Appendix C: Optimization model for water use

Setup

To understand the mechanisms through which the SES structure impacts provincial water use, we developed a dynamic marginal benefits analysis to analyze how institutional mismatch could have led to the changes in water use, especially among provinces with high incentives for excess water use. Specifically, we modeled individual provincial decision-making in water resources before quota execution.

We proposed three intuitive and general assumptions:

Assumption 1. *(Water-dependent production) Because of irreplaceability, water is assumed to be the only input of the production function with two types of production efficiency. The production function of a high-incentive province is $A_H F(x)$, and the production function of a low-incentive province is $A_L F(x)$ ($A_H > A_L$). F(x) is continuous, $F'(0) = \infty$, $F'(\infty) = 0$, $F'(x) > 0$, and $F''(x) < 0$. The production output is under perfect competition, with a constant unit price of P.*

Assumption 2. *(Ecological cost allocation) Under the assumption that the ecology is a single entity for the whole basin involved in N provinces, the cost of water use is equally assigned to each province under any water use. The unit cost of water is a constant C.*

Assumption 3. *(Multi-period settings) There are infinite periods with a constant discount factor β lying in (0,1). There is no cross-period smoothing in water use.*

Under the above assumptions, we can demonstrate three cases to simulate the water use decision-making and water use patterns in a whole basin.

Under the above assumptions, we can demonstrate three cases consisting of local governments in a whole basin to simulate their water use decision-making and water use patterns.

Case 1. *Dentralized decision: This case corresponds to a situation without any high-level water allocation institution.*

When each province independently decides on its water use, the optimal water use x_i^* in province i satisfies:

$AF'(x) = \frac{C}{P}$,

where A_H and A_L denote high-incentive and low-incentive provinces, respectively.

When the decisions in different periods are independent, for $t = 0, 1, 2 \cdots$, then:

$x_{it}^* = x_i^*$

Case 2. *Mismatched institution: This case corresponds to an SES structure where fragmented stakeholders are linked to unified river reaches.*

The water quota is determined at t=0 and imposed in t=1,2,... Under the subjective expectation of each province that current water use may influence the future water allocation determined by high-level authorities, the total quota is a constant denoted as Q, and the quota for province i is determined in a proportional form:

$$Q_i = Q \cdot \frac{x_i}{x_i + \sum x_{-i}}.$$

Under a scenario with decentralized decision-making with a water quota, given other provinces' decisions on water use remain unchanged, the optimal water use of province i at t=0 satisfies:

$$AF'(x_{i,0}) = \frac{C}{P \cdot N} - \frac{\beta}{1-\beta} \cdot A \cdot f(Q \cdot \frac{x_{i,0}}{x_{i,0} + \sum x_{-i,0}}) \cdot Q \cdot \frac{\sum x_{-i,0}}{(x_{i,0} + \sum x_{-i,0})^2},$$

where A_H denotes a high-incentive province and A_L denotes a low-incentive province.

Case 3. *Matched institution: This case corresponds to the institution under which water use in a basin is centrally managed.*

When the N provinces decide on water use as a unified whole (e.g., the central government completely decides and controls the water use in each province), the optimal water use x_i^* of province i satisfies:

$$F'(x) = \frac{C}{P}.$$

We propose Proposition 1 and Proposition 2:

Proposition 1: Compared with the decentralized institution, a matched institution with unified management decreases total water use.

The optimal water use under the three cases implies that mismatched institutions cause incentive distortions and lead to resource overuse.

Proposition 2: Water overuse is higher among provinces with high water use incentives than low- water use incentives under a mismatched institution.

The intuition for this proposition is straightforward in that all provinces would use up their allocated quota under a relatively small Q. As production efficiency increases, the marginal benefits of a unit quota increase, and the quota would provide higher future benefits for a pre-emptive water use strategy. Provinces with high production efficiency have higher optimal water use values under the decentralized decision. The divergence in water use would be exaggerated when the water quota is expected to be implemented with greater competition.

Extensions of the model are shown in Supplementary Material S3.

Appendix: Water Use Optimization

Case 1. *Centralized decision*

When the N provinces decide on water uses as a unity, the marginal cost is C, equal to its fixed unit cost. The water use of province i aims to maximize $P \cdot A \cdot F(x) - C$. *Hence,* x_i^* *satisfies* $P \cdot A \cdot F'(x) = C$, *i.e.,* $AF'(x) = \frac{C}{P}$, *where A denotes* A_H *for a high-incentive province and* A_L *for a low-incentive province.*

Case 2. *Decentralized decision*

When each of the N provinces independently decides on its water use, the marginal cost of water use would be $\frac{C}{N}$ *as a result of cost-sharing with others. Hence, the optimal water use in province i at period t, denoted as* $\hat{x}_i^*$, *satisfies* $P \cdot A \cdot F'(x_{it}) = \frac{C}{N}$, *i.e.,* $A \cdot F'(x) = \frac{C}{P \cdot N}$. *Since* F' *is monotonically decreasing,* $\hat{x}_{it}^* > x_i^*$.

