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The Longest Lasting Sandcastle

Abstract

Sandcastles built on sand beaches are inevitably subject to waves and tides, but different shapes are certainly subject to waves. The purpose of this paper is to construct a 3D model system based on 3D cellular automata. By simulating the effect of wave erosion force on sandcastle and adjusting the parameters of sandcastle, we analyzes the water model which has the longest existence time under the erosion of water. The full text can be divided into seven main parts: Data acquisition and preprocessing; Model representation; Parameter estimation; Model analysis; Stability analysis; Problem solving; Advantage and disadvantage analysis.

First we identify all the data we want to use in the ideal state. However, since we do not have access to a subset of the ideal data, we must build the model using a different set of data than we expected. Incomplete, but sufficient to justify the main points of our model. Second, in order to make the paper readable and understandable, we focus only on the representation and semantics of the model and ignore the abstruse mathematical details. Firstly, we set up a 3D cellular automata and set up 3D models of different shapes by setting parameters. The 3D cellular automata was constructed to simulate the forces of waves on different positions of sandcastle according to probability theory. Third, we introduce the water-sand ratio and model slope angle parameters to establish an easily understood relationship between the basic principles and the probability theory model, and other issues are thoroughly discussed. During this process, we built four models, namely the Sandcastle Foundation Model (SFM), Wave Sine Model (WSM), Sand Stress Model (SSM), and Rain-Sand Erosion Model (RSEM), and integrated them into our final result.

In the process, we come to a conclusion:

1. The sandcastle of Elliptic cylinder has the best stability. At the same time, when all other conditions of the sandcastle remain unchanged, the higher the height of the sandcastle under the same shape condition, the better the stability of the sandcastle.
2. The sandcastle of Elliptic cylinder has the best performance when the sand-to-water mixture proportion is about 1.1%. In other words, the most stable sandcastle's water content does not need a very high concentration, as long as sand-to-water mixture proportion up to 1.1% sandcastle can achieve very high stability.
3. In the case of rain, the sandcastle of Elliptic cylinder type is still the most stable sandcastle shape. Moreover, in the case of rain, optimizing a conical spire on the sandcastle of Elliptic cylinder cannot optimize the model, but will play an opposite role.
4. Building a sandcastle far from the sea can effectively reduce the sea wave erosion, using other materials at the bottom of the sandcastle to put a layer of insulation layer to cushion the sand castle can also effectively extend the sandcastle survival time.

To sum up, the analysis of three-dimensional cellular automata model and fluid-layer water-sand ratio model is correct and reasonable. By modeling through simulation, our model creates a great deal of flexibility, making it easy to adapt to changes in the situation.

Key words: Three Dimensional Cellular Automata, Physical, Sandcastle, Wave Erosion

Contents

1 Introduction

1.1 Problem Background

Sandcastles are a popular pastime anywhere in the world where there are recreational beaches. Behind the sandcastle, what kind of water-sand ratio can build the most stable sandcastle? What kind of sandcastle has the most stable physical structure? These are issues that we need to address. In addition, the problem behind this is not only the simple problem of building sandcastles, the study of the most stable sandcastle model is also conducive to the analysis of the stability of breakthrough along the river, the analysis of the stability of quicksand and the analysis of the stability of coastal sculpture. Therefore, it is of great significance to construct the simulation sandcastle stability model.

1.2 Our Method

The overall objectives of our model are as follows:

1. We built the **Sandcastle Foundation Model (SFM)** with different structures and simulated the subsequent studies on the stability of the Sandcastle, so as to represent the structure of each part of the Sandcastle as accurately as possible.
2. Use the **Wave Sine Model (WSM)** to simulate the Wave sine model and build the Wave sine model to simulate the Wave erosion of the sandcastle.[5]
3. A **Sand Stress Model (SSM)** was constructed to simulate the sand erosion process of sandcastle.[3]
4. Build a **Rain-Sand Erosion Model (RSEM)** of sandcastle under the action of falling forces of Rain water, and realize the simulation of Rain erosion of sandcastle.

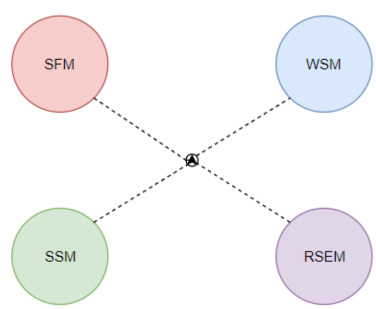


Figure 1: The structure of our model

The structure of the model we built is shown above. First, we built the SFM to simulate the shape of the sandcastle, so as to further study the disappearance process of the sandcastle in the process of erosion and decay. After that, we established the WSM to realize the simplified simulation of wave forces, so as to analyze the wave forces. Third, we established the SSM to simulate the process of sand being hit by waves. Finally, on the basis of the established model, we built the RSEM to analyze the impact of rain on sandcastle.

2 Assumption and Acronyms

2.1 Assumption

1. The movement of waves is stable, and there will be no major tsunami in the area.
2. The sandcastle is located in a suitable and stable environment, free from natural disasters such as major storms.
3. The data source is reliable.

