Discovering first-principle of behavioural change in disease transmission dynamics by deep learning

Pengfei, Song and Yanni, Xiao and Jianhong, Wu

Abstract Human behaviors are key determinants of the course, duration and outcomes of disease outbreaks, and disease transmission dynamics models incorporating behavioural changes have proved to be a power tool in guiding public health measures in epidemic control. Nonetheless, how to develop computationally tractable first-principle behavioural change models with good generalization and predictability remains a challenge. Neural networks, despite the perception of being a black-box uniform approximator and difficult to interpret, have an incredible effectiveness in learning unknown mechanisms in differential equations with bless of dimensionality. In this study, we propose an epidemiology informed two-step recovering-explaining method which balance the learning ability and interprecity, and apply the method to learn the behavioural change mechanisms in transmission dynamics and represent the mechanisms in closed form by formulas as simple as possible. The proposed method can also give new insights to patterns of disease outbreaks in modern era.

Key words: Human behavioural change models, transmission dynamics, Deep learning, Data driven methods, neural differential equations
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1 Introduction

Infectious diseases can have a large impact on society as they can negatively affect, among others, morbidity, mortality, unemployment and inequality. The ongoing COVID-19 pandemic since first officially reported in Wuhan, China, in late 2019, poses continuing threat on human's health ([30]). Controlling emerging viral infectious disease will depend critically on non-pharmaceutical interventions including social distancing, mask wearing, contact tracing, isolation, quarantine, border control and pharmaceutical measures such as vaccination and antiviral drugs ([3]). The successful implementation of public health interventions is greatly subject to human behaviors as the key determinant of the course, duration and outcomes of disease outbreaks ([53, 2, 47, 48]). Therefore, there is a great interest to incorporate behavioural change in response to disease-related information into models for infectious disease transmission dynamics.

Behavioural change transmission dynamics models share a tremendous popularity in recent years, see reviews [18, 47, 44] for more details. To characterize the human behavioural change in transmission dynamics models, two ways are common. The first way is to add more compartments representing risk-aware and risk-unaware subpopulations, see [16, 17] for example. The second way is to change the incidence rate by including a function which decreases as infection increases. For example, [31] introduced a media function $\beta e^{-\alpha_1 E-\alpha_2 I-\alpha_3 H}$ into the transmission coefficient, where E, I, and H are the numbers of exposed, infectious, and hospitalized individuals, respectively. The study [49] extended these media functions by assuming that the function depends on both the case number and its rate of change, and obtained that media impact switches on and off in a highly nonlinear fashion. The subsequent work [50] further extended these models by including extra compartment, i.e., the level of media coverage M, and consequently media impact is modeled by including the function $e^{-\mu M}$ with $\mu > 0$ in the incidence.

However, these behavioural change transmission dynamics models are heavily experts-based or assumptions-based. The successful applications of these models in epidemic control are greatly dependent on good assumptions. Simple assumptions make transmission dynamics models theoretically tractable, but are not prone to fit the real epidemic data; complicated assumptions can fit the real epidemic data better, but often make transmission dynamics computations intensive and may not have good generalization and predictability. How to characterize the behavioural changes in transmission dynamics model with a simpler and more data driven way is still a challenge. Behavioural change models must strike a delicate balance, harnessing available data and theory on complex social and behavioral phenomena, while keeping important modeling properties, such as computational tractability and simplicity.

In recent years, deep neural networks ([29, 19]), as a universal approximator for unknown mappings ([23, 27]) and nonlinear operators ([32]), shows incredible effectiveness ([41]) in pattern recognition, learning unknown mechanisms and solving traditional difficult problems such as image recognition ([20]), natural language processing ([14]). Recently, deep learning methods have been used to aid mathematicians in discovering new conjectures and theorems ([12]), and physicians in finding new physical laws ([11]). Moreover, to better understand and improve the performance of deep neural networks on scientific problems in physics, chemistry and epidemiology, increasing attentions have been paid to coupling or embedding differential equations and deep neural network. One important idea is regarding deep neural network (DNN) as discretization of differential equation, which inspires researchers to redesign traditional sequential neural architectures based on numerical discretization schemes ([34, 9, 33, 38]) or to replace DNN by a continuous model characterized by differential equations ([8, 13, 37]). Another revolutionary idea is training DNN from the view of optimal control. This extends the idea of backpropagation ([19]) to include adjoint sensitivity analysis ([7]), which opens the new research frontiers in differential programming ([1]) and inspires a lot of architectures coupling neural networks and differential equations, such as neural differential equation ([8]) and universal differential equation ([35]).

