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MCM/ICM
Summary Sheet

In this paper, we focus on evaluating the self-driving, cooperating cars' influence to the existing highway traffic flow. We build a uniform speed traffic flow model while carefully discussing the interacting mode of the cars, aiming to analyze the change of highway capacity caused by those autonomous cars. Then, we adjust the measure, Level of Service (LOS), into Ratio-based Level of Service (RLOS) to evaluate the effects on traffic flow. At last, we use greedy strategy to produce a plan of dedicated-lanes.

First, according to the different relative position of autonomous cars and the others in a lane, we design different strategies for the cars to keep safe distance from the preceding car. Then, we derive highway capacity from the calculated average safe distance. We find out that at 60 miles per hour limited speed, the capacity first grows slowly (quasi-equilibria) but then begin to grow exponentially after the percentage of autonomous cars reaches nearly 55%.

Second, to better evaluate the performance of the whole traffic network, we use the ratio of the demanded volume of cars to the highway capacity instead of the density of cars to decide the level of service (RLOS-Measure). What's more, a synthetic index is proposed to quantify the performance. The result verifies a bad performance of the present traffic network and forecast considerable amelioration when the percentage of self-driving cars grows.

Third, we adopt a greedy strategy (changing the number of dedicated lanes) to design a plan which maximizes the capacity of each road especially during peak travel hours. We then use the RLOS and the synthetic index to evaluate the plan. We discover a stable improvement of the traffic condition when the percentage of self-driving cars is between 25% and 90%.

At last, we analyze the sensitivity of our model to confirm its robustness as well as discuss the advantages and disadvantages of our model.

Contents

1 A Letter to the Governor's Office

Dear Sirs,

In response to your questions regarding the effect-analysis of autonomous cars on current traffic flow, we are writing to inform you of our work.

Since self-driving cars are still rare now and no cooperating system has been available commercially, we build a Uniform Speed Model (USM) to evaluate the self-driving, cooperating cars influence to the existing highway traffic flow. We have calculated theoretical capacity under these different condition, which is reliable compared with data in Highway Capacity Manual 2010. We find out that at normal 60 miles per hour limited speed, the capacity first grows slowly but then begin to grow exponentially after the percentage of autonomous cars reaches nearly 55%. This indicates that only when the percentage of self-driving car reach a relatively high level will the traffic condition gain obvious amelioration. But regarding to the benefits that these improvement does not company the increasing of lanes , the solution is still the most effective. So we suggest a long-term aid plan to assist those enterprise relating to the autonomous car.

More creatively, we develop an advanced evaluation system to evaluate the traffic flow effects. We call this system Ratio-based Level of Service (RLOS), which is based on Level of Service (LOS) and is extended to situations with self-driving cars. In this evaluation system we use the capacity of highways (calculated using the USM) and the actual traffic volume to calculate the service level. And the result verifies a bad performance of the present traffic network and forecast considerable amelioration when the percentage of self-driving cars grows to nearly 60%

Whats more, we find out all the highways that need to set up dedicated-lanes for autonomous cars under different proportion by a greedy strategy. For example, under the proportion of 40%, 192 out of 224 highways in the sample data should set up dedicated-lanes during peak hours. And we have attached the more detailed plan to the appendix. By the plan of dedicated-lanes, to attain the same performance of the traffic network, we will only need less proportion of the autonomous cars, like from 60% down to 45%. And it is essential to point out that when there is no more than 25% of autonomous cars, this dedicated-lanes plan will be useless because of the low utilization of the dedicated-lanes.

After the sensitivity test, we are convinced that our models are reasonable and reliable. To summarize, we hope that our work can be helpful when the Governors office propose new policies.

Yours, sincerely
MCM team members

2 Introduction

2.1 Background

The low speed of traffic flow in many regions during peak traffic hours are caused by the volume of traffic exceed the designed capacity of the road networks. This problem is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520. We are asked by the Governor of the state of Washington to analyze the effects of allowing self-driving, cooperating cars on the roads listed above in Thurston, Pierce, King, and Snohomish counties. The behavior of self-driving, cooperating cars are not well understood.

2.2 Restatement of Problem

In this article, we will specifically anlyze these problems:

- In particular, how do the effects change as the percentage of self-driving cars increases from 10% to 50% to 90%?
- Do equilibria exist?
- Is there a tipping point where performance changes markedly? Under what conditions, if any, should lanes be dedicated to these cars?
- What policy changes our model suggests?

3 Uniform Speed Model

3.1 Model Introduction

To solve the problems above, we decide to first model for a single lane, than use the model to analyze the whole road networks to see what strategies can be taken to improve the traffic capacity. The model focuses on the speed, density, volume and the percentage of self-driving cars of a lane, calculates how these parameters influence each other.

3.2 Simplifying Assumptions

We start to build the model by first making several assumptions. These assumptions are aimed to simplify the model as long as they don't affect the accuracy seriously.

1. Considering the data in the list is an average number during a year, we assume each part the roads are in a stable state. All cars on one part of the roads have the same, constant speed and run without changing lanes.
2. To simulate the distribution of the cars on each parts of the road, we assume that all cars remain safe distances with their preceding cars.
3. All self-driving, cooperating cars can cooperate, so we use 'self-driving' represent 'self-driving,cooperating' for simplicity.

4. The breaking abilities of cars are varied due to the different shapes, weights, qualities of cars and road conditions, which means cars have different absolute values of acceleration when applying emergency brakes. To simplify the calculation, we assume that the absolute values of acceleration has an uniform distribution between $5m/s^2 - 8.5m/s^2$.[1]

5. Drivers can reasonably estimate their safe distances to preceding cars depending on their experience. So the safe distances they remain have a relation with the a of their cars. For example, a truck driver knows that his car can not brake as hard as a sedan, so he will keep a longer safe distance than a sedan driver under the same speed.

6. Self-driving cars know their absolute values of acceleration when applying emergency brakes. It may be reliable constants or functions (with respect to the weight of the cars or external environment) which had been inputted into their memories.

7. The reaction times of drivers have a normal distribution, but the variance is small, so we use the mean to calculate in our model. The mean is $1.1s$.[1]

8. The mean of the time a self-driving car needs to sense the deceleration of preceding car is $0.245s$, to communicate is $0.081s$, to begin decelerating is $0.1s$. [1]

9. To avoid accident, a driver always considers the preceding car has the best braking ability when keeping the safe distance.

10. Average car length is $4.3m$. [1]

3.3 Nomenclature

Abbreviation	Description
$\bar{D}$	Expectation of the distance a car remains to preceding car
P_H	Percentage of human-driving cars on lane
$\bar{D}_H$	Expectation of the distance a driver remains to preceding car
P	Percentage of self-driving, cooperating cars on lane
$\bar{D}_S$	Expectation of the distance a self-driving car remains to preceding car
$\bar{D}_{S1}$	Expectation of the distance a self-driving car remains to a human-driving preceding car
$\bar{D}_{S2}$	Expectation of the distance a self-driving car remains to a self-driving preceding car
T_0	Human reaction time
V	Speed of the cars on lane
A	A random variable represent the absolute value of the acceleration of a car applying emergency brake
a_{max}	Maximum absolute value of the acceleration of a car applying emergency brake
a_{min}	Minimum absolute value of the acceleration of a car applying emergency brake
$f(a)$	Probability density function of the absolute value of the acceleration of a car applying emergency brake
T_S	Time a self-driving car needs to sense the deceleration of preceding car.
T_C	Time a self-driving car needs to communicate when deceleration happens.
T_B	Time a self-driving car needs to begin decelerating.
l	Average car length
Q	Highway capacity. Usually we use C to represent highway capacity, but in uniform speed model, the traffic volume is the same number as highway capacity, so we use Q
K	Density of traffic flow

4 Self-Driving Cars

All self-driving cars have sensors to perceive the speed of preceding cars. Self-driving cars communicate with their adjacent self-driving cars as what is shown in Figure 1. It should be declared that the communication between two nonadjacent self-driving cars is possible, but the human-driving car between them has too much uncertainty, so the information from the front self-driving car can hardly be used by the last self-driving car. Therefore, we assume that two nonadjacent self-driving car can not communicate.