2.2 Acronyms

Symbols	Definition
L	The length of the sandcastle
V_x	The horizontal velocity of a wave
V_y	The velocity of a wave in the vertical direction
V_w	The speed of the water along the side of the sandcastle
Δt	A very short time
f	Adhesion between grains of sand
S	The distance that the sea moves in time Δt
λ_s	The surface density of seawater
r	Sand-to-water mixture proportion
r_0	The original sand-to-water mixture proportion of the sandcastle
n	The number of grains of sand that waves can wash away at a time
$f(x)$	The equation of wave fluctuation over time at sandcastle
θ	The tilt angle of the sandcastle facade
A	Wave amplitude
γ	The wave frequency

3 Data Processing

3.1 Data Collection and Augmentation

To determine the wave vibration function, we used data provided by the global ocean wave factor seasonal variation study. The wavelength, wave propagation speed, average wave amplitude, effective wave height and average wave direction distribution involved in the statistical analysis were included. The selected data information included data of multiple coastal areas of the United States in different seasons. The data we can find includes only wind and wave information from the nine coastal areas of the United States, but fortunately, the data information in these nine coastal areas is stable and there is no significant data change, which basically ensures that the data is consistent with the facts. At the same time, through the analysis of wave data, we can fit the motion model of wave which basically satisfies the sine function, so we use sine wave here to fit the motion of wave.[1]

3.2 Data Normalization

We sorted out the collected data and classified the wave information according to the coastline length of different regions. Then, we took the coastline length of each coast as the weight and weighted average each data to obtain the data used in our model.

Let x_i be the original data obtained by us, while r_i is the data obtained by us after standardization, and c_i is the proportion of the length of each coastline in the total coastline. $i = 1, 2, 3... n$

So from the above, we can get the following equation:

$$r_i = \sum_{i=1}^{n} c_i * x_i$$

So we can get standardized data r_i, which is the data that we use when we simulate the model.

4 Model Representation

4.1 Sandcastle Foundation Model (SFM)

4.1.1 Overview

In order to find out the sandcastle that can last the longest on the coast, we used several most possible sandcastle models to simulate the wave erosion process through 3D cellular automata. To simulate the shape of the sandcastle and its motion in the waves, we used a three-dimensional cellular automata.

4.1.2 Three Dimensional Cellular Automata

Cellular automata algorithm is one of the most widely concerned algorithms in the world. In recent decades, after continuous development and improvement, it has become an algorithm with a broad application prospect. It takes the thought of system unit as the application foundation, and at the same time integrates the objective law of system process into the algorithm, which has the incomparable characteristics and advantages of traditional algorithm, and has excellent universality and stability. The advantage of cellular automata is that it can describe the collective behavior and time evolution of multiple systems with local interactions.[4] This method firstly divides the space into regular lattice composed of common features or cells. Each lattice point or cell corresponds to a finite set of values to describe the state of the unitary cell. These values correspond to a certain time, and these values change synchronously with the time step of separation according to a certain time rule, which is determined by the environment we need to simulate and the purpose we want to achieve. If the cells are distributed on a straight line equidistant, it is called one-dimensional cellular automata. If it is regularly distributed in space, it becomes a binary cellular automata. In our modeling work, the cellular automata is evenly dispersed in space to build a three-dimensional cellular automata model about sandcastle and sea waves.[7]

4.1.3 Implementation of the sandcastle model

We use the mathematical model, 3D Cellular Automata based on MATLAB tool to simulate the real situation. In our 3D Cellular Automation model, We discretize the continuous feature such as time, space and state, and we also use a certain number of cellular particles to fit the real macroscopic object like the sandcastle and waves, which, we think, is feasible and reasonable to some extent. Besides, using this model, the complex process can be simplify and have clear rules for change without leading the overall effect to be too far from reality. Moreover, It makes it easy to visualize the effects of the simulation process! In our 3D Cellular Automation model, We transform the real objects, such as sandcastles, to three-dimensional grid space in a specific proportion. Then, according to this proportion, the real related physical quantities are converted into parameters that are easy to simulate and calculate. In this way, it is beneficial to solve the deviation between the simulation effect and the real effect caused by the small size of model. In fact, it's a pity that we can't get enough computing resources to simulate larger models in this short period of time.

Because we've discretized the dynamic process, we define each state of the whole model as one unit time, and then in the simulation process, we describe the duration of sandcastle by cumulatively calculating the number of states that sandcastle can maintain. Figure 2,3,4 and 5 show the SFM rendering.

