or $\cot(A+B) = \cot(\pi - C)$

or $\dfrac{\cot A \cdot \cot B - 1}{\cot A + \cot B} = -\cot C$

or $\cot A \cdot \cot B - 1 = -\cot A \cdot \cot C - \cot B \cdot \cot C$

or $\cot A \cdot \cot B + \cot A \cdot \cot C + \cot B \cdot \cot C = -1$

or $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A = 1$

Proved.

---

c. $\cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2} = \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}$

Solution:

We have,

$A + B + C = \pi$

or $A + B = \pi - C$

or $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

or $\cot\left(\frac{A}{2} + \frac{B}{2}\right) = \cot\left(\frac{\pi}{2} - \frac{C}{2}\right)$

or, $\dfrac{\cot A/2 \cdot \cot B/2 - 1}{\cot A/2 + \cot B/2} = \tan \frac{C}{2}$

or, $\dfrac{\cot \frac{A}{2} \cdot \cot \frac{B}{2} - 1}{\cot \frac{A}{2} + \cot \frac{B}{2}} = \dfrac{1}{\tan \frac{C}{2}}$

or $\cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2} - \cot \frac{C}{2} = \cot \frac{A}{2} + \cot \frac{B}{2}$

or $\cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2} = \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}$

Proved.

d. $\tan\frac{A}{2}\cdot\tan\frac{B}{2} + \tan\frac{B}{2}\cdot\tan\frac{C}{2} + \tan\frac{C}{2}\cdot\tan\frac{A}{2} = 1$

Solution:

we have,

$A + B + C = \pi$

or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

or, $\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$

or, $\frac{\tan A/2 + \tan B/2}{1 - \tan A/2 \cdot \tan B} = \cot C/2$

or, $\frac{\tan A/2 + \tan B/2}{1 - \tan A/2 \cdot \tan B/2} = \frac{1}{\tan C/2}$

or, $\tan A/2 \cdot \tan C/2 + \tan B/2 \cdot \tan C/2 = 1 - \tan A/2 \cdot \tan B/2$

or, $\tan A/2 \cdot \tan C/2 + \tan B/2 \cdot \tan C/2 + \tan A/2 \cdot \tan B/2 = 1$

or, $\tan A/2 \cdot \tan B/2 + \tan B/2 \cdot \tan C/2 + \tan C/2 \cdot \tan A/2 = 1$

Proved,,

---

e. $\tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C$

Solution:

We have,

$A + B + C = \pi$

or, $2A + 2B = 2\pi - 2C$

or, $\tan(2A + 2B) = \tan(2\pi - 2C)$

or, $\frac{\tan 2A + \tan 2B}{1 - \tan 2A \cdot \tan 2B} = -\tan 2C$

or, $\tan 2A + \tan 2B = -\tan 2C + \tan 2A \cdot \tan 2B \cdot \tan 2C$

or, $\tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C$

2. If $A+B+C = \pi^c$, prove that

a. $\sin 2A + \sin 2B + \sin 2C = 4\sin A . \sin B . \sin C$

Solution :

Here,

$A+B+C = \pi$

or $A+B = \pi - C$

or $\sin(A+B) = \sin(\pi - C) = \sin C$

or $\cos(A+B) = \cos(\pi - C) = -\cos C$

Now,

LHS

$\sin 2A + \sin 2B + \sin 2C$

$= 2\sin\frac{2A+2B}{2} . \cos\frac{2A-2B}{2} + \sin 2C$

$= 2\sin(A+B) . \cos(A-B) + 2\sin C . \cos C$

$= 2\sin C . \cos(A-B) + 2\sin C . \{-\cos(A+B)\}$

$= 2\sin C \{\cos(A-B) - \cos(A+B)\}$

$= 2\sin C . 2 \overset{\sin}{\cancel{\cos}} \frac{A-B+A+B}{2} . \sin\frac{A+B-(A-B)}{2}$

$= 4\sin C . 2\sin\frac{2A}{2} . \sin\frac{2B}{2}$

$= 4\sin A . \sin B . \sin C$

RHS proved.

---

b. $\sin 2A + \sin 2B - \sin 2C = 4\cos A . \cos B . \sin C$

Solution :

we have,

$A+B+C = \pi$

or, $A+B = \pi - C$

or, $\sin(A+B) = \sin(\pi - C) = \sin C$

or, $\cos(A+B) = \cos(\pi - C) = -\cos C$

LHS

$\sin 2A + \sin 2B - \sin 2C$

$2\sin\left(\frac{2A+2B}{2}\right)\cdot\cos\left(\frac{2A-2B}{2}\right) - \sin 2C$

$= 2\sin(A+B)\cdot\cos(A-B) - \sin 2C$

$= 2\sin C\cdot\cos(A-B) - 2\sin C\cdot\cos C$

$= 2\sin C\,[\cos(A-B) - \cos C]$

$= 2\sin C\,[\cos(A-B) + \cos(A+B)]$

$= 2\sin C\cdot 2\cos A\cdot\cos B$

$= 4\cos A\cdot\cos B\cdot\cancel{\cos C}\,\sin C$

RHS proved

---

c. $\cancel{2}\sin\cancel{B}2A - \cancel{2}\sin 2B + \cancel{2}\sin 2C = 4\cos A\cdot\sin B\cdot\cos C$

Solution:

we have,

$A+B+C = \pi$

or, $A+B = \pi - C$

or, $\sin(A+B) = \sin(\pi-C) = \sin C$

or, $\cos(A+B) = \cos(\pi-C) = -\cos C$

Now,

LHS

~~2 sin~~ $\sin 2A - \sin 2B + \sin 2C$

$= 2\cos\left(\frac{2A+2B}{2}\right)\cdot\sin\left(\frac{2A-2B}{2}\right) + \sin 2C$

$= 2\cos(A+B)\cdot\sin(A-B) + 2\sin C\cdot\cos C$

$= -2\cos C\cdot\sin(A-B) + 2\sin C\cdot\cos C$

$= 2\cos C\,[-\sin(A-B) + \sin C]$

$= 2\cos C\,[-\sin(A-B) + \sin(A+B)]$

$= 2\cos C\,[\sin(A+B) - \sin(A-B)]$

$= 2\cos C\cdot 2\cos A\cdot\sin B$

$= 4\cos A\cdot\sin B\cdot\cos C$ proved

d. $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cdot \cos B \cdot \cos C$

Solution:

We have,

$A + B + C = \pi$

or, $\sin(A+B) = \sin(\pi - C) = \sin C$

or, $\cos(A+B) = \cos(\pi - C) = -\cos C$

Now,

LHS

$\cos 2A + \cos 2B + \cos 2C$

$= 2\cos\left(\frac{2A+2B}{2}\right) \cdot \cos\left(\frac{2A-2B}{2}\right) + \cos 2C$

