RHS proved.

Exercise - 7.5

General section:

1. Solve the following equations ($0^\circ \leq \theta \leq 180^\circ$)

a. $\sqrt{2}\sin\theta = 1$

Solution:

$\sqrt{2}\sin\theta = 1$

or $\sin\theta = \frac{1}{\sqrt{2}}$

or $\sin\theta = \sin 45^\circ$

$\therefore \theta = 45^\circ$

Here, θ = +ve

So, $\theta = (180^\circ - \theta 45) = 135^\circ$

b. $2\cos\theta - 1 = 0$

or $\cos\theta = \frac{1}{2}$

or $\cos\theta = \cos 60^\circ$

$\therefore \theta = 60^\circ$

Also,

$\theta =$

c. $\sqrt{3}\ \tan\theta - 1 = 0$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^\circ$

$\therefore \theta = 30^\circ$

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d. $\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = -1$

$\Rightarrow \tan\theta = -\tan 45^\circ$

So,

$\theta = (180^\circ - 45^\circ) = 135^\circ$

e. $2\cos\theta + 1 = 0$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \cos\theta = -\cos 60^\circ$

Here, θ = -ve, it lies in 2nd and 3rd quadranth.

$\theta = (180^\circ - 60^\circ) = 120^\circ$

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f. $\sin^2\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \pm\sqrt{1/2}$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$

Taking (+ve) sign,

$\sin\theta = \frac{1}{2}$

$\sin\theta = \sin 30$

$\therefore \theta = 30^\circ$

Also,

$\theta = (180^\circ - 30^\circ) = 150^\circ$

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g. $4\cos^2\theta = 3$

or, $\cos^2\theta = \frac{3}{4}$

or, $\cos\theta = \pm\sqrt{3/4}$

or, $\cos\theta = \pm\frac{\sqrt{3}}{2}$

Taking (+ve) sign

$\cos\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \cos 30^\circ$

$\therefore \theta = 30^\circ$

~~As, θ = +ve~~

~~θ = {~~

h. $\tan^2\theta = 1$

or, $\tan\theta = \sqrt{1}$

or, $\tan\theta = 1$

or, $\tan\theta = \tan 45^\circ$

$\therefore \theta = 45^\circ$

i. $\sqrt{2}\sec\theta + 2 = 0$

or, $\sec\theta = \frac{-2}{\sqrt{2}}$

or, $\sec\theta = -\frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}}$

or, $\sec\theta = -\sqrt{2}\,(-45^\circ)$

Here, $\sec\theta$ = -ve, θ lies in 2nd and 3rd quadrant.

$\theta = (180^\circ - 45^\circ) = 135^\circ$

j. $2 + \sqrt{3}\,\mathrm{cosec}\,\theta = 0$

or, $\mathrm{cosec}\,\theta = -\frac{2}{\sqrt{3}}$

or, The value of θ is not possible within $0^\circ < \theta \leq 180^\circ$

2. Solve: $(0^\circ \leq \theta \leq 180^\circ)$

a. $4\sin\theta = 3\operatorname{cosec}\theta$

or $4\sin\theta = 3\dfrac{1}{\sin\theta}$

or $4\sin^2\theta = 3$

or $\sin\theta = \sqrt{3/4}$

or $\sin\theta = \pm\dfrac{\sqrt{3}}{2}$

or, $\sin\theta = \sin 60^\circ$ (+ve only)

$\theta = 60^\circ$

θ is +ve, So,

$\theta = (180^\circ - 60) = 120^\circ$

$\therefore \theta = 60, 120^\circ$

b. ~~$4\sin^2\theta - 8\cos\theta + 1 = 6$~~ $\cos 2\theta = \sin\theta$

Solution:

