Exercise - 6.1

General Section

1.a. Matrix A has 2 rows and 3 Columns whereas matrix B has 3 rows and 3 columns. Can BA be multiplied? Can AB be multiplied? Give reason for your answer.

Solution:

Here, $A = A_{2\times3}$

$B = B_{3\times3}$

for AB, No. of Column of matrix A = No. of rows of matrix B. So, AB is possible.

for BA, No. of column of matrix ~~A~~B ≠ No. of ~~A~~ rows of matrix ~~B~~A. So, BA is not possible.

2. Find the product of the matrices given belows.

a. $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$. find AB

Solution

$$AB = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times3+3\times6 & 1\times2+3\times4 \\ 2\times3+4\times6 & 2\times2+4\times4 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 14 \\ 30 & 20 \end{pmatrix}$$

b. $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$. find AB and BA

Solution:

$$AB = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times1 + 5\times-3 & 2\times-1 + 5\times2 \\ 1\times1 + 3\times-3 & 1\times-1 + 3\times2 \end{pmatrix}$$

$$= \begin{pmatrix} 2-15 & -2+10 \\ 1-9 & -1+6 \end{pmatrix}$$

$$= \begin{pmatrix} -13 & 8 \\ -8 & 5 \end{pmatrix}$$

Also,

$$BA = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times2 + (-1\times1) & 1\times5 + (-1\times3) \\ -3\times2 + 2\times1 & -3\times5 + 2\times3 \end{pmatrix}$$

$$= \begin{pmatrix} 2-1 & 5-3 \\ -6+2 & -15+6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ -4 & -9 \end{pmatrix}$$

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c. $A = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$. find AB and BA.

Solution:

$$AB = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2 - 2\times3 + 1\times1 & 1\times1 - 2\times2 + 1\times1 \\ 2\times2 + 1\times3 + 3\times1 & 2\times1 + 1\times2 + 3\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$$

$$= \begin{bmatrix} 2\times1 + 1\times2 & 2\times-2 + 1\times1 & 2\times1 + 1\times3 \\ 3\times1 + 2\times2 & 3\times-2 + 2\times1 & 3\times1 + 2\times3 \\ 1\times1 + 1\times2 & 1\times-2 + 1\times1 & 1\times1 + 1\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 5 \\ 7 & -4 & 9 \\ 3 & -1 & 4 \end{bmatrix}$$

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d. $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. Find AB and BA if possible otherwise give reason for your answer.

Solution:

$$AB = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times3 + 2\times2 + 3\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \end{bmatrix}$$

Also,

$$BA = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times1 & 3\times2 & 3\times3 \\ 2\times1 & 2\times2 & \cancel{1\times3}\ 2\times3 \\ 1\times1 & 1\times2 & 1\times3 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

3.a. If $A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$ then prove that $A^2 = A$.

Solution:

Here,

$$A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$$

Also,

$$A^2 = A \times A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2 + 1\times-2 & 2\times1 + 1\times-1 \\ -2\times2 + -1\times-2 & -2\times1 + -1\times-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$$

$$= A$$

$$\therefore A^2 = A$$

b. If $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find P^3 and hence show that $P^3 = P$.

Solution:

Here,

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Also,

$$P^3 = P \times P \times P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 0\times0 & 1\times0 + 0\times1 \\ 0\times1 + 1\times0 & 0\times0 + 1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 0\times0 & 1\times0 + 0\times1 \\ 0\times1 + 1\times0 & 0\times1 + 1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= P$

$\therefore P^3 = P$

Proved//

c. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ show that $A^2 - 2A = O$, where O is a null matrix of the same order as of A.

Solution:

Here,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Now,

$$A^2 - 2A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 1\times1 & 1\times1 + 1\times1 \\ 1\times1 + 1\times1 & 1\times1 + 1\times1 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 2-2 \\ 2-2 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Proved//

d. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ show that $A^2 - 5A = 14I$, where I is an identity matrix.