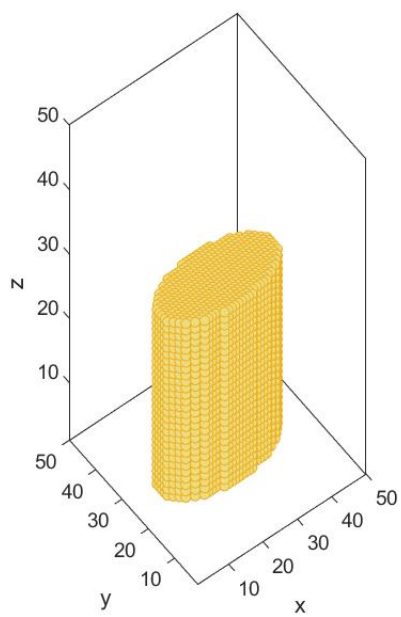


Figure 2: Elliptic cylinder

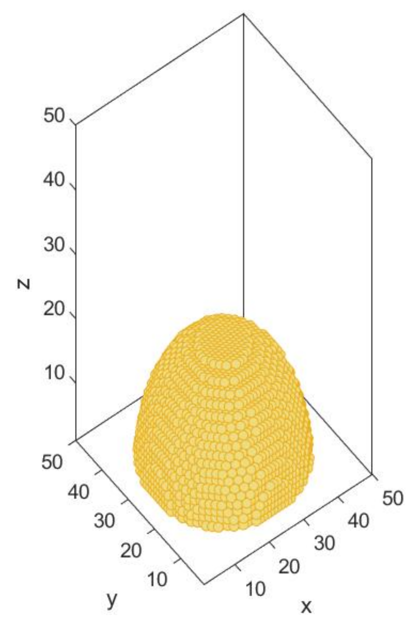


Figure 3: Hemispheroid

4.2 Wave Sine Model (WSM)

In order to truly describe the waves flow phenomenon, many researchers trying to find some methods to construct the waveform from the field of fluid mechanics, such as the Navier - Stokes equations (NSEs), while the NSEs is very time-consuming to solve, very sensitive to the time step, can easily lead to the result. In regular wave model, the simulation application of the theory of the wave energy

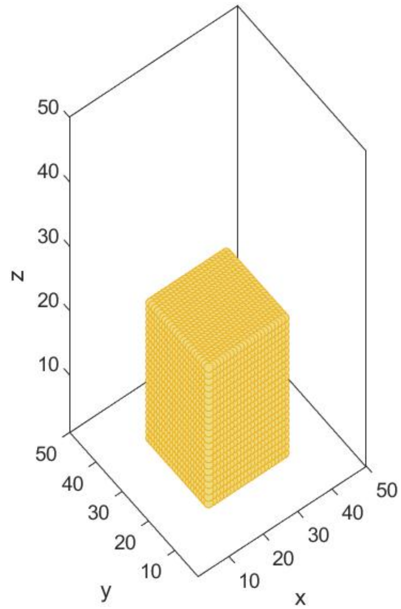


Figure 4: Cube

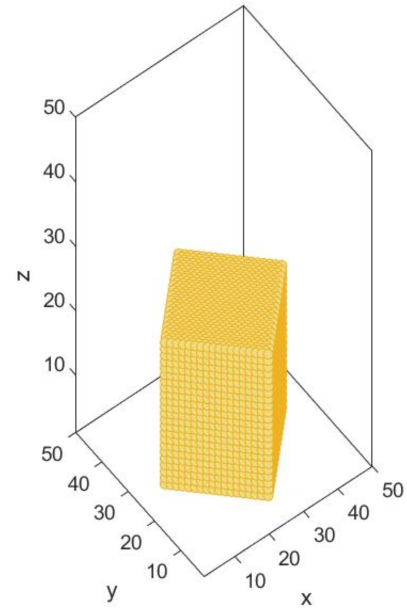


Figure 5: Prism

spectrum of ocean waves, the waves as by multiple amplitude, wavelength, completely different Jane harmonic superposition and to reflect the randomness of ocean waves, and in our model, we adopt the law of single-wave harmonic model to simulate the wave movement, for the sea erosion of sandcastle is coastal part of the simple harmonic, this part of the sea laws and stable, so we can believe that this assumption is reasonable.

Therefore, the wave model at sandcastle adopted in the model changes with time as:

$$f(x) = Asin(\gamma x) \tag{1}$$

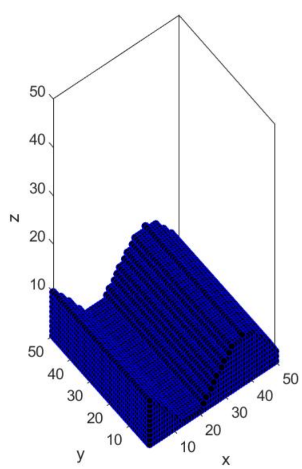


Figure 6: WSM rendering

4.3 Sand Stress Model (SSM)

In this part, we use the method of kinetic energy theorem to analyze the impact of each wave. First we do a fixed parameter analysis, in the following same case:

- Different sandcastle in the face of the waves of the sea on one side of the tilt angle is consistent, without considering the vertical (under the condition of vertical angle of ninety degrees)
- Keep the initial sand-to-water mixture proportion of the different sandcastle same
- Under the impact of the waves, change rule of sand-to-water mixture proportion is same in different shapes sandcastles
- In sandcastles of different shapes, the friction between the grains remains the same