Self-driving cars can share the information of their braking ability, the acceleration when applying emergency brake. If a preceding car decides to decelerate, it will send a message to the following car at the same time it makes the brake. The relative time gaps are shown in Figure 2.

5 Safe Distances

In order to calculate the traffic capacity on a lane, we must use the density of the traffic flow. The density is a function with respect to the average length of cars and the average safe distance

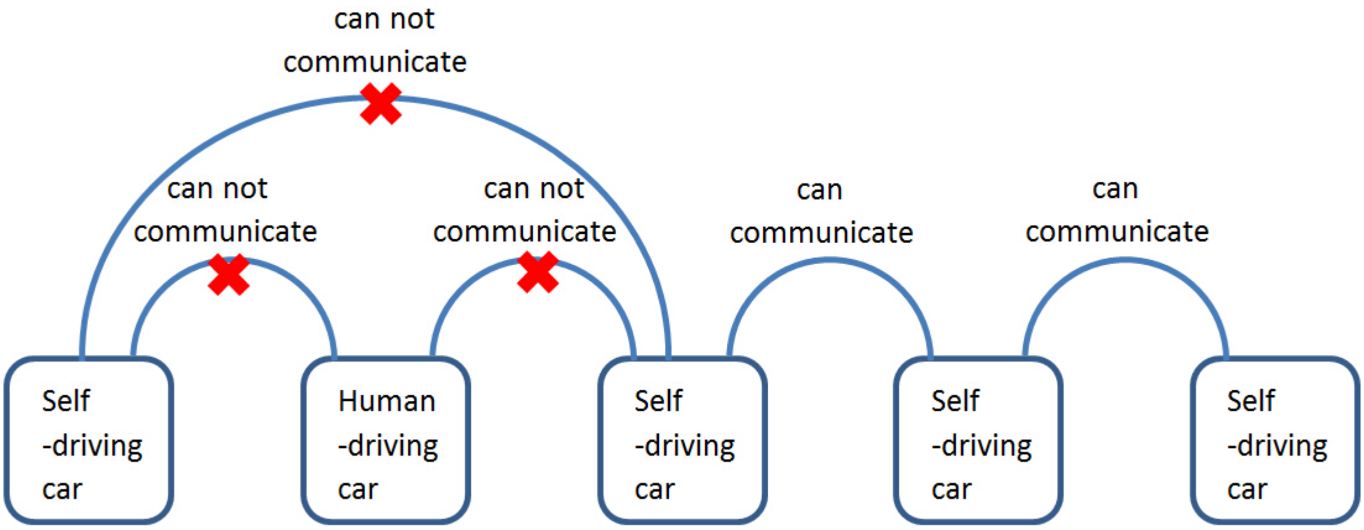

Figure 1: How Self-Driving Cars Communicate with Each Other

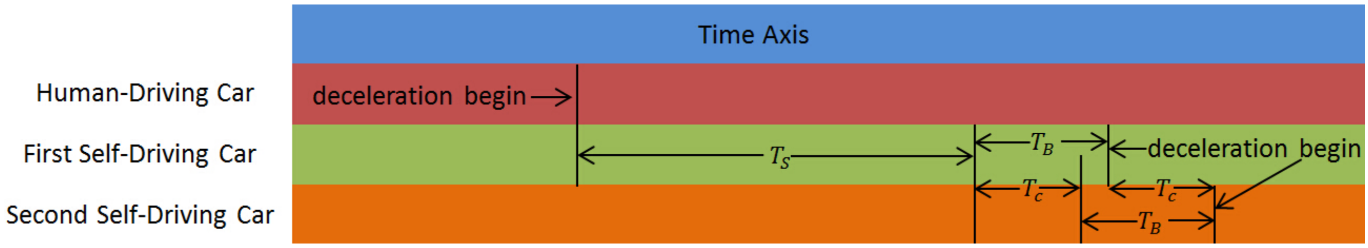

Figure 2: Description of Relative Time Gaps

between cars. So now we must calculate the expectation of safe distances.

Now analyzing a car on a lane, it has a probability P_H to be a human-driving car and a probability P to be a self-driving car. $\bar{D}$ is the average safe distance on lane, $\bar{D}_H$ is the average safe distance for a human-driving car, $\bar{D}_S$ is the average safe distance for a self-driving car. So we have the equation below.

$$\bar{D} = P_H \bar{D}_H + P \bar{D}_S \tag{1}$$

We have already assumed that the acceleration of cars when applying emergency brakes have a uniform distribution. a_{max} is the maximum of the distribution, a_{min} is the minimum of the distribution. $f(a)$ is the probability density function of A. So we have:

$$f(a) = \frac{a - a_{min}}{a_{max} - a_{min}} \tag{2}$$

According to the assumptions, human drivers decides their safe distances according to their experience and knowledge of their cars. So the expectation of safe distance of human-driving cars can be calculated in a reasonable way. It consists of three parts. The first one is the distance that the car runs before the brake is applied while the driver is reacting. The second one is the distance the car runs after applying the brake. This part should be calculated as a expectation using integral considering the absolute value of the acceleration of a car applying emergency brake have a uniform distribution. The third one is the distance the preceding car runs in the process. We use a_{max} to calculate the distance, because we assume a driver would always consider the preceding car has the best braking ability when keeping the safe distance. The sum of the first and second parts minus the third parts is the distance the car should keeps to preceding car. Because a driver do not know the braking ability of preceding car, we use a_{max} as the absolute value of the acceleration of preceding car when applying emergency brake in calculation. $\bar{D}_H$ is the average safe distance of human-driving cars, V is the speed of the cars on lane, T_0 is the mean of human reaction time. So we have:

$$\bar{D}_H = T_0 V + \int_{a_{min}}^{a_{max}} \frac{V^2}{2a} f(a)\,\mathrm{d}x - \frac{V^2}{2a_{max}} \tag{3}$$

The next thing is to calculate $\bar{D}_S$.

$$\bar{D}_S = P_H \bar{D}_{S1} + P\bar{D}_{S2} \tag{4}$$

There are two situations.

