Supplementary Material of "An Evolution Path Based Reproduction Operator for Many-Objective Optimization"

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***Abstract*—This is the supplementary material to the paper entitled "An Evolution Path Based Reproduction Operator for Many-Objective Optimization", written by Xiaoyu He, Yuren Zhou, and Zefeng Chen. This material provides a detailed description of the experimental settings and the parameter sensitivity test. Additional experimental results are also presented in this material.**

S-I. Experimental Settings

A. Test Problems

In total, 20 widely used benchmark problems are tested to evaluate the performance of the proposed reproduction operator. Among them, WFG1 to WFG9 are from the WFG test suite [1], DTLZ1 to DTLZ7 are unconstrained problems from DTLZ test suite [2], and DTLZ1^{-1} to DTLZ4^{-1} are the minus version of DTLZ1 to DTLZ4 from the Minus-DTLZ test suite [3]. For each problem, the number of objectives is set to 3, 5, 8, 10, and 15, respectively. According to the suggestions from [2], the number of decision variables is set to $D = M+r-1$ for DTLZ test suite, where $r = 5$ for DTLZ1, $r = 10$ for DTLZ2 to DTLZ6, and $r = 20$ for DTLZ7. Also, the number of decision variables is set to $D = k + l$ for WFG test suite as suggested in [1], where $k = 2 \cdot (M - 1)$ is the number of position-related variables and $l = 20$ is the number of distance-related variables. Problems in the Minus-DTLZ test suite use the same settings as their original versions.

As demonstrated in recent study [3], whether the PF shape is consistent with the distribution of reference vectors significantly influences the performance of decomposition based MaOEAs. Thus, it is essential to understand the PF shapes of these problems before conducting the experiments. DTLZ1 to DTLZ4 and WFG4 to WFG9 have regular PFs since their PFs are hyperspheres or hyperplanes. On the contrary, the other ten problems have irregular PFs. Specifically, DTLZ7 and WFG2 have disconnected PFs while the PFs of WFG1, WFG3, DTLZ5, and DTLZ6 are very complicated[1]. DTLZ1^{-1} to DTLZ4^{-1} are slightly modified from DTLZ1 to DTLZ4, respectively, but they have significantly different properties: The PF concavity of DTLZ2^{-1} to DTLZ4^{-1} is changed, while the PF of DTLZ1^{-1} is rotated[2]. According to the PF regularity, these problems are classified into two groups. When testing the effectiveness of the proposed operator, the experiments are carried out on these two groups separately.

B. Performance Metrics

In the numerical experiments, hypervolume (HV) [5] is employed to provide a joint measurement of both the convergence and diversity of the obtained solutions. It measures the volume of the objective space between the obtained solutions and a pre-defined reference point $\boldsymbol{z}_{rp}$. Obviously, a larger HV value is better. We set $\boldsymbol{z}_{rp} = (3, 5, \ldots, 2M + 1)^T$ for WFG1 to WFG9, $\boldsymbol{z}_{rp} = (1.5, 1.5, \ldots, 1.5)^T$ for DTLZ1, and $\boldsymbol{z}_{rp} = (2, 2, \ldots, 2)^T$ for DTLZ2 to DTLZ4. For DTLZ5 to DTLZ7, $\boldsymbol{z}_{rp}$ is set to the nadir point of the set of non-dominated solutions obtained by all the comparative algorithms. The HV value is calculated exactly using the WFG algorithm [6], [7] for problems with $M < 5$ and approximated by the Monte Carlo simulation [8] for problems with $M \geq 5$. When performing the simulation, 1,000,000 points are sampled to ensure the accuracy.

For the Minus-DTLZ test suite, many algorithms have difficulties in converging to the PF especially in high-dimensional objective space. That is, the HV value may be unavailable. So, the averaged Hausdorff distance (Δ_p) [9] is used as a secondary performance indicator. This indicator can be considered as a combination of the generational distance (GD) [10] and the inverted generational distance (IGD) [11]. Thus, a small Δ_p value is desirable. Compared with IGD and GD, it prefers evenly spread solutions along the PF and is able to handle the outlier tradeoff. Note that, the PF of Minus-DTLZ problems are known (hyperspheres or hyperplanes), thus, the reference points in calculating Δ_p are uniformly sampled on the PFs. In this paper, the parameter p in calculating Δ_p is set to 1.

C. Algorithms for Comparison

First, four reproduction operators including SBX [12], BLX-α [13], DE [14], and HOP [15] are incorporated in the original version of MOEA/D [16] to compare with MOEA/D-EP. The corresponding algorithms are referred to as MOEA/D-SBX, MOEA/D-BLX-α, MOEA/D-DE, and MOEA/D-HOP,

[1]WFG3, DTLZ5, and DTLZ6 are original designed as problems with degenerate PFs. However, this is only true in low-dimensional objective space as pointed out in [1], [4]. In spite of that, their PFs are very different from those of other problems. So they are also considered as having irregular PFs.

[2]The PFs of DTLZ1^{-1} to DTLZ4^{-1} are also hyperplanes or hyperspheres. But their intersections with the reference vectors used in this study are not uniformly distributed. Therefore, their PFs are considered as being irregular.

respectively. The settings of these operators are summarized as follows:

- For SBX, the crossover probability p_c is set to 1 and the distribution index η_c is set to 20.
- For BLX-α, α is set to 0.5.
- For DE, the crossover probability Cr is set to 1 and the scaling factor F is set to 0.5.
- For HOP, the probability of using interpolation p_{inter} is set to 0.67 and the probability of performing traditional DE operations p_c is set to 0.75.

