3D Computer Vision (Spring 2023)

HW0

Deadline: 11:59pm March 24th (Friday), 2023

1 Instruction

In this homework, you are encouraged to study basic tools of linear algebra. Check the following concepts and address the meanings, algorithms, applications, and so on. You may search on the internet, but you should not copy and paste whole sentences. Please make your answers as concise as possible, with only essential parts (each answer should be within a few lines).

- **Make sure to submit HW0**. Students who do not submit HW0 are considered not taking the course, and will be dropped.
- Homeworks need to be submitted electronically on ETL. **Only PDF generated from LaTeX is accepted.**
- Collaboration on solving the homework is allowed. Discussions are encouraged but you should think about the problems on your own.
- If you do collaborate with someone or use a book or website, you are supposed to write up your solution independently.
- Make sure you cite your work/collaborators at the end of the homework.
- **Honor Code**: This document is exclusively for Spring 2023, M1522.007100 students with Professor Hanbyul Joo at Seoul National University. Any student who references this document outside of that course during that semester (including any student who retakes the course in a different semester), or who shares this document with another student who is not in that course during that semester, or who in any way makes copies of this document (digital or physical) without consent of Professor Hanbyul Joo is guilty of cheating, and therefore subject to penalty according to the Seoul National University Honor Code.

2 SVD (Singular Value Decomposition) (7pts)

- Briefly describe SVD: its input, outputs, basic properties, and applications.

- Describe the relation between SVD and Eigenvalue Decomposition
- Describe Principal Component Analysis (PCA) and its relation with SVD.
- Apply SVD on the following example matrix and show the results, $A = U\Sigma V^T$. Note that you are allowed to use any existing libraries (e.g., svd() in Matlab). Include both your code and output in your pdf submission (Note that the code should be very simple, at most a couple of lines):

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 5 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$$

- In above outputs, verify that both U and V are orthogonal matrices.
- Generate any 5x3 matrix by yourself and apply SVD again (show the output). Also verify whether U and V are orthogonal matrices.
- Generate any 3x4 matrix by yourself and apply SVD again (show the output). Also verify whether U and V are orthogonal matrices.

3 QR Decomposition (3pts)

- Briefly describe QR decomposition: its input, outputs, basic properties, and applications.
- Briefly describe RQ decomposition. In many cases, RQ decomposition is not available in libraries. Then, address how we can modify QR decomposition to RQ decomposition.
- Find the QR decomposition output of the following matrix, using any libraries you prefer (e.g., Matlab or python). Verify Q is an orthogonal matrix in your output.:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

4 Cholesky Decomposition (2pts)

- Briefly describe Cholesky decomposition, its input, outputs, and basic properties.
- Find the Cholesky decomposition output of the following matrix, using any libraries you prefer (e.g., Matlab or python):

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 5 & -4 \\ 2 & -4 & 6 \end{bmatrix}$$

5 LU Decomposition (2pts)

- Briefly describe LU decomposition, its input, outputs, and basic properties.
- Find the LU decomposition output of the following matrix, using any libraries you prefer (e.g., Matlab or python):

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{bmatrix}$$

6 Ax=0 (3pts)

- Describe what Null space means given a Matrix **A**. What happen when a matrix A is full-rank? When does a matrix **A** have the non-trivial null space?
- Describe an algorithm to solve the homogeneous linear system $\mathbf{Ax} = 0$ via SVD, given **A**.
- Find a non-trivial solution ($\mathbf{x} \neq 0$) of $\mathbf{Ax} = 0$ (submit both code and outputs) via SVD, where:

$$A = \begin{bmatrix} -3.0975 & -1.9844 & 3.9311 \\ 14.0057 & 7.9991 & -17.9960 \\ 3.9054 & 2.0152 & -5.0669 \\ 1.9940 & 78.0010 & -1.0043 \end{bmatrix}$$

  Note that you should use svd, and are not allowed to use built-in function to solve linear system in the libraries (e.g., linsolve). Make sure you got near zero values from **Ax**.

7 $\mathbf{Ax} = \mathbf{b}$ (2pts)

- Describe how to compute the pseudo inverse of a matrix A using SVD
- Find the solution of the following by computing the pseudo inverse of A:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 0 & 3 \\ 2 & 1 & 2 \\ 1 & 8 & 0 \end{bmatrix}, b = \begin{bmatrix} 8.2593 \\ 17.9659 \\ 15.9216 \\ 3.1024 \end{bmatrix}$$

  Note that you should use svd to compute pseudo inverse, and are not allowed to use the built-in functions to solve linear system in the libraries (e.g., pinv or linsolve). In fact, you can compare your output with the output of linsolve(A,b) or pinv(A) in matlab to double check. Or, simply compute Ax to see whether you got b.

8 Levenberg–Marquardt algorithm (1pts)

- Briefly describe Levenberg–Marquardt (LM) algorithm. How the LM method differs from the gradient descent and Gauss–Newton algorithm