HW

Exercise - 8.1

1. Find the equations of the tangents and normal to the following circles.

a. $x^2 + y^2 = 8$ at $(2,2)$

Solution:

Comparing $x^2+y^2=8$ with $x^2+y^2=a^2$, we get

$a^2 = 8$

Equation of the tangent is :

$xx_1 + y.y_1 = a^2$

or, $x.2 + y.2 = 8$

or, $2x + 2y = 8$

$\therefore x + y - 4 = 0$

Again,

Equation of normal is given by;

$x - y + k = 0$

Since, the normal passes through $(2,2)$ then,

$2 - 2 + k = 0$

$\therefore k = 0$

Hence, the equation of normal is $x - y = 0$

b. $x^2+y^2=36$ at $(-6,0)$

Solution:

Comparing $x^2+y^2=36$ with $x^2+y^2=a^2$, we get

$a = 36$

Now, Equation of tangent is given by;

$xx_1 + y.y_1 = a^2$

or, $x(-6) + y.0 = 36$

or, $-6x = 36$

or, $x + 6 = 0$ is the req. equation.

Again,

Equation of normal is given by

$0 \cdot x + y + k = 0$

Since the normal passes through $(-6, 0)$ then

$0 \cdot (-6) + 0 + k = 0$

$\therefore k = 0$

Hence, the equation of normal is $y = 0$.

c. $x^2 + y^2 - 6x - 8y - 4 = 0$ at $(8, 6)$

Solution:

Given, $x^2 + y^2 - 6x - 8y - 4 = 0$ —— (1)

Comparing eq (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = -6$ $\quad$ $2f = -8$ $\quad$ $c = -4$

$\therefore g = -3$ $\quad$ $\therefore f = -4$

Now,

Equation of tangent is given by

$x \cdot x_1 + y \cdot y_1 + g(x + x_1) + f(y + y_1) + c = 0$

or, $x \cdot 8 + y \cdot 6 + [-3(x + 8)] - 4(y + 6) - 4 = 0$

or, $8x + 6y - 3x - 24 - 4y - 24 - 4 = 0$

or, $5x + 2y - 52 = 0$

Also,

Equation of normal is given by;

$2x - 5y + k = 0$

Since, it passes through $(8, 6)$ then

$2 \times 8 - 5 \times 6 + k = 0$

$\therefore k = 14$

Hence, the equation of normal is $2x - 5y + 14 = 0$

d. $x^2 + y^2 - 3x + 10y - 15 = 0$ at $(4, -11)$

Solution:

Given, $x^2 + y^2 - 3x + 10y - 15 = 0$ —— (1)

Comparing eq (i) with $x^2 + y^2 + 2gx + \cancel{2y}\ 2fy + c = 0$, we get

$g = -3/2$, $\quad$ $f = 5$ $\quad$ $c = -15$

Equation of Tangent is given by:

$$\cancel{x^2 +}\ x.x_1 + y.y_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$\text{or, } x.4 - y.11 - \frac{3}{2}(x+4) + 5(y-11) - 15 = 0$$

$$\text{or, } 8x - 22y - 3x - 12 + 10y - 110 - 30 = 0$$

$$\text{or, } 5x - 12y - 152 = 0$$

Also,

Equation of the normal is given by:

$$+12x + 5y + k = 0$$

Since it passes through $(4, -11)$ then,

$$+12 \times 4 - 5 \times 11 + k = 0$$

$$\therefore k = 7$$

Hence, the equation of normal is $12x + 5y + 7 = 0$

e. $x^2 + y^2 = 40$ at the point whose i) abscissa is 2 and ii) ordinate is -6.

Solution:

On comparing $x^2 + y^2 = a^2$, we get

$$a^2 = 40$$

The equation of tangent is given by:

$$x.x_1 + y.y_1 = a^2$$

$$\text{or, } x.2 + y.6 = 40$$

$$\text{or, } 2x + 6y = 40$$

$$\text{or, } x + 3y - 20 = 0$$

Now,

Equation of normal is given by

$$3x - y + k = 0$$

Since, it passes through $(2, 6)$ then,

$$3 \times 2 - 6 + k = 0$$

$$\therefore k = 0$$

Hence, req. equation is $3x - y = 0$

i. Given, Abscissa $(x) = 2$ then,
Using the value of x in $x^2+y^2=40$ to get y.

$2^2 + y^2 = 40$

$\therefore y = 6$

Hence, the coordinates are $(2, 6)$
Now, the ~~equat~~ equation of the tangent is given by;

$x \cdot x_1 + y \cdot y_1 = a^2$

or, $x \cdot 2 + y \cdot 6 = 40$

or, $2x + 6y = 40$

$\therefore x + 3y = 20$ —— (i)

Again,
The equation of normal is given by;

$3x - y + k = 0$

Since, it passes through $(2, 6)$ then,

$3 \times 2 - 6 + k = 0$

$\therefore k = 0$

Hence, equation of normal is ~~$x + 3y$~~ $3x - y = 0$

ii. Ordinate is -6
When, ordinate i.e. $y = -6$, using it in $x^2+y^2=40$.
to get x.