Then, we compare NSGA-III-EP with NSGA-III. Finally, NSGA-III-EP and MOEA/D-EP are compared with five state-of-the-art MaOEAs including MOEA/DD [17], MOMBI2 [18], MOEA/D-DU [19], RVEA [20], and VaEA [21]. Several common parameters are set as follows:

- In MOEA/D-SBX, MOEA/D-BLX-α, MOEA/D-DE, MOEA/D-HOP, MOEA/D-EP, MOEA/DD, and MOEA/D-DU, the neighborhood size T is set to $\lceil N/10 \rceil$. The PBI approach with a penalty parameter $\theta = 5$ is employed.
- The polynomial mutation with probability $p_m = 1/D$ and distribution index $\eta_m = 20$ is used in all algorithms.
- The SBX crossover with probability $p_c = 1$ and distribution index $\eta_m = 30$ is used in NSGA-III, MOEA/DD, MOMBI2, MOEA/D-DU, RVEA, and VaEA.
- In MOEA/D-EP and NSGA-III-EP, HL is set to $\lceil N/10 \rceil$ and S is set to 4.

Some algorithms have the following specific parameters:

- In MOEA/DD, the probability of choosing parents from the neighborhood δ is set to 0.9.
- In MOEA/D-DU, the parameter K is set to 5.
- In RVEA, the rate α is set to 2 and the adaptation frequency fr is set to 0.1.
- In MOMBI2, the threshold of variances α is set to 0.5, the record length is set to 5, and the tolerance threshold is set to 0.001. The modified Tchebycheff approach [22] is adopted to calculate the aggregation values.

All these algorithms are implemented in the open source software jMetal [23].

D. General Experimental Settings

Some general settings for the numerical experiments are listed as follows:

1) Termination criterion: The only one termination criterion in our experiments is the maximum number of function evaluations allowed (maxFEs). The settings of maxFEs on each problem for different numbers of objectives are summarized in Table V.

2) Statistical method: All algorithms are independently run 20 times on each test instance. The median and interquartile range (IQR) are utilized to measure the average value and variability of the result data, respectively. To compare the performance between two algorithms, the Wilcoxon signed rank test [24] is carried out to test whether there is a statistical difference at a 0.05 significance level.

TABLE V
THE SETTINGS OF MAXFES FOR DIFFERENT NUMBERS OF OBJECTIVES

Problem	$M = 3$	$M = 5$	$M = 8$	$M = 10$	$M = 15$
DTLZ1 DTLZ1^{-1}	36400	126000	117000	275000	202500
DTLZ2,5,6 DTLZ2^{-1}	22750	73500	78000	206250	135000
DTLZ3,7 DTLZ3^{-1}	91000	210000	156000	412500	270000
DTLZ4 DTLZ4^{-1}	54600	210000	195000	550000	405000
WFG1-9	36400	157500	234000	550000	405000

TABLE VI
THE SETTINGS OF H FOR DIFFERENT NUMBERS OF OBJECTIVES

M	3	5	8	10	15
H	91	210	156	275	135

3) Reference Vectors and Population Size: The set of well-spread reference vectors is produced using the Das and Dennis' systematic approach [25]. For problems having more than five objectives, the two layers of weight vectors [26] are adopted to avoid producing too much boundary vectors. Table VI summaries the settings of the number of reference vectors (H). In all algorithms, we set the population size $N = H$.

S-II. PARAMETER SENSITIVITY

In this section, we discuss the parameters in the proposed operator. The first parameter, denoted by S, is the number of evolution paths corresponding to each reference vector. Though S could be any positive integer, a guideline of its setting is provided as follows: Remember that, before generating an offspring, a potential solution is produced firstly by extrapolating a polynomial (see Eq. (4)). Since this polynomial is built using Lagrage interpolation, $S - 1$ is exactly equal to the order of this polynomial. It is known that when using extrapolating, its accuracy may decrease significantly when the order increases. Therefore, we suggest that the value of S should be small (i.e., $S \leq 4$).

The second parameter is the length of historical memories HL. Here we describe the motivation why we introduce this parameter. After a potential solution is produced, it is injected into a DE operator to produce the final offspring (see Eq. (5) and (6)). The traditional DE operator employs two parameters F and Cr and extensive studies have demonstrated that their values significantly influence the algorithm performance. So a historical memory based self-adaptation mechanism is designed to control them automatically, and HL is an internal parameter of this mechanism.

To investigate the impacts of S and HL on the performance, we have tested 30 combinations of ten values of HL (i.e., $HL/H \in \{0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5\}$) and three values of S (i.e., $2, 3$ and 4) in MOEA/D-EP. Four 15-objective problems including DTLZ3, DTLZ4, WFG8,

TABLE VII
MEDIAN AND IQR OF HV RESULTS ON THE WFG TEST SUITE. THE BEST AND THE SECOND BEST RESULTS FOR EACH TEST INSTANCE ARE SHOWN WITH DARK AND LIGHT GRAY BACKGROUND, RESPECTIVELY.

