Chapter-4
Dynamics

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Linear Momentum

Linear Momentum is the quantity of motion contained in a body, it is measured by the product of mass and velocity. It is a vector quantity and denoted by P. It is given by:

$$\vec{P} = M.\vec{v}$$

Dimension formula of P = $[MLT^{-1}]$

SI unit of P = $kg\,ms^{-1}$

The force ~~of~~ in terms of linear momentum is given by the rate of change of linear momentum with respect to time.

i.e. $F = \frac{dp}{dt}$

or, $F = \frac{d(Mv)}{dt}$

or, $F = m\frac{dv}{dt}$

$\therefore F = ma$

Note:

$V = \frac{dx}{dt}$

or $\frac{dv}{dt} = a$

Now,

$\frac{dP}{dt} = ma = F$

or, $dP = Fdt$

Change in momentum = Force × time

Impulse

Imp SQ

If a large force acts on a body for a very short interval of time then the force is called impulse force. The product of impulsive force and the time interval for which it acts is called impulse.

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Imp # Give Reasons:

a. A cricketer lower his hand while catching cricket ball. Why?

Ans: The impulse of a force is, Impulse $= f \times t =$ Change in linear momentum which remains constant. Lowering the hands helps to increase time due to which reactional force decreases. Hence, hands are not hurt severely.

b. China Wares or glass wares are wrapped with straws or foams. Why?

Ans: China wares or glass wares are wrapped with straws or ~~for~~ foams because straws and foams absorbs force by increasing the time in case of event of fall. Because of increase in time, force is reduced as we know, Impulse $= f \times t$.

c. A man get hurts while jumping on the cemented floor but not mud or straw heap. Why?

Ans: The impulse of a force is, Impulse $= f \times t =$ change in linear momentum which remains constant. Jumping on the cemented floor takes very short time due to which reactional force is increased but jumping on sand increases time which helps to decrease reactional force. Hence, person is not hurt while jumping on the mud.

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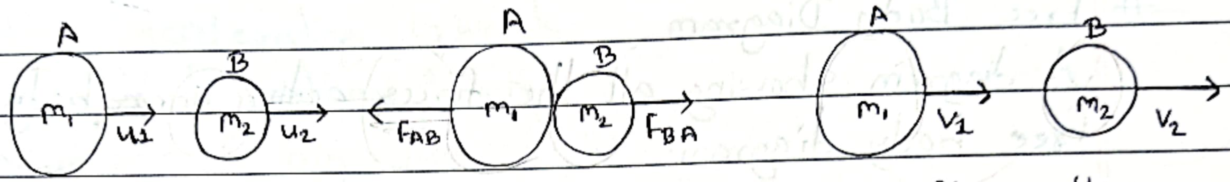
Principle of Conservation of Linear Momentum

It states that, "If the vector sum of external forces acting on a system is zero, then its total linear momentum remains conserved or constant."

Derivations:



Before collision   During collision   After collision

Let two particles A and B having respective masses m_1 and m_2 are moving with respective velocities u_1 and u_2 in a straight line before collision such that $u_1 > u_2$ as shown in the figure.

Let v_1 and v_2 be their respective velocities after collision.

Let, Δt = small time of collision.

Before Collision,

Momentum of A = $m_1 u_1$

Momentum of B = $m_2 u_2$

Total momentum = $m_1 u_1 + m_2 u_2$

After collision

Momentum of A = $m_1 V_1$

Momentum of B = $m_2 V_2$

Total momentum = $m_1 V_1 + m_2 V_2$

Therefore,

Change in momentum of A = $m_1 V_1 - m_1 u_1$

Change in momentum of B = $m_2 V_2 - m_2 u_2$

Let, F_{AB} = Force exerted by B on A during collision

F_{BA} = Force exerted by A on B during collision.

From Newton's 3rd Law of Motion

$F_{AB} = -F_{BA}$

or, $\dfrac{\text{change in momentum of A}}{\text{time of collision}} = - \dfrac{\text{change in momentum of B}}{\text{time of collision}}$

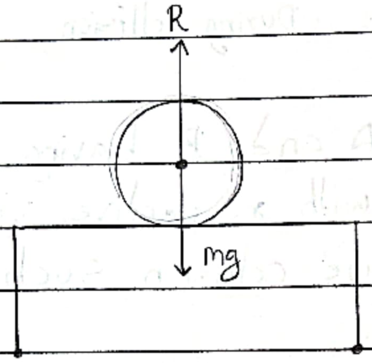
$$\text{or, } \frac{m_1v_1 - m_1u_1}{\Delta t} = -\left(\frac{m_2v_2 - m_2u_2}{\Delta t}\right)$$

$$\text{or, } m_1v_1 + m_2v_2 = m_2u_2 + m_1u_1$$

$\therefore$ Initial momentum = final momentum

Free Body Diagram

A diagram showing all the forces acting on a body is called free body diagram.



