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Chapter-6
Quadratic Equations

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Quadratic Equation

$f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots + a_{n-1}x + a_n \quad (a_0 \neq 0)$ —— ①

Where n is the non-negative integer, $a_0, a_1, \ldots\ldots\ldots, a_{n-1}, a_n$ are all the constants is called a rational ~~number~~ Function or polynomial of degree n.

Roots of Quadratic Equation : $ax^2 + bx + c = 0$

We know,

$$ax^2 + bx + c = 0$$

or, $x^2 + \frac{bx}{a} + \frac{c}{a} = 0$

or, $x^2 + \frac{2bx}{2a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$

or, $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

or, $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Taking roots both side; we get

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

or, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Two roots of quadratic equation are;

(i) $\frac{-b + \sqrt{b^2-4ac}}{2a}$

(ii) $\frac{-b - \sqrt{b^2-4ac}}{2a}$

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Nature of roots of quadratic equation.

i. If $b^2 - 4ac > 0$ then roots of quadratic equation are real and distinct.

ii. If $b^2 - 4ac = 0$ then two roots are real and identical.

iii. If $b^2 - 4ac < 0$ then two roots are imaginary and distinct.

Exercise - 6.1

1. Determine the nature of the roots of each of the following equations.

a. $x^2 - 6x + 5 = 0$

Solution:

Comparing $x^2 - 6x + 5 = 0$ with $ax^2 + bx + c = 0$, we get

$a = 1 \quad b = -6 \quad c = 5$

Now, $b^2 - 4ac = (-6)^2 - 4.1.5$

$= 36 - 20$

$= 16 > 0$

$\therefore$ The roots are real and distinct.

b. $x^2 - 4x - 3 = 0$

Solution:

Comparing $x^2 - 4x - 3 = 0$ with $ax^2 + bx + c = 0$, we get

$a = 1 \quad b = -4 \quad c = -3$

Now, $b^2 - 4ac = (-4)^2 - 4.1.(-3)$

$= 16 + 12$

$= 28 > 0$

$\therefore$ The roots are real and distinct.

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c. $x^2 - 6x + 9 = 0$

Solution:

Comparing $x^2 - 6x + 9 = 0$ with $ax^2 + bx + c = 0$ we get;

$a = 1 \quad b = -6 \quad c = 9$

So, $b^2 - 4ac = (-6)^2 - 4.1.9$

$= 36 - 36$

$= 0$

$\therefore$ The roots are real and identical.

d. $4x^2 - 4x + 1 = 0$

Solution:

Comparing $4x^2 - 4x + 1 = 0$ with $ax^2 + bx + c = 0$, we get

$a = 4, \quad b = -4 \quad c = 1$

So, $b^2 - 4ac = (-4)^2 - 4.4.1$

$= 16 - 16$

$= 0$

$\therefore$ The roots are real and identical.

e. $2x^2 - 9x + 35 = 0$

Solution:

Comparing $2x^2 - 9x + 35 = 0$ with $ax^2 + bx + c = 0$, we get

$a = 2 \quad b = -9 \quad c = 35$

So, $b^2 - 4ac = (-9)^2 - 4.2.35$

$= 81 - 280$

$= -199 < 0$

$\therefore$ The roots are imaginary and distinct.

f. $4x^2 + 8x - 5 = 0$

Comparing $4x^2 + 8x - 5 = 0$ with $ax^2 + bx + c = 0$, we get

$a = 4 \quad b = 8 \quad c = -5$

So, $b^2 - 4ac = 8^2 - 4 \cdot 4 \cdot (-5)$

$= 64 + 80$

$= 144 > 0$

$\therefore$ The roots are real and distinct.

2. For what values of p will the equation $5x^2 - px + 45 = 0$ have equal roots?

Solution

Given;

$5x^2 - px + 45 = 0 \quad$ — ①

Comparing equation ① with $ax^2 + bx + c = 0$, we get

$a = 5 \quad b = -p \quad c = 45$

By question;

Equation have equal roots

i.e. $b^2 - 4ac = 0$

or, $(-p)^2 - 4 \cdot 5 \cdot 45 = 0$

or, $p^2 = 900$

$\therefore p = \pm 30$

$\therefore$ The required value of p is ± 30.

