Unit-1

Physical Quantities

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Fundamental and Derived Units

There are two type of quantities;

i. Measurable quantities
ii. Non-measurable quantities

Those quantities which can be measured are called physical quantities. Physical quantities are categorized into two classes;

a. Fundamental quantities
b. Derived quantities

Fundamental Quantities

Those quantities which are independent to the other quantities are called fundamental quantities. There are 7 types of fundamental quantities and two types of supplement quantities.

	Fundament quantities	Units
1.	Mass (M)	kg
2.	Length (L)	m
3.	Time (T)	s
4.	Current (A)	A
5.	Temperature (K)	k
6.	Luminous Intensity (I)	cd
7.	Amount of substance (s)	mol

	Supplementary quantities	Units
1.	Plane angle	Radian (Rad)
2.	Solid angle	Steradian (Sr.)

Derived quantities

There are many types of derived quantities. for example; Area, volume, force, velocity, density etc.

Derived quantities are those quantities which depends on other fundamental quantities. For example force = mass × acceleration, ~~mass~~ Dimension of force (f) = $M^1L^1T^{-2}$

Defination of Dimension

The dimension of a physical quantity is the power raised to the fundamental quantities. The dimension of force is (1, 1, -2). Similarily, dimension of area is (0,2,0) i.e. Dimension of A = $[M^0L^2T^0]$, dimension of density is (1,-3,0) i.e. dimension of d = $[M^1L^{-3}T^0]$

Units of ~~fundamet~~ fundamental quantities are called fundamental units.

Units of derived fundamental quantities are called ~~fundame~~ derived units. Example;

Unit of force = $kg\,ms^{-2}$

Unit of density = $kg\,m^{-3}$

Precise Measurement

The measurement in which the observed value of a physical quantity can be reproduced again and again by repeated experiment and procedure is called precise measurement.

If the reading are very close to each other than they are called precise reading. Eg: The thickness of glass plate measured by spherometer are 2.43 mm, 2.44mm and 2.45 mm. These reading are very close to each other. So they are precise measurement.

Accurate Measurement

The measurement in which observed value of any physical quantity is closer to the standard value of that physical quantity is called accurate measurement.

For example: Standard value of 'g' in lab is $9.8 \ m/s^2$. The measured value of 'g' is obtained as $9.67 \ m/s^2$ and $9.79 \ m/s^2$. The value $9.79 \ m/s^2$ is very closed to the standard value. So, it is ~~mass~~ more accurate than the value $9.67 \ m/s^2$.

Significant Figure

The meaningful digits of a number are called significant figure. Significant figure depends on the least count of a ~~measurement~~ measuring device.

Example: Let us take the length of rod by ruler for 3 times. Readings are obtained as 10.2 cm, 10.3 cm and 10.3 cm. So mean length;

$$= \frac{10.2 + 10.3 + 10.3}{3}$$

$$= 10.266666 \text{ cm}$$

Here all digits are not significant, only first three digits are significant. The length of ~~rod~~ rod is taken as 10.2 cm or 10.3 cm

Error in a measurement

The error in a measurement is equal to the difference between true value and measured value of quantity.

i.e. Error = True value − measured value

Types of Error

1. Systematic Error
2. Random Error
3. Least Count Error
4. Gross Error

1. Systematic Error : The error which tends to occur in one direction, either positive or negative is called systematic errors.

Source of Systematic errors:

a. Instrumental errors : It arise due to imperfect design of measuring instrument.

b. Imperfection in experimental Technique : To determine the temperature of a human body, a thermometer placed under the armpit will always give a lower temperature than actual body temperature

c. Personal Errors : It arise due to inexperience of the observer. Eg: Lack of proper setting of the apparatus.

d. Errors due to external cause : The external condition such as changes in temperature, pressure humidity etc. during the experiment

2. Random errors : The errors which occurs irregularly and at random in magnitude are called random errors. Such errors occur by chance and arise due to slight variation in the attention of observer while taking the readings.

3. Least Count error : The smallest value that can be measured by the measuring instrument is called least count. This error is associated with the resolution of the instrument.

4. Gross error : These errors occurs due to the carelessness of the person or due to improper ~~adjustmen~~ adjustment of the apparatus.