Elevator (Lift)

a. Lift is at rest

Equation: $R - mg = ma$

or, $R - mg = m.0$ $\quad [\because a = 0]$

$\therefore R = Mg$

b. Lift moves upwards uniformly

For upward motion: $R > mg$

Equation: $R - mg = ma$

or $R - mg = m.0$ $\quad [\because a = 0, \text{ as } v = u = \text{uniform}]$

$\therefore R = mg$

c. Lift moves downwards uniformly

For downward motion: $Mg > R$

Equation: $Mg - R = ma$

or $Mg = R$ $[\because a = 0]$

$\therefore R = mg$

d. Lift is accelerating upwards

For upward motion: $R > mg$

Equation: $R - mg = ma$

or $R = ma + mg$

$\therefore R = m(a + g)$

e. Lift is accelerating downward

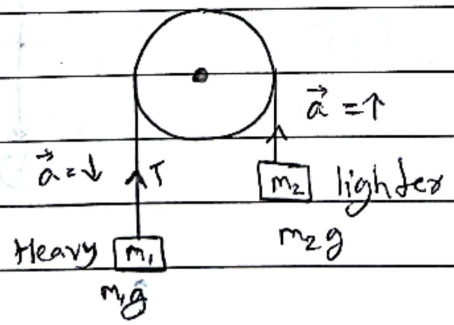
For downward : $Mg > R$

Equation: $Mg - R = ma$

or $Mg - ma = R$

$\therefore R = m(g - a)$

Mass and Pully



Note:

Weight is directed towards centre of earth.

→ Tension in a string or rope is always directed towards the point of suspence.

→ Tension in a ~~string~~ single string always equal.

→ Magnitude of acceleration in a single string always equal.

i. Equation of Heavy mass body;
for downward motion of M_1,
$(\because M_1 g > T)$

$M_1 g - T = M_1 a$

or, $T = M_1 g - M_1 a$

$\therefore T = M_1 (g - a)$ ----- (i)

ii. Equation of motion of light mass body;
for upward motion of M_2, $T > M_2 g$

$T - M_2 g = M_2 a$

or, $T = M_2 a + M_2 g$

$\therefore T = M_2 (a + g)$ ----- (ii)

Numerical

a. A lift moves (i) up and (ii) down with an acceleration of $2 m/s^2$ in each case. Calculate the reaction of the floor on a man of mass 50 kg standing in the lift.

Solution:

Mass (m) = 50 kg

Acceleration (a) = $2 m/s^2$

i. For upward motion;

$R > mg$

or, $R - mg = ma$

or, $R = mg + ma$

or, $R = m(a + g)$

or, $R = 50(2 + 10)$

$\therefore R = 600 N$

ii. For downward motion;

$mg > R$

$mg - R = ma$

or, $R = mg - ma$

or, $R = m(g-a)$

or, $R = 50(10-2)$

$\therefore R = 400\,N$

b. A box of mass 50kg is pulled up from a hold of a ship with an acceleration of $1\,m/s^2$ by a vertical rope attached to it. Find the tension in the rope and find tension when the box moves up with an uniform velocity of $1\,m/s^2$.

Solution:

Given,

Mass of a box $(m) = 50\,kg$

Acceleration $(a) = 1\,m/s^2$

Tension in rope $(T) = ?$

When box moves upward, $(T > mg)$

we have,

$T - mg = ma$

or, $T = ma + mg$

or, $T = m(a+g)$

or, $T = 50(1+10)$

$\therefore T = 550\,N$

If the box moves upwith a uniform velocity tension in the rope, $T' = ?$

In this case : $T > mg$

or, $T' - mg = ma$

or, $T' - mg = 0$ $\quad [\because a = 0]$

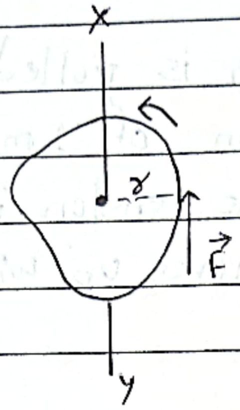
or, $T' = mg$

or, $T' = 50 \times 10$

$\therefore T' = 500\,N$

Moment of Force

The product of force and perpendicular distance acting on a body such that it produces rotational effect on the body is called moment of force or torque.

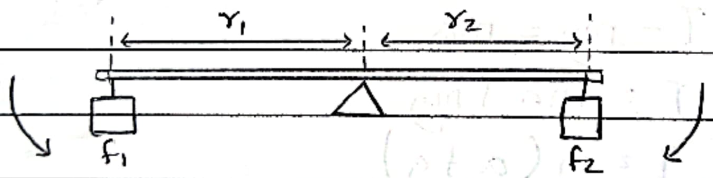


Moment of force $= \vec{r} \times \vec{F}$

or, $\vec{\tau} = \vec{r} \times \vec{F}$ $\{\tau = \text{Tau}\}$

Unit of τ = Nm = Joule

Principle of Moment



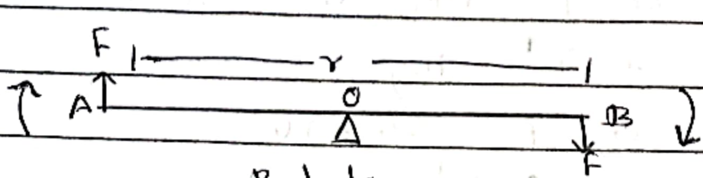
Anticlockwise moment $= r_1 \times f_1$

Clockwise moment $= r_2 \times f_2$

Now,

Principle of moment = Sum of clockwise moment = sum of anticlockwise moment for balance cond.

Torque due to couple of force



$\tau = OA \times F + OB \times F$

or, $\tau = (OA + OB) \times F$

$= AB \times f$

$= r \times f$

Two unequal and unlike parallel force with different lines of action produces couple.

Equilibrium

A body is said to be equilibrium if net force or net torque acting on a body is equal to zero.

