SEC1: Document Preparation and Presentation Software

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In this section, we will introduce the important notion of the limit of a function. The intuitive idea of the function f having a limit **L** at the point c is that the values $f(x)$ are close to **L** when x is close to (but different from) c. But it is necessary to have a technical way of working with the idea of "Close to" and this is accomplished in the $\varepsilon - \delta$ definition given below.
In order for the idea of the limit of a function f at a point c to be meaningful,it is necessary that f be defined at points near c. It need not be defined at the point c,but it should be defined at enough points close to c to make the study interesting . This is the reason for the following definition.

The "lim" tells us we're looking for a limit value, not a function value.

This tells us which function we're working with.

$$\lim_{x \to 6} f(x) = 4$$

This tells us what the variable is, and what it is approaching.

This is the value the function is approaching.

Figure: Basic Terms in Limits

Let $A \subset \mathbf{R}$. A point $c \in \mathbf{R}$ is a **Cluster point** of A if for every $\delta > 0$ there exists at least one point $x \in A, x \neq c$ such that mod $x - c < \delta$. This definition is rephrased in the language of neighborhoods as follows : A point c is a cluster point of the set A if every δ-neighborhood $V_\delta(c) = (c - \delta, c + \delta)$ of c contains at least one point of A distinct from c.

Note The point c may or may not be a member of A, but even if it is in A,it is ignored when deciding whether it is a cluster point of A or not, since we explicitly require that there be points in $V_\delta(c) \cap A$ distinct from c in order for c to be a cluster point of A.

For example, if $A := 1, 2$,then the point 1 is a cluster point of A, since choosing $\delta := \frac{1}{2}$ gives a neighborhood of 1 that contains no points of A distinct from 1. The same is true for the point 2, so we see that A has no cluster points.

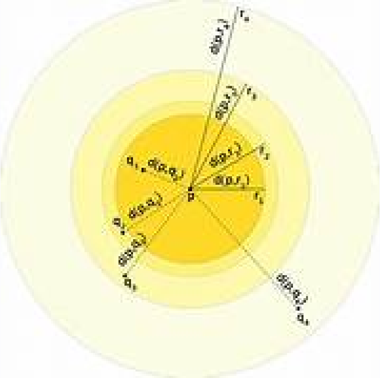

Figure: Cluster points

Enumerate

1. For the open interval $A_1 := (0, 1)$, every point of the closed interval $[0, 1]$ is a cluster point of A_1. Note that the points 0,1 are cluster points of A_1, but do not belong to A_1. All the points of A_1 are cluster points of A_1.
2. A finite set has no cluster points.
3. The infinite set N has no cluster points.
4. The set $A_4 := 1/n : n \in N$ has only the point) as a cluster point. None of the points in A_4 is a cluster point of A_4.
5. If $I := [0, 1]$, then the set $A_5 := I \cap Q$ consists of all the rational numbers in I. It follows from the Density Theorem 2.4.8 that every point in I is a cluster point of A_5.

Having made this brief detour, we now return to the concept of the limit of a function at a cluster point of its domain.

Itemize

- $\lim_{x \to c} f(x) = L$.
- *Given any ϵ-neighborhood $V_\epsilon(L$ of L, there exists a δ-neighborhood $V_\delta(c)$ of c such that if $x \neq c$ is any point in $V_\delta(c) \cap A$, then f(x) belongs to $V_\epsilon(L)$.*

We now state the precise definition of the limit of a function f at a point c. It is important to note that in this definition, it is immaterial whether f is defined at c or not. In any case, we exclude c from consideration in the determination of the limit.

Let $A \subset R$, and let c be a cluster point of A. For a function $f\colon A \to R$, a real number L is said to be a **limit of f at c** if, given any $\epsilon > 0$, there exists a $\delta > 0$ such that if $x \in A$ and $0 < |x - c| < \delta$, then $|f(x) - \mathbf{L}| < \epsilon$. **Remarks**

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1}\end{aligned}$$

Equations

1. Let $x = (x_1, \cdots, x_n)$, where the x_i are non-negative real numbers. Set

$$M_r(x) = (\frac{x_1^r + x_2^r + \cdots + x_n^r}{n})^{1/r}, \ \ r \in \mathbf{R} \setminus \{0\}$$

and

$$M_0(x) = (x_1 x_2 \cdots x_n)^{1/n}.$$

We call $M_r(x)$ the *rth power mean* of x. Claim:

$$\lim_{r \to 0} M_r(x) = M_0(x)$$

2. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde matrix* of order n.
Claim:

$$\det V_n = \prod_{1 \leq i \leq j \leq n} (x_j - x_i).$$

$$
\begin{aligned}
1+2 &= 3 \\
4+5+6 &= 7+8 \\
9+10+11+12 &= 13+14+15 \\
16+17+18+19+20 &= 21+22+23+24 \\
25+26+27+28+29+30 &= 31+32+33+34+35
\end{aligned}
$$

$$
\begin{aligned}
(a+b)^2 &= (a+b)a+(a+b)b \\
&= a(a+b)+b(a+b) \\
&= a^2+ab+ba+b^2 \\
&= a^2+ab+ab+b^2 \\
&= a^2+2ab+b^2
\end{aligned}
$$

$$
\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}
$$

Tables in LaTeX can be created through a combination of the table environment and the tabular environment. The table environment part contains the caption and defines the float for our table, i.e. where in our document the table should be positioned and whether we want it to be displayed centered.

Country List		
Country Name or Area Name	ISO ALPHA 2 Code	ISO ALPHA 3
Afghanistan	AF	AFG
Aland Islands	AX	ALA
Albania	AL	ALB
Algeria	DZ	DZA
American Samoa	AS	ASM
Andorra	AD	AND
Angola	AO	AGO

Ex.2

<table>
<tr><td rowspan="2">gnats</td><td>gram</td><td>$13.65</td></tr>
<tr><td>each</td><td>.01</td></tr>
<tr><td>gnu</td><td rowspan="2">stuffed</td><td>92.50</td></tr>
<tr><td>emu</td><td>33.33</td></tr>
<tr><td>armadillo</td><td>frozen</td><td>8.99</td></tr>
</table>

Ex.3

<table>
<tr><th>coll</th><th>col2</th><th>col3</th></tr>
<tr><td rowspan="3">Multiple row</td><td>cell2</td><td>cell3</td></tr>
<tr><td>cell5</td><td>cell6</td></tr>
<tr><td>cell8</td><td>cell9</td></tr>
</table>

LATEX provides several options to handle images and make them look exactly what you need. Latex can not manage images by itself, so we need to use the graphicx package. To use it, we include the following line in the preamble: graphicx. Plus we can angle the image we want to add in the document.

[1, 2, 3]

A.Einstein.Zur Elektrodynamik bewegter Korper.(German) [On the electrodynamics of moving bodies].*Annalen der physik*,32(10):891-921,1905.doi: http://dx.doi.org/10.1002/andp.19053221004.

M.Goossens, F.Mittlebach, and A.Samarin. The LaTeX Companion.Addison-Wesley,Reading,Massachusetts,1993.

D.Knuth. Knuth: Computers and typesetting. URL http://www-cs-faculty.stanford.edu/ uno/abcde.html.

Thank You