MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1. (a) It is because printing money can lead to inflation and economic instability, while issuing bonds provide a controlled borrowing mechanism without devaluing the currency.
   (b) The long-term part of a yield curve might flatten if investors expect slower economic growth and lower future interest rates, leading them to buy long-term bonds to lock in current rate, which drives their yields closer to short-term yields.
   (c) Quantitative easing is monetary policy where a central bank purchases securities on a open market, where Fed purchased 56% of the Treasury issuance of securities during the pandemic [1] to lower long-term interest rates and stimulate the economy.
2. I selected the following 10 bonds:

Name	**ISIN**	**Coupon (%)**	**Maturity Date**	**Maturity Bucket**
CAN 3.500 Aug 01	CA135087Q640	3.500	2025-08-01	0-1Y
CAN 3.000 Oct 01	CA135087P246	3.000	2025-10-01	0-1Y
CAN 3.000 Apr 01	CA135087P816	3.000	2026-04-01	1-2Y
CAN 3.250 Nov 01	CA135087S398	3.250	2026-11-01	1-2Y
CAN 3.000 Feb 01	CA135087S547	3.000	2027-02-01	2-3Y
CAN 3.245 Aug 24	CA135087P733	3.245	2027-08-24	2-3Y
CAN 2.750 Sept 01	CA135087N837	2.750	2027-09-01	2-3Y
CAN 3.500 Mar 01	CA135087P576	3.500	2028-03-01	3-4Y
CAN 3.250 Sept 01	CA135087Q491	3.250	2028-09-01	3-4Y
CAN 3.500 Sept 01	CA135087R895	3.500	2029-09-01	4-5Y

   The selection includes a diverse mix of maturities, all within five years from January 20, 2025, to avoid clustering, and also prioritizes consistent coupon rates (primarily 3.000%, 3.250%, and 3.500%), ensuring smooth yield and spot curve construction. This strategy balances maturity variety with comparable yields, enhancing liquidity, market representation, and the accuracy of interpolation.
3. Eigenvalues indicate how much variance each principal component captures, with larger values representing more variance. Additionally, eigenvectors define the directions of maximum data variation, with the first few components capturing the most significant trends along the stochastic curve.

Empirical Questions - 75 points

4. (a) Yield to Maturity (YTM) for each of 10 selected bonds are calculated using their **dirty prices** (clean price plus accrued interest). The YTM is computed with the **semi-annual coupon** formula using the **Newton-Raphson method** for solving. After calculating YTMs for each bond on different dates, the **PCHIP interpolation** method is used to smooth the data and generate continuous yield curves to ensure a smooth, monotonic curve without unrealistic oscillations.

The result is a plot showing how the 5-year yield curve evolves over time, with each date's curve superimposed in Figure 1.

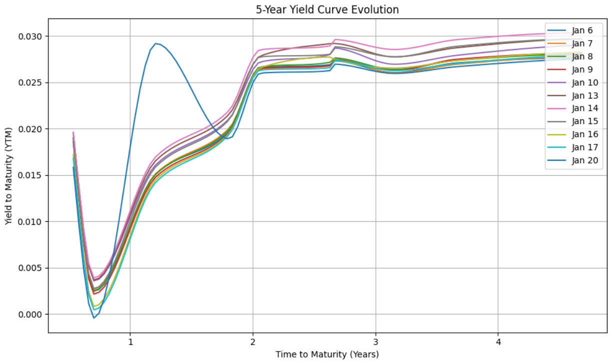

Figure 1: Yield-to-Maturity (YTM) Curves

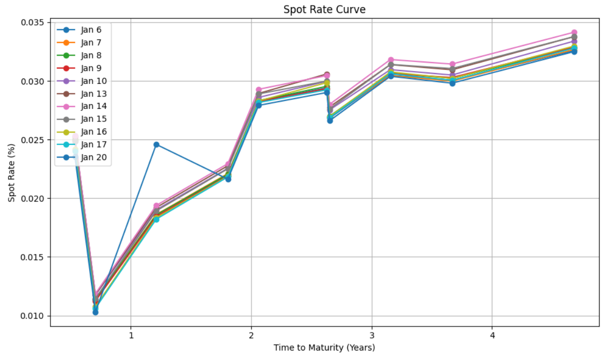

Figure 2: Spot Rate Curves

(b) To derive the spot curve using bootstrapping, it is started by calculating the spot rate for the shortest maturity bond (typically a 1-year bond) using its **dirty price** and **cash flows**. For longer maturity bonds, use the previously calculated spot rates to discount the earlier cash flows, and solve for the spot rate for the remaining maturity. This process is repeated for each bond and maturity, progressively calculating the spot rates for longer maturities based on earlier ones. The resulting spot rates for each date are stored in a dictionary, which can then be plotted to visualize the spot rate curve for each day, with maturities on the x-axis and spot rates on the y-axis as in Figure 2.

(c) To derive the 1-year forward curve with terms ranging from 2-5 years, begin by extracting the relevant spot rates for maturities from 1 to 5 years. For each date, calculate the forward rate for each period (`1y1y`, `1y2y`, `1y3y`, and `1y4y`) by using the formula:

$$F_{t,t+n} = \left(\frac{(1+S_{t+n})^{2(t+n)}}{(1+S_t)^{2t}} \right)^{\frac{1}{2n}} - 1$$

Where S_t is the spot rate for year t and S_{t+n} is the spot rate for year $t+n$. This formula computes the forward rate between year t and $t+n$. Repeat this process for each date, storing the calculated forward rates. Finally, organize the results into a dataframe with dates as rows and forward rates (`1y1y`, `1y2y`, etc.) as columns, then plot the 1-year forward rate curves for each day superimposed on each other as in Figure 3.

