Construction of High-Risk and Low-Risk Investment Portfolios Using Current Stock Data

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May 19, 2025

1 Objective

This project aims to construct two potentially profitable investment portfolios using current stock market data. One that is higher risk and one that is lower risk.

- A **high-risk portfolio** targeting higher potential returns but tolerating greater volatility.
- A **low-risk portfolio** focused on preserving capital while providing stable returns.

We use historical daily closing prices of stocks, compute essential risk metrics, and apply portfolio optimization techniques to assign weights under constraint-based definitions of **high** and **low** risk.

2 Data Collection

We sourced stock data of Apple (AAPL), Microsoft (MSFT), Google (GOOGL), Tesla (TSLA), Nvidea (NVDA), Coca-Cola (KO), Johnson and Johnson (JNG), P & G (PG), Pepsico (PEP), and Exxon Mobil (XOM) using the Python package yfinance. We used closing prices of the last 2 years to measure the risk metrics for the above-mentioned 10 stocks. In addition, we extracted Price-to-Earnings (P/E) ratios using Yahoo Finance fundamentals via `Ticker().info['trailingPE']`.

3 Methodology

This section outlines how key financial metrics were calculated and how optimization was performed to construct the high-risk and low-risk portfolios.

A. Daily Log Returns

For each stock, daily log returns were calculated using the formula:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where P_t is the adjusted closing price at time t.

B. Annualized Metrics

The following annualized statistics were computed from daily log returns:

- **Expected Return**:
$$\mu_{\text{annual}} = \bar{r}_{\text{daily}} \times 252$$
- **Volatility (Standard Deviation)**:
$$\sigma_{\text{annual}} = \text{std}(r_{\text{daily}}) \times \sqrt{252}$$
- **Covariance Matrix**:
$$\Sigma = \text{Cov}(r_i, r_j) \times 252$$

C. Beta

The beta coefficient measures the sensitivity of a stock to market movements. It was calculated using:

$$\beta_i = \frac{\text{Cov}(r_i, r_m)}{\text{Var}(r_m)}$$

where r_i is the log return of the stock and r_m is the log return of the market proxy (SPY ETF).

D. Price-to-Earnings (P/E) Ratio

The trailing Price-to-Earnings (P/E) ratio for each stock was collected using the Yahoo Finance API:

$$\text{P/E} = \frac{\text{Price per Share}}{\text{Earnings per Share (TTM)}}$$

The portfolio-level P/E ratio was computed as a weighted average:

$$\text{P/E}_{\text{portfolio}} = \sum_{i=1}^{n} w_i \cdot \text{P/E}_i$$

where w_i is the portfolio weight of stock i.

E. Sharpe Ratio

The optimization objective was to maximize the portfolio's Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where:

- R_p = Expected return of the portfolio
- R_f = Risk-free rate (assumed to be 2%)
- σ_p = Standard deviation of portfolio returns

F. Optimization Procedure

Portfolio weights were optimized using Sequential Least Squares Programming (SLSQP), as implemented in `scipy.optimize.minimize()`. The optimization sought to:

$$\min_{w} \left(-\frac{w^\top \mu - R_f}{\sqrt{w^\top \Sigma w}} \right)$$

subject to:

- Full investment constraint: $\sum_i w_i = 1$
- Bounds: $w_i \in [0.05, 1.0]$
- Risk-specific constraints for high-risk and low-risk portfolio definitions

4 Portfolio Constraints

The optimization was subject to distinct constraint sets that reflect different investment philosophies:

A. High-Risk Portfolio Constraints

The high-risk portfolio is designed to target aggressive return and market exposure. The constraints are:

- **Full Investment:** $\sum_{i=1}^{n} w_i = 1$
- **Minimum Volatility:** $\sqrt{w^\top \Sigma w} \geq 20\%$
- **Minimum Expected Return:** $w^\top \mu \geq 12\%$
- **High-Beta Allocation:** At least 30% of the total weight must be allocated to high-beta stocks: TSLA, NVDA, and GOOGL

- **Minimum Portfolio P/E:** Weighted portfolio P/E must be at least 25:

$$\sum_{i=1}^{n} w_i \cdot \text{P/E}_i \geq 25$$

- **Minimum Diversification:** Each stock must have at least 5% weight: $w_i \geq 0.05$