$\cos 2\theta = \sin\theta$

or $1 - 2\sin^2\theta = \sin\theta$

or $\sin\theta + 2\sin^2\theta - 1 = 0$

or $2\sin^2\theta + \sin\theta - 1 = 0$

or $2\sin^2\theta + (2-1)\sin\theta - 1 = 0$

or $2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

or $2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$

or $(2\sin\theta - 1)(\sin\theta + 1) = 0$

Either

$2\sin\theta - 1 = 0$

or, $\sin\theta = \dfrac{1}{2}$

$\therefore \theta = 30^\circ$

OR

$\sin\theta + 1 = 0$

or $\sin\theta = -1$

$\therefore \theta = -90^\circ$

~~(not possible)~~

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As, θ is (+ve), it lies in 2nd quadrant

or, $\theta = (180^\circ - 30^\circ) = 150^\circ$

c. ~~Sin2θ~~ $\sin 2\theta - \sin\theta = 0$

or, $2\sin\theta \cdot \cos\theta - \sin\theta = 0$

or, $\sin\theta (2\cos\theta - 1) = 0$

or, $2\cos\theta - 1 = 0$

or, $\cos\theta = \frac{1}{2}$

or, $\cos\theta = \cos 60^\circ$

$\therefore \theta = 60^\circ$

d. $\tan\theta - \sin\theta = 0$

or, $\frac{\sin\theta}{\cos\theta} - \sin\theta = 0$

or, $\frac{\sin\theta - \sin\theta \cdot \cos\theta}{\cos\theta} = 0$

or, $\sin\theta (1 - \cos\theta) = 0$

or, $1 - \cos\theta = 0$

or, $\cos\theta = 1$

or, $\cos\theta = \cos 0^\circ$

$\therefore \theta = 0^\circ$

e. $3\cot\theta - \tan\theta = 2$

or, $3\frac{1}{\tan\theta} - \tan\theta = 2$

or, $\frac{3 - \tan^2\theta}{\tan\theta} = 2$

or, $3 - \tan^2\theta = 2\tan\theta$

or, $\tan^2\theta + 2\tan\theta - 3 = 0$

or, $\tan^2\theta + 3\tan\theta - \tan\theta - 3 = 0$

or, $(\tan\theta + 3)(\tan\theta - 1) = 0$

Either $\tan\theta = -3$ (Not possible)

OR $\tan\theta = 1$

or, $\tan\theta = \tan 45^\circ$

$\therefore \theta = 45^\circ$

f. $\tan\theta + \cot\theta = 2$

or, $\tan\theta + \frac{1}{\tan\theta} = 2$

or, $\frac{\tan^2\theta + 1}{\tan\theta} = 2$

or, $\tan^2\theta - 2\tan\theta + 1 = 0$

or, $(\tan\theta - 1)^2 = 0^2$

or, $\tan\theta - 1 = 0$

or, $\tan\theta = 1$

or, $\tan\theta = \tan 45^\circ$

$\therefore \theta = 45^\circ$

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g. $\sec\theta \cdot \tan\theta = \sqrt{2}$

or $\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = \sqrt{2}$

or, $\sin\theta = \sqrt{2}\cos^2\theta$

or $\sqrt{2}\cos^2\theta - \sin\theta = 0$

or, $\sqrt{2}(1 - \sin^2\theta) - \sin\theta = 0$

or $\sqrt{2} - \sqrt{2}\sin^2\theta - \sin\theta = 0$

or $\sqrt{2}\sin^2\theta + \sin\theta - \sqrt{2} = 0$

or, $\sqrt{2}\sin^2\theta + (2-1)\sin\theta - \sqrt{2} = 0$

or $\sqrt{2}\sin^2\theta + 2\sin\theta - \sin\theta - \sqrt{2} = 0$

or $\sqrt{2}\sin\theta(\sin\theta + \sqrt{2}) - 1(\sin\theta + \sqrt{2}) = 0$

or, $(\sin\theta + \sqrt{2})(\sqrt{2}\sin\theta - 1) = 0$

Either

$\sin\theta + \sqrt{2} = 0$

or $\sin\theta = -\sqrt{2}$

[Impossible]