Case 3. *Forward-looking decentralized decision under quota restrictions*

When the water quota would constrain future water use, the dynamic optimization problem of province i is shown as follows. In $t = 1, 2, \cdots$, *there would be no relevant cost when the quota is bound that each province takes ongoing costs of* $\frac{P \cdot Q}{N}$ *regardless of the allocation. Therefore, it is sufficient to consider only the total water quota is less than total water use in Case 2 since a "too large" quota doesn't make sense for ecological policies.*

$$max \quad P \cdot A \cdot F(x_{i,0}) - \frac{C \cdot \sum x_{i,0} + x_{-i,0}}{N} + \beta P \cdot A \cdot F(x_{i,1}) + \beta^2 P \cdot A \cdot F(x_{i,2}) + ...$$

$$= P \cdot A \cdot F(x_{i,0}) - C \cdot \frac{x_{i,0} + \sum x_{-i,0}}{N} + \frac{\beta}{1-\beta} P \cdot A \cdot F(Q \cdot \frac{x_{i,0}}{x_{i,0} + \sum x_{-i,0}})$$

First-order condition: $P \cdot A \cdot F'(x_{i,0}) - \frac{C}{N} + \frac{\beta}{1-\beta}[P \cdot A \cdot f(Q \cdot \frac{x_{i,0}}{x_{i,0} + \sum x_{-i,0}}) \cdot Q \cdot \frac{\sum x_{-i,0}}{(x_{i,0} + \sum x_{-i,0})^2}] = 0$

where $f(\cdot)$ *is the differential function of* $F(\cdot)$.

The optimal water use in province i at t=0 $\widetilde{x}_{i,0}^*$ *satisfies* $P \cdot A \cdot F'(x_{i,0}) = \frac{C}{N} - \frac{\beta}{1-\beta} \cdot P \cdot A \cdot f(Q \cdot \frac{x_{i,0}}{x_{i,0} + \sum x_{-i,0}}) \cdot Q \cdot \frac{\sum x_{-i,0}}{(x_{i,0} + \sum x_{-i,0})^2}$, *i.e.,* $A \cdot F'(x_{i,0}) = \frac{C}{P \cdot N} - \frac{\beta}{1-\beta} \cdot A \cdot f(Q \cdot \frac{x_{i,0}}{x_{i,0} + \sum x_{-i,0}}) \cdot Q \cdot \frac{\sum x_{-i,0}}{(x_{i,0} + \sum x_{-i,0})^2}$.

Since $F' > 0$ *and* $F'' < 0$, $\widetilde{x}_i^* > \hat{x}_i^* > x_i^*$, *taken others' water use* $x_{-i,0}$ *as given. Since the provincial water use decisions are exactly symmetric, total water use would increase when each province has higher incentives for current water use.*

Proof of Proposition 1:

Because $F' > 0$ and $F''(x) < 0$ is monotonically decreasing, based on a comparison of costs and benefits for stakeholders (provinces) in the three cases,

$\widetilde{x}_i^* > \hat{x}_i^* > x_i^*$.

The result of $\hat{x}_i^* > x_i^*$ indicates that individual rationality would deviate from collective rationality under unclear property rights where a water user is fully responsible for the relevant costs. The result of $\hat{x}_i^* > x_i^*$

The difference between x_i^* and $\hat{x}_i^*$ stems from two parts: the effect of the marginal returns and the effect of the marginal costs. First, the "shadow value" provides additional marginal returns of water use in $t = 0$, which increases the incentives of water overuse by encouraging bargaining for a larger quota. Second, the future cost of water use would be degraded from $\frac{P}{N}$ to an irrelevant cost.

Proof of Proposition 2:

Since $A_H > A_L$, $F'(x_H) < F'(x_L)$, Eq.(xxx) implies a positive relation between x_{i0} and A, when β, P, C, Q, and other provinces' water use are taken as given.

The difference between $\widetilde{x}_i^*$ and $\hat{x}_i^*$ (i.e., $\frac{\beta}{1-\beta} \cdot A \cdot f(Q \cdot \frac{x_{i,0}}{x_{i,0} + \sum x_{-i,0}}) \cdot Q \cdot \frac{\sum x_{-i,0}}{(x_{i,0} + \sum x_{-i,0})^2}$) represents the incentive of water overuse derived from an expectation of water quota allocation. The incentive of water overuse increases by A.