Dec 27th

We define quadratic variation of the random walk $\sum_{j=1}^{i} (S_j - S_{j-1})^2$

In the coin game $|S_j - S_{j-1}| = 1$ so the sum is i for coin prob

- Brownian motion. We define a game where we restrict t time for 6 tosses. Second the bet size isn't \$1 but $\sqrt{\frac{t}{6}}$. $\sum_{i=1}^{6} (S_j - S_{j-1})^2 = 6 \cdot \left(\sqrt{\frac{t}{6}}\right)^2 = t$

As n increase we do more & more ~~[illegible]~~

As $n \to \infty$ $E[S(t)] = 0$ and a variance of $E[S(t)^2] = t$

For a population $\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$

Properties of brownian motion: continuous, indep increments, markov process, martingale process, quadratic variation:

$$[X]_T = \lim_{|\Pi| \to 0} \sum_{i=1}^{n} (X_{t_i} - X_{t_{i+1}})^2 = t$$

$$\Pi = \{0 = t_0 < t_1 < \dots < t_n = T\}$$

$|\Pi| \to 0$ partition becomes infinitely ~~A~~

Normality For $v > 0$ the increment

$B_{t+v} - B_v$ is normally distr

with mean $= 0$ Variance $= v$

$$B_{t+v} - B_t \sim N(0, v)$$

make brownian motion a

gaussian process

We define stochastic integral by

$$W(t) = \int_0^t f(\tau)\, dX(\tau) =$$

$$\lim_{n \to \infty} f(t_{j-1}) \left(X(t_j) - X(t_{j-1}) \right)$$

$$t_j = \frac{jt}{n}$$

$X(t)$ is our stochastic Process

f is our function due to time

say f is how much of an asset

we hold $X(t)$ model fluctuations