Exercise- 9.1

1. Find the scalar product of the following pair of vector.

a. Find $\vec{a} \cdot \vec{b}$ where $|\vec{a}| = 2$, $|\vec{b}| = 3$ and angle between $\vec{a}$ and $\vec{b} = 60°$

Solution:

$|\vec{a}| = 2$

$|\vec{b}| = 3$

$\theta = 60°$

We know,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\text{or, } \cos 60° = \frac{\vec{a} \cdot \vec{b}}{2 \times 3}$$

$$\text{or, } \frac{1}{2} = \frac{\vec{a} \cdot \vec{b}}{6}$$

$$\therefore \vec{a} \cdot \vec{b} = 3$$

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b. Find $\vec{a} \cdot \vec{b}$ where $|\vec{a}| = 2$, $|\vec{b}| = 5$ and angle between the $\vec{a}$ and $\vec{b} = 0$

Solution:

$|\vec{a}| = 2$

$|\vec{b}| = 5$

$\theta = 0°$

We know

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\text{or, } \cos 0° = \frac{\vec{a} \cdot \vec{b}}{2 \times 5}$$

$$\therefore \vec{a} \cdot \vec{b} = 10$$

c. Find $\vec{a}.\vec{b}$ where $|\vec{a}| = 15$, $|\vec{b}| = 25$ and angle between $\vec{a}$ and $\vec{b} = 120°$.

Solution:

$|\vec{a}| = 15$

$|\vec{b}| = 25$

$\theta = 120°$

We know

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$$

or $$\cos 120° = \frac{\vec{a}.\vec{b}}{15 \times 25}$$

or $$-\frac{1}{2} = \frac{\vec{a}.\vec{b}}{375}$$

$$\therefore \vec{a}.\vec{b} = \frac{-375}{2}$$

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2. Find the scalar product of the following pair of vector.

a. $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Solution:

We know,

$$\vec{a}.\vec{b} = a_1.b_1 + a_2.b_2$$
$$= 3 \times 2 + 4 \times 1$$
$$= 6 + 4$$
$$= 10$$

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b. $\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

We know,

$$\vec{a}.\vec{b} = a^1.b^1 + a^2.b^2$$

$= 1 \times 3 + (2 \times -1)$

$= 3 - 2$

$= 1$

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c. $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = \vec{i} + \vec{j}$.

Soln,

$\vec{a} = 3\vec{i} + 4\vec{j} = (3, 4)$

$\vec{b} = \vec{i} + \vec{j} = (1, 1)$

We know,

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$

$= 3 \times 1 + 4 \times 1$

$= 7$

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d. $\vec{a} = 4\vec{i} + 2\vec{j}$ and $\vec{b} = -\vec{i} + 2\vec{j}$

Soln,

$\vec{a} = 4\vec{i} + 2\vec{j} = (4, 2)$

$\vec{b} = -\vec{i} + 2\vec{j} = (-1, 2)$

We know,

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$

$= 4 \times -1 + 2 \times 2$

$= -4 + 4$

$= 0$

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e. $\vec{a} = 2\vec{i} - 3\vec{j}$ and $\vec{b} = -3\vec{i} + \vec{j}$

Solution:

$\vec{a} = 2\vec{i} - 3\vec{j} = (2, -3)$

$\vec{b} = -\vec{i} + \vec{j} = (-3, 1)$

We know,

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$

$= 2 \times -3 + (-3 \times 1)$

$= -6 - 3 = -9$

f. $\vec{a} = 5\vec{i}$ and $\vec{b} = 3\vec{j}$

Solution:

$\vec{a} = 5\vec{i} + 0\vec{j} = (5, 0)$

$\vec{b} = 0\vec{i} + 3\vec{j} = (0, 3)$

We know,

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$

$= 5 \times 0 + 0 \times 3$

$= 0$

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3. If $\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$, find the value of the following.

a. $\vec{a} \cdot \vec{b}$

Solution:

$\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Now, $\vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$= 2 \times 2 + (-3 \times 1)$

$= 4 - 3$

$= 1$

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b. $\vec{a} \cdot \vec{c}$

Solution:

$\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Now, $\vec{a} \cdot \vec{c} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$= 2 \times 0 + (-2 \times 1) = -2$

c. $\vec{b} \cdot \vec{c}$

Solution:

$\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Now, $\vec{b} \cdot \vec{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$= 2 \times 0 + (-3 \times -2)$

$= 6$

d. a^2

Solution:

$\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Now, $a^2 = \vec{a} \cdot \vec{a}$

$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= 2 \times 2 + 1 \times 1$

$= 5$

e. b^2

Solution:

$\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Now, $b^2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$= 2 \times 2 + (-3 \times -3)$

$= 4 + 9$

$= 13$

f. c^2

Solution:

$\vec{c} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Now $c^2 = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$= 0 \times 0 + (-2 \times -2)$

$= 4$

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4. Find the angle between the following pair of vectors.

a. $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Here,

$\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Let θ be the angle between them

Now,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

or, $$\cos\theta = \frac{(2 \times 1) + (-3 \times -2)}{\sqrt{4+9} \cdot \sqrt{1+4}}$$

or, $$\cos\theta = \frac{2+6}{\sqrt{13} \cdot \sqrt{5}}$$

or, $$\cos\theta = \frac{8}{\sqrt{65}}$$

$$\therefore \theta = \cos^{-1}(8/\sqrt{65})$$

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b. $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution:

$\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Let θ be the angle between them.

Now,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

or, $\cos\theta = \frac{(3\times1)+(4\times1)}{\sqrt{3^2+4^2}\cdot\sqrt{1^2+1^2}}$

or, $\cos\theta = \frac{3+4}{\sqrt{25}\cdot\sqrt{2}}$

or $\cos\theta = \frac{7}{5\sqrt{2}}$

$\therefore \theta = \cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

c. $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = 5\vec{i} - 12\vec{j}$

Solution:

$\vec{a} = 3\vec{i} + 4\vec{j} = (3,4)$

$\vec{b} = 5\vec{i} - 12\vec{j} = (5,-12)$

Let θ be the angle between them.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

or, $\cos\theta = \frac{(3\times5)+(4\times-12)}{\sqrt{3^2+4^2}\cdot\sqrt{5^2+(-12)^2}}$

or $\cos\theta = \frac{15-48}{\sqrt{25}\cdot\sqrt{169}}$

or $\cos\theta = \frac{-33}{65}$

$\therefore \theta = \cos^{-1}\left(-\frac{33}{65}\right)$

d. $\vec{a} = 2\vec{i} + \vec{j}$ and $\vec{b} = \vec{i} + 2\vec{j}$

Solution:

$\vec{a} = 2\vec{i} + \vec{j} = (2,1)$

$\vec{b} = \vec{i} + 2\vec{j} = (1,2)$

Let θ be the angle between them.

Now,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

or $\cos\theta = \frac{(2\times1)+(2\times1)}{\sqrt{2^2+1^2}\cdot\sqrt{2^2+1^2}}$

or, $\cos\theta = \frac{2+2}{\sqrt{5}\cdot\sqrt{5}}$

or, $\cos\theta = \frac{4}{5}$

$\therefore \theta = \cos^{-1}(4/5)$

5.a. If $\vec{a} = \vec{i} + 3\vec{j}$ and $b = 2\vec{i} + \vec{j}$, find the angle between $\vec{a}$ and $\vec{b}$.

Solution:

$\vec{a} = \vec{i} + 3\vec{j} = (1,3)$

$\vec{b} = 2\vec{i} + \vec{j} = (2,1)$

Let θ be the angle between them.

Now

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

or $\cos\theta = \frac{(2\times1)+(3\times1)}{\sqrt{1^2+3^2}\cdot\sqrt{2^2+1^2}}$

or $\cos\theta = \frac{5}{\sqrt{10}\cdot\sqrt{5}}$

or $\cos\theta = \frac{\cancel{5}\,1}{\cancel{5}\sqrt{2}}$

or $\cos\theta = 1/\sqrt{2}$

or $\cos\theta = \cos 45°$

$\therefore \theta = 45°$

b. If $|\vec{OA}| = 6$, $|\vec{OB}| = 7$ and $\vec{OA} \cdot \vec{OB} = 21$. find the value of angle of $\angle AOB$.

Solution:

$|\vec{OA}| = 6 = 6\vec{i} + 0\vec{j}$

$|\vec{OB}| = 7 = 7\vec{i} + 0\vec{j}$

$\vec{OA} \cdot \vec{OB} = 21$

Let θ be the angle of $\angle AOB$.

Now

$$\cos\theta = \frac{\vec{OA} \cdot \vec{OB}}{|\vec{OA}| \cdot |\vec{OB}|}$$

$$\text{or, } \cos\theta = \frac{21}{\sqrt{6^2} \cdot \sqrt{7^2}}$$

$$\text{or, } \cos\theta = \frac{21}{42}$$

$$\text{or, } \cos\theta = \frac{1}{2}$$

$$\text{or, } \cos\theta = \cos 60°$$

$$\therefore \theta = 60°$$

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c. If $\vec{OA} = \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix}$. Calculate angle AOB

Solution:

Here, $\vec{OA} = \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix} = \sqrt{3}\vec{i} + (-1\vec{j})$

$\vec{OB} = \begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix} = \sqrt{3}\vec{i} + 3\vec{j}$

Now,

$$\cos\theta = \frac{\vec{OA} \cdot \vec{OB}}{|\vec{OA}| \cdot |\vec{OB}|}$$

$$\text{or, } \cos\theta = \frac{(\sqrt{3} \times \sqrt{3}) + (-1 \times 3)}{\sqrt{(\sqrt{3})^2 + (-1)^2} \cdot \sqrt{(\sqrt{3})^2 + (3)^2}}$$

or, $\cos\theta = \frac{3-3}{\sqrt{4}\cdot\sqrt{12}}$

or, $\cos\theta = \frac{0}{2\sqrt{12}}$

or, $\cos\theta = 0$

or, $\cos\theta = \cos 90°$

$\therefore \theta = 90°$

6. Show that the following pairs of vectors are perpendicular to each other.

a. $\vec{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ and $\vec{b} \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

Solution:

$\vec{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

Now,

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$

or, $\cos\theta = \frac{(6\times -1)+(6\times 1)}{\sqrt{36+1}\cdot\sqrt{1+36}}$

or, $\cos\theta = \frac{-6+6}{\sqrt{37}\cdot\sqrt{37}}$

or, $\cos\theta = 0$

or, $\cos\theta = \cos 90°$

$\therefore \theta = 90°$

Thus, the ~~bot~~ pair of vector are perpendicular to each other.

b $\vec{a} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Solution:

Here, $\vec{a} = (-6, 2)$ and $\vec{b} = (1, 3)$

Now,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

or $\cos\theta = \dfrac{(-6 \times 1) + (2 \times 3)}{\sqrt{36+4} \cdot \sqrt{1+9}}$

or $\cos\theta = \dfrac{-6+6}{\sqrt{40} \cdot \sqrt{10}}$

or $\cos\theta = \dfrac{0}{\sqrt{40} \cdot \sqrt{10}}$

or $\cos\theta = 0$

or $\cos\theta = \cos 90°$

$\therefore \theta = 90°$

Thus, they are perpendicular to each other.

c. $\vec{a} = 3\vec{i} + 7\vec{j}$ and $\vec{b} = -7\vec{i} + 3\vec{j}$

Solution:

$\vec{a} = 3\vec{i} + 7\vec{j} = (3, 7)$

$\vec{b} = -7\vec{i} + 3\vec{j} = (-7, 3)$

Now,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

or $\cos\theta = \dfrac{-7 \times 3 + 7 \times 3}{\sqrt{3^2 + 7^2} \cdot \sqrt{(-7)^2 + 3^2}}$

or $\cos\theta = \dfrac{0}{\sqrt{58} \cdot \sqrt{58}}$

or $\cos\theta = 0°$

or $\cos\theta = \cos 90°$

$\therefore \theta = 90^\circ$

Thus, they are perpendicular to each other.

d. $\vec{a} = 4\vec{i} + 2\vec{j}$ and $\vec{b} = -\vec{i} + 2\vec{j}$

Solution:

$\vec{a} = 4\vec{i} + 2\vec{j} = (4,2)$

$\vec{b} = -\vec{i} + 2\vec{j} = (-1,2)$

Now,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

$$\text{or } \cos\theta = \frac{(4\times -1) + (2\times 2)}{\sqrt{4^2+2^2}\cdot\sqrt{-1^2+2^2}}$$

$$\text{or } \cos\theta = \frac{0}{\sqrt{16+4}\cdot\sqrt{1+4}}$$

or, $\cos\theta = 0$

or, $\cos\theta = \cos 90^\circ$

$\therefore \theta = 90^\circ$

Thus, they are perpendicular to each other.

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7. Find the value of m if the following pair of vectors are orthogonal.

a. $\vec{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} m \\ -3 \end{pmatrix}$

Solution:

$\vec{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} m \\ -3 \end{pmatrix}$

Also,

$\vec{a}$ and $\vec{b}$ are perpendicular, then

$\vec{a}\cdot\vec{b} = 0$

or, $1\cdot m + (-2\times -3) = 0$

or $m + 6 = 0$

or, $m = -6$

$\therefore m = -6$

b. $\vec{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 6 \\ m \end{pmatrix}$

Solution:

$\vec{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 6 \\ m \end{pmatrix}$

Since, $\vec{a}$ and $\vec{b}$ are perpendicular to each other

$\vec{a} \cdot \vec{b} = 0$

or $4 \times 6 + (-3 \times m) = 0$

or $24 - 3m = 0$

or $24 = 3m$

or, $m = \frac{24}{3}$

$\therefore m = 8$

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c. $\vec{a} = 2\vec{i} + m\vec{j}$ and $\vec{b} = 3\vec{i} - 2\vec{j}$

Solution:

$\vec{a} = 2\vec{i} + m\vec{j} = (2, m)$

$\vec{b} = 3\vec{i} - 2\vec{j} = (3, -2)$

Since, they are perpendicular to each other.

$\vec{a} \cdot \vec{b} = 0$

or, $(2 \times 3) + (m \times -2) = 0$

or, $6 - 2m = 0$

or $6 = 2m$

or $m = \frac{6}{2}$

$\therefore m = 3$

d. $\vec{a} = m\vec{i} + 3\vec{j}$ and $\vec{b} = -3\vec{i} + 4\vec{j}$

Solution

$\vec{a} = m\vec{i} + 3\vec{j} = (m, 3)$

$\vec{b} = -3\vec{i} + 4\vec{j} = (-3, 4)$

Since, they are perpendicular to each other.

$\vec{a} \cdot \vec{b} = 0$

or, $(m \times -3) + (4 \times 3) = 0$

or, $-3m + 12 = 0$

or, $3m = 12$

or, $m = \frac{12}{3}$

$\therefore m = 4$

8a. for what value of x are the two vectors $3\vec{i} - 2\vec{j}$ and $x\vec{i} + 3\vec{j}$ are perpendicular to each other?

Solution:

Let $\vec{a}$ be $3\vec{i} - 2\vec{j}$ and $\vec{b}$ be $x\vec{i} + 3\vec{j}$

Now,

$\vec{a} = 3\vec{i} - 2\vec{j} = (3, -2)$

$\vec{b} = x\vec{i} + 3\vec{j} = (x, 3)$

Since, they are perpendicular;

$\vec{a} \cdot \vec{b} = 0$

or, $(3 \cdot x) + (-2 \times 3) = 0$

or, $3x - 6 = 0$

or, $3x = 6$

or, $x = \frac{6}{3}$

$\therefore x = 2$

$\therefore$ The value of x should be 2.

8. If $\vec{x} = 3\vec{i} + m\vec{j}$ and $\vec{y} = 4\vec{i} - 2\vec{j}$ are perpendicular, find the value of m.

Solution:

$\vec{x} = 3\vec{i} + m\vec{j} = (3, m)$

$\vec{y} = 4\vec{i} - 2\vec{j} = (4, -2)$

As, $\vec{x}$ and $\vec{y}$ are perpendicular to each other.

$\vec{x} \cdot \vec{y} = 0$

or, $(3 \times 4) + (m \times -2) = 0$

or, $12 - 2m = 0$

or, $12 = 2m$

or, $m = \frac{12}{2}$

$\therefore m = 6$

$\therefore$ The value of m is 6.

9. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then, show that $\vec{a}$ is perpendicular to $\vec{b}$.

Solution:

Here, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

Now, squaring both side;

$(|\vec{a} + \vec{b}|)^2 = (|\vec{a} - \vec{b}|)^2$

or, $|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$

or, $a^2 + 2\vec{a}\cdot\vec{b} + b^2 = a^2 - 2\vec{a}\cdot\vec{b} + b^2$

or, $2\vec{a}\cdot\vec{b} + 2\vec{a}\cdot\vec{b} = 0$

or, $4\vec{a}\cdot\vec{b} = 0$

or, $\vec{a}\cdot\vec{b} = 0$

$\therefore \vec{a}\cdot\vec{b} = 0$

Hence, $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

10. Find the angle between $\vec{a}$ and $\vec{b}$.

a. $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 4$, $|\vec{b}| = 3$ and $|\vec{c}| = \sqrt{37}$

Solution

Let angle between $\vec{a}$ and $\vec{b}$ be θ.

Also,

$\vec{a} + \vec{b} + \vec{c} = 0$

or $\vec{a} + \vec{b} = -\vec{c}$

S.b.s.

or $(\vec{a} + \vec{b})^2 = (-\vec{c})^2$

or $a^2 + 2ab + b^2 = c^2$

or, $(4)^2 + 2\vec{a}\vec{b} + (3)^2 = (\sqrt{37})^2$

or, $16 + 2\vec{a}\vec{b} + 9 = 37$

or $2\vec{a}\vec{b} = 37 - 9 - 16$

or $\vec{a}.\vec{b} = \frac{12}{2}$

$\therefore \vec{a}.\vec{b} = 6$

Now,

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

or $\cos\theta = \frac{6}{4.3}$

or $\cos\theta = \frac{6}{12}$

or $\cos\theta = \frac{1}{2}$

or $\cos\theta = \cos 60^\circ$

$\therefore \theta = 60^\circ$

b. $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=6$, $|\vec{b}|=7$ and $|\vec{c}|=\sqrt{106}$

Solution:

Let angle between $\vec{a}$ and $\vec{b}$ be θ

Now,

$|\vec{a}|=6$

$|\vec{b}|=7$

$|\vec{c}|=\sqrt{106}$

Also,

$\vec{a}+\vec{b}+\vec{c}=0$

or $\vec{a}+\vec{b}=-\vec{c}$

s.b.s

$(\vec{a}+\vec{b})^2=(-\vec{c})^2$

or $a^2+2\vec{a}\vec{b}+b^2=c^2$

or, $6^2+2\vec{a}\cdot\vec{b}+7^2=(\sqrt{106})^2$

or, $36+2\vec{a}\cdot\vec{b}+49=106$

or, $\vec{a}\cdot\vec{b}=\frac{106-49-36}{2}$

$\therefore \vec{a}\cdot\vec{b}=\frac{21}{2}$

Now, $\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$

or $\cos\theta=\frac{\frac{21}{2}}{6\times7}$

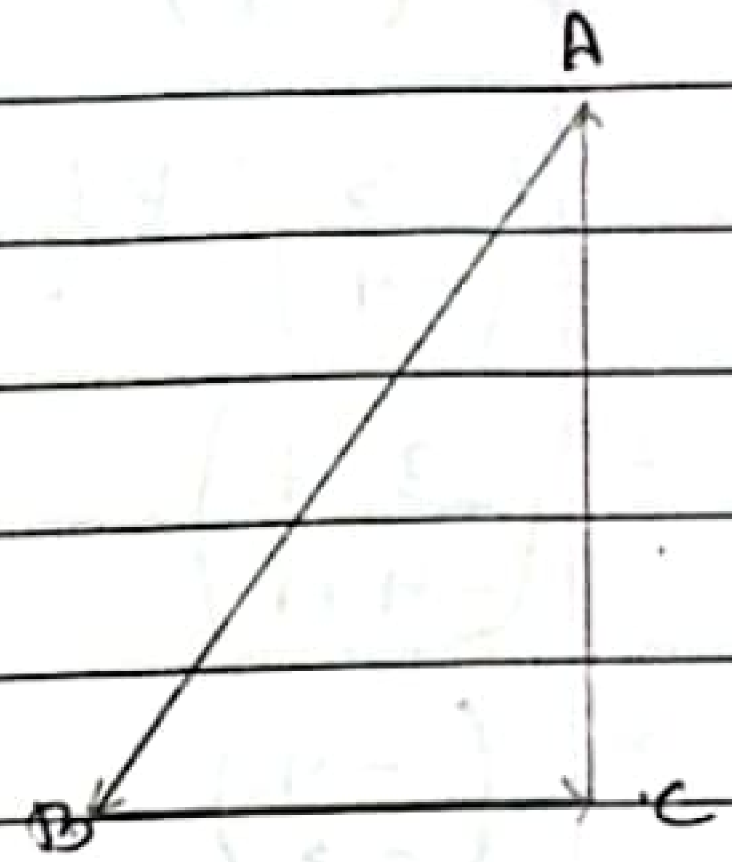
or $\cos\theta=\frac{21}{2}\times\frac{1}{42}$

or $\cos\theta=\frac{1}{4}$

or $\cos\theta=\cancel{\frac{1}{\sqrt{2}}}$ ~~co~~ $^1/_4$

$\therefore \theta=$ ~~co~~ $\cos^{-1}(^1/_4)$

11.a. In ΔABC, if $\vec{AB} = 5\vec{i} - 9\vec{j}$ and $\vec{BC} = 4\vec{i} + 14\vec{j}$. Show that the triangle is right angle triangle.



