1. Find the equation of the circle with the data given below:

a. Center at (4,5) and radius 3 units

Solution:

Here, Center $(h, k) = (4,5)$

radius $(r) = 3$

Now,

$r^2 = (x-h)^2 + (y-k)^2$

or, $3^2 = (x-4)^2 + (y-5)^2$

or, $9 = x^2 - 8x + 16 + y^2 - 10y + 25$

or, $x^2 + y^2 - 8x - 10y + 32 = 0$ is the required equation.

---

b. Center at (-3,-4) and radius 6 units.

Here,

Center $(h,k) = (-3,-4)$

Radius $(r) = 6$ units

Now

$r^2 = (x-h)^2 + (y-k)^2$

or, $36 = (x+3)^2 + (y+4)^2$

or, $36 = x^2 + 6x + 9 + y^2 + 8y + 16$

or, $x^2 + y^2 + 6x + 8y - 11 = 0$ is the required equation.

---

c. Center at (0,0) and radius $\sqrt{2}$ units.

Here,

centre $(h,k) = (0,0)$

radius $(r) = \sqrt{2}$

Now,

$r^2 = (x-h)^2 + (y-k)^2$

or, $(\sqrt{2})^2 = (x-0)^2 + (y-0)^2$

or, $2 = x^2 + y^2$

or, $x^2 + y^2 - 2 = 0$ is the required equation.

d. Center at $(-5,2)$ and diameter 8 units.

Here,

Center $(h,k) = (-5,2)$

Diameter $(d) = 8$

So, radius $(r) = \frac{8}{2} = 4$

Now,

$(x-h)^2 + (y-k)^2 = r^2$

or $(x+5)^2 + (y-2)^2 = (4)^2$

or $x^2 + 10x + 25 + y^2 - 4y + 4 = 16$

or, $x^2 + y^2 + 10x - 4y + 13 = 0$ is the required equation.

e. Center at $(1,2)$ and circumference at 44 units.

Here,

Center $(h,k) = (1,2)$

Circumference $= 44$

or $2\pi r = 44$

or $2 \times \frac{22}{7} \times r = 44$

or $r = \frac{44 \times 7}{44}$

$\therefore r = 7$

Now,

$(x-h)^2 + (y-k)^2 = r^2$

or $(x-1)^2 + (y-2)^2 = 7^2$

or, $x^2 - 2x + 1 + y^2 - 4y + 4 = 49$

or $x^2 + y^2 - 2x - 4y$ ~~$-44 = 0$~~ $-44 = 0$ is the req. equatio

2. Find the equation of circle when the co-ordinate of the ends of diameter are

a. $(0,0)$ and $(4,7)$

Here,

$(x_1, y_1) = (0,0)$

$(x_2, y_2) = (4,7)$

Now

Equation of circle of the given ~~diamte~~ diameter is;

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

or $(x-0)(x-4) + (y-0)(y-7) = 0$

or $x^2 - 4x + y^2 - 7y = 0$

or $x^2 + y^2 - 4x - 7y = 0$

b. $(3,-4)$ and $(-5,2)$

Here,

$(x_1, y_1) = (3,-4)$

$(x_2, y_2) = (-5,2)$

Now,

The equation of circle in diameter form is given by;

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

or $(x-3)(x+5) + (y+4)(y-2) = 0$

or, $x^2 + 5x - 3x - 15 + y^2 - 2y + 4y - 8 = 0$

or $x^2 + y^2 + 2x + 2y - 23 = 0$

c. $(a,0)$ and $(-a,0)$

Here,

$(x_1, y_1) = (a,0)$

$(x_2, y_2) = (-a,0)$

Now

Equation of a circle is given by;

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

or, $(x-a)(x+a)+(y-0)(y-0)=0$

or, $x^2+ax-ax-a^2+y^2=0$

or, $x^2+y^2-a^2=0$

---

d. $(-a,-a)$ and (a,a)

Here,

$(x_1,y_1)=(-a,-a)$

$(x_2,y_2)=(a,a)$

Now,

equation of circle is given by;

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

or, $(x+a)(x-a)+(y+a)(y-a)=0$

or, $x^2-ax+ax-a^2+y^2-ay+ay-a^2=0$

or, $x^2+y^2-2a^2=0$

---

3. Find the center and the radius of the circles given below:

a. $(x-4)^2+(y-2)^2=36$

or, $(x-4)^2+(y-2)^2=6^2$

Now,

Comparing it with $(x-h)^2+(y-k)^2=r^2$

So,

centre $(h,k)=(4,2)$

Radius $(r)=6$

---

b. $(x+5)^2+(y-3)^2=5$

or, $\{x-(-5)\}^2+(y-3)^2=(\sqrt{5})^2$

Comparing it with $(x-h)^2+(y-k)^2=r^2$

So, center $(h,k)=(-5,3)$

radius $(r)=\sqrt{5}$

c. $(x-2)^2 + (y+3)^2 = 9$

or, $(x-2)^2 + \{y-(-3)\}^2 = 3^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$, we get