Although neural differential equations and universal differential equations allow efficient training of complex models on high dimensional datasets and show incredible learning effectiveness, black-box terms are still unwelcome. One still cares about what the exact equation should be, and wish the equation to be as simple as possible, rather than represented as a neural network and difficult to interpret in differential equations. Data driven methods like symbolic regression, sparse identification of nonlinear dynamical systems ([6, 39, 25, 5]), dynamic mode decomposition ([45, 15, 5]), have been proposed in recent years for this purpose. Unlike deep neural networks, data driven methods are interpretable equation-search methods with the purpose to find the simplest analytic formulas to describe science, engineering and real world data. However, compared to deep learning methods, data-driven methods show the weakness in training efficiency in handling high dimension differential equations and are difficult to learn the dynamics when data observation is limited to small number of samples.

In this work, we propose a two-step recovering-explaining framework which combines deep learning methods and data driven methods, and use the framework to discover the exact expression of the unknown behavioural change mechanisms in transmission dynamics models. We mention that the proposed framework balances the learning ability and interprecity of the transmission dynamics models with neural networks embedded, and can be used to address other relevant issues where hidden mechanisms may be explored with limited epidemic data.

The rest of the paper is organized as follows. An expert-based human behavioural change disease transmission dynamics model is proposed in Section 2 to show that a good assumption for behavioural change transmission dynamics model is theoretically tractable and can have good predictability. We will introduce two-step recovering-explaining framework in Section 3. In Section 4, the framework is applied to Ontario COVID-19 data to discover the behavioural change mechanisms in the province. Discussions and conclusions will be in Section 5.

2 Expert-based Behavioural Change Transmission Dynamics Models

In this section, we propose a novel simple SIR (suspected-infected-recovered) model incorporated with behavioural change mechanisms, and show that the simple behavioural change SIR model is theoretically tractable and have good generalization and predictability in the first wave of Covid-19 pandemic in the province of Ontario, Canada. This simple behavioural change SIR model reveals the importance of good assumptions in expert-based models and necessities to use deep learning methods to discover unknown mechanisms and find good assumptions in transmission dynamics models.

We extend the classic SIR model ([26]) to include a compartment to describe the dynamics of human behaviors as follows:

$$\begin{cases} \frac{\mathrm{dS}}{\mathrm{dt}} = -\beta c S I, \\ \frac{\mathrm{dI}}{\mathrm{dt}} = \beta c S I - \gamma I, \\ \frac{\mathrm{dR}}{\mathrm{dt}} = \gamma I, \\ \frac{\mathrm{d\ln c}}{\mathrm{dt}} = -\theta \frac{\mathrm{d\ln I}}{\mathrm{dt}} - \delta I, \end{cases} \tag{1}$$

where S and I denote the number of susceptible and infectious individuals, respectively, β is the baseline infection rate, γ the is recovery rate, and c is the average number of disease transmission effective contacts to describe the human behavioural change effect. Here we assume that the changing rate of human behaviors depends on the prevalence and changing rate of prevalence.

2.1 Calculation of the Final Epidemic Size

To start with, by using

$$I = \frac{1}{\gamma} \frac{\mathrm{dR}}{\mathrm{dt}},$$

the third equation can be written as

$$\frac{\mathrm{d}(\ln \mathrm{c} + \theta \ln \mathrm{I} + \delta \mathrm{R}/\gamma)}{dt} = 0,$$

which implies

$$\ln\left(cI^{\theta}\right) + \delta R/\gamma = \ln\left(c_0 I_0^{\theta}\right) + \delta R_0/\gamma := M_0,$$

i.e.,

$$c = I^{-\theta} \exp(M_0 - \delta R/\gamma).$$

Thus, the equation becomes

$$\begin{cases} \dfrac{\mathrm{dS}}{\mathrm{dt}} = -\beta \exp(M_0 - \delta R(t)/\gamma) S I^{1-\theta}, \\ \dfrac{\mathrm{dI}}{\mathrm{dt}} = \beta \exp(M_0 - \delta R(t)/\gamma) S I^{1-\theta} - \gamma I, \\ \dfrac{\mathrm{dR}}{\mathrm{dt}} = \gamma I. \end{cases}$$

