Dynamic Agent Allocation Problem in Multi-Agent Target Defense Differential Games

Arunaditya Dash P (210005010), Harsh Soni (210003035),
Shubhneet Tiwari (210002073)
Supervisor: Dr. Sharad Kumar Singh

1 Introduction

Pursuit-evasion games are a fundamental area of study in differential game theory, modeling scenarios where agents with conflicting objectives interact dynamically [1]. In recent years, the introduction of a third player, namely the defender, has expanded these interactions into more complex Target-Attacker-Defender (TAD) differential games, which find crucial applications in military operations, security systems, and autonomous vehicle coordination [1, 2]

TAD games present significant challenges in developing strategies for each player type due to their dynamic and multi-agent nature. As modern defense scenarios become increasingly complex and autonomous systems more prevalent, the need for sophisticated, real-time decision-making strategies in multi-agent environments has grown substantially. Traditional static assignment methods often fail to adapt to rapidly changing game states, potentially leading to suboptimal outcomes [3, 4].

The complexity of TAD games arises from several factors. First, the continuous-time nature of the game requires strategies that can adapt in real-time to changing conditions. Second, the presence of multiple agents with conflicting objectives creates a highly interdependent decision space, where each agent's actions affect the optimal strategies of all others. Third, the need for defenders to coordinate their efforts while dealing with potentially uncoordinated attackers adds another layer of strategic complexity [5, 6].

This report investigates dynamic assignment strategies in multi-agent TAD differential games, focusing on scenarios with a moving target, multiple attackers, and multiple defenders. The primary objective is to develop an optimal strategy for solving the Dynamic Agent Allocation Problem, which involves continuously reassigning defenders to attackers in real-time to maximize the likelihood of successful target defense.

In the following sections, we will delve deeper into the motivation behind this work, outline our project objectives, and provide a brief literature survey to contextualize our research within the broader field of TAD games.

1.1 Motivation and Theme of Work

The dynamic interplay between offense and defense in complex systems motivates the study of Target-Attacker-Defender (TAD) differential games. These games serve as powerful models for a wide range of real-world scenarios, from military engagements to cyber-security challenges. Our work is driven by the need to develop more sophisticated, adaptive strategies in these multi-agent environments. The increasing complexity of modern defense scenarios and the rise of autonomous systems necessitate the development of advanced, real-time decision-making strategies for multi-agent environments. Current static assignment methods often struggle to adapt to rapidly evolving game states, potentially resulting in suboptimal outcomes. Dynamic assignment strategies offer the potential for improved performance by continuously updating defender-attacker pairings based on the current game state.

This research has significant real-world applications across various domains. In military operations, dynamic assignment strategies could enhance missile defense systems, improving their ability to intercept multiple incoming threats. For critical infrastructure protection, these strategies could be applied to security systems for more efficient allocation of security resources in response to potential threats. In the realm of autonomous vehicle coordination, such strategies could be employed to coordinate swarms of drones for tasks like search and rescue operations or environmental monitoring, where adaptability to changing conditions is crucial.

In the next section, we will outline the specific objectives of our project, which aim to address these challenges and advance the field of TAD differential games.

1.2 Project Objective

The primary objectives of this project are:

(a) To develop and evaluate optimal strategies for solving the Dynamic Agent Allocation Problem in Multi-Agent Target Defense Differential Games. An optimal strategy maximizes the defenders' chances of protecting the target while minimizing resource expenditure.

(b) To minimize the overall distance between paired defenders and attackers to maximize the likelihood of successful interceptions.

(c) To investigate the impact of assignment strategies on the overall performance of the defense system, considering factors such as the number of agents, their relative speeds, and the geometry of their initial positions.

While our full project scope aims to handle multiple targets, attackers, and defenders, our current scenario focuses on a situation with one target, three attackers, and three defenders. This simplified model allows us to refine our core algorithms before expanding to more complex scenarios. Ultimately, our goal is to contribute to the field of differential games by providing effective strategies for dynamic agent allocation in complex, multi-target defense scenarios. These strategies have potential applications in military operations, security systems, and autonomous vehicle coordination, where real-time decision-making in complex multi-agent environments is crucial. Through our research, we hope to advance defensive tactics for dynamic, multi-agent systems and provide valuable insights for both theoretical development and practical implementation in real-world scenarios.