Centre $(h,k) = (2,-3)$

radius $(r) = 3$

d. $(x+6)^2 + (y-1)^2 = 50$

$\{x-(-6)\}^2 + (y-1)^2 = (5\sqrt{2})^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$, we get

Centre $(h,k) = (-6,1)$

radius $(r) = (5\sqrt{2})$

e. $x^2 + (y+2)^2 = 49$

or, $(x-0)^2 + (y+2)^2 = 7^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$, we get

Centre $(h,k) = (0,-2)$

radius $(r) = 7$

f. $(x+5)^2 + y^2 = 121$

or, $\{x-(-5)\}^2 + (y-0)^2 = 11^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$, we get

Centre $(h,k) = (-5,0)$

radius $(r) = 11$

g. $x^2 + y^2 - 2y = 24$

or, $(x-0)^2 + y^2 - 2\cdot y\cdot 1 + 1^2 = 24+1$

or, $(x-0)^2 + (y-1)^2 = 25$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$

Centre $(h,k) = (0,1)$

radius $(r) = 5$

h. $2x - 6y - x^2 - y^2 = 1$

or, $-(x^2 + y^2 - 2x + 6y) = 1$

or, $x^2 + y^2 - 2x + 6y = -1$

or, $x^2 - 2x + y^2 + 6y = -1$

or, $x^2 - 2.x.1 + 1^2 + y^2 + 2.y.3 + 9 = -1 + 1 + 9$

or, $(x-1)^2 + (y+3)^2 = 3^2$

or, $(x-1)^2 + \{y-(-3)\}^2 = 3^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$

so,

Centre $(h,k) = (1,-3)$

radius $(r) = 3$

---

i. $x^2 + y^2 - 4x - 6y - 12 = 0$

or, $x^2 - 4x + y^2 - 6y - 12 = 0$

or, $x^2 - 4x + y^2 - 6y = 12$

or, $x^2 - 2.x.2 + 2^2 + y^2 - 2.y.3 + 3^2 = 12 + 4 + 9$

or, $(x-2)^2 + (y-3)^2 = 5^2$

Now

Comparing it with $(x-h)^2 + (y-k)^2 = 0$

Centre $(h,k) = (2,3)$

radius$(r) = 5$

---

j. $2x^2 + 2y^2 + 4x - 2y + 1 = 0$

or, $2(x^2 + y^2 + 2x - y + \frac{1}{2}) = 0$

or, $x^2 + 2.x.1 + 1^2 + y^2 - 2.y.\frac{1}{2} + (\frac{1}{2})^2 + \frac{1}{2} = 1 + \frac{1}{4}$

or, $(x+1)^2 + (y - 1/2)^2 = 1 + \frac{1}{4} - \frac{1}{2}$

or, $(x+1)^2 + (y - 1/2)^2 = (\sqrt{3}/2)^2$

Centre $(h,k) = (-1, 1/2)$

radius $(r) = \sqrt{3}/2$

k. $9x^2+9y^2-36x+6y=107$

or $(3x)^2-2.3x.6+6^2+(3y)^2+2.3y.1+1^2=107+36+1$

or $(3x-6)^2+(3y+1)^2=144$

or $(3x-6)^2+\{3y-(-1)\}^2=(12)^2$

Also,

Comparing it with $(x-h)^2+(y-k)^2=r^2$, we get

Centre $(h,k)=(6,-1)$

Radius $(r)=12$

l. $x^2+y^2-2ax\cos\theta-2ay\sin\theta=0$

or $(x)^2-2x.a\cos\theta+(a\cos\theta)^2+y^2-2y.a\sin\theta+(a\sin\theta)^2$

$=a^2\cos^2\theta+a^2\sin^2\theta$

or $(x-a\cos\theta)^2+\cancel{y-y\sin\theta}\,(y-a\sin\theta)^2=a^2(\cos^2\theta+\sin^2\theta)$