$x^2 + (-6)^2 = 40$

or $x = \sqrt{4}$

$\therefore x = 2$ $\therefore x, y = (2, -6)$

The equation of tangent is given by

$x \cdot x_1 + y \cdot y_1 = a^2$

or, $x \cdot 2 - y \cdot 6 = 40$

or $2x - 6y = 40$

or $x - 3y = 20$ —— (ii)

Again,
The ~~for~~ equation of normal is given by;

$3x + y + k = 0$

Since, it passes through $(2, -6)$ then

$3 \times 2 - 6 + k = 0$

$\therefore k = 0$

Hence, the equation of normal is $3x + y = 0$.

2.a. Find the equation of the tangent to the circle $2x^2 + 2y^2 = 9$ which makes angle $45°$ with the x-axis.

Solution:

Given, $2x^2 + 2y^2 = 9$

or, $x^2 + y^2 = \frac{9}{2}$

Comparing it with $x^2 + y^2 = a^2$, we get

$a = \frac{3}{\sqrt{2}}$

Also, $\theta = 45°$

$\therefore m = \tan 45°$

$= 1$

Now, Equation of the tangent is given by;

$y = mx + c$

or, $y = 1 \times x \pm a\sqrt{1 + m^2}$

or, $y = x \pm \frac{3}{\sqrt{2}} \sqrt{1 + 1^2}$

or, $y = x \pm \frac{3}{\sqrt{2}} . \sqrt{2}$

$\therefore y = x \pm 3$

b. Find the equation of the normal to the circle $2x^2 + 2y^2 = 9$ which makes an angle $45°$ with the x-axis.

Solution:

Given, $2x^2 + 2y^2 = 9$

or, $x^2 + y^2 = \frac{3}{\sqrt{2}}$

On comparing it with $x^2+y^2 = a^2$, we get

$a = \frac{3}{\sqrt{2}}$

Also, $\theta = 45°$

$\therefore m = \tan 45°$

$= 1$

Eq. of the tangent is given by;

$y = mx + c$

or $y = mx \pm a\sqrt{1+m^2}$

or, $y = x \pm \frac{3}{\sqrt{2}}\sqrt{1+1^2}$

or, $y = x \pm 3$

Either $y = x - 3$ or, $y = x + 3$

i.e. $y - x + 3 = 0$ i.e. $x - y + 3 = 0$

Equation of the normal is given by

Now,

The equation of normal line is given by.

$y - y_1 = m(x - x_1)$

or, $y - 0 = 1(x - 0)$

or, $y = x$

$\therefore x - y = 0$

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3.a. Show that the line $3x-4y=25$ and the circle $x^2+y^2=25$ intersect in two coincident points.

Solution:

Given equation of the line is $3x-4y=25$

$$\text{or, } x = \frac{4y+25}{3} \quad \text{--- (i)}$$

Also,

Equation of circle is $x^2+y^2=25$ --- (ii)

$$\text{or, } \left(\frac{4y+25}{3}\right)^2 + y^2 = 25$$

$$\text{or, } \frac{16y^2+200y+625}{9} + y^2 = 25$$

$$\text{or, } 25y^2+200y+625 = 225$$

$$\text{or, } y^2+8y+16=0$$

$$\text{or, } (y+4)(y+4)=0$$

Either $y+4=0$ $\therefore y=-4$

OR $y+4=0$ $\therefore y=-4$

Putting the value of y in equation (i), we get;

$$x = \frac{4(-4)+25}{3}$$

$$\text{or, } x = \frac{-16+25}{3} = 3$$

$\therefore x = 3, 3$ [We got same for eq. (ii) also]

Since,

x and y are same for both equation of line and for the circle. This implies, they are intersecting in two coincident points.

b. Prove that the line $5x + 12y + 78 = 0$ is tangent to the circle $x^2 + y^2 = 36$.

Solution:

Given, $x^2 + y^2 = 36$ —— (i)

Comparing eq. (i) with $x^2 + y^2 = a^2$, we get

$a = 6$

Now, $5x + 12y + 78 = 0$

or, $y = -\frac{5}{12}x - \frac{78}{12}$

$\therefore y = -\frac{5}{12}x - \frac{13}{2}$ —— (ii)

On comparing equation (ii) with $y = mx + c$, we get

$m = -\frac{5}{12}$ and $c = -\frac{13}{2}$

We know, $c = \pm a\sqrt{1+m^2}$

or, $-\frac{13}{2} = \pm 6\sqrt{1 + (-5/12)^2}$

or, $\frac{169}{4} = 36\left(1 + \frac{25}{144}\right)$

$\therefore \frac{169}{4} = \frac{169}{4}$

Hence, $5x + 12y + 78 = 0$ is a tangent to the circle $x^2 + y^2 = 36$.

c. Prove that the tangent to the circle $x^2 + y^2 = 5$ at the point $(1, -2)$ also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ and find the point of contact.

