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CORDIC: Cartesian to Polar Coordinate Transformation

Authors:

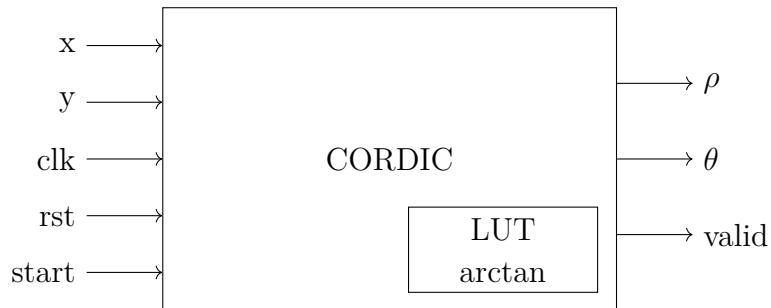
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1 Introduction



It is required to design a digital circuit for implementing the transformation from cartesian coordinates into polar coordinates using the CORDIC algorithm in Vectoring mode. It is implemented with these recursive equations:

$$\begin{aligned} x_{i+1} &= x_i - y_i \cdot d_i \cdot 2^{-i} \\ y_{i+1} &= y_i + x_i \cdot d_i \cdot 2^{-i} \\ z_{i+1} &= z_i - d_i \cdot \arctan(2^{-i}) \end{aligned} \tag{1.1}$$

where $d_i = -1$ if $y_i > 0$, $+1$ otherwise. After n iterations, the equations converge to:

$$x_n = A_n \cdot \sqrt{x_0^2 + y_0^2}$$

$$y_n = 0$$

$$z_n = z_0 + \arctan\left(\frac{y_0}{x_0}\right)$$

$$A_n = \prod_{i=0}^n \sqrt{1 + 2^{-2i}}$$

1.1 Circuit Applications

The CORDIC (COordinate Rotation DIgital Computer) algorithm is an iterative method for performing vector rotations and solving mathematical functions such as trigonometric, hyperbolic, exponential, logarithmic, and square root operations. It was introduced by Jack E. Volder in 1959 to simplify the computation of these

functions in hardware with limited resources.

CORDIC is widely used because it eliminates the need for multiplication and division, relying instead on shift and addition operations. This makes it highly efficient for hardware implementations, particularly in devices with limited computational power, such as embedded systems, calculators, and digital signal processors (DSPs).

For example the Intel 8087, a floating-point coprocessor introduced in the early 1980s, utilized the CORDIC algorithm to perform efficient trigonometric and hyperbolic computations, such as sine, cosine, and arctangent, without relying on hardware multipliers. Similarly, during the Apollo Lunar Module missions, a precursor concept to CORDIC was employed in the Apollo Guidance Computer (AGC) to perform real-time navigation calculations. The AGC used iterative methods to determine angles and distances for lunar landings, efficiently converting Cartesian spacecraft coordinates to polar forms to ensure precise trajectory adjustments during descent.

The CORDIC algorithm can operate in two different modes: **rotation mode** and **vectoring mode**

- **Rotation mode:** rotates a vector by a specified angle, used for calculating trigonometric functions or vector transformations
- **Vectoring mode:** the input vector will be rotated towards the x-axis until the y-component is reduced to zero. After a fixed number of iterations, the modulus of the vector can be retrieved on the x-axis, while its phase corresponds to the angle of rotation.

In this context, this project will focus on developing the vectoring mode of the CORDIC algorithm to convert Cartesian coordinates into polar form.

2 Architecture

2.1 Fix Step

By default, the CORDIC algorithm converges only for input angles within the range $(-99.7^\circ, 99.7^\circ)$. To address this limitation and enable the algorithm to handle arbitrary input angles, we introduced an **initial correction step**. This step adjusts the input angle to bring it into the principal range of the algorithm. The adjustment ensures that the CORDIC algorithm works seamlessly for all input angles, not just those within the default convergence range.

$$x_0 = -y_{\text{input}} \cdot d_{\text{input}}$$

$$y_0 = x_{\text{input}} \cdot d_{\text{input}}$$

$$z_0 = -\frac{\pi}{2} \cdot d_{\text{input}}$$

Additionally, since the iterative process of the CORDIC algorithm introduces a scaling factor A_n , we normalize the final result by dividing ρ (the magnitude) by A_n .

2.2 Algorithm implementation

With reference to the Equations 1.1, the combinatorics of a single step is given by the following block diagram:

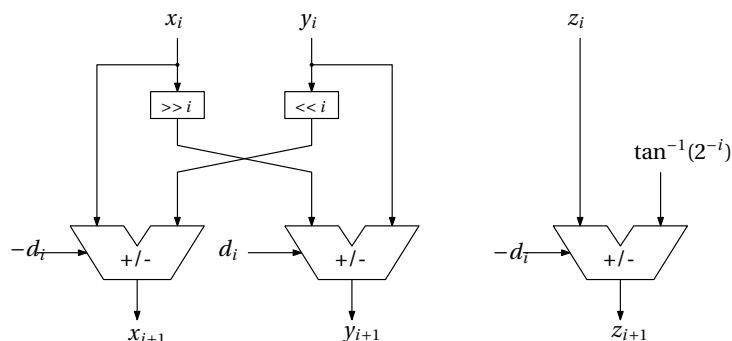


Figure 2.1: Block diagram of a single step of CORDIC

Since the CORDIC algorithm typically converges at a rate of one bit per iteration, the number of steps is predetermined at design time based on data representation. For this reason, a possible implementation of CORDIC involves a series of n combinatorics as shown in Figure 2.1. The resulting combinatorics between input and output registers will assume the following form:

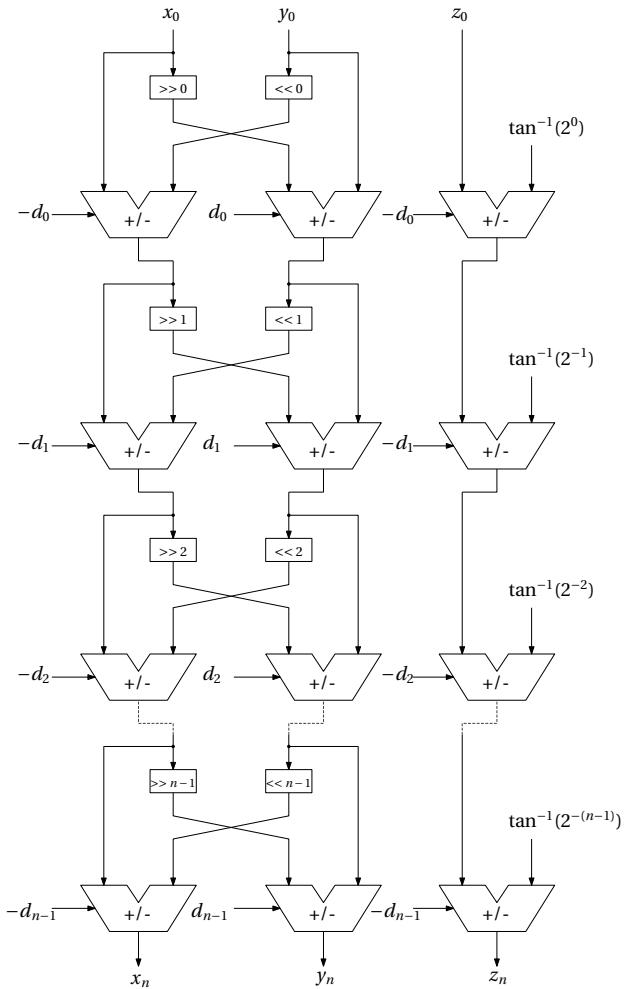


Figure 2.2: Block diagram of the full combinatorics of CORDIC algorithm

Assuming the shift operation has no cost, the logic required will be composed by $3 \cdot N$ adders, where N represents the number of iterations needed for convergence. Additionally, the function will go through N levels of logic. Solutions designed in this way tend to have a long critical path, resulting in a low clock frequency for the network.

