

Q3(c):

Suppose that $d_0 > 0$, $P = \overbrace{(0, 0, \dots, 0)}^p$ and $Q = \overbrace{(d_0, 0, \dots, 0)}^p$
then making a simplifying assumption that error is multivariate normal, we get:

$$\begin{aligned} P^m &= P + \varepsilon & \varepsilon &\sim N_p(0, I) & \varepsilon &= (\varepsilon_i)_{i=1, p} \\ Q^m &= Q + \eta & \eta &\sim N_p(0, I) & \eta &= (\eta_i)_{i=1, p} \end{aligned}$$

The points P^m and Q^m have "measurement error."

$$\begin{aligned} E[d(P^m, Q^m)^2] &= E[\|Q^m - P^m\|^2] \\ &= E[\|Q + \eta - (P + \varepsilon)\|^2] \\ &= E[(d_0 + \eta_1 - \varepsilon_1)^2 + (\eta_2 - \varepsilon_2)^2 + \dots + (\eta_p - \varepsilon_p)^2] \\ &= E[d_0^2 + d_0(\eta_1 - \varepsilon_1) + (\eta_1 - \varepsilon_1)^2 + \dots + (\eta_p - \varepsilon_p)^2] \\ &= d_0^2 + E[(\eta_1 - \varepsilon_1)^2 + \dots + (\eta_p - \varepsilon_p)^2] \\ &= d_0^2 + \text{Var}(\eta_1 - \varepsilon_1) + \text{Var}(\eta_2 - \varepsilon_2) + \dots + \text{Var}(\eta_p - \varepsilon_p) \\ &> d_0^2 \end{aligned}$$

Also notice that if we assume $(\eta_i - \varepsilon_i) = N(0, c)$ for $i=1, \dots, p$
then

$$\begin{aligned} E[d(P^m, Q^m)^2] &= d_0^2 + \sqrt{c} E[X] & X &\sim \chi_p^2 \\ &= d_0^2 + \sqrt{c} p. & \text{As } E[X] &= p. \end{aligned}$$