or $(x-a\cos\theta)^2+(y-a\sin\theta)^2=a^2$

Now,

Comparing it with $(x-h)^2+(y-k)^2=r^2$, we get

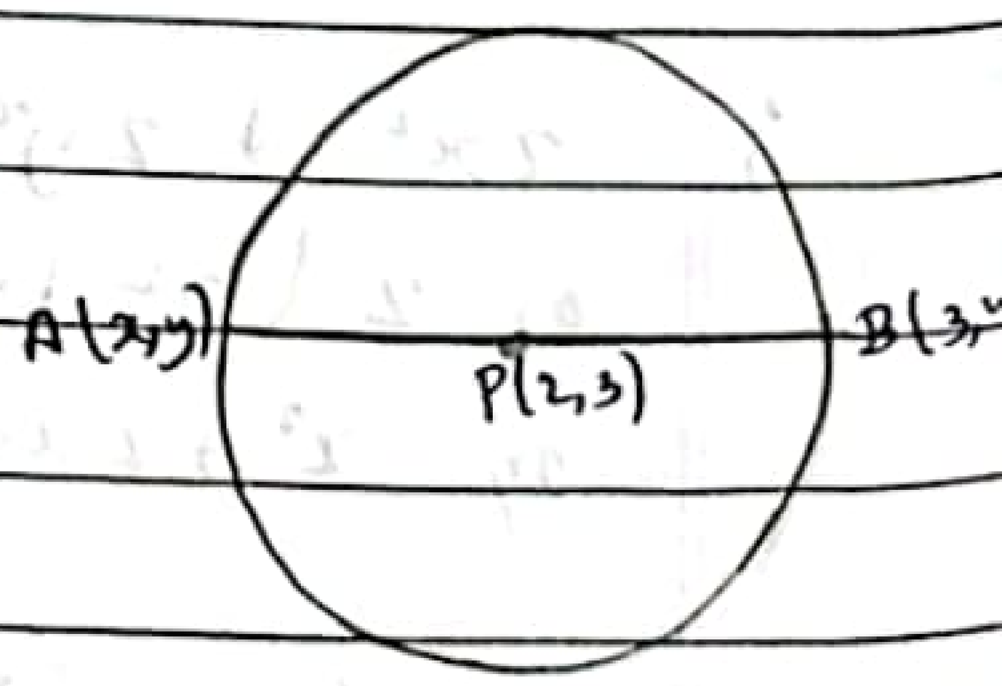
Centre $(h,k)=(a\cos\theta,a\sin\theta)$

Radius $(r)=a$

4. If one end of a diameter of a circle $x^2+y^2-4x-6y+11=0$ is $(3,4)$. find the co-ordinate of the other side.

Solution:

Here, equation of circle is;



$x^2+y^2-4x-6y=-11$

or $x^2-2.2x+2^2+y^2-2y.3+3^2=4+9-11$

or $(x-2)^2+(y-3)^2=(\sqrt{2})^2$

Now,

Comparing it with $(x-h)^2+(y-k)^2=r^2$, we get

Centre $(h,k)=(2,3)$

radius $(r)=\sqrt{2}$

Here, $P(2,3)$ is the mid-point of AB. So,

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = 2, 3$$

or $\frac{x+3}{2} = 2$, $\frac{y+4}{2} = 3$

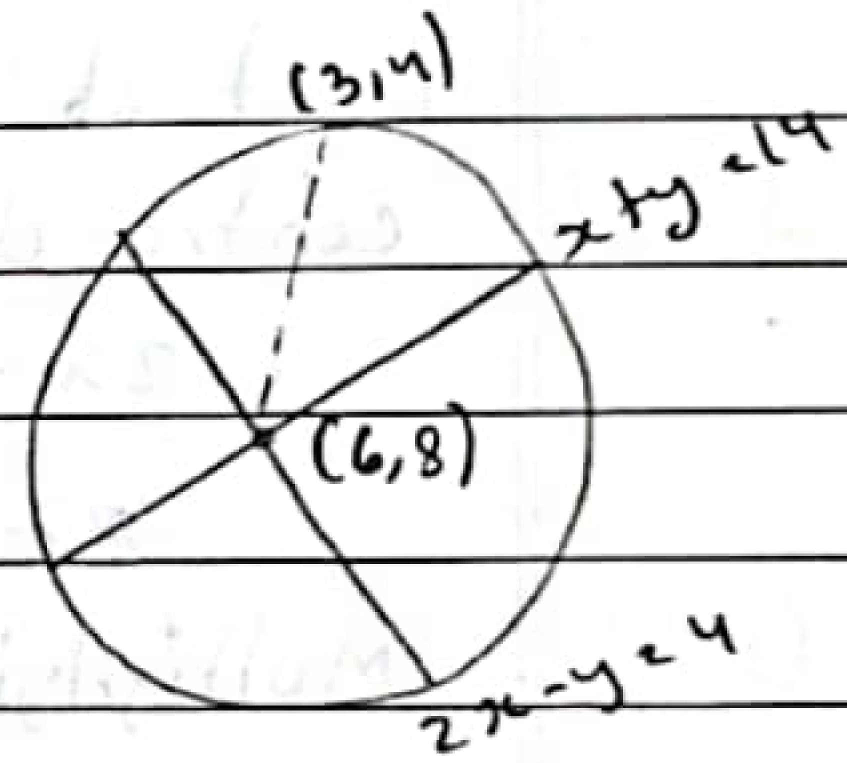
or $x+3 = 4$, $y+4 = 6$

or $x = 1$, $y = 2$

Hence,

the coordinate of other end $A(x,y)$ is $(1,2)$

5.a. The equation of two diameter of a circle passing through the point $(3,4)$ are $(x+y=14)$ and $2x-y=4$. Find the equation of circle.



