Student ID: _____

AUT UNIVERSITY COMPUTER + MATHEMATICAL SCIENCES

716181 – Algorithm Design and Analysis
Semester 2, 2015

Final Examination (Mock)

Time Allowed Two hours (plus reading time).

Instructions

- This is a closed-book exam.
- Please answer all questions in the space provided.
- Calculator is allowed for this exam.

1. Determine whether each statement is TRUE or FALSE. [10 marks]

   (i) Statement: If $f(n)$ is $O(h(n))$ and $g(n)$ is $O(h(n))$, then $f(n)+g(n)$ is $O(h(n))$.

   TRUE or FALSE?

   (ii) Statement: Strassen's algorithm uses the divide-and-conquer strategy to multiply two n-digit integers in time $O(n^{\log_2(3)})$

   TRUE or FALSE?

   (iii) Statement: If $f(n)=n^2$ and $g(n)=(n+1)\log_2 n$, then $f(n)$ is $\Omega(g(n))$.

   TRUE or FALSE?

   (iv) Statement: An undirected graph has at most $n(n+1)$ edges where n is the order of the graph.

   TRUE or FALSE?

   (v) Statement: Kuskal's algorithm uses a greedy strategy to solve the minimal spanning tree problem.

   TRUE or FALSE?

   (vi) Statement: If $T(n)=3T(n/3)+n\log n$, then master theorem implies that $T(n)$ is $\Theta(n\log n)$

   TRUE or FALSE?

   (vii) Statement: A directed graph is linearisable if and only if it is a tree.

   TRUE or FALSE?

   (viii) Statement: The complexity of Dijkstra's algorithm depends on specific implementation of priority queues.

   TRUE or FALSE?

2. Answer the following questions:

   (a) Describe an algorithm for computing the nth Fibonacci number F_n. Your algorithm should take time $O(\log n)$. (5 *marks*)

(b) Consider the following instance of the fractional knapsack problem: There are n items $x_1, x_2, \ldots, x_6$, whose weights and values are specified as follows: (5 *marks*)

Item	x_1	x_2	x_3	x_4	x_5	x_6
Weight	10	20	25	15	10	20
Value	65	40	55	90	45	140

The total weight capacity is 320. We would like to select (possible fractions of) items and maximise the total value, while not exceeding the weight capacity. Apply a greedy algorithm to solve this problem. Describe the algorithm and show all your steps.

3. Answer the following questions:

   (a) State the edit distance between the following pairs of words and give optimal alignment of these words. (2 *marks*)

   - `MICHAEL` and `MICHELL`

   - `UNITED` and `STATES`

   (b) Complete the following table for computing the longest increasing subsequence of the following array of integers: (3 *marks*)

23	12	15	7	26	19	8	21

   (c) Describe a dynamic programming algorithm that computes the edit distance between two words. (5 *marks*)

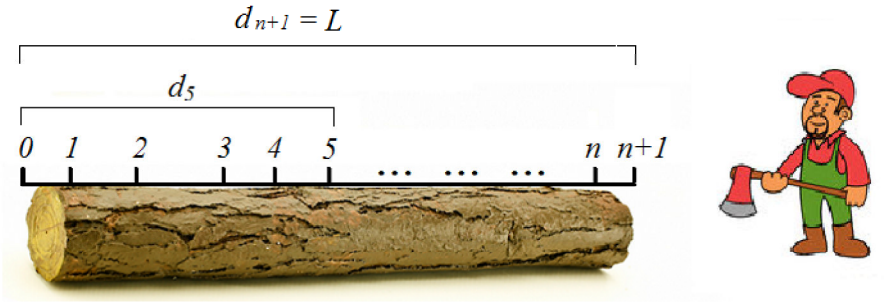
4. [**The Coin Counterfeiter**] You are given $n > 1$ coins, among which one (and only one) is a counterfeit. We know that the counterfeit coin may be either heavier or lighter than the rest, while all the other coins weigh the same. You have a scale and can put an arbitrary number of coins on either side of the scale for each weighing. Give a divide-and-conquer algorithm to find the counterfeit coin in $O(\log n)$ weighings. You may assume that n is a power of 2.