To address this limitation, pipelined solutions are generally preferred. By introducing three registers between each step of the process, the throughput of the design remains unchanged. However, the critical path reduces to:

$$T_{\text{critical}} = T_{c \rightarrow q} + T_{\text{prop}} + T_{\text{setup}},$$

where $T_{c \rightarrow q}$ is the clock-to-Q delay of the flip-flop, T_{prop} is the propagation delay of the combinatorial logic which, this time, is just a single level of adder, and T_{setup} is the setup time of the next flip-flop.

Although this modification improves the clock speed, it introduces new trade-offs: The delay for processing a single input becomes $N \cdot T_{\text{clk}}$. More critically, the cost in terms of area occupation and number of components increases significantly, as $3 \cdot N$ additional registers are inserted into the design. This also negatively impacts power consumption. Hybrid solutions, where registers are placed every 2 or 3 levels of logic, are also commonly adopted nowadays to balance clock speed, area efficiency, and power consumption.

The solution we adopted takes advantage of the recursive nature of the CORDIC equations by slightly modifying the single step and incorporating a feedback loop. This design choice ensures a small critical path, allowing for a high clock frequency while minimizing area occupation, cost, and power consumption:

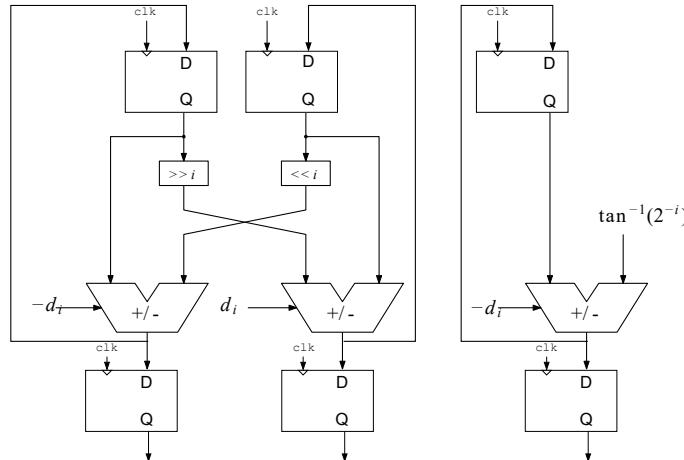


Figure 2.3: Block diagram of a single step CORDIC with feedback loop

This approach was chosen with a specific scenario in mind: the CORDIC algorithm acting as an on-board accelerator. In such a scenario, a smaller design with lower power consumption is highly desirable, as it facilitates integration without significantly affecting the board's utilization, heat dissipation, or energy requirements.

In order to achieve this, we will adopt a Finite State Machine (FSM) with a control part, which includes a status register, and an operational part dedicated to performing the necessary calculations.

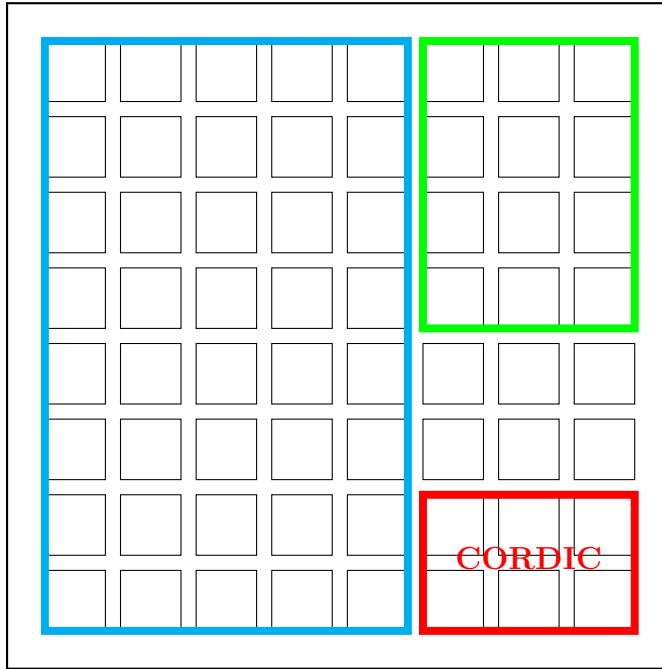


Figure 2.4: Schematic representation of an FPGA. The grid represents the available logic blocks, with coloured outlines indicating resource usage. The CORDIC block, highlighted in red, is a small portion of the total FPGA area.

It is true that this design results in a delay of $N \cdot T_{\text{clk}}$ for each operation and reduces throughput to $1/N$ (processing one input every N clock cycles). However, these trade-offs are justified by the need to prioritize compactness and efficiency over throughput, aligning with the intended use case of the CORDIC as an auxiliary accelerator.

2.3 Data Representation

To implement the CORDIC algorithm, we used fixed-point arithmetic for the input and intermediate values. Specifically:

- x, y : signed 16-bit fixed-point representation with 8 bits allocated for the fractional part.
- ρ : unsigned 16-bit fixed-point representation with 8 bits allocated for the fractional part.
- z, θ : signed 16-bit fixed-point representation with 13 bits allocated for the fractional part. This decision was made because they always lie within the range $[-\pi, \pi]$, and the additional fractional bits ensure high angular precision.
- For the intermediate calculations a 24-bit representation was used to minimize truncation errors during the iterative process.

The number of iterations for the CORDIC algorithm was set to 16, as this provides a

high level of precision while balancing computational efficiency. Beyond 16 iterations, the precision gained decreases significantly.

2.4 Module Precision

The parameter ρ is represented as unsigned with a UQ8.8 fixed-point format. During intermediate calculations, it uses a Q10.14 format in order to prevent overflow. Specifically, the two additional integer bits account for cases like $x = 127$ and $y = 127$, where the output, before normalization, is

$$\text{Output} = A_n \cdot \sqrt{2} \cdot 127,$$

with $A_n \approx 1.6467605$. Substituting, the value is approximately

$$\text{Output} \approx 1.6467605 \cdot 1.4142135 \cdot 127 \approx 295.342.$$

This value exceeds the UQ8.16 range but fits within Q10.14. After normalization, the output is scaled by $\frac{1}{A_n} \approx 0.607252$ and then converted to the UQ8.8 format by truncation of the least significant bits (LSBs).

2.5 Phase Precision and Error Analysis

Focusing on the phase, θ is represented as signed with Q3.13 fixed-point format. During intermediate calculations it uses a Q3.21 fixed-point format. Assuming no error on the input, the values of $\text{atan}(2^{-1})$ in Q3.21 format have an absolute error ϵ_a of 2^{-21} . Over the course of 16 iterations, the accumulated error ϵ_M becomes:

$$\epsilon_M = \epsilon_a * 16 = 2^{-21} * 2^4 = 2^{-17}$$

Since the final result is truncated to Q3.13, which involves discarding the LSBs, the accumulated error from the iterative sums becomes negligible. This ensures that the phase computation maintains high precision.

