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Chapter - 15

Limits and Continuity

Exercise - 15.1

1. Find the following limits.

a. $\lim_{x \to 2} (2x^2 + 3x - 14)$

Solution

$= 2 \cdot 2^2 + 3 \cdot 2 - 14$

$= 8 + 6 - 14$

$= 0$

b. $\lim_{x \to 5} (x^2 + 2x - 9)$

Solution:

$= 5^2 + 2 \cdot 5 - 9$

$= 25 + 10 - 9$

$= 26$

c. $\lim_{x \to 1} \dfrac{3x^2 + 2x - 4}{x^2 + 5x - 4}$

Solution:

$= \dfrac{3 \cdot 1^2 + 2 \cdot 1 - 4}{1^2 + 5 \cdot 1 - 4}$

$= \dfrac{1}{2}$

d. $\lim_{x \to 3} \dfrac{6x^2 + 3x - 12}{2x^2 + x + 2}$

Solution:

$= \dfrac{6 \cdot 3^2 + 3 \cdot 3 - 12}{2 \cdot 3^2 + 3 + 1}$

$= \dfrac{51}{22}$

2. Compute the following limits.

a. $\lim_{x \to 0} \dfrac{4x^3 - x^2 + 2x}{3x^2 + 4x}$

Solution.

$= \lim_{x \to 0} \dfrac{x(4x^2 - x + 2)}{x(3x + 4)}$

$= \dfrac{4 \cdot 0^2 - 0 + 2}{3 \cdot 0 + 4}$

$= \dfrac{1}{2}$

b. $\lim_{x \to 4} \dfrac{x^3 - 64}{x^2 - 16}$

Solution:

$= \lim_{x \to 4} \dfrac{(x-4)(x^2 + 4x + 16)}{(x+4)(x-4)}$

$= \dfrac{4^2 + 4 \cdot 4 + 16}{4 + 4}$

$= 6$

c. $\lim_{x \to a} \dfrac{x^{2/3} - a^{2/3}}{x - a}$

Solution:

$= \lim_{x \to a} \dfrac{(x^{1/3})^2 - (a^{1/3})^2}{(x^{1/3})^3 - (a^{1/3})^3}$

$= \lim_{x \to a} \dfrac{(x^{1/3} - a^{1/3})(x^{1/3} + a^{1/3})}{(x^{1/3} - a^{1/3})(x^{2/3} + x^{1/3} \cdot a^{1/3} + a^{2/3})}$

$= \lim_{x \to a} \dfrac{(x^{1/3} + a^{1/3})}{x^{2/3} + x^{1/3} \cdot a^{2/3} + a^{2/3}}$

$= \dfrac{a^{1/3} + a^{1/3}}{a^{2/3} + a^{2/3} + a^{2/3}}$

$= \dfrac{2a^{1/3}}{3a^{2/3}}$

$= \dfrac{2a}{3a^{1/3}} = \dfrac{2}{3} a^{-1/3}$

d. $\lim_{x \to 1} \dfrac{x^2 + 3x - 4}{x - 1}$

$= \lim_{x \to 1} \dfrac{(x+4)(x-1)}{(x-1)}$

$= \lim_{x \to 1} (x+4)$

$= 1 + 4$

$= 5$

e. $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - x - 2}$

$= \lim_{x \to 2} \dfrac{(x-3)(x-2)}{(x+1)(x-2)}$

$= \lim_{x \to 2} \dfrac{(x-3)}{(x+1)}$

$= \dfrac{2-3}{2+1}$

$= -1/3$

f. $\lim_{x \to 2} \dfrac{x^2 - 4x + 4}{x^2 - 7x + 10}$

$= \lim_{x \to 2} \dfrac{(x-2)(x-2)}{(x-2)(x-5)}$

$= \lim_{x \to 2} \dfrac{(x-2)}{(x-5)}$

$= \dfrac{2-2}{2-5}$

$= 0$

g. $\lim_{x \to a} \dfrac{\sqrt{3x} - \sqrt{2x+a}}{2(x-a)}$

$= \lim_{x \to a} \dfrac{\sqrt{3x} - \sqrt{2x+a}}{2(x-a)} \times \dfrac{\sqrt{3x} + \sqrt{2x+a}}{\sqrt{3x} + \sqrt{2x+a}}$

