Dec 28th

Review of (∇f) gradient, divergence $(\nabla \cdot \vec{F})$ & curl $(\nabla \times \vec{F})$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

is the vector that points in steepest ascent of a functn

Divergence is of a vector field $\vec{F} = (F_x, F_y, F_z)$

$$(\nabla \cdot \vec{F}) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Curl $(\nabla \times \vec{F})$ measures the rotation around a point. If zero then no rotation where $|\nabla \times \vec{F}| > 0$ where the value indicates strength of rot

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

The linear first-order PDE

$$\frac{\partial u}{\partial t} + c \cdot \frac{\partial u}{\partial x} = 0$$

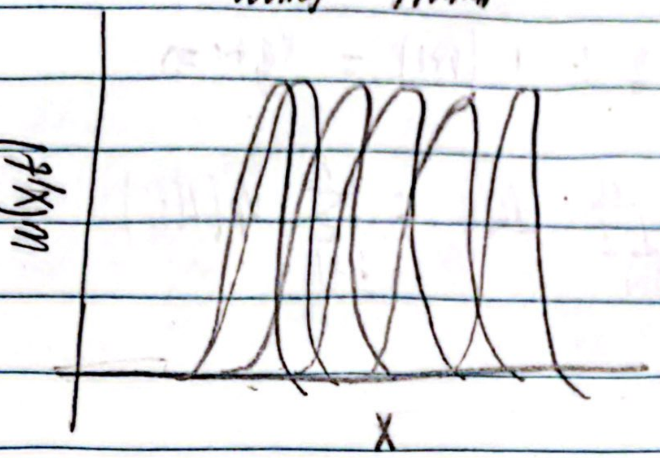
$u(x,t)$ unknown functn of position x & time t

c is a constant representing wave speed

If $c > 0$ it's moving right with speed c

If $c < 0$ left with $|c|$

Wave fronts



Visualization of $c = 2$

$x(t)$ is the characteristic curve

The equation $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$ is hard we ask there is a path in x-t plane along which $u(x,t)$ is constant

$$\frac{du}{dt} = 0 = \frac{\partial u}{\partial t} + \frac{dx}{dt} \cdot \frac{\partial u}{\partial x}$$

direct contribution of t

what a c [?]

If $\frac{dx}{dt} = c$ the slope of the characteristic curves satisfies the u stays const

$$\frac{dx}{dt} = c \Rightarrow x - ct = \text{const}$$