According to the current motion equation of the wave (1), the current vertical velocity of the wave, V_y, can be obtained. Meanwhile, the velocity in the horizontal direction, V_x, can be retrieved according to the data. According to the velocity decomposition and synthesis under the orthogonal basis, the partial velocity of the wave along the sandcastle slope direction can be obtained.[2]

$$V_w = V_x cos\theta + V_y sin\theta \tag{2}$$

Assuming that all the kinetic energy of the wave is used to overcome the friction between the sand particles to do work, then we can get equation (3) according to the kinetic energy theorem:

$$\frac{1}{2} m_w V_w^2 = nfS \tag{3}$$

By analyzing the mass of a wave moving in a very short period of time, we can obtain the following equation by making waves into micro-elements:

$$m_w = LV_w \Delta t \lambda_s \tag{4}$$

The distance the wave has traved in Δt is:

$$S = V_w \Delta t \tag{5}$$

By combining the equations (2), (3), (4) and (5), we can get the following equation:

$$\frac{1}{2} L\lambda_s (V_x cos\theta + V_y sin\theta)^2 = n_0 f \tag{6}$$

According to the hypothesis we have expounded before and the data obtained from the data collection section, the change relation of sand-to-water mixture proportion be obtained:

$$r = -kr_0 V_w + 1.7r_0 \tag{7}$$

The following relationship exists between the friction between sand grains and sand-to-water mixture proportion:

$$f = k_0(r - r_0)^2 \tag{8}$$

So if we plug in the reorganization we get the final expression:

$$n_0 = \frac{\frac{1}{2}L\lambda_s(V_x cos\theta + V_y sin\theta)^2}{k_0(-kr_0V_w + 0.7r_0)^2} \tag{9}$$

According to this equation, we can get the number of sand grains that can be washed off by the part of the sandcastle that is hit by the waves. For the plane model, each part is hit by the waves in the same way, so the number of sand grains that are separated from each part under the impact of the waves is n obtained from the above equation.[6] For the non-plane model, we calculated that the impact of waves at all angles can be fitted with normal distribution, and the results are as follows, where R is the radius of the sandcastle model:

$$n = \frac{1}{\sqrt{2\pi}R}e^{-\frac{x^2}{2R^2}}n_0 \tag{10}$$

4.4 Rain-Sand Erosion Model (RSEM)

The RSEM describes the effect of the action between rain and sand on the stability of sandbags. The RSEM is similar to the SSM, except that the speed of rain is different from that of sea water, and the rest of the hand patterns are the same, so we will not go into details here.

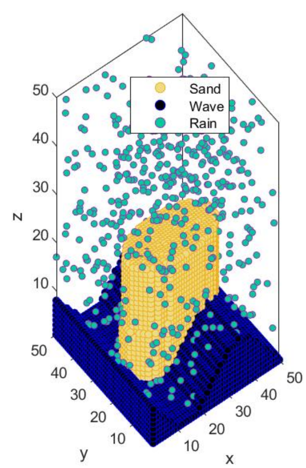


Figure 7: The comprehensive model

4.5 Comprehensive analysis of models

The erosion of waves by sandcastles is the result of multiple factors at the same time, so it is clearly wrong to build separate models without putting them together. So we put the model together to simulate the erosion of the sandcastle in the natural environment.

When we study the best shape, we do not consider the rain, only consider the erosion effect of waves on the sandcastle.

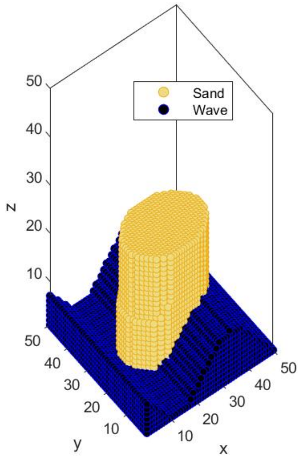


Figure 8: Elliptic cylinder

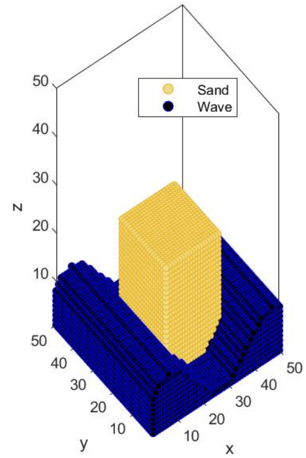


Figure 9: Cube

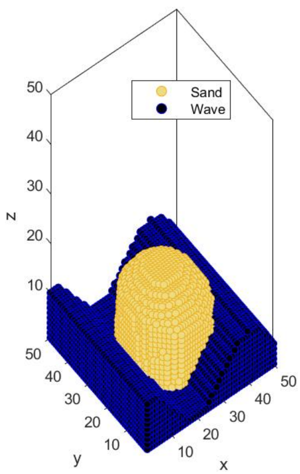


Figure 10: Hemispheroid

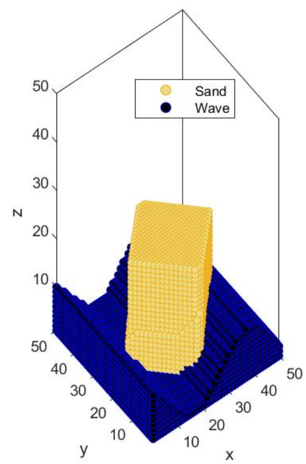


Figure 11: Prism

5 Parameter Estimation

5.1 Initial sand-to-water mixture proportion analysis

The initial sand-to-water mixture proportion is an important parameter of the model. Among the most stable sandcastle 3D shapes, the value of the optimal sand-to-water mixture proportion is uncertain, so we need to select an initial sand-to-water mixture proportion in the model test. The selection of this parameter requires that there is enough friction between the sand grains to keep the sandcastle for a long time and it cannot disappear under the impact of sand waves. As the parameter sand-to-water mixture proportion is very sensitive, the initial estimation of this parameter is very difficult. The method we adopted is to specify the extreme value of the parameter range of sand-to-water mixture proportion, and test the difference of the time that the mixture proportion of different sand-to-water can adhere to. It is found that the performance of the mixture proportion of sand-to-water is the most stable at about 1%. Therefore, the initial sand-to-water mixture proportion parameter was selected as 1% to realize the model mapping.