To contextualize our work within the broader field of TAD games, the next section provides a brief survey of relevant literature, highlighting key developments and current research trends.

1.3 Brief Literature Survey

The study of TAD games has evolved significantly over the past few decades, progressing from simple scenarios to complex multi-agent systems. Isaacs [7] laid the foundation for differential games, including pursuit-evasion scenarios, which form the basis for modern TAD game theory. His seminal work introduced the concept of value function and optimal strategies in a continuous-time setting, providing a mathematical framework for analyzing conflicts between multiple agents. Basar and Olsder [8] extended this work, offering a comprehensive treatment of dynamic noncooperative game theory and introducing the concept of information structures in differential games.

Recent research has focused on increasingly complex multi-agent scenarios. Garcia et al. [2] explored multiple pursuer-multiple evader differential games, while Singh et al. [1] studied multiple target defense using receding horizon-based switching strategies. These works, along with contributions from Shima [4], Ratnoo and Shima [5], and Pachter et al. [6], have significantly advanced our understanding of cooperative strategies, guidance laws, and optimal defensive tactics in TAD games. The application of TAD games to missile defense scenarios has been a significant area of focus, with researchers like Venkatesan and Sinha [9] proposing new guidance laws for the defense of non-maneuverable aircraft. These studies have not only expanded the theoretical foundations of TAD games but also addressed the practical challenges of implementing such strategies in real-world systems. Our work builds upon these foundations, particularly drawing inspiration from the multi-agent approaches and dynamic strategy formulations, while aiming to bridge the gap between theoretical models and practical implementation in complex, real-time defense scenarios.

In the next section, we will define the game setup and dynamics, providing a formal mathematical framework for our TAD differential game.

2 Game Definition

We consider a differential game with one target (T), three defenders (D), and three attackers (A) on a 2D plane. Before delving into the specific dynamics, it's important to understand how these players interact within the game framework.

2.1 Players' Interactions in the Differential Game

The game models a scenario where each player type has distinct objectives. The target (T) aims to evade capture, maximizing its distance from attackers while potentially seeking proximity to defenders for protection. Attackers (A) pursue the target, attempting to minimize their distance to it and ultimately capture it. Defenders (D) have a dual objective: they seek to intercept attackers before they can reach the target, while also maintaining a strategic position relative to the target for effective defense.

In this context, optimal assignments refer to the pairing of defenders with attackers that minimizes the overall distance between paired agents. This optimization aims to maximize the defenders' chances of intercepting attackers before they reach the target. The dynamic nature of the game requires continuous reassessment and updating of these assignments as the game state evolves.

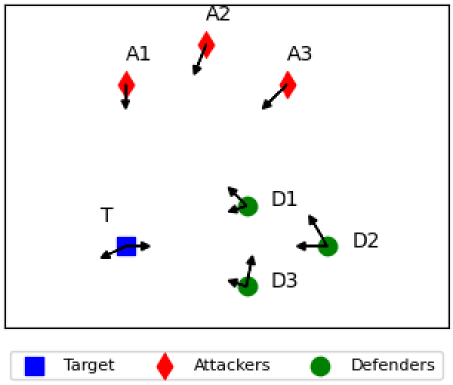


Figure 1: Players' interaction in the game

In the following subsection, we will define the specific dynamics governing the movement of each player type in our game.

2.2 Dynamics of Players

We consider two approaches for modeling player dynamics. The first approach uses a simplified model with direct velocity control, while the second employs a more sophisticated linear system model with control inputs. This dual approach allows us to compare the performance of a straightforward, computationally efficient method against a more complex, potentially more accurate model.