Solution:

Here,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

Now,

$$A^2 - 5A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times3 + -5\times-4 & 3\times-5 + -5\times2 \\ -4\times3 + \cancel{2}\times-4 & -4\times-5 + 2\times2 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15 & -25+25 \\ -20+20 & 24-10 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 14I$$

Proved//

e. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ show that $A^3 - 3A - 2I = 0$

Solution:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Now,

$$A^3 - 3A - \cancel{2A}\ 2I$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 3 \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0+1\times1+1\times1 & 0\times1+1\times0+1\times1 & 0\times1+1\times1+1\times0 \\ 1\times0+0\times1+1\times1 & 1\times1+0\times0+1\times1 & 1\times1+0\times1+1\times0 \\ 1\times0+1\times1+0\times1 & 1\times1+1\times0+0\times1 & 1\times1+1\times1+0\times0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$- \begin{bmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times0+1\times1+1\times1 & 2\times1+1\times0+1\times1 & 2\times1+1\times1+1\times0 \\ 1\times0+2\times1+1\times1 & 1\times1+2\times0+1\times1 & 1\times1+2\times1+1\times0 \\ 1\times0+1\times1+2\times1 & 1\times1+1\times0+2\times1 & 1\times1+1\times1+2\times0 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= O$$

Proved

f. If $A = \begin{bmatrix} -2 & 4 \\ 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ -2 & 5 \end{bmatrix}$, show that $(AB)' = B'A'$.

Soln,

$$AB = \begin{bmatrix} -2 & 4 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times 3 + 4\times -2 & -2\times 0 + 4\times 5 \\ 3\times 3 + 7\times -2 & 3\times 0 + 7\times 5 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 20 \\ -5 & 35 \end{bmatrix}$$

Also,

$$(AB)' = \begin{bmatrix} -14 & -5 \\ 20 & 35 \end{bmatrix}$$

$$A' = \begin{bmatrix} -2 & 3 \\ 4 & 7 \end{bmatrix} \qquad B' = \begin{bmatrix} 3 & -2 \\ 0 & 5 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} -2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times 3 + 3\times 0 & -2\times -2 + 3\times 5 \\ 4\times 3 + 7\times 0 & 4\times -2 + 7\times 5 \end{bmatrix}$$

$$= \begin{bmatrix} -\cancel{0}14 & \cancel{20}\ -5 \\ 20 & 35 \end{bmatrix}$$

Hence,

$$(AB)' = B'A'$$

4. a. If $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 5 \\ 3 & -2 \end{pmatrix}$ show that $AB \neq BA$

Solution:

$$AB = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -1 & 5 \\ 3 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times-1 + 4\times3 & 2\times5 + 4\times-2 \\ 1\times-1 + 3\times3 & 1\times5 + 3\times-2 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 2 \\ 8 & -1 \end{pmatrix}$$

Also,

$$BA = \begin{pmatrix} -1 & 5 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1\times2 + 5\times1 & -1\times4 + 5\times3 \\ 3\times2 + -2\times1 & 3\times4 + -2\times3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 11 \\ 4 & 6 \end{pmatrix}$$

Hence,

$AB \neq BA$ //

b. If $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$, show that $AB = BA$

Solution:

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times1 + 0\times0 & 1\times0 + 0\times4 \\ 0\times1 + 2\times0 & 0\times0 + 2\times4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 8 \end{pmatrix}$$

Again,

$$BA = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times1 + 0\times0 & 1\times0 + 0\times2 \\ 0\times1 + 4\times0 & 0\times0 + 4\times2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 8 \end{pmatrix}$$

Hence,

$AB = BA$, proved

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c. If $A = \begin{pmatrix} 5 & 3 \\ 2 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix}$, show that $AB = BA = I$.

