10^{-34}\,\mathrm{J\cdot s})}{(22)(1.7\times10^{-27}\mathrm{kg})(590\times10^{-9}\mathrm{m})} = 3\times10^{-2}\mathrm{m/s}.$$

(c) Emitting another photon will slow the sodium by about the same amount.

E45-42 (a) $(430\,\mathrm{m/s})/(0.15\,\mathrm{m/s}) \approx 2900$ interactions.

(b) If the argon averages a speed of 220 m/s, then it requires interactions at the rate of

$$(2900)(220\,\mathrm{m/s})/(1.0\,\mathrm{m}) = 6.4\times10^5/\mathrm{s}$$

if it is going to slow down in time.

P45-1 The radiant intensity is given by Eq. 45-3, $I = \sigma T^4$. The power that is radiated through the opening is $P = IA$, where A is the area of the opening. But energy goes both ways through the opening; it is the *difference* that will give the net power transfer. Then

$$P_{\mathrm{net}} = (I_0 - I_1)A = \sigma A\left(T_0^4 - T_1^4\right).$$

Put in the numbers, and

$$P_{\mathrm{net}} = (5.67\times10^{-8}\mathrm{W/m^2\cdot K^4})(5.20\times10^{-4}\mathrm{m^2})\left((488\,\mathrm{K})^4 - (299\,\mathrm{K})^4\right) = 1.44\,\mathrm{W}.$$

P45-2 (a) $I = \sigma T^4$ and $P = IA$. Then $T^4 = P/\sigma A$, or

$$T = \sqrt[4]{\frac{(100\,\mathrm{W})}{(5.67\times10^{-8}\mathrm{W/m^2\cdot K^4})\pi(0.28\times10^{-3}\mathrm{m})(1.8\times10^{-2}\mathrm{m})}} = 3248\,\mathrm{K}.$$

That's 2980°C.

(b) The rate that energy is radiated off is given by $dQ/dt = mC\,dT/dt$. The mass is found from $m = \rho V$, where V is the volume. This can be combined with the power expression to yield

$$\sigma A T^4 = -\rho V C dT/dt,$$

which can be integrated to yield

$$\Delta t = \frac{\rho V C}{3\sigma A}(1/T_2^3 - 1/T_1^3).$$

Putting in numbers,

$$\begin{aligned}\Delta t &= \frac{(19300\mathrm{kg/m^3})(0.28\times10^{-3}\mathrm{m})(132\mathrm{J/kgC})}{3(5.67\times10^{-8}\mathrm{W/m^2\cdot K^4})(4)}[1/(2748\,\mathrm{K})^3 - 1/(3248\,\mathrm{K})^3],\\ &= 20\,\mathrm{ms}.\end{aligned}$$

P45-3 Light from the sun will "heat-up" the thin black screen. As the temperature of the screen increases it will begin to radiate energy. When the rate of energy radiation from the screen is equal to the rate at which the energy from the sun strikes the screen we will have equilibrium. We need first to find an expression for the rate at which energy from the sun strikes the screen.

The temperature of the sun is T_{S}. The radiant intensity is given by Eq. 45-3, $I_{\mathrm{S}} = \sigma {T_{\mathrm{S}}}^4$. The total power radiated by the sun is the product of this radiant intensity and the surface area of the sun, so

$$P_{\mathrm{S}} = 4\pi {r_{\mathrm{S}}}^2 \sigma {T_{\mathrm{S}}}^4,$$

where r_S is the radius of the sun.

Assuming that the lens is on the surface of the Earth (a reasonable assumption), then we can find the power incident on the lens if we know the intensity of sunlight at the distance of the Earth from the sun. That intensity is

$$I_E = \frac{P_S}{A} = \frac{P_S}{4\pi {R_E}^2},$$

where R_E is the radius of the Earth's orbit. Combining,

$$I_E = \sigma {T_S}^4 \left(\frac{r_S}{R_E}\right)^2$$

The total power incident on the lens is then

$$P_{\text{lens}} = I_E A_{\text{lens}} = \sigma {T_S}^4 \left(\frac{r_S}{R_E}\right)^2 \pi {r_l}^2,$$

where r_l is the radius of the lens. All of the energy that strikes the lens is focused on the image, so the power incident on the lens is also incident on the image.

The screen radiates as the temperature increases. The radiant intensity is $I = \sigma T^4$, where T is the temperature of the screen. The power radiated is this intensity times the surface area, so

$$P = IA = 2\pi {r_i}^2 \sigma T^4.$$

The factor of "2" is because the screen has two sides, while r_i is the radius of the image. Set this equal to P_{lens},

$$2\pi {r_i}^2 \sigma T^4 = \sigma {T_S}^4 \left(\frac{r_S}{R_E}\right)^2 \pi {r_l}^2,$$

or

$$T^4 = \frac{1}{2} {T_S}^4 \left(\frac{r_S\, r_l}{R_E\, r_i}\right)^2.$$

The radius of the image of the sun divided by the radius of the sun is the magnification of the lens. But magnification is also related to image distance divided by object distance, so

$$\frac{r_i}{r_S} = |m| = \frac{i}{o},$$

Distances should be measured from the lens, but since the sun is so far from the Earth, we won't be far off in stating $o \approx R_E$. Since the object is so far from the lens, the image will be very, very close to the focal point, so we can also state $i \approx f$. Then

$$\frac{r_i}{r_S} = \frac{f}{R_E},$$

so the expression for the temperature of the thin black screen is considerably simplified to

$$T^4 = \frac{1}{2} {T_S}^4 \left(\frac{r_l}{f}\right)^2.$$

Now we can put in some of the numbers.

$$T = \frac{1}{2^{1/4}} (5800\,\text{K}) \sqrt{\frac{(1.9\ \text{cm})}{(26\ \text{cm})}} = 1300\,\text{K}.$$

P45-4 The derivative of R with respect to λ is

$$-10\,\frac{\pi\,c^2\,h}{\lambda^6\,(e^{(\frac{h\,c}{\lambda\,k\,T})}-1)}+\frac{2\,\pi\,c^3\,h^2\,e^{(\frac{h\,c}{\lambda\,k\,T})}}{\lambda^7\,(e^{(\frac{h\,c}{\lambda\,k\,T})}-1)^2\,k\,T}.$$

Ohh, that's ugly. Setting it equal to zero allows considerable simplification, and we are left with

$$(5-x)e^x = 5,$$

where $x = hc/\lambda kT$. The solution is found numerically to be $x = 4.965114232$. Then

$$\lambda = \frac{(1240\text{ eV}\cdot\text{nm})}{(4.965)(8.62\times 10^{-5}\text{eV/K})T} = \frac{2.898\times 10^{-3}\text{m}\cdot\text{K}}{T}.$$

$\boxed{\textbf{P45-5}}$ (a) If the planet has a temperature T, then the radiant intensity of the planet will be $I\sigma T^4$, and the rate of energy radiation from the planet will be

$$P = 4\pi R^2\sigma T^4,$$

where R is the radius of the planet.

A steady state planet temperature requires that the energy from the sun arrive at the same rate as the energy is radiated from the planet. The intensity of the energy from the sun a distance r from the sun is

$$P_{\text{sun}}/4\pi r^2,$$

and the total power incident on the planet is then

$$P = \pi R^2\frac{P_{\text{sun}}}{4\pi r^2}.$$

Equating,

$$\begin{aligned} 4\pi R^2\sigma T^4 &= \pi R^2\frac{P_{\text{sun}}}{4\pi r^2}, \\ T^4 &= \frac{P_{\text{sun}}}{16\pi\sigma r^2}. \end{aligned}$$

(b) Using the last equation and the numbers from Problem 3,

$$T = \frac{1}{\sqrt{2}}(5800\,\text{K})\sqrt{\frac{(6.96\times 10^8\text{m})}{(1.5\times 10^{11}\text{m})}} = 279\,\text{K}.$$

That's about 43° F.

P45-6 (a) Change variables as suggested, then $\lambda = hc/xkT$ and $d\lambda = -(hc/x^2kT)dx$. Integrate (note the swapping of the variables of integration picks up a minus sign):

$$\begin{aligned} I &= \int \frac{2\pi c^2 h}{(hc/xkT)^5}\frac{(hc/x^2kT)dx}{e^x-1}, \\ &= \frac{2\pi k^4T^4}{h^3c^2}\int\frac{x^3\,dx}{e^x-1}, \\ &= \frac{2\pi^5k^4}{15h^3c^2}T^4. \end{aligned}$$

P45-7 (a) $P = E/t = nhf/t = (hc/\lambda)(n/t)$, where n/t is the rate of photon emission. Then

$$n/t = \frac{(100\,\text{W})(589\times10^{-9}\text{m})}{(6.63\times10^{-34}\text{J}\cdot\text{s})(3.00\times10^{8}\text{m/s})} = 2.96\times10^{20}/\text{s}.$$

(b) The flux at a distance r is the rate divided by the area of the sphere of radius r, so

$$r = \sqrt{\frac{(2.96\times10^{20}/\text{s})}{4\pi(1\times10^{4}/\text{m}^2\cdot\text{s})}} = 4.8\times10^{7}\text{m}.$$

(c) The photon density is the flux divided by the speed of light; the distance is then

$$r = \sqrt{\frac{(2.96\times10^{20}/\text{s})}{4\pi(1\times10^{6}/\text{m}^3)(3\times10^{8}\text{m/s})}} = 280\,\text{m}.$$

(d) The flux is given by

$$\frac{(2.96\times10^{20}/\text{s})}{4\pi(2.0\,\text{m})^2} = 5.89\times10^{18}/\text{m}^2\cdot\text{s}.$$

The photon density is

$$(5.89\times10^{18}\text{m}^2\cdot\text{s})/(3.00\times10^{8}\text{m/s}) = 1.96\times10^{10}/\text{m}^3.$$

P45-8 Momentum conservation requires

$$p_\lambda = p_e,$$

while energy conservation requires

$$E_\lambda + mc^2 = E_e.$$

Square both sides of the energy expression and

$$\begin{aligned} E_\lambda^2 + 2E_\lambda mc^2 + m^2c^4 &= E_e^2 = p_e^2c^2 + m^2c^4, \\ E_\lambda^2 + 2E_\lambda mc^2 &= p_e^2c^2, \\ p_\lambda^2c^2 + 2E_\lambda mc^2 &= p_e^2c^2. \end{aligned}$$

But the momentum expression can be used here, and the result is

$$2E_\lambda mc^2 = 0.$$

Not likely.

P45-9 (a) Since $qvB = mv^2/r$, $v = (q/m)rB$ The kinetic energy of (non-relativistic) electrons will be

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{q^2(rB)^2}{m},$$

or

$$K = \frac{1}{2}\frac{(1.6\times10^{-19}\text{C})}{(9.1\times10^{-31}\text{kg})}(188\times10^{-6}\text{T}\cdot\text{m})^2 = 3.1\times10^{3}\ \text{eV}.$$

(b) Use the results of Exercise 45-1,

$$\phi = \frac{(1240\ \text{eV}\cdot\text{nm})}{(71\times10^{-3}\text{nm})} - 3.1\times10^{3}\text{eV} = 1.44\times10^{4}\ \text{eV}.$$

P45-10

$\boxed{\textbf{P45-11}}$ (a) The maximum value of $\Delta\lambda$ is $2h/mc$. The maximum energy lost by the photon is then

$$\begin{aligned} \Delta E &= \frac{hc}{\lambda_\mathrm{f}} - \frac{hc}{\lambda_\mathrm{i}}, \\ &= hc\left(\frac{1}{\lambda_\mathrm{i} + \Delta\lambda} - \frac{1}{\lambda_\mathrm{i}}\right), \\ &= hc\frac{-2h/mc}{\lambda(\lambda + 2h/mc)}, \end{aligned}$$

where in the last line we wrote λ for λ_i. The energy given to the electron is the negative of this, so

$$K_\mathrm{max} = \frac{2h^2}{m\lambda(\lambda + 2h/mc)}.$$

Multiplying through by $1^2 = (E\lambda/hc)^2$ we get

$$K_\mathrm{max} = \frac{2E^2}{mc^2(1 + 2hc/\lambda mc^2)}.$$

or

$$K_\mathrm{max} = \frac{E^2}{mc^2/2 + E}.$$

(b) The answer is

$$K_\mathrm{max} = \frac{(17.5\ \mathrm{keV})^2}{(511\ \mathrm{eV})/2 + (17.5\ \mathrm{keV})} = 1.12\ \mathrm{keV}.$$

$\boxed{\textbf{E46-1}}$ (a) Apply Eq. 46-1, $\lambda = h/p$. The momentum of the bullet is

$$p = mv = (0.041\,\text{kg})(960\,\text{m/s}) = 39\text{kg}\cdot\text{m/s},$$

so the corresponding wavelength is

$$\lambda = h/p = (6.63\times10^{-34}\text{J}\cdot\text{s})/(39\text{kg}\cdot\text{m/s}) = 1.7\times10^{-35}\,\text{m}.$$

(b) This length is much too small to be significant. How much too small? If the radius of the galaxy were one meter, this distance would correspond to the diameter of a proton.

E46-2 (a) $\lambda = h/p$ and $p^2/2m = K$, then

$$\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{(1240\text{ eV}\cdot\text{nm})}{\sqrt{2(511\text{ keV})}\sqrt{K}} = \frac{1.226\text{ nm}}{\sqrt{K}}.$$

(b) $K = eV$, so

$$\lambda = \frac{1.226\text{ nm}}{\sqrt{eV}} = \sqrt{\frac{1.5\text{ V}}{V}}\text{nm}.$$

E46-3 For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$.

(a) For the electron,

$$\lambda = \frac{(1240\text{ eV}\cdot\text{nm})}{\sqrt{2(511\text{ keV})(1.0\text{ keV})}} = 0.0388\text{ nm}.$$

(c) For the neutron,

$$\lambda = \frac{(1240\text{ MeV}\cdot\text{fm})}{\sqrt{2(940\text{ MeV})(0.001\text{ MeV})}} = 904\text{ fm}.$$

(b) For ultra-relativistic particles $K \approx E \approx pc$, so

$$\lambda = \frac{hc}{E} = \frac{(1240\text{ eV}\cdot\text{nm})}{(1000\text{ eV})} = 1.24\text{ nm}.$$

E46-4 For non-relativistic particles $p = h/\lambda$ and $p^2/2m = K$, so $K = (hc)^2/2mc^2\lambda^2$. Then

$$K = \frac{(1240\text{ eV}\cdot\text{nm})^2}{2(5.11\times10^6\text{eV})(589\text{ nm})^2} = 4.34\times10^{-6}\text{ eV}.$$

$\boxed{\textbf{E46-5}}$ (a) Apply Eq. 46-1, $p = h/\lambda$. The proton speed would then be

$$v = \frac{h}{m\lambda} = c\frac{hc}{mc^2\lambda} = c\frac{(1240\text{ MeV}\cdot\text{fm})}{(938\text{ MeV})(113\text{ fm})} = 0.0117c.$$

This is good, because it means we were justified in using the non-relativistic equations. Then $v = 3.51\times10^6\text{m/s}$.

(b) The kinetic energy of this electron would be

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(938\text{ MeV})(0.0117)^2 = 64.2\text{ keV}.$$

The potential through which it would need to be accelerated is 64.2 kV.

E46-6 (a) $K = qV$ and $p = \sqrt{2mK}$. Then

$$p = \sqrt{2(22)(932\ \text{MeV}/c^2)(325\ \text{eV})} = 3.65\times10^6\ \text{eV}/c.$$

(b) $\lambda = h/p$, so

$$\lambda = \frac{hc}{pc} = \frac{(1240\ \text{eV}\cdot\text{nm})}{(3.65\times10^6\ \text{eV}/c)c} = 3.39\times10^{-4}\text{nm}.$$

E46-7 (a) For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$. For the alpha particle,

$$\lambda = \frac{(1240\ \text{MeV}\cdot\text{fm})}{\sqrt{2(4)(932\ \text{MeV})(7.5\ \text{MeV})}} = 5.2\ \text{fm}.$$

(b) Since the wavelength of the alpha is considerably smaller than the distance to the nucleus we can ignore the wave nature of the alpha particle.

E46-8 (a) For non-relativistic particles $p = h/\lambda$ and $p^2/2m = K$, so $K = (hc)^2/2mc^2\lambda^2$. Then

$$K = \frac{(1240\ \text{keV}\cdot\text{pm})^2}{2(511\text{keV})(10\ \text{pm})^2} = 15\ \text{keV}.$$

(b) For ultra-relativistic particles $K \approx E \approx pc$, so

$$E = \frac{hc}{\lambda} = \frac{(1240\ \text{keV}\cdot\text{pm})}{(10\ \text{pm})} = 124\ \text{keV}.$$

E46-9 The relativistic relationship between energy and momentum is

$$E^2 = p^2c^2 + m^2c^4,$$

and if the energy is very large (compared to mc^2), then the contribution of the mass to the above expression is small, and

$$E^2 \approx p^2c^2.$$

Then from Eq. 46-1,

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{E} = \frac{(1240\ \text{MeV}\cdot fm)}{(50\times10^3\ \text{MeV})} = 2.5\times10^{-2}\ \text{fm}.$$

E46-10 (a) $K = 3kT/2$, $p = \sqrt{2mK}$, and $\lambda = h/p$, so

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{3mkT}} = \frac{hc}{\sqrt{3mc^2kT}},\\ &= \frac{(1240\ \text{MeV}\cdot fm)}{\sqrt{3(4)(932\text{MeV})(8.62\times10^{-11}\text{MeV/K})(291\,\text{K})}} = 74\ \text{pm}.\end{aligned}$$

(b) $pV = NkT$; assuming that each particle occupies a cube of volume $d^3 = V_0$ then the inter-particle spacing is d, so

$$d = \sqrt[3]{V/N} = \sqrt[3]{\frac{(1.38\times10^{-23}\text{J/K})(291\,\text{K})}{(1.01\times10^5\text{Pa})}} = 3.4\ \text{nm}.$$

E46-11 $p = mv$ and $p = h/\lambda$, so $m = h/\lambda v$. Taking the ratio,

$$\frac{m_e}{m} = \frac{\lambda v}{\lambda_e v_e} = (1.813\times10^{-4})(3) = 5.439\times10^{-4}.$$

The mass of the unknown particle is then

$$m = \frac{(0.511\ \text{MeV}/c^2)}{(5.439\times10^{-4})} = 939.5\ \text{MeV}.$$

That would make it a neutron.

E46-12 (a) For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$.

For the electron,

$$\lambda = \frac{(1240\ \text{eV}\cdot\text{nm})}{\sqrt{2(5.11\times10^5\ \text{eV})(1.5\ \text{eV})}} = 1.0\ \text{nm}.$$

For ultra-relativistic particles $K \approx E \approx pc$, so for the photon

$$\lambda = \frac{hc}{E} = \frac{(1240\ \text{eV}\cdot\text{nm})}{(1.5\ \text{eV})} = 830\ \text{nm}.$$

(b) Electrons with energies that high are ultra-relativistic. Both the photon and the electron will then have the same wavelength;

$$\lambda = \frac{hc}{E} = \frac{(1240\ \text{MeV}\cdot\text{fm})}{(1.5\ \text{GeV})} = 0.83\ \text{fm}.$$

$\boxed{\textbf{E46-13}}$ (a) The classical expression for kinetic energy is

$$p = \sqrt{2mK},$$

so

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2K}} = \frac{(1240\ \text{keV}\cdot pm)}{\sqrt{2(511\ \text{keV})(25.0\ \text{keV})}} = 7.76\ \text{pm}.$$

(a) The relativistic expression for momentum is

$$pc = sqrtE^2 - m^2c^4 = \sqrt{(mc^2+K)^2 - m^2c^4} = \sqrt{K^2 + 2mc^2K}.$$

Then

$$\lambda = \frac{hc}{pc} = \frac{(1240\ \text{keV}\cdot pm)}{\sqrt{(25.0\ \text{keV})^2 + 2(511\ \text{keV})(25.0\ \text{keV})}} = 7.66\ \text{pm}.$$

E46-14 We want to match the wavelength of the gamma to that of the electron. For the gamma, $\lambda = hc/E_\gamma$. For the electron, $K = p^2/2m = h^2/2m\lambda^2$. Combining,

$$K = \frac{h^2}{2mh^2c^2}E_\gamma^2 = \frac{E_\gamma^2}{2mc^2}.$$

With numbers,

$$K = \frac{(136\text{keV})^2}{2(511\ \text{keV})} = 18.1\ \text{keV}.$$

That would require an accelerating potential of 18.1 kV.

E46-15 First find the wavelength of the neutrons. For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$. Then

$$\lambda = \frac{(1240 \text{ keV} \cdot \text{pm})}{\sqrt{2(940\times 10^3 \text{ keV})(4.2\times 10^{-3} \text{ keV})}} = 14 \text{ pm}.$$

Bragg reflection occurs when $2d \sin\theta = \lambda$, so

$$\theta = \arcsin(14 \text{ pm})/2(73.2 \text{ pm}) = 5.5^\circ.$$

E46-16 This is merely a Bragg reflection problem. Then $2d \sin\theta = m\lambda$, or

$$\begin{aligned}\theta &= \arcsin(1)(11 \text{ pm})/2(54.64 \text{ pm}) = 5.78^\circ,\\ \theta &= \arcsin(2)(11 \text{ pm})/2(54.64 \text{ pm}) = 11.6^\circ,\\ \theta &= \arcsin(3)(11 \text{ pm})/2(54.64 \text{ pm}) = 17.6^\circ.\end{aligned}$$

$\boxed{\textbf{E46-17}}$ (a) Since $\sin 52^\circ = 0.78$, then $2(\lambda/d) = 1.57 > 1$, so there is no diffraction order other than the first.

(b) For an accelerating potential of 54 volts we have $\lambda/d = 0.78$. Increasing the potential will increase the kinetic energy, increase the momentum, and decrease the wavelength. d won't change. The kinetic energy is increased by a factor of $60/54 = 1.11$, the momentum increases by a factor of $\sqrt{1.11} = 1.05$, so the wavelength changes by a factor of $1/1.05 = 0.952$. The new angle is then

$$\theta = \arcsin(0.952 \times 0.78) = 48^\circ.$$

E46-18 First find the wavelength of the electrons. For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$. Then

$$\lambda = \frac{(1240 \text{ keV} \cdot \text{pm})}{\sqrt{2(511 \text{ keV})(0.380 \text{ keV})}} = 62.9 \text{ pm}.$$

This is now a Bragg reflection problem. Then $2d \sin\theta = m\lambda$, or

$$\begin{aligned}\theta &= \arcsin(1)(62.9 \text{ pm})/2(314 \text{ pm}) = 5.74^\circ,\\ \theta &= \arcsin(2)(62.9 \text{ pm})/2(314 \text{ pm}) = 11.6^\circ,\\ \theta &= \arcsin(3)(62.9 \text{ pm})/2(314 \text{ pm}) = 17.5^\circ,\\ \theta &= \arcsin(4)(62.9 \text{ pm})/2(314 \text{ pm}) = 23.6^\circ,\\ \theta &= \arcsin(5)(62.9 \text{ pm})/2(314 \text{ pm}) = 30.1^\circ,\\ \theta &= \arcsin(6)(62.9 \text{ pm})/2(314 \text{ pm}) = 36.9^\circ,\\ \theta &= \arcsin(7)(62.9 \text{ pm})/2(314 \text{ pm}) = 44.5^\circ,\\ \theta &= \arcsin(8)(62.9 \text{ pm})/2(314 \text{ pm}) = 53.3^\circ,\\ \theta &= \arcsin(9)(62.9 \text{ pm})/2(314 \text{ pm}) = 64.3^\circ.\end{aligned}$$

But the odd orders vanish (see Chapter 43 for a discussion on this).

E46-19 Since $\Delta f \cdot \Delta t \approx 1/2\pi$, we have

$$\Delta f = 1/2\pi(0.23\,\text{s}) = 0.69/\text{s}.$$

E46-20 Since $\Delta f \cdot \Delta t \approx 1/2\pi$, we have

$$\Delta f = 1/2\pi(0.10\times 10^{-9}\text{s}) = 1.6\times 10^{10}/\text{s}.$$

The bandwidth wouldn't fit in the frequency allocation!

$\boxed{\textbf{E46-21}}$ Apply Eq. 46-9,

$$\Delta E \geq \frac{h}{2\pi\Delta t} = \frac{4.14\times 10^{-15}\ \text{eV}\cdot\text{s})}{2\pi(8.7\times 10^{-12}\text{s})} = 7.6\times 10^{-5}\ \text{eV}.$$

This is *much* smaller than the photon energy.

E46-22 Apply Heisenberg twice:

$$\Delta E_1 = \frac{4.14\times 10^{-15}\ \text{eV}\cdot\text{s})}{2\pi(12\times 10^{-9}\text{s})} = 5.49\times 10^{-8}\ \text{eV}.$$

and

$$\Delta E_2 = \frac{4.14\times 10^{-15}\ \text{eV}\cdot\text{s})}{2\pi(23\times 10^{-9}\text{s})} = 2.86\times 10^{-8}\ \text{eV}.$$

The sum is $\Delta E_{\text{transition}} = 8.4\times 10^{-8}\text{eV}$.

E46-23 Apply Heisenberg:

$$\Delta p = \frac{6.63\times 10^{-34}\text{J}\cdot\text{s})}{2\pi(12\times 10^{-12}\text{m})} = 8.8\times 10^{-24}\text{kg}\cdot\text{m/s}.$$

E46-24 $\Delta p = (0.5\,\text{kg})(1.2\,\text{s}) = 0.6\,\text{kg}\cdot\text{m/s}$. The position uncertainty would then be

$$\Delta x = \frac{(0.6\,\text{J/s})}{2\pi(0.6\,\text{kg}\cdot\text{m/s})} = 0.16\,\text{m}.$$

$\boxed{\textbf{E46-25}}$ We want $v \approx \Delta v$, which means $p \approx \Delta p$. Apply Eq. 46-8, and

$$\Delta x \geq \frac{h}{2\pi\Delta p} \approx \frac{h}{2\pi p}.$$

According to Eq. 46-1, the de Broglie wavelength is related to the momentum by

$$\lambda = h/p,$$

so

$$\Delta x \geq \frac{\lambda}{2\pi}.$$

E46-26 (a) A particle confined in a (one dimensional) box of size L will have a position uncertainty of no more than $\Delta x \approx L$. The momentum uncertainty will then be no less than

$$\Delta p \geq \frac{h}{2\pi\Delta x} \approx \frac{h}{2\pi L}.$$

so

$$\Delta p \approx \frac{(6.63\times 10^{-34}\text{J}\cdot\text{s})}{2\pi(\times 10^{-10}\,\text{m})} = 1\times 10^{-24}\text{kg}\cdot\text{m/s}.$$

(b) Assuming that $p \approx \Delta p$, we have

$$p \geq \frac{h}{2\pi L},$$

and then the electron will have a (minimum) kinetic energy of

$$E \approx \frac{p^2}{2m} \approx \frac{h^2}{8\pi^2 m L^2}.$$

or

$$E \approx \frac{(hc)^2}{8\pi^2 mc^2 L^2} = \frac{(1240 \text{ keV} \cdot \text{pm})^2}{8\pi^2(511 \text{ keV})(100 \text{ pm})^2} = 0.004 \text{ keV}.$$

$\boxed{\textbf{E46-27}}$ (a) A particle confined in a (one dimensional) box of size L will have a position uncertainty of no more than $\Delta x \approx L$. The momentum uncertainty will then be no less than

$$\Delta p \geq \frac{h}{2\pi \Delta x} \approx \frac{h}{2\pi L}.$$

so

$$\Delta p \approx \frac{(6.63\times 10^{-34}\text{J}\cdot\text{s})}{2\pi(\times 10^{-14}\,\text{m})} = 1\times 10^{-20}\text{kg}\cdot\text{m/s}.$$

(b) Assuming that $p \approx \Delta p$, we have

$$p \geq \frac{h}{2\pi L},$$

and then the electron will have a (minimum) kinetic energy of

$$E \approx \frac{p^2}{2m} \approx \frac{h^2}{8\pi^2 m L^2}.$$

or

$$E \approx \frac{(hc)^2}{8\pi^2 mc^2 L^2} = \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{8\pi^2(0.511 \text{ MeV})(10 \text{ fm})^2} = 381 \text{ MeV}.$$

This is so large compared to the mass energy of the electron that we must consider relativistic effects. It will be very relativistic ($381 \gg 0.5$!), so we can use $E = pc$ as was derived in Exercise 9. Then

$$E = \frac{hc}{2\pi L} = \frac{(1240 \text{ MeV}\cdot\text{fm})}{2\pi(10 \text{ fm})} = 19.7 \text{ MeV}.$$

This is the *total* energy; so we subtract 0.511 MeV to get $K = 19$ MeV.