(a) For our first approach, we use a simplified model where the dynamics of the players are as follows:

$$\begin{aligned} \text{Target:} \quad & \dot{x}_T = v_T \hat{d}_T \\ \text{Defenders:} \quad & \dot{x}_{D_i} = v_D \frac{x_{A_j} - x_{D_i}}{\|x_{A_j} - x_{D_i}\|} \\ \text{Attackers:} \quad & \dot{x}_{A_i} = v_A \frac{x_T - x_{A_i}}{\|x_T - x_{A_i}\|} \end{aligned} \tag{1}$$

where $i, j = 1, 2, \ldots, n$, and v_T, v_D, and v_A are the speeds of the target, defenders, and attackers, respectively, and $\hat{d}_T$ is the unit vector in the direction of the target's movement.

(b) For our second approach, we employ a more sophisticated linear system model:

$$\dot{X}(t) = \sum_{i=1}^{n} B_{D_i} u_{D_i}(t) + \sum_{i=1}^{n} B_{A_i} u_{A_i}(t) + B_T u_T(t) \tag{2}$$

where $X(t) \in \mathbb{R}^{2(2n+1)}$ is the state vector containing the positions of all agents, $u_i(t) \in \mathbb{R}^2$ are control inputs for each agent, and $B_i \in \mathbb{R}^{2(2n+1)\times 2}$ are input matrices defined as:

$$B_T = \begin{bmatrix} I_2 \\ 0_{4n\times 2} \end{bmatrix}, \quad B_{D_i} = \begin{bmatrix} 0_{2i\times 2} \\ I_2 \\ 0_{(4n-2i)\times 2} \end{bmatrix}, \quad B_{A_i} = \begin{bmatrix} 0_{(6+2i)\times 2} \\ I_2 \\ 0_{(4n-2i-6)\times 2} \end{bmatrix} \tag{3}$$

where I_2 is the 2×2 identity matrix and $0_{p\times q}$ is a $p \times q$ zero matrix. The index i in these matrices represents the corresponding agent (defender or attacker).

This formulation allows for more nuanced control strategies and interactions between players, capturing the complex interdependencies present in real-world scenarios.

In the next subsection, we will define the termination conditions for our game, which determine when the game ends and how the outcome is decided.

2.3 Termination Condition

The game terminates when one of the following conditions is met:

$$\exists i : \|x_T(t) - x_{A_i}(t)\| \leq \sigma_a \quad \text{or} \quad \exists i, j : \|x_{D_i}(t) - x_{A_j}(t)\| \leq \sigma_d \tag{4}$$

An attacker captures the target when the distance between any attacker and the target becomes less than or equal to a predefined capture radius σ_a. This represents a failure scenario for the defenders. Alternatively, a defender intercepts an attacker when the distance between any defender and any attacker becomes less than or equal to a predefined interception radius σ_d. The game continues until either an attacker captures the target or a defender intercepts an attacker.

In the next section, we will analyze the game in the interception mode, focusing on the defenders' strategies to intercept attackers before they can reach the target.

3 Analysis in Interception Mode

In the interception mode, we focus on the defender's strategy to intercept attackers before they can reach the target. For the game theory-based approach, we model it as a linear quadratic differential game (LQDG).

3.1 Cost Functions

The performance metric to be minimized by player $i \in \mathcal{P}$, where $\mathcal{P} = \{T, D_1, D_2, D_3, A_1, A_2, A_3\}$, is given by:

$$J_i = G_i(T) + \int_0^T L_i(t)\, dt \tag{5}$$

where $G_i(T)$ and $L_i(t)$ denote the terminal and running costs, respectively. The specific forms of these costs for each player type are:

$$G_T(T) = \frac{1}{2}\sum_{j=1}^{3}\left(\|x_T(T) - x_{D_j}(T)\|^2_{Q_{TD_j}} - \|x_T(T) - x_{A_j}(T)\|^2_{Q_{TA_j}}\right) \tag{6}$$

$$L_T(t) = \frac{1}{2}u_T^T(t)R_T u_T(t) + \frac{1}{2}\sum_{j=1}^{3}\left(\|x_T(t) - x_{D_j}(t)\|^2_{Q_{TD_j}} - \|x_T(t) - x_{A_j}(t)\|^2_{Q_{TA_j}}\right) \tag{7}$$