Uses of dimensional analysis

a. Density

We know,

$$D = \frac{mass}{Volume}$$

Dimensional formula of Density (D) = $[M^1 L^{-3} T^0]$

b. Momentum

we know, momentum = m×Velocity

Dimensional formula of momentum (P) = $[M^1 L^0 T^{-1}]$

c. Frequency

We know,

Momentum of frequency (f) = $[T^{-1}] = \frac{1}{\text{Time period}}$

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d. Work

We know, Work(W) $= f \times d$

$= m \times a \times d$

$= m \times \frac{d}{t^2} \times d$

Dimensional formula of Work (W) $= [M^1 L^2 T^{-2}]$

e. Power

We know, Power (P) $= \frac{W}{t}$

$= \frac{ma \times d}{t}$

$= m \times \frac{d}{t^2} \times \frac{1}{t} \times d$

Dimensional formula of Power (P) $= [M^1 L^2 T^{-3}]$

f. $\sin\theta$

We know, $\sin\theta = \frac{p}{h} = \frac{\text{Length}}{\text{Length}}$

$= \frac{[M^0 L^1 T^0]}{[M^0 L^1 T^0]}$

$= [M^0 L^0 T^0]$

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Uses of Dimensional Equation

1.i Convert 1 Joule into erg?

Solution:

We know that, Joule and Erg are the units of energy in MKS and CGS system respectively.

Since, Work (W) = $[ML^2T^{-2}]$. So $a = 1, b = 2$ and $c = -2$

Here,

Given, system (SI)	New system (CGS)
$n_1 = 1$	$n_2 = ?$
$M_1 = 1\,kg$	$M_2 = 1\,g$
$L_1 = 1\,m$	$L_2 = 1\,cm$
$T_1 = 1\,s$	$T_2 = 1\,sec.$

Now,

Using formula

$$n_2 = n_1\left[\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c\right]$$

$$= 1\left[\left(\frac{1\,kg}{1\,g}\right)^1\left(\frac{1\,m}{1\,cm}\right)^2\left(\frac{1\,s}{1\,s}\right)^{-2}\right]$$

$$= 1\left[\left(\frac{1000\,g}{1\,g}\right)^1\left(\frac{100\,cm}{1\,cm}\right)^2\left(\frac{1\,s}{1\,s}\right)^{-2}\right]$$

$$= 1000 \times 10000 \times 1$$

$$= 10^7\ erg$$

$\therefore$ 1 Joule $= 10^7$ erg

ii. Convert 1 Newton into dynes.

Solution:

We know, Newton is the unit of force.

Now, Dimensional formula of force (F) = $[MLT^{-2}]$

Here, $a = 1$, $b = 1$ and $c = -2$

Using formula;

$$n_2 = n_1 \left[\left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c \right]$$

$$= 1 \left[\left(\frac{1000g}{1g}\right)^1 \left(\frac{100\,cm}{1\,cm}\right)^1 \left(\frac{1s}{1s}\right)^{-2} \right]$$

$$= 1000 \times 100 \times 1^{-2}$$

$$= 10^5 \text{ dyne}$$

$\therefore$ 1 Newton $= 10^5$ dyne

2. To check the correctness of physical relation.

i. $F = ma$

Solution:

Dimension of LHS = Dimension of RHS → Principle of homogeneity

Dimension of F = $[M L^1 T^{-2}]$

Dimension of ma = $[M L^0 T^0][M^0 L^1 T^{-2}]$

$= [M^1 L^1 T^{-2}]$

$\therefore$ LHS = RHS

ii. $KE = \frac{1}{2} mv^2$

Solution:

LHS = KE = $[M^1 L^2 T^{-2}]$

RHS = $\frac{1}{2} mv^2$

$= [M^1 L^0 T^0] \cancel{[M^0 L^2 T^{-2}]} [M^0 L^2 T^{-2}]$

$= [M^1 L^2 T^{-2}]$

$\therefore$ LHS = RHS

iii. $T = 2\pi\sqrt{g/l}$

Solution:

LHS

$T = [M^0 L^0 T^1]$

RHS

$2\pi\sqrt{g/L} = \sqrt{g/L}$

$= \left[\dfrac{M^0 L^1 T^{-2}}{M^0 L^1 T^0}\right]^{1/2}$

$= M^0 L^0 T^{-2}$

$\therefore$ LHS $\neq$ RHS

Hence, The dimensional formula is not correct.

iii. $PE = mgh$

Solution:

Here, L.H.S

$PE = [M L^2 T^{-2}]$

Also,

RHS $= Mgh = [M^1][LT^{-2}][L^1]$

$= [M L^2 T^{-2}]$

$\therefore$ LHS = RHS

iv. $S = vt + \frac{1}{2}at^2$

Solution:

LHS

S = Displacement $= [M^0 L^1 T^0]$

RHS

$vt + \frac{1}{2}at^2 = [\ \cancel{vt =}\ [M^0 L^1 T^0] + [M^0 L T^{-2}][M^0 L^0 T^2]$

$= [M^0 L^1 T^{-2}][M^0 L^0 T^2]$

$= [M^0 L^1 T^0]$

$\therefore$ LHS = RHS

3. To determine the dimension of Constant

a. Find the dimension of gravitational Constant G.

Solution:

We know,

$$F = \frac{G m_1 m_2}{d^2}$$

$$\text{or, } \frac{Fd^2}{m_1 \cdot m_2} = G$$

$$\text{or, } G = \frac{Fd^2}{m_1 m_2} = \frac{[M^1 L^1 T^{-2}][M^0 L^2 T^0]}{[M^2 L^0 T^0]}$$

$$= \frac{[M^1 L^3 T^{-2}]}{[M^2 L^0 T^0]}$$

$$= [M^{-1} L^3 T^{-2}]$$

b. Find the dimension of gas constant.

Solution:

We know,

$$PV = nRT$$

Where,

P = Pressure

V = Volume

n = no. of mole

T = Temperature

Now,

$$R = \frac{PV}{nT}$$

$$= \frac{\frac{F}{A} \times V}{nT}$$

$$= \frac{\frac{m \times a}{A} \times V}{nT}$$

$$= \frac{\dfrac{m \times \dfrac{d}{t^2}}{l^2} \times L^3}{mol \cdot kelvin(k)}$$

$$= \frac{M^1 L^2 T^{-2}}{Mol. k}$$

$$= [M^1 L^2 T^{-2} mol^{-1} k^{-1}]$$

4. To derive relation between various physical quantity.

Solution:

Let 't' be the time period of simple pendulum. Assume that its time period is proportional to ~~ists~~ its length (l^a), its mass (m^b) and acceleration due to gravity (g^c) where a, b and c are dimensional constant which has to be determined.

Then,

We have, $t \propto l^a m^b g^c$

or, $t = k\, l^a m^b g^c$

Experimentally, k be the calculated having value 2π.

$\therefore T = 2\pi\, l^a m^b g^c$ —— (i)

Now,

Dimension of LHS : $(T) = [M^0 L^0 T^1]$

Also,

Dimension of RHS $= [M^0 L^1 T^0]^a [M^1 L^0 T^0]^b [M^0 L^1 T^{-2}]^c$

$= [M^b L^{a+c} T^{-2c}]$

Here,

Comparing LHS and RHS we get

$0 = b$, $0 = a+c$, $1 = -2c$

or, $b = 0$; $a = -c$, $c = -1/2$

or, $b = 0$, $a = 1/2$, $c = -1/2$ —— (ii)

From equation (i) and (ii) we get.

$T = 2\pi L^{1/2} M^0 g^{-1/2}$

$= 2\pi \sqrt{L/g}$

Q.i. A student writes an expression of the force causing the a body of mass (m) to move in a circular section with a velocity (v) as $F = mv^2$. Use the dimensional method to check its correctness.

Solution:

Here, $F = Mv^2$

Now, LHS:

i.e. $F = [MLT^{-2}]$

Also,

$$RHS = Mv^2$$
$$= [M^1L^0T^0][LT^{-1}]^2$$
$$= [M^1L^0T^0][L^2T^{-2}]$$
$$= [ML^2T^{-2}]$$

Here, LHS ≠ RHS. so, the student is dimensionally wrong.

ii. Convert 10 erg into Joule.

