Testing Normality in Log Returns of Financial Assets

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Overview

This project investigates the assumption of normality in the log returns of financial assets, a common premise in many mathematical finance models. A series of statistical tests and empirical experiments are conducted to examine the distributional properties of stock returns and portfolios, with particular attention to how trimming extreme values and combining assets affect normality.

Task 1: Testing for Normality of Log Returns in Individual Stocks

Methodology:

- Collected daily adjusted close prices for 10 stocks from 2018 to 2024 using `yfinance`.
- Computed log returns: $r_t = \log(P_t/P_{t-1})$.
- Applied Shapiro-Wilk, Jarque-Bera, and Anderson-Darling tests.
- Generated plots: histograms with KDE, Q-Q plots, and boxplots.

Results:

- All 10 stocks (AAPL, MSFT, GOOGL, TSLA, NVDA, KO, JNJ, PG, PEP, XOM) failed all three tests.
- Plots showed heavy tails, skewness, and significant outliers.

Conclusion: Full-period log returns are not normally distributed.

Task 2: Identifying Time Periods with Normally Distributed Log Returns

Methodology:

- Applied a 126-day rolling window to each stock's log return series.
- Performed the Anderson-Darling test in each window.
- Computed and visualized the percentage of windows passing the test.

Results:

- XOM passed in over 80% of windows.
- AAPL, MSFT, and NVDA passed in 55–61% of windows.
- GOOGL and consumer staples (KO, JNJ, PG, PEP) passed in fewer than 45%.

Conclusion: While full-period returns are non-normal, some stocks show frequent periods of local normality.

Task 3: Testing Normality After Trimming Extremes

Methodology:

- Removed top and bottom 1% and 3% of log returns.
- Re-ran the same three normality tests.
- Compared pre- and post-trimming histograms and Q-Q plots.

Results:

- With 1% trimming, only XOM passed the AD test.
- With 3% trimming, JNJ, KO, NVDA, TSLA, and XOM passed the AD test.
- Q-Q plots became more linear; histograms became bell-shaped.

Conclusion: Trimming reduces extreme values and improves normality, especially at the 3% level.

Task 4: Constructing a Portfolio with Normally Distributed Log Returns

Methodology:

- Selected the 5 stocks that passed the AD test after trimming.
- Trimmed 3% extremes from each stock.
- Constructed an equal-weighted portfolio.
- Applied all three normality tests to the portfolio's returns.

- Visualized the final portfolio histogram and Q-Q plot.

Results:

- Shapiro-Wilk p-value: 0.3058
- Jarque-Bera p-value: 0.9508
- Anderson-Darling test: Passed
- Visuals showed a clean bell curve and linear Q-Q plot.

Conclusion: A portfolio built from trimmed returns of select stocks yields a distribution that closely approximates normality.

Final Remarks

This project shows that financial return distributions often deviate from normality due to outliers and heavy tails. By trimming and analyzing rolling periods, we find that normality can be recovered under certain conditions. Carefully selected portfolios constructed from well-behaved assets can produce return distributions consistent with Gaussian assumptions.

Future Work

- Analyze return distributions during different market regimes.
- Extend the study to intraday or weekly return intervals.
- Explore t-distributions and extreme value theory as alternatives.

Tools Used: Python, Pandas, NumPy, Matplotlib, Seaborn, Scipy, Statsmodels, yfinance