The Derivatives

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Derivative

The derivative or defferential coefficient of a function $y = f(x)$ is defined and denoted by;

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

where,

Δx is the sufficiently small increment in x and Δy is the corresponding increment in y.

1. Derivative of x^n

Let $y = x^n$

Let Δx be the small increment in x and Δy be the corresponding increment in y.

Then,

$$y + \Delta y = (x + \Delta x)^n$$

$$\text{or, } \Delta y = (x + \Delta x)^n - y$$

$$\text{or, } \Delta y = (x + \Delta x)^n - x^n$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{(x + \Delta x)^n - x^n}{\Delta x}$$

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^n - x^n}{\Delta x}$$

$$\text{or, } \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^n - x^n}{(x + \Delta x) - x}$$

$$\text{or, } \frac{dy}{dx} = nx^{n-1}$$

$$\therefore \frac{dx^n}{dx} = nx^{n-1}$$

formula used:

$$\frac{x^n - a^n}{x - a} = na^{n-1}$$

2. Derivation of $(ax+b)^n$

Solution:

Let $y = (ax+b)^n$

Let Δx be the small increment in x and Δy be the increment in y.

Then,

$$y + \Delta y = \{a(x+\Delta x)+b\}^n$$

or, $\Delta y = (ax + a\cdot\Delta x + b)^n - y$

or, $\Delta y = (ax + a\cdot\Delta x + b)^n - (ax+b)^n$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{(ax + a\cdot\Delta x + b)^n - (ax+b)^n}{\Delta x}$

or, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{(ax + a\cdot\Delta x + b)^n - (ax+b)^n}{a\cdot\Delta x} \times a$

or, $\dfrac{dy}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{(ax + a\cdot\Delta x + b)^n - (ax+b)^n}{(ax + a\cdot\Delta x + b) - (ax+b)} \times a$

or, $\dfrac{dy}{dx} = n(ax+b)^{n-1} \times a$

or, $\dfrac{d(ax+b)^n}{dx} = na(ax+b)^{n-1}$

Exercise - 16.1

1. Find, from defination, the derivatives of the following:

i. $3x^2$

Solution

Let $y = 3x^2$

Let Δx and Δy be the small increment in x and y respectively. Then,

$$y + \Delta y = 3(x+\Delta x)^2$$

or, $\Delta y = 3(x+\Delta x)^2 - y$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{3(x+\Delta x)^2 - 3x^2}{\Delta x}$

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or, $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 3 \lim_{\Delta x \to 0} \frac{x^2 + 2.x.\Delta x + \Delta x^2 - x^2}{\Delta x}$

or, $\frac{dy}{dx} = 3 \lim_{\Delta x \to 0} \frac{(2x.\Delta x + \Delta x^2)}{\Delta x}$

or, $\frac{dy}{dx} = 3 \lim_{\Delta x \to 0} (2x + \Delta x)$

or $\frac{dy}{dx} = 3(2x)$

or, $\frac{dy}{dx} = 6x$

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ii. $x^2 - 2$

Solution:

Let $y = x^2 - 2$

Let Δx and Δy be the small increment in x and y respectively

Then,

$y + \Delta y = (x + \Delta x)^2 - 2$

or, $\Delta y = x^2 + 2.x.\Delta x + \Delta x^2 - 2 - y$

or, $\Delta y = x^2 + 2x.\Delta x + \Delta x^2 - 2 - x^2 + 2$

or, $\frac{\Delta y}{\Delta x} = \frac{2x.\Delta x + \Delta x^2}{\Delta x}$

or, $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$

or, $\frac{dy}{dx} = \lim_{\Delta x \to 0} 2x + \Delta x$

or, $\frac{dy}{dx} = 2x$

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iii. $x^2 + 5x - 3$

Solution:

Let $y = x^2 + 5x - 3$

Let Δx and Δy be the small increment in x and y respectively.