OR

$\sqrt{2}\sin\theta - 1 = 0$

or $\sin\theta = \frac{1}{\sqrt{2}}$

or $\sin\theta = \sin 45^\circ$

$\therefore \theta = 45^\circ$

Also, $\theta = (180^\circ - 45^\circ) = 135^\circ$

h. $\text{Cosec}\theta - 2\sin\theta = 1$

or $\frac{1}{\sin\theta} - 2\sin\theta = 1$

or, $1 - 2\sin^2\theta = \sin\theta$

or $2\sin^2\theta + \sin\theta - 1 = 0$

or $2\sin^2\theta + (2-1)\sin\theta - 1 = 0$

or $2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

or $2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$

or $(\sin\theta + 1)(2\sin\theta - 1) = 0$

Either

$\sin\theta + 1 = 0$

or $\sin\theta = -1$

or $\sin\theta = -\sin 90^\circ$

$\therefore \theta = 90^\circ$

OR

$2\sin\theta - 1 = 0$

or, $\sin\theta = \frac{1}{2}$

or $\sin\theta = \sin 30^\circ$

$\therefore \theta = 30^\circ$

Also,

$\theta = (180^\circ - 30^\circ) = 150^\circ$

Also,

$\theta =$

---

i. $\sin\theta - \cos\theta = 0$

or $\sin\theta = \cos\theta$

or $\frac{\sin\theta}{\cos\theta} = 1$

or, $\tan\theta = 1$

or $\tan\theta = \tan 45^\circ$

$\therefore \theta = 45^\circ$

j. $\frac{\sqrt{2}}{\cos\theta} + 2 = 0$

or, $\frac{\sqrt{2} + 2\cos\theta}{\cos\theta} = 0$

or, $\sqrt{2} + 2\cos\theta = 0$

or, $\cos\theta = \frac{-\sqrt{2}}{2}$

or, $\cos\theta = \frac{-1}{\sqrt{2}}$

or $\cos\theta = -45^\circ$

Also,

$\theta = (180^\circ - 45^\circ) = 135^\circ$

k. $\cos 7\theta = \cos\theta - \sin 4\theta$

or, $\cos\theta - \cos 7\theta = \sin 4\theta$

or, $2\sin\frac{\theta+7\theta}{2} \cdot \sin\frac{7\theta-\theta}{2} = \sin 4\theta$

or, $2\sin 4\theta \cdot \sin 3\theta = \sin 4\theta$

or, $\sin 3\theta = \frac{\sin 4\theta}{2\sin 4\theta}$

or, $\sin 3\theta = \frac{1}{2}$

or, $\sin 3\theta = \sin 30^\circ$

or, $3\theta = 30^\circ$

or, $\theta = 10^\circ$

As, θ = +ve

$\theta = (180 - 10^\circ) = 170^\circ$

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1. $\sin\theta + \sin 2\theta + \sin 3\theta = 0$

or, $\sin\theta + \sin 3\theta = -\sin\theta$

or, $2\sin\frac{\theta+3\theta}{2} \cdot \cos\frac{\theta-3\theta}{2} = -\sin 2\theta$

or, $2\sin 2\theta \cdot \cos\theta = -\sin\theta - \sin 2\theta$

or, $\cos\theta = \frac{-\sin 2\theta}{2\sin 2\theta}$

or, $\cos\theta = -\frac{1}{2}$

or, $\cos\theta = -\cos 60^\circ$

$\therefore \theta = -60^\circ$

As, θ = -ve

Also, $\theta = (180^\circ - 60^\circ) = 120^\circ$

Creative Section:

3. Solve: $(0^\circ \leq \theta \leq 360^\circ)$

a. $7\sin^2\theta + 3\cos^2\theta - 4 = 0$

or, $7\sin^2\theta + 3(1-\sin^2\theta) - 4 = 0$

or, $7\sin^2\theta + 3 - 3\sin^2\theta - 4 = 0$

or, $4\sin^2\theta - 1 = 0$

or, $\sin^2\theta = \frac{1}{4}$

or, $\sin\theta = \pm\sqrt{\frac{1}{4}}$

or, $\sin\theta = \pm\frac{1}{2}$

or, $\sin\theta = \sin 30^\circ$

$\therefore \theta = 30^\circ$

Taking (+ve)

$\theta = (180^\circ - 30^\circ) = 150^\circ$

Taking (-ve)

