Assignment 2

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$\dot{x} = Ax + Bu$
$y = Cx$

$V_a = 580\ km/h$

Problem 1

a)

$v_a = v_g - v_w$
$v_w = 0 \Rightarrow v_g = v_a = 580\ km/h$

b)

Ok

c)

$$x = \begin{bmatrix} \beta \\ \phi \\ p \\ r \\ \delta_a \end{bmatrix}, \qquad u = [\delta_a^c], \qquad y = \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix}$$

$$A = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -7.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7.5 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$Y_v = -0.322$
$6.87 = N_v V_a^* \cos\beta$
$\Rightarrow N_v = \dfrac{6.87}{V_a^* \cos\beta} = \dfrac{6.87}{580/3.6} = 0.0426$

$N_r = -0.32$

$-1.12 = \dfrac{Y_r}{V_a^* \cos\beta^*}$
$\Rightarrow Y_r = -1.12 \cdot \dfrac{580}{3.6} = -180.44$

$s^2 + (-Y_v - N_r)s + (Y_v N_r - N_v Y_r) = 0$
$s^2 + 2\zeta\omega s + \omega^2 = 0$

$\Rightarrow Y_v N_r - N_v Y_r = \omega^2$
$\Rightarrow \omega = \sqrt{Y_v N_r - N_v Y_r} = 2.79$

$2\zeta\omega = -Y_v - N_r$

$$\Rightarrow \zeta = \frac{-Y_v - N_r}{2\omega} = 0.114679$$

$$\lambda_{dutch-roll} = \frac{Y_v + N_r}{2} \pm \sqrt{\left(\frac{Y_v + N_r}{2}\right)^2 - (Y_v N_r - N_v Y_r)}$$

$$\lambda_{dutch-roll} = -0.321 \pm 2.77i$$

d)

$$L_v = -\frac{10.6}{V_a^* \cos\beta^*} = -\frac{10.6}{\frac{580}{3.6}} = -0.06579$$

$$L_r = 0.46$$

$$\lambda_{spiral} = \frac{N_r L_v - N_v L_r}{L_v} = -0.02221$$

e)

$$L_p = -2.87$$

$$\lambda_{roll} = -2.87$$

Problem 2

a)

$$\delta_a \left(\frac{a_{\phi_2}}{s + a_{\phi_1}}\right) = p \Rightarrow \frac{p}{\delta_a} = \frac{a_{\phi_2}}{s + a_{\phi_1}}$$

$$\dot{p} = -10.6\beta - 2.87p + 0.46r - 0.65\delta_a$$

$$\dot{p} = -2.87p - 0.65\delta_a \Rightarrow p(s + 2.87) = -0.65\delta_a \Rightarrow \frac{p}{\delta_a} = \frac{-0.65}{s + 2.87}$$

$$a_{\phi 1} = 2.87$$

$$a_{\phi 2} = -0.65 \; feil? \; 7.5?$$

$$k_{p\phi} = \frac{\delta_a^{max}}{e_\phi^{max}} sgn(a_{\phi 2}) = \frac{30}{15} = -2$$

$$\omega_{n_\phi} = \sqrt{|a_{\phi 2}| \frac{\delta_a^{max}}{e_\phi^{max}}} = \sqrt{1.3}$$

$$k_{d\phi} = \frac{2\zeta_\phi \omega_{n_\phi} - a_{\phi 1}}{a_{\phi 2}} = \frac{2 \cdot 0.707 \cdot \sqrt{1.3} - 2.87}{-0.65} = 1.935$$

$$k_{d\phi} = -12.8406 \text{ før?}$$

$$k_{i\phi} = 2$$

$$k_{p\phi} + k_{i\phi}\frac{1}{s} + \frac{\frac{a_{\phi_2}}{s + a_{\phi_1}}}{1 + k_{d\phi}\frac{a_{\phi_2}}{s + a_{\phi_1}}} \cdot \frac{1}{s} = k_{p\phi} + k_{i\phi}\frac{1}{s} + \frac{a_{\phi_2}}{s + a_{\phi_1} + k_{d\phi}a_{\phi_2}} \cdot \frac{1}{s}$$

$$= \frac{k_{p\phi}\left(s + a_{\phi_1} + k_{d\phi}a_{\phi_2}\right)s + \frac{k_{i\phi}}{s}\left(s + a_{\phi_1} + k_{d\phi}a_{\phi_2}\right)s + a_{\phi_2}}{\left(s + a_{\phi_1} + k_{d\phi}a_{\phi_2}\right)s}$$

$$\omega_{n\chi} = \frac{1}{W_\chi}\omega_{n\phi}$$

$$k_{p\chi} = \frac{2\zeta_\chi \omega_{n\chi} V_g}{g}$$

$$k_{i\chi} = \frac{\omega_{n\chi}^2 V_g}{g}$$