Algorithms Design and Analysis

Exercises
Graphs Basics and DFS

2015, AUT - CJLU

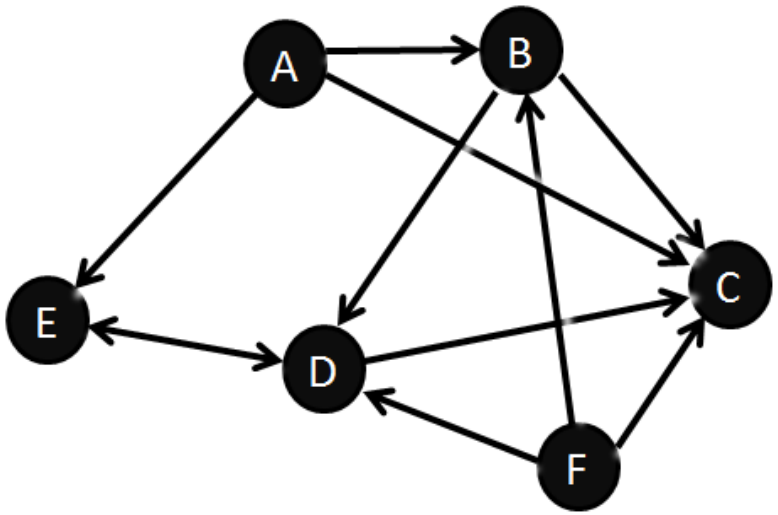
Question 1.

The following is the adjacency list of a digraph with 6 nodes $\{0, 1, 2, 3, 4, 5\}$. Draw the digraph out and then write down its adjacency matrix.
Draw the digraph beside the adjacency list:

1 2 5
2
0 3
2 4
5
0

Question 2.

Perform DFS on the digraph below starting from the node A. Do the following:



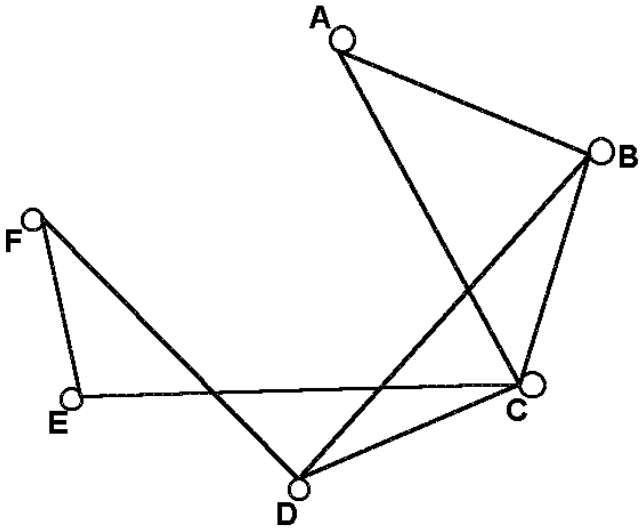
- Beside each node, correctly label the pre/post times.
- Draw the DFS forest beside the digraph
- Identify the forward edges, back edges and cross edges.

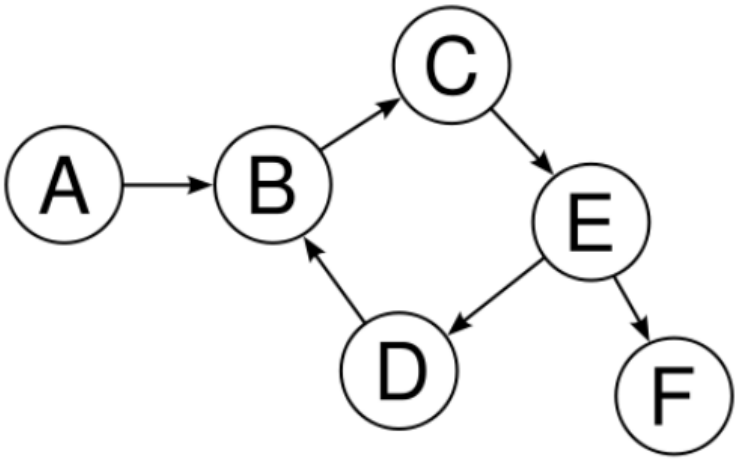
Question 3.