$$G_{D_i}(T) = \frac{1}{2}\sum_{j=1}^{3} A_{ij}(T)\|x_{D_i}(T) - x_{A_j}(T)\|^2_{Q_{D_iA_j}} + \|x_{D_i}(T) - x_T(T)\|^2_{Q_{D_iT}} \tag{8}$$

$$L_{D_i}(t) = \frac{1}{2}u_{D_i}^T(t)R_{D_i}u_{D_i}(t) + \frac{1}{2}\sum_{j=1}^{3} A_{ij}(t)\|x_{D_i}(t) - x_{A_j}(t)\|^2_{Q_{D_iA_j}} + \|x_{D_i}(t) - x_T(t)\|^2_{Q_{D_iT}} \tag{9}$$

$$G_{A_i}(T) = \frac{1}{2}\|x_{A_i}(T) - x_T(T)\|^2_{Q_{A_iT}} \tag{10}$$

$$L_{A_i}(t) = \frac{1}{2}u_{A_i}^T(t)R_{A_i}u_{A_i}(t) + \frac{1}{2}\|x_{A_i}(t) - x_T(t)\|^2_{Q_{A_iT}} \tag{11}$$

Here, Q_{ij} and R_i are weighting matrices that can be tuned to adjust the relative importance of different objectives. The matrices $\tilde{Q}_i$, $\tilde{Q}_{iT}$, and R_i are constructed as follows: $\tilde{Q}_T = \tilde{Q}_{TT} = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix}$, $Q_1 = \sum_{j=1}^{3}(Q_{TD_j} - Q_{TA_j})$, $Q_2 = [-Q_{TD_1}, -Q_{TD_2}, -Q_{TD_3}, Q_{TA_1}, Q_{TA_2}, Q_{TA_3}]$, and $Q_3 = \text{diag}(Q_{TD_1}, Q_{TD_2}, Q_{TD_3}, -Q_{TA_1}, -Q_{TA_2}, -Q_{TA_3})$. For the defenders, $\tilde{Q}_{D_i} = \tilde{Q}_{D_iT} = \begin{bmatrix} Q_{D_iT} & Q_4 \\ Q_4^T & Q_5 \end{bmatrix}$, where $Q_4 = [q_1, \ldots, q_{2n}]$, with $q_j = -Q_{D_iT}$ if $i = j$ and 0 otherwise, for $j = 1, 2, \ldots, 2n$. $Q_5 = \begin{bmatrix} Q_{5_1} & Q_{5_2} \\ Q_{5_2}^T & Q_{5_3} \end{bmatrix}$, where $Q_{5_1} = \text{diag}(q_1, \ldots, q_n)$, with $q_j = \sum_{k=1}^{3} A_{jk}Q_{D_jA_k} + Q_{D_jT}$ if $i = j$ and $\sum_{k=1}^{3} A_{jk}Q_{D_jA_k}$ otherwise. $Q_{5_2} = [q_{jk}]$, with $q_{jk} = -A_{jk}Q_{D_jA_k}$ for $j, k = 1, \ldots, n$, and $Q_{5_3} = \text{diag}(q_1, \ldots, q_n)$, with $q_j = \sum_{k=1}^{3} A_{jk}Q_{D_jA_k}$. For the attackers, $\tilde{Q}_{A_i} = \tilde{Q}_{A_iT} = \begin{bmatrix} Q_{A_iT} & Q_6 \\ Q_6^T & Q_7 \end{bmatrix}$, where $Q_6 = [q_1, \ldots, q_{2n}]$, with $q_j = -Q_{A_iT}$ if $i = j - n$ and 0 otherwise, for $j = 1, 2, \ldots, 2n$, and $Q_7 = \text{diag}(q_1, \ldots, q_n)$, with $q_j = Q_{A_iT}$ if $i = j - n$ and 0 otherwise, for $j = 1, 2, \ldots, 2n$.

