clw

Chapter-18
Antiderivatives

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The reverse process of derivative is called anti-derivative or integration.

Rules of integration

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

2. $\int \frac{1}{x} \, dx = \log x + c$

3. $\int e^x \, dx = e^x + c$

4. $\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$

5. $\int \cos x \, dx = \sin x + c$
6. $\int \sin x \, dx = -\cos x + c$
7. $\int \sec x . \tan x \, dx = \sec x + c$
8. $\int \sec^2 x \, dx = \tan x + c$
9. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
10. $\int \operatorname{cosec} x . \cot x \, dx = -\operatorname{cosec} x + c$

i. $\int \cos ax \, dx = \frac{\sin ax}{a} + c$

ii. $\int \sin ax . dx = \frac{-\cos ax}{a} + c$

iii. $\int \sec^2 ax . dx = \frac{\tan ax}{a} + c$

iv. $\int \sec ax . \tan ax . dx = \frac{\sec ax}{a} + c$

v. $\int \operatorname{cosec}^2 ax . dx = \frac{-\cot ax}{a} + c$

vi. $\int \operatorname{cosec} ax . \cot ax . dx = -\frac{\operatorname{cosec} ax}{a} + c$

vii. $\int dx = x + c$

Exercise - 18.1

Find the indefinite integrals:

1\. i. $\int 5x^3\, dx$

Solution:

$I = \int 5x^3\, dx$

$= 5 \dfrac{x^{3+1}}{3+1} + c$

$= 5 \dfrac{x^4}{4} + c$

ii. $\int 7x^{5/2}\, dx$

Solution:

$I = \int 7x^{5/2}\, dx$

$= 7 \dfrac{x^{5/2+1}}{\frac{5}{2}+1} + c$

$= 2x^{7/2} + c$

iii. $\int 2x^{-7/2}\, dx$

Solution:

$I = \int 2x^{-7/2}\, dx$

$= 2\int x^{-7/2}\, dx$

$= -\dfrac{4}{5} x^{-5/2} + c$

iv. $\int 4x^{-5}\, dx$

Solution:

$I = \int 4x^{-5}\, dx$

$= 4\int x^{-5}\, dx$

$= 4 \dfrac{x^{-5+1}}{-5+1} + c$

$= -x^4 + c$

2\. i. $\int (x^2+2)\, dx$

Solution:

$I = \int (x^2+2)\, dx$

$= \int x^2\, dx + 2\int dx$

$= \dfrac{x^{2+1}}{2+1} + 2x + c$

$= \dfrac{x^3}{3} + 2x + c$

$= \dfrac{1}{3}x^3 + 2x + c$

ii. $\int (3x^2 + 2x + 1)\,dx$

Solution:

$$I = \int (3x^2 + 2x + 1)\,dx$$

$$= 3\int x^2\,dx + 2\int x\,.dx + 1\int dx$$

$$= 3\,\frac{x^{2+1}}{2+1} + 2\,\frac{x^{1+1}}{1+1} + 1x + c$$

$$= x^3 + x^2 + x + c$$

iii. $\int (x^{3/4} + x^{1/2} + 4x^{1/3})\,dx$

Solution:

$$I = \int (x^{3/4} + x^{1/2} + 4x^{1/3})\,dx$$

$$= \int x^{3/4}\,dx + \int x^{1/2}\,dx + 4\int x^{1/3}\,dx$$

$$= \frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1} + \frac{x^{1/2+1}}{1/2+1} + 4\,.\frac{x^{1/3+1}}{1/3+1}$$

$$= \frac{x^{7/4}}{7/4} + \frac{x^{3/2}}{3/2} + 4\,\frac{x^{4/3}}{4/3}$$

$$= \frac{4}{7}x^{7/4} + \frac{2}{3}x^{3/2} + 3x^{4/3}$$

iv. $\int (2x+1)(3x+2)\,dx$

Solution:

$$I = \int (2x+1)(3x+2)\,dx$$

$$= \int (6x^2 + 7x + 2)\,dx$$

$$= 6\int x^2\,.dx + 7\int x\,.dx + 2\int dx$$

$$= 6\,\frac{x^{2+1}}{2+1} + 7\,\frac{x^{1+1}}{1+1} + 2x + c$$

$$= 2x^3 + \frac{7}{2}x^2 + 2x + c$$

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v. $\int \left(x^2 - \frac{1}{x^2}\right) dx$