E46-28 We want to find L when $T = 0.01$. This means solving

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)e^{-2kL}, \\
(0.01) &= 16\frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\left(1 - \frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\right)e^{-2k'L}, \\
&= 2.22e^{-2k'L}, \\
\ln(4.5\times 10^{-3}) &= 2(5.12\times 10^9/\text{m})L, \\
5.3\times 10^{-10}\text{m} &= L.
\end{aligned}$$

$\boxed{\textbf{E46-29}}$ The wave number, k, is given by

$$k = \frac{2\pi}{hc}\sqrt{2mc^2(U_0 - E)}.$$

(a) For the proton $mc^2 = 938$ MeV, so

$$k = \frac{2\pi}{(1240 \text{ MeV}\cdot\text{fm})}\sqrt{2(938 \text{ MeV})(10 \text{ MeV} - 3.0 \text{ MeV})} = 0.581 \text{ fm}^{-1}.$$

The transmission coefficient is then

$$T = 16\frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\left(1 - \frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\right)e^{-2(0.581 \text{ fm}^{-1})(10 \text{ fm})} = 3.0\times10^{-5}.$$

(b) For the deuteron $mc^2 = 2 \times 938$ MeV, so

$$k = \frac{2\pi}{(1240 \text{ MeV}\cdot\text{fm})}\sqrt{2(2)(938 \text{ MeV})(10 \text{ MeV} - 3.0 \text{ MeV})} = 0.821 \text{ fm}^{-1}.$$

The transmission coefficient is then

$$T = 16\frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\left(1 - \frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\right)e^{-2(0.821 \text{ fm}^{-1})(10 \text{ fm})} = 2.5\times10^{-7}.$$

E46-30 The wave number, k, is given by

$$k = \frac{2\pi}{hc}\sqrt{2mc^2(U_0 - E)}.$$

(a) For the proton $mc^2 = 938$ MeV, so

$$k = \frac{2\pi}{(1240 \text{ keV}\cdot\text{pm})}\sqrt{2(938 \text{ MeV})(6.0 \text{ eV} - 5.0 \text{ eV})} = 0.219 \text{ pm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned} T &= 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)e^{-2kL}, \\ &= 16\frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\left(1 - \frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\right)e^{-2(0.219\times10^{12})(0.7\times10^{-9})}, \\ &= 1.6\times10^{-133}. \end{aligned}$$

A current of 1 kA corresponds to

$$N = (1\times10^3\text{C/s})/(1.6\times10^{-19}\text{C}) = 6.3\times10^{21}/\text{s}$$

protons per seconds. The time required for one proton to pass is then

$$t = 1/(6.3\times10^{21}/\text{s})(1.6\times10^{-133}) = 9.9\times10^{110}\text{s}.$$

That's 10^{104} years!

$\boxed{\textbf{P46-1}}$ We will interpret low energy to mean non-relativistic. Then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_\mathrm{n}K}}.$$

The diffraction pattern is then given by

$$d\sin\theta = m\lambda = mh/\sqrt{2m_\mathrm{n}K},$$

where m is diffraction order while m_n is the neutron mass. We want to investigate the spread by taking the derivative of θ with respect to K,

$$d\cos\theta\, d\theta = -\frac{mh}{2\sqrt{2m_\mathrm{n}K^3}}\, dK.$$

Divide this by the original equation, and then

$$\frac{\cos\theta}{\sin\theta}d\theta = -\frac{dK}{2K}.$$

Rearrange, change the differential to a difference, and then

$$\Delta\theta = \tan\theta\frac{\Delta K}{2K}.$$

We dropped the negative sign out of laziness; but the angles are in radians, so we need to multiply by $180/\pi$ to convert to degrees.

P46-2

P46-3 We want to solve

$$T = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL},$$

for E. Unfortunately, E is contained in k since

$$k = \frac{2\pi}{hc}\sqrt{2mc^2(U_0-E)}.$$

We can do this by iteration. The maximum possible value for

$$\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$$

is $1/4$; using this value we can get an estimate for k:

$$\begin{aligned}
(0.001) &= 16(0.25)e^{-2kL},\\
\ln(2.5\times10^{-4}) &= -2k(0.7\text{ nm}),\\
5.92/\text{ nm} &= k.
\end{aligned}$$

Now solve for E:

$$\begin{aligned}
E &= U_0 - (hc)^2k^2 8mc^2\pi^2,\\
&= (6.0\text{ eV}) - \frac{(1240\text{ eV}\cdot\text{nm})^2(5.92/\text{nm})^2}{8\pi^2(5.11\times10^5\text{eV})},\\
&= 4.67\text{ eV}.
\end{aligned}$$

Put this value for E back into the transmission equation to find a new k:

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL},\\
(0.001) &= 16\frac{(4.7\text{ eV})}{(6.0\text{ eV})}\left(1-\frac{(4.7\text{ eV})}{(6.0\text{ eV})}\right)e^{-2kL},\\
\ln(3.68\times10^{-4}) &= -2k(0.7\text{ nm}),\\
5.65/\text{ nm} &= k.
\end{aligned}$$

Now solve for E using this new, improved, value for k:

$$\begin{aligned}
E &= U_0 - (hc)^2k^2 8mc^2\pi^2,\\
&= (6.0\text{ eV}) - \frac{(1240\text{ eV}\cdot\text{nm})^2(5.65/\text{nm})^2}{8\pi^2(5.11\times10^5\text{eV})},\\
&= 4.78\text{ eV}.
\end{aligned}$$

Keep at it. You'll eventually stop around $E = 5.07$ eV.

P46-4 (a) A one percent increase in the barrier height means $U_0 = 6.06$ eV.
For the electron $mc^2 = 5.11\times10^5$ eV, so

$$k = \frac{2\pi}{(1240\text{ eV}\cdot\text{nm})}\sqrt{2(5.11\times10^5\text{ eV})(6.06\text{ eV}-5.0\text{ eV})} = 5.27\text{ nm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL},\\
&= 16\frac{(5.0\text{ eV})}{(6.06\text{ eV})}\left(1-\frac{(5.0\text{ eV})}{(6.06\text{ eV})}\right)e^{-2(5.27)(0.7)},\\
&= 1.44\times10^{-3}.
\end{aligned}$$

That's a 16% decrease.
(b) A one percent increase in the barrier thickness means $L = 0.707$ nm.
For the electron $mc^2 = 5.11\times10^5$ eV, so

$$k = \frac{2\pi}{(1240\text{ eV}\cdot\text{nm})}\sqrt{2(5.11\times10^5\text{ eV})(6.0\text{ eV}-5.0\text{ eV})} = 5.12\text{ nm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL},\\
&= 16\frac{(5.0\text{ eV})}{(6.0\text{ eV})}\left(1-\frac{(5.0\text{ eV})}{(6.0\text{ eV})}\right)e^{-2(5.12)(0.707)},\\
&= 1.59\times10^{-3}.
\end{aligned}$$

That's a 8.1% decrease.
(c) A one percent increase in the incident energy means $E = 5.05$ eV.
For the electron $mc^2 = 5.11\times10^5$ eV, so

$$k = \frac{2\pi}{(1240\text{ eV}\cdot\text{nm})}\sqrt{2(5.11\times10^5\text{ eV})(6.0\text{ eV}-5.05\text{ eV})} = 4.99\text{ nm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned} T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL}, \\ &= 16\frac{(5.05\text{ eV})}{(6.0\text{ eV})}\left(1-\frac{(5.05\text{ eV})}{(6.0\text{ eV})}\right)e^{-2(4.99)(0.7)}, \\ &= 1.97\times10^{-3}. \end{aligned}$$

That's a 14% increase.

P46-5 First, the rule for exponents

$$e^{i(a+b)} = e^{ia}\,e^{ib}.$$

Then apply Eq. 46-12, $e^{i\theta} = \cos\theta + i\sin\theta$,

$$\cos(a+b) + i\sin(a+b) = (\cos a + i\sin a)(\sin b + i\sin b).$$

Expand the right hand side, remembering that $i^2 = -1$,

$$\cos(a+b) + i\sin(a+b) = \cos a\cos b + i\cos a\sin b + i\sin a\cos b - \sin a\sin b.$$

Since the real part of the left hand side must equal the real part of the right and the imaginary part of the left hand side must equal the imaginary part of the right, we actually have *two* equations. They are

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

and

$$\sin(a+b) = \cos a\sin b + \sin a\cos b.$$

P46-6

$\boxed{\textbf{E47-1}}$ (a) The ground state energy level will be given by

$$E_1 = \frac{h^2}{8mL^2} = \frac{(6.63\times10^{-34}\,\mathrm{J\cdot s})^2}{8(9.11\times10^{-31}\mathrm{kg})(1.4\times10^{-14}\,\mathrm{m})^2} = 3.1\times10^{-10}\,\mathrm{J}.$$

The answer is correct, but the units make it almost useless. We can divide by the electron charge to express this in electron volts, and then $E = 1900$ MeV. Note that this is an extremely relativistic quantity, so the energy expression loses validity.

(b) We can repeat what we did above, or we can apply a "trick" that is often used in solving these problems. Multiplying the top and the bottom of the energy expression by c^2 we get

$$E_1 = \frac{(hc)^2}{8(mc^2)L^2}$$

Then

$$E_1 = \frac{(1240\ \mathrm{MeV\cdot fm})^2}{8(940\ \mathrm{MeV})(14\ \mathrm{fm})^2} = 1.0\ \mathrm{MeV}.$$

(c) Finding an neutron inside the nucleus seems reasonable; but finding the electron would not. The energy of such an electron is considerably larger than binding energies of the particles in the nucleus.

E47-2 Solve

$$E_n = \frac{n^2(hc)^2}{8(mc^2)L^2}$$

for L, then

$$\begin{aligned} L &= \frac{nhc}{\sqrt{8mc^2E_n}}, \\ &= \frac{(3)(1240\ \mathrm{eV\cdot nm})}{\sqrt{8(5.11\times10^5\mathrm{eV})(4.7\ \mathrm{eV})}}, \\ &= 0.85\ \mathrm{nm}. \end{aligned}$$

E47-3 Solve for $E_4 - E_1$:

$$\begin{aligned} E_4 - E_1 &= \frac{4^2(hc)^2}{8(mc^2)L^2} - \frac{1^2(hc)^2}{8(mc^2)L^2}, \\ &= \frac{(16-1)(1240\ \mathrm{eV\cdot nm})^2}{8(5.11\times10^5)(0.253\ \mathrm{nm})^2}, \\ &= 88.1\ \mathrm{eV}. \end{aligned}$$

E47-4 Since $E \propto 1/L^2$, doubling the width of the well will lower the ground state energy to $(1/2)^2 = 1/4$, or 0.65 eV.

E47-5 (a) Solve for $E_2 - E_1$:

$$\begin{aligned} E_2 - E_1 &= \frac{2^2h^2}{8mL^2} - \frac{1^2h^2}{8mL^2}, \\ &= \frac{(3)(6.63\times10^{-34}\mathrm{J\cdot s})^2}{8(40)(1.67\times10^{-27}\mathrm{kg})(0.2\,\mathrm{m})^2}, \\ &= 6.2\times10^{-41}\mathrm{J}. \end{aligned}$$

(b) $K = 3kT/2 = 3(1.38\times10^{-23}\mathrm{J/K})(300\,\mathrm{K})/2 = 6.21\times10^{-21}$. The ratio is 1×10^{-20}.
(c) $T = 2(6.2\times10^{-41}\mathrm{J})/3(1.38\times10^{-23}\mathrm{J/K}) = 3.0\times10^{-18}\mathrm{K}$.

E47-6 (a) The fractional difference is $(E_{n+1} - E_n)/E_n$, or

$$\begin{aligned} \frac{\Delta E_n}{E_n} &= \left[(n+1)^2\frac{h^2}{8mL^2} - n^2\frac{h^2}{8mL^2}\right] \Big/ \left[n^2\frac{h^2}{8mL^2}\right], \\ &= \frac{(n+1)^2 - n^2}{n^2}, \\ &= \frac{2n+1}{n^2}. \end{aligned}$$

(b) As $n \to \infty$ the fractional difference goes to zero; the system behaves as if it is continuous.

$\boxed{\textbf{E47-7}}$ (a) We will take advantage of the "trick" that was developed in part (b) of Exercise 47-1. Then

$$E_n = n^2\frac{(hc)^2}{8mc^2\,L} = (15)^2\frac{(1240\text{ eV}\cdot\text{nm})^2}{8(0.511\times10^6\text{ eV})(0.0985\text{ nm})^2} = 8.72\text{ keV}.$$

(b) The magnitude of the momentum is *exactly* known because $E = p^2/2m$. This momentum is given by

$$pc = \sqrt{2mc^2E} = \sqrt{2(511\text{ keV})(8.72\text{ keV})} = 94.4\text{ keV}.$$

What we don't know is in which direction the particle is moving. It is bouncing back and forth between the walls of the box, so the momentum could be directed toward the right or toward the left. The uncertainty in the momentum is then

$$\Delta p = p$$

which can be expressed in terms of the box size L by

$$\Delta p = p = \sqrt{2mE} = \sqrt{\frac{n^2h^2}{4L^2}} = \frac{nh}{2L}.$$

(c) The uncertainty in the position is 98.5 pm; the electron could be *anywhere* inside the well.

E47-8 The probability distribution function is

$$P_2 = \frac{2}{L}\sin^2\frac{2\pi x}{L}.$$

We want to integrate over the central third, or

$$\begin{aligned} P &= \int_{-L/6}^{L/6}\frac{2}{L}\sin^2\frac{2\pi x}{L}dx, \\ &= \frac{1}{\pi}\int_{-\pi/3}^{\pi/3}\sin^2\theta\,d\theta, \\ &= 0.196. \end{aligned}$$

E47-9 (a) Maximum probability occurs when the argument of the cosine (sine) function is $k\pi$ ($[k+1/2]\pi$). This occurs when

$$x = NL/2n$$

for odd N.

(b) Minimum probability occurs when the argument of the cosine (sine) function is $[k+1/2]\pi$ ($k\pi$). This occurs when

$$x = NL/2n$$

for even N.

E47-10 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

(a) The Lyman series is the series which ends on E_1. The least energetic state starts on E_2. The transition energy is

$$E_2 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/2^2) = 10.2 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(10.2 \text{ eV})} = 121.6 \text{ nm}.$$

(b) The series limit is

$$0 - E_1 = (13.6 \text{ eV})(1/1^2) = 13.6 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(13.6 \text{ eV})} = 91.2 \text{ nm}.$$

$\boxed{\textbf{E47-11}}$ The ground state of hydrogen, as given by Eq. 47-21, is

$$E_1 = -\frac{me^4}{8\epsilon_0^2 h^2} = -\frac{(9.109 \times 10^{-31}\,\text{kg})(1.602 \times 10^{-19}\,\text{C})^4}{8(8.854 \times 10^{-12}\,\text{F/m})^2(6.626 \times 10^{-34}\,\text{J} \cdot \text{s})^2} = 2.179 \times 10^{-18}\,\text{J}.$$

In terms of electron volts the ground state energy is

$$E_1 = -(2.179 \times 10^{-18}\,\text{J})/(1.602 \times 10^{-19}\,\text{C}) = -13.60 \text{ eV}.$$

E47-12 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

(c) The transition energy is

$$\Delta E = E_3 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/3^2) = 12.1 \text{ eV}.$$

(a) The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(12.1 \text{ eV})} = 102.5 \text{ nm}.$$

(b) The momentum is

$$p = E/c = 12.1 \text{ eV}/c.$$

E47-13 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

(a) The Balmer series is the series which ends on E_2. The least energetic state starts on E_3. The transition energy is

$$E_3 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/3^2) = 1.89 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(1.89 \text{ eV})} = 656 \text{ nm}.$$

(b) The next energetic state starts on E_4. The transition energy is

$$E_4 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/4^2) = 2.55 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(2.55 \text{ eV})} = 486 \text{ nm}.$$

(c) The next energetic state starts on E_5. The transition energy is

$$E_5 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/5^2) = 2.86 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(2.86 \text{ eV})} = 434 \text{ nm}.$$

(d) The next energetic state starts on E_6. The transition energy is

$$E_6 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/6^2) = 3.02 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(3.02 \text{ eV})} = 411 \text{ nm}.$$

(e) The next energetic state starts on E_7. The transition energy is

$$E_7 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/7^2) = 3.12 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(3.12 \text{ eV})} = 397 \text{ nm}.$$

E47-14 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

The transition energy is

$$\Delta E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(121.6 \text{ nm})} = 10.20 \text{ eV}.$$

This *must* be part of the Lyman series, so the higher state must be

$$E_n = (10.20 \text{ eV}) - (13.6 \text{ eV}) = -3.4 \text{ eV}.$$

That would correspond to $n = 2$.

E47-15 The binding energy is the energy required to remove the electron. If the energy of the electron is negative, then that negative energy is a measure of the energy required to set the electron free.

The first excited state is when $n = 2$ in Eq. 47-21. It is *not* necessary to re-evaluate the constants in this equation every time, instead, we start from

$$E_n = \frac{E_1}{n^2} \text{ where } E_1 = -13.60 \text{ eV}.$$

Then the first excited state has energy

$$E_2 = \frac{(-13.6 \text{ eV})}{(2)^2} = -3.4 \text{ eV}.$$

The binding energy is then 3.4 eV.

E47-16 $r_n = a_0 n^2$, so

$$n = \sqrt{(847 \text{ pm})/(52.9 \text{ pm})} = 4.$$

E47-17 (a) The energy of this photon is

$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(1281.8 \text{ nm})} = 0.96739 \text{ eV}.$$

The final state of the hydrogen must have an energy of no more than -0.96739, so the largest possible n of the final state is

$$n < \sqrt{13.60 \text{ eV}/0.96739 \text{ eV}} = 3.75,$$

so the final n is 1, 2, or 3. The initial state is only slightly higher than the final state. The jump from $n = 2$ to $n = 1$ is *too* large (see Exercise 15), any other initial state would have a larger energy difference, so $n = 1$ is *not* the final state.

So what level might be above $n = 2$? We'll try

$$n = \sqrt{13.6 \text{ eV}/(3.4 \text{ eV} - 0.97 \text{ eV})} = 2.36,$$

which is *so* far from being an integer that we don't need to look farther. The $n = 3$ state has energy $13.6 \text{ eV}/9 = 1.51$ eV. Then the initial state could be

$$n = \sqrt{13.6 \text{ eV}/(1.51 \text{ eV} - 0.97 \text{ eV})} = 5.01,$$

which is close enough to 5 that we can assume the transition was $n = 5$ to $n = 3$.

(b) This belongs to the Paschen series.

E47-18 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

(a) The transition energy is

$$\Delta E = E_4 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/4^2) = 12.8 \text{ eV}.$$

(b) All transitions $n \to m$ are allowed for $n \leq 4$ and $m < n$. The transition energy will be of the form

$$E_n - E_m = (13.6 \text{ eV})(1/m^2 - 1/n^2).$$

The six possible results are 12.8 eV, 12.1 eV, 10.2 eV, 2.55 eV, 1.89 eV, and 0.66 eV.

E47-19 $\Delta E = h/2\pi\Delta t$, so

$$\Delta E = (4.14\times10^{-15} \text{ eV} \cdot \text{s})/2\pi(1\times10^{-8}\text{s}) = 6.6\times10^{-8}\text{eV}.$$

E47-20 (a) According to electrostatics and uniform circular motion,

$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 hn}.$$

Putting in the numbers,

$$v = \frac{(1.6\times 10^{-19}\mathrm{C})^2}{2(8.85\times 10^{-12}\mathrm{F/m})(6.63\times 10^{-34}\mathrm{J\cdot s})n} = \frac{2.18\times 10^6\mathrm{m/s}}{n}.$$

In this case $n = 1$.

(b) $\lambda = h/mv$,

$$\lambda = (6.63\times 10^{-34}\mathrm{J\cdot s})/(9.11\times 10^{-31}\mathrm{kg})(2.18\times 10^6\mathrm{m/s}) = 3.34\times 10^{-10}\mathrm{m}.$$

(c) $\lambda/a_0 = (3.34\times 10^{-10}\mathrm{m})/(5.29\times 10^{-11}) = 6.31 \approx 2\pi$. Actually, it is exactly 2π.

E47-21 In order to have an inelastic collision with the 6.0 eV neutron there must exist a transition with an energy difference of less than 6.0 eV. For a hydrogen atom in the ground state $E_1 = -13.6$ eV the nearest state is

$$E_2 = (-13.6 \text{ eV})/(2)^2 = -3.4 \text{ eV}.$$

Since the difference is 10.2 eV, it will *not* be possible for the 6.0 eV neutron to have an inelastic collision with a ground state hydrogen atom.

E47-22 (a) The atom is originally in the state n given by

$$n = \sqrt{(13.6 \text{ eV})/(0.85 \text{ eV})} = 4.$$

The state with an excitation energy of 10.2 eV, is

$$n = \sqrt{(13.6 \text{ eV})/(13.6 \text{ eV} - 10.2 \text{ eV})} = 2.$$

The transition energy is then

$$\Delta E = (13.6 \text{ eV})(1/2^2 - 1/4^2) = 2.55 \text{ eV}.$$

E47-23 According to electrostatics and uniform circular motion,

$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 hn}.$$

The de Broglie wavelength is then

$$\lambda = \frac{h}{mv} = \frac{2\epsilon_0 hn}{me^2}.$$

The ratio of λ/r is

$$\frac{\lambda}{r} = \frac{2\epsilon_0 hn}{me^2 a_0 n^2} = kn,$$

where k is a constant. As $n \to \infty$ the ratio goes to zero.

E47-24 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

The transition energy is

$$\Delta E = E_4 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/4^2) = 12.8 \text{ eV}.$$

The momentum of the emitted photon is

$$p = E/c = (12.8 \text{ eV})/c.$$

This is the momentum of the recoiling hydrogen atom, which then has velocity

$$v = \frac{p}{m} = \frac{pc}{mc^2}c = \frac{(12.8 \text{ eV})}{(932 \text{ MeV})}(3.00\times10^8\text{m/s}) = 4.1\,\text{m/s}.$$

$\boxed{\textbf{E47-25}}$ The first Lyman line is the $n = 1$ to $n = 2$ transition. The second Lyman line is the $n = 1$ to $n = 3$ transition. The first Balmer line is the $n = 2$ to $n = 3$ transition. Since the photon frequency is proportional to the photon energy ($E = hf$) and the photon energy is the energy difference between the two levels, we have

$$f_{n\to m} = \frac{E_m - E_n}{h}$$

where the E_n is the hydrogen atom energy level. Then

$$\begin{aligned}
f_{1\to3} &= \frac{E_3 - E_1}{h}, \\
&= \frac{E_3 - E_2 + E_2 - E_1}{h} = \frac{E_3 - E_2}{h} + \frac{E_2 - E_1}{h}, \\
&= f_{2\to3} + f_{1\to2}.
\end{aligned}$$

E47-26 Use

$$E_n = -Z^2(13.6 \text{ eV})/n^2.$$

(a) The ionization energy of the ground state of He^+ is

$$E_n = -(2)^2(13.6 \text{ eV})/(1)^2 = 54.4 \text{ eV}.$$

(b) The ionization energy of the $n = 3$ state of Li^{2+} is

$$E_n = -(3)^2(13.6 \text{ eV})/(3)^2 = 13.6 \text{ eV}.$$

$\boxed{\textbf{E47-27}}$ (a) The energy levels in the He^+ spectrum are given by

$$E_n = -Z^2(13.6 \text{ eV})/n^2,$$

where $Z = 2$, as is discussed in Sample Problem 47-6. The photon wavelengths for the $n = 4$ series are then

$$\lambda = \frac{hc}{E_n - E_4} = \frac{hc/E_4}{1 - E_n/E_4},$$

which can also be written as

$$\begin{aligned}\lambda &= \frac{16hc/(54.4 \text{ eV})}{1-16/n^2},\\ &= \frac{16hcn^2/(54.4 \text{ eV})}{n^2-16},\\ &= \frac{Cn^2}{n^2-16},\end{aligned}$$

where $C = hc/(3.4 \text{ eV}) = 365$ nm.

(b) The wavelength of the first line is the transition from $n = 5$,

$$\lambda = \frac{(365 \text{ nm})(5)^2}{(5)^2 - (4)^2} = 1014 \text{ nm}.$$

The series limit is the transition from $n = \infty$, so

$$\lambda = 365 \text{ nm}.$$

(c) The series starts in the infrared (1014 nm), and ends in the ultraviolet (365 nm). So it must also include some visible lines.

E47-28 We answer these questions out of order!

(a) $n = 1$.
(b) $r = a_0 = 5.29\times10^{-11}$m.
(f) According to electrostatics and uniform circular motion,

$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 hn}.$$

Putting in the numbers,

$$v = \frac{(1.6\times10^{-19}\text{C})^2}{2(8.85\times10^{-12}\text{F/m})(6.63\times10^{-34}\text{J}\cdot\text{s})(1)} = 2.18\times10^6\text{m/s}.$$

(d) $p = (9.11\times10^{-31}\text{kg})(2.18\times10^6\text{m/s}) = 1.99\times10^{-24}\text{kg}\cdot\text{m/s}$.
(e) $\omega = v/r = (2.18\times10^6\text{m/s})/(5.29\times10^{-11}\text{m}) = 4.12\times10^{16}\text{rad/s}$.
(c) $l = pr = (1.99\times10^{-24}\text{kg}\cdot\text{m/s})(5.29\times10^{-11}\text{m}) = 1.05\times10^{-34}\text{J}\cdot\text{s}$.
(g) $F = mv^2/r$, so

$$F = (9.11\times10^{-31}\text{kg})(2.18\times10^6\text{m/s})^2/(5.29\times10^{-11}\text{m}) = 8.18\times10^{-8}\text{N}.$$

(h) $a = (8.18\times10^{-8}\text{N})/(9.11\times10^{-31}kg) = 8.98\times10^{22}\text{m/s}^2$.
(i) $K = mv^2/r$, or

$$K = \frac{(9.11\times10^{-31}\text{kg})(2.18\times10^6\text{m/s})^2}{2} = 2.16\times10^{-18}\text{J} = 13.6 \text{ eV}.$$

(k) $E = -13.6$ eV.
(j) $P = E - K = (-13.6 \text{ eV}) - (13.6 \text{ eV}) = -27.2$ eV.

E47-29 For each r in the quantity we have a factor of n^2.

(a) n is proportional to n.
(b) r is proportional to n^2.
(f) v is proportional to $\sqrt{1/r}$, or $1/n$.
(d) p is proportional to v, or $1/n$.
(e) ω is proportional to v/r, or $1/n^3$.
(c) l is proportional to pr, or n.
(g) f is proportional to v^2/r, or $1/n^4$.
(h) a is proportional to F, or $1/n^4$.
(i) K is proportional to v^2, or $1/n^2$.
(j) E is proportional to $1/n^2$.
(k) P is proportional to $1/n^2$.

E47-30 (a) Using the results of Exercise 45-1,

$$E_1 = \frac{(1240 \text{ eV}\cdot\text{nm})}{(0.010 \text{ nm})} = 1.24\times 10^5 \text{ eV}.$$

(b) Using the results of Problem 45-11,

$$K_{\text{max}} = \frac{E^2}{mc^2/2 + E} = \frac{(1.24\times 10^5 \text{ eV})^2}{(5.11\times 10^5 \text{ eV})/2 + (1.24\times 10^5 \text{ eV})} = 40.5\times 10^4 \text{ eV}.$$

(c) This would likely knock the electron way out of the atom.