The first one is that the preceding car is a human-driving car. The conditional probability is P_H. The self-driving car can only use its sensor to perceive the preceding cars deceleration under this circumstance, because the human-driving car can not send massage to it. The safe distance consists of three parts. The first part is the distance that the car runs before deceleration during the process of sensing and beginning to brake. The second and the third parts are the same as what have been discussed in the calculation of $\bar{D}_H$. T_S is the time a self-driving car needs to sense the deceleration of preceding car. T_B is the time a self-driving car needs to begin decelerating. We obtain the expectation of the distance which a self-driving car keeps to a human-driving preceding car in the equation below.

$$\bar{D}_{S1} = (T_S + T_B)V + \int_{a_{min}}^{a_{max}} \frac{V^2}{2a} f(a)\,\mathrm{d}x - \frac{V^2}{2a_{max}} \tag{5}$$

The second situation is that the preceding car is a self-driving car. The conditional probability is P. Before we calculate the distance, we should first make sure how self-driving cars decide their safe distances. Since self-driving car know the braking ability of preceding car, it does not need to assume that the preceding car has the best braking ability. A self-driving car can adjust the safe distance depends on the braking ability of itself and the preceding car. There are two different cases in calculation. They are shown in Figure 3, in which the length of l_{AB} represents T_C, T_C is the time a self-driving car needs to communicate when deceleration happens, l_{ACE} represents the preceding car and l_{ABCE} or l_{ABD} represents the following car. l_{BE} is the boundary condition between two cases. When

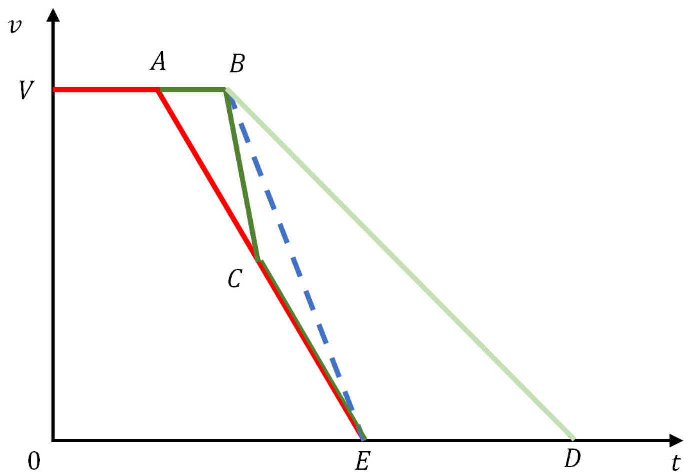

Figure 3: How Self-Driving Cars Decide Safe Distance

$$a_2 < \frac{V}{\frac{V}{a_1} - T_C} \tag{6}$$

the area of trapezoid $ABDE$ is the safe distance a self-driving car should keep to preceding car. When

$$a_2 \geq \frac{V}{\frac{V}{a_1} - T_C} \tag{7}$$

$S_{\Delta ABC}$ is the safe distance a self-driving car should keep to preceding car.

To calculate the area of trapezoid $ABDE$, considering the relation between speed, acceleration and time, we can easily obtain

$$t_{DE} = T_C + \frac{V}{a_2} - \frac{V}{a_1} \tag{8}$$

then we have the area of trapezoid $ABDE$:

$$S_t = \frac{V}{2}(T_C + T_C + \frac{V}{a_2} - \frac{V}{a_1}) \tag{9}$$

Use $t_{AB} = T_C$, $k_{AC} = a_1$, $k_{BC} = a_2$, we can obtain the y-coordinate of C.

$$y_C = V + \frac{a_1 a_2 T_C}{a_1 - a_2} \tag{10}$$

Then we have $S_{\Delta ABC}$.

$$S_{\Delta ABC} = \frac{1}{2} T_C (V - y_C) \tag{11}$$

Considering the uniform distribution of A, we have the equation of expectation of the distance a self-driving car remains to a self-driving preceding car.

$$\begin{aligned}\bar{D}_{S2} = \int_{a_{min}}^{a_{max}} \frac{1}{a_{max} - a_{min}} \Big[\int_{a_{min}}^{\frac{V}{\frac{V}{a_1} - T_C}} \frac{V}{2}(2T_C + \frac{V}{a_2} - \frac{V}{a_1}) \frac{1}{a_{max} - a_{min}} \mathrm{d}a_2 \\ + \int_{\frac{V}{\frac{V}{a_1} - T_C}}^{a_{max}} \frac{a_1 a_2 T_C^2}{2(a_2 - a_1)} \frac{1}{a_{max} - amin} \mathrm{d}a_2 \Big] \mathrm{d}a_1\end{aligned} \tag{12}$$

In order to calculate the equation in computer, we simplify it to the form below.

$$\begin{aligned}\bar{D}_{S2} = \frac{1}{(a_{max} - a_{min})^2} \int_{a_{min}}^{a_{max}} \frac{V}{2}(2T_C - \frac{V}{a_1})(\frac{V}{\frac{V}{a_1} - T_C} - a_{min}) + \frac{V^2}{2} \ln \frac{\frac{V}{\frac{V}{a_1} - T_C}}{a_{min}} \\ + \frac{a_1 T_C^2}{2} \Big[(a_{max} - \frac{V}{\frac{V}{a_1} - T_C}) + a_1 \ln \frac{a_{max} - a_1}{\frac{V}{\frac{V}{a_1} - T_C} - a_1} \Big] \mathrm{d}a_1\end{aligned} \tag{13}$$

Notice,

$$P_H = 1 - P \tag{14}$$

combine equation 1, 3, 4, 5, 13, 14, we can obtain a equation about $\bar{D}$ with respect to V and P.

6 Highway Capacity

In this section, we will calculate the highway capacity using the parameters we have gained before. l is the average length of a car. K is the density of a traffic flow, which is equal to the number of cars per kilometer per lane. First, we calculate K.

$$K = \frac{1000}{\bar{D} + l} \tag{15}$$

Then, we can obtain the highway capacity Q, which is the number of cars pass a cross-section per hour per lane.

$$Q = 3.6VK \tag{16}$$

$3.6V$ is converting V from m/s^2 to km/h.

7 Analysis using uniform speed model(USM)

We have already have a equation about Q only with respect to two parameters, V and P. So we can draw a three-dimentional surface graph Figure 4. From Figure 4, we can see the highway

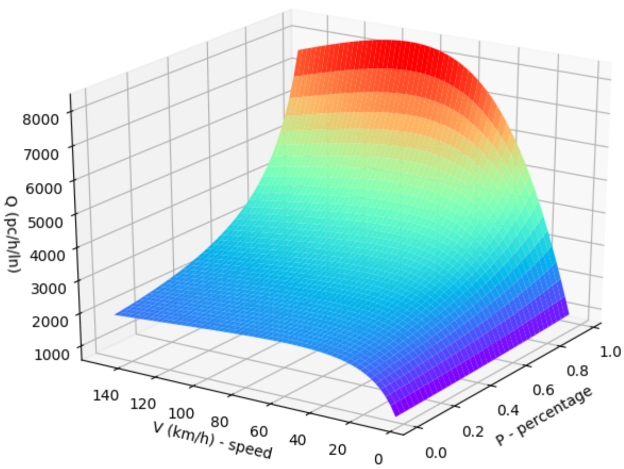

Figure 4: Relation between V, Q, P

capacity increases, under each fixed V, as P enlarges. Then, to simulate the real speed limit, we fix V at $60 miles/h$($96.56 km/h$) and draw the plot of Q and P in Figure 4. From Figure **??**, we can see there is a tipping point around $P = 0.5 \sim 0.6$. When $P < 0.5$, the curve changes gently. When $P > 0.6$, the curve changes markedly. There is a quasi-equilibria when $P < 0.5$. The traffic condition will become better and better as the percentage of self-driving cars increasing.