1. Translation Equilibrium

A body is said to be in translation equilibrium if net force acting on a body is equal to zero.

i.e. $\Sigma F = 0$

i.e. acceleration $= 0$

i.e. body is at rest or moving with constant velocity

→ If $\Sigma F = 0$ and body is at rest, it is called or said to be static translation equilibrium.

→ If $\Sigma F = 0$ and body is moving with constant velocity, it is said to be dynamic translation equilibrium.

2. Rotational Equilibrium

A body is said to be in rotational equilibrium if net torque acting on a body is equal to zero.

i.e. $\Sigma T = 0$

i.e. angular acceleration $(\alpha) = 0$

i.e. body is at rest or moving with uniform velocity.

Friction

The force which comes between two surface in contact and tends to oppose the relative motion between them is called frictional force.

Types of friction

a. Static Friction

The force of friction acting between two surface when they are at rest is called static friction. The maximum value of static friction is called limiting friction.

b. Kinetic or dynamic friction

The force of friction acting between two surface when they are at motion is called kinetic or dynamic friction.

Laws of friction

a. The force of friction between two surface depends upon the nature of surface.

b. The force of limiting friction is directly proportional to the normal reaction. i.e $F \propto R$

c. The force of limiting friction is independent to the area of contact between two surface.

d. The kinetic friction is independent to the relative velocity of two surfaces.

c. Sliding friction

The force of friction which is developed between the surfaces in contact when body is sliding over another is called sliding friction.

d. Rolling friction

Rolling friction is the force of friction developed between two bodies surface when one body is rolling on the surface of another body.

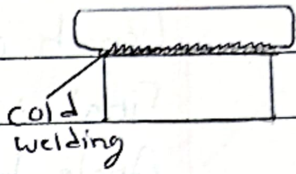
Classical View

According to this, the main cause of friction is the interlocking of projections of surface.

Modern View

Intermolecular force of attraction between the surface in contact is the cause of friction.

There is interatomic or intermolecular force of attraction which comes into play on two contact surface. There is cold welding between two surfaces due to this force of attraction.



Give reasons:

a. Why kinetic friction is less than limiting friction?

Ans: Kinetic friction is less than limiting friction because ~~limiting friction is the~~ kinetic friction is the friction that occurs when two surfaces are already in motion whereas limiting friction is the maximum amount of friction that can be generated between two surfaces that are not yet in motion. (static)

b. Why rolling is easier than sliding?

Ans: Frictional force depends on the area of contact between two surfaces. As the area of contact is less in the case of rolling than in sliding. Due to this, rolling is easier than sliding.

Numericals

1. An ice block of 8 kg, released from rest at the top of a 1.5 m long frictionless ramp, sliding downhill, reaching a speed of 2.5 m/s at the bottom. What is the angle between the ramp and the horizontal?

Solution:

Mass of block (m) = 8 kg

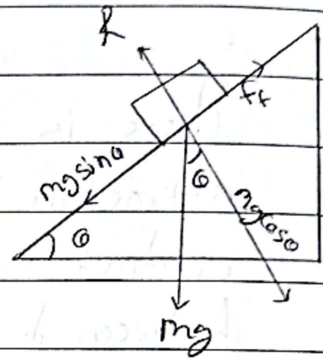
Initial Velocity (u) = 0

Length of the ramp (s) = 1.5 m

Final velocity (v) = 2.5 m/s

Angle between ramp and horizontal (θ) = ?

For frictionless ramp, $f_f = 0$



Using,

$$v^2 = u^2 + 2as$$

or, $(2.5)^2 = 0^2 + 2a \times 1.5$

or, $a = \frac{6.25}{1.5}$

$\therefore a = 2.08\ m/s^2$

Now,

For the downhill motion of the ramp, the equation of motion is,

$$mg\sin\theta - f_f = ma$$

or, $8 \times 10 \times \sin\theta - 0 = 8 \times 2.08$

or, $\sin\theta = \frac{16.67}{80}$

or, $\theta = \sin^{-1}(0.208)$

$\therefore \theta = 12.02°$

2\. An iron block of mass 10 kg rests on a wooden plane inclined at 30° to the horizontal. It is found that the least force parallel to the plane which causes the block to slide up the plane is 100N. Calculate the coefficient of friction between the two surfaces.

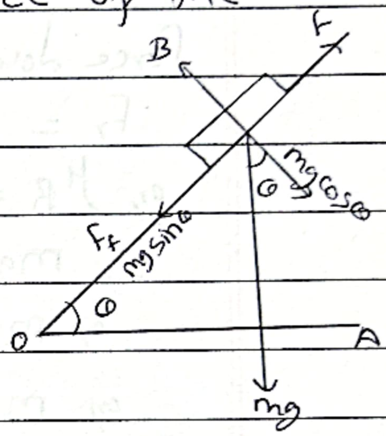
Solution:

Mass of iron block (m) = 10 kg

Angle of inclination (θ) = 30°

Least force parallel to plane required to slide up the Plane (F) = 100N

Coefficient of sliding friction (μ) = ?



