c/w

Chapter-14

Anti-derivatives

Page No.

Date: / /

Standard Integral (I)

1. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

2. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$

3. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) + c$

4. $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left(x+\sqrt{x^2+a^2}\right) + c$

5. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left(x+\sqrt{x^2-a^2}\right) + c$

6. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

Solved Questions

1. $\int \frac{1}{x^2+a^2}\,dx$

Solution:

Put $x = a\tan\theta$

Then, $dx = a\sec^2\theta \cdot d\theta$

Now,

$$I = \int \frac{a\sec^2\theta \cdot d\theta}{a^2\tan^2\theta + a^2}$$

$$= \int \frac{a\sec^2\theta \cdot d\theta}{a^2(\tan^2\theta + 1)}$$

$$= \frac{1}{a}\int \frac{\sec^2\theta \cdot d\theta}{\tan^2\theta + 1}$$

$$= \frac{1}{a}\int d\theta$$

$$= \frac{1}{a}\cdot\theta$$

$$= \frac{1}{a}\cdot \tan^{-1}\left(\frac{x}{a}\right)$$

2. $\displaystyle\int \frac{1}{\sqrt{x^2+a^2}}\,dx$

Solution:

Put $x = a\tan\theta$

Then, $dx = a\sec^2\theta \cdot d\theta$

Now, $\displaystyle I = \int \frac{a\sec^2\theta\cdot d\theta}{\sqrt{a^2 \cancel{\sec^2\theta} \tan^2\theta + a^2}}$

$$= \int \frac{a\sec^2\theta\cdot d\theta}{a\sqrt{\tan^2\theta+1}}$$

$$= \int \frac{\sec^2\theta\cdot d\theta}{\sec\theta}$$

$$= \int \sec\theta\cdot d\theta$$

$$= \log(\sec\theta + \tan\theta) + c$$

$$= \log\left(\sqrt{\tan^2\theta+1} + \tan\theta\right) + c$$

$$= \log\left(\frac{\sqrt{x^2+a^2}}{a} + \frac{x}{a}\right) + c$$

$$= \log\left(\sqrt{x^2+a^2} + x\right) - \log a + c$$

$$= \log\left(x + \sqrt{x^2+a^2}\right) + c$$

c/w

Exercise - 14.1

Integrate

1. i. $\int \frac{dx}{x^2+49}$

$= \int \frac{dx}{x^2+7^2}$

$= \frac{1}{7} \operatorname{Tan}^{-1}\left(\frac{x}{7}\right) + c$

ii. $\int \frac{dx}{4x^2+25}$

$= \int \frac{dx}{(2x)^2+5^2}$

$= \frac{1}{10} \operatorname{Tan}^{-1}\left(\frac{2x}{5}\right) + c$

iii. $\int \frac{x \cdot dx}{x^4+3}$

$= \int \frac{x \cdot dx}{(x^2)^2+(\sqrt{3})^2}$

Put $y = x^2$

Then $dy = 2x \cdot dx$

$= \frac{1}{2} \int \frac{2x \cdot dx}{y^2+(\sqrt{3})^2}$

$= \frac{1}{2} \int \frac{dy}{y^2+(\sqrt{3})^2}$

$= \frac{1}{2\sqrt{3}} \operatorname{Tan}^{-1}\left(\frac{y}{\sqrt{3}}\right) + c$

$= \frac{1}{2\sqrt{3}} \operatorname{Tan}^{-1}\left(\frac{x^2}{\sqrt{3}}\right) + c$

iv. $\int \frac{(2x+3)\,dx}{4x^2+1}$

$= \frac{1}{4} \int \frac{(8x)\,dx + 3\,dx}{(4x^2+1)}$

$= \frac{1}{4} \int \frac{8x \cdot dx}{4x^2+1} + 3 \int \frac{dx}{4x^2+1}$

$= \frac{1}{4} \operatorname{Log}(4x^2+1) + \frac{3}{2} \operatorname{Tan}^{-1}(2x$