$= \lim_{x \to a} \dfrac{3x - 2x - a}{2(x-a)(\sqrt{3x} + \sqrt{2x+a})}$

$= \lim_{x \to a} \dfrac{(x-a)}{2(x-a)(\sqrt{3x} + \sqrt{2x+a})}$

$= \lim_{x \to a} \dfrac{1}{2(\sqrt{3x} + \sqrt{2x+a})}$

$= \dfrac{1}{2(\sqrt{3a} + \sqrt{3a})}$

$= \dfrac{1}{4\sqrt{3a}}$

h. $\lim_{x \to a} \dfrac{\sqrt{2x} - \sqrt{3x-a}}{\sqrt{x} - \sqrt{a}}$

$= \lim_{x \to a} \dfrac{\sqrt{2x} - \sqrt{3x-a}}{\sqrt{x} - \sqrt{a}} \times \dfrac{\sqrt{2x} + \sqrt{3x-a}}{\sqrt{2x} + \sqrt{3x-a}} \times \dfrac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$

$= \lim_{x \to a} \dfrac{(2x) - (3x-a)}{(x-a)} \times \dfrac{\sqrt{x} + \sqrt{a}}{\sqrt{2x} + \sqrt{3x-a}}$

$= \lim_{x \to a} \dfrac{(2x-3x+a)(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{2x}+\sqrt{3x-a})}$

$= \lim_{x \to a} \dfrac{-(x-a)(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{2x}+\sqrt{3x-a})}$

$= \lim_{x \to a} \dfrac{-(\sqrt{x}+\sqrt{a})}{\sqrt{2x}+\sqrt{3x-a}}$

$= \dfrac{-(\sqrt{a}+\sqrt{a})}{\sqrt{2a}+\sqrt{2a}}$

$= \dfrac{-2\sqrt{a}}{\cancel{2\sqrt{a}}\ 2\sqrt{2a}}$

$= \dfrac{-1}{\sqrt{2}}$

i. $\lim_{x \to 1} \dfrac{\sqrt{2x} - \sqrt{3-x^2}}{x-1}$

$= \lim_{x \to 1} \dfrac{\sqrt{2x} - \sqrt{3-x^2}}{(x-1)} \times \dfrac{\sqrt{2x} + \sqrt{3-x^2}}{\sqrt{2x} + \sqrt{3-x^2}}$

$= \lim_{x \to 1} \dfrac{(2x) - (3-x^2)}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \lim_{x \to 1} \dfrac{x^2+2x-3}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \lim_{x \to 1} \dfrac{(x+3)(x-1)}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \dfrac{1+3}{\sqrt{2}+\sqrt{2}}$

$= \sqrt{2}$

j. $\lim_{x \to 2} \dfrac{\sqrt{x} - \sqrt{6-x^2}}{x-2}$

$= \lim_{x \to 2} \dfrac{\sqrt{x} - \sqrt{6-x^2}}{x-2} \times \dfrac{\sqrt{x} + \sqrt{6-x^2}}{\sqrt{x} + \sqrt{6-x^2}}$

$= \lim_{x \to 2} \dfrac{x - 6 + x^2}{(x-2)(\sqrt{x} + \sqrt{6-x^2})}$

$= \lim_{x \to 2} \dfrac{(x-2)(x+3)}{(x-2)(\sqrt{x} + \sqrt{6-x^2})}$

$= \lim_{x \to 2} \dfrac{x+3}{\sqrt{x} + \sqrt{6-x^2}}$

$= \dfrac{2+3}{\sqrt{2}+\sqrt{2}}$

$= \dfrac{5}{2\sqrt{2}}$

k. $\lim_{x \to 64} \dfrac{\sqrt[6]{x} - 2}{\sqrt[3]{x} - 4}$

$= \lim_{x \to 64} \dfrac{x^{1/6} - 2}{(\sqrt[6]{x})^2 - 2^2}$

$= \lim_{x \to 64} \dfrac{(\sqrt[6]{x} - 2)}{(\sqrt[6]{x} - 2)(\sqrt[6]{x} + 2)}$