An undirected graph with 11 nodes $v_2, v_3, \ldots, v_{12}$ has edges (v_i, v_j) iff the greatest common divisor of i and j exceeds one. Draw this graph. How many connected components does this graph have?

Question 4.

Write down the formal definition of the following (di)graphs:

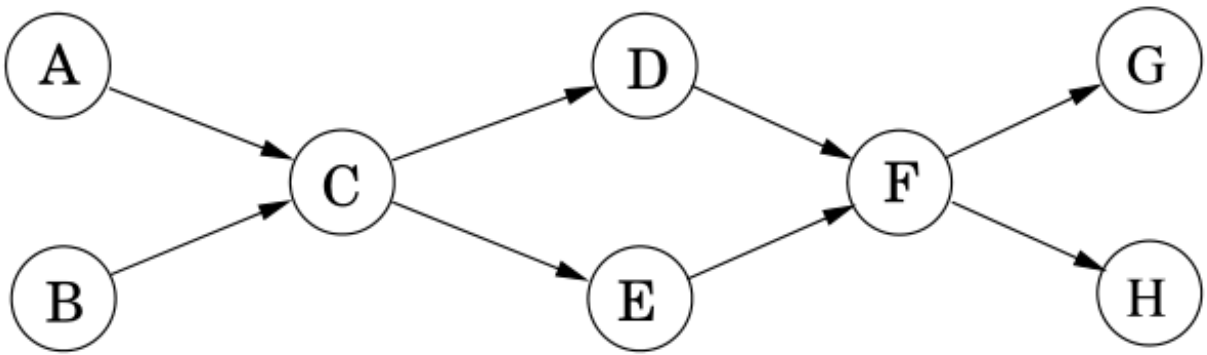




- In the first graph, write down a walk from A to F that is not a path. Then write down the degree of all vertices.
- In the second graph, draw the underlying graph. Then write down the out-degree and in-degree of all vertices.

Question 5.

Run the DFS-based linearization algorithm on the following graph. Whenever you have a choice of nodes to explore, always pick the one that is alphabetically first.



(a) Indicate the pre and post numbers of the nodes.

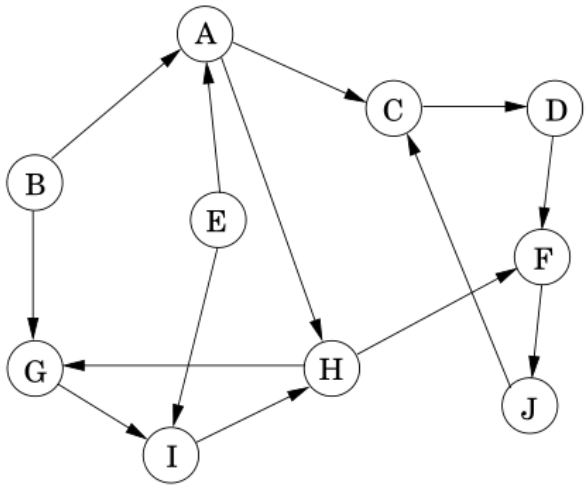
(b) What linearization is found by the algorithm?

(c) How many linearizations does this graph have?

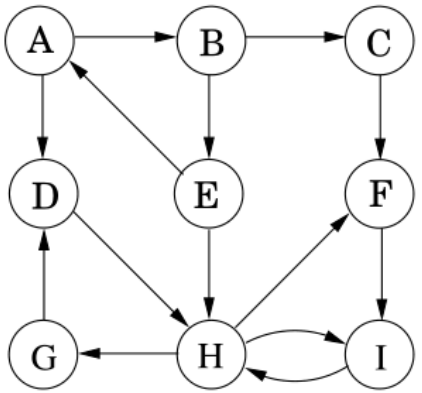
Question 6.

Run the strongly connected components algorithm on the following directed graph G.

(i)



(ii)



In each case answer the following questions:

(a) In what order are the strongly connected components (SCCs) found?

(b) Which are the source SCCs and which are the sink SCCs?

(c) Draw the "metagraph" (each meta-node is an SCC of G)