$= \frac{1}{4} \operatorname{Log}(4x^2+1) + \frac{3}{2} \operatorname{Tan}^{-1}(2x) + c$

v. $\int \frac{(6x+1)\,dx}{x^2+9}$

$= \int \left\{ \frac{(2x)\,3 + 1}{x^2+9} \right\} dx$

$= 3 \int \frac{2x \cdot dx}{x^2+9} + 1 \int \frac{dx}{x^2+3^2}$

$= 3 \operatorname{Log}(x^2+9) + \frac{1}{3} \operatorname{Tan}^{-1}\left(\frac{x}{3}\right) + c$

vi. $\int \frac{dx}{e^x + e^{-x}}$

$= \int \frac{dx}{e^x + \frac{1}{e^x}}$

$= \int \frac{e^x . dx}{e^{2x} + 1}$

$= \int \frac{e^x . dx}{(e^x)^2 + 1^2}$

Put $y = e^x$

Then, $dy = e^x . dx$

$= \int \frac{dy}{y^2 + 1^2}$

$= Tan^{-1} y + C$

$= Tan^{-1} e^x + C$

vii. $\int \frac{dx}{4x^2 - 9}$

$= \int \frac{dx}{(2x)^2 - 3^2}$

$= \frac{1}{2.3.2} Log\left(\frac{2x-3}{2x+3}\right) + C$

$= \frac{1}{12} Log\left(\frac{2x-3}{2x+3}\right) + C$

viii. $\int \frac{dx}{5x^2 - 4}$

$= \int \frac{dx}{(\sqrt{5}x)^2 - 2^2} = \frac{1}{4\sqrt{5}} Log\left(\frac{\sqrt{5}x - 2}{\sqrt{5}x + 2}\right) + C$

ix. $\int \frac{dx}{x^2+6x+8}$

$= \int \frac{dx}{(x+3)^2-1^2}$

$= \frac{1}{2} \text{Log}\left(\frac{x+3-1}{x+3+1}\right) + C$

$= \frac{1}{2} \text{Log}\left(\frac{x+2}{x+4}\right) + C$

x. $\int \frac{dx}{1+x+x^2}$

$= \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \int \frac{dx}{\left(\frac{2x+1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{2}{\sqrt{3}} \text{Tan}^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

xi. $\int \frac{dx}{1+x-x^2}$

$= \int \frac{dx}{-(x^2-x-1)}$

$= -\int \frac{dx}{\left(\frac{2x-1}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2}$

$= \int \frac{dx}{\left(\frac{\sqrt{5}}{2}\right)^2 - \left(\frac{2x-1}{2}\right)^2}$

$= \frac{1}{\sqrt{5}} \text{Log}\left(\frac{\sqrt{5}+2x-1}{\sqrt{5}-2x+1}\right) + C$

xii. $\displaystyle\int \frac{\cos x . dx}{\sin^2 x + 4\sin x + 5}$

Solution:

$$= \int \frac{\cos x . dx}{(\sin x)^2 + 2.\sin x \times 2 + 4 - 4 + 5}$$

$$= \int \frac{\cos x . dx}{(\sin x + 2)^2 + 1^2}$$

Put $y = \sin x + 2$

Then, $dy = \cos x . dx$

$$= \int \frac{dy}{y^2 + 2^2}$$

$$= \tan^{-1} y$$

$$= \tan^{-1}(\sin x + 2) + c$$

xiii. $\displaystyle\int \frac{x . dx}{x^2 + 4x - 12}$

$$= \int \frac{(2x+4) . \frac{1}{2} - 2}{x^2 + 4x - 12}$$

$$= \frac{1}{2}\int \frac{(2x+4) . dx}{x^2 + 4x - 12} - 2\int \frac{dx}{(x+2)^2 - 4^2}$$

$$= \frac{1}{2}\log(x^2 + 4x - 12) - 2 . \frac{1}{2.4}\log\left(\frac{x+2-4}{x+2+4}\right) + c$$

$$= \frac{1}{2}\log(x^2 + 4x - 12) - \frac{1}{4}\log\left(\frac{x-2}{x+6}\right) + c$$

xiv. $\displaystyle\int \frac{x . dx}{x^2 + 4x + 40}$

$$= \int \frac{(2x+4) . \frac{1}{2} - 2 . dx}{x^2 + 4x + 40}$$

$$= \frac{1}{2}\int \frac{(2x+4)\, dx}{x^2 + 4x + 40} - 2\int \frac{dx}{(x+2)^2 + 6^2}$$