$= 2\cos(A+B) \cdot \cos(A-B) + 2\cos^2 C - 1$

$= -1 + 2\{-\cos C\} \cdot \cos(A-B) + 2\cos C \cdot \cos C$

$= -1 - 2\cos C \cdot \cos(A-B) + 2\{-\cos(A+B)\} \cdot \cos C$

$= -1 - 2\cos C \cdot \cos(A-B) - 2\cos(A+B) \cdot \cos C$

$= -1 - 2\cos C [\cos(A-B) + \cancel{\cos C\}}\ \cos(A+B)]$

$= -1 - 2\cos C [\cos(A-B) + \cos(A+B)]$

$= -1 - 2\cos C \cdot 2\cos A \cdot \cos B$

$= -1 - 4\cos A \cdot \cos B \cdot \cos C$

RHS proved.

---

e. $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A \cdot \sin B \cdot \cos C$

Solution:

We have.

$A + B + C = \pi$

or, $\sin(A+B) = \sin(\pi - C) = \sin C$

or, $\cos(A+B) = \cos(\pi - C) = -\cos C$

Now,

LHS

$\cos 2A + \cos 2B - \cos 2C$

$2\cos\left(\frac{2A+2B}{2}\right) \cdot \cos\left(\frac{2B-2B}{2}\right) - \cos 2C$

$= 2\cos(A+B)\cdot\cos(A-B) - 2\cos^2C + 1$

$= 1 + \{-\cos C\}\cdot\cos(A-B) - 2\cos C\cdot\cos C$

$= 1 - 2\cos C\cdot\cos(A-B) + 2\cos(A+B)\cdot\cos C$

$= 1 - 2\cos C\,[\cos(A-B) - \cos(A+B)]$

$= 1 - 2\cos C\cdot 2\sin A\cdot\sin B$

$= 1 - 4\sin A\cdot\sin B\cdot\sin C$

RHS proved//

---

3. If A, B and C are the angles of a triangle, prove that:

a. $\sin A - \sin B + \sin C = 4\sin\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\sin\frac{C}{2}$

Solution.

We have, $A+B+C = 180°$

or, $\sin\left(\frac{A}{2}+\frac{B}{2}\right) = \sin\left(\frac{180}{2}-\frac{C}{2}\right) = \cos\frac{C}{2}$

And, $\cos\left(\frac{A}{2}+\frac{B}{2}\right) = \cos\left(\frac{180}{2}-\frac{C}{2}\right) = \sin\frac{C}{2}$

LHS

$\sin A - \sin B + \sin C$

or, $2\cos\frac{A+B}{2}\cdot\sin\frac{A-B}{2} + \sin 2\cdot\frac{C}{2}$

or, $2\sin\frac{C}{2}\;\sin\frac{A-B}{2} + 2\sin\frac{C}{2}\cdot\cos\frac{C}{2}$

or, $2\sin\frac{C}{2}\left[\sin\left(\frac{A-B}{2}\right) + \sin\left(\frac{A+B}{2}\right)\right]$

or, $2\sin\frac{C}{2}\left[\sin\left(\frac{A}{2}-\frac{B}{2}\right) + \sin\left(\frac{A}{2}+\frac{B}{2}\right)\right]$

or, $2\sin\frac{C}{2}\cdot 2\sin\frac{A}{2}\cdot\cos\frac{B}{2}$

or, $4\sin A/2\cdot\cos B/2\cdot\sin C/2$ Proved//

b. $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2} \cdot \cos\frac{B}{2} \cdot \sin\frac{C}{2}$

Soln,,

$A + B + C = 180$

or, $\sin\left(\frac{A+B}{2}\right) = \sin\left(90 - \frac{C}{2}\right) = \cancel{\sin\frac{C}{2}}\ \cos\frac{C}{2}$

or, $\cos\left(\frac{A+B}{2}\right) = \cos\left(90 - \frac{C}{2}\right) = \sin\frac{C}{2}$

Now,

LHS

$\cos A + \cos B - \cos C$

$= 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) - \cos 2 \cdot \frac{C}{2}$

$= 2\sin\frac{C}{2} \cdot \cos\left(\frac{A-B}{2}\right) - \left(1 - 2\sin^2\frac{C}{2}\right)$

$= 2\sin\frac{C}{2} \cdot \cos\left(\frac{A-B}{2}\right) - 1 + 2\sin\frac{C}{2} \cdot \sin\frac{C}{2}$

$= 2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right] - 1$

$= -1 + 2\sin\frac{C}{2} \cdot 2\cos\frac{A}{2} \cdot \cos\frac{B}{2}$

$= -1 + 4\cos\frac{A}{2} \cdot \cos\frac{B}{2} \cdot \sin\frac{C}{2}$

Proved,,

---

c. $\cos B + \cos C - \cos A = 4\sin\frac{A}{2} \cdot \cos\frac{B}{2} \cdot \cos\frac{C}{2} - 1$

Soln,,

We have,

$A + B + C = 180°$

or, $\sin\left(\frac{\cancel{A+B}\ B+C}{2}\right) = \sin\left(90° - A/2\right) = \cancel{\cos C/2}\ \cos A/2$

And, $\cos\left(\frac{B+C}{2}\right) = \cos\left(90 - \frac{A}{2}\right) = \sin\frac{A}{2}$

Now, LHS

$= \cos B + \cos C - \cos A$

$= 2\cos\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right) - \cos 2\cdot\frac{A}{2}$

$= 2\sin\frac{A}{2}\cdot\cos\left(\frac{B-C}{2}\right) - 1 + 2\sin^2\frac{A}{2}$

$= 2\sin\frac{A}{2}\left[\cos\left(\frac{B-C}{2}\right) + \cos\left(\frac{B+C}{2}\right)\right] - 1$

$= 2\sin\frac{A}{2}\cdot 2\cos\frac{B}{2}\cdot\cos\frac{C}{2} - 1$

$= 4\sin\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2} - 1$

RHS proved.

