# CSE 216 Home Work 1
# Name: Son Tran
# Id: 115797806

## Part 1:

### 1.

incr 2 = (λz.λf.λy.f(zfy))2

(Using **β-reduction,** replace z with 2: )
= (λf.λy.f(2fy))   [z->2] (λf.λy.f(zfy))

=> incr 2 = (λf.λy.f(2fy))

2fy = (λf.λy.f(fy))fy

(Using **β-reduction,** applying f to f: )
= λy.f(fy))y     [f->f](λy.f(fy))fy

(Using **β-reduction,** applying y to y: )

= = f(fy)     [y->y] f(fy))y

=> incr 2 = (λf.λy.f(f(fy)))  = 3

2.

a)(((λx.λy.λz.((xy)z)(λu.λv.u))A)B)

b)(((λx.λy.λz.((xy)z)(λu.λv.v))A)B)

Answer:

Let:
true=(λu.λv.u)
false=(λu.λv.v)

If true then A else B

a)

(((λx.λy.λz.((xy)z)(λu.λv.u))A)B)

Apply **β-reduction** to replace **x, y, ,z** with **(λu.λv.u), A, B** respectively:

(((λx.λy.λz.((xy)z)(λu.λv.u))A)B)
= (((λy.λz.(((λu.λv.u)y)z))A)B) [x-> λu.λv.u] ((λy.λz.((xy)z)A)B)
= (((λz.(((λu.λv.u)A)z))B) [y-> A] (λz.((xy)z)B)

= (((λu.λv.u)A)B) [z-> A] ((xy)z)

Apply **β-reduction** to replace **u,v** with **A, B** respectively:

((λu.λv.u)A)B
= λv.A [u->A] (λv.u)B
= A [v->B] A

So this expression is right because we can see that

If true then A is executed.

b)

(((λx.λy.λz.((xy)z)(λu.λv.v))A)B)

Apply **β-reduction** to replace **x, y, ,z** with **(λu.λv.v), A, B** respectively:

(((λx.λy.λz.((xy)z)(λu.λv.v))A)B)
= (((λy.λz.(((λu.λv.v)y)z))A)B) [x-> λu.λv.v] ((λy.λz.((xy)z)A)B)
= (((λz.(((λu.λv.v)A)z))B) [y-> A] (λz.((xy)z)B)

= (((λu.λv.v)A)B) [z-> A] ((xy)z)

Apply **β-reduction** to replace **u,v** with **A, B** respectively:

(((λu.λv.v)A)B
= (λv.v)B [u->A] (λv.v)B
= B [v->B] v

If false then B

=>So this expression is right because we can see that

If true then B is executed.