Questions

October 30, 2018

1st :
对$\forall \varepsilon > 0$, 设$x_0 = m, x_1 = m + \varepsilon \sin x_0, \ldots x_n = m + \varepsilon \sin x_{n-1}$. 证明：数列$\{x_n\}$ 收敛.

2nd :
若$x_n > 0 (n = 1, 2, 3, \ldots)$ 且$\overline{\lim\limits_{n\to\infty}} x_n \cdot \overline{\lim\limits_{n\to\infty}} \frac{1}{x_n} = 1$. 证明：数列$\{x_n\}$ 收敛.

3rd :
证明：对任意正数序列$\{x_n\}$, 有$\overline{\lim\limits_{n\to\infty}} n(\frac{1+x_{n+1}}{x_n} - 1) \geq 1$.

4th :
设$a_n = \sin 1 + \frac{\sin 2}{2!} + \cdots + \frac{\sin n}{n!}, n \in \mathbf{N}_+$, 证明：数列$\{a_n\}$ 收敛，但不单调.

5th :
设$0 < x_1 < 1, x_{n+1} = x_n(1 - x_n), n = 1, 2, \cdots$. 证明：1) $\lim\limits_{n\to\infty} x_n = 0$;　2) $\lim\limits_{n\to\infty} nx_n = 1$.

6th :
设$y_n = x_n + 2x_{n+1}, n \in \mathbf{N}_+$, 证明：在$\{y_n\}$ 收敛时, $\{x_n\}$ 也收敛.