---

d. $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

Solution:

We have,

$A + B + C = 180$

or, $\sin\left(\frac{A+B}{2}\right) = \sin\left(90 - \frac{C}{2}\right) = \cos C/2$

And, $\cos\left(\frac{A+B}{2}\right) = \cos\left(90 - C/2\right) = \sin C/2$

Now

LHS

$\cos A + \cos B + \cos C$

$= 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) + \cos 2\frac{C}{2}$

$= 2\sin\frac{C}{2}\cdot\cos\left(\frac{A-B}{2}\right) + 1 - 2\sin\frac{C}{2}\cdot\sin\frac{C}{2}$

$= 2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right] + 1$

$= 2\sin\frac{C}{2}\cdot 2\sin\frac{A}{2}\cdot\sin\frac{B}{2} + 1$

$= 4\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

RHS proved.

e. $\dfrac{\sin A+\sin B-\sin C}{\sin A\cdot\sin B} = \sec\frac{A}{2}\cdot\sec\frac{B}{2}\cdot\cancel{\sec\frac{C}{2}}\ \cos\frac{C}{2}$

Solution:

We have,

$A+B+C = 180°$

or, $\sin\left(\frac{A+B}{2}\right) = \sin\left(90-\frac{C}{2}\right) = \cos\frac{C}{2}$

And, $\cos\left(\frac{A+B}{2}\right) = \cos\left(90-\frac{C}{2}\right) = \sin\frac{C}{2}$

LHS

$\dfrac{\sin A+\sin B-\sin C}{\sin A\cdot\sin B}$

$= \dfrac{2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) - \cancel{\sin C}\ \sin 2\frac{C}{2}}{2\sin A/2\cdot\cos A/2\cdot 2\sin B/2\cdot\cos B/2}$

$= \dfrac{2\cos\frac{C}{2}\cdot\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cdot\cos\frac{C}{2}}{\cancel{2\cos C}\ \ 2\sin\frac{A}{2}\cdot\cos\frac{A}{2}\cdot\sin B/2\cdot\cos B/2}$

$= \dfrac{2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]}{2\sin\frac{A}{2}\cdot\cos\frac{A}{2}\cdot 2\cos B/2\cdot\sin B/2}$

$$= \frac{\cancel{2}\cos\frac{C}{2}\cdot 2\sin\frac{A}{2}\cdot\sin\frac{B}{2}}{\cancel{2}\cdot 2\sin\frac{A}{2}\cdot\sin\frac{B}{2}} \times \sec\frac{A}{2}\cdot\sec\frac{B}{2}$$

$$= \frac{\cos\frac{C}{2}\cdot \cancel{2\sin\frac{A}{2}}\cdot\cancel{\sin\frac{B}{2}}}{\cancel{2\sin\frac{A}{2}}\cdot\cancel{\sin\frac{B}{2}}} \times \sec\frac{A}{2}\cdot\sec\frac{B}{2}$$

$$= \cos\frac{C}{2}\cdot\sec\frac{A}{2}\cdot\sec\frac{B}{2}$$

$$= \sec\frac{A}{2}\cdot\sec\frac{B}{2}\cdot\cos\frac{C}{2}$$

RHS proved.

4. If A, B and C are the angles of a triangle, prove that:

a. $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A\cdot\cos B\cdot\cos C$

Solution:

We have,

$A + B + C = 180°$

or, $\sin(A+B) = \sin(180° - C)$

or, $\sin(A+B) = \sin C$

Also,

$\cos(A+B) = \cos(180° - C)$

$= -\cos C$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A = \frac{1-\cos 2A}{2}$

Now,

LHS

$\sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} + \sin^2 C$

$= \frac{2 - \cos 2A - \cos 2B}{2} + \sin^2 C$

$= \frac{2 - (\cos 2A + \cos 2B)}{2} + \sin^2 C$

$= \frac{2 - 2\cos\frac{2A+2B}{2} \cdot \cos\frac{2A-2B}{2}}{2} + \sin^2 C$

$= \frac{2 - 2\cos(A+B) \cdot \cos(A-B)}{2} + \sin^2 C$

$= \frac{\cancel{2}\{1 - \cos(A+B) \cdot \cos(A-B)\}}{\cancel{2}} + \sin^2 C$

$= 1 + \cos C \cdot \cos(A-B) + 1 - \cos^2 C$

$= 2 + \cos C \cdot \cos(A-B) - \cos C \cdot \cos C$

$= 2 + \cos C \{\cos(A-B) - \cos C\}$

$= 2 + \cos C \{\cos(A-B) + \cos(A+B)\}$

$= 2 + \cos C \cdot 2\cos A \cdot \cos B$

$= 2 + 2\cos A \cdot \cos B \cdot \cos C$

RHS proved.

b. $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A \cdot \sin B \cdot \cancel{\sin C}\ \cos C$

Solution:

we know

$A + B + C = 180°$

$\sin(A+B) = \sin(180 - C) = \sin C$

Also, $\cos(A+B) = \cos(180 - C) = -\cos C$

Now,

LHS

$\sin^2 A + \sin^2 B - \sin^2 C$

$= \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} - \sin^2 C$

$= \dfrac{2 - \cos 2A - \cos 2B}{2} - \sin^2 C$

$= \dfrac{2 - (\cos 2A + \cos 2B)}{2} - \sin^2 C$

$= \dfrac{2 - 2\cos\frac{2A+2B}{2} \cdot \cos\frac{2A-2B}{2}}{2} - \sin^2 C$

$= \dfrac{2 - 2\cos(A+B) \cdot \cos(A-B)}{2} - \sin^2 C$

$= \dfrac{\cancel{2}\{1 - \cos(A+B) \cdot \cos(A-B)\}}{\cancel{2}} - \sin^2 C$

$= 1 + \cos C \cdot \cos(A-B) - 1 + \cos^2 C$

$= \cos C \cdot \cos(A-B) + \cos C \cdot \cos C$

$= \cos C \{\cos(A-B) - \cos(A+B)\}$

$= \cos C \cdot \{2 \cdot \sin A \cdot \sin B\}$

$= 2 \sin A \cdot \sin B \cdot \cos C$

RHS proved//

---

c. $\sin^2 A - \sin^2 B - \sin^2 C = -2\cos A \cdot \sin B \cdot \sin C$

Solution:

We know,

$\sin(A+B) = \sin(180° - C) = \sin C$

or $\cos(A+B) = \cos(180 - C) = -\cos C$

Now,

LHS

$\sin^2 A - \sin^2 B - \sin^2 C$

$= \dfrac{1 - \cos 2A}{2} - \dfrac{1 - \cos 2B}{2} - \sin^2 C$

$= \dfrac{-\cos 2A + \cos 2B}{2} - \sin^2 C$

$= -\frac{1}{2}(\cos 2A - \cos 2B) - \sin^2 C$

$= -\frac{1}{\cancel{2}} \cdot \cancel{2} \sin \frac{2A+2B}{2} \cdot \sin \frac{2B-2A}{2} - \sin^2 C$