E47-31 The energy of the photon in the series limit is given by

$$E_{\text{limit}} = (13.6 \text{ eV})/n^2,$$

where $n = 1$ for Lyman, $n = 2$ for Balmer, and $n = 3$ for Paschen. The wavelength of the photon is

$$\lambda_{\text{limit}} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(13.6 \text{ eV})} n^2 = (91.17 \text{ nm})n^2.$$

The energy of the longest wavelength comes from the transition from the nearest level, or

$$E_{\text{long}} = \frac{(-13.6 \text{ eV})}{(n+1)^2} - \frac{(-13.6 \text{ eV})}{n^2} = (13.6 \text{ eV})\frac{2n+1}{[n(n+1)]^2}.$$

The wavelength of the photon is

$$\lambda_{\text{long}} = \frac{(1240 \text{ eV}\cdot\text{nm})[n(n+1)]^2}{(13.6 \text{ eV})n^2} = (91.17 \text{ nm})\frac{[n(n+1)]^2}{2n+1}.$$

(a) The wavelength interval $\lambda_{\text{long}} - \lambda_{\text{limit}}$, or

$$\Delta\lambda = (91.17 \text{ nm})\frac{n^2(n+1)^2 - n^2(2n+1)}{2n+1} = (91.17 \text{ nm})\frac{n^4}{2n+1}.$$

For $n = 1$, $\Delta\lambda = 30.4$ nm. For $n = 2$, $\Delta\lambda = 292$ nm. For $n = 3$, $\Delta\lambda = 1055$ nm.

(b) The frequency interval is found from

$$\Delta f = \frac{E_{\text{limit}} - E_{\text{long}}}{h} = \frac{(13.6 \text{ eV})}{(4.14\times 10^{-15}\text{eV}\cdot\text{s})}\frac{1}{(n+1)^2} = \frac{(3.29\times 10^{15}/\text{s})}{(n+1)^2}.$$

For $n = 1$, $\Delta f = 8.23\times 10^{14}$Hz. For $n = 2$, $\Delta f = 3.66\times 10^{14}$Hz. For $n = 3$, $\Delta f = 2.05\times 10^{14}$Hz.

E47-32

$\boxed{\textbf{E47-33}}$ (a) We'll use Eqs. 47-25 and 47-26. At $r = 0$

$$\psi^2(0) = \frac{1}{\pi a_0^3} e^{-2(0)/a_0} = \frac{1}{\pi a_0^3} = 2150\ \text{nm}^{-3},$$

while

$$P(0) = 4\pi(0)^2\psi^2(0) = 0.$$

(b) At $r = a_0$ we have

$$\psi^2(a_0) = frac1\pi a_0^3 e^{-2(a_0)/a_0} = \frac{e^{-2}}{\pi a_0^3} = 291\ \text{nm}^{-3},$$

and

$$P(a_0) = 4\pi(a_0)^2\psi^2(a_0) = 10.2\ \text{nm}^{-1}.$$

E47-34 Assume that $\psi(a_0)$ is a reasonable estimate for $\psi(r)$ everywhere inside the small sphere. Then

$$\psi^2 = \frac{e^{-2}}{\pi a_0^3} = \frac{0.1353}{\pi a_0^3}.$$

The probability of finding it in a sphere of radius $0.05a_0$ is

$$\int_0^{0.05a_0} \frac{(0.1353)4\pi r^2\, dr}{\pi a_0^3} = \frac{4}{3}(0.1353)(0.05)^3 = 2.26\times 10^{-5}.$$

E47-35 Using Eq. 47-26 the ratio of the probabilities is

$$\frac{P(a_0)}{P(2a_0)} = \frac{(a_0)^2 e^{-2(a_0)/a_0}}{(2a_0)^2 e^{-2(2a_0)/a_0}} = \frac{e^{-2}}{4e^{-4}} = 1.85.$$

E47-36 The probability is

$$\begin{aligned} P &= \int_{a_0}^{1.016a_0} \frac{4r^2 e^{-2r/a_0}}{a_0^3} dr, \\ &= \frac{1}{2}\int_2^{2.032} u^2 e^{-u} du, \\ &= 0.00866. \end{aligned}$$

$\boxed{\textbf{E47-37}}$ If $l = 3$ then m_l can be 0, ± 1, ± 2, or ± 3.

(a) From Eq. 47-30, $L_z = m_l h/2\pi$.. So L_z can equal 0, $\pm h/2\pi$, $\pm h/\pi$, or $\pm 3h/2\pi$.

(b) From Eq. 47-31, $\theta = \arccos(m_l/\sqrt{l(l+1)})$, so θ can equal 90°, 73.2°, 54.7°, or 30.0°.

(c) The magnitude of $\vec{\mathbf{L}}$ is given by Eq. 47-28,

$$L = \sqrt{l(l+1)}\frac{h}{2\pi} = \sqrt{3}h/\pi.$$

E47-38 The maximum possible value of m_l is 5. Apply Eq. 47-31:

$$\theta = \arccos\frac{(5)}{\sqrt{(5)(5+1)}} = 24.1^\circ.$$

E47-39 Use the hint.

$$\begin{aligned}\Delta p \cdot \Delta x &= \frac{h}{2\pi},\\ \Delta p \frac{r}{r} \Delta x &= \frac{h}{2\pi},\\ \Delta p \cdot r\frac{\Delta x}{r} &= \frac{h}{2\pi},\\ \Delta L \cdot \Delta\theta &= \frac{h}{2\pi}.\end{aligned}$$

E47-40 Note that there is a typo in the formula; $P(r)$ must have dimensions of one over length. The probability is

$$\begin{aligned}P &= \int_0^\infty \frac{r^4 e^{-r/a_0}}{24a_0^5} dr,\\ &= \frac{1}{24}\int_0^\infty u^4 e^{-u} du,\\ &= 1.00\end{aligned}$$

What does it mean? It means that if we look for the electron, we will find it somewhere.

E47-41 (a) Find the maxima by taking the derivative and setting it equal to zero.

$$\frac{dP}{dr} = \frac{r(2a-r)(4a^2-6ra+r^2)}{8a_0^6}e^{-r} = 0.$$

The solutions are $r = 0$, $r = 2a$, and $4a^2 - 6ra + r^2 = 0$. The first two correspond to minima (see Fig. 47-14). The other two are the solutions to the quadratic, or $r = 0.764a_0$ and $r = 5.236a_0$.

(b) Substitute these two values into Eq. 47-36. The results are

$$P(0.764a_0) = 0.981\ \mathrm{nm}^{-1}.$$

and

$$P(5.236a_0) = 3.61\ \mathrm{nm}^{-1}.$$

E47-42 The probability is

$$\begin{aligned}P &= \int_{5.00a_0}^{5.01a_0} \frac{r^2(2-r/a_0)^2 e^{-r/a_0}}{8a_0^3} dr,\\ &= 0.01896.\end{aligned}$$

E47-43 $n = 4$ and $l = 3$, while m_l can be any of

$$-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,$$

while m_s can be either $-1/2$ or $1/2$. There are 14 possible states.

E47-44 n must be greater than l, so $n \geq 4$. $|m_l|$ must be less than or equal to l, so $|m_l| \leq 3$. m_s is $-1/2$ or $1/2$.

$\boxed{\textbf{E47-45}}$ If $m_l = 4$ then $l \geq 4$. But $n \geq l + 1$, so $n > 4$. We only know that $m_s = \pm 1/2$.

E47-46 There are $2n^2$ states in a shell n, so if $n = 5$ there are 50 states.

E47-47 Each is in the $n = 1$ shell, the $l = 0$ angular momentum state, and the $m_l = 0$ state. But one is in the state $m_s = +1/2$ while the other is in the state $m_s = -1/2$.

E47-48 Apply Eq. 47-31:

$$\theta = \arccos \frac{(+1/2)}{\sqrt{(1/2)(1/2+1)}} = 54.7^\circ$$

and

$$\theta = \arccos \frac{(-1/2)}{\sqrt{(1/2)(1/2+1)}} = 125.3^\circ.$$

E47-49 All of the statements are true.

E47-50 There are n possible values for l (start at 0!). For each value of l there are $2l+1$ possible values for m_l. If $n = 1$, the sum is 1. If $n = 2$, the sum is $1+3 = 4$. If $n = 3$, the sum is $1+3+5 = 9$. The pattern is clear, the sum is n^2. But there are two spin states, so the number of states is $2n^2$.

P47-1 We can simplify the energy expression as

$$E = E_0 \left(n_x^2 + n_y^2 + n_z^2\right) \text{ where } E_0 = \frac{h^2}{8mL^2}.$$

To find the lowest energy levels we need to focus on the values of n_x, n_y, and n_z.

It doesn't take much imagination to realize that the set $(1, 1, 1)$ will result in the smallest value for $n_x^2 + n_y^2 + n_z^2$. The next choice is to set one of the values equal to 2, and try the set $(2, 1, 1)$.

Then it starts to get harder, as the next lowest might be either $(2, 2, 1)$ or $(3, 1, 1)$. The only way to find out is to try. I'll tabulate the results for you:

n_x	n_y	n_z	$n_x^2+n_y^2+n_z^2$	Mult.	n_x	n_y	n_z	$n_x^2+n_y^2+n_z^2$	Mult.
1	1	1	3	1	3	2	1	14	6
2	1	1	6	3	3	2	2	17	3
2	2	1	9	3	4	1	1	18	3
3	1	1	11	3	3	3	1	19	3
2	2	2	12	1	4	2	1	21	6

We are now in a position to state the five lowest energy levels. The fundamental quantity is

$$E_0 = \frac{(hc)^2}{8mc^2L^2} = \frac{(1240 \text{ eV}\cdot\text{nm})^2}{8(0.511\times10^6 \text{ eV})(250 \text{ nm})^2} = 6.02\times10^{-6} \text{ eV}.$$

The five lowest levels are found by multiplying this fundamental quantity by the numbers in the table above.

P47-2 (a) Write the states between 0 and L. Then all states, odd or even, can be written with probability distribution function

$$P(x) = \frac{2}{L}\sin^2\frac{n\pi x}{L},$$

we find the probability of finding the particle in the region $0 \leq x \leq L/3$ is

$$\begin{aligned} P &= \int_0^{L/3} \frac{2}{L} \cos^2 \frac{n\pi x}{L} dx, \\ &= \frac{1}{3}\left(1 - \frac{sin(2n\pi/3)}{2n\pi/3}\right). \end{aligned}$$

(b) If $n = 1$ use the formula and $P = 0.196$.
(c) If $n = 2$ use the formula and $P = 0.402$.
(d) If $n = 3$ use the formula and $P = 0.333$.
(e) Classically the probability distribution function is uniform, so there is a 1/3 chance of finding it in the region 0 to L/3.

P47-3 The region of interest is small compared to the variation in $P(x)$; as such we can approximate the probability with the expression $P = P(x)\Delta x$.

(b) Evaluating,

$$\begin{aligned} P &= \frac{2}{L} \sin^2 \frac{4\pi x}{L} \Delta x, \\ &= \frac{2}{L} \sin^2 \frac{4\pi(L/8)}{L} (0.0003L), \\ &= 0.0006. \end{aligned}$$

(b) Evaluating,

$$\begin{aligned} P &= \frac{2}{L} \sin^2 \frac{4\pi x}{L} \Delta x, \\ &= \frac{2}{L} \sin^2 \frac{4\pi(3L/16)}{L} (0.0003L), \\ &= 0.0003. \end{aligned}$$

P47-4 (a) $P = \Psi^*\Psi$, or

$$P = A_0^2 e^{-2\pi m\omega x^2/h}.$$

(b) Integrating,

$$\begin{aligned} 1 &= A_0^2 \int_{-\infty}^{\infty} e^{-2\pi m\omega x^2/h} dx, \\ &= A_0^2 \sqrt{\frac{h}{2\pi m\omega}} \int_{-\infty}^{\infty} e^{-u^2} du, \\ &= A_0^2 \sqrt{\frac{h}{2\pi m\omega}} \sqrt{pi}, \\ \sqrt[4]{\frac{2m\omega}{h}} &= A_0. \end{aligned}$$

(c) $x = 0$.

$\boxed{\textbf{P47-5}}$ We will want an expression for

$$\frac{d^2}{dx^2}\psi_0.$$

Doing the math one derivative at a time,

$$\begin{aligned}
\frac{d^2}{dx^2}\psi_0 &= \frac{d}{dx}\left(\frac{d}{dx}\psi_0\right), \\
&= \frac{d}{dx}\left(A_0(-2\pi m\omega x/h)e^{-\pi m\omega x^2/h}\right), \\
&= A_0(-2\pi m\omega x/h)^2 e^{-\pi m\omega x^2/h} + A_0(-2\pi m\omega/h)e^{-\pi m\omega x^2/h}, \\
&= \left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right) A_0 e^{-\pi m\omega x^2/h}, \\
&= \left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right) \psi_0.
\end{aligned}$$

In the last line we factored out ψ_0. This will make our lives easier later on.

Now we want to go to Schrödinger's equation, and make some substitutions.

$$\begin{aligned}
-\frac{h^2}{8\pi^2 m}\frac{d^2}{dx^2}\psi_0 + U\psi_0 &= E\psi_0, \\
-\frac{h^2}{8\pi^2 m}\left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right)\psi_0 + U\psi_0 &= E\psi_0, \\
-\frac{h^2}{8\pi^2 m}\left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right) + U &= E,
\end{aligned}$$

where in the last line we divided through by ψ_0. Now for some algebra,

$$\begin{aligned}
U &= E + \frac{h^2}{8\pi^2 m}\left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right), \\
&= E + \frac{m\omega^2 x^2}{2} - \frac{h\omega}{4\pi}.
\end{aligned}$$

But we are given that $E = h\omega/4\pi$, so this simplifies to

$$U = \frac{m\omega^2 x^2}{2}$$

which looks like a harmonic oscillator type potential.

P47-6 Assume the electron is originally in the state n. The classical frequency of the electron is f_0, where

$$f_0 = v/2\pi r.$$

According to electrostatics and uniform circular motion,

$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m r}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 h n}.$$

Then

$$f_0 = \frac{e^2}{2\epsilon_0 h n}\frac{1}{2\pi}\frac{\pi m e^2}{\epsilon_0 h^2 n^2} = \frac{m e^4}{4\epsilon_0^2 h^3 n^3} = \frac{-2E_1}{h n^3}$$

Here $E_1 = -13.6$ eV.

Photon frequency is related to energy according to $f = \Delta E_{nm}/h$, where ΔE_{nm} is the energy of transition from state n down to state m. Then

$$f = \frac{E_1}{h}\left(\frac{1}{n^2} - \frac{1}{m^2}\right),$$

where $E_1 = -13.6$ eV. Combining the fractions and letting $m = n - \delta$, where δ is an integer,

$$\begin{aligned}
f &= \frac{E_1}{h}\frac{m^2 - n_2}{m^2 n^2}, \\
&= \frac{-E_1}{h}\frac{(n-m)(m+n)}{m^2 n^2}, \\
&= \frac{-E_1}{h}\frac{\delta(2n+\delta)}{(n+\delta)^2 n^2}, \\
&\approx \frac{-E_1}{h}\frac{\delta(2n)}{(n)^2 n^2}, \\
&= \frac{-2E_1}{hn^3}\delta = f_0\delta.
\end{aligned}$$

P47-7 We need to use the reduced mass of the muon since the muon and proton masses are so close together. Then

$$m = \frac{(207)(1836)}{(207) + (1836)} m_\mathrm{e} = 186 m_\mathrm{e}.$$

(a) Apply Eq. 47-20 1/2:

$$a_\mu = a_0/(186) = (52.9 \text{ pm})/(186) = 0.284 \text{ pm}.$$

(b) Apply Eq. 47-21:

$$E_\mu = E_1(186) = (13.6 \text{ eV})(186) = 2.53 \text{ keV}.$$

(c) $\lambda = (1240 \text{ keV} \cdot \text{pm})/(2.53 \text{ pm}) = 490$ pm.

P47-8 (a) The reduced mass of the electron is

$$m = \frac{(1)(1)}{(1) + (1)} m_\mathrm{e} = 0.5 m_\mathrm{e}.$$

The spectrum is similar, except for this additional factor of 1/2; hence

$$\lambda_\mathrm{pos} = 2\lambda_\mathrm{H}.$$

(b) $a_\mathrm{pos} = a_0/(186) = (52.9 \text{ pm})/(1/2) = 105.8$ pm. This is the distance between the particles, but they are both revolving about the center of mass. The radius is then half this quantity, or 52.9 pm.

$\boxed{\textbf{P47-9}}$ This problem isn't really that much of a problem. Start with the magnitude of a vector in terms of the components,

$$L_x^2 + L_y^2 + L_z^2 = L^2,$$

and then rearrange,

$$L_x^2 + L_y^2 = L^2 - L_z^2.$$

According to Eq. 47-28 $L^2 = l(l+1)h^2/4\pi^2$, while according to Eq. 47-30 $L_z = m_l h/2\pi$. Substitute that into the equation, and

$$L_x^2 + L_y^2 = l(l+1)h^2/4\pi^2 - m_l^2 h^2/4\pi^2 = \left(l(l+1) - m_l^2\right)\frac{h^2}{4\pi^2}.$$

Take the square root of both sides of this expression, and we are done.

The maximum value for m_l is l, while the minimum value is 0. Consequently,

$$\sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m_l^2}\, h/2\pi \leq \sqrt{l(l+1)}\, h/2\pi,$$

and

$$\sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m_l^2}\, h/2\pi \geq \sqrt{l}\, h/2\pi.$$

P47-10 Assume that $\psi(0)$ is a reasonable estimate for $\psi(r)$ everywhere inside the small sphere. Then

$$\psi^2 = \frac{e^{-0}}{\pi a_0^3} = \frac{1}{\pi a_0^3}.$$

The probability of finding it in a sphere of radius 1.1×10^{-15}m is

$$\int_0^{1.1\times10^{-15}\text{m}} \frac{4\pi r^2\, dr}{\pi a_0^3} = \frac{4}{3}\frac{(1.1\times10^{-15}\text{m})^3}{(5.29\times10^{-11}\text{m})^3} = 1.2\times10^{-14}.$$

P47-11 Assume that $\psi(0)$ is a reasonable estimate for $\psi(r)$ everywhere inside the small sphere. Then

$$\psi^2 = \frac{(2)^2 e^{-0}}{32\pi a_0^3} = \frac{1}{8\pi a_0^3}.$$

The probability of finding it in a sphere of radius 1.1×10^{-15}m is

$$\int_0^{1.1\times10^{-15}\text{m}} \frac{4\pi r^2\, dr}{8\pi a_0^3} = \frac{1}{6}\frac{(1.1\times10^{-15}\text{m})^3}{(5.29\times10^{-11}\text{m})^3} = 1.5\times10^{-15}.$$

P47-12 (a) The wave function squared is

$$\psi^2 = \frac{e^{-2r/a_0}}{\pi a_0^3}$$

The probability of finding it in a sphere of radius $r = xa_0$ is

$$\begin{aligned} P &= \int_0^{xa_0} \frac{4\pi r^2 e^{-2r/a_0}\, dr}{\pi a_0^3}, \\ &= \int_0^x 4x^2 e^{-2x}\, dx, \\ &= 1 - e^{-2x}(1 + 2x + 2x^2). \end{aligned}$$

(b) Let $x = 1$, then

$$P = 1 - e^{-2}(5) = 0.323.$$

P47-13 We want to evaluate the difference between the values of P at $x = 2$ and $x = 2$. Then

$$\begin{aligned} P(2) - P(1) &= \left(1 - e^{-4}(1 + 2(2) + 2(2)^2)\right) - \left(1 - e^{-2}(1 + 2(1) + 2(1)^2)\right), \\ &= 5e^{-2} - 13e^{-4} = 0.439. \end{aligned}$$

P47-14 Using the results of Problem 47-12,

$$0.5 = 1 - e^{-2x}(1 + 2x + 2x^2),$$

or

$$e{-}2x = 1 + 2x + 2x^2.$$

The result is $x = 1.34$, or $r = 1.34a_0$.

P47-15 The probability of finding it in a sphere of radius $r = xa_0$ is

$$\begin{aligned} P &= \int_0^{xa_0} \frac{r^2(2 - r/a_0)^2 e^{-r/a_0}\, dr}{8a_0^3} \\ &= \frac{1}{8}\int_0^x x^2(2-x)^2 e^{-x}\, dx \\ &= 1 - e^{-x}(y^4/8 + y^2/2 + y + 1). \end{aligned}$$

The minimum occurs at $x = 2$, so

$$P = 1 - e^{-2}(2 + 2 + 2 + 1) = 0.0527.$$

$\boxed{\textbf{E48-1}}$ The highest energy x-ray photon will have an energy equal to the bombarding electrons, as is shown in Eq. 48-1,

$$\lambda_{\min} = \frac{hc}{eV}$$

Insert the appropriate values into the above expression,

$$\lambda_{\min} = \frac{(4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{eV} = \frac{1240 \times 10^{-9} \text{ eV} \cdot \text{m}}{eV}.$$

The expression is then

$$\lambda_{\min} = \frac{1240 \times 10^{-9} \text{ V} \cdot \text{m}}{V} = \frac{1240 \text{ kV} \cdot \text{pm}}{V}.$$

So long as we are certain that the "V" will be measured in units of kilovolts, we can write this as

$$\lambda_{\min} = 1240 \text{ pm}/V.$$

E48-2 $f = c/\lambda = (3.00\times10^8\text{m/s})/(31.1\times10^{-12}\text{m}) = 9.646\times10^{18}/\text{s}$. Planck's constant is then

$$h = \frac{E}{f} = \frac{(40.0 \text{ keV})}{(9.646\times10^{18}/\text{s})} = 4.14\times10^{-15} \text{ eV} \cdot \text{s}.$$

E48-3 Applying the results of Exercise 48-1,

$$\Delta V = \frac{(1240\text{kV} \cdot \text{pm})}{(126 \text{ pm})} = 9.84 \text{ kV}.$$

E48-4 (a) Applying the results of Exercise 48-1,

$$\lambda_{\min} = \frac{(1240\text{kV} \cdot \text{pm})}{(35.0 \text{ kV})} = 35.4 \text{ pm}.$$

(b) Applying the results of Exercise 45-1,

$$\lambda_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(25.51 \text{ keV}) - (0.53 \text{ keV})} = 49.6 \text{ pm}.$$

(c) Applying the results of Exercise 45-1,

$$\lambda_{K_\alpha} = \frac{(1240\text{keV} \cdot \text{pm})}{(25.51 \text{ keV}) - (3.56 \text{ keV})} = 56.5 \text{ pm}.$$

$\boxed{\textbf{E48-5}}$ (a) Changing the accelerating potential of the x-ray tube will decrease $\lambda_{\min}$. The new value will be (using the results of Exercise 48-1)

$$\lambda_{\min} = 1240 \text{ pm}/(50.0) = 24.8 \text{ pm}.$$

(b) λ_{K_β} doesn't change. It is a property of the atom, not a property of the accelerating potential of the x-ray tube. The only way in which the accelerating potential might make a difference is if $\lambda_{K_\beta} < \lambda_{\min}$ for which case there would not be a λ_{K_β} line.

(c) λ_{K_α} doesn't change. See part (b).

E48-6 (a) Applying the results of Exercise 45-1,

$$\Delta E = \frac{(1240\text{keV} \cdot \text{pm})}{(19.3 \text{ pm})} = 64.2 \text{ keV}.$$

(b) This is the transition $n = 2$ to $n = 1$, so

$$\Delta E = (13.6 \text{ eV})(1/1^2 - 1/2^2) = 10.2 \text{ eV}.$$

E48-7 Applying the results of Exercise 45-1,

$$\Delta E_\beta = \frac{(1240\text{keV} \cdot \text{pm})}{(62.5 \text{ pm})} = 19.8 \text{ keV}.$$

and

$$\Delta E_\alpha = \frac{(1240\text{keV} \cdot \text{pm})}{(70.5 \text{ pm})} = 17.6 \text{ keV}.$$

The difference is

$$\Delta E = (19.8 \text{ keV}) - (17.6 \text{ keV}) = 2.2 \text{ eV}.$$

E48-8 Since $E_\lambda = hf = hc/\lambda$, and $\lambda = h/mc = hc/mc^2$, then

$$E_\lambda = hc/\lambda = mc^2.$$

or $\Delta V = E_\lambda/e = mc^2/e = 511$ kV.

E48-9 The 50.0 keV electron makes a collision and loses half of its energy to a photon, then the photon has an energy of 25.0 keV. The electron is now a 25.0 keV electron, and on the next collision again loses loses half of its energy to a photon, then this photon has an energy of 12.5 keV. On the third collision the electron loses the remaining energy, so this photon has an energy of 12.5 keV. The wavelengths of these photons will be given by

$$\lambda = \frac{(1240 \text{ keV} \cdot \text{pm})}{E},$$

which is a variation of Exercise 45-1.

E48-10 (a) The x-ray will need to knock free a K shell electron, so it must have an energy of at least 69.5 keV.

(b) Applying the results of Exercise 48-1,

$$\lambda_{\text{min}} = \frac{(1240\text{kV} \cdot \text{pm})}{(69.5 \text{ kV})} = 17.8 \text{ pm}.$$

(c) Applying the results of Exercise 45-1,

$$\lambda_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(69.5 \text{ keV}) - (2.3 \text{ keV})} = 18.5 \text{ pm}.$$

Applying the results of Exercise 45-1,

$$\lambda_{K_\alpha} = \frac{(1240\text{keV} \cdot \text{pm})}{(69.5 \text{ keV}) - (11.3 \text{ keV})} = 21.3 \text{ pm}.$$

E48-11 (a) Applying the results of Exercise 45-1,

$$E_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(63 \text{ pm})} = 19.7 \text{ keV}.$$

Again applying the results of Exercise 45-1,

$$E_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(71 \text{ pm})} = 17.5 \text{ keV}.$$

(b) Zr or Nb; the others will not significantly absorb either line.

E48-12 Applying the results of Exercise 45-1,

$$\lambda_{K_\alpha} = \frac{(1240\text{keV} \cdot \text{pm})}{(8.979 \text{ keV}) - (0.951 \text{ keV})} = 154.5 \text{ pm}.$$

Applying the Bragg reflection relationship,

$$d = \frac{\lambda}{2\sin\theta} = \frac{(154.5 \text{ pm})}{2\sin(15.9^\circ)} = 282 \text{ pm}.$$

E48-13 Plot the data. The plot should look just like Fig 48-4. Note that the vertical axis is $\sqrt{f}$, which is related to the wavelength according to $\sqrt{f} = \sqrt{c/\lambda}$.

E48-14 Remember that the m in Eq. 48-4 refers to the electron, not the nucleus. This means that the constant C in Eq. 48-5 is the same for all elements. Since $\sqrt{f} = \sqrt{c/\lambda}$, we have

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{Z_2 - 1}{Z_1 - 1}\right)^2.$$

For Ga and Nb the wavelength ratio is then

$$\frac{\lambda_{\text{Nb}}}{\lambda_{\text{Ga}}} = \left(\frac{(31) - 1}{(41) - 1}\right)^2 = 0.5625.$$

$\boxed{\textbf{E48-15}}$ (a) The ground state question is fairly easy. The $n = 1$ shell is completely occupied by the first two electrons. So the third electron will be in the $n = 2$ state. The lowest energy angular momentum state in any shell is the s sub-shell, corresponding to $l = 0$. There is only one choice for m_l in this case: $m_l = 0$. There is no way at this level of coverage to distinguish between the energy of either the spin up or spin down configuration, so we'll arbitrarily pick spin up.

(b) Determining the configuration for the first excited state will require some thought. We could assume that one of the K shell electrons ($n = 1$) is promoted to the L shell ($n = 2$). Or we could assume that the L shell electron is promoted to the M shell. Or we could assume that the L shell electron remains in the L shell, but that the angular momentum value is changed to $l = 1$. The question that we would need to answer is which of these possibilities has the lowest energy.

The answer is the last choice: increasing the l value results in a small increase in the energy of multi-electron atoms.

E48-16 Refer to Sample Problem 47-6:

$$r_1 = \frac{a_0(1)^2}{Z} = \frac{(5.29\times10^{-11}\text{m})}{(92)} = 5.75\times10^{-13}\,\text{m}.$$

$\boxed{\textbf{E48-17}}$ We will assume that the ordering of the energy of the shells and sub-shells is the same. That ordering is

$$\begin{gathered} 1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p \\ < 6s < 4f < 5d < 6p < 7s < 5f < 6d < 7p < 8s. \end{gathered}$$

If there is no spin the s sub-shell would hold 1 electron, the p sub-shell would hold 3, the d sub-shell 5, and the f sub-shell 7. inert gases occur when a p sub-shell has filled, so the first three inert gases would be element 1 (Hydrogen), element $1 + 1 + 3 = 5$ (Boron), and element $1 + 1 + 3 + 1 + 3 = 9$ (Fluorine).

