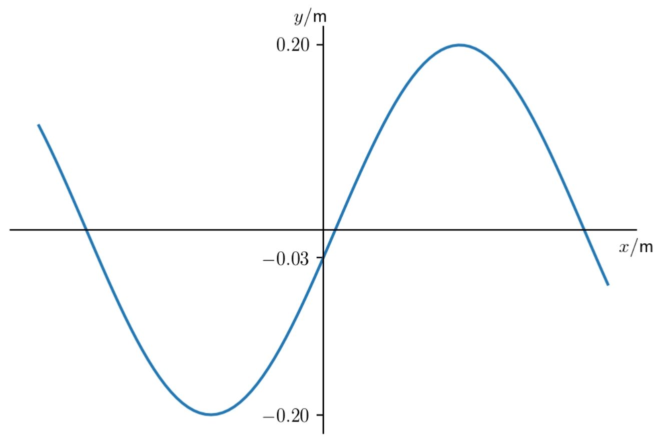
Homework 6

1 Problem 1:

(a)



(b) According to the description, $A = 0.2m$, $\lambda = 0.35m$, $f = 12\text{Hz}$:
the angular wave number:

$$k = \frac{2\pi}{\lambda} = \frac{40}{7}\pi = 5.71\pi = 17.94m^{-1} \tag{1}$$

period:

$$T = \frac{1}{f} = \frac{1}{12}\text{s} = 0.083s \tag{2}$$

angular frequency:

$$w = 2\pi f = 24\pi\text{rad/s} = 75.36rad \cdot s^{-1} \tag{3}$$

wave speed :

$$v = \frac{\lambda}{T} = \lambda f = 4.2m/s \tag{4}$$

(c) Because the wave travels in the $-x$ direction:

$$y(x,t) = A\sin[kx + \omega t + \phi_0] \tag{5}$$

$$y(0,0) = -0.03 \Longrightarrow \sin\phi_0 = -0.15$$

Therefore:

$$y(x,t) = 0.2\sin[\frac{40\pi}{7}x + 12t + \arcsin(-0.15)](\mathrm{m}) \tag{6}$$

2 Problem 2:

The speed of the tidal wave is

$$v = \frac{4450}{9.5}km/h = 468.42km/h = 130.1\mathrm{m/s} \tag{7}$$

Therefore,the depth of the water is:

$$d \approx \frac{v^2}{g} = 1693m \tag{8}$$

3 Problem 3:

(a) The wave length of the wave is:

$$\lambda = \tfrac{v}{f} = 16m \tag{9}$$

$$\Delta x = 8m$$

Therefore,the receiver records a minimum in sound intensity.
(b) To make the intensity remain at a minimum, we have:

$$|\sqrt{(x+5)^2+y^2} - \sqrt{(x-5)^2+y^2}| = \frac{\lambda}{2}(2k+1) \tag{10}$$

where k is an integer. And according to the question, we know:

$$\sqrt{(x+5)^2+y^2} - \sqrt{(x-5)^2+y^2} = 8 \Longrightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

4 Problem 4:

(a) For the vibration of a wire fixed at both ends:

$$L = n\frac{\lambda}{2} \Longrightarrow \lambda = \frac{2L}{n}(n = 1, 2, 3, \cdots)$$

the speed of the wave:

$$v = \frac{\omega}{k} = \sqrt{\frac{F}{m/L}} = 20\text{m/s}$$

so the frequency:

$$f = \frac{v}{\lambda} = \frac{nv}{2L}(n = 1, 2, 3, \cdots)$$

for the first three allowed modes:

$$f_1 = \frac{v}{2L} = 5\text{Hz}$$

$$f_2 = \frac{2v}{2L} = 10\text{Hz}$$

$$f_3 = \frac{3v}{2L} = 15\text{Hz}$$

(b) in this case we have:

$$0.4\text{m} = n'\frac{\lambda}{2} \Longrightarrow \lambda = \frac{2 \times 0.4}{n'}(n' = 1, 2, 3, \cdots)$$

$$\lambda = \frac{4}{n} = \frac{0.8}{n'} \Longrightarrow n = 5n'$$

so it's in the modes with $n = 5n'$, and the frequency:

$$f = \frac{v}{\lambda} = 25n'$$

where $n' = 1, 2, 3, \cdots$.

5 Problem 5:

(a) The wave speed in the string is:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}} = \lambda f = \frac{2L}{n} \cdot f \tag{11}$$

where n is the mode number, therefore:

$$\frac{n_2}{n_1} = \sqrt{\frac{m_1}{m_2}} = \frac{4}{5} \tag{12}$$

because no standing waves are observed with any mass between m_1 and m_2:

$$|n_1 - n_2| = 1 \tag{13}$$

So:

$$n_1 = 5, n_2 = 4 \tag{14}$$

Thus:

$$f = \sqrt{\frac{m_1 g}{\mu}} \cdot \frac{n_1}{2L} = 250\sqrt{2}\text{Hz} \tag{15}$$

(b) When $n = 1$:

$$m = m_{\max} = \frac{n_1^2}{n} m_1 = 400\text{kg} \tag{16}$$