The matrices R_i are defined as $R_T = r_T I_2$, $R_{D_i} = r_{D_i} I_2$, and $R_{A_i} = r_{A_i} I_2$, where I_2 is the 2×2 identity matrix, and r_T, r_{D_i}, and r_{A_i} are positive scalars that penalize the control effort of the target, defenders, and attackers, respectively.

In the next subsection, we will discuss the concept of Nash equilibrium in the context of our game and how it is computed.

3.2 Nash Equilibrium

In game theory, a Nash equilibrium represents a state where no player can unilaterally improve their outcome by changing their strategy, given that other players maintain their current strategies. In the context of our LQDG framework, we seek to find Nash equilibrium strategies for all players.

A strategy profile $(u_T^*, u_{D_1}^*, u_{D_2}^*, u_{D_3}^*, u_{A_1}^*, u_{A_2}^*, u_{A_3}^*)$ is a Nash equilibrium if, for each player i, u_i^* minimizes the cost J_i given that all other players are using their equilibrium strategies u_{-i}^*. Mathematically, this can be expressed as:

$$J_i(u_i^*(.), u_{-i}^*(.), t_0, T) \leq J_i(u_i(.), u_{-i}^*(.), t_0, T) \quad \forall u_i(.). \tag{12}$$

The Nash equilibrium strategies are obtained by solving a system of coupled Riccati differential equations. For our 7-player game (1 target, 3 defenders, 3 attackers), let there exist a solution set $\{P_i(t), i \in \mathcal{P}\}$ to the following coupled Riccati differential equations:

$$\dot{P}_i(t) = P_i(t)\left(S_T P_T(t) + \sum_{j=1}^{3}[S_{D_j}P_{D_j}(t) + S_{A_j}P_{A_j}(t)]\right) - \tilde{Q}_i \tag{13}$$

where $P_i(T) = \tilde{Q}_{iT}$ and $S_i = B_i R_i^{-1} B_i^T$ for $i \in \{T, D_1, D_2, D_3, A_1, A_2, A_3\}$.

Here, $\tilde{Q}_i$ and $\tilde{Q}_{iT}$ are the state cost matrices as defined in the previous section, B_i are the input matrices, and R_i are the control cost matrices for each player.

In the next subsection, we will introduce the receding horizon strategy, which allows for real-time adaptation of player strategies.

3.3 Receding Horizon Strategy

The Receding Horizon Strategy is a method for breaking down the continuous-time problem into a series of finite-horizon optimization problems. This approach allows for real-time adaptation and decision-making in dynamic environments.

To enable real-time adaptation, we employ a receding horizon strategy. At each time step t_k, players solve the following optimization problem:

$$J_i^{RH} = J_i(u_T(.), u_{D_1}(.), u_{D_2}(.), u_{D_3}(.), u_{A_1}(.), u_{A_2}(.), u_{A_3}(.), t_k, T) \tag{14}$$

The open-loop Nash equilibrium strategy is computed over $[t_k, t_k+T]$, but only implemented for $[t_k, t_{k+1})$. The state at t_{k+1} is updated as:

$$X(t_{k+1}) = [I_{2N}\ 0_{2N}\ \cdots\ 0_{2N}]e^{M(t_{k+1}-T)}[I_{2N}\ Q_T^T\ \cdots\ Q_{A_3}^T]^T H^{-1}(T)X(t_k) \tag{15}$$

where N is the total number of agents, and M is defined as $M = \begin{bmatrix} 0_{2N} & -S \\ -Q & 0_{2N} \end{bmatrix}$, and $H(T)$ is given by $H(T) = [I_{2N}\ 0_{2N}\ \cdots\ 0_{2N}]e^{-MT}[I_{2N}\ Q_T^T\ \cdots\ Q_{A_3}^T]^T$. This process repeats at each time step until termination criteria are met, allowing for dynamic reassignment and adaptation to changing game states.

In the next section, we will discuss the dynamic assignment strategies that form the core of our approach to solving the Dynamic Agent Allocation Problem.

