

Worksheet 2

Math/Stat 561, Algebraic and Geometric Methods in Statistics

23 January 2023

Group members: Write your names here.

1 Conditional independence ideals

Definition

Proposition (4.1.6.) & Definition (4.1.7.): If X is a discrete random vector $X = (X_1, \dots, X_m)$, then the CI statement $X_A \perp\!\!\!\perp X_B | X_C$ is equivalent to

$$p_{i_A, i_B, i_C, +} \cdot p_{j_A, j_B, i_C, +} - p_{i_A, j_B, i_C, +} \cdot p_{j_A, i_B, i_C, +} = 0$$

for all possible states of the variables i_A, j_A, i_B, j_B , and i_C .

The CI ideal $I_{A \perp\!\!\!\perp B | C}$ is the set of polynomials generated by all quadratic polynomials above.

Task

Verify that the following polynomials are the correct polynomials for the ideal of the statement $gender \perp\!\!\!\perp hair | soccer$ from lecture 4.

$$\begin{aligned} & -p_{1,2,1,1}p_{2,1,1,1} - p_{1,2,1,2}p_{2,1,1,1} - p_{1,2,1,1}p_{2,1,1,2} - p_{1,2,1,2}p_{2,1,1,2} + p_{1,1,1,1}p_{2,2,1,1} + \\ & p_{1,1,1,2}p_{2,2,1,1} + p_{1,1,1,1}p_{2,2,1,2} + p_{1,1,1,2}p_{2,2,1,2}, \\ & -p_{1,2,1,1}p_{3,1,1,1} - p_{1,2,1,2}p_{3,1,1,1} - p_{1,2,1,1}p_{3,1,1,2} - p_{1,2,1,2}p_{3,1,1,2} + p_{1,1,1,1}p_{3,2,1,1} + \\ & p_{1,1,1,2}p_{3,2,1,1} + p_{1,1,1,1}p_{3,2,1,2} + p_{1,1,1,2}p_{3,2,1,2}, \\ & -p_{2,2,1,1}p_{3,1,1,1} - p_{2,2,1,2}p_{3,1,1,1} - p_{2,2,1,1}p_{3,1,1,2} - p_{2,2,1,2}p_{3,1,1,2} + p_{2,1,1,1}p_{3,2,1,1} + \\ & p_{2,1,1,2}p_{3,2,1,1} + p_{2,1,1,1}p_{3,2,1,2} + p_{2,1,1,2}p_{3,2,1,2}, \\ & -p_{1,2,2,1}p_{2,1,2,1} - p_{1,2,2,2}p_{2,1,2,1} - p_{1,2,2,1}p_{2,1,2,2} - p_{1,2,2,2}p_{2,1,2,2} + p_{1,1,2,1}p_{2,2,2,1} + \\ & p_{1,1,2,2}p_{2,2,2,1} + p_{1,1,2,1}p_{2,2,2,2} + p_{1,1,2,2}p_{2,2,2,2}, \\ & -p_{1,2,2,1}p_{3,1,2,1} - p_{1,2,2,2}p_{3,1,2,1} - p_{1,2,2,1}p_{3,1,2,2} - p_{1,2,2,2}p_{3,1,2,2} + p_{1,1,2,1}p_{3,2,2,1} + \\ & p_{1,1,2,2}p_{3,2,2,1} + p_{1,1,2,1}p_{3,2,2,2} + p_{1,1,2,2}p_{3,2,2,2}, \\ & -p_{2,2,2,1}p_{3,1,2,1} - p_{2,2,2,2}p_{3,1,2,1} - p_{2,2,2,1}p_{3,1,2,2} - p_{2,2,2,2}p_{3,1,2,2} + p_{2,1,2,1}p_{3,2,2,1} + \\ & p_{2,1,2,2}p_{3,2,2,1} + p_{2,1,2,1}p_{3,2,2,2} + p_{2,1,2,2}p_{3,2,2,2}. \end{aligned}$$