Exercise 2.1

1. Find the product of:

a. $3x^3 + 5x^2 + 9x + 8$ and $3x + 4$

$= (3x^3 + 5x^2 + 9x + 8)(3x + 4)$

$= 3x(3x^3 + 5x^2 + 9x + 8) + 4(3x^3 + 5x^2 + \cancel{9x}\ 9x + 8)$

$= 9x^4 + 15x^3 + 27x^3 + 24x + 12x^3 + 20x^2 + 9x^4 + 27x^3 + 47x^2 + 60x$
$+ 32$

$= 9x^4 + 27x^3 + 47x^2 + 60x + 32$

b. $4x^2 + 5x + 7$ and $x + 1$

Solution:

$(x+1)(4x^2 + 5x + 7)$

$= 4x^3 + 5x^2 + 7x + 4x^2 + 5x + 7$

$= 4x^3 + 9x^2 + 12x + 7$

c. $3x^3 - 2x^2 + 5x - 1$ and $2x - 3$

Solution:

$(2x - 3)(3x^3 - 2x^2 + 5x - 1)$

$= 6x^4 - 4x^3 + 10x^2 - 2x - 9x^3 + 6x^2 - 15x + 3$

$= 6x^4 - 13x^3 + 16x^2 - 17x + 3$

2. Divide the polynomial $f(x)$ by $g(x)$ if.

a. $f(x) = 8x^4 + 2x^3 - 3x - 8$

$g(x) = 2x^2 - 3$

Solution:

$2x^2 - 3 \,\big)\, 8x^4 + 2x^3 - 3x - 8 \,\big(\, 4x^2 + x + 6$

$8x^4 \quad - 12x^2$

$(-) \quad (+)$

$2x^3 + 12x^2$

$2x^3 - 3x$
$(-) \quad (+)$

$12x^2 + 3x$
$12x^2 - 18$
$- \quad (+)$

$\cancel{3x} + 18 - \cancel{3x}$
$18 - 8$
10

b. $f(x) = 10x^2 - 3 - 6x + 3x^3 \quad g(x) = 4-x$
Solution:

~~$4-x \,\big)\, 10x^2 - 3 - 6x + 3x^3 \,\big($~~

$4-x \,\big)\, 3x^3 + 10x^2 - 6x - 3 \,\big(\, -3x^2 - 22x - 82$
$3x^3 - 12x^2$
$(-) \quad (+)$

$22x^2 - 6x$
$22x^2 - 88x$
$(-) \quad (+)$

$82x - 3$
$82x - 328$
$(-) \quad (+)$

325

Quotient = $-3x^2 - 22x - 82$
Remainder = 325

c. $f(x) = 4x^2 + 32x + 15 \qquad g(x) = 4x + 3$

Solution:

$$4x+3 \overline{) \; 4x^2 + 32x + 15 \;} \; \left(x + \frac{29}{4} \right.$$

$$\cancel{4x^2} + 3x$$

$$(-) \qquad (-)$$

$$29x + 15$$

$$29x + \frac{87}{4}$$

$$(-) \qquad (-)$$

$$15 - \frac{87}{4}$$

$$0 \quad -\frac{27}{4}$$

$\therefore$ Quotient $= x + \frac{29}{4}$

Remainder $= -\frac{27}{4}$

3a. If $2x^3 - 9x^2 + 5x - 5 = (2x-3)\, q(x) + R$, find quotient and rem

Solution

$$2x-3 \overline{) \; 2x^3 - 9x^2 + 5x - 5 \;} \; \left(x^2 - 3x - 2 \right.$$

$$\cancel{2x^3} - 3x^2$$

$$(-) \qquad (+)$$

$$\cancel{-6x^2} + 5x$$

$$-\cancel{6x^2} + 9x$$

$$(+) \qquad (-)$$

$$-\cancel{4x} - 5$$

$$-\cancel{4x} + 6$$

$$(+) \qquad (-)$$

$$-11$$

$\therefore$ Quotient $= x^2 - 3x - 2$

$\therefore$ Remainder $= -11$

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b. Find $q(x)$ and R if $3x^4 - 8x^2 - 20 = (x^2-2)\,q(x) + R$

Solution:

$$x^2-2 \,\big)\, 3x^4 - 8x^2 - 20 \,\big(\, 3x^2 - 2$$

$$3x^4 - 6x^2$$

$$(-) \quad (+)$$

$$-2x^2 - 20$$

$$-2x^2 + 4$$

$$(+) \quad (-)$$

$$-24$$

Quotient $= 3x^2 - 2$

Remainder $= -24$

Exercise - 2.2

General Section

1. Divide the following and hence find $q(x)$ and R

a. $x^2 + 5x + 6$ by $x + 2$

Solution

$$x+2 \,\big)\, x^2 + 5x + 6 \,\big(\, x + 3$$

$$x^2 + 2x$$

$$(-) \quad (-)$$

$$3x + 6$$

$$3x + 6$$

$$(-) \quad (-)$$

$$0$$

Quotient $= x + 3$

Remainder $= 0$

b. $3x^3 - 2x^2 - 13x + 10$ by $x - 2$

Solution

$$x-2 \,\big)\, 3x^3 - 2x^2 - 13x \,\big(\, 3x^2 + 4x - 5$$

$$3x^3 - 6x^2$$

$$(-) \quad (+)$$

$$4x^2 - 13x$$

$$4x^2 - 8x$$

$$(-) \quad (+)$$

$$-5x$$

$$-5x$$

$$(+)$$

$$0$$

Quotient = $3x^2 + 4x - 5$

Remainder = 0

---

c. $8x^4 + 2x^3 - 3x - 8$ by $2x^2 - 3$

Solution

$$2x^2 - 3 \,\big)\, 8x^4 + 2x^3 - 3x - 8 \,\big(\, 4x^2 + x + 6$$

$$8x^4 - 12x^2$$

$$(-) \quad (+)$$

$$2x^3 + 12x^2 - 3x - 8$$

$$2x^3 - 3x$$

$$(-) \quad (+)$$

$$12x^2 + \cancel{3x} - \cancel{3x} - 8$$

$$12x^2 - 8$$

$$12x^2 - 18$$

$$(-) \quad (+)$$

$$10$$

Quotient = $4x^2 + x + 6$

Remainder = 10

2. Using synthetic division.

a. divide $f(x) = 3x^2 - 4x + 6$ by $g(x) = x - 3$ and hence calculate the quotient and the remainder.

Solution:

$f(x) = 3x^2 - 4x + 6$

$g(x) = x - 3$

comparing $x-3$, with $x-a$, $a = 3$

Now,

$$\begin{array}{r|rrr} & 3 & -4 & 6 \\ 3 & & 9 & 15 \\ \hline & 3 & 5 & 21 \end{array}$$

Quotient $= 3x + 5$

Remainder $= 21$

b. Divide $f(y) = 2y^4 + 5y^3 + 6$ by $g(y) = y + 2$ and hence find quotient and the remainder.

Solution:

$f(y) = 2y^4 + 5y^3 + 6$, $g(y) = y + 2$

Comparing $y+2$ with $x-a$, $a = -2$

Now,

$$\begin{array}{r|rrrrr} & 2 & 5 & 0 & 0 & 6 \\ -2 & & -4 & -2 & 4 & -8 \\ \hline & 2 & 1 & -2 & 4 & -2 \end{array}$$

Quotient $= 2y^3 + y^2 - 2y + 4$

Remainder $= -2$

c. Divide $f(z) = z^3 - 19z - 30$ by $g(z) = z - 5$ and hence calculate the quotient and remainder.

Solution:

Comparing $z-5$ with $x-a$, $a = 5$

Now,

5	1	0	-19	-30
		5	25	30
	1	5	6	0

Quotient $= z^2 + 5z + 6$

Remainder $= 0$

---

3. Using synthetic division, find the quotient and the remainder.

a. $4x^3 + 2x^2 - 4x + 3$ is divided by $2x+3$

Solution:

Comparing $2x+3$ with $x-a$, $a = \frac{-3}{2}$

$-3/2$	4	2	-4	3
		-6	6	-3
	4	-4	2	0

Quotient $= 4x^2 - 4x + 2$

Remainder $= 0$

---

b. $2x^3 - 9x^2 + 4x - 6$ is divided by $2x - 3$

Solution:

Comparing $2x-3$ with $x-a$, $a = -3/2$

Now,

$-3/2$	2	-9	4	-6
		3	18	-33
	2	-12	22	-39

$$\begin{array}{c|cccc} \frac{3}{2} & 2 & -9 & 4 & -6 \\ & \downarrow & 3 & -9 & -\frac{15}{2} \\ \hline & 2 & -6 & -5 & -\frac{27}{2} \end{array}$$

Quotient = $2x^2 - 6x - 5$

Remainder = $\frac{-27}{2}$

---

c. $12x^6 - 9x^2 - 5$ is divided by $2x^2 + 1$

Solution:

~~Com~~ Let,

$f(x) = 12x^6 - 9x^2 - 5$

$g(x) = 2x^2 + 1$

Let 'x^2' be y

$f(y) = \cancel{12y}\ 12y^3 - 9y - 5$

$g(y) = 2y + 1$

Now,

$g(y) = 2\left(y + \frac{1}{2}\right)$

Comparing $y + \frac{1}{2}$ with $y - a$, we get $a = \frac{-1}{2}$

$$\begin{array}{c|ccccccc} & 12 & 0 & 0 & 0 & -9 & -0 & -5 \\ \frac{-1}{2} & \downarrow & -6 & 3 & \frac{-3}{2} & -\frac{3}{4} & \frac{39}{8} & -\frac{39}{16} \\ \hline & 12 & -6 & 3 & \frac{-3}{2} & -\frac{39}{4} & \frac{39}{8} & \end{array}$$

$$\begin{array}{c|cccc} \frac{-1}{2} & 12 & 0 & -9 & -5 \\ & \downarrow & -6 & 3 & 3 \\ \hline & 12 & -6 & -6 & -2 \end{array}$$

Now,

Quotient (Q) = $\frac{1}{2}(12y^2 - 6y - 6)$

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$= 6y^2 - 3y - 3$

$= 6(x^2)^2 - 3(x^2) - 3$

$= 6x^4 - 3x^2 - 3$

Also,

Remainder (R) $= -2$

4a. Find the remainder when $3x^3 - 5x^2 + 2x - 3$ is divided by $x - 2$ with the help of remainder theorem.