$= \lim_{x \to 64} \dfrac{1}{\sqrt[6]{x} + 2}$

$= \dfrac{1}{\sqrt[6]{64} + 2}$

$= \dfrac{1}{4}$

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1. $$\lim_{x \to a} \frac{\sqrt{3a-x} - \sqrt{x+a}}{4(x-a)}$$

$$= \lim_{x \to a} \frac{\sqrt{3a-x} - \sqrt{x+a}}{4(x-a)} \times \frac{\sqrt{3a-x} + \sqrt{x+a}}{\sqrt{3a-x} + \sqrt{x+a}}$$

$$= \lim_{x \to a} \frac{(3a-x-x-a)}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$$

$$= \lim_{x \to a} \frac{-2(x-a)}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$$

$$= \lim_{x \to a} \frac{-1}{2(\sqrt{3a-x} + \sqrt{x+a})}$$

$$= \frac{-1}{2(\sqrt{2a} + \sqrt{2a})}$$

$$= \frac{-1}{4\sqrt{2a}}$$

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3. Calculate the following limits.

a. $$\lim_{x \to \infty} \frac{2x^2}{3x^2+2}$$

$$= \lim_{x \to \infty} \frac{2}{3 + \frac{2}{x^2}}$$

$$= \frac{2}{3+0}$$

$$= \frac{2}{3}$$

b. $$\lim_{x \to \infty} \frac{3x^2-4}{4x^2}$$

$$= \lim_{x \to \infty} \frac{\frac{3x^2-4}{4x^2}}{\frac{4x^2}{4x^2}}$$

$$= \lim_{x \to \infty} \frac{3}{4} - \frac{1}{x^2}$$

$$= \frac{3}{4} - \frac{1}{\infty}$$

$$= \frac{3}{4}$$

c. $\lim_{x\to\infty} \dfrac{4x^2+3x+2}{5x^2+4x-3}$

$= \lim_{x\to\infty} \dfrac{4+\dfrac{3}{x}+\dfrac{2}{x^2}}{5+\dfrac{4}{x}-\dfrac{3}{x^2}}$

$= \dfrac{4+\dfrac{3}{\infty}+\dfrac{2}{\infty}}{5+\dfrac{4}{\infty}-\dfrac{3}{\infty}}$

$= \dfrac{4}{5}$

d. $\lim_{x\to\infty} \dfrac{5x^2+2x-7}{3x^2+5x+2}$

$= \lim_{x\to\infty} \dfrac{5+\dfrac{2}{x}-\dfrac{7}{x^2}}{3+\dfrac{5}{x}+\dfrac{2}{x^2}}$

$= \dfrac{5+\dfrac{2}{\infty}-\dfrac{7}{\infty}}{3+\dfrac{5}{\infty}+\dfrac{2}{\infty}}$

$= \dfrac{5}{3}$

4. Calculate the following limits.

a. $\lim_{x \to \infty} \left(\sqrt{x} - \sqrt{x-3}\right)$

$= \lim_{x \to \infty} \sqrt{x} - \sqrt{x-3} \times \frac{\sqrt{x} + \sqrt{x-3}}{\sqrt{x} + \sqrt{x-3}}$

$= \lim_{x \to \infty} \frac{(x - x + 3)}{\sqrt{x} + \sqrt{x-3}}$

$= \lim_{x \to \infty} \frac{3}{\sqrt{x} + \sqrt{x-3}}$

$= \frac{3}{\infty + \infty}$

$= 0$

b. $\lim_{x \to \infty} \left(\sqrt{x-a} - \sqrt{x-b}\right)$

$= \lim_{x \to \infty} \sqrt{x-a} - \sqrt{x-b} \times \frac{\sqrt{x-a} + \sqrt{x-b}}{\sqrt{x-a} + \sqrt{x-b}}$

$= \lim_{x \to \infty} \frac{(x - a - x + b)}{\sqrt{x-a} + \sqrt{x-b}}$

$= \lim_{x \to \infty} \frac{b-a}{\sqrt{x-a} + \sqrt{x-b}}$

$= \frac{b-a}{\infty + \infty}$

$= 0$

c. $\lim_{x \to \infty} \left(\sqrt{3x} - \sqrt{x-5}\right)$

$= \lim_{x \to \infty} \sqrt{3x} - \sqrt{x-5} \times \frac{\sqrt{3x} + \sqrt{x-5}}{\sqrt{3x} + \sqrt{x-5}}$