Soln,

$$AB = \begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 5\times-1 + 3\times2 & 5\times3 + 3\times-5 \\ 2\times-1 + 1\times2 & 2\times3 + 1\times-5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1\times5 + 3\times2 & -1\times3 + 3\times1 \\ 2\times5 + -5\times2 & 2\times3 + -5\times1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Hence,

$AB = BA = I$ proved

d. If $A = \begin{pmatrix} 2 & -3 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 0 \\ 4 & 0 \end{pmatrix}$, show that $AB = 0$.

Solution:

$$AB = \begin{pmatrix} 2 & -3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 0 \\ 4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times6+-3\times4 & 2\times0+-3\times0 \\ 0\times6+0\times4 & 0\times0+0\times0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$$

proved

5. a. If $\begin{bmatrix} 1 & 2 \\ x & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} y & 2 \\ 4 & -3 \end{bmatrix}$, find the values of x and y.

Solution:

$$\begin{bmatrix} 1 & 2 \\ x & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} y & 2 \\ 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+2\times0 & 1\times0+2\times1 \\ x\times1+-3\times0 & x\times0+-3\times1 \end{bmatrix} = \begin{bmatrix} y & 2 \\ 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ x & -3 \end{bmatrix} = \begin{bmatrix} y & 2 \\ 4 & -3 \end{bmatrix}$$

Now, Comparing the corresponding value of equal matrix.

$\therefore y = 1$ and $x = 4$

b. If $\begin{pmatrix} 4 & 1 \\ 7 & -3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} x & -1 \\ 4 & y \end{pmatrix}$, then find the value of x and y.

Solution:

$$\begin{pmatrix} 4 & 1 \\ 7 & -3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} x & -1 \\ 4 & y \end{pmatrix}$$

$$= \begin{pmatrix} 4\times1 + 1\times1 & 4\times-1 + 1\times3 \\ 7\times1 + -3\times1 & 7\times-1 + -3\times3 \end{pmatrix} = \begin{pmatrix} x & -1 \\ 4 & y \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 \\ 4 & -16 \end{pmatrix} = \begin{pmatrix} x & -1 \\ 4 & y \end{pmatrix}$$

Now,

Comparing the corresponding values.

$\therefore x = 5$ and $y = -16$

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c. Given that $A = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ and $AB = A+B$ then find the value of a, b and c.

Solution:

$$AB = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

$$= \begin{pmatrix} 3\times a + 0\times0 & 3\times b + 0\times c \\ 0\times a + 4\times0 & 0\times b + 4\times c \end{pmatrix}$$

$$= \begin{pmatrix} 3a & 3b \\ 0 & 4c \end{pmatrix}$$

Also,

$$A + B = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

$$= \begin{pmatrix} 3+a & b \\ 0 & 4+c \end{pmatrix}$$

Since,

$AB = A+B$ then,

$$\begin{pmatrix} 3a & 3b \\ 0 & 4c \end{pmatrix} = \begin{pmatrix} 3+a & b \\ 0 & 4+c \end{pmatrix}$$

Comparing the corresponding elements.

$3a = 3+a$

or $3a - a = 3$

or $2a = 3$

$\therefore a = 3/2$

$3b = b$

or $3b - b = 0$

or $2b = 0$

$\therefore b = 0$

$4c = 4+c$

or $4c - c = 4$

or $c = 4/3$

Hence, $a = 3/2$, $b = 0$ and $c = 4/3$

d. If $\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, find the matrix $\begin{pmatrix} x \\ y \end{pmatrix}$

Solution:

Here, $\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Or, $\begin{pmatrix} -1 \times x + 0 \times y \\ 0 \times x + -2 \times y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

or $\begin{pmatrix} -x + 0 \\ 0 + -2y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

or $\begin{pmatrix} -x \\ -2y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Comparing the corresponding elements.

$-x = -2$ $\qquad$ $-2y = 4$

$\therefore x = 2$ $\qquad$ $\therefore y = -2$

Creative Section

a. If $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 4 \\ 2 & 4 \end{pmatrix}$ and if $AC = B$, find the matrix C.

