Q:
For what values of $b$ does

$$\boxed{2x_1^2 + 2x_2^2 + 2b x_1 x_2} \leq 3$$

define an ellipse?

A:

$$\boxed{\phantom{xxxx}} = x^T Q x \text{ for } Q = \begin{bmatrix} 2 & b \\ b & 2 \end{bmatrix}.$$

$Q \succ 0$ when $4 - b^2 > 0$.

Thus $2x_1^2 + 2x_2^2 + 2b x_1 x_2 \leq 3$ defines an elipse when $b^2 < 4$ or $b \in (-2, 2)$.

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Q: The constraint

$$(*) \; (2x_1 - x_2 + 1)^2 + (-x_1 + x_2 + 1)^2 \leq (4x_1 - x_2 + 2)^2$$

is NOT a SOC constraint.

A:

let $y_1 = 2x_1 - x_2 + 1$

$y_2 = -x_1 + x_2 + 1$

$t = 4x_1 - x_2 + 2$

Add constraint $t \geq 0$.

Then $(*)$ is equivalent to

$(t, y) \in \mathcal{L}$ or $t \geq \|y\|_2$,

$t \geq \sqrt{y_1^2 + y_2^2}$

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Q: $\|x-a\| + \|y-b\|^2 \leq 0$

A: only feasible solution (since $\|\cdot\|$ always $\geq 0$) is $x = a$, $y = b$. [Linear constraints]

Q: $\|x-a\|_2^2 - r^2 \leq 0$, $r \geq 0$

A: This is a SOC:

$$r \geq \|x-a\|_2$$

Note that $r \geq 0$ is important!

Q: $\|x-a\|_2 \leq \|x-b\|_2$

A: $\|x-a\|_2 \leq \|x-b\|_2 \Leftrightarrow \|x-a\|^2 \leq \|x-b\|^2$

$\Leftrightarrow \quad (x-a)^T(x-a) \leq (x-b)^T(x-b)$

$$x^Tx - 2a^Tx + a^Ta \leq x^Tx - 2b^Tx + b^Tb$$

$$2(b-a)^Tx \leq b^Tb - a^Ta$$

# Linear Constraint

**Q:** prove for $\lambda > 0$ it is impossible to find $x$ such that $J_1(x) < J_1(x_\lambda)$ AND $J_2(x) < J_2(x_\lambda)$

**A:** Suppose for a contradiction that such an $x$ existed, then we have

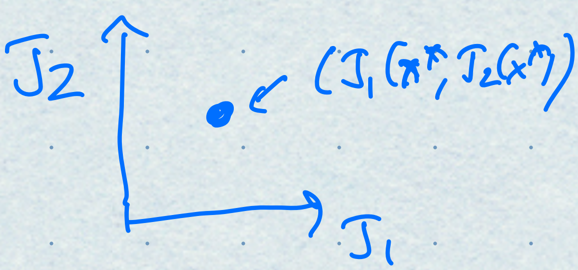
$$J_1(x) + \lambda J_2(x) < J_1(x_\lambda) + \lambda J_2(x_\lambda)$$

when $\lambda > 0$, but this means than $x_\lambda$ was not the minimizer ∎

**Q:** suppose $\arg\min_x J_1(x) = \arg\min_x J_2(x) =: x^*$

what does pareto curve look like?

**A:** whole pareto curve is a single point,




suppose $\exists\ x = x^*$ with

$$J_1(x) + \lambda J_2(x) < J_1(x^*) + \lambda J_2(x^*)$$

we know that $J_1(x^*) \le J_1(x) \quad \forall x$

$J_2(x^*) \le J_2(x) \quad \forall x,$

so such an $x$ cannot exist.

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Q: $$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} & Q & \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \le 1 \qquad (*)$$

$$Q = \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix} \begin{bmatrix} 4/25 & 0 \\ 0 & 4/25 \end{bmatrix} \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}^T$$

A: This is an ellipsoid which is a convex set

[ Ellipsoid since eigenvalues all $> 0$ ]

picture

2)

Q: Find $A, b$ such that (*) can be written like $\left\| A \begin{pmatrix} x \\ y \end{pmatrix} \right\| \le b$