Problem	M	MOEA/D-EP	NSGA-III-EP	MOEA/DD	MOMBI2	MOEA/D-DU	RVEA	VaEA
WFG1	3	4.93e−1(8.6e−3)	5.22e−1(1.2e−2)	5.64e−1(3.2e−2)	5.51e−1(4.0e−2)	5.47e−1(2.5e−2)	5.52e−1(2.6e−2)	5.59e−1(2.7e−2)
	5	7.27e−1(3.9e−2)	5.41e−1(2.1e−2)	6.74e−1(2.6e−2) ◇	8.10e−1(2.1e−2)	7.65e−1(2.1e−2)	7.69e−1(2.4e−2)	5.57e−1(3.9e−2) ◇
	8	9.31e−1(5.4e−3)	5.14e−1(5.1e−2)	7.40e−1(4.0e−2) ◇	8.88e−1(2.3e−2) ◇	8.93e−1(1.5e−2) ◇	8.49e−1(3.4e−2) ◇	8.21e−1(2.4e−2) ◇
	10	9.39e−1(2.2e−3)	6.61e−1(4.5e−2)	8.80e−1(2.0e−2) ◇	9.35e−1(9.2e−3)	9.20e−1(3.8e−3) ◇	8.96e−1(1.3e−2) ◇	8.94e−1(7.0e−3) ◇
	15	8.86e−1(4.3e−2)	6.40e−1(6.7e−2)	7.69e−1(6.8e−2) ◇	7.51e−1(1.2e−1) ◇	8.75e−1(2.9e−2)	8.01e−1(5.5e−2) ◇	8.99e−1(4.2e−3)
WFG2	3	8.90e−1(2.0e−2)	9.06e−1(9.9e−3)	9.08e−1(6.4e−2)	8.47e−1(5.2e−2) ‡	8.74e−1(6.4e−2)	8.79e−1(6.8e−2)	9.15e−1(5.9e−2)
	5	9.66e−1(4.0e−3)	9.46e−1(5.4e−3)	9.56e−1(4.7e−2)	9.29e−1(8.1e−2)	9.55e−1(4.7e−2)	9.85e−1(3.2e−3)	9.83e−1(1.7e−3)
	8	9.58e−1(2.9e−3)	9.82e−1(3.7e−3)	9.14e−1(6.8e−2) ‡	9.27e−1(8.5e−2)	9.18e−1(6.5e−2) ‡	9.03e−1(8.3e−2) †	9.70e−1(5.0e−2)
	10	9.63e−1(2.6e−3)	9.92e−1(2.9e−3)	9.63e−1(3.1e−3) †	9.37e−1(8.4e−2)	9.51e−1(3.8e−2) ‡	9.49e−1(6.8e−2) †	9.82e−1(3.8e−2) †
	15	9.59e−1(3.7e−3)	9.89e−1(3.2e−3)	9.34e−1(3.4e−2) ‡	8.73e−1(7.9e−2) ‡	9.29e−1(4.8e−2) ‡	9.06e−1(7.8e−2) †	9.57e−1(7.1e−2)
WFG3	3	7.23e−1(7.7e−3)	7.15e−1(1.1e−2)	7.11e−1(9.5e−3) ◇	7.10e−1(2.0e−2) ◇	7.25e−1(7.9e−3)	7.27e−1(7.6e−3)	7.39e−1(4.8e−3)
	5	6.48e−1(1.0e−2)	7.04e−1(7.8e−3)	6.69e−1(1.1e−2) †	6.64e−1(1.6e−2) †	6.98e−1(7.5e−3) †	7.11e−1(5.4e−3)	6.92e−1(9.6e−3) †
	8	5.02e−1(1.5e−2)	5.92e−1(2.2e−2)	5.81e−1(1.2e−2) †	2.63e−1(1.7e−1) ‡	5.95e−1(1.5e−2)	3.95e−1(9.5e−2) ‡	6.55e−1(1.5e−2)
	10	3.36e−1(4.6e−2)	5.95e−1(1.6e−2)	5.70e−1(1.1e−2) †	5.59e−1(1.4e−1)	5.60e−1(3.0e−2) †	4.19e−1(1.1e−1) †	6.51e−1(1.8e−2)
	15	2.47e−1(1.3e−3)	6.02e−1(4.4e−2)	4.39e−1(1.3e−2) †	3.99e−1(2.3e−2) †	2.41e−1(1.8e−3) ‡	2.34e−1(7.0e−3) ‡	6.10e−1(2.2e−2)
WFG4	3	7.41e−1(4.0e−3)	7.58e−1(4.5e−3)	7.82e−1(1.1e−3)	7.55e−1(8.8e−3)	7.63e−1(3.9e−3)	7.77e−1(2.5e−3)	7.73e−1(2.8e−3)
	5	8.52e−1(1.2e−2)	8.86e−1(4.5e−3)	9.06e−1(2.8e−3)	8.57e−1(1.2e−2) †	9.00e−1(3.0e−3)	9.03e−1(3.7e−3)	8.69e−1(3.9e−3) †
	8	6.93e−1(2.3e−2)	9.47e−1(2.2e−3)	9.23e−1(5.4e−3) †	8.70e−1(3.5e−2) †	8.73e−1(1.5e−2) †	9.36e−1(1.4e−2) †	9.15e−1(7.0e−3) †
	10	7.40e−1(3.3e−2)	9.71e−1(1.2e−3)	9.30e−1(8.3e−3) †	9.17e−1(2.7e−2) †	8.70e−1(1.6e−2) †	9.68e−1(6.4e−3)	9.22e−1(4.9e−3) †
	15	6.20e−1(6.3e−2)	9.79e−1(2.4e−3)	9.02e−1(4.1e−2) †	6.63e−1(7.2e−2) †	7.98e−1(5.5e−2) †	9.64e−1(1.8e−2) †	9.29e−1(7.6e−3) †
WFG5	3	7.46e−1(8.6e−4)	7.50e−1(1.3e−3)	7.51e−1(2.5e−3)	7.27e−1(3.4e−3) ‡	7.41e−1(2.1e−3) ‡	7.51e−1(3.0e−3)	7.53e−1(4.0e−3)
	5	8.51e−1(2.8e−3)	8.72e−1(1.5e−3)	8.65e−1(1.7e−3) †	8.42e−1(1.1e−2) ‡	8.63e−1(2.0e−3) †	8.76e−1(2.6e−3)	8.57e−1(4.3e−3) †
