Math 440 – Real Analysis II

Homework 2

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Problem 1 Test the following series for convergence.

(a) $\sum\limits_{k=1}^{\infty} \frac{k+1}{2k^3-k}$ Does converge

Proof: Let $a_k = \frac{1}{k^2}$, then $\lim\limits_{k\to\infty} \frac{s_k}{a_k} = \lim\limits_{k\to\infty} \frac{(k+1)(k^2)}{2k^3-k} = \frac{1}{2}$

Since $\frac{1}{2} \in \mathbb{R}^+$, $\sum\limits_{k=1}^{\infty} \frac{k+1}{2k^3-k}$ converges by the Limit Comparison Test.

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(b) $\sum\limits_{n=1}^{\infty} \frac{p^n}{n!}$, $p \in \mathbb{R}$ Does converge

Proof: Using the ratio test, let $L = \lim\limits_{k\to\infty} (\frac{p^{k+1}}{(k+1)!})(\frac{k!}{p^k}) = \lim\limits_{k\to\infty} \frac{p}{k+1} = 0$

Since $L < 1$, the series $\sum\limits_{n=1}^{\infty} \frac{p^n}{n!}$ does converge.

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Problem 2 Suppose $a_k \geqslant 0$ for all $k \in \mathbb{N}$ and $\sum a_k < \infty$. Prove that the following converge or give an example where the series does not.

(a) $\sum\limits_{k=1}^{\infty} {a_k}^2$ is guaranteed to converge

Proof: By assumption $a_k \geqslant 0$ for all $k \in \mathbb{N}$, so $a_k = |a_k|$ and $\sum |a_k| = \sum a_k$. The series is thus absolutely convergent, so for some $m = max\{a_1, a_2, \ldots\}$, $m \sum a_k = \sum m a_k < \infty$. Since $a_k \leqslant m$, ${a_k}^2 \leqslant m a_k$, the series $\sum\limits_{k=1}^{\infty} {a_k}^2$ converges by the Weierstrass M-test.

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(b) $\sum\limits_{k=1}^{\infty} \frac{{a_k}^2}{1+a_k}$

Proof: pf

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