Homework 3

1 Problem 1:

$$W(OAC) = \int_{OA} \vec{F}_i dx + \int_{AC} \vec{F}_j dy = \int_0^5 25 dy = 125J \tag{1}$$

$$W(OBC) = \int_{OB} \vec{F}_j dy + \int_{BC} \vec{F}_i dx = \int_0^5 10 dy = 50J \tag{2}$$

$$W(OC) = \int_{OC} (x^2 + 2x) dx = \frac{200}{3} J \tag{3}$$

note that $x = y$ along OC.
$\boldsymbol{F}$ is nonconservative, because work done by $\boldsymbol{F}$ between two points depends on the path.

2 Problem 2:

Choose the potential energy at point C to be zero , the total energy at the top is

$$E = mgR + \frac{1}{2} m v_i^2 = \frac{3}{2} mgR \tag{4}$$

Therefore, the kinetic energy at θ is

$$E_{k\theta} = \frac{3}{2} mgR + mgR cos\theta = \frac{1}{2} m v_\theta^2 \tag{5}$$

the time the ball need to reach C is:

$$t = -\frac{R sin\theta}{v_\theta cos\theta} \tag{6}$$

and the direction in the horizon:

$$R cos\theta = v_\theta sin\theta \cdot t - \frac{1}{2} g t^2 \tag{7}$$

thus

$$cos\theta = \frac{\sqrt{6}-3}{3} \tag{8}$$

3 Problem 3:

The momentum is conserved when the bullet passes through bob.

$$mv = Mv_M + \frac{1}{2}mv \tag{9}$$

$$v_M = \frac{mv}{2M} \tag{10}$$

when the bob get to the top,v could be zero and the energy conservation gives:

$$\frac{1}{2}Mv_M^2 = 2Mgl \tag{11}$$

thus:

$$v_M = \sqrt{4gl} \tag{12}$$

$$v_{min} = \frac{2v_M M}{m} = \frac{4M}{m}\sqrt{gl} \tag{13}$$

4 Problem 4:

(a)According to the momentum conservation:

$$3Mv_1 + Mv_2 = 0 \tag{14}$$

therefore:

$$v_2 = -3v_1 \tag{15}$$

We know $v_1 = 2m/s$, to the right.

Therefore $v_2 = 6m/s$, to the left.

(b)since the energy is conserved in such a system:

$$E = \frac{1}{2}Mv_2^2 + \frac{1}{2} \cdot 3Mv_1^2 = 8.4J \tag{16}$$

5 Problem 5:

Assume the struck ball's velocity is $\vec{v}_1$,the other ball's velocity is $\vec{v}_2$.

Therefore:

$$m\vec{v}_0 = m\vec{v}_1 + m\vec{v}_2 \tag{17}$$

so:

$$\vec{v}_1 = \vec{v}_0 - \vec{v}_2 \tag{18}$$

we know

$$\vec{v_0} = 5\vec{i}m/s, \tag{19}$$

$$\vec{v_1} = 4.33 \times (\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}) \tag{20}$$

So,

$$\vec{v}_2 = (5 - 4.33 \times \frac{\sqrt{3}}{2})\vec{i} - 4.33 \times \frac{1}{2}\vec{j} \tag{21}$$

thus:

$$|v_2| \approx 2.5m/s \tag{22}$$

$$cos\theta = \frac{5 - 4.33 \times \frac{\sqrt{3}}{2}}{2.5} \approx \frac{1}{2}, \theta \approx \frac{\pi}{3} \tag{23}$$

Therefore,$v_2 = 2.5m/s$ at an angle of $60.0°$ with the respect to the original line of motion.

6 Problem 6:

The velocity of the center of mass is defined as

$$\vec{v}_c = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = -\frac{2\sqrt{2}}{15}\vec{i} + \frac{2\sqrt{2}}{5}\vec{j} = -0.189\vec{i} + 0.566\vec{j}. \tag{24}$$

The magnitutde:

$$v = \frac{4}{3\sqrt{5}}\text{m/s} = 0.596\text{m/s}$$

The direction:

$$\frac{\vec{v}}{v} = -\frac{\sqrt{10}}{10}\vec{i} + \frac{3\sqrt{10}}{10}\vec{j}$$

The position vector is:

$$\vec{s} = \vec{v}_c t = -\frac{2\sqrt{2}}{15}t\vec{i} + \frac{2\sqrt{2}}{5}t\vec{j}. \tag{25}$$