1-(d). $x_1 \cdots x_n$ 이 서로 다를 때 determinant A가 non-zero 이다

1-(e) $w = A^{-1} y$.

Handwriting ~~project~~ Assignment DATE. NO.

20230359 송도원

1-(a) :

Size of w : $d+1$

Size of y : n

1-(b)

$$A = \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^d \\ 1 & x_2 & x_2^2 & \cdots & x_2^d \\ 1 & x_3 & \vdots & \ddots & x_3^d \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^d \end{pmatrix}$$

Size of matrix A : $n \times (d+1)$

1-(c).

$$\det A = \begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \ddots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{vmatrix} = \begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 0 & x_2-x_1 & x_2^2-x_1^2 & \cdots & x_2^{n-1}- \\ 0 & x_3-x_1 & x_3^2-x_1^2 & & \\ \vdots & \vdots & \vdots & & \\ 0 & x_n-x_1 & x_n^2-x_1^2 & \cdots & x_n \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & & & (x_2-x_1)x_2^{n-2} \\ \vdots & & \ddots & & \vdots \\ 0 & x_n-x_1 & \cdots & & (x_n-x_1) \times x_n^{n-2} \end{vmatrix}$$

$$\prod_{i=2}^{n} (x_i - x_1) \begin{vmatrix} 1 & x_2 & \cdots & x_2^{n-2} \\ 1 & x_3 & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^{n-2} \end{vmatrix}$$ (반복)

$$\therefore \det A = \prod_{1 \le j < i \le n} (x_i - x_j)$$