Is there a pattern? Yes. The new inert gases have *half* of the atomic number of the original inert gases. The factor of one-half comes about because there are no longer two spin states for each set of n, l, m_l quantum numbers.

We can save time and simply divide the atomic numbers of the remaining inert gases in half: element 18 (Argon), element 27 (Cobalt), element 43 (Technetium), element 59 (Praseodymium).

E48-18 The pattern is

$$2 + 8 + 8 + 18 + 18 + 32 + 32 + ?$$

or

$$2(1^2 + 2^2 + 2^2 + 3^3 + 3^3 + 4^2 + 4^2 + x^2)$$

The unknown is probably $x = 5$, the next noble element is probably

$$118 + 2 \cdot 5^2 = 168.$$

E48-19 (a) Apply Eq. 47-23, which can be written as

$$E_n = \frac{(-13.6 \text{ eV})Z^2}{n^2}.$$

For the valence electron of sodium $n = 3$,

$$Z = \sqrt{\frac{(5.14 \text{ eV})(3)^2}{(13.6 \text{ eV})}} = 1.84,$$

while for the valence electron of potassium $n = 4$,

$$Z = \sqrt{\frac{(4.34 \text{ eV})(4)^2}{(13.6 \text{ eV})}} = 2.26,$$

(b) The ratios with the actual values of Z are 0.167 and 0.119, respectively.

E48-20 (a) There are three m_l states allowed, and two m_s states. The first electron can be in any one of these six combinations of M_1 and m_2. The second electron, given no exclusion principle, could also be in any one of these six states. The total is 36. Unfortunately, this is wrong, because we can't distinguish electrons. Of this total of 36, six involve the electrons being in the same state, while 30 involve the electron being in different states. But if the electrons are in different states, then they could be swapped, and we won't know, so we must divide this number by two. The total number of distinguishable states is then

$$(30/2) + 6 = 21.$$

(b) Six. See the above discussion.

E48-21 (a) The Bohr orbits are circular orbits of radius $r_n = a_0 n^2$ (Eq. 47-20). The electron is orbiting where the force is

$$F_n = \frac{e^2}{4\pi\epsilon_0 r_n^2},$$

and this force is equal to the centripetal force, so

$$\frac{mv^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2}.$$

where v is the velocity of the electron. Rearranging,

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}.$$

The time it takes for the electron to make one orbit can be used to calculate the current,

$$i = \frac{q}{t} = \frac{e}{2\pi r_n/v} = \frac{e}{2\pi r_n}\sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}.$$

The magnetic moment of a current loop is the current times the area of the loop, so

$$\mu = iA = \frac{e}{2\pi r_n}\sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}\pi r_n^2,$$

which can be simplified to

$$\mu = \frac{e}{2}\sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}} r_n.$$

But $r_n = a_0 n^2$, so

$$\mu = n\frac{e}{2}\sqrt{\frac{a_0 e^2}{4\pi\epsilon_0 m}}.$$

This might not look right, but $a_0 = \epsilon_0 h^2/\pi m e^2$, so the expression can simplify to

$$\mu = n\frac{e}{2}\sqrt{\frac{h^2}{4\pi^2 m^2}} = n\left(\frac{eh}{4\pi m}\right) = n\mu_{\mathrm{B}}.$$

(b) In reality the magnetic moments depend on the angular momentum quantum number, not the principle quantum number. Although the Bohr theory correctly predicts the magnitudes, it does not correctly predict when these values would occur.

E48-22 (a) Apply Eq. 48-14:

$$F_z = \mu_z \frac{dB_z}{dz} = (9.27\times10^{-24}\,\mathrm{J/T})(16\times10^{-3}\,\mathrm{T/m}) = 1.5\times10^{-25}\,\mathrm{N}.$$

(b) $a = F/m$, $\Delta z = at^2/2$, and $t = y/v_y$. Then

$$\Delta z = \frac{Fy^2}{2mv_y^2} = \frac{(1.5\times10^{-25}\,\mathrm{N})(0.82\,\mathrm{m})^2}{2(1.67\times10^{-27}\mathrm{kg})(970\,\mathrm{m/s})^2} = 3.2\times10^{-5}\mathrm{m}.$$

E48-23 $a = (9.27\times10^{-24}\mathrm{J/T})(1.4\times10^3\mathrm{T/m})/(1.7\times10^{-25}\mathrm{kg}) = 7.6\times10^4\mathrm{m/s^2}$.

E48-24 (a) $\Delta U = 2\mu B$, or

$$\Delta U = 2(5.79\times10^{-5}\text{eV/T})(0.520\,\text{T}) = 6.02\times10^{-5}\text{eV}.$$

(b) $f = E/h = (6.02\times10^{-5}\text{eV})(4.14\times10^{-15}\text{eV}\cdot\text{s}) = 1.45\times10^{10}\text{Hz}.$
(c) $\lambda = c/f = (3\times10^{8}\text{m/s})/(1.45\times10^{10}\text{Hz}) = 2.07\times10^{-2}\text{m}.$

$\boxed{\textbf{E48-25}}$ The energy change can be derived from Eq. 48-13; we multiply by a factor of 2 because the spin is completely flipped. Then

$$\Delta E = 2\mu_z B_z = 2(9.27\times10^{-24}\,\text{J/T})(0.190\,\text{T}) = 3.52\times10^{-24}\text{J}.$$

The corresponding wavelength is

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})(3.00\times10^{8}\text{m/s})}{(3.52\times10^{-24}\text{J})} = 5.65\times10^{-2}\text{m}.$$

This is somewhere near the microwave range.

E48-26 The photon has an energy $E = hc/\lambda$. This energy is related to the magnetic field in the vicinity of the electron according to

$$E = 2\mu B,$$

so

$$B = \frac{hc}{2\mu\lambda} = \frac{(1240\ \text{eV}\cdot\text{nm})}{2(5.79\times10^{-5}\text{J/T})(21\times10^{7}\text{nm})} = 0.051\,\text{T}.$$

E48-27 Applying the results of Exercise 45-1,

$$E = \frac{(1240\ \text{eV}\cdot\text{nm})}{(800\text{nm})} = 1.55\ \text{eV}.$$

The production rate is then

$$R = \frac{(5.0\times10^{-3}\text{W})}{(1.55\ \text{eV})(1.6\times10^{-19}\text{J/eV})} = 2.0\times10^{16}/\text{s}.$$

E48-28 (a) $x = (3\times10^{8}\text{m/s})(12\times10^{-12}\text{s}) = 3.6\times10^{-3}\text{m}.$
(b) Applying the results of Exercise 45-1,

$$E = \frac{(1240\ \text{eV}\cdot\text{nm})}{(694.4\text{nm})} = 1.786\ \text{eV}.$$

The number of photons in the pulse is then

$$N = (0.150\text{J})/(1.786\ \text{eV})(1.6\times10^{-19}\text{J/eV}) = 5.25\times10^{17}.$$

$\boxed{\textbf{E48-29}}$ We need to find out how many 10 MHz wide signals can fit between the two wavelengths. The lower frequency is

$$f_1 = \frac{c}{\lambda_1} = \frac{(3.00\times10^{8}\ \text{m/s})}{700\times10^{-9}\ \text{m})} = 4.29\times10^{14}\ \text{Hz}.$$

The higher frequency is

$$f_1 = \frac{c}{\lambda_1} = \frac{(3.00 \times 10^8 \,\mathrm{m/s})}{400 \times 10^{-9} \,\mathrm{m})} = 7.50 \times 10^{14} \text{ Hz}.$$

The number of signals that can be sent in this range is

$$\frac{f_2 - f_1}{(10 \text{ MHz})} = \frac{(7.50 \times 10^{14} \text{ Hz}) - (4.29 \times 10^{14} \text{ Hz})}{(10 \times 10^6 \text{ Hz})} = 3.21 \times 10^7.$$

That's quite a number of television channels.

E48-30 Applying the results of Exercise 45-1,

$$E = \frac{(1240 \text{ eV} \cdot \text{nm})}{(632.8\text{nm})} = 1.960 \text{ eV}.$$

The number of photons emitted in one minute is then

$$N = \frac{(2.3\times 10^{-3}\mathrm{W})(60\,\mathrm{s})}{(1.960 \text{ eV})(1.6\times 10^{-19}\mathrm{J/eV})} = 4.4\times 10^{17}.$$

E48-31 Apply Eq. 48-19. $E_13 - E_11 = 2(1.2 \text{ eV})$.. The ratio is then

$$\frac{n_13}{n_11} = e^{-(2.4 \text{ eV})/(8.62\times 10^{-5}\mathrm{eV/K})(2000 \text{ K})} = 9\times 10^{-7}.$$

E48-32 (a) Population inversion means that the higher energy state is more populated; this can only happen if the ratio in Eq. 48-19 is greater than one, which can only happen if the argument of the exponent is positive. That would require a negative temperature.

(b) If $n_2 = 1.1 n_1$ then the ratio is 1.1, so

$$T = \frac{(-2.26 \text{ eV}}{(8.62\times 10^{-5}\mathrm{eV/K})\ln(1.1)} = -2.75\times 10^5\mathrm{K}.$$

$\boxed{\textbf{E48-33}}$ (a) At thermal equilibrium the population ratio is given by

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-\Delta E/kT}.$$

But ΔE can be written in terms of the transition photon wavelength, so this expression becomes

$$N_2 = N_1 e^{-hc/\lambda kT}.$$

Putting in the numbers,

$$N_2 = (4.0\times 10^{20})e^{-(1240 \text{ eV}\cdot\text{nm})/(582 \text{ nm})(8.62\times 10^{-5} \text{ eV/K})(300\,\mathrm{K}))} = 6.62\times 10^{-16}.$$

That's effectively *none.*

(b) If the population of the upper state were 7.0×10^{20}, then in a single laser pulse

$$E = N\frac{hc}{\lambda} = (7.0\times 10^{20})\frac{(6.63\times 10^{-34}\mathrm{J\cdot s})(3.00\times 10^8\mathrm{m/s})}{(582\times 10^{-9}\mathrm{m})} = 240\,\mathrm{J}.$$

E48-34 The allowed wavelength in a standing wave chamber are $\lambda_n = 2L/n$. For large n we can write

$$\lambda_{n+1} = \frac{2L}{n+1} \approx \frac{2L}{n} - \frac{2L}{n^2}.$$

The wavelength difference is then

$$\Delta\lambda = \frac{2L}{n^2} = \frac{\lambda_n^2}{2L},$$

which in this case is

$$\Delta\lambda = \frac{(533\times10^{-9}\text{m})^2}{2(8.3\times10^{-2}\text{m})} = 1.7\times10^{-12}\text{m}.$$

E48-35 (a) The central disk will have an angle as measured from the center given by

$$d\sin\theta = (1.22)\lambda,$$

and since the parallel rays of the laser are focused on the screen in a distance f, we also have $R/f = \sin\theta$. Combining, and rearranging,

$$R = \frac{1.22 f\lambda}{d}.$$

(b) $R = 1.22(3.5\text{ cm})(515\text{ nm})/(3\text{ mm}) = 7.2\times10^{-6}\text{m}$.
(c) $I = P/A = (5.21\,\text{W})/\pi(1.5\text{ mm})^2 = 7.37\times10^5\text{W/m}^2$.
(d) $I = P/A = (0.84)(5.21\,\text{W})/\pi(7.2\,\mu\text{m})^2 = 2.7\times10^{10}\text{W/m}^2$.

E48-36

P48-1 Let λ_1 be the wavelength of the first photon. Then $\lambda_2 = \lambda_1 + 130$ pm. The total energy transfered to the two photons is then

$$E_1 + E_2 = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = 20.0\text{ keV}.$$

We can solve this for λ_1,

$$\begin{aligned}\frac{20.0\text{ keV}}{hc} &= \frac{1}{\lambda_1} + \frac{1}{\lambda_1 + 130\text{ pm}},\\ &= \frac{2\lambda_1 + 130\text{ pm}}{\lambda_1(\lambda_1 + 130\text{ pm})},\end{aligned}$$

which can also be written as

$$\begin{aligned}\lambda_1(\lambda_1 + 130\text{ pm}) &= (62\text{ pm})(2\lambda_1 + 130\text{ pm}),\\ \lambda_1^2 + (6\text{ pm})\lambda_1 - (8060\text{ pm}^2) &= 0.\end{aligned}$$

This equation has solutions

$$\lambda_1 = 86.8\text{ pm and } -92.8\text{ pm}.$$

Only the positive answer has physical meaning. The energy of this first photon is then

$$E_1 = \frac{(1240\text{ keV}\cdot\text{pm})}{(86.8\text{ pm})} = 14.3\text{ keV}.$$

(a) After this first photon is emitted the electron still has a kinetic energy of

$$20.0\text{ keV} - 14.3\text{ keV} = 5.7\text{ keV}.$$

(b) We found the energy and wavelength of the first photon above. The energy of the second photon *must* be 5.7 keV, with wavelength

$$\lambda_2 = (86.8\text{ pm}) + 130\text{ pm} = 217\text{ pm}.$$

P48-2 Originally,

$$\gamma = \frac{1}{\sqrt{1-(2.73\times10^8\mathrm{m/s})^2/(3\times10^8\mathrm{m/s})^2}} = 2.412.$$

The energy of the electron is

$$E_0 = \gamma mc^2 = (2.412)(511\text{ keV}) = 1232\text{ keV}.$$

Upon emitting the photon the new energy is

$$E = (1232\text{ keV}) - (43.8\text{ keV}) = 1189\text{ keV},$$

so the new gamma factor is

$$\gamma = (1189\text{ keV})/(511\text{ keV}) = 2.326,$$

and the new speed is

$$v = c\sqrt{1-1/(2.326)^2} = (0.903)c.$$

P48-3 Switch to a reference frame where the electron is originally at rest.

Momentum conservation requires

$$0 = p_\lambda + p_e = 0,$$

while energy conservation requires

$$mc^2 = E_\lambda + E_e.$$

Rearrange to

$$E_e = mc^2 - E_\lambda.$$

Square both sides of this energy expression and

$$\begin{aligned} E_\lambda^2 - 2E_\lambda mc^2 + m^2c^4 &= E_e^2 = p_e^2c^2 + m^2c^4, \\ E_\lambda^2 - 2E_\lambda mc^2 &= p_e^2c^2, \\ p_\lambda^2c^2 - 2E_\lambda mc^2 &= p_e^2c^2. \end{aligned}$$

But the momentum expression can be used here, and the result is

$$-2E_\lambda mc^2 = 0.$$

Not likely.

P48-4 (a) In the Bohr theory we can assume that the K shell electrons "see" a nucleus with charge Z. The L shell electrons, however, are shielded by the one electron in the K shell and so they "see" a nucleus with charge $Z-1$. Finally, the M shell electrons are shielded by the one electron in the K shell and the eight electrons in the K shell, so they "see" a nucleus with charge $Z-9$.

The transition wavelengths are then

$$\begin{aligned} \frac{1}{\lambda_\alpha} &= \frac{\Delta E}{hc} = \frac{E_0(Z-1)^2}{hc}\left(\frac{1}{2^2}-\frac{1}{1^2}\right), \\ &= \frac{E_0(Z-1)^2}{hc}\frac{3}{4}. \end{aligned}$$

and

$$\begin{aligned} \frac{1}{\lambda_\beta} &= \frac{\Delta E}{hc} = \frac{E_0}{hc}\left(\frac{1}{3^2}-\frac{1}{1^2}\right), \\ &= \frac{E_0(Z-9)^2}{hc}\frac{-8}{9}. \end{aligned}$$

The ratio of these two wavelengths is

$$\frac{\lambda_\beta}{\lambda_\alpha} = \frac{27}{32}\frac{(Z-1)^2}{(Z-9)^2}.$$

Note that the formula in the text has the square in the wrong place!

P48-5 (a) $E = hc/\lambda$; the energy difference is then

$$\begin{aligned}
\Delta E &= hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right), \\
&= hc\frac{\lambda_2 - \lambda_1}{\lambda_2\lambda_1}. \\
&= \frac{hc}{\lambda_2\lambda_1}\Delta\lambda.
\end{aligned}$$

Since λ_1 and λ_2 are *so* close together we can treat the product $\lambda_1\lambda_2$ as being either λ_1^2 or λ_2^2. Then

$$\Delta E = \frac{(1240 \text{ eV} \cdot \text{nm})}{(589 \text{ nm})^2}(0.597 \text{ nm}) = 2.1\times10^{-3}\text{eV}.$$

(b) The same energy difference exists in the $4s \to 3p$ doublet, so

$$\Delta\lambda = \frac{(1139 \text{ nm})^2}{(1240 \text{ eV} \cdot \text{nm})}(2.1\times10^{-3}\text{eV}) = 2.2 \text{ nm}.$$

P48-6 (a) We can assume that the K shell electron "sees" a nucleus of charge $Z-1$, since the other electron in the shell screens it. Then, according to the derivation leading to Eq. 47-22,

$$r_K = a_0/(Z-1).$$

(b) The outermost electron "sees" a nucleus screened by all of the other electrons; as such $Z = 1$, and the radius is

$$r = a_0$$

$\boxed{\textbf{P48-7}}$ We assume in this crude model that one electron moves in a circular orbit attracted to the helium nucleus but repelled from the other electron. Look back to Sample Problem 47-6; we need to use some of the results from that Sample Problem to solve this problem.

The factor of e^2 in Eq. 47-20 (the expression for the Bohr radius) and the factor of $(e^2)^2$ in Eq. 47-21 (the expression for the Bohr energy levels) was from the Coulomb force between the single electron and the single proton in the nucleus. This force is

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}.$$

In our approximation the force of attraction between the one electron and the helium nucleus is

$$F_1 = \frac{2e^2}{4\pi\epsilon_0 r^2}.$$

The factor of two is because there are two protons in the helium nucleus.

There is also a repulsive force between the one electron and the other electron,

$$F_2 = \frac{e^2}{4\pi\epsilon_0 (2r)^2},$$

where the factor of $2r$ is because the two electrons are on opposite side of the nucleus.

The net force on the first electron in our approximation is then

$$F_1 - F_2 = \frac{2e^2}{4\pi\epsilon_0 r^2} - \frac{e^2}{4\pi\epsilon_0 (2r)^2},$$

which can be rearranged to yield

$$F_{\text{net}} = \frac{e^2}{4\pi\epsilon_0 r^2}\left(2 - \frac{1}{4}\right) = \frac{e^2}{4\pi\epsilon_0 r^2}\left(\frac{7}{4}\right).$$

It is apparent that we need to substitute $7e^2/4$ for every occurrence of e^2.

(a) The ground state radius of the helium atom will then be given by Eq. 47-20 with the appropriate substitution,

$$r = \frac{\epsilon_0 h^2}{\pi m(7e^2/4)} = \frac{4}{7}a_0.$$

(b) The energy of *one* electron in this ground state is given by Eq. 47-21 with the substitution of $7e^2/4$ for every occurrence of e^2, then

$$E = -\frac{m(7e^2/4)^2}{8\epsilon_0^4 h^2} = -\frac{49}{16}\frac{me^4}{8\epsilon_0^4 h^2}.$$

We already evaluated all of the constants to be 13.6 eV.

One last thing. There are *two* electrons, so we need to double the above expression. The ground state energy of a helium atom in this approximation is

$$E_0 = -2\frac{49}{16}(13.6\text{ eV}) = -83.3eV.$$

(c) Removing one electron will allow the remaining electron to move closer to the nucleus. The energy of the remaining electron is given by the Bohr theory for He^+, and is

$$E_{\text{He}^+} = (4)(-13.60\text{ eV}) = 54.4\text{ eV},$$

so the ionization energy is 83.3 eV - 54.4 eV = 28.9 eV. This compares well with the accepted value.

P48-8 Applying Eq. 48-19:

$$T = \frac{(-3.2\text{ eV})}{(8.62\times10^{-5}\text{eV/K})\ln(6.1\times10^{13}/2.5\times10^{15})} = 1.0\times10^4\text{K}.$$

P48-9 $\sin\theta \approx r/R$, where r is the radius of the beam on the moon and R is the distance to the moon. Then

$$r = \frac{1.22(600\times10^{-9}\text{m})(3.82\times10^{8}\text{m})}{(0.118\,\text{m})} = 2360\,\text{m}.$$

The beam diameter is twice this, or 4740 m.

P48-10 (a) $N = 2L/\lambda_n$, or

$$N = \frac{2(6\times10^{-2}\text{m})(1.75)}{(694\times10^{-9})} = 3.03\times10^5.$$

(b) $N = 2nLf/c$, so

$$\Delta f = \frac{c}{2nL} = \frac{(3\times 10^8 \mathrm{m/s})}{2(1.75)(6\times 10^{-2}\mathrm{m})} = 1.43\times 10^9/\mathrm{s}.$$

Note that the travel time to and fro is $\Delta t = 2nL/c$!

(c) $\Delta f/f$ is then

$$\frac{\Delta f}{f} = \frac{\lambda}{2nL} = \frac{(694\times 10^{-9})}{2(1.75)(6\times 10^{-2}\mathrm{m})} = 3.3\times 10^{-6}.$$

E49-1 (a) Equation 49-2 is

$$n(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2} = \frac{8\sqrt{2}\pi (mc^2)^{3/2}}{(hc)^3}E^{1/2}.$$

We can evaluate this by substituting in all known quantities,

$$n(E) = \frac{8\sqrt{2}\pi(0.511 \times 10^6 \text{ eV})^{3/2}}{(1240 \times 10^{-9} \text{ eV} \cdot \text{m})^3}E^{1/2} = (6.81 \times 10^{27}\,\text{m}^{-3} \cdot \text{eV}^{-3/2})E^{1/2}.$$

Once again, we simplified the expression by writing hc wherever we could, and then using $hc = 1240 \times 10^{-9}$ eV $\cdot$ m.

(b) Then, if $E = 5.00$ eV,

$$n(E) = (6.81 \times 10^{27}\,\text{m}^{-3} \cdot \text{eV}^{-3/2})(5.00 \text{ eV})^{1/2} = 1.52 \times 10^{28}\,\text{m}^{-3} \cdot \text{eV}^{-1}.$$

E49-2 Apply the results of Ex. 49-1:

$$n(E) = (6.81 \times 10^{27}\,\text{m}^{-3} \cdot \text{eV}^{-3/2})(8.00 \text{ eV})^{1/2} = 1.93 \times 10^{28}\,\text{m}^{-3} \cdot \text{eV}^{-1}.$$

E49-3 Monovalent means only one electron is available as a conducting electron. Hence we need only calculate the density of atoms:

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{(19.3\times10^3\text{kg/m}^3)(6.02\times10^{23}\text{mol}^{-1})}{(0.197\,\text{kg/mol})} = 5.90\times10^{28}/\text{m}^3.$$

E49-4 Use the ideal gas law: $pV = NkT$. Then

$$p = \frac{N}{V}kT = (8.49\times10^{28}\text{m}^3)(1.38\times10^{-23}\text{J}/\cdot\text{K})(297\,\text{K}) = 3.48\times10^8\text{Pa}.$$

E49-5 (a) The approximate volume of a single sodium atom is

$$V_1 = \frac{(0.023\,\text{kg/mol})}{(6.02\times10^{23}\text{part/mol})(971\,\text{kg/m}^3)} = 3.93\times10^{-29}\text{m}^3.$$

The volume of the sodium ion sphere is

$$V_2 = \frac{4\pi}{3}(98\times10^{-12}\,\text{m})^3 = 3.94\times10^{-30}\,\text{m}^3.$$

The fractional volume available for conduction electrons is

$$\frac{V_1 - V_2}{V_1} = \frac{(3.93\times10^{-29}\text{m}^3) - (3.94\times10^{-30}\,\text{m}^3)}{(3.93\times10^{-29}\text{m}^3)} = 90\%.$$

(b) The approximate volume of a single copper atom is

$$V_1 = \frac{(0.0635\,\text{kg/mol})}{(6.02\times10^{23}\text{part/mol})(8960\,\text{kg/m}^3)} = 1.18\times10^{-29}\text{m}^3.$$

The volume of the copper ion sphere is

$$V_2 = \frac{4\pi}{3}(96\times10^{-12}\,\text{m})^3 = 3.71\times10^{-30}\,\text{m}^3.$$

The fractional volume available for conduction electrons is

$$\frac{V_1 - V_2}{V_1} = \frac{(1.18\times10^{-29}\text{m}^3) - (3.71\times10^{-30}\,\text{m}^3)}{(1.18\times10^{-29}\text{m}^3)} = 69\%.$$

(c) Sodium, since more of the volume is available for the conduction electron.

E49-6 (a) Apply Eq. 49-6:

$$p = 1/\left[e^{(0.0730\text{ eV})/(8.62\times10^{-5}\text{eV/K})(0\,\text{K})} + 1\right] = 0.$$

(b) Apply Eq. 49-6:

$$p = 1/\left[e^{(0.0730\text{ eV})/(8.62\times10^{-5}\text{eV/K})(320\,\text{K})} + 1\right] = 6.62\times10^{-2}.$$

E49-7 Apply Eq. 49-6, remembering to use the energy *difference*:

$$p = 1/\left[e^{(-1.1)\text{ eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 1.00,$$

$$p = 1/\left[e^{(-0.1)\text{ eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.986,$$

$$p = 1/\left[e^{(0.0)\text{ eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.5,$$

$$p = 1/\left[e^{(0.1)\text{ eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.014,$$

$$p = 1/\left[e^{(1.1)\text{ eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.0.$$

(b) Inverting the equation,

$$T = \frac{\Delta E}{k\ln(1/p-1)},$$

so

$$T = \frac{(0.1\text{ eV})}{(8.62\times10^{-5}\text{eV/K})\ln(1/(0.16)-1)} = 700\,\text{K}$$

E49-8 The energy differences are equal, except for the sign. Then

$$\begin{aligned}
\frac{1}{e^{+\Delta E/kt}+1} + \frac{1}{e^{-\Delta E/kt}+1} &= \;, \\
\frac{e^{-\Delta E/2kt}}{e^{+\Delta E/2kt}+e^{-\Delta E/2kt}} + \frac{e^{+\Delta E/2kt}}{e^{-\Delta E/2kt}+e^{+\Delta E/2kt}} &= \;, \\
\frac{e^{-\Delta E/2kt}+e^{+\Delta E/2kt}}{e^{-\Delta E/2kt}+e^{+\Delta E/2kt}} &= 1.
\end{aligned}$$

$\boxed{\textbf{E49-9}}$ The Fermi energy is given by Eq. 49-5,

$$E_{\text{F}} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3},$$

where n is the density of conduction electrons. For gold we have

$$n = \frac{(19.3\text{ g/cm}^3)(6.02\times10^{23}\text{part/mol})}{(197\text{ g/mol})} = 5.90\times10^{22}\text{ elect./cm}^3 = 59\text{ elect./nm}^3$$

The Fermi energy is then

$$E_{\text{F}} = \frac{(1240\text{ eV}\cdot\text{nm})^2}{8(0.511\times10^6\text{ eV})}\left(\frac{3(59\text{ electrons/nm}^3)}{\pi}\right)^{2/3} = 5.53\text{ eV}.$$

E49-10 Combine the results of Ex. 49-1 and Eq. 49-6:

$$n_o = \frac{C\sqrt{E}}{e^{\Delta E/kt}+1}.$$

Then for each of the energies we have

$$n_o = \frac{(6.81\times10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(4\,\mathrm{eV})}}{e^{(-3.06\,\mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 1.36\times10^{28}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_o = \frac{(6.81\times10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(6.75\,\mathrm{eV})}}{e^{(-0.31\,\mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 1.72\times10^{28}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_o = \frac{(6.81\times10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(7\,\mathrm{eV})}}{e^{(-0.06\,\mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 9.02\times10^{27}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_o = \frac{(6.81\times10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(7.25\,\mathrm{eV})}}{e^{(0.19\,\mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 1.82\times10^{27}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_o = \frac{(6.81\times10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(9\,\mathrm{eV})}}{e^{(1.94\,\mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 3.43\times10^{18}/\mathrm{m}^3\cdot\mathrm{eV}.$$

E49-11 Solve

$$E_n = \frac{n^2(hc)^2}{8(mc^2)L^2}$$

for $n = 50$, since there are two electrons in each level. Then

$$E_f = \frac{(50)^2(1240\ \mathrm{eV}\cdot\mathrm{nm})^2}{8(5.11\times10^5\mathrm{eV})(0.12\ \mathrm{nm})^2} = 6.53\times10^4\mathrm{eV}.$$

E49-12 We need to be much higher than $T = (7.06\,\mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K}) = 8.2\times10^4\mathrm{K}$.