Solution:

The point of intersection of two diameter is the centre of circle which is obtained by;

$$x+y = 14$$
$$2x - y = 4$$
$$3x = 18$$
$$\therefore x = 6$$

Also,

$$6+y = 14$$
$$\therefore y = 8$$

Hence, centre (h,k) is $(6,8)$

Also,

Radius of the circle is the distance between $(6,8)$ and $(3,4)$

~~$r^2 = (x-h)^2 + (y-k)^2$~~

~~or, $r^2 = (6-3)^2 + 18$.~~

$$r^2 = (3-6)^2 + (4-8)^2$$

or $r^2 = 9+16$

or, $r^2 = 25 = 5$

Again,

Equation of circle is given by;

$$(x-h)^2 + (y-k)^2 = r^2$$

or $(x-6)^2 + (y-8)^2 = 5^2$

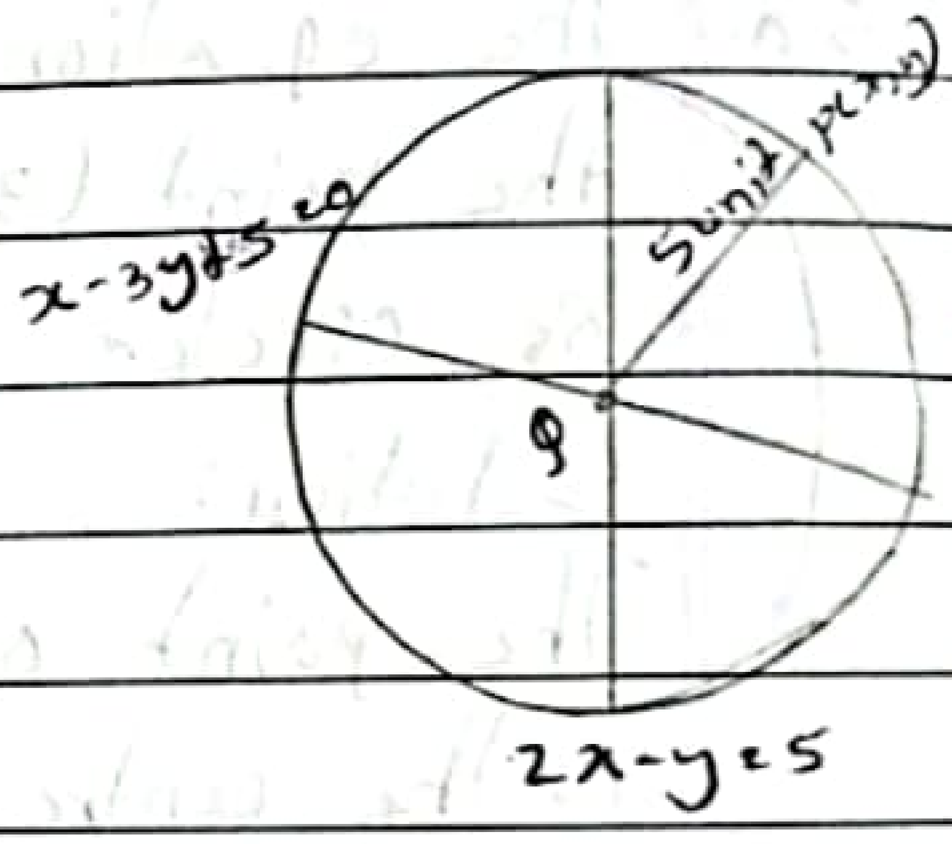
or, $x^2 - 12x + 36 + y^2 - 16y + 64 - 25 = 0$

or, $x^2 - 12x - 16y + 75 = 0$ is the required equation.

---

b. A circle has radius 5 units and the equation of its two diameter are $2x - y = 5$ and $x - 3y + 5 = 0$. Find the equation of the circle and show that it passes through the origin.

Solution:



Here,

Point of intersection of diameter is the centre of circle.

$2x - y = 5$ —(i)

$\cancel{2}x - 3y = -5$ —(ii)

Multiplying equation (ii) by (2)

Now,

$$2\cancel{x} - y = 5$$
$$- \cancel{2x} - 6y = -10$$
$$\quad (+) \qquad (+)$$

$$5y = 15$$
$$\therefore y = 3$$

Also,

$$2x - 3 = 5$$
$$\therefore x = 4$$

Hence, Centre $(h,k) = \cancel{(3,4)}\ (4,3)$

radius $(r) = 5$ units

Now,

Equation of the circle is given by;

$$(x-h)^2 + (y-k)^2 = r^2$$

or $(x-4)^2 + (y-3)^2 = 25$

or $x^2 - 8x + 16 + y^2 - 6x + 9 = 25$

or, $x^2 - 8x - 6x + y^2 + 25 = 25$

or $x^2 + y^2 - 8x - 6y = 25 - 25$

or $x^2 + y^2 - 8x - 6y = 0$ is the required equation.

Also,

Equation of circle is $x^2 + y^2 - 8x - 6y = 0$

Now,

substituting $(0,0)$ in the equation.

or, $0^2 + 0^2 - 8\times 0 - 6\times 0 = 0$

or $0 = 0$

So, it passes through the origin.