Solution:

Here, The dimensional formula of Energy is given by;

$$E = F \times d$$
$$= [ML^1T^{-2}][L]$$
$$= [ML^2T^{-2}]$$

$\therefore a = 1, b = 2, c = -2$

Now, using formula;

$$n_2 = n_1 \left[\left(\frac{M_1}{M_2}\right)^a \cdot \left(\frac{L_1}{L_2}\right)^b \left(\frac{T^1}{T_2}\right)^c\right]$$
$$= 10\left[\left(\frac{1gm}{1kg}\right)^a \left(\frac{1cm}{1m}\right)^b \left(\frac{1s}{1s}\right)^c\right]$$
$$= 10\left[\left(\frac{1gm}{1000gm}\right)^1 \left(\frac{1cm}{100cm}\right)^2 \left(\frac{1s}{1s}\right)^{-2}\right]$$
$$= 10 \times 10^{-3} \times 10^{-2\times2} \times 1^{-2}$$
$$= 10^{1-3-4}$$
$$= 10^{-6} \text{ Joule.}$$

Chapter-1
Physical Quantities

Gravitational Constant Dimension:

We know,

$$F = \frac{G m_1 m_2}{d^2}$$

$$\text{or, } \frac{F d^2}{\cancel{}} \times \frac{1}{m_1 . m_2} = G$$

$$\text{or, } G = \frac{F d^2}{m_1 . m_2}$$

$$\text{or, } G = \frac{[MLT^{-2}][L^2]}{M^2}$$

$$\text{or, } G = [M^{-1} L^3 T^{-2}]$$

1. To check the correctness of physical relation.

The density rho of the earth is given by $\rho = \frac{3g}{4\pi R G}$

where g = acc. due to gravity

G = Universal Gravitation Constant.

R = Radius of earth.

Check the dimensional consistency of this relation

Solution:

Dimension of $g = [M^0 L T^{-2}]$

$R = [M^0 L T^0]$

$G = [M^{-1} L^3 T^{-2}]$

$\rho = [ML^{-3} T^0]$

Now,

Dimension of L.H.S (i.e ρ) = $ML^{-3}T^0$

Dimension of RHS i.e $\frac{3g}{4\pi RG}$

$$= \frac{M^0 L T^{-2}}{(M^0 L T^0)(M^{-1} L^3 T^{-2})}$$

$$= \frac{M^0 L T^{-2}}{M^{-1} L^4 T^{-2}}$$

$$= [M L^{-3} T^0]$$

Hence, the given equation is dimensionally correct.

2. Check the correctness of formula;

$$t = 2\pi \sqrt{\frac{L}{g}}$$

Where, t = time period
L = Length of pendulum
g = acceleration due to gravity

Solution:

Dimensional formula of $t = [M^0 L^0 T]$
$L = [M^0 L T^0]$
$g = [M^0 L T^{-2}]$

Now,

Dimension of LHS i.e. $t = [M^0 L^0 T]$

Dimension of RHS i.e. $\left(\frac{L}{g}\right)^{1/2} = \left[\frac{M^0 L T^0}{M^0 L T^{-2}}\right]^{1/2}$

$$= [M^0 L^0 T]$$

3. A student writes $\sqrt{\frac{R}{2Gm}}$ for escape velocity of the earth. Check the correctness of the formula by using dimensional analysis.

Solution:

Given equation $= V = \sqrt{\frac{R}{2Gm}}$

Dimensional formula of $V = [M^0 L T^{-1}]$

$R = [M^0 L T^0]$

$G = [M^{-1} L^3 T^{-2}]$

$m = [M L^0 T^0]$

Now,

Dimensional formula of $\left(\frac{R}{Gm}\right)^{1/2} = \left[\frac{M^0 L T^0}{M^0 L^3 T^{-2}}\right]^{1/2}$

$= [M^0 L^{-2} T^2]^{1/2}$

$= [M^0 L^{-1} T^1]$

Hence, it is dimensionally not correct.

Determine the time(t) period of a simple pendulum which depend upon mass (m) of pendulum, Length (l) and acceleration due to gravity (g).

Solution:

By question:

$t \propto m^a$

$t \propto l^b$

$t \propto g^c$

Combining we get;

$t \propto m^a l^b g^c$

or $t = k\, m^a l^b g^c$ —— ①

where k is dimensionless constant

Now,

$$[M^0 L^0 T] = [M L^0 T^0]^a [M^0 L T^0]^b [M^0 L^1 T^{-2}]^c$$

or, $[M^0 L^0 T] = [M^a L^{b+c} T^{-2c}]$

Equating both side, we get;