3 VHDL code

3.1 CORDIC

The following code shows the implementation of the CORDIC in vectoring mode for Cartesian to polar conversion. This implementation supports the fixed step for accepting inputs in all 4 quadrants, and also normalizes ρ . As said in Section 2.2, we followed a Finite State Machine (FSM) approach where the control part uses a status register represented as `current_state`.

```
1 LIBRARY ieee;
2 USE ieee.std_logic_1164.ALL;
3 USE ieee.numeric_std.ALL;
4 ENTITY CORDIC IS
5
6   GENERIC (
7     M : POSITIVE := 24; -- internal representation size
8     N : POSITIVE := 16; -- input size
9     ITERATIONS : POSITIVE := 16; -- CORDIC algorithm iterations
10    ITER_BITS : POSITIVE := 4 -- number of bits needed to represent
11      → iterations
12  );
13  PORT (
14    clk : IN STD_LOGIC;
15    rst : IN STD_LOGIC;
16    x : IN STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
17    y : IN STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
18    start : IN STD_LOGIC;
19    rho : OUT STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
20    theta : OUT STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
21    valid : OUT STD_LOGIC
22  );
23
24 END ENTITY;
25
26 ARCHITECTURE behavioral OF CORDIC IS
27
28   CONSTANT k : UNSIGNED(M - 1 DOWNTO 0) :=
29     → to_unsigned(INTEGER(0.6072528458 * (2 ** (M - 1))), M);
```

```

28 CONSTANT HALF_PI : SIGNED(M - 1 DOWNTO 0) :=
29   &gt; to_signed(INTEGER(1.570796327 * (2 ** (M - 3))), M);
30
31 -- internal registers
32 SIGNAL x_t : SIGNED(M - 1 DOWNTO 0);
33 SIGNAL y_t : SIGNED(M - 1 DOWNTO 0);
34 SIGNAL z_t : SIGNED(M - 1 DOWNTO 0);
35
36 -- output registers
37 SIGNAL x_out : STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
38 SIGNAL z_out : STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
39
40 -- atan table address and output
41 SIGNAL address : STD_LOGIC_VECTOR(ITER_BITS - 1 DOWNTO 0);
42 SIGNAL atan_out : STD_LOGIC_VECTOR(M - 1 DOWNTO 0);
43
44 SIGNAL sign : STD_LOGIC;
45
46 -- atan table
47 COMPONENT ATAN_LUT IS
48   PORT (
49     address : IN STD_LOGIC_VECTOR(ITER_BITS - 1 DOWNTO 0);
50     lut_out : OUT STD_LOGIC_VECTOR(M - 1 DOWNTO 0)
51   );
52 END COMPONENT;
53
54 -- state type and registers
55 TYPE state_t IS (WAITING, FIX, COMPUTING, FINISHED);
56 SIGNAL current_state : state_t;
57
58 -- counter for iterations
59 SIGNAL counter : UNSIGNED(ITER_BITS - 1 DOWNTO 0);
60
61 BEGIN
62 -----
63 -- INSTANTIATE ALL COMPONENTS
64 -----
65
66 -- output assignment
67 rho <= x_out;
68 theta <= z_out;
69
70 -- sign bit
71 sign <= y_t(M - 1);
72
73 -- atan table
74 atan_lut_inst : ATAN_LUT
75 PORT MAP(
76   address => address,
77   lut_out => atan_out
78 );
79
80 -- atan table address

```

```

80 address <= STD_LOGIC_VECTOR(counter);
81
82 -----  

83 -- CONTROL PART  

84 -----
85
86 control : PROCESS (clk, rst)
87 BEGIN
88   IF (rising_edge(clk)) THEN
89     IF rst = '1' THEN
90       current_state <= WAITING;
91     ELSE
92
93       CASE current_state IS
94         WHEN WAITING =>
95           IF start = '1' THEN
96             current_state <= FIX;
97           ELSE
98             current_state <= WAITING;
99           END IF;
100
101      WHEN FIX =>
102        current_state <= COMPUTING;
103
104      WHEN COMPUTING =>
105        IF counter = ITERS - 1 THEN
106          current_state <= FINISHED;
107        ELSE
108          current_state <= COMPUTING;
109        END IF;
110
111      WHEN FINISHED =>
112        current_state <= WAITING;
113
114      WHEN OTHERS =>
115        current_state <= WAITING;
116
117    END CASE;
118  END IF;
119 END IF;
120 END PROCESS;
121
122 -----  

123 -- OPERATIONAL PART  

124 -----
125
126 OPEATIONAL : PROCESS (clk, rst)
127 BEGIN
128   IF (rising_edge(clk)) THEN
129     IF rst = '1' THEN
130       valid <= '0';
131       x_out <= (OTHERS => '0');
132       z_out <= (OTHERS => '0');

```

```

133     counter <= (OTHERS => '0');
134     x_t <= (OTHERS => '0');
135     y_t <= (OTHERS => '0');
136     z_t <= (OTHERS => '0');
137 ELSE
138
139     -- Default assignment
140     x_t <= (OTHERS => '-');
141     y_t <= (OTHERS => '-');
142     z_t <= (OTHERS => '-');
143     x_out <= (OTHERS => '-');
144     z_out <= (OTHERS => '-');
145     counter <= (OTHERS => '0');
146
147 CASE current_state IS
148     WHEN WAITING =>
149         -- x_t and y_t have 2 more bits for the integer compared to
150         -- x and y
151         -- this avoids overflows during the computation
152         x_t <= shift_left(resize(signed(x), M), (M - N - 2));
153         y_t <= shift_left(resize(signed(y), M), (M - N - 2));
154         z_t <= to_signed(0, M);
155         valid <= '1';
156         x_out <= x_out;
157         z_out <= z_out;
158
159     WHEN FIX =>
160         IF sign = '1' THEN
161             x_t <= - y_t;
162             y_t <= x_t;
163             z_t <= z_t - HALF_PI;
164         ELSE
165             x_t <= y_t;
166             y_t <= - x_t;
167             z_t <= z_t + HALF_PI;
168         END IF;
169
170         valid <= '0';
171         counter <= (OTHERS => '0');
172
173     WHEN COMPUTING =>
174         IF sign = '1' THEN
175             x_t <= x_t - shift_right(y_t, to_integer(counter));
176             y_t <= y_t + shift_right(x_t, to_integer(counter));
177             z_t <= z_t - signed(atan_out);
178         ELSE
179             x_t <= x_t + shift_right(y_t, to_integer(counter));
180             y_t <= y_t - shift_right(x_t, to_integer(counter));
181             z_t <= z_t + signed(atan_out);
182         END IF;
183
184         counter <= counter + 1;

```

```

185         valid <= '0';
186
187     WHEN FINISHED =>
188         x_out <= STD_LOGIC_VECTOR(resize(shift_right(unsigned(x_t) *
189             <- k, M - 1), M)(M - 1 - 2 DOWNTO M - N - 2));
190         z_out <= STD_LOGIC_VECTOR(z_t(M - 1 DOWNTO M - N));
191
192         valid <= '1';
193
194     END CASE;
195     END IF;
196     END IF;
197 END PROCESS;
198 END ARCHITECTURE;

```

Listing 3.1: CORDIC.vhd

3.2 Atan LUT

The values of the LUT table were calculated using the formula:

$$\text{LUT}_i = \left\lfloor \text{atan}(2^{-i}) * 2^{21} \right\rfloor$$

```

1 library IEEE;
2 use IEEE.std_logic_1164.all;
3 use IEEE.numeric_std.all;
4
5 ENTITY ATAN_LUT IS
6   port (
7     address : in std_logic_vector(3 downto 0);
8     lut_out : out std_logic_vector(23 downto 0)
9   );
10 end entity;
11
12 architecture rtl of ATAN_LUT is
13
14   type LUT_t is array (natural range 0 to 15) of integer;
15   constant LUT: LUT_t := (
16     0 => 1647099,
17     1 => 972339,
18     2 => 513757,
19     3 => 260791,
20     4 => 130901,
21     5 => 65514,
22     6 => 32765,
23     7 => 16383,

```

```
24      8 => 8191,
25      9 => 4095,
26      10 => 2047,
27      11 => 1023,
28      12 => 511,
29      13 => 255,
30      14 => 127,
31      15 => 63
32    );
33
34 begin
35
36 PROCESS (address)
37 BEGIN
38   lut_out <=
39     STD_LOGIC_VECTOR(to_signed(LUT(to_integer(unsigned(address))), 
40                               24));
41 END PROCESS;
42
43 end architecture;
```

