



Construct3D: A Virtual Reality Application for Mathematics and Geometry Education

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Construct3D is a three dimensional geometric construction tool based on the collaborative augmented reality system 'Studierstube'. Our setup uses a stereoscopic head mounted display (HMD) and the Personal Interaction Panel (PIP) - a two-handed 3D interaction tool that simplifies 3D model interaction. Means of application in mathematics and geometry education at high school as well as university level are being discussed. A pilot study summarizes the strengths and possible extensions of our system. Anecdotal evidence supports our claim that the use of Construct3D is easy to learn and encourages experimentation with geometric constructions.

Keywords: virtual reality; geometry education; enhancing spatial abilities; user-interface design; 3D modelling.

Introduction

Spatial abilities present an important component of human intelligence. The term spatial abilities covers five components, spatial perception, spatial visualization, mental rotations, spatial relations and spatial orientation (Maier, 1994). Generally, the main goal of geometry education is to improve these spatial skills. As shown in various studies (Osberg, 1997; Rizzo et al. 1998) spatial abilities can be improved by virtual reality (VR) technology.

In order to provide mathematics and geometry education with an applicable tool, this paper presents a three dimensional geometric construction tool called Construct3D that can be used in high school and university education.

Construct3D is based on the collaborative augmented reality system 'Studierstube' (Schmalstieg et al. 1996; Szalavári et al. 1998; Fuhrmann and Schmalstieg, 1999). Basic techniques for working within Construct3D are described to show how users intuitively use and work with the Personal Interaction Panel (PIP), a two-handed 3D interaction tool composed of instrumented hand-help props—a pen and a pad equipped with position and orientation trackers. This user interface is only possible in augmented reality. Rather than forcing a user to deal exclusively with a virtual environment, augmented reality embeds

virtual reality into the real world. The 'Studierstube' system allows multiple users to work and interact in the same environment. We use this feature in Construct3D for teacher-student interaction which proves to be very sensible. In addition, we introduce an audio help system.

We discuss what was learned from the difficulties that arose during the implementation process. Finally, the results of our pilot study with geometry students actually using Construct3D are presented with an emphasis on suggested improvements and new techniques that can be applied within other virtual worlds. A discussion with emphasis on the implications and practical significance of the findings and their limitations follows.

Our aim was not to create a professional 3D modelling package but rather a simple and intuitive 3D construction tool in an immersive virtual environment for educational purposes. Similar to the CAD3D package co-developed by the third author, which won the German-Austrian academic software award in 1993, our main goal was to keep the user interface as simple as possible to facilitate learning and efficient use. The main areas of application of our system in mathematics and geometry education are vector analysis, descriptive geometry and geometry in general. These areas have not been specifically addressed by previous systems.

The main advantage of Construct3D to student learning in mathematics and geometry education is that students actually see three dimensional objects which they until now had to calculate and construct with traditional methods. Virtual reality provides them with a nearly tangible picture of complex three dimensional objects and scenes. It enhances, enriches and complements the mental pictures that students form in their minds when working with three dimensional objects. By working directly in 3D space complex spatial problems and relationships can be comprehended better and faster than with traditional methods.

Related Work

A large body of work has been done on 3D modelling. Although 3D input devices with six degrees of freedom (6DOF) have been used to enhance modellers, little modelling has been done in immersive virtual reality systems. In the following we refer to immersive virtual reality systems using head-mounted displays (HMDs) as HMD based systems. Only minimally immersive VR systems (fishtank VR) or fully immersive VR systems but no pure desktop systems are being mentioned.

Some previous uses of HMD based systems have concentrated more on exploration of virtual worlds rather than creating or modifying them directly in virtual reality (Fisher et al. 1986; Bricken and Byrne, 1993). A very good overview of 3D modelling systems with 6DOF input devices can be found in the work of Mine (1996).

One of the earliest interactive design systems that used an immersive head-mounted display was the one built by Clark (1976). This pioneering system for use in the interactive design of free form surfaces addressed many of the issues that face developers of interactive design systems today.