Solution:

$\vec{AB} = 5\vec{i} - 9\vec{j} = (5, -9)$

$\vec{BC} = 4\vec{i} + 14\vec{j} = (4, 14)$

Also,

$\vec{AC} = \vec{AB} + \vec{BC}$

$= 5\vec{i} - 9\vec{j} + 4\vec{i} + 14\vec{j}$

$= 9\vec{i} + 5\vec{j}$

$= (9, 5)$

Now,

angle between $\vec{AB}$ and $\vec{AC}$ is;

$$\cos\theta = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|}$$

or $$\cos\theta = \frac{(5\times 9) + (-9\times 5)}{\sqrt{5^2+(-9)^2} \cdot \sqrt{9^2+5^2}}$$

or $$\cos\theta = \frac{0}{\sqrt{25+81} \cdot \sqrt{81+25}}$$

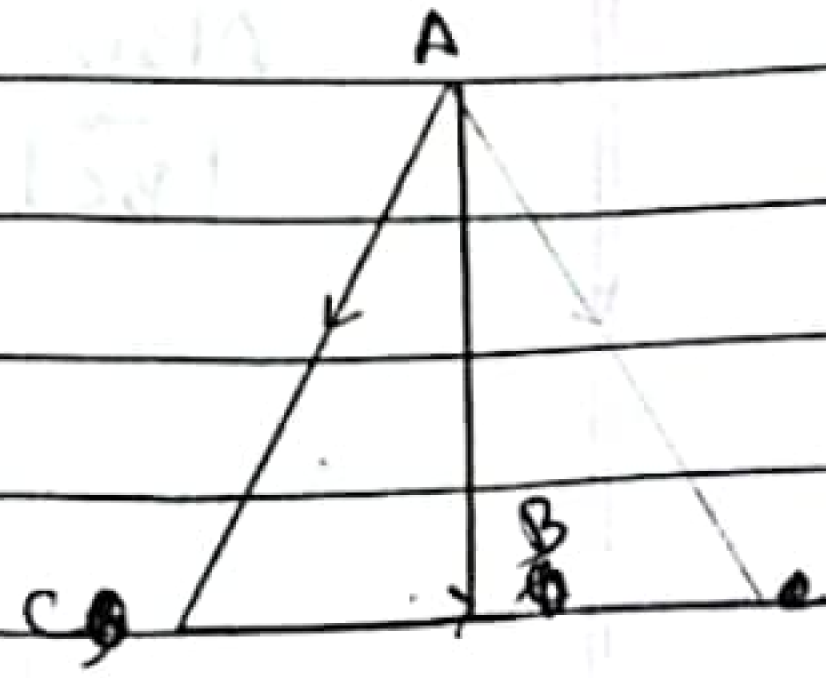
or $\cos\theta = 0$

or $\cos\theta = \cos 90°$

$\therefore \theta = 90°$

Thus, the triangle ABC is right angled triangle.

b. If $\vec{AB} = -3\vec{i} + 4\vec{j}$ and $\vec{AC} = -7\vec{i} + \vec{j}$, prove that ΔABC is an isosceles right angled triangle.



Solution:

$\vec{AB} = -3\vec{i} + 4\vec{j} = (-3, 4)$

$\vec{AC} = -7\vec{i} + j = (-7, 1)$

Now,

$\vec{BC} = \vec{BA} + \vec{AC}$ [$\because$ from Δ Law of vector addition]

$= -(AB) + \vec{AC}$

$= -\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} -7 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} -7 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3-7 \\ -4+1 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

Now, Let angle between $\vec{AB}$ and $\vec{BC}$ be θ

$\cos\theta = \frac{\vec{AB} \cdot \vec{BC}}{|\vec{AB}| \cdot |\vec{BC}|}$

or $\cos\theta = \frac{(-3 \times -4) + (4 \times -3)}{\sqrt{(-3)^2 + 4^2} \cdot \sqrt{(-4)^2 + (-3)^2}}$

or $\cos\theta = \frac{12 - 12}{\sqrt{9+16} \cdot \sqrt{16+9}}$

or $\cos\theta = 0$

or $\cos\theta = \cos 90^\circ$

$\therefore \theta = 90^\circ$

Since, $\vec{AB} \cdot \vec{BC} = 0$, $\measuredangle ABC = 90^\circ$

Again,

$|\vec{AB}| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

Also,

$|\vec{BC}| = \sqrt{(-4)^2 + (-3)^2}$

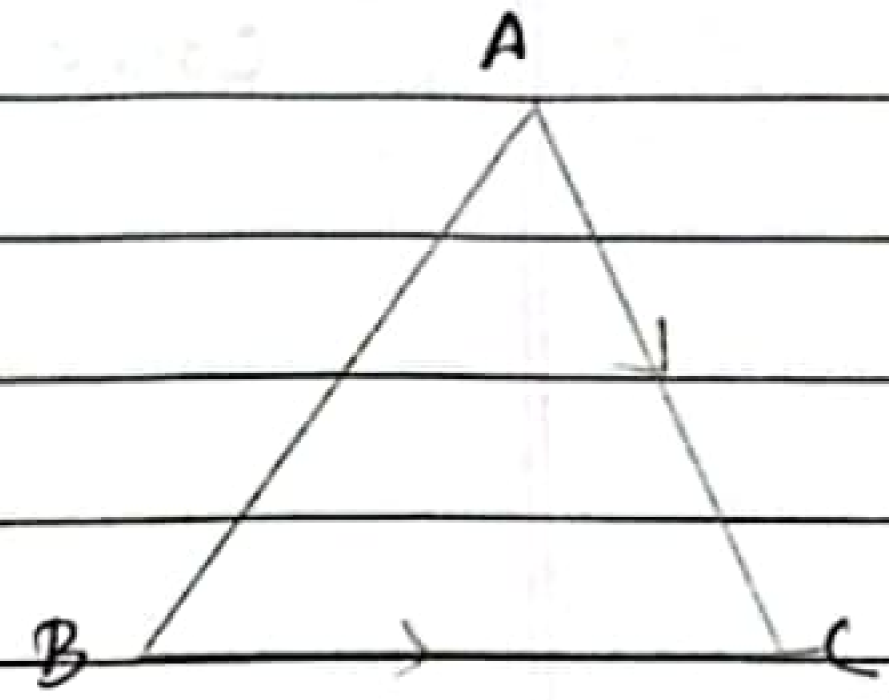
$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

$\therefore |\vec{AB}| = |\vec{BC}|$ and $\measuredangle ABC = 90^\circ$, so ΔABC is an isosceles right angled triangle.

12. In ΔABC, if $\vec{AC} = 2\vec{i} - 2\vec{j}$ and $\vec{BC} = 4\vec{i}$ then show that $\angle BAC = 2\angle ABC$



Solution:

$\vec{AC} = 2\vec{i} - 2\vec{j} = (2, -2)$

$\vec{BC} = 4\vec{i} = (4, 0)$

$\vec{AB} = \vec{AC} + \vec{CB}$

$= \vec{AC} + \cancel{(-AB)}\ (-\vec{BC})$

$= \cancel{\vec{AC}} \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \left\{-\begin{pmatrix} 4 \\ 0 \end{pmatrix}\right\}$

$= \begin{pmatrix} 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

Again,

$\vec{BA} = \vec{BC} + \vec{CA}$

$= \vec{BC} - (\vec{AC})$

$= \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Now,

$\cos \angle BAC = \dfrac{\vec{AC} \cdot \vec{AB}}{|\vec{AC}| \cdot |\vec{AB}|}$

or $\cos \angle BAC = \dfrac{(2 \times -2) + (-2 \times -2)}{|\vec{AC}| \cdot |\vec{AB}|}$

or $\cos \angle BAC = \dfrac{0}{|\vec{AC}| \cdot |\vec{AB}|}$

$= 0$

or $\cos \angle BAC = \cos 90^\circ$

$\therefore \angle BAC = 90^\circ$

Again,

$$\cos\angle ABC = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\cdot|\vec{BC}|}$$

$$= \frac{2.4 + 2.0}{\sqrt{2^2+2^2}\cdot\sqrt{4^2+0}}$$

$$= \frac{\cancel{8}\,2}{\sqrt{8}\cdot\cancel{4}}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{2}}$$

or, $\cos\angle ABC = \frac{1}{\sqrt{2}}$

~~$\therefore \angle ABC = 45°$~~

or, $\cos\angle ABC = \cos 45°$

$\therefore \angle ABC = 45°$

Hence, $\angle BAC = 2\,ABC$

Exercise - 9.2

1. a. The position vectors of the points A and B are $5\vec{i} + 2\vec{j}$ and $3\vec{i} + 6\vec{j}$ respectively. Find the position vector of the mid-point M of the line AB.

