Homework 8

1 Problem 1:

(a) Since the fetus's ventricular wall moves in simple harmonic motion, the maximum linear speed of the heart wall is:

$$v_{wall} = A\omega = A \cdot 2\pi f = 0.0217\text{m/s} \tag{1}$$

(b) Moving observer:

$$f_{max} = \frac{v_d + v_{wall}}{v_d} \cdot f_d = 2.0000288 \times 10^6\text{Hz} \tag{2}$$

(c) Moving source:

$$f'_{max} = \frac{v_d}{v_d - v_{wall}} \cdot f_{max} = 2.0000578 \times 10^6\text{Hz} \tag{3}$$

2 Problem 2:

Considering the velocity addition rules for two different beams, we find:

$$t_1 = \frac{2L}{\sqrt{c^2 - v^2}} \tag{4}$$

$$t_2 = \frac{L}{c-v} + \frac{L}{c+v} \tag{5}$$

therefore:

$$\Delta = |t_1 - t_2| = \frac{2Lv^2}{(c^2 - v^2)(\sqrt{c^2 - v^2} + c)} \approx \frac{2Lv^2}{2c^3} = \frac{Lv^2}{c^3} \tag{6}$$

where

$$v \ll c \tag{7}$$

3 Problem 3:

In the frame of the train:

$$\frac{u}{c} = \frac{1-f}{1+f} \Longrightarrow f = \frac{c-u}{c+u}$$

and in the frame of the track:

$$\begin{cases} D = cT_0 - wT_0 \\ L = cT_0 - vT_0 \end{cases} \Longrightarrow \frac{D}{c-w} = \frac{L}{c-v} \tag{8}$$

$$\begin{cases} D = cT_1 + wT_1 \\ fL = cT_1 + vT_1 \end{cases} \Longrightarrow \frac{D}{c+w} = \frac{fL}{c+v} \tag{9}$$

divide (9) by (8):

$$\frac{c-w}{c+w} = (\frac{c-u}{c+u})(\frac{c-v}{c+v})$$

4 Problem 4:

from problem 3, we know that:

$$\frac{c-w}{c+w} = (\frac{c-u}{c+u})(\frac{c-v}{c+v}) \Longrightarrow c-w = (c+w)(\frac{c-u}{c+u})(\frac{c-v}{c+v}) \tag{10}$$

$$\Longrightarrow w = \frac{u+v}{1+(\frac{u}{c})(\frac{v}{c})} \tag{11}$$

In the frame of the big ball, the velocity u of the small ball:

$$u = \frac{c}{2} = v$$

where v is the speed which the big ball is thrown at respect to the ground. In the frame of the ground, the velocity w of the small ball:

$$w = \frac{u+v}{1+(\frac{u}{c})(\frac{v}{c})} = \frac{\frac{c}{2}+\frac{c}{2}}{1+(\frac{1}{2})(\frac{1}{2})} = \frac{4}{5}c$$

Similarly, when the big ball is thrown at a faster speed v:

$$w = \frac{u+v}{1+(\frac{u}{c})(\frac{v}{c})} = \frac{2v}{1+\frac{v^2}{c^2}} = \frac{2c}{\frac{c}{v}+\frac{v}{c}} < c$$

where

$$v < c \Rightarrow \frac{c}{v} + \frac{v}{c} > 2$$

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5 Problem 5:

According to Problem 4,we have:

$$\begin{aligned} w = \frac{\frac{c}{n} + v}{1 + \frac{\frac{c}{n}}{c} \cdot \frac{v}{c}} = \frac{\frac{c}{n} + v}{1 + \frac{v}{cn}} \\ \approx (\frac{c}{n} + v)(1 - \frac{v}{cn}) \\ = \frac{c}{n} + v - \frac{v}{n^2} - \frac{v^2}{cn} \\ \approx \frac{c}{n} + v(1 - \frac{1}{n^2}) \end{aligned}$$

where

$$v \ll c \tag{12}$$