3. If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k.

Solution:

Comparing $x^2 + 2(k+2)x + 9k = 0$ with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 2(k+2)$ $c = 9$

By Question:

$b^2 - 4ac = 0 \qquad [\because$ roots are equal$]$

or, $4(k+2)^2 = 4 \cdot 1 \cdot 9k$

or, $k^2 + 4k + 4 = 9k$

or, $k^2 - 5x + 4 = 0$

or, $(k-1)(k-4) = 0$

Either $\quad$ OR

$k = 1 \quad\quad k = 4$

4. For what value of 'a' will the equation $x^2 - (3a-1)x + 2(a^2-1) = 0$ have equal roots?

Solution:

Comparing $x^2 - (3a-1)x + 2(a^2-1) = 0$ with $ax^2 + bx + c = 0$ we get;

$a = 1 \quad b = -(3a-1) \quad c = 2(a^2-1)$

By question:

$b^2 - 4ac = 0 \quad$ [$\because$ for equal roots]

or, $[-(3a-1)]^2 - 4 \cdot 1 \cdot 2(a^2-1) = 0$

or, $(3a-1)^2 - 8(a^2-1) = 0$

or, $9a^2 - 6a + 1 - 8a^2 + 8 = 0$

or, $a^2 - 6a + 9 = 0$

or, $(a-3)^2 = 0$

$\therefore a = 3$

5. If the roots of the equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, then $\frac{a}{b} = \frac{c}{d}$.

Solution:

Given, $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ —— (i)

Comparing eq. (i) with $Ax^2 + Bx + C = 0$, we get;

$A = (a^2+b^2) \quad B = -2(ac+bd) \quad C = (c^2+d^2)$

By question:

$B^2 - 4AC = 0 \quad$ [$\because$ Condition for equal roots]

or, $[-2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2) = 0$

or, $4(ac+bd)^2 - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$

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or, $4(a^2c^2 + 2acbd + b^2d^2) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$

or, $4a^2c^2 + 8abcd + 4b^2d^2 - 4a^2c^2 - 4a^2d^2 - 4b^2c^2 - 4b^2d^2 = 0$

or, $8abcd - 4a^2d^2 - 4b^2c^2 = 0$

or, $(ad - bc)^2 = 0$

or, $ad - bc = 0$

or, $ad = bc$

or, $\frac{a}{b} = \frac{c}{d}$

$\therefore \frac{a}{b} = \frac{c}{d}$ proved//

6. Show that the roots of the equation $(a^2 - bc)x^2 + 2(b^2 - ca)x + (c^2 - ab) = 0$ will be equal, if either $b = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$.

Solution:

Comparing the given equation with $Ax^2 + Bx + C = 0$, we get

$A = (a^2 - bc)$ $B = 2(b^2 - ca)$ $C = (c^2 - ab)$

For equal roots;

$B^2 - 4AC = 0$

or, $[2(b^2 - ca)]^2 - 4(a^2 - bc)(c^2 - ab) = 0$

or, $4(b^2 - ca)^2 - 4(a^2 - bc)(c^2 - ab) = 0$

or, $4(b^4 - 2b^2 \cdot ca + c^2a^2) - 4(a^2c^2 - a^3b - bc^3 + ab^2c) = 0$

or, $b^4 - 2b^2ca + c^2a^2 - a^2c^2 + a^3b + bc^3 - ab^2c = 0$

or, $b^4 - 3b^2ca + a^3b + bc^3 = 0$

or, $b(b^3 - 3abc + a^3 + c^3) = 0$

Either OR

$b = 0$ $a^3 + b^3 + c^3 - 3abc = 0$

7. If a, b, c are rational and $a+b+c=0$, show that the roots of $(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$ are rational.

