H/W

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Exercise - 1.1

1. Which of the following sentences are the statements? Find the truth values of those sentences which are statements

a. Kathmandu is the capital of Nepal
⇒ Statement : T

b. Nepal exports oil
⇒ Statement : T

c. $3 + 5 = 8$
⇒ Statement : T

d. Where do you live?
⇒ Not a statement

e. Understand Logic
⇒ Not a statement

f. Oh! how beautiful the scene is?
⇒ Not a statement

2. Let p : Demand is increasing and q : Supply is decreasing. Express each of the following statement into words.

a. $\sim p$
⇒ Demand is not increasing

b. $\sim q$
⇒ Supply is not decreasing

c. $p \wedge q$
⇒ Demand is increasing and supply is decreasing

d. $p \vee q$

⇒ Demand is increasing or supply is decreasing

e. $\sim p \wedge q$

⇒ Demand is not increasing and supply is decreasing

f. $p \vee \sim q$

⇒ Demand is increasing or supply is not decreasing.

g. $\sim p \wedge \sim q$

⇒ Neither demand is increasing nor supply is decreasing

h. $\sim (p \wedge q)$

⇒ It is false that demand is increasing and supply is decreasing.

3. Express each of the following statement into symbolic form.

a. p : temperature is increasing;
q : length is expanding
Temperature is increasing and length is expanding.
Solution:
$p \wedge q$

b. p : Pressure is decreasing;
q : Volume is increasing
Pressure is decreasing or volume is increasing
Solution:
$p \vee q$

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c. p : demand is increasing

q : Price is increasing

Neither demand is increasing nor price is decreasing (not increasing)

Solution:

$\sim p \wedge \sim q$

d. p : Pradeep is bold

q : Sandeep is handsome

It is false that Pradeep is bold or Sandeep is handsome.

Solution:

$\sim (p \vee q)$

4. Construct the truth tables for the following compound statements.

a. $(\sim p) \wedge q$

Solution:

p	q	$\sim p$	$(\sim p) \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

b. $(\sim p) \vee (\sim q)$

Solution:

p	q	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

c. $\sim(p \wedge q)$

Solution:

p	q	$(p \wedge q)$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

d. $\sim[p \vee (\sim q)]$

Solution:

p	q	$\sim q$	$p \vee (\sim q)$	$\sim[p \vee (\sim q)]$
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

e. $(p \to q) \wedge (q \to p)$

Solution:

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q - p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

f. $(\sim p \wedge q) \to (p \vee q)$

Solution:

p	q	$\sim p$	$(\sim p \wedge q)$	$p \vee q$	$(\sim p \wedge q) \to (p \vee q)$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	T

g. $[p \wedge (p \to q)] \to q$

Solution:

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

h. $(p \to q) \leftrightarrow (\sim p \vee q)$

Solution:

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	$(p \to q) \leftrightarrow (\sim p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

i. $(p \leftrightarrow q) \vee (q \leftrightarrow r)$

Solution:

p	q	r	$p \leftrightarrow q$	$q \leftrightarrow r$	$(p \leftrightarrow q) \vee (q \leftrightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	T	T

5. Let P, q, r and s be four simple statements. If P is true and, q is false, r is true and s is false, find the truth value of the following compound.

a. $p \wedge q$

Solution:

Here, P is true and q is false

So, $p \wedge q$ is false

b. $p \vee (\sim q)$

Solution:

Here, P is true and q is false

So, $p \vee (\sim q)$ is true

c. $(\sim p) \wedge (\sim q)$

Solution:

Here, P is true and q is false.

So, $(\sim p) \wedge (\sim q)$ is False

d. $q \vee (p \wedge s)$

Solution:

Here, q is false, P is true and s is false

So, $p \wedge s$ = False

Also, $q \vee (p \wedge s)$ = False

e. $\sim(\sim p)$

Solution:

Here, P is true

So, $\sim(\sim p)$ is also true

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f. $(p \vee q) \wedge (r \vee s)$

Solution:

Here, p is true, q is false, r is true and s is false.

Now, $p \vee q$ = ~~False~~ True

$r \vee s$ = True

$(p \vee q) \wedge (r \vee s)$ = True

6. If p and q are any two statements, prove that

a. $p \wedge (\sim p) \equiv C$ where C is a contradiction.