Solution:

$f(x) = 3x^3 - 5x^2 + 2x - 3$

$g(x) = x - 2$

Comparing $x-2$ with $x-a$, $a = 2$

Now,

$f(a) = f(2) = 3\times2^3 - 5\times2^2 + 2\times2 - 3$

$= 3\times8 - 5\times4 + 4 - 3$

$= 24 - 20 + 1$

$= 5$

$\therefore$ Remainder (R) $= 5$

---

b. Find the remainder when $5x^3 + 7x^2 - 3x + 2$ is divided by $x+3$ with the help of remainder theorem.

Solution:

$f(x) = 5x^3 + 7x^2 - 3x + 2$

$g(x) = x+3$

Comparing, $x+3$ with $x-a$, $a = -3$

Now,

$f(a) = f(-3) = 5\times(-3)^3 + 7\times(-3)^2 - 3\cdot(-3) + 2$

$= -135 + 63 + 9 + 2$

$= -61$

$\therefore$ Remainder (R) $= -61$

5.a. Use factor theorem to show $(x-3)$ is a factor of $x^3 - 4x^2 + x + 6$

Solution:

$f(x) = x^3 - 4x^2 + x + 6$

Also,

$x - a = x - 3$

i.e. $a = 3$

Now,

$f(a) = f(3)$

$= 3^3 - 4 \times (3)^2 + 3 + 6$

$= 27 - 36 + 9$

$= 0$

Hence, $(x-3)$ is a factor of $x^3 - 4x^2 + x + 6$

b. Prove that $(2x+1)$ is a factor of $6x^3 + 13x^2 + 17x + 6$

Solution:

$F(x) = 6x^3 + 13x^2 + 17x + 6$

Also,

$x - a = 2x + 1$

i.e. $a = -1/2$

Now, $f(a) = f(3)$

$= 6 \times \left(\frac{-1}{2}\right)^3 + 13 \times \left(\frac{-1}{2}\right)^2 + 17 \times \left(\frac{-1}{2}\right) + 6$

$= \frac{-6}{8} + \frac{13}{4} - \frac{17}{2} + 6$

$= \frac{-6 + 26 - 68}{8} + 6$

$= \frac{-48 + 48}{8}$

$= 0$

Hence, $(x-3)$ is a factor of $6x^3 + 13x^2 + 17x + 6$

c. Use factor theorem to determine whether $g(x) = x+3$ is a factor of the polynomial $f(x) = x^3 - 4x^2 - 18x + 9$

Solution:

$f(x) = x^3 - 4x^2 - 18x + 9$

$g(x) = x+3$

Comparing $x+3$ with $x-a$, $a = -3$

Now,

$$\begin{aligned} f(a) &= f(-3) \\ &= -3^3 - 4(-3)^2 - 18(-3) + 9 \\ &= -27 - 4\times 9 + 54 + 9 \\ &= -63 + 63 \\ &= 0 \end{aligned}$$

Hence, $x+3$ is a factor of $x^3 - 4x^2 - 18x + 9$

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d. Prove that $f(x) = x(x+1)(x+3)(x+4) - 4$ has a factor $(x+2)$.

Solution:

$f(x) = x(x+1)(x+3)\cancel{x+4}(x+4) - 4$

$g(x) = x+2$

Comparing $x+2$ with $x-a$, $a = -2$

Now,

$$\begin{aligned} f(a) = f(-2) &= -2(-2+1)(-2+3)(-2+4) - 4 \\ &= -2\times(-1)(1)(2) - 4 \\ &= 2\times 2 - 4 \\ &= 4 - 4 \\ &= 0 \end{aligned}$$

Hence, $x(x+1)(x+3)(x+4) - 4$ has a factor $(x+2$

a. If $x^3 + ax^2 - x + 7$ leaves a remainder 4 when divided by $x-3$, find the value of a.

Solution:

$f(x) = x^3 + ax^2 - x + 7$

$g(x) = x - 3$

Comparing $x-3$ with $x-a$, $a = 3$

Now,

$f(a) = f(3) = 4$

or, $3^3 + a \cdot 3^2 - 3 + 7 = 4$

or, $27 + 9a + 4 = 4$

or, $9a = -27$

$\therefore a = -3$

b. The polynomial $f(x) = kx^3 + 9x^2 + 4x - 8$ when divided by $(x+3)$ leaves a remainder -20. Find the value of k.

