H/W

Chapter-5

Second Law of Thermodynamics

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Second Law of Thermodynamics

i. Kelvin - Plank's statement

It states that 'It is impossible to design a device that works on a cycle and produces no other effect than heat transfer from a single body for the production of work.'

ii. Clausius Statement

It states that 'It is impossible for a self acting machine working in a cyclic process, unaided by any external agency, to transfer heat from cold body to hot body.

Heat Engine

Heat engine is a device which converts heat energy into mechanical work. It consist of 3 parts. They are:

a. Source:

The source is a ~~body~~ hot body at constant high temperature from which the heat engine can draw heat.

b. Working Substance

It is an ideal gas which on being supplied with heat performs mechanical work.

c. Sink

Sink is a cold body at low temperature to which any amount of heat is rejected.

Efficiency of Heat Engine

It is defined as the ratio of amount of work done to the total heat taken from the source.

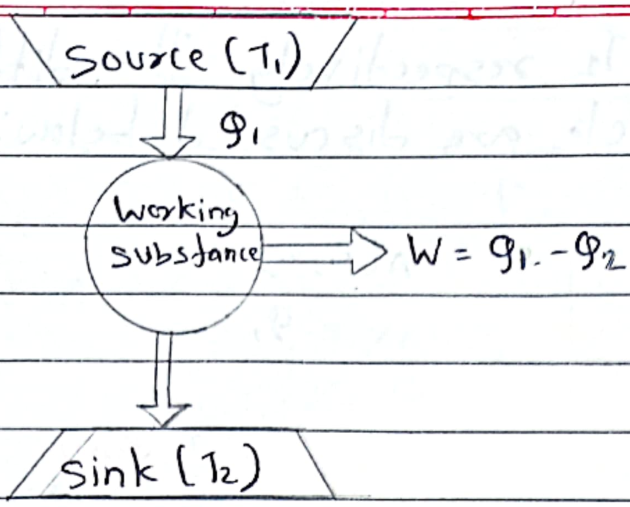

Fig: Heat Engine

$$\text{Efficiency}(\eta) = \frac{\text{External Work done}}{\text{Energy Absorbed}} \times 100\%$$

$$= \frac{W_1}{q_1} \times 100\%$$

If q_1 is the amount of heat taken from source and q_2 is the heat rejected to the sink. Then,

$$\text{Work done } (W) = q_1 - q_2$$

$$\therefore \eta = \frac{q_1 - q_2}{q_1} \times 100\%$$

Carnot Engine

Carnot Engine is an ideal heat engine having maximum efficiency which can work in a cycle of operation. It cannot be realized in actual practice.

Carnot Cycle

The carnot engine performs a work in four cycle of operation having two isothermal and two adiabatic process such as cycle of operation is called carnot cycle.

Consider n-mole of an ideal gas enclosed in a cylinder having initial Pressure, volume and Temperature

P_1, V_1 and T_1 respectively. The different process in a carnot cycle are discussed below:

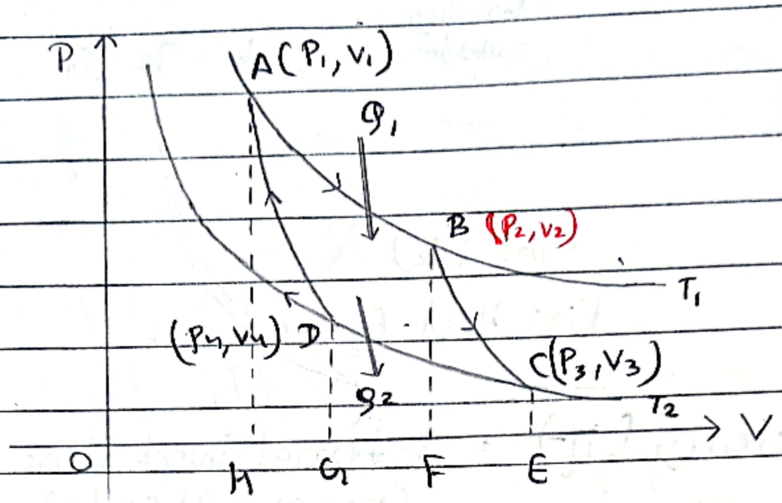

Fig: P-V diagram of carnot cycle

a. Isothermal Expansion

Initially the cylinder is placed on hot source and gas absorbs q amount of heat at constant temperature and gas expands isothermally from $A(P_1, V_1)$ to $B(P_2, V_2)$ which is represented by curve AB in a PV-diagram.

$$\text{Work done } (W_1) = q_1 = \cancel{AR} \int_{V_1}^{V_2} P\,dV$$

$$= nRT_1 \ln \frac{V_2}{V_1} \quad \text{——— (1)}$$

$$= \text{Area of } ABFHA$$

b. Isothermal Compression

Now the ~~gas~~ engine is placed on a sink and is compressed isothermally from $C(P_3, V_3)$ to $D(P_4, V_4)$. It is represented by ~~curve~~ curve CD in P-V diagram.