$\boxed{\textbf{E49-13}}$ Equation 49-5 is

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3},$$

and if we collect the constants,

$$E_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3} n^{3/2} = An^{3/2},$$

where, if we multiply the top and bottom by c^2

$$A = \frac{(hc)^2}{8mc^2}\left(\frac{3}{\pi}\right)^{2/3} = \frac{(1240\times10^{-9}\ \mathrm{eV}\cdot\mathrm{m})^2}{8(0.511\times10^6\ \mathrm{eV})}\left(\frac{3}{\pi}\right)^{2/3} = 3.65\times10^{-19}\,\mathrm{m}^2\cdot\mathrm{eV}.$$

E49-14 (a) Inverting Eq. 49-6,

$$\Delta E = kT\ln(1/p-1),$$

so

$$\Delta E = (8.62\times10^{-5}\mathrm{eV/K})(1050\,\mathrm{K})\ln(1/(0.91)-1) = -0.209\ \mathrm{eV}.$$

Then $E = (-0.209\,\mathrm{eV}) + (7.06\,\mathrm{eV}) = 6.85\,\mathrm{eV}$.

(b) Apply the results of Ex. 49-1:

$$n(E) = (6.81\times10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})(6.85\ \mathrm{eV})^{1/2} = 1.78\times10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}.$$

(c) $n_o = np = (1.78\times10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1})(0.910) = 1.62\times10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}$.

E49-15 Equation 49-5 is

$$E_{\mathrm{F}} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3},$$

and if we rearrange,

$$E_{\mathrm{F}}{}^{3/2} = \frac{3h^3}{16\sqrt{2}\pi m^{3/2}}n,$$

Equation 49-2 is then

$$n(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2} = \frac{3}{2}nE_{\mathrm{F}}{}^{-3/2}E^{1/2}.$$

E49-16 $p_{\mathrm{h}} = 1 - p$, so

$$\begin{aligned} p_{\mathrm{h}} &= 1 - \frac{1}{e^{\Delta E/kT}+1}, \\ &= \frac{e^{\Delta E/kT}}{e^{\Delta E/kT}+1}, \\ &= \frac{1}{1+e^{-\Delta E/kT}}. \end{aligned}$$

$\boxed{\textbf{E49-17}}$ The steps to solve this exercise are equivalent to the steps for Exercise 49-9, except now the iron atoms each contribute 26 electrons and we have to find the density.

First, the density is

$$\rho = \frac{m}{4\pi r^3/3} = \frac{(1.99\times10^{30}\mathrm{kg})}{4\pi(6.37\times10^6\mathrm{m})^3/3} = 1.84\times10^9\mathrm{kg/m^3}$$

Then

$$\begin{aligned} n &= \frac{(26)(1.84\times10^6\ \mathrm{g/cm^3})(6.02\times10^{23}\mathrm{part/mol})}{(56\ \mathrm{g/mol})} = 5.1\times10^{29}\ \mathrm{elect./cm^3}, \\ &= 5.1\times10^8\ \mathrm{elect./nm^3} \end{aligned}$$

The Fermi energy is then

$$E_{\mathrm{F}} = \frac{(1240\ \mathrm{eV\cdot nm})^2}{8(0.511\times10^6\ \mathrm{eV})}\left(\frac{3(5.1\times10^8\ \mathrm{elect./nm^3})}{\pi}\right)^{2/3} = 230\ \mathrm{keV}.$$

E49-18 First, the density is

$$\rho = \frac{m}{4\pi r^3/3} = \frac{2(1.99\times10^{30}\mathrm{kg})}{4\pi(10\times10^3\mathrm{m})^3/3} = 9.5\times10^{17}\mathrm{kg/m^3}$$

Then

$$n = (9.5\times10^{17}\mathrm{kg/m^3})/(1.67\times10^{-27}\mathrm{kg}) = 5.69\times10^{44}/\mathrm{m^3}.$$

The Fermi energy is then

$$E_{\mathrm{F}} = \frac{(1240\ \mathrm{MeV\cdot fm})^2}{8(940\ \mathrm{MeV})}\left(\frac{3(5.69\times10^{-1}/\mathrm{fm^3})}{\pi}\right)^{2/3} = 137\ \mathrm{MeV}.$$

E49-19

E49-20 (a) $E_{\rm F} = 7.06\,\text{eV}$, so

$$f = \frac{3(8.62\times10^{-5}\text{eV}\cdot\text{K})(0\,\text{K})}{2(7.06\,\text{eV})} = 0,$$

(b) $f = 3(8.62\times10^{-5}\text{eV}\cdot\text{K})(300\,\text{K})/2(7.06\,\text{eV}) = 0.0055.$
(c) $f = 3(8.62\times10^{-5}\text{eV}\cdot\text{K})(1000\,\text{K})/2(7.06\,\text{eV}) = 0.0183.$

$\boxed{\textbf{E49-21}}$ Using the results of Exercise 19,

$$T = \frac{2fE_{\rm F}}{3k} = \frac{2(0.0130)(4.71\text{ eV})}{3(8.62\times10^{-5}\text{eV}\cdot\text{K})} = 474\,\text{K}.$$

E49-22 $f = 3(8.62\times10^{-5}\text{eV}\cdot\text{K})(1235\,\text{K})/2(5.5\,\text{eV}) = 0.029.$

E49-23 (a) Monovalent means only one electron is available as a conducting electron. Hence we need only calculate the density of atoms:

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{(10.5\times10^3\text{kg/m}^3)(6.02\times10^{23}\text{mol}^{-1})}{(0.107\,\text{kg/mol})} = 5.90\times10^{28}/\text{m}^3.$$

(b) Using the results of Ex. 49-13,

$$E_{\rm F} = (3.65\times10^{-19}\,\text{m}^2\cdot\text{eV})(5.90\times10^{28}/\text{m}^3)^{2/3} = 5.5\,\text{eV}.$$

(c) $v = \sqrt{2K/m}$, or

$$v = \sqrt{2(5.5\,\text{eV})}(5.11\times10^5\text{eV}/c^2) = 1.4\times10^8\text{m/s}.$$

(d) $\lambda = h/p$, or

$$\lambda = \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})}{(9.11\times10^{-31}\text{kg})(1.4\times10^8\text{m/s})} = 5.2\times10^{-12}\text{m}.$$

E49-24 (a) Bivalent means two electrons are available as a conducting electron. Hence we need to double the calculation of the density of atoms:

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{2(7.13\times10^3\text{kg/m}^3)(6.02\times10^{23}\text{mol}^{-1})}{(0.065\,\text{kg/mol})} = 1.32\times10^{29}/\text{m}^3.$$

(b) Using the results of Ex. 49-13,

$$E_{\rm F} = (3.65\times10^{-19}\,\text{m}^2\cdot\text{eV})(1.32\times10^{29}/\text{m}^3)^{2/3} = 9.4\,\text{eV}.$$

(c) $v = \sqrt{2K/m}$, or

$$v = \sqrt{2(9.4\,\text{eV})}(5.11\times10^5\text{eV}/c^2) = 1.8\times10^8\text{m/s}.$$

(d) $\lambda = h/p$, or

$$\lambda = \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})}{(9.11\times10^{-31}\text{kg})(1.8\times10^8\text{m/s})} = 4.0\times10^{-12}\text{m}.$$

E49-25 (a) Refer to Sample Problem 49-5 where we learn that the mean free path λ can be written in terms of Fermi speed $v_{\rm F}$ and mean time between collisions τ as

$$\lambda = v_{\rm F}\tau.$$

The Fermi speed is

$$v_{\rm F} = c\sqrt{2E_F/mc^2} = c\sqrt{2(5.51\text{ eV})/(5.11\times10^5\text{ eV})} = 4.64\times10^{-3}c.$$

The time between collisions is

$$\tau = \frac{m}{ne^2\rho} = \frac{(9.11\times10^{-31}\text{kg})}{(5.86\times10^{28}\text{m}^{-3})(1.60\times10^{-19}\text{C})^2(1.62\times10^{-8}\Omega\cdot\text{m})} = 3.74\times10^{-14}\text{s}.$$

We found n by looking up the answers from Exercise 49-23 in the back of the book. The mean free path is then

$$\lambda = (4.64\times10^{-3})(3.00\times10^8\text{m/s})(3.74\times10^{-14}\text{s}) = 52\text{ nm}.$$

(b) The spacing between the ion cores is approximated by the cube root of volume per atom. This atomic volume for silver is

$$V = \frac{(108\text{ g/mol})}{(6.02\times10^{23}\text{part/mol})(10.5\text{ g/cm}^3)} = 1.71\times10^{-23}\text{cm}^3.$$

The distance between the ions is then

$$l = \sqrt[3]{V} = 0.257\text{ nm}.$$

The ratio is

$$\lambda/l = 190.$$

E49-26 (a) For $T = 1000\,\text{K}$ we can use the approximation, so for diamond

$$p = e^{-(5.5\,\text{eV})/2(8.62\times10^{-5}\text{eV/K})(1000\,\text{K})} = 1.4\times10^{-14},$$

while for silicon,

$$p = e^{-(1.1\,\text{eV})/2(8.62\times10^{-5}\text{eV/K})(1000\,\text{K})} = 1.7\times10^{-3},$$

(b) For $T = 4\,\text{K}$ we can use the same approximation, but now $\Delta E \gg kT$ and the exponential function goes to zero.

E49-27 (a) $E - E_{\rm F} \approx 0.67\,\text{eV}/2 = 0.34\,\text{eV}$.. The probability the state is occupied is then

$$p = 1/\left[e^{(0.34)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(290\,\text{K})} + 1\right] = 1.2\times10^{-6}.$$

(b) $E - E_{\rm F} \approx -0.67\,\text{eV}/2 = -0.34\,\text{eV}$.. The probability the state is *unoccupied* is then $1-p$, or

$$p = 1 - 1/\left[e^{(-0.34)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(290\,\text{K})} + 1\right] = 1.2\times10^{-6}.$$

E49-28 (a) $E - E_{\rm F} \approx 0.67\,\text{eV}/2 = 0.34\,\text{eV}$.. The probability the state is occupied is then

$$p = 1/\left[e^{(0.34)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(289\,\text{K})} + 1\right] = 1.2\times10^{-6}.$$

$\boxed{\textbf{E49-29}}$ (a) The number of silicon atoms per unit volume is

$$n = \frac{(6.02\times10^{23}\text{part/mol})(2.33\text{ g/cm}^3)}{(28.1\text{ g/mol})} = 4.99\times10^{22}\text{ part./cm}^3.$$

If one out of $1.0eex7$ are replaced then there will be an additional charge carrier density of

$$4.99\times10^{22}\text{ part./cm}^3/1.0\times10^7 = 4.99\times10^{15}\text{ part./cm}^3 = 4.99\times10^{21}\text{m}^{-3}.$$

(b) The ratio is

$$(4.99\times10^{21}\text{m}^{-3})/(2\times1.5\times10^{16}\text{m}^{-3}) = 1.7\times10^5.$$

The extra factor of two is because *all* of the charge carriers in silicon (holes and electrons) are charge carriers.

E49-30 Since one out of every 5×10^6 silicon atoms needs to be replaced, then the mass of phosphorus would be

$$m = \frac{1}{5\times10^6}\frac{30}{28} = 2.1\times10^{-7}\text{ g}.$$

E49-31 $l = \sqrt[3]{1/10^{22}/\text{m}^3} = 4.6\times10^{-8}\text{m}.$

E49-32 The atom density of germanium is

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{(5.32\times10^3\text{kg/m}^3)(6.02\times10^{23}\text{mol}^{-1})}{(0.197\,\text{kg/mol})} = 1.63\times10^{28}/\text{m}^3.$$

The atom density of the impurity is

$$(1.63\times10^{28}/\text{m}^3)/(1.3\times10^9) = 1.25\times10^{19}.$$

The average spacing is

$$l = \sqrt[3]{1/1.25\times10^{19}/\text{m}^3} = 4.3\times10^{-7}\text{m}.$$

$\boxed{\textbf{E49-33}}$ The first one is an insulator because the lower band is filled and band gap is so large; there is no impurity.

The second one is an extrinsic n-type semiconductor: it is a semiconductor because the lower band is filled and the band gap is small; it is extrinsic because there is an impurity; since the impurity level is close to the top of the band gap the impurity is a donor.

The third sample is an intrinsic semiconductor: it is a semiconductor because the lower band is filled and the band gap is small.

The fourth sample is a conductor; although the band gap is large, the lower band is *not* completely filled.

The fifth sample is a conductor: the Fermi level is above the bottom of the upper band.

The sixth one is an extrinsic p-type semiconductor: it is a semiconductor because the lower band is filled and the band gap is small; it is extrinsic because there is an impurity; since the impurity level is close to the bottom of the band gap the impurity is an acceptor.

E49-34 $6.62\times10^5\text{eV}/1.1\,\text{eV} = 6.0\times10^5$ electron-hole pairs.

E49-35 (a) $R = (1\,\text{V})/(50\times10^{-12}\text{A}) = 2\times10^{10}\Omega.$

(b) $R = (0.75\,\text{V})/(8\,\text{mA}) = 90\Omega.$

E49-36 (a) A region with some potential difference exists that has a gap between the charged areas.

(b) $C = Q/\Delta V$. Using the results in Sample Problem 49-9 for q and ΔV,

$$C = \frac{n_0 e A d/2}{n_0 e d^2/4\kappa\epsilon_0} = 2\kappa\epsilon_0 A/d.$$

E49-37 (a) Apply that ever so useful formula

$$\lambda = \frac{hc}{E} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(5.5 \text{ eV})} = 225 \text{ nm}.$$

Why is this a *maximum?* Because longer wavelengths would have *lower* energy, and so not enough to cause an electron to jump across the band gap.

(b) Ultraviolet.

E49-38 Apply that ever so useful formula

$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(295 \text{ nm})} = 4.20\,\text{eV}.$$

E49-39 The photon energy is

$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(140 \text{ nm})} = 8.86\,\text{eV}.$$

which is enough to excite the electrons through the band gap. As such, the photon will be absorbed, which means the crystal is opaque to this wavelength.

E49-40

P49-1 We can calculate the electron density from Eq. 49-5,

$$\begin{aligned} n &= \frac{\pi}{3}\left(\frac{8mc^2 E_\text{F}}{(hc)^2}\right)^{3/2}, \\ &= \frac{\pi}{3}\left(\frac{8(0.511\times10^6 \text{ eV})(11.66 \text{ eV})}{(1240 \text{ eV}\cdot\text{nm})^2}\right)^{3/2}, \\ &= 181 \text{ electrons/nm}^3. \end{aligned}$$

From this we calculate the number of electrons per particle,

$$\frac{(181 \text{ electrons/nm}^3)(27.0 \text{ g/mol})}{(2.70 \text{ g/cm}^3)(6.02\times10^{23} \text{ particles/mol})} = 3.01,$$

which we can reasonably approximate as 3.

P49-2 At absolute zero all states below E_F are filled, an none above. Using the results of Ex. 49-15,

$$\begin{aligned}
E_\mathrm{av} &= \frac{1}{n}\int_0^{E_\mathrm{F}} En(E)\,dE, \\
&= \frac{3}{2}{E_\mathrm{F}}^{-3/2}\int_0^{E_\mathrm{F}} E^{3/2}\,dE, \\
&= \frac{3}{2}{E_\mathrm{F}}^{-3/2}\frac{2}{5}{E_\mathrm{F}}^{5/2}, \\
&= \frac{3}{5}E_\mathrm{F}.
\end{aligned}$$

P49-3 (a) The total number of conduction electron is

$$n = \frac{(0.0031\,\mathrm{kg})(6.02\times10^{23}\mathrm{mol}^{-1})}{(0.0635\,\mathrm{kg/mol})} = 2.94\times10^{22}.$$

The total energy is

$$E = \frac{3}{5}(7.06\,\mathrm{eV})(2.94\times10^{22}) = 1.24\times10^{23}\mathrm{eV} = 2\times10^{4}\mathrm{J}.$$

(b) This will light a 100 W bulb for

$$t = (2\times10^4\,\mathrm{J})/(100\,\mathrm{W}) = 200\,\mathrm{s}.$$

P49-4 (a) First do the easy part: $n_\mathrm{c} = N_\mathrm{c}p(E_\mathrm{c})$, so

$$\frac{N_\mathrm{c}}{e^{(E_\mathrm{c}-E_\mathrm{F})/kT}+1}.$$

Then use the results of Ex. 49-16, and write

$$n_\mathrm{v} = N_\mathrm{v}[1-p(E_\mathrm{v})] = \frac{N_\mathrm{v}}{e^{-(E_\mathrm{v}-E_\mathrm{F})/kT}+1}.$$

Since each electron in the conduction band must have left a hole in the valence band, then these two expressions must be equal.

(b) If the exponentials dominate then we can drop the +1 in each denominator, and

$$\begin{aligned}
\frac{N_\mathrm{c}}{e^{(E_\mathrm{c}-E_\mathrm{F})/kT}} &= \frac{N_\mathrm{v}}{e^{-(E_\mathrm{v}-E_\mathrm{F})/kT}}, \\
\frac{N_\mathrm{c}}{N_\mathrm{v}} &= e^{(E_\mathrm{c}-2E_\mathrm{F}+E_\mathrm{v})/kT}, \\
E_\mathrm{F} &= \frac{1}{2}\left(E_\mathrm{c}+E_\mathrm{v}+kT\ln(N_\mathrm{c}/N_\mathrm{v})\right).
\end{aligned}$$

$\boxed{\textbf{P49-5}}$ (a) We want to use Eq. 49-6; although we don't know the Fermi energy, we do know the differences between the energies in question. In the un-doped silicon $E - E_\mathrm{F} = 0.55$ eV for the bottom of the conduction band. The quantity

$$kT = (8.62\times10^{-5}\ \mathrm{eV/K})(290\,\mathrm{K}) = 0.025\ \mathrm{eV},$$

which is a good number to remember— at room temperature kT is 1/40 of an electron-volt.

Then

$$p = \frac{1}{e^{(0.55\ \text{eV})/(0.025\ \text{eV})} + 1} = 2.8\times 10^{-10}.$$

In the doped silicon $E - E_{\text{F}} = 0.084$ eV for the bottom of the conduction band. Then

$$p = \frac{1}{e^{(0.084\ \text{eV})/(0.025\ \text{eV})} + 1} = 3.4\times 10^{-2}.$$

(b) For the donor state $E - E_{\text{F}} = -0.066$ eV, so

$$p = \frac{1}{e^{(-0.066\ \text{eV})/(0.025\ \text{eV})} + 1} = 0.93.$$

P49-6 (a) Inverting Eq. 49-6,

$$E - E_{\text{F}} = kT\ \ln(1/p - 1),$$

so

$$E_{\text{F}} = (1.1\,\text{eV} - 0.11\,\text{eV}) - (8.62\times 10^{-5}\ \text{eV/K})(290\,\text{K})\ln(1/(4.8\times 10^{-5}) - 1) = 0.74\,\text{eV}$$

above the valence band.

(b) $E - E_{\text{F}} = (1.1\,\text{eV}) - (0.74\,\text{eV}) = 0.36\,\text{eV}$, so

$$p = \frac{1}{e^{(0.36\ \text{eV})/(0.025\ \text{eV})} + 1} = 5.6\times 10^{-7}.$$

P49-7 (a) Plot the graph with a spreadsheet. It should look like Fig. 49-12.

(b) $kT = 0.025\,\text{eV}$ when $T = 290\,\text{K}$. The ratio is then

$$\frac{i_{\text{f}}}{i_{\text{r}}} = \frac{e^{(0.5\,\text{eV})/(0.025\,\text{eV})} + 1}{e^{(-0.5\,\text{eV})/(0.025\,\text{eV})} + 1} = 4.9\times 10^{8}.$$

P49-8

$\boxed{\textbf{E50-1}}$ We want to follow the example set in Sample Problem 50-1. The distance of closest approach is given by

$$
\begin{aligned}
d &= \frac{qQ}{4\pi\epsilon_0 K_\alpha},\\
&= \frac{(2)(29)(1.60\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{Nm}^2)(5.30\text{MeV})(1.60\times10^{-13}\text{J/MeV})},\\
&= 1.57\times10^{-14}\,\text{m}.
\end{aligned}
$$

That's pretty close.

E50-2 (a) The gold atom can be treated as a point particle:

$$
\begin{aligned}
F &= \frac{q_1 q_2}{4\pi\epsilon_0 r^2},\\
&= \frac{(2)(79)(1.60\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{Nm}^2)(0.16\times10^{-9}\text{m})^2},\\
&= 1.4\times10^{-6}\text{N}.
\end{aligned}
$$

(b) $W = Fd$, so

$$
d = \frac{(5.3\times10^{6}\text{eV})(1.6\times10^{-19}\text{J/eV})}{(1.4\times10^{-6}\text{N})} = 6.06\times10^{-7}\text{m}.
$$

That's 1900 gold atom diameters.

E50-3 Take an approach similar to Sample Problem 50-1:

$$
\begin{aligned}
K &= \frac{qQ}{4\pi\epsilon_0 d},\\
&= \frac{(2)(79)(1.60\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{Nm}^2)(8.78\times10^{-15}\text{m})(1.60\times10^{-19}\text{J/eV})},\\
&= 2.6\times10^{7}\text{eV}.
\end{aligned}
$$

E50-4 All are stable except ^{88}Rb and ^{239}Pb.

$\boxed{\textbf{E50-5}}$ We can make an estimate of the mass number A from Eq. 50-1,

$$
R = R_0 A^{1/3},
$$

where $R_0 = 1.2$ fm. If the measurements indicate a radius of 3.6 fm we would have

$$
A = (R/R_0)^3 = ((3.6\text{ fm})/(1.2\text{ fm}))^3 = 27.
$$

E50-6

E50-7 The mass number of the sun is

$$
A = (1.99\times10^{30}\text{kg})/(1.67\times10^{-27}\text{kg}) = 1.2\times10^{57}.
$$

The radius would be

$$
R = (1.2\times10^{-15}\text{m})\sqrt[3]{1.2\times10^{57}} = 1.3\times10^{4}\text{m}.
$$

E50-8 ^{239}Pu is composed of 94 protons and $239 - 94 = 145$ neutrons. The combined mass of the free particles is

$$M = Zm_{\rm p} + Nm_{\rm n} = (94)(1.007825\ {\rm u}) + (145)(1.008665\ {\rm u}) = 240.991975\ {\rm u}.$$

The binding energy is the difference

$$E_{\rm B} = (240.991975\ {\rm u} - 239.052156\ {\rm u})(931.5\ {\rm MeV/u}) = 1806.9\ {\rm MeV},$$

and the binding energy per nucleon is then

$$(1806.9\ {\rm MeV})/(239) = 7.56\ {\rm MeV}.$$

$\boxed{\textbf{E50-9}}$ ^{62}Ni is composed of 28 protons and $62 - 28 = 34$ neutrons. The combined mass of the free particles is

$$M = Zm_{\rm p} + Nm_{\rm n} = (28)(1.007825\ {\rm u}) + (34)(1.008665\ {\rm u}) = 62.513710\ {\rm u}.$$

The binding energy is the difference

$$E_{\rm B} = (62.513710\ {\rm u} - 61.928349\ {\rm u})(931.5\ {\rm MeV/u}) = 545.3\ {\rm MeV},$$

and the binding energy per nucleon is then

$$(545.3\ {\rm MeV})/(62) = 8.795\ {\rm MeV}.$$

E50-10 (a) Multiply each by 1/1.007825, so

$$m_{^1{\rm H}} = 1.00000,$$

$$m_{^{12}{\rm C}} = 11.906829,$$

and

$$m_{^{238}{\rm U}} = 236.202500.$$

E50-11 (a) Since the binding energy per nucleon is fairly constant, the energy must be proportional to A.

(b) Coulomb repulsion acts between pairs of protons; there are Z protons that can be chosen as first in the pair, and $Z-1$ protons remaining that can make up the partner in the pair. That makes for $Z(Z-1)$ pairs. The electrostatic energy must be proportional to this.

(c) Z^2 grows faster than A, which is roughly proportional to Z.

E50-12 Solve

$$(0.7899)(23.985042) + x(24.985837) + (0.2101 - x)(25.982593) = 24.305$$

for x. The result is $x = 0.1001$, and then the amount ^{26}Mg is 0.1100.

$\boxed{\textbf{E50-13}}$ The neutron confined in a nucleus of radius R will have a position uncertainty on the order of $\Delta x \approx R$. The momentum uncertainty will then be no less than

$$\Delta p \geq \frac{h}{2\pi \Delta x} \approx \frac{h}{2\pi R}.$$

Assuming that $p \approx \Delta p$, we have

$$p \geq \frac{h}{2\pi R},$$

and then the neutron will have a (minimum) kinetic energy of

$$E \approx \frac{p^2}{2m} \approx \frac{h^2}{8\pi^2 m R^2}.$$

But $R = R_0 A^{1/3}$, so

$$E \approx \frac{(hc)^2}{8\pi^2 mc^2 R_0^2 A^{2/3}}.$$

For an atom with $A = 100$ we get

$$E \approx \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{8\pi^2 (940 \text{ MeV})(1.2 \text{ fm})^2 (100)^{2/3}} = 0.668 \text{ MeV}.$$

This is about a factor of 5 or 10 less than the binding energy per nucleon.

E50-14 (a) To remove a proton,

$$E = [(1.007825) + (3.016049) - (4.002603)]\,(931.5 \text{ MeV}) = 19.81 \text{ MeV}.$$

To remove a neutron,

$$E = [(1.008665) + (2.014102) - (3.016049)]\,(931.5 \text{ MeV}) = 6.258 \text{ MeV}.$$

To remove a proton,

$$E = [(1.007825) + (1.008665) - (2.014102)]\,(931.5 \text{ MeV}) = 2.224 \text{ MeV}.$$

(b) $E = (19.81 + 6.258 + 2.224)\text{MeV} = 28.30$ MeV.
(c) $(28.30 \text{ MeV})/4 = 7.07$ MeV.

E50-15 (a) $\Delta = [(1.007825) - (1)](931.5 \text{ MeV}) = 7.289$ MeV.
(b) $\Delta = [(1.008665) - (1)](931.5 \text{ MeV}) = 8.071$ MeV.
(c) $\Delta = [(119.902197) - (120)](931.5 \text{ MeV}) = -91.10$ MeV.

E50-16 (a) $E_\text{B} = (Zm_\text{H} + Nm_\text{N} - m)c^2$. Substitute the definition for mass excess, $mc^2 = Ac^2 + \Delta$, and

$$\begin{aligned} E_\text{B} &= Z(c^2 + \Delta_\text{H}) + N(c^2 + \Delta_\text{N}) - Ac^2 - \Delta, \\ &= Z\Delta_\text{H} + N\Delta_\text{N} - \Delta. \end{aligned}$$

(b) For ^{197}Au,

$$E_\text{B} = (79)(7.289 \text{ MeV}) + (197 - 79)(8.071 \text{ MeV}) - (-31.157 \text{ MeV}) = 1559 \text{ MeV},$$

and the binding energy per nucleon is then

$$(1559 \text{ MeV})/(197) = 7.92 \text{ MeV}.$$

E50-17 The binding energy of ^{63}Cu is given by

$$M = Zm_{\rm p} + Nm_{\rm n} = (29)(1.007825\text{ u}) + (34)(1.008665\text{ u}) = 63.521535\text{ u}.$$

The binding energy is the difference

$$E_{\rm B} = (63.521535\text{ u} - 62.929601\text{ u})(931.5\text{ MeV/u}) = 551.4\text{ MeV}.$$

The number of atoms in the sample is

$$n = \frac{(0.003\,\text{kg})(6.02\times 10^{23}\text{mol}^{-1})}{(0.0629\,\text{kg/mol})} = 2.87\times 10^{22}.$$

The total energy is then

$$(2.87\times 10^{22})(551.4\text{ MeV})(1.6\times 10^{-19}\text{J/eV}) = 2.53\times 10^{12}\text{J}.$$

E50-18 (a) For ultra-relativistic particles $E = pc$, so

$$\lambda = \frac{(1240\text{ MeV}\cdot\text{fm})}{(480\text{ MeV})} = 2.59\text{ fm}.$$

(b) Yes, since the wavelength is smaller than nuclear radii.

