TTK4190 Guidance and Control of Vehicles

Assignment 2

Aircraft Autopilot Design with State Estimation

Fall 2018

Deadline part 1: Monday the 8th of October at 23:59
Deadline part 2: Friday the 26th of October at 23:59

Objective

In this assignment you will apply linear theory and design an autopilot for course control for an aircraft. You will also learn about state estimation using a Kalman filter. You will mostly refer to the book and lecture notes from Beard and McLain (2012) [1] in the first part, but the book by Fossen [2] is relevant in the second part.

Grading

This assignment must be passed to get access to the final exam. The overall impression of how well you have understood the problems will be the basis for the evaluation. You need at least 60 % of the assignment correct to pass. **You are encouraged/supposed to work in groups of 2-4 people, but are allowed to do the assignment individually if that is preferred for some reason**. Note that the grading will be equally severe if you do the work individually. The participants in the group will receive the same feedback. Because someone might want to change groups, a new set of groups for the second assignment should be used and are available on Blackboard. Use the sign-up sheet called "Assignment 2 - Group" when you register in a group. It is obviously allowed to cooperate with the same people as on the first assignment, but you still need to register again to get access to the delivery. Both parts will be evaluated together after the second deadline.

Deadline and Delivery Details

Part 1 of the assignment must be handed in **by 23:59 on Monday the 8th of October and the second part on the 26th of October at 23:59**. If you use Simulink to solve the simulations, they should be started via a m-file and not the Simulink window. Matlab code and Simulink models should not be included in the report. Use degrees as the unit in all of your figures (degrees for angles and degrees/s for the rates). Further information regarding the Matlab code is given in the corresponding tasks. You need to deliver a written report that will be evaluated. The report has to be handed in **via Blackboard**. You are strongly urged to write the report on your PC using your favorite editor (LaTeX, Word, Pages...). You can hand in a scanned version (not recommended), but in the end it has to be a **PDF** document. Paper versions are not accepted.

Aircraft Model

The linear lateral model of an aircraft controlled with ailerons is given as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned} \tag{1}$$

where

$$\mathbf{x} = \begin{bmatrix} \beta \text{ (rad)} \\ \phi \text{ (rad)} \\ p \text{ (rad/s)} \\ r \text{ (rad/s)} \\ \delta_a \text{ (rad)} \end{bmatrix}, \mathbf{u} = \begin{bmatrix} \delta_a^c \text{ (rad)} \end{bmatrix}, \mathbf{y} = \begin{bmatrix} \beta \text{ (rad)} \\ \phi \text{ (rad)} \\ p \text{ (rad/s)} \\ r \text{ (rad/s)} \end{bmatrix} \tag{2}$$

The actuator dynamics of the aileron are augmented to the lateral equations of motion as a state δ_a, which is modeled as a low-pass filter ($H_l(s)$) with time constant $T_l = 1/7.5$ s. Moreover,

$$H_l(s) = \frac{\delta_a}{\delta_a^c} = \frac{1}{T_l s + 1} = \frac{7.5}{s + 7.5} \tag{3}$$

The control input for the augmented model is denoted δ_a^c. The aileron has a maximum deflection of ± 30 degrees. <u>All of your simulations need to make sure that this constraint is satisfied</u>.

During a coordinated turn the course angle χ satisfies the *bank-to-turn* equation:

$$\dot{\chi} = \frac{g}{V_g} \tan(\phi) \cos(\chi - \psi) \tag{4}$$

Assume that the wind speed is zero (no wind).

Continuous-time State-space matrices and trim condition

Airspeed $\quad V_a = 580$ km/h

$$\mathbf{A} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -7.5 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7.5 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{5}$$

Part 1 (50 %)

The first part of the assignment starts here and should be handed in by 23:59 on Monday the 8th of October.

Problem 1 - Open-loop analysis

When investigating the modes in this problem, use the definitions from the book [1].

a) What is the ground speed of the aircraft (numerical value) in the absence of wind?

b) Write down two expressions for the sideslip (crab) angle β in the absence of wind. One expression should depend on aircraft velocity and the other on aircraft heading.

c) Compute the Dutch-roll natural frequency and relative damping ratio for the aircraft. Can you, very briefly with your own words, describe how the Dutch roll mode affect the yaw and roll motion? How would the motion change with increased relative damping ratio?

d) Compute the spiral-divergence mode. Is the mode unstable?

e) Compute the roll mode. Is the roll mode faster or slower than the spiral-divergence mode?

Problem 2 - Autopilot for course hold using aileron and successive loop closure

Figure 1 shows the block diagram for a lateral autopilot using successive loop closure. The autopilot is based on a simplified version of the coordinated turn equation and a linearization of the roll dynamics from the book [1]. This represents an autopilot for course hold using aileron δ_a as control input. The disturbance d in Figure 1 models a bias in the coordinated-turn equation, and should have a constant value of 1.5 degrees. It should not be included in the roll feedback loop.