	8	7.27e−1(2.4e−2)	8.97e−1(1.1e−2)	8.66e−1(7.3e−3) †	8.67e−1(6.7e−3) †	8.27e−1(1.3e−2) †	9.26e−1(7.7e−4)	9.02e−1(4.1e−3)
	10	7.59e−1(2.1e−2)	9.15e−1(9.8e−3)	8.70e−1(7.3e−3) †	8.93e−1(4.3e−3) †	8.27e−1(1.7e−2) †	9.37e−1(3.6e−4)	9.06e−1(5.3e−3) †
	15	6.04e−1(2.7e−2)	9.11e−1(9.3e−3)	7.79e−1(2.1e−2) †	8.51e−1(6.5e−2) †	7.16e−1(2.5e−2) †	9.37e−1(6.6e−5)	9.02e−1(2.9e−3) †
WFG6	3	7.31e−1(1.5e−3)	7.40e−1(2.4e−4)	7.57e−1(3.9e−3)	7.43e−1(5.3e−3)	7.42e−1(6.3e−3)	7.56e−1(3.8e−3)	7.56e−1(5.5e−3)
	5	8.35e−1(7.7e−3)	8.56e−1(1.9e−4)	8.79e−1(5.4e−3)	8.39e−1(2.3e−2) †	8.72e−1(5.5e−3)	8.81e−1(4.3e−3)	8.55e−1(6.9e−3)
	8	6.23e−1(5.9e−2)	8.87e−1(4.0e−4)	9.02e−1(1.1e−2)	8.80e−1(1.2e−2) †	8.39e−1(1.5e−2) †	9.24e−1(8.1e−3)	9.20e−1(8.2e−3)
	10	6.88e−1(3.9e−2)	8.94e−1(6.2e−4)	8.94e−1(1.2e−2)	9.01e−1(7.6e−3)	8.18e−1(2.2e−2) †	9.41e−1(8.3e−3)	9.24e−1(9.2e−3)
	15	5.34e−1(7.2e−2)	8.86e−1(1.7e−3)	8.04e−1(2.7e−2) †	8.69e−1(4.2e−2)	6.42e−1(6.1e−2) †	9.24e−1(1.9e−2)	9.26e−1(1.1e−2)
WFG7	3	7.27e−1(7.4e−3)	7.34e−1(6.8e−3)	7.87e−1(1.6e−3)	7.69e−1(2.7e−3)	7.50e−1(9.3e−3)	7.84e−1(1.7e−3)	7.82e−1(2.0e−3)
	5	8.49e−1(1.3e−2)	8.85e−1(5.5e−3)	9.13e−1(1.3e−3)	8.83e−1(7.1e−3)	9.11e−1(1.6e−3)	9.20e−1(1.3e−3)	8.95e−1(2.3e−3)
	8	6.51e−1(4.3e−2)	9.56e−1(4.6e−3)	9.47e−1(3.3e−3) †	9.07e−1(2.7e−2) †	8.97e−1(9.8e−3) †	9.45e−1(1.3e−2) †	9.56e−1(2.3e−3)
	10	7.18e−1(3.0e−2)	9.82e−1(1.7e−3)	9.57e−1(2.0e−3) †	9.39e−1(1.7e−2) †	9.10e−1(1.0e−2) †	9.79e−1(3.1e−3) †	9.65e−1(1.8e−3) †
	15	6.07e−1(1.6e−3)	9.92e−1(3.4e−3)	9.39e−1(3.3e−2) †	8.40e−1(2.8e−2) †	8.67e−1(9.0e−3) †	9.93e−1(1.7e−3)	9.65e−1(3.1e−3) †
WFG8	3	6.84e−1(9.0e−3)	6.91e−1(5.3e−3)	7.31e−1(2.5e−3)	7.06e−1(6.3e−3)	7.22e−1(4.3e−3)	7.21e−1(4.2e−3)	7.17e−1(2.3e−3)
	5	7.46e−1(4.7e−2)	8.16e−1(7.4e−3)	8.38e−1(5.2e−3)	6.88e−1(7.6e−3) ‡	8.45e−1(4.3e−3)	8.32e−1(5.6e−3)	7.92e−1(5.5e−3) †
	8	4.60e−1(5.7e−2)	8.71e−1(1.2e−2)	8.10e−1(3.3e−2) †	7.43e−1(1.3e−2) †	7.11e−1(5.6e−2) †	6.53e−1(6.7e−2) †	8.05e−1(6.5e−3) †
	10	5.05e−1(6.3e−2)	9.14e−1(1.0e−2)	8.35e−1(5.0e−2) †	7.78e−1(1.6e−2) †	7.50e−1(5.4e−2) †	7.26e−1(9.0e−2) †	8.33e−1(8.5e−3) †
	15	4.16e−1(9.2e−2)	8.84e−1(2.6e−2)	8.63e−1(9.6e−2)	6.83e−1(2.4e−2) †	5.59e−1(8.1e−2) †	6.18e−1(2.4e−1) †	8.77e−1(8.5e−3)
WFG9	3	7.08e−1(1.3e−3)	7.12e−1(1.2e−3)	7.21e−1(1.5e−2)	7.07e−1(1.7e−2)	7.04e−1(6.2e−3) ‡	7.21e−1(1.3e−2)	7.26e−1(1.4e−2)
	5	7.97e−1(1.7e−3)	8.05e−1(1.9e−3)	8.12e−1(1.5e−2)	6.99e−1(1.8e−2) ‡	8.15e−1(1.1e−2)	8.30e−1(1.4e−2)	7.84e−1(3.0e−3) ‡
	8	6.19e−1(1.4e−2)	8.01e−1(1.1e−2)	7.69e−1(3.0e−2) †	7.73e−1(1.6e−2) †	7.29e−1(1.5e−2) †	8.43e−1(1.4e−2)	7.83e−1(6.2e−3) †
	10	5.59e−1(7.3e−2)	8.09e−1(7.0e−3)	7.62e−1(2.5e−2) †	7.88e−1(2.6e−2) †	6.75e−1(1.1e−2) †	8.62e−1(8.1e−3)	7.84e−1(1.3e−2) †
	15	4.84e−1(1.5e−1)	7.68e−1(1.1e−2)	6.11e−1(5.3e−2) †	6.60e−1(5.5e−2) †	5.07e−1(3.5e−2) †	8.05e−1(4.2e−2)	7.39e−1(2.2e−2) †

