Chapter-13
Measures of Dispersion

c/w

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Mean
i. Direct Method

$$\bar{x} = \frac{\Sigma fx}{N} \quad \text{or} \quad \bar{x} = \frac{\Sigma x}{N}$$

ii. Assuming Method

$$\bar{x} = A + \frac{\Sigma fd}{N}$$

A = Assumed Mean

$d = x - A$

iii. Step-deviation Method

$$\bar{x} = A + \frac{\Sigma fd'}{N} \times h$$

where, A = Assumed Mean

$$d' = \frac{x - A}{h}$$

h = common factor

Combined Mean

$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Partional Value
i. Individual and discrete Series.

$$Q = \left(\frac{N+1}{4}\right)^{th} \text{term}$$

$$Q_3 = 3\frac{(N+1)}{4}^{th} \text{item}$$

$$Md = \left(\frac{N+1}{2}\right)^{th} \text{item}$$

ii. Continuous Series

Class of $Q_1 = \frac{N}{4}$ th term

$$Q_1 = L + \frac{N/4 - cf}{f} \times h$$

Class of $Q_3 = \frac{3N}{4}$ th ~~item~~ term

$$Q_3 = L + \frac{3N/4 - cf}{f} \times h$$

Standard Deviation

Standard deviation is defined as the positive square root of the mean of the square of deviation taken from mean. It is denoted by 'σ'

A. Individual Series;

i. Actual mean method

$$S.D\ (\sigma) = \sqrt{\frac{\Sigma f(x-\bar{x})}{N}}$$

ii. Direct Method

$$S.D = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

iii. Short-Cut Method

$$S.D\ (\sigma) = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

where, $d = x - A$

A = Assumed Mean

iv. Step deviation Method

$$S.D\ (\sigma) = \sqrt{\frac{\Sigma d'^2}{N} - \left(\frac{\Sigma d'}{N}\right)^2}$$

Where, $d' = \frac{x - A}{h}$, A = Assumed Mean
h = Common Factor

B. Discrete Series

i. Actual ~~Method~~ Mean Method,

$$S.D\ (\sigma) = \sqrt{\frac{\Sigma f\ (x - \bar{x})}{N}}$$

ii. Direct Method,

$$S.D = \sqrt{\frac{\Sigma f d'^2}{N} - \left(\frac{\Sigma f d'}{N}\right)^2}$$

iii. Short cut Method

$$S.D\ (\sigma) = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2}$$

Where, $d = x - A$
A = Assumed Mean

iv. Step Deviation Method

$$S.D\ (\sigma) = \sqrt{\frac{\Sigma f d'^2}{N} - \left(\frac{\Sigma f d'}{N}\right)^2}$$

Where, $d' = \frac{x - A}{h}$, A = Assumed Mean
h = Common factor

c. Continuous Series

i. Actual Mean Method:

$$S.D(\sigma) = \sqrt{\frac{\Sigma f(M-\bar{x})}{N}}$$

ii. Direct Method

$$S.D = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$$

iii. Short cut Method

$$S.D(\sigma) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

where, $d = m - A$

A = Assumed mean

iv. Step-deviation Method

$$S.D(\sigma) = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2}$$

$d' = \frac{m-A}{h}$, A = Assumed Mean.
h = common factor

Variance

The square of standard deviation is called Variance and it is denoted by $(\sigma)^2$.

Coefficient of Variation

The relative measure of dispersion based on standard deviation is called Coefficient of Variation.

$$C.V = \frac{S.D}{\bar{x}} \times 100\% \quad \left(\text{Coeff. of } S.D = \frac{S.D}{\bar{x}}\right)$$

Note:

Two distribution can be compared with the help of C.V for their variability. Less the C.V, more will be the uniformity, consistensity, stability etc. and vice versa

Exercise - ~~13~~·1

1.a. Find the standard deviation from the following set of observation.

