Assignment 5

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```
knitr::opts_chunk$set(echo = TRUE)
knitr::opts_chunk$set(eval=FALSE)
library(mvnfast)
library(RMTstat)
library(future.apply)
```

```
## Loading required package: future
```

```
library(knitr)
library(expm)
```

```
## Loading required package: Matrix

##
## Attaching package: 'expm'

## The following object is masked from 'package:Matrix':
##
##     expm
```

Question 2

In this question, we shall consider high-dimensional sample covariance matrices of data that is sampled from an elliptical distribution. We say that a random vector $\mathbf{x}$ with zero mean follows an elliptical distribution if (and only if) it has the stochastic representation

$$\mathbf{x} = \xi A \mathbf{u}, \quad (\star)$$

where the matrix $A \in \mathbb{R}^{p \times p}$ is nonrandom and $\text{rank}(A) = p$, $\xi \geq 0$ is a random variable representing the radius of $\mathbf{x}$, and $u \in \mathbb{R}^p$ is the random direction, which is independent of ξ and uniformly distributed on the unit sphere S_{p-1} in $\mathbb{R}^p$, denoted by $\mathbf{u} \sim \text{Unif}(S_{p-1})$. The class of elliptical distributions is a natural generalization of the multivariate normal distribution, and contains many widely used distributions as special cases including the multivariate t-distribution, the symmetric multivariate Laplace distribution, and the symmetric multivariate stable distribution.

(a) Write a function runifsphere(n,p) that samples n observations from the distribution $\text{Unif}(S_{p-1})$ using the fact that if $\mathbf{z} \sim N_p(0, I_p)$ then $\frac{\mathbf{z}}{\|\mathbf{z}\|} \sim \text{Unif}(S_{p-1})$. Check your results by:

set $p = 25$, $n = 50$ and show that the (Euclidean) norm of each observation is equal to 1.

Answer to question 2 (a) (1):

Now we show that the (Euclidean) norm of each observation is equal to 1 in (1):