$M^0 = M^a$

$\therefore a = 0$

Also,

$L^0 = L^{b+c}$

or $0 = b + c$

or $0 = b - \frac{1}{2}$

$\therefore b = \frac{1}{2}$

$T^1 = T^{-2c}$

or $1 = -2c$

$\therefore c = -\frac{1}{2}$

Using the value of a, b and c in eqn ① we get

$t = k\, m^0 l^{1/2} g^{-1/2}$

or $t = k \frac{l^{1/2}}{g^{1/2}}$

or $t = k\sqrt{\frac{L}{g}}$

$\therefore t = 2\pi\sqrt{\frac{L}{g}}$ [$\because k = 2\pi$, found experimentally]

Using the method of dimension, derive an expression for the centripetal force (f) acting on an particle of mass (m) moving with velocity (v) in a circle of radius (r).

Solution :

By question:

$F \propto m^a$

$F \propto v^b$

$F \propto r^c$

Combining all we get;

$F \propto m^a v^b r^c$

or $F = k m^a v^b r^c$ ——— (1)

where k is dimensionless constant

Now,

$[MLT^{-2}] = [ML^0T^0]^a [M^0LT^{-1}]^b [M^0LT^0]^c$

or, $[MLT^{-2}] = [M^a L^{b+c} T^{-b}]$

Equating both side we get;

$M^1 = M^a$

$\therefore a = 1$

$T^{-2} = T^{-b}$

$\therefore b = 2$

$L^1 = L^{b+c}$

or $1 = \cancel{b+c}$ $b+c$

or $1 = \cancel{1+c}$ $2+c$

$\therefore c = -1$

Using the value of a, b and c in equation ①.

$$F = kMv^2 r^{-1}$$

or, $F = k\frac{Mv^2}{r}$

$\therefore F = \frac{Mv^2}{r}$

The frequency an of vibration of a stretch string is a function of its tension (F), Length (L) and mass per unit length (m). From the knowledge of dimension, prove that $n \propto \frac{1}{L}\sqrt{\frac{F}{M}}$

Solution:

By Question

$n \propto F^a$

$n \propto L^b$

$n \propto m^c$

Combining all we get;

$n \propto F^a L^b m^c$

Now,

Dimensional formula of $n = [M^0 L^0 T^{-1}]$

Dimensional formula of $F^a = [MLT^{-2}]^a$

Dimensional formula of $L^b = [M^0 L T^0]^b$

Dimensional formula of $M^c = [M^1 L T^0]^c$

Equating corresponding value :

$a + c = 0$

or, $\frac{1}{2} + c = 0$

$\therefore c = -\frac{1}{2}$

Again,

$-2a = -1$

$\therefore a = \frac{1}{2}$

Again,

$a + b - c = 0$

or, $\frac{1}{2} + b + \frac{1}{2} = 0$

$\therefore b = -1$

Using the value of a, b and c in equation ①

$n \propto F^{1/2} L^{-1} M^{-1/2}$

or, $n \propto \frac{1}{L} \frac{\sqrt{F}}{\sqrt{M}}$

or, $n \propto \frac{1}{L} \sqrt{\frac{F}{M}}$

Hence, proved#

Sphere of radius 'a' moving through a fluid of density 's' with a velocity 'v' experiences a retarding force 'F' which is given by ; $F = k a^x s^y v^z$ where k is non-dimensional coefficient. Use the method of dimension to find the value of x, y and z.

Solution :

By question :

$$F = k a^x s^y v^z$$

or, $[MLT^{-2}] = [M^0 L T^0]^x [M L^{-3} T^0]^y [M^0 L^1 T^{-1}]^z$

Now,

~~Equating both sides, we get~~

$[MLT^{-2}] = [M^y L^{x-3y+z} T^{-z}]$

Equating both we get ;

$M^1 = M^y$

$\therefore y = 1$

$L^1 = L^{x-3y+z}$

or, $1 = x - 3y + z$

or, $1 = x - 3 \cdot 1 + 2$

or, ~~$x = x$~~ $1 = x - 1$

$\therefore x = 2$

$T^{-2} = T^{-z}$

or, $-2 = -z$

$\therefore z = 2$

Hence, the value of x, y and z are 2, 1 and 2 respectively.

To convert the value of physical quantity from one system to another.