6.a.b. Find the ~~equation~~ radius of the circle which passes through the point $A(-4,-2)$, $B(2,6)$ and $C(2,-2)$

Solution:

Since, the radius of circle passes through $A(-4,-2)$, $B(2,6)$ and $C(2,-2)$.

The equation of circle is given by;

$(-4-h)^2 + (-2-k)^2 = r^2$ —— (i)

$(2-h)^2 + (6-k)^2 = r^2$ —— (ii)

$(2-h)^2 + (-2-k)^2 = r^2$ —— (iii)

Now,

from equation (i) and (ii)

$(-4-h)^2 + (-2-k)^2 = (2-h)^2 + (6-k)^2$

or $16 + 8h + \cancel{h^2} + 4 + 4k + \cancel{k^2} = 4 - 4h + \cancel{h^2} + 36 - 12k + \cancel{k^2}$

or, $20 + 8h + 4k = 40 - 4h - 12k$

or $8h + 4h + 4k + 12k = 40 - 20$

or, $12h + 16k = 20$

or, $4(3h + 4k) = 20$

or $3h + 4k = 5$ —— (iv)

Again, from (ii) and (iii)

$(2-h)^2 + (6-k)^2 = (2-h)^2 + (-2-k)^2$

or $(6-k)^2 = (-2-k)^2$

or $36 - 12k + \cancel{k^2} = 4 + 4k + \cancel{k^2}$

or $36 - 4 = 12k + 4k$

or $32 = 16k$

$\therefore k = 2$

Again,

from (iv)

$3h + 4k = 5$

or, $3h + 4 \times 2 = 5$

or, $3h = 5 - 8$

$\therefore h = -1$

Hence,

Centre $(h,k) = (-1,2)$

Now,

radius of the circle is the distance between $(-1,2)$ and $(2,6)$

or $r = \sqrt{(2+1)^2 + (6-2)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$ units

---

b. Find the equation of the circle which passes through the points $(1,2)$, $(3,-4)$ and $(5,-6)$. Hence, find its radius.

Solution:

Hence, the equation of circle passes through $(1,2)$, $(3,-4)$ and $(5,-6)$.

The equation is given by;

$(1-h)^2 + (2-k)^2 = r^2$ —— (i)

$(3-h)^2 + (-4-k)^2 = r^2$ —— (ii)

$(5-h)^2 + (-6-k)^2 = r^2$ —— (iii)

Here,

from (i) and (ii)

$(1-h)^2 + (2-k)^2 = (3-h)^2 + (-4-k)^2$

or, $1 - 2h + \cancel{h^2} + 4 - 4k + \cancel{k^2} = 9 - 6h + \cancel{h^2} + 16 + 8k + \cancel{k^2}$

or, $5 - 2h - 4k = 25 - 6h + 8k$

or, $5 - 25 = -6h + 2h + 8k + 4k$

or $-20 = -4h + 12k$

or $-20 = -4(h - 3k)$

or, $h - 3k = \frac{-20}{-4}$

$\therefore h - 3k = 5$ —— (iv)

Again,

from (ii) and (iii)

$(3-h)^2 + (-4-k)^2 = (5-h)^2 + (-6-k)^2$

or, $9 - 6h + h^2 + 16 + 8k + k^2 = 25 - 10h + h^2 + 36 + 12k + \cancel{k^2}$

or, $25 - 6h + 8k = 61 - 10h + 12k$

or, $-6h + 10h + 8k - 12k = 61 - 25$

or, $4h - 4k = 36$

or $(h-k) = \frac{36}{4}$

or $h - k = 9$

or, $h = k + 9$ —— (iv)

Now,

Putting the value of h in equation (iv)

$h - 3k = 5$

or, $k + 9 - 3k = 5$

or $9 - 5 = 2k$

$\therefore k = 2$

Also,

Putting the value of k in equation (iv)

$h - 3k = 5$

or $h - 3\times2 = 5$

$\therefore h = 11$

Hence,

Centre $(h,k) = (11,2)$

Now radius of the circle is the distance between $(11,2)$ and $(1,2)$

or, $r = \sqrt{(1-11)^2 + (2-2)^2}$

or, $r = \sqrt{(10)^2}$

$r = 10$

Here,

centre $(h,k) = (11,2)$

radius $(r) = 10$

Equation of circle which passes through $(2,2)$, $(3,-4)$ and $(5,-6)$ is given by;

$(x-h)^2 + (y-k)^2 = r^2$

or $(x-11)^2 + (y-2)^2 = (10)^2$

or $x^2 - 22x + 121 + y^2 - 4y + 4 = 100$

or $x^2 - 22x + y^2 - 4y + 125 - 100 = 0$

or $x^2 + y^2 - 22x - 4y + 25 = 0$ is the required equation.