B. Low-Risk Portfolio Constraints

The low-risk portfolio aims for stability, income preservation, and defensive positioning:

- **Full Investment:** $\sum_{i=1}^{n} w_i = 1$
- **Maximum Volatility:** $\sqrt{w^\top \Sigma w} \leq 15\%$
- **Minimum Expected Return:** $w^\top \mu \geq 6\%$
- **High-Volatility Cap:** Combined allocation to TSLA and NVDA must not exceed 10%
- **Defensive Exposure:** At least 40% of the portfolio must be invested in KO, JNJ, PG, and PEP
- **Maximum Portfolio P/E:** Weighted portfolio P/E must not exceed 20:

$$\sum_{i=1}^{n} w_i \cdot \text{P/E}_i \leq 20$$

- **Minimum Diversification:** Each stock must have at least 5% weight: $w_i \geq 0.05$

5 Results

5.1 With Minimum Weight Allocation

The optimization process successfully produced two distinct portfolios that met their respective constraints consider minimum weight of 5% for each stock.

A. Portfolio Performance

- **High-Risk Portfolio**
  - Annualized Return: 29.54%
  - Annualized Volatility: 20.00%
- **Low-Risk Portfolio**
  - Annualized Return: 6.55%
  - Annualized Volatility: 13.28%

B. Optimized Portfolio Weights

Ticker	High-Risk (%)	Low-Risk (%)
AAPL	5.00	5.00
MSFT	5.00	5.00
GOOGL	5.00	5.00
TSLA	29.58	5.00
NVDA	5.00	5.00
KO	30.42	25.00
JNJ	5.00	5.00
PG	5.00	5.00
PEP	5.00	5.00
XOM	5.00	35.00

Table 1: Optimized portfolio weights for high-risk and low-risk portfolios. Values are rounded and reflecting minimum weight constraints.

5.2 Without Minimum Weight Allocation

The optimization was also executed without enforcing a minimum weight per stock, allowing to allocate capital freely under the defined high-risk and low-risk constraints. As a result, some stocks received zero weight in the final portfolios.

A. Portfolio Performance

- **High-Risk Portfolio**
  - Annualized Return: 33.77%
  - Annualized Volatility: 20.00%
- **Low-Risk Portfolio**
  - Annualized Return: 18.49%
  - Annualized Volatility: 15.00%

B. Optimized Portfolio Weights

Ticker	**High-Risk (%)**	**Low-Risk (%)**
AAPL	0.00	0.00
MSFT	0.00	0.00
GOOGL	0.00	28.14
TSLA	64.10	0.00
NVDA	0.00	0.00
KO	35.90	21.41
JNJ	0.00	6.48
PG	0.00	8.86
PEP	0.00	3.25
XOM	0.00	31.86

Table 2: Optimized portfolio weights without minimum allocation constraint.

6 Conclusion

When a minimum weight constraint of 5% per stock was enforced, the optimizer was required to include every asset in both the high-risk and low-risk portfolios. This led to more diversified portfolios that better reflect real-world investment practice. The high-risk portfolio still favored growth-oriented and high-beta stocks (like TSLA), while the low-risk portfolio emphasized defensive and value stocks (such as KO, and XOM).

When portfolio weights were optimized without requiring a minimum allocation to each stock, the solutions were highly concentrated. The high-risk portfolio allocated over 64% of capital to Tesla (TSLA) and the remaining to Coca-Cola (KO), entirely excluding the remaining 8 assets. The low-risk portfolio was more diversified but still excluded AAPL, MSFT, NVDA, and TSLA entirely, favoring defensive or value-oriented names like GOOGL, XOM, and KO.

This result illustrates a key trade-off in portfolio construction:

- Optimization algorithms naturally favor concentrated portfolios when minimizing risk or maximizing return.
- Without minimum diversification constraints, theoretically optimal portfolios may ignore many available assets.

While these solutions are valid under the optimization framework, they may not be desirable in practice due to concentration risk and lack of diversification. In real-world scenarios, portfolio managers typically impose diversification floors or maximum caps to ensure exposure across multiple sectors and avoid over-reliance on a small number of securities.