$= \frac{1}{2} \operatorname{Log}(x^2+4x+40) - 2 \cdot \frac{1}{6} \operatorname{Tan}^{-1}\left(\frac{x+2}{6}\right) + C$

$= \frac{1}{2} \operatorname{Log}(x^2+4x+40) - \frac{1}{3} \operatorname{Tan}^{-1}\left(\frac{x+2}{6}\right) + C$

XV. $\int \frac{2x+3}{x^2+4x+5} \cdot dx$

Solution:

$= \int \frac{\{(2x+4)-1\}}{x^2+4x+5} dx$

$= \int \frac{(2x+4)\,dx}{x^2+4x+5} - 1\int \frac{dx}{x^2+4x+5}$

$= \operatorname{Log}(x^2+4x+5) - 1\int \frac{dx}{(x+2)^2+1^2}$

$= \operatorname{Log}(x^2+4x+5) - \operatorname{Tan}^{-1}(x+2) + C$

XVi. $\int \frac{(2x+2)\,dx}{3+2x-x^2}$

$= \int \frac{(-2x+2)\times \cancel{(-1)} + 4}{3+2x-x^2}$

$= -\int \frac{\{(2-2x)+4\}}{3+2x-x^2} dx$

$= -\int \frac{(2-2x)\,dx}{3+2x-x^2} + 4\int \frac{dx}{3+2x-x^2}$

$= -\operatorname{Log}(3+2x-x^2) - 4\int \frac{dx}{x^2-2x-3}$

$= -\operatorname{Log}(3+2x-x^2) + 4\int \frac{dx}{-(x+1)^2+2^2}$

$= -\operatorname{Log}(3+2x-x^2) + \frac{4}{2 \cdot 2} \operatorname{Log}\left(\frac{\cancel{x+1-2}\ 2+x}{2-x+1}\right) + C$

$= \operatorname{Log}\left(\frac{2+x}{3-x}\right) - \operatorname{Log}(3+2x-x^2) + C$

xvii. $\int \frac{(4x+3)\,dx}{4x^2+12x+5}$

$= \int \frac{\{(8x+12)\frac{1}{2}-3\}\,dx}{(4x^2+12x+5)}$

$= \frac{1}{2}\int \frac{(8x+12)\,dx}{4x^2+12x+5} - 3\int \frac{dx}{4x^2+12x+5}$

$= \frac{1}{2}\operatorname{Log}(4x^2+12x+5) - 3\int \frac{dx}{(2x+3)^2-2^2} + c$

$= \frac{1}{2}\operatorname{Log}(4x^2+12x+5) - \frac{3}{2.2.2}\operatorname{Log}\left(\frac{2x+3-2}{2x+3+2}\right) + c$

$= \frac{1}{2}\operatorname{Log}(4x^2+12x+5) - \frac{3}{8}\operatorname{Log}\left(\frac{2x+1}{2x+5}\right) + c$

xviii. $\int \frac{(6x+2)\,dx}{9x^2+6x+26}$

$= \int \frac{(18x+6)\frac{1}{3}.\,dx}{9x^2+6x+26}$

$= \frac{1}{3}\int \frac{(18x+6)\,dx}{9x^2+6x+26}$

$= \frac{1}{3}\operatorname{Log}(9x^2+6x+26) + c$

2. i. $\int \frac{dx}{\sqrt{x^2-16}}$

$= \int \frac{dx}{\sqrt{x^2-4^2}}$

$= \operatorname{Log}(x+\sqrt{x^2-4^2}) + c$

ii. $\int \frac{dx}{\sqrt{(4x)^2+9}}$

$= \int \frac{dx}{\sqrt{(2x)^2+3^2}} = \frac{1}{2}\operatorname{Log}(2x+\sqrt{2x+9}) + c$

