Prerequisites:

At first we must recall the following summation formulas/rules:

- $\sum_{i=1}^{n}(c) = nc$
- $\sum_{i=1}^{n}(i) = \frac{n(n+1)}{2}$
- $\sum_{i=1}^{n}(x_i \pm y_i) = \sum_{i=1}^{n}(x_i) \pm \sum_{i=1}^{n}(y_i)$
- $\sum_{i=1}^{n}(ci) = c\sum_{i=1}^{n}(i)$
- $\sum_{i=1}^{n}(i^2) = \frac{n(n+1)(2n+1)}{6}$

Solution:

Now let us evaluate the expression:

1n + 2(n-1) + 3(n-2) + 4 (n-3) + … + n

=n + 2n - 2 + 3n - 6 + 4n + 12 … + n(n-(n-1))

=(1n + 2n + 3n + … + n^2)

-(2 + 6 + 12 + … + n(n-1))

=n(1 + 2 + 3 + … + n)

- 2(1 + 3 + 6 + … + $\frac{n(n-1)}{2}$)

$=n\sum_{i=1}^{n}(i) - 2\left(\sum_{i=1}^{1}(i) + \sum_{i=1}^{2}(i) + \sum_{i=1}^{3}(i) + \ldots + \sum_{i=1}^{n-1}(i)\right)$

$=n\sum_{i=1}^{n}(i) - 2\sum_{j=1}^{n-1}\left(\sum_{i=1}^{j}(i)\right)$

$=n\sum_{i=1}^{n}(i) - 2\sum_{j=1}^{n-1}\left(\frac{j(j+1)}{2}\right)$

$=n\sum_{i=1}^{n}(i) - 2\sum_{j=1}^{n-1}\left(\frac{1}{2}j + \frac{1}{2}j^2\right)$

$$= n \sum_{i=1}^{n} (i) \;-\; 2.\frac{1}{2} \sum_{j=1}^{n-1} (j) \;-\; 2.\frac{1}{2} \sum_{j=1}^{n-1} (j^2)$$

$$= n\frac{n(n+1)}{2} \;-\; \frac{n(n-1)}{2} \;-\; \frac{n(n-1)(2n-1)}{6}$$

$$= \frac{n^3+3n^2+2n}{6}$$

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