8 Level-of-Service in North America and Plan for Dedicated Lanes

8.1 Level-of-Service in North America

Level of service (LOS) is a qualitative measure used to relate the quality of traffic service [2]. LOS on a basic freeway segment is defined by density. And LOS are defined to represent reasonable

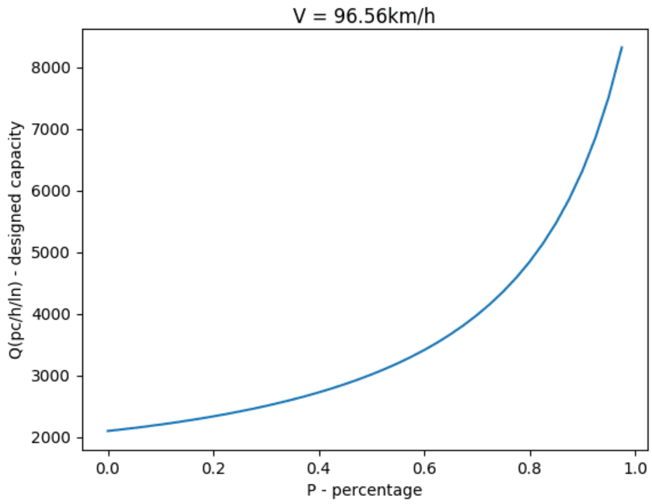

Figure 5: Relation between Q and P when V is $60 miles/h$

ranges in the three critical flow variables: speed, density, and flow rate. [3] The following Figure **??** shows the different LOS Examples. [3]

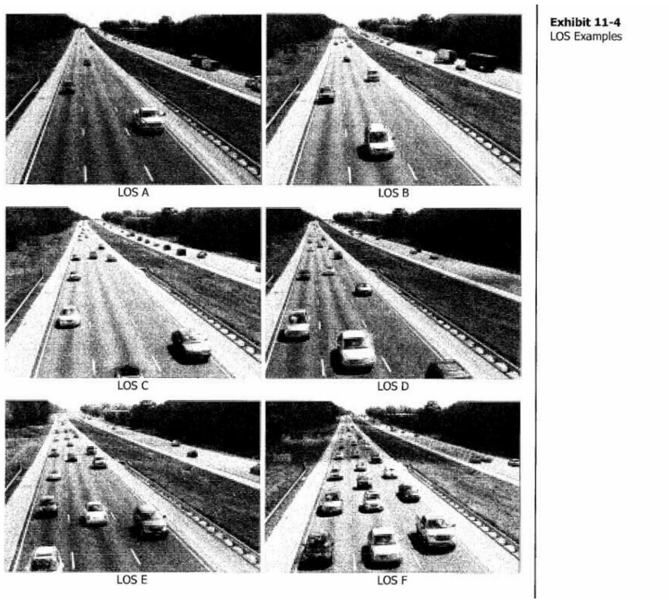

Figure 6: Different LOS Examples

Brief description of LOS:

- A: free flow. Traffic flows at or above the posted speed limit and motorists have complete mobility between lanes. LOS A generally occurs late at night in urban areas and frequently in rural areas.
- B: reasonably free flow. LOS A speeds are maintained, maneuverability within the traffic stream is slightly restricted. Motorists still have a high level of physical and psychological comfort.

- C: stable flow, at or near free flow. Ability to maneuver through lanes is noticeably restricted and lane changes require more driver awareness. This is the target LOS for some urban and most rural highways.
- D: approaching unstable flow. Speeds slightly decrease as traffic volume slightly increase. Freedom to maneuver within the traffic stream is much more limited. It is a common goal for urban streets during peak hours, as attaining LOS C would require prohibitive cost and societal impact in bypass roads and lane additions.
- E: unstable flow, operating at capacity. Flow becomes irregular and speed varies rapidly Any incident will create serious delays. This is a common standard in larger urban areas, where some roadway congestion is inevitable.
- F: forced or breakdown flow. A road in a constant traffic jam is at this LOS.

According to the former statement, our target is to improve the synthetic LOS of the particular roads which refer to the problem-C. Especially we would try to increase the number of the roads which are at the A~C levels and to reduce the number of roads which are at the D~E levels. However, we will not directly use the rules of LOS. Because LOS depends on the density of vehicles on the roads. It is suitable for our model only when $P = 0$. If there are much more self-driving cars on the roads, even if the density of cars is increasing, the speed of the traffic flow does not reduce significantly because of the benefits brought by those self-driving cars(Self-driving cars will keep a shorter safe distance and remain a relative high speed). So we choose $\frac{V}{Q}$(the rate of the traffic volume to the capacity of the road) as the main factors to assess the LOS of the roads. And we call this method as RLOS(R refer to the rate). And when $P = 0$, RLOS can still have the exact same assessment result compared to the standard LOS in North America. The data of A F standard $\frac{V}{Q}$ rate we used is shown in Table 1.

FFS	Target Level of Service					
(mi/h)	A	B	C	D	E	F
60	660	1080	1560	2010	2300	
	Rate(Demanding Volume/Capacity)					
60	$\frac{660}{2300} = 0.287$	$\frac{1080}{2300} = 0.470$	$\frac{1560}{2300} = 0.678$	$\frac{2010}{2300} = 0.874$	$\frac{2300}{2300} = 1$	>1

Table 1: Maximum Service Flow Rate

Applying our model to the data in the list of Problem-C, evaluating the result using the standard LOS, we have Figure 7.

In Figure 7, P is the percentage of self-driving cars. We can see as P grows, the LOS of all roads is getting better. When there is no self-driving cars on roads ($P = 0$), there are about 70% roads in the $D \sim F$ LOS. When $P = 10\%$, there are still about 70% roads in the $D \sim F$ LOS. When $P = 50\%$, there are about 30% roads in the $D \sim F$ LOS. When $P = 90\%$, all roads are in $A \sim C$ LOS.

8.2 Synthetic Index of the Level of the Traffic Network Service

To quantify the effects on the traffic flow of the number of lanes, peak traffic volume, and percentage of vehicles using self-driving, we use the following rules to calculate the scores of the level of the whole traffic network service.

$$S = \sum_{i=1}^{224} \frac{Q_i w_i}{Q_s} \tag{17}$$

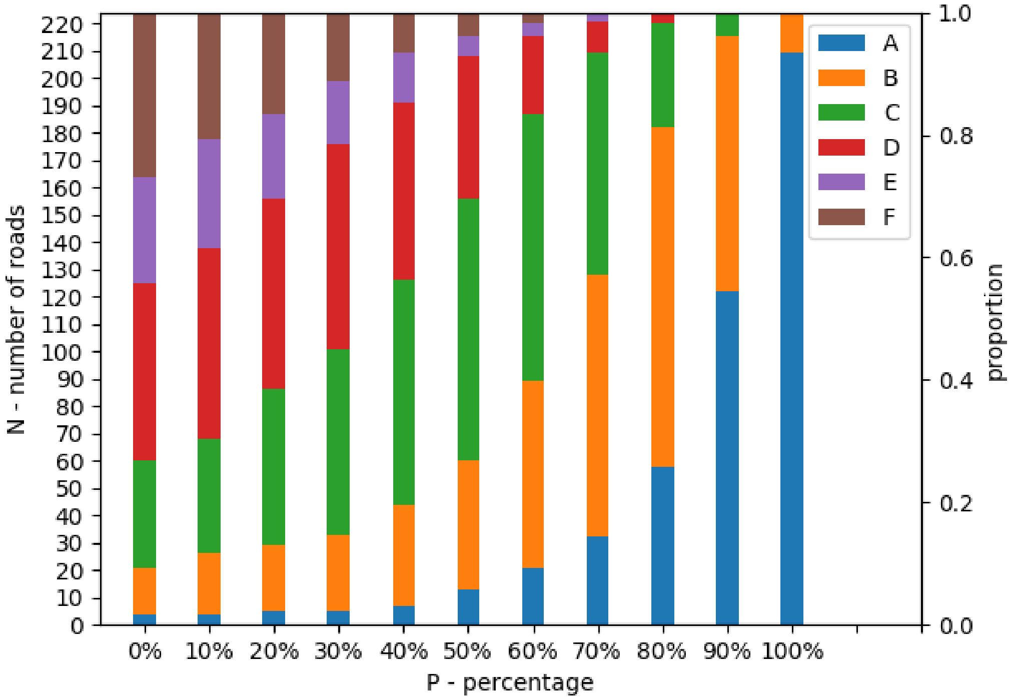

Figure 7: LOS of Interested Roads before Setting Dedicated Lanes

S is the the scores of the level of the whole traffic network service, 224 is the whole number of the roads we are interested in, Q_i is the highway capacity of one part of the roads, w_i is the weight of each part of the roads (w_i depends on the ratio of the traffic volume to the highway capacity of each level in LOS), Q_s is the sum of the traffic volume in all the roads we are considering. w_i has been assigned following the rules in Table 2. Use the rules of the scores of the level of the whole

RLOS	A	B	C	D	E	F
w_i	1	0.874	0.678	0.470	0.287	0

Table 2: Weights of Roads Corresponding to RLOS

traffic network service, we can draw a plot of the relation between S and P, in Figure 8.