Then

$$y + \Delta y = (x + \Delta x)^2 + 5x - 3$$

$$\text{or, } \Delta y = x^2 + 2 \cdot x \cdot \Delta x + \Delta x^2 + 5x - 3 - y$$

$$\text{or, } \Delta y = \cancel{x^2} + 2x \cdot \Delta x + \Delta x^2 + \cancel{5x} - \cancel{3} - \cancel{x^2} - \cancel{5x} + \cancel{3}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{\cancel{x^2 +} 2x \cdot \Delta x + \Delta x^2}{\Delta x}$$

$$y + \Delta y = (x + \Delta x)^2 + 5(x + \Delta x) - 3$$

$$\text{or, } \Delta y = (x + \Delta x)^2 + 5(x + \Delta x) - 3 - y$$

$$\text{or, } \Delta y = x^2 + \Delta x \cdot 2 \cdot x + \Delta x^2 + 5x + 5\Delta x - 3 - (x^2 + 5x - 3)$$

$$\text{or, } \Delta y = x^2 + 2x \cdot \Delta x + \Delta x^2 + 5x + 5\Delta x - 3 - x^2 - 5x + 3$$

$$\text{or, } \Delta y = 2x \cdot \Delta x + \Delta x^2 + 5 \cdot \Delta x$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{\Delta x (2x + \Delta x + 5)}{\Delta x}$$

$$\text{or, } \frac{dy}{dx} \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x + 5)$$

$$\text{or, } \frac{dy}{dx} = 2x + 5$$

$$\therefore \frac{dy}{dx} = 2x + 5$$

iv. $3x^2 - 2x + 1$

Solution:

Let $y = 3x^2 - 2x + 1$

Let Δx and Δy be the small increment in x and y respectively

Then,

$$y + \Delta y = 3(x + \Delta x)^2 - 2(x + \Delta x) + 1$$

$$\text{or, } \Delta y = 3(x + \Delta x)^2 - 2(x + \Delta x) + 1 - y$$

or, $\Delta y = 3(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (3x^2 - 2x + 1)$

or, $\Delta y = 3(x^2 + 2x\cdot\Delta x + \Delta x^2) - 2x - 2\Delta x + 1 - 3x^2 + 2x - 1$

or, $\Delta y = \cancel{3x^2} + 6x\cdot\Delta x + 3\Delta x^2 - \cancel{2x} - 2\Delta x - \cancel{3x^2} + \cancel{2x}$

or, $\Delta y = 6x\cdot\Delta x + 3\Delta x^2 - 2\Delta x$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{6x\cdot\Delta x + 3\Delta x^2 - 2\Delta x}{\Delta x}$

or, $\lim\limits_{\Delta x\to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{\Delta x\,(6x + 3\cdot\Delta x - 2)}{\Delta x}$

or, $\dfrac{dy}{dx} = \lim\limits_{\Delta x\to 0} (6x + 3\cdot\Delta x - 2)$

$\therefore \dfrac{dy}{dx} = 6x - 2$

V. $\dfrac{1}{x}$

Solution:

Let $y = 1/x$

Let Δx and Δy be the small increment in x and y respectively. Then

$y + \Delta y = \dfrac{1}{x+\Delta x}$

or, $\Delta y = \dfrac{1}{x+\Delta x} - y$

or, $\Delta y = \dfrac{1}{x+\Delta x} - \dfrac{1}{x}$

or, $\Delta y = \dfrac{x - (x+\Delta x)}{x(x+\Delta x)}$

or, $\Delta y = \dfrac{x - x - \Delta x}{x(x+\Delta x)}$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{-\Delta x}{x(x+\Delta x)} \times \dfrac{1}{\Delta x}$

or, $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)}$

or, $\frac{dy}{dx} = \lim\limits_{\Delta x \to 0} \frac{-1}{x^2 + x\cdot\Delta x}$

$\therefore \frac{dy}{dx} = \frac{-1}{x^2}$

vi. $\frac{3}{2x^2}$

Solution:

Let $y = 3/2x^2$

Let Δx and Δy be the small increment in x and y respectively

Then,

$y + \Delta y = \frac{3}{2(x+\Delta x)^2}$

or, $\Delta y = \frac{3}{2(x+\Delta x)^2} - \frac{3}{2x^2}$

or, $\Delta y = \frac{3\cdot x^2 - 3(x+\Delta x)^2}{2x^2(x+\Delta x)^2}$

or, $\Delta y = \frac{3x^2 - 3(x^2 + 2x\cdot\Delta x + \Delta x^2)}{2x^2(x^2 + 2x\cdot\Delta x + \Delta x^2)}$

or, $\Delta y = \frac{3x^2 - 3x^2 - 6x\cdot\Delta x - 3\Delta x^2}{2x^4 + 4x^3\cdot\Delta x + 2x^2\cdot\Delta x^2}$