Solution:

Given,

$\vec{OA} = 5\vec{i} + 2\vec{j}$

$\vec{OB} = 3\vec{i} + 6\vec{j}$

Now,

Let M be the mid-point, then

$$\vec{OM} = \frac{\vec{OA} + \vec{OB}}{2}$$

$$= \frac{5\vec{i} + 2\vec{j} + 3\vec{i} + 6\vec{j}}{2}$$

$$= 4\vec{i} + 4\vec{j}$$

b. Find the position vector of the mid-point M on the line segment AB if the coordinate of points A and B are (3,5) and (9,-11) respectively.

Solution:

Position vector of A = $\vec{OA} = 3\vec{i} + 5\vec{j}$

Position vector of B = $\vec{OB} = 9\vec{i} - 11\vec{j}$

Also,

Let M be the mid-point, then

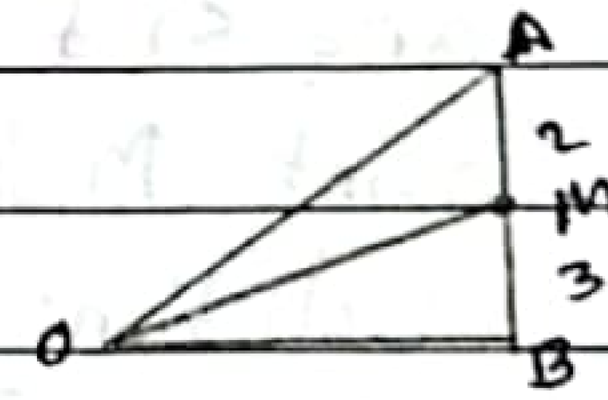
$$\vec{OM} = \frac{\vec{OA} + \vec{OB}}{2}$$

$$= \frac{3\vec{i} + 5\vec{j} + 9\vec{i} - 11\vec{j}}{2}$$

$$= \frac{12\vec{i} - 6\vec{j}}{2}$$

$$= 6\vec{i} - 3\vec{j}$$

2.a. The position vectors of the points A and B are $2\vec{i} - 3\vec{j}$ and $3\vec{i} + 2\vec{j}$ respectively. Find the position vector of the point M which divides the line segment AB internally in the ratio 2:3.



Solution:

Position vector of A, $\vec{OA} = 2\vec{i} - 3\vec{j}$

Position vector of B, $\vec{OB} = 3\vec{i} + 2\vec{j}$

If M divides the line segment AB in the ratio 2:3 then

$$\vec{OM} = \frac{3\cdot\vec{OA} + 2\vec{OB}}{3+2}$$

$$= \frac{3(2\vec{i} - 3\vec{j}) + 2(3\vec{i} + 2\vec{j})}{5}$$

$$= \frac{6\vec{i} - 9\vec{j} + 6\vec{i} + 4\vec{j}}{5}$$

$$= \frac{12\vec{i}}{5} - \vec{j}$$

b. Find the position vector of the point M which divides the line segment joining A(5,2) and (3,6) internally in the ratio 2:3.

Solution

Position vector of A, $\vec{OA} = (5\vec{i} + 2\vec{j})$

Position vector of B, $\vec{OB} = (3\vec{i} + 6\vec{j})$

Let M be the ~~mid~~ point dividing AB in the ratio 2:3.

We know,

$$\vec{OM} = \frac{3.\vec{OA} + 2.\vec{OB}}{2+3}$$

$$= \{3(5\vec{i} + 2\vec{j}) + 2.(3\vec{i} + 6\vec{j})\}/2+3$$

$$= \frac{15\vec{i} + 6\vec{j} + 6\vec{i} + 12\vec{j}}{2+3}$$

$$= \frac{21\vec{i} + 18\vec{j}}{5}$$

$$= \frac{21}{5}\vec{i} + \frac{18}{5}\vec{j}$$

3.a. The position vector of the points A and B are $2\vec{i} - 3\vec{j}$ and $5\vec{i} + 4\vec{j}$ respectively. Find the position vector of the point M which divides AB externally in the ratio 3:2

Solution:

Given, $\vec{OA} = (2\vec{i} - 3\vec{j})$

$\vec{OB} = (5\vec{i} + 4\vec{j})$

Let M be the point dividing AB externally in the ratio 3:2

We know,

$$\vec{OM} = \frac{2.\vec{OA} - 3(\vec{OB})}{3-2}$$

$$= \frac{2(2\vec{i} - 3\vec{j}) - 3(5\vec{i} + 4\vec{j})}{1}$$

$$= \frac{\cancel{4\vec{i} - 6\vec{j} - 15\vec{i} - 12\vec{j}}}{\cancel{1}}$$

$$\vec{OM} = \frac{\qquad}{3-2}$$

$$= \frac{\qquad}{1}$$

$$= \frac{\qquad}{}$$

$=$

---

6. Find the position vector of the point M which divides the line segment joining A(-4,8) and B(3,7) externally in the ratio 4:3.

Solution:

Given, $\vec{OA} = (-4\vec{i} + 8\vec{j})$

$\vec{OB} = (3\vec{i} + 7\vec{j})$

Let M be the mid-point dividing AB in the ratio 4:3 externally.

We know, $\vec{OM} = \dfrac{4\vec{OB} - 3\vec{OA}}{4-3}$

$= \dfrac{4(3\vec{i} + 7\vec{j}) - 3(-4\vec{i} + 8\vec{j})}{1}$

$= 12\vec{i} + 28\vec{j} + 12\vec{i} - 24\vec{j}$

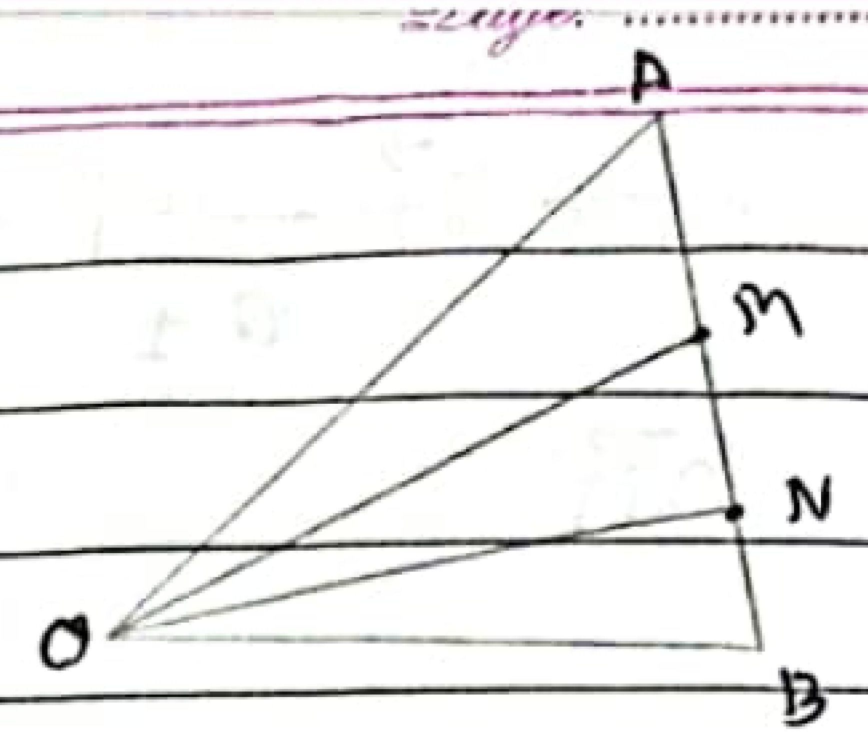
$= 24\vec{i} + 4\vec{j}$

4.a. Find the position vector of the point M and N which trisect the line joining the points A(2,-2) and B(-1,4).

Solution:

$A = (2,-2)$

$\vec{OA} = (2\vec{i} - 2\vec{j}$



$B(-1, 4)$

$\vec{OB} = (-1\vec{i} + 4\vec{j})$

Here,

M and N are the point of trisection.

As, M divides AB in the ratio 1:2, then

$$\vec{OM} = \frac{2 \cdot \vec{OA} + 1 \cdot \vec{OB}}{1+2}$$

$$= \frac{2(2\vec{i} - 2\vec{j}) + (-1\vec{i} + 4\vec{j})}{3}$$

$$= \frac{4\vec{i} - \cancel{4\vec{j}} - 1\vec{i} + \cancel{4\vec{j}}}{3}$$

$$= \frac{3\vec{i}}{3}$$

$$= 1\vec{i} + 0\vec{j}$$

Again,

As, N divides AB in the ratio 2:1, then

$$\vec{ON} = \frac{1 \cdot \vec{OA} + 2\vec{OB}}{2+1}$$

$$= \frac{2\vec{i} - 2\vec{j} + 2(-1\vec{i} + 4\vec{j})}{3}$$

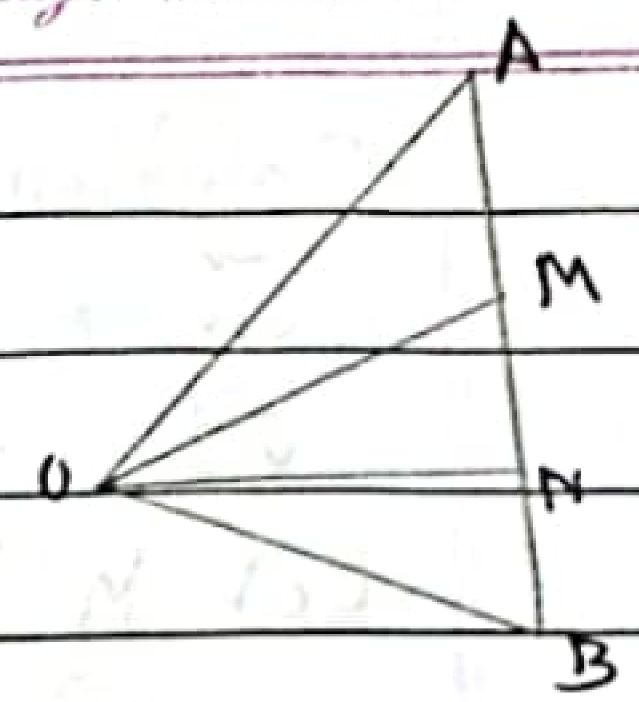
$$= \frac{\cancel{2\vec{i}} - 2\vec{j} - \cancel{2\vec{i}} + 8\vec{j}}{3}$$

$$= \frac{6\vec{j}}{3}$$

$$= 2\vec{j}$$

---

b. Position vector of A and B are $6\vec{i} - 9\vec{j}$ and $-6\vec{i} - 9\vec{j}$ respectively. Find the position vector of two points M and N which trisect the line segment AB.