8.3 Dedicated Lanes Model

In this model we have the assumptions:

1. If necessary, dedicate lanes should be set in the both side of the roads.
2. Human-driving cars is forbidden to drive on the dedicated roads when it is during the peak hours.
3. Self-driving cars will definitely drive on the dedicated roads when it is during the peak hours.
4. The roads will keep at least a lane (on one side) for the human-driving cars to drive on.

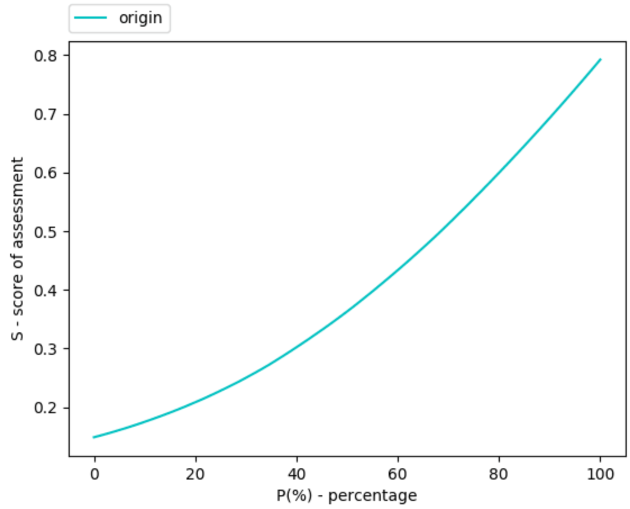

Figure 8: Scores of Assessment before Setting Dedicated Lanes

5. We may not use the plan if it is not in the peak hours.

8.4 Strategy

When it comes to a certain road, we enumerate the numbers of the dedicated lanes (according to the assumptions, like 0,2,4,6,8). And then we select the numbers of the dedicated lanes which can produce the maximum capacity of the lane.

8.5 Plan for Dedicated Lanes

We recalculate the RLOS of the new model in the Figure 9. Compared to the Figure 7 ,we can simply realize that for the same P value, the proportion of the roads of $A \sim C$ level has increased in this new model.

We calculate the score of the dedicated-lane-model in Figure 10. And we also draw the curves which indicate the total number of the different dedicated-lanes roads(CNT). From the Figure 10 we can find out that the dedicated-lane-model do improve the score of the traffic network. By this solution, we can actually attain a higher score with the same P compared to the original one.

We can also find out a reasonable phenomenon that when P is less than 20%, to set the dedicated lanes can not improve the capacity of the roads, because there are too little self-driving cars and thus the utilization of dedicated lanes is actually low. So according to our algorithm, at this kind of situation($p < 20\%$), dedicated-lanes solution is useless. And when P is more than 95%, to set dedicated lanes is unnecessary because even a normal lane can be a dedicated lane if nearly all cars are self-driving. So we advise the government to use this solution when P is between 25% and 90%.

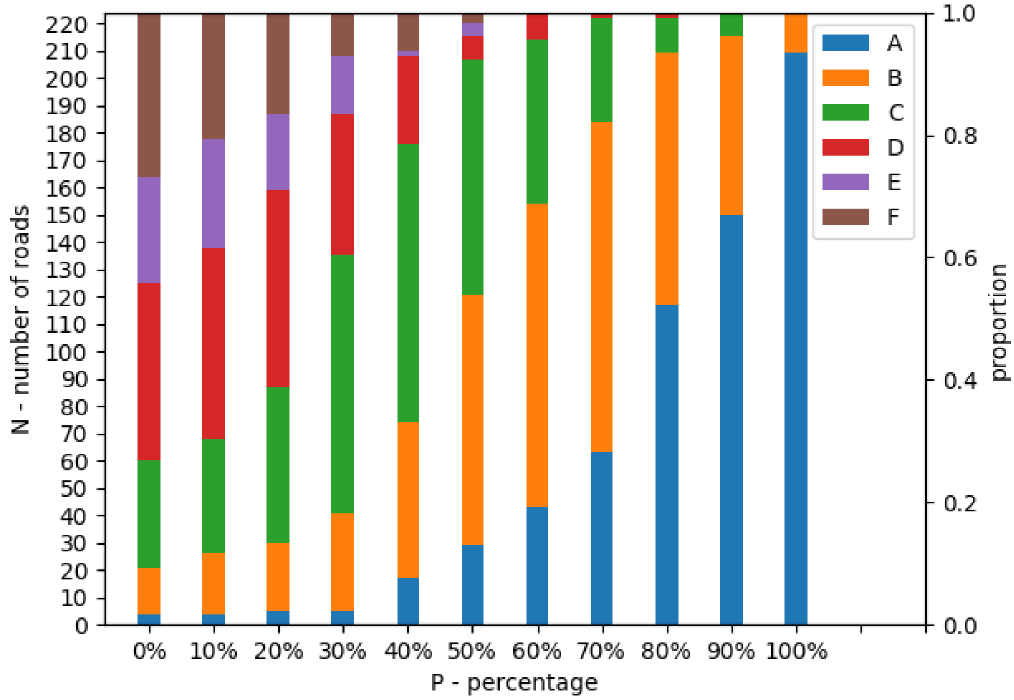

Figure 9: LOS of Interested Roads after Setting Dedicated Lanes

9 Sensitivity Analysis

9.1 Human Reaction Time

We use $T_0 = 1.1s$ as the mean of human reaction time in our calculation, however, it may have some error. So we make T_0 change from $1.1s \times 95\%$ to $1.1s \times 105\%$ and see how Q changes in Figure 11, in which Q' is the highway capacity when T_0 does not change, ΔQ is the variation of Q when T_0 changes and $\frac{\Delta Q}{Q'}\%$ is the percentage Q changes. From Figure 11, we can see when T_0 changes within $\pm 5\%$, Q changes within $\pm 3\%$. So the inaccuracy of T_0 does not affect the reliability of the model. Figure 11 is also reasonable. Because when $P = 0$, the cars on roads are all human-driving cars, T_0 has a biggest influence on Q. As P increasing, the effect decreases.

9.2 Maximum Braking Ability of A Car

We use $a_{max} = 8.5m/s^2, a_{min} = 5m/s^2$ in our calculation. Now we make a_{max} change within $8.5m/s^2 \pm (8.5-5) \times 5\%$ and see how Q changes in Figure 12. From Figure 12, we can see when a_{max} changes within $\pm 5\% \times (8.5-5)$, Q changes within $\pm 1\%$. So the variance of a_{max} does not affect the reliability of the model. Figure 12 can be explained as follow: the variation becomes bigger when P increases because self-driving cars uses a_{max} to calculate their safe distances more accurately than human who estimates safe distances.

