Ex- 6.1

1a) Sol^n

$LHS = a(b\cos C - c\cos B)$

$= ab\cos C - ac\cos B$

$= ab\dfrac{(a^2+b^2-c^2)}{2ab} - ac\dfrac{(a^2+c^2-b^2)}{2ca}$

$= \dfrac{abc(a^2+b^2-c^2) - abc(a^2+c^2-b^2)}{2abc}$

$= \dfrac{a^2+b^2-c^2-a^2-c^2+b^2}{2}$

$= b^2-c^2$

b) Sol^n, To prove;

$\dfrac{\cos A}{a} + \dfrac{a}{bc} = \dfrac{\cos B}{b} + \dfrac{b}{ca} = \dfrac{\cos C}{c} + \dfrac{c}{ab}$

$LS = \dfrac{\cos A}{a} + \dfrac{a}{bc}$

$= \dfrac{b^2+c^2-a^2+2a^2}{2abc}$

$= \dfrac{a^2+b^2+c^2}{2abc}$

$MS = \dfrac{\cos B}{b} + \dfrac{b}{ac}$

$$= \frac{a^2+c^2-b^2+2b^2}{2abc}$$

$$= \frac{a^2+b^2+c^2}{2abc}$$

$$RS = \frac{\cos C}{c} + \frac{c}{ab}$$

$$= \frac{a^2+b^2-c^2+2c^2}{2abc}$$

$$= \frac{a^2+b^2+c^2}{2abc}$$

$LS = MS = RS$ proved.

20. Sol^n

$LHS = \cancel{a^3\left(\frac{b}{2R}\right.}$

$$a^3(\sin^3 B - \sin^3 C) + b^3(\sin^3 C - \sin^3 A) + c^3(\sin^3 A - \sin^3 B) = 0$$

$$LHS = \left[a^3\left\{\left(\frac{b}{2R}\right)^3 - \left(\frac{c}{2R}\right)^3\right\} + b^3\left\{\left(\frac{c}{2R}\right)^3 - \left(\frac{a}{2R}\right)^3\right\}\right.$$

$$\left. + c^3\left\{\left(\frac{a}{2R}\right)^3 - \left(\frac{b}{2R}\right)^3\right\}\right]$$

$$= \frac{1}{8R^3}\left[a^3b^3 - a^3c^3 + b^3c^3 - a^3b^3 + a^3c^3 - c^3b^3\right]$$

$= 0$ RHS proved.

(c) Soln

$$\frac{a \sin(B-C)}{b^2-c^2} = \frac{b \sin(C-A)}{c^2-a^2} = \frac{c \sin(A-B)}{a^2-b^2}$$

$$LS = \frac{a \sin(B-C)}{b^2-c^2}$$

$$= \frac{2R \sin A \sin(B-C)}{4R^2 \sin^2 B - 4R^2 \sin^2 C}$$

$$= \frac{\sin(B+C) \sin(B-C)}{2R \{\sin^2 B - \sin^2 C)}$$

$$= \frac{(\sin^2 B - \sin^2 C)}{2R (\sin^2 B - \sin^2 C)}$$

$$= \frac{1}{2R}$$

$$MS = \frac{b \sin(C-A)}{c^2-a^2}$$

$$= \frac{2R \sin B \cdot \sin(C-A)}{4R^2 (\sin^2 C - \sin^2 A)}$$

$$= \frac{\sin^2 C - \sin^2 A}{2R (\sin^2 C - \sin^2 A)}$$

$$= 1/2R$$

$$RS = \frac{c \sin(A-B)}{a^2-b^2}$$

$$= \frac{2R \cdot \sin C \cdot \sin(A-B)}{4R^2 (\sin^2 A - \sin^2 B)}$$

$$= \frac{(\sin^2 A - \sin^2 B)}{2R (\sin^2 A - \sin^2 B)}$$

$$= \frac{1}{2R}$$

$\therefore LS = MS = RS$

QNO 3

a sol^n

$(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$

$LHS = b\cos A + c\cos A + c\cos B + a\cos B + a\cos C + b\cos C$

$= b\cos A + a\cos B + c\cos A + a\cos C + c\cos B + b\cos C$

$= c + b + a$ proved.