iii. $\int \frac{dx}{\sqrt{x^2+4x-5}}$

$= \int \frac{dx}{\sqrt{(x+2)^2-3^2}}$

$= \text{Log}\left(x+2+\sqrt{(x+2)^2-9}\right) + C$

$= \text{Log}\left(x+2+\sqrt{x^2+4x-5}\right) + C$

iv. $\int \frac{dx}{\sqrt{x^2+6x+10}}$

$= \int \frac{dx}{\sqrt{(x+3)^2+1^2}}$

$= \text{Log}\left(x+3+\sqrt{(x+3)^2+1^2}\right) + C$

$= \text{Log}\left(x+3+\sqrt{x^2+6x+10}\right) + C$

v. $\int \frac{dx}{\sqrt{2ax+x^2}}$

$= \int \frac{dx}{\sqrt{(x+a)^2-a^2}}$

$= \text{Log}\left(x+a+\sqrt{(x+a)^2-a^2}\right) + C$

$= \text{Log}\left(x+a+\sqrt{2ax+x^2}\right) + C$

vi. $\int \frac{dx}{\sqrt{2ax-x^2}}$

$= \int \frac{dx}{\sqrt{-(x^2-2ax+a^2-a^2)}}$

$= \int \frac{dx}{\sqrt{-\{(x-a)^2-a^2\}}}$

$= \int \frac{dx}{\sqrt{a^2-(x-a)^2}}$

$= \sin^{-1}\left(\frac{x-a}{a}\right) + C$

vii. $$\int \frac{dx}{\sqrt{2x^2+3x+4}}$$

$$= \int \frac{dx}{\sqrt{2\left(x^2+\frac{3x}{2}+2\right)}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\left(x^2+\frac{3x}{2}+2\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\left(x+3/4\right)^2+\sqrt{23}/4}$$

$$= \frac{1}{\sqrt{2}} \operatorname{Log}\left(x+\frac{3}{4}+\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{23}}{2}\right)^2}\right)+C$$

$$= \frac{1}{\sqrt{2}} \operatorname{Log}\left(x+\frac{3}{4}+\sqrt{x^2+\frac{3}{2}x+2}\right)+C$$

viii. $$\int \frac{x.dx}{\sqrt{a^4+x^4}}$$

$$= \int \frac{4x.\frac{1}{4}.dx}{\sqrt{x^4+a^4}}$$

$$= \frac{1}{4} \int \frac{4x.dx}{\sqrt{x^4+a^4}}$$

$$= \frac{1}{4} \int \frac{4x}{\sqrt{(x^2)^2+(a^2)^2}}$$

Put $y = x^2$

Then, $dy = 2x$

$$= \frac{1}{2} \int \frac{dy}{\sqrt{y^2+(a^2)^2}}$$

$$= \frac{1}{2} \operatorname{Log}\left(y+\sqrt{y^2+(a^2)^2}\right)+C$$

$$= \frac{1}{2} \operatorname{Log}\left(x^2+\sqrt{x^4+a^4}\right)+C$$

ix. $\int \frac{x \cdot dx}{\sqrt{x^4+2x^2+10}}$

$= \int \frac{\cancel{4x + 4x}\ x \cdot dx}{\sqrt{(x^2)^2 + 2 \cdot x^2 \cdot + \cancel{4-4+10}\ 1-1+10}}$

$= \int \frac{x \cdot dx}{\sqrt{(x^2+1)^2 + 3^2}}$

Put $x^2+1 = y$

Then, $dy = 2x \cdot dx$

$= \frac{1}{2} \int \frac{2x \cdot dx}{\sqrt{y^2+3^2}}$

$= \frac{1}{2} \int \frac{dy}{\sqrt{y^2+3^2}}$

$= \frac{1}{2} \operatorname{Log}\left(y + \sqrt{y^2+9}\right) + c$

$= \frac{1}{2} \operatorname{Log}\left(x+1 + \sqrt{(x^2+1)^2+9}\right) + c$

$= \frac{1}{2} \operatorname{Log}\left\{(x+1) + \sqrt{x^4+2x^2+10}\right\} + c$

x. $\int \frac{x \cdot dx}{\sqrt{x^2+4x+5}}$

$= \int \frac{\cancel{x}\ dx}{\sqrt{(x+2)^2+1^2}}$

~~put y =~~

$= \int \frac{\left\{(2x+4)\frac{1}{2} - 2\right\} dx}{\sqrt{(x^2+4x+5)}}$

$= \frac{1}{2}\int \frac{(2x+4)\, dx}{\sqrt{x^2+4x+5}} - 2\int \frac{dx}{\sqrt{x^2+4x+5}}$