Answer the following questions below:

(a) Explain your algorithm by clearly describing the subproblems and how you would combine the solutions of the subproblems into a solution of the original problem (5 *marks*)

(b) Give a recurrence of the running time of the algorithm and use the Master theorem to prove that your algorithm runs in time $O(\log n)$. (5 *marks*)

5. **[Cutting Costs]** Woody the woodcutter will cut a given log of wood, at any place you choose, for a price equal to the length of the given log. Suppose you have a log of length L, marked to be cut in n different locations labeled $1, 2, 3, \ldots, n$. For simplicity, let indices 0 and $n+1$ denote the left and right endpoints of the original log of length L. Let d_i denote the distance of mark i from the left end of the log, and assume that $0 = d_0 < d_1 < d_2 < \ldots < d_n < d_{n+1} = L$.



The **wood-cutting problem** is the problem of determining the sequence of cuts to the log that will cut the log at all the marked places and minimize your total payment. Design an algorithm to solve the wood-cutting problem using dynamic programming.

(a) For $0 \leq i < j \leq n+1$, let $c(i, j)$ denote the minimal cost of cutting a log with left endpoint at location i and right endpoint at location j, at all its marked locations. Write a recurrence expressing the value of $c(i, j)$. (4 *marks*)

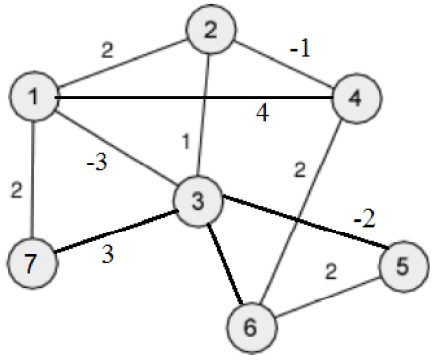
(b) Based on the recurrence you wrote in (a), design an algorithm for solving the wood-cutting problem using dynamic programming. (4 *marks*)

(c) What is the time complexity of your algorithm in (b)? (2 *marks*)

6. Answer the following questions:

(a) Describe an algorithm that solves the single-sourced shortest path problem on graphs that may contain negatively weighted edges. (2 *marks*)

(b) Run Floyd-Warshall algorithm on the directed graph below. Fill out the following tables (for f_0 and f_1) (4 *marks*)



$f_0(u,v)$	$v=1$	$v=2$	$v=3$	$v=4$	$v=5$	$v=6$	$v=7$
$u=1$	0	2	-3	4	∞	∞	2
$u=2$	2	0	1	-1	∞	∞	∞
$u=3$							
$u=4$							
$u=5$							
$u=6$							
$u=7$							

$f_1(u,v)$	$v=1$	$v=2$	$v=3$	$v=4$	$v=5$	$v=6$	$v=7$
$u=1$							
$u=2$							
$u=3$							
$u=4$							
$u=5$							
$u=6$							
$u=7$							

7. Answer the following questions:

   (a) Why doesn't the degree of nodes always give a faithful indication of the centrality of nodes in a network? Given your answer using an example. (3 *marks*)

   (b) What is the eigenvector centrality of a complete graph? (4 *marks*)

   (c) What is the "Web graph"? (2 *marks*)

   (d) Describe the algorithm for computing the Page rank of nodes in a directed graph. (4 *marks*)

11. [**Indiana Jones and the Tunnel of Doom**] Indiana Jones and three of his friends want to cross a tunnel in an ancient tomb; they all begin on the same side of the tunnel. They have 17 minutes to get them all across to the other side. It is very dark, and they have only one torch. A maximum of two people can cross the tunnel at one time. Any party that crosses, either one or two people, must have the torch with them. The torch must be walked back and forth.

    Due to different physical abilities, the four people would take different times to cross the tunnel:

People	Times to Cross
Indiana Jones	1 minute
Mutt	2 minutes
Willie	5 minutes
Short Round	10 minutes

    A pair must walk together at the rate of the slower person's pace. For example, if Indiana Jones and Short Round walk across the tunnel, 10 minutes would elapse when they get to the other side. If Short Round returns the flashlight, a total of 20 minutes have passed and they have failed the mission.