Ranking Diversity

Refer: http://www-personal.umich.edu/~qmei/pub/kdd10-divrank.pdf

Vertex Reinforced Random Walk

$$p_T(u, v) = (1 - \lambda)p^*(v) + \lambda \frac{p_0(u, v)N_T(v)}{D_T(u)}$$

$$D_T(u) = \sum_{v \in V} p_0(u, v) N_T(v)$$

$$p_0(u,v) = \begin{cases} \alpha \frac{w(u,v)}{\deg(u)}, & \text{if } u \neq v \\ 1-\alpha, & \text{otherwise} \end{cases}$$

$$E[N_{T+1}(v)] = E[N_T(v)] + p_{T+1}(v)$$

$$p_{T}(v) = \sum_{(u,v)\in E} p(u,v)p_{T-1}(u)$$

$$p_{T}(v) = \sum_{(u,v)\in E} p_{T}(u,v)p_{T-1}(u)$$

CumDivRank

$$E[N_T(v)] \propto \sum_{t=0}^{t=T} p_t(v)$$

PointwiseDivRank

$$E[N_T(v)] \propto p_T(v)$$

[Standard Random Walk]

PointWise DivRank

$$p_T(u, v) = (1 - \lambda)p^*(v) + \lambda \frac{p_0(u, v)N_T(v)}{D_T(u)}$$

$$D_T(u) = \sum_{v \in V} p_0(u, v) N_T(v)$$

$$p_0(u,v) = \begin{cases} \alpha \frac{w(u,v)}{\deg(u)}, & \text{if } u \neq v \\ 1-\alpha, & \text{otherwise} \end{cases}$$

$$E[N_{T+1}(v)] = E[N_T(v)] + p_{T+1}(v)$$

$$E[N_T(v)] \propto p_T(v)$$

$$p_T(u, v) = (1 - \lambda)p^*(v) + \lambda \frac{p_0(u, v)p_T(v)}{D_T(u)}$$

$$D_T(u) = \sum_{u \in V} p_0(u, v) p_T(v)$$

Cumulative DivRank

$$p_T(u, v) = (1 - \lambda)p^*(v) + \lambda \frac{p_0(u, v)N_T(v)}{D_T(u)}$$

$$D_T(u) = \sum_{v \in V} p_0(u, v) N_T(v)$$

$$p_0(u,v) = \begin{cases} \alpha \frac{w(u,v)}{\deg(u)}, & \text{if } u \neq v \\ 1-\alpha, & \text{otherwise} \end{cases}$$

$$E[N_{T+1}(v)] = E[N_T(v)] + p_{T+1}(v)$$

$$p_T(v) = \sum_{(u,v) \in E} p_T(u,v) p_{T-1}(u)$$

$$E[N_T(v)] \propto \sum_{t=0}^{t=T} p_t(v)$$

$$p_T(u,v) = (1-\lambda)p^*(v) + \lambda \frac{p_0(u,v)\sum_{t=0}^{t=T}p_t(v)}{D_T(u)}$$

$$D_T(u) = \sum_{v \in V} p_0(u,v)\sum_{t=0}^{t=T}p_t(v)$$
 Both the terms need to be normalised, to handle overflow

$$N_T(v) = \frac{\sum_{t=0}^{t=T} p_t(v)}{\sum_{u \in V} \sum_{t=0}^{t=T} p_t(u)}$$