7.a. Find the ~~radius~~ equation of the circle which passes through the point ~~A(-4,-2)~~ $(-2,0)$ and $(0,-2)$ and its center lies on the straight line $2x - 3y + 1 = 0$

Solution:

Equation of circle is

$(x-h)^2 + (y-k)^2 = r^2$ —— (i)

It passes through the point $(-2,0)$ and $(0,-2)$ then

$(-2-h)^2 + (0-k)^2 = r^2$ —— (ii)

$(0-h)^2 + (-2-k)^2 = r^2$ —— (iii)

From (ii) and (iii)

$(0-h)^2 + (-2-k)^2 = (-2-h)^2 + (0-k)^2$

or $\cancel{h^2} + 4 + 4k + \cancel{k^2} = 4 + 4h + \cancel{h^2} + \cancel{k^2}$

or $\cancel{4} + 4k = \cancel{4} + 4h$

or $4k = 4h$

$\therefore k = h$ —— (iv)

Since,

the centre lies on the line;

$2x - 3y + 1 = 0$

or, $2h - 3k = -1$

or $2h - 3h = -1$

or $-h = -1$

$\therefore h = 1 = k$

$\therefore$ Centre $(h,k) = (1,1)$

Hence,

radius is the distance between $(1,1)$ and $(0,-2)$ then

$r^2 = (0-1)^2 + (-2-1)^2$

or $r^2 = 1 + 9$

$\therefore r^2 = 10$

Also,

Equation of the circle is;

$(x-h)^2 + (y-k)^2 = r^2$

or $(x-1)^2 + (y-1)^2 = 10$

or, $x^2 - 2x + 1 + y^2 - 2y + 1 = 10$

or $x^2 - 2x - 2y + 2 + y^2 = 10$

or, $x^2 - 2x - 2y + y^2 = 10 - 2$

or $x^2 + y^2 - 2x - 2y - 8 = 0$ is the required equation.

6. The centre of the circle passing through the origin and the point (4,2) lies on the line $x+y=1$. Find the equation of the circle.

Solution:

Equation of circle is given by

$(x-h)^2+(y-k)^2=r^2$ —(i)

Since,

~~The centre of the~~ It passes through (0,0) and (4,2). then

$(0-h)^2+(0-k)^2=r^2$ —(ii)

$(4-h)^2+(2-k)^2=r^2$ —(iii)

from (ii) and (iii)

$(0-h)^2+(0-k)^2=\cancel{r^2}\ (4-h)^2+(2-k)^2$

or, $h^2+k^2=\cancel{16+}\ 16-8h+h^2+4-4k+k^2$

or, $h^2-h^2+k^2-k^2=16-8h-4k+4$

or, $20-8h-4k=0$ —(iv)

Here,

the centre (h,k) lies on the line $x+y=1$, then

$h+k=1$

or $h=1-k$ —(v)

Also,

Putting the value of h in equation (iv)

or $20-8(1-k)-4k=0$

or, $20-8+8k-4k=0$

or, $12+4k=0$

$\therefore k=-3$

Also,

Putting the value of k in equation (v)

$h=1+3$

$\therefore h=4$

Hence, Centre $(h,k)=(4,-3)$

Now,

radius of the circle is the distance between (4,-3) and (0,0)

$r^2 = (0-4)^2 + (0+3)^2$

$r^2 = 16+9$

$r^2 = 25$

Also,

The equation of the circle is given by;

$(x-h)^2 + (y-k)^2 = r^2$

or $(x-4)^2 + (y+3)^2 = 25$

or $x^2 - 8x + 16 + y^2 + 6y + 9 = 25$

or $x^2 + y^2 - 8x + 6y + 25 = 25$

or $x^2 + y^2 - 8x + 6y = 25 - 25$

or $x^2 + y^2 - 8x + 6y = 0$ is the required equation.

8. Find the equation of a circle which is concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passes through (5,4).

Solution.

Here, equation of the circle is;

$x^2 + y^2 - 8x + 12y + 15 = 0$

or $x^2 - 2.x.4 + 4^2 + y^2 + 2.y.6 + 6^2 = -15 + 16 + 36$

or $(x-4)^2 + (y-6)^2 = 37$

or $(x-4)^2 + (y+6)^2 = (\sqrt{37})^2$

Now,

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$

Centre $(h,k) = (4,-6)$

radius $(r) = \sqrt{37}$

Also,

Equation of circle which is concentric with the circle $x^2+y^2 - 8x + 12y + 15 = 0$ with centre $(h,k) = (4,-6)$

$\therefore$ Equation is given by;

$(x-4)^2 + (y+6)^2 = r^2$

Since, it passes through (5,4), then

$(x-4)^2 + (y+6)^2 = r^2$

or, $(5-4)^2 + (4+6)^2 = r^2$

or, $1 + 100 = r^2$

$\therefore r^2 = 101$

Again,

The required equation is given by;

$(x-4)^2 + (y+6)^2 = 101$

or, $x^2 - 8x + 16 + y^2 + 12y + 36 = 101$

or, $x^2 + y^2 - 8x + 12y = 101 - 16 - 36$

or, $x^2 + y^2 - 8x + 12y - 49 = 0$ is the required equation

9.a Find the equation of a circle which touches the x-axis at a point (3,0) and passing through the point (1,2).

