AMS 597 Spring 2021
Homework 7 Due 05/04/2021 10:00 AM

Instruction: Submit your homework via Blackboard. If you have difficulty submitting via Blackboard, email the TA directly. No late homework will be accepted, based on the time the email is sent out. If you are submitting via email (not recommended), put this as the subject of your email
AMS597 2021: Homework/Exam #7. ID: XXXXXXXXX. Name: XXX XXX

1. Estimate the bias, standard error, standard normal bootstrap CI, basic bootstrap CI and bootstrap t-interval at 95% of the sample median for `x` where

   ```
   > set.seed(123)
   > x <- rnorm(50)
   ```

2. Another test statistic for two group comparison, i.e., $H_0 : F = G$ vs $H_a : F \neq G$ in the univariate case is the Cramer-von Mises statistic

   $$W_2 = \frac{mn}{(m+n)^2}\left[\sum_{i=1}^{n}(F_n(x_i) - G_m(x_i))^2 + \sum_{j=1}^{m}(F_n(y_j) - G_m(y_j))^2\right]$$

   where F_n is the ecdf of $x_1, ..., x_n$ and G_m is the ecdf of $y_1, ..., y_m$.
   Implement the two group comparison using Cramer-von Mises test statistic but obtaining the p-value from permutation test.

3. Implement the bivariate Spearman rank correlation test as a permutation test. Compare the achieved significance level of the permutation test with the p-value reported by `cor.test` on the following samples:

   ```
   > set.seed(123)
   > x <- rnorm(50)
   > y <- 0.2*x+rnorm(50)
   ```

4. (a) Derive the Newton-Raphson iteration for solving

   $$e^{2x} = x + 6$$

   Implement your iteration as a R function and report the solution to the equation.

   (b) Compare your answer to the solution using `uniroot` function (i.e., Brent's method).

5. (a) Let $x_1, ..., x_n$ be a random sample from a $Poisson(\lambda)$. Find the maximum likelihood estimator of λ using the function `optimize` in R. Apply this to

   ```
   set.seed(123)
   lambda.true <- 10
   x <- rpois(1000,lambda.true)
   ```

(b) Let $x_1, ..., x_n$ be a random sample from a $Beta(a, b)$. Find the maximum likelihood estimator of $\theta = (a, b)$ using the function `optim` and `optimx` in R. Apply this to

```
set.seed(123)
a.true <- 5
b.true <- 2
x <- rbeta(1000,shape1=a.true,shape2=b.true)
```