# Zhejiang University
## Department of Physics

General Physics (H) Solution to Problem Set #13

1. *In order to take a nice warm bath, you mix 50 liters of hot water at 55°C with 25 liters of cold water at 10°C. How much new entropy have you created by mixing the water?*

   **Solution** We first need to figure out the final temperature of the water; this is obtained by the equality of the heat that flows out of the hot water and that flows into the cold one, or (we assume the specific heat is a constant within the range of temperature of interest)

   $$c_w m_h (T_{ih} - T_f) = c_w m_c (T_f - T_{ic}). \tag{1}$$

   If we neglect the volume change of the water, which is small, we immediately get $T_f = 40^\circ$ C = 313 K. The new entropy introduced is the sum of the entropy changes in hot and cold water.

   $$\begin{aligned} \Delta S &= \int_{T_{ih}}^{T_f} c_w m_h \frac{dT}{T} + \int_{T_{ic}}^{T_f} c_w m_c \frac{dT}{T} \\ &= c_w m_h \ln\left(\frac{T_f}{T_{ih}}\right) + c_w m_c \ln\left(\frac{T_f}{T_{ic}}\right) \\ &= \frac{4.2J}{(g\cdot K)} \times (50\, l \times 1000\frac{g}{l}) \ln(\frac{313}{328}) + \frac{4.2J}{(g\cdot K)} \times (25\, l \times 1000\frac{g}{l}) \ln(\frac{313}{283}) \\ &\approx 750\ J/K. \end{aligned} \tag{2}$$

2. *Calculate the change in entropy for a process in which 2 moles of an ideal gas undergoes a free expansion to three times its initial volume.*

   **Solution** The free expansion is an irreversible process. Nevertheless, we can calculate the entropy change for an equivalent reversible expansion, i.e., an isothermal expansion. For the equivalent isothermal expansion, $\Delta U = 0$, the work done by the gas during the expansion from $V$ to $3V$ is

   $$W = \int_{V_0}^{3V_0} P dV = \int_{V_0}^{3V_0} \frac{nRT}{V} dV = nRT \ln(3V_0/V_0).$$

   According to the first law of thermodynamics, the gas must absorb heat of amount $Q = W$ from its environment. Therefore, the entropy change is $\Delta S = Q/T = 2R\ln 3 = 18.3$ J/K.

3. *Experimental measurements of the heat capacity of aluminum at low temperatures (below about 50 K) can be fit to the formula*

$$C_V = aT + bT^3,$$

*where $C_V$ is the heat capacity of one mole of aluminum, and the constants $a$ and $b$ are approximately $a = 0.00135$ J/K$^2$ and $b = 2.48 \times 10^{-5}$ J/K$^4$. From this data, find a formula for the entropy of a mole of aluminum as a function of temperature (assuming $S = 0$ at 0 K). Evaluate your formula at $T = 1$ K and at $T = 10$ K.*

**Solution** The heat capacity $C_v = T(\partial S/\partial T)_v$, hence

$$\begin{aligned} S - S_0 &= \int_{T_0}^{T} C_v(T')dT'/T' = \int_{T_0}^{T} (a + bT'^2)dT \\ &= a(T - T_0) + (b/3)(T^3 - T_0^3) \end{aligned} \tag{3}$$

If we assume $S(T_0 = 0) = 0$, i.e., no residual entropy, we obtain $S = aT + bT^3/3$. For $T = 1$ K, $S = 0.00136$ J/K $= 9.84 \times 10^{19} k$. For $T = 10$ K, $S = 0.0218$ J/K $= 1.58 \times 10^{21} k$.

4. *Derive the efficiency of the Otto cycle*

$$e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1},$$

*where $V_1/V_2$ is the compression ratio and $\gamma$ is the adiabatic exponent.*

**Solution** In an Otto cycle, heat only enters or leaves the gas during the ignition ($2 \to 3$) and the exhaust ($4 \to 1$) processes. The compression ($1 \to 2$) and power ($3 \to 4$) processes are adiabatic.

During ignition, $Q_1 = C_v(T_3 - T_2)$. During exhaust, $Q_2 = C_V(T_1 - T_4)$. As in a Carnot cycle,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = \frac{T_3}{T_4}, \tag{4}$$

where $\gamma$ is the adiabatic exponent. Therefore, the efficiency of an Otto engine is

$$\epsilon = 1 - \frac{|Q_2|}{|Q_1|} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}, \tag{5}$$

where $V_1/V_2$ is called the compression ratio.

5. *A bit of computer memory is some physical object that can be in two different states, often interpreted as 0 and 1. A byte is eight bits, a kilobyte is 1024 = $2^{10}$ bytes, a megabyte is 1024 kilobytes, and a gigabyte is 1024 megabytes. (i) Suppose that your computer erases or overwrites one gigabyte of memory, keeping no record of the information that was stored. Explain why this process must create a certain*

*amount of entropy, and calculate how much. (ii) If the entropy is dumped into an environment at room temperature, how much heat must come along with it? Is this amount of heat significant?*

**Solution** (a) One gigabyte is $8 \times 2^{30} = 2^{33}$ bits. Erasing a bit of information, we have an uncertainty of 0 or 1. Hence the entropy increase for one bit is $k \ln 2$. Therefore, we must create at least $\Delta S = 2^{33} \ln 2k = 8.22 \times 10^{-14}$ J/K.

(b) At room temperature, the heat coming along with it is $Q = T_{RT} \Delta S = 2.5 \times 10^{-11}$ J. This is not significant. Heat generated in your computer is mostly the energy dissipation of electrons traveling in narrow channel of wires (a few tens of nanometers wide).