B.E. B.Tech. DEGREE EXAMINATION, MAY / JUNE 2007.

Answer ALL questions

PART - A - (10 x 2 = 20 marks)

1. ***Express the statement "Good food is not cheap" in symbolic form.***

Answer

P : food is good; q : food is cheap

Symbolic form : $P \to 7q$

2. ***Obtain PDNF for $7P \vee Q$.***

Answer

$$7P \vee Q = (7P \wedge (Q \vee 7Q)) \vee ((P \vee 7P) \wedge Q)$$

$$= (P \wedge Q) \vee (7P \wedge Q) \vee (7P \wedge 7Q) \text{ which is the PDNF.}$$

3. ***Define simple statement function.***

Answer

A simple statement function of one variable is defined to be an expression consisting of a Predicate symbol and an individual variable.

4. ***Express the statement "For every x there exist a y such that $x^2 + y^2 \geq 100$" in symbolic form.***

Answer

$$(\forall x)(\exists y)\left(x^2 + y^2 \geq 100\right)$$

5. ***Give an example of a relation which is symmetric, transitive but not reflexive on $\{a,b,c\}$***

Answer

R = {(a, b) (b, a) (a, a) (b, b)}

6. ***Define partially ordered set.***

Answer

A set with a partially ordering relation is called a Poset or partially ordered set.

7. ***If A has 3 elements and B has 2 elements, how many functions are there from A to B?***

Answer

$2^3 = 8$ functions

8. ***Define Characteristic function.***

Solution

Let U be a universal set and A be any arbitrary sub set of U. The function $f_A = U \to [0, 1]$ defined by $f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$, is called a characteristic function.

9. ***Give an example of sub semi-group.***

Answer

For the semi group (N, +), where N is the set of natural number, the

10. Define normal subgroup of a group.

Answer

A subgroup H of a group G is called a normal subgroup if for every $a \in G$, aH = Ha ie left coset of H = right coset of H.

PART - B (5x16 = 80 marks)

11 (a)(i) Without using truth tables, show that $Q \vee (P \wedge 7Q) \vee (7P \wedge 7Q)$ is a tautology

Solution

$Q \vee (P \wedge 7Q) \vee (7P \wedge 7Q)$

$\Leftrightarrow \quad Q \vee ((P \vee 7P) \wedge 7Q)$

$\Leftrightarrow \quad Q \vee (T \wedge 7Q)$

$\Leftrightarrow \quad Q \vee 7Q \Leftrightarrow T$

(OR)

$Q \vee (P \wedge 7Q) \vee (7P \wedge 7Q) \Leftrightarrow ((Q \vee P) \wedge (Q \vee 7Q)) \vee (7P \wedge 7Q)$

$\Leftrightarrow \quad ((Q \vee P) \wedge T) \vee (7P \wedge 7Q)$

$\Leftrightarrow \quad (P \vee Q) \vee 7(P \vee Q)$

$\Leftrightarrow$ T where T is a Tautology.

(ii) Obtain the PDNF for $(P \wedge Q) \vee (7P \wedge Q) \vee (Q \wedge R)$

Answer

$(P \wedge Q) \vee (7P \wedge Q) \vee (Q \wedge R)$

$\Leftrightarrow ((P \wedge Q) \wedge (R \vee 7R)) \vee ((7P \wedge Q) \wedge (R \vee 7R)) \vee ((Q \wedge R) \wedge (P \vee 7P))$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge 7R) \vee (7P \wedge Q \wedge R) \vee (7P \wedge Q \wedge 7R)$ The RHS is the PDNF.

(b) (i) Show that S is valid inference from the premises

$P \rightarrow \sim Q, Q \vee R, \sim S \rightarrow P$ and $\sim R$

Solution

Steps	Premises	Reason
(1)	$Q \vee R$	P, Given premise
(2)	7R	P, Given premise
(3)	Q	T, (1), (2) $(P \vee Q \wedge 7Q \Rightarrow 7P)$
(4)	$P \rightarrow 7Q$	P, Given premise
(5)	7P	T, (3),(4)modus Tollens $(P \rightarrow Q) \wedge 7Q \Rightarrow 7P$
(6)	$7S \rightarrow P$	P, Given premise
(7)	$7P \rightarrow S$	T, (6) $(P \rightarrow Q) \Leftrightarrow 7Q \rightarrow 7P$
(8)	S	T, (5), (7) modus Ponens