Here,

$F = F_f + mg\sin\theta$ —— (I)

And, $R = mg\cos\theta$ —— (II)

Also,

$$\mu = \frac{F_f}{R}$$

$$= \frac{F - mg\sin\theta}{mg\cos\theta} \quad [\text{from (I) and (II)}]$$

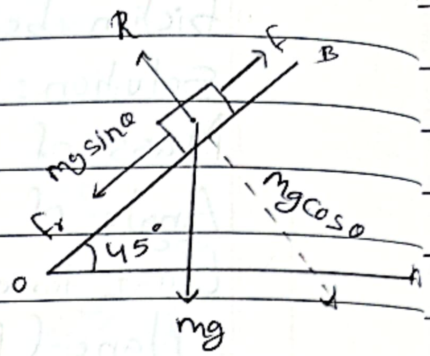
$$= \frac{100 - 10 \times 10 \times \sin 30^\circ}{10 \times 10 \times \cos 30^\circ}$$

$$= \frac{50}{86.60}$$

$\therefore \mu = 0.577$

3. What would be the acceleration of a block sliding down an inclined plane that makes an angle of $45°$ with the horizontal if the coefficient of sliding friction between two surfaces is 0.3?

Solution:



Acceleration due to gravity $(a) = ?$

Angle of inclination $(\theta) = 45°$

Coefficient of linear friction $(\mu) = 0.3$

Now,

force down the inclined plane;

$F_r = mg\sin\theta + f$

or, $\mu R = mg\sin\theta + ma \quad [\because F = ma, \ \& \ f_r = \mu R]$

or, $ma = mg\sin\theta - \mu R$

or, $ma = mg\sin\theta - \mu(mg\cos\theta)$

or, $ma = mg(\sin\theta - \mu\cos\theta)$

or, $a = 10(\sin 45 - 0.3 \times \cos 45)$

$\therefore a = 4.95 \ m/s^2$

4. A box rests on a frozen pond, which serves as a frictionless horizontal surface. If a fisherman applies a horizontal force with magnitude $48N$ to the box and produces an acceleration of magnitude $3 m/s^2$, What is the mass of the box?

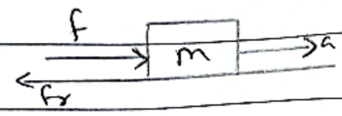
Solution:

Horizontal Force $(f) = 48N$

Acceleration $(a) = 3 m/s^2$

Mass of the box $(m) = ?$

For a frictionless surface, $(f_f) = 0$



from Newton's second law of motion;

$f - f_f = ma$

or $f - 0 = ma$

or $480 = m \times 3$

$\therefore m = 16 \ kg$

5. A box of mass 15kg placed on horizontal floor is pulled by a horizontal force. What will be the work done by the force if the coefficient of sliding friction between the box and the surface of the floor is 0.3 and body moves a unit distance.

Solution:

Given, Mass of box (m) = 15 kg

Coefficient of sliding friction (μ) = 0.3

Distance travelled (d) = 1m

Work done (W) = ?

We know,

$ma = F - f_r$

or, $m \times 0 = F - f_r$

or, $F = f_r$

Now,

$f_r = \mu R$

or, $F = 0.3 \times mg$

$= 0.3 \times 15 \times 10$

$= 45N$

Again,

Work done (W) = F.d

$= 45 \times 1$

$= 45$ Joule

∴ The work done will be 45 joule.

6. In the physics lab experiment, a 6kg box is pushed across a flat table by a horizontal force (F).

i. If the box is moving at a constant speed of 0.35 m/s and the coefficient of kinetic friction is 0.12. What is the magnitude of F?

ii. If the box is speeding up with a constant acceleration of $0.18 m/s^2$, what will be the magnitude of F?

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Solution:

i. Mass of the box (m) = 6 kg

Speed (s) = 0.35 m/s

Coefficient of kinetic friction (μ) = 0.12

Acceleration (a) = 0 m/s² [∵ Speed is constant]

Now,

Force (F) = ?

We know,

$$ma = f - fr$$

or $m \times 0 = f - fr$

or $0 = f - \mu mg$

or $f = 0.12 \times 6 \times 10$

$\therefore f = 7.2\ N$

ii. If the box is speeding up with constant acceleration of 0.18 m/s².

Then, $ma = f - fr$

or $6 \times 0.18 = f - \mu mg$

or, $6 \times 0.18 = f - 0.12 \times 6 \times 10$

or, $1.08 = f - 7.2$

$\therefore f = 8.28\ N$

c/w

Chapter-4

Dynamics.

1. A car of mass of 1000kg is accelerating at $2 m/s^2$. What resultant force acts on the car, if the resistance to the motion is 1000 N. What is the force due to engine.

Solution:

Mass of car (m) = 1000 kg

Acceleration (a) = $2 m/s^2$

Frictional force (f_r) = 1000 N

Force of engine (F_e) = ?

We know,

$$F = ma$$
$$= 1000 \times 2$$
$$= 2000 \text{ N}$$

Now,

$$F_e = F + f_r$$
$$= 2000 + 1000$$
$$= 3000 \text{ N}$$

2. A lift moves (i) up and (ii) down with an acceleration $2 m/s^2$. In each case calculate the reaction of the floor on a man of mass 50 kg standing in the lift.

Solution:

Acceleration (a) = $2 m/s^2$

Mass of man (m) = 50 kg

1^{st} case

$R > mg$

i.e $R - mg = ma$

or, $R - 50 \times 10 = 50 \times 2$

or, $R = 100 + 500$

$\therefore R = 600\ N$

2nd case

~~R~~ $mg > R$

$R = mg - ma$

or, $R = m(g-a)$

$= 50(10-2)$

$= 400\ N$

3. A box of mass 50kg is pulled up from the hold of a ship with an acceleration $1\ m/s^2$ by a vertical rope attached to it. Find the tension in the rope. What is the tension in the rope when the box moves up with an uniform velocity of $1\ m/s$?