$= \lim_{x \to \infty} \frac{(3x - x + 5)}{\sqrt{3x} + \sqrt{x-5}}$

$= \lim_{x \to \infty} \frac{2x + 5}{\sqrt{3x} + \sqrt{x-5}}$

$$= \lim_{x \to \infty} \frac{2 + \frac{5}{x}}{\sqrt{3} + \sqrt{1 - 5/x}}$$

$$= \frac{2 + 0}{\sqrt{3} + \infty}$$

$$= \infty$$

d. $\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x} - \sqrt{x-a}\right)$

$$= \lim_{x \to \infty} \sqrt{x}\left(\sqrt{x} - \sqrt{x-a}\right) \times \frac{\sqrt{x} + \sqrt{x-a}}{\sqrt{x} + \sqrt{x-a}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}\,(x - x + a)}{\sqrt{x} + \sqrt{x-a}}$$

$$= \lim_{x \to \infty} \frac{a\sqrt{x}}{\sqrt{x} + \sqrt{x-a}}$$

$$= \lim_{x \to \infty} \frac{a}{1 + \sqrt{1 - a/x}}$$

$$= \frac{a}{1 + \sqrt{1}}$$

$$= \frac{a}{2}$$

e. $\lim_{x \to \infty} \left(\sqrt{x-a} - \sqrt{bx}\right)$

$$= \lim_{x \to \infty} \left(\sqrt{x-a} - \sqrt{bx}\right) \times \frac{\sqrt{x-a} + \sqrt{bx}}{\sqrt{x-a} + \sqrt{bx}}$$

$$= \cancel{x-a} \lim_{x \to \infty} \frac{(x - a - bx)}{\sqrt{x-a} + \sqrt{bx}}$$

$$= \lim_{x \to \infty} \frac{x(1-b) - a}{\sqrt{x-a} + \sqrt{bx}}$$

$$= \lim_{x \to \infty} \frac{(1-b) - a/x}{\sqrt{\frac{1}{x} - a/x^2} + \sqrt{b/x}} \quad \text{[Dividing by 'x']}$$

$$= \lim_{x\to\infty} \frac{(1-b) - a/x}{\sqrt{1/x - a/x^2} + \sqrt{b/x}} = \infty \text{ if } b \neq 1$$

If $b = 1$ then,

$$\lim_{x\to\infty} \frac{(1-b)(x-a)}{\sqrt{x-a} + \sqrt{bx}} = \frac{-a}{\infty + \infty} = 0$$

5.a $$\lim_{x\to 2} \frac{x - \sqrt{8-x^2}}{\sqrt{x^2+12} - 4}$$

$$= \lim_{x\to 2} \frac{x - \sqrt{8-x^2}}{\sqrt{x^2+12} - 4} \times \frac{x + \sqrt{8-x^2}}{x + \sqrt{8-x^2}} \times \frac{\sqrt{x^2+12} + 4}{\sqrt{x^2+12} + 4}$$

$$= \lim_{x\to 2} \frac{(x^2 - 8 + x^2)(\sqrt{x^2+12} + 4)}{(x^2 + 12 - 16)(x + \sqrt{8-x^2})}$$

$$= \lim_{x\to 2} \frac{2(x^2-4)(\sqrt{x^2+12} + 4)}{(x^2-4)(x + \sqrt{8-x^2})}$$

$$= \lim_{x\to 2} \frac{2\sqrt{x^2+12} + 4}{x + \sqrt{8-x^2}}$$

$$= \frac{2\sqrt{4+12} + 4}{2 + \sqrt{8-4}}$$

$$= 4$$

b $$\lim_{x\to 1} \frac{x - \sqrt{2-x^2}}{2x - \sqrt{2x+2x^2}}$$

$$= \lim_{x\to 1} \frac{x - \sqrt{2-x^2}}{2x - \sqrt{2x+2x^2}} \times \frac{x + \sqrt{2-x^2}}{x + \sqrt{2-x^2}} \times \frac{2x + \sqrt{2x+2x^2}}{2x + \sqrt{2x+2x^2}}$$

$$= \lim_{x\to 1} \frac{(x^2 - 2 + x^2)}{(4x^2 - 2 - 2x^2)} \times \frac{2x + \sqrt{2+2x^2}}{x + \sqrt{2-x^2}}$$

$$= \lim_{x\to 1} \frac{(2x^2-2)}{(2x^2-2)} \times \frac{(2x + \sqrt{2+2x^2})}{(x + \sqrt{2-x^2})}$$

$$= \frac{2\cdot 1 + \sqrt{2+2\cdot 1}}{1 + \sqrt{2-1^2}}$$

$$= 2$$