"◇" indicates that the result is significantly outperformed by MOEA/D-EP.
"†" indicates that the result is significantly outperformed by NSGA-III-EP.
"‡" indicates that the result is significantly outperformed by both MOEA/D-EP and NSGA-III-EP.

and WFG9 are chosen to test the algorithm performance. All the other parameters remain the same as in the above experiments. Fig. 16 shows the median HV values obtained through 20 independent runs on each combination. On DTLZ3 and DTLZ4, no matter what the values the parameters are, the obtained HV values are equal to 1. On WFG8 and WFG9, it seems that setting S to 4 may result to a better result, but the difference is not significant. In summary, the proposed operator is insensitive to both S and HL.

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TABLE VIII

MEDIAN AND IQR OF HV RESULTS ON THE DTLZ TEST SUITE. THE BEST AND THE SECOND BEST RESULTS FOR EACH TEST INSTANCE ARE SHOWN WITH DARK AND LIGHT GRAY BACKGROUND, RESPECTIVELY.

Problem	M	MOEA/D-EP	NSGA-III-EP	MOEA/DD	MOMBI2	MOEA/D-DU	RVEA	VaEA
DTLZ1	3	9.62e − 1(1.0e − 1)	8.71e − 1(1.6e − 1)	9.94e − 1(5.5e − 5)	9.94e − 1(3.0e − 4)	9.94e − 1(5.5e − 5)	9.94e − 1(4.5e − 5)	9.91e − 1(1.0e − 3)
	5	1.00e + 0(1.3e − 5)	9.90e − 1(1.3e − 2)	1.00e + 0(7.9e − 6)	1.00e + 0(8.7e − 6) ◇	1.00e + 0(8.2e − 6)	1.00e + 0(7.9e − 6)	1.00e + 0(9.6e − 5) ◇
	8	1.00e + 0(6.9e − 7)	2.10e − 1(3.0e − 1)	1.00e + 0(9.0e − 6) ◇	1.00e + 0(1.0e − 6) ◇	1.00e + 0(6.8e − 6) ◇	1.00e + 0(6.2e − 6) ◇	1.00e + 0(3.9e − 5) ◇
	10	1.00e + 0(0.0e + 0)	4.47e − 1(3.3e − 1)	1.00e + 0(3.8e − 6) ◇	1.00e + 0(3.7e − 9) ◇	1.00e + 0(3.1e − 6) ◇	1.00e + 0(2.4e − 6) ◇	1.00e + 0(3.2e − 5) ◇
	15	1.00e + 0(0.0e + 0)	6.51e − 2(1.7e − 1)	1.00e + 0(4.1e − 5) ◇	1.00e + 0(4.7e − 7) ◇	1.00e + 0(1.1e − 5) ◇	1.00e + 0(3.2e − 6) ◇	1.00e + 0(9.2e − 5) ◇
DTLZ2	3	9.43e − 1(2.0e − 4)	9.44e − 1(6.5e − 5)	9.45e − 1(1.2e − 5)	9.44e − 1(7.9e − 4)	9.45e − 1(1.7e − 5)	9.45e − 1(3.5e − 5)	9.44e − 1(1.5e − 4) †
	5	9.93e − 1(2.0e − 4)	9.94e − 1(6.4e − 5)	9.94e − 1(7.8e − 5)	9.93e − 1(4.2e − 4) †	9.94e − 1(7.7e − 5)	9.94e − 1(7.3e − 5)	9.93e − 1(1.1e − 4) †
	8	1.00e + 0(1.5e − 5)	1.00e + 0(2.0e − 5)	1.00e + 0(1.7e − 5)	9.99e − 1(5.1e − 5) ‡	1.00e + 0(1.6e − 5)	1.00e + 0(1.8e − 5)	1.00e + 0(4.2e − 5) ‡
	10	1.00e + 0(6.8e − 6)	1.00e + 0(5.4e − 6)	1.00e + 0(5.3e − 6)	1.00e + 0(1.3e − 5) ‡	1.00e + 0(5.0e − 6)	1.00e + 0(4.1e − 6)	1.00e + 0(1.9e − 5) ‡
	15	1.00e + 0(5.9e − 7)	1.00e + 0(4.4e − 7)	1.00e + 0(9.0e − 7) ‡	9.99e − 1(2.2e − 3) ‡	1.00e + 0(7.0e − 7) ‡	1.00e + 0(1.1e − 6) ‡	1.00e + 0(6.2e − 6) ‡
DTLZ3	3	3.64e − 1(4.3e − 1)	4.42e − 1(3.6e − 1)	9.45e − 1(2.7e − 4)	9.44e − 1(5.7e − 4)	9.44e − 1(5.6e − 4)	9.44e − 1(6.4e − 4)	9.44e − 1(5.2e − 4)
	5	2.69e − 1(4.0e − 1)	3.53e − 1(3.8e − 1)	9.94e − 1(8.1e − 5)	9.94e − 1(1.6e − 4)	9.94e − 1(7.9e − 5)	9.94e − 1(7.0e − 5)	9.93e − 1(2.0e − 4)
	8	5.56e − 1(4.8e − 1)	0.00e + 0(0.0e + 0)	1.00e + 0(1.5e − 5)	1.00e + 0(3.0e − 5)	1.00e + 0(2.1e − 5)	1.00e + 0(2.4e − 5)	9.04e − 1(2.3e − 1)
	10	9.98e − 1(8.3e − 3)	0.00e + 0(0.0e + 0)	1.00e + 0(6.3e − 6)	1.00e + 0(8.1e − 6) ◇	1.00e + 0(5.5e − 6)	1.00e + 0(6.2e − 6)	9.94e − 1(2.4e − 2) ◇