$= \cancel{\frac{1}{2} \operatorname{Log}(x^2 -}$

Put $y = x^2+4x+5$

Then, $dy = 2x+4$

Also,

~~Put z =~~

$= \frac{1}{2}\int \frac{dy}{\sqrt{y}} - 2\int \frac{dx}{\sqrt{(x+2)^2+1^2}}$

$= \frac{1}{2}\int y^{-\frac{1}{2}}\cdot dy - 2\,\mathrm{Log}\left(x+2+\sqrt{(x+2)^2+1^2}\right)+C$

$= \frac{2}{2}\sqrt{y} - 2\,\mathrm{Log}\left(x+2+\sqrt{x^2+4x+5}\right)+C$

$= \sqrt{x^2+4x+5} - 2\,\mathrm{Log}\left(x+2+\sqrt{x^2+4x+5}\right)+C$

xi. $\int \frac{(2x+3)\,dx}{\sqrt{x^2+4x+20}}$

$= \int \frac{\{(2x+4)-1\}\,dx}{\sqrt{x^2+4x+20}}$

$= \int \frac{(2x+4)\,dx}{\sqrt{x^2+4x+20}} - 1\int \frac{dx}{\sqrt{x^2+4x+20}}$

= Put $y = x^2+4x+20$

Then, $dy = 2x+4$

$= \int \frac{dy}{\sqrt{y}} - \int \frac{dx}{\sqrt{(x+2)^2+2^2}}$

$= \int y^{-1/2}\cdot dy - \int \frac{dx}{\sqrt{(x+2)^2+4}}$

$= 2\sqrt{y} - \mathrm{Log}\left(x+2+\sqrt{(x+2)^2+4}\right)+C$

$= 2\sqrt{x^2+4x+20} - \mathrm{Log}\left(x+2+\sqrt{x^2+4x+20}\right)+C$

xii. $\int \frac{x+2}{\sqrt{x^2+x+1}}$

$= \int \frac{\{(2x+1)\cdot\frac{1}{2}\} + 3/2}{\sqrt{x^2+x+1}}$

$= \frac{1}{2}\int \frac{(2x+1)\,dx}{\sqrt{x^2+x+1}} + \frac{3}{2}\int \frac{dx}{\sqrt{x^2+x+1}}$

= Put $y = \cancel{2x+1}\; x^2+x+1$

$\therefore dy = 2x+1$

Now,

$= \frac{1}{2}\int \frac{dy}{\sqrt{y}} + \frac{3}{2}\int \frac{dx}{\sqrt{(x+\frac{1}{2})^2 + \frac{5}{4}}}$

$= \frac{1}{2}\int y^{-\frac{1}{2}}\cdot dy + \frac{3}{2}\int \frac{dx}{\sqrt{(x+\frac{1}{2})^2 + (\sqrt{5}/2)^2}}$

$= \sqrt{y} + \frac{3}{2} \operatorname{Log}\left(x+\frac{1}{2} + \sqrt{\left(x+\frac{1}{2}\right)^2 + \frac{5}{4}} \right) + c$

$= \sqrt{x^2+x+1} + \frac{3}{2} \operatorname{Log}\left(\frac{2x+1}{2} + \sqrt{x^2+x+1} \right) + c$

xiii. $\int \frac{(x-2)\,dx}{\sqrt{2x^2-8x+5}}$

$= \int \frac{(4x-8)\frac{1}{4}\cdot dx}{\sqrt{2x^2-8x+5}}$

$= \frac{1}{4}\int \frac{(4x-8)\,dx}{\sqrt{2x^2-8x+5}}$

Put $y = 2x^2-8x+5$

Then, $dy = 4x-8$

$= \frac{1}{4}\int \frac{dy}{\sqrt{y}}$

$= \frac{1}{4}\int y^{-\frac{1}{2}}\cdot dy$

$= \frac{1}{2}\sqrt{y} = \frac{1}{2}\sqrt{2x^2-8x+5} + C$

xiv. $\int \frac{dx}{\sqrt{x^2-7x+12}}$

$= \int \frac{dx}{\sqrt{x^2-2.x.7/2+49/4-49/4+12}}$

$= \int \frac{dx}{\sqrt{(x-7/2)^2-1/4}}$

$= Log\left(x-\frac{7}{2}+\sqrt{\left(x-\frac{7}{2}\right)^2-\frac{1}{4}}\right.$