Solution:

Equation of the tangent of the circle $x^2 + y^2 \cancel{+5} = 5$ at the point $(1, -2)$ is given by:

$x.x_1 + y.y_1 = a^2$

or, $x \cdot 1 - 2 \cdot y = 5$

or, $x - 2y = 5$ ——— (i)

Also,

$x^2 + y^2 - 8x + 6y + 20 = 0$ ——— (ii)

Comparing eq (ii) we $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = -8$ $\quad$ $2f = 6$ $\quad$ $c = 20$

$\therefore g = -4$ $\quad$ $\therefore f = 3$

$\therefore$ Centre $(h, k) = (-g, -f)$

$= (4, -3)$

Radius $(r) = \sqrt{g^2 + f^2 - c}$

$= \sqrt{16 + 9 - 20} = \sqrt{5}$

Again,

~~Perpendicular length of eq (ii)~~

$CT = \pm \dfrac{x - 2y - 5}{\sqrt{1^2 + (-2)^2}}$

$= \dfrac{4 - 2(-3) - 5}{\sqrt{1 + 4}}$

$= \dfrac{10 - 5}{\sqrt{5}}$

$= \sqrt{5}$

Since,

CT = radius of circle of eq (ii).

Hence, the tangent to circle (i) at point $(1, -2)$ also touches the circle (ii).

For point of contact using the value of x from (i) in (ii)

$(5 + 2y)^2 + y^2 - 8(5 + 2y) + 6y + 20 = 0$

or, $25 + 20y + 4y^2 + y^2 - 40 - 16y + 6y + 20 = 0$

or, $y^2 + 2y + 1 = 0$

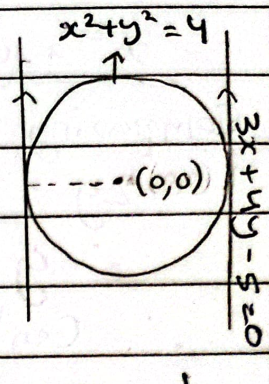
or, $(y + 1)(y + 1) = 0$

On solving we get, $x = 3$ and $y = -1$

Hence, point of contact $= (3, -1)$

4. a. Find the equation of the tangents to the circle $x^2+y^2=4$, which are parallel to $3x+4y-5=0$

Solution:



Given eq: $x^2+y^2=4$ —(i)

Comparing eq. (i) with $x^2+y^2=a^2$, we get

$a^2=4$

Equation of the line is;

$3x+4y-5=0$ —(ii)

The equation of the line // to eq (ii) is given by:

$3x+4y+k=0 \rightarrow \{ax+by+c=0\}$

Since,

The line is tangent to the circle.

$$p = \pm\left(\frac{ax+by+c}{\sqrt{a^2+b^2}}\right)$$

i.e. $$2 = \pm\left(\frac{3\times0+4\times0+k}{\sqrt{3^2+4^2}}\right)$$

or $$2 = \pm\frac{k}{\sqrt{25}}$$

or, $2\times5 = \pm k$

$\therefore k = \pm10$

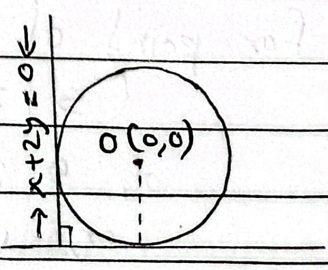
$\therefore$ The equation of the tangent is $3x+4y\pm10=0$.

b. $x^2+y^2=5$, which is perpendicular to $x+2y=0$

Solution:

Equation of circle: $x^2+y^2=5$ —(i)

radius (a) $=\sqrt{5}$



Equation of line : $x+2y=0$ —(ii)

Equation of the tangent perpendicular to eq (ii) is given by:

$2x-y+k=0$

Since, this line is also the tangent to the circle. The perpendicular distance should be equal to the radius.

i.e. $\sqrt{5} = \pm \left(\frac{2\times 0 - 0 + k}{\sqrt{2^2+1^2}} \right)$

or, $\sqrt{5} = \pm \frac{k}{\sqrt{5}}$

or, $k = \pm 5$

Hence,

The equation of the tangent is $2x - y \pm 5 = 0$.

c. $x^2 + y^2 - 6x + 4y = 12$, which are parallel to the line $4x + 3y + 5 = 0$.