Solution:

p	$\sim p$	$p \wedge (\sim p)$
T	F	F
F	T	F

Here,

$p \wedge (\sim p)$ has all truth value false. So, $p \wedge (\sim p) \equiv C$.

b. $(p \vee q) \equiv (q \vee p)$

Solution:

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Now, from the truth value table given above, we can say that $(p \vee q) \equiv (q \vee p)$

~

c. $\sim(p \vee (\sim q)) \equiv (\sim p) \wedge q$

Solution:

p	q	$\sim p$	$\sim q$	$p \vee (\sim q)$	$(\sim p) \wedge q$	$\sim(p \vee (\sim q))$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	F	F

Now,
from the truth table we can say that, $\sim(p \vee (\sim q)) \equiv (\sim p) \wedge q$

d. $\sim((\sim p) \wedge q) \equiv p \vee (\sim q)$

Solution:

p	q	$\sim p$	$\sim q$	$(\sim p) \wedge q$	$\sim((\sim p) \wedge q)$	$p \vee (\sim q)$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Now,
from above truth table we can say $\sim((\sim p) \wedge q) \equiv p \vee (\sim q)$

7. Find the negation of each of the following statements:

a. Light travels in a straight line

Solution:

Light doesn't travels in a straight line.

b. Rivers can be used to produce electricity

Solutions:

Rivers cannot be used to produce electricity.

c. $x > 0$

$\Rightarrow x < 0$

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d. Some students are weak in Mathematics

Solution:

No students are weak in Mathematics.

e. All teachers are labourous.

Solutions:

~~All~~ some teachers are not labourous.

8. Find the truth value and the negation of the following statements.

a. $3 + 2 = 5$ or 6 is a multiple of 5.

Solution:

Truth value = True

Negation: $3+2 \neq 5$ and 6 is not a multiple of 5.

b. 8 is a prime number and 4 is even number

Solution:

Truth value : False

Negation : 8 is not a prime number or 4 is not even number

c. If $3 > 0$ ~~and 4+6~~ then $4+6 = 10$

Solution:

Truth Value : True

Negation : If $3 > 0$ and $4+6 \neq 10$

d. If 2 is odd or 3 is a natural number then $2+3 = 8$

Solution:

Truth value : False

Negation : 2 is odd or 3 is a natural number and $2+3 \neq 8$.

e. If $2 \times 3 = 5 \Rightarrow 3 > 1$ then 6 is even.

Solution:

Truth value : True

Negation : If $2 \times 3 = 5 \rightarrow 3 > 1$ and 6 is odd

f. A triangle ABC is right angled at B if and only if $AB^2 + BC^2 = AC^2$

Solution:

Truth value : True

Negation : A triangle ABC is right angled at B and $AB^2 + BC^2 \neq AC^2$

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g. Some statement are given below:

a. If 3 is a natural number then $1/3$ is a rational number.

b. $x^2 = 4$ at whenever $x = 2$

c. If the battery is low then the mobile does not work properly.

Find the;

i) Antecedent and consequent of all statement.

~~Solution:~~

ii. Converse, inverse and contrapositive

Solutions:

a. If 3 is a natural number then $1/3$ is a rational number.

i. Antecedent = 3 is a natural number

~~Cons~~ Consequent = $1/3$ is a rational number

ii. Converse = If $1/3$ is a rational number then 3 is a natural number.

Inverse = If 3 is not a ~~ra~~ natural number then $1/3$ is not a rational number

Contrapositive = If $1/3$ is not a rational number then 3 is not a natural no.

b. $x^2 = 4$ whenever $x = 2$

Solution:

i. Antecedent = $x = 2$

Consequent = $x^2 = 4$

ii. Converse = If $x^2 = 4$ then $x = 2$

Inverse = If $x \neq 2$ then $x^2 \neq 4$

Contrapositive = If $x^2 \neq 4$ then $x \neq 2$.

c. If the battery is low then ~~mob~~ the mobile does not work well.

Solution:

i. Antecedent = The battery is low

Consequent = The mobile does not work, well.

ii. Converse = If the mobile does not work well then the battery is low.

Inverse = If the battery is not low then the mobile work well.

Contrapositive = If the mobile work well then the battery is ~~low~~ not low.