Liang and Green (1994) developed JDCAD, an interactive 3D modelling system with a non-stereo head tracked display and a single 6DOF input device. Their focus was an improvement in ergonomics and precision for engineering tasks. Development of JDCAD system (now called JDCAD+) continues at the University of Alberta with the addition of new animation editing and scene composition functions.

One of the few HMD based modelling systems called 3DM was presented by Butterworth *et al.* (1992). It includes several grid and snap functions, an extrusion tool for surface creation and some other interesting features. It lacks, however, many of the other aids and constraints that since have been found necessary for accomplishing precise work, as rated by Mine (1996), who presented the Chapel Hill Immersive Modeling Program CHIMP. It is a testbed for various interaction techniques as well as its precursor ISAAC (Mine, 1997; Mine, 1996).

These systems belong to a number of 3D modelling and design systems studying user interaction techniques. They are also used in university education with design or architecture students. DesignSpace by Chapin (1994) is one of them. Another example is Bowman's Conceptual Design Space (Bowman, 1996)—a real-time, interactive virtual environment application which attempts to address the issue of 3D design in general and immersive design in particular. The Virtual Reality Aided Modeler (VRAM) by the faculty of architecture at the Bauhaus-University Weimar, is an ongoing testbed for the application of 3D user interface techniques on a conceptual design tool for architects and industrial designers that runs with VRML97 (Regenbrecht *et al.* 2000). SeamlessDesign by Kiyokawa (2000) is another system with similar goals.

On the other hand, a great number of people have been working on virtual reality applications for pure educational use (Bell and Fogler, 1995; Bricken and Byrne, 1993; Byrne, 1996; Winn, 1997; and many others). For instance Bricken and Byrne (1993) worked with 60 students age 10–15 in a summer camp for 7 weeks and taught them to build their own 3D worlds with traditional 3D modellers. The students had regular access to the university VR lab to test their worlds. Video observations, opinion and informal surveys are supplied.

Many related publications can be found on the web page of the Human Interface Technology Laboratory, University of Washington (<http://www.hitl.washington.edu/>). Contributions have been made by the Support Initiative for Multimedia Applications (SIMA) project (<http://www.man.ac.zk/MVC//SIMA/>) based at the Manchester Visualization Center. Various project reports and workshops such as 'The Potential of VR for UK Higher Education' in 1995 are examples.

Construct3D is a construction tool in an immersive virtual environment which addresses the specific needs of mathematics and geometry education. This combination cannot be found in previous systems.

Implementation

We are using the collaborative augmented reality system 'Studierstube' (Schmalstieg *et al.* 1996; Szalavári *et al.* 1998; Fuhrmann and Schmalstieg, 1999). As an augmented reality

system, it allows an integration of the virtual world into the real world. Users can partly see the real world and interact with it—a feature that proved to be very useful for our application. In addition this VR system provides us with very useful built-in features such as multi-user capabilities and support of the Personal Interaction Panel (PIP). We added sound support to the ‘Studierstube’.

We chose to use the Personal Interaction Panel, (Szalavári and Gervautz, 1997), a two-handed 3D interface composed of a position tracked pen and pad to control the application. It allows the straightforward integration of conventional 2D interface elements like buttons, sliders, dials etc. as well as novel 3D interaction widgets. The haptic feedback from the physical props guides the user when interacting with the PIP, while the overlaid graphics allows the props to be used as multi-functional tools (Figure 1). Every application displays its own interface in the form of a PIP ‘sheet’, which appears on the PIP. The pen and pad are our primary interaction devices. The pen has two buttons, a front—also referred as the primary—button and a back or secondary button. We use both.

User Interface

Our source of inspiration for designing a user interface for this HMD-based application is based on various ideas, problems and suggestions from such diverse areas as user interfaces, user centered design, usability engineering, human computer interaction in general (Szalavári and Gervautz, 1997; Mine *et al.* 1997; Baudisch, 1996; Pierce *et al.* 1997) as well as current software used for 3D modelling.

The PIP sheet of Construct3D represents the menu system. We try to keep the menu very simple. Large, textured 3D buttons are used with meaningful 3D icons floating above the buttons to allow easy and fast selection of the menu elements (Figure 2).