5.2 Analysis of sandcastle tilt angle

In the model of sandcastle, the slope of sandcastle's oblique side is obviously not invariable, so at what value does the sandbag model have the highest stability? That's what we're going to talk about in this section. We know that the velocity of the sea satisfies this equation:

$$V_w = V_x cos\theta + V_y sin\theta$$

The number of grains of sand being washed away by the water wave satisfies the following equation:

$$n_0 = \frac{\frac{1}{2}L\lambda_s(V_x cos\theta + V_y sin\theta)^2}{k_0(-kr_0V_w + 0.7r_0)^2}$$

Taking V_w as the independent variable to do the maximum analysis on the expression n_0, we can get the value of the value of theta when n_0 takes the minimum value of ground, so this value is the initial value of the sandcastle inclination Angle we expect to get.

6 Model Analysis

6.1 Analysis of different shapes of sandcastles holding time

In our model, we by changing the sandbags model of different parameters, including the top and bottom of the ratio of the area, the number of sandcastle stare blankly, going a while at the radius and the minimum Angle and direction of the wave movement, basal area and the ratio of the height, etc., to construct the different shapes of sandcastle by the sea erosion, statistics the same amount of the same sand under the same frequency wave erosion of the length of time, This led to the sandcastle foundation that will last the longest period of time on a stretch that will take knocks waves.

According to the effect analysis in Figure 12, we can get the results in the four shapes of Cube, Elliptic cylinder, Prism, and Prism pheroid. Elliptic cylinder is the most stable shape, which lasts the longest under the same wave erosion, and the reduction of sand content is the slowest. Hemispheres pheroid is the most unstable shape, with the shortest duration and the fastest sediment reduction. In the remaining two shapes of Cube and Prism, Cube shows better performance than Prism in the early stage, but Prism is obviously more stable than Cube in the later stage.

6.2 Analysis of different height of sandcastles holding time

From the above analysis, we can see that Elliptic cylinder is the most stable sandcastle foundation shape. Then for the sandcastle foundation of Elliptic cylinder shape, what other factors affect the persistence of sandcastle under wave erosion? In this section we discuss the effect of the height of the sandcastle on the time it takes for the sandcastle to remain under the waves. According to the experimental results, the sandcastle foundation with the same volume and higher height has better stability. In other words, in a certain height range, the more slender the sandcastle is, the better the stability will be.

In Figure 13, it can be clearly seen that as the height of the sandcastle increases, the time that the sandcastle can hold up under the impact of waves increases significantly and the proportion of time increase is consistent with that of height increase.

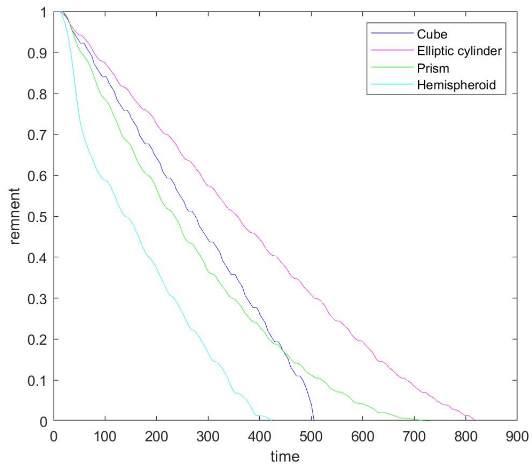


Figure 12: The holding time that sandcastles of different shapes

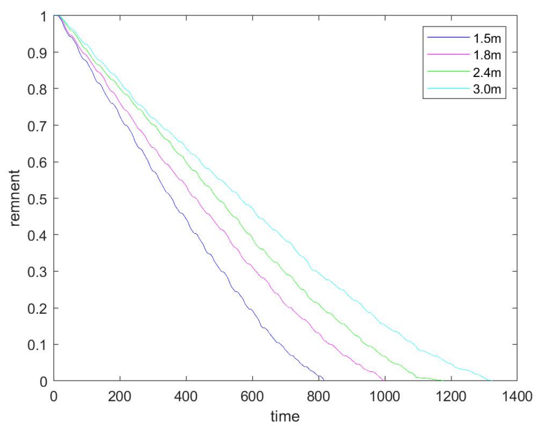


Figure 13: The influence of sandcastles' height on the stability of sandbags

6.3 Analysis of different ratio of the long axis to the short axis sandcastles holding time

We have determined that Elliptic cylinder is the most stable shape under the same conditions, so the proportion of Elliptic and long and short axes has become one of the elements we will discuss here. In other words, we're talking about whether sandcastles are flatter and more stable or rounder and more stable. By adjusting the ratio of the long axis and the short axis of the sandcastle ground ellipse, and controlling that the base area and height of the ellipse remain unchanged, we can obtain the graph of Figure 14.