Formula: $n_1 u_1 = n_2 u_2$

10 Newton = ? dyne

We know,

Dimension of force = $[MLT^{-2}]$

For SI system

$n_1 = 10N$

$M_1 = kg$

$L_1 = M$

$T_1 = sec$

For CGS system

$n_2 = ?$

$M_2 = gm$

$L_2 = cm$

$T_2 = sec.$

We know,

$$n_1 u_1 = n_2 u_2$$

or $10 [M_1 L_1 T_1^{-2}] = n_2 [M_2 L_2 T^{-2}]$

or $n_2 = 10 \frac{[M_1 L_1 T_1^{-2}]}{[M_2 L_2 T_2^{-2}]}$

$$= 10 \left[\frac{M_1}{m_2}\right] \left[\frac{L_1}{L_2}\right] \left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 10 \left[\frac{kg}{g}\right] \left[\frac{m}{cm}\right] \left[\frac{sec}{sec}\right]^{-2}$$

$$= 10 \left[\frac{1000g}{g}\right] \left[\frac{100\ cm}{cm}\right] \left[\frac{1}{1}\right]^{-2}$$

$\therefore n_2 = 10^6$ dyne

Convert 5 Joule into erg.

Solution:

We know, Joule is SI unit of work.

Dimensional formula of work = $[ML^2T^{-2}]$

Now,

For SI unit

$n_1 = 5$

$m_1 = kg$

$L_1 = m$

$T_1 = sec$

For cgs unit

$n_2 = ?$

$m_2 = kg$

$L_2 = m$

$T_2 = sec$

Here,

$$n_1 u_1 = n_2 u_2$$

or $5[M_1L_1^2T_1^{-2}] = n_2[M_2L_2^2T_2^{-2}]$

or $n_2 = \frac{5[M_1L_1^2T_1^{-2}]}{[M_2L_2^2T_2^{-2}]}$

$$= 5\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 5\left[\frac{1000kg}{kg}\right]\left[\frac{100cm}{cm}\right]^2\left[\frac{1sec}{1sec}\right]^{-2}$$

$\therefore n_2 = 5 \times 10^7$ erg

Convert density of water 1gm/cm³ into kg/m³

Solution:

We know,

gm/cm³ is the cgs unit of density.

Dimensional formula of density = $[ML^{-3}T^{0}]$

For ~~SI~~ cgs unit

$n_1 = 1$

$m_1 = gm$

$L_1 = cm$

$T_1 = sec$

For SI unit

$n_2 = ?$

$m_2 = kg$

$L_2 = m$

$T_2 = sec$

We know,

$$n_1 u_1 = n_2 u_2$$

$$\text{or, } 1[M_1 L_1^{-3} T_1^{0}] = n_2 [M_2 L_2^{-3} T_2^{0}]$$

$$\text{or, } n_2 = \frac{[M_1 L_1^{-3} T_1^{0}]}{[M_2 L_2^{-3} T_2^{0}]}$$

$$= \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^{-3}$$

$$= \left[\frac{gm}{1000gm}\right]\left[\frac{cm}{100cm}\right]$$

$$= \left[\frac{\frac{1}{1000}kg}{1kg}\right]\left[\frac{\frac{1}{100}m}{1m}\right]^{-3}$$

$$= \left[\frac{1}{1000}\right]\left[\frac{1}{100}\right]^{-3}$$

$$= 10^{-3} \times (10^{-2})^{-3}$$

$$= 10^{-3} \times 10^{6}$$

$$= 10^{3}\ kg/m^{3}$$

Write down the dimension of latent heat and specific heat capacity.

Solution:

For Latent heat;

$$Q = ML$$

$$\text{or } L = \frac{Q}{M}$$

$$\text{or } L = \frac{[ML^2T^{-2}]}{[ML^0T^0]}$$

$$\therefore L = [L^2T^{-2}]$$

For specific heat capacity:

$$Q = Ms\,dt$$

$$\text{or, } s = \frac{Q}{M\,dt}$$

$$= \frac{[ML^2T^{-2}]}{[ML^0T^0][M^0L^0K^1]}$$

$$= [L^2T^{-2}K^{-1}]$$

c/w

Find the dimension of the constant a and b in the relation $P = \left(\frac{b - x^2}{at}\right)$, where P is power, x is distance and 't' is time.