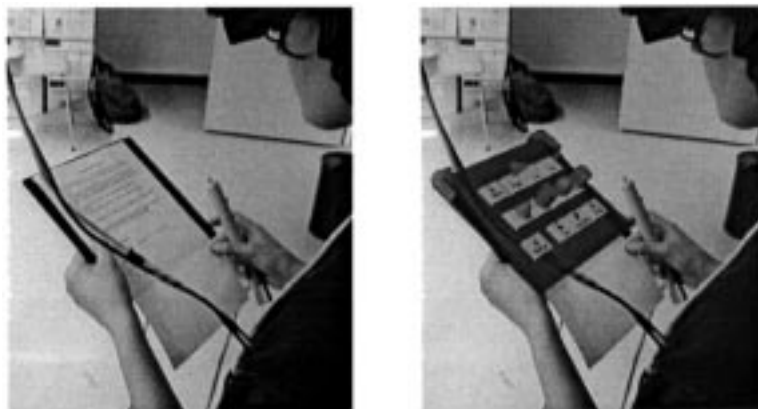


Figure 1. Working with the PIP in augmented reality. By looking out from underneath the head mounted display at the menu panel, the instructions on a sheet of paper can be read (left), by looking through the HMD the menu system with 3D buttons can be seen (right).



Figure 2. PIP sheet and menu of Construct3D.

Menu selection is achieved by moving the pen over the appropriate 3D button until it is highlighted and clicking on the primary button of the pen (a highlighted 3D button can be seen in Figure 3). The menu button turns red when clicked and appears pressed down—a selection technique known to most users from 2D window interfaces.

The representation of the pen in the virtual world is a small green cylinder with a spherical red end and a conical blue tip as can be seen in Figure 3. Two functions are assigned to the two buttons of the pen and can be activated at any time during the construction process. By clicking the primary button the user places a point at the current position—a point is represented by a small cube instead of a sphere, to keep the number of polygons low for its presentation in the virtual world.

The secondary button selects the geometric element that is nearest to the current position of the pen. Once an element is selected it is highlighted in red (e.g. the cutting plane in Figure 3). In order to be able to select points on surfaces for instance, points have

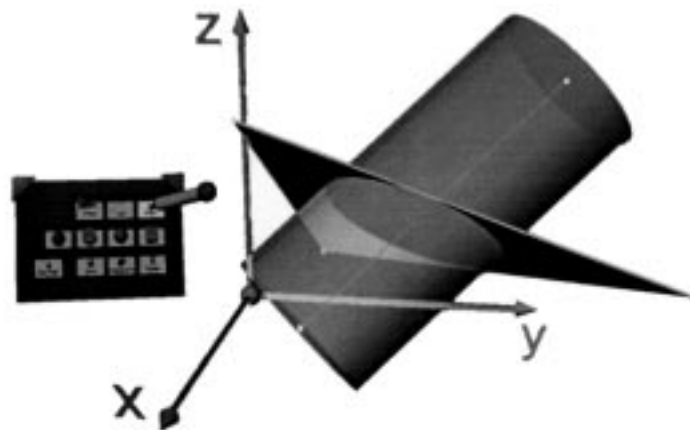


Figure 3. Sample construction as seen by a user through the HMD. The highlighted plane intersects the semi-transparent cylinder in an ellipse.

selection priority over other elements by default. Selecting a selected element again deselects it.

Orientation and Navigation

When starting Construct3D the user faces a three dimensional coordinate system centered in its workspace, in the middle of the room. For easier orientation in the three dimensional coordinate system we attach a very simple 'grid' to the tip of the pen (Baudisch, 1996). This grid consists of three axis parallel lines, pointing from the current position to the foot in the corresponding coordinate planes xy, xz and yz.

In addition, the current coordinates in the given coordinate system are displayed left of the pen in centimeters with millimetre precision. Coordinates are written line by line, the text always faces the user. This enables construction of given points by their coordinates.