Solution:

Equation of circle: $x^2 + y^2 - 6x + 4y = 12$ —— (I)

Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$g = -3 \quad f = 2 \quad c = -12$

Now,

Centre (C) $= (-g, -f)$
$= (3, -2)$

Radius (r) $= \sqrt{g^2 + f^2 - c}$
$= \sqrt{(-3)^2 + 2^2 - (-12)}$
$= 5$

Now,

Equation of the line parallel to $4x + 3y + 5 = 0$ is given by

$4x + 3y + k = 0$ —— (II)

Since,

This line is also the tangent to the circle. So,

$5 = \pm \left(\frac{4\times 3 + 3(-2) + k}{\sqrt{4^2+3^2}} \right)$

or, $5 = \pm \frac{(6 + k)}{5}$

or, $25 = \pm(6+k)$

Taking '+ve' $\therefore k = -31$

Taking '-ve' $\therefore k = 19$

Using the value of k in equation (ii), we get the equation of tangent : $4x+3y+19=0$

: $4x+3y-31=0$

d. $x^2+y^2-2x-4y-4=0$, which are perpendicular to the line $3x-4y=1$.

Solution:

Equation of circle : $x^2+y^2-2x-4y-4=0$ —— (i)

On comparing it with $x^2+y^2+2gx+2fy+c=0$, we get

$g=-1$ $\quad f=-2$ $\quad c=-4$

So,

$$\text{Radius } (r) = \sqrt{g^2+f^2-c}$$
$$= \sqrt{(-1)^2+(-2)^2+4}$$
$$= 3$$

$$\text{Centre } (c) = (-g,-f)$$
$$= (1,2)$$

Equation of line : $3x-4y=1$ —— (ii)

Equation of another line perpendicular to eq (ii) is given by $4x+3y+k=0$ —— (iii)

Since,

This line is the tangent to the circle. So,

$$3 = \pm\left(\frac{4\times1+3\times2+k}{\sqrt{4^2+3^2}}\right)$$

or, $3\times5 = \pm(10+k)$

Taking '+ve' $\therefore k=-25$

Taking '-ve' $\therefore k=5$

Now,

Using the value of 'k' in equation (iii) we get;

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The tangents are: $4x + 3y - 25 = 0$ and
$4x + 3y + 5 = 0$

5. Find the value of 'k' so that:
a. the line $4x + 3y + k = 0$ may touch the circle $x^2+y^2-4x + 10y + 4 = 0$.

Solution:
Given, equation of circle: $x^2 + y^2 - 4x + 10y + 4 = 0$
Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$g = -2$ $f = 5$ $c = 4$
Hence,

$$\text{Centre (c)} = (-g, -f)$$
$$= (2, -5)$$
$$\text{Radius}(r) = \sqrt{g^2 + f^2 - c}$$
$$= \sqrt{2^2 + (-5)^2 - 4}$$
$$= 5$$

Since,
The line $4x + 3y + k = 0$ touches the circle. Then the perpendicular distance is given by:

$$5 = \pm\left(\frac{4\times2 + 3\times(-5) + k}{\sqrt{4^2+3^2}}\right)$$

or, $5 \times 5 = \pm(-7 + k)$

Now,

Taking '+ve'
$25 = -7 + k$
$\therefore k = 32$

Taking '-ve'
$25 = 7 - k$
$\therefore k = -18$

Hence,
The value of k is 32 or -18.

b. The line $2x - y + 4k = 0$ touches the circle $x^2+y^2 -2x - 2y - 3 = 0$.

Solution:

Equation of circle: $x^2+y^2-2x-2y-3=0$

Comparing it with $x^2+y^2+2gx+2fy+c=0$ we get

$$g=-1, \quad f=-1, \quad c=-3$$

Now,

~~Radius (r)~~ $= (-g, -f) =$ Centre (c)

$\therefore c = (1, 1)$

~~Cent~~ Radius (r) $= \sqrt{g^2+f^2-c}$

$= \sqrt{1+1+3}$

$= \sqrt{5}$

Equation of line: $2x - y + 4k = 0$.

Since, it touches the circle, the perpendicular distance is given by:

$$\sqrt{5} = \pm \left(\frac{2\times 1 - 1 + 4k}{\sqrt{2^2+1}} \right)$$

or, $5 = \pm (1+4k)$

Taking '+ve sign'

$5 = \cancel{-1}\ 1 + 4k$

$\therefore k = 1$

Taking '-ve sign'

$5 = -1 - 4k$

$\therefore k = -\frac{3}{2}$

Hence, The value of k is 1 or $-\frac{3}{2}$

6a. Find the condition that the line $px + qy = r$ is ~~tangen~~ tangent to the circle $x^2+y^2=a^2$.