$\theta = (180^\circ + 30^\circ) = 210^\circ$

Also, $\theta = (360^\circ - 30^\circ) = 330^\circ$

b. $4\sin^2\theta - 8\cos\theta + 1 = 0$

or, $4(1-\cos^2\theta) - 8\cos\theta + 1 = 0$

or, $4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

or, $4\cos^2\theta + 8\cos\theta - 5 = 0$

or, $4\cos^2\theta + (10-2)\cos\theta - 5 = 0$

or, $4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$

or, $2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$

or, $(2\cos\theta + 5)(2\cos\theta - 1) = 0$

Either

$\cos\theta = -\frac{5}{2}$

(Impossible)

OR

$2\cos\theta = 1$

or, $\cos\theta = \frac{1}{2}$ $(\cos 60^\circ)$

$\therefore \theta = 60^\circ$, Also, $\theta = (360^\circ - 60^\circ) = 300^\circ$

c. $6\sin^2\theta + 4\cos^2\theta = 5$

or, $6(1-\cos^2\theta) + 4\cos^2\theta = 5$

or, $6 - 6\cos^2\theta + 4\cos^2\theta = 5$

or, $6 - 2\cos^2\theta = 5$

or, $2\cos^2\theta = 6-5$

or, $\cos\theta = \pm\sqrt{\frac{1}{2}}$

or, $\cos\theta = \pm\frac{1}{\sqrt{2}}$

or, $\cos\theta = \cos 45^\circ$

$\therefore \theta = 45^\circ$

Taking (-ve)

As, $\theta = (+ve)$

$\theta = (180^\circ - 45^\circ) = 135^\circ$

$\theta = (360^\circ - 45^\circ) = 315^\circ$

$\theta = (180^\circ + 45^\circ) = 225^\circ$

d. $\cos^2\theta - \sin\theta = \frac{1}{4}$

or, $1 - \sin^2\theta - \sin\theta = \frac{1}{4}$

or, $1 - \frac{1}{4} = \sin^2\theta + \sin\theta$

or, $\sin^2\theta + \sin\theta - \frac{1}{4} = 0$

or, $4\sin^2\theta + 4\sin\theta - 1 = 0$

or, $4\sin^2\theta + (2+2)\sin\theta - 1 = 0$

or, $4\sin^2\theta + 2\sin\theta + 2\sin\theta - 1 = 0$

or, $2\sin\theta(2\sin\theta + 1)$

d. $\cos^2\theta - \sin\theta = \frac{1}{4}$

or, $1 - \sin^2\theta - \sin\theta = \frac{1}{4}$

or, $4 - 4\sin^2\theta - 4\sin\theta = 1$

or, $4\sin^2\theta + 4\sin\theta - 3 = 0$

or, $4\sin^2\theta + (6-2)\sin\theta - 3 = 0$

or, $4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

or, $2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

or, $(2\sin\theta + 3)(2\sin\theta - 1) = 0$

Either

$2\sin\theta + 3 = 0$

or, $\sin\theta = -\frac{3}{2}$

[Impossible]