	15	9.57e − 1(1.3e − 1)	0.00e + 0(0.0e + 0)	1.00e + 0(8.2e − 7)	1.00e + 0(9.9e − 4) ◇	1.00e + 0(8.9e − 7)	1.00e + 0(7.4e − 7)	8.58e − 1(3.0e − 1) ◇
DTLZ4	3	9.45e − 1(2.5e − 5)	9.45e − 1(2.6e − 5)	9.45e − 1(4.6e − 7)	9.28e − 1(4.0e − 2)	9.45e − 1(1.2e − 6)	9.45e − 1(3.3e − 6)	9.44e − 1(1.9e − 4) ‡
	5	9.94e − 1(6.8e − 5)	9.94e − 1(8.8e − 5)	9.94e − 1(6.1e − 5)	9.94e − 1(6.0e − 5)	9.94e − 1(6.1e − 5)	9.94e − 1(6.2e − 5)	9.93e − 1(1.3e − 4) ‡
	8	1.00e + 0(1.8e − 5)	1.00e + 0(1.7e − 5)	1.00e + 0(1.4e − 5)	1.00e + 0(2.5e − 4)	1.00e + 0(1.4e − 5)	1.00e + 0(1.4e − 5)	1.00e + 0(3.6e − 5) ‡
	10	1.00e + 0(4.6e − 6)	1.00e + 0(4.6e − 6)	1.00e + 0(6.1e − 6)	1.00e + 0(1.4e − 5)	1.00e + 0(6.0e − 6)	1.00e + 0(6.2e − 6)	1.00e + 0(8.5e − 6) ‡
	15	1.00e + 0(3.7e − 7)	1.00e + 0(5.7e − 7)	1.00e + 0(1.1e − 6) ‡	1.00e + 0(2.0e − 6) ‡	1.00e + 0(1.1e − 6) ‡	1.00e + 0(1.0e − 6) ‡	1.00e + 0(1.3e − 6) ‡
DTLZ5	3	1.83e − 1(3.7e − 4)	1.91e − 1(1.2e − 3)	1.81e − 1(8.7e − 4) ‡	1.90e − 1(2.0e − 4) †	1.64e − 1(1.9e − 3) ‡	1.49e − 1(9.9e − 3) ‡	1.99e − 1(1.9e − 4)
	5	1.27e − 1(3.6e − 4)	1.10e − 1(6.6e − 3)	1.09e − 1(5.9e − 3) ◇	1.06e − 1(6.9e − 3) ‡	1.20e − 1(2.7e − 4) ◇	1.06e − 1(4.1e − 3) ‡	9.90e − 2(5.2e − 3) ‡
	8	1.04e − 1(2.8e − 4)	9.30e − 2(1.9e − 3)	5.78e − 2(2.0e − 2) ‡	9.69e − 2(2.8e − 3) ◇	9.98e − 2(2.7e − 4) ◇	9.07e − 2(6.4e − 4) ‡	8.86e − 2(1.5e − 3) ‡
	10	1.00e − 1(3.4e − 4)	9.14e − 2(6.8e − 4)	8.34e − 2(5.3e − 3) ‡	9.56e − 2(1.8e − 3) ◇	9.66e − 2(3.2e − 4) ◇	9.09e − 2(8.5e − 5) ‡	8.81e − 2(2.9e − 3) ‡
	15	9.44e − 2(3.6e − 4)	9.11e − 2(2.3e − 4)	8.34e − 2(6.7e − 3) ‡	9.13e − 2(7.5e − 4) ◇	9.25e − 2(3.0e − 4) ◇	9.09e − 2(1.6e − 4) ‡	9.10e − 2(3.8e − 4) ◇
DTLZ6	3	1.82e − 1(6.4e − 6)	1.88e − 1(1.8e − 3)	0.00e + 0(0.0e + 0) ‡	1.18e − 4(3.3e − 4) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡
	5	1.27e − 1(3.5e − 4)	1.06e − 1(6.3e − 3)	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡
	8	1.04e − 1(3.1e − 4)	9.10e − 2(1.7e − 4)	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡
	10	1.00e − 1(3.2e − 4)	8.65e − 2(2.0e − 2)	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	7.89e − 8(2.3e − 7) ‡	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡
	15	9.44e − 2(2.8e − 4)	9.10e − 2(3.6e − 4)	0.00e + 0(0.0e + 0) ‡	0.00e + 0(0.0e + 0) ‡	1.97e − 2(2.0e − 2) ‡	1.83e − 7(6.7e − 7) ‡	0.00e + 0(0.0e + 0) ‡
DTLZ7	3	2.50e − 1(1.6e − 2)	2.47e − 1(6.2e − 3)	2.58e − 1(5.0e − 4)	2.68e − 1(1.6e − 2)	2.55e − 1(1.4e − 3)	2.65e − 1(1.3e − 3)	2.71e − 1(1.8e − 2)
	5	1.42e − 1(1.1e − 3)	2.21e − 1(7.7e − 3)	1.43e − 1(2.2e − 3) †	2.60e − 1(3.0e − 3)	1.45e − 1(6.6e − 4) †	2.18e − 1(4.3e − 3)	2.50e − 1(2.8e − 3)
	8	3.62e − 5(7.9e − 6)	1.46e − 1(9.5e − 3)	1.40e − 3(1.1e − 3) †	1.85e − 1(6.6e − 3)	1.44e − 4(1.3e − 4) †	1.35e − 1(2.2e − 2)	1.56e − 1(6.6e − 3)
	10	2.29e − 6(7.1e − 7)	1.37e − 1(9.5e − 3)	2.39e − 4(2.0e − 4) †	1.72e − 1(4.6e − 3)	2.37e − 6(8.4e − 7) †	1.23e − 1(1.7e − 2) †	1.37e − 1(9.3e − 3)
	15	3.18e − 6(7.3e − 6)	1.31e − 1(3.9e − 3)	6.31e − 7(9.4e − 7) †	1.17e − 1(1.4e − 2) †	6.70e − 9(7.9e − 9) ‡	1.23e − 1(4.7e − 3) †	9.11e − 2(6.6e − 4) †