Solution:

$\vec{OA} = 6\vec{i} - 9\vec{j}$

$\vec{OB} = -6\vec{i} - 9\vec{j}$

Also,

~~Here~~ M ~~are~~ and N are the point of trisection.

As, M divides AB in the ratio, 1:2 then

$$\vec{OM} = \frac{2\vec{OA} + 1\vec{OB}}{1+2}$$

$$= \frac{2(6\vec{i} - 9\vec{j}) + (-6\vec{i} - 9\vec{j})}{3}$$

$$= \frac{12\vec{i} - 18\vec{j} - 6\vec{i} - 9\vec{j}}{3}$$

$$= \frac{6\vec{i} - 27\vec{j}}{3}$$

$$= 2\vec{i} - 9\vec{j}$$

Again,

N divides AB in the ratio, ~~10~~ 2:1 then

$$\vec{ON} = \frac{1 \cdot \vec{OA} + 2 \cdot \vec{OB}}{1+2}$$

$$= \frac{6\vec{i} - 9\vec{j} + 2(-6\vec{i} - 9\vec{j})}{3}$$

$$= \frac{6\vec{i} - 9\vec{j} - 12\vec{i} - 18\vec{j}}{3}$$

$$= \frac{-6\vec{i} - 27\vec{j}}{3}$$

$$= -2\vec{i} - 9\vec{j}$$

6a. Position vector of A and B are $2\vec{a} - 3\vec{b}$ and $-\vec{a} - 2\vec{b}$ respectively. find the position vector of the mid-point of the line segment AB.

Solution:

$\vec{OA} = 2\vec{a} - 3\vec{b}$

$\vec{OB} = -\vec{a} - 2\vec{b}$

Let M be the mid-point of AB, then

$$\vec{OM} = \frac{\vec{OA} + \vec{OB}}{2}$$

$$= \frac{2\vec{a} - 3\vec{b} - \vec{a} - 2\vec{b}}{2}$$

$$= \frac{\vec{a} - 5\vec{b}}{2}$$

$$= \frac{\vec{a}}{2} - \frac{5\vec{b}}{2}$$

---

b. Position vector of A and B are $2\vec{a} - 3\vec{b}$ and $5\vec{a} - 4\vec{b}$ respectively. Find the position vector of the point that divides AB internally in the ratio 3:2

Solution:

$\vec{OA} = 2\vec{a} - 3\vec{b}$

$\vec{OB} = 5\vec{a} - 4\vec{b}$

As, Let M be the point that divides AB in the ratio 3:2 internally, then,

$$\vec{OM} = \frac{2\vec{OA} + 3\vec{OB}}{3+2}$$

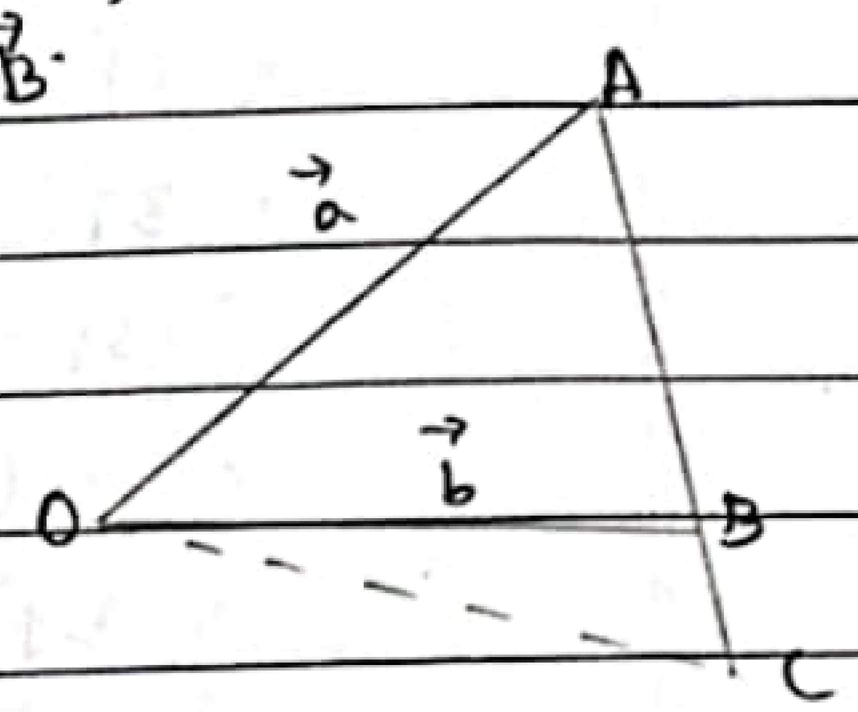
$$= \frac{2(2\vec{a} - 3\vec{b}) + 3(5\vec{a} - 4\vec{b})}{5}$$

$$= \frac{4\vec{a} - 6\vec{b} + 15\vec{a} - 12\vec{b}}{5}$$

$$= \frac{19\vec{a} - 18\vec{b}}{5}$$

$$= \frac{19\vec{a}}{5} - \frac{18\vec{b}}{5}$$

7.a. $\vec{a}$ and $\vec{b}$ are the position vector of the point A and B respectively. Find the position vector of a point C in AB produced externally such that $\vec{AC} = 3\vec{AB}$.



Solution:

Given, $\vec{OA} = \vec{a}$

$\vec{OB} = \vec{b}$

By question:

$\vec{AC} = 3\vec{AB}$

$\vec{AO} + \vec{OC} = 3(\vec{AO} + \vec{OB})$

or $-\vec{OA} + \vec{OC} = 3\cancel{(-\vec{a}+\vec{b})}(-\vec{OA} + \vec{OB})$

or $-\vec{a} + \vec{OC} = 3\{(-a) + \vec{b}\}$

or $-\vec{a} + \vec{OC} - 3\vec{a} + 3\vec{b}$

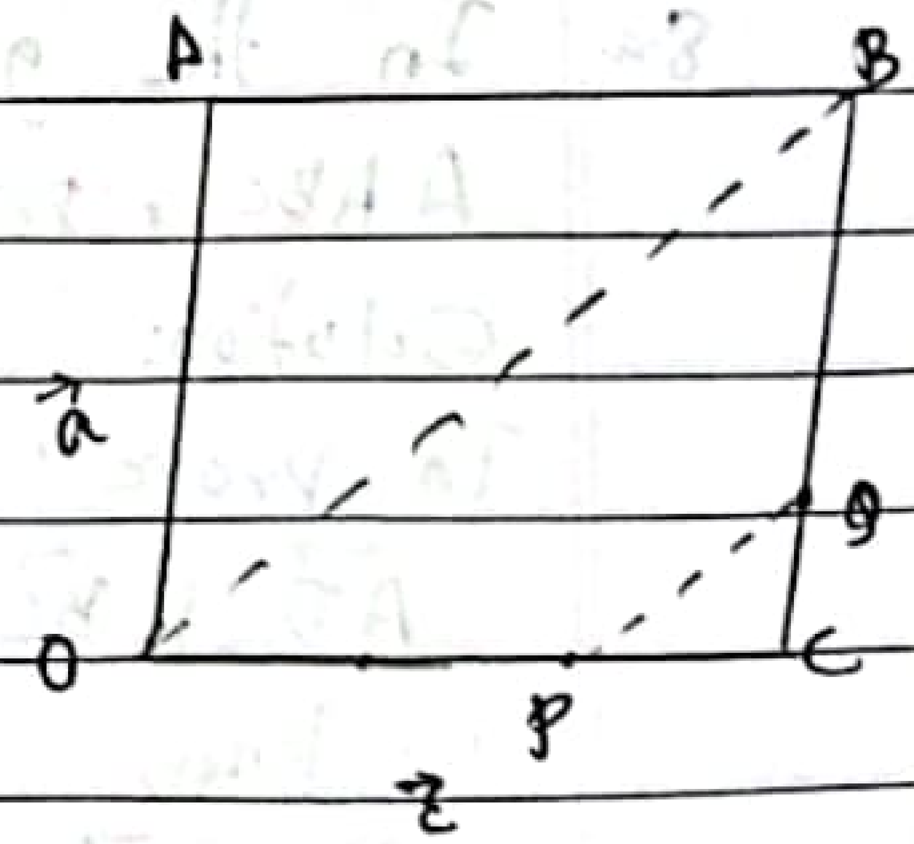
or ~~$\vec{a} + 3\vec{a}$~~ $\vec{OC} = \vec{a} - 3\vec{a} + 3\vec{b}$

or $\vec{OC} = -2\vec{a} + 3\vec{b}$

$\therefore \vec{OC} = -2\vec{a} + 3\vec{b}$

---

b. OABC is a parallelogram. P and Q divides OC and BC in the ratio, CP : PO = CQ : QB = 1 : 3. If $\vec{OA} = \vec{a}$ and $\vec{OC} = \vec{c}$ find the vector $\vec{PQ}$ and show that PQ ∥ ~~QB~~ OB



Solution:

$\vec{OA} = \vec{a}$

$\vec{OC} = \vec{c}$

CP : PO = 1 : 3

or, $\frac{CP}{PO} = \frac{1}{3}$

$\therefore 3CP = PO \Rightarrow OP = 3PC$

Also,

CQ : QB = 1 : 3

or $\frac{CQ}{QB} = \frac{1}{3}$

$\therefore 3CQ = QB$

Now,

$\vec{OB} = \vec{OC} + \vec{CB}$

$= \vec{OP} + \vec{PC} + \vec{CQ} + \vec{QB}$

$= 3\vec{PC} + \vec{PC} + \vec{CQ} + 3\vec{CQ}$

$= 3\vec{PC} + 3\vec{CQ} + \vec{PC} + \vec{CQ}$

$= 3(\vec{PC} + \vec{CQ}) + \vec{PQ}$

$= 3\vec{PQ} + \vec{PQ}$

$= 4\vec{PQ}$

$\therefore \vec{OB} = 4\vec{PQ}$

$\therefore \vec{OB} \parallel \vec{PQ}$

Again,

$\vec{OB} = \vec{OA} + \vec{AB}$

or $\vec{OB} = \vec{a} + \vec{b}$

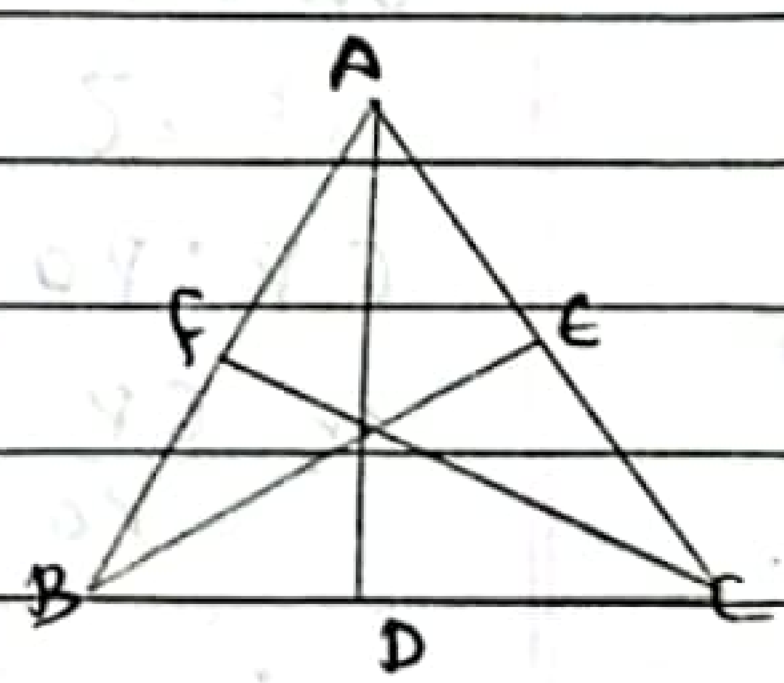
or $4\vec{PQ} = \vec{a} + \vec{b}$ (from above value)

or $\vec{PQ} = \frac{\vec{a} + \vec{b}}{4}$

$\therefore \vec{PQ} = \frac{\vec{a}}{4} + \frac{\vec{b}}{4}$

8.a. In the given figure AD, BE and CF are the medians of a ΔABC, prove that $\vec{AD} + \vec{BE} + \vec{CF} = 0$

Solution:



To prove:

$\vec{AD} + \vec{BE} + \vec{CF} = 0$

we know,

$\vec{AD} = \vec{AB} + \vec{BD}$

$= \vec{AB} + \frac{1}{2}\vec{BC}$

$\vec{BE} = \vec{BC} + \vec{CE}$

$= \vec{BC} + \frac{1}{2}\vec{CA}$

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$\vec{CF} = \vec{CA} + \vec{AF}$
$= \vec{CA} + \frac{1}{2}\vec{AB}$

Now,
$\vec{AB} + \vec{BE} + \vec{CF} = \vec{AB} + \frac{1}{2}\vec{BC} + \vec{BC} + \frac{1}{2}\vec{CA} + \vec{CA} + \frac{1}{2}\vec{AB}$

$= \vec{AB} + \vec{BC} + \vec{CA} + \frac{1}{2}\vec{BC} + \frac{1}{2}\vec{CA} + \frac{1}{2}\vec{AB}$

$= 0 + \frac{1}{2}(\vec{BC} + \vec{CA} + \vec{AB})$

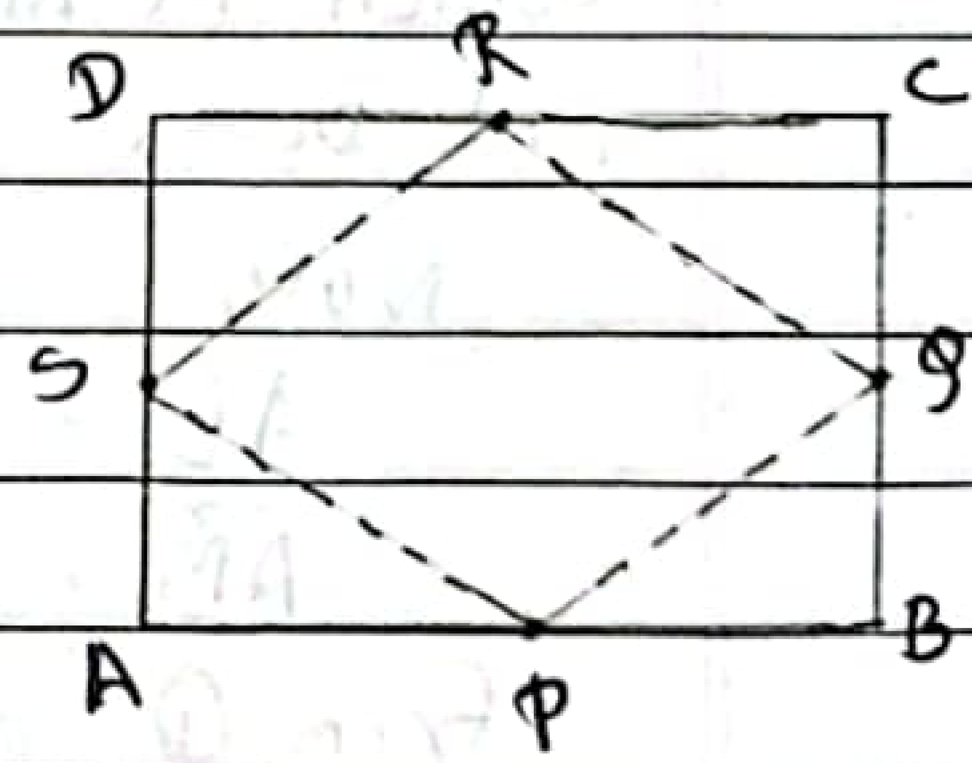
$= 0 + \frac{1}{2} \times 0$

$= 0$

RHS proved.

b. P, Q, R, and S are the mid-point of the sides AB, AD, CD and BC respectively of the quadrilateral ABCD. Prove that PR and QS bisect each other.

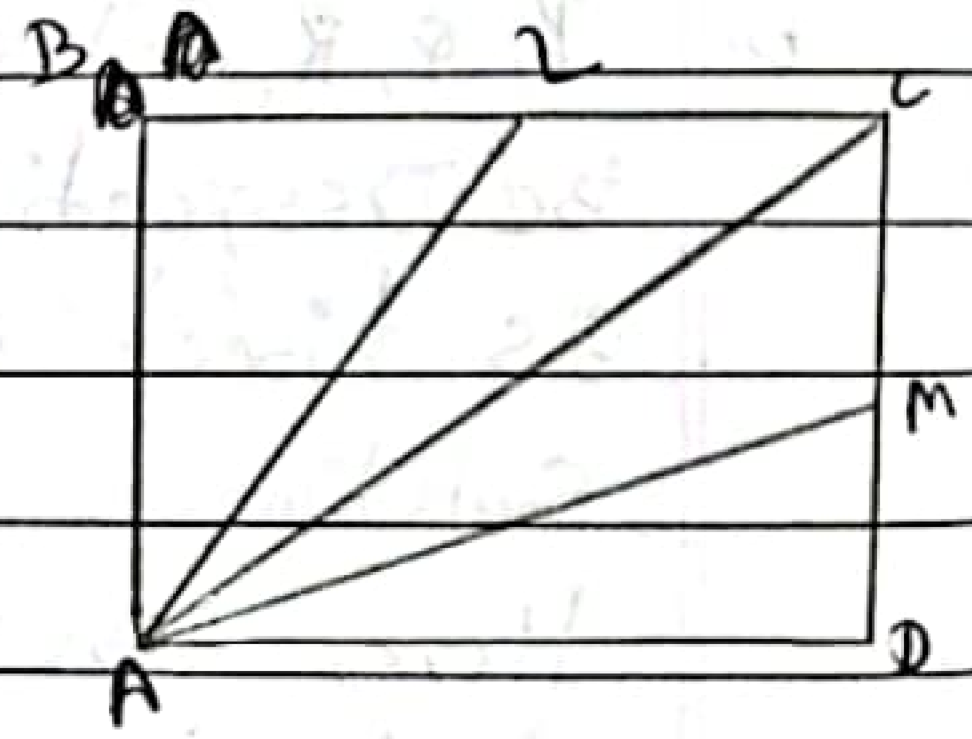
Solution:
Here, ABCD is a quadrilateral in which P, Q, R and S are the mid. point of



c. In the given figure ABCD is a parallelogram. ~~Let~~ L and M are the mid-point of sides BC and CD respectively. Prove that : $\vec{AL} + \vec{AM} = \frac{3}{2}\vec{AC}$

Solution:

Here, ABCD is a parallelogram in which M and L are the mid point of CD and BC.