Solution:

Here, $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 4 \\ 2 & 4 \end{pmatrix}$

Let the matrix C be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

According to question

$AC = B$ then,

$$\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & 4 \end{pmatrix}$$

or $\begin{pmatrix} 2\times a + 1\times c & 2\times b + 1\times d \\ 5\times a + 3\times c & 5\times b + 3\times d \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & 4 \end{pmatrix}$

or, $\begin{pmatrix} 2a + c & 2b + d \\ 5a + 3c & 5b + 3d \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & 4 \end{pmatrix}$

Comparing the corresponding elements;

$2a + c = 3$ —— (i)

$5a + 3c = 2$ —— (ii)

$2b + d = 4$ —— (iii)

$5b + 3d = 4$ —— (iv)

Now,

Multiplying equation (i) by 3 and subtracting equation (ii) from (i)

$6a + 3\cancel{c} = 9$

$5a + 3\cancel{c} = 2$

$(-) \quad (-) \quad\quad (-)$

$a = 7$

putting the value of a in equation (ii)

$5 \times 7 + 3c = 2$

or $35 + 3c = 2$

or $c = \frac{2-35}{3}$

$\therefore c = -11$

Again,

Multiplying equation (iii) by 3 and subtracting (iv) from (iii)

$6b + 3\cancel{d} = 12$

$5b + 3\cancel{d} = 4$

$(-) \quad (-) \quad\quad (-)$

$b = 8$

putting the value of b in equation (v)

$5 \times 8 + 3d = 4$

or $d = \frac{4-40}{3}$

$\therefore d = -12$

Hence

$$C = \begin{bmatrix} 7 & 8 \\ -11 & -12 \end{bmatrix}$$

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b. If $\begin{pmatrix} -1 & 2 \\ 2 & -2 \end{pmatrix} \times A = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, find matrix A.

Soln,

Let matrix A be $\begin{pmatrix} x \\ y \end{pmatrix}$

Now $\begin{pmatrix} -1 & 2 \\ 2 & -2 \end{pmatrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

or $\begin{pmatrix} -1\times x + 2\times y \\ 2\times x + -2\times y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

or $\begin{pmatrix} -x + 2y \\ 2x - 2y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Now,

Comparing corresponding elements

$-x + 2y = -2$ —— (i)

$2x - 2y = 4$ —— (ii)

Adding (i) and (ii)

$-x + \cancel{2y} = -2$

$2x - \cancel{2y} = 4$

$x = 2$

putting the value of x in equation (ii)

$2\times 2 - 2y = 4$

or $y = \frac{4-4}{0}$

$\therefore y = 0$

Hence, $A = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

Which matrix post multiplies to $\begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix}$ to get $\begin{pmatrix} 19 & -3 \\ 25 & 13 \end{pmatrix}$

Solution:

Let the required matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

By question:

$$\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 19 & -3 \\ 75 & 13 \end{pmatrix}$$

or, $$\begin{pmatrix} 1\times a+3\times c & 1\times b+3\times d \\ 3\times a+4\times c & 3\times b+4\times d \end{pmatrix} = \begin{pmatrix} 19 & -3 \\ 75 & 13 \end{pmatrix}$$

or, $$\begin{pmatrix} a+3c & b+3d \\ 3a+4c & 3b+4d \end{pmatrix} = \begin{pmatrix} 19 & -3 \\ 75 & 13 \end{pmatrix}$$

Comparing the corresponding elements.