4 Dynamic Assignment Strategies

We propose two strategies for dynamic assignment in multi-agent target defense differential games: a heuristic approach and a game theory based approach using Linear Quadratic Differential Game (LQDG) theory. In the next section, we detail these approaches and their implementation.

4.1 Heuristic Approach

In this approach, we use a well-known assignment algorithm to optimally assign defenders to attackers at each time step. The cost matrix C is calculated as:

$$C_{ij} = \|x_{D_i} - x_{A_j}\| \tag{16}$$

where x_{D_i} and x_{A_j} are the positions of defender i and attacker j, respectively.

The optimal assignment is then found by solving:

$$\min_{\pi} \sum_{j} C_{j,\pi(j)} \tag{17}$$

where π is a permutation representing the assignment.

The algorithm for the heuristic approach is as follows:

Algorithm 1 Heuristic Dynamic Assignment Algorithm

Initialize player positions and parameters
while game not terminated **do**
    Compute cost matrix $C_{ij} = \|x_{D_i} - x_{A_j}\|$
    Solve assignment problem: $\pi = \arg\min_{\pi} \sum_j C_{j,\pi(j)}$
    Update player positions according to dynamics
    for each attacker **do**
        if distance to target $\leq$ capture radius **then**
            Target captured, end game
        end if
    end for
    for each defender **do**
        if distance to assigned attacker $\leq$ interception radius **then**
            Remove attacker from game
        end if
    end for
    if all attackers removed **then**
        All attackers intercepted, end game
    end if
end while

In the next subsection, we will discuss our game theory based approach, which incorporates the LQDG formulation with dynamic reassignment.

4.2 Game Theory Based Approach: LQDG with Dynamic Reassignment

Our primary focus is on this game theory based approach, which combines the LQDG formulation with dynamic reassignment. The assignment matrix A is updated at each time step based on the current state:

$$A_{ij}(t) = \begin{cases} 1 & \text{if defender } i \text{ is assigned to attacker } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

The cost functions and other parameters are then modified to incorporate this assignment, as shown in the cost function equations in Section 3.1.

The algorithm for the LQDG approach with dynamic reassignment is as mentioned in the next page.

In the next section, we will present and discuss the results of our simulations using these dynamic assignment strategies.

5 Simulation Results and Discussion

We implemented both strategies and simulated various scenarios. For comparison, we used identical initial conditions and parameters for both approaches. The target was initially positioned at (5, 5), with attackers at (0, 0), (10, 0), and (5, 10), and defenders at (0, 10), (10, 10), and (5, 0). We set the capture radii σ_a and σ_d to 0.1, and used speeds of $v_T = 1.0$ for the target, $v_A = 1.2$ for attackers, and $v_D = 0.8$ for defenders. The simulation ran for 10 seconds with a time step of 0.1 seconds. The simulation results are shown in figure 2.

Let's consider the initial positions of the defenders and attackers. At the start of the simulation ($t = 0$), the target was positioned at (5, 5), with attackers at (0, 0), (10, 0), and (5, 10), and defenders at (0, 10), (10, 10), and (5, 0). Using these positions, we calculate the cost matrix C using equation 16: $C_{ij} = \|x_{D_i} - x_{A_j}\|$.

This results in the matrix $C = \begin{bmatrix} 10.00 & 14.14 & 10.00 \\ 10.00 & 10.00 & 14.14 \\ 5.00 & 5.00 & 10.00 \end{bmatrix}$.

Algorithm 2 LQDG with Dynamic Reassignment

Require: Initial state $X(0)$, planning horizon T, time step δt
Ensure: Nash equilibrium strategies