Tools and Functions

Seven basic objects can be constructed by clicking the appropriate 3D button on the PIP: point, line, plane, box, sphere, cone and cylinder (Figure 2). The four solids box, sphere, cone and cylinder are drawn semi-transparent in order to see objects inside and behind solids (e.g. in Figure 3 the axis and the point of the plane behind the cylinder).

There are four other menu buttons. Clicking the 'Delete' button deletes all selected objects, 'Save' saves the whole scene as an Open Inventor file which can be loaded and manipulated afterwards with desktop tools. 'New Start' deletes all objects and resets the environment to initial state. The 'Deselect All' button deselects all selected objects. For every new construction new elements must be selected, therefore deselecting all elements at once is faster than deselecting objects one by one. We implemented a gestural action (as suggested by Mine et al. 1997) to speed up and simplify this action. If all objects are outside a certain range of the pen and the user clicks the secondary button (selection button) all objects are deselected. The easiest gestural action related to this algorithm is to move the arm down, parallel to the body and press the secondary button.

Unless generating a point, base elements must be selected before a new shape can be constructed. A line requires two points as input elements. A plane can be generated by three points, one point and one line or two lines. E.g.: If a user selects a line and a point and clicks on the plane button, the system recognizes the selected elements and calls the according procedure to draw a new plane.

A box requires the lower left and upper right corner as input. To construct a sphere the midpoint and one point on the sphere or the midpoint and two points indicating the radius of the sphere must be given. A cylinder can either be generated by two points (their connecting line representing the axis) and a point on the surface or by a line (the axis) and a point on the surface or by two lines, the first one being the axis, the second one being a tangent to the cylinder. A cone requires three input points: midpoint of the base circle,

apex and a point on the cone. When more than one point is needed, the order of the selection is relevant.

Last but not least, the 'Normal' menu button has been assigned construction of normal lines and normal planes to given elements. Input of a point not lying on a plane, sphere, cylinder and cone results in a line perpendicular to the selected surface. Selecting a point on the surface of a sphere, cylinder or cone results in a tangent plane.

The 'Point' button is used for calculating intersection points. Intersections of line with line, plane, box, sphere, cylinder and cone are implemented.

Audio Help System

If the user selects a menu button without having selected the correct base elements, an audio help system, which otherwise remains in the background, gives feedback. A realistic, computer generated, female voice offers help such as 'Please select axis and point on cylinder or a tangent' when trying to construct a cylinder.

Hardware and Software Setup

Construct3D was developed using semi-transparent Virtual IO i-glasses display devices and Ascension Flock of Birds 6DOF magnetic trackers to track the head, pen and panel. The images are rendered on an SGI Indigo2 Impact or Intergraph Wildcat workstation using the Open Inventor toolkit with 'Studierstube' specific libraries (Fuhrmann and Schmalstieg, 1999).

User Studies

Our system is still under development therefore we cannot provide a formal evaluation study at this time. However, an informal pilot study has been conducted to evaluate the efficiency of Construct3D and its value for mathematics and geometry education.

Subjects

Our 14 participants (6 female, 8 male), aged 22–34, are students in Vienna; 13 of them had and enjoyed geometry education (descriptive geometry) in high school, 9 are students of mathematics and geometry with the aim of becoming high school teachers. On average, they have basic computer skills and good working knowledge of traditional CAD packages.

Methods

The test session consists of two parts. The first part requires each participant to solve a construction example from mathematics education with the help of a tutor in Construct3D. The example stems from vector analysis as taught in 10th grade in Austria. For high school students, calculating the results would be lengthy and rather complex.

In the second part all subjects complete a brief survey. The survey contains an informal section about VR in general and questions about Construct3D.

The following task was assigned to all participants at the beginning of the test: A sphere is given by its midpoint M and a tangent plane A, B, C . Construct the sphere. The line $[A, Q]$ intersects the given sphere. Draw the tangent plane to the sphere in the highest intersection point. The line and the backmost point of the plane span a new plane. Select this plane and interpret its intersection with the sphere. Save the file.