10. If p and q be the statements, prove that

a. $p \vee \sim(p \wedge q)$ is a tautology

Solution:

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Here,

All truth value are true. So, it is tautology

b. $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Solution:

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Here,
All the truth value are true. So the statement is tautology.

c. $\sim(p \vee q) \wedge q$ is a contradiction

Solution:

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim(p \vee q) \wedge q$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

Here,
All the truth value ~~is false~~ are false. So, the given statement is contradiction.

d. $(p \wedge q) \wedge \sim(p \vee q)$ is a contradiction

Solution:

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Here,
All the truth value are false. So, the given statement is contradiction.

e. $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$ is a tautology.

Solution:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim q \wedge (p \rightarrow q) \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Here,
The truth value of given statement is true, so it is tautology.

f. $[(p \wedge q) \rightarrow p] \rightarrow (q \wedge \sim q)$ is a contradiction.

Solution:

~~p~~	~~q~~	~~$\sim p$~~	~~$p \wedge q$~~	~~$(p \wedge q) \rightarrow p$~~	~~$q \wedge \sim p$~~	~~$[(p \wedge q) \rightarrow p] \rightarrow (q \wedge$~~
~~T~~	~~T~~	~~F~~	~~T~~	~~T~~	~~F~~	
~~T~~	~~F~~	~~F~~	~~F~~	~~T~~	~~F~~	
~~F~~	~~T~~	~~T~~	~~F~~	~~T~~	~~T~~	
~~F~~	~~F~~	~~T~~	~~F~~	~~T~~	~~F~~	

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$	$\sim q$	$(q \wedge \sim q)$	$[(p \wedge q) \rightarrow p] \rightarrow (q \wedge \sim$
T	T	T	T	F	F	F
T	F	F	T	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	F	F

Here,
The truth value are false so the given statement is contradiction.

Exercise - 1.2

1. If $A \cap B = \phi$, prove that.

a. $A \subseteq \bar{B}$

Solution:

Let x be the element of A

Then, $x \in A \Rightarrow x \notin B \quad (\because A \cap B = \phi)$

$\Rightarrow x \in \bar{B}$

$\therefore x \in A \Rightarrow x \in \bar{B}$

$\therefore A \subseteq \bar{B}$

b. $B \cap \bar{A} = B$

Solution:

$B \cap \bar{A} = \{x : x \in B \text{ and } x \in \bar{A}\}$

$= \{x : x \in B \text{ and } x \notin A\}$

$= \{x : x \in B\} \quad (\because A \cap B = \phi)$

$= B$ proved

c. $A \cup \bar{B} = \bar{B}$

Solution:

$A \cup \bar{B} = \{x : x \in A \text{ or } x \in \bar{B}\}$

$= \{x : x \notin B \text{ or } x \in \bar{B}\} \quad (\because A \cap B = \phi)$

$= \{x : x \in \bar{B} \text{ or } x \in \bar{B}\}$

$= \{x : x \in \bar{B}\}$

$= \bar{B}$ proved

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2. If $A \subseteq B$, prove that.

a. $\bar{B} \subseteq \bar{A}$

Solution:

Let x be the element of $\bar{B}$

Then, $x \in \bar{B} \Rightarrow x \notin B$

$\Rightarrow x \notin A \quad (\because A \subseteq B)$

$\Rightarrow x \in \bar{A}$

or, $x \in \bar{B} \Rightarrow x \in \bar{A}$, $\therefore \bar{B} \subseteq \bar{A}$

b. $A \cup B = B$

Solution:

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$= \{x : x \in B \text{ or } x \in B\}$ $(\because A \subseteq B)$, $x \in A \rightarrow x \in B$ then $A \subseteq B$

$= \{x : x \in B\}$

$= B$ proved

c. $A \cap B = A$

Solution:

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$= \{x : x \in A \text{ and } x \in A\}$ $(\because A \subseteq B)$

$= \{x : x \in A\}$

$= A$ proved

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3. Prove that;

a. $B - A = B \cap \bar{A}$

Solution:

$B - A = \{x : x \in B \text{ and } x \notin A\}$

$= \{x : x \in B \text{ and } x \in \bar{A}\}$

$= \{x : x \in (B \cap \bar{A})\}$

$= B \cap \bar{A}$ proved

b. $A - \bar{B} = A \cap B$

Solution:

$A - \bar{B} = \{x : x \in A \text{ and } x \notin \bar{B}\}$

$= \{x : x \in A \text{ and } x \in B\}$

$= \{x : x \in (A \cap B)\}$

$= A \cap B$ proved

c. $A - B = \bar{B} - \bar{A}$

Solution:

$A - B = \{x : x \in A \text{ and } x \notin B\}$

$= \{x : x \notin \bar{A} \text{ and } x \in \bar{B}\}$

$= \{x : x \in \bar{B} \text{ and } x \notin \bar{A}\}$

$= \bar{B} - \bar{A}$ proved.

4. If A, B and C are the subsets of a universal set U, prove that;

a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

$A \cup (B \cap C) = \{x : x \in A \text{ or } \cancel{\text{and}}\ x \in (B \cap C)\}$

$= \{x : x \in A\ \cancel{\text{and}} \text{ or } (x \in B \text{ and } x \in C)\}$

$= \{x : (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)\}$

$= \{x : x \in (A \cup B) \text{ and } x \in (A \cup C)\}$

$= (A \cup B) \cap (A \cup C)$ proved.

b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution:

$A \cap (B \cup C) = \{x : x \in A \text{ and } x \in (B \cup C)\}$

$= \{x : x \in A \text{ and } (x \in B \text{ or } x \in C)\}$

$= \{x : (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)\}$

$= \{x : x \in (A \cap B) \text{ or } x \in (A \cap C)\}$

$= (A \cap B) \cup (A \cap C)$ proved.

5. If A, B and C are the subsets of a universal set U, prove that

a. $(A \cup B)' = A' \cap B'$

Solution:

$(A \cup B)' = \{x : x \notin (A \cup B)\}$

$= \{x : x \notin A \text{ and } x \notin B\}$

$= \{x : x \in A' \text{ and } x \in B'\}$

$= A' \cap B'$ proved

b. $(A \cap B)' = A' \cup B'$

Solution:

$$(A \cap B)' = \{x : x \notin (A \cap B)\}$$
$$= \{x : x \notin A \text{ or } x \notin B\}$$
$$= \{x : x \in A' \text{ or } x \in B'\}$$
$$= A' \cup B' \text{ proved}$$

c. $A - (B \cup C) = (A - B) \cap (A - C) = (A - B) - C$

Solution:

$$A - (B \cup C) = \{x : x \in A \text{ and } x \notin (B \cup C)\}$$
$$= \{x : x \in A \text{ and } (x \notin B \text{ and } x \notin C)\}$$
$$= \{x : (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C)\}$$
$$= \{x : x \in (A - B) \text{ and } x \in (A - C)\}$$
$$= (A - B) \cap (A - C)$$

~~Also~~ Again,

$$(A - B) - C = \{x : x \in (A - B) \text{ and } x \notin C\}$$
$$= \{x : x \in A \text{ and } x \notin B \text{ and } x \notin C\}$$
$$= \{x : x \in A \text{ and } x \notin (B \cup C)\}$$
$$= \cancel{\{x : x}$$
$$= A - (B \cup C) \text{ proved}$$

d. $A - (B \cap C) = (A - B) \cup (A - C)$

Solution:

$$A - (B \cap C) = \{x : x \in A \text{ and } x \notin (B \cap C)\}$$
$$= \{x : x \in A \text{ and } (x \notin B \text{ or } x \notin C)\}$$
$$= \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)\}$$
$$= \{x : x \in (A - B) \text{ or } x \in (A - C)\}$$
$$= (A - B) \cup (A - C)$$

proved

Exercise - 1.3

1. Evaluate:

a. $|-2| + 4$

$= 2 + 4$

$= 6$

b. $|-5| + |-2| - 3$

$= 5 + 2 - 3$

$= 4$

c. $2 + |-3| - |-5|$

$= 2 + 3 - 5$

$= 0$

d. $|3 - |-5||$

$= 3 - 5$

$= 2$

2. Let (i) $x = 2, y = 3$ (ii) $x = 2, y = -3$ verify each of the followings.

a. $|x+y| \leq |x| + |y|$

Solution:

When $x = 2$ and $y = 3$

$|x+y| = |2+3|$

$= |5|$

$= 5$

Also,

$|x| + |y| = |2| + |3|$

$= 2 + 3$

$= 5$

$\therefore |x+y| = |x| + |y|$

Again,

When $x = 2$ and $y = -3$

$|x+y| = |2-3|$

$= |-1|$

$= 1$

$|x| + |y| = |2| + |-3|$

$= 2 + 3$

$= 5$ $\therefore |x+y| < |x| + |y|$

b. $|x-y| \geq |x| - |y|$

Solution:

When $x = 2$ and $y = 3$

$|x-y| = |2-3|$

$= |-1|$

$= 1$

$|x| - |y| = |2| - |3|$

$= 2 - 3$

$= -1$

$\therefore |x-y| > |x| - |y|$

Again,

When $x = 2$ and $y = -3$

$|x-y| = |2+3|$

$= |5|$

$= 5$

$|x| - |y| = |2| - |-3|$

$= \cancel{|2| - 1}\ 2 - 3$

$= -1$

$\therefore |x-y| > |x| - |y|$

c. $|xy| = |x| . |y|$

Solution

When $x = 2$ and $y = 3$

$|xy| = |2 . 3|$

$= |6|$

$= 6$

$|x| . |y| = |2| . |3|$

$= 2 \times 3$

$= 6$

$\therefore |xy| = |x| . |y|.$

Again,

When $x = 2$ and $y = -3$

$$|xy| = |2 \times -3|$$
$$= |-6|$$
$$= 6$$
$$|x| \cdot |y| = |2| \cdot |-3|$$
$$= 2 \times 3$$
$$= 6$$
$$\therefore |xy| = |x| \cdot |y|$$

d. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

Solution:

When $x = 2$ and $y = 3$

$$\left|\frac{x}{y}\right| = \left|\frac{2}{3}\right|$$
$$= \frac{2}{3}$$
$$\frac{|x|}{|y|} = \frac{|2|}{|3|}$$
$$= \frac{2}{3}$$
$$\therefore \left|\frac{x}{y}\right| = \frac{|x|}{|y|}$$

Again, When $x = 2$ and $y = -3$

$$\left|\frac{x}{y}\right| = \left|\frac{2}{-3}\right|$$
$$= \frac{2}{3}$$
$$\frac{|x|}{|y|} = \frac{|2|}{|-3|}$$
$$= \frac{2}{3}$$
$$\therefore \left|\frac{x}{y}\right| = \frac{|x|}{|y|}$$

3. Solve the following inequalities.

a. $x - 1 > 2$

Solution:

$x - 1 > 2$

or, $x - 1 + 1 > 2 + 1$

or, $x > 3$

$\therefore x > 3$

b. $x - 3 \le 5$

Solution:

$x - 3 \le 5$

or, $x - 3 + 3 \le 5 + 3$

or, $x \le 5 + 3$

$\therefore x \le 8$

c. $-1 < x - 2 < 3$

Solution:

$-1 < x - 2 < 3$

or, $-1 + 2 < x - 2 + 2 < 3 + 2$

or, $1 < x < 5$

$\therefore 1 < x < 5$

d. $-3 \le 2x - 1 \le 5$

Solution:

$-3 \le 2x - 1 \le 5$

or, $-3 + 1 \le 2x - 1 + 1 \le 5 + 1$

or, $-2 \le 2x \le 6$

$\therefore -1 \le x \le 3$

e. $x^2 - 2x > 0$

Solution:

$x^2 = 2x > 0$

~~or,~~ The corresponding equation of $x^2 - 2x > 0$ is;

$x^2 - 2x = 0$

or, $x(x-2) = 0$

$\therefore x = 0, 2$

These two points 0 and 2 divides the real line in 3 different intervals.

Lets observe the result from the following table.