Initialize Q_{ij}, R_i, B_i matrices
Define $S_i = B_i R_i^{-1} B_i^T$
while game not terminated **do**
    Compute assignment matrix $A(t)$ using Hungarian algorithm
    Update cost matrices Q_i using $A(t)$
    Construct $M = \begin{bmatrix} 0 & -S \\ -Q & 0 \end{bmatrix}$
    $H(T) \leftarrow [I, 0, ..., 0]e^{-MT}[I, Q_T^T, Q_{D1}^T, ..., Q_{A3}^T]^T$
    if $H(T)$ is not invertible **then**
        Adjust T or modify cost matrices
        continue
    end if
    $Y \leftarrow e^{-MT} Q H(T)^{-1} X(0)$
    Apply strategies for time step δt
    Update state: $X(t + \delta t) \leftarrow Y[: 14]$
    if $\exists i : \|x_T - x_{A_i}\| \leq \sigma_a$ **then**
        return "Target captured by attacker"
    else if $\forall j : \exists i : \|x_{D_i} - x_{A_j}\| \leq \sigma_d$ **then**
        return "All attackers captured by defenders"
    end if
    $t \leftarrow t + \delta t$
end while

This cost matrix represents the Euclidean distances between each defender and each attacker. Using the Hungarian algorithm to solve equation 17, $\min_\pi \sum_j C_{j,\pi(j)}$, we obtain the initial assignment $\pi = [2, 1, 0]$. This optimal assignment minimizes the total distance between paired defenders and attackers.

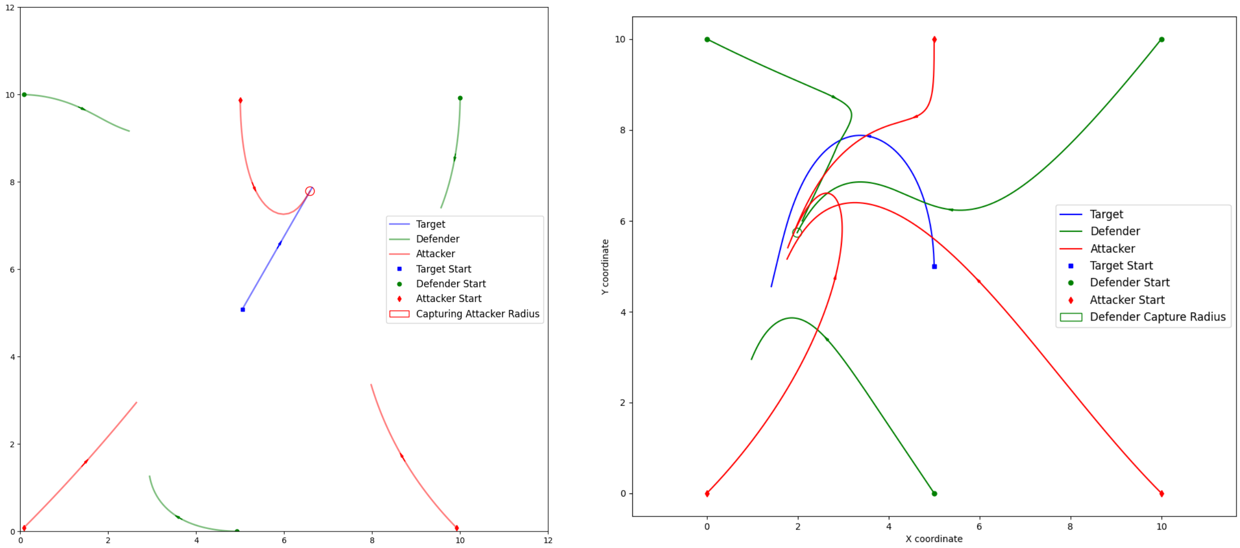
The assignment π corresponds to the following assignment matrix A (equation 18): $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, where $A_{ij}(t) = 1$ if defender i is assigned to attacker j at time t, and 0 otherwise. This matrix A indicates that the first defender is assigned to the third attacker, the second defender to the second attacker, and the third defender to the first attacker.

As the simulation progresses, these assignments are continuously reevaluated and updated. At $t \approx 0.144$, we observed that the assignment matrix changed to: $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. This change in the assignment demonstrates the dynamic nature of the allocation strategy, allowing defenders to adapt their strategies based on the evolving game state.