$= -\frac{1}{1} \cdot \sin(A+B) \cdot \sin B-A - \sin^2 C$

$= -\sin(A+B) \cdot \sin(B-A) - \sin C \cdot \sin C$

$= \cancel{-\sin C\, [\sin(A+B) + \sin C]}$

$= \cancel{-\sin C\, [\sin(A+B) + \sin}$

$= -\sin C[\sin(B-A) + \sin C]$

$= -\sin C\,[\sin(B-A) + \sin(A+B)]$

$= -\sin C \cdot 2 \sin \frac{B-A+A+B}{2} \cdot \cos \frac{B-A-A-B}{2}$

$= -2\sin C \cdot \sin \frac{2B}{2} \cdot \cos \frac{-2A}{2}$

$= -2\sin C \cdot \sin B \cdot \cancel{\sin(-A}\ \cos(-A))$

$= -2\sin C \cdot \sin B \cdot \cos A$

$= -2\cos A \cdot \sin B \cdot \sin C$

RHS proved,,

---

d. $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \cdot \sin B \cdot \cos C$

Solution:

We know,

$\sin(A+B) = \sin C$

$\cos(A+B) = -\cos C$

LHS

$\cos^2 A + \cos^2 B - \cos^2 C$

$= 1 - \sin^2 A + 1 - \sin^2 B - \cos^2 C$

$= -(\sin^2 A + \sin^2 B) - \cos^2 C$

$= -\left\{ \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} \right\} - \cos^2 C$

H/w

Date:

Page:

LHS

$\cos^2 A + \cos^2 B - \cos^2 C$

$= \frac{\cos 2A + 1}{2} + \frac{\cos 2B + 1}{2} - \cos^2 C$

$\cos 2A = 2\cos^2 A - 1$

$\cos^2 A = \frac{\cos 2A + 1}{2}$

$= \frac{2 + \cos 2A + \cos 2B}{2} - \cos^2 C$

$= \frac{2 + 2\cos(A+B)\cdot\cos(A-B)}{2} - \cos^2 C$

$= \frac{\cancel{2}\{1 + \cos(A+B)\cdot\cos(A-B)\}}{\cancel{2}} - \cos^2 C$

$= 1 + \cos(A+B)\cdot\cos(A-B) - \cos^2 C$

$= 1 + \{-\cos C\}\cdot\cos(A-B) - \cos^2 C$

$= 1 - \cos C\cdot\cos(A-B) - \cos^2 C$

$= 1 - \cos C\{\cos(A-B) + \cos C\}$

$= 1 - \cos C\cdot\{\cos(A-B) + (-\cos A+B)\}$

$= 1 - \cos C\cdot\{\cos(A-B) - \cos(A+B)\}$

$= \cancel{1 - \cos C\cdot 2\cos A\cdot\cos B}$

$= \cancel{1 - 2\cos}$

$= 1 - \cos C\cdot 2\sin A\cdot\sin B$

$= 1 - 2\sin A\cdot\sin B\cdot\cos C$

RHS proved

e. $\cos^2 2A + \cos^2 2B + \cos^2 2C = 1 + \cos 2A \cdot \cos 2B \cdot \cos 2C$

Solution:

$A + B + C = 180°$

or, ~~$\sin(A+B)$~~

or, $2A + 2B = 2 \times 180° - 2C$

or, $\sin(2A+2B) = \sin(360° - 2C) = -\sin 2C$

or, $\cos(2A+2B) = \cos(360° - 2C) = \cos 2C$

Now

LHS

$\cos^2 2A + \cos^2 2B + \cos^2 2C$

$\cos 4A = \cos 2.2A$

or $\cos 4A = 2\cos^2 A - 1$

or $\frac{\cos 4A + 1}{2} = \cos^2 A$

$= \frac{\cos 4A + 1}{2} + \frac{\cos 4B + 1}{2} + \cos^2 2C$

$= \frac{2 + \cos 4A + \cos 4B}{2} + \cos^2 2C$

$= \frac{2 + 2\cos\left(\frac{4A+4B}{2}\right) \cdot \cos\left(\frac{4A-4B}{2}\right)}{2} + \cos^2 2C$

$= \frac{2 + 2\cos(2A+2B) \cdot \cos(2A-2B)}{2} + \cos^2 2C$

$= \frac{\cancel{2}\{1 + \cos(2A+2B) \cdot \cos(2A-2B)\}}{\cancel{2}} + \cos^2 2C$

$= 1 + \cos 2C \cdot \cos(2A-2B) + \cos 2C \cdot \cos 2C$

$= 1 + \cos 2C \{\cos(2A-2B) + \cos(2A+2B)\}$

$= 1 + \cos 2C \cdot 2\cos 2A \cdot \cos 2B$

$= 1 + 2\cos 2A \cdot \cos 2B \cdot \cos 2C$

RHS proved.

5. If $A + B + C = \pi^c$, prove that

a. $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2\sin\frac{A}{2} \cdot \sin\frac{B}{2} \cdot \sin\frac{C}{2}$

Solution:

$A + B + C = \pi$

or $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

or $\sin\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = \sin\frac{C}{2}$

LHS.

$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$\cos A = \cos 2.\frac{A}{2}$

$= \frac{1-\cos A}{2} + \frac{1-\cos B}{2} + \sin^2\frac{C}{2}$

$\cos A = 1 - 2\sin^2\frac{A}{2}$

or $\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$

$= \frac{2 - (\cos A + \cos B)}{2} + \sin^2\frac{C}{2}$

$= \frac{2 - \left\{2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)\right\}}{2} + \sin^2\frac{C}{2}$

$= \frac{\cancel{2}\left\{1 - \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)\right\}}{\cancel{2}} + \sin^2\frac{C}{2}$

$= 1 - \sin\frac{C}{2} \cdot \cos\frac{A-B}{2} + \sin\frac{C}{2} \cdot \sin\frac{C}{2}$

$= 1 - \sin\frac{C}{2}\left\{\cos\frac{A-B}{2} - \sin\frac{C}{2}\right\}$

$= 1 - \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\left(\frac{A}{2} + \frac{B}{2}\right)\right\}$

$= 1 - \sin\frac{C}{2} \cdot 2\sin\frac{A}{2} \cdot \cancel{\cos B}\ \sin\frac{B}{2}$

$= 1 - 2\sin\frac{A}{2} \cdot \sin\frac{B}{2} \cdot \sin\frac{C}{2}$

proved.

b. $\sin^2\frac{A}{2} - \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\cos\frac{C}{2}$

We know,

$A+B+C = 180°$

or, $\sin\left(\frac{A+C}{2}\right) = \sin\left(\frac{180-B}{2}\right) = \cancel{\cos\frac{B}{2}}\ \cos\frac{B}{2}$