Solution:

$f(x) = kx^3 + 9x^2 + 4x - 8$

$g(x) = x + 3$

Comparing $x+3$ with $x-a$, $a = -3$

Now,

$f(a) = -20$

or, $f(-3) = -20$

or, $k(-3)^3 + 9(-3)^2 + 4(-3) - 8 = -20$

or, $-27k + 81 ~~+~~ -12 - 8 = -20$

or, $-27k = -20 - 61$

or, $-27k = -81$

$\therefore k = 3$

c. If $2x^3 - 4x^2 + 6x - k$ leaves a remainder 18 when divided by $(x-2)$. Find the value of k.

Solution:

If $f(x) = 2x^3 - 4x^2 + 6x - k$

$g(x) = x - 2$

Comparing $x-2$ with $x-a$, $a = 2$

$f(a) = 18$

or, $f(2) = 18$

or, $2 \times (2)^3 - 4 \times 2^2 + 6 \times 2 - k = 18$

or, $16 - 16 + 12 - k = 18$

or, $-k = 18 - 12$

or, $-k = 6$

$\therefore k = -6$

7.a. Use factor theorem to find the factor when $f(x) = x^3 - 2x^2 + 1$

Solution:

$f(x) = x^3 - 2x^2 + 1$

when, $x = 1$

$f(x) = 1^3 - 2 \times 1^2 + 1$

$= 2 - 2$

$= 0$

Hence, $(x-1)$ is the factor of $x^3 - 2x^2 + 1$

b. If $\cancel{(x+1)}$ $(x-2)$ is a factor of the polynomial $x^2 - 3x + 5a$, find the value of a.

Solution:

$f(x) = x^3 - 3x + 5a$

$g(x) = x - 2$

i.e. $a = 2$

$f(2) = 2^2 - 3 \times 2 + 5a$

or, $4 - 6 + 5a = 0$

or, $-2 + 5a = 0$

$\therefore a = \frac{2}{5}$

c. If $(x+1)$ is a factor of $2x^3 - kx^2 - 8x + 5$, find the Value of a.

Solution:

$f(x) = 2x^3 - kx^2 - 8x + 5$

$g(x) = x+1$

comparing, $x+1$, with $x-a$, $a = -1$

$f(a) = f(-1)$

or, $0 = 2(-1)^3 - k(-1)^2 - 8\times(-1) + 5$

or, $0 = -2 - \cancel{0}k + 8 + 5$

or, $0 = -11 + k$

or, $k = 11$

$\therefore k = 11$

d. If $2x+1$ is a factor of $2x^3 + ax^2 + x + 2$, Find a

Solution:

$f(x) = 2x^3 + ax^2 + x + 2$

$g(x) = 2x + 1$

$= 2(x + 1/2)$

Comparing $x + \frac{1}{2}$ with $x - a$, $a = -\frac{1}{2}$

$f(-\frac{1}{2}) = 2x^3 + ax^2 + x + 2$

i.e. $= 2\times\left(\frac{-1}{2}\right)^3 + a\left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + 2 = 0$

or, $-\frac{2}{8} + \frac{a}{4} - \frac{1}{2} + 2 = 0$

or, $-\frac{2}{8} \cancel{-\frac{4}{16}} - \frac{1}{2} + 2 = -\frac{a}{4}$

or, $\cancel{\frac{-2-4+4}{8}}$

or, $-\frac{2}{8} - \frac{1+4}{2} = -\frac{a}{4}$

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or, $\frac{-2+3\times4}{8} = -\frac{a}{4}$

or, $\frac{-2+12}{8} = -\frac{a}{4}$

or, $\frac{10}{\cancel{8}2} = -\frac{a}{\cancel{4}}$

$\therefore a = -5$

8.a. It is given that $2x^3-7x^2+10 = (x-1)\cdot Q(x)+R$. Find the quotient and remainder.

Solution:

Comparing, $x-1$ with $x-a$, $a=1$

	2	-7	0	10
1	↓	2	-5	-5
	2	-5	-5	5

Hence,

Quotient $= 2x^2-5x-5$

Remainder (R) $= 5$

b. To form a polynomial whose factors are $(x+1)$, $(x+2)$ and $(x-3)$.

Solution:

$(x+1)(x+2)(x-3)$

$= (x^2+x+2x+2)(x-3)$

$= x^3+x^2+2x^2+2x-3x^2-3x-6x-6$

$= x^3+3x^2-3x^2+2x-3x-6x-6$

$= x^3-7x-6$

Hence, the required polynomial is (x^3-7x-6)

Creative Section

9. Factorize:

a. $x^3 - 7x^2 + 7x + 15$

Solution:

The factor of 15 ~~is~~ are $\pm 1, \pm 3, \pm 5, \pm 15$

when, $f(x) = (-1)^3 - 7(-1)^2 + (7x-1) + 15$

$= 0$

Hence, $x = -1$, so, $(x+1)$ is a factor.