Solution:

$$I = \int \left(x^2 - \frac{1}{x^2} \right) dx$$

$$= \int x^2 . dx - \int \frac{1}{x^2} . dx$$

$$= \cancel{\frac{x^{2+1}}{2+1}} - \left(\cancel{-2+1} + c \right)$$

$$= \frac{x^{2+1}}{2+1} - \left(\frac{x^{-2+1}}{-2+1} \right) + c$$

$$= \frac{x^3}{3} + \frac{1}{x} + c$$

$$= \frac{1}{3} x^3 + \frac{1}{x} + c$$

vi. $\int \left(\sqrt{x} - 1/\sqrt{x} \right) dx$

Solution:

$$I = \int x^{1/2} . dx - \int x^{-1/2} dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

$$= \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2x^{3/2}}{3} - 2x^{1/2} + c$$

vii. $\int \sqrt{x}\,(x^2-5)\,dx$

Solution:

$I = \int x^{\frac{5}{2}}.dx - 5\int x^{\frac{1}{2}}$

$= \dfrac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} - 5\,\dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= \dfrac{x^{7/2}}{7/2} - \dfrac{5x^{3/2}}{3/2} + c$

$= \dfrac{2}{7}x^{7/2} - \dfrac{10}{3}x^{3/2} + c$

viii. $\int (x-3)^2\,dx$

Solution:

$I = \int (x^2 - 6x + 9).dx$

$= \int x^2.dx - 6\int x.dx + 9\int dx$

$= \dfrac{x^{2+1}}{3} - \dfrac{6x^2}{2} + 9x + c$

$= \dfrac{1}{3}x^3 - 3x^2 + 9x + c$

ix. $\int \dfrac{3x^2 - 5x + 2}{x}$

Solution:

$I = \int \left(\dfrac{3x^2}{x} - \dfrac{5x}{x} + \dfrac{2}{x}\right) dx$

$= 3\int x.dx - 5\int dx + 2\int \dfrac{1}{x}\,dx$

$= \frac{3 \cdot x^2}{2} - 5x + 2\log x + c$

x. $\int \frac{ax^2 + bx + c}{x^2} dx$

Solution:

$I = \int \left(\frac{ax^2 + bx + c}{x^2} \right) dx$

$= a\int dx + b \int \frac{1}{x} dx + c \int x^{-2} dx$

$= ax + b \cdot \log x + \frac{cx^{-1}}{-1} + k$

$= ax + b\log x - \frac{c}{x} + k$

xi. $\int (x^2 + 3x + 5) x^{-\frac{1}{3}} dx$

Solution:

$I = \int \left(x^{2-\frac{1}{3}} + 3x^{1-\frac{1}{3}} + 5x^{-\frac{1}{3}} \right) dx$

$= \int \left(x^{\frac{5}{3}} + 3x^{\frac{2}{3}} + 5x^{-\frac{1}{3}} \right) dx$

$= \int x^{5/3} dx + 3 \int x^{\frac{2}{3}} \cdot dx + 5 \int x^{-1/3} \cdot dx$

$= \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + \frac{3x^{2/3+1}}{\frac{2}{3}+1} + \frac{5x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c$

$= \frac{3}{8} x^{8/3} + \frac{9}{5} x^{5/3} + \frac{15}{2} x^{2/3} + c$

3(i). $\int (3x+5)^4 dx$

Solution:

$I = \int (3x+5)^4 dx$

Let $y = 3x+5$

or, $\frac{dy}{dx} = 3$

$\therefore dx = \frac{dy}{3}$

Now, $I = \int y^4 \frac{dy}{3}$

$= \frac{1}{3} \int y^4 dy$

$= \frac{1}{3} \frac{y^{4+1}}{4+1} + c$

$= \frac{1}{3} \frac{y^5}{5} + c$

$= \frac{1}{3} \frac{(3x+5)^5}{5} + c$

ii. $\int (a-bx)^5 dx$

Solution:

Let $y = (a-bx)$

or, $\frac{dy}{dx} = a-bx$

$\therefore dx = \frac{dy}{(a-bx)}$

Now, $I = \int y^5 \cdot \frac{dy}{(a-bx)}$

ii. $\int (a-bx)^5 \, dx$

Solution:

formula:

$$(ax+b)^n = \frac{(ax+b)^{n+1}}{a(n+1)}$$

$$I = \int (a-bx)^5 \, dx$$

$$= \int (-bx+a)^5 \, dx$$

$$= \frac{(-bx+a)^{5+1}}{(5+1)(-b)} + c$$

$$= -\frac{(a \cancel{+} -bx)^6}{6b} + c$$

iii. $\int (c+dx)^{-3/2} \, dx$

Solution:

$$I = \int (c+dx)^{-3/2} \, dx$$

$$= \int (dx+c)^{-3/2} \, dx$$

$$= \frac{(dx+c)^{-3/2+1}}{\left(-\frac{3}{2}+1\right)d} + k$$

$$= \frac{-2(c+dx)^{-1/2}}{d} + k$$

iv. $\int \frac{dx}{\sqrt{2x+7}}$

Solution:

$$I = \int (2x+7)^{-1/2} \, dx$$

$$= \cancel{\int} \frac{(2x+7)^{-1/2+1}}{\left(\frac{-1}{2}+1\right)2} + c$$

$$= \sqrt{2x+7} + c$$

v. $$\int \left[x + \frac{1}{(x+3)^2} \right] dx$$

Solution:

$$I = \int x + (x+3)^{-2}\, dx$$

$$= \int x \cdot dx + \int (x+3)^{-2}\, dx$$

$$= \frac{x^2}{2} + \frac{(x+3)^{-2+1}}{-2+1} + c$$

$$= \frac{x^2}{2} - \frac{1}{(x+3)} + c$$

vi. $$\int \left[4 + \frac{1}{(5x+1)^2} \right. dx$$

Solution:

$$I = \int 4 + (5x+1)^{-2}\, dx$$

$$= 4 \int dx + \int (5x+1)^{-2}\, dx$$

$$= 4x + \frac{(5x+1)^{-2+1}}{(-2+1)\,5} + c$$

$$= 4x - \frac{1}{5(5x+1)} + c$$

vii. $\int \frac{x+3}{x-3}\,dx$

Solution:

$$I = \int \frac{(x-3)+6}{x-3}\,dx$$

$$= \int \frac{(x-3)}{(x-3)} + \frac{6}{(x-3)}\,dx$$

$$= 1\int dx + 6\int \frac{1}{(x-3)}\,dx$$

$$= x + 6\log(x-3) + c$$

viii. $\int \frac{3x-1}{x-2}\,dx$

Solution:

$$I = \int \frac{3x-6+5}{x-2}\,dx$$

$$= \int \frac{3(x-2)}{(x-2)} + \frac{5}{(x-2)}\,dx$$

$$= 3\int dx + 5\int \frac{1}{(x-2)}\,dx$$

$$= 3x + 5\log(x-2) + c$$

ix. $\int \frac{x^2+3x+3}{x+1}\,dx$

Solution:

$$I = \int \frac{x(x+1)+2(x+1)+1}{(x+1)}\,dx$$

$$= \int \frac{(x+1)(x+2)+1}{(x+1)}\,dx$$

$$= \int \left\{(x+2) + \frac{1}{(x+1)}\right\} dx$$

$$= \int x\cdot dx + 2\int dx + \int \frac{1}{(x+1)}\,dx$$

$$= \frac{x^2}{2} + 2x + \text{Log}(x+1) + c$$

x. $\int \frac{x^2+5}{x+2}\,dx$

Solution:

$$I = \int \frac{x^2-4+9}{x+2}\,dx$$

$$= \int \frac{(x+2)(x-2)+9}{x+2}\,dx$$

$$= \int (x-2) + \int \frac{9}{x+2}\,dx$$

$$= \frac{x^2}{2} - 2x + 9\,\text{Log}(x+2) + c$$

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4 i. $\int x\sqrt{x+1}\,dx$

Solution:

$$I = \int (x+1-1)(x+1)^{1/2}\,dx$$

$$= \int \{(x+1)-1\}\{x+1\}^{1/2}\,dx$$

$$= \int \{(x+1)^{3/2} - (x+1)^{1/2}\}\,dx$$

$$= \int (x+1)^{3/2}\,dx - \int (x+1)^{1/2}\,dx$$

$$= \frac{(x+1)^{5/2}}{\frac{5}{2}} - \frac{(x+1)^{3/2}}{\frac{3}{2}} + c$$

$$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$$

ii. $\int x\sqrt{ax+b}\,dx$

Solution:

$$I = \int \frac{ax}{a}\sqrt{ax+b}\,dx$$

$$= \frac{1}{a}\int (ax+b)-b\,(ax+b)^{1/2}\,dx$$

$$= \frac{1}{a}\int (ax+b)^{3/2}\,dx - \frac{b}{a}\int (ax+b)^{1/2}\,dx$$

$$= \frac{1}{a}\frac{(ax+b)^{5/2}}{\frac{5}{2}a} - \frac{b}{a}\frac{(ax+b)^{3/2}}{\frac{3}{2}a} + c$$

$$= \frac{1}{5a^2}(ax+b)^{5/2} - \frac{2b}{3a^2}(ax+b)^{3/2} + c$$

iii. $\int 2x\sqrt{2x+3}\,dx$

Solution:

$$I = \int \{(2x+3)-3\}(2x+3)^{1/2}\,dx$$

$$= \int \{(2x+3)^{3/2} - 3(2x+3)^{1/2}\}\,dx$$

$$= \frac{(2x+3)^{5/2}}{\frac{5}{2}\times 2} - \frac{3(2x+3)^{1/2+1}}{\frac{3}{2}\times 2} + C$$

$$= \frac{1}{5}(2x+3)^{5/2} - (2x+3)^{3/2} + C$$

iv. $\int 5x\sqrt{5x+2}\,dx$

Solution:

$$I = \int \{(5x+2)-2\}(5x+2)^{1/2}\,dx$$

$$= \int (5x+2)^{3/2}\,dx - 2\int (5x+2)^{1/2}\,dx$$

$$= \frac{(5x+2)^{5/2}}{\frac{5}{2}\times 5} - \frac{2(5x+2)^{3/2}}{\frac{3}{2}\times 5} + C$$

$$= \frac{2}{25}(5x+2)^{5/2} - \frac{4(5x+2)^{3/2}}{15} + C$$

v. $\int (x+2)\sqrt{3x+2}\,dx$

Solution:

$$I = \int \left\{\frac{(3x+2)+4}{3}\right\}(3x+2)^{1/2}\,dx$$

$$= \frac{1}{3}\int \{(3x+2)^{3/2} + 4(3x+2)^{1/2}\}\,dx$$

$$= \frac{1}{3}\frac{(3x+2)^{5/2}}{\frac{5}{2}\times 3} + 4\frac{(3x+2)^{3/2}}{\frac{3}{2}\times 3} + c$$

$$= \frac{2}{45}(3x+2)^{5/2} + 8\frac{(3x+2)^{3/2}}{27} + c$$

$$= \frac{2}{9}\left[\frac{1}{5}(3x+2)^{5/2} + \frac{4}{3}(3x+2)^{3/2}\right] + c$$

vi. $\int (2x+3)\sqrt{3x+1}\,dx$

Solution:

$$I = \int \frac{3(2x+3)}{3}(3x+1)^{1/2}\,dx$$

$$= \frac{1}{3}\int (6x+9)(3x+1)^{1/2}\,dx$$

$$= \frac{1}{3}\int \{(6x+2)+7\}.(3x+1)^{1/2}\,dx$$

$$= \frac{1}{3}\int \{2(3x+1)+7\}(3x+1)^{1/2}\,dx$$

$$= \frac{1}{3}\times 2\int \{(3x+1)^{3/2} + 7(3x+1)^{1/2}\}\,dx$$

$$= \frac{2}{3}\frac{(3x+1)^{5/2}}{\frac{5}{2}\times 3} + 7\frac{(3x+1)^{3/2}}{\frac{3}{2}\times 3} + c$$

$$= \frac{4}{45}(3x+1)^{5/2} + \frac{14}{27}(3x+1)^{3/2} + c$$

$$= \frac{2}{9}\left[\frac{2}{5}(3x+1)^{5/2} + \frac{7}{3}(3x+1)^{3/2}\right] + c$$

vii. $\int (5x+3)\sqrt{4x+1}\,dx$

Solution:

$$I = \int \frac{5x\cdot 4 + 12}{4} \sqrt{4x+1}\,dx$$

$$= \frac{1}{4}\int (20x+5+7)(4x+1)^{1/2}\,dx$$

$$= \frac{1}{4}\int \{5(4x+1)+7\}\frac{(4x+1)^{1/2}}{4}\,dx$$

$$= \frac{5}{4}\int \left\{(4x+1)^{3/2} + \frac{7(4x+1)^{1/2}}{4}\right\}dx$$

$$= \frac{5}{4}\frac{(4x+1)^{5/2}}{\frac{5}{2}\times 4} + \frac{7}{4}\frac{(4x+1)^{3/2}}{\frac{3}{2}\times 4} + C$$

$$= \frac{10}{80}(4x+1)^{5/2} + \frac{14}{48}(4x+1)^{3/2} + C$$

$$= \frac{1}{8}(4x+1)^{5/2} + \frac{7}{24}(4x+1)^{3/2} + C$$

$$= \frac{1}{8}\left[(4x+1)^{5/2} + \frac{7}{3}(4x+1)^{3/2}\right] + C$$

viii. $\int \frac{x+2}{\sqrt{x+1}}\,dx$

Solution:

$$I = \int [\{x+1+1\}(x+1)^{-1/2}]\,dx$$

$$= \int [\{(x+1)+1\}(x+1)^{-1/2}]\,dx$$

$$= \int [(x+1)^{1/2} + (x+1)^{-1/2}]\,dx$$

$$= \frac{2(x+1)^{3/2}}{3} + \frac{2(x+1)^{1/2}}{1} + C$$

ix. $\displaystyle\int \frac{3x+4}{\sqrt{x+1}}\,dx$

Solution:

$$I = \int (3x+3+1)(x+1)^{-1/2}\,dx$$

$$= \int \{3(x+1)+1\}(x+1)^{-1/2}\,dx$$

$$= 3\int \left[(x+1)^{1/2} + (x+1)^{-1/2}\right] dx$$

$$= 3\int (x+1)^{1/2}dx + 3\int (x+1)^{-1/2}\,dx$$

$$= \frac{3\times 2(x+1)^{3/2}}{3} + \frac{\cancel{3\times}2(x+1)^{1/2}}{1} + c$$

$$= 2(x+1)^{3/2} + 2(x+1)^{1/2} + c$$

x. $\displaystyle\int \frac{(2x+1)}{\sqrt{3x+2}}\,dx$

Solution:

$$I = \frac{1}{3}\int \{3(2x+1)\}(3x+2)^{-1/2}\,dx$$

$$= \frac{1}{3}\int (6x+3)(3x+2)^{-1/2}\,dx$$

$$= \frac{1}{3}\int (6x+4-1)(3x+2)^{-1/2}\,dx$$

$$= \frac{1}{3}\int \left[\{2(3x+2)-1\}(3x+2)^{-1/2}\,dx\right.$$

$$= \frac{2}{3}\int \left[(3x+2)^{1/2} - (3x+2)^{-1/2}\right] dx$$

$$= \frac{2}{3}\ \frac{(3x+2)^{3/2}}{3/2\times 3} - \frac{2(3x+2)^{1/2}}{3\times 3} + c$$

$$= \frac{4}{9\times 3}(3x+2)^{3/2} - \frac{2}{3\times 3}(3x+2)^{1/2} + c$$

$$= \frac{2}{9}\left[\frac{2}{3}(3x+2)^{3/2} - (3x+2)^{1/2}\right] + c$$

5.i. $\int \frac{1}{\sqrt{x+1}-\sqrt{x}}\,dx$

Solution:

$$I = \int \frac{1}{\sqrt{x+1}-\sqrt{x}} \times \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\,dx$$

$$= \int \frac{(\sqrt{x+1}+\sqrt{x})}{x+1-x}\,dx$$

$$= \int (x+1)^{1/2} + \int x^{1/2}\,dx$$

$$= \frac{2(x+1)^{3/2}}{3} + \frac{2}{3}x^{3/2} + c$$

ii. $\int \frac{dx}{\sqrt{x+a}-\sqrt{x-a}}$

solution:

$$I = \int \frac{\sqrt{x+a}+\sqrt{x-a}}{x+a-x+a}\,dx$$

$$= \frac{1}{2a}\left[\int (x+a)^{1/2}\,dx + \int (x-a)^{1/2}\,dx\right]$$

$$= \frac{1}{2a}\times\frac{2(x+a)^{3/2}}{3} + \frac{2(x-a)^{3/2}}{3\times 2a} + c$$

$$= \frac{1}{3a}\left[(x+a)^{3/2} + (x-a)^{3/2}\right] + c$$

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iii. $\displaystyle\int \frac{1}{\sqrt{2x+1}-\sqrt{2x-3}}\,dx$