By using $I = \frac{1}{\gamma}\frac{\mathrm{dR}}{\mathrm{dt}}$, the first equation becomes

$$\frac{\mathrm{d}\ln \mathrm{S}}{\mathrm{dt}} = -\beta \exp(M_0 - \delta R(t)/\gamma) \left(\frac{1}{\gamma}\frac{\mathrm{dR}}{\mathrm{dt}}\right)^{1-\theta}.$$

Denote the final epidemic size as r. It can be easily proved that as $t \to \infty$,

$$S \to 1 - r, \quad I \to 0, \quad R \to r.$$

If $\theta = 0$, then

$$\frac{\mathrm{d}\ln \mathrm{S}}{\mathrm{dt}} = \frac{\beta}{\delta}\left(\frac{\mathrm{d}\,(\exp(\mathrm{M_0} - \delta \mathrm{R(t)}/\gamma))}{\mathrm{dt}}\right),$$

which implies

$$\ln(1 - r) = \frac{\beta}{\delta}\exp(M_0 - \delta r/\gamma) - \ln c_0.$$

If $\delta = 0$, we have

$$\frac{\mathrm{d}\ln \mathrm{S}}{\mathrm{dt}} = -\beta c_0 I_0^{\theta}\left(\frac{1}{\gamma}\frac{\mathrm{dR}}{\mathrm{dt}}\right)^{1-\theta}.$$

In summary, we have the following theorem:

Theorem 1 *Denote the final epidemic size of the behavioural change disease transmission dynamics model* (1) *as r. We have*
(i) *if $\theta = 0$ and $\delta \neq 0$, i.e., changing rate of human behaviors only depends on prevalence, then the final epidemic size satisfies*

$$\ln(1 - r) = \frac{\beta}{\delta}\exp(M_0 - \delta r/\gamma) - \ln c_0.$$

(ii) *if $\theta \neq 0$ and $\delta = 0$, i.e., the change rate of human behaviors only depends on the changing rate of prevalence, then the final epidemic size satisfies*

$$\frac{\mathrm{d}\ln \mathrm{S}}{\mathrm{dt}} = -\beta c_0 I_0^{\theta}\left(\frac{1}{\gamma}\frac{\mathrm{dR}}{\mathrm{dt}}\right)^{1-\theta}.$$

Remark: we note from (i) that if $\theta = \delta = 0$, i.e., without behavioural changes, then the final epidemic size satisfies

$$\ln(1-r) = -\mathcal{R}_0 r, \quad \mathcal{R}_0 = \frac{\beta c_0}{\gamma}.$$

2.2 Applications to the Ontario's First COVID-19 Pandemic Wave

In this part, model (1) is applied on the Ontario's First COVID-19 pandemic wave. In the province, the outbreak began on Feb 25, 2020 and the first wave continued for about 150 days. The case data is shown in Fig. 2.

To start with, we use a simple SIR model to investigate the first wave data, we find that even with full 150 days training data, a simple SIR model can not fit well, which implies a simple SIR model without considering human behavioural changes is not enough to capture the evolution of epidemic. The results are shown in Fig. 1.

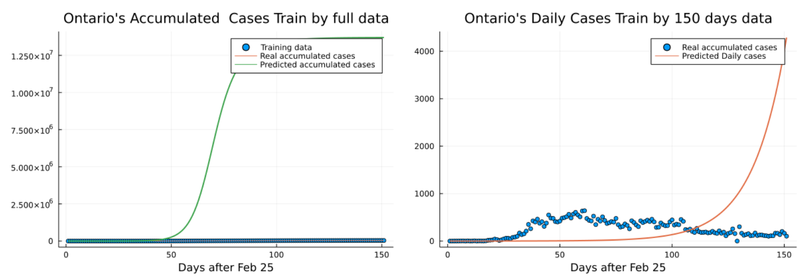

Fig. 1 Failures of simple SIR model to fit first wave Ontario COVID-19 even with full observation data.

