

Identifying Subnetworks

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Abstract

Studies that dissect the hidden representations of deep neural networks are commonplace in machine learning research. A long-standing problem within the field has been to determine to what extent these representations are modular and specialized. For example, if a model is trained on an arithmetic dataset, is there a part of the model that computes addition, and is this part distinguishable from the part that computes subtraction? Recently, the field of mechanistic interpretability has become a popular approach for answering such questions. This is typically done by identifying so-called circuits, which are subnetworks that correspond to the individual operations involved in solving a task. This functional definition of modularity, however, is also found under other names and by other methods than those typically used in mechanistic interpretability. This document tries to synthesize existing methods for identifying such subnetworks under a shared vocabulary, with an emphasis on applications within NLP.

1 Introduction

The work surveyed in this article are all, in some way or another, concerned with the following question: does deep learning models learn solutions to problems that are modular and specialized? And if so, can we find evidence of this if we inspect the latent space? This is especially relevant for tasks where there exists a compositional solution that follows from the application and combination of a set of individual functions or smaller units of computation, such as in different types of reasoning, multi-hop question answering and code verification. This question can also be formulated more formally: Given a parameterized model M_θ and a target task T that is composed of a set of i distinguishable subtasks ST_i , is there a subnetwork in M_θ , $\hat{\theta} \subset \theta$, that can solve ST_i with a comparable performance to the overall model, so that $P(x|M_\theta) \approx P(x|M_{\hat{\theta}})$ for $x \sim ST_i$?

In the literature there exists multiple terms for describing $\hat{\theta}$ and multiple ways of separating it from θ . In the following sections we will try to synthesize these terms and methodologies using a notation that relates back to this question.

2 We have many names for the things we love

This section presents common ways of describing $\hat{\theta}$.

Subnetwork Csordás et al. (2020) uses the terms *module* and *subnetwork* interchangeably to refer to $\hat{\theta}$. A subnetwork is defined as a subset of θ that is responsible for performing a specific function within an overall task. The same functional definition is used in Lepori et al. (2023) but there exclusively under the name *subnetwork*.

Cluster Watanabe (2019) uses the same functional definition of modularity, but uses the term *cluster* to refer to a set of feature vectors that are the most influential on the output of a model for a set of specific inputs. This term is also used by Casper et al. (2022), who defines a cluster to be a subset of the network when viewed as a computational graph (with neurons being the node abstraction). These clusters are analyzed with respect to their *local specialization*, where goal of the analysis is to determine to what extent certain clusters translate to functional abstractions from the target task.

Subset In contrast to the functional definition, Ansell et al. (2022) uses the term *subset* to refer to the parameters of M that are the most influential on a finetuning task. This definition is closely related to works on efficient finetuning, such as adapters (Houlsby et al., 2019), where additional parameters are inserted into M . As these parameters are not part of the original model, these methods fall out of scope for this survey.

Circuits The term *circuit* is the standard in the field of mechanistic interpretability and is commonly described as a subset of of a network when viewed as a computational graph (Conmy et al., 2023; Nanda et al., 2023; Wang et al., 2023). As most works in mechanistic interpretability focuses on the Transformer, this graph is a DAG, and a circuit will consequently be a path along the residual stream of the model. The nodes of the graph typically represent entire attention heads and individual nodes from the intermediate MLPs, i.e model components.

3 Identification methods

In this section we discuss existing methods for identifying $\hat{\theta}$ from θ .

3.1 Masks

3.1.1 Differentiable weight masks

Csordás et al. (2020) proposes a method for training binary weight masks over θ . Their method requires a set of subtasks ST_i that correspond to the functions required to solve T . The first step is to train M_θ on samples from T . Next, they train a mask m on samples from ST_i while keeping θ frozen. The resulting mask reveals the parameters $\hat{\theta}_i$ responsible for solving the functionality for the samples in ST_i .

The mask m is initialized as a set of learnable logits $l_i \in \mathbb{R}$, where $i \in [1, N]$ for N weights in θ . l_i is initially set to 0.9 for each i in order to have a high probability of keeping weights. During training, l_i is regularized such that the probability for weight θ_i not being masked out during inference is high if θ_i is necessary for solving ST_i . The regularization term r is set as $r = \alpha \sum_i l_i$, where α is a hyperparameter that controls the strength of the regularization. The mask training procedure is based on sampling. For each l_i , a sample $s_i \in [0, 1]$ is drawn from the mask as follows:

$$s_i = \sigma((l_i - \log(\log U_1 / \log U_2) / \tau)), \quad (1)$$

with $U_1, U_2 \sim U(0, 1)$, and where τ is a hyperparameter and σ is the sigmoid function. s_i is then gated to become the final binary mask, b_i . This is done with a straight-through estimator, which allows for estimating the gradient of threshold functions—like the one needed here to turn the continuous s_i into the discrete b_i .¹ The authors sample 4-8 binary masks per batch and apply it to different parts of the batch. After training, the mask is applied to M_θ through elementwise multiplication of the mask with the original weights: $\theta_i \odot b_i$, revealing $\hat{\theta}$ as those parameters that are not set to zero from this multiplication.

Lepori et al. (2023) uses almost the exact same approach as Csordás et al. (2020) but with a different and simpler masking technique. Their approach relies on a pruning technique called *continuous sparsification* (Savarese et al., 2020), which the authors claim is both deterministic and better at finding sparser subnetworks than the one used in Csordás et al. (2020). This method

¹There was a lot of details here that I did not quite understand, but I think this should explain the gist of it at least

uses l_0 regularization, where the training incentivize a weight mask to have as many zero-elements as possible. Given a model M_θ that is trained to solve T , the first step is to initialize the mask m as a set of parameters with the same dimensionality as θ . The next step is to train m on samples from ST_i while keeping θ frozen. This mask training optimizes the following objective function:

$$\min_{m \in \mathbb{R}^d} L(M_{\theta_i \odot m_i}(x)) + \lambda * \|\sigma(\beta * m_i)\|_1, \quad (2)$$

where L is the cross entropy, $x \sim ST_i$, and λ and β are hyperparameters that effectively control the balance between the loss and the number of zero-elements in θ . After mask training, the mask is made binary and applied elementwise with the original network through a heaviside function, substituting $\sigma(\beta * m_i)$ with:

$$H(S) = \left\{ \begin{array}{l} 0, s < 0 \\ 1, s > 0 \end{array} \right\}, \quad (3)$$

which gives us the final subnetwork responsible for computing ST_i : $\hat{\theta}_i = M_{\theta_i \odot H(m_i)}$

3.1.2 Lottery Ticket Sparse Fine-Tuning

Another way of identifying masks is the Lottery Ticket Sparse Fine-Tuning approach proposed by [Ansell et al. \(2022\)](#). After finetuning a pretrained network on a target task, they identify the subset of parameters that changed the most during this training phase. Given a pretrained model M parameterized by θ^0 , finetuning M on a target task yields the parameters θ^1 . Parameters are then ranked according to their greatest absolute difference: $|\theta_i^1 - \theta_i^0|$. A binary mask is then constructed by selecting the top K parameters and setting all elements in $\theta_{i \in K}$ to 1 and $\theta_{i \notin K}$ to 0, which gives $\hat{\theta}$. A similar approach is also used in [Frankle and Carbin \(2019\)](#).

3.2 Clustering

[Watanabe \(2019\)](#); [Casper et al. \(2022\)](#)

3.3 Circuits

([Conmy et al., 2023](#); [Nanda et al., 2023](#); [Wang et al., 2023](#))

4 Subnetwork composition

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