Hypothesis Testing of Standard Assumptions Theoretical Financial Mathematics

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Introduction

In the theory of mathematical finance and modeling, it's often assumed that log returns of stock/index are normally distributed. This mini project investigates if the log returns of stocks of our chosen are normally distributed.

We will consider 5 years' daily log returns of 10 real-world stocks and explore the followings:

1. Test if there are periods of time when the log-returns of a stock have evidence of normal distribution.
2. Test if removing extremal return data creates a distribution with evidence of being normal.
3. Create a personalized portfolio of stocks with historical log return data that is normally distributed.
4. Test if the portfolio created in Mini Project 1 has significant periods of time with normally distributed log returns.
5. Perform full-period normality tests on several individual stocks to identify any that exhibit normal log return behavior.

1 Preliminaries

Data Used: We downloaded adjusted closing prices for 10 large-cap stocks: AAPL, MSFT, GOOGL, TSLA, NVDA, KO, JNJ, PG, PEP, and XOM. Daily log returns were calculated as:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$

Normality Test: Throughout we test normality with

- Shapiro–Wilk (SW),
- Jarque–Bera (JB),
- Anderson–Darling (AD)[1].

[1] Accept normality at 5% significance if AD statistic $<$ critical value$_{5\%}$.

2 Rolling 6-Month Normality Test

For each stock we applied the AD test to a rolling 126-day window.

Ticker	% windows normal	Comment
XOM	80.3 %	Most Gaussian-like
AAPL, NVDA, MSFT	59–61 %	Frequent local normality
GOOGL	33.6 %	Rarely normal
Consumer-staple set	$\approx$ 42 %	Intermittent normality

Conclusion. Some stocks do exhibit local periods of normal behavior.

3 Normality After Removing Extremes

Define the α-trimmed return sample

$$r_t^{(\alpha)} = \begin{cases} r_t & \text{if } q_\alpha \le r_t \le q_{1-\alpha}, \\ \text{(discard)} & \text{otherwise,} \end{cases}$$

where q_α is the empirical α-quantile.

- With $\alpha = 1\%$, only **XOM** passed AD.
- With $\alpha = 3\%$, five stocks passed: **JNJ, KO, NVDA, TSLA, XOM**.

Conclusion. Fat tails drive most rejections of normality; removing 3% of extremes makes several equities Gaussian-like.

4 Constructing a Normal Portfolio

Using the trimmed data from Task 2, we selected the 5 stocks with best normality test results and created an equal-weight portfolio. Portfolio returns were computed as:

$$R_t = \sum_{i=1}^{5} w_i \, r_{i,t}^{(\text{trimmed})} \quad \text{with } w_i = 0.2$$

Test Results on Portfolio:

- Shapiro–Wilk $p = 0.3058$
- Jarque–Bera $p = 0.9508$
- Anderson–Darling: Passed at 5% significance

Conclusion: A well-chosen and trimmed stock portfolio can produce log returns that are approximately normally distributed.

5 Task 4: Testing Mini Project 1 Portfolio

We took the portfolio weights from Mini Project 1 for both high-risk and low-risk portfolios and tested their rolling normality using 126-day windows.

Portfolio Details:

- **High-Risk:** TSLA, KO, PG, PEP, XOM
- **Low-Risk:** GOOGL, KO, JNJ, PG, PEP, XOM

Results:

- High-Risk Portfolio passed in 80.85% of periods.
- Low-Risk Portfolio passed in 69.86% of periods.

Conclusion: Both portfolios from Mini Project 1 exhibit frequent periods of normal log return behavior.

6 Full-Period Normality for Individual Stocks

We tested the entire 5-year log return series for each of the 10 stocks using:

- Shapiro–Wilk Test
- Jarque–Bera Test
- Anderson–Darling Test

Summary:

- None of the 10 stocks passed all three tests.
- Most had low p-values, indicating deviation from normality.
- Visual inspection (histograms and Q–Q plots) supported heavy tails and skewness.

Conclusion: Over long time horizons, individual stock log returns do not follow a normal distribution.

7 Conclusion

- Log returns are rarely normal over long periods, but often locally normal.
- Removing outliers and constructing portfolios can improve normality.
- Statistical tests and visual tools are both essential for diagnosing normality.