Solution:

Mass of box $(m) = 50\ kg$

Acceleration $(a) = 1\ m/s^2$

Tension $(T) = ?$

Here,

$T > mg$

$T - mg = ma$

or, $T = mg + ma$

or, $T = m(g+a)$

or, $T = 50(10+1)$

$\therefore T = 550\ N$

2^{nd} case:

Acceleration (a) = 0 m/s² [∵ velocity is uniform]

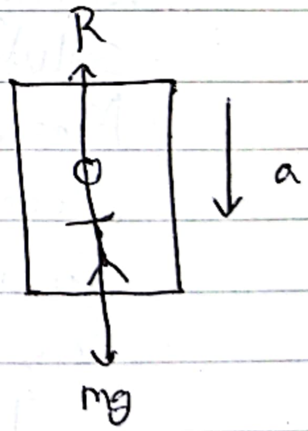
Tension (T') = ?

We know,

$$T' - mg = ma$$
$$\text{or, } T' = mg + ma$$
$$\text{or, } T' = 50 \times 10 + 0$$
$$\therefore T' = 500N$$

4. A 550 N physics student stands on the bathroom scale in an elevator. As the elevator starts moving, the scale reads ~~44~~ 450N. Draw a free body diagram and find the magnitude and direction of the acceleration of the elevator.

Solution:



Weight of the student (W) = 550 N

Acceleration (a) = ?

When moving downward.

Reactional force (R) = 450 N

We know,

$$W = mg$$
$$\text{or, } 550 = m \times 10$$
$$\therefore m = 55\text{kg}$$

Since, R < W then the elevator is moving downward with acceleration 'a'.

$$\text{So, } mg - R = ma$$
$$\text{or, } 55 \times 10 - 450 = 55 \times a$$

or, $a = \frac{550 - 450}{55}$

$\therefore a = 1.8\ m/s^2$

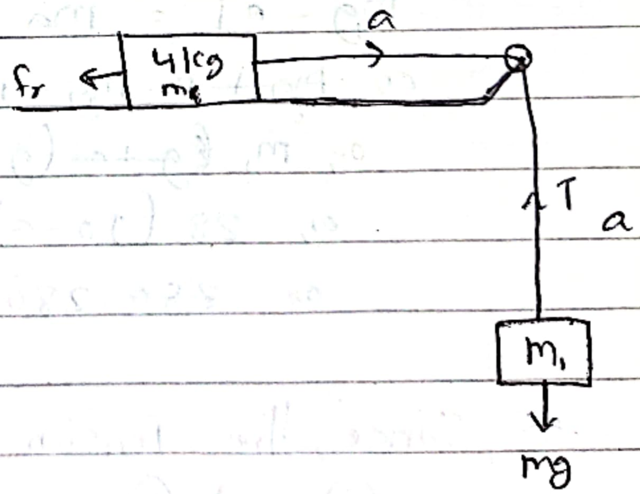
5. A light rope is attached to a block with mass 4 kg that rests on a frictionless horizontal surface. The horizontal rope ~~throug o~~ passes over a frictionless pulley and a block with mass 'm' is suspended from the other end. When the block is released the tension in the rope is 10N. Draw a free body diagram and calculate the acceleration of either block and the mass 'm' of the hanging block.

Solution.

Let common acc. be 'a'

Tension (T) = 10N

Let mass of hanging block ~~(m₁) =~~ be (m_1) = ?



For 4 kg mass

$T - f_r = m_1 a$

or, $10 - 0 = 4 \times a$

$\therefore a = 2.5\ m/s^2$

For hanging block

$mg - T = ma$

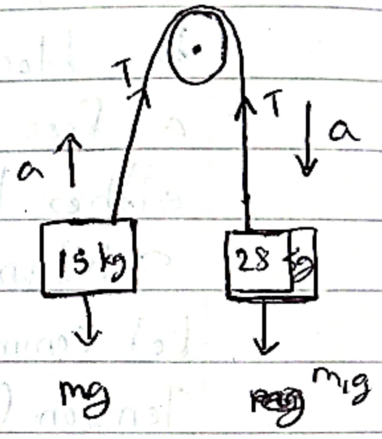
or $mg - ma = T$

or $m(10 - 2.5) = 10$

$\therefore m = 1.33\ kg.$

6. A 15 kg load of a brick hangs from one end of a load that passes over a frictionless pulley. A 28 kg counter weight is suspended from the other end of the rope as shown in the figure. The system is released from rest. Using free body diagram method, find the magnitude of upward acc. of the load and the tension in the rope while the load is moving.

Solution:



For the load of 15 kg

$T - mg = ma$

or, $T = mg + ma$

or, $T = m(g+a)$

or, $T = 15(10+a)$

or, $T = 150 + 15a$ —— (1)

For the load of 28 kg

$Mg - T = ma$

or, $mg - ma = T$

or, $m_1(g-a) = T$

or, $28(10-a) = T$

or, $280 - 28a = T$ —— (II)

Since, the Tension for both case are same. Equating eq. (i) and (II)

$150 + 15a = 280 - 28a$

or, $43a = 130$

$\therefore a = 3.02\ m/s^2$

Putting the value of 'a' in equation ① we get.