Listing 3.2: ATAN_LUT.vhd

4 Verification and testing

4.1 Testbench

The testbench CORDIC_tb verifies the basic functionality of the circuit by performing a power on reset and iteratively gives one input and waits for the corresponding output. For debugging reasons the testbench also print the output values on the console.

```
1 LIBRARY IEEE;
2 USE IEEE.STD_LOGIC_1164.ALL;
3 USE IEEE.NUMERIC_STD.ALL;
4 USE ieee.math_real.ALL;
5
6 ENTITY CORDIC_TB IS
7   GENERIC (
8     N : POSITIVE := 16;
9     floating : INTEGER := 8
10    );
11 END CORDIC_TB;
12
13 ARCHITECTURE Behavioral OF CORDIC_TB IS
14   -- Component declaration for CORDIC
15
16   COMPONENT CORDIC
17     PORT (
18       clk : IN STD_LOGIC;
19       rst : IN STD_LOGIC;
20       x : IN STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
21       y : IN STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
22       start : IN STD_LOGIC;
23       rho : OUT STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
24       theta : OUT STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
25       valid : OUT STD_LOGIC
26     );
27   END COMPONENT;
28
29   -- Signals for CORDIC inputs and outputs
30
31   SIGNAL clk : STD_LOGIC := '0';
32   SIGNAL reset : STD_LOGIC := '0';
```

```

33  SIGNAL x : STD_LOGIC_VECTOR(N - 1 DOWNTO 0) := (OTHERS => '0');
34  SIGNAL y : STD_LOGIC_VECTOR(N - 1 DOWNTO 0) := (OTHERS => '0');
35  SIGNAL start : STD_LOGIC := '0';
36  SIGNAL rho : STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
37  SIGNAL theta : STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
38  SIGNAL valid : STD_LOGIC := '0';
39
40  SIGNAL run_simulation : STD_LOGIC := '1';
41
42  -- Clock period definition
43  CONSTANT T_clk : TIME := 20 ns;
44
45  -- Coordinate type
46  TYPE Coordinate IS RECORD
47      x : real;
48      y : real;
49  END RECORD;
50  CONSTANT n_coordinates : NATURAL := 9;
51
52  TYPE CoordinateArray IS ARRAY (0 TO n_coordinates - 1) OF Coordinate;
53
54  -- Array of coordinates to test
55  CONSTANT Coordinates : CoordinateArray := (
56
57      (1.0, 1.0),
58      (10.0, 10.0),
59      (0.0, 0.0),
60      (-0.1, -4.0),
61      (-1.0, 1.0),
62      (-1.0, 0.0),
63      (-1.0, 0.1),
64      (31.0, 31.0),
65      (127.0, 127.0)
66  );
67
68  -- function to print values in report
69  FUNCTION to_real(val : unsigned(N - 1 DOWNTO 0); fraction_bits :
70      INTEGER) RETURN real IS
71  BEGIN
72      RETURN real(to_integer(val)) / 2.0 ** fraction_bits;
73  END FUNCTION;
74
75  -- function to print values in report
76  FUNCTION to_real(val : signed(N - 1 DOWNTO 0); fraction_bits :
77      INTEGER) RETURN real IS
78  BEGIN
79      RETURN real(to_integer(val)) / 2.0 ** fraction_bits;
80  END FUNCTION;
81
82  PROCEDURE echo(arg : IN STRING := "") IS
83  BEGIN
84      std.textio.write(std.textio.output, arg & LF); -- LF ensures a
85      -- newline after each message

```

```

83      END PROCEDURE echo;
84
85  BEGIN
86    -- Instantiate the CORDIC component
87    cordic_inst : CORDIC
88    PORT MAP(
89      clk => clk,
90      rst => reset,
91      x => x,
92      y => y,
93      start => start,
94      rho => rho,
95      theta => theta,
96      valid => valid
97    );
98
99    -- todo fix behavior if cordic is not ready / does not work
100   clk <= (NOT(clk) AND run_simulation) AFTER T_clk / 2;
101
102  -- test process
103  test : PROCESS
104  BEGIN
105    reset <= '1';
106    start <= '0';
107    WAIT FOR 2 * T_clk;
108
109    reset <= '0';
110    FOR i IN 0 TO n_coordinates - 1 LOOP
111
112      IF valid = '0' THEN
113        WAIT UNTIL valid = '1';
114      END IF;
115
116      x <= STD_LOGIC_VECTOR(to_signed(INTEGER(Coordinates(i).x * 2.0 **
117        → floating), N));
117      y <= STD_LOGIC_VECTOR(to_signed(INTEGER(Coordinates(i).y * 2.0 **
118        → floating), N));
119      start <= '1';
120
121      WAIT FOR 5 * T_clk;
122
123      start <= '0';
124      IF valid = '0' THEN
125        WAIT UNTIL valid = '1';
126      END IF;
127
128      WAIT FOR 1 * T_clk;
129      -- Osservazione del valore del modulo e della fase
130      echo("");
131      echo("Test " & INTEGER'image(i) & " X: " &
→ real'image(Coordinates(i).x) & " Y: " &
→ real'image(Coordinates(i).y));
      echo("-----");

```

```

132      echo("Module (Q8.8) = " & real'image(to_real(unsigned(rho), 8)));
133      echo("Phase  (Q3.13) = " & real'image(to_real(signed(theta),
134          ↵ 13)));
135      echo("-----");
136      WAIT FOR 10 * T_clk;
137
138      END LOOP;
139
140      run_simulation <= '0';
141      WAIT;
142      END PROCESS;
143  END ARCHITECTURE;

