Вар 16 (10391) Кузнецова Софья 3374

1. $3335x - 3128y = 46$

3335		1	-0
3128		-0	1
207	1	1	-1
23	15	-15	16
0	9	136	-145
		3335	3128

$$\begin{cases} x = -15 \cdot \frac{46}{23} = -15 \cdot 2 + 136k \\ -y = 16 \cdot \frac{46}{23} = 16 \cdot 2 - 145k \end{cases}, k \in \mathbb{Z}$$

Ответ: $\longrightarrow$ $\begin{cases} x = -30 + 136k \\ y = -32 + 145k \end{cases}, k \in \mathbb{Z}$

2. $\sqrt{300} = 17 + (\sqrt{300} - 17) = 17 + \frac{1}{\alpha_1}$

$$\alpha_1 = \frac{1}{\sqrt{300}-17} = \frac{\sqrt{300}+17}{300-289} = \frac{\sqrt{300}+17}{11} = \frac{17+(\sqrt{300}-17)+17}{11} = \frac{34+(\sqrt{300}-17)}{11} = 3 + \frac{1+\sqrt{300}-17}{11} = 3 + \frac{\sqrt{300}-16}{11}$$

$= 3 + \frac{1}{\alpha_2}$

$$\alpha_2 = \frac{11}{\sqrt{300}-16} = \frac{\cancel{11}(\sqrt{300}+16)}{\cancel{44}\,4} = \frac{\sqrt{300}+16}{4} = \frac{32+(\sqrt{300}-16)}{4}$$

$= 8 + \frac{\sqrt{300}-16}{4} = 8 + \frac{1}{\alpha_3}$

$$\alpha_3 = \frac{4}{\sqrt{300}-16} = \frac{\cancel{4}(\sqrt{300}+16)}{\cancel{44}\,11} = \frac{\sqrt{300}+16}{11} = \frac{17+(\sqrt{300}-16)+16}{11} = \frac{33+(\sqrt{300}-16)}{11} = 3 + \frac{\sqrt{300}-17}{11} = 3 + \frac{1}{\alpha_4}$$

$$\alpha_4 = \frac{11}{\sqrt{300}-17} = \frac{\cancel{11}(\sqrt{300}+17)}{\cancel{11}} = \sqrt{300}+17 = 34 + (\sqrt{300}-17)$$

Ответ: $(17/3, 8, 3, 34))$

4. $43^{5^{57}} \pmod{46}$

$\varphi(46) = \varphi(23)\varphi(2) = 22 \cdot 1 = 22$

$43^{22} \underset{22}{\equiv} 1$

$43^{22k+r} \underset{22}{\equiv} \underbrace{43^{22k}}_{1} \cdot 43^r = 43^r$

$\varphi(22) = \varphi(11)\varphi(2) = 10 \cdot 2 = 20$

$5^{20} \underset{20}{\equiv} 1 \equiv 5^{20k} \underset{20}{\equiv} 1$

$5^{20k+r} \underset{20}{\equiv} 5^r$

$5^{57} \underset{22}{\equiv} 17 \cdot 9 \cdot 17 \cdot 5 \underset{22}{\equiv} 7$

$17 \cdot 9 = 153 \underset{22}{\equiv} 11$

$11 \cdot 17 = 187 \underset{22}{\equiv} 15$

$15 \cdot 5 = 75 \underset{22}{\equiv} 7$

$43^{5^{57}} \equiv 43^7 \pmod{46}$

$43^7 \underset{46}{\equiv} (43^2)^2 \cdot 43^2 \cdot 43 \underset{46}{\equiv} 25 \cdot 19 \cdot 43 \underset{46}{\equiv} 1$

$25 \cdot 19 = 475 \underset{46}{\equiv} 31$

$31 \cdot 43 = 1333 \underset{46}{\equiv} 1$

Ответ: 1

5. $p(1) = 5$ x_0
$p(2) = -10$ x_1
$p(-1) = 11$ x_2
$p(-2) = 26$ x_3
$p(-3) 25$ x_4

$P_0(x) = y_0$

$P_1(x) = P_0(x) + \frac{(x - x_0)}{(x_1 - x_0)}(y_1 - P_0(x_1))$

...

$P_k(x) = P_k(x) + \prod_{i=1}^{k-1} \frac{x - x_i}{x_k - x_i}(y_k - P_{k-1}(x_k))$

$P_0(x) = 5$

$P_1(x) = 5 + \frac{x-1}{2-1}(-10-5) = 20-15x$

$P_2(x) = 20-15x + \frac{(x-1)(x-2)}{(-1-5)(-1-2)}(11-35) =$
$= -4x^2-3x+12$

$P_3(x) = -4x^2-3x+12 + \frac{(x-1)(x-2)(x+1)}{(-2-1)(-2-2)(-2+1)}(26-2) =$

$= -2x^3-x+8$

$P_4(x) = -2x^3-x+8 + \frac{(x-1)(x-2)(x+1)(x+2)}{(-3-1)(-3-2)(-3+1)(-3+2)}(25-65) =$

$= 5x^4-2x^3-25x^2-x+28$

Ответ:

6. $x^4-5x^3-6x^2+7x-2$

$\{\pm 1, \pm 2\}$ – делители

	1	-5	-6	7	-2
2	1	-7	8	-9	16 ≠ 0

	1	-5	-6	7	-2
2	1	-3	-12	17	-36 ≠ 0

	1	-5	-6	7	-2
-1	1	-6	0	7	-9 ≠ 0

	1	-5	-6	7	-2
1	1	-4	-10	-3	-5 ≠ 0

Ни один из делителей не явл-ся корнем многочлена => он не имеет рациональных корней

7. $7x + 43 = 425$

$7_9 = 7_{10}$

$43_9 = 4\cdot 9^1 + 3\cdot 9^0 = 36+3 = 39_{10}$

$425_9 = 4\cdot 9^2 + 2\cdot 9^1 + 5\cdot 9^0 = 4\cdot 81 + 2\cdot 9 + 5 = 324+18+5 = 347_{10}$

$7x + 39 = 347$
$7x = 308$
$x = 44$

$44_{10} = 4\cdot 9^1 + 8\cdot 9^0 = 48_9$

Ответ: в 9-й сс – 48_9, в 10-й сс. – 44_{10}

8. 13 | 46.

$$99$$

$$\frac{13}{46} = \frac{x}{1} \qquad 46x \underset{99}{\equiv} 13$$

$$46x \underset{99}{\equiv} -1472$$

$$x \underset{99}{\equiv} -32$$

$$x \underset{99}{\equiv} 67$$

Ответ: $x \underset{99}{\equiv} 67$

9. $$\frac{363}{166} = 2\frac{31}{166} = 2 + \frac{1}{\frac{166}{31}} = 2 + \frac{1}{5 + \frac{11}{31}} = 2 + \frac{1}{5 + \frac{1}{\frac{31}{11}}} =$$

$$= 2 + \frac{1}{5 + \frac{1}{2 + \frac{9}{11}}} = 2 + \frac{1}{5 + \frac{1}{2 + \frac{1}{\frac{11}{9}}}} = 2 + \frac{1}{5 + \frac{1}{2 + \frac{1}{1 + \frac{2}{9}}}} =$$

$$= 2 + \frac{1}{5 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\frac{9}{2}}}}} = 2 + \frac{1}{5 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4 + \frac{1}{2}}}}}$$

Ответ: $(2; 5, 2, 1, 4, 2)$

10. $4x^5 + 6x^4 + 6x^3 + 5x^2 + 4x \;|\; 6x^3 + 4x^2 + 5x + 3$ в $\mathbb{Z}/_7\mathbb{Z}[x]$

$-\underline{4x^5 + 5x^4 + x^3 + 2x^2} \;|\; 3x^2 + 6x + 5$

$x^4 + 5x^3 + 3x^2 + 4x$

$-\underline{x^4 + 3x^3 + 2x^2 + 4x}$

$2x^3 + x^2$

$-\underline{2x^3 + 6x^2 + 4x + 1}$

$-5x^2 - 4x - 1 = \boxed{2x^2 + 3x + 6}$

Ответ: $2x^2 + 3x + 6$

53. $x \equiv 14 \bmod 20$; $x \equiv 0 \bmod 21$; $x \equiv 17 \bmod 19$; $x \equiv 2 \bmod 11$

$\Sigma (M/M_k) y_k b_k \pmod{M}$

$M = 20 \cdot 21 \cdot 19 \cdot 11 = 87780$

$\frac{M}{M_1} = 21 \cdot 19 \cdot 11 = 4389$ $\qquad$ $\frac{M}{M_3} = 20 \cdot 21 \cdot 11 = 4620$

$\frac{M}{M_2} = 20 \cdot 19 \cdot 11 = 4180$ $\qquad$ $\frac{M}{M_4} = 20 \cdot 21 \cdot 19 = 7980$

$4389 y_1 \equiv 1 \pmod{20}$ $\qquad$ $4620 y_3 \equiv 1 \pmod{19}$

$4180 y_2 \equiv 1 \pmod{21}$ $\qquad$ $7980 y_4 = 1 \pmod{11}$

Запишем в виде:

$(219 \cdot 20 + 9) y_1 \equiv 1 \pmod{20}$ $\qquad$ $(243 \cdot 19 + 3) y_3 = 1 \pmod{19}$

$(199 \cdot 21 + 1) y_2 \equiv 1 \pmod{21}$ $\qquad$ $(725 \cdot 11 + 5) y_4 = 1 \pmod{11}$

т.к. $am + b = b \pmod{m}$, то система имеет вид:

$9 y_1 \equiv 1 \pmod{20}$ $\qquad$ $3 y_3 \equiv 1 \pmod{19}$

$1 y_2 \equiv 1 \pmod{21}$ $\qquad$ $5 y_4 \equiv 1 \pmod{11}$

$y_1 = 9, \ y_2 = 1, \ y_3 = 13, \ y_4 = 9$

$x \equiv 4389 \cdot 9 \cdot 14 + 4180 \cdot 1 \cdot 0 + 4620 \cdot 13 \cdot 17 + 7980 \cdot 9 \cdot 2$
$\pmod{87780} = 553014 + 1021020 + 143640 =$
$= 1717674 \pmod{87780}$

Т.к. $1717674 = 19 \cdot 87780 + 49854$, то <b>$x = 49854$
$\pmod{87780}$

Ответ