However, by using the novel behavioural change model, we find that using only 50 days of data is enough to predict trend of the epidemic in the first wave, see Fig. 1 for details.

Here models are calibrated by Non-U-Turn Hamiltonian Monte Carlo method [21], which is implemented in Julia 1.7.3, an open-source software. We kept the same range of parameter values when we fit the models without and with behavioural change. More details on parameter values, implementation of parameter estimation, and confidence intervals can be seen in Github repo.

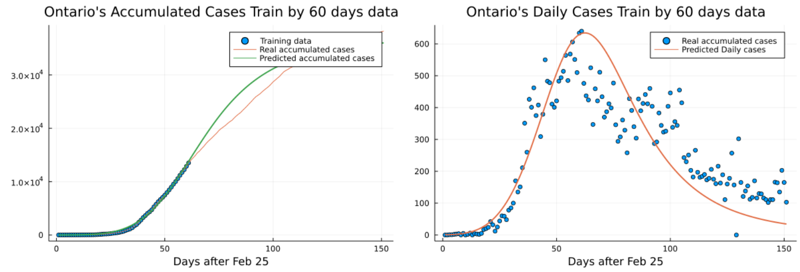

Fig. 2 Calibration of a transmission dynamics model incorporating behavioural changes to the 150 days of incidence data during the first wave of COVID-19 in the Province of Ontario, where only the first 60 days of the data is used to produce a good prediction for the remaining 90 days of the first wave.

3 Two-step Recovering-Explaining Framework

For expert-based transmission dynamics models, describing unknown mechanisms such as human behavioural changes, greatly depends on intuitions and good assumptions. However, finding a good assumption to describe unknown mechanisms in dynamic systems is a challenge. Here we propose a deep learning method based two-step recovering-explaining framework to discover unknown mechanisms and make good assumptions.

3.1 Universal Differential Equations

We start with a short introduction of the state-of-the-art methodology, universal differential equations (UDEs) ([35]), which embeds universal approximators in differential equations for scientifically-based learning, and can be used to discover previously unknown mechanisms, accurately extrapolate beyond the original data. Before the introduction on UDEs, we introduce neural differential equations ([8]) first, which were proposed before UDEs, inspired the ideas in UDEs and can be regarded as a special case of UDEs. Neural differential equations are initial value problems with the following form

$$u' = \mathrm{NN}_\theta(u, t), \tag{2}$$

where the right-hand side NN is a deep neural network receiving $[u, t]$ as input. Neural differential equations make use of scientific structures as a modeling basis, and because the embedded function is a universal approximator. Thus, neural differential equations can learn to approximate any sufficiently regular differential equation. From the perspective of deep learning, neural differential equations are redesigned sequential neural networks based on numerical schemes of differential equation, and they can be regarded as continuous-depth or "infinitely deep" ResNet-like deep learning models. From the perspective of optimal control, the training of neural differential equations can be regarded as optimal control problems, which extend the idea of backpropagation ([19]) in differential programming to include adjoint sensitivity analysis ([7]).

However, the resulting neural differential model is defined without direct incorporation of known mechanisms. UDEs extends the previous data-driven neural ODE approaches to directly utilize mechanistic modeling simultaneously with universal approximators in order to allow for arbitrary data-driven model extensions. UDEs are initial value problems with the following forms:

$$u' = f_{\theta_2}(u, t, UA_{\theta_1}(u, t)), \tag{3}$$

where f is a known mechanistic model and UA denotes the missing or unknown terms which can be represented by universal approximators such as neural networks, Gaussian process. Throughout the rest of this study, we choose neural networks as universal approximator. θ_1 and θ_2 are parameters of known mechanisms and neural networks, respectively, which can be estimated simultaneously. UDEs has strong explainability than deep neural networks or neural differential equations, since they keep known mechanisms in physics, chemistry or epidemiology. UDEs have been proved to be methods with good generalization and can be trained with less sample data ([35]).