(ii) Without using truth tables, show that

$$(7P\wedge(7Q\wedge R))\vee(Q\wedge R)\vee(P\wedge R)\Leftrightarrow R$$

Answer

$$(7P\wedge(7Q\wedge R))\vee(Q\wedge R)\vee(P\wedge R) \Leftrightarrow (7(P\vee Q)\wedge R) \vee ((P\vee Q)\wedge R)$$

$$\Leftrightarrow (7(P\vee Q)\wedge R) \vee ((P\vee Q)\wedge R)$$

$$\Leftrightarrow (7(P\vee Q)\vee(P\vee Q))\wedge R$$

$$\Leftrightarrow T\wedge R$$

$$\Leftrightarrow R$$

12.(a) *(i) Show that*

$$(x)(P(x)\to Q(x))\wedge(x)(Q(x)\to R(x))\Rightarrow(x)(P(x)\to R(x))$$

Solution

Steps	premises	Reason
(1)	$(x)\ (P(x)\to Q(x))$	Given premise
(2)	$P(y)\to Q(y)$	Us (1)
(3)	$(x)\ (Q(x)\to R(x))$	Given premise
(4)	$Q(y)\to R(y)$	Us (3)
(5)	$P(y)\to R(y)$	(2), (4) Hyp. syllogusm.

(ii) Is the following conclusion validly derivable from the premises given? If $(\forall x)(P(x)\to Q(x)),(\exists y)P(y)$ then $(\exists z)Q(z)$.

Solution

Steps	Premises	Reason
(1)	$7(\exists z)Q(z)$	Premise (assumed)
(2)	$(\forall z)\ 7\ Q(z)$	(1) $7(\exists x)\ P(x) \Leftrightarrow (x)\ 7P(x)$
(3)	$(\exists y)\ P(y)$	Premise
(4)	P(a)	(1), Es
(5)	7Q(a)	(2), Us
(6)	$P(a)\wedge 7Q(a)$	(4), (5) $P, Q \Rightarrow P\wedge Q$
(7)	$7\ (P(a)\ \to\ Q(a))$	(6), $7(P\to Q) = P\wedge 7Q$
(8)	$(\forall x)(P(x)\to Q(x))$	Premise
(9)	$P(a)\to Q(a)$	(8) Us
(10)	$(P(a)\to Q(a))\wedge$ $7(P(a)\to Q(a))$	(7), (8) $P,\ Q \Rightarrow P\wedge Q$
	$\equiv F$	

Hence the conclusion $(\exists z)\ Q(z)$ is validly derivable from the premises $(\forall x)\ (P(x)\to Q(z))$ and $(\exists y)\ P(y)$

(b) (i) Verify the validity of the inference. If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than Peter. Therefore, John is not successful than Peter.

Solution

Universe : All persons

S(x, y) : x is more successful than y

W(x, y) : x has worked harder than y to deserve success

a : John ; b : Peter

To prove $(\forall x)(\forall y)\,[S(x,y) \to W(x,y)],\quad 7W(a,b) \Rightarrow 7S(a,b)$

Proof Sequence

Steps	**Premises**	**Reason**
(1)	7 W (a, b)	Given premise
(2)	$(\forall x)(\forall y)\,[S(x,y) \to W(x,y)]$	Given premise
(3)	$(\forall y)\,[S(a,y) \to W(a,y)]$	Us (2)
(4)	$S(a,b) \to W(a,b)$	Us (3)
(5)	$7S(a,b)$	T, (1) (4) Modus Tollens

(ii) Show that $R \vee S$ is a valid conclusion from the premises $C \vee D, (C \vee D) \to 7H, 7H \to (A \wedge 7B)$ and $(A \wedge 7B) \to (R \vee S)$

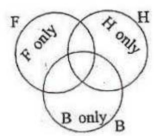
Solution

Steps	Premises	Reason
(1)	$C \vee D$	Given premise
(2)	$C \vee D \to 7H$	premise
(3)	7H	(1), (2) Modus Ponens
(4)	$7H \to (A \wedge 7B)$	premise
(5)	$A \wedge 7B$	(3), (4) Modus Ponens
(6)	$(A \wedge 7B) \to R \vee S$	premise
(7)	$R \vee S$	(5), (6) Modus Ponens

13.(a)(i) A survey of 500 television watchers produced the following information: 285 watch football games; 195 watch hockey games: 115 watch Basket ball games: 45 watch foot ball and basket ball games: 70 watch foot ball and hockey games: 50 watch hockey and basket ball games: 50 donot watch any of the three games: How many people watch exactly one of the three games?