OR

$2\sin\theta - 1 = 0$

or, $\sin\theta = \frac{1}{2}$

$\therefore \theta = 30^\circ$

As, θ = +ve

$\theta = (180^\circ - 30^\circ) = 150^\circ$

e. $2\sin^2\theta + 3\cos\theta = 3$

or, $2(1 - \cos^2\theta) + 3\cos\theta = 3$

or, $2 - 2\cos^2\theta + 3\cos\theta - 3 = 0$

or, $2\cos^2\theta - 3\cos\theta + 1 = 0$

or, $2\cos^2\theta - (2+1)\cos\theta + 1 = 0$

or, $2\cos^2\theta - 2\cos\theta - \cos\theta + 1 = 0$

or, $2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$

or, $(\cos\theta - 1)(2\cos\theta - 1) = 0$

Either

$\cos\theta - 1 = 0$

or, $\cos\theta = 1 \ (\cos 0^\circ)$

$\therefore \theta = 0^\circ$

Also, $\theta = (90^\circ + 0) = 90^\circ$

OR

$2\cos\theta - 1 = 0$

or, $\cos\theta = \frac{1}{2} \ (\cos 60^\circ)$

$\theta = 60^\circ$

Also, $\theta = (360^\circ - 60) = 300^\circ$

f. $\tan^2\theta - 3\sec\theta + 3 = 0$

or, $\sec^2\theta - 1 + 3 - 3\sec\theta = 0$

or, $\sec^2\theta - 3\sec\theta + 2 = 0$

or, $\sec^2\theta - (2+1)\sec\theta + 2 = 0$

or, $\sec^2\theta - 2 - \sec\theta + 2 = 0$

or, $\sec\theta(\sec\theta - 2) - 1(\sec\theta - 2) = 0$

or, $(\sec\theta - 1)(\sec\theta - 2) = 0$

Either

$\sec\theta - 1 = 0$

or, $\sec\theta = 1$

or, $\sec\theta = \sec 0^\circ$

$\therefore \theta = 0^\circ$

As, θ = tvc

$\theta = (90^\circ + 0^\circ) = 90^\circ$

$\theta = (360^\circ - 0^\circ) = 360^\circ$

OR

$\sec\theta - 2 = 0$

or, $\sec\theta = 2$ ~~[Impossible]~~

or, $\sec\theta = \sec 60^\circ$

$\therefore \theta = 60^\circ$

Also, $\theta = 180^\circ - (360^\circ - 60^\circ) = 300^\circ$

g. $1 + \cos\theta = 2\sin^2\theta$

or, $1 + \cos\theta = 2(1 - \cos^2\theta)$

or, $2 - 2\cos^2\theta = 1 + \cos\theta$

or, $2\cos^2\theta + \cos\theta - 1 = 0$

or, $2\cos^2\theta + (2-1)\cos\theta - 1 = 0$

or, $2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$

or, $2\cos\theta(\cos\theta + 1) - 1(\cos\theta + 1) = 0$

or, $(\cos\theta + 1)(2\cos\theta - 1) = 0$

Either

$\cos\theta + 1 = 0$

or, $\cos\theta = -1$

or, $\cos\theta = -\cos 0^\circ$

$\therefore \theta = -0^\circ$

OR

$2\cos\theta - 1 = 0$

or, $\cos\theta = \frac{1}{2}$

or, $\cos\theta = \cos 60^\circ$

$\therefore \theta = 60^\circ$

Also $\theta = (360^\circ - 60) = 300^\circ$

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h. $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$

or, $1 - \cos^2\theta - 2\cos\theta + \frac{1}{4}$

or, $4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

or, $4\cos^2\theta + 8\cos\theta - 5 = 0$

or, $4\cos^2\theta + (10-2)\cos\theta - 5 = 0$

or, $4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$

or, $2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$

or, $(2\cos\theta + 5)(2\cos\theta - 1) = 0$

Either

$\cos\theta = -\frac{5}{2}$

[Impossible]

OR

$\cos\theta = \frac{1}{2}$

or $\cos\theta = \cos 60^\circ$

$\therefore \theta = 60^\circ$

As, θ is positive

$\theta = (360^\circ - 60^\circ) = 300^\circ$

i. $3\cos^2\theta - 1 = 2\sin^2\theta$

$3(1 - \sin^2\theta) - 1 - 2\sin^2\theta = 0$

or, $3 - 3\sin^2\theta - 1 - 2\sin^2\theta = 0$

or, $2 - 5\sin^2\theta = 0$

or, $5\sin^2\theta = 2$

or, $\sin\theta = \sqrt{2/5}$

$\therefore \sin\theta = \sqrt{2/5}$

[No. Solution]

$3\cos^2\theta - 1 = 2\sin^2\theta$

or, $3\cos^2\theta - 1 = 2(1 - \cos^2\theta)$

or $3\cos^2\theta - 1 - 2 + 2\cos^2\theta = 0$

or $5\cos^2\theta - 3 = 0$

or $5\cos^2\theta = 3$

or $\cos^2\theta = \frac{3}{5}$

or, $\cos\theta = \sqrt{3/5}$

[No solution]

j. $\cot^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta = -1$

or, $\cot^2\theta + \sqrt{3}\cot\theta + \frac{1}{\sqrt{3}}\cot\theta + 1 = 0$

or, $\cot^2\theta + \sqrt{3}\cot\theta + \frac{1}{\sqrt{3}}\cot\theta + 1.\frac{\sqrt{3}}{\sqrt{3}} = 0$

or, $\cot\theta\left(\cot\theta + \sqrt{3}\right) + \frac{1}{\sqrt{3}}\left(\cot\theta + \sqrt{3}\right) = 0$

or $\left(\cot\theta + \sqrt{3}\right)\left(\cot\theta + \frac{1}{\sqrt{3}}\right) = 0$