b: sol^n

$b^2 \sin 2C + c^2 \sin 2B = 2ab\sin C$

$LHS = b^2 \times 2\sin C\cos C + c^2 \times 2\sin B\cos B$

$= 2b^2 \sin C \left(\frac{a^2+b^2-c^2}{2ab}\right) + 2c^2 \sin B \times \left(\frac{a^2+c^2-b^2}{2ac}\right)$

$$= \frac{2b^2 \sin C\,(a^2+b^2-c^2)}{2ab} + \frac{2c^2 \sin B\,(a^2+c^2-b^2)}{2ac}$$

$$= \frac{2R\sin B\,\sin C\,(a^2+b^2-c^2)}{2R\sin A} + \frac{2R\sin C\,\sin B\,(a^2+c^2-b^2)}{2R\sin A}$$

$$= \frac{\sin B\,\sin C}{\sin A}\,(a^2+b^2-c^2+a^2+c^2-b^2)$$

$$= \frac{\sin B\,\sin C}{\sin A} \times 2 \times 4R^2 \sin^2 A$$

$$= 8R^2 \sin B\,\sin C\,\sin A$$

$$= 2R\sin A \times 2R\sin B \times 2\sin C$$

$$= 2ab\,\sin C = \text{RHS proved.}$$

(c) Sol^n

$$c^2\cos^2 B + b^2\cos^2 C + bc\cos(B-C) = \frac{1}{2}(a^2+b^2+c^2)$$

$$\text{LHS} = \cos^2 B\, c^2 + b^2\cos^2 C + bc\cos(B-C)$$

$$= (c\cos B)^2 + (b\cos C)^2 + bc\cos(B-C)$$

$$= (c\cos B + b\cos C)^2 - 2bc\cos B\cdot\cos C + bc\cos(B-C)$$

$$= a^2 - 2bc\cos B\cdot\cos C + bc\cos B\cos C + bc\sin B\sin C$$

$$= a^2 - bc\cos B\cos C + bc\sin B\sin C$$

$= a^2 - bc\{\cos B\cos C - \sin B\sin C\}$

$= a^2 - bc\,\cos(B+C)$

$= a^2 + bc\cos A$

$= a^2 + bc \cdot \dfrac{b^2+c^2-a^2}{2bc}$

$= \dfrac{2a^2+b^2+c^2-a^2}{2}$

$= \dfrac{1}{2}(a^2+b^2+c^2)$ proved

d. Solⁿ

$$\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$$

$$\text{LHS} = \frac{c-b\cos A}{b-c\cos A}$$

$$= \frac{a\cos B + b\cos A - b\cos A}{c\cos A + a\cos C - c\cos A}$$

$$= \frac{a\cos B}{a\cos C}$$

$= \dfrac{\cos B}{\cos C}$ RHS proved.

e. Solⁿ

$$\frac{a-b\cos C}{c-b\cos A} = \frac{\sin C}{\sin A}$$

Date ______
Page ______

$LHS = \frac{a - b\cos C}{c - b\cos A}$

$= \frac{b\cos C + c\cos B - b\cos C}{b\cos A + a\cos B - b\cos A}$

$= \frac{c}{a}$

$= \frac{2R\sin C}{2R\sin A}$

$= \frac{\sin C}{\sin A}$ proved #.

@NOH

b) Sol^{n}

$\frac{b^2 - c^2}{a^2}\sin 2A + \frac{c^2 - a^2}{b^2}\sin 2B + \frac{a^2 - b^2}{c^2}\sin 2C = 0$

$LHS = \frac{b^2 - c^2}{a^2}\sin 2A + \frac{c^2 - a^2}{b^2}\sin 2B + \frac{a^2 - b^2}{c^2}\sin 2C$

$= \frac{b^2 - c^2}{a^2} 2\sin A\cos A + \frac{c^2 - a^2}{b^2} 2\sin B\cos B + \frac{a^2 - b^2}{c^2} 2\sin C\cos C$

$= \frac{b^2 - c^2}{a^2} \times 2 \times \frac{a}{2R}\cos A + \frac{c^2 - a^2}{b^2} \times 2 \times \frac{b}{2R} \times \cos B +$

$\frac{a^2 - b^2}{c^2} \times 2 \times \frac{c}{2R} \times \cos C$

$$= \frac{\alpha}{2R}\left[\frac{b^2-c^2}{a^2} \times a \times \cos A + \frac{c^2-a^2}{b^2} \times b \times \cos B + \frac{a^2-b^2}{c^2} \times c \times \cos C \right]$$

$$= \frac{1}{R}\left[\frac{(b^2-c^2)(b^2+c^2-a^2)}{2abc} + \frac{(c^2-a^2)(a^2+c^2-b^2)}{2abc} + \frac{(a^2-b^2)(a^2+b^2-c^2)}{2abc} \right]$$

$$= \frac{1}{2Rabc}\left[b^4+b^2c^2-b^2a^2-b^2c^2-c^4+a^2c^2+a^2c^2+c^4-c^2b^2-a^4-a^2c^2+a^2b^2+a^4+a^2b^2-a^2c^2-a^2b^2-b^4+b^2c^2 \right]$$