Solution:

Given, Equation of circle: $x^2+y^2=a^2$ — (i)

Equation of line: $px + qy = r$ — (ii)

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We know, inorder to be the tangent, the distance from the Centre to the point of contact and radius should be equal.

i.e. $a = \pm\left(\frac{px + qy - r}{\sqrt{p^2+q^2}}\right)$

or, $a\sqrt{p^2+q^2} = \pm(p\times 0 + q\times 0 - r)$

or, $a\sqrt{p^2+q^2} = \pm r$

S.b.s

$a^2(p^2+q^2) = r^2$ which is the req. condition for the line to ~~p~~ be the tangent.

b. Deduce the condition that $lx + my + n = 0$ may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Solution:

Given equation of circle: $x^2 + y^2 + 2gx + 2fy + c = 0$ —①

from this equation, we get $g = g$, $f = f$ and $c = c$

So,

Centre (C) = $(-g, -f)$

radius (r) = $\sqrt{g^2+f^2-c}$

Now,

Inorder to be a tangent, the perpendicular distance should be equal to the radius.

i.e. $r = \pm\left(\frac{Lx + my + n}{\sqrt{L^2+m^2}}\right)$

or, $\sqrt{g^2+f^2-c} = \pm\left(\frac{-Lg - mf + n}{\sqrt{L^2+m^2}}\right)$

S.b.s

$g^2+f^2-c = \frac{(\pm n - Lg - mf)^2}{L^2+m^2}$

or, $(L^2+m^2)(g^2+f^2-c) = (n - \cancel{gx}\ Lg - mf)^2$

Hence, it is the required condition for the line to be a tangent.

c. Find the condition that the line $Lx + my + n = 0$ should be normal to the circle $x^2+y^2+2gx+2fy+c=0$

Solution:

Equation of the circle: $x^2+y^2+2gx+2fy+c=0$.

Here, Centre (C) = $(-g, -f)$

We know,

every normal passes through centre of circle.

So, if $Lx+my+n=0$ is normal to the circle then $(-g, -f)$ should lie on $Lx+my+n=0$.

~~i.e.~~ So, $L(-g) + m(-f) + n = 0$

$\therefore$ $Lg + mf - n = 0$ is the required condition.

d. Find the condition that the circle $x^2+y^2+2gx+2fy+c=0$ touches (i) The x-axis and ii) the y-axis.

Solution:

Given, equation of circle: $x^2+y^2+2gx+2fy+c=0$ — (1)

From eq. (i), we get $g=g$, $f=f$ and $c=c$

So, Centre (C) = $(-g, -f)$

radius (r) = $\sqrt{g^2+f^2-c}$

i. When it touches x-axis then,

Centre = $-g, -f$

Point of contact = $-g, 0$

We know,

radius = distance

i.e. $\sqrt{g^2+f^2-c} = \sqrt{(-g+g)^2+(-f-0)^2}$

S.b.s

$g^2+f^2-c = f^2$

$\therefore g^2 = c$

ii. When it ~~cov~~ touches y-axis

~~Coord~~ Centre C(c) = $(-g, -f)$

Point of contact = $(0, -f)$

We know,

radius = distance

$$\sqrt{g^2+f^2-c} = \sqrt{(-g-0)^2+(-f+f)^2}$$

s.b.s

$$g^2+f^2-c = g^2$$

$$\therefore f^2 = c$$

7a. Show that the tangents to the circle $x^2+y^2=100$ at the points $(6,8)$ and $(8,-6)$ are perpendicular to each other.

Solution:

Given equation of the circle: $x^2+y^2=100$

Now, Equation of tangent at $(6,8)$ is given by:

$$x \cdot x_1 + y \cdot y_1 = a^2$$

or, $x \cdot 6 + y \cdot 8 = 100$

$\therefore 6x + 8y = 100$ —— (i)

Also,

Equation of the tangent at $(8,-6)$ is given by:

$$x \cdot x_1 + y \cdot y_1 = a^2$$

or, $x \cdot 8 - y \cdot 6 = 100$

or, $8x - 6y = 100$ —— (ii)

Now,

slope of eq. (i) i.e. $m_1 = \dfrac{-6}{8}$

slope of eq(ii) i.e. $m_2 = \dfrac{-8}{-6}$

Then, $m_1 \cdot m_2 = \dfrac{-6}{8} \times \dfrac{-8}{-6}$

$= -1$

Since, the product of slope of two tangents is equal to -1, which proves that they are perpendicular.

b. Prove that the tangents to the circle $x^2+y^2+4x+8y+2=0$ at the point $(1,-1)$ and $(-5,-7)$ are parallel.

Solution:

Given, Equation of circle : $x^2+y^2+4x+\cancel{8x}\ 8y+2=0$

~~From this we get; g = 2~~

Equation of tangent at $(1,-1)$ is given by.