TABLE IX

MEDIAN AND IQR OF Δ_p RESULTS ON THE MINUS-DTLZ TEST SUITE. THE BEST AND THE SECOND BEST RESULTS FOR EACH TEST INSTANCE ARE SHOWN WITH DARK AND LIGHT GRAY BACKGROUND, RESPECTIVELY.

Problem	M	MOEA/D-EP	NSGA-III-EP	MOEA/DD	MOMBI2	MOEA/D-DU	RVEA	VaEA
DTLZ1^{-1}	3	4.34e + 1(2.0e + 0)	3.60e + 1(7.9e − 1)	9.95e + 1(5.7e + 0) ‡	3.59e + 1(1.2e + 0)	4.16e + 1(1.2e + 0) †	4.65e + 1(1.6e + 0) ‡	2.44e + 1(3.6e − 1)
	5	6.90e + 1(2.1e − 1)	8.17e + 1(1.3e + 0)	1.07e + 2(2.0e + 0) ‡	1.24e + 2(1.3e + 0) ‡	9.25e + 1(1.9e + 0) ‡	1.40e + 2(6.3e + 0) ‡	5.89e + 1(4.1e − 1)
	8	2.44e + 2(5.8e + 0)	1.69e + 2(1.5e + 0)	1.92e + 2(4.3e + 0) †	1.80e + 2(1.9e + 0) †	1.90e + 2(5.7e + 0) †	3.16e + 2(3.4e + 1) ‡	1.15e + 2(6.7e − 1)
	10	2.56e + 2(6.2e + 0)	1.72e + 2(1.9e + 0)	1.94e + 2(2.8e + 0) †	1.86e + 2(2.3e + 0) †	1.94e + 2(2.7e + 0) †	3.21e + 2(3.8e + 1) ‡	1.21e + 2(6.0e − 1)
	15	3.22e + 2(7.2e + 0)	1.89e + 2(2.1e + 0)	2.15e + 2(2.3e + 0) †	2.20e + 2(5.3e + 0) †	2.16e + 2(2.9e + 0) †	3.69e + 2(4.1e + 1) ‡	1.66e + 2(1.1e + 0)
DTLZ2^{-1}	3	2.52e − 1(5.0e − 3)	2.77e − 1(6.4e − 3)	6.02e − 1(3.6e − 2) ‡	7.90e − 1(7.6e − 2) ‡	2.82e − 1(1.0e − 2) ◇	2.84e − 1(4.6e − 3) ‡	2.48e − 1(7.4e − 3)
	5	8.12e − 1(1.5e − 3)	8.19e − 1(1.3e − 2)	1.35e + 0(1.7e − 2) ‡	1.12e + 0(2.5e − 2) ‡	8.27e − 1(3.9e − 3) ‡	9.52e − 1(7.5e − 3) ‡	6.02e − 1(6.3e − 3)
	8	2.12e + 0(3.5e − 3)	1.93e + 0(3.5e − 2)	1.98e + 0(3.3e − 2) †	2.04e + 0(1.1e − 2) †	1.98e + 0(2.6e − 2) †	2.13e + 0(3.3e − 2) ‡	1.28e + 0(7.4e − 3)
	10	2.42e + 0(1.5e − 2)	2.15e + 0(3.9e − 2)	2.17e + 0(2.4e − 2) †	2.38e + 0(1.4e − 2) †	2.17e + 0(2.0e − 2) †	2.36e + 0(2.3e − 2) †	1.43e + 0(6.5e − 3)
	15	3.05e + 0(2.0e − 2)	2.87e + 0(2.9e − 2)	2.71e + 0(3.6e − 2)	3.02e + 0(1.5e − 2) †	2.83e + 0(4.3e − 2)	2.98e + 0(2.0e − 2) †	2.05e + 0(1.0e − 2)
DTLZ3^{-1}	3	2.17e + 3(9.3e + 0)	2.16e + 3(9.1e + 0)	1.80e + 3(1.6e + 1)	2.19e + 3(9.0e + 0) ‡	2.20e + 3(1.7e + 0) ‡	2.19e + 3(4.6e + 0) ‡	2.19e + 3(3.6e + 0) ‡
	5	2.19e + 3(5.6e + 0)	2.07e + 3(1.4e + 1)	1.33e + 3(1.5e + 1)	2.20e + 3(3.0e + 0) ‡	2.05e + 3(3.2e + 1)	2.19e + 3(4.1e + 0) †	2.17e + 3(9.4e + 0) †
	8	2.16e + 3(1.9e + 1)	2.07e + 3(1.5e + 1)	1.33e + 3(1.7e + 1)	2.20e + 3(1.8e + 0) ‡	2.07e + 3(3.4e + 1)	2.19e + 3(7.1e + 0) ‡	2.19e + 3(5.0e + 0) ‡
	10	2.19e + 3(2.4e + 0)	2.09e + 3(1.4e + 1)	1.23e + 3(2.3e + 1)	2.20e + 3(3.3e − 1) ‡	1.94e + 3(4.9e + 1)	2.19e + 3(9.0e + 0) †	2.19e + 3(6.1e + 0) †
	15	2.20e + 3(4.7e − 1)	2.13e + 3(1.2e + 1)	1.28e + 3(2.8e + 1)	2.20e + 3(9.3e − 1) †	1.93e + 3(6.4e + 1)	2.20e + 3(5.6e + 0) †	2.19e + 3(2.9e + 0) †
DTLZ4^{-1}	3	2.54e − 1(6.5e − 3)	2.71e − 1(6.9e − 3)	6.17e − 1(2.5e − 2) ‡	7.14e − 1(1.1e − 1) ‡	2.57e − 1(3.1e − 3) ◇	2.85e − 1(3.2e − 3) ‡	2.54e − 1(8.2e − 3)
	5	8.16e − 1(1.8e − 3)	7.81e − 1(9.4e − 3)	1.35e + 0(1.5e − 2) ‡	1.10e + 0(2.7e − 2) ‡	8.31e − 1(2.2e − 3) ‡	9.62e − 1(2.7e − 2) ‡	6.15e − 1(3.9e − 3)
	8	2.13e + 0(7.4e − 3)	1.57e + 0(4.1e − 2)	2.14e + 0(4.7e − 2) †	2.02e + 0(6.4e − 3) †	2.04e + 0(4.1e − 2) †	2.42e + 0(1.4e − 1) ‡	1.30e + 0(3.5e − 3)
	10	2.48e + 0(2.1e − 2)	1.73e + 0(2.3e − 2)	2.27e + 0(3.5e − 2) †	2.38e + 0(1.7e − 2) †	2.21e + 0(4.0e − 2) †	2.74e + 0(4.0e − 1) ‡	1.45e + 0(5.5e − 3)
	15	3.06e + 0(8.2e − 3)	2.28e + 0(2.3e − 2)	2.65e + 0(5.0e − 2) †	2.97e + 0(5.3e − 2) †	2.78e + 0(5.9e − 2) †	3.45e + 0(1.2e − 1) ‡	2.08e + 0(8.9e − 3)

