Homework 5

1 Problem 1:

In the view of the angular momentum:

$$mv_i \frac{d}{2} = (I_m + I_M)w \tag{1}$$

$$w = \frac{6mv_i}{3md + Md} \tag{2}$$

The loss in mechanical energy is:

$$\Delta E_k = \frac{1}{2}mv_i^2 - \frac{1}{2}(I_m + I_M)w^2 = \frac{Mmv_i^2}{6m + 2M} \tag{3}$$

2 Problem 2:

The moments are in balance at the statical equbibrium, then

$$Fr = mg \cdot 3r \tag{4}$$

where

$$F = \frac{1}{2}Mg \cdot \cos\theta \tag{5}$$

So,

$$m = \frac{250\sqrt{2}}{2} = 176.8\text{kg} \tag{6}$$

3 Problem 3

(a) When the turntable rotates, we have

$$Iw_t = rmv \tag{7}$$

thus

$$w_t = \frac{rmv}{I} = 0.36\text{rad} \cdot \text{s}^{-1} \tag{8}$$

That is counter-clockwise.
(b) the work can be expressed as:

$$W = \frac{1}{2}Iw_t^2 + \frac{1}{2}mv^2 = 99.9J \tag{9}$$

4 Problem 4:

This is the case when the center of mass is just above the edge. For the upper book, the distance it exceeds the lower one is:

$$x_1 = L/2$$

Now for the system of these two books, the center of mass is $L/4$ from the front of the lower book. Thus:

$$x = x_1 + L/4 = 3L/4$$

5 Problem 5:

The force balance is

$$2000 \cdot \cos 65^\circ \cdot L + 1200 \cdot \cos 65^\circ \cdot \frac{L}{2} = T \cdot \frac{3}{4}L \tag{10}$$

Thus:

$$T = 1465N \tag{11}$$

the reaction force is

$$F_x = T \cdot \cos 25^\circ = 1327.74\text{N} \tag{12}$$

$$F_y = 2000\text{N} + 1200\text{N} - T \cdot \sin 25^\circ = 2580.86\text{N} \tag{13}$$

and

$$F = \sqrt{F_x^2 + F_y^2} = 2902N \tag{14}$$

6 Problem 6:

$$ML^2\frac{d^2\theta}{dt^2} = -(L \cdot Mg\sin\theta + h \cdot kh\tan\theta) \tag{15}$$

For small θ, $\sin\theta \approx \theta, \tan\theta \approx \theta$. So,

$$\frac{d^2\theta}{dt^2} = -(\frac{g}{L} + \frac{kh^2}{ML^2})\theta \tag{16}$$

therefore , the frequency is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L} + \frac{kh^2}{ML^2}} \tag{17}$$