The coordinates of all points are given. Because of inaccuracies with measuring positions in our virtual world (caused by tracker hardware), a tolerance of 2 centimeters per coordinate is acceptable. The result can be seen in Figure 4.

One of the reasons why we chose such a traditional example was to demonstrate how VR can be integrated into today's mathematics education without any changes to the curriculum—though we believe that curricula will change once VR is integrated. The reason why we chose words such as ‘backmost’ and ‘highest’ was to see if students would have problems with spatial relationships in the virtual world.

Results

10 of our subjects experienced VR for the first time. All received a one minute introduction into the system consisting of an explanation how to put on the HMD, how to use the pen and the menu. In total, our students took between 6 and 13 minutes to solve the task. This reflects the time they spent using the system. While working on their first construction, a tutor was inside the virtual world with them to help when problems occurred and to answer



Figure 4. Working in Construct3D. The tutor assists the student while working on the model (Image composition).

any questions. The assistant gave help regarding the interface, problems that arose because of a lack of geometrical knowledge occurred only once. The goal of the assistant was to help overcoming a certain fear and disorientation when having first contact with the new medium.

It was very gratifying for us to see users work with Construct3D in such a constructive manner. It was obvious that they did not need a long introduction to the system but applied their experience with 2D user interfaces to our 3D interface. After generating the first plane, a normal to the plane through the midpoint, the intersection point with the plane and the sphere, all students could complete their work without further assistance.

The students' interactions with the system were interesting to watch. Using both buttons of the pen proved to be difficult due to ergonomic reasons. After completing their task, some walked around the object, viewing it from different sides. It was clear that they were proud of what they 'built'. One student even got down on his knees and looked at the object from below.

The first part of the survey confirmed our observations. All students want to experience VR again and all rate it as a very good playground for experiments. All think that VR presents a rather good learning environment though questions arose of how to work with larger groups of students.

Six students felt a little bit dizzy – some of them during, most of them after leaving the virtual world – probably a light form of cybersickness. Ten participants think that it is easier to view a three dimensional world in VR than on a flat screen. Two of them find using a screen easier. These two had difficulties with the small field of view of the HMD, with orientation and spatial relationships in the virtual world. Two others remark that screen and VR are two totally different categories and argue that VR is great but many technical difficulties are related to it. It is interesting that the same people give the same answers when asked if viewing a three dimensional world in VR gives them a better spatial understanding compared to viewing a world on the screen from different views.

The second part of the survey covered questions about Construct3D.

Eight participants think that for a first contact working with a CAD package on-screen is easier than working with Construct3D. However, all except one can imagine working with Construct3D without ever having worked with a traditional CAD package. It is interesting to note that all of them have worked with other CAD packages before. The only person who had never done so and never had descriptive geometry education in school, says that she would prefer more basic knowledge about geometry before working with Construct3D to have a better understanding of what she is doing. A similar pattern applies to the next question: if participants can imagine constructing in VR without having learned the basics of descriptive geometry – to construct in two views such as front and top view. 11 persons think that they can construct in VR without any knowledge about descriptive geometry. This is especially interesting for geometry education in Austria where classical descriptive geometry education has a long tradition. This needs further thought and discussion.

All students reported problems with setting points at given coordinates and we received a number of good suggestions for improvements. Hand-eye coordination proved to be very difficult when spotting a point accurately in 3D space, freely without any haptic feedback, without constraints. The only feedback the students had were the coordinates of the current position. In addition a light flicker of the pen is produced by measuring inaccuracies. Most

students suggested implementing an invisible grid with automatic snapping to the grid points with variable grid width. Another suggestion was to lock specific axis while trying to locate a point. This is similar to the idea of fixing a point in a coordinate plane e.g. xy plane first and adding its third coordinate afterwards. Bowman (1999) suggests in his extensive work about interaction techniques for common tasks in immersive virtual environments to restrict user movement to less than 3 dimensions. This corresponds to the suggestion of our students.

Though we never mentioned any technical aspects of Construct3D, all participants see technical difficulties when asked about the relevance of constructing in VR for future geometry education. All agree that the idea is good but one student points out that it can only be an addition to geometry education and never a substitution, though if used would be highly motivating and very interesting. The authors agree with that.

