Homework 4

1 Problem 1:

The moments of inertia for these identical rods are:

$$I_1 = \int_0^L \tfrac{m}{L} x^2 dx = \tfrac{1}{3} mL^2 \tag{1}$$

$$I_2 = m \cdot (\tfrac{L}{2})^2 = \tfrac{1}{4} mL^2 \tag{2}$$

$$I_3 = 2 \cdot \int_0^{\frac{L}{2}} \tfrac{m}{L} (\tfrac{L^2}{4} + x^2) dx = \tfrac{1}{3} mL^2 \tag{3}$$

Therefore,The moment of inertia is:

$$I = I_1 + I_2 + I_3 = \frac{11}{12} mL^2 \tag{4}$$

2 Problem 2:

According to the conservation of energy

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} Iw^2 \tag{5}$$

Due to:

$$w = \frac{v}{r} \tag{6}$$

So,

$$I = mr^2 (\frac{2gh}{v^2} - 1) \tag{7}$$

3 Problem 3:

(a) The rotational energy of the Earth is:

$$E_k = \tfrac{1}{2} I w^2 = \tfrac{4}{5} M R^2 \cdot \tfrac{\pi^2}{T^2} = 2.56 \times 10^{29} J \tag{8}$$

(b) Due to $\mathrm{d}T = 10\mu s$, we have:

$$\mathrm{d}E = \frac{1}{2} I \cdot 2\omega \mathrm{d}\omega = \frac{1}{2} I \cdot 2\omega \cdot -\frac{2\pi}{T^2}\mathrm{d}T = -I \cdot \frac{4\pi^2}{T^3}\mathrm{d}T = -5.94 \times 10^{19} J$$

So, in one day,

$$\Delta E' = \tfrac{\mathrm{d}E}{365} = -1.63 \times 10^{17} J. \tag{9}$$

4 Problem 4:

According to the conservation of energy:

$$\frac{1}{2} m v^2 + \frac{1}{2} I w^2 = mgh$$

i.e.

$$m v \frac{dv}{dt} + I w \frac{dw}{dt} = mg \frac{dh}{dt}$$

$$m v \frac{dv}{dt} + \frac{I}{R^2} v \frac{dv}{dt} = mgvsin\theta$$

thus:

$$\frac{dv}{dt} = \frac{gsin\theta}{1 + \frac{I}{mR^2}}$$

$$I_{\text{sphere}} = \frac{2}{5} m R^2, \frac{dv}{dt} = \frac{g \sin\theta}{1 + 2/5}$$

$$I_{\text{sCylinder}} = \frac{1}{2} m R^2, \frac{dv}{dt} = \frac{g \sin\theta}{1 + 1/2}$$

$$I_{\text{hCylinder}} = \frac{1}{2} m (R^2 + r^2), \frac{dv}{dt} = \frac{g \sin\theta}{1 + \frac{1}{2}(1 + r^2/R^2)}$$

Therefore , sphere reaches the bottom first and hollow cylinder reaches it last.

5 Problem 5:

In the view of theorem of torque, we have:

$$Fr_1 - fr_2 = I\alpha = mr_2^2\alpha, \tag{10}$$

$$F = \frac{1}{r_1}(fr_2 + mr_2^2\alpha) = 871.7N \tag{11}$$

Considering

$$F'r' = Fr_1 \tag{12}$$

We find

$$F' = \frac{Fr_1}{r'} = 1401N \tag{13}$$

6 Problem 6:

The cross-product of two vectors is perpendicular to both of these two vectors. But

$$(2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}) \cdot (4\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}) = 8 - 9 - 4 = -5$$

So I don't believe this claim.