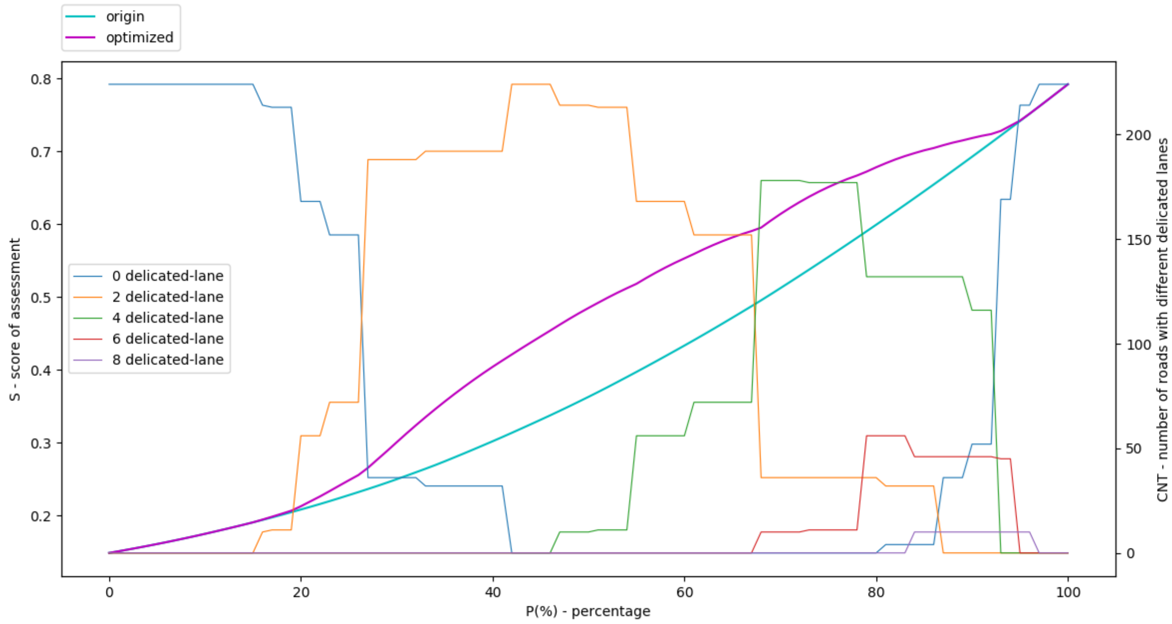

Figure 10: Scores of Assessment after Setting Dedicated Lanes

9.3 Minimum Braking Ability of A Car

We make a_{min} change within $5m/s^2 \pm (8.5-5) \times 5\%$ and see how Q changes in Figure 13. From Figure 13, we can see when a_{min} changes within $\pm 5\% \times (8.5-5)$, Q changes within $\pm 3\%$. So the variance of a_{min} does not affect the reliability of the model. Figure 13 can be explained in the same way like Figure 12 .

9.4 Average Car Length

We use $l = 4.3m$ as the mean of car length in our calculation. So we make l change from $4.3 \times 95\%$ to $4.3 \times 105\%$ and see how Q changes in Figure 14. From Figure 14, we can see when l changes within $\pm 5\%$, Q changes within $\pm 1.5\%$. So the inaccuracy of l does not affect the reliability of the model. Figure 13 can be explained as that the little change of car length will not cause much attention of drivers, but it will make a difference to the self-driving cars' calculation system.

9.5 Time A Self-Driving Car Needs to Begin Decelerating

We use $T_B = 0.1s$ as the time a self-driving car needs to begin decelerating in our calculation. So we make T_B change from $0.1 \times 95\%$ to $0.1 \times 105\%$ and see how Q changes in Figure 15. From Figure 15, we can see when T_B changes within $\pm 5\%$, Q changes within $\pm 0.1\%$. So the inaccuracy of T_B does not affect the reliability of the model. Figure 15 can be explained. When P is low, which means there are a little self-driving cars on road, the influence is small, because the proportion of self-driving cars is too small to affect the average safe distance. When P is high, which means there are many self-driving cars on road, the influence is also not large, since most self-driving car use T_C to decide their safe distances, not T_B, as what has been shown in equation 13.

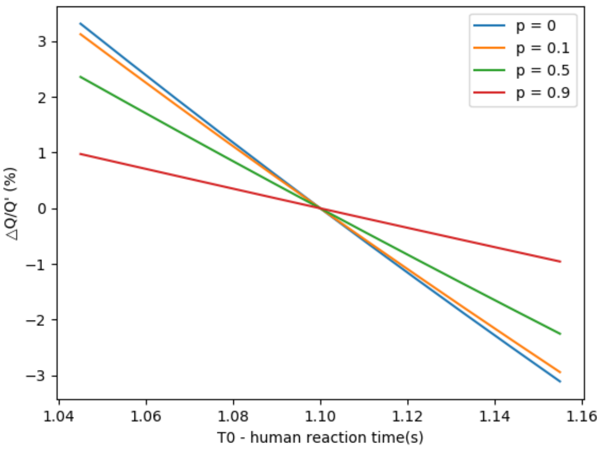

Figure 11: Sensitivity Analysis of T_0

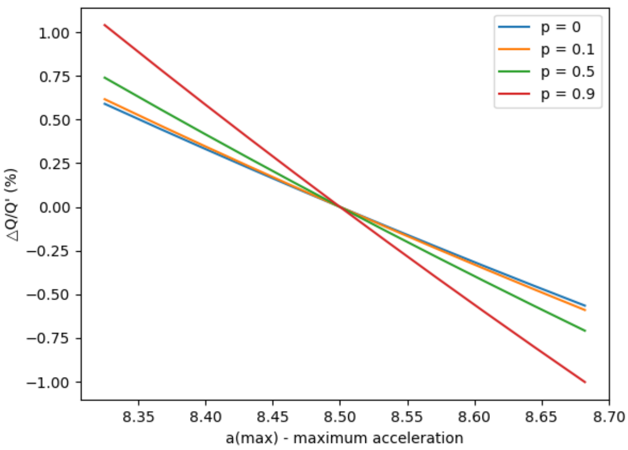

Figure 12: Sensitivity Analysis of a_{max}

9.6 Time A Self-Driving Car Needs to Sense

We use $T_S = 0.245s$ as the time a self-driving car needs to sense the deceleration of preceding car in our calculation. So we make T_S change from $0.245 \times 95\%$ to $0.245 \times 105\%$ and see how Q changes in Figure 16. From Figure 16, we can see when T_S changes within $\pm 5\%$, Q changes within $\pm 0.25\%$. So the inaccuracy of T_S does not affect the reliability of the model. Figure 16 can be explained in the same way as Figure 15, because only when a self-driving car needs to use its sensor, it will calculate its safe distance using T_B and T_S, as in equation 5.