A very encouraging result is that all participants can imagine the use of Construct3D in its current state (with small additions such as measuring distances and angles and an invisible raster) in mathematics and geometry education for solving simple problems. The following application areas are mentioned: interactive conic sections, vector analysis, enhancing spatial abilities, intersection problems, experiencing space (for very young students) and building three dimensional worlds from two dimensional views. If technical difficulties could be neglected even more application areas come to mind such as: elementary geometry, visualization of constructions, geometry didactics—learning by doing, training of spatial abilities by viewing objects from different sides.

Finally we asked all students what they liked best and least about Construct3D and what they would like to change. On VR in general, people commented having really enjoyed this new experience, the spatial feeling and body movement. About constructing in VR, they liked walking around and inside objects, the ‘playful’ way of constructing, that spatial relationships and complex three dimensional situations are directly visible. Regarding Construct3D the clearness of the menu system and the audio help system were mentioned positively.

Technical aspects caused problems that reflect in the participants’ comments such as slow rendering speed, bad calibration of the whole system which resulted in small difficulties clicking menu buttons, inaccuracy of the pen due to position tracking inaccuracies and a small field of view caused by our HMDs. One person felt disoriented in the virtual world, another one experienced some form of cybersickness, another one said that constructing was strenuous. Concerning Construct3D and its user interface, people criticize that they had difficulties with choosing the right buttons on the pen. Further they suggested to implement a grid or snapping method, transformation of objects with constraints, intersections between primitives, different colors for basic elements and to label elements. One user did not like the transparency we used for solids.

Discussion

One of the difficulties that arose during the design and implementation process was the use of transparency for solids. On the one hand, transparent solids are often needed to see other

objects inside these solids. On the other hand, shading or color differences can hardly be seen on objects that are too transparent and a complex construction just looks like a transparent blob. Object intersections can only be seen on opaque objects. Therefore we will assign variable transparency to the objects in future versions that can be manipulated by the user on the PIP.

One of the big problems is inaccuracy of our constructions. Compared to paper and pencil a tolerance of 2 centimeters per coordinate seems enormous though it is not that large considering that the work range in the virtual world is at least 4 times larger than with paper and pencil. We hope to get rid of these problems with new tracking hardware in the near future in addition to an effective grid implementation.

During the evaluation process other interesting results emerged. The gestural action that we implemented for deselecting all objects proved to be easy and fast for all users. It was impressing to see how quickly they adapted to this new working environment and completed a task in a very short time that would have taken them more than half an hour with paper and pencil.

The suggestions of the students are very useful for future improvements of our system. We are definitely going to implement most of the suggested changes in order to make our system more user friendly and easier to handle. Regarding functionality, a 'switchable' PIP will be implemented in order to switch between different menu sheets, to be flexible if new functions are added. A measuring tool for measuring distances, angles and radii is important for many application areas and often needed.

To improve the selection method, a configurable snapping method which only allows points or lines or planes or points and planes etc. to be snapped will be used. To enhance construction of points by given coordinates a configurable, invisible grid will be implemented. This will allow, for instance, restriction of coordinate values to integers only and other specific grid widths can be selected on the PIP.

On the pedagogic side, a separate PIP sheet for student and teacher is possible giving additional functions to the teacher that should stay hidden from the student for pedagogic reasons.

In addition to the application areas mentioned by the participants of our study it would be possible to load instructions or partial constructions into Construct3D so a student could finish them. One area of application could be to load difficult geometric situations into the workspace for visualization purposes. Students could add simple geometric constructions to control and understand properties e.g. the functionality of the GPS system could be explained that way. Another very fruitful application area is three dimensional dynamic geometry e.g. selecting an axis and a skew line to watch the one rotate around the axis, generating a hyperboloid of revolution.

After this pilot study the next logical step would be to test our system with a small group of high school students in regular mathematics or geometry education. As hardware improves and costs drop, VR will become a means of education sooner or later. To cite Kimberly Osberg (1993) 'The technology is moving ahead, regardless of what we as educators may wish. So we can either become a part of the research and development effort, adding the cognitive component to the mix, or we can sit back and let technology take the educational process by storm. I for one, choose to explore further'.

