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Campinghedgehog

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P 1 1

Define

$$f(x) = \begin{cases} e^{-1/x^2}, & (x \neq 0) \\ 0, & (x = 0) \end{cases} \tag{1}$$

Prove that f has derivaties of all orders at $x = 0$, and that $f^{(n)}(0) = 0$ for $n = 1, 2, 3, \ldots$.

(sol) 1.1

P 2 2

Let a_{ij} be the number in the ith row and jth column of the array

$$\begin{array}{ccccc} -1 & 0 & 0 & 0 & \ldots \\ \frac{1}{2} & -1 & 0 & 0 & \ldots \\ \frac{1}{4} & \frac{1}{2} & -1 & 0 & \ldots \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & -1 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{array}$$

so that

$$a_{ij} = \begin{cases} 0, & (i < j) \\ -1, & (i = j) \\ 2^{j-1}, & (i > j) \end{cases} \tag{2}$$

Prove that

$$\sum_i \sum_j a_{ij} = -2, \qquad \sum_j \sum_i a_{ij} = 0$$

(sol) **2.1**

P 3

(sol) **3.1**