We can see in Figure 14, Elliptic cylinder has the highest stability when the ratio of the length of axis is 2:1, while it has the lowest stability when the ratio of the length of axis is 1:1. At the same time, the ratio of The curve of Elliptic cylinder with the ratio of 4:1 and 1.5:1 is basically coincident with the curve of 2:1 and 1:1, this shows a flat or round Elliptic cylinder is not a stable model. But when the

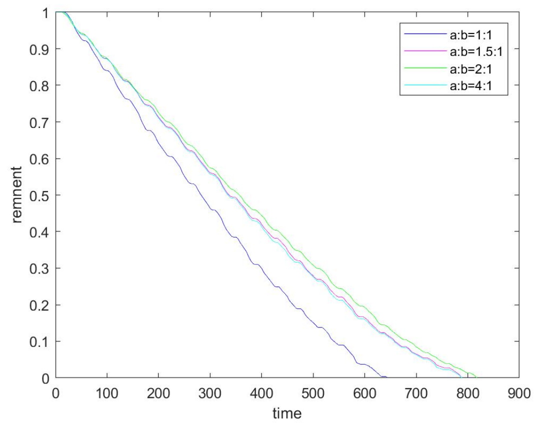


Figure 14: The influence of the ratio of long axis and short axis on the stability of sand castle

ratio of the long axis and short axis is close to 2:1, the sandcastle in the shape of Elliptic cylinder has the highest stability. At the same time, we found that the duration of Elliptic cylinder with a ratio of 1:1 was significantly shorter than the other three, so we could conclude that the Elliptic cylinder with a ratio of 1:1 was significantly shorter than the other three.

6.4 Analysis of the effect of rainfall conditions to holding time

Rainfall is also a major factor in sandcastles on the beach, and there is no doubt that rainfall will have an impact on the stability of sandcastles. We use a similar method to the SSM model to construct the RSEM. By giving certain velocity and energy to the raindrops, we judge whether it is an effective impact through the physical model. After the impact, we can get the RSEM by substituting the formula derived from the SSM model.

7 Problem Solving

7.1 Best sandcastle model

According to the above description, we can conclude that the sandcastle in the shape of Elliptic cylinder has the best stability, and on the basis of the Elliptic cylinder, the stability is the best when the ratio of the long axes and short axes of the Elliptic cylinder on the ground is 2:1. Other conditions being the same, we can also know that the higher the height of Elliptic cylinder, the better the stability.

7.2 Best sand-to-water mixture proportion

In our model, it can be seen that the initial sand-to-water mixture proportion in formula n is reflected in our model as a parameter, so we can adjust the initial sand-to-water mixture proportion by adjusting the time that the statistical model can adhere to under different sand-to-water mixture proportion. The figure is shown in Figure 15. According to the image, we can see that the sand-to-water mixture proportion reached a maximum with a value of 1.1%.When the sand-to-water mixture proportion is less than 1.1%, sandcastle model's holding time grows as it increases, and the rate of this process is very fast. While the proportion exceeds 1.1%, the time sandcastle can hold gradually declines, but the rate is far less than the speed of increase. Therefore, it can be concluded that when sand-to-water mixture proportion is at 1.1%, the internal friction between the grains is the greatest, and the best sand-to-water mixture proportion is 1.1%.

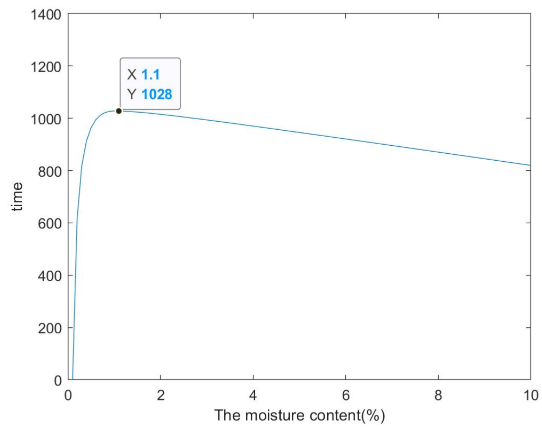


Figure 15: Different sand-to-water mixture proportion sandcastles' holding time

7.3 The effect of rainfall on sandcastle model

Rainfall will accelerate the erosion of the model, and the more severe the rainfall, the more obvious the erosion of the model. We discussed the model of adding a cone cover to the top of Elliptic cylinder, that is, the green curve in our figure. The fact shows that the addition of the cone cover cannot slow down the erosion rate of the sandcastle, but will accelerate the erosion of the sandcastle. Therefore, we can conclude that the Elliptic cylinder model we obtained in the first question is still the best model under the conditions of rainfall.