$T = 150 + 15 \times 3.02$

$T = 195.30$ N

7. Suppose you try to move a carate by tieing a rope around it and pulling the rope at angle 30° above the horizontal. What is the tension required to keep the carat moving with constant velocity. [Coef. of dynamic friction = 0.40]

Solution.

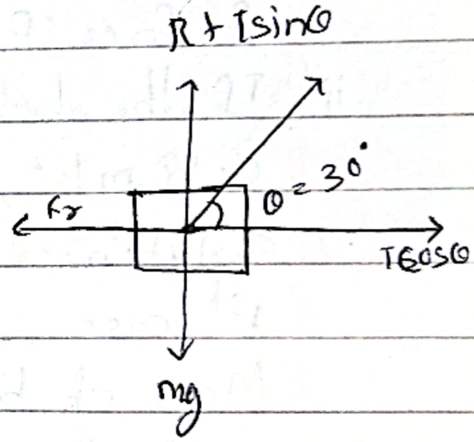
Given,

Weight of Carate (W) = 500N

Coefficient of friction (μ) = 0.40

Angle made by rope with horizontal (θ) = 30°

Tension (T) = ?



We know,

$F_r = \mu R$

or, $\mu = \frac{f_r}{R}$

or, $0.40 = \frac{T\cos\theta}{R}$ —①

Also,

$R + T\sin\theta = mg$

or, $R + T\sin 30° = 500$

or, $R = 500 - T\sin\theta$ —⑪

from ① and ⑪

$\frac{T\cos\theta}{0.40} = 500 - T\sin\theta$

or, $\frac{T\cos\theta}{0.40} - T\sin\theta \cdot 0.40 = 500$

or, $T\cos 30^\circ - T\sin 30^\circ \cdot 0.40 = 200$

or $T\left(\frac{\sqrt{3}}{2} - \frac{1}{5}\right) = 200$

$\therefore T = 188.6\ N$

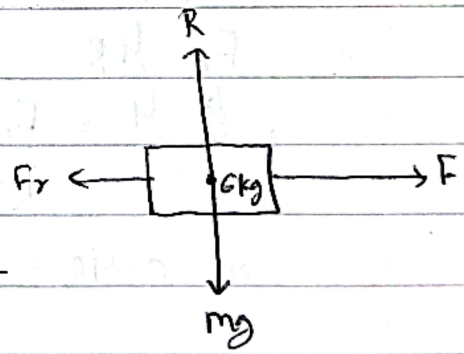
8. In a physics lab experiment a 6 kg box is pushed across a flat table by a horizontal force F.

i. If the box is moving at constant speed, 0.35 m/s and the coefficient of kinetic friction is 0.12, find the magnitude of force 'F'.

ii. If the box is speeding up with a constant acceleration of $0.28\ m/s^2$, what will be the magnitude of force 'F'

Solution:

1st case



Mass of box (m) = 6kg

Constant speed (s) = 0.35 m/s

Co-efficient of kinetic friction (μ) = 0.12

Force (F) = ?

We know,

$F - f_r = ma$

or, $F - f_r = m \times 0$ [speed = constant]

or $F = f_r$

or, $F = \mu R$

or, $F = 0.12 \times (m \times g)$

or, $F = 0.12 \times 6 \times 10$

$\therefore F = 7.2$ N

Also,

2^{nd} case

Acceleration (a) = ~~0.33~~ 0.18 m/s^2

~~Frictional force (f) = 7.2 N~~

Force (F') = ?

We know,

$F' - f_r = ma$

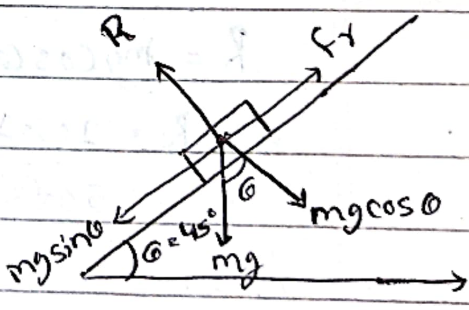
or, $F' - \mu R = ma$

or, $F' = 0.12 \times 6 \times 10 + 6 \times 0.18$

$\therefore F' = 8.28$ N

9. What would be the acceleration of a block sliding down an ~~a~~ inclined plane that makes an angle 45° with the horizontal. If the coefficient of sliding friction between two surfaces is 0.3.

Solution:



Angle of inclination (θ) = 45°

Coefficient of sliding friction (μ) = 0.3

Acceleration (a) = ?

We know,

$mg\sin\theta - f_r = ma$

or $mg\sin\theta - \mu R = ma$

or, $mg\sin\theta - \mu . mg\cos\theta = ma$

or, ~~g sin~~ $10 \times \sin 45° - 0.3 \times 10 \times \cos 45° = a$ (~~cos 30~~)

or, ~~5 - 2.5980~~ $5\sqrt{2} - \frac{3\sqrt{2}}{2}$

$\therefore a = 4.94$ m/s^2

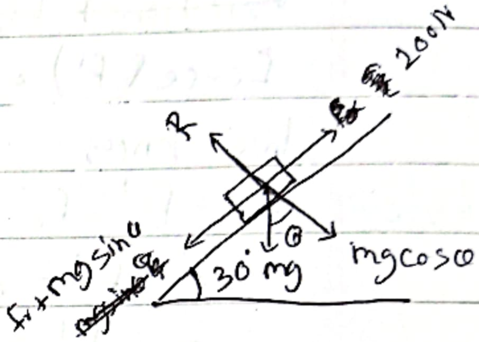
c/w

10. An iron block of mass 10 kg rest on wooden plane inclined at 30° to the horizontal. It is found that the least parallel force to the plane which causes the block to slide up the plane is 100 N. Calculate the co-efficient of sliding friction, between wood and iron

Solution:

Angle of inclination (θ) = 30°

Coefficient of friction (μ) = ?