Now,

$$\vec{AL} = \vec{AC} + \vec{CL} \quad \text{—— (i)}$$

$$\vec{AM} = \vec{AC} + \vec{CM} \quad \text{—— (ii)}$$

From (i) and (ii)

$$\vec{AL} + \vec{AM} = \vec{AC} + \vec{CL} + \vec{AC} + \vec{CM}$$

$$= 2\vec{AC} + \frac{1}{2}\vec{CB} + \frac{1}{2}\vec{CD}$$

$$= 2\vec{AC} - \frac{1}{2}\vec{BC} - \frac{1}{2}\vec{DC}$$

$$= 2\vec{AC} - \frac{1}{2}(\vec{BC} + \vec{DC})$$

$$= 2\vec{AC} - \frac{1}{2}(\vec{AB} + \vec{DC})$$

$= 2\vec{AC} - \frac{1}{2}\vec{AC}$

$= \frac{4\vec{AC} - \vec{AC}}{2}$

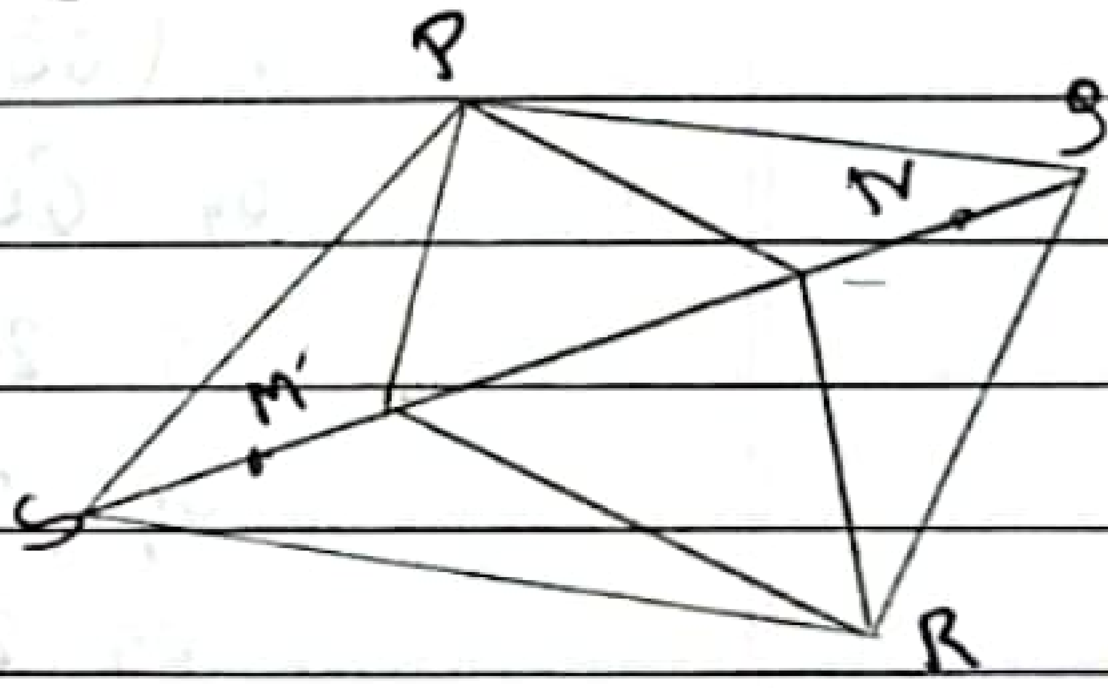
$= \frac{3}{2}\vec{AC}$

$\therefore \vec{AL} + \vec{AM} = \frac{3}{2}\vec{AC}$

Proved//

d. In the given figures PQRS is a parallelogram. M and N are two points on the diagonal SQ. If SM = NQ, prove by vector method that PMRN is a parallelogram.

Solution:



Here, PQRS is a parallelogram where M and N are two points on diagonal SQ.

By question:

SM = NQ

we know,

$\vec{PM} = \vec{PS} + \vec{SM}$

$= \vec{PS} + \vec{NQ}$

$= \vec{QR} + \vec{NQ}$

$= \vec{NQ} + \vec{QR}$

$= \vec{NR}$

similarily,

$\vec{PN} = \vec{PQ} + \vec{QN}$

$= \vec{SR} + \vec{MS}$

$= \vec{MS} + \vec{SR}$

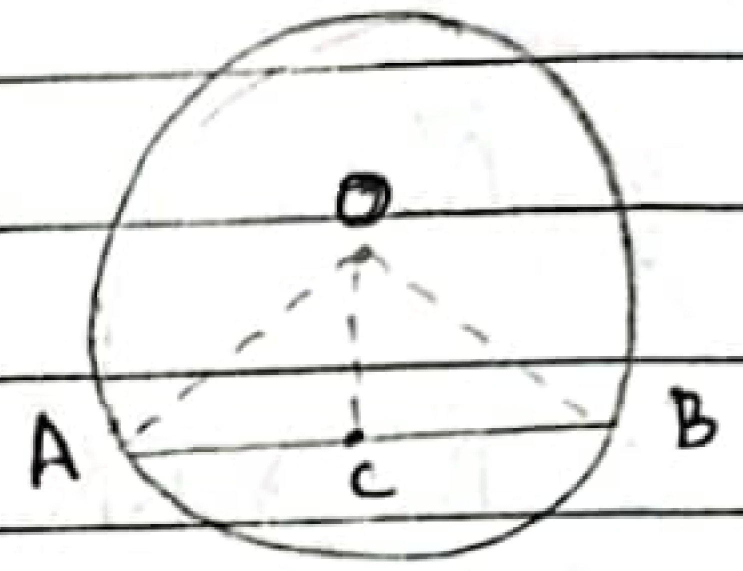
$= \vec{MR}$

Hence, $\vec{PM} = \vec{NR}$ and $\vec{PN} = \vec{MR}$

$\therefore$ PMRN is a parallelogram.

e. If a line is drawn from the centre of a circle to the mid-point of a chord, prove by vector method that the line is perpendicular to the chord.

Solution:



O is the centre and AB be the chord where C is the mid-point of AB.

Now,

$\vec{OA} = \cancel{\vec{AC} + \vec{CO}}\ \vec{OC} + \vec{CA}$ —— (i)

$\vec{OB} = \vec{OC} + \vec{CB}$ —— (ii)

from (i) and (ii)

$(\vec{OA})^2 = (\vec{OB})^2$

or, $(\vec{OC} + \vec{CA})^2 = (\vec{OC} + \vec{CB})^2$

or, $\cancel{\vec{OC}^2} + 2\vec{OC}\cdot\vec{CA} + \vec{CA}^2 = \cancel{\vec{OC}^2} + 2\vec{OC}\cdot\vec{CB} + \vec{CB}^2$

or, $2\vec{OC}\cdot\vec{CA} + \vec{CA}^2 = 2\vec{OC}\cdot\vec{CB} + \vec{CB}^2$

or, $2\vec{OC}\cdot\vec{CA} + \vec{CA}^2 = 2\vec{OC}\cdot\vec{AC} + \vec{AC}^2$

or, $2\vec{OC}\cdot\vec{CA} + \vec{CA}^2 = 2\vec{OC}(-\vec{CA}) + (-\vec{CA})^2$

or, $2\vec{OC}\cdot\vec{CA} + \vec{CA}^2 = -2\vec{OC}\cdot\vec{CA} + \vec{CA}^2$

or, $2\vec{OC}\cdot\vec{CA} + 2\vec{OC}\cdot\vec{CA} = 0$

or, $4\vec{OC}\cdot\vec{CA} = 0$

or, $\vec{OC}\cdot\vec{CA} = 0$

Hence, $\vec{OC} \perp \vec{CA}$.

f. Prove by vector method that the median of a trapezium is parallel to the parallel side and is half of the sum of parallel sides.

ABCD is a parallelogram and O is the origin. If $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{OC} = \vec{c}$. find $\vec{OD}$

Here,

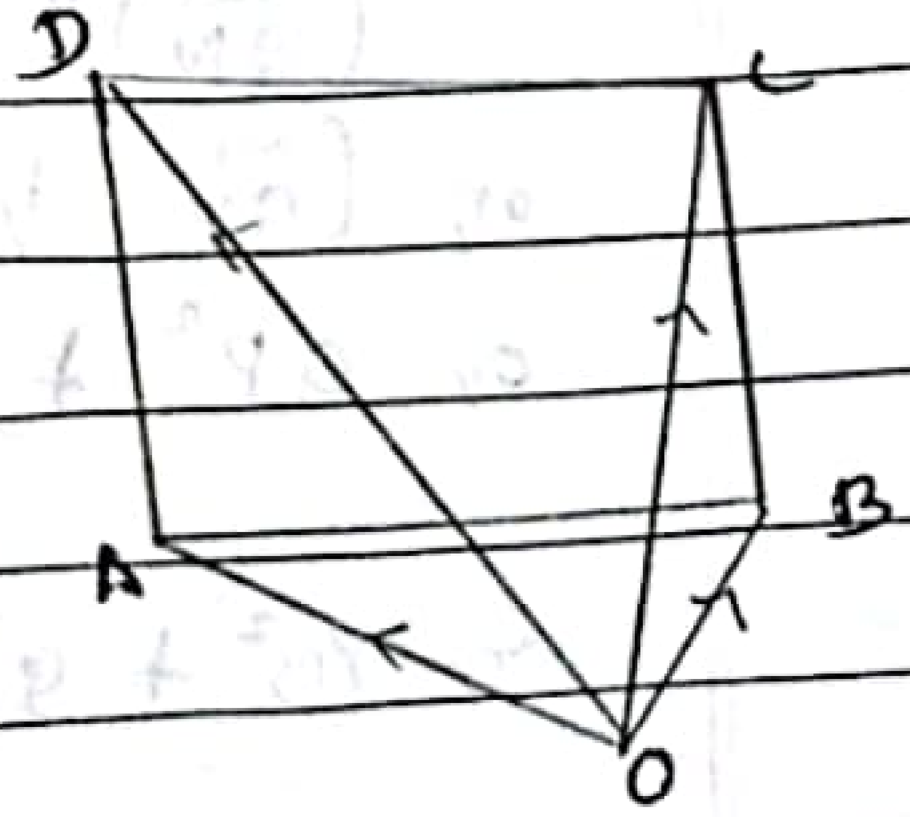
ABCD is a parallelogram.