9.7 Time A Self-Driving Car Needs to Communicate

We use $T_C = 0.081s$ as time a self-driving car needs to communicate in our calculation. So we make T_C change from $0.081 \times 95\%$ to $0.081 \times 105\%$ and see how Q changes in Figure 17. From Figure 17, we can see when T_C changes within $\pm 5\%$, Q changes within $\pm 0.35\%$. So the inaccuracy of T_C does not affect the reliability of the model. Figure 17 can be explained as the more self-driving

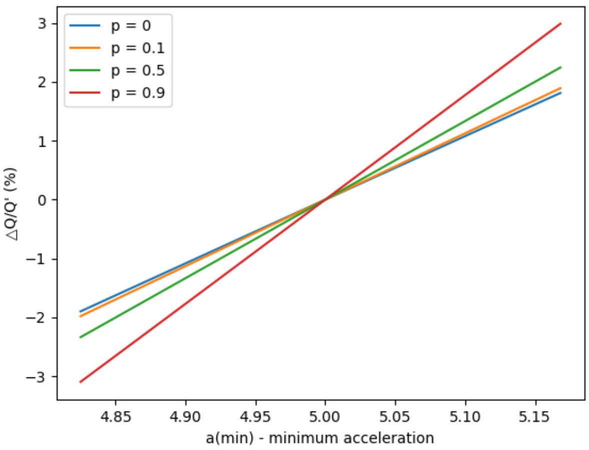

Figure 13: Sensitivity Analysis of a_{min}

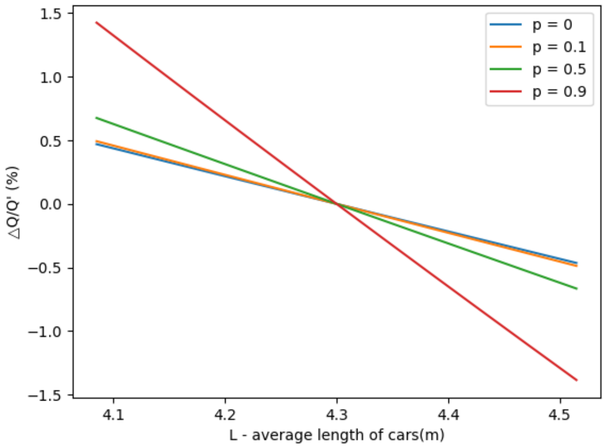

Figure 14: Sensitivity Analysis of l

cars the more influence T_C has.

10 Model Analysis

10.1 Advantages

1. The calcution in this model is efficient, being compared to cellular automaton.
2. This model has strong robustness because it has low sensitivity.
3. The model has a clear principle, so it is convinient to add more elements or adjust parameters.

10.2 Disadvantages

1. Parameter can be more accurate if have more time and energy to do surveys.

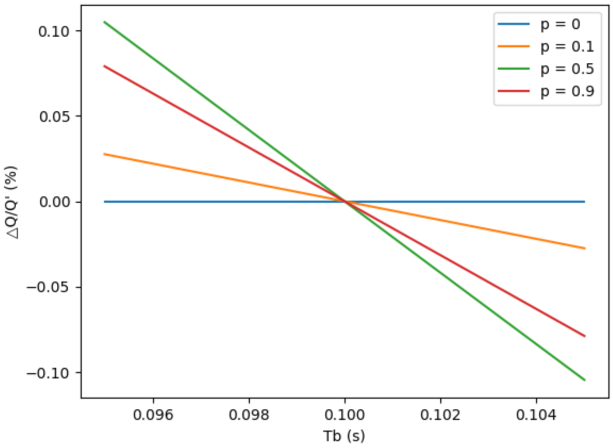

Figure 15: Sensitivity Analysis of T_B

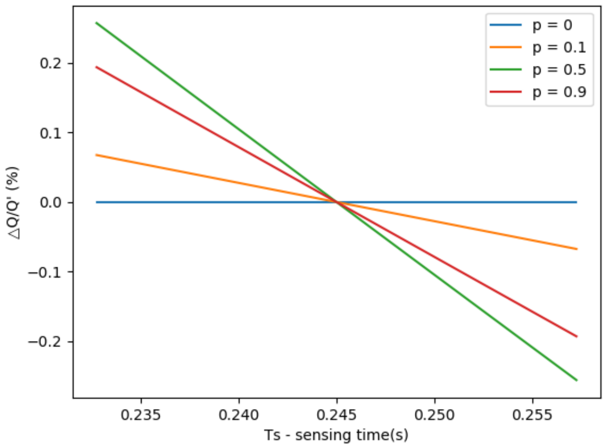

Figure 16: Sensitivity Analysis of T_S

2. The model use the expectation to include the variation of speed. However, in some low-speed road, the wave of traffic flow may cause error.

3. The mode self-driving cars cooperate with each other still have room to be improved.

11 Conclusion

When the percentage of self-driving car on roads is 10%, the effect of allowing self-driving car is small. There are still about 70% roads in the $D \sim F$ LOS. When half cars on roads are self-driving cars, the effect has become much stronger. There are about 30% roads in the $D \sim F$ LOS. When the percentage is 90%, the effect is so powerful that all roads are in $A \sim C$ LOS. A quasi-equilibria exists when the percentage is under 50%. However, once the percentage is bigger than 60%, the effect will grow much faster. We suggest that the governor should set dedicated lanes when the percentage of the self-driving cars is between 25% and 90%.

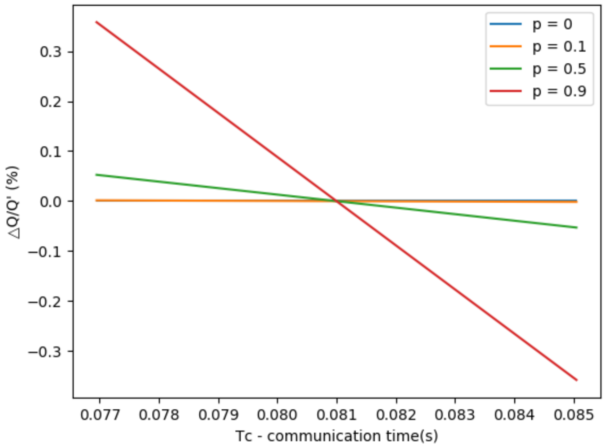

Figure 17: Sensitivity Analysis of T_C

References

[1] Patcharinee Tientrakool, Ya-Chi Ho, and Nicholas F. Maxemchuk, Department of Electrical Engineering, Columbia University, Highway Capacity Benefits from Using Vehicle-to-Vehicle Communication and Sensors for Collision Avoidance

[2] https://en.wikipedia.org/wiki/Level_ of_ service

[3] Highway Capacity Manual 2010 (HCM2010)