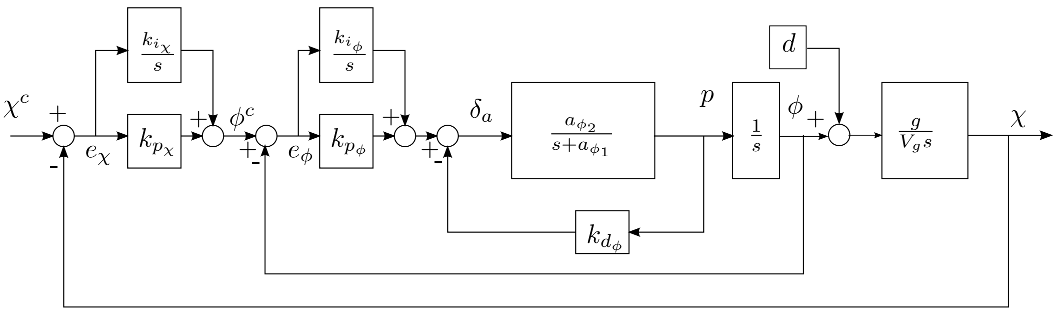

Figure 1: Successive loop closure for autopilot design.

a) Find numerical values for a_{ϕ_1} and a_{ϕ_2} based on the state-space model (1).

b) Find numerical values for the five gains in Figure 1 associated with the lateral autopilot using successive loop closure. Use root-locus analysis to propose an interval where the integral gain in the roll loop (k_{i_ϕ}) should be chosen within in order to maintain system stability. Include a figure of the root-locus analysis in your report and remember to specify the input range for the gain somewhere in the figure (use both positive and negative gain values). Justify how you choose the design parameters in the course loop.

Hint: *When designing the roll loop, choose* $\delta_a^{max} = 30°$, $e_\phi^{max} = 15°$, $\zeta_\phi = 0.707$

c) Can you come up with a reason for why it in some cases may be beneficial to remove the integral action from the roll loop? You are free to remove the integral action from the roll loop in the rest of the assignment if you think that is appropriate, but write why if you choose to do so.

d) Present simulation results for the system in Figure 1 with course changing maneuvers: choose a series of steps (you may choose yourself the desired values). Comment on the results. You should not have larger steps than ± 15 degrees. Include figures of the course and the aileron input in the report.

e) Present simulation results for course changing maneuvers (choose the same desired values as in Problem 2d) with the complete state-space model (1). Keep your control gains from Problem 2d). The aircraft model must be the complete state-space model, including the actuator dynamics, and the simplified version of the coordinated turn equation must be replaced by the real coordinated turn equation (4). Moreover, the system dynamics (1) should replace the simplified roll dynamics in Figure 1. Remember to add the bias d at the appropriate place. Compare the results with the results obtained in Problem 2d). Would you say that the simplified model in Problem 2d) accurately reproduce the true dynamics? Include figures of the course and aileron input in the report.

f) The integral action in the course loop compensates for the disturbance d. However, if the aileron input is saturated, integrator windup can occur. Would you claim that integrator windup is a problem in your simulations? If that is the case, can you propose a solution that would limit the problem? Show simulation results (course changing maneuvers) with the system from Problem 2e) with your suggested action to avoid integrator windup. Attach a figure of the course and input in the report if you choose to do so.

Part 2 (50 %)

The second part of the assignment starts here and should be handed in by 23:59 on Friday the 26th of October.

Problem 3 - State Estimation using a Kalman filter

In Problem 2e/2f), it was assumed that all states could be measured perfectly and were available for feedback. In practice, accurate measurements of all states are usually not available and state feedback cannot be used directly. Therefore, it may be necessary to design a state estimator that can be used for feedback in the control system. In this problem, you will implement a Kalman filter to estimate the states based on measurements affected by Gaussian white noise. The bias d affecting the coordinated-turn equation can be removed in this part of the assignment.

The goal is to estimate sideslip, roll, roll rate and yaw rate (β, ϕ, p and r). Measurements contaminated by Gaussian white noise are available for p and r (sideslip and roll cannot be measured). The set-point of the aileron input is obviously also available, but the aileron dynamics should be neglected and considered to be unknown in the Kalman filter. Therefore, the continuous-time Kalman filter matrices are (the subscript k indicates that it belongs to the Kalman filter)

$$\begin{aligned}
\mathbf{A}_k &= \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 \\ 0 & 0 & 1 & -0.001 \\ -10.6 & 0 & -2.87 & 0.46 \\ 6.87 & 0 & -0.04 & -0.32 \end{bmatrix}, \quad \mathbf{B}_k = \begin{bmatrix} 0.002 \\ 0 \\ -0.65 \\ -0.02 \end{bmatrix}, \\
\mathbf{C}_k &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{E}_k = \mathbf{I}_{4x4}
\end{aligned} \tag{6}$$

and corresponds to the system

$$\begin{aligned}
\dot{\hat{\mathbf{x}}} &= \mathbf{A}_k\hat{\mathbf{x}} + \mathbf{B}_k\delta_a^c + \mathbf{E}_k\mathbf{w} \\
\mathbf{y} &= \mathbf{C}_k\hat{\mathbf{x}} + \mathbf{v}
\end{aligned} \tag{7}$$

where $\hat{\mathbf{x}} = [\beta, \phi, p, r]^\top$ is the state vector. The input to the system is the desired aileron angle δ_a^c. $\mathbf{w}$ and $\mathbf{v}$ are Gaussian white noise processes in the state space model and measurement model, respectively. Note that the input to the coordinated-turn equation should be from the state-space model and not the Kalman filter since the true states are given by the output of the state-space model.

