2019-2020春夏学期微分几何第十二周作业

P_{70}

2. **证明** 因为$2\pi = \int_0^L k_r ds$,由已知，$\int_0^L k_r ds \leq \frac{1}{R} = \frac{L}{R}$,
所以$L \geq 2\pi R$.

3. **证明** 以AB为弦，做一圆，使得由AB分得的两段圆弧弧长分别为$L, \tilde{L}$,两个弓形面积分别为S_1, S_2, 固定弧长$\tilde{L}$的圆弧,其与AB围成的面积为S_2，考率周长为$L+\tilde{L}$的简单闭曲线，则当S_1取得最大值时，S_1+S_2达到最大值，而由等周不等式$S_1+S_2 \leq \frac{(L+\tilde{L})^2}{4\pi}$而此时，$L_1$为圆弧。

5. **解** $\frac{ds}{dt} = |\frac{dx}{dt}| = (sin^2 t + 4cos^2 2t)^{\frac{1}{2}}$,其且映射为$\varphi(t) = (x^1(t), x^2(t))$, 直接计算可得$deg\varphi = \frac{1}{2\pi}\int_0^{2\pi}(x^1(x^2)' - x^2(x^1)')dt = 0$,从而$i_r(C) = 0, \int_0^{2\pi} k_r ds = 0$.也可通过观察其图像，其切线正向反向各旋转一周，所以其旋转指标为0。

7. **证明**设$\bar{\varphi}(t) = \frac{x(t)}{|x(t)|}$，平移了圆心$o$到$a$，从而$\omega = deg\varphi = deg\bar{\varphi} = \frac{1}{2\pi}\int_0^l \frac{x^1(x^2)' - x^2(x^1)'}{|x(t)|^2}dt$

8.**解** (1)$l_{\bar{C}} = \int_0^l |\dot{x}(s) - a\dot{N}_r(s)|ds = \int_0^l |(1+ak_r)T(s)|ds = l_C + 2\pi a$.
(2)设$\bar{s}$是曲线$\bar{C}$的弧长参数.

$$\bar{T}(s) = \frac{d\bar{x}(s)}{d\bar{s}} = \frac{d\bar{x}(s)}{ds} \cdot \frac{ds}{d\bar{s}} = T(s)$$

可得$\frac{ds}{d\bar{s}} = \frac{1}{1+ak_r}$, 于是

$$\bar{k}(s) = |\frac{d\bar{T}}{d\bar{s}}| = |\frac{dT}{ds} \cdot \frac{ds}{d\bar{s}}| = \frac{k_r}{1+ak_r}.$$

(3)

$$\begin{aligned}
\bar{A} &= \int_0^l \bar{x}_1 \dot{\bar{x}}_2 ds \\
&= \int_0^l (x_1 + a\dot{x}_2) \cdot (\dot{x}_2 - a\ddot{x}_1) ds \\
&= \int_0^l (x_1\dot{x}_2 - ax_1\ddot{x}_1 + a\dot{x}_2\dot{x}_2 - a^2\dot{x}_2\ddot{x}_1) ds
\end{aligned}$$

因为

$$\int_0^l (-ax_1\ddot{x}_1 + a\dot{x}_2\dot{x}_2) ds = \int_0^l (a\dot{x}_1\dot{x}_1 + a\dot{x}_2\dot{x}_2) = al,$$

$$\int_0^l (-a^2\dot{x}_2\ddot{x}_1) ds = \frac{1}{2}\int_0^l (a^2\dot{x}_1\ddot{x}_2 - a^2\dot{x}_2\ddot{x}_1) ds = \int_0^l a^2 k_r (\dot{x}_1^2 + \dot{x}_2^2) ds = a^2\pi.$$

于是

$$\bar{A} = A + al + a^2\pi.$$