Appendix

A Number of Dedicated Lanes

Route_ID	startMilepost	endMilepost	Number of Delicated Lanes during Peak Hours (each direction has the half)	Route_ID	startMile post	endMile post	Number of Delicated Lanes during Peak Hours(each direction has the half)	Route_ID	startMile post	endMile post	Number of Delicated Lanes during Peak Hours (each direction has the half)	Route_ID	startMile post	endMile post	Number of Delicated Lanes during Peak Hours (each direction has the half)
5	100.93	101.87	2	90	1.94	2.04	2	405	0	0.09	2	520	0	0.36	0
5	101.87	103.17	2	90	2.04	2.4	0	405	0.09	0.37	2	520	0.36	0.85	0
5	103.17	103.42	2	90	2.4	2.54	0	405	0.37	0.54	2	520	0.85	1.43	0
5	103.42	104.81	2	90	2.54	2.79	0	405	0.54	0.75	2	520	1.43	1.63	0
5	104.81	105.63	2	90	2.79	3.94	2	405	0.75	1.22	2	520	1.63	4.4	0
5	105.63	106.23	2	90	3.94	5.82	2	405	1.22	2.26	2	520	4.4	5.39	0
5	106.23	107.09	2	90	5.82	6.56	2	405	2.26	2.8	2	520	5.39	5.6	0
5	107.09	107.74	2	90	6.56	6.85	2	405	2.8	3.3	2	520	5.6	6.52	0
5	107.74	108.46	2	90	6.85	7.64	2	405	3.3	3.69	0	520	6.52	6.93	0
5	108.46	108.94	2	90	7.64	8.7	2	405	3.69	4.5	0	520	6.93	9.6	0
5	108.94	109.52	2	90	8.7	9.61	2	405	4.5	4.79	0	520	9.6	9.72	0
5	109.52	111.43	2	90	9.61	9.87	2	405	4.79	5.19	0	520	9.72	11.35	0
5	111.43	112.43	2	90	9.87	10.15	2	405	5.19	5.89	0	520	11.35	12.05	0
5	112.43	113.85	2	90	10.15	11.64	2	405	5.89	6.34	0	520	12.05	12.38	0
5	113.85	114.65	2	90	11.64	12.34	2	405	6.34	6.72	0	520	12.38	12.83	0
5	114.65	116.42	2	90	12.34	13.3	2	405	6.72	7.2	0				
5	116.42	117.25	2	90	13.3	14.32	2	405	7.2	7.69	0				
5	117.25	117.78	2	90	14.32	15.36	2	405	7.69	8.98	0				
5	117.78	118.4	2	90	15.37	16.31	2	405	8.98	9.59	0				
5	118.4	118.67	2	90	16.31	16.85	2	405	9.59	9.96	0				
5	118.67	119.38	2	90	16.85	18	2	405	9.96	10.56	0				
5	119.38	120.46	2	90	18	18.38	2	405	10.56	10.93	0				
5	120.46	121.4	2	90	18.38	19.97	2	405	10.93	11.69	2				
5	121.4	122.4	2	90	19.97	20.75	2	405	11.69	12.44	2				
5	122.4	123.09	2	90	20.75	22.22	2	405	12.44	12.99	2				
5	123.09	123.28	2	90	22.22	22.86	2	405	12.99	13.42	2				
5	123.28	123.94	2	90	22.86	25.37	2	405	13.42	13.78	2				
5	123.94	124.33	2					405	13.78	14.83	2				
5	124.33	125.09	2					405	14.83	15.76	2				
5	125.09	125.6	2					405	15.76	17.05	2				
5	125.6	126.2	2					405	17.05	17.62	2				

Route_ID	startMilepost	endMilepost	Number of Delicated Lanes during Peak Hours (each direction has the half)	Route_ID	startMile post	endMilep ost	Number of Delicated Lanes during Peak Hours(each direction has the half)	Route_ID	startMile post	endMilep ost	Number of Delicated Lanes during Peak Hours (each direction has the half)	Route_ID	startMile post	endMilep ost	Number of Delicated Lanes during Peak Hours (each direction has the half)
5	126.2	127.13	2	5	157.7	157.85	2	5	175.28	175.52	2	405	17.62	17.78	2
5	127.13	128.06	2	5	157.85	158.32	2	5	175.52	175.79	2	405	17.78	18.6	2
5	128.06	128.59	2	5	158.32	160.74	2	5	175.79	176.55	2	405	18.6	19.51	2
5	128.59	129.36	2	5	160.74	162.24	2	5	176.55	177.17	2	405	19.51	20.27	2
5	129.36	130.02	2	5	162.24	162.79	2	5	177.17	178.61	2	405	20.27	21.28	2
5	130.02	130.31	2	5	162.79	163.36	2	5	178.61	179.17	2	405	21.28	22.32	2
5	130.31	131.22	2	5	163.36	163.48	2	5	179.17	179.61	2	405	22.32	23.47	2
5	131.22	131.35	2	5	163.48	164.22	2	5	179.61	180.36	2	405	23.47	23.88	2
5	131.35	132.89	2	5	164.22	165.29	2	5	180.36	181.31	2	405	23.88	24.09	2
5	132.89	133.66	2	5	165.29	165.67	2	5	181.31	182.59	2	405	24.09	24.28	2
5	133.66	134.18	2	5	165.67	165.96	2	5	182.59	183.19	2	405	24.28	24.83	2
5	134.18	134.61	2	5	165.96	166.15	2	5	183.19	183.59	2	405	24.83	26.31	2
5	134.61	134.97	2	5	166.15	166.66	2	5	183.59	184.47	2	405	26.31	27.4	2
5	134.97	135.32	2	5	166.66	167.35	2	5	184.47	186.18	2	405	27.4	29.88	2
5	135.32	136.51	2	5	167.35	167.39	2	5	186.18	186.87	2	405	29.88	30.21	2
5	136.51	137.15	2	5	167.39	167.8	2	5	186.87	187.55	2	405	30.21	30.32	2
5	137.15	138.04	2	5	167.8	168.31	2	5	187.55	188.69	2				
5	138.04	141.64	2	5	168.31	168.56	2	5	188.69	189.97	2				
5	141.64	142.76	2	5	168.56	169.01	2	5	189.97	192.47	2				
5	142.76	143.67	2	5	169.01	169.71	2	5	192.47	192.94	2				
5	143.67	144.61	2	5	169.71	170.04	2	5	192.94	193.35	2				
5	144.61	146.49	2	5	170.04	170.5	2	5	193.35	193.65	2				
5	146.49	147.28	2	5	170.5	170.91	2	5	193.65	194.44	2				
5	147.28	149.83	2	5	170.91	171.22	2	5	194.44	194.56	2				
5	149.83	151.5	2	5	171.22	171.76	2	5	194.56	198.61	2				
5	151.5	152.03	2	5	171.76	172.52	2	5	198.61	198.95	2				
5	152.03	152.72	2	5	172.52	173.15	2	5	198.95	199.58	2				
5	152.72	155.18	2	5	173.15	173.51	2	5	199.58	200.25	2				
5	155.18	155.59	2	5	173.51	174.27	2	5	200.25	201.19	2				
5	155.59	156.29	2	5	174.27	175.1	2								
5	156.29	157.7	2	5	175.1	175.28	2								

B Core Code Calculating Highway Capacity

```
import matplotlib.pyplot as plt
import math
import numpy as np
import random
import sympy
from mpl_toolkits.mplot3d import Axes3D
import csv

t0=1.1
a_max=8.5
a_min=5
Tm=0.1
Ts=0.245
Tc=0.081
l=4.3

def s_func(a,V):
vp=V/3.6
u=vp/(vp/a-Tc)
aa1=vp/2*((Tc*2-vp/a)*(u-a_min))/(a_max-a_min)**2
aa3=np.log(u/a_min)/(a_max-a_min)**2*vp**2/2
aa4=(a_max*a-u*a)/(a_max-a_min)**2*Tc**2/2
aa5=a**2*np.log((a_max-a)/(u-a))/(a_max-a_min)**2*Tc**2/2
# aa6=-a**2*np.log(Tc*a)/(a_max-a_min)**2*Tc**2/2
# aa7=a**2*np.log(vp/a-Tc)/(a_max-a_min)**2*Tc**2/2
return aa1+aa3+aa4+aa5#+aa6+aa7
```

```
def integ(V):
x=np.linspace(a_min,a_max-0.0000001,100)
tmp=s_func(x,V)
return np.trapz(tmp,x)

def calc_Q(P,V):
D_H  = t0*V/3.6 + V**2.0/(2*3.6**2)*np.log(a_max/a_min)
/(a_max-a_min) - V**2/(2*3.6**2*a_max)
D_C1 = ((Ts+Tm)*V/3.6 + V**2.0/(2*3.6**2)*np.log(a_max/a_min)
/(a_max-a_min) - V**2/(2*3.6**2*a_max))
if (isinstance(V,np.ndarray)):
D_C2 = []
for i in V:
if (isinstance(i,np.ndarray)):
D_C2.append([])
for j in i:
D_C2[-1].append(integ(j))
else:
D_C2.append(integ(i))
D_C2 = np.array(D_C2)
else:
D_C2 = integ(V)
D_C  = (1-P) * D_C1 + P * D_C2
D_   = (1.0-P) * D_H + P * D_C
Q    = 1000*V/(D_+l)
return Q
```