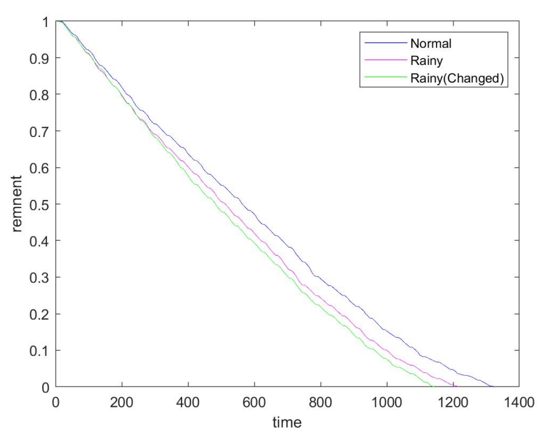


Figure 16: The influence of rainfall on the stability of sandcastle

7.4 Strategies to extend the life of sandcastles

Our team thought that if the sandcastle's foundation was built a bit further away from the waves, its survival time would be well extended. As the distance between the sandcastle and the waves increases, the waves will take longer to reach the sandcastle. Due to the presence of natural resistance, wave amplitude and wave speed will decrease. Accordingly, after we reduced the amplitude and horizontal wave velocity of the simulated waves in our model, the survival time of the sandcastle was also extended, as well as in the rainy conditions in Problem 3. (Figure 17).

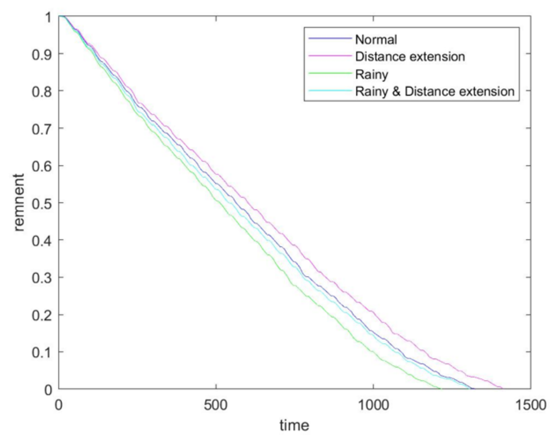


Figure 17: The relationships among sandcastle holding time, residual ratio and four different environmental conditions

Here the four environmental conditions are: Normal, Distance extension, Rainy, Rainy & Distance extension. The parameter effect of the distance extension is reflected in the amplitude and velocity of the wave decreases by 10%.

Similarly, using other materials to support the sandcastle with a layer of insulation at the bottom of the sandcastle, equivalent of reducing the amplitude of the wave, can reduce the impact of waves on the sandcastle. Due to the length of this article, it will not be repeated here.

8 Sensitivity Analysis

In the Sensitivity Analysis part, the amplitude of waves fluctuates within a limited range to observe the behavior changes of the model, so as to improve the stability of the model. The amplitude of the wave here represents the display meaning of the wave intensity, so here we analyze whether our model still has the universal optimization survival results under different wave intensities.

The figure above shows the model curve of our optimal model and Cube's sandcastle maintenance time at standard wave strength and when the wave strength is increased to the original 120 %. According to the figure above, after the amplitude increases by 20%, the duration of our model is still higher than that of Cube, and the performance ratio between them has changed from 240% to 260%. This result

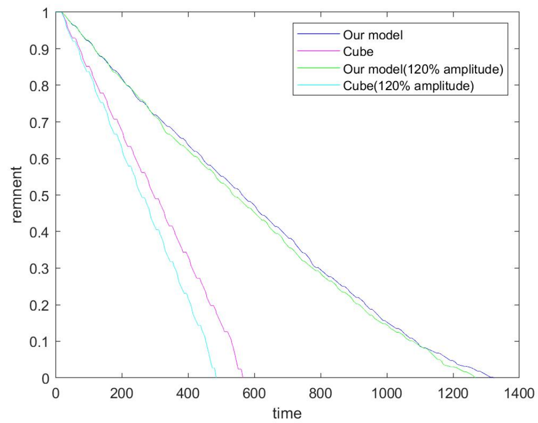


Figure 18: Models' holding time at different amplitudes

not only shows that the wave is a relatively sensitive factor to the model, but also shows that our model is an effective and stable model with significant stability to the change of environmental variables.

9 Strengths and Weaknesses

9.1 Strengths

1. In this model, both sandcastle and seawater are discretized. One cell is used to represent the sand grain model, and the sand grain parameters are given to satisfy the realistic situation to the greatest extent. Meanwhile, the discretization of seawater effectively realizes the simulation of collision between seawater and sand.

2. The change of sandcastle sand-to-water mixture proportion caused by the impact of each wave was considered, and the change of sandcastle sand-to-water mixture proportion would affect the viscosity coefficient of sand particles, which had a significant effect on the stability of sandcastle. Considering the change of sandcastle sand-to-water mixture proportion caused by each wave, this model can simulate the process of sandcastle erosion more effectively.

3. The automaticity of 3D cellular automata is effectively used to make the model run automatically once the initial value is set, the simulation results are true and reliable, and the workload is greatly reduced.

9.2 Weaknesses

1. The data of the model is small and the application range of the results is very low. The model only discusses the optimal model under the four conditions of Elliptic cylinder, Cube, Prism and Hemispheroid, and does not discuss the performance of the model under the more complex irregular shape, so the conclusion of this model is of low applicability.

2. The force model is not comprehensive enough. This model does not consider the interaction between the supporting structures of sand grains, but simply considers the interaction between seawater and

sandcastle. This approach is relatively simple, resulting in an important premise of the model to make the sand model stable, in addition to the erosion of the sea will not collapse phenomenon.