A: $x^T Q x \le 1 \iff \| Q^{\frac{1}{2}} x \| \le 1$

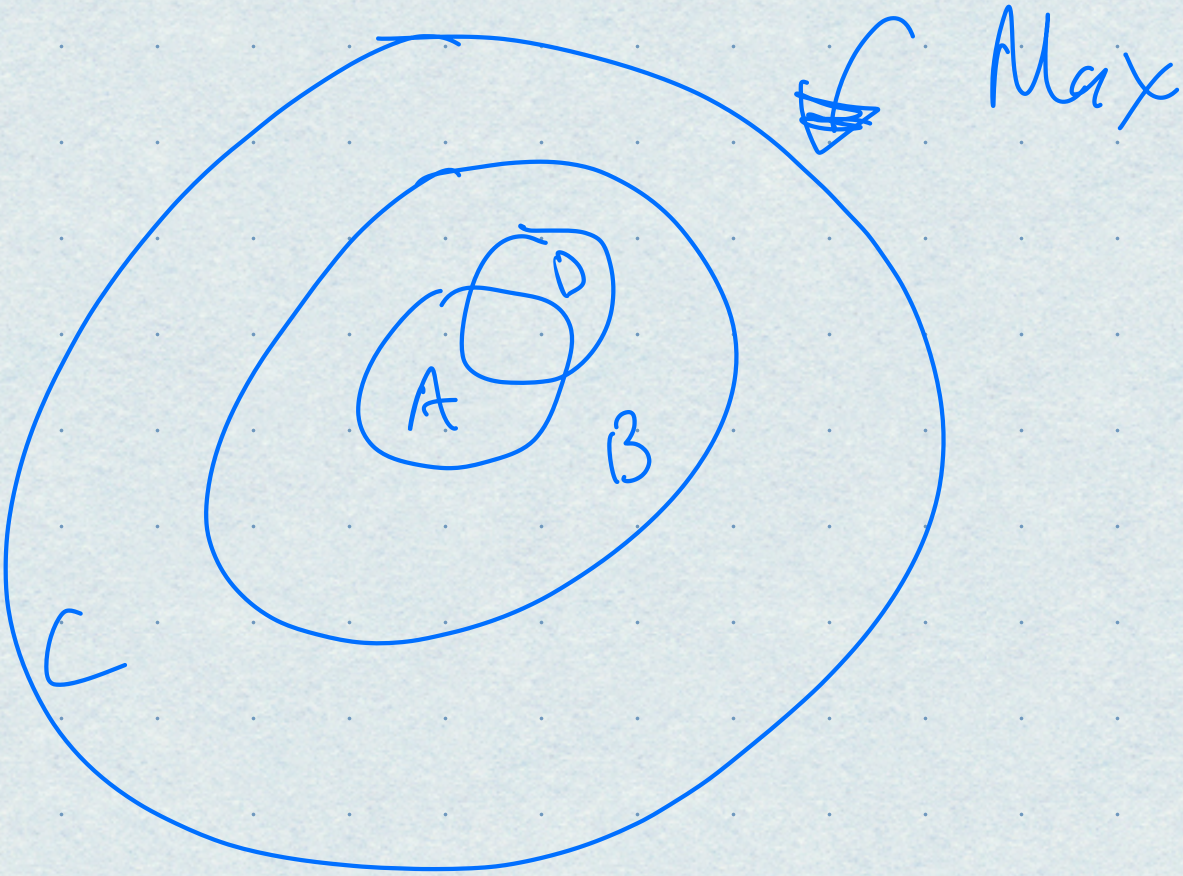
$Q = U \Lambda U^T$, so $Q^{\frac{1}{2}} = U \Lambda^{\frac{1}{2}}$

$$\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 1/5 & 0 \\ 0 & 2/5 \end{bmatrix} = \begin{bmatrix} \frac{3}{25} & -\frac{8}{25} \\ \frac{4}{25} & \frac{6}{25} \end{bmatrix}$$

$$A = U \Lambda^{\frac{1}{2}} = \frac{1}{25} \begin{bmatrix} 3 & -8 \\ 4 & 6 \end{bmatrix}$$

$b = 1.$

Q: Relaxations:




E: Same feas region as A, except 'min'

F: Dual LP to ~~A~~ C.

A:

$$Z_A \le Z_B$$

$$Z_A \times Z_D$$

$$Z_A \ge Z_E$$

$$Z_F = Z_C$$

$$Z_D \le Z_B$$

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Beer Location:

2.1 $\min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} h_i z_i$

2.2

$$\sum_{i \in I} x_{ij} \geq b_j \quad \forall\ j \in J$$

Note this implements "fixed cost"

2.3

$$\sum_{j \in J} x_{ij} \leq M_i z_i \quad \forall\ i \in I$$

where $M_i,\ i \in I$ is upper bound on $\sum_{j \in J} x_{ij}$, say

$$M_i = \sum_{j \in J} b_j \quad \forall\ i \in I.$$

2.4

$$x_{ij} \geq 0 \quad \forall\ i \in I,\ j \in J$$

$$z_i \in \{0, 1\} \quad \forall\ i \in I$$