Future Work

While designing and implementing Construct3D we learned a lot about all different areas that combine in building an easy to use educational application. Human computer interaction and virtual reality are relatively young sciences but especially in psychology a lot of research has been done recently (Osberg, 1997; Rizzo *et al.* 1998; Larson *et al.* 1999). Rizzo and Larson are working into this direction but the effects of VR technology on all components of spatial intelligence need further and more detailed investigation. This is vital for the understanding of the full potential that virtual reality can bring our children especially in mathematics and geometry education.

Though a lot of work still has to be done to improve Construct3D as a three dimensional construction tool, Construct3D in its current state is an intermediate step to our long term goal – Construction in n Dimensions: An educational VR application for construction of higher dimensional geometry.

In descriptive higher dimensional geometry we generally use a number of orthogonal projections into 2-dimensional space to visualize higher dimensional geometry. E.g., descriptive geometry in 4-dimensional space can be visualized with two 2-dimensional views, where each view represents two coordinates of the object. In contrast to conventional 2D computer graphics, virtual reality enables us to use 3 dimensions for all visualization processes.

We will introduce the idea of using two 3-dimensional views to visualize up to 6 dimensions. Similar to the geometric concept of using a pair of orthogonal projections from 3D into 2D such as front- and top-view, we propose to use a pair of orthogonal projections from 4- or 5-dimensional space into 3-dimensional space. This will enable us to directly construct and work with geometric objects in 4D and 5D with the help of a pair of 3D views.

The use of additional dimensions for visualizing 4 and 5-dimensional geometry will give students as well as teachers a better understanding of higher dimensional geometry.

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References

- Baudisch P. (1996). The Cage: Efficient Construction in 3D using a Cubic Adaptive Grid, in *Proceedings of the 9th ACM Symposium on User Interface Software and Technology* (UIST '96), 171–172.