$= 0 =$ RHS proved.

e. Soln

$$\frac{\cos B - \cos C}{\cos A + 1} = \frac{c-b}{a}$$

$$\text{LHS} = \frac{\cos B - \cos C}{\cos A + 1}$$

$$= \frac{\frac{a^2+c^2-b^2}{2ac} - \frac{a^2+b^2-c^2}{2ab}}{\frac{b^2+c^2-a^2}{2bc} + 1}$$

$$= \frac{b(a^2+c-b^2) - c(a^2+b^2-c^2)}{a(b^2+c^2-a^2+2bc)}$$

$$= \frac{a^2b + cb^2 - b^3 - a^2c + b^2c - c^3}{a\{(b+c)^2 - (a)^2\}}$$

$$= \frac{a^2(b-c) - bc(b-c) - (b-c)(b^2+bc+c^2)}{a\{(b+c)^2 - a^2\}}$$

$$= \frac{(b-c)(a^2 - bc - b^2 - bc - c^2)}{a\{(b+c)^2 - a^2\}}$$

$$= \frac{(b-c)\{a^2 - (b^2+2bc+c^2)\}}{a\{(b+c)^2 - a^2\}}$$

$$= \frac{b-c}{-a}$$

$$= \frac{c-b}{a} \text{ proved.}$$

7. Sol^n.

$$\frac{a\sin A + b\sin B + c\sin C}{a\cos A + b\cos B + c\cos C} = \frac{R}{abc}(a^2+b^2+c^2)$$

$$LHS = \frac{a\sin A + b\sin B + c\sin C}{a\cos A + b\cos B + c\cos C}$$

$$= \frac{a \times \frac{a}{2R} + b \times \frac{b}{2R} + c \times \frac{c}{2R}}{2R\sin A\cos A + 2R\sin B\cos B + 2R\sin C\cos C}$$

$$= \frac{1/2R\,(a^2+b^2+c^2)}{R\,(\sin 2A + \sin 2B + \sin 2C)}$$

$$= \frac{(a^2+b^2+c^2)}{2R^2 \times 4\sin A \sin B \sin C}$$

$$= \frac{(a^2+b^2+c^2)}{2R^2 \times 4 \times \frac{a}{2R} \times \frac{b}{2R} \times \frac{c}{2R}}$$

$$= \frac{R(a^2+b^2+c^2)}{(abc)}$$

= RHS proved

QNO 5

b. Sol^n

$$b\cos^2 \tfrac{1}{2}A + a\cos^2 \tfrac{1}{2}B = \frac{(a+b+c)}{2}$$

$$LHS = b\cos^2\frac{A}{2} + a\cos^2\frac{B}{2}$$

$$= b \times \frac{s(s-a)}{bc} + a\,\frac{s(s-b)}{ac}$$

$$= \frac{s^2-sa}{c} + \frac{s^2-sb}{c}$$

$$= \frac{s^2-sa+s^2-sb}{c}$$

$$= \frac{2s^2-sa-sb}{c}$$

$$= s\,\frac{(2s-a-b)}{c}$$

$$= \frac{(a+b+c)\ (a+b+c-a-b)}{2c}$$

$$= \frac{(a+b+c)}{2} = \text{RHS proved.}$$

1\. Sol^n

$$\frac{b-c}{a}\cos^2\frac{A}{2} + \frac{c-a}{b}\cos^2\frac{B}{2} + \frac{a-b}{c}\cos^2\frac{C}{A} = 1$$

$$LHS = \frac{b-c}{a}\ \frac{s(s-a)}{bc} + \frac{c-a}{b}\ \frac{s(s-b)}{ac} + \frac{a-b}{c} \cdot \frac{s(s-c)}{ab}$$

$$= \frac{(b-c)(s^2-sa)}{abc} + \frac{(c-a)(s^2-sb)}{abc} + \frac{(a-b)(s^2-sc)}{abc}$$

$$= \frac{bs^2 - bsa - s^2c + csa + s^2c - sbc - s^2a + abs + s^2a - sac - s^2b + sbc}{abc}$$

$= 0 =$ RHS proved.