Solution

Let A (h,k) be the centre of the circle.

Since the circle touches x-axis at (3,0), then

$h = 3$ and radius $(r) = k$

So,

the equation of the circle is

$(x-h)^2 + (y-k)^2 = r^2$

or, $(x-3)^2 + (y-k)^2 = k^2$ — (i)

Since,

It passes through the point (1,2)

so, $(1-3)^2 + (2-k)^2 = k^2$

or, $4 + 4 - 4k + \cancel{k^2} = \cancel{k^2}$

or, $8 - 4k = 0$

or, $k = 2$

Then from (i) equation of the circle is

$(x-3)^2 + (y-k)^2 = k^2$

or, $(x-3)^2 + (y-2)^2 = 2^2$

or $x^2 - 6x + 9 + y^2 - 4y + \cancel{4} = \cancel{4}$

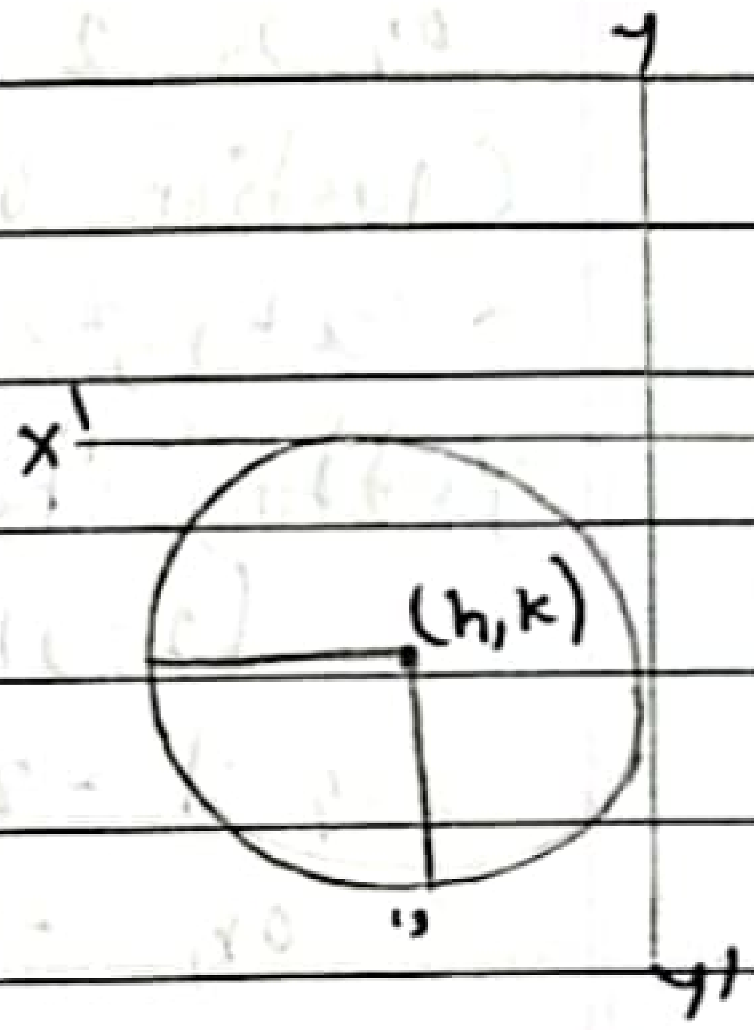
or, $x^2 + y^2 - 6x - 4y = 0$ is the required equation.

b. Find the equation of the circle which touches both the axes and has the centre on the line $x - 2y = 3$.

Solution:

Let (h,k) be the centre of the circle.

As, the circle touches both axes and lies on the line $x-2y=3$, then



$$x - 2y = 3$$

$$\text{or, } h - 2k = 3 \quad \text{—(i)}$$

Since,

the circle touches both the axes, so,

$$h = k = r$$

Now, the equation of circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\text{or, } (x-h)^2 + (y-h)^2 = h^2$$

$$\text{or, } x - 2xh + h^2 + y^2 - 2yh + \cancel{h^2} = \cancel{h^2}$$

$$\text{or } x^2 - 2hx + h^2 + y^2 - 2hy = 0 \quad \text{—(ii)}$$

Also,

(h,k) lies on $x - 2y = 3$

i.e. $h - 2k = 3$

or $h - 2h = 3$

$\therefore h = -3 = k$

From (ii)