Here,

$F_r + mg\sin\theta = 100$

or, $\mu R + mg\sin\theta = 100$

or, $\mu mg\cos\theta + mg\sin\theta = 100$

or, $\mu \times 10 \times 10 \times \frac{\sqrt{3}}{2} + 10 \times 10 \times \frac{1}{2} = 100$

or, $\mu \times 100\sqrt{3} + 100 = 400$

$\therefore \mu = 1.7$

$R = mg\cos\theta$ —— ①

or, $R = 10 \times 10 \times \cos 30°$

$= 50\sqrt{3}$

Also,

$F_r + mg\sin\theta = 100$

or, $F_r = 100 - mg\sin\theta$

or, $F_r = 50$

or, $\mu R = 50$

or, $\mu \times 50\sqrt{3} = 50$

$\therefore \mu = 0.577$ #

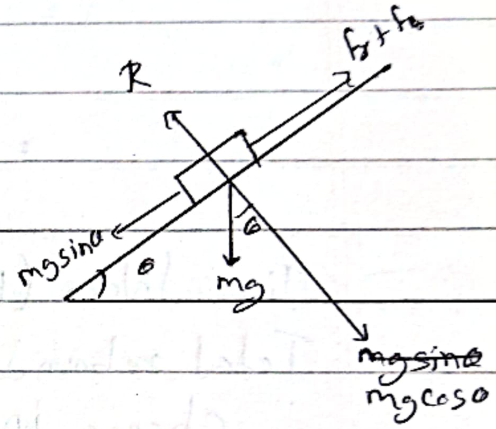
c/w

11. A block of wood of mass 150 gm rest on a inclined plane as in figure. If the coefficient of static friction between the surface in contact is 0.30, find

a. The greatest angle to which the plane may be ~~tit~~ tilted without the block sliping.

b. The force parallel to the plane necessary to prevent sliping when the angle of the plane with the horizontal is 30°.

Solution:



Case 1:

For the block not to slip;

$f_r = mg\sin\theta$

or, $\mu R = mg\sin\theta$

or, $\mu\, mg\cos\theta = mg\sin\theta$

or $\mu = \tan\theta$

or, $0.30 = \tan\theta$

$\therefore \theta = 16.6$

Case 2:

We know,

$mg\sin\theta = f_r + f$

or $mg\sin\theta = \mu R + f$

or $mg\sin\theta = f + \cancel{\mu mg\cos\theta}\ \mu mg\cos\theta$

or $mg\sin\theta - \mu mg\cos\theta = f$

or, $0.15\times 10\times\sin 30^\circ - 0.30\times 0.15\times 10\times\cos 30^\circ = f$

$\therefore f =$

12. A ball of mass 0.05kg is strikes a smooth wall normally 4 times in 2sec. with a velocity of 10m/s. Each time the ball rebounch with a same speed of 10m/s. Calculate the average force on the wall.

Solution:

We know,

$$f = ma$$
$$= m\left(\frac{v-u}{t}\right)$$
$$= \frac{mv - mu}{t}$$

Time taken (t) = ~~4sec~~ 2 sec

Total rebound = 4

∴ Change in momentum in 1 rebound

$$= mv - mu$$
$$= 0.05(v-u)$$
$$= 0.05[10-(-10)]$$
$$= 1Ns$$

Also,

Change in momentum in 4 sec = 4×1

$= 4N$

Now,

$$\text{Average force} = \frac{\text{change in Momentum in 4 rebound}}{\text{time}}$$
$$= \frac{4}{2}$$
$$= 2N$$

13. A ball 'A' of mass 0.1 kg moving with a velocity of 6 m/s collides directly with a ball 'B' of mass 0.2 kg at rest. Calculate their common velocity if both balls move off together. If 'A' had rebound with a velocity 2 m/s in opposite direction after collision, what would be the new velocity of B ?

Solution:

Mass of ball A $(m_1) = 0.1$ kg

Initial velocity of A $(u_1) = 6$ m/s

Initial velocity of B $(u_2) = 0$ m/s

Common final velocity $(V) = ?$

From the principal of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

or, $0.1 \times 6 + 0.2 \times 0 = 0.1 \cdot V + 0.2 \cdot V$

or, $\dfrac{0.1 \times 6}{0.1 + 0.2} = V$

$\therefore V = 2$ m/s

Case II

Final velocity of ball ~~of~~ A $(V_1) = -2$ m

We know,

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

or, $0.1 \times 6 + 0.2 \times 0 = 0.1 \times (-2) + 0.2 \times V_2$

or, $\dfrac{0.6 + 0.2}{0.2} = V_2$

$\therefore V_2 = 4$ m/s

c/w

14. A bullet of mass 20gm fired horizontally into a suspended stationary wooden block of mass 380 gm with a velocity of 200 m/s. What is the common velocity of the bullet and block, if the bullet is embedded in the block. If the block and the bullet experience a constant opposing force of 2N, find the time taken by them, to come to rest.