Our simulations revealed several key insights into the performance of both approaches. The heuristic approach demonstrated its ability to provide quick, optimal assignments at each time step. This method excels in scenarios where rapid decision-making is crucial and computational resources are limited. However, its main limitation lies in its myopic nature, as it optimizes only for the current state without considering the overall game dynamics.

In contrast, the game theory based LQDG approach showed superior performance, particularly in complex scenarios involving multiple attackers and defenders. The LQDG approach incorporates more sophisticated cost functions that capture the complexities of the multi-agent interactions. These functions consider not just distances, but also control efforts and relative positions of all agents simultaneously. By computing Nash equilibrium strategies, this method accounts for the strategic interactions between all players, leading to more robust decision-making that considers the potential actions of other agents.

Furthermore, the receding horizon implementation, although not predictive, allows for a form of look-ahead by solving the game over a finite time horizon at each step. This provides a balance between immediate actions and longer-term considerations. The LQDG approach's ability to capture the complex interplay between agents and balance multiple objectives simultaneously resulted in more sophisticated and effective strategies compared to the simpler heuristic method.



(a) Heuristic Approach (Target Captured) (b) Game Theory Based Approach (Attackers Captured)

Figure 2: Comparison of Heuristic and Game Theory Based Approaches

In the next section, we will outline our planned workflow for extending this research to more complex scenarios with multiple targets and larger numbers of attackers and defenders.

6 Planned Workflow

Our roadmap outlines a systematic approach to expanding and refining our game theory-based model for multi-agent target defense differential games. The planned workflow consists of three main stages:

(a) Extension to multiple attackers and multiple defenders for a single target: Scale the state space representation to accommodate the increased number of agents. Modify the cost functions to account for multiple agents.

(b) Incorporation of multiple targets: Extend the state space and control inputs to account for multiple targets. Modify the objective functions to consider multiple target priorities. Adapt the assignment problem for the multi-target scenario.

(c) Comprehensive analysis of strategies: Study emergent behaviors and patterns in different scenarios. Examine the effectiveness of various defensive formations. Investigate the impact of different parameter settings on game outcomes.

This staged approach will allow us to systematically address the challenges that arise with increased complexity, ensuring that our methods remain computationally tractable and practically applicable as we move towards more sophisticated models.

References

[1] S. K. Singh, P. V. Reddy, and B. Vundurthy, "Study of Multiple Target Defense Differential Games Using Receding Horizon-Based Switching Strategies," IEEE Trans. Control Syst. Technol., vol. 30, no. 4, pp. 1403-1414, July 2022.

[2] E. Garcia, D. W. Casbeer, A. Von Moll, and M. Pachter, "Multiple Pursuer Multiple Evader Differential Games," IEEE Trans. Autom. Control, vol. 66, no. 5, pp. 2345-2360, May 2021.

[3] Y. Shi, C. Wang, and Z. Ding, "Strategy Analysis and Design of Target-Attacker-Defender Game with Non-zero Capture Radius for Attacker Vehicle," IEEE Trans. Intell. Veh., vol. 8, no. 5, pp. 3690-3701, May 2023.

[4] T. Shima, "Optimal cooperative pursuit and evasion strategies against a homing missile," J. Guid., Control, Dyn., vol. 34, no. 2, pp. 414–425, 2011.

[5] A. Ratnoo and T. Shima, "Line-of-sight interceptor guidance for defending an aircraft," J. Guid., Control, Dyn., vol. 34, no. 2, pp. 522–532, Mar. 2011.

[6] M. Pachter, E. Garcia, and D. W. Casbeer, "Differential game of guarding a target," J. Guid., Control, Dyn., vol. 40, no. 11, pp. 2991–2998, 2017.

[7] R. Isaacs, "Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization," John Wiley and Sons, 1965.

[8] T. Basar and G. J. Olsder, "Dynamic Noncooperative Game Theory," SIAM, 1999.

[9] R. H. Venkatesan and N. K. Sinha, "A new guidance law for the defense missile of nonmaneuverable aircraft," IEEE Trans. Control Syst. Technol., vol. 23, no. 6, pp. 2424-2431, Nov. 2015.