Solution:

$$I = \int \frac{\sqrt{2x+1}+\sqrt{2x-3}}{2x+1-2x+3}\,dx$$

$$= \frac{1}{4}\int \left[(2x+1)^{1/2}+(2x-3)^{1/2}\right]dx$$

$$= \frac{1}{4}\left[\frac{2(2x+1)^{3/2}}{3\times 2}+\frac{2(2x-3)^{3/2}}{3\times 2}\right]+c$$

$$= \frac{1}{12}\left[(2x+1)^{3/2}+(2x-3)^{3/2}\right]+c$$

iv. $\displaystyle\int \frac{dx}{\sqrt{x+a}-\sqrt{x-b}}$

Solution:

$$I = \int \frac{\sqrt{x+a}+\sqrt{x-b}}{x+a-x+b}\,dx$$

$$= \frac{1}{(a+b)}\int \left[(x+a)^{1/2}+(x-b)^{1/2}\right]dx$$

$$= \frac{1}{(a+b)}\left[\frac{2(x+a)^{3/2}}{3}+\frac{2(x-b)^{3/2}}{3}\right]+c$$

$$= \frac{2}{3(a+b)}\left[(x+a)^{3/2}+(x-b)^{3/2}\right]+c$$

6\. i. $\int \sin 5x \cdot dx$

Solution:

$$I = \int \sin 5x \, dx$$

$$= -\frac{1}{5} \cos 5x + c$$

ii. $\int \sin(ax+b)\, dx$

solution:

$$I = \int \sin(ax+b)\, dx$$

$$= \frac{-\cos(ax+b)}{a} + c$$

iii. $\int \cos(a^2x+b)\, dx$

Solution:

$$I = \int \cos(a^2x+b)\, dx$$

$$= \frac{\sin(a^2x+b)}{a^2} + c$$

iv. $\int \sec^2(2x+3)\, dx$

solution:

$$I = \int \sec^2(2x+3)\, dx$$

$$= \frac{\tan(2x+3)}{2} + c$$

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v. $\int \sin^2 ax \, dx$

Solution:

$$I = \int \sin^2 ax \, dx$$

$$= \int \left(\frac{1 - \cos 2ax}{2}\right) dx$$

$$= \frac{1}{2} \int (1 - \cos 2ax) \, dx$$

$$= \frac{1}{2} \left[x - \frac{\sin 2ax}{2a} \right] + c$$

vi. $\int \cos^2 bx \, dx$

Solution:

$$I = \int \left(\frac{1 + \cos 2bx}{2}\right) dx$$

$$= \frac{1}{2} \int (1 + \cos 2bx) \, dx$$

$$= \frac{1}{2} \left[x + \frac{\sin 2bx}{2b} \right] + c$$

vii. $\int \tan^2 ax \, dx$

Solution:

$$I = \int (\sec^2 ax - 1) \, dx$$

$$= \frac{\tan ax}{a} - x + c$$

viii. $\int \sin^4 x \, dx$

Solution:

$$I = \int (\sin^2 x)^2 \, dx$$

$$= \int \left(\frac{1 - \cos 2x}{2} \right)^2 dx$$

$$= \frac{1}{4} \int \left[1 - 2\cos 2x + \cos^2 2x \right] \cancel{dx + c}$$

$$= \frac{1}{4} \int \left[1 - 2\cos 2x + \left(\frac{1 + \cos 4x}{2} \right) \right] dx$$

$$= \frac{1}{4} \left[x - \frac{2\sin 2x}{2} + \frac{1}{2}x + \frac{1}{2} \frac{\sin 4x}{4} \right] + c$$

$$= \frac{1}{4} \left[\frac{8x - 8\sin 2x + 4x + \sin 4x}{8} \right] + c$$

$$= \frac{1}{32} \left[12x - 8\sin 2x + \sin 4x \right] + c$$

ix. $\int \cos^4 nx \, dx$

Solution:

$$I = \int \left(\frac{1 + \cos 2nx}{2} \right)^2 dx$$

$$= \frac{1}{4} \int \left(1 + 2\cos 2nx + \cos^2 2nx \right) dx$$

$$= \frac{1}{4}\int\left[1 + 2\cos 2nx + \left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$= \frac{1}{4}\left[x + \frac{2\sin 2nx}{2n} + \frac{1}{2}x + \frac{1}{2}\frac{\sin 4x}{4x}\right] + c$$

$$= \frac{1}{4}\left[\frac{8nx + 8\sin 2nx + 4nx + \sin 4x}{8n}\right] + c$$

$$= \frac{1}{32n}\left[12nx + 8\sin 2nx + \sin 4x\right] + c$$

x. $\int \frac{1}{\cos^2 x \cdot \sin^2 x}\, dx$

Solution:

$$I = \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \cdot \sin^2 x}\, dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx$$

$$= \int (\sec^2 x + \operatorname{cosec}^2 x)\, dx$$

$$= \cancel{-\cot x}\ \tan x - \cot x + c$$

xi. $\int \frac{1}{\sec^2 x \cdot \tan^2 x}\, dx$

Solution:

$$I = \int \frac{\sec^2 x - \tan^2 x}{\sec^2 x \cdot \tan^2 x}\, dx$$

$$= \int \left(\frac{1}{\tan^2 x} - \frac{1}{\sec^2 x} \right) dx$$

$$= \int \left(\cot^2 x - \cos^2 x \right) dx$$

$$= \int \left(\csc^2 x - 1 - \cos^2 x \right) dx$$

$$= \int \left[\csc^2 x - 1 - \left(\frac{1 + \cos 2x}{2} \right) \right] dx$$

$$= \int \left[\csc^2 x - 1 - \frac{1}{2} - \frac{1}{2} \cos 2x \right] dx$$

$$= \int \left[\csc^2 x - \frac{3}{2} - \frac{1}{2} \cos 2x \right] dx$$

$$= -\cot x - \frac{3}{2} x - \frac{1}{2} \sin 2x + c$$

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7. $\int \sqrt{1 + \cos nx} \, dx$

Solution:

$$I = \int \sqrt{1 + 2\cos^2 nx/2 - 1} \, dx$$

$$= \sqrt{2} \int \cos \frac{nx}{2} \, dx$$

$$= \sqrt{2} \, \frac{\sin nx/2}{n/2} + c$$

$$= 2\sqrt{2} \, \frac{\sin nx/2}{n}$$

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ii. $\int \sqrt{1 - \cos px}\ dx$

Solution:

$$I = \int \left(\sqrt{2 \sin^2 px/2}\right) dx$$

$$= \sqrt{2} \int \sin \frac{px}{2}\ dx$$

$$= \sqrt{2}\left(-\frac{\cos px/2}{p/2}\right) + c$$

$$= -\frac{2\sqrt{2}}{p} \cos \frac{px}{2} + c$$

iii. $\int \sqrt{1 + \sin 2ax}\ dx$

Solution:

$$I = \int \sqrt{(\sin^2 ax + \cos^2 ax) + 2 \sin ax \cdot \cos ax}\ dx$$

$$= \int \sqrt{(\sin ax + \cos^2 ax)^2}\ dx$$

$$= \int (\sin ax + \cos ax)\, dx$$

$$= -\frac{\cos ax}{a} + \frac{\sin ax}{a} + c$$

$$= \frac{1}{a}(\sin ax - \cos ax) + c$$

iv. $\int \frac{dx}{1 + \cos mx}$

Solution:

$$I = \int \frac{1 - \cos mx}{(1 + \cos mx)(1 - \cos mx)}\ dx$$

$$= \int \frac{1 - \cos mx}{\sin^2 mx} dx$$

$$= \int \operatorname{cosec}^2 mx \, . dx - \int \cot mx \, . \cancel{dx} \operatorname{cosec} mx . dx$$

$$= -\frac{\cot mx}{m} - \left(-\frac{\operatorname{cosec} mx}{m} \right) + c$$

$$= \frac{\operatorname{cosec} mx}{m} - \frac{\cot mx}{m} + c$$

v. $\int \frac{dx}{1 - \cos nx}$

Solution:

$$I = \int \frac{dx}{2 \sin^2 nx/2}$$

$$= \frac{1}{2} \int \operatorname{cosec}^2 nx/2 \, . dx$$

$$= \frac{1}{2} \left(\frac{-\cot nx/2}{n/2} \right) + c$$

$$= -\frac{1}{n} \cot \frac{nx}{2} + c$$

vi) $\int \frac{dx}{1 - \sin ax}$

Solution:

$$I = \int \frac{1 + \sin ax}{1 - \sin^2 ax} dx$$

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$$= \int \frac{1 + \sin ax}{\cos^2 ax} dx$$

$$= \int \sec^2 ax \cdot dx + \int \tan ax \cdot \cancel{\cos} \cdot \sec ax \cdot dx$$

$$= \frac{\tan ax}{a} + \frac{\sec ax}{a} + c$$

$$= \frac{1}{a} (\tan ax + \sec ax) + c$$

8.i. $\int \sin 6x \cdot \cos 2x \, dx$

Solution:

$$I = \frac{1}{2} \int 2 \sin 6x \cdot \cos 2x \, dx$$

$$= \frac{1}{2} \int (\sin 8x + \sin 4x) \, dx$$

$$= \frac{1}{2} \left[-\frac{\cos 8x}{8} - \frac{\cos 4x}{4} \right] + C$$

$$= \frac{-\cos 8x}{16} - \frac{\cos 4x}{8} + C$$

ii. $\int \sin 7x \cdot \sin 5x \, dx$

Solution:

$$I = \frac{1}{2} \int 2 \sin 7x \cdot \sin 5x \, dx$$

$$= \frac{1}{2} \int \left[\sin (7x - 5x) - \cos (7x + 5x) \right] dx$$

$$= \frac{1}{2}\int (\sin 2x - \cos 12x)\, dx$$

$$= \frac{1}{2}\left[\frac{-\cos 2x}{2} - \frac{\sin 12x}{12}\right] + c$$

$$= \frac{-\cos 2x}{4} - \frac{\sin 12x}{24} + c$$

iii. $\int \sin 6x \cos 8x \, dx$

Solution:

$$I = \frac{1}{2}\int 2 \sin 6x \cdot \cos 8x \, dx$$

$$= \frac{1}{2}\int \left[\sin(6x+8x) + \sin(6x-8x)\right] dx$$

$$= \frac{1}{2}\int (\sin 14x - \sin 2x)\, dx$$

$$= \frac{1}{2}\left[\frac{-\cos 14x}{14} + \frac{\cos 2x}{2}\right] + c$$

$$= \frac{\cos 2x}{4} - \frac{\cos 14x}{28} + c$$

iv. $\int \cos px \cdot \cos qx \, dx$

Solution:

$$I = \frac{1}{2}\int 2 \cos px \cdot \cos qx \cdot dx$$

$$= \frac{1}{2}\int \left[\cos(px+qx) + \cos(px-qx)\right] dx$$

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$= \frac{1}{2} \int \cos(p+q)x \cdot dx + \int \cos(p-q)x \cdot dx$

$= \frac{1}{2} \left[\frac{\sin(p+q)}{(p+q)} + \frac{\sin(p-q)}{(p-q)} \right] + c$

9.i. $\int (e^{px} + e^{-qx})\, dx$

Solution:

$I = \int e^{px} \cdot dx + \int e^{-qx} \cdot dx$

$= \frac{e^{px}}{p} + \frac{e^{-qx}}{-q} + c$

$= \frac{e^{px}}{p} - \frac{e^{-qx}}{q} + c$

ii. $\int (e^{px} + e^{-px})^2\, dx$

Solution:

$I = \int \{ (e^{px})^2 + 2 + (e^{-px})^2 \}\, dx$

$= \int (e^{2px} + 2 + e^{-2px})\, dx$

$= \frac{e^{2px}}{2p} + 2x + \frac{e^{-2px}}{-2p} + c$

$= \frac{e^{2px}}{2p} + 2x - \frac{e^{-2px}}{2p} + c$

HW

iii. $\int \frac{e^{2x} + e^{x} + 1}{e^{x}} dx$

Solution:

$I = \int (e^{x} + 1 + e^{-x}) dx$

$= e^{x} + x - e^{-x} + c$

iv. $\int e^{x} (e^{2x} + 1) dx$

Solution:

$I = \int (e^{3x} + e^{x}) dx$

$= \frac{e^{3x}}{3} + e^{x} + c$