Solution

U : Total numbers of Television watchers



F : Person who is watching foot ball game

H : Person who is watching hockey game

B : Person who is watching basket ball game

$\therefore$ $n(U) = 500$; $n(F) = 285$; $n(H) = 195$;

$n(B) = 115$; $n(F \cap B) = 45$; $n(F \cap H) = 70$;

$n(H \cap B) = 50$ $n(\overline{F} \cap \overline{H} \cap \overline{B}) = 50$

$$n(F \cup H \cup B) = n(U) - n\left(\overline{F} \cap \overline{H} \cap \overline{B}\right)$$

$$= 450$$

$$\therefore \quad 450 = 285 + 195 + 115 - 45 - 70 - 50 + n(F \cap B \cap C)$$

$$\Rightarrow \quad n(F \cap H \cap B) = 20$$

$$n(F\ only) = n(F) - n(F \cap H) - n(F \cap B) + n(F \cap H \cap B)$$

$$= 285 - 70 - 45 + 20 = 190$$

$$n(H\ only) = n(H) - n(F \cap H) - n(H \cap B) + n(F \cap H \cap B)$$

$$= 195 - 70 - 50 + 20 = 95$$

$$n(B\ only) = n(B) - n(F \cap B) - n(H \cap B) + n(F \cap H \cap B)$$

$$= 115 - 45 - 50 + 20 = 40$$

$\therefore$ No. of people watch exactly one of the three games

$$= 190 + 95 + 40 = 325$$

(OR) Number of people watch exactly one of the three games =

$$n(F \cap H \cap B) - n(F \cap B) - n(H \cap B) - n(F \cap H) + 2n(F \cap H \cap B)$$

$$= 325$$

(ii) ***In a Boolean algebra L, prove that***

$(a \wedge b)' = a' \vee b'$ **for all $a, b \in L$**

Solution

Consider $(a \cap b) \vee (a^1 \vee b^1) = \left(a \vee \left(a^1 \vee b^1\right)\right) \wedge \left(b \vee a^1 \vee b^1\right)$

$$= \left(\left(a \vee a^1\right) \vee b^1\right) \wedge \left(a^1 \vee \left(b \vee b^1\right)\right)$$

$$= \left(1 \vee b^1\right) \wedge \left(a^1 \vee 1\right)$$

$$= 1 \wedge 1 = 1$$

and $(a \wedge b) \wedge (a^1 \vee b^1) = \left(a \wedge b \wedge a^1\right) \vee \left(a \wedge b \wedge b^1\right)$

$$= \left(b \wedge \left(a \wedge a^1\right)\right) \vee \left(a \wedge \left(b \wedge b^1\right)\right)$$

$$= (b \wedge 0) \vee (a \wedge 0) = 0 \vee 0 = 0$$

Hence $(a \wedge b)' = (a' \vee b')$

(b) (i)Let the relation R be defined on the set of all real numbers by 'if x, y are real numbers, $xRy \Leftrightarrow x - y$ is a rational number'. Show that R is an equivalence relation.

Solution

A.Y. U.Q Nov Dec 2010

$x - x = 0$, is a rational number for all real number x

$\therefore$ R is reflexive

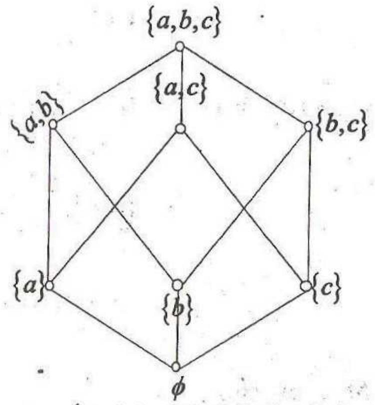
Let x Ry. Then $x - y = z$ is a rational number

Non $y - x = -z$ is also a rational number

So y R x $\Rightarrow$ R is symmetic

Let x Ry and y Rz. Then $x - y = a$ and $y - z = b$ where a and b are rational numbers. $x - z = (x - y) + (y - z) = a + b$ is also a rational number $\Rightarrow$ x Rz and so R is transitive. Hence the relation R is an equivalence relation.

(ii) ***Draw the Hasse diagram of the Lattice*** L ***of all subsets of*** $\{a,b,c\}$ ***under intersection and union.***



Since $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ where $A = \{a,b,c\}$.

14.(a)(i) ***Let the function*** f ***and*** g ***be defined*** $f(x) = 2x+1$ ***and*** $g(x) = x^2 - 2$***. Determine the composition function*** $f \circ g$ ***and*** $g \circ f$***.***

Solution

$$(f \circ g)(x) = f[g(x)]$$

$$= f(x^2-2) = 2(x^2-2)+1 = 2x^2-3$$

$$(g \circ f)(x) = g[f(x)]$$

$$= g(2x+1) = (2x+1)^2 - 2 = 4x^2+4x-1$$

(ii) ***Let*** a ***and*** b ***be positive integers and suppose*** Q ***is defined recursively as follows:***

$$Q(a,b) = \begin{cases} 0, & \text{if } a < b \\ Q(a-b,b)+1, & \text{if } b \le a \end{cases}$$

Find $Q(2,5), Q(12,5), Q(5861,7)$.

