Zhejiang University
Department of Physics

General Physics (H) Solution to Problem Set #12

1. *If we apply the highly successful kinetic theory of gases to a metal, considered as a gas of electrons (in fact, back in 1900 Drude constructed the theory, hence the Drude theory of metals), and assume that the electron velocity distribution is given by the Maxwell distribution, what would the most probable speed, average speed, and rms speed for electrons at room temperature? Compare to those of the H_2 gas.*

   Solution From the Maxwell-Boltzmann distribution

   $$f(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv^2}{2kT}\right], \tag{1}$$

   The most probable speed, the average speed, and the rms speed are

   $$v_{max} = \sqrt{\frac{2kT}{m}} \tag{2}$$
   $$\overline{v} = \sqrt{\frac{8kT}{\pi m}} \tag{3}$$
   $$v_{rms} = \sqrt{\frac{3kT}{m}}, \tag{4}$$

   respectively. The difference between hydrogen molecules and electrons is their mass. The mass of a proton is about 2000 times larger than an electron. A hydrogen molecule has two protons, which means its $\sqrt{m}$ is about 60 times larger than an electron. So all corresponding velocities are about 60 times smaller.

2. *How much heat is needed to convert 1 kg of ice at -10°C to steam at 100°C? Look up the necessary constants from books or the Internet yourself.*

   Solution The total heat is the sum of the following terms: (1) heating ice to 0°C, (2) melting ice to water, (3) heating water to 100°C, and (4) boiling water into steam.

   During the phase transitions (2) and (4), the heat can be evaulated by $Q = mL$, where the latenhe temperature is always such that rotational degrees of freedom are t heat is 333 J/g for melting ice and 2260 J/g for boling water.

   During (1) and (3), the heat necessary to warm a homogeneous material is $Q = cm\Delta T$, where the temperature change ΔT is in Kelvin (the same as in Celsius), the specific heat c is 4.18 J/(g·K) for water and 2.05 J/(g·K) for ice.

Therefore, for *one gram* of the ice/water/steam, we have

$$Q = 2.05 \times 10 + 333 + 4.18 \times 100 + 2260 \approx 3030J \tag{5}$$

For 1 Kg, we need 3030 KJ of heat.

3. *An iron rod (with heat conductivity being 80 W/m·K) of length 1 m and radius 2 cm has one end dipped into an ice-water mixture and the other in boiling water. What is the heat flow $Q/\Delta t$?*

   Solution The heat flow through a material with thermal conductivity κ, length δx, cross-sectional area A, and a temperature difference of ΔT is:

$$\frac{dQ}{dt} = -\kappa A \frac{\Delta T}{\Delta x} \tag{6}$$

   where the minus sign ensures that heat flows from the hot side to the cold side. Since the area of the cross section is $A = \pi r^2$, the temperature difference between boiling water and ice water is 100 K, and the thermal conductivity of iron is 80 $W/(m \cdot K)$, we have

$$\frac{dQ}{dt} = 80W/(m \cdot K) \times \pi(0.02m)^2 \frac{100}{1} = 10W \tag{7}$$

4. *An ideal diatomic gas, in a cylinder with a movable piston, undergoes the rectangular cyclic process shown in Figure 1 (see PS#14). Assume that the temperature is always such that rotational degrees of freedom are active, but vibrational modes are "frozen out." Also assume that the only type of work done on the gas is quasistatic compression-expansion work. (i) For each of the four steps A through D, compute the work done on the gas, the heat added to the gas, and the change in the internal energy of the gas. Express all answers in terms of P_1, P_2, V_1, and V_2. (ii) Describe in words what is physically being done during the four steps; for example, during step A, heat is added to the gas (from an external source) while the piston is held fixed. (iii) Compute the net work done on the gas, the net heat added to the gas, and the net change in the internal energy of the gas during the entire cycle. Are the results as you expected? Explain briefly.*

   Solution (a) Process A:

$$W = -\int^{A} PdV = 0 \tag{8}$$

$$\Delta U = \frac{5}{2} nR\Delta T = \frac{5}{2}(P_2 - P_1)V_1 \tag{9}$$

$$Q = \Delta U - W = \frac{5}{2}(P_2 - P_1)V_1 \tag{10}$$

Process B:

$$W = -\int^{B} PdV = -P_2(V_2 - V_1) \tag{11}$$

$$\Delta U = \frac{5}{2}nR\Delta T = \frac{5}{2}P_2(V_2 - V_1) \tag{12}$$

$$Q = \Delta U - W = \frac{7}{2}P_2(V_2 - V_1) \tag{13}$$

Process C:

$$W = -\int^{C} PdV = 0 \tag{14}$$

$$\Delta U = \frac{5}{2}nR\Delta T = \frac{5}{2}(P_1 - P_2)V_2 \tag{15}$$

So

$$Q = \Delta U - W = \frac{5}{2}(P_1 - P_2)V_2 \tag{16}$$

Process D:

$$W = -\int^{D} PdV = -P_1(V_1 - V_2) \tag{17}$$

$$\Delta U = \frac{5}{2}nR\Delta T = \frac{5}{2}P_1(V_1 - V_2) \tag{18}$$

$$Q = \Delta U - W = \frac{7}{2}P_1(V_1 - V_2) \tag{19}$$

(b) During step A: heat is added to the gas, while the piston is held fixed.

During step B: heat is added to the gas, although the work is negative.

During step C: heat is released from the gas, while the piston is held fixed.

During step D: heat is released from the gas, although the work is positive.

(c)

$$\begin{aligned} W_{total} &= P_2(V_2 - V_1) + P_1(V_1 - V_2) = -(P_2 - P_1)(V_2 - V_1) && (20) \\ \Delta U_{total} &= 0 && (21) \\ Q_{total} &= (P_2 - P_1)(V_2 - V_1) && (22) \end{aligned}$$

In the cycle, the gas absorbs heat and converts into work.

5. *In a Diesel engine, atomospheric air is quickly compressed to about 1/20 of its original volume. Estimate the temperature of the air after compression, and explain why a Diesel engine does not require spark plug?*

Solution We assume the atmospheric air at 300 K. The quick compression is an adiabatic process, which obeys

$$VT^{f/2} = \text{constant}, \tag{23}$$

or

$$V^{2/f}T = \text{constant}, \tag{24}$$

We assume the atmospheric air (dominated by N_2 and O_2) has an $f = 5$ at room temperature and the vibrational degrees of freedom remain frozen. After a compression to 1/20 of its volume, the air temperature

$$T_f = 300 \text{ K} \times 20^{2/f} = 994 \text{ K}. \tag{25}$$

The autoignition temperature of diesel is 210° C or about 500 K. Hence a Diesel engine does not require spark plugs to ignite the fuel.