$\cos\left(\frac{A+C}{2}\right) = \cos\left(\frac{180-B}{2}\right) = \cancel{\sin\frac{B}{2}}\ \sin\frac{B}{2}$

LHS

$\sin^2\frac{A}{2} - \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

Also,

$\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$

$= \frac{1-\cos A}{2} + \frac{1-\cos C}{2} - \sin^2\frac{B}{2}$

$= \frac{2-(\cos A + \cos C)}{2} - \sin^2\frac{B}{2}$

$= \frac{2-\left\{2\cos\frac{A+C}{2}\cdot\cos\frac{A-C}{2}\right\}}{2} - \sin^2\frac{B}{2}$

$= \frac{\cancel{2}\left(1-\cos\frac{A+C}{2}\cdot\cos\frac{A-C}{2}\right)}{\cancel{2}} - \sin^2\frac{B}{2}$

$= 1 - \sin\frac{B}{2}\cdot\cos\frac{A-C}{2} - \sin^2\frac{B}{2}$

$= 1 - \sin\frac{B}{2}\left(\cos\frac{A-C}{2} + \cos\frac{A+C}{2}\right)$

$= 1 - \sin\frac{B}{2}\cdot 2\cos\frac{A}{2}\cdot\cos\frac{C}{2}$

$= 1 - 2\cos\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\cos\frac{C}{2}$

RHS proved//

c. $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + 2\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

Solution:

We know,

$A + B + C = 180°$

or $\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{180-C}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{180-C}{2}\right) = \sin\frac{C}{2}$

LHS

$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2}$

$= \frac{\cos A + 1}{2} + \frac{\cos B + 1}{2} + \cos^2\frac{C}{2}$

$= \frac{2 + \cos A + \cos B}{2} + \cos^2\frac{C}{2}$

$= \frac{2 + 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}}{2} + \cos^2\frac{C}{2}$

$= \frac{\cancel{2}\left(1 + \cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}\right)}{\cancel{2}} + \cos^2\frac{C}{2}$

$= 1 + \cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2} + 1 - \sin^2\frac{C}{2}$

$= 2 + \sin\frac{C}{2}\cdot\cos\frac{A-B}{2} - \sin^2\frac{C}{2}$

$= 2 + \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}$

$= 2 + \sin\frac{C}{2}\cdot 2\sin\frac{A}{2}\cdot\sin\frac{B}{2}$

$= 2 + 2\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$ proved//

d. $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = 2\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\sin\frac{C}{2}$

Solution:

$A + B + C = 180^\circ$

or, $\sin\left(\frac{A+B}{2}\right) = \left(\frac{180-C}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{180-C}{2}\right) = \sin\frac{C}{2}$

Also,

$\cos A = \cos 2\cdot\frac{A}{2}$

$\cos A = 2\cos^2\frac{A}{2} - 1$

or, $\cos^2\frac{A}{2} = \frac{\cos A + 1}{2}$

LHS

$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2}$

$= \frac{\cos A + 1}{2} + \frac{\cos B + 1}{2} - \cos^2\frac{C}{2}$

$= \frac{2 + \cos A + \cos B}{2} - \cos^2\frac{C}{2}$

$= \frac{2 + 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)}{2} - \cos^2\frac{C}{2}$

$= \frac{\cancel{2}\left(1 + \cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}\right)}{\cancel{2}} - \cos^2\frac{C}{2}$

$= \cancel{1} + \cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2} - \cancel{1} + \sin^2\frac{C}{2}$

$= \sin\frac{C}{2}\cdot\cos\frac{A-B}{2} + \sin^2\frac{C}{2}$

$= \sin\frac{C}{2}\left(\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right)$

$= \sin\frac{C}{2}\cdot 2\cos\frac{A}{2}\cdot\cos\frac{B}{2}$

$= 2\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\sin\frac{C}{2}$

Proved

Creative Section

6. If $A+B+C = \pi^c$, show that

a. $\cos A \cdot \sin A + \cos B \cdot \sin B + \cos C \cdot \sin C = 2\sin A \cdot \sin B \cdot \sin C$

Solution;

$A+B+C = \pi$

or, $\sin(A+B) = \sin(\pi - C) = \sin C$

$\cos(A+B) = \cos(\pi - C) = -\cos C$

Now,

LHS

$\cos A \cdot \sin A + \cos B \cdot \sin B + \cos C \cdot \sin C$

$= \frac{2\cos A \cdot \sin A}{2} + \frac{2\cos B \cdot \sin B}{2} + \cos C \cdot \sin C$

$= \frac{\sin 2A + \sin 2B}{2} + \cos C \cdot \sin C$

$= \frac{2\sin(A+B) \cdot \cos(A-B)}{2} + \cos C \cdot \sin C$

$= \frac{\cancel{2}\{\sin C \cdot \cos(A-B)\}}{\cancel{2}} + \cos C \cdot \sin C$

$= \sin C\{\cos(A-B) - \cos(A+B)\}$

$= \sin C \cdot 2\sin A \cdot \sin B$

$= 2\sin A \cdot \sin B \cdot \sin C$

RHS proved.

b. $\sin A \cdot \sin B \cdot \cos C + \sin A \cdot \cos B \cdot \sin C + \cos A \cdot \sin B \cdot \sin C = 1 + \cos A \cdot \cos B \cdot \cos C$

Solution:

$A+B+C = 180°$

or, $\sin(A+B) = \cancel{(180} \sin(180 - C) = \sin C$

$\cos(A+B) = \cos(180 - C) = -\cos C$

LHS

$\sin A \cdot \sin B \cdot \cos C + \sin A \cdot \cos B \cdot \sin C + \cos A \cdot \sin B \cdot \sin C$

$= \sin A\ (\sin B \cdot \cancel{\sin C} + \cos C + \cos B \cdot \sin C) + \cos A \cdot \sin B \cdot \sin C$

$= \sin A \cdot \sin(B+C) + \cos A \cdot \sin B \cdot \sin C$

$= \sin A \cdot \sin A + \cos A \cdot \sin B \cdot \sin C$

$= \sin^2 A + \cos A \cdot \sin B \cdot \sin C$

$= 1 - \cos^2 A + \cos A \cdot \sin B \cdot \sin C$

$= 1 - \cos A\ (\cos A - \sin B \cdot \sin C)$

$= 1 - \cos A\ [-\cos(B+C) - \sin B \cdot \sin C]$

$= 1 - \cos A\ [-\{\cos B \cdot \cos C - \sin B \cdot \sin C\} + \sin B \cdot \sin C]$

$= 1 + \cos A\ \{\cos B \cdot \cos C - \cancel{\sin B \cdot \sin C} + \cancel{\sin B \cdot \sin C}\}$

$= 1 + \cos A \cdot \cos B \cdot \cos C$

RHS Proved.