$x.x_1+y.y_1+g(x+x_1)+f(y+y_1)+c=0$

or, $x.1-y.1+g(x+1)+f(y-1)+c=0$

or, $x-y+2(x+1)+4(y-1)+2=0$

or, $x-y+2x+2+4y-4+2=0$

or, $3x+3y=0$

or, $x+y=0$ —— (i)

Again,

Equation of tangent at $(-5,-7)$ is given by:

$x.x_1+y.y_1+\cancel{2f}\ \cancel{2g}\ g(x+x_1)+f(y+y_1)+c=0$

or, $-x.5-7.y+2(x-5)+4(y-7)+2=0$

or, $-5x-7y+2x-10+4y-28+2=0$

or, $-3x-3x-36=0$

or, $-3(x+y)=36$

$\therefore x+y=-12$ —— (ii)

Now,

slope of eq. (i) $m_1=-1$

slope of eq. (ii) $m_2=-1$

Here,

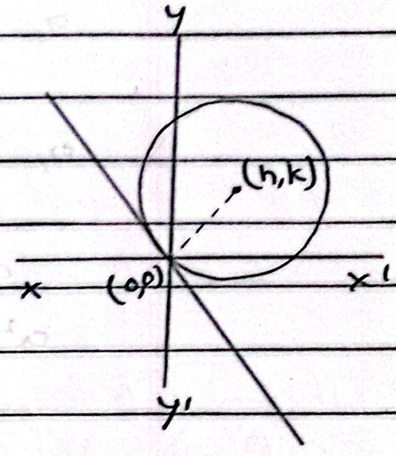
$m_1=m_2$. Hence, it is proved that the tangents are parallel to each other.

8. Find the equation of the circle whose centre is at the point (h, k) and which passes through the origin and prove that the equation of the tangent at the origin is $hx + ky = 0$.

Solution:

Here, centre (c) = (h, k)

Point = (0, 0)



Now, radius $(r) = \sqrt{(h-0)^2 + (k-0)^2}$

$= \sqrt{h^2 + k^2}$

Also,

Equation of the circle is given by:

$(x-h)^2 + (y-k)^2 = r^2$

or, $x^2 - 2xh + h^2 + y^2 - 2yk + k^2 = h^2 + k^2$

or, $x^2 - 2xh + y^2 - 2yk = 0$

or, $x^2 + y^2 - 2xh - 2yk = 0$ ——— (i)

Comparing eq (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get:

$g = -h$, $f = -k$ $c = 0$

Now,

Equation of the tangent at the origin is:

$x.x_1 + y.y_1 + g(x + x_1) + f(y + y_1) + c = 0$

or, $x.0 + y.0 - h(x+0) - k(y+0) + 0 = 0$

or, $-hx - ky = 0$

$\therefore hx + ky = 0$

Proved

9. If the line $lx + my = 1$ touches the circle $x^2 + y^2 = a^2$, prove that the point (l, m) lies on a circle whose radius is $1/a$.

Solution:

Given, equation of circle : $x^2 + y^2 = a^2$, where, $r = a$, $C = (0,0)$

Equation of line : $lx + my - 1 = 0$

Since, the line touches the circle, it means radius is equal to the perpendicular distance.

i.e. $a = \pm \left(\frac{Lx + my - 1}{\sqrt{L^2 + m^2}} \right)$

or, $a = \pm \left(\frac{L \times 0 + m \cdot 0 - 1}{\sqrt{L^2 + m^2}} \right)$

or, $a = \pm \frac{(-1)}{\sqrt{L^2 + m^2}}$

s.b.s

$a^2 = \frac{1}{L^2 + m^2}$

or, $L^2 + m^2 = \frac{1}{a^2}$ —— ①

Here, eq (i) is in the form of $x^2 + y^2 = a^2$ where a is the radius.

i.e. $r^2 = \frac{1}{a^2}$

$\therefore r = \frac{1}{a}$

Hence, it proves that the point (L, m) lies on circle, whose radius is $\frac{1}{a}$.

10.a. Find the conditions for the two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ to touch:

i. Externally ii. Internally

Solution:

Equation of 1st circle

$x^2 + y^2 = a^2$ —— ①

Centre $(C_1) = (0, 0)$

Radius $(r_1) = \cancel{a^2}\ a$

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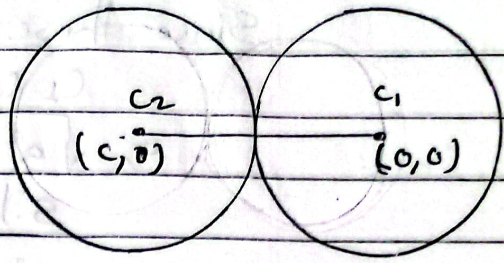
Equation of 2nd circle:

$(x-c)^2 + y^2 = b^2$

Centre $(c_2) = (c, 0)$

radius $(r_2) = b$



i. When the circle touches externally then,

$C_1 C_2 = r_1 + r_2$

or, $\sqrt{(c-0)^2 + (0-0)^2} = a + b$

or, $\sqrt{c^2} = a+b$

$\therefore c = a+b$

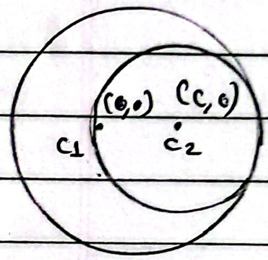
ii. When the circle touches internally then,

