Assessment 1

Due by Monday 8 August 2023 09:00

Question 1 [2 marks]

Show that the determinant of the $p \times p$ diagonal matrix $\mathbb{A} = (a_{ij})$ with $a_{ij} = 0$, $i \neq j$, is given by the product of the diagonal elements; that is, $|\mathbb{A}| = a_{11}a_{22} \cdots a_{pp}$.

Question 2 [2 marks]

Show that the determinant of a square symmetric $p \times p$ matrix $\mathbb{A}$ can be expressed as the product of its eigenvalues λ_1, λ_2, ..., λ_p; that is, $|\mathbb{A}| = \prod_{i=1}^{p} \lambda_i$.

Question 3 [6 marks]

A closed-form solution for the volume of an n-dimensional ball exists and is given in terms of the gamma function $\Gamma(x)$.

[2]   (a) Write down the volume of an n-dimensional ball as a mathematical formula (i.e., no proof necessary) and then implement this formula in the R programming language.

[2]   (b) Write some code to plot the volume of an n-dimensional unit ball in the R programming language where the dimension n is on the x-axis and the volume is on the y-axis. Plot the volume for $n = 1, 2, 3, \ldots, N$ for some large N of your choice.

[2]   (c) Comment on how the plot obtained in (b) relates to the discussions in Lecture 1.

This homework is to be submitted through Wattle in <u>*digital form only*</u> *as per ANU policy. The R code for any computational question must be supplied. If you use any references (note: this will never count against you), please clearly indicate which ones.*