Coordinator
Daniel Balint

Probability Theory

Exercise Sheet 7

Exercise 7.1 Let X and Y be two independent Bernoulli distributed random variables with parameter p. Define $Z = 1_{\{X+Y=0\}}$ and $\mathcal{G} = \sigma(Z)$. Find $E[X|\mathcal{G}]$ and $E[Y|\mathcal{G}]$. Are these random variables also independent?

Exercise 7.2 Let X and Y be random variables whose joint distribution is the uniform distribution on the triangle $\{(x, y) \in \mathbb{R}^2 : \ 0 \le y \le x \le 1\}$.

(a) Compute the distribution of Y/X.

(b) Show that Y/X and X are independent.

(c) Compute the conditional expectation $E[Y|X]$.

Exercise 7.3 Let S be a random variable with $P[S > t] = e^{-t}$, for all $t > 0$. Calculate the following conditional expectations for arbitrary $t > 0$:

(a) $E\Big[S \ \Big| S \wedge t\Big]$, where $S \wedge t := \min(S, t)$;

(b) $E\Big[S \ \Big| S \vee t\Big]$, where $S \vee t := \max(S, t)$.

Exercise 7.4 (Optional.) In this exercise we prove that in Theorem 1.37 (Kolmogorov's Three Series Theorem) (1.4.16) $\Rightarrow$ (1.4.17).

Consider X_k, $k \ge 1$ independent random variables and $A > 0$. Set $Y_k := X_k 1_{|X_k| \le A}$, $k \ge 1$. Assume that $\sum_k X_k$ converges P-a.s.

(a) Show that $P[\liminf_k \{X_k = Y_k\}] = 1$.

(b) Deduce from (a) that $\sum_k P[|X_k| > A] < \infty$ and $\sum_k Y_k$ converges P-a.s.

(c) Show that $\sum_k \mathrm{Var}(Y_k) < \infty$. (**Hint:** use Exercise 6.4.)

(d) Show that $\sum_k E[Y_k]$ converges. (**Hint:** use Theorem 1.34, moreover (c) and (b).)

Submission: until 14:15, Nov 12., during exercise class or in the tray outside of HG G 53.

Office hours (Präsenz): Mon. and Thu., 12:00-13:00 in HG G 32.6.

Class assignment:

Students	Time & Date	Room	Assistant
Afa-Fül	Tue 13-14	HG F 26.5	Angelo Abächerli
Gan-Math	Tue 13-14	ML H 41.1	Zhouyi Tan
Meh-Schu	Tue 14-15	HG F 26.5	Angelo Abächerli
Schü-Zur	Tue 14-15	ML H 41.1	Dániel Bálint