$\boxed{\textbf{E50-19}}$ We will do this one the easy way because we can. This method won't work except when there is an integer number of half-lives. The activity of the sample will fall to one-half of the initial decay rate after one half-life; it will fall to one-half of one-half (one-fourth) after two half-lives. So two half-lives have elapsed, for a total of (2)(140 d) = 280 d.

E50-20 $N = N_0(1/2)^{t/t_{1/2}}$, so

$$N = (48\times 10^{19})(0.5)^{(26)/(6.5)} = 3.0\times 10^{19}.$$

E50-21 (a) $t_{1/2} = \ln 2/(0.0108/\text{h}) = 64.2$ h.

(b) $N = N_0(1/2)^{t/t_{1/2}}$, so

$$N/N_0 = (0.5)^{(3)} = 0.125.$$

(c) $N = N_0(1/2)^{t/t_{1/2}}$, so

$$N/N_0 = (0.5)^{(240)/(64.2)} = 0.0749.$$

E50-22 (a) $\lambda = (-dN/dt)/N$, or

$$\lambda = (12/\text{s})/(2.5\times 10^{18}) = 4.8\times 10^{-18}/\text{s}.$$

(b) $t_{1/2} = \ln 2/\lambda$, so

$$t_{1/2} = \ln 2/(4.8\times 10^{-18}/\text{s}) = 1.44\times 10^{17}\text{s},$$

which is 4.5 billion years.

$\boxed{\textbf{E50-23}}$ (a) The decay constant for ^{67}Ga can be derived from Eq. 50-8,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(2.817\times10^5\,\mathrm{s})} = 2.461\times10^{-6}\mathrm{s}^{-1}.$$

The activity is given by $R = \lambda N$, so we want to know how many atoms are present. That can be found from

$$3.42\ \mathrm{g}\left(\frac{1\ \mathrm{u}}{1.6605\times10^{-24}\ \mathrm{g}}\right)\left(\frac{1\ \mathrm{atom}}{66.93\ \mathrm{u}}\right) = 3.077\times10^{22}\ \mathrm{atoms}.$$

So the activity is

$$R = (2.461\times10^{-6}/\mathrm{s}^{-1})(3.077\times10^{22}\ \mathrm{atoms}) = 7.572\times10^{16}\ \mathrm{decays/s}.$$

(b) After 1.728×10^5 s the activity would have decreased to

$$R = R_0 e^{-\lambda t} = (7.572\times10^{16}\ \mathrm{decays/s})e^{-(2.461\times10^{-6}/\mathrm{s}^{-1})(1.728\times10^5\,\mathrm{s})} = 4.949\times10^{16}\ \mathrm{decays/s}.$$

E50-24 $N = N_0 e^{-\lambda t}$, but $\lambda = \ln 2/t_{1/2}$, so

$$N = N_0 e^{-\ln 2t/t_{1/2}} = N_0(2)^{-t/t_{1/2}} = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}.$$

E50-25 The remaining ^{223} is

$$N = (4.7\times10^{21})(0.5)^{(28)/(11.43)} = 8.6\times10^{20}.$$

The number of decays, each of which produced an alpha particle, is

$$(4.7\times10^{21}) - (8.6\times10^{20}) = 3.84\times10^{21}.$$

E50-26 The amount remaining after 14 hours is

$$m = (5.50\ \mathrm{g})(0.5)^{(14)/(12.7)} = 2.562\ \mathrm{g}.$$

The amount remaining after 16 hours is

$$m = (5.50\ \mathrm{g})(0.5)^{(16)/(12.7)} = 2.297\ \mathrm{g}.$$

The difference is the amount which decayed during the two hour interval:

$$(2.562\ \mathrm{g}) - (2.297\ \mathrm{g}) = 0.265\ \mathrm{g}.$$

$\boxed{\textbf{E50-27}}$ (a) Apply Eq. 50-7,

$$R = R_0 e^{-\lambda t}.$$

We first need to know the decay constant from Eq. 50-8,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(1.234\times10^6\,\mathrm{s})} = 5.618\times10^{-7}\mathrm{s}^{-1}.$$

And the the time is found from

$$\begin{aligned} t &= -\frac{1}{\lambda}\ln\frac{R}{R_0}, \\ &= -\frac{1}{(5.618\times10^{-7}\mathrm{s}^{-1})}\ln\frac{(170\ \mathrm{counts/s})}{(3050\ \mathrm{counts/s})}, \\ &= 5.139\times10^6\,\mathrm{s} \approx 59.5\ \mathrm{days}. \end{aligned}$$

Note that counts/s is *not* the same as decays/s. Not all decay events will be picked up by a detector and recorded as a count; we are assuming that whatever scaling factor which connects the initial count rate to the initial decay rate is valid at later times as well. Such an assumption is a reasonable assumption.

(b) The purpose of such an experiment would be to measure the amount of phosphorus that is taken up in a leaf. But the activity of the tracer decays with time, and so without a correction factor we would record the wrong amount of phosphorus in the leaf. That correction factor is R_0/R; we need to multiply the measured counts by this factor to correct for the decay.

In this case

$$\frac{R}{R_0} = e^{\lambda t} = e^{(5.618\times10^{-7}\text{s}^{-1})(3.007\times10^5\,\text{s})} = 1.184.$$

E50-28 The number of particles of ^{147}Sm is

$$n = (0.15)\frac{(0.001\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(0.147\,\text{kg/mol})} = 6.143\times10^{20}.$$

The decay constant is

$$\lambda = (120/\text{s})/(6.143\times10^{20}) = 1.95\times10^{-19}/\text{s}.$$

The half-life is

$$t_{1/2} = \ln 2/(1.95\times10^{-19}/\text{s}) = 3.55\times10^{18}\text{s},$$

or 110 Gy.

E50-29 The number of particles of ^{239}Pu is

$$n_0 = \frac{(0.012\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(0.239\,\text{kg/mol})} = 3.023\times10^{22}.$$

The number which decay is

$$n_0 - n = (3.025\times10^{22})\left[1-(0.5)^{(20000)/(24100)}\right] = 1.32\times10^{22}.$$

The mass of helium produced is then

$$m = \frac{(0.004\,\text{kg/mol})(1.32\times10^{22})}{(6.02\times10^{23}\text{mol}^{-1})} = 8.78\times10^{-5}\text{kg}.$$

E50-30 Let $R_{33}/(R_{33}+R32) = x$, where $x_0 = 0.1$ originally, and we want to find out at what time $x = 0.9$. Rearranging,

$$(R_{33}+R32)/R_{33} = 1/x,$$

so

$$R32/R_{33} = 1/x - 1.$$

Since $R = R_0(0.5)^{t/t_{1/2}}$ we can write a ratio

$$\frac{1}{x} - 1 = \left(\frac{1}{x_0} - 1\right)(0.5)^{t/t_{32}-t/t_{33}}.$$

Put in some of the numbers, and

$$\ln[(1/9)/(9)] = \ln[0.5]t\left(\frac{1}{14.3} - \frac{1}{25.3}\right),$$

which has solution $t = 209$ d.

E50-31

E50-32 (a) $N/N_0 = (0.5)^{(4500)/(82)} = 3.0\times 10^{-17}$.
(b) $N/N_0 = (0.5)^{(4500)/(0.034)} = 0$.

$\boxed{\textbf{E50-33}}$ The Q values are

$$\begin{aligned} Q_3 &= (235.043923 - 232.038050 - 3.016029)(931.5\text{ MeV}) = -9.46\text{ MeV}, \\ Q_4 &= (235.043923 - 231.036297 - 4.002603)(931.5\text{ MeV}) = 4.68\text{ MeV}, \\ Q_5 &= (235.043923 - 230.033127 - 5.012228)(931.5\text{ MeV}) = -1.33\text{ MeV}. \end{aligned}$$

Only reactions with positive Q values are energetically possible.

E50-34 (a) For the ^{14}C decay,

$$Q = (223.018497 - 208.981075 - 14.003242)(931.5\text{ MeV}) = 31.84\text{ MeV}.$$

For the ^{4}He decay,

$$Q = (223.018497 - 219.009475 - 4.002603)(931.5\text{ MeV}) = 5.979\text{ MeV}.$$

E50-35 $Q = (136.907084 - 136.905821)(931.5\text{ MeV}) = 1.17\text{ MeV}$.

E50-36 $Q = (1.008665 - 1.007825)(931.5\text{ MeV}) = 0.782\text{ MeV}$.

$\boxed{\textbf{E50-37}}$ (a) The kinetic energy of this electron is significant compared to the rest mass energy, so we *must* use relativity to find the momentum. The total energy of the electron is $E = K + mc^2$, the momentum will be given by

$$\begin{aligned} pc &= \sqrt{E^2 - m^2c^4} = \sqrt{K^2 + 2Kmc^2}, \\ &= \sqrt{(1.00\text{ MeV})^2 + 2(1.00\text{ MeV})(0.511\text{ MeV})} = 1.42\text{ MeV}. \end{aligned}$$

The de Broglie wavelength is then

$$\lambda = \frac{hc}{pc} = \frac{(1240\text{ MeV}\cdot\text{fm})}{(1.42\text{ MeV})} = 873\text{ fm}.$$

(b) The radius of the emitting nucleus is

$$R = R_0 A^{1/3} = (1.2\text{ fm})(150)^{1/3} = 6.4\text{ fm}.$$

(c) The longest wavelength standing wave on a string fixed at each end is twice the length of the string. Although the rules for standing waves in a box are slightly more complicated, it is a fair assumption that the electron could not exist as a standing a wave in the nucleus.

(d) See part (c).

E50-38 The electron is relativistic, so

$$\begin{aligned} pc &= \sqrt{E^2 - m^2c^4}, \\ &= \sqrt{(1.71 \text{ MeV} + 0.51 \text{ MeV})^2 - (0.51 \text{ MeV})^2}, \\ &= 2.16 \text{ MeV}. \end{aligned}$$

This is also the magnitude of the momentum of the recoiling ^{32}S. Non-relativistic relations are $K = p^2/2m$, so

$$K = \frac{(2.16 \text{ MeV})^2}{2(31.97)(931.5 \text{ MeV})} = 78.4\,\text{eV}.$$

E50-39 $N = mN_A/M_r$ will give the number of atoms of ^{198}Au; $R = \lambda N$ will give the activity; $\lambda = \ln 2/t_{1/2}$ will give the decay constant. Combining,

$$m = \frac{NM_r}{N_A} = \frac{Rt_{1/2}M_r}{\ln 2 N_A}.$$

Then for the sample in question

$$m = \frac{(250)(3.7\times10^{10}/\text{s})(2.693)(86400\,\text{s})(198 \text{ g/mol})}{\ln 2(6.02\times10^{23}/\text{mol})} = 1.02\times10^{-3}\text{g}.$$

E50-40 $R = (8722/60\,\text{s})/(3.7\times10^{10}/\text{s}) = 3.93\times10^{-9}\text{Ci}.$

E50-41 The radiation absorbed dose (rad) is related to the roentgen equivalent man (rem) by the quality factor, so for the chest x-ray

$$\frac{(25 \text{ mrem})}{(0.85)} = 29 \text{ mrad}.$$

This is well beneath the annual exposure average.

Each rad corresponds to the delivery of 10^{-5} J/g, so the energy absorbed by the patient is

$$(0.029)(10^{-5} \text{ J/g})\left(\frac{1}{2}\right)(88\,\text{kg}) = 1.28\times10^{-2}\text{ J}.$$

E50-42 (a) $(75\,kg)(10^{-2}\text{J/kg})(0.024 \text{ rad}) = 1.8\times10^{-2}\text{J}.$

(b) $(0.024 \text{ rad})(12) = 0.29$ rem.

E50-43 $R = R_0(0.5)^{t/t_{1/2}}$, so

$$R_0 = (3.94\,\mu\text{Ci})(2)^{(6.048\times10^5\text{S})/(1.82\times10^5\text{S})} = 39.4\,\mu\text{Ci}.$$

E50-44 (a) $N = mN_A/M_R$, so

$$N = \frac{(2\times10^{-3}\text{g})(6.02\times10^{23}/\text{mol})}{(239 \text{ g/mol})} = 5.08\times10^{18}.$$

(b) $R = \lambda N = \ln 2N/t_{1/2}$, so

$$R = \ln 2(5.08\times10^{18})/(2.411\times10^4\text{y})(3.15\times10^7\text{s/y}) = 4.64\times10^6/\text{s}.$$

(c) $R = (4.64\times10^6/\text{s})/(3.7\times10^{10} \text{ decays/s}\cdot\text{Ci}) = 1.25\times10^{-4}\text{Ci}.$

E50-45 The hospital uses a 6000 Ci source, and that is all the information we need to find the number of disintegrations per second:

$$(6000 \text{ Ci})(3.7\times10^{10} \text{ decays/s}\cdot\text{Ci}) = 2.22\times10^{14} \text{ decays/s}.$$

We are told the half life, but to find the number of radioactive nuclei present we want to know the decay constant. Then

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(1.66\times10^{8}\,\text{s})} = 4.17\times10^{-9}\,\text{s}^{-1}.$$

The number of ^{60}Co nuclei is then

$$N = \frac{R}{\lambda} = \frac{(2.22\times10^{14} \text{ decays/s})}{(4.17\times10^{-9}\,\text{s}^{-1})} = 5.32\times10^{22}.$$

E50-46 The annual equivalent does is

$$(12\times10^{-4}\text{rem/h})(20 \text{ h/week})(52 \text{ week/y}) = 1.25 \text{ rem}.$$

E50-47 (a) $N = mN_A/M_R$ and $M_R = (226) + 2(35) = 296$, so

$$N = \frac{(1\times10^{-1}\text{g})(6.02\times10^{23}/\text{mol})}{(296 \text{ g/mol})} = 2.03\times10^{20}.$$

(b) $R = \lambda N = \ln 2N/t_{1/2}$, so

$$R = \ln 2(2.03\times10^{20})/(1600 \text{ y})(3.15\times10^{7}\text{s/y}) = 2.8\times10^{9}\text{Bq}.$$

(c) $(2.8\times10^{9})/(3.7\times10^{10}) = 76$ mCi.

E50-48 $R = \lambda N = \ln 2N/t_{1/2}$, so

$$N = \frac{(4.6\times10^{-6})(3.7\times10^{10}/\text{s})(1.28\times10^{9}\text{y})(3.15\times10^{7}\text{s/y})}{\ln 2} = 9.9\times10^{21},$$

$N = mN_A/M_R$, so

$$m = \frac{(40 \text{ g/mol})(9.9\times10^{21})}{(6.02\times10^{23}/\text{mol})} = 0.658 \text{ g}.$$

E50-49 We can apply Eq. 50-18 to find the age of the rock,

$$\begin{aligned} t &= \frac{t_{1/2}}{\ln 2}\ln\left(1+\frac{N_F}{N_I}\right), \\ &= \frac{(4.47\times10^{9}\text{y})}{\ln 2}\ln\left(1+\frac{(2.00\times10^{-3}\text{g})/(206 \text{ g/mol})}{(4.20\times10^{-3}\text{g})/(238 \text{ g/mol})}\right), \\ &= 2.83\times10^{9}\text{y}. \end{aligned}$$

E50-50 The number of atoms of ^{238}U originally present is

$$N = \frac{(3.71\times 10^{-3}\text{g})(6.02\times 10^{23}/\text{mol})}{(238\text{ g/mol})} = 9.38\times 10^{18}.$$

The number remaining after 260 million years is

$$N = (9.38\times 10^{18})(0.5)^{(260\text{ My})/(4470\text{ My})} = 9.01\times 10^{18}.$$

The difference decays into lead (eventually), so the mass of lead present should be

$$m = \frac{(206\text{ g/mol})(0.37\times 10^{18})}{(6.02\times 10^{23}/\text{mol})} = 1.27\times 10^{-4}\text{ g}.$$

E50-51 We can apply Eq. 50-18 to find the age of the rock,

$$\begin{aligned} t &= \frac{t_{1/2}}{\ln 2}\ln\left(1+\frac{N_F}{N_I}\right), \\ &= \frac{(4.47\times 10^{9}\text{y})}{\ln 2}\ln\left(1+\frac{(150\times 10^{-6}\text{g})/(206\text{ g/mol})}{(860\times 10^{-6}\text{g})/(238\text{ g/mol})}\right), \\ &= 1.18\times 10^{9}\text{y}. \end{aligned}$$

Inverting Eq. 50-18 to find the mass of ^{40}K originally present,

$$\frac{N_\text{F}}{N_\text{I}} = 2^{t/t_{1/2}} - 1,$$

so (since they have the same atomic mass) the mass of ^{40}K is

$$m = \frac{(1.6\times 10^{-3}\text{ g})}{2^{(1.18)/(1.28)} - 1} = 1.78\times 10^{-3}\text{g}.$$

E50-52 (a) There is an excess proton on the left and an excess neutron, so the unknown must be a deuteron, or d.

(b) We've added two protons but only one (net) neutron, so the element is Ti and the mass number is 43, or ^{43}Ti.

(c) The mass number doesn't change but we swapped one proton for a neutron, so ^{7}Li.

E50-53 Do the math:

$$Q = (58.933200 + 1.007825 - 58.934352 - 1.008665)(931.5\text{ MeV}) = -1.86\text{ MeV}.$$

E50-54 The reactions are

$$\begin{gathered} ^{201}\text{Hg}(\gamma,\alpha)^{197}\text{Pt}, \\ ^{197}\text{Au}(\text{n},\text{p})^{197}\text{Pt}, \\ ^{196}\text{Pt}(\text{n},\gamma)^{197}\text{Pt}, \\ ^{198}\text{Pt}(\gamma,\text{n})^{197}\text{Pt}, \\ ^{196}\text{Pt}(\text{d},\text{p})^{197}\text{Pt}, \\ ^{198}\text{Pt}(\text{p},\text{d})^{197}\text{Pt}. \end{gathered}$$

$\boxed{\textbf{E50-55}}$ We will write these reactions in the same way as Eq. 50-20 represents the reaction of Eq. 50-19. It is helpful to work backwards before proceeding by asking the following question: what nuclei will we have if we subtract one of the allowed projectiles?

The goal is ^{60}Co, which has 27 protons and $60 - 27 = 33$ neutrons.

1. Removing a proton will leave 26 protons and 33 neutrons, which is ^{59}Fe; but that nuclide is unstable.
2. Removing a neutron will leave 27 protons and 32 neutrons, which is ^{59}Co; and that nuclide is stable.
3. Removing a deuteron will leave 26 protons and 32 neutrons, which is ^{58}Fe; and that nuclide is stable.

It looks as if only ^{59}Co(n)^{60}Co and ^{58}Fe(d)^{60}Co are possible. If, however, we allow for the possibility of other daughter particles we should also consider some of the following reactions.

1. Swapping a neutron for a proton: ^{60}Ni(n,p)^{60}Co.
2. Using a neutron to knock out a deuteron: ^{61}Ni(n,d)^{60}Co.
3. Using a neutron to knock out an alpha particle: ^{63}Cu(n,α)^{60}Co.
4. Using a deuteron to knock out an alpha particle: ^{62}Ni(d,α)^{60}Co.

E50-56 (a) The possible results are ^{64}Zn, ^{66}Zn, ^{64}Cu, ^{66}Cu, ^{61}Ni, ^{63}Ni, ^{65}Zn, and ^{67}Zn.

(b) The stable results are ^{64}Zn, ^{66}Zn, ^{61}Ni, and ^{67}Zn.

E50-57

E50-58 The resulting reactions are ^{194}Pt(d,α)^{192}Ir, ^{196}Pt(d,α)^{194}Ir, and ^{198}Pt(d,α)^{196}Ir.

E50-59

E50-60 Shells occur at numbers 2, 8, 20, 28, 50, 82. The shells occur separately for protons and neutrons. To answer the question you need to know both Z and $N = A - Z$ of the isotope.

(a) Filled shells are ^{18}O, ^{60}Ni, ^{92}Mo, ^{144}Sm, and ^{207}Pb.

(b) One nucleon outside a shell are ^{40}K, ^{91}Zr, ^{121}Sb, and ^{143}Nd.

(c) One vacancy in a shell are ^{13}C, ^{40}K, ^{49}Ti, ^{205}Tl, and ^{207}Pb.

$\boxed{\textbf{E50-61}}$ (a) The binding energy of this neutron can be found by considering the Q value of the reaction ^{90}Zr(n)^{91}Zr which is

$$(89.904704 + 1.008665 - 90.905645)(931.5\text{ MeV}) = 7.19\text{ MeV}.$$

(b) The binding energy of this neutron can be found by considering the Q value of the reaction ^{89}Zr(n)^{90}Zr which is

$$(88.908889 + 1.008665 - 89.904704)(931.5\text{ MeV}) = 12.0\text{ MeV}.$$

This neutron is bound more tightly that the one in part (a).

(c) The binding energy per nucleon is found by dividing the binding energy by the number of nucleons:

$$\frac{(40\times 1.007825 + 51\times 1.008665 - 90.905645)(931.5\text{ MeV})}{91} = 8.69\text{ MeV}.$$

The neutron in the outside shell of ^{91}Zr is less tightly bound than the average nucleon in ^{91}Zr.

$\boxed{\textbf{P50-1}}$ Before doing anything we need to know whether or not the motion is relativistic. The rest mass energy of an α particle is

$$mc^2 = (4.00)(931.5\text{ MeV}) = 3.73\text{ GeV},$$

and since this is much greater than the kinetic energy we can assume the motion is non-relativistic, and we can apply non-relativistic momentum and energy conservation principles. The initial velocity of the α particle is then

$$v = \sqrt{2K/m} = c\sqrt{2K/mc^2} = c\sqrt{2(5.00\text{ MeV})/(3.73\text{ GeV})} = 5.18\times10^{-2}c.$$

For an elastic collision where the second particle is at originally at rest we have the final velocity of the first particle as

$$v_{1,\text{f}} = v_{1,\text{i}}\frac{m_2 - m_1}{m_2 + m_1} = (5.18\times10^{-2}c)\frac{(4.00\text{u}) - (197\text{u})}{(4.00\text{u}) + (197\text{u})} = -4.97\times10^{-2}c,$$

while the final velocity of the second particle is

$$v_{2,\text{f}} = v_{1,\text{i}}\frac{2m_1}{m_2 + m_1} = (5.18\times10^{-2}c)\frac{2(4.00\text{u})}{(4.00\text{u}) + (197\text{u})} = 2.06\times10^{-3}c.$$

(a) The kinetic energy of the recoiling nucleus is

$$\begin{aligned} K = \frac{1}{2}mv^2 &= \frac{1}{2}m(2.06\times10^{-3}c)^2 = (2.12\times10^{-6})mc^2 \\ &= (2.12\times10^{-6})(197)(931.5\text{ MeV}) = 0.389\text{ MeV}. \end{aligned}$$

(b) Energy conservation is the fastest way to answer this question, since it is an elastic collision. Then

$$(5.00\text{ MeV}) - (0.389\text{ MeV}) = 4.61\text{ MeV}.$$

P50-2 The gamma ray carries away a mass equivalent energy of

$$m_\gamma = (2.2233\text{ MeV})/(931.5\text{ MeV/u}) = 0.002387\text{ u}.$$

The neutron mass would then be

$$m_\text{N} = (2.014102 - 1.007825 + 0.002387)\text{u} = 1.008664\text{ u}.$$

P50-3 (a) There are four substates: m_j can be +3/2, +1/2, -1/2, and -3/2.
(b) $\Delta E = (2/3)(3.26)(3.15\times10^{-8}\text{eV/T})(2.16\text{ T}) = 1.48\times10^{-7}\text{eV}$.
(c) $\lambda = (1240\text{ eV}\cdot\text{nm})/(1.48\times10^{-7}\text{eV}) = 8.38\text{ m}$.
(d) This is in the radio region.

P50-4 (a) The charge density is $\rho = 3Q/4\pi R^3$. The charge on the shell of radius r is $dq = 4\pi r^2\rho\,dr$. The potential at the surface of a solid sphere of radius r is

$$V = \frac{q}{4\pi\epsilon_0 r} = \frac{\rho r^2}{3\epsilon_0}.$$

The energy required to add a layer of charge dq is

$$dU = V\,dq = \frac{4\pi\rho^2 r^4}{3\epsilon_0}dr,$$

which can be integrated to yield

$$U = \frac{4\pi\rho^2 R^5}{3\epsilon_0} = \frac{3Q^2}{20\pi\epsilon_0 R}.$$

(b) For ^{239}Pu,

$$U = \frac{3(94)^2(1.6\times10^{-19}\mathrm{C})}{20\pi(8.85\times10^{-12}\mathrm{F/m})(7.45\times10^{-15}\mathrm{m})} = 1024\times10^6\mathrm{eV}.$$

(c) The electrostatic energy is 10.9 MeV per proton.

$\boxed{\textbf{P50-5}}$ The decay rate is given by $R = \lambda N$, where N is the number of radioactive nuclei present. If R exceeds P then nuclei will decay faster than they are produced; but this will cause N to decrease, which means R will decrease until it is equal to P. If R is less than P then nuclei will be produced faster than they are decaying; but this will cause N to increase, which means R will increase until it is equal to P. In either case equilibrium occurs when $R = P$, and it is a stable equilibrium because it is approached no matter which side is larger. Then

$$P = R = \lambda N$$

at equilibrium, so $N = P/\lambda$.

P50-6 (a) $A = \lambda N$; at equilibrium $A = P$, so $P = 8.88\times10^{10}/\mathrm{s}$.
(b) $(8.88\times10^{10}/\mathrm{s})(1-e^{-0.269t})$, where t is in hours. The factor 0.269 comes from $\ln(2)/(2.58) = \lambda$.
(c) $N = P/\lambda = (8.88\times10^{10}/\mathrm{s})(3600\ \mathrm{s/h})/(0.269/\mathrm{h}) = 1.19\times10^{15}$.
(d) $m = NM_r/N_A$, or

$$m = \frac{(1.19\times10^{15})(55.94\ \mathrm{g/mol})}{(6.02\times10^{23}/\mathrm{mol})} = 1.10\times10^{-7}\mathrm{g}.$$

P50-7 (a) $A = \lambda N$, so

$$A = \frac{\ln 2 m N_A}{t_{1/2}M_r} = \frac{\ln 2(1\times10^{-3}\mathrm{g})(6.02\times10^{23}/\mathrm{mol})}{(1600)(3.15\times10^7\mathrm{s})(226\ \mathrm{g/mol})} = 3.66\times10^7/\mathrm{s}.$$

(b) The rate *must* be the same if the system is in secular equilibrium.
(c) $N = P/\lambda = t_{1/2}P/\ln 2$, so

$$m = \frac{(3.82)(86400\,\mathrm{s})(3.66\times10^7/\mathrm{s})(222\ \mathrm{g/mol})}{(6.02\times10^{23}/\mathrm{mol})\ln 2} = 6.43\times10^{-9}\mathrm{g}.$$

P50-8 The number of water molecules in the body is

$$N = (6.02\times10^{23}/\mathrm{mol})(70\times10^3\mathrm{g})/(18\ \mathrm{g/mol}) = 2.34\times10^{27}.$$

There are ten protons in each water molecule. The activity is then

$$A = (2.34\times10^{27})\ln 2/(1\times10^{32}\mathrm{y}) = 1.62\times10^{-5}/\mathrm{y}.$$

The time between decays is then

$$1/A = 6200\ \mathrm{y}.$$

$\boxed{\textbf{P50-9}}$ Assuming the ^{238}U nucleus is originally at rest the total initial momentum is zero, which means the magnitudes of the final momenta of the α particle and the ^{234}Th nucleus are equal.