Either

$\cot\theta = -\sqrt{3}$

or $\cot\theta = -\cot 30^\circ$

$\therefore \theta = (180 - 30) = 150^\circ$

Also, $360^\circ - 30^\circ = 330^\circ$

OR

$\cot\theta = -\frac{1}{\sqrt{3}}$

or $\cot\theta = -\cot 60^\circ$

$\therefore \theta = (180^\circ - 60^\circ) = 120^\circ$

$\theta = (360^\circ - 60^\circ) = 300^\circ$

k. $\tan^2\theta - (1 + \sqrt{3})\tan\theta + \sqrt{3} = 0$

or, $\tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$

or, $\tan\theta(\tan\theta - 1) - \sqrt{3}(\tan\theta - 1) = 0$

or, $(\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$

Either

$\tan\theta - 1 = 0$

or $\tan\theta = 1$

$\therefore \theta = 45^\circ$

Also,

$\theta = (180 + 45^\circ)$

$= 225^\circ$

OR

$\tan\theta - \sqrt{3} = 0$

or $\tan\theta = \sqrt{3}$

$\therefore \theta = 60^\circ$

Also,

$\theta = (180^\circ + 60^\circ)$

$= 240^\circ$

L. $3\tan^2\theta + 2\sqrt{3}\tan\theta - 3 = 0$

or, $3\tan^2\theta + (3\sqrt{3} - \sqrt{3})\tan\theta - 3 = 0$

or $3\tan^2\theta + 3\sqrt{3}\tan\theta - \sqrt{3}\tan\theta - 3 = 0$

or $3\tan\theta(\tan\theta + \sqrt{3}) - \sqrt{3}(\tan\theta + \sqrt{3}) = 0$

Or, $(\tan\theta + \sqrt{3})(3\tan\theta - \sqrt{3})$

Either

$\tan\theta + \sqrt{3} = 0$

or $\tan\theta = -\sqrt{3}$

or, $\tan\theta = -\tan 60^\circ$

$\therefore \theta = 60^\circ$

Also, $\theta = (180^\circ - 60^\circ) = 120^\circ$

$\theta = (360^\circ - 60^\circ) = 300^\circ$

OR

or, $3\tan\theta - \sqrt{3} = 0$

or $\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

or $\tan\theta = \tan 30^\circ$

$\therefore \theta = 30^\circ$

Also, $\theta = (180^\circ + 30^\circ) = 210^\circ$

4. Solve: $(0^\circ \leq \theta \leq 360^\circ)$

a. $\sin\theta + \cos\theta = \frac{1}{\sqrt{2}}$

or $(\sin\theta + \cos\theta)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$

or, $\sin^2\theta + 2\cdot\sin\theta\cdot\cos\theta + \cos^2\theta = \frac{1}{2}$

or $1 + \sin 2\theta = \frac{1}{2}$

or, $\sin 2\theta = \frac{1}{2} - 1$

or $\sin 2\theta = -\frac{1}{2}$

or $\sin 2\theta = \sin(180^\circ + 30^\circ)$

or, $\sin 2\theta = \sin 210^\circ$

or, $\theta = \frac{210}{2}$

$\therefore \theta = 105^\circ$

Also, $\sin 2\theta = \sin(360^\circ - 30^\circ)$

$\therefore \theta = 165^\circ$

Also, $\theta = (180^\circ + 105^\circ) = 285^\circ$

b. $\sin\theta + \cos\theta = \sqrt{2}$

or, $(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$

or, $\sin^2\theta + 2\cdot\sin\theta\cdot\cos\theta + \cos^2\theta = 2$

or, $1 + \sin 2\theta = 2$

or, $\sin 2\theta = 2 - 1$

or, $\sin 2\theta = 1$

or, $\sin 2\theta = \sin 90^\circ$

or, $2\theta = 90^\circ$

or, $\theta = 45^\circ$

As, θ = +ve

$\theta = (180^\circ - 45^\circ) = 135^\circ$

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c. $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