Solution

$Q(2, 5) = 0$ since $2 < 5$

$Q(12, 5) = Q(7,5) + 1$

$= Q(2,5) + 1 + 1 = 0 + 1 + 1 = 2$

Q (a,b) is nothing but quotient when a is divisible by b.

$\therefore Q(5861, 7) = 837$

(b) (i) ***Let $f: R \to R$ be defined by $f(x) = 2x - 3$. Find a formula for f^{-1}.***

Solution

$$f(x) = f(y) \Rightarrow 2x - 3 = 2y - 3$$

$$\Rightarrow x = y$$

$\therefore$ f is one - to - one

$$f(x) = y \Rightarrow y = 2x - 3$$

$$\Rightarrow x = \frac{y+3}{2} \in R$$

For every $y \in R$, $\exists x \in R$ such that $f(x) = 2\frac{(y+3)}{2} - 3 = y$

$\Rightarrow$ f is onto

$\therefore$ f is a bijection and hence f^{-1} exists

$f^{-1}: R \to R$ is difined by $f^{-1}(x) = \frac{x+3}{2} \forall x \in R$

(ii) ***Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by using characteristic function.***

Solution

$$\phi_{A \cap (B \cup C)}(x) = \phi_A(x)\left[\phi_{B \cup C}(x)\right]$$

$$= \phi_A(x)\left[\phi_B(x) + \phi_c(x) - \phi_{B \cap C}(x)\right]$$

$$= \phi_A(x)\left[\phi_B(x) + \phi_c(x) - \phi_B(x)\phi_c(x)\right]$$

$$= \phi_A(x)\phi_B(x) + \phi_A(x)\phi_c(x) - \phi_A(x)\phi_B(x)\phi_c(x)$$

$$= \phi_{A \cap B}(x) + \phi_{A \cap C}(x) - \phi_{A \cap B \cap C}(x)$$

$$= \phi_{(A \cap B) \cup (A \cap C)}(x)$$

Hence $A \cap B \cup C = (A \cap B) \cup (A \cap C)$

15. (a)(i) ***Show that the mapping g from the algebraic system $(S,+)$ to the system (T,X) defined by $g(a) = 3^a$, where S is the set of all rational numbers under addition + and T is the set of non-zero real numbers under multiplication operation *, is a homomorphism but not an isomorphism.***

Solution

$$g(a+b) = 3^{a+b} = 3^a * 3^b = g(a) * g(b)$$

$\therefore g(a+b) = g(a) * g(b) \quad \forall\ a, b \in S$

$\Rightarrow$ g is a homomorphism

Further $g(a) = g(b) \Rightarrow 3^a = 3^b$

$$\Rightarrow a = b$$

$\therefore$ g is one to - one.

But g is not onto, since range of g has no negative numbers

Hence g is not an isomorphism

(ii) ***Show that (2, 5) encoding function defined by*** $e(00)=00000$, $e(01)=01110$, $e(10)=10101$, $e(11)=11011$ ***is a group code.***

Solution

Let N be the set of code words

ie. N = { 00000, 01110, 10101, 11011} = $\{x^{(0)}, x^1, x^2, x^3\}$ (say)

N is closed for the operation $\oplus$, addition modulo 2 and the table is given below

$\oplus$	x°	x^1	x^2	x^3
x°	x°	x^1	x^2	x^3
x^1	x^1	x°	x^3	x^2
x^2	x^2	x^3	x°	x^1
x^3	x^3	x^2	x^1	x°

From the above table, e is a group code.

(b) (i) Find the minimum distance of the encoding function $e: B^2 \to B^4$ ***given by*** $e(00)=0000$, $e(10)=0110$, $e(01)=1011$, $e(11)=1100$

Solution

Let a = e (00) = 0000

b = e(10) = 0110

c = e(01) = 1011 and d = e(11) = 1100

δ (a, b) = 2 ; δ (a, c) = 3 ; δ (a, d) = 2

δ (b, c) = 3 ; δ (b, d) = 2 ; δ (c, d) = 3

The mininum of all distances = 2

$\therefore$ The minimum distance of e = 2

(ii) ***The intersection of any two subgroups of a group G is again a sub-group of G. - Prove.***

Solution

Let H and K be two subgroups of (G, *) and e be the identity element of G.

$e \in H$ and $e \in K \Rightarrow e \in H \cap K$

Hence $H \cap K$ is non - empty

Let a, b $\in H \cap K$. Then a, $b \in H$ and a, $b \in K$

Since H is a subgroup, $a \in H$, $b \in H \Rightarrow a * b^{-1} \in H$

Ill ly K is a subgroup, $a \in K$, $b \in K \Rightarrow a * b^{-1} \in K$

$\therefore$ a, b $\in H \cap K \Rightarrow a * b^{-1} \in H \cap K$

This shows that $H \cap K$ is also a subgroup of G.