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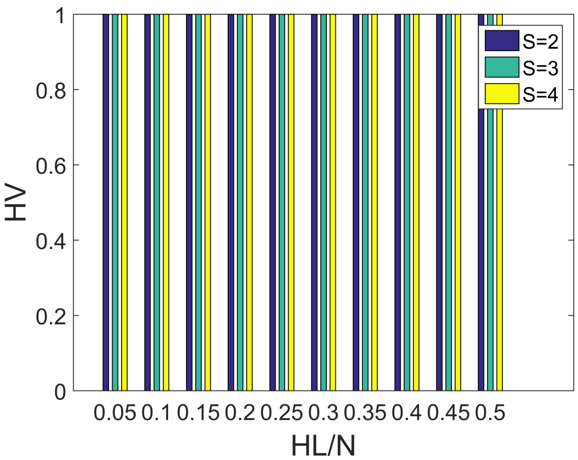
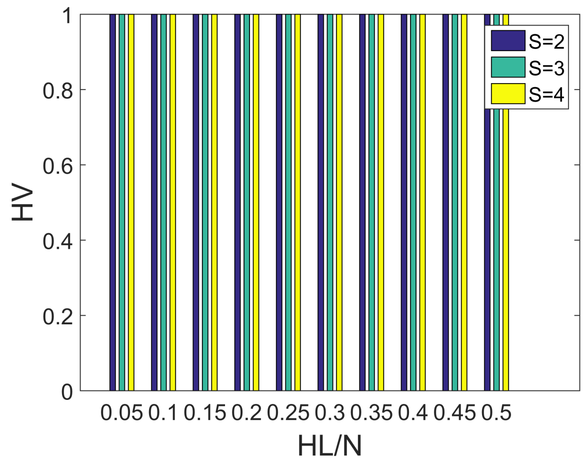
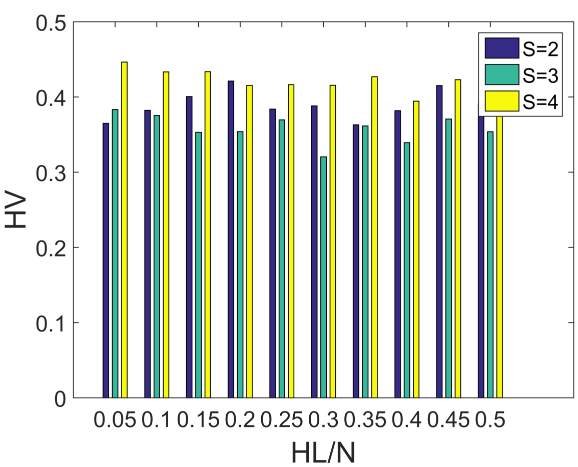


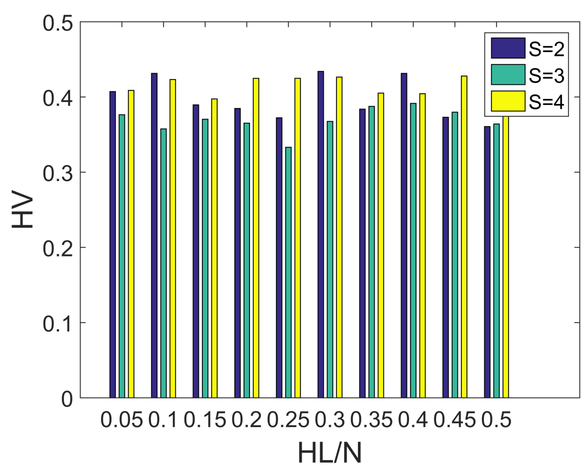
(a) DTLZ3



(b) DTLZ4



(c) WFG8



(d) WFG9

Fig. 16. Median HV values obtained by MOEA/D-EP with different combinations of HL and S values on 15-objective test instances.