$a+3c = 19$ —— (i)

$3a+4c = 75$ —— (ii)

$b+3d = -3$ —— (iii)

$3b+4d = 13$ —— (iv)

Here,

Multiplying equation (i) by ~~three~~ 3 and subtracting to equation (ii)

$$\begin{aligned} \cancel{3a} + 9c &= 57 \\ \cancel{3a} + 4c &= 75 \\ (-)\quad (-)\quad &\quad (-) \\ \hline 5c &= -18 \end{aligned}$$

$\therefore c = -18/5$

putting the value of c in equation (ii)

$3a + 4\times -\frac{18}{5} = 75$

or $3a - \frac{72}{5} = 75$

or $3a = 75 + \frac{72}{5}$

or $a = \frac{89.4}{3}$

$\therefore a = 149/5$

Multiplying equation (iii) by 3 and subtracting equation ~~(iii) from~~ (iv)

$$\cancel{3b} + \cancel{4d}\ 9d = -9$$
$$\cancel{3b} + 4d = 13$$
$$(-)\quad (-)\quad (-)$$

$$5d = -22$$
$$\therefore d = -22/5$$

putting the value of d in equation (iv)

$$\cancel{3\times\frac{-22}{5}} + \quad 3b + 4\times\frac{-22}{5} = 13$$

$$\text{or } 15b - 88 = 13\times5$$
$$\text{or } b = \frac{65+88}{15}$$
$$\therefore b = \frac{153}{15} = \frac{51}{5}$$

Hence, required matrix is;

$$\begin{pmatrix} 149/5 & 51/5 \\ -18/5 & -22/5 \end{pmatrix} \qquad \cancel{\begin{pmatrix} 149/3 & 153/25 \\ -18/5 & 22/5 \end{pmatrix}}$$

a. Which matrix pre multiplies the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ to get $\begin{pmatrix} 4 & 5 \\ 6 & 2 \end{pmatrix}$?

Solution:

Let the required matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

By question:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 6 & 2 \end{pmatrix}$$

$$\text{or } \begin{pmatrix} a\times1 + b\times3 & a\times1 + b\times4 \\ c\times1 + d\times3 & c\times1 + d\times4 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 6 & 2 \end{pmatrix}$$

$$\begin{pmatrix} a + 3b & a + 4b \\ c + 3d & c + 4d \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 6 & 2 \end{pmatrix}$$

Comparing the corresponding elements.

$a + 3b = 4$ —— (i)

$a + 4b = 5$ —— (ii)

$c + 3d = 6$ —— (iii)

$c + 4d = 2$ —— (iv).

Now,

~~Multiplying~~ subtracting equation (i) from (ii)

$$\begin{array}{rcrcl} \cancel{a} & + & 4b & = & 5 \\ \cancel{a} & + & 3b & = & 4 \\ (-) & & (-) & & (-) \\ \hline \end{array}$$

$\therefore b = 1$

putting the value of b in equation (i)

$a + 4 \times 1 = 5$

$\therefore b = 1$

Subtracting equation (iii) from (iv)

$$\begin{array}{rcrcl} \cancel{c} & + & 4d & = & 2 \\ \cancel{c} & + & 3d & = & 6 \\ (-) & & (-) & & (-) \\ \hline \end{array}$$

$\therefore d = -4$

putting the value of d in equation (i)

$c - 4 \times 4 = 2$

or, $c - 16 = 2$

$\therefore c = 18$

Hence, the required matrix is $\begin{pmatrix} 1 & 1 \\ 18 & -4 \end{pmatrix}$

b. Which matrix pre multiplies to $\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$ to get $\begin{bmatrix} 14 & 20 \end{bmatrix}$

Solution:

Let the required matrix be $\begin{bmatrix} a & b \end{bmatrix}$

By question:

$$\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} = \begin{bmatrix} 14 & 20 \end{bmatrix}$$

or $\begin{pmatrix} a\times2+b\times6 & a\times4+b\times8 \end{pmatrix} = \begin{pmatrix} 14 & 20 \end{pmatrix}$

or $\begin{pmatrix} 2a+6b & 4a+8b \end{pmatrix} = \begin{pmatrix} 14 & 20 \end{pmatrix}$

Comparing the corresponding elements.