20, 25, 30, 36, 32, 43

Solution:

x	x^2
20	400
25	625
30	900
32	~~1296~~ 1024
36	~~1024~~ 1296
43	1849
$\Sigma x = 186$	$\Sigma x^2 = 6094$

Here, $\Sigma x = 186$, $\Sigma x^2 = 6094$, $n = 6$, $\sigma = ?$

We know,

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$= \sqrt{\frac{6094}{6} - \left(\frac{186}{6}\right)^2}$$

$$= 7.39$$

Hence, the req. S.D = 7.39.

b. Daily expenditure of 6 families are given below. Find S.D
Rs. 240, Rs. 180, Rs 320, Rs 160, Rs 260, Rs 400.

Solution:

x	x^2
~~24~~160	25,600
180	32,400
240	57,600
260	67,600
320	102,400
400	16,0000
$\Sigma x = 1560$	$\Sigma x^2 = 445600$

Here, $\Sigma x = 1560$, $\Sigma x^2 = 44,5600$, $n = 6$, $\sigma = ?$

We know,

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{445600}{6} - \left(\frac{1560}{6}\right)^2}$$

$$= \sqrt{74266.667 - 67600}$$

$$= 81.64$$

Hence, the required S.D is 81.64

2.a. The following table represents the distribution of the bonous of 50 workers. Find mean and standard deviation.

Bonus	5	10	15	20	25
Frequency:	12	20	8	6	4

Solution:

Arranging the data in a table:

x	f	$d = (x-A)$	fd	fd^2
5	12	-10	-120	1200
10	20	-5	-200	1,000
15	8	0	0	0

20	6	5	30	150
25	4	10	40	400
	N = 50		$\Sigma fd = -250$	$\Sigma fd^2 = 2750$

Now,

$$\bar{x} = A + \frac{\Sigma fd}{N}$$

$$= 15 + \frac{(-250)}{50}$$

$$= 10$$

Also,

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{2750}{50} - \left(\frac{-250}{50}\right)^2}$$

$$= \sqrt{55 - 25}$$

$$= 5.47$$

b. Find the standard deviation from the following data.

x	10	20	30	40	50
f	8	12	15	9	6

Solution:

Arranging the data in a table.

x	f	d = (x - A)	fd	fd^2
10	8	-20	-160	3200
20	12	-10	-120	1200
30	15	0	0	0
40	9	10	90	900
50	6	20	120	2400
	N = 50		$\Sigma fd = -70$	$\Sigma fd^2 = 7,700$

We know,

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{7700}{50} - \left(\frac{-70}{50}\right)^2}$$

$$= \sqrt{154 - 1.96}$$

$$= 12.33$$

c. Find the variance from the following data.

Variable (x)	10	12	15	18	20
Frequency (f)	2	7	10	8	3

Solution:

Arranging the data in a table.

x	f	$d = (x-A)$	fd	fd^2
10	2	-5	-10	50
12	7	-3	-21	63
15	10	0	0	0
18	8	3	24	72
20	3	5	15	75
	$N = 30$		$\Sigma fd = 5$	$\Sigma fd^2 = 260$

We know,

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{260}{30} - \left(\frac{5}{30}\right)^2}$$

$$= \sqrt{8.64}$$

Now,

$$(\sigma)^2 = \left(\sqrt{8.64}\right)^2$$

$$= 8.64$$

$\therefore$ The variance $(\sigma^2) = 8.64$

3.a. Find the standard deviation of the following frequency distribution.

Age (in yrs):	2-4	4-6	6-8	8-10
Frequency:	6	5	7	2

Solution:

Arranging the given data in a table.

x	m	f	fm	fm^2
2-4	3	6	18	54
4-6	5	5	25	125
6-8	7	7	49	343
8-10	9	2	18	162
		$N = 20$	$\Sigma fm = 110$	$\Sigma fm^2 = 684$

Now,

$$\sigma = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$$

$$= \sqrt{\frac{684}{20} - \left(\frac{110}{20}\right)^2}$$

$$= \sqrt{34.2 - 30.25}$$

$$= 1.987$$

$$= 1.99 \text{ [equivalent]}$$

b. Calculate the mean, standard deviation and the coefficient of variation from the following frequency distribution.