1\. Sol^n

$$\tan^2\frac{A}{2} \times \tan^2\frac{B}{2} \times \tan^2\frac{C}{2} = \left(\frac{s-a}{s}\right)\left(\frac{s-b}{s}\right)\left(\frac{s-c}{s}\right)$$

$$LHS = \tan^2\frac{A}{2}\ \tan^2\frac{B}{2}\ \tan\frac{C}{2}$$

$$= \frac{(s-b)(s-c)}{s(s-a)} \times \frac{(s-c)(s-a)}{s(s-b)} \times \frac{(s-a)(s-b)}{s(s-c)}$$

$$= \frac{(s-a)(s-c)(s-b)}{s^3}$$

$$= \left(\frac{s-a}{s}\right)\left(\frac{s-c}{s}\right)\left(\frac{s-b}{s}\right) = \text{RHS proved.}$$

$\sqrt{}$ 2.0

QNo 6.

Soln.

here $a^4+b^4+c^4-2c^2(a^2+b^2)=0$

or, $(a^2)^2+(b^2)^2+(c^2)^2-2c^2(a^2+b^2)=0$

or, $(a^2+b^2)^2-2a^2b^2-2c^2(a^2+b^2)+(c^2)^2=0$

or, $(a^2+b^2)^2-2c^2(a^2+b^2)+(c^2)^2=2a^2b^2$

or, $(a^2+b^2-c^2)^2=(\sqrt{2}ab)^2$

or, $\frac{a^2+b^2-c^2}{ab}=\pm\sqrt{2}$

or, $\frac{a^2+b^2-c^2}{2ab}=\pm\frac{\sqrt{2}}{2}$

+ve

$\cos C=\frac{1}{\sqrt{2}}$

$C=45°$

− ve

$\cos C=\cos 135$

$C=135$

$C=45°$ or $135°$

QNO7.

Soln

$(a+b+c)(b+c-a) = 3bc$

or, $ab+ac-a^2+b^2+bc-ab+bc+c^2-ac = 3bc$

or, $b^2+c^2-a^2+2bc = 3bc$

or, $b^2+c^2-a^2 = bc$

or, $\frac{b^2+c^2-a^2}{2bc} = \frac{1}{2}$

or, $\cos A = 1/2$

or, $\cos A = \cos 60'$

$\therefore A = 60'$

QNO8

8. Soln

$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

or, $\frac{b+c+a+c}{(a+c)(b+c)} = \frac{3}{a+b+c}$

or, $\frac{(a+b+c)(a+b+2c)}{ab+ac+bc+c^2} = 3$

or, $\frac{ab+a^2+2ac+b^2+ab+2bc+bc+ac+2c^2}{3ab+3ac+3bc+3c^2} = 1$

or, $2ab + a^2 + 3ac + b^2 + 3bc + 2c^2 = 3ab + 3ac + 3bc + 3c^2$

or, $a^2 + 2ab + b^2 + 2c^2 = 3ab + 3c^2$

or, $\frac{a^2 + b^2 - c^2}{ab} = 1$

or, $\frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2}$

or, $\cos C = \cos 60^\circ$

$\therefore C = 60^\circ$ proved

Q.NO 9

Soln

$\frac{\cos A}{\cos B} = \frac{a}{b}$

or, $b \cos A = a \cos B$

or, $b \times \frac{b^2 + c^2 - a^2}{2bc} = a \times \frac{a^2 + c^2 - b^2}{2ac}$

or, $b^2 + c^2 - a^2 = a^2 + c^2 - b^2$

or, $2b^2 = 2a^2$

or, $b = a$

$\therefore$ Given Δ is Isosceles Δ.

Q NO 10

$$\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$$

$\Rightarrow \sin C(\cos A + 2\cos C) = \sin B(\cos A + 2\cos B)$

or, $\cos A \sin C + 2\sin C \cos C = \cos A \sin B + 2\sin B \cos B$

or, $\cos A \sin C - \sin B \cos A = 2\sin B \cos B - 2\sin C \cos C$

or, $\cos A(\sin C - \sin B) + \sin 2C - \sin 2B = 0$

or, $\cos A \times 2\cos\frac{B+C}{2}\sin\frac{C-B}{2} - 2\cos\frac{2C+2B}{2}\sin\frac{2C-2B}{2} = 0$

or, $\cos A \, 2\sin\frac{A}{2}\sin\frac{C-B}{2} - 2\cos A \times 2\sin\left(\frac{C-B}{2}\right)\cos\left(\frac{C-B}{2}\right) = 0$

or, $2\cos A\left[\sin\frac{A}{2}\sin\frac{(C-B)}{2} - 2\sin\frac{(C-B)}{2}\cos\frac{(C-B)}{2}\right] = 0$