Now,

$x^3 - 7x^2 + 7x + 15$

or, $x^3 + x^2 - 8x^2 - 8x + 15x + 15$

or, $x^2(x+1) - 8x(x+1) + 15(x+1)$

or, $(x+1)(x^2 - 8x + 15)$

or, $(x+1)(x^2 - 5x - 3x + 15)$

or, $(x+1)\{x(x-5) - 3(x-5)\}$

or, $(x+1)(x-3)(x-5)$

b. $2x^3 - 3x^2 - 3x + 2$

solution:

The factor of 2 are $\pm 1, \pm 2$

when, $f(x) = -1$

$2(-1)^3 - 3(-1)^2 - 3(-1) + 2$

$= -2 - 3 + 3 + 2$

$= 0$

Hence, $(x+1)$ is a factor.

Now,

$2x^3 - 3x^2 - 3x + 2$

$= 2x^3 + 2x^2 - 3x^2 - 3x + 2x + 2$

$= 2x^2(x+1) - 3x(x+1) + 2(x+1)$

$= (x+1)(2x^2 - 3x + 2)$

$\cancel{(x+1)\ )\ 2x^2-4x}$

Comparing $(x+1)$ with $x-a$, $a=-1$

$$\begin{array}{r|rrrr} & 2 & -3 & -3 & 2 \\ -1 & \downarrow & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$

Here,

Quotient $= 2x^2-5x+2$

Now, $2x^2-5x+2 = 2x^2-4x-x+2$

$= 2x(x-2)-1(x-2)$

$= (x-2)(2x-1)$

Hence, $2x^3-3x^2-3x+2 = (x+1)(x-2)(2x-1)$

c) $x^3-19x-30$

Solution:

The factor of 30 is $\pm1, \pm2, \pm3, \pm5, \pm15$

when, $x = (-3)^3-19(-3)-30$

$= 0$

Here, $(x+3)$ is a factor of $x^3-19x-30$

Using synthesis division:

$$\begin{array}{r|rrrr} & 1 & 0 & -19 & -30 \\ -3 & \downarrow & -3 & 9 & 30 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

Here,

Quotient $= x^2-3x-10$

Now,

$x^2-3x-10 = x^2-(5-2)x-10$

$= x^2-5x+2x-10$

$= x(x-5)+2(x-5)$

$= (x+2)(x-5)$

Hence, $x^3-19x-30 = (x+2)(x+3)(x-5)$

d. $3x^3 - 13x^2 + 16$

Solution:

The factors of 16 are $\pm 1, \pm 2, \pm 4, \pm 16$

when, $f(x) = -1$

$= 3(-1)^3 - 13(-1)^2 + 16$

$= -3 - 13 + 16$

$= 0$

Hence, $(x+1)$ is a factor of $3x^3 - 13x^2 + 16$

Now,

$$\begin{array}{r|rrrr} & 3 & -13 & 0 & 16 \\ -1 & \downarrow & -3 & 16 & -16 \\ \hline & 3 & -16 & 16 & 0 \end{array}$$

Quotient $= 3x^2 - 16x + 16$

$= 3x^2 - (12+4)x + 16$

$= 3x^2 - 12x - 4x + 16$

$= 3x(x-4) - 4(x-4)$

$= (x-4)(3x-4)$

Hence,

Factor of $3x^3 - 13x^2 + 16 = (x+1)(x-4)(x - 4/3)$

e. $x^4 - x^3 - 19x^2 + 49x - 30$

Solution:

factor of 30 are $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 30$

when, $x = 2$

$= (2)^4 - (2)^3 - 19(2)^2 + 49 \times 2 - 30$

$= 0$

Hence, $(x-1)$ is a factor of $x^4 - x^3 - 19x^2 + 49x - 30$

Now,

$$\begin{array}{r|rrrrr} & 1 & -1 & -19 & 49 & -30 \\ 2 & \downarrow & 2 & 2 & -34 & 30 \\ \hline & 1 & 1 & -17 & 15 & 0 \end{array}$$