```

Listing 4.1: CORDIC_tb.vhd

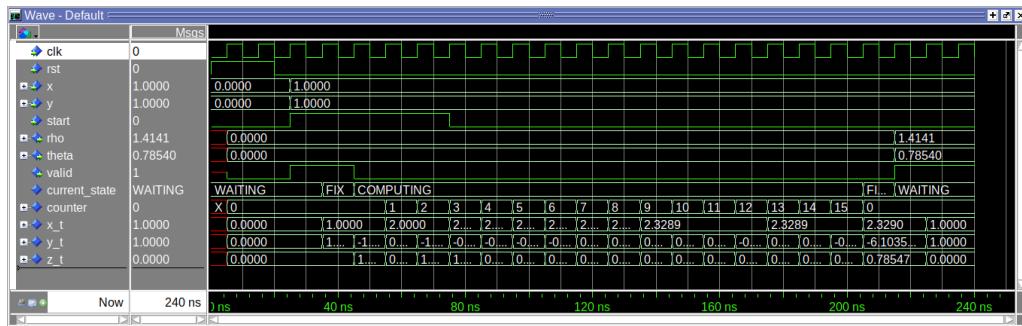


Figure 4.1: Simulation of a single iteration of the testbench

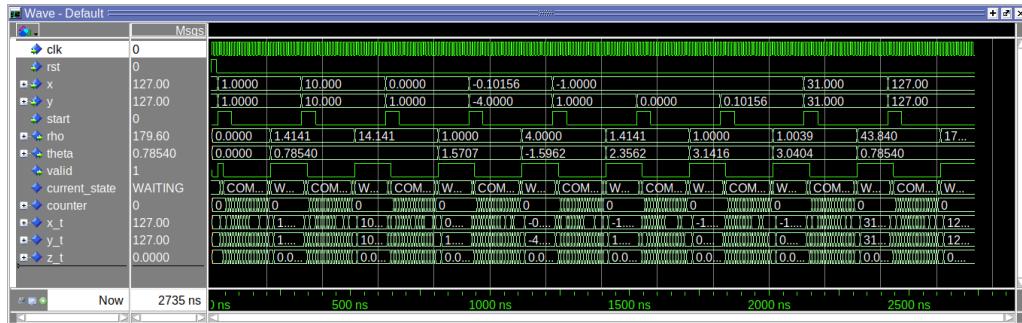


Figure 4.2: Full simulation of the testbench

4.2 Graph testbench

This testbench serves the purpose of ensuring the functionality of the circuit over the range of possible inputs. The test was performed on 512×512 input points uniformly distributed within the range $(-128, 128)$ for both x and y .

```

1 LIBRARY ieee;
2 USE ieee.std_logic_1164.ALL;
3 USE ieee.numeric_std.ALL;
4 USE ieee.math_real.ALL;
5 USE std.textio.ALL;
6
7 ENTITY GRAPH_tb IS
8   GENERIC (
9     N : POSITIVE := 16;
10    floating : INTEGER := 8 -- (Q8.8)
11  );
12 END ENTITY;
13
14 ARCHITECTURE Behavioral OF GRAPH_tb IS
15
16   -----
17   -- CORDIC component
18   -----
19
20   COMPONENT CORDIC
21     PORT (
22       clk : IN STD_LOGIC;
23       rst : IN STD_LOGIC;
24       x : IN STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
25       y : IN STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
26       start : IN STD_LOGIC;
27       rho : OUT STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
28       theta : OUT STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
29       valid : OUT STD_LOGIC
30     );
31   END COMPONENT;
32
33   -----
34   -- Signals
35   -----
36
37   SIGNAL clk : STD_LOGIC := '0';
38   SIGNAL rst : STD_LOGIC := '0';
39   SIGNAL x_in : STD_LOGIC_VECTOR(N - 1 DOWNTO 0) := (OTHERS => '0');
40   SIGNAL y_in : STD_LOGIC_VECTOR(N - 1 DOWNTO 0) := (OTHERS => '0');
41   SIGNAL start_in : STD_LOGIC := '0';
42   SIGNAL rho_out : STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
43   SIGNAL theta_out : STD_LOGIC_VECTOR(N - 1 DOWNTO 0);
44   SIGNAL valid_out : STD_LOGIC := '0';
45
46   -----
47   -- Simulation parameters
48   -----
49
50   CONSTANT T_clk : TIME := 20 ns; -- clock period = 20 ns
51   CONSTANT SAMPLES : INTEGER := 512; -- samples per dimension
52
53   -----

```

```
54  -- Output file
55  -----
56
57  FILE results_file : text OPEN write_mode IS
58  <-- "cordic_results_512x512.txt";
59
60  -----
61  -- Conversion function from std_logic_vector to real
62
63  FUNCTION to_real(val : unsigned(N - 1 DOWNTO 0); fraction_bits :
64  <-- INTEGER) RETURN real IS
65  BEGIN
66      RETURN real(to_integer(val)) / 2.0 ** fraction_bits;
67  END FUNCTION;
68
69  FUNCTION to_real(val : signed(N - 1 DOWNTO 0); fraction_bits :
70  <-- INTEGER) RETURN real IS
71  BEGIN
72      RETURN real(to_integer(val)) / 2.0 ** fraction_bits;
73  END FUNCTION;
74
75  -----
76  -- CORDIC declaration
77
78  UUT : CORDIC
79  PORT MAP(
80      clk => clk,
81      rst => rst,
82      x => x_in,
83      y => y_in,
84      start => start_in,
85      rho => rho_out,
86      theta => theta_out,
87      valid => valid_out
88  );
89
90  -----
91  -- CLK process
92
93  clk_process : PROCESS
94  BEGIN
95      WHILE run_simulation = '1' LOOP
96          clk <= '1';
97          WAIT FOR T_clk/2;
98          clk <= '0';
99          WAIT FOR T_clk/2;
100     END LOOP;
101     WAIT; -- end process when run_simulation = '0'
102  END PROCESS clk_process;
```

```

104
105  -- Test process
106
107 test_process : PROCESS
108   VARIABLE L : line;
109   VARIABLE real_x, real_y : real;
110   VARIABLE mod_val, phase_val : real;
111 BEGIN
112
113  -- Power on reset
114
115  rst <= '1';
116  start_in <= '0';
117  WAIT FOR 5 * T_clk;
118  rst <= '0';
119  WAIT FOR 5 * T_clk;
120
121
122  -- Loop over (i, j) to test all values
123
124 FOR i IN 0 TO SAMPLES - 1 LOOP
125   FOR j IN 0 TO SAMPLES - 1 LOOP
126
127     real_x := - 128.0 + real(i) * 256.0 / real(SAMPLES);
128     real_y := - 128.0 + real(j) * 256.0 / real(SAMPLES);
129
130     -- Q8.8 Conversion (signed)
131     x_in <= STD_LOGIC_VECTOR(to_signed(INTEGER(floor(real_x * 2.0 **
132       floating)), N));
132     y_in <= STD_LOGIC_VECTOR(to_signed(INTEGER(floor(real_y * 2.0 **
133       floating)), N));
134
135     start_in <= '1';
136     WAIT FOR 2*T_clk;
137     start_in <= '0';
138
139     WAIT UNTIL valid_out = '1';
140     WAIT FOR 10 ns;
141
142     mod_val := to_real(unsigned(rho_out), 8); -- Q8.8
143     phase_val := to_real(signed(theta_out), 13); -- Q3.13
144
145  -- Write on text file: x, y, rho, theta
146
147  -- Comma separated values
148  write(L, real_x, RIGHT, 0, 6);
149  write(L, STRING','');
150  write(L, real_y, RIGHT, 0, 6);
151  write(L, STRING','');
152  write(L, mod_val, RIGHT, 0, 6);
153  write(L, STRING','');
154  write(L, phase_val, RIGHT, 0, 6);

```

```

155     writeline(results_file, L);
156
157     WAIT FOR 1 * T_clk;
158
159     END LOOP;
160 END LOOP;
161
162 -----
163 -- End simulation
164 -----
165 run_simulation <= '0';
166 WAIT;
167 END PROCESS test_process;
168
169 END Behavioral;

```

Listing 4.2: GRAPH_tb.vhd

4.3 Error verification

The implementation of the CORDIC algorithm in vectoring mode for Cartesian-to-polar conversion demonstrates high accuracy in both module (ρ) and phase (θ) outputs. Using a fixed-point representation of $UQ8.8$ for ρ and $Q3.13$ for θ , the errors were summarized as follows:

Module Error: Max = 0.004030 units, Mean = 0.001889 units

Phase Error: Max = 0.000154 rad, Mean = 0.000062 rad

Module Error: $\log_2(\text{Max error}) \approx -7.955$

Phase Error: $\log_2(\text{Max error}) \approx -12.664$

The observed errors are well within the precision limits of 2^{-8} for ρ and 2^{-13} for θ . For the most part, discrepancies between the expected values and the output values are primarily caused by the conversion of fixed-point representations into real numbers for plotting and analysis.

The 3D plots of module and phase errors visually confirm the bounded and negligible nature of the errors across all tested input combinations. Assuming no error on the input, this analysis validates that the algorithm introduces at most 1 bit error in the output.

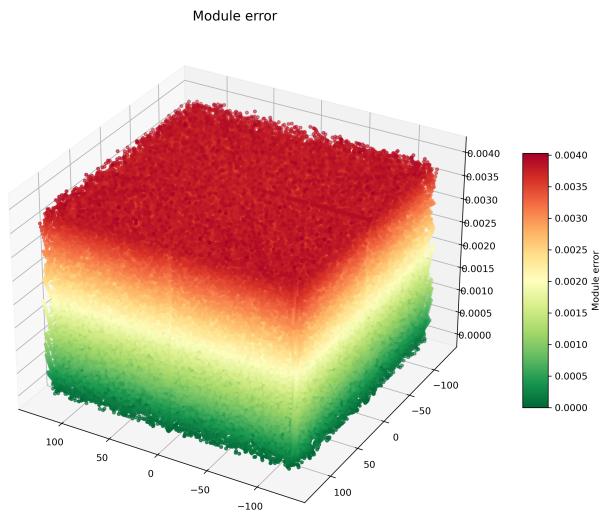


Figure 4.3: 3D plot of module error (ρ).