Solution:

Given, $P = \frac{b - x^2}{at}$

or $P = \frac{b}{at} - \frac{x^2}{at}$

From principal of homogenity;

Dimension of P = Dimension of $\left(\frac{b}{at}\right)$ —— (I)

Dimension of P = Dimension of $\frac{x^2}{at}$ —— (II)

We know,

Dimension of P = $[ML^2T^{-3}]$

Dimension of x = $[M^0L^1T^0]$

Dimension of t = $[M^0L^0T]$

Using the first relation:

$$P = \frac{b}{at}$$

or, $[ML^2T^{-3}] = \frac{b}{[a][M^0L^0T]}$ —— (III)

Using second relation

$$P = \frac{x^2}{at}$$

or $$[ML^2T^{-3}] = \frac{[M^0LT^0]^2}{[a][M^0L^0T]}$$

or $$a = \frac{[M^0L^2T^0]}{[ML^2T^{-2}]}$$

$$\therefore a = [M^{-1}L^0T^2]$$

Using, the value of 'a' in equation (iii)

or $$[ML^2T^{-3}] = \frac{b}{[M^{-1}L^0T^2][M^0L^0T]}$$

or $$[ML^2T^{-3}] = \frac{b}{[M^{-1}L^0T^3]}$$

$$\therefore b = [M^0L^2T^0]$$

If $y = a + bt + ct^2$ where y is the distance and t is time. What is the dimension of a, b and c.

Solution:

Given, $y = a + bt + ct^2$

From principle of homogenity:

Dimension of y = Dimension of a

Dimension of y = Dimension of bt

Dimension of y = Dimension of ct^2

We know,

Dimension of $y = [M^0 L T^0]$

Dimension of $t = [M^0 L^0 T]$

Now,

Using 1^{st} relation:

$y = a$

or, $[M^0 L T^0] = a$

Again,

Using 2^{nd} relation:

$y = bt$

or $[M^0 L T^0] = [b][M^0 L^0 T]$

$\therefore b = [M^0 L T^{-1}]$

Also,

using 3^{rd} relation

$y = ct^2$

or $[M^0 L T^0] = [c][M^0 L^0 T]^2$

$\therefore c = [M^0 L T^{-2}]$

Assuming length [L], Mass [M] and force [F] as fundamental units, find the dimension of time.

Solution:

We know,

Dimension of $T = [F^a M^b L^c]$ —①

Dimension of $T = [M^0 L^0 T]$

Dimension of $F = [M L T^{-2}]$

Dimension of $M = [M L^0 T^0]$

Dimension of $L = [M^0 L T^0]$

Using the value of T, F, M and L in equation ①

i.e. $[M^0L^0T] = [F]^a[M]^b[L]^c$

or, $[M^0L^0T] = [MLT^{-2}]^a[ML^0T^0]^b[M^0LT^0]^c$

or, $[M^0L^0T] = [M^{a+b}L^{a+c}T^{-2a}]$

Equating both sides;

$a+b=0$ $\qquad a+c=0$ $\qquad -2a=1$

or, $-\frac{1}{2}+b=0$ $\qquad$ or, $-\frac{1}{2}+c=0$ $\qquad \therefore a=\frac{1}{2}$

$\therefore b=\frac{1}{2}$ $\qquad \therefore c=\frac{1}{2}$

Using the value of a, b and c in eq. (i)

$T=[F^{-1/2}M^{1/2}L^{1/2}]$

\# Taking force, Length and time as fundamental quantities. Find the dimensional formula for density.

Solution:

By question:

$D=[F^aL^bT^c]$ —— ①

We know,

Dimension of $D = [ML^{-3}T^0]$

Dimension of $F = [MLT^{-2}]$

Dimension of $L = [M^0LT^0]$

Dimension of $T = [M^0L^0T]$

Using the value of F, L, T in equation ①

i.e. $[ML^{-3}T^0] = [MLT^{-2}]^a[M^0LT^0]^b[M^0L^0T]^c$

or, $[ML^{-3}T^0] = [M^aL^{a+b}T^{-2a+c}]$

Equating both side we get:

$a = 1$ $\qquad$ $-3 = a + b$ $\qquad$ $0 = -2a + c$

$\therefore a = 1$ $\qquad$ or $-4 = b$ $\qquad$ or $0 = -2 \times 1 + c$

$\therefore -4 = b$ $\qquad$ $\therefore c = +2$

Now

Putting the value of a, b and c in equation (1)

$$D = [F^{1} L^{-4} T^{2}]$$