3.2 Data Driven Methods or Equation Searching Methods

Although neural differential equations and universal differential equations allow efficient training of complex models on high dimensional datasets and show incredible learning effectiveness, black-box terms are still unwelcome. One still cares about what the exact equation should be, and wish the equation to be as simple as possible, rather than represented as neural network and difficult to interpret in differential equations. Data driven methods like sparse identification of nonlinear dynamical systems ([6, 39, 25, 5]), dynamic mode decomposition ([45, 15, 5]) have been proposed in recent years for this purpose. Unlike deep neural network, data driven methods are interpretable equation-search methods with the purpose to find the simplest analytic expressions to describe science, engineering and real world data. In what follows, we will give a brief introduction of two famous data driven methods: symbolic regression and sparse identification of nonlinear dynamics (SINDy).

3.2.1 Symbolic Regression

Symbolic Regression (SR) is a type of regression analysis that searches the space of mathematical expressions to find the model that best fits a given dataset, both in terms of accuracy and simplicity. SR uses binary-tree to represent a function, and no particular model is provided as a starting point to the algorithm. Instead, initial expressions are formed by randomly combining mathematical building blocks such as

- binary mathematical operators: $+, -, *, /$;
- unary analytic functions: $\sin, \cos, \exp, \tanh, \ldots$;
- constants;
- state variables.

Usually, a subset of these primitives will be specified by the person operating it, but that's not a requirement of the technique. SR uses genetic programming ([40]), as well as more recently methods utilizing bayesian methods ([24]) and deep learning methods ([46]) to discover the equations.

By not requiring *a priori* specification of a model, symbolic regression is not affected by human bias, or unknown gaps in domain knowledge. It attempts to uncover the intrinsic relationships of the dataset, by letting the patterns in the data itself reveal the appropriate models, rather than imposing a model structure that is deemed mathematically tractable from a human perspective. The fitness function that drives the evolution of the models takes into account not only error metrics (to ensure the models accurately predict the data), but also special complexity measures, thus ensuring that the resulting models reveal the data's underlying structure in a way that's understandable from a human perspective.

3.2.2 Sparse Identification of Nonlinear Dynamics (SINDy)

The SINDy (sparse identification of nonlinear dynamics) algorithm ([6]) provides a principled, data-driven discovery method for nonlinear dynamics of the form

$$u' = f(u) \tag{4}$$

where $u(t) = [u_1(t); u_2(t); \cdots ; u_n(t)] \in \mathbb{R}^n$ is system states represented as a row vector. SINDy applies a set of candidate functions that would characterize the right-hand side of the governing equations. Candidate model terms form the library

$$\mathbf{\Theta}(\mathbf{U}) = \left[\theta_1(\mathbf{U}), \theta_2(\mathbf{U}), \cdots, \theta_p(\mathbf{U})\right] \in \mathbb{R}^{m\times p}$$

of potential right-hand side terms, where

$$\mathbf{U} = \begin{bmatrix} u^T(t_1) \\ u^T(t_2) \\ \cdots \\ u^T(t_m) \end{bmatrix} = \begin{bmatrix} u_1(t_1) & u_2(t_1) & \cdots & u_n(t_1) \\ u_1(t_2) & u_2(t_2) & \cdots & u_n(t_2) \\ \cdots & \cdots & \ddots & \cdots \\ u_1(t_m) & u_2(t_m) & \cdots & u_n(t_m) \end{bmatrix}$$

is an $\mathbb{R}^{m\times n}$ matrix. $\theta_i(\mathbf{U})$ can be any candidate basis function that may describe the system dynamics $f(u(t))$ such as trigonometric functions $\theta_i(\mathbf{U}) = cos(\mathbf{U})$ or polynomial functions $\theta_i(\mathbf{U}) = \mathbf{U}^2$. This then allows for the formulation of a regression problem to select only the few candidate terms necessary to describe the dynamics:

$$\arg\min_{\Xi} \|\dot{\mathbf{U}} - \Theta(\mathbf{U})\Xi\|_2 + \lambda\|\boldsymbol{\Xi}\|_0$$