The α particle has a final velocity of

$$v = \sqrt{2K/m} = c\sqrt{2K/mc^2} = c\sqrt{2(4.196 \text{ MeV})/(4.0026\times 931.5 \text{ MeV})} = 4.744\times 10^{-2}c.$$

Since the magnitudes of the final momenta are the same, the ^{234}Th nucleus has a final velocity of

$$(4.744\times 10^{-2}c)\left(\frac{(4.0026 \text{ u})}{(234.04 \text{ u})}\right) = 8.113\times 10^{-4}c.$$

The kinetic energy of the ^{234}Th nucleus is

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{1}{2}m(8.113\times 10^{-4}c)^2 = (3.291\times 10^{-7})mc^2 \\ &= (3.291\times 10^{-7})(234.04)(931.5 \text{ MeV}) = 71.75 \text{ keV}. \end{aligned}$$

The Q value for the reaction is then

$$(4.196 \text{ MeV}) + (71.75 \text{ keV}) = 4.268 \text{ MeV},$$

which agrees well with the Sample Problem.

P50-10 (a) The Q value is

$$Q = (238.050783 - 4.002603 - 234.043596)(931.5 \text{ MeV}) = 4.27 \text{ MeV}.$$

(b) The Q values for each step are

$$Q = (238.050783 - 237.048724 - 1.008665)(931.5 \text{ MeV}) = -6.153 \text{ MeV},$$

$$Q = (237.048724 - 236.048674 - 1.007825)(931.5 \text{ MeV}) = -7.242 \text{ MeV},$$

$$Q = (236.048674 - 235.045432 - 1.008665)(931.5 \text{ MeV}) = -5.052 \text{ MeV},$$

$$Q = (235.045432 - 234.043596 - 1.007825)(931.5 \text{ MeV}) = -5.579 \text{ MeV}.$$

(c) The total Q for part (b) is -24.026 MeV. The difference between (a) and (b) is 28.296 MeV. The binding energy for the alpha particle is

$$E = [2(1.007825) + 2(1.008665) - 4.002603](931.5 \text{ MeV}) = 28.296 \text{ MeV}.$$

P50-11 (a) The emitted positron leaves the atom, so the mass must be subtracted. But the daughter particle now has an extra electron, so that must also be subtracted. Hence the factor $-2m_\text{e}$.

(b) The Q value is

$$Q = [11.011434 - 11.009305 - 2(0.0005486)](931.5 \text{ MeV}) = 0.961 \text{ MeV}.$$

P50-12 (a) Capturing an electron is equivalent to negative beta decay in that the total number of electrons is accounted for on both the left and right sides of the equation. The loss of the K shell electron, however, must be taken into account as this energy may be significant.

(b) The Q value is

$$Q = (48.948517 - 48.947871)(931.5 \text{ MeV}) - (0.00547 \text{ MeV}) = 0.596 \text{ MeV}.$$

$\boxed{\textbf{P50-13}}$ The decay constant for ^{90}Sr is

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(9.15\times 10^8\,\text{s})} = 7.58\times 10^{-10}\text{s}^{-1}.$$

The number of nuclei present in 400 g of ^{90}Sr is

$$N = (400\text{ g})\frac{(6.02\times 10^{23}/\text{mol})}{(89.9\text{ g/mol})} = 2.68\times 10^{24},$$

so the overall activity of the 400 g of ^{90}Sr is

$$R = \lambda N = (7.58\times 10^{-10}\text{s}^{-1})(2.68\times 10^{24})/(3.7\times 10^{10}/\text{Ci}\cdot\text{s}) = 5.49\times 10^4\text{ Ci}.$$

This is spread out over a 2000 km^2 area, so the "activity surface density" is

$$\frac{(5.49\times 10^4\text{ Ci})}{(20006\,\text{m}^2)} = 2.74\times 10^{-5}\text{ Ci/m}^2.$$

If the allowable limit is 0.002 mCi, then the area of land that would contain this activity is

$$\frac{(0.002\times 10^{-3}\text{ Ci})}{(2.74\times 10^{-5}\text{ Ci/m}^2)} = 7.30\times 10^{-2}\text{m}^2.$$

P50-14 (a) $N = mN_A/M_r$, so

$$N = (2.5\times 10^{-3}\text{g})(6.02\times 10^{23}/\text{mol})/(239\text{ g/mol}) = 6.3\times 10^{18}.$$

(b) $A = \ln 2N/t_{1/2}$, so the number that decay in 12 hours is

$$\frac{\ln 2(6.3\times 10^{18})(12)(3600\,\text{s})}{(24100)(3.15\times 10^7\text{s})} = 2.5\times 10^{11}.$$

(c) The energy absorbed by the body is

$$E = (2.5\times 10^{11})(5.2\text{ MeV})(1.6\times 10^{-19}\text{J/eV}) = 0.20\,\text{J}.$$

(d) The dose in rad is $(0.20\,\text{J})/(87\,\text{kg}) = 0.23$ rad.
(e) The biological dose in rem is $(0.23)(13) = 3$ rem.

P50-15 (a) The amount of ^{238}U per kilogram of granite is

$$N = \frac{(4\times 10^{-6}\text{kg})(6.02\times 10^{23}/\text{mol})}{(0.238\text{ kg/mol})} = 1.01\times 10^{19}.$$

The activity is then

$$A = \frac{\ln 2(1.01\times 10^{19})}{(4.47\times 10^9\text{y})(3.15\times 10^7\text{s/y})} = 49.7/\text{s}.$$

The energy released in one second is

$$E = (49.7/\text{s})(51.7\text{ MeV}) = 4.1\times 10^{-10}\text{J}.$$

The amount of ^{232}Th per kilogram of granite is

$$N = \frac{(13\times 10^{-6}\text{kg})(6.02\times 10^{23}/\text{mol})}{(0.232\text{ kg/mol})} = 3.37\times 10^{19}.$$

The activity is then

$$A = \frac{\ln 2(3.37\times10^{19})}{(1.41\times10^{10}\text{y})(3.15\times10^{7}\text{s/y})} = 52.6/\text{s}.$$

The energy released in one second is

$$E = (52.6/\text{s})(42.7\text{ MeV}) = 3.6\times10^{-10}\text{J}.$$

The amount of ^{40}K per kilogram of granite is

$$N = \frac{(4\times10^{-6}\text{kg})(6.02\times10^{23}/\text{mol})}{(0.040\text{ kg/mol})} = 6.02\times10^{19}.$$

The activity is then

$$A = \frac{\ln 2(6.02\times10^{19})}{(1.28\times10^{9}\text{y})(3.15\times10^{7}\text{s/y})} = 1030/\text{s}.$$

The energy released in one second is

$$E = (1030/\text{s})(1.32\text{ MeV}) = 2.2\times10^{-10}\text{J}.$$

The total of the three is 9.9×10^{-10}W per kilogram of granite.
(b) The total for the Earth is 2.7×10^{13} W.

P50-16 (a) Since only a is moving originally then the velocity of the center of mass is

$$V = \frac{m_a v_a + m_X(0)}{m_X + m_a} = v_a\frac{m_a}{m_a + m_X}.$$

No, since momentum is conserved.

(b) Moving to the center of mass frame gives the velocity of X as V, and the velocity of a as $v_a - V$. The kinetic energy is now

$$\begin{aligned}
K_{\text{cm}} &= \frac{1}{2}\left(m_X V^2 + m_a(v_a - V)^2\right), \\
&= \frac{v_a^2}{2}\left(m_X\frac{m_a^2}{(m_a+m_X)^2} + m_a\frac{m_X^2}{(m_a+m_X)^2}\right), \\
&= \frac{m_a v_a^2}{2}\frac{m_a m_X + m_X^2}{(m_a+m_X)^2}, \\
&= K_{\text{lab}}\frac{m_X}{m_a+m_X}.
\end{aligned}$$

Yes; kinetic energy is not conserved.

(c) $v_a = \sqrt{2K/m}$, so

$$v_a = \sqrt{2(15.9\text{ MeV})/(1876\text{ MeV})}c = 0.130c.$$

The center of mass velocity is

$$V = (0.130c)\frac{(2)}{(2)+(90)} = 2.83\times10^{-3}c.$$

Finally,

$$K_{\text{cm}} = (15.9\text{ MeV})\frac{(90)}{(2)+(90)} = 15.6\text{ MeV}.$$

P50-17 Let $Q = K_{\text{cm}}$ in the result of Problem 50-16, and invert, solving for K_{lab}.

P50-18 (a) Removing a proton from ^{209}Bi:

$$E = (207.976636 + 1.007825 - 208.980383)(931.5 \text{ MeV}) = 3.80 \text{ MeV}.$$

Removing a proton from ^{208}Pb:

$$E = (206.977408 + 1.007825 - 207.976636)(931.5 \text{ MeV}) = 8.01 \text{ MeV}.$$

(b) Removing a neutron from ^{209}Pb:

$$E = (207.976636 + 1.008665 - 208.981075)(931.5 \text{ MeV}) = 3.94 \text{ MeV}.$$

Removing a neutron from ^{208}Pb:

$$E = (206.975881 + 1.008665 - 207.976636)(931.5 \text{ MeV}) = 7.37 \text{ MeV}.$$

E51-1 (a) For the coal,

$$m = (1\times 10^9\text{J})/(2.9\times 10^7\text{J/kg}) = 34\,\text{kg}.$$

(b) For the uranium,

$$m = (1\times 10^9\text{J})/(8.2\times 10^{13}\text{J/kg}) = 1.2\times 10^{-5}\text{kg}.$$

E51-2 (a) The energy from the coal is

$$E = (100\,\text{kg})(2.9\times 10^7\text{J/kg}) = 2.9\times 10^9\text{J}.$$

(b) The energy from the uranium in the ash is

$$E = (3\times 10^{-6})(100\,\text{kg})(8.2\times 10^{13}\text{J}) = 2.5\times 10^{10}\text{J}.$$

$\boxed{\textbf{E51-3}}$ (a) There are

$$\frac{(1.00\,\text{kg})(6.02\times 10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times 10^{24}$$

atoms in 1.00 kg of ^{235}U.

(b) If each atom releases 200 MeV, then

$$(200\times 10^6\ \text{eV})(1.6\times 10^{-19}\text{J/ eV})(2.56\times 10^{24}) = 8.19\times 10^{13}\,\text{J}$$

of energy could be released from 1.00 kg of ^{235}U.

(c) This amount of energy would keep a 100-W lamp lit for

$$t = \frac{(8.19\times 10^{13}\,\text{J})}{(100\,\text{W})} = 8.19\times 10^{11}\text{s} \approx 26,000\ \text{y!}$$

E51-4 $2\,\text{W} = 1.25\times 10^{19}\text{eV/s}$. This requires

$$(1.25\times 10^{19}\text{eV/s})/(200\times 10^6\text{eV}) = 6.25\times 10^{10}/\text{s}$$

as the fission rate.

E51-5 There are

$$\frac{(1.00\,\text{kg})(6.02\times 10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times 10^{24}$$

atoms in 1.00 kg of ^{235}U. If each atom releases 200 MeV, then

$$(200\times 10^6\ \text{eV})(1.6\times 10^{-19}\text{J/ eV})(2.56\times 10^{24}) = 8.19\times 10^{13}\,\text{J}$$

of energy could be released from 1.00 kg of ^{235}U. This amount of energy would keep a 100-W lamp lit for

$$t = \frac{(8.19\times 10^{13}\,\text{J})}{(100\,\text{W})} = 8.19\times 10^{11}\text{s} \approx 30,000\ \text{y!}$$

E51-6 There are

$$\frac{(1.00\,\text{kg})(6.02\times 10^{23}\text{mol}^{-1})}{(239\text{g/mol})} = 2.52\times 10^{24}$$

atoms in 1.00 kg of ^{239}Pu. If each atom releases 180 MeV, then

$$(180\times 10^6\ \text{eV})(1.6\times 10^{-19}\text{J/ eV})(2.52\times 10^{24}) = 7.25\times 10^{13}\,\text{J}$$

of energy could be released from 1.00 kg of ^{239}Pu.

$\boxed{\textbf{E51-7}}$ When the ^{233}U nucleus absorbs a neutron we are given a total of 92 protons and 142 neutrons. Gallium has 31 protons and around 39 neutrons; chromium has 24 protons and around 28 neutrons. There are then 37 protons and around 75 neutrons left over. This would be rubidium, but the number of neutrons is *very* wrong. Although the elemental identification is correct, because we must conserve proton number, the isotopes are *wrong* in our above choices for neutron numbers.

E51-8 Beta decay is the emission of an electron from the nucleus; one of the neutrons changes into a proton. The atom now needs one more electron in the electron shells; by using atomic masses (as opposed to nuclear masses) then the beta electron is accounted for. This is *only* true for negative beta decay, not for positive beta decay.

E51-9 (a) There are

$$\frac{(1.0\text{ g})(6.02\times10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times10^{21}$$

atoms in 1.00 g of ^{235}U. The fission rate is

$$A = \ln 2N/t_{1/2} = \ln 2(2.56\times10^{21})/(3.5\times10^{17}\text{y})(365\text{d/y}) = 13.9/\text{d}.$$

(b) The ratio is the inverse ratio of the half-lives:

$$(3.5\times10^{17}\text{y})/(7.04\times10^{8}\text{y}) = 4.97\times10^{8}.$$

E51-10 (a) The atomic number of Y must be $92-54=38$, so the element is Sr. The mass number is $235+1-140-1=95$, so Y is ^{95}Sr.

(b) The atomic number of Y must be $92-53=39$, so the element is Y. The mass number is $235+1-139-2=95$, so Y is ^{95}Y.

(c) The atomic number of X must be $92-40=52$, so the element is Te. The mass number is $235+1-100-2=134$, so X is ^{134}Te.

(d) The mass number difference is $235+1-141-92=3$, so $b=3$.

E51-11 The Q value is

$$Q = [51.94012 - 2(25.982593)](931.5\text{ MeV}) = -23\text{ MeV}.$$

The negative value implies that this fission reaction is not possible.

E51-12 The Q value is

$$Q = [97.905408 - 2(48.950024)](931.5\text{ MeV}) = 4.99\text{ MeV}.$$

The two fragments would have a very large Coulomb barrier to overcome.

$\boxed{\textbf{E51-13}}$ The energy released is

$$(235.043923 - 140.920044 - 91.919726 - 2\times1.008665)(931.5\text{ MeV}) = 174\text{ MeV}.$$

E51-14 Since $E_n > E_b$ fission is possible by thermal neutrons.

E51-15 (a) The uranium starts with 92 protons. The two end products have a total of $58+44=102$. This means that there must have been ten beta decays.

(b) The Q value for this process is

$$Q = (238.050783 + 1.008665 - 139.905434 - 98.905939)(931.5\text{ MeV}) = 231\text{ MeV}.$$

E51-16 (a) The other fragment has $92-32=60$ protons and $235+1-83=153$ neutrons. That element is ^{153}Nd.

(b) Since $K=p^2/2m$ and momentum is conserved, then $2m_1K_1=2m_2K_2$. This means that $K_2=(m_1/m_2)K_1$. But $K_1+K_2=Q$, so

$$K_1\frac{m_2+m_1}{m_2}=Q,$$

or

$$K_1=\frac{m_2}{m_1+m_2}Q,$$

with a similar expression for K_2. Then for ^{83}Ge

$$K=\frac{(153)}{(83+153)}(170\text{ MeV})=110\text{ MeV},$$

while for ^{153}Nd

$$K=\frac{(83)}{(83+153)}(170\text{ MeV})=60\text{ MeV},$$

(c) For ^{83}Ge,

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(110\text{ MeV})}{(83)(931\text{ MeV})}}c=0.053c,$$

while for ^{153}Nd

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(60\text{ MeV})}{(153)(931\text{ MeV})}}c=0.029c.$$

E51-17 Since ^{239}Pu is one nucleon heavier than ^{238}U only one neutron capture is required. The atomic number of Pu is two *more* than U, so two beta decays will be required. The reaction series is then

$$\begin{aligned}
{}^{238}\text{U}+\text{n} &\rightarrow {}^{239}\text{U},\\
{}^{239}\text{U} &\rightarrow {}^{239}\text{Np}+\beta^-+\bar{\nu},\\
{}^{239}\text{Np} &\rightarrow {}^{239}\text{Pu}+\beta^-+\bar{\nu}.
\end{aligned}$$

E51-18 Each fission releases 200 MeV. The total energy released over the three years is

$$(190\times10^6\text{W})(3)(3.15\times10^7\text{s})=1.8\times10^{16}\text{J}.$$

That's

$$(1.8\times10^{16}\text{J})/(1.6\times10^{-19}\text{J/eV})(200\times10^6\text{eV})=5.6\times10^{26}$$

fission events. That requires

$$m=(5.6\times10^{26})(0.235\text{ kg/mol})/(6.02\times10^{23}/\text{mol})=218\,\text{kg}.$$

But this is only half the original amount, or 437 kg.

E51-19 According to Sample Problem 51-3 the rate at which non-fission thermal neutron capture occurs is one quarter that of fission. Hence the mass which undergoes non-fission thermal neutron capture is one quarter the answer of Ex. 51-18. The total is then

$$(437\,\text{kg})(1+0.25)=546\,\text{kg}.$$

E51-20 (a) $Q_{\text{eff}} = E/\Delta N$, where E is the total energy released and ΔN is the number of decays. This can also be written as

$$Q_{\text{eff}} = \frac{P}{A} = \frac{Pt_{1/2}}{\ln 2N} = \frac{Pt_{1/2}M_r}{\ln 2N_A m},$$

where A is the activity and P the power output from the sample. Solving,

$$Q_{\text{eff}} = \frac{(2.3\,\text{W})(29\text{ y})(3.15\times10^7\text{s})(90\text{ g/mol})}{\ln 2(6.02\times10^{23}/\text{mol})(1\text{ g})} = 4.53\times10^{-13}\text{J} = 2.8\text{ MeV}.$$

(b) $P = (0.05)m(2300\,\text{W/kg})$, so

$$m = \frac{(150\,\text{W})}{(0.05)(2300\,\text{W/kg})} = 1.3\,\text{kg}.$$

E51-21 Let the energy released by one fission be E_1. If the average time to the next fission event is t_{gen}, then the "average" power output from the one fission is $P_1 = E_1/t_{\text{gen}}$. If every fission event results in the release of k neutrons, each of which cause a later fission event, then after every time period t_{gen} the number of fission events, and hence the average power output from *all* of the fission events, will increase by a factor of k.

For long enough times we can write

$$P(t) = P_0 k^{t/t_{\text{gen}}}.$$

E51-22 Invert the expression derived in Exercise 51-21:

$$k = \left(\frac{P}{P_0}\right)^{t_{\text{gen}}/t} = \left(\frac{(350)}{(1200)}\right)^{(1.3\times10^{-3}\text{S})/(2.6\,\text{S})} = 0.99938.$$

E51-23 Each fission releases 200 MeV. Then the fission rate is

$$(500\times10^6\text{W})/(200\times10^6\text{eV})(1.6\times10^{-19}\text{J/eV}) = 1.6\times10^{19}/s$$

The number of neutrons in "transit" is then

$$(1.6\times10^{19}/\text{s})(1.0\times10^{-3}\text{s}) = 1.6\times10^{16}.$$

E51-24 Using the results of Exercise 51-21:

$$P = (400\text{ MW})(1.0003)^{(300\,\text{S})/(0.03\,\text{S})} = 8030\text{ MW}.$$

E51-25 The time constant for this decay is

$$\lambda = \frac{\ln 2}{(2.77\times10^9\text{ s})} = 2.50\times10^{-10}\text{s}^{-1}.$$

The number of nuclei present in 1.00 kg is

$$N = \frac{(1.00\,\text{kg})(6.02\times10^{23}\text{ mol}^{-1})}{(238\text{ g/mol})} = 2.53\times10^{24}.$$

The decay rate is then

$$R = \lambda N = (2.50\times10^{-10}\text{s}^{-1})(2.53\times10^{24}) = 6.33\times10^{14}\text{s}^{-1}.$$

The power generated is the decay rate times the energy released per decay,

$$P = (6.33\times10^{14}\text{s}^{-1})(5.59\times10^6\text{ eV})(1.6\times10^{-19}\text{ J/eV}) = 566\,\text{W}.$$

E51-26 The detector detects only a fraction of the emitted neutrons. This fraction is

$$\frac{A}{4\pi R^2} = \frac{(2.5\,\mathrm{m}^2)}{4\pi(35\,\mathrm{m})^2} = 1.62\times10^{-4}.$$

The total flux out of the warhead is then

$$(4.0/\mathrm{s})/(1.62\times10^{-4}) = 2.47\times10^4/\mathrm{s}.$$

The number of ^{239}Pu atoms is

$$N = \frac{A}{\lambda} = \frac{(2.47\times10^4/\mathrm{s})(1.34\times10^{11}\mathrm{y})(3.15\times10^7\mathrm{s/y})}{\ln 2(2.5)} = 6.02\times10^{22}.$$

That's one tenth of a mole, so the mass is $(239)/10 = 24$ g.

E51-27 Using the results of Sample Problem 51-4,

$$t = \frac{\ln[R(0)/R(t)]}{\lambda_5 - \lambda_8},$$

so

$$t = \frac{\ln[(0.03)/(0.0072)]}{(0.984 - 0.155)(1\times10^{-9}/\mathrm{y})} = 1.72\times10^9\mathrm{y}.$$

E51-28 (a) $(15\times10^9\mathrm{W}\cdot\mathrm{y})(2\times10^5\mathrm{y}) = 7.5\times10^4\mathrm{W}$.
(b) The number of fissions required is

$$N = \frac{(15\times10^9\mathrm{W}\cdot\mathrm{y})(3.15\times10^7\mathrm{s/y})}{(200\ \mathrm{MeV})(1.6\times10^{-19}\mathrm{J/eV})} = 1.5\times10^{28}.$$

The mass of ^{235}U consumed is

$$m = (1.5\times10^{28})(0.235\mathrm{kg/mol})/(6.02\times10^{-23}/\mathrm{mol}) = 5.8\times10^3\mathrm{kg}.$$

$\boxed{\textbf{E51-29}}$ If ^{238}U absorbs a neutron it becomes ^{239}U, which will decay by beta decay to first ^{239}Np and then ^{239}Pu; we looked at this in Exercise 51-17. This can decay by alpha emission according to

$$^{239}\mathrm{Pu} \rightarrow ^{235}\mathrm{U} + \alpha.$$

E51-30 The number of atoms present in the sample is

$$N = (6.02\times10^{23}/\mathrm{mol})(1000\,\mathrm{kg})/(2.014\mathrm{g/mol}) = 2.99\times10^{26}.$$

It takes two to make a fusion, so the energy released is

$$(3.27\ \mathrm{MeV})(2.99\times10^{26})/2 = 4.89\times10^{26}\mathrm{MeV}.$$

That's 7.8×10^{13}J, which is enough to burn the lamp for

$$t = (7.8\times10^{13}\mathrm{J})/(100\,\mathrm{W}) = 7.8\times10^{11}\mathrm{s} = 24800\ \mathrm{y}.$$

E51-31 The potential energy at closest approach is

$$U = \frac{(1.6\times10^{-19}\mathrm{C})^2}{4\pi(8.85\times10^{-12}\mathrm{F/m})(1.6\times10^{-15}\mathrm{m})} = 9\times10^5\mathrm{eV}.$$

E51-32 The ratio can be written as

$$\frac{n(K_1)}{n(K_2)} = \sqrt{\frac{K_1}{K_2}} e^{(K_2-K_1)/kT},$$

so the ratio is

$$\sqrt{\frac{(5000\,\text{eV})}{(1900\,\text{eV})}} e^{(-3100\,\text{eV})/(8.62\times10^{-5}\text{eV/K})(1.5\times10^{7}\text{K})} = 0.15.$$

E51-33 (a) See Sample Problem 51-5.

E51-34 Add up *all* of the Q values in the cycle of Fig. 51-10.

$\boxed{\textbf{E51-35}}$ The energy released is

$$(3\times4.002603 - 12.0000000)(931.5\text{ MeV}) = 7.27\text{ MeV}.$$

E51-36 (a) The number of particle of hydrogen in $1\,\text{m}^3$ is

$$N = (0.35)(1.5\times10^5\text{kg})(6.02\times10^{23}/\text{mol})/(0.001\text{ kg/mol}) = 3.16\times10^{31}$$

(b) The density of particles is $N/V = p/kT$; the ratio is

$$\frac{(3.16\times10^{31})(1.38\times10^{-23}\text{J/K})(298\,\text{K})}{(1.01\times10^5\text{Pa})} = 1.2\times10^6.$$

E51-37 (a) There are

$$\frac{(1.00\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(1\text{g/mol})} = 6.02\times10^{26}$$

atoms in 1.00 kg of ^{1}H. If four atoms fuse to releases 26.7 MeV, then

$$(26.7\text{ MeV})(6.02\times10^{26})/4 = 4.0\times10^{27}\text{MeV}$$

of energy could be released from 1.00 kg of ^{1}H.

(b) There are

$$\frac{(1.00\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times10^{24}$$

atoms in 1.00 kg of ^{235}U. If each atom releases 200 MeV, then

$$(200\text{ MeV})(2.56\times10^{24}) = 5.1\times10^{26}\text{MeV}$$

of energy could be released from 1.00 kg of ^{235}U.

E51-38 (a) $E = \Delta mc^2$, so

$$\Delta m = \frac{(3.9\times10^{26}\text{J/s})}{(3.0\times10^8\text{m/s})^2} = 4.3\times10^9\text{kg/s}.$$

(b) The fraction of the Sun's mass "lost" is

$$\frac{(4.3\times10^9\text{kg/s})(3.15\times10^7\text{s/y})(4.5\times10^9\text{y})}{(2.0\times10^{30}\text{kg})} = 0.03\,\%.$$

$\boxed{\textbf{E51-39}}$ The rate of consumption is 6.2×10^{11}kg/s, the core has 1/8 the mass but only 35% is hydrogen, so the time remaining is

$$t = (0.35)(1/8)(2.0\times10^{30}\text{kg})/(6.2\times10^{11}\text{kg/s}) = 1.4\times10^{17}\text{s},$$

or about 4.5×10^9 years.

E51-40 For the first two reactions into one:

$$Q = [2(1.007825) - (2.014102)](931.5 \text{ MeV}) = 1.44 \text{ MeV}.$$

For the second,

$$Q = [(1.007825) + (2.014102) - (3.016029)](931.5 \text{ MeV}) = 5.49 \text{ MeV}.$$

For the last,

$$Q = [2(3.016029) - (4.002603) - 2(1.007825)](931.5 \text{ MeV}) = 12.86 \text{ MeV}.$$

E51-41 (a) Use $mN_A/M_r = N$, so

$$(3.3\times10^7\text{J/kg})\frac{(0.012 \text{ kg/mol})}{(6.02\times10^{23}/\text{mol})}\frac{1}{(1.6\times10^{-19}\text{J/eV})} = 4.1 \text{ eV}.$$

(b) For every 12 grams of carbon we require 32 grams of oxygen, the total is 44 grams. The total mass required is then 40/12 that of carbon alone. The energy production is then

$$(3.3\times10^7\text{J/kg})(12/44) = 9\times10^6\text{J/kg}.$$

(c) The sun would burn for

$$\frac{(2\times10^{30}\text{kg})(9\times10^6\text{J/kg})}{(3.9\times10^{26}\text{W})} = 4.6\times10^{10}\text{s}.$$

That's only 1500 years!