Then,

$$\text{Work done } (W_3) = q_2 = -\int_3^4 P\,dV$$

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b. Adiabatic Expansion

Now, the cylinder is kept on a isolating stand and a gas is expanded adiabatically from $B(P_2, V_2)$ to $C(P_3, V_3)$ and temperature changes from T_1 to T_2 which is shown by the curve BC in the PV diagram.

Now,

$$\text{Work done } (W_2) = \int_{V_2}^{V_3} P\,dv = \frac{nR(T_1 - T_2)}{\gamma - 1} \quad \text{--- (ii)}$$

= Area of BCEFB

c. Isothermal Compression

Now, the engine is placed on a sink and is compressed isothermally from $C(P_3, V_3)$ to $D(P_4, V_4)$ which is represented by the curve CD in the PV diagram.

Then,

$$\text{Work done } (W_3) = Q_2 = -\int_{V_3}^{V_4} P\,dv$$

$$= \int_{V_4}^{V_3} P\,dv = nRT_2 \ln \frac{V_3}{V_4} \quad \text{--- (iii)}$$

= Area of CEGDC

d. Adiabatic Compression

Now, the cylinder is kept on a insulating stand and compressed the gas adiabatically from $D(P_4, V_4)$ to $A(P_1, V_1)$ which is shown by the curve DA in the PV-diagram. The temperature changes from T_2 to T_1. Then,

$$W_4 = -\int_{V_4}^{V_1} P\,dv = \frac{nR(T_1 - T_2)}{\gamma - 1} \quad \text{--- (iv)}$$

= Area of ADGHA

Therefore total amount of Workdone during one Complete cycle is;

$$W = W_1 + W_2 - (W_4 + W_3)$$

$$= nRT_1 \ln\left(\frac{V_2}{V_1}\right) + \frac{nR(T_1 - T_2)}{\gamma - 1} - nR_2 \ln\left(\frac{V_3}{V_4}\right) - \frac{nR(T_1 - T_2)}{\gamma - 1}$$

$$= nRT_1 \ln\frac{V_2}{V_1} - nRT_2 \ln\frac{V_3}{V_4} \quad \text{—— (V)}$$

Similarily,

In Terms of Area;

W = Area ABFHA + Area BCEFB − Area of CEGDC − Area of ADGHA

= Area of ABCDA

So, Total work done during the cyclic process is the area enclosed by the cycle in the PV diagram.

Efficiency of Carnot's Cycle

We know,

$$\eta = \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

$$\text{or, } \eta = \left[1 - \frac{Q_2}{Q_1}\right] \times 100\%$$

$$\text{or, } \eta = \left[1 - \frac{RT_2 \operatorname{Log}(V_3/V_4)}{RT_1 \operatorname{Log}(V_2/V_1)}\right] \times 100 \quad [\because \text{ from eq. (I) and (III)}]$$

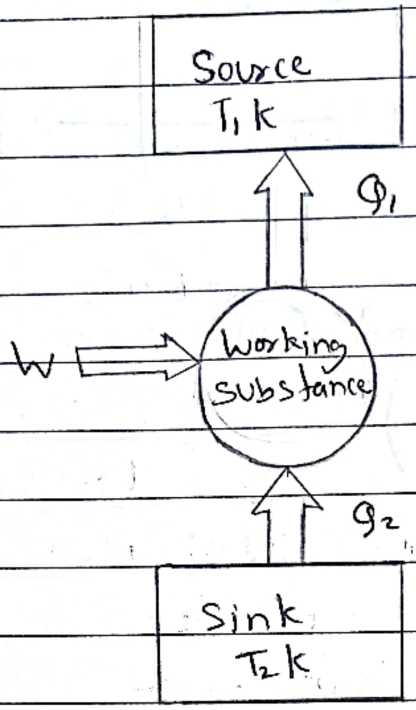
$$\text{or, } \eta = \left[1 - \frac{T_2 \operatorname{Log}(V_3/V_4)}{T_1 \operatorname{Log}(V_2/V_1)}\right] \times 100\% \quad \text{—— (VI)}$$

from Adiabatic Curve BC

$$T_1 V_2^{\gamma - 1} = T_2 V_3^{\gamma - 1}$$

$$\text{or, } \frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma - 1} \quad \text{—— (VII)}$$

Refrigerator



Now

Coefficient of performance $(\beta) = \frac{Q_2}{W}$

or $\beta = \frac{Q_2}{Q_1 - Q_2}$

or $\beta = \frac{\frac{Q_1}{Q_2}}{\frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}} = \frac{\frac{Q_2}{Q_1}}{1 - \frac{Q_2}{Q_1}}$

In Carnot's cycle, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

So, $\beta = \frac{\frac{T_2}{T_1}}{1 - \frac{T_2}{T_1}}$

$\therefore \beta = \frac{T_2}{T_1 - T_2}$

From Adiabatic curve AD,

$$T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$$

or, $$\frac{T_2}{T_1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{------------} \quad (VIII)$$

From equation (VII) and (VIII)

$$\left(\frac{V_3}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$$

or, $$\frac{V_3}{V_2} = \frac{V_4}{V_1}$$

or, $$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Now,

Substituting these values in equation (VI), we get

$$\eta = \left[1 - \frac{T_2}{T_1}\right] \times 100\%$$