Income (Rs):	300-400	400-500	500-600	600-700	700-800
No. of Persons:	8	12	20	6	4

Solution:

Arranging the data in a table.

Rs	x	f	$d' = \frac{x-550}{10}$	fd'	$(fd'$
300-400	350	8	-20	-160	3200
400-500	450	12	-10	-120	1200
500-600	550	20	0	0	0

600-700	650	6	10	60	600
700-800	750	4	20	80	1600
		N = 50		$\Sigma fm' = -140$	$\Sigma fm'^2 = 6600$

We know,

$$\bar{x} = A + \frac{\Sigma fd'}{N} \times h$$

$$= 550 + \frac{(-140)}{50} \times 10$$

$$= \text{Rs } 522$$

Again,

$$\sigma = h \times \sqrt{\frac{(\Sigma fd'^2)}{N} - \left(\frac{\Sigma fd'}{N}\right)^2}$$

$$= 10 \times \sqrt{\frac{6600}{50} - \left(\frac{-140}{50}\right)^2}$$

$$= \text{Rs } 111.43$$

Also,

$$\text{coefficient of Variation (C.V)} = \frac{\sigma}{\bar{x}} \times 100\ \%$$

$$= \frac{111.43}{522} \times 100\ \%$$

$$= 21.34\ \%$$

c. Determine the mean, standard deviation and the coefficient of variation from the following data.

Profit (in Rs):	0-10	10-20	20-30	30-40	40-50
No. of shops:	8	13	16	8	5

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Solution:

Profit	x	f	$d' = \frac{x-25}{10}$	fd'	fd'^2
0-10	5	8	-2	-16	32
10-20	15	13	-1	-13	13
20-30	25	16	0	0	0
30-40	35	8	1	8	8
40-50	45	5	2	10	20
		$N=50$		$\Sigma fd' = -11$	$\Sigma fd'^2 = 73$

We know,

$$\bar{x} = A + \frac{\Sigma fd'}{N} \times h$$

$$= 25 + \frac{(-11)}{50} \times 10$$

$$= \text{Rs } 22.8$$

Again,

$$\sigma = h \times \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2}$$

$$= 10 \times \sqrt{\frac{73}{50} - \left(\frac{-11}{50}\right)^2}$$

$$= \text{Rs } 11.88$$

Also,

Coefficient of Variation (C.V) $= \frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{11.88}{22.8} \times 100\%$$

$$= 52.1\%$$

d. Find the mean, Variance and the coefficient of Variation from the following frequency distribution.

Age (in yrs) :	0-9	10-19	20-29	30-39	40-49
Frequency :	5	8	12	9	6

Solution:

Arranging the given data in a table.

Age	x	f	$d' = \frac{x-a}{h}$	fd'	fd'^2
0-9.5	4.75	5	-1.975	-9.875	19.50
9.5-19.5	14.5	8	-1	-8	8
19.5-29.5	24.5	12	0	0	0
29.5-39.5	34.5	9	1	9	9
39.5-49.5	44.5	6	2	12	24
		N = 40		$\Sigma fd' = 3.125$	$\Sigma fd'^2 = 60.5$

Now,

$$\bar{x} = a + \frac{\Sigma fd'}{N} \times h$$

$$= 24.5 + \frac{3.125}{40} \times 10$$

$$= 25.28$$

Also,

$$\sigma = h \times \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2}$$

$$= 10 \times \sqrt{\frac{60.5}{40} - \left(\frac{3.125}{40}\right)^2} = 12.2735$$

Also, $(\sigma^2) = (12.2735)^2$

$$= 150.63$$

Again,

$$C.V = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{12.2735}{25.28} \times 100\% = 48.55\%$$

4.a. The coefficient of variation of two series are 24% and 40% and their respective standard deviations are 6 and 8, find the respective mean.