- Bell, J.T. and Fogler, H.S. (1995). The Investigation and Application of Virtual Reality as an Education Tool. In *Proceedings of the American Society for Engineering Education 1995 Annual Conference*, Session number 2513, Anaheim, CA.
- Bowman, D.A. (1996). Conceptual Design Space—Beyond Walk-through to Immersive Design. In Bertol, D., *Designing Digital Space*, John Wiley & Sons, New York.
- Bowman, D.A. (1999). Interaction Techniques for Common Tasks in Immersive Virtual Environments: Design, Evaluation, and Application. Ph.D. thesis, Virginia Polytechnic & State University.
- Bricken, M. and Byrne, C. (1993). Summer Students in Virtual Reality: A Pilot Study on Educational Applications of VR Technology. In *Virtual Reality, Applications and Explorations* A. Wexelblat (ed). Cambridge, MA: Academic Press Professional.
- Butterworth, J., Davidson, A., Hench, S. and Olano, T.M. (1992). 3DM: A Three Dimensional Modeler Using a Head-Mounted Display. *Computer Graphics. Proceedings 1992 Symposium on Interactive 3D Graphics* **25**(2), pp. 135–138.
- Byrne, C.M. (1996). Water on Tap: The Use of Virtual Reality as an Educational Tool. Unpublished Ph.D. Dissertation, University of Washington, College of Engineering.
- Clark, J.H. Designing Surfaces in 3-D. *Communications of the ACM* (1976). **19**(8), pp. 454–460.
- Chapin, W.L., Lacey, T.A. and Leifer, L. (1994) DesignSpace: a Manual Interaction Environment of Computer Aided Design. In *Proceedings of the ACM's SIGCHI 1994 Conference: CHI'94 Human Factors In Computing Systems*.
- Fisher, S.S., McGreevy, M., Humphries, J. and Robinett, W. (1986). Virtual Environment Display Systems. In *Proceedings of the Workshop on Interactive 3D Graphics*. Sponsored by ACM SIGGRAPH.
- Fuhrmann, A. and Schmalstieg, D. (1999). Concept and Implementation of a Collaborative Workspace for Augmented Reality, *GRAPHICS'99*, Volume 18, No. 3.
- Kiyokawa, K., Takemura, H. and Yokoya, N. (2000). SeamlessDesign for 3D Object Creation, *IEEE MultiMedia magazine*, **7**(1), pp. 22–33. Online: <http://www.crl.go.jp/ts221/kiyo/cr/ieeemultimedia.pdf>
- Larson, P., Rizzo, A.A., Buckwalter, J.G., van Rooyen, A., Kratz, K., Neumann, U., Kesselman, C., Thiebaut, M. and van der Zaag, C. Gender Issues in the Use of Virtual Environments. In *CyberPsychology and Behavior*, (1999). **2**(2).
- Liang, J. and Green M. (1994). JDCAD: a highly interactive 3D modeling system. *Computers & Graphics* **18**(4), pp. 499–506.
- Maier, P.H. (1994). *Räumliches Vorstellungsvermögen*. Peter Lang GmbH, Europäische Hochschulschriften: Reihe 6, Bd. 493, Frankfurt am Main.
- Mine, M. (1996). Working in a Virtual World: Interaction Techniques Used in the Chapel Hill Immersive Modeling Program., UNC Chapel Hill Computer Science Technical Report TR96-029.
- Mine, M. (1997). ISAAC: A Meta-CAD System for Virtual Environments. *Computer-Aided Design*, **29**(8).
- Mine, M., Brooks, F.P.Jr. and Sequin, C.H. (1997) Moving Objects in Space: Exploiting Proprioception in Virtual-Environment Interaction. *Proceedings of SIGGRAPH 97*, Los Angeles, CA.
- Osberg, K. (1993). Virtual Reality and Education: A Look at Both Sides of the Sword, *Technical R-93-7*. Seattle: Human Interface Technology Lab. Online: <http://www.hitl.washington.edu/publications/r-93-7/>
- Osberg, K. (1997). Spatial Cognition in the Virtual Environment, *Technical R-97-18*. Seattle: Human Interface Technology Lab. Online: <http://www.hitl.washington.edu/publications/r-97-18/>
- Pierce, J.S., Forsberg, A., Conway, M.J., Hong, S., Zeleznik, R. and Mine M.R. (1997). Image Plane Interaction Techniques in 3D Immersive Environments.
- Regenbrecht, H., Kruijff, E., Donath, D., Seichter, H. and Beetz, J. (2000). Virtual Reality Aided Modeler – 3DUI testbed results. Paper submitted to EGVE'00.
- Rizzo, A.A., Buckwalter, J.G., Neumann, U., Kesselman, C., Thiebaut, M., Larson, P. and van Rooyen, A. (1998). The Virtual Reality Mental Rotation Spatial Skills Project. In *CyberPsychology and Behavior*, **1**(2), pp. 113–120.
- Schmalstieg, D., Fuhrmann, A., Szalavári, Z., and Gervautz, M. (1996). 'Studierstube' – An Environment for Collaboration in Augmented Reality Extended abstract in *Proceedings of Collaborative Virtual Environments '96*, Nottingham, UK.

- Szalavári, Z., and Gervautz M. (1997). The Personal Interaction Panel—A Two-Handed Interface for Augmented Reality, *Computer Graphics Forum*, **16**, 3, pp. 335–346.
- Szalavári, Z., Schmalstieg, D., Fuhrmann, A. and Gervautz, M. (1998). ‘Studierstube’ An Environment for Collaboration in Augmented Reality. In *Virtual Reality—Systems, Development and Applications*, **3**(1), pp. 37–49.
- Winn, W., and Bricken, W. (1992). Designing virtual worlds for use in mathematics education: The example of experiential algebra. *Education Technology*, **32**(12), pp. 12–19.
- Winn, W. (1997). The Impact of Three-Dimensional Immersive Virtual Environments on Modern Pedagogy. HITL Technical Report R-97-15. Discussion paper for NSF Workshop. Human Interface Technology Laboratory, University of Washington.