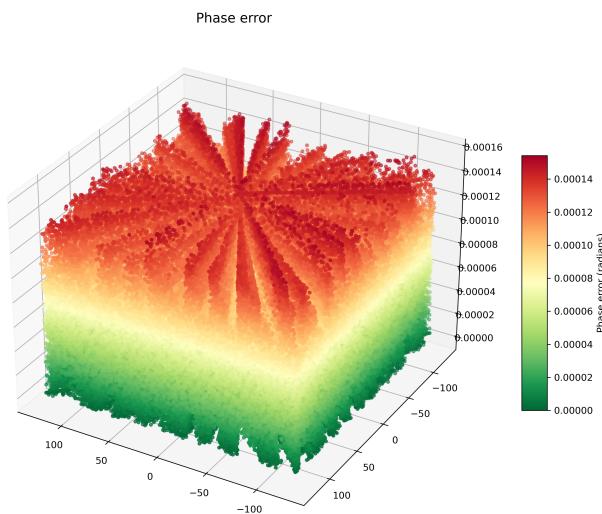


Figure 4.4: 3D plot of phase error (θ).

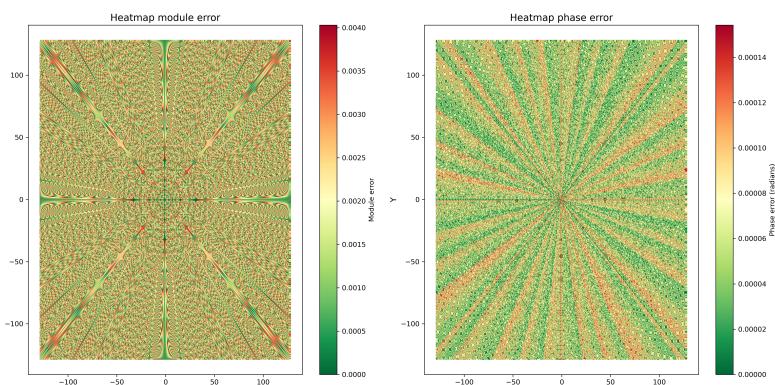


Figure 4.5: Heatmap for phase and module errors.

4.4 Corner cases

When the `valid` signal is set to 1, it indicates that the output data is valid and that the CORDIC module is ready to accept a new computation request. However, if CORDIC is currently performing a computation, as we can notice from the control part of the Code 3.1, any incoming `start` command will be ignored. This ensures that the ongoing computation is not interrupted. The new request will only be accepted once the CORDIC transitions back to the `WAITING` state, signaling with `valid = '1'` that the current computation has been completed.

Regarding the specific input case of the point $(0, 0)$, no modifications are required in the VHDL code. The output for this scenario is already defined as $\rho = 0$ and $\theta = 3.3139$ rad. Since with zero modulus vectors ($\rho = 0$) the phase (θ) is irrelevant, this behavior is both mathematically consistent and functionally correct. This means the implementation handles such cases correctly without the need of extra logic.

Finally, as discussed in Section 2.1, thanks to the `FIX_STEP` showed in Code 3.1, the CORDIC algorithm converges for all possible input values and not only values in the 1st quadrant and 2nd quadrant.

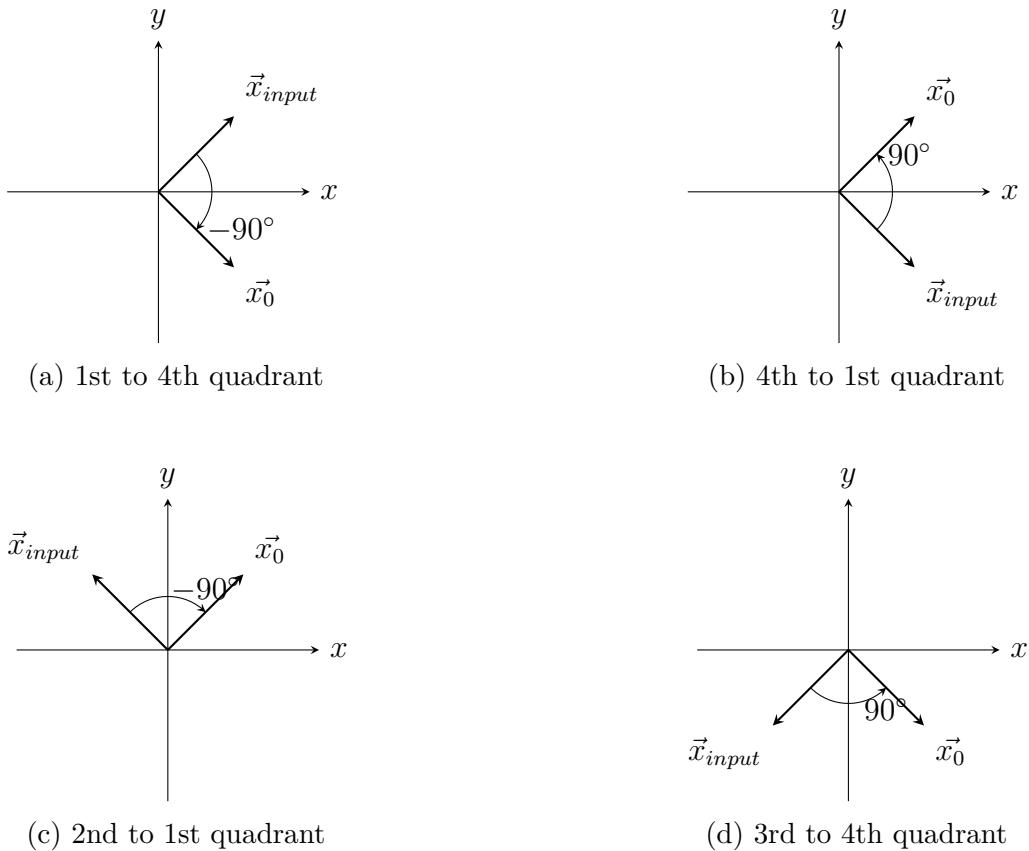


Figure 4.6: Vector transformations across quadrants

5 Vivado results

After completing the verification phase, the circuit design was synthesized and implemented using the Vivado Tool, specifically targeting the Zybo Zynq-7010 development board.

5.1 Vivado Design flow

For the implementation of the CORDIC algorithm in vectoring mode for Cartesian-to-Polar conversion, the Vivado design flow was employed using Xilinx/AMD Vivado software. This process involved RTL Elaboration, Synthesis, and Implementation on the target FPGA, along with the application of design constraints and the extraction of power and timing reports. To ensure accurate and reliable timing analysis, all combinational logic paths were structured to follow a Register-Logic-Register configuration.

5.2 RTL

Vivado produced a logic network made of:

1. 144 cells (e.g. multiplexers, DFFs, adders)
2. 68 I/O ports ($16_x + 16_y + 16_\rho + 16_\theta + 1_{clk} + 1_{rst} + 1_{start} + 1_{valid}$)
3. 802 nets (for connecting all the components)

The RTL Analysis generated the Elaborated Design shown in Figure 5.1, consistent with the expected structure of the system.

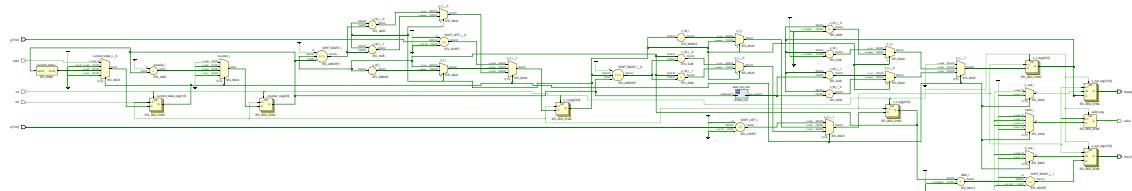


Figure 5.1: Elaborated RTL design.