Solution:

Given, Coefficient of Variation $(C.V)_1 = 24\%$

Coefficient of variation $(C.V)_2 = 40\%$

Standard deviation $(S.D)_1 = 6$

Standard deviation $(S.D)_2 = 8$

1st Case:

We know, $C.V_1 = \frac{S.D_1}{\bar{x}} \times 100\%$

or, $24\% = \frac{6}{\bar{x}} \times 100\%$

$\therefore \bar{x} = 25$

Also,

2nd Case:

$C.V_2 = \frac{S.D_2}{\bar{x}} \times 100\%$

or, $40\% = \frac{8}{\bar{x}} \times 100\%$

$\therefore \bar{x} = 20$

b. The coefficient of variation of two series are 45% and 30% and their respective means are 16 and 40, find their respective standard deviation.

Solution:

1st Case:

Coefficient of Variation $(C.V) = 45\%$

Mean $(\bar{x}) = 16$

Standard deviation $(\sigma) = ?$

We know,

$$C.V_1 = \frac{\sigma_1}{\bar{x}} \times 100\%$$

$$\text{or, } 45 = \frac{\sigma_1}{16} \times 100$$

$$\therefore \sigma = 7.2$$

Also,

2^{nd} case:

Coefficient of Variation $(C.V)_2 = 30\%$

Standard deviation $(\sigma') = ?$

Mean $(\bar{x}_1) = 40$

We know,

$$C.V_2 = \frac{\sigma'}{\bar{x}'} \times 100\%$$

$$\text{or, } 30\% = \frac{\sigma'}{40} \times 100\%$$

$$\therefore \sigma' = 12$$

5.a. Following are the information about the marks of two students A and B.

	A	B
Average Marks :	84	92
Variance of Marks :	16	25

Examine who has got the uniform marks.

Solution:

Mean of A $(\bar{x}) = 84$

Mean of B $(\bar{y}) = 92$

Variance $(\sigma_x^2) = 16$

Variance of B $(\sigma_y^2) = 2.5$

Now,

S.D of A $(\sigma_x) = 4$

S.D of B $(\sigma_y) = 5$

Also, C.V of A = $\frac{S.D_x}{\bar{x}} \times 100\%$

$= \frac{4}{84} \times 100\%$

$= 4.76\%$

Also,

C.V of B = $\frac{S.D_y}{\bar{y}} \times 100\%$

$= \frac{5}{92} \times 100\%$

$= 5.43\%$ Since, $C.V_A < C.V_B$. Ans is A.

b. From the following information, examine which of the firm A or B has the greatest variability of distribution of the wage. Also, find combined mean.

	firm A	firm B
No. of workers	25	15
Avg. monthly wage	Rs.6400	Rs.7500
Standard deviation of wage	4.5	5.4

Solution:

For firm A

$n_1 = 25$

$\bar{x}_1 = 6400$

$S.D(\sigma) = 4.5$

$C.V_A = \frac{\sigma}{\bar{x}_1} \times 100\%$

$= \frac{4.5}{6400} \times 100\%$

$= 0.070\%$

For firm B.

$n_2 = 15$

$\bar{x}_2 = 7500$

$\sigma = 5.4$

Now,

$$C.V_B = \frac{\sigma}{\bar{x}_2} \times 100\%$$

$$= \frac{5.4}{7500} \times 100\%$$

$$= 0.072\ \%$$

Since, $C.V_A\% < C.V_B$, so the distribution of the wage in firm B has greater variability.

Also,

$$\text{Combined Mean} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{25 \times 6400 + 15 \times 7500}{25 + 15}$$

$$= 6812.5$$

$\therefore$ The req. combined mean is 6812.5.

c. The average weekly wage in a factory has increased from Rs. 4200 to Rs. 4800 and the standard deviation has increased from Rs. 5 to Rs. 8. Can you conclude that the wage has increased uniformly?