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c. $\dfrac{\sin A}{\cos B \cdot \cos C} + \dfrac{\sin B}{\cos C \cdot \cos A} + \dfrac{\sin C}{\cos A \cdot \cos B} = 2 \tan A \cdot \tan B \cdot \tan C$

Solution:

$A + B + C = \pi$

$\sin(A+B) = \sin(\pi - C) = \sin C$

$\cos(A+B) = \cos(\pi - C) = -\cos C$

LHS

$\dfrac{\sin A}{\cos B \cdot \cos C} + \dfrac{\sin B}{\cos C \cdot \cos A} + \dfrac{\sin C}{\cos A \cdot \cos B}$

$= \dfrac{\sin A \cdot \cos A + \sin B \cdot \cos B + \sin C \cdot \cos C}{\cos A \cdot \cos B \cdot \cos C}$

$= \dfrac{\dfrac{2 \sin A \cdot \cos A + 2 \sin B \cdot \cos B}{2} + \sin C \cdot \cos C}{\cos A \cdot \cos B \cdot \cos C}$

$= \dfrac{\dfrac{\sin 2A + \sin 2B}{2} + \sin C \cdot \cos C}{\cos A \cdot \cos B \cdot \cos C}$

$= \dfrac{\dfrac{\cancel{2} \sin(A+B) \cdot \cos(A-B)}{\cancel{2}} + \sin C \cdot \cos C}{\cos A \cdot \cos B \cdot \cos C}$

$= \dfrac{\sin C \cdot \cos(A-B) + \sin C \cdot \cos C}{\cos A \cdot \cos B \cdot \cos C}$

$= \dfrac{\sin C \{\cos(A-B) - \cos(A+B)\}}{\cos A \cdot \cos B \cdot \cos C}$

$= \cancel{\sin C \cdot 2 \sin A}$

$= \dfrac{\sin C \cdot 2 \sin A \cdot \sin B}{\cos A \cdot \cos B \cdot \cos C}$

$= 2 \tan A \cdot \tan B \cdot \tan C$

RHS proved,

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7. If A, B and C are the ~~triangles of a~~ angles of a triangle, prove that:

a. $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\frac{B+C}{4} \cdot \sin\frac{C+A}{4} \cdot \sin\frac{A+B}{4}$

Solution:

Here, $A+B+C = 180^\circ$

$\sin(A+B) = \sin(180^\circ - C) = \sin C$

$\cos(A+B) = \cos(180^\circ - C) = -\cos C$

RHS

$1 + 4\sin\frac{B+C}{4} \cdot \sin\frac{C+A}{4} \cdot \sin\frac{A+B}{4}$

$= 1 + 2\left[2\sin\frac{B+C}{4} \cdot \sin\frac{C+A}{4}\right] \cdot \sin\frac{A+B}{4}$

$= 1 + 2\left[\cos\left(\frac{B+C}{4} - \frac{C+A}{4}\right) - \cos\left(\frac{B+C}{4} + \frac{C+A}{4}\right)\right] \cdot \sin\frac{A+B}{4}$

$= 1 + 2\left[\cos\left(\frac{B+C-C-A}{4}\right) - \cos\left(\frac{B+C+C+A}{4}\right)\right] \cdot \sin\frac{A+B}{4}$

$= 1 + 2\left[\cos\left(\frac{B-A}{4}\right) - \cos\left(\frac{2C+A+B}{4}\right)\right] \cdot \sin\frac{A+B}{4}$

$= 1 + 2\cos\frac{B-A}{4} \cdot \sin\frac{A+B}{4} - \cos\frac{2C+A+B}{4} \cdot \sin\frac{A+B}{4}$

$= 1 + \left[\sin\left\{\frac{B-A+A+B}{4}\right\} - \sin\left\{\frac{B-A-A-B}{4}\right\}\right] - \left\{\sin\frac{2C+A+B+A+B}{4}\right.$

$\left. - \sin\frac{2C+A+B-A-B}{4}\right\}$

$= 1 + \left\{\sin\frac{B}{2} - \sin\left(\frac{-2A}{4}\right)\right\} - \left\{\sin\frac{A+B+C}{2} - \sin\frac{C}{2}\right\}$

$= 1 + \sin\frac{B}{2} + \sin\frac{A}{2} - \sin\frac{180}{2} + \sin\frac{C}{2}$

$= 1 + \sin\frac{B}{2} + \sin\frac{A}{2} - 1 + \sin\frac{C}{2}$

$= \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$

RHS Proved.

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b. $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\frac{\pi-A}{4} \cdot \cos\frac{\pi-B}{4} \cdot \cos\frac{\pi-C}{4}$

Solution:

$A + B + C = 180° = \pi$

RHS

$4\cos\frac{\pi-A}{4} \cdot \cos\frac{\pi-B}{4} \cdot \cos\frac{\pi-C}{4}$

$= 2\left[2\cos\frac{\pi-A}{4} \cdot \cos\frac{\pi-B}{4}\right] \cdot \cos\frac{\pi-C}{4}$

$= 2\left[\cos\frac{\pi-A+\pi-B}{4} + \cos\frac{\pi-A-\pi+B}{4}\right] \cdot \cos\frac{\pi-C}{4}$

$= 2\left[\cos\frac{2\pi-(A+B)}{4} + \cos\frac{B-A}{4}\right] \cdot \cos\frac{\pi-C}{4}$

$\cancel{= 2\left[\cos\frac{A+B}{4} + \cos\frac{B-A}{4}\right] \cdot \cos\frac{\pi-C}{4}}$

$\cancel{= 2\left[\cos\frac{A+B}{4} + \cos\frac{B-A}{4}\right] \cdot \cos\frac{\pi-C}{4}}$

$= \cancel{2\cos\frac{A+B}{4}\cdot\cos\frac{\pi-C}{4}}$

$= 2\left[\cos\frac{2\pi-(\pi-C)}{4} + \cos\frac{B-A}{4}\right]\cdot\cos\frac{\pi-C}{4}$

$= 2\cos\frac{\pi+C}{4}\cdot\cos\frac{\pi-C}{4} + 2\cos\frac{B-A}{4}\cdot\cos\frac{\pi-C}{4}$

