Math 440 – Homework 1

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Problem 1 Find the limit inferior and limit superior of each of the following sequences $\{s_n\}$.

(a)
$$s_n = 2 - \frac{1}{n}$$

$$\overline{\lim}\{s_n\} = \underline{\lim}\{s_n\} = \lim_{n \to \infty} s_n = 2$$

(b) $s_n = n \mod 4$

For all $k \in \mathbb{N}$ the sequence $\{s_k, s_{k+1}, s_{k+2}, \ldots\}$ contains only the values $\{0, 1, 2, 3\}$. Thus $\forall k, 0 \leq s_k \leq 3$, so

$$\overline{\lim}\{s_n\} = 3$$

And

$$\underline{\lim}\{s_n\} = 0$$

(c)
$$s_n = \begin{cases} n & \text{if } n \text{ is even} \\ \frac{1}{n} + \cos \pi n & \text{if } n \text{ is odd} \end{cases}$$