Sol.

or, $\frac{\sqrt{3}}{2}\cdot\sin\theta - \frac{1}{2}\cos\theta = \frac{\sqrt{2}}{2}$

or, $\cos 30^\circ\cdot\sin\theta - \sin 30^\circ\cdot\cos\theta = \frac{1}{\sqrt{2}}$

or, $\sin\theta\cdot\cos 30^\circ - \cos\theta\cdot\sin 30 = \frac{1}{\sqrt{2}}$

or, $\sin(\theta - 30^\circ) = 45^\circ \sin 45^\circ$

or, $\theta - 30^\circ = 45^\circ$

$\therefore \theta = 75^\circ$

Also, $\theta = (180^\circ - 75^\circ) = 105^\circ$

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d. $\sqrt{2}\sec\theta + \tan\theta = 1$

or, $(\sqrt{2}\sec\theta)^2 = (1 - \tan\theta)^2$

or, $2\sec^2\theta = 1 - 2\tan\theta + \tan^2\theta$

or, $2(1 + \tan^2\theta) = 1 - 2\tan\theta + \tan^2\theta$

or, $2 + 2\tan^2\theta = 1 - 2\tan\theta + \tan^2\theta$

or, $2\tan^2\theta - \tan^2\theta + 2\tan\theta + 2 - 1 = 0$

$\Rightarrow \tan^2\theta + 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta + 1)^2 = 0^2$

$\Rightarrow \tan\theta = -1$

$\Rightarrow \tan\theta = -\tan 45^\circ$

$\Rightarrow \tan\theta = \tan(180^\circ - 45^\circ)$

$\Rightarrow \theta = 180^\circ - 45^\circ$

$= 135^\circ$

Also, $\theta = (360^\circ - 45^\circ)$

$= 315^\circ$

---

e. $\sin\theta + \cos\theta = 1$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 1^2$