$= Log\left(\frac{2x-7}{2}+\sqrt{x^2-7x+12}\right)+c$

xv. $\int \frac{dx}{\sqrt{8-2x-x^2}}$

$= \int \frac{dx}{\sqrt{-(x^2+2x-8)}}$

$= \int \frac{dx}{\sqrt{-\{(x+1)^2-3^2\}}}$

$= \int \frac{dx}{\sqrt{9-(x+1)^2}}$

$= Sin^{-1}\left(\frac{x+1}{3}\right)+c$

xvi. $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$ $(\beta>\alpha)$

$=$ Put $x-\alpha=y^2$

Then, $dx = 2y\,dy$

$= \int \frac{2y.dy}{\sqrt{y^2(y^2+\alpha-\beta)}}$

$= \int \frac{2y.dy}{y\sqrt{y^2+\alpha-\beta}}$

$$= 2\int \frac{dy}{\sqrt{y^2+\alpha-\beta}}$$

$$= 2\int \frac{dy}{\sqrt{y^2+(\sqrt{\alpha-\beta})^2}}$$

$$= 2\, \text{Log}\left(y+\sqrt{y^2+\alpha-\beta}\right)$$

$$= 2\, \text{Log}\left(\sqrt{x-a}+\sqrt{x-\beta}\right)+c$$

xvii. $\displaystyle\int \frac{dx}{(4x+3)\sqrt{x+3}}$

= Let $y^2 = x+3$

Then, $dx = 2y \cdot dy$

Now,

$$4x+3 = 4(y^2-3)+3$$

$$= 4y^2-12+3$$

$$= 4y^2-9$$

Then, $\displaystyle\int \frac{2y \cdot dy}{(4y^2-9)y}$

$$= 2\int \frac{dy}{(2y)^2-3^2}$$

$$= 2 \cdot \frac{1}{2\cdot 3\cdot 2}\, \text{Log}\left(\frac{2y-3}{2y+3}\right)+c$$

$$= \frac{1}{6}\, \text{Log}\, \frac{2\sqrt{x+3}-3}{2\sqrt{x+3}+3}+c \quad [\because y = \sqrt{x+3}]$$

xviii. $\displaystyle\int \frac{dx}{(2x+1)\sqrt{4x+3}}$

= Put $z^2 = 4x+3$

Then, ~~$dz = 4 \cdot dx$~~ $dx =$ ~~$2z+4$~~ $2z \cdot dz$

Now,

$I = \displaystyle\int \cancel{\frac{2z \cdot dz}{2(z^2-3)z}} \quad \frac{2z \cdot dz}{(4\ {}^2-12+3)z}$

xviii. $\int \frac{dx}{(2x+1)\sqrt{4x+3}}$

Put $z^2 = 4x+3$

$x = \frac{z^2-3}{4}$

$\therefore dx = \frac{z}{2} \cdot dz$

Now, $I = \frac{1}{2}\int \frac{z \cdot dz}{\left[2\left(\frac{z^2-3}{4}\right)+1\right] z}$

$= \frac{1}{2} \times 2 \int \frac{dz}{z^2-1}$

$= \frac{1}{2 \cdot 1} \text{Log} \frac{z-1}{z+1} + C$

$= \frac{1}{2} \text{Log} \frac{\sqrt{4x+3}-1}{\sqrt{4x+3}+1} + C$

xix. $\sqrt{\frac{1+x}{\cancel{1+x}\, 1-x}}\, dx$

$= \int \frac{(1+x)\, dx}{\sqrt{1-x^2}}$

$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x \cdot dx}{\sqrt{1-x^2}}$

$= \sin^{-1} x - \frac{1}{2} \int \frac{-2x \cdot dx}{\sqrt{1-x^2}}$

$= \sin^{-1} x - \frac{2}{2} \sqrt{1-x^2} + C$

$= \sin^{-1} x - \sqrt{1-x^2} + C$

Process:

Put $z = 1-x^2$

Then $dz = -2x \cdot dx$

$= -\frac{1}{2} \int \frac{dz}{\sqrt{z}}$

$= -1 \sqrt{1-x^2}$