5. Put $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto -\log x$. It is clear that $\mathbb{R}$ is convex.

Then, $f''(x) = \frac{1}{x^2} > 0$.

By Taylor, we get
$$\begin{cases} f(x_1) > f(\eta x_1 + (1-\eta)x_2) + f'(\eta x_1 + (1-\eta)x_2)(1-\eta)(x_1 - x_2) \\ f(x_2) > f(\eta x_1 + (1-\eta)x_2) + f'(\eta x_1 + (1-\eta)x_2)\,\eta\,(x_2 - x_1) \end{cases}, \quad x_1 \neq x_2$$

$\Rightarrow \eta f(x_1) + (1-\eta) f(x_2) > f(\eta x_1 + (1-\eta)x_2)$, $\eta \in (0,1)$, $\forall x_1, x_2 \in \mathbb{R}$, $x_1 \neq x_2$.

$\therefore f(x) = -\log x$ is strictly convex.

Now, if $p \neq q$, then r.v. $\frac{p_I}{q_I}$ is not constant when $p(I=i) = p_i$.

$$\begin{aligned} \therefore D_{KL}(p \| q) &= \mathbb{E}_I \left( \log \frac{p_I}{q_I} \right) \\ &= \mathbb{E}_I \left( -\log \frac{q_I}{p_I} \right) \qquad \text{By Strict Jensen's,} \\ &> -\log \mathbb{E}_I \left( \frac{q_I}{p_I} \right) \\ &= -\log \sum_i \cancel{p_i} \frac{q_i}{\cancel{p_i}} \\ &= -\log 1 \\ &= 0 \end{aligned}$$

$\square$