Solution:

Case I

$\bar{x}_1 = \text{Rs } 4200$

$\sigma_1 = 5$

Now, $C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100\%$

$$= \cancel{4200}\ \frac{5}{4200} \times 100\%$$

= 0.11%

Case II

$\bar{x}_2 = 4800$

$\sigma_2 = 8$

Now, $C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100\%$

$= \frac{8}{4800} \times 100\%$

$= 0.16\%$

Here,

$C.V_1 \neq C.V_2$, so we cannot conclude that the wage has increased uniformly.

d. Examine which variable x, the length (in cm) or Y, the weight (in kg) show the greater variability from the following data.

$\Sigma x = 280$, $\Sigma x^2 = 8250$

$\Sigma Y = 565$, $\Sigma Y^2 = 36400$; $n = 10$

Solution:

Case I

$\Sigma x = 280$, $\Sigma x^2 = 8250$, $n = 10$

Now,

$\bar{x}_1 = \frac{\Sigma x}{n} = \frac{280}{10} = 28$

Also,

$S.D(\sigma_1) = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$= \sqrt{\frac{8250}{10} - \left(\frac{280}{10}\right)^2}$

$= 6.4$

Also, $C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100\%$

$= \frac{6.4}{28} \times 100\%$

$= 22.85\%$

Case II

$\Sigma y = 565$

$\Sigma y^2 = 36400$

$n = 10$

Now,

$\bar{x}_2 = \frac{\Sigma y}{n} = \frac{565}{10} = 56.5$

Also, $\sigma_2 = \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$

$= \sqrt{\frac{36400}{10} - \left(\frac{565}{10}\right)^2}$

$= 21.16$

Again,

$C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100\%$

$= \frac{21.16}{56.5} \times 100\%$

$= 37.45\%$

Here,

$C.V_1 < C.V_2$, so ~~K(~~ Y, the weight (in kg) show the greater Variability.

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6.a. Following are the marks obtained by the students X and Y in 6 test of 100 marks each.

Test	1	2	3	4	5	6
X:	56	72	48	69	64	81
Y:	63	74	45	57	82	63

If the consistency of the performance is the criteria for awarding a prize, who should get the prize?

Solution:

For X			For Y		
x	$d = x - a$	d^2	Y	$d = Y - B$	d^2
56	-13	169	63	6	36
72	3	9	74	17	289
48	-21	441	45	-12	144
69	0	0	57	0	0
64	-5	25	82	25	625
81	12	144	63	6	36
~~Σx~~	$\Sigma d = -24$	$\Sigma d^2 = 788$	$\Sigma Y = 384$	$\Sigma d = 42$	$\Sigma d^2 = 1130$

Now, for x

$$\sigma_1 = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{788}{6} - \left(\frac{-24}{6}\right)^2}$$

$$= 10.73$$

$$\bar{x} = A + \frac{\Sigma d}{N}$$

$$= 69 - \frac{24}{6}$$

$$= 65$$

Also,

$$C.V_1 = \frac{\sigma_1}{\bar{x}} \times 100\%$$

$$= \frac{10.73}{65} \times 100\%$$

$$= 16.50\%$$

Again, for Y

~~Sse~~ $\bar{Y} = \frac{\Sigma Y}{N}$

$= \frac{384}{6}$

$= 64$

$\sigma_2 = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$

$= \sqrt{\frac{1130}{6} - \left(\frac{42}{6}\right)^2}$

$= 11.80$

Now,

$C.V_2 = \frac{\sigma_2}{\bar{Y}} \times 100\%$

$= \frac{11.80}{64} \times 100\%$

$= 18.43\%$

Here, $C.V_1 < C.V_2$. So, the prize should be given to X. As lower C.V shows consistency.

b. The goals scored by two teams X and Y in a football season were as follows:

No. of goals scored in a match	No. of matches X	No. of matches Y
0	27	17
1	9	9
2	8	6
3	5	5
4	4	3

Find out which team is consistent.