5.3 Synthesis timing report

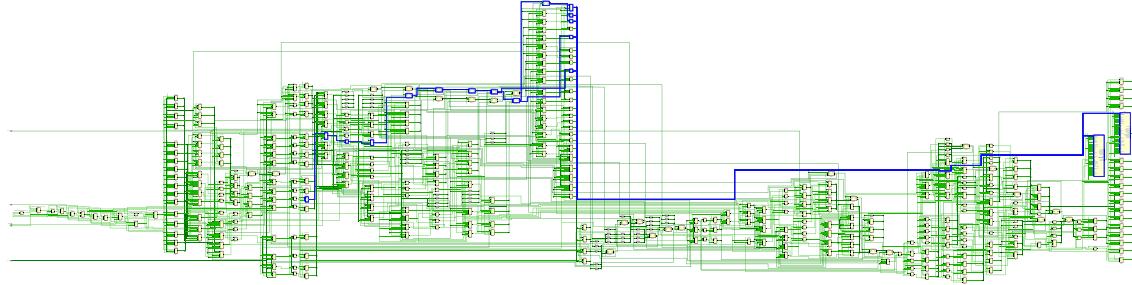


Figure 5.2: Figure showing the elaborated RTL design with the main critical paths for set-up-time-violation found in synthesis step highlighted in blue.

Name	Slack	Levels	Routes	From	To	Total Delay
Path 1	12.78	10	6	y_t_reg[14]/C	ARG/A[23]	6.77
Path 2	12.78	10	6	y_t_reg[14]/C	ARG_0/A[23]	6.77
Path 3	12.90	9	6	y_t_reg[14]/C	ARG/A[19]	6.66
Path 4	12.90	9	6	y_t_reg[14]/C	ARG_0/A[19]	6.66
Path 5	13.02	8	6	y_t_reg[14]/C	ARG/A[15]	6.54
Path 6	13.02	8	6	y_t_reg[14]/C	ARG_0/A[15]	6.54
Path 7	13.03	10	6	y_t_reg[14]/C	ARG/A[22]	6.53
Path 8	13.03	10	6	y_t_reg[14]/C	ARG_0/A[22]	6.53
Path 9	13.08	10	6	y_t_reg[14]/C	ARG/A[21]	6.47
Path 10	13.08	10	6	y_t_reg[14]/C	ARG_0/A[21]	6.47

Table 5.1: Table showing data of the main critical paths found in synthesis step

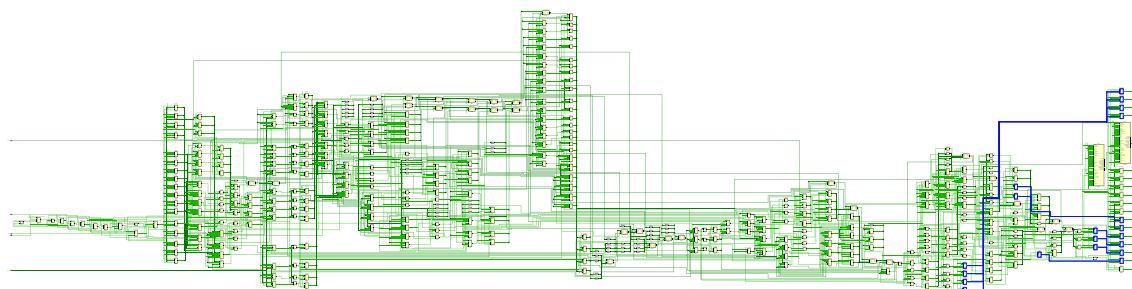


Figure 5.3: Figure showing the elaborated RTL design with the main critical paths for hold-time-violation found in synthesis step highlighted in blue.

Name	Slack	Levels	Routes	From	To	Total Delay
Path 11	0.28	0	1	z_t_reg[23]/C	z_out_reg[15]/D	0.30
Path 12	0.28	0	1	z_t_reg[8]/C	z_out_reg[0]/D	0.31
Path 13	0.28	0	1	z_t_reg[18]/C	z_out_reg[10]/D	0.31
Path 14	0.28	0	1	z_t_reg[19]/C	z_out_reg[11]/D	0.31
Path 15	0.28	0	1	z_t_reg[20]/C	z_out_reg[12]/D	0.31
Path 16	0.28	0	1	z_t_reg[21]/C	z_out_reg[13]/D	0.31
Path 17	0.28	0	1	z_t_reg[22]/C	z_out_reg[14]/D	0.31
Path 18	0.28	0	1	z_t_reg[9]/C	z_out_reg[1]/D	0.31
Path 19	0.28	0	1	z_t_reg[10]/C	z_out_reg[2]/D	0.31
Path 20	0.28	0	1	z_t_reg[11]/C	z_out_reg[3]/D	0.31

Table 5.2: Table showing the characteristics regarding critical paths for hold-time-violation found in synthesis step.

5.4 Implementation timing report

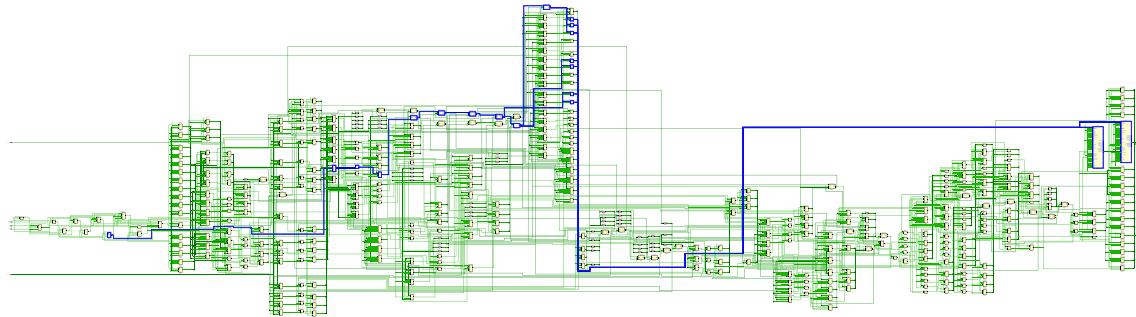


Figure 5.4: Figure showing the elaborated RTL design with the main critical paths for set-up-time-violation found during implementation step highlighted in blue.

Name	Slack	Levels	Routes	From	To	Total Delay
Path 1	10.19	10	6	counter_reg[2]/C	ARG/A[20]	9.16
Path 2	10.38	10	6	counter_reg[2]/C	ARG_0/A[20]	8.97
Path 3	10.43	10	6	counter_reg[2]/C	ARG_0/A[21]	9.13
Path 4	10.43	10	6	counter_reg[2]/C	ARG/A[22]	8.92
Path 5	10.57	8	6	counter_reg[2]/C	ARG_0/A[12]	8.78
Path 6	10.59	9	6	counter_reg[2]/C	ARG_0/A[17]	8.97
Path 7	10.60	9	6	counter_reg[2]/C	ARG/A[16]	8.75
Path 8	10.61	8	6	counter_reg[2]/C	ARG_0/A[13]	8.94
Path 9	10.61	10	6	counter_reg[2]/C	ARG/A[21]	8.94
Path 10	10.62	10	6	counter_reg[2]/C	ARG_0/A[22]	8.73

Table 5.3: Table showing the main critical paths for the setup-time-violation during the implementation step.

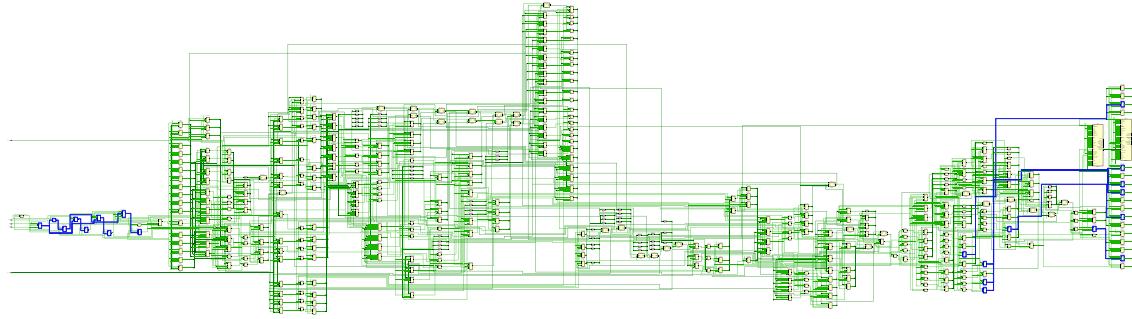


Figure 5.5: Figure showing the elaborated RTL design with the main critical paths for hold-time-violation found during implementation step highlighted in blue.