Interval	Signs of		
	x	$x-2$	$x(x-2)$
$-\infty$ to 0	$-$	$-$	$+$
0 to 2	$+$	$-$	$-$
2 to ∞	$+$	$+$	$+$

From the above table, the possible positive intervals are $(-\infty, 0)$ and $(2, \infty)$

$\therefore$ The required solution is $x \in (-\infty, 0) \cup (2, \infty)$

f. $6 + 5x - x^2 \geq 0$

Solution:

The corresponding equation of $6 + 5x - x^2 \geq 0$ is,

$6 + 5x - x^2 = 0$

or, $\cancel{x^2 - 5x - 6 = 0}$ $6 + 6x - x - x^2 = 0$

or, $\cancel{(x-6)(x+1) = 0}$ $(1+x)(6-x) = 0$

$\therefore x = \cancel{6, 1}$ $\cancel{6, -1}$ $-1, 6$

These two points 6 and -1 divides the real line into 3 ~~pa~~ intervals.

Lets observe the result from the given table.

Intervals	Signs of		
	$6-x$	$x+1$	$(x-6)(x+1)$
$-\infty$ to -1	+	-	-
-1 to 6	+	+	+
6 to ∞	-	+	-

Hence,

the required solution is $(-1, 6)$

i.e. $x \in [-1, 6]$

g. $\dfrac{x(x+2)}{x-1} \leq 0$

Solution:

The corresponding equation of given inequality is;

$\dfrac{x(x+2)}{x-1} = 0, \quad x \neq 1$

or, $x(x+2) = 0$

$\therefore x = 0, -2$

~~Lets Observe~~

Here, the critical points are -2, 0 and 1

Now, these points divide the number line into 4 intervals.

Lets observe the result from the table.

Intervals	Signs of			$\frac{x(x+2)}{x-1}$
	x	$x+2$	$\cancel{x(x+2)}\ \frac{1}{x-1}$	
$-\infty$ to -2	$-$	$-$	$-$	$-$
-2 to 0	$-$	$+$	$-$	$+$
0 to 1	$+$	$+$	$-$	$-$
1 to ∞	$+$	$+$	$+$	$+$

Hence,

The required solution is $(-\infty$ to $-2)$ and $(0$ to $1)$

i.e. $x \in (-\infty, -2) \cup \cancel{(0 \text{ to } 1)}\ (0, 1)$

4a. Let $A = [-3, 1)$ and $B = [-2, 4]$. Perform the indicated operations.

i. $A \cup B$

$= \{x : -3 \le x < 1\} \cup \{x : -2 \le x \le 4\}$

$= \{x : -3 \le x \le 4\}$

$= [-3, 4]$

ii. $A \cap B$

$= \cancel{\{x : -2 \le x \le 4\} \cap}$

$= [-3, 1) \cap [-2, 4]$

$= \{x : -3 \le x < 1\} \cap \{x : -2 \le x \le 4\}$

$= \{x : -2 \le x < 1\}$

$= [-2, 1)$

iii) $A - B$

Solution

$= (-3,1) - [-2,4]$

$= \{x : -3 \le x < 1\} - \{x : -2 \le x \le 4\}$

$= \cancel{\{x : -3 \le x \le -2\}}\ \{x : -3 \le x < -2\}$

$= \cancel{[-3,-2]}\ [-3,-2)$

b. If $A = (-1,4)$ and $B = [3,5)$, find $A \cup B$, $A \cap B$ and $A - B$.

Solution:

$A \cup B = (-1,4) \cup [3,5)$

$= \{x : -1 < x < 4\} \cup \{x : 3 \le x < 5\}$

$= \{x : -1 < x < 5\}$

$= (-1,5)$

$A \cap B = (-1,4) \cap [3,5)$

$= \{x : -1 < x < 4\} \cap \{x : 3 \le x < 5\}$

$= \{x : 3 \le x < 4\}$

$= [3,4)$

$A - B = (-1,4) - [3,5)$

$= \{x : -1 < x < 4\} - \{x : 3 \le x < 5\}$

$= \{x : -1 < x < 3\}$

$= (-1,3)$

c. If $U = R = (-\infty,\infty)$, $A = (-1,3]$ and $B = [0,5)$, find $A \cup B$, $A \cap B$, $A - B$, $B - A$, $\bar{A}$ and $\bar{B}$.

