Homework 2

1 Problem 1:

$$a_x = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = 10\mathrm{m/s}^2,$$

$$a_y = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = 18t\mathrm{m/s}^2 = 36\mathrm{m/s}^2$$

$$F = m\sqrt{a_x^2 + a_y^2} = 112.09\mathrm{N}$$

2 Problem 2:

$$ma = \mu mg$$

$$a = \mu g = 8\mathrm{m/s}^2$$

and no need to know her mass.

3 Problem 3:

$$ma = F = -kmv^2$$

$$\mathrm{d}v/\mathrm{d}t = -kv^2$$

$$\frac{\mathrm{d}v}{v^2} = -k\mathrm{d}t$$

$$\int_{v_f}^{v} \frac{\mathrm{d}v}{v^2} = \int_0^t -k\mathrm{d}t$$

$$-\frac{1}{v} + \frac{1}{v_f} = -kt$$

$$v = \frac{v_f}{1 + ktv_f}$$

4 Problem 4:

In the direction which tangent to the cylinder, we have

$$0 = F - mgcos\theta. \tag{1}$$

Using the boundary condition $W(\theta = 0) = 0$, the work done by the mass:

$$W = \int F \cdot d\vec{s} = \int_0^{\frac{\pi}{2}} mgcos\theta R \mathrm{d}\theta = mgR \tag{2}$$

5 Problem 5:

For the energy-efficient lightbulb, the energy that is transformed in its lifetime is

$$W_e = 28 \cdot 10^{-3} \cdot 10000 kw \cdot h = 280 kw \cdot h \tag{3}$$

and the cost is

$$M_e = 280 \cdot 0.08 + 17 = 39.4; \tag{4}$$

For the conventional lightbulb, the energy that is transformed in its lifetime is:

$$W_c = 100 \cdot 10000 \cdot 10^{-3} kw \cdot h = 1000 kw \cdot h \tag{5}$$

the number of bulbs must be used is

$$n_c = \frac{10000}{750} = 13.3 \tag{6}$$

then the cost is

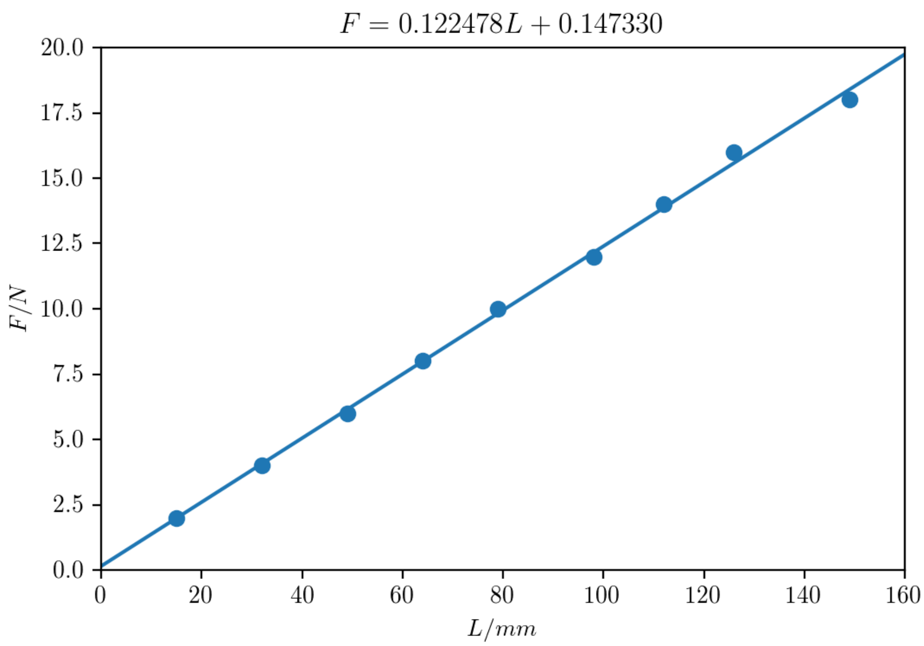
$$M_c = 1000 \cdot 0.08 + 13.3 \cdot 0.42 = 85.6 \tag{7}$$

Therefore the total savings is

$$M_c - M_e = 46.2 \tag{8}$$

6 Problem 6:

With the help of computer, we can get the least square fitting result:



Or, we can assume that,

$$F = kL + b$$

Then,

$$\bar{L} = \frac{1}{n}\sum_{i=1}^{n} L_i$$

$$\bar{F} = \frac{1}{n}\sum_{i=1}^{n} F_i$$

$$k = \frac{\sum_{i=1}^{n} F_i L_i - n\bar{F}\bar{L}}{\sum_{i=1}^{n} L_i^2 - n\bar{L}^2} = 122.5\mathrm{N/m}$$

$$b = \bar{F} - k\bar{L} = 0.147\mathrm{N}$$

Thus,

$$F = 0.1225 \times 105 + 0.147 = 13.0\mathrm{N}$$