$2a+6b = 14$ — (i)

$4a+8b = 20$ — (ii)

Multiplying equation (i) by 2 and subtracting equation (ii)

$\cancel{4a} + 12b = 28$

$\cancel{4a} + 8b = 20$

(-) (-) (-)

or $4b = 8$

$\therefore b = 2$

putting the value of b in equation (i)

$2\times a + 6\times2 = 14$

or $a = \frac{14-12}{2}$

$\therefore a = 1$

Hence, required matrix is $\begin{pmatrix} 1 & 2 \end{pmatrix}$

8.a If $\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$. Prove that $a=b$.

Solution:

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$(a\times 2 + b\times 3) = (2\times a + 4\times b)$

or, $(2a + 3b) = (a + 4b)$

or, $2a - a = 4b - 3b$

or, $a = b$

$\therefore a = b$ proved.

b. If $(2a \;\; -1)\begin{bmatrix} -3 \\ -a \end{bmatrix} = (10)$, find the value of a.

Solution:

$(2a \;\; -1)\begin{bmatrix} -3 \\ -a \end{bmatrix} = (10)$

or, $(2a\times -3 + -a\times -1) = (10)$

or, $(-6a + a) = 10$

or, $-5a = 10$

$\therefore a = -2$

c. If $(m \;\; n)\begin{pmatrix} m \\ n \end{pmatrix} = (13)$ and $m = n - 1$, find m and n.

Solution:

$m^2 + n^2 = 13$

or, $(n-1)^2 + n^2 = 13$

or, $n^2 - 2n + 1 + n^2 = 13$

or, $2n^2 - 2n + 1 = 13$

or, $2n^2 - 2n - 12 = 0$

or, $2n^2 - 6n + 4n - 12 = 0$

or, $2n(n-3) + 4(n-3) = 0$

or, $(n-3)(2n+4) = 0$

Either $n = 3$ OR $n = -2$

when $n = 3$, $m = 2$

when $n = -2$, $m = -3$

2.a. A is a matrix of order $(2x+1)\times 2$ and B is another matrix of order $(3y-1)\times 3$. If AB and BA both exists then find the value of x and y.

Solution:

Given,

$B = (3y-1)\times 3$ and $A = (2x+1)\times 2$

Now,

If AB is possible, then no. of column of A = no. of row of B.

i.e. $2 = 3y-1$

or $2+1 = 3y$

$\therefore y = 1$

Again,

If BA is possible, then no. of column of B = no. of row of A

i.e. $3 = 2x+1$

or $3-1 = 2x$

$\therefore x = 1$

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b. Matrix P has x rows and $(11-x)$ columns. Matrix Q has y rows and $(y+5)$ columns. Find the values of x and y when PQ and QP both exists.

Solution:

$P = x\times(11-x)$ and $Q = (y)\times(y+5)$

Now,

If PQ is possible then, ~~rows~~ no. of column of P = no. of rows of Q.

i.e. $11-x = y$

or $x+y = 11$ —— (i)

Again,

If QP is possible then, column of Q = row of P i.e. $y+5 = x$

or $x-y=5$ — (ii)

Also,

Adding equation (i) and (ii)

$$x+y = 11$$
$$x-y = 5$$
$$2x = 16$$
$$\therefore x = 8$$

putting the value of x in equation (i)

$$8+y = 11$$
$$\therefore y = 3$$

Exercise - 6.2

General section:

1. Evaluate the value of the following determinants.

a. $\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix}$

Soln,

$= 2\times 3 - 4\times 1$

$= 6-4$

$= 2$

b. $\begin{vmatrix} 1 & \cos^2\theta \\ -1 & \sin^2\theta \end{vmatrix}$

Soln,

$= 1\times\sin^2\theta - (-1\times\cos^2\theta)$

$= \sin^2\theta + \cos^2\theta$

$= 1$

c. $\begin{vmatrix} a & b^2 \\ b & a^2 \end{vmatrix}$

Soln,

$= a\times a^2 - b\times b^2$

$= a^3 - b^3$

d. $\begin{vmatrix} a+b & 2b \\ 2a & a+b \end{vmatrix}$

Soln,

$= (a+b)\times(a+b) - 2a\times 2$

$= (a+b)^2 - 4ab$