Homework 1

1 Problem 1:

Consider that

$$[a] = LT^{-2}, \quad [t] = T, \tag{1}$$

Then we can get:

$$[a^m t^n] = L = [s], \tag{2}$$

So the expression $s = ka^m t^n$ is satisfied if we choose m = 1, n = 2.
However, this analysis can not give the value of k, because k is dimensionless.

2 Problem 2:

Since

$$\rho = \frac{3}{4\pi}\frac{M}{r^3} = 1.608 \cdot 10^3 kg/m^3 \tag{3}$$

there are two kinds of answers for calculating the uncertainty, one is

$$\delta\rho = \frac{3}{4\pi}\frac{\delta Mr - 3\delta rM}{r^4} = 131.06 kg/m^3 \tag{4}$$

thus $\rho = 1608.03 \pm 131.06 kg/m^3$
the other is

$$ln\rho = lnM - 3lnr + ln\frac{3}{4\pi} \tag{5}$$

$$\frac{\partial ln\rho}{\partial M} = \frac{1}{M} \quad , \quad \frac{\partial ln\rho}{\partial r} = -\frac{3}{r} \tag{6}$$

$$\frac{\delta\rho}{\rho} = \sqrt{\left(\frac{\partial ln\rho}{\partial M}\right)^2 \times (\delta M)^2 + \left(\frac{\partial ln\rho}{\partial r}\right)^2 \times (\delta r)^2} = 0.093 \tag{7}$$

$$\delta\rho = \rho \times 0.093 = 149.54 kg/m^3 \tag{8}$$

thus $\rho = 1608.03 \pm 149.54 kg/m^3$

3 Problem 3:

since

$$v = 3 - 8t \tag{9}$$

$$x = 2.00 + 3.00t - 4.00t^2 \tag{10}$$

the instant it changes direction is at $t = \frac{3}{8}$s, thus

$$x = 2.5625m. \tag{11}$$

Solve the equation

$$3t - 4t^2 = 0, \tag{12}$$

we can find the time when it returns to the position is $t = \frac{3}{4}s$, thus according to Eq.(12)

$$v = -3m/s. \tag{13}$$

4 Problem 4:

(a)

$$a_x = a_{xi} + Jt \tag{14}$$

$$v_x = v_{xi} + \int_0^t a_x \mathrm{d}t' = v_{xi} + a_{xi}t + \frac{1}{2}Jt^2 \tag{15}$$

$$x = x_i + \int_0^t v_x \mathrm{d}t' = x_i + v_{xi}t + \frac{1}{2}at^2 + \frac{1}{6}Jt^3 \tag{16}$$

(b)

$$a_x + a_{xi} = \frac{2(v_x - v_{xi})}{t} \tag{17}$$

$$a_x - a_{xi} = Jt \tag{18}$$

$$a_x^2 - a_{xi}^2 = (a_x + a_{xi})(a_x - a_{xi}) = 2J(v_x - v_{xi}) \tag{19}$$

5 Problem 5:

(a)

$$v_i \cos\theta_i t = d\cos\phi \Rightarrow t = \frac{d\cos\phi}{v_i\cos\theta_i} \tag{20}$$

$$v_i \sin\theta_i t - \frac{1}{2}gt^2 = d\sin\phi \tag{21}$$

from Eq.(20) and Eq.(21):

$$d = \frac{2v_i^2\cos\theta_i\sin(\theta_i - \phi)}{g\cos^2\phi}$$

(b)

$$d = \frac{2v_i^2\cos\theta_i\sin(\theta_i - \phi)}{g\cos^2\phi} = \frac{v_i^2[\sin(2\theta_i - \phi) - \sin\phi]}{g\cos^2\phi} \tag{22}$$

so when $2\theta_i - \phi = \frac{\pi}{2}$, d is a maximum.
i.e.

$$d_{\max} = \frac{v_i^2[1-\sin\phi]}{g\cos^2\phi}, \text{ when } \theta_i = \frac{\phi}{2} + \frac{\pi}{4}$$

6 Problem 6:

$$\bar{v} = \frac{l_{ABC}}{t} = 6.53m/s. \quad R = \frac{l_{ABC}}{\frac{\pi}{2}} = 149.6m. \quad \overrightarrow{a_n} = \frac{v^2}{R}. \tag{23}$$

So,the acceleration is

$$\overrightarrow{a} = \frac{v^2}{R}(-\cos 35^\circ \overrightarrow{i} + \sin 35^\circ \overrightarrow{j}) = -0.233 \overrightarrow{i} + 0.163 \overrightarrow{j} \tag{24}$$

there are two kinds of answers for average speed, one is

$$\bar{v} = \frac{l_{ABC}}{t} = 6.53m/s. \tag{25}$$

the other is

$$\overrightarrow{\bar{v}} = \frac{\triangle \overrightarrow{r}}{\triangle t} = \frac{R}{t} \overrightarrow{i} + \frac{R}{t} \overrightarrow{j} = \left(4.16 \overrightarrow{i} + 4.16 \overrightarrow{j}\right) \quad m/s^2 \tag{26}$$

and the average acceleration is

$$\overrightarrow{a} = \frac{\triangle \overrightarrow{v}}{\triangle t} = -\frac{v}{t} \overrightarrow{i} + \frac{v}{t} \overrightarrow{j} = \left(-0.181 \overrightarrow{i} + 0.181 \overrightarrow{j}\right) \quad m/s^2 \tag{27}$$