where the matrix

$$\Xi = [\xi_1, \xi_2, \cdots, \xi_n] \in \mathbb{R}^{p\times n}$$

is comprised of the sparse vectors $\xi_1 \in \mathbb{R}^p$ that select candidate model terms. The amount of sparsity promotion is controlled by the parameter λ, which determines the penalization by the ℓ_0-norm. By solving system (4), we can identify a model of system dynamics

$$u' = f(u) \approx \Theta(u)\Xi. \tag{5}$$

3.3 Two-step Recovering-Explaining Methods

However, compared to deep learning methods, data-driven methods show the weakness in training efficiency in handling high dimension differential equations and is difficult to learn the dynamics when observed data is limited to small number of samples. Can we combine these two areas' methods? Two-step Recovering-Explaining framework can be an answer.

In epidemiology, researchers care about both learning ability and interpreticity. We want to efficiently learn the missing mechanisms in transmission dynamics models, which implies UDEs ([35]) and physics informed neural networks (PINNs, [36]) are good choices. However, we do not wish the missing mechanisms represented as black-box neural network and difficult to interpret in epidemiology, which implies data-driven methods are good choices. Recall that observed data in epidemiology are noisy, partial observed, sparsely sampled, and with heterogeneity between datasets, which gives a lot of challenges on data driven methods such as SINDy ([6]). Is there a way to balance the learning ability and interprecity, efficiently learn the missing mechanisms in transmission dynamics and represent the mechanisms as simple as possible? In what follows, we propose a two-step recovering-explaining method as a solution. The key points of two-step recovering-explaining methods are

- Firstly we use machine learning methods like UDEs ([35]) or PINNs([36]) to recover the partial observed, sparsely sampled and noisy data to fully observed, continuous, differentiable data.
- Then we apply the recovered data to find the unknown simple equations in transmission dynamics by data driven methods like symbolic regression (([40])) or SINDy ([6]).

For epidemic models, the observed data such as death, accumulated, daily confirmed cases are not exact solutions of the differential equations. The state in transmission dynamics models are often regarded as the latent state of the observed data. Thus, it is difficult to apply data-driven methods like SINDy to discover the unknown mechanisms. Deep learning methods based on differential programming does not need the case data in epidemiology to be fully observed and differentiable. Moreover, deep learning methods can be regarded as an advanced data smoothing method showing incomparable performance to other traditional approaches such as moving average. Data assimilation methods like particle filter ([28]) can recover fully observed, differentiable data, but knowledge on the unknown mechanisms is required *a priori*. Here we remark that two-step methods can also be trained simultaneously, if the algorithms in data driven methods are differential programming based, for example OccamNet ([10]).

The processes of two-step recovering-explaining methods are shown (Fig. 3) as follows:

- Step one: data reprocessing ([4]), such as data smoothing, outlier detection ([51]);
- Step two: data recovering: using machine learning methods like UDEs ([35]) or PINNs([36]) to recover the partial observed, sparsely sampled and noisy data to fully observed, continuous, differentiable data.
- Step three: data explaining: find the unknown simple equations in transmission dynamics models by data driven methods like symbolic regression ([40]) or SINDy ([6])
- Step four: handling uncertainties: keep the formula found by data driven methods, use the parameters in the equations as prior knowledge, and using bayesian inference methods such as Non-U-turn hamiltonian monte carlo to handle the uncertainties.

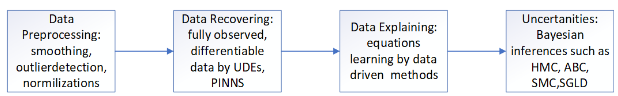

Fig. 3 A schematic illustration of the four major steps in the proposed two-step recovering-explaining method.