E51-42 The rate of fusion events is

$$\frac{(5.3\times10^{30}\text{W})}{(7.27\times10^6\text{eV})(1.6\times10^{-19}\text{J/eV})} = 4.56\times10^{42}/\text{s}.$$

The carbon is then produced at a rate

$$(4.56\times10^{42}/\text{s})(0.012 \text{ kg/mol})/(6.02\times10^{23}/\text{mol}) = 9.08\times10^{16}\text{kg/s}.$$

The process will be complete in

$$\frac{(4.6\times10^{32}\text{kg})}{(9.08\times10^{16}\text{kg/s})(3.15\times10^7\text{s/y})} = 1.6\times10^8\text{y}.$$

E51-43 (a) For the reaction d-d,

$$Q = [2(2.014102) - (3.016029) - (1.008665)](931.5 \text{ MeV}) = 3.27 \text{ MeV}.$$

(b) For the reaction d-d,

$$Q = [2(2.014102) - (3.016029) - (1.007825)](931.5 \text{ MeV}) = 4.03 \text{ MeV}.$$

(c) For the reaction d-t,

$$Q = [(2.014102) + (3.016049) - (4.002603) - (1.008665)](931.5 \text{ MeV}) = 17.59 \text{ MeV}.$$

E51-44 One liter of water has a mass of one kilogram. The number of atoms of ^{2}H is

$$(0.00015\,\text{kg})\frac{(6.02\times10^{23}/\text{mol})}{(0.002\text{ kg/mol})} = 4.5\times10^{22}.$$

The energy available is

$$(3.27\times10^6\text{eV})(1.6\times10^{-19}\text{J/eV})(4.5\times10^{22})/2 = 1.18\times10^{10}\text{J}.$$

The power output is then

$$\frac{(1.18\times10^{10}\text{J})}{(86400\,\text{s})} = 1.4\times10^5\text{W}$$

$\boxed{\textbf{E51-45}}$ Assume momentum conservation, then

$$p_\alpha = p_\text{n} \text{ or } v_\text{n}/v_\alpha = m_\alpha/m_\text{n}.$$

The ratio of the kinetic energies is then

$$\frac{K_\text{n}}{K_\alpha} = \frac{m_\text{n}v_\text{n}^2}{m_\alpha v_\alpha^2} = \frac{m_\alpha}{m_\text{n}} \approx 4.$$

Then $K_\text{n} = 4Q/5 = 14.07$ MeV while $K_\alpha = Q/5 = 3.52$ MeV.

E51-46 The Q value is

$$Q = (6.015122 + 1.008665 - 3.016049 - 4.002603)(931.5\text{ MeV}) = 4.78\text{ MeV}.$$

Combine the two reactions to get a net $Q = 22.37$ MeV. The amount of ^{6}Li required is

$$N = (2.6\times10^{28}\text{MeV})/(22.37\text{ MeV}) = 1.16\times10^{27}.$$

The mass of LiD required is

$$m = \frac{(1.16\times10^{27})(0.008\text{ kg/mol})}{(6.02\times10^{23}/\text{mol})} = 15.4\,\text{kg}.$$

$\boxed{\textbf{P51-1}}$ (a) Equation 50-1 is

$$R = R_0A^{1/3},$$

where $R_0 = 1.2$ fm. The distance between the two nuclei will be the sum of the radii, or

$$R_0\left((140)^{1/3} + (94)^{1/3}\right).$$

The potential energy will be

$$\begin{aligned}
U &= \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r},\\
&= \frac{e^2}{4\pi\epsilon_0 R_0}\frac{(54)(38)}{\left((140)^{1/3} + (94)^{1/3}\right)},\\
&= \frac{(1.60\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{Nm}^2)(1.2\text{ fm})}211,\\
&= 253\text{ MeV}.
\end{aligned}$$

(b) The energy will eventually appear as thermal energy.

P51-2 (a) Since $R = R_0\sqrt[3]{A}$, the surface area a is proportional to $A^{2/3}$. The fractional change in surface area is

$$\frac{(a_1 + a_2) - a_0}{a_0} = \frac{(140)^{2/3} + (96)^{2/3} - (236)^{2/3}}{(236)^{2/3}} = 25\,\%.$$

(b) Nuclei have a constant density, so there is no change in volume.

(c) Since $U \propto Q^2/R$, $U \propto Q^2/\sqrt[3]{A}$. The fractional change in the electrostatic potential energy is

$$\frac{U_1 + U_2 - U_0}{U_0} = \frac{(54)^2(140)^{-1/3} + (38)^2(96)^{-1/3} - (92)^2(236)^{-1/3}}{(92)^2(236)^{-1/3}} = -36\,\%.$$

P51-3 (a) There are

$$\frac{(2.5\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(239\text{g/mol})} = 6.29\times10^{24}$$

atoms in 2.5 kg of ^{239}Pu. If each atom releases 180 MeV, then

$$(180\text{ MeV})(6.29\times10^{24})/(2.6\times10^{28}\text{MeV/megaton}) = 44\text{ kiloton}$$

is the bomb yield.

P51-4 (a) In an elastic collision the nucleus moves forward with a speed of

$$v = v_0\frac{2m_n}{m_n + m},$$

so the kinetic energy when it moves forward is

$$\Delta K = \frac{m}{2}v_0^2\frac{4m_n^2}{(m + m_n)^2} = K\frac{m_n m}{(m_n + m)^2},$$

where we can write ΔK because in an elastic collision whatever energy kinetic energy the nucleus carries off had to come from the neutron.

(b) For hydrogen,

$$\frac{\Delta K}{K} = \frac{4(1)(1)}{(1+1)^2} = 1.00.$$

For deuterium,

$$\frac{\Delta K}{K} = \frac{4(1)(2)}{(1+2)^2} = 0.89.$$

For carbon,

$$\frac{\Delta K}{K} = \frac{4(1)(12)}{(1+12)^2} = 0.28.$$

For lead,

$$\frac{\Delta K}{K} = \frac{4(1)(206)}{(1+206)^2} = 0.019.$$

(c) If each collision reduces the energy by a factor of $1 - 0.89 = 0.11$, then the number of collisions required is the solution to

$$(0.025\,\text{eV}) = (1\times10^6\text{eV})(0.11)^N,$$

which is $N = 8$.

P51-5 The radii of the nuclei are

$$R = (1.2\text{ fm})\sqrt[3]{7} = 2.3\text{ fm}.$$

The using the derivation of Sample Problem 51-5,

$$K = \frac{(3)^2(1.6\times10^{-19}\text{C})^2}{16\pi(8.85\times10^{-12}\text{F/m})(2.3\times10^{-15}\text{m})} = 1.4\times10^6\text{eV}.$$

P51-6 (a) Add up the six equations to get

$$\begin{aligned} {}^{12}\text{C} \quad & + {}^{1}\text{H} + {}^{13}\text{N} + {}^{13}\text{C} + {}^{1}\text{H} + {}^{14}\text{N} + {}^{1}\text{H} + {}^{15}\text{O} + {}^{15}\text{N} + {}^{1}\text{H} \rightarrow \\ & {}^{13}\text{N} + \gamma + {}^{13}\text{C} + \text{e}^+ + \nu + {}^{14}\text{N} + \gamma + {}^{15}\text{O} + \gamma + {}^{15}\text{N} + \text{e}^+ + \nu + {}^{12}\text{C} + {}^{4}\text{He}. \end{aligned}$$

Cancel out things that occur on both sides and get

$${}^{1}\text{H} + {}^{1}\text{H} + {}^{1}\text{H} + {}^{1}\text{H} \rightarrow \gamma + \text{e}^+ + \nu + \gamma + \gamma + \text{e}^+ + \nu + {}^{4}\text{He}.$$

(b) Add up the Q values, and then add on 4(0.511 MeV for the annihilation of the two positrons.

$\boxed{\textbf{P51-7}}$ (a) Demonstrating the consistency of this expression is considerably easier than deriving it from first principles. From Problem 50-4 we have that a uniform sphere of charge Q and radius R has potential energy

$$U = \frac{3Q^2}{20\pi\epsilon_0 R}.$$

This expression was derived from the fundamental expression

$$dU = \frac{1}{4\pi\epsilon_0}\frac{q\,dq}{r}.$$

For gravity the fundamental expression is

$$dU = \frac{Gm\,dm}{r},$$

so we replace $1/4\pi\epsilon_0$ with G and Q with M. But like charges repel while all masses attract, so we pick up a negative sign.

(b) The initial energy would be zero if $R = \infty$, so the energy released is

$$U = \frac{3GM^2}{5R} = \frac{3(6.7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\text{kg})^2}{5(7.0\times10^8\text{m})} = 2.3\times10^{41}\text{J}.$$

At the current rate (see Sample Problem 51-6), the sun would be

$$t = \frac{(2.3\times10^{41}\text{J})}{(3.9\times10^{26}\text{W})} = 5.9\times10^{14}\text{ s},$$

or 187 million years old.

P51-8 (a) The rate of fusion events is

$$\frac{(3.9\times10^{26}\text{W})}{(26.2\times10^{6}\text{eV})(1.6\times10^{-19}\text{J/eV})} = 9.3\times10^{37}/\text{s}.$$

Each event produces two neutrinos, so the rate is

$$1.86\times10^{38}/\text{s}.$$

(b) The rate these neutrinos impinge on the Earth is proportional to the solid angle subtended by the Earth as seen from the Sun:

$$\frac{\pi r^2}{4\pi R^2} = \frac{(6.37\times10^{6}\text{m})^2}{4(1.50\times10^{11}\text{m})^2} = 4.5\times10^{-10},$$

so the rate of neutrinos impinging on the Earth is

$$(1.86\times10^{38}/\text{s})(4.5\times10^{-10}) = 8.4\times10^{28}/\text{s}.$$

P51-9 (a) Reaction A releases, for each d

$$(1/2)(4.03\text{ MeV}) = 2.02\text{ MeV},$$

Reaction B releases, for each d

$$(1/3)(17.59\text{ MeV}) + (1/3)(4.03\text{ MeV}) = 7.21\text{ MeV}.$$

Reaction B is better, and releases

$$(7.21\text{ MeV}) - (2/02\text{ MeV}) = 5.19\text{ MeV}$$

more for each N.

P51-10 (a) The mass of the pellet is

$$m = \frac{4}{3}\pi(2.0\times10^{-5}\text{m})^3(200\,\text{kg/m}^3) = 6.7\times10^{-12}\text{kg}.$$

The number of d-t pairs is

$$N = \frac{(6.7\times10^{-12}\text{kg})(6.02\times10^{23}/\text{mol})}{(0.005\text{ kg/mol})} = 8.06\times10^{14},$$

and if 10% fuse then the energy release is

$$(17.59\text{ MeV})(0.1)(8.06\times10^{14})(1.6\times10^{-19}\text{J/eV}) = 230\,\text{J}.$$

(b) That's

$$(230\,\text{J})/(4.6\times10^{6}\text{J/kg}) = 0.05\,\text{kg}$$

of TNT.

(c) The power released would be $(230\,\text{J})(100/\text{s}) = 2.3\times10^{4}\text{W}$.

$\boxed{\textbf{E52-1}}$ (a) The gravitational force is given by Gm^2/r^2, while the electrostatic force is given by $q^2/4\pi\epsilon_0 r^2$. The ratio is

$$\begin{aligned}\frac{4\pi\epsilon_0 Gm^2}{q^2} &= \frac{4\pi(8.85\times10^{-12}\mathrm{C^2/Nm^2})(6.67\times10^{-11}\mathrm{Nm^2/kg^2})(9.11\times10^{-31}\mathrm{kg})^2}{(1.60\times10^{-19}\mathrm{C})^2},\\ &= 2.4\times10^{-43}.\end{aligned}$$

Gravitational effects would be swamped by electrostatic effects at *any* separation.
(b) The ratio is

$$\begin{aligned}\frac{4\pi\epsilon_0 Gm^2}{q^2} &= \frac{4\pi(8.85\times10^{-12}\mathrm{C^2/Nm^2})(6.67\times10^{-11}\mathrm{Nm^2/kg^2})(1.67\times10^{-27}\mathrm{kg})^2}{(1.60\times10^{-19}\mathrm{C})^2},\\ &= 8.1\times10^{-37}.\end{aligned}$$

E52-2 (a) $Q = 938.27$ MeV $- 0.511$ MeV) $= 937.76$ MeV.
(b) $Q = 938.27$ MeV $- 135$ MeV) $= 803$ MeV.

E52-3 The gravitational force from the lead sphere is

$$\frac{Gm_eM}{R^2} = \frac{4\pi G\rho m_e R}{3}.$$

Setting this equal to the electrostatic force from the proton and solving for R,

$$R = \frac{3e^2}{16\pi^2\epsilon_0 G\rho m_e a_0^2},$$

or

$$\frac{3(1.6\times10^{-19}\mathrm{C})^2}{16\pi^2(8.85\times10^{-12}\mathrm{F/m})(6.67\times10^{-11}\mathrm{Nm^2/kg^2})(11350\,\mathrm{kg/m^3})(9.11\times10^{-31}\mathrm{kg})(5.29\times10^{-11}\mathrm{m})^2}$$

which means $R = 2.85\times10^{28}$m.

E52-4 Each γ takes half the energy of the pion, so

$$\lambda = \frac{(1240\ \mathrm{MeV\cdot fm})}{(135\ \mathrm{MeV})/2} = 18.4\ \mathrm{fm}.$$

$\boxed{\textbf{E52-5}}$ The energy of one of the pions will be

$$E = \sqrt{(pc)^2+(mc^2)^2} = \sqrt{(358.3\ \mathrm{MeV})^2+(140\ \mathrm{MeV})^2} = 385\ \mathrm{MeV}.$$

There are two of these pions, so the rest mass energy of the ρ_0 is 770 MeV.

E52-6 $E = \gamma mc^2$, so

$$\gamma = (1.5\times10^6\mathrm{eV})/(20\,\mathrm{eV}) = 7.5\times10^4.$$

The speed is given by

$$v = c\sqrt{1-1/\gamma^2} \approx c - c/2\gamma^2,$$

where the approximation is true for large γ. Then

$$\Delta v = c/2(7.5\times10^4)^2 = 2.7\times10^{-2}\mathrm{m/s}.$$

E52-7 $d = c\Delta t = hc/2\pi\Delta E$. Then

$$d = \frac{(1240\ \mathrm{MeV}\cdot\mathrm{fm})}{2\pi(91200\ \mathrm{MeV})} = 2.16\times 10^{-3}\mathrm{fm}.$$

E52-8 (a) Electromagnetic.
(b) Weak, since neutrinos are present.
(c) Strong.
(d) Weak, since strangeness changes.

$\boxed{\textbf{E52-9}}$ (a) Baryon number is conserved by having two "p" on one side and a "p" and a Δ^0 on the other. Charge will only be conserved if the particle x is positive. Strangeness will only be conserved if x is strange. Since it can't be a baryon it must be a meson. Then x is K^+.

(b) Baryon number on the left is 0, so x must be an anti-baryon. Charge on the left is zero, so x must be neutral because "n" is neutral. Strangeness is everywhere zero, so the particle must be $\bar{\mathrm{n}}$.

(c) There is one baryon on the left and one on the right, so x has baryon number 0. The charge on the left adds to zero, so x is neutral. The strangeness of x must also be 0, so it must be a π^0.

E52-10 There are two positive on the left, and two on the right. The anti-neutron must then be neutral. The baryon number on the right is one, that on the left would be two, unless the anti-neutron has a baryon number of minus one. There is no strangeness on the right or left, except possible the anti-neutron, so it must also have strangeness zero.

E52-11 (a) Annihilation reactions are electromagnetic, and this involves $\mathrm{s}\bar{\mathrm{s}}$.
(b) This is neither weak nor electromagnetic, so it must be strong.
(c) This is strangeness changing, so it is weak.
(d) Strangeness is conserved, so this is neither weak nor electromagnetic, so it must be strong.

E52-12 (a) $\mathrm{K}^0 \rightarrow \mathrm{e}^+ + \nu_\mathrm{e}$,
(b) $\mathrm{K}^0 \rightarrow \pi^+ + \pi^0$,
(c) $\mathrm{K}^0 \rightarrow \pi^+ + \pi^+ + \pi^-$,
(d) $\mathrm{K}^0 \rightarrow \pi^+ + \pi^0 + \pi^0$,

E52-13 (a) $\bar{\Delta}^0 \rightarrow \bar{\mathrm{p}} + \pi^+$.
(b) $\bar{\mathrm{n}} \rightarrow \bar{\mathrm{p}} + \mathrm{e}^+ + \nu_\mathrm{e}$.
(c) $\tau^+ \rightarrow \mu^+ + \nu_\mu + \bar{\nu_\tau}$.
(d) $\mathrm{K}^- \rightarrow \mu^- + \bar{\nu_\mu}$.

E52-14

E52-15 From top to bottom, they are Δ^{*++}, Δ^{*+}, Δ^{*0}, Σ^{*+}, Ξ^{*0}, Σ^{*0}, Δ^{*-}, Σ^{*-}, Ξ^{*-}, and Ω^-.

E52-16 (a) This is not possible.
(b) uuu works.

$\boxed{\textbf{E52-17}}$ A strangeness of +1 corresponds to the existence of an $\bar{\mathrm{s}}$ anti-quark, which has a charge of +1/3. The only quarks that can combine with this anti-quark to form a meson will have charges of -1/3 or +2/3. It is only possible to have a net charge of 0 or +1. The reverse is true for strangeness -1.

E52-18 Put bars over everything. For the anti-proton, $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}Z$, for the anti-neutron, $\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{d}}$.

E52-19 We'll construct a table:

quarks	Q	S	C	particle
$\mathrm{u}\bar{\mathrm{c}}$	0	0	-1	$\overline{\mathrm{D}}^0$
$\mathrm{d}\bar{\mathrm{c}}$	-1	0	-1	D^-
$\mathrm{s}\bar{\mathrm{c}}$	-1	-1	-1	D_s^-
$\mathrm{c}\bar{\mathrm{c}}$	0	0	0	η_c
$\mathrm{c}\bar{\mathrm{u}}$	0	0	1	D^0
$\mathrm{c}\bar{\mathrm{d}}$	1	0	1	D^+
$\mathrm{c}\bar{\mathrm{s}}$	1	1	1	D_s^+

E52-20 (a) Write the quark content out then cancel out the parts which are the same on both sides:

$$\mathrm{dds} \to \mathrm{udd} + \mathrm{d}\bar{\mathrm{u}},$$

so the fundamental process is

$$\mathrm{s} \to \mathrm{u} + \mathrm{d} + \bar{\mathrm{u}}.$$

(b) Write the quark content out then cancel out the parts which are the same on both sides:

$$\mathrm{d}\bar{\mathrm{s}} \to \mathrm{u}\bar{\mathrm{d}} + \mathrm{d}\bar{\mathrm{u}},$$

so the fundamental process is

$$\bar{\mathrm{s}} \to \mathrm{u} + \bar{\mathrm{d}} + \bar{\mathrm{u}}.$$

(c) Write the quark content out then cancel out the parts which are the same on both sides:

$$\mathrm{u}\bar{\mathrm{d}} + \mathrm{uud} \to \mathrm{uus} + \mathrm{u}\bar{\mathrm{s}},$$

so the fundamental process is

$$\bar{\mathrm{d}} + \mathrm{d} \to \mathrm{s} + \bar{\mathrm{s}}.$$

(d) Write the quark content out then cancel out the parts which are the same on both sides:

$$\gamma + \mathrm{udd} \to \mathrm{uud} + \mathrm{d}\bar{\mathrm{u}},$$

so the fundamental process is

$$\gamma \to \mathrm{u} + \bar{\mathrm{u}}.$$

E52-21 The slope is

$$\frac{(7000\ \mathrm{km/s})}{(100\ \mathrm{Mpc})} = 70\ \mathrm{km/s} \cdot \mathrm{Mpc}.$$

E52-22 $c = Hd$, so

$$d = (3\times 10^5\ \mathrm{km/s})/(72\ \mathrm{km/s}\cdot\mathrm{Mpc}) = 4300\ \mathrm{Mpc}.$$

E52-23 The question should read "What is the..."

The speed of the galaxy is

$$v = Hd = (72\ \mathrm{km/s}\cdot\mathrm{Mpc})(240\ \mathrm{Mpc}) = 1.72\times 10^7 \mathrm{m/s}.$$

The red shift of this would then be

$$\lambda = (656.3\mathrm{nm})\frac{\sqrt{1-(1.72\times 10^7\mathrm{m/s})^2/(3\times 10^8\mathrm{m/s})^2}}{1-(1.72\times 10^7\mathrm{m/s})/(3\times 10^8\mathrm{m/s})} = 695\ \mathrm{nm}.$$

E52-24 We can approximate the red shift as

$$\lambda = \lambda_0/(1 - u/c),$$

so

$$u = c\left(1 - \frac{\lambda_0}{\lambda}\right) = c\left(1 - \frac{(590\text{ nm})}{(602\text{ nm})}\right) = 0.02c.$$

The distance is

$$d = v/H = (0.02)(3\times10^8\text{m/s})/(72\text{ km/s}\cdot\text{Mpc}) = 83\text{ Mpc}.$$

$\boxed{\textbf{E52-25}}$ The minimum energy required to produce the pairs is through the collision of two 140 MeV photons. This corresponds to a temperature of

$$T = (140\text{ MeV})/(8.62\times10^{-5}\text{eV/K}) = 1.62\times10^{12}\text{K}.$$

This temperature existed at a time

$$t = \frac{(1.5\times10^{10}\text{s}^{1/2}\text{K})^2}{(1.62\times10^{12}\text{K})^2} = 86\,\mu\text{s}.$$

E52-26 (a) $\lambda \approx 0.002\,\text{m}$.
(b) $f = (3\times10^8\text{m/s})/(0.002\,\text{m}) = 1.5\times10^{11}\text{Hz}$.
(c) $E = (1240\text{ eV}\cdot\text{nm})/(2\times10^6\text{nm}) = 6.2\times10^{-4}\text{eV}$.

E52-27 (a) Use Eq. 52-3:

$$t = \frac{(1.5\times10^{10}\sqrt{s}\text{K})^2}{(5000\,\text{K})^2} = 9\times10^{12}\text{s}.$$

That's about 280,000 years.
(b) $kT = (8.62\times10^{-5}\text{eV/K})(5000\,\text{K}) = 0.43\,\text{eV}$.
(c) The ratio is

$$\frac{(10^9)(0.43\text{ eV})}{(940\times10^6\text{eV})} = 0.457.$$

$\boxed{\textbf{P52-1}}$ The total energy of the pion is $135 + 80 = 215$ MeV. The gamma factor of relativity is

$$\gamma = E/mc^2 = (215\text{ MeV})/(135\text{ MeV}) = 1.59,$$

so the velocity parameter is

$$\beta = \sqrt{1 - 1/\gamma^2} = 0.777.$$

The lifetime of the pion as measured in the laboratory is

$$t = (8.4\times10^{-17}\text{ s})(1.59) = 1.34\times10^{-16}\text{s},$$

so the distance traveled is

$$d = vt = (0.777)(3.00\times10^8\text{m/s})(1.34\times10^{-16}\text{s}) = 31\text{ nm}.$$

P52-2 (a) $E = K + mc^2$ and $pc = \sqrt{E^2 - (mc^2)^2}$, so

$$pc = \sqrt{(2200 \text{ MeV} + 1777 \text{ MeV})^2 - (1777 \text{ MeV})^2} = 3558 \text{ MeV}.$$

That's the same as

$$p = \frac{(3558 \times 10^6 \text{eV})}{(3 \times 10^8 \text{m/s})}(1.6 \times 10^{-19} \text{J/eV}) = 1.90 \times 10^{-18} \text{kg} \cdot \text{m/s}$$

.

(b) $qvB = mv^2/r$, so $p/qB = r$. Then

$$r = \frac{(1.90 \times 10^{-18} \text{kg} \cdot \text{m/s})}{(1.6 \times 10^{-19} \text{C})(1.2 \text{ T})} = 9.9 \text{ m}.$$

P52-3 (a) Apply the results of Exercise 45-1:

$$E = \frac{(1240 \text{ MeV} \cdot \text{fm})}{(2898 \, \mu\text{m} \cdot \text{K})T} = (4.28 \times 10^{-10} \text{MeV/K})T.$$

(b) $T = 2(0.511 \text{ MeV})/(4.28 \times 10^{-10} \text{MeV/K}) = 2.39 \times 10^9 \text{K}$.

P52-4 (a) Since

$$\lambda = \lambda_0 \frac{\sqrt{1-\beta^2}}{1-\beta},$$

we have

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\sqrt{1-\beta^2}}{1-\beta} - 1,$$

or

$$z = \frac{\sqrt{1-\beta^2} + \beta - 1}{1-\beta}.$$

Now invert,

$$\begin{aligned} z(1-\beta) + 1 - \beta &= \sqrt{1-\beta^2}, \\ (z+1)^2(1-\beta)^2 &= 1-\beta^2, \\ (z^2+2z+1)(1-2\beta+\beta^2) &= 1-\beta^2, \\ (z^2+2z+2)\beta^2 - 2(z^2+2z+1)\beta + (z^2+2z) &= 0. \end{aligned}$$

Solve this quadratic for β, and

$$\beta = \frac{z^2+2z}{z^2+2z+2}.$$

(b) Using the result,

$$\beta = \frac{(4.43)^2 + 2(4.43)}{(4.43)^2 + 2(4.43) + 2} = 0.934.$$

(c) The distance is

$$d = v/H = (0.934)(3 \times 10^8 \text{m/s})/(72 \text{ km/s} \cdot \text{Mpc}) = 3893 \text{ Mpc}.$$

P52-5 (a) Using Eq. 48-19,

$$\Delta E = -kT \ln \frac{n_1}{n_2}.$$

Here $n_1 = 0.23$ while $n_2 = 1 - 0.23$, then

$$\Delta E = -(8.62\times10^{-5}\text{eV/K})(2.7\,\text{K}) \ln(0.23/0.77) = 2.8\times10^{-4}\text{eV}.$$

(b) Apply the results of Exercise 45-1:

$$\lambda = \frac{(1240\text{ eV}\cdot\text{nm})}{(2.8\times10^{-4}\text{eV})} = 4.4\,\text{mm}.$$

P52-6 (a) Unlimited expansion means that $v \geq Hr$, so we are interested in $v = Hr$. Then

$$\begin{aligned} Hr &= \sqrt{2GM/r}, \\ H^2r^3 &= 2G(4\pi r^3\rho/3), \\ 3H^2/8\pi G &= \rho. \end{aligned}$$

(b) Evaluating,

$$\frac{3[72\times10^3\text{m/s}\cdot(3.084\times10^{22}\text{m})]^2}{8\pi(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)}\frac{(6.02\times10^{23}/\text{mol})}{(0.001\text{ kg/mol})} = 5.9/\text{m}^3.$$

$\boxed{\textbf{P52-7}}$ (a) The force on a particle in a spherical distribution of matter depends only on the matter contained in a sphere of radius *smaller* than the distance to the center of the spherical distribution. And then we can treat all that relevant matter as being concentrated at the center. If M is the total mass, then

$$M' = M\frac{r^3}{R^3},$$

is the fraction of matter contained in the sphere of radius $r < R$. The force on a star of mass m a distance r from the center is

$$F = GmM'/r^2 = GmMr/R^3.$$

This force is the source of the centripetal force, so the velocity is

$$v = \sqrt{ar} = \sqrt{Fr/m} = r\sqrt{GM/R^3}.$$

The time required to make a revolution is then

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{R^3/GM}.$$

Note that this means that the system rotates as if it were a solid body!

(b) If, instead, all of the mass were concentrated at the center, then the centripetal force would be

$$F = GmM/r^2,$$

so

$$v = \sqrt{ar} = \sqrt{Fr/m} = \sqrt{GM/r},$$

and the period would be

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{r^3/GM}.$$

P52-8 We will need to integrate Eq. 45-6 from 0 to $\lambda_{\rm min}$, divide this by $I(T)$, and set it equal to $z = 0.2\times 10^{-9}$. Unfortunately, we need to know T to perform the integration. Writing what we do know and then letting $x = hc/\lambda kT$,

$$\begin{aligned} z &= \frac{15c^2h^3}{2\pi^5k^4T^4}\int_0^{\lambda_{\rm m}} \frac{2\pi c^2 h}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}, \\ &= \frac{15c^2h^3}{2\pi^5k^4T^4}\frac{2\pi k^4T^4}{h^3c^2}\int_\infty^{x_{\rm m}} \frac{x^3\,dx}{e^x-1}, \\ &= \frac{15}{\pi^4}\int_{x_{\rm m}}^\infty \frac{x^3\,dx}{e^x-1}. \end{aligned}$$

The result is a small number, so we expect that $x_{\rm m}$ is fairly large. We can then ignore the -1 in the denominator and then write

$$z\pi^4/15 = \int_{x_{\rm m}}^\infty x^3 e^{-x} dx$$

which easily integrates to

$$z\pi^4/15 \approx {x_{\rm m}}^3 e^{x_{\rm m}}.$$

The solution is

$$x \approx 30,$$

so

$$T = \frac{(2.2\times 10^6 \text{eV})}{(8.62\times 10^{-5}\text{eV/K})(30)} = 8.5\times 10^8 \text{K}.$$