MA 2160
Spring 2023
Quiz 3

Name:______________________________________

1 Using the Simpson's rule approximate the integral $\int_1^2 \sqrt{x^3-1}\,dx$ with $n = 10$, round your answer to six decimal place. [5 pts]

$\Delta x = \frac{2-1}{10} = 0.1$

$$\int_1^2 \sqrt{x^3-1}\,dx \approx \frac{\Delta x}{3}\Big[f(1) + 4f(1.1) + 2f(1.2) + 4f(1.3) + 2f(1.4) + 4f(1.5) + 2f(1.6) + 4f(1.7) + 2f(1.8) + 4f(1.9) + f(2)\Big]$$

$$\approx 1.511519$$

2 Identify whether the integral $\int_1^\infty \frac{ln(x)}{x}\,dx$ converges or diverges? [5 pts]

$$\therefore \int_1^\infty \frac{\ln(x)}{x} = \lim_{t\to\infty} \int_1^t \frac{\ln(x)}{x}\,dx$$

$\ln(x) = u$

$\frac{1}{x}\,dx = du$

When $x=1,\ u=0$

$x=\infty,\ u=\infty$

$$= \lim_{t\to\infty} \int_0^\infty u\,du = \lim_{t\to\infty} \frac{u^2}{2}\Big|_0^\infty$$

$$= \infty$$

So, $\int_1^\infty \frac{\ln(x)}{x}\,dx$ diverges

3 Find the arc length of the curve $y = ln(1 - x^2)$ from $x = 0$ to $x = 1/2$. [5 pts]

$$L = \int_0^{1/2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \quad , \quad \frac{dy}{dx} = \frac{-2x}{1-x^2}$$

$$\therefore L = \int_0^{1/2} \sqrt{1+\left(\frac{-2x}{1-x^2}\right)^2}\, dx = \int_0^{1/2} \sqrt{1+\frac{4x^2}{(1-x^2)^2}}\, dx$$

$$= \int_0^{1/2} \sqrt{\frac{(1-x^2)^2+4x^2}{(1-x^2)^2}}\, dx = \int_0^{1/2} \frac{\sqrt{1-2x^2+x^4+4x^2}}{(1-x^2)}\, dx$$

$$= \int_0^{1/2} \frac{\sqrt{1+2x^2+x^4}}{1-x^2}\, dx = \int_0^{1/2} \frac{\sqrt{(1+x^2)^2}}{1-x^2}\, dx$$

$$= \int_0^{1/2} \frac{1+x^2}{1-x^2}\, dx = \int_0^{1/2} \frac{(x^2-1)+2}{1-x^2}\, dx$$

$$= \int_0^{1/2} \left(\frac{-(1-x^2)}{1-x^2} + \frac{2}{1-x^2}\right) dx = -\int_0^{1/2} dx + 2\int_0^{1/2} \frac{dx}{1-x^2}$$

$$= -x\Big|_0^{1/2} + 2\underbrace{\int_0^{1/2} \frac{dx}{(1+x)(1-x)}}_{\text{Do partial fraction}} = -\frac{1}{2} + 2\cdot\frac{1}{2}\int_0^{1/2} \left(\frac{1}{1+x}+\frac{1}{1-x}\right) dx$$

$$= -\frac{1}{2} + \int_0^{1/2} \frac{dx}{1+x} + \int_0^{1/2} \frac{dx}{1-x} = -\frac{1}{2} + \ln|1+x|\Big|_0^{1/2} - \ln|1-x|\Big|_0^{1/2}$$

$$= -\frac{1}{2} + \ln|1+\tfrac{1}{2}| - \ln|1+0| - \ln|1-\tfrac{1}{2}| + \ln|1-0|$$

$$= -\frac{1}{2} + \ln|\tfrac{3}{2}| - \underbrace{\ln|1|}_{=0} - \ln|\tfrac{1}{2}| + \underbrace{\ln|1|}_{=0}$$

$$= -\frac{1}{2} + \ln\left(\frac{3}{2}\right) - \ln\left(\frac{1}{2}\right) = -\frac{1}{2} + \ln\left(\frac{3}{2} \div \frac{1}{2}\right)$$

$$= \ln(3) - \frac{1}{2}$$

Ans