~~Quotient = $x^3 - 2x^2 - 21x + 28$~~
~~Remainder = 2~~ 0

$(x^3 + x^2 - 17x + 15)\ (x-2)$

when $x = 1$

$f(x) = 1^3 + 1^2 - 17 \times 1 + 15$

$= 0$

so, $(x-1)$ is also a factor

Using synthesis ~~#~~ division

	1	1	-17	15
1	↓	1	2	-15
	1	2	-15	0

$= (x^2 + 2x - 15)(x-1)$

$= (x^2 + 5x - 3x - 15)\ (x-1)$

$= x(x+5) - 3(x+5)\ (x-1)$

$= (x-1)\ (x-3)\ (x-5)$ ~~are the~~

#

g. $(x+1)(x+3)\ (x+5)\ (x+7) + 16$

$= (x+1)\ (x+7)\ (x+3)(x+5) + 16$

$= (x^2 + 7x + x + 7)\ (x^2 + 5x + 3x + 15) + 16$

$= (x^2 + 8x + 7)\ (x^2 + 8x + 15) + 16$

Let $x^2 + 8x$ be 'a' then

$(a+7)\ (a+15) + 16$

$= a^2 + 15a + 7a + 105 + 16$

$= a^2 + 22a + 121$

$= a^2 + 2 \cdot a \cdot 11 + (11)^2$

$= (a+11)^2$ —— ①

Putting the value of x in eqn ①

$(a+11)^2$

$= (x^2 + 8x + 11)^2$

h. $x(x+1)(x+2)(x+3)+1$

$= (x+1)(x+2)\,x(x+3)+1$

$= (x^2+2x+x+2)\,(x^2+3x)+1$

$= (x^2+3x+2)\,(x^2+3x)+1$

Let 'x^2+3x' be 'a', then

$(a+2)(a)+1$

$= a^2+2a+1$

$= a^2+2\cdot a\cdot 1+1^2$

$= (a+1)^2$ — (1)

Putting the value of a in eqn (1)

$(a+1)^2$

$= (x^2+3x+1)^2$

10. Solve the following polynomials.

a. $x^3-7x^2+7x+15=0$

When $x=-1$, the value of given f'x is zero.

$(x+1)$ is a factor.

Using synthesis division:

	1	-7	7	15
-1	↓	-1	8	-15
	1	-8	15	0

So,

$(x^2-8x+15)(x+1)=0$

or, $(x^2-5x-3x+15)(x+1)=0$

or, $x(x-5)-3(x-5)(x+1)=0$

or, $(x-3)(x+1)(x-5)=0$

Hence,

$x=-1,3,5$

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b. $x^3 - 4x^2 - 7x + 10 = 0$

When $x = -2$

The given $f(x)$ is zero. So, $(x+2)$ is a factor

Now,

$x^3 - 4x^2 - 7x + 10 = 0$

or, $x^3 + 2x^2 - 6x^2 - 12x + 5x + 10 = 0$

or, $x^2(x+2) - 6x(x+2) + 5(x+2) = 0$

or, $(x+2)(x^2 - 6x + 5) = 0$

or, $(x+2)(x^2 - 5x - x + 5) = 0$

or, $(x+2)\ x(x-5) - 1(x-5) = 0$

or, $(x+2)(x-1)(x-5) = 0$

Hence,

$x = -2, 1, 5$

c. $x^3 - 4x^2 + x + 6 = 0$

when, $x = -1$

The value of $f(x)$ will be zero. So, $(x+1)$ is a factor

Now,

$x^3 - 4x^2 + x + 6 = 0$

or, $x^3 + x^2 - 5x^2 - 5x + 6x + 6 = 0$

or, $x^2(x+1) - 5x(x+1) + 6(x+1) = 0$

or, $(x+1)(x^2 - 5x + 6) = 0$

or, $(x+1)[x^2 - 3x - 2x + 6] = 0.$

or, $(x+1)[x(x-3) - 2(x-3)] = 0$

or, $(x+1)(x-2)(x-3) = 0$

Hence,

$x = -1, 2, 3$

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d. $x^3 - 9x^2 + 24x - 20 = 0$

when $x = 2$

The given $f(x)$ will be zero, so, $(x-2)$ is a factor.

Now,

$x^3 - 9x^2 + 24x - 20 = 0$

or $x^3 - 2x^2 - 7x^2 + 14x + 10x - 20 = 0$

or $x^2(x-2) - 7x(x-2) + 10(x-2) = 0$

or $(x-2)(x^2 - 7x + 10) = 0$

or $(x-2)[x^2 - 5x - 2x + 10] = 0$

or $(x-2)(x-2)(x-5) = 0$

Hence,

$x = 2, 2, 5$

e. $3x^3 - 13x^2 + 16 = 0$

When $x = -1$

The given $f(x)$ will be zero. So $(x+1)$ is a factor.

Now,

$3x^3 - 13x^2 + 16 = 0$

or $3x^3 + 3x^2 - 16x^2 - 16x + 16x + 16 = 0$

or $3x^2(x+1) - 16x(x+1) + 16(x+1) = 0$

or $(x+1)(3x^2 - 16x + 16) = 0$

or $(x+1)[3x^2 - (12+4)x + 16) = 0$

or, $(x+1)[3x^2 - 12x - 4x + 16] = 0$

or, $(x+1) \cdot 3x(x-4) - 4(x-4) = 0$

or, $(x+1)(x-4)(3x-4) = 0$

Hence,

$x = -1, 4, \frac{4}{3}$

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f. $x^3 - 21x - 20 = 0$

when $x = -4$

The given fx will be zero. So $(x+4)$ is a factor.

Now,

$x^3 - 21x - 20 = 0$

or, $x^3 + 4x^2 - 4x^2 - 16x - 5x - 20 = 0$

or, $x^2(x+4) - 4x(x+4) - 5(x+4) = 0$

or, $(x+4)(x^2 - 4x - 5) = 0$

or, $(x+4)(x^2 - 5x + x - 5) = 0$

or, $(x+4)(x-5)(x+1) = 0$

Hence,

$x = -4, 5, -1$

g. $z^3 - 19z - 30 = 0$

when $x = -3$

The given fx will be zero. So, $(z+3)$ is a factor.