4 Deep Learning Based Behavioural Change Transmission Dynamics Models

In this section, the two-step recovering-explaining framework is used to discover the unknown behavioural change mechanisms. In particular, we will use the case data from the first and second wave of the COVID-19 in Ontario to fit the following neural differential equation model:

$$
\begin{aligned}
\frac{\mathrm{dS}}{\mathrm{dt}} &= -\mathrm{abs}(NN(I,R))S/N,\\
\frac{\mathrm{dI}}{\mathrm{dt}} &= -\mathrm{abs}(NN(I,R))S/N - \gamma I,\\
\frac{\mathrm{dR}}{\mathrm{dt}} &= \gamma I,\\
\frac{\mathrm{dH}}{\mathrm{dt}} &= abs(NN(I,R))S/N,
\end{aligned}
$$

where $NN(I,R)$ denotes the neural network to learn the media impact function, and H denotes the accumulated cases. Deep learning methods and symbolic regression methods are implemented in open source Julia language 1.7.3 (Bezanson et al. (2017)), Laptop Y7000P with i5-9300HF CPU, 16G RAM. The training time of deep neural networks and symbolic regression methods are approximately one hour, 30 seconds, respectively. All the details of the algorithms and codes can be found in Github repo.

4.1 The Behavioural Change Laws

To start with, we recover the data by UDEs, the results are shown in Fig. 4.

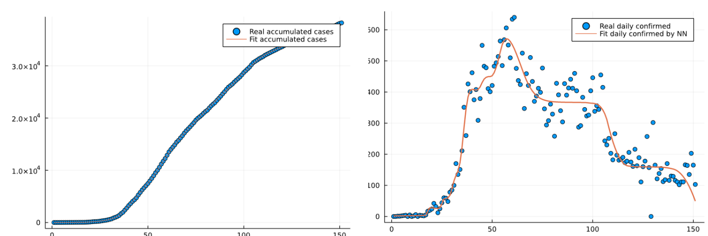

Fig. 4 Learn Ontario first wave data (150 days after Feb 25, 2020) by universal differential equations.

After recovering the data, we use symbolic regression to find the simplest equation to fit $\mathrm{abs}(NN(I,R))$, and the equation found is kind of saturated function

$$\mathrm{abs}(NN(I,R)) \approx \frac{aI+b}{R+d},$$

which implies that

$$c' = -\frac{bc}{I(aI+b)}I' - \frac{\gamma I^2 c^2}{aI+b}.$$

The result are shown in Fig. 5.

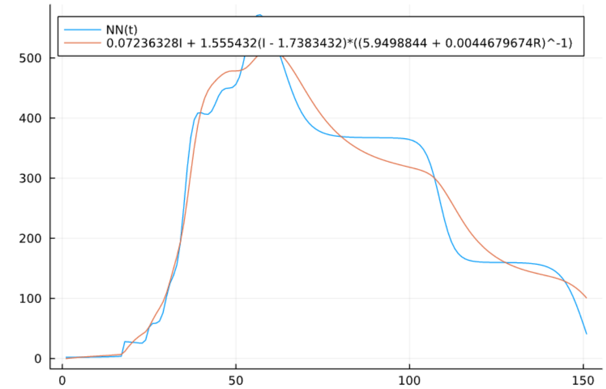

Fig. 5 Symbolic regression to find the simplest equation to fit $\mathrm{abs}(NN(I,R))$.

Using the same framework, we also fit Ontario's second COVID-19 pandemic wave data and explore the evolution of human behavior pattern. We first recover the data by UDEs, the results are shown in Fig. 6.

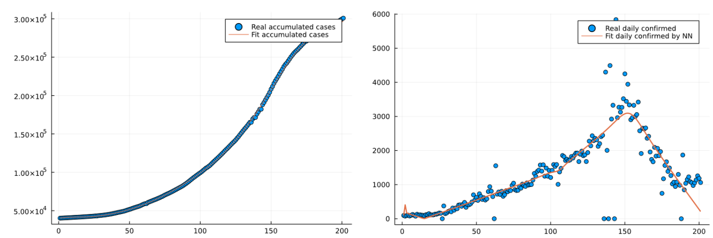

Fig. 6 Learn Ontario's second COVID-19 pandemic wave data (200 days after July 25, 2021) by universal differential equations.