Either $2\cos A = 0$

$\cos A = 0$

$A = 90°$

OR

$\sin\frac{(C-B)}{2}\left[\sin\frac{A}{2} - 2\cos\frac{C-B}{2}\right] = 0$

Either $\sin\frac{(C-B)}{2} = 0$ or $\sin\frac{A}{2} - 2\cos\frac{(C-B)}{2} = 0$

$\sin\frac{(C-B)}{2} = \sin 0$ $\quad$ $\sin A = 2\cos\frac{(C-B)}{2}$

$\frac{C-B}{2} = 0$ $\quad$ not possible

$C = B$

Hence proved.

Q NO 11

Soln

$2\cos A = \frac{\sin B}{\sin C}$

or, $2\cos(A)\sin(C) = \sin B$

or, $2 \times \frac{b^2+c^2-a^2}{2bc} \times \frac{c}{2R} = \frac{b}{2R}$

or, $(b^2+c^2-a^2) = b^2$

or $a^2 = c^2$

or, $a = c$

$\therefore$ The Δ is Isosceles Δ.

Q NO 12

Soln

Here,
$$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$

or, $\sin A \sin(B-C) = \sin C \sin(A-B)$

or, $\sin(B+C)\sin(B-C) = \sin(A+C)\sin(A-B)$

or, $\sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$

or, $\frac{b^2}{4R^2} - \frac{c^2}{4R^2} = \frac{a^2}{4R^2} - \frac{b^2}{4R^2}$

or, $b^2 - c^2 = a^2 - b^2$

or, $b^2 - b^2 = c^2 - b^2$ [proved]

QNO 13

a. Soln

$a^2 \cot A + b^2 \cot B + c^2 \cot C = 4\Delta$

LHS $= (2R\sin A)^2 \cdot \frac{\cos A}{\sin A} + (2R\sin B)^2 \cdot \frac{\cos B}{\sin B} + (2R\sin C)^2 \cdot \frac{\cos C}{\sin C}$

$= 4R^2 \sin A \cdot \cos A + 4R^2 \sin B \cos B + 4R^2 \sin C \cos C$

or, $= 2R^2 (\sin 2A + \sin 2B + \sin 2C)$

$= 2R^2 \times 4 \sin A \sin B \sin C$

$= 8R^2 \times \frac{a}{2R} \times \frac{b}{2R} \times \frac{c}{2R}$

$= \frac{abc}{R} = 4\Delta$

$\left[\begin{array}{l} \Delta = abc/4R \\ 4\Delta = abc/R \end{array}\right]$

c. Sol^n.

$\sin A + \sin B + \sin C = \frac{s}{R}$

$LHS = \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$

$= \frac{1}{2R}(a+b+c)$

$= \frac{s}{R}$ proved.

QNo 14

Sol^n.

$8R^2 = a^2 + b^2 + c^2$

or, $8R^2 = (2R\sin A)^2 + (2R\sin B)^2 + (2R\sin C)^2$

or, $2 = \sin^2 A + \sin^2 B + \sin^2 C$

or, $2 = 1 - \cos^2 A + 1 - \cos^2 B + 1 - \cos^2 C$

or, $2 = 3 - (\cos^2 A + \cos^2 B + \cos^2 C)$

or, $2 = 3 - (1 - 2\cos A \cos B \cos C)$

or $2 = 3 - 1 + 2\cos A \cos B \cos C$

or $0 = 2\cos A \cos B \cos C$

or $\cos A \cos B \cos C = 0$.

Either

	OR	OR
$\cos A = 0$	$\cos B = 0$	$\cos C = 0$
$A = 90^\circ$	$B = 90^\circ$	$C = 90^\circ$

QNO 15 (V.V.V.V.V.V.V.V.V.V. Imp)

Soln.

here, $a = 10$

$b = 8$

$c = 6$.

$$s = \frac{a+b+c}{2} = \frac{10+8+6}{2} = 12$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$= \sqrt{\frac{(12-6)(12-10)}{6\times10}}$$

$$= \frac{1}{\sqrt{5}}$$

$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12\times2\times4\times6}$$

$$= 24\ cm^2$$

$$R = \frac{abc}{4\Delta} = \frac{10\times8\times6}{4\times24} = 5\ cm.$$

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9/15