2. 设 $\{\sigma_1, \cdots, \sigma_m\}$ 是 m 维向量空间 V 的基, $\varphi = \sigma_1 \wedge \cdots \wedge \sigma_p (0 < p < m)$. 试证: 若 $\omega \in V$ 满足 $\omega \wedge \varphi = 0$, 则 ω 是 $\sigma_1, \cdots, \sigma_p$ 的线性组合.

3. 设 $\omega = \sum\limits_{1 \leqslant \alpha < \beta \leqslant m} a_{\alpha\beta} \, \mathrm{d} u^\alpha \wedge \mathrm{d} u^\beta, a_{\alpha\beta} + a_{\beta\alpha} = 0$. 求证:

$$\mathrm{d}\omega = \sum_{1 \leqslant \alpha < \beta < \gamma \leqslant m} \left(\frac{\partial a_{\alpha\beta}}{\partial u^\gamma} + \frac{\partial a_{\beta\gamma}}{\partial u^\alpha} + \frac{\partial a_{\gamma\alpha}}{\partial u^\beta} \right) \mathrm{d} u^\alpha \wedge \mathrm{d} u^\beta \wedge \mathrm{d} u^\gamma.$$

4. 设 $\varphi = yz \, \mathrm{d} x + \mathrm{d} z, \psi = xz \, \mathrm{d} y + \cos y \, \mathrm{d} x, \eta = xy \, \mathrm{d} z - \sin z \, \mathrm{d} y$, 计算: (1) $\varphi \wedge \psi, \psi \wedge \eta, \eta \wedge \varphi$; (2) $\mathrm{d}\varphi$, $\mathrm{d}\psi$, $\mathrm{d}\eta$.

5. 设 $x = x(u,v), y = y(u,v)$, 证明:

$$\mathrm{d} x \wedge \mathrm{d} y = \frac{\partial(x,y)}{\partial(u,v)} \mathrm{d} u \wedge \mathrm{d} v.$$

6. 求证: 1 形式 $\omega = yz \, \mathrm{d} x + zx \, \mathrm{d} y + xy \, \mathrm{d} z$ 是闭形式, 并且找出函数 f 使得 $\mathrm{d} f = \omega$.

7. 设 $E(u,v), F(u,v), G(u,v)$ 是曲面的第一基本量. 作参数变换 $\widetilde{u} = \widetilde{u}(u,v), \widetilde{v} = \widetilde{v}(u,v)$, 并记 $\widetilde{E}, \widetilde{F}, \widetilde{G}$ 是新参数下的第一基本量. 求证:

$$\sqrt{\widetilde{E}\widetilde{G} - \widetilde{F}^2} \, \mathrm{d} \widetilde{u} \wedge \mathrm{d} \widetilde{v} = \sqrt{EG - F^2} \, \mathrm{d} u \wedge \mathrm{d} v.$$