$$x^2 - 2hx + h^2 + y^2 - 2hy = 0$$

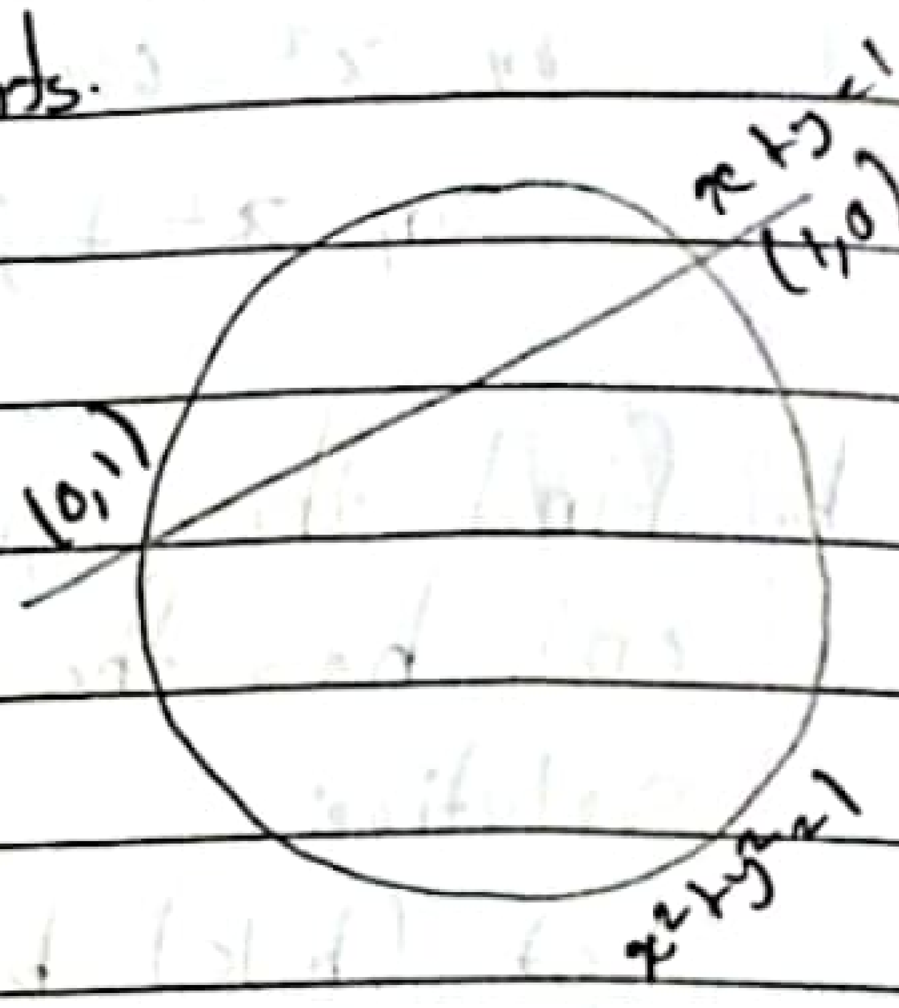
$$\text{or, } x^2 - 2(-3)x + (-3)^2 + y^2 - 2(-3)y = 0$$

$$\text{or } x^2 + 6x + 9 + y^2 + 6y = 0$$

or $x^2 + y^2 + 6x + 6y + 9 = 0$ is the required equation.

10.a. If a line $x+y=1$ cuts a circle $x^2+y^2=1$ at two points. Find the distance between two points.

Solution:



Equation of line is

$x+y=1$

or, $x=1-y$ —— (i)

Equation of circle is

$x^2+y^2=1$ —— (ii)

Putting the value of 'x' in equation (ii)

$(1-y)^2+y^2=1$

or, $1-2y+y^2+y^2=1$

or, $-2y+2y^2=0$

or $-2y(1-y)=0$

~~So, $y_1=0$ -1 $y_2=0$~~

So, $y_1=1$

$y_2=0$

When,

$y=-1$, then

$x=1-y$

or $x=1-1$

$\therefore x_1=0$

Also,

when $y=0$, then

$x=1-y$

or $x=1-0$

$\therefore x_2=1$

Now, the two points are ~~(1,0~~ (0,1) and (1,0).

Distance between them $=\sqrt{(1-0)^2+(0-1)^2}$

$=\sqrt{1+1}$

$=\sqrt{2}$ units.

b. Show that the two circles $x^2+y^2=100$ and $x^2+y^2-24x-10y+160=0$ touch externally.

Solution:

Equation of first circle is;

$x^2+y^2=100$

or, $(x-0)^2+(y-0)^2=10^2$

Comparing it with $(x-h)^2+(y-k)^2=r^2$

Centre $(h,k)=(0,0)$

radius$(r)=10$

Again,

equation of second circle is;

$x^2+y^2-24x-10y+160=0$

or $x^2-2.x.12+(12)^2+y^2-2.y.5+5^2=-160+25+144$

or $(x-12)^2+(y-5)^2=3^2$

Now,

Comparing it with $(x-h)^2+(y-k)^2=r^2$, then

Centre $(h,k)=(12,5)$

radius$(r)=3$

Also,

Distance between $(0,0)$ and $(12,5)$ is

$\sqrt{(12-0)^2+(5-0)^2}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$=13$

Also,

Sum of the radius $=10+3$

$=13$

Hence,

Distance between centre = Sum of radius. So, the circle touches externally.