Solution:

$A \cup B = (-1,3] \cup [0,5)$

$= \{x : -1 < x \le 3\} \cup \{x : 0 \le x < 5\}$

$= \{x : -1 < x < 5\}$

$= \cancel{(-1,5]}\ (-1,5)$

$A \cap B = (-1, 3] \cap [0, 5)$

$= \{x : -1 < x \leq 3\} \cap \{x : 0 \leq x < 5\}$

$= \{x : 0 \leq x \leq 3\}$

$= [0, 3]$

$A - B = (-1, 3] - [0, 5)$

$= \{x : -1 < x \leq 3\} - \{x : 0 \leq x < 5\}$

$= \{x : \cancel{0 \leq x} -1 < x < 0\}$

$= (-1, 0)$

$B - A = [0, 5) - (-1, 3]$

$= \{x : 0 \leq x < 5\} - \{x : -1 < x \leq 3\}$

$= \{x : 3 < x < 5\}$

$= (3, 5)$

$\bar{A} = (-\infty, \infty) - (-1, 3]$

$= \{x : -\infty < x < \infty\} - \{x : -1 < x \leq 3\}$

$= \{x : -\infty < x < \infty\} \cup \{x : -1 < x \leq 3\}$

$= (-\infty, -1] \cup \cancel{(\infty, 3)}\ (3, \infty)$

$\bar{B} = \cancel{B -}\ U - B$

$= (-\infty, \infty) - [0, 5)$

$= \{x : -\infty < x < \infty\} - \{x : 0 \leq x < 5\}$

$= \{x : -\infty < x < \infty \cup x : 0 \leq x < 5\}$

$= (-\infty, 0) \cup [5, \infty)$

5. Write the following without using absolute value sign.

a. $|x| < 4$

Solution:

$-4 < x < 4$ $\quad$ $[\because |x| < a \text{ is } -a < x < a]$

b. $|x - 3| < 2$

Solution

$|x - 3| < 2$

or, $-2 < x - 3 < 2$

or, $-2 + 3 < x - 3 + 3 < 2 + 3$

or, $-1 < x < 5$

$\therefore x \in (-1, 5)$

c. $|2x + 1| \leq 3$

Solution:

$|2x + 1| \leq 3$

or, $-3 \leq 2x + 1 \leq 3$

or, $-3 - 1 \leq 2x + 1 - 1 \leq 3 - 1$

or, $-4 \leq 2x \leq 2$

or, $-2 \leq x \leq 1$

$\therefore x \in [-2, 1]$

d. $|2x - 1| \leq 5$

Solution:

$|2x - 1| \leq 5$

or, $-5 \leq 2x - 1 \leq 5$

or, $-5 + 1 \leq 2x - 1 + 1 \leq 5 + 1$

or, $-4 \leq 2x \leq 6$

or, $-2 \leq x \leq 3$

$\therefore x \in [-2, 3]$

6. Rewrite the following inequalities using absolute value sign.

a. $-5 < x < 7$

Solution:

$-5 < x < 7$

or, $-5-1 < x-1 < 7-1$

or, $-6 < x-1 < 6$

~~or~~ $\therefore |x-1| < 6$

b. $-3 \leq x \leq -1$

Solution:

$-3 \leq x \leq -1$

or, $-3+2 \leq x+2 \leq -1+2$

or, $-1 \leq x+2 \leq 1$

$\therefore |x+2| \leq 1$

c. $-3 < x < 4$

Solution:

$-3 < x < 4$

or, $-6 < 2x < 8$

or, $-6-1 < 2x-1 < 8-1$

or, $-7 < 2x-1 < 7$

$\therefore |2x-1| < 7$

d. $-4 \leq x \leq -2$

Solution:

$-4 \leq x \leq -1$

or, $-8 \leq 2x \leq -2$

or, $-8+5 \leq 2x+5 \leq \cancel{2x} -2+5$

or, $-3 \leq 2x+5 \leq 3$

$\therefore |2x+5| \leq 3$

7. Solve the following inequalities. Also, draw the graphs of the inequalities.

a. $|x+2| < 4$

Solution:

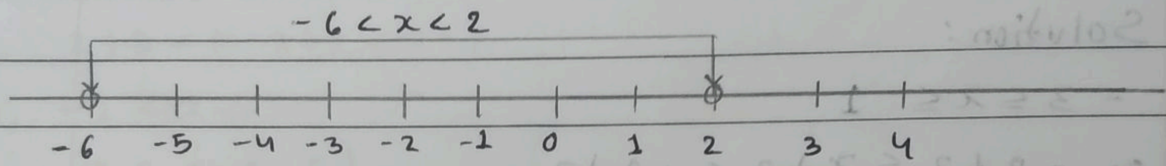
$|x+2| < 4$

or, $-4 < x+2 < 4$

or, $-4-2 < x+2-2 < 4-2$

or, $-6 < x < 2$

$\therefore$ The solution is $\{x : -6 < x < 2\}$



b. $|x-1| \leq 2$

Solution:

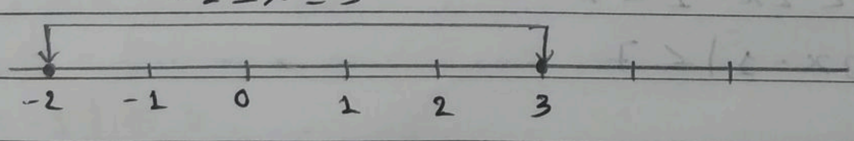
$|x-1| \leq 2$

or, $-2 \leq x-1 \leq 2$

or, $-2+1 \leq x-1+1 \leq 2+1$

or, $-1 \leq x \leq 3$

$\therefore$ The solution is $\{x : -1 \leq x \leq 3\}$



c. $|2x+3| \leq 1$

solution:

$|2x+3| \leq 1$

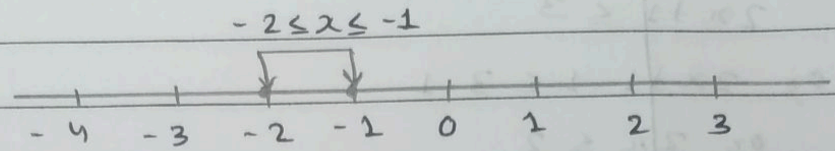
or, $-1 \leq 2x+3 \leq 1$

or, $-1-3 \leq 2x+3-3 \leq 1-3$

or, $-4 \leq 2x \leq -2$

or, $-2 \leq x \leq -1$

$\therefore$ The solution is $\{x : -2 \le x \le -1\}$



d. $|x-1| > 1$

Solution:

if $(x-1) > 0$, then

$(x-1) > 1$

or, $x - 1 + 1 > 1 + 1$

or, $x > 2$

$\therefore x \in (2, \infty)$

Again,

When $(x-1) < 0$, then

$-(x-1) > 1$

or, $x - 1 < -1$

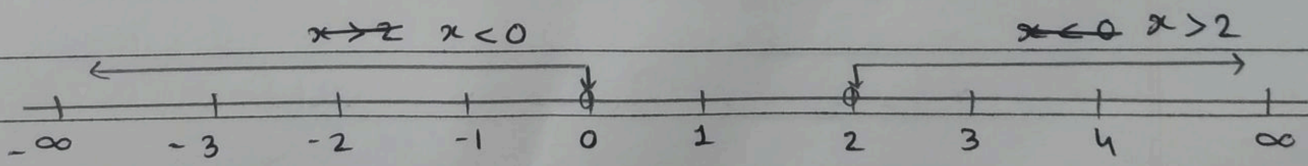
or, $x - 1 + 1 < -1 + 1$

or, $x < 0$

$\therefore x \in (-\infty, 0)$

$\therefore$ The required solution is $\{x : 2 > x \text{ or } x < 0\}$

i.e. $x \in (2, \infty) \cup (-\infty, 0)$



e. $|2x+1| \ge 3$

Solution:

When $(2x+1) > 0$ then

$(2x+1) \ge 3$

or, $2x + 1 - 1 \ge 3 - 1$

or, $x \ge 1$

$\therefore x \in [1, \infty)$

When $(2x+1) \geq 0$, then

$2x+1 \leq 3$

or, $2x+1-1 \leq 3-1$

or, $2x \leq 2$

or, $x \leq 1$

When,

$(2x+1) < 0$ then

or, $-(2x+1) \geq 3$

or, $2x+1 \leq -3$

or, $2x+1-1 \leq -3-1$

or, $2x \leq -4$

or, $x \leq -2$

$\therefore x \in (-\infty, -2]$

$\therefore$ The required solution of the inequality is ~~$\{x : x \leq -2$ or x~~
$\{x : x \leq -2 \text{ or } x \geq 1\}$