$C_1 C_2 = r_1 - r_2$

or, $\sqrt{(c-0)^2 + (0-0)^2} = a-b$

or, $\sqrt{c^2} = a-b$

$\therefore c = a-b$



b. Prove that the two circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

Solution:

Equation of 1st circle: $x^2+y^2+2ax+c^2=0$ —— (i)

from (i), we get Centre $(c_1) = (-g, -f)$

$= (-a, 0)$

radius $(r_1) = \sqrt{g^2+f^2-c}$

$= \sqrt{(-a)^2 - c^2}$

$= \sqrt{a^2-c^2}$

Also,

Equation of 2nd circle: $x^2+y^2+2by+c^2=0$ —— (ii)

from (ii), we get, Centre $(c_2) = (-g, -f)$

$= \cancel{(-b, 0)}\ (0, -b)$

radius $(r_2) = \sqrt{b^2-c^2}$

Since, these two circle touches each other.

$C_1 C_2 = r_1 + r_2$

or, $\sqrt{a^2+b^2} = \sqrt{a^2-c^2} + \sqrt{b^2-c^2}$

S.b.s

$a^2+b^2 = a^2-c^2 + 2\sqrt{a^2-c^2}\cdot\sqrt{b^2-c^2} + b^2-c^2$

or, $0 = -2c^2 + \cancel{2\sqrt{a^2-b}}\ 2\sqrt{a^2-c^2}\cdot\sqrt{b^2-c^2}$

or, $2c^2 = 2\sqrt{a^2-c^2}\cdot\sqrt{b^2-c^2}$

or, $c^2 = \sqrt{a^2-c^2}\cdot\sqrt{b^2-c^2}$

S.b.s

$c^4 = (a^2-c^2)(b^2-c^2)$

or, $c^4 = a^2b^2 - a^2c^2 - b^2c^2 + c^4$

or, $a^2c^2 + b^2c^2 = a^2b^2$

or, $c^2(a^2+b^2) = a^2b^2$

Dividing both side by $a^2b^2c^2$, we get

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$$ proved

11 Find the equations of the tangents drawn from the point (11, 3) to the circle $x^2+y^2=65$. Also, find the angle between the two tangents.

Solution:

Equation of circle: $x^2+y^2=65$ —①

from ①, Centre (C) = (0,0)

Radius (r) = $\sqrt{65}$

The equation of the line through the point (11, 3) is

$y = mx + c$

or, $y = mx \pm r\sqrt{m^2+1}$

or, $3 = m.11 \pm \sqrt{65}\sqrt{1+m^2}$

or, $3 = 11m \pm \sqrt{65 + 65m^2}$

or, $3 - 11m = \pm\sqrt{65+65m^2}$

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S.b.s

$9 - 66m + 121m^2 = 65 + 65m^2$

or, $9 - 65 - 66m = -56m^2$

or, $56m^2 = 56 + 66m$

or, $56m^2 - 66m - 56 = 0$

or, $28m^2 - 33m - 28 = 0$

or, $28m^2 - (49-16)m - 28 = 0$

or, $28m^2 - 49m + 16m - 28 = 0$

or, $7m(4m-7) + 4(4m-7) = 0$

or, $(4m-7)(7m+4) = 0$

Either $4m - 7 = 0$ $\therefore m = \frac{7}{4}$

OR $7m + 4 = 0$ $\therefore m = \frac{-4}{7}$

Multiplying these two slopes we get;

i.e. $\frac{7}{4} \times \frac{-4}{7} = -1$

It implies that these two tangents are perpendicular to each other, and angle between them is 90°.

~~Angle between them is; $\tan\theta = -1$~~

~~or, $\theta = \tan^{-1}(-1)$~~

~~$\therefore \theta = 90$~~

Forgot to find the equation. use $y - y_1 = m(x - x_1)$ to find it. [$\because m = \frac{7}{4}$ and $-\frac{4}{7}$]

12. Prove that the straight line $y = x + a\sqrt{2}$ touches the circle $x^2 + y^2 = a^2$ and find it's point of contact.

Solution:

Equation of circle: $x^2 + y^2 = a^2$ —— ①

from ①, we get;

Centre (C) = (0,0)

radius (r) = a^2

If the st. line touches the circle, then the length of perpendicular should be equal to the radius.

i.e. $a = \pm \left(\frac{x - y + a\sqrt{2}}{\sqrt{1^2 + (-1)^2}} \right)$.

or $a = \pm \left(\frac{0 - 0 + a\sqrt{2}}{\sqrt{2}} \right)$

or $a = \pm \frac{a\sqrt{2}}{\sqrt{2}}$

$\therefore a = a$ [radius can't be -ve]

It is proved that the st. line touches the circle.
If it touches then the equation must be satisfied.