For X			For Y		
X	$d = x - a$	d^2	Y	$d = Y - B$	d^2
27	19	361	17	11	121
9	1	1	9	3	9
8	0	0	6	0	0
5	-3	9	5	-1	1
4	-4	16	3	-3	9
$\Sigma x = 53$	$\Sigma d = 13$	$\Sigma d^2 = 387$	$\Sigma Y = 40$	$\Sigma d = 10$	$\Sigma d^2 = 140$

For Team X

$n = 5$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{53}{5} = 10.6$$

Now, $$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{387}{5} - \left(\frac{13}{5}\right)^2}$$

$$= 8.40$$

Also,

$$C.V_x = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{8.40}{10.6} \times 100\%$$

$$= 79.24\%$$

For Team Y

$n = 5$

$$\bar{Y} = \frac{\Sigma Y}{N} = \frac{40}{5} = 8$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{140}{5} - \left(\frac{10}{5}\right)^2}$$

$= 4.89$

So,

$$C.V_y = \frac{\sigma}{\bar{y}} \times 100\%$$

$$= \frac{4.89}{8} \times 100\%$$

$$= 61.125\%$$

Here,

$C.V_y < C.V_x$, which means Team y is consistent as it has low c.v.

c. Ananda obtained samples of CFL bulbs from two suppliers. He got the samples tested in his laboratory for the length of the life. The results of the test are given below.

Length of life (in hours)	No. of bulbs	
	Supplier A	Supplier B
400 - 500	8	6
500 - 600	20	24
600 - 700	16	12
700 - 800	6	8

Which suppliers bulb shows greater variability in length of life.

Solution:

Here, Computing the values of mean and S.D for supplier A.

Length of life	x	f	$d = x - A$	fd	fd^2
400-500	450	8	-200	-1600	320000
500-600	550	20	-100	-2000	200000
600-700	650	16	0	0	0
700-800	750	6	100	600	60000
		$N = 50$		$\Sigma fd = -3k$	$\Sigma fd^2 = 580k$

Now,

$$\bar{x} = A + \frac{\Sigma fd}{N}$$

$$= 650 - \frac{3000}{50}$$

$$= 590$$

Also, $$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{580,000}{50} - \left(\frac{-3000}{50}\right)^2}$$

$$= 89.44$$

Also

$$C.V_A = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{89.44}{590} \times 100\%$$

$$= 15.15\%$$

Also,

For supplier B.

x	f	$d = x - A$	fd	fd^2
450	6	-200	-1200	240,000
550	24	-100	-2400	240000
650	12	0	0	0
750	8	100	800	80000
	$N = 50$		$\Sigma fd = -2800$	$\Sigma fd^2 = 560k$

Now, $\bar{x} = A + \frac{\Sigma fd}{N}$

$= 650 - \frac{2800}{50}$

$= 594$

Also,

$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$= \sqrt{\frac{560,000}{50} - \left(\frac{-2800}{50}\right)^2}$

$= 89.79$

Also,

$C.V_B = \frac{\sigma}{\bar{x}} \times 100\%$

$= \frac{89.79}{594} \times 100\%$

$= 15.11\%$

Here, $C.V_A > C.V_B$. So, $C.V_A$ ~~has~~ shows greater variability in length of life.

7.a. The average score of 200 students of college A is 70 with standard deviation 12 and the average score of 300 students of college B is observed to be 60 with S.D 15. What are the combined mean and the combined standard deviation of score of two colleges combined together?

Solution:

For college A

$N_1 = 200$

$\bar{x}_A = 70$

$\sigma_A = 12$

For College B

$N_2 = 300$

$\bar{x}_B = 60$

$\sigma_B = 15$

We know,

Combined Mean $(\bar{x}_{AB}) = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \dfrac{200 \times 70 + 300 \times 60}{200 + 300}$

$= 64$

Now,

For combined standard deviation.

$d_1 = \bar{x}_A - \bar{x}_{AB}$

$= 70 - 64$

$= 6.$

$d_2 = \bar{x}_B - \bar{x}_{AB}$

$= 60 - 64$

$= -4$

Also,

$n_1(\sigma_A^2 + d_1^2) + n_2(\sigma_B^2 + d_2^2)$

$= 200(144 + 36) + 300(225 + 16)$

$= 108300$

So,

Combined S.D $(\sigma_{AB}) = \sqrt{\dfrac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$

$= \sqrt{\dfrac{108300}{500}}$

$= 14.71$

b. The monthly wages paid to the workers of two firm A and B belonging to the same industry have presented below.