5. These are the two covariance matrices for both yield log-returns and forward rate log-returns:

Covariance Matrix for Yield Log-Returns:

	1 Years	2 Years	3 Years	4 Years	5 Years
1 Years	0.054782	0.000189	0.000169	0.000126	−0.000161
2 Years	0.000189	0.000530	0.000566	0.000580	0.000535
3 Years	0.000169	0.000566	0.000611	0.000628	0.000574
4 Years	0.000126	0.000580	0.000628	0.000677	0.000593
5 Years	−0.000161	0.000535	0.000574	0.000593	0.000577

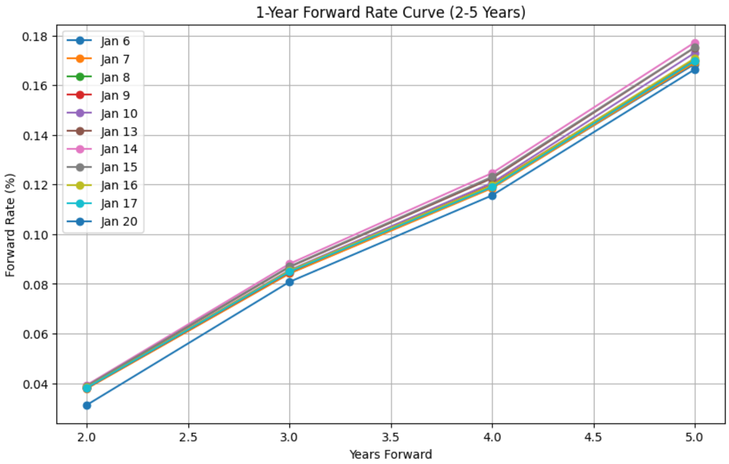

Figure 3: Forward Rate Curves

Covariance Matrix for Forward Rate Log-Returns:

	1y1y	1y2y	1y3y	1y4y
1y1y	0.004143	0.001111	0.000700	0.000540
1y2y	0.001111	0.000377	0.000291	0.000241
1y3y	0.000700	0.000291	0.000263	0.000219
1y4y	0.000540	0.000241	0.000219	0.000201

6. These are the eigenvalues and eigenvectors for the two matrices.

Eigenvalues for Yield Covariance Matrix:

$$[5.47835140 \times 10^{-2}, 2.34048327 \times 10^{-3}, 3.12646046 \times 10^{-5}, 1.89860905 \times 10^{-5}, 2.36196315 \times 10^{-6}]$$

Eigenvectors for Yield Covariance Matrix:

$$\begin{bmatrix} -9.99981942 \times 10^{-1} & 3.11490647 \times 10^{-3} & -3.75866521 \times 10^{-3} & 3.48205066 \times 10^{-3} & -4.00732877 \times 10^{-4} \\ -3.50922352 \times 10^{-3} & -4.72450271 \times 10^{-1} & -5.77998261 \times 10^{-2} & -5.70515943 \times 10^{-1} & 6.69290046 \times 10^{-1} \\ -3.15398914 \times 10^{-3} & -5.08416877 \times 10^{-1} & 8.26281548 \times 10^{-2} & -4.46003664 \times 10^{-1} & -7.31953313 \times 10^{-1} \\ -2.36509421 \times 10^{-3} & -5.30026123 \times 10^{-1} & 6.58095766 \times 10^{-1} & 5.19774486 \times 10^{-1} & 1.25742445 \times 10^{-1} \\ 2.87393499 \times 10^{-3} & -4.87201835 \times 10^{-1} & -7.46141812 \times 10^{-1} & 4.53226930 \times 10^{-1} & -2.19967876 \times 10^{-2} \end{bmatrix}$$

Eigenvalues for Forward Rate Covariance Matrix:

$$[4.66158800 \times 10^{-3}, 3.09171104 \times 10^{-4}, 2.87808034 \times 10^{-6}, 1.00616930 \times 10^{-5}]$$

Eigenvectors for Forward Rate Covariance Matrix:

$$\begin{bmatrix} 0.93916729 & 0.31551034 & 0.13048043 & 0.03732123 \\ 0.26303726 & -0.41244323 & -0.86577012 & -0.10556556 \\ 0.17368179 & -0.61697301 & 0.42465216 & -0.63941339 \\ 0.13642149 & -0.59134433 & 0.23040725 & 0.76066652 \end{bmatrix}$$

Explanation: The first eigenvalue of the yield covariance matrix (0.054783514) represents the largest variance component, and its associated eigenvector indicates the principal direction in which the yield curve exhibits the greatest variation. This implies that the associated eigenvector reflects the dominant risk factor driving the yield curve's behavior.

References

[1] Congressional Research Service. The federal reserve's response to covid-19: Policy issues. Technical report, Congressional Research Service, 2020. (Pages 28–30 of PDF).

GitHub Link: `https://github.com/Soniazdp/APM466`