$\Rightarrow \sin^2\theta + 2\sin\theta \cdot \cos\theta + \cos^2\theta = 1$

$\Rightarrow 1 + \sin 2\theta = 1$

$\Rightarrow \sin 2\theta = 1 - 1$

$\Rightarrow \sin 2\theta = 0$

$\Rightarrow \sin 2\theta = \sin 0^\circ$

$\Rightarrow 2\theta = 0^\circ$

$\therefore \theta = 0^\circ$

Again,

$\sin 2\theta = \sin 0^\circ$

$\Rightarrow \sin 2\theta = \sin(360^\circ + 0^\circ)$

$\Rightarrow 2\theta = 360^\circ$

$\therefore \theta = 180^\circ$

Also,

$\sin 2\theta = \sin 0^\circ$

$\Rightarrow \sin 2\theta = \sin(180 - \theta)$

$\Rightarrow 2\theta = 180^\circ$

$\therefore \theta = 90^\circ$

f. $\cos\theta - \sqrt{3}\sin\theta = 1$

or, $\cos\theta - 1 = \sqrt{3}\sin\theta$

or, $(\cos\theta - 1)^2 = (\sqrt{3}\sin\theta)^2$

or, $\cos^2\theta - 2\cos\theta + 1 = 3\sin^2\theta$

or, $\cos^2\theta - 3\sin^2\theta - 2\cos\theta + 1 = 0$

or, $\cos^2\theta - 3(1 - \cos^2\theta) - 2\cos\theta + 1 = 0$

or, $\cos^2\theta - 3 + 3\cos^2\theta - 2\cos\theta + 1 = 0$

or, $4\cos^2\theta - 2\cos\theta - 2 = 0$

or, $4\cos^2\theta - (4-2)\cos\theta - 2 = 0$

or, $4\cos^2\theta - 4\cos\theta + 2\cos\theta - 2 = 0$

or, $4\cos\theta(\cos\theta - 1) + 2(\cos\theta - 1) = 0$

or, $(\cos\theta - 1)(4\cos\theta + 2) = 0$

Either

$\cos\theta - 1 = 0$

or $\cos\theta = 1$

or $\cos\theta = \cos 0^\circ$

$\therefore \theta = 0^\circ$

Also,

$\theta = (360^\circ - 0^\circ) = 360^\circ$

OR

$4\cos\theta + 2 = 0$

or $\cos\theta = \frac{-2}{4}$

or $\cos\theta = -1/2$

or $\cos\theta = -\cos 60^\circ$

$\therefore \theta = 60^\circ$

~~Also, $\theta = (360^\circ - 60^\circ$~~

Also, $\theta = (180^\circ - 60^\circ) = 120^\circ$

$\theta = (180 + 60) = 240^\circ$

g. $\sqrt{3}\cos\theta + \sin\theta = \sqrt{3}$

or, $(\sqrt{3}\cos\theta - \sqrt{3})^2 = (-\sin\theta)^2$

or, $3\cos^2\theta - 6\cos\theta + 3 = \sin^2\theta$

or, $3\cos^2\theta - 6\cos\theta + 3 = 1 - \cos^2\theta$

or $3\cos^2\theta - 6\cos\theta + 3 - 1 + \cos^2\theta = 0$

or, $4\cos^2\theta - 6\cos\theta + 2 = 0$

or, $4\cos^2\theta - (4+2)\cos\theta + 2 = 0$

or, $4\cos^2\theta - 4\cos\theta - 2\cos\theta + 2 = 0$

Or, $4\cos\theta(\cos\theta-1)-2(\cos\theta-1)=0$

Or, $(\cos\theta-1)(4\cos\theta-2)=0$

Either

$\cos\theta-1=0$

Or, $\cos\theta=1$

Or, $\cos\theta=\cos 0^\circ$

$\therefore \theta=0$

Also, $\theta=(360^\circ-0)$

$=360^\circ$

OR

$4\cos\theta-2=0$

Or, $\cos\theta=\frac{2}{4}$

Or, $\cos\theta=\frac{1}{2}$

Or, $\cos\theta=\cos 60^\circ$

$\therefore \theta=60^\circ$

Also, $\theta=(180^\circ-60^\circ)=120^\circ$

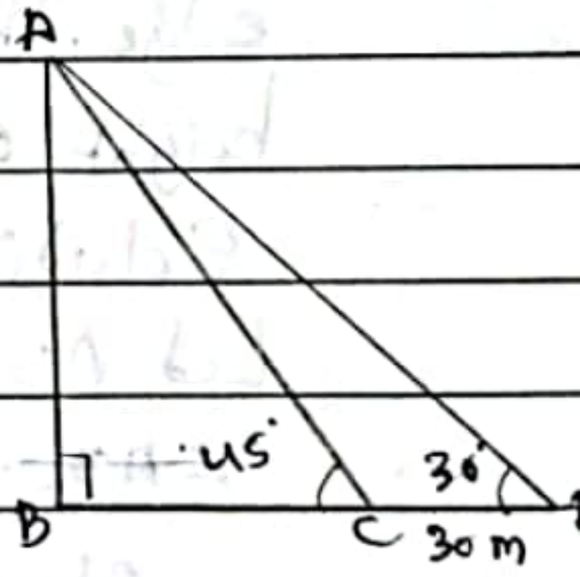
$\theta=(360^\circ-60^\circ)=300^\circ$

Height and Distance

Exercise - 7.6

1.a. The angle of elevation of the top of a tower from a point was observed to be 45°. On walking 30m away from that point it was found to be 30°. find the height of the tower.

Solution



Here,

Let AB be the tower;

Angle of elevation, $\angle ACB=45^\circ$, $\angle ADB=30^\circ$

$CD=30m$

Now, from right angled triangle, $\triangle ABC$

$$\tan\theta=\frac{AB}{BC}$$

Or, $\tan 45^\circ=\frac{AB}{BC}$

Or, $1=\frac{AB}{BC}$

$\therefore AB=BC$ —— (i)