Solution:

Given, $(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$ —(1)

Comparing eq(i) with $Ax^2+Bx+C=0$, we get;

$A = (b+c-a)$

$B = (c+a-b)$

$C = (a+b-c)$

Now,

$B^2-4AC = (c+a-b)^2-4(b+c-a)(a+b-c)$

$= (-b-b)^2-4(-a-a)(-c-c)$ [$\because a+b+c=0$]

$= (-2b)^2-4(-2a)(-2c)$

$= 4b^2-16ac$

$= 4(b^2-4ac)$

$= 4\{b^2-4a(-a-b)\}$ [$\because a+b+c=0$]

$= 4[b^2+4a^2+4ab]$

$= 4[2a+b]^2$

$= [2(2a+b)]^2 > 0$

$\therefore$ The roots are rational.

8. Prove that the roots of the equation $(x-a)(x-b)=k^2$ are real ~~and~~ for all values of k.

Solution:

Given, $(x-a)(x-b)=k^2$

or, $(x^2-ax-bx+ab)-k^2=0$

or, $x^2-x(a+b)+ab-k^2=0$

or, $x^2-(a+b)x+(ab-k^2)=0$

Now,

$B^2-4AC = [-(a+b)]^2-4.1.(ab-k^2)$

$= (a+b)^2-4(ab-k^2)$

$= a^2 + 2ab + b^2 - 4ab + 4k^2$

$= a^2 - 2ab + b^2 + 4k^2$

$= (a-b)^2 + 4k^2 > 0$

$\therefore$ The roots of the given equation are real.

9\. Show that the roots of the equation $x^2 - 4abx + (a^2+2b^2)^2 = 0$ ~~Solution:~~ are imaginary.

Solution:

Comparing the given equation with $Ax^2 + Bx + C = 0$, we get;

$A = 1 \quad B = -4ab \quad C = (a^2+2b^2)^2$

Now,

$$\begin{aligned} B^2 - 4AC &= (-4ab)^2 - 4 \cdot 1 (a^2+2b^2)^2 \\ &= 16a^2b^2 - 4(a^4 + 4a^2b^2 + 4b^4) \\ &= 16a^2b^2 - 4a^4 - 16a^2b^2 - 16b^4 \\ &= -4a^4 - 16b^4 \\ &= -4(a^4 + b^4 \cdot 4) < 0 \end{aligned}$$

$\therefore$ The roots of the given equation are imaginary.

10\. If the roots of the quadratic equation $9x^2 + 2px + 2q = 0$ are real and unequal, prove that the roots of the equation $(p+q)x^2 + 2qx + (p-q) = 0$ are imaginary.

Solution:

Given, $9x^2 + 2px + 2q = 0$ has a real and unequal roots

Then,

$(2p)^2 - 4q \cdot 2q > 0$

or, $4p^2 - 8q^2 > 0$

or, $p^2 - 2q^2 > 0 \quad$ — (1)

Now,

$(p+q)x^2 + 2qx + (p-q) = 0 \rightarrow Ax^2 + Bx + C$

Then, $B^2 - 4AC = (2q)^2 - 4 \cdot (p+q)(p-q)$

$= 4q^2 - 4(p^2 - q^2)$

$= 4q^2 - 4p^2 + 4q^2$

$= 8q^2 - 4p^2$

$= -4p^2 + 8q^2$

$= -4(p^2 - 2q^2) < 0 \quad [\because p^2 - 2q > 0]$

$\therefore$ It has imaginary roots.

Relations Between Roots and Coefficients

We know,

$$ax^2 + bx + c = 0$$

$$\text{or, } x^2 + \frac{bx}{a} + \frac{c}{a} = 0 \quad \text{—} (1)$$

Let α and β be two roots of the equation.

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

By addition:

$$\alpha + \beta = \frac{-2b}{2a}$$

$$= \frac{-b}{a}$$

By Multiplication;

$$\alpha . \beta = \frac{c}{a}$$

Put the value of $\frac{b}{a}$ and $\frac{c}{a}$ in equation (1), then,

$$x^2 - (\alpha + \beta)x + \alpha . \beta = 0$$