	Firm A	Firm B
No. of workers	50	40
Avg. monthly wage	Rs. 63	Rs. 54
Variance of wages	81	36

Determine the combined mean and combined SD of the combined group of 90 workers.

Solution:

For firm A

$n_1 = 50$

$\bar{x}_1 = 63$

$\sigma_1^2 = 81$

For firm B

$n_2 = 40$

$\bar{x}_2 = 54$

$\sigma_2^2 = 36$

Now,

Combined mean $(\bar{x}_{12}) = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \dfrac{50 \times 63 + 40 \times 54}{50 + 40}$

$= 59$

For combined S.D

$d_1 = \bar{x}_1 - \bar{x}_{12}$

$= 63 - 59$

$= 4$

$d_2 = \bar{x}_2 - \bar{x}_{12}$

$= \cancel{58}\ 54 - 59$

$= -5$

So, $\sigma_{12} = \sqrt{\dfrac{n_1(d_1^2 + \sigma_1^2) + n_2(d_2^2 + \sigma_2^2)}{n_1 + n_2}}$

$= \sqrt{\dfrac{50(16 + 81) + 40(25 + 36)}{50 + 40}}$

$= 9$

S.A. The arithmetic mean and the standard ~~mea~~ deviation of a series of 20 items are calculated by a student were 20 cm and 5 cm resp. But while calculating an item 13 were misread as 30. Find the correct mean and S.D (σ).

Solution:

Here, $n = 20$

$\bar{x} = 20$ cm

$\sigma = 5$ cm

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow 20 = \frac{\Sigma x}{20}$$

$$\therefore \Sigma x = 400$$

Now,

Correct value of Σx $(\Sigma x_c) = 400 - 30 + 13$

$= 383$

Now,

Correct value of $\bar{x}$ $(\bar{x}_c) = \frac{\Sigma x_c}{n} = \frac{383}{20}$

$= 19.15$ cm

Again,

$$\sigma^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$$

$$\text{or } 25 = \frac{\Sigma x^2}{20} - (20)^2$$

$$\therefore \Sigma x^2 = 8500$$

Correct value of σ $(\sigma_c) = \sqrt{\frac{\Sigma x_c^2}{n_c} - \left(\frac{\Sigma x_c}{n_c}\right)^2}$

$$= \sqrt{\frac{8500 - 30^2 + 13^2}{20} - \left(\frac{383}{20}\right)^2}$$

$= 4.66$ cm

b) The arithmetic mean and standard deviation of 21 observation are 40 and 8 respectively. At the time of calculation, if one item 30 is wrongly recorded. Find the new standard deviation of the items when the wrongly recorded item is omitted.

Solution:

Here, $n = 21$

$\bar{x} = 40$

$\sigma = 8$

Now,

$\bar{x} = \frac{\Sigma x}{N}$

or, $40 = \frac{\Sigma x}{21}$

$\therefore \Sigma x = 840$

$S.D = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$

or, $8 = \sqrt{\frac{\Sigma x^2}{21} - (40)^2}$

or, $64 = \frac{\Sigma x^2}{21} - 1600$

$\therefore \Sigma x^2 = 34944$

Now, for corrected data:

$N_c = 21 - 1 = 20$

$\Sigma x_c = 840 - 30$

$= 810$

So, $\bar{x}_c = \frac{\Sigma x_c}{N_c} = \frac{810}{20} = 40.5$

Also,

$\sigma_c = \sqrt{\frac{\Sigma x_c^2}{N_c} - \left(\frac{\Sigma x_c}{N_c}\right)^2}$

$= \sqrt{\frac{34944 - 30^2}{20} - (40.5)^2}$

$= 7.87$