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Chapter-19
Numerical Computation

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Exercise - 19.1

1. Apply the method of successive bisection to find the
a. square root of 3 within 2 places of decimal in (1,2).

Solution:

Let $x = \sqrt{3}$

or, $x^2 - 3 = 0$

Let $f(x) = x^2 - 3$ ——①

Here, $a = 1$, $b = 2$ then,

$f(1) = 1^2 - 3 = -2 < 0$

$f(2) = 2^2 - 3 = 1 > 0$

Now,

a	b	$m = \frac{a+b}{2}$	$f(m) = m^2 - 3$
1	2	1.5	-0.75
1.5	2	1.75	0.0625
1.5	1.75	1.625	-0.359375
1.625	1.75	1.6875	-0.1523437 5
1.6875	1.75	1.71875	-0.045898
1.71875	1.75	1.73437	0.0080
1.71875	1.73437	1.72656	-0.01899
1.72656	1.73437	1.73046	-0.00547
1.73046	1.73437	1.732415	0.00126

Hence, the required root is 1.73.

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b. Square root of 123 within 2 places of decimal in (11, 12)

Solution:

Let $x = \sqrt{123}$

Then, $x^2 - 123 = 0$ ——①

Now,

a	b	$m = \frac{a+b}{2}$	f(a)	f(b)	f(m)
11	12	11.5	-2	21	9.25
11	11.5	11.25	-2	9.25	3.5625
11	11.25	11.125	-2	3.56	0.76
11	11.125	11.0625	-2	0.76	-0.62
11.0625	11.125	11.09375	-0.62	0.76	0.071
11.0625	11.09375	11.078125	-0.62	0.071	-0.27
11.078125	11.09375	11.0859375	-0.27	0.071	-0.101
11.0859375	11.09375	11.08984375	-0.101	0.071	-0.01
11.08984375	11.09375	11.09179688	-0.01	0.071	0.02
11.08984375	11.09179688	11.09082032	-0.01	0.02	0.006
11.08984375	11.09082032	11.09033204	-0.01	0.006	-0.004
11.09033204	11.09082032	11.09057618	-0.004	0.006	0.0008

Hence, the required root is 11.090

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c. the approximate value of $\sqrt{2}$ within an error of 10^{-3}.

Solution:

Let, $x = \sqrt{2}$

or, $x^2 - 2 = 0$

Let $f(x) = x^2 - 2$ ——— ①

Now, $f(1) = 1^2 - 2 = -1$

$f(2) = 2^2 - 2 = 2$

and $f(1) \cdot f(2) = -1 \times 2 = -2 < 0$

So, the root lies between 1 and 2

Now,

a	b	$m = \frac{a+b}{2}$	$f(m) = m^2 - 2$
1	2	1.5	0.25
1	1.5	1.25	-0.4375
1.25	1.5	1.375	-0.109
1.375	1.5	1.4375	0.0664
1.375	1.4375	1.40625	-0.0224
1.40625	1.4375	1.42185	0.0217
1.40625	1.42185	1.4140625	-0.000427

Here, $|f(m)| = |-0.000427|$

$= 0.000427 < 10^{-3} (0.001)$

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Newton-Raphson Method

Let us consider $f(x) = x^2 - a$, then Newton's interation equation are given by;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

i.e. $x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n}$

1.d. the value of $\sqrt[3]{10}$ with error not more than 0.03.

Solution:

Let $x = \sqrt[3]{10}$

or, $x^3 - 10 = 0$

Now, Let $f(x) = x^3 - 10 = 0$ —①

Then, $f(2) = 2^3 - 10 = -2$

$f(3) = 27 - 10 = 17$

and $f(2).f(3) = -34 < 0$

∴ The root lies between 2 and 3.

a	b	$m = \frac{a+b}{2}$	f(a)	f(b)	f(m)	b-a
2	3	2.5	-2	17	5.625	1
2	2.5	2.25	-2	5.625	1.3906	
2	2.25	2.125	-2	1.3906	-0.4042	
2.125	2.25	4.375	-0.4042	1.3906	73.7402	
2.125	4.375	~~3.25~~ 6.5 ~~73.7402~~	-0.4042	73.7402		

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2. Apply the method of successive bisection to find the root of the equation.

a. $x^2 + x - 4 = 0$ in (1, 2) correct to two places of decimal.

Solution:

Let $f(x) = x^2 + x - 4$ then,

$f(1) = 1^2 + 1 - 4 = -2$

$f(2) = 2^2 + 2 - 4 = 2$

So, $f(1) \cdot f(2) = -4 < 0$

∴ The root lies between 1 and 2

a	b	m = $\frac{a+b}{2}$	f(a)	f(b)	f(m)
1	2	1.5	-2	2	-0.25
1.5	~~1.5~~ 2	1.75	-0.25	2	0.8125
1.5	1.75	1.625	-0.25	0.8125	0.2656
1.5	1.625	1.5625	-0.25	0.2656	0.0039
1.5	1.5625	1.53125	-0.25	0.0039	-0.124
1.53125	1.5625	1.546875	-0.124	0.0039	-0.060
1.546875	1.5625	1.5546875	-0.0603	0.0039	-0.028
1.5546875	1.5625	1.55859375	-0.0282	0.0039	-0.012
1.55859375	1.5625	1.560546875	-0.0121	0.0039	-0.004
1.560546875	1.5625	1.561523438	-0.00414		-0.00012

Hence, the required root is 1.56

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b. $2x^2 - x - 5 = 0$ Correct to 4 places of decimals within an error of 0.05.

Solution:

Let, $f(x) = 2x^2 - x - 5 = 0$ then,

$f(1) = 2 \cdot 1^2 - 1 - 5 = -4$

$f(2) = 2 \cdot 2^2 - 2 - 5 = 1$

So, $f(1) \cdot f(2) = -4 < 0$

∴ The root lies between 1 and 2

a	b	$m = \frac{a+b}{2}$	f(a)	f(b)	f(m)
1	2	1.5	-4	1	-2
1.5	2	1.75	-2	1	-0.62
1.75	2	1.875	-0.625	1	0.15
1.75	1.875	1.8125	-0.625	0.156	-0.24
1.875	1.8125	1.84375	0.156	-0.24	-0.04
1.84375	1.8125	1.828125	-0.044	-0.24	-0.14
1.828125	1.8125	1.8203125	-0.144	-0.24	-0.19
1.8203125	1.8125	1.81640625	-0.19	-0.24	-0.21
1.81640625	1.8125	1.814453125	-0.21	-0.24	-0.22
1.814453125	1.8125	1.813476563	-0.22	-0.24	-0.23
1.813476563	1.8125	1.812988282	-0.236	-0.24	-0.23