After recovering the data, we use symbolic regression to find the simplest equation to fit $\mathrm{abs}(NN(I,R))$, and the equation found is as follows:

$$\mathrm{abs}(NN(I,R)) \approx \left| \frac{aI}{(R+b)^c} - d \right|,$$

which implies that

$$c' = \frac{c}{I} + \frac{I^{1.004}(cI^{1.004}+520)^2}{26} - \left(\frac{c}{I} + 520I^{0.004}\right)$$

The results are shown in Fig. 7.

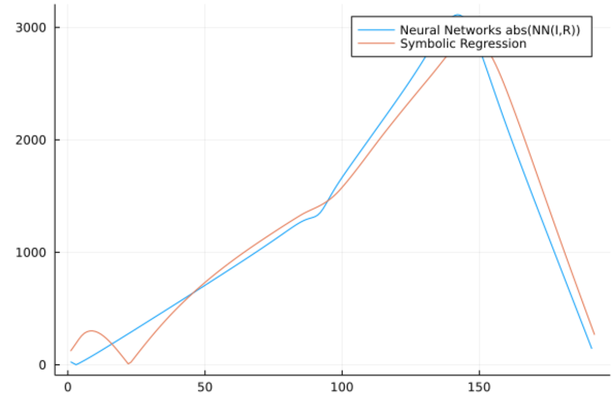

Fig. 7 Symbolic regression to find the simplest equation to fit abs$(NN(I,R))$ of Ontario's second COVID-19 pandemic wave data.

5 Discussions and Conclusions

Human behaviors can shape the spread of infectious disease, shift the epidemics away from peaks and toward plateaus, shoulders, and oscillations. Study of human behavioural change disease transmission dynamics models can guide public health measures, such as reports of mass medias. Nonetheless, the unknown behavioural change mechanisms restrict the ability of disease transmission dynamics in epidemic control. To characterize the human behavior effect on infectious diseases, traditional epidemic models greatly rely on proper assumptions. Models with good assumptions are theoretically tractable, generalizing well, interpretable and easy to communicate. However, transmission dynamics models without good assumptions often fail to replicate, fail to generalize, fail to predict outcomes of interest, and fail to offer solutions to real-world epidemiology problems ([22]). How to discover unknown mechanisms and make proper assumptions become critical issues.

Deep neural networks ([29, 19]), as a universal approximator for unknown mappings ([23, 27]) and nonlinear operators ([32]), shows incredible effectiveness ([41]) in pattern recognition. Particularly, deep neural networks can be embedded in differential equations ([8, 35]) and used to learn unknown mechanisms and solve traditional difficult problems with new perspectives. Data driven methods proposed in recent years like symbolic regression, sparse identification of nonlinear dynamical systems ([6, 39, 25, 5]), dynamic mode decomposition, ([45, 15, 5]) can find the simplest analytic expressions to describe science, engineering and real world data. Both deep learning methods and data-driven methods are defensible on their own terms, and have generated large, productive scientific literatures; however, both approaches have also been subjected to serious criticism. Deep learning methods can only output black-box terms which are hard to interpret; data-driven methods need fully observed high quality data, which is impossible in emerging infectious disease transmission dynamics because of difficulties of tracking the number of suspected and asymptomatic infected individuals. The proposed two-step recovering-explaining framework combines these two areas together, handles the weakness of black-box terms in deep learning methods and the weakness of requiring high quality observed data in equation-search methods. More explorations and applications of this two-step recovering-explaining method can be found in [43, 42, 52].

Due to the limitation of computation abilities, we have not tested the behavioural change disease transmission dynamics models in different settings. We have not added human behavioural change data such as media reports, search data in Google, Baidu. These data are most meaningful and can give a clear picture on evolution of human behaviors patterns in infectious disease. Moreover, much remains to be done in the future to improve the performance of the two-step recovering-explaining framework. A first step forward maybe to discover specific neural network architectures for specific mechanisms in epidemiology models. The second step forward may be to improve the trainability of the two-step framework. How to simultaneously train the recovering and explaining steps repay much more theoretical studies. Coupling transmission dynamics models and deep learning with combining explanation and prediction methods is a promising research field in mathematical epidemiology, and any progress in this filed can provide new insights on epidemiological phenomena such as evolution of human mobility pattern, shifting of contact matrix and mutation of variants.

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