$\vec{OA} = \vec{a}$

$\vec{OB} = \vec{b}$

$\vec{OC} = \vec{c}$

$\vec{OD} = ?$



we know,

$$\vec{AD} = \vec{AO} + \vec{OD}$$

or $\vec{BC} = -\vec{OA} + \vec{OD}$

or $\vec{BO} + \vec{OC} = -\vec{OA} + \vec{OD}$

or $-\vec{OB} + \vec{c} = -\vec{a} + \vec{OD}$

or $-\vec{b} + \vec{c} + \vec{a} = \vec{OD}$

$\therefore \vec{OD} = \vec{a} + \vec{c} - \vec{b}$

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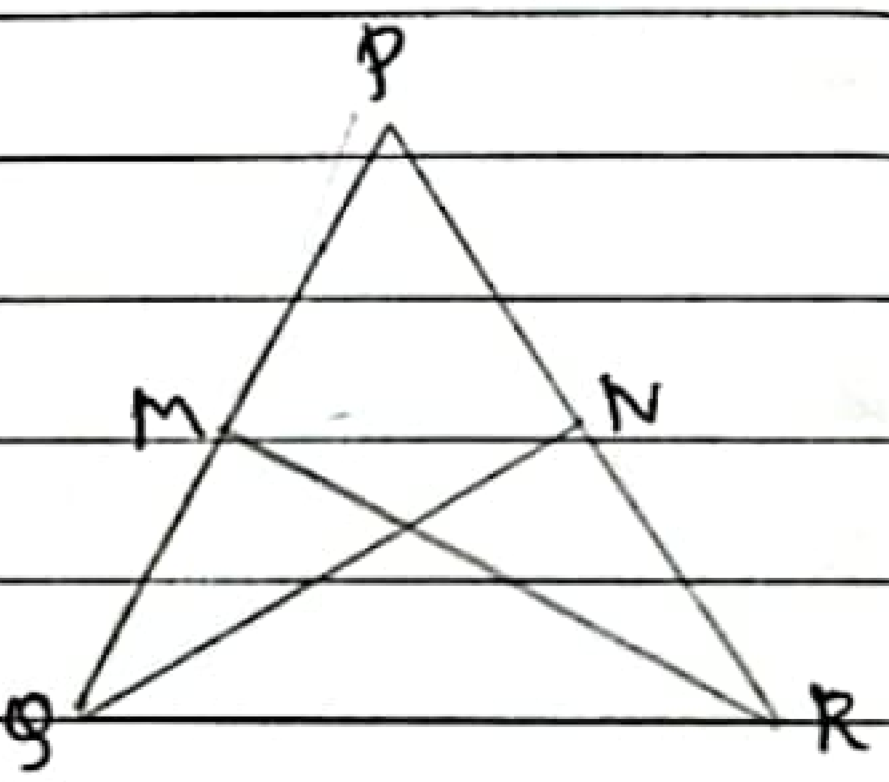
10. In the adjoining figure, RM and QN are the medians of the ΔPQR. If RM = QN, prove by vector method that ΔPQR is an isosceles.

Solution:



PQR is a triangle where QN and RM are medians.

Also,

RM = QN

Now,

$$\vec{QN} = \vec{QP} + \vec{PN}$$
$$= \vec{QP} + \frac{1}{2}\vec{PR} \quad \text{—(i)}$$

Also,

$$\vec{RM} = \vec{RP} + \vec{PM}$$
$$= \vec{RP} + \frac{1}{2}\vec{PQ} \quad \text{—(ii)}$$

From (i) and (ii)

$$(\vec{QN})^2 = (\vec{RM})^2$$

or, $(\vec{QP} + \frac{1}{2}\vec{PR})^2 = (\vec{RP} + \frac{1}{2}\vec{PQ})^2$

or $QP^2 + 2\vec{QP}\cdot\frac{1}{2}\vec{PR} + \frac{1}{4}PR^2 = RP^2 + 2\vec{RP}\cdot\frac{1}{2}\vec{PQ} + \frac{1}{4}PQ^2$

or, $PQ^2 + \vec{QP}\cdot\vec{PR} + \frac{1}{4}PR^2 = RP^2 + \vec{RP}\cdot\vec{PQ} + \frac{2}{4}PQ^2$

or, $PQ^2 - \frac{1}{4}PQ^2 + \vec{QP}\cdot\vec{PR} = PR^2 - \frac{1}{4}PR^2 + (-\vec{PR})\cdot(-\vec{QP})$

or, $\frac{3PQ^2}{4} + \vec{QP}\cdot\vec{PR} = \frac{3PR^2}{4} + \vec{PR}\cdot\vec{QP}$

or, $\frac{3PQ^2}{4} = \frac{3PR^2}{4}$

or, $PQ^2 = PR^2$

$\therefore PQ = PR$

So, ~~they are~~ the sides are equal. It means it is an isosceles triangle.

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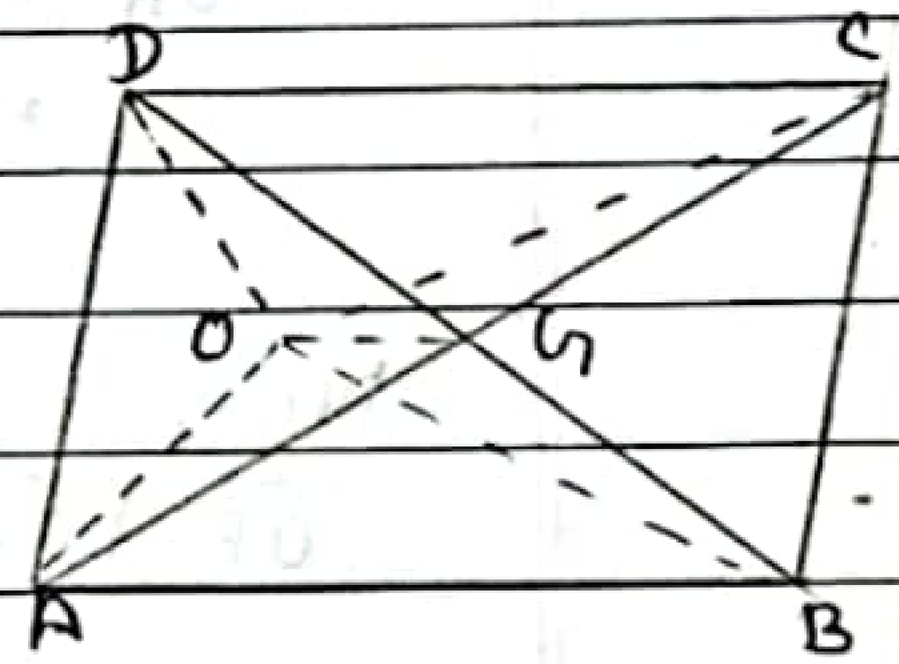
11. In the adjoining figure ABCD is a parallelogram and G is the point of intersection of its diagonals. If O is any point prove that:
$\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$



Solution:
In the given diagram, ABCD is a parallelogram in which G is the point of intersection of diagonals. Then,

$\vec{OA} = \vec{OG} + \vec{GA}$

$\vec{OB} = \vec{OG} + \vec{GB}$

$\vec{OC} = \vec{OG} + \vec{GC}$

$\vec{OD} = \vec{OG} + \vec{GD}$

Now,

$\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG} + \vec{GA} + \vec{GB} + \vec{GC} + \vec{GD}$

$= 4\vec{OG} + \vec{GA} + \vec{GB} - \vec{CG} - \vec{DG}$

$= 4\vec{OG} + \vec{GA} + \vec{GB} - \vec{GA} - \vec{GB}$

$= 4\vec{OG}$

$\therefore \vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$

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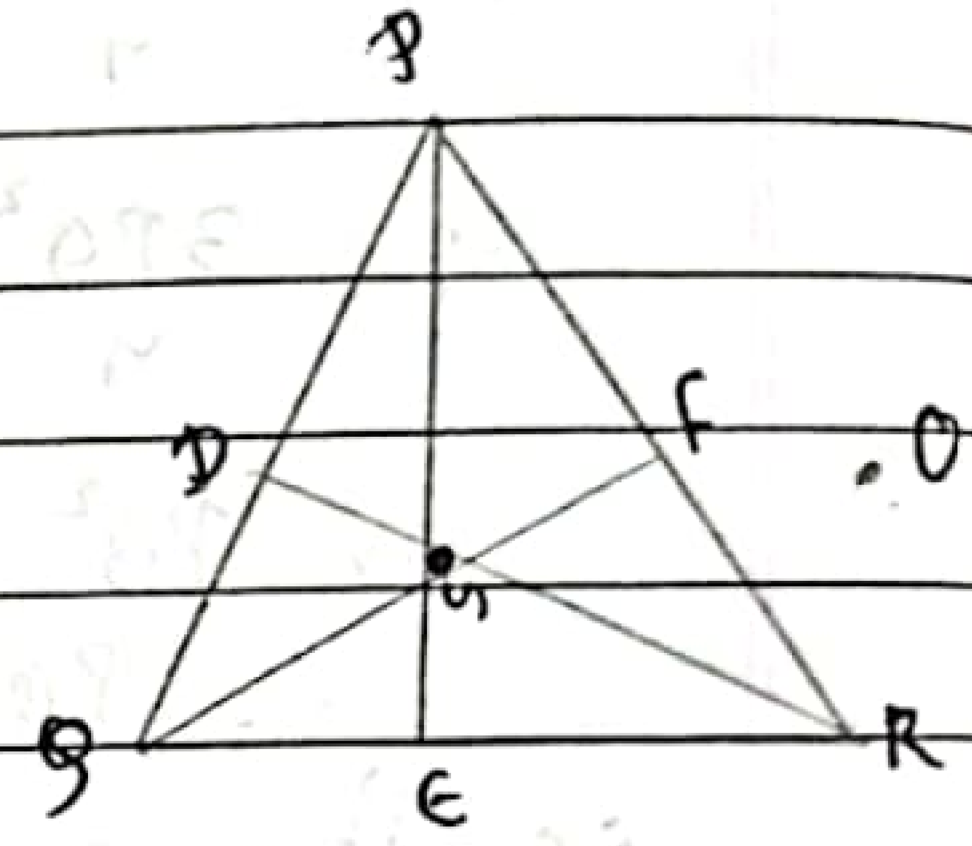
12. In the adjoining figure D, E and F are the mid-point of the side ΔPQR and O is any point, prove that:
$\vec{OP} + \vec{OQ} + \vec{OR} = \vec{OD} + \vec{OE} + \vec{OF}$



Solution:
In the given figure, PQR is a triangle in which PE, QF and RD are medians.

Then,

$\vec{OP} = \vec{OD} + \vec{DP}$

$= \vec{OD} + \frac{1}{2} \vec{QP}$

$\vec{OQ} = \vec{OE} + \vec{EQ}$

$= \vec{OE} + \frac{1}{2} \vec{RQ}$

$\vec{OR} = \vec{OF} + \vec{FR}$

$= \vec{OF} + \frac{1}{2} \vec{PR}$

Now,

$\vec{OP} + \vec{OQ} + \vec{OR} = \vec{OD} + \frac{1}{2}\vec{QP} + \vec{OE} + \frac{1}{2}\vec{RQ} + \vec{OF} + \frac{1}{2}\vec{PR}$

$= \vec{OD} + \vec{OE} + \vec{OF} + \frac{1}{2}(\vec{QP} + \vec{PR} + \vec{RQ})$

$= \vec{OD} + \vec{OE} + \vec{OF} + \frac{1}{2} \times 0$

$= \vec{OD} + \vec{OE} + \vec{OF}$

Proved.