$= \cos\left(\frac{\pi+C+\pi-C}{4}\right) + \cos\left(\frac{\pi+C-\pi+C}{4}\right) + \cos\left(\frac{B-A+\pi-C}{4}\right) + \cos\left(\frac{B-A-\pi+C}{4}\right)$

$= \cos\frac{2\pi}{4}\cdot\cos\frac{2C}{4} + \cos\left\{\frac{\pi+B-(A+C)}{4}\right\} + \cos\left\{\frac{(B+C)-\pi-A}{4}\right\}$

$= \cos\frac{2\pi}{4}\cdot\cos\frac{2C}{4} + \cos\frac{\pi+B-\pi+B}{4} + \cos\frac{\pi-A-\pi-A}{4}$

$= \cos\frac{\pi}{2}\cdot\cos\frac{C}{2} + \cos\frac{2B}{2} + \cos\left(\frac{-2A}{4}\right)$

$= \cos\frac{\pi}{2} + \cos\frac{C}{2} + \cos\frac{B}{2} + \cos\frac{A}{2}$

$= 0 + \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$

$= \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$

~~RHS~~ LHS proved,,

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c. $\cos\frac{A}{2} + \cos\frac{B}{2} - \cos\frac{C}{2} = 4\cos\frac{\pi+A}{4}\cdot\cos\frac{\pi+B}{4}\cdot\cos\frac{\pi-C}{4}$

Solution:

$A+B+C = \pi$

or $\cos(A+C) = \cos(\pi-C)$

or $\cos(B+C) = \cos(\pi-A)$

RHS

$$4\cos\frac{\pi+A}{4}\cdot\cos\frac{\pi+B}{4}\cdot\cos\frac{\pi-C}{4}$$

$$= 2\left\{2\cos\frac{\pi+A}{4}\cdot\cos\frac{\pi+B}{4}\right\}\cdot\cos\frac{\pi-C}{4}$$

$$= 2\left[\cos\left(\frac{\pi+A}{4}+\frac{\pi+B}{4}\right)+\cos\left(\frac{\pi+A}{4}-\frac{\pi+B}{4}\right)\right]\cdot\cos\frac{\pi-C}{4}$$

$$= 2\left[\cos\frac{\pi+A+\pi+B}{4}+\cos\frac{\pi+A-\pi-B}{4}\right]\cdot\cos\frac{\pi-C}{4}$$

$$= 2\left[\cos\frac{2\pi+(A+B)}{4}+\cos\frac{A-B}{4}\right]\cdot\cos\frac{\pi-C}{4}$$

$$= 2\left[\cos\frac{2\pi+\pi-C}{4}+\cos\frac{A-B}{4}\right]\cdot\cos\frac{\pi-C}{4}$$

$$= 2\cos\frac{3\pi-C}{4}\cdot\cos\frac{\pi-C}{4}+2\cos\frac{A-B}{4}\cdot\cos\frac{\pi-C}{4}$$

$$= \cos\frac{3\pi-C+\pi-C}{4}+\cos\frac{3\pi-C-\pi+C}{4}+\cos\frac{A-B+\pi-C}{4}+\cos\frac{A-B-\pi+C}{4}$$

$$= \cos\frac{4\pi-2C}{4}+\cos\frac{2\pi}{4}+\cos\frac{\pi+A-(B+C)}{4}+\cos\frac{(A+C)-B-\pi}{4}$$

$$= \cos\frac{2(2\pi-C)}{4}+\cos\frac{\pi}{2}+\cos\frac{\pi+A-\pi+A}{4}+\cos\frac{\pi-B-B-\pi}{4}$$

$$= \cos\left(\frac{2\pi}{2}-\frac{C}{2}\right)+0+\cos\frac{2A}{4}+\cos\left(\frac{-2B}{4}\right)$$

$$= \cos\frac{C}{2}+\cos\frac{A}{2}+\cos\frac{B}{2}$$

$$= \cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}$$

LHS proved

8. If $A+B+C=\pi$, show that.

a. $\cos\frac{A}{2}\cdot\sin\frac{B-C}{2}+\cos\frac{B}{2}\cdot\sin\frac{C-A}{2}+\cos\frac{C}{2}\cdot\sin\frac{A-B}{2}=0$

Solution:

$A+B+C=\pi$

$\cos\frac{A}{2}=\cos\left\{\frac{\pi-(B+C)}{2}\right\}=\sin\frac{(B+C)}{2}$

LHS

$\cos\frac{A}{2}\cdot\sin\frac{B-C}{2}+\cos\frac{B}{2}\cdot\sin\frac{C-A}{2}+\cos\frac{C}{2}\cdot\sin\frac{A-B}{2}$

$=\sin\frac{B+C}{2}\cdot\sin\frac{B-C}{2}+\sin\frac{A+C}{2}\cdot\sin\frac{C-A}{2}+\sin\frac{A+B}{2}\cdot\sin\frac{A-B}{2}$

$=\frac{1}{2}\left[2\sin\frac{B+C}{2}\cdot\sin\frac{B-C}{2}+2\sin\frac{A+C}{2}\cdot\sin\frac{C-A}{2}+2\sin\frac{A+B}{2}\cdot\sin\frac{A-B}{2}\right]$

$=\frac{1}{2}\left[\cos\frac{B+C-B+C}{2}-\cos\frac{B+C+B-C}{2}+\cos\frac{A+C-C+A}{2}-\cos\frac{A+C+C-A}{2}\right.$

$\left.+\cos\frac{A+B-A+B}{2}-\cos\frac{A+B+A-B}{2}\right]$

$=\frac{1}{2}\left[\cos\frac{2C}{2}-\cos\frac{2B}{2}+\cos\frac{2A}{2}-\cos\frac{2C}{2}+\cos\frac{2B}{2}-\cos\frac{2A}{2}\right]$

$=\frac{1}{2}\left[\cancel{\cos C}-\cancel{\cos B}+\cancel{\cos A}-\cancel{\cos C}+\cancel{\cos B}-\cancel{\cos A}\right]$

$=\frac{1}{2}\times 0$

$=0$

RHS proved.

b $\cos\frac{A}{2}\cdot \cos\frac{B-C}{2}+\cos\frac{B}{2}\cdot\cos\frac{C-A}{2}+\cos\frac{C}{2}\cdot\cos\frac{A-B}{2}$

$=\sin A+\sin B+\sin C$

Solution:

$A+B+C=\pi$

$\cos\frac{A}{2}=\cos\left\{\frac{\pi-(A+B)}{2}\right\}=\sin\left(\frac{A+B}{2}\right)$

$\cos\frac{B}{2}=\cos\left\{\frac{\pi-(A+C)}{2}\right\}=\sin\left(\frac{A+C}{2}\right)$