Name	Slack	Levels	From	To	Total Delay
Path 11	0.17	1	current_state_reg[1]/C	counter_reg[2]/D	0.33
Path 12	0.18	1	current_state_reg[1]/C	counter_reg[0]/D	0.32
Path 13	0.18	1	current_state_reg[1]/C	counter_reg[1]/D	0.32
Path 14	0.19	0	z_t_reg[22]/C	z_out_reg[14]/D	0.29
Path 15	0.22	0	z_t_reg[12]/C	z_out_reg[4]/D	0.29
Path 16	0.22	0	z_t_reg[14]/C	z_out_reg[6]/D	0.30
Path 17	0.23	0	z_t_reg[15]/C	z_out_reg[7]/D	0.26
Path 18	0.23	0	z_t_reg[10]/C	z_out_reg[2]/D	0.33
Path 19	0.24	1	counter_reg[0]/C	counter_reg[3]/D	0.38
Path 20	0.24	0	z_t_reg[18]/C	z_out_reg[10]/D	0.32

Table 5.4: Table showing the main critical paths for hold-time-violation found in the implementation step.

5.5 Utilization Report

Resource	Utilization (%)	Description
Slice LUTs	1.59%	Look-Up Tables used as logic
Slice Registers	0.27%	Registers used in the design
Slice	1.93%	Total slices utilized
LUT as Logic	1.59%	LUTs specifically used as logic
DSPs	2.50%	Digital Signal Processing blocks
Bonded IOB	0.00%	Bonded Input/Output Blocks
BUFGCTRL	0.00%	Global Clock Buffers

Table 5.5: Resource utilization for the CORDIC design (only non-zero values shown)

5.6 Power Report

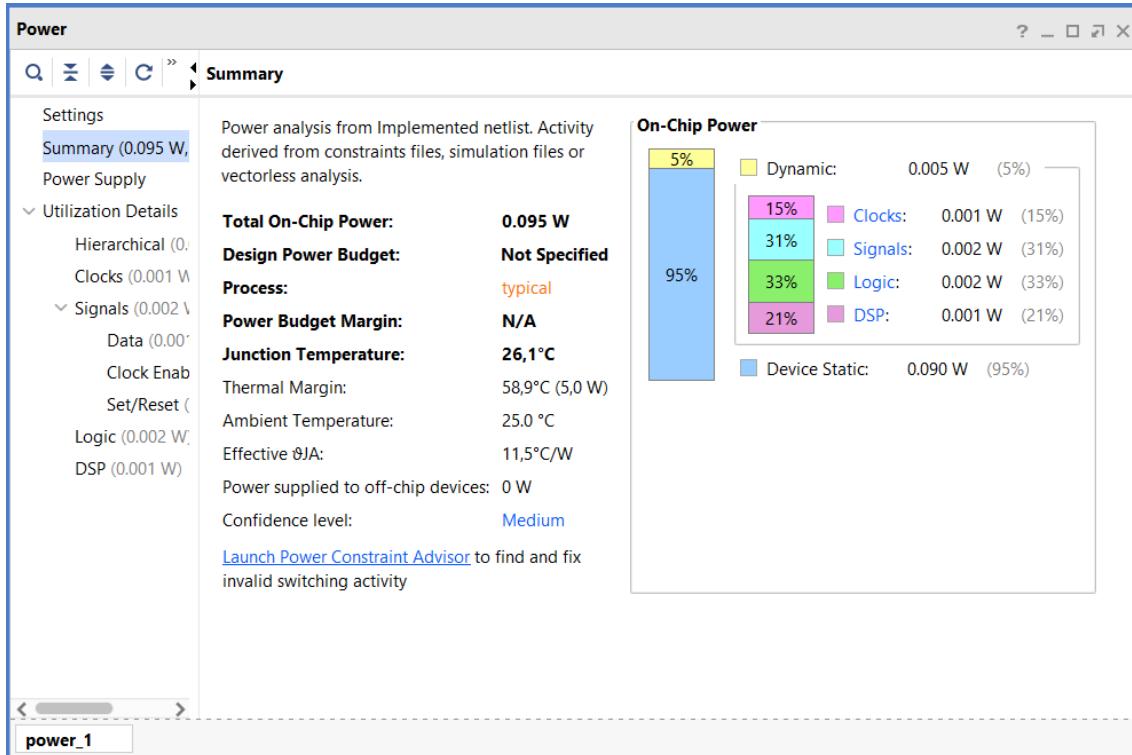


Figure 5.6: Vivado Power Report after implementation.

The power report highlights a total on-chip power consumption of 0.095 W, with 95% attributed to static power (0.090 W) and 5% to dynamic power (0.05 W). The dynamic power is distributed among different components: clocks (15%), signals (31%), logic (33%), DSP (21%), with signals and logic being the highest consumers of dynamic power.

The high value of static power over dynamic power is largely to be attributed to the fact that the design is using very little of the available hardware. With reference to the Utilization Report in Chapter 4, resources like Slice LUTs and Registers are used only for 1.59% and 0.27%, this makes the overall switching activity of the chip very small.

5.7 Conclusions

During the implementation phase, various warnings were observed. These warnings arise from the fact that the design was developed in an out-of-context mode, which is standard practice for proof-of-concept implementations.

Warnings

- **DRC 23-814** Not all possible (connectivity-based) DRCs may have been run because this design is seen as Out of Context.
- **Route 35-197** Clock port "clk" does not have an associated HD.CLK_SRC. Without this constraint, timing analysis may not be accurate, and upstream checks cannot be done to ensure correct clock placement.
- **Route 35-198** Port "rst" does not have an associated HD.PARTPIN_LOCS, which will prevent the partial routing of the signal "rst". Without this partial route, timing analysis to/from this port will not be accurate, and no routing information for this port can be exported.
- **Timing 38-242** The property HD.CLK_SRC of clock port "clk" is not set. In out-of-context mode, this prevents timing estimation for clock delay/skew.

In our particular study case presented in Section 2.2, these warnings are not a cause for any concern. They arise because the design was intentionally tested in isolation, by just applying a single clock constraint with a 50MHz frequency. As a result, issues like incomplete connectivity checks and timing inaccuracies are reported.

Moreover, applying I/O pin constraints or testing the design with external device constraints would be meaningless in this context. As discussed in 2.2, this device is intended to be an onboard accelerator for other devices. Its primary purpose is to function as part of a larger system, and its inputs and outputs will be internally routed. Therefore, testing with external I/O constraints would not provide any meaningful insights.

By observing Table 5.3 we can notice that the Worst Negative Slack (WNS) for setup timing is reported as 10.194 ns.

[Trenitalia: need to draw conclusions on the worst paths, clock obtained and theoretical maximum clock. Also the fact that this implementation respect all the requirements set in 2.2 in terms of low area occupation and low power consumption]

6 Final considerations

The design process adopted in this work demonstrates an efficient approach to implementing the CORDIC algorithm. The iterative methodology ensures high accuracy while maintaining low resource utilization, as confirmed by the Vivado synthesis and power analysis reports.

It is important to note that while the implemented circuit is tailored for the constraints of the Zybo board, the modularity of the design makes it adaptable to other hardware platforms. The testing and verification phase, which included testbenches and graphical analysis, validated the functionality and robustness of the system across a wide range of input conditions.