a) Describe the purpose of the matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{P}$ in the Kalman filter and state their dimension.

b) Measurements of roll rate and yaw rate are available. What type of sensors would you use to measure these states and how? Write down a typical measurement model for the sensor you have chosen. What kind of noise would your sensors be affected by in practice? Can you come up with an example of a situation where the white noise assumption in the Kalman filter may be problematic?

c) Under which conditions are the Kalman filter the optimal linear state estimator (unbiased and minimizes the variance)? Are all of these conditions met in this case?

d) Design a Matlab function that implements a Kalman filter. If you want to use Simulink, the Kalman filter should still be implemented as a Matlab function. You cannot use the Kalman filter block in Simulink when you do simulations later. Add the Kalman filter to your simulation setup from Problem 2e)/2f). The Kalman filter should estimate β, ϕ, p and r and be based on the matrices and measurements stated in Problem 3a).

   Hint: *Check the block diagram on Blackboard to see how the whole system can look like with the Kalman filter. This is the structure of the system that should be simulated in 3f).*

e) Simulate the course loop with the Kalman filter included. You need to add process noise to the state-space model (code for how this can be achieved is available on Blackboard). Use the desired course from Problem 2. Moreover, use the noise-contaminated measurement of roll rate for feedback to the control loop. The roll angle feedback should be from the Kalman filter since roll cannot be measured. In other words, you need to generate the noise-contaminated measurements at the output of the state-space model by adding Gaussian white noise to the true output for p and r. Measurements should be available at a rate of 100 Hz. The Gaussian white measurement noise should have the following variance (the measurements are uncorrelated):

Measurement	Variance
Roll rate (p)	$(0.2 \text{ deg/s})^2$
Yaw rate (r)	$(0.2 \text{ deg/s})^2$

The process noise in the state-space model should have covariance (discrete-time)

$$\mathbf{Q} = h * 10^-6 * \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where h is the step length of the discretization. More information about that can be seen in the code and figure on Blackboard. A step length of $0.01s$ is recommended and used in the example on Blackboard. You can choose the initial conditions in the Kalman filter yourself. Comment on the results and attach the following figures in the report:

- Course together with desired course.
- Aileron input.
- The estimated sideslip angle together with the true value (given as the output from the state-space).
- The estimated roll angle together with the true value.
- The noise-contaminated measurements of the roll rate together with the estimated state and the true value.
- The noise-contaminated measurements of the yaw rate together with the estimated state and the true value.

Hint: *The discrete-time matrices can be calculated as*

$$\begin{aligned} \mathbf{A}_d &= \mathbf{I} + h\mathbf{A}_k \\ \mathbf{B}_d &= h\mathbf{B}_k \\ \mathbf{E}_d &= \mathbf{E}_k \end{aligned}$$

where $\mathbf{E}_d = \mathbf{E}_k$ since the process noise $\mathbf{Q}$ above is given directly in discrete-time. You can check the variance of the noise by sending it to the workspace and verify it after the simulation to see if it is implemented correctly. The variance should comply with the given covariance $\mathbf{Q}$ and $\mathbf{R}$.

Hint 2: *As mentioned earlier, the input to the coordinated-turn equation must be from the state-space model and not the Kalman filter since the true states are given by the state-space model.*

f) In the previous problem, we used the noise-contaminated measurement of roll rate directly for feedback. In this problem, you will replace the feedback with the corresponding estimate from the Kalman filter. Simulate the system from Problem 3e) with the feedback from the Kalman filter for both roll and roll rate. Comment on the performance of the course control compared to the control system in Problem 3e)? Would you say that the course control is better than in the previous task? In theory, how should the bandwidth of the estimator be compared to the bandwidth of the control loop?

Attach the same set of figures as in Problem 3e).

g) The Kalman filter can also be used for prediction in situations where measurements are unavailable. Assume that a sensor failure is happening in the middle of the simulation so that no measurements are available in the last half. Simulate the system in this case and compare the results with what you achieved in Problem 3f).

Hint: *A sensor failure can be simulated by increasing* $\boldsymbol{R}$ *towards infinity at the time of the failure and throughout the rest of the simulation. In other words, choose a suitable time for sensor failure and set* $\boldsymbol{R}$ *to a very large value at that time. The Kalman filter will not use measurements to update the estimates in such a situation.*

References

[1] R. W. Beard and T. W. McLain, *Small unmanned aircraft: Theory and practice.* Princeton university press, 2012.

[2] T. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control.* John Wiley & Sons, 2011.