$y = x + a\sqrt{2}$ ——— (ii)

Using it in eq. (i), we get

$x^2 + (x + a\sqrt{2})^2 = a^2$

$x^2 + x^2 + 2ax\sqrt{2} + 2a^2 = a^2$

$2x^2 + 2ax\sqrt{2} + \cancel{2a^2} \cancel{- a} \; a^2 = 0$

or, $(\sqrt{2}x)^2 + 2.\sqrt{2}.x.a + a^2 = 0$

or, $(\sqrt{2}x + a)^2 = 0$

or, $x = \frac{-a}{\sqrt{2}}$

Using the value of 'x' in eq. (ii)

$y = \frac{-a}{\sqrt{2}} + a\sqrt{2}$

or $y = \frac{-a + 2a}{\sqrt{2}}$

$\therefore y = \frac{a}{\sqrt{2}}$

Hence, the point of contact ~~are~~ is $\left(\frac{-a}{\sqrt{2}}, \frac{a}{\sqrt{2}} \right)$

13. Tangents are drawn from the origin to the circle $\cancel{x^2+y}$ $x^2+y^2+10x+10y+40=0$. find their equation.

Solution:

Equation of circle: $x^2+y^2+10x+10y+40=0$ —— ①

Comparing eq. (i) with $x^2+y^2+2gx+2fy+c=0$, we get

$g=5$, $f=5$, $c=40$

Centre (c) $= (-g,-f)$
$= (-5,-5)$

radius $(r) = \sqrt{g^2+f^2-c}$
$= \sqrt{25+25-40}$
$= \sqrt{10}$

We know,

Tangent is a st. line and equation of st. line through origin is given by:

$y-y_1 = m(x-x_1)$

or, $y-0 = m(x-0)$

or, $y = mx$

$\therefore mx-y=0$

Also,

Length of perpendicular radius of circle

or $\sqrt{10} = \frac{\pm(mx-y)}{\sqrt{m^2+1^2}}$

or $\sqrt{10} = \frac{\pm(-5m+5)}{(\sqrt{m^2+1})}$

S.b.s

$10 = 25m^2-50m+25$

or, $-15m^2+50m-15=0$

or $-5(3m^2-10m-3)=0$

or, $3m^2-9m-1m+3=0$

or $3m(m-3)-1(m-3)=0$

or $(3m-1)(m-3)=0$

Either $m = \frac{1}{3}$ or $m = 3$

When $m = \frac{1}{3}$

The equation of tangent is $\frac{x}{3} - y = 0$

$\therefore x - 3y = 0$

When, $m = 3$

The equation of tangent is $3x - y = 0$

14. Determine the length of the tangent to the circle.

a. $x^2 + y^2 = 25$ from $(3,5)$

Solution:

Equation of circle: $x^2 + y^2 = 25$ —(i)

from eq (i), we get, $a = 5$

Now,

The length of the tangent to the circle is given by:

$$L = \sqrt{x_1^2 + y_1^2 - a^2}$$
$$= \sqrt{3^2 + 5^2 - 25}$$
$$= 3$$

b. $x^2 + y^2 + 4x + 6y - 19 = 0$ from $(6,4)$

Solution:

Comparing $x^2 + y^2 + 4x + 6y - 19 = 0$ with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get;

$2g = 4$ $\quad$ $2f = 6$ $\quad$ $c = -19$

$\therefore g = 2$ $\quad$ $\therefore f = 3$

Now,

The length of tangent to the circle is given by:

$$L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$
$$= \sqrt{6^2 + 4^2 + 2 \times 2 \times 6 + 2 \times 3 \times 4 - 19}$$

c/w

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$= 9$

15. Determine the value of k so that the length of the tangents from (5,4) to the circle $x^2+y^2+2ky=0$ is 5.

Solution:

Here, $(x_1, y_1) = (5,4)$

Equation of circle : $x^2+y^2 = -2ky$ —— (i)

Now,

$\therefore a^2 = -2ky$

$$L = \sqrt{(x_1^2) + (y_1)^2 - a^2}$$

or, $5 = \sqrt{5^2 + 4^2 - (-2k.y)}$

or, $5 = \sqrt{25+16-(-2k.4)^2}$

or, $5 = \sqrt{41+8k}$

S.b.s

$25 = 41 + 8k$

or, $-16 = 8k$

$\therefore k = -2$

16. Show that the length of the tangent drawn from any point on the circle $x^2+y^2+2gx+2fy+c=0$ to the circle $x^2+y^2+2gx+2fy+c_1=0$ is $\sqrt{c_1-c}$

Solution:

Given, equation of circle : $x^2+y^2+2gx+2fy+c=0$ —— (i)

$x^2+y^2+2gx+2fy+c_1=0$ —— (ii)