Now,

$z^3 - 19z - 30 = 0$

or, $z^3 + 3z^2 - 3z^2 - 9z - 10z - 30 = 0$

or, $z^2(z+3) - 3z(z+3) - 10(z+3) = 0$

or, $(z+3)(z^2 - 3z - 10) = 0$

or, $(z+3)(z^2 - 5z + 2z - 10) = 0$

or, $(z+3)\ z(z-5) + 2(z-5) = 0$

or, $(z+3)(z+2)(z-5) = 0$

Hence,

$z = -3, -2, 5$

h. $x^3 - 3x - 2 = 0$

when $x = -1$

The given $f(x)$ will be zero. So, $(x+1)$ is a factor

Now,

$x^3 - 3x - 2 = 0$

or, $x^3 + x^2 - x^2 - x - 2x - 2 = 0$

or, $x^2(x+1) - x(x+1) - 2(x+1) = 0$

or, $(x+1)(x^2 - x - 2) = 0$

or, $(x+1)(x^2 - 2x + x - 2) = 0$

or, $(x+1)[x(x-2) + 1(x-2)] = 0$

or, $(x+1)(x+1)(x-2) = 0$

Hence,

$x = -1, -1, 2$

i. $x^4 - x^3 - 19x^2 + 49x - 30 = 0$

When $x = 1$

The given $f(x)$ will be zero. So, $(x-1)$ is a factor.

$x^4 - x^3 - 19x^2 + 49x - 30 = 0$

or, $\cancel{x^3(x-1) \quad (x-1)}$

or, $x^4 - x^3 - 19x^2 + 19x + 30x - 30 = 0$

or, $x^3(x-1) - 19x(x-1) + 30(x-1) = 0$

or, $(x-1)(x^3 - 19x + 30) = 0$

~~or, (x-1)~~

~~or, (x-1) [~~

Also, when $x = -5$, $(x^3 - 19x + 30)$ will be zero. So $(x+5)$ is a factor.

$x^3 - 19x + 30 = 0$

or, $x^3 - 5x^2 \; 5x^2 + 25x - 44x + 220$

or, $x^2(x-5) - 5x(x-5) - 44(x-5)$

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$x^3 - 19x + 30 = 0$

or, $x^3 + 5x^2 - 5x^2 - 25x + 6x + 30 = 0$

or, $x^2(x+5) - 5x(x+5) + 6(x+5) = 0$

or, $(x+5)(x^2 - 5x + 6) = 0$

or, $(x+5)(x^2 - 3x - 2x + 6) = 0$

or, $(x+5)(x-3)(x-2) = 0$

here,

$(x-1)(x-2)(x-3)(x+5) = 0$

So, $x = 1, 2, 3, -5$

j. $(x-1)(2x^2 + 15x + 15) - 21 = 0$

when

k. $4x^3 - 3x - 1 = 0$

when $x = -1/2$

The given fx will be zero. So, $(x + 1/2)$ is a factor

$4x^3 - 3x - 1 = 0$

or, $4x^3 + 2x^2 - 2x^2 - x - 2x - 1 = 0$

or, $4x^2\left(x + \frac{1}{2}\right) - 2x\left(x + \frac{1}{2}\right) - 2\left(x + \frac{1}{2}\right) = 0$

or, $\left(x + \frac{1}{2}\right)(4x^2 - 2x - 2) = 0$

or, $\left(x + \frac{1}{2}\right)(4x^2 - 4x + 2x - 2) = 0$

or, $\left(x + \frac{1}{2}\right)(4x + 2)(x - 1) = 0$

Hence, $x = -\frac{1}{2}, -\frac{1}{2}, 1$

11. Find the values of a and b in each of the following cases.

a. $(x-1)$ and $(x-2)$ are the factors of $x^3 - ax^2 + bx - 8$

Solution:

Here, $(x-1)$ is a factor of $x^3 - ax^2 + bx - 8$

So, $f(1) = 0$

i.e. $1^3 - a.1^2 + b.1 - 8 = 0$

or, $1 - a + b - 8 = 0$

or, $-a + b = 7$ ———— (1)

Also,

$(x-2)$ is a factor of $x^3 - ax^2 + bx - 8$

then $f(2) = 0$

ie. $2^3 - a.2^2 + b.2 - 8 = 0$

or, $8 - 4a + 2b - 8 = 0$

or, $2b = 4a$

$\therefore b = 2a$ —— (ii)

from (i) and (ii)

$-a + 2a = 7$

$a = 7$ —— (iii)

from (ii) and (iii)

$b = 2 \times a$

$= 2 \times 7$

$= 14$

b. $2x^3 + ax^2 + bx - 2$ has a factor $(x+2)$ and leaves a remainder 7 when divided by $2x-3$.

