

## LinelandNavigation Writeup

In my algorithm, I represented Lineland as a digraph. The nodes of the graphs were locations in Lineland, while the edges were forwards or backwards jumps (by which I mean legal movements) that LePoint could make between these locations. I kept a boolean array of all possible locations that LePoint might visit (from 0 to a bit past the goal). Whenever a location was declared to be a mine through a call to `addMinedLineSegment()`, I flipped its element in the array to `true`.

I implemented `solveIt()` as a Breadth-First Search. I did so by building a BFS tree and determining the depth of each location as it was added to the tree; in a BFS tree, depth represents the minimum number of edges needed to reach the node. When I found the goal, I returned its depth. Since each edge is a single jump, the depth of the goal is the minimum number of jumps necessary to reach it. To ensure that the BFS tree would not contain any mines, I marked each location as it was mined in a boolean array (as described above), and then used that same array in my BFS to determine if a node was already visited, ensuring that mined nodes would not be visited, even if they would otherwise have led to a shorter path.

In order to reduce the cost of the algorithm, I made some modifications to BFS. Rather than creating a graph ahead of time and passing it to the algorithm, I created the graph as I traversed it. Whenever I visit a node `location`, I add `location + m` to the BFS queue (the forward jump from that node). I then add backwards jumps, starting at `location - 1`, but as soon as I find a node that has already been added to the queue (as proven by its depth being nonzero, so it isn't confused with a mine), I stop looking for more edges incident on `location`.

The reason I can do this is that my algorithm has an invariant that all nodes before a node added to the queue were also added to the queue (unless they were mines). This can be proven using (\*sigh\*) induction. In the base case of jump 0 (before the first jump), there are no (valid) nodes before node 0, so all the nodes before it are on the queue. In the inductive case of jump  $n$ , I only need to add nodes working backwards until I reach a node already on the queue, which we will call node  $q$ . Since all the locations before  $q$  were already added to the queue (using strong induction), all the nodes before  $n$  have now been added to the queue (unless they were mines).

I added this order of growth analysis before I realized you didn't care, but I don't want to take it out after spending time on it. The order of growth of any BFS is  $O(V + E)$ .  $V$  in this case is  $g$ , because no node before 0 can be considered, and only one node at or beyond  $g$  can be

considered before the algorithm returns.  $E$  in this case is proportional to  $V$ . I can perform up to  $V$  forward jumps, because each valid vertex (non-mine) has exactly one forward jump edge. Additionally, the number of back jump edges (by which I mean forward edges pointing backwards on the number line, not actual back edges) is also proportional to  $V$ . Each vertex can be examined for the first time, and be added to the queue, once. Once I examine an edge and discover its vertex to have already been added to the queue, I do not check any edges before it. I can examine a number of edges already on the queue to the number of vertices for which I am examining back jump edges. If a vertex is a mine, it can also be examined no more than twice: once for the first time, whether from a forward or backward jump; and once when being revisited when checking for nodes from the non-mine vertex immediately before the mine. All other edges are not even examined, and so might as well not exist. Because of this, `solveIt()`'s order of growth is proportional to the number of vertices, or  $O(g)$ .