LHS

$\cos\frac{A}{2}\cdot\cos\frac{B-C}{2}+\cos\frac{B}{2}\cdot\cos\frac{C-A}{2}+\cos\frac{C}{2}\cdot\cos\frac{A-B}{2}$

$=\sin\frac{B+C}{2}\cdot\cos\frac{B-C}{2}+\sin\frac{A+C}{2}\cdot\cos\frac{C-A}{2}+\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$

$=\frac{1}{2}\left[2\sin\frac{B+C}{2}\cdot\cos\frac{B-C}{2}+2\sin\frac{A+C}{2}\cdot\cos\frac{C-A}{2}+2\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}\right]$

$=\frac{1}{2}\left[\sin\frac{B+C+B-C}{2}+\sin\frac{B+C-B+C}{2}+\sin\frac{A+C+C-A}{2}+\sin\frac{A+C-C+A}{2}\right.$

$\left.+\sin\frac{A+B+A-B}{2}+\sin\frac{A+B-A+B}{2}\right]$

$=\frac{1}{2}\left[\sin\frac{2B}{2}+\sin\frac{2C}{2}+\sin\frac{2C}{2}+\sin\frac{2A}{2}+\sin\frac{2A}{2}\right.$

$\left.+\sin\frac{2B}{2}\right]$

$=\frac{1}{2}\left[\sin B+\sin C+\sin C+\sin A+\sin A+\sin B\right]$

$=\frac{1}{2}\times 2\sin A+\sin B+\sin C$

$=\sin A+\sin B+\sin C$

proved

c. $\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A = 1 + \sec A \cdot \sec B \cdot \sec C$

Solution:

$A + B + C = \pi$

$\sin(A+C) = \sin(\pi - B) = \sin B$

$\cos(A+C) = \cos(\pi - B) = -\cos C$

LHS

$\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A$

$= \frac{\sin A \cdot \sin B}{\cos A \cdot \cos B} + \frac{\sin B \cdot \sin C}{\cos B \cdot \cos C} + \frac{\sin C \cdot \sin A}{\cos C \cdot \cos A}$

$= \frac{\sin A \cdot \sin B \cdot \cos C + \sin B \cdot \sin C \cdot \cos A + \sin C \cdot \sin A \cdot \cos B}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{\sin B\,(\sin A \cdot \cos C + \sin C \cdot \cos A) + \sin C \cdot \sin A \cdot \cos B}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{\sin B \cdot \sin(A+C) + \sin C \cdot \sin A \cdot \cos B}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{\sin B \cdot \sin B + \sin C \cdot \sin A \cdot \cos B}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{\sin^2 B + \sin C \cdot \sin A \cdot \cos B}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{1 - \cos^2 B + \sin C \cdot \sin A \cdot \cos B}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{1 - \cos B\,[\cos B - \sin A \cdot \sin C]}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{1 - \cos B\,[-\cos(A+C) - \sin A \cdot \sin C]}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{1 + \cos B\,[\cos(A+C) + \sin A \cdot \sin C]}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{1 + \cos B\,[\cos A \cdot \cos C - \sin A \cdot \sin C + \sin A \cdot \sin C]}{\cos A \cdot \cos B \cdot \cos C}$

$= \frac{1 + \cos B \cdot \cos A \cdot \cos C}{\cos A \cdot \cos B \cdot \cos C}$

$= \dfrac{1}{\cos A \cdot \cos B \cdot \cos C} + \dfrac{\cos A \cdot \cos B \cdot \cos C}{\cos A \cdot \cos B \cdot \cos C}$

$= \sec A \cdot \sec B \cdot \sec C + 1$

$= 1 + \sec A \cdot \sec B \cdot \sec C$

RHS proved.

d. $\sin(B+2C) + \sin(C+2A) + \sin(A+2B) = 4\sin\left(\frac{B-C}{2}\right) \cdot \sin\left(\frac{C-A}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

Solution:

$A+B+C = \pi$

or, $\sin B = \sin \pi - A - C$

$\sin C = \sin \pi - A - B$

$\sin A = \sin \pi - B - C$

LHS

$\sin(B+2C) + \sin(C+2A) + \sin(A+2B)$

$= \sin(\pi - A - C + 2C) + \sin(\pi - B - A + 2A) + \sin(\pi - B - C + 2B)$

$= \sin(\pi - A + C) + \sin(\pi - B + A) + \sin(\pi - C + B)$

$= \sin\{\pi - (A-C)\} + \sin\{(\pi - (B-A)\} + \sin\{(\pi - (C-B)\}$

$= \sin(A-C) + \sin(B-A) + \sin(C-B)$

$= 2\sin\frac{A-C+B-A}{2} \cdot \cos\frac{A-C-B+A}{2} + \sin 2\frac{(C-B)}{2}$

$= 2\sin\frac{B-C}{2} \cdot \cos\frac{2A-B-C}{2} - 2\sin\frac{B-C}{2} \cdot \cos\frac{B-C}{2}$

$= 2\sin\frac{B-C}{2}\left[\cos\frac{2A-B-C}{2} - \cos\frac{B-C}{2}\right]$

$= 2\sin\frac{B-C}{2} \cdot 2\sin\frac{2A-B-C+B-C}{4} \cdot -\sin\frac{2A-B-C-B+C}{4}$

$= 2\sin\frac{B-C}{2} \cdot 2\sin\frac{2A-2C}{4} \cdot -\sin\frac{2A-2B}{4}$

Page:

$= 4 \sin\frac{B-C}{2} \cdot \sin\frac{\cancel{4}(A-C)}{2\cancel{4}} \cdot -\sin\frac{\cancel{4}(A-B)}{2\cancel{4}}$

$= 4 \sin\frac{B-C}{2} \cdot -\sin\frac{(C-A)}{2} \cdot -\sin\frac{(A-B)}{2}$

$= 4 \sin\left(\frac{B-C}{2}\right) \cdot \sin\left(\frac{C-A}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

RHS proved.

Exercise - 7.5

General Section:

1. Solve the following equations $(0° \le \theta \le 180°)$

a. $\sqrt{2}\sin\theta = 1$

Solution:

$\sqrt{2}\sin\theta = 1$

or $\sin\theta = \frac{1}{\sqrt{2}}$

or $\sin\theta = \sin 45°$

$\therefore \theta = 45°$

Here, $\theta =$ +ve

So, $\theta = (180° - 45) = 135°$

b. $2\cos\theta - 1 = 0$

or $\cos\theta = \frac{1}{2}$

or $\cos\theta = \cos 60°$

$\therefore \theta = 60°$

~~Also,~~

~~$\theta =$~~