Solution:

Given;

$f(x) = 2x^3 + ax^2 + bx - 2$

$g(x) = x + 2$

Comparing $(x+2)$ with $x-a$, Then $a = -2$

Now,

$f(x) = 2x^3 + ax^2 + bx - 2$

or, $0 = 2(-2)^3 + a(-2)^2 + b(-2) - 2$

or, $0 = 2 \times -8 + a \times 4 - 2b - 2$

or, $0 = -16 + 4a - 2b - 2$

or, $0 = 4a - 2b - 18$

or, $0 = 2(2a - b - 9)$

or, $2a - b - 9 = 0$ —— (i)

Again,

Remainder $(r) = 7$

$g(x) = 2x - 3$, $a = \frac{3}{2}$

Now, $f(x) = 2x^3 + ax^2 + bx - 2$

or, $7 = 2 \times (3/2)^3 + a(3/2)^2 + b(3/2) - 2$

or, $7 = 2 \times \frac{27}{84} + a\left(\frac{9}{4}\right) + \frac{3b}{2} - 2$

or, $7 + 2 = \frac{27}{4} + \frac{9a}{4} + \frac{3b}{2}$

or, $9 = \frac{27 + 9a + 6b}{4}$

or, $36 = 27 + 9a + 6b$

or, $3 = 3a + 2b$ —— (ii)

Here,

Solving eq (i) and (ii) also multiplying eqn (i) by 2

$$4a - 2b = 18$$
$$3a + 2b = 3$$
$$7a = 21$$
$$a = 3$$

putting the value of a in eqn (i)

$9 = 2a - b$

or, $9 = 2 \times 3 - b$

or, $9 - 6 = -b$

$\therefore b = -3$

Hence, The value of a is 3 and b is -3

c. $x^2 - 1$ is a factor of $x^3 + ax^2 - x + b$ and leaves a remainder 15 when divided by $x - 2$.

Solution:

Given, $f(x) = x^3 + ax^2 - x + b$

$g(x) = x^2 - 1$

Comparing $x^2 - 1$ with $x - a$, $a = 1$

Now,

$f(x) = x^3 + ax^2 - x + b$

or, $0 = 1^3 + a \cdot 1^2 - 1 + b$

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or, $0 = 1 + a - 1 + b$

or, $a + b = 0$ — (i)

Again,

$f(x) = x^3 + ax^2 - x + b$

$g(x) = x - 2$

Remainder $(R) = 15$

Comparing $x-2$, with ~~a = 2~~ $x-a$, $a = 2$

Now,

$f(x) = x^3 + ax^2 - x + b$

or, $15 = 2^3 + a.2^2 - 2 + b$

or, $15 = 8 + 4a - 2 + b$

or, $15 = 6 + 4a + b$

or, $15 - 6 = 4a + b$

or, $4a + b = 9$ — (ii)

Here,

Solving eqn (i) and (ii)

$$\begin{array}{rcl} 4a + b &=& 9 \\ a + b &=& 0 \\ (-) \quad (-) && (-) \\ \hline 3a &=& 9 \end{array}$$

$\therefore a = 3$

putting the value of a in eqn (i)

$a + b = 0$

or, $3 + b = 0$

$\therefore b = -3$

Hence, $a = 3$, $b = -3$

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d. $2x^3 + ax^2 - 11x + b$ leaves a remainder 0 and 42 when divided by $(x-2)$ and $(x-3)$

Solution:

1st case:

$f(x) = 2x^3 + ax^2 - 11x + b$

$g(x) = x - 2$

Comparing $x-2$, with $x-a$, $a = 2$

Now,

$f(x) = 2x^3 + ax^2 - 11x + b$

or, $0 = 2 \times (2)^3 + a \cdot 2^2 - 11 \times 2 + b$

or, $0 = 16 + 4a - 22 + b$

or, $6 = 4a + b$

or, $4a + b = 6$ —(i)

Case II

$f(x) = 2x^3 + ax^2 - 11x + b$

Remainder (R) = 42

$g(x) = x - 3$

Comparing $x-3$ with $x-a$, $a = 3$

Now,

$f(x) = 2x^3 + ax^2 - 11x + b$

or, $42 = 2 \times 3^3 + a \times 3^2 - 11 \times 3 + b$

or, $42 = 54 + 9a - 33 + b$

or, $42 + 33 - 54 = 9a + b$

or, $21 = 9a + b$

or, $9a + b = 21$ —(ii)

Here,

Solving eqn (i) and (ii)

Solving eqn (i) and (ii)

$$9a + b = 21$$
$$4a + b = 6$$
$$(-) \quad (-) \quad (-)$$

$$5a = 15$$
$$\therefore a = 3$$

Also,

putting the value of a in eqn (i)

$$4a + b = 6$$
or $4 \times 3 + b = 6$
or, $b = 6 - 12$
$$\therefore b = -6$$

Hence, $a = 3$ and $b = -6$