10 Article in the vacation magazine: Fun in the Sun

On many recreational beaches, it's common to see adults and children stacking sandcastles. Before they are built, they usually stack the sand into an initial shape, and then polish the sandcastle foundation into the final sandcastle carefully.

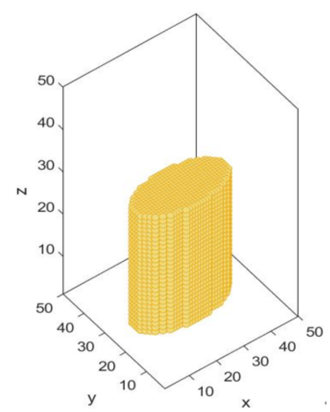
Besides paying attention to the appearance of the sandcastle, people also care about how the sandcastle can survive longer on the beach under the influence of the waves and possibly the rain. Our team first studied this aspect of the initial sandcastle foundation.

We simulated some common 3D shapes: we used Matlab software to generate virtual sandcastles and waves, as well as possible rain, then observed and recorded the survival time of sandcastles. At the same time, the most suitable ratio of water to sand and other factors were considered to explore how to extend the survival time of sandcastle.

Here are our findings:

1. Considering only the influence of waves, under the same control volume, the survival time of the elliptic cylinder and prism is longer than that of the hemispheroid and cube. After controlling the same height, the duration of the elliptic cylinder is about 12% longer than that of the prism, and about 90% longer than that of the hemispheroid.

   For the same size, different height of elliptic cylinder, the higher the height, the longer the duration. Based on the average height of waves in the US sea of 0.8m[1] , the duration of an ellipsoidal column with a height of 3m is about 61% longer than that with a height of 1.5m. Sandcastles above 3m are too difficult to build to be considered here.

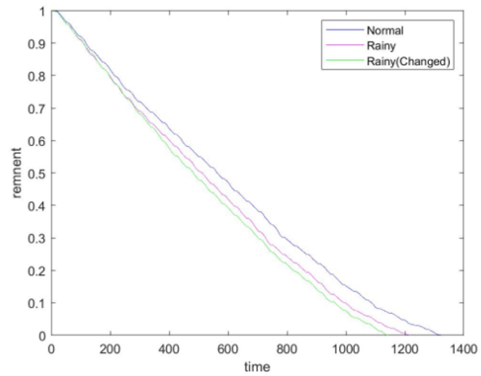
   

   Model of an elliptic cylindrical sandcastle

   For the elliptic cylinder with the same volume and height, different ellipse shapes, the ratio of length to width equal to 2:1 is the most suitable shape. However, if the ratio is slightly off, the impact of the duration is insignificant, so it's not necessary to be exacting during construction.

2. We adjusted the water-sand ratio of the simulated sandcastle and found that the sandcastle lasted the longest when the water content of the sand was around 1.1%. So it doesn't take much water to build a sandcastle.

3. Considering the rain, the duration of the sandcastle is reduced to about 91%, while the flat top ellipsoid is still the most suitable. For example, the duration was reduced a bit by changing the top of the sandcastle by adding a spire.



Changes after rain and changing the top

4. On the basis of the above, other measures can be taken to extend the duration of the sandcastle. For example, building the foundation of the sandcastle slightly away from the waves, and using other materials to put a layer of insulation at the bottom of the sandcastle to raise it. These are the ways to reduce the impact of the waves on the sandcastle.

5. In general, in order to extend the duration of the sandcastle, it can be considered to design the sandcastle foundation into the shape of an ellipse cylinder, with the height is as high as possible, and the ratio of length to width is about 2:1.

Thank you very much for reading our results! We sincerely hope this article can help you!

11 Reference

[1] ZHUANG Xiaoxiao, LIN Yihua, et al. 2014. Seasonal Variation of Global Ocean Wave [J]. Chinese Journal of Atmospheric Sciences (in Chinese), (2): 251-260.

[2] Nian ShengCheng,Adrian Wing KeungLaw.Exponential formula for computing effective viscosity[J].Powder Technology,2003,129(1-3):156-160.

[3] Tomoya Shibayama,Nguyen Ngoc An.A Visco-Elastic-Plastic Model for Wave-Mud Interaction[J].Coastal Engineering in Japan,1993,36(1):67-89.

[4] W.B.Guan,S.C.Kotb,E.Wolanskic.3-D fluid-mud dynamics in the Jiaojiang Estuary, China[J].Estuarine, Coastal and Shelf Science,2005,65(4):747-762.

[5] CHONG Bin, CUI HongLin, LIU zhen.The establishment and simulation of wave model[N].Journal of Xi'an Technological University,Vol. 29 No. 5 Oct. 2009(01).

[6] Zhongfan Zhu , Hongrui Wang and Dingzhi Peng.Dependence of Sediment Suspension Viscosity on Solid Concentration: A Simple General Equation[J].MDPI,2017,9(474):1-17.

[7]Kai Lu.Research on temperature field algorithm of milling cutter based on cellular automata[J].Journal of Harbin university of commerce (natural science),2019,35(06):689-692.