Solution:

Mass of bullet $(m_1) = 0.02$ kg

Mass of block $(m_2) = 0.38$ kg

Initial velocity of bullet $(u_1) = 200$ m/s

Initial Velocity of block $(u_2) = 0$ m/s

Common Velocity $(v) = ?$

Time taken to come to rest $(t) = ?$

From principal of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or, $0.02 \times 200 + 0.38 \times 0 = 0.02 \cdot V + 0.38 \cdot V$

or, $V = \dfrac{0.02 \times 200 + 0}{0.02 + 0.38}$

$\therefore V = 10$ m/s

Also,

we know,

~~Acceleration given (a) =~~ $F = ma$

or, $F = (m_1 + m_2) \cdot a$

or, $a = \dfrac{2}{0.38 + 0.02}$

$\therefore a = 5$ m/s

We know,

$a = \dfrac{V - v}{t}$

or, $t = \dfrac{0 - 10}{-5}$

$\therefore t = 2$ sec.

15. A bullet of mass 10gm travelling horizontally with a Velocity of 300 m/s strike a block of wood of mass 290 gm which rests on rough horizontal floor. After impact, the block and the bullet move together and come to rest when the block has travelled the distance of 15 m. Calculate the coefficient of sliding friction between the block and the floor.

Solution:

Mass of bullet $(m_1) = 0.01$ kg

Mass of wood $(m_2) =$ ~~0.3 kg~~ 0.29 kg

Initial velocity of bullet $(u_1) = 300$ m/s

Initial velocity of block $(u_2) = 0$ m/s

Common velocity of block and bullet $(v) = ?$

We know,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

or, $0.01 \times 300 + \cancel{0.29 \times 0}\ 0.29 \times 0 = m_1 . v + m_2 . v$ [both have same velocity]

or, $3 = (0.01 + 0.29) . v$

$\therefore v =$ ~~9.67 m/s~~ 10 m/s

Now,

from Conservation of Energy, we have

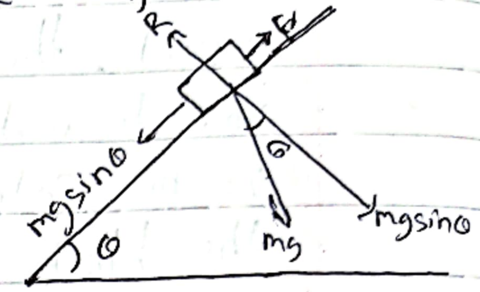
$$\frac{1}{2} mv^2 = \mu M . g . d$$

or, $\frac{1}{2} v^2 = \cancel{\mu\, g . d}\ \mu . g . d$

or, $\frac{1}{2} \times 100 = \mu . 10 \times 15$

$\therefore \mu = 0.33$

16. A 8 kg of ice, released from rest at the top of a 1.50 m long frictionless ramp, reaching a speed of 2.50 m/s at the bottom. What is the angle between the ramp and the horizontal.



Solution:

Mass of ice (m) = 8 kg

Initial velocity (u) = 0 m/s

Final velocity (v) = 2.50 m/s

Length of ramp (s) = 1.50 m

Angle of inclination (θ) = ?

friction (f) = 0

We know,

$$V^2 = u^2 + 2as$$

or, $(2.50)^2 = 0^2 + 2 \times a \times 1.50$

or, $\frac{6.25}{2 \times 1.50} = a$

$\therefore a = 2.08 \ m/s^2$

Now,

$mg\sin\theta - f_r = ma$

or, $8 \times 10 \times \sin\theta = 8 \times a$

or, $10\sin\theta = 2.08$

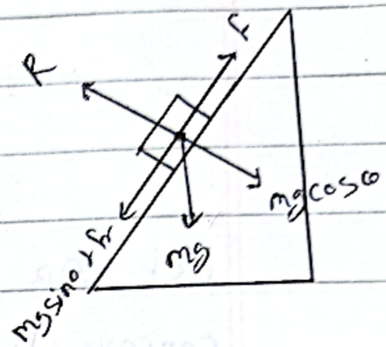
or, $\sin\theta = 0.208$

or, $\theta = \sin^{-1}(0.208)$

$\therefore \theta = 12.02°$

12. A 650 kW power engine of a vehicle of mass 1.5×10^5 kg is rising on an inclined plane of inclination 1 in 100 with a constant speed of 60 km/hr. Find the frictional force between the wheels of the vehicle and plane.

Solution:



Given,

Power (P) = 650 kW

$= 650000$ W

Mass of vehicle (m) = 1.5×10^5 kg

Uniform velocity (v) = 60 km/hr

$= \frac{60 \times 1000}{60 \times 60}$

$= 16.667$ m/s

~~Angle of inclination (θ) = 1/100 0.57~~ $\sin\theta = 1/100$

Frictional force (fr) = ?

We know,

Power (P) = $F \times v$

or $650000 = (mg\sin\theta + f_r) \times v$

or, $650000 = (1.5 \times 10^5 \times 10 \times \sin 1/100 + f_r) \times 16.667$ (0.01)

or, $\frac{650000}{16.667} - 1.5 \times 10^5 \times 10 \times \cancel{\sin(1/100)} = \cancel{f_r \sin(0.57)} = f_r$

$\therefore f_r = \cancel{2399.92\ N}$ 23992.2 N