$y + \Delta y = \frac{3}{2(x+\Delta x)^2}$

or, $\Delta y = \frac{3}{2(x+\Delta x)^2} - \frac{3}{2x^2}$

or, $\Delta y = \frac{3}{2}\left[\frac{x^2 - (x+\Delta x)^2}{x^2(x+\Delta x)^2}\right]$

or, $\Delta y = \frac{3}{2} \times \frac{x^2 - x^2 - 2x\cdot\Delta x - \Delta x^2}{x^2(x+\Delta x)^2}$

$$\frac{\Delta y}{\Delta x} = \frac{-\frac{3}{2} \times \Delta x\,(2x + \Delta x)}{\Delta x.\left[x^2 (x + \Delta x)^2\right]}$$

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-3}{2} \lim_{\Delta x \to 0} \frac{(2x + \Delta x)}{x^2 (x + \Delta x)^2}$$

$$\text{or, } \frac{dy}{dx} = \frac{-\frac{3}{2} \times 2x}{x^2 . x^2}$$

$$\therefore \frac{dy}{dx} = -\frac{3}{x^3}$$

vii. $\dfrac{1}{x-1}$

Solution:

Let $y = \dfrac{1}{x-1}$

Let Δx and Δy be the small increment in x and y resp.

Then,

$$y + \Delta y = \frac{1}{(x+\Delta x) - 1}$$

$$\text{or, } \Delta y = \frac{1}{x + \Delta x - 1} - \frac{1}{x-1}$$

$$\text{or, } \Delta y = \frac{(x-1) - (x + \Delta x - 1)}{(x + \Delta x - 1)\,(x-1)}$$

$$\text{or, } \Delta y = \frac{x - 1 - x - \Delta x + 1}{(x + \Delta x - 1)\,(x-1)}$$

$$\text{or, } \Delta y = \frac{-\Delta x}{(x + \Delta x - 1)(x-1)}$$

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\Delta x}{(x + \Delta x - 1)\,(x-1)} \times \frac{1}{\Delta x}$$

$$\text{or, } \frac{dy}{dx} = \frac{-1}{(x-1)(x-1)}$$

$$\therefore \frac{dy}{dx} = \frac{-1}{(x-1)^2}$$

viii. $\frac{1}{5-x}$

Solution:

Let $y = \frac{1}{5-x}$

Let Δx and Δy be the small increment in x and y respectively. Then,

$$y + \Delta y = \frac{1}{5-(x+\Delta x)}$$

$$\text{or, } \Delta y = \frac{1}{(5-x-\Delta x)} - \frac{1}{(5-x)}$$

$$\text{or, } \Delta y = \frac{5-x-5+x+\Delta x}{(5-x-\Delta x)(5-x)}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{\Delta x}{(5-x-\Delta x)(5-x)} \times \frac{1}{\Delta x}$$

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{1}{(5-x-\Delta x)(5-x)}$$

$$\text{or, } \frac{dy}{dx} = \frac{1}{(5-x)(5-x)}$$

$$\therefore \frac{dy}{dx} = \frac{1}{(5-x)^2}$$

ix. $\frac{1}{2x+3}$

Solution:

Let $y = \frac{1}{2x+3}$

Let Δx and Δy be the small increment in x and y respectively.
Then,

$$y + \Delta y = \frac{1}{2(x+\Delta x)+3}$$

or, $$\Delta y = \frac{1}{(2x+2\Delta x+3)} - \frac{1}{(2x+3)}$$

or, $$\Delta y = \frac{2x+3 - 2x - 2\Delta x - 3}{(2x+2\Delta x+3)(2x+3)}$$

or, $$\frac{\Delta y}{\Delta x} = \frac{-2\Delta x}{(2x+2\Delta x+3)(2x+3)}$$

or, $$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-2\Delta x}{(2x+2\Delta x+3)(2x+3)} \times \frac{1}{\Delta x}$$

or, $$\frac{dy}{dx} = \frac{-2}{(2x+3)(2x+3)}$$

$$\therefore \frac{dy}{dx} = \frac{-2}{(2x+3)^2}$$

x. $\frac{ax+b}{x}$

Solution:

Let $y = \frac{ax+b}{x}$

Let Δx and Δy be the small increment in x and y resp.
Then,

$$y + \Delta y = \frac{a(x+\Delta x)+b}{x+\Delta x}$$

or, $$\Delta y = \frac{a(x+\Delta x)+b}{(x+\Delta x)} - \frac{ax+b}{x}$$

or, $$\Delta y = \frac{(ax+a.\Delta x+b)}{(x+\Delta x)} - \frac{(ax+b)}{x}$$

or, $$\Delta y = \frac{ax^2+ax.\Delta x+bx - (ax+b)(x+\Delta x)}{x(x+\Delta x)}$$

$$\text{or, } \Delta y = \frac{ax^2 + ax.\Delta x + bx - ax^2 - bx - ax.\Delta x - b\Delta x}{x(x+\Delta x)}$$

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{-b.\Delta x}{x(x+\Delta x)} \times \frac{1}{\Delta x}$$

$$\text{or, } \frac{dy}{dx} = \frac{-b}{x.x}$$

$$\therefore \frac{dy}{dx} = \frac{-b}{x^2}$$

xi. $x^{1/2}$

Solution:

Let $y = x^{1/2} = \sqrt{x}$

Let Δx and Δy be the small increment in x and y respectively.

Then,

$$y + \Delta y = \sqrt{x + \Delta x}$$

$$\text{or, } \Delta y = \sqrt{x+\Delta x} - y$$

$$\text{or, } \Delta y = \sqrt{x+\Delta x} - \sqrt{x}$$

$$\text{or, } \Delta y = \sqrt{x+\Delta x} - \sqrt{x} \times \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\text{or, } \Delta y = \frac{x + \Delta x - x}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{\Delta x}{\sqrt{x+\Delta x} + \sqrt{x}} \times \frac{1}{\Delta x}$$

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\text{or, } \frac{dy}{dx} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

xii. $x + \sqrt{x}$

Solution:

Let $y = x + \sqrt{x}$

Let Δx and Δy be the small increment in x and y respectively.

Then,

$$y + \Delta y = x + \Delta x + \sqrt{x + \Delta x}$$

$$\text{or, } \Delta y = x + \Delta x + \sqrt{x + \Delta x} - x - \sqrt{x}$$

$$\text{or, } \Delta y = \Delta x + \left(\sqrt{x + \Delta x} - \sqrt{x}\right)$$

$$\text{or, } \Delta y = \Delta x + \left(\sqrt{x + \Delta x} - \sqrt{x}\right) \times \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$\text{or, } \Delta y = \Delta x + \frac{x + \Delta x - x}{\sqrt{x + \Delta x} + \sqrt{x}}$$

Dividing both side by Δx

$$\text{or, } \frac{\Delta y}{\Delta x} = 1 + \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 1 + \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$\text{or, } \frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = 1 + \frac{1}{2} x^{-1/2}$$

xiii. $(1 + x)^{1/2}$

Solution:

Let $y = (1 + x)^{1/2} = \sqrt{1 + x}$

Let Δx and Δy be the increment in x and y respectively.

Then,

$$y + \Delta y = \sqrt{1 + x + \Delta x}$$

$$\text{or, } \Delta y = \sqrt{1 + x + \Delta x} - \sqrt{1 + x}$$

$$\text{or, } \Delta y = \sqrt{1 + x + \Delta x} - \sqrt{1 + x} \times \frac{\sqrt{1 + x + \Delta x} + \sqrt{1 + x}}{\sqrt{1 + x + \Delta x} + \sqrt{1 + x}}$$

or, $\Delta y = \dfrac{1 + x + \Delta x - 1 - x}{\sqrt{1+x+\Delta x} + \sqrt{1+x}}$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{\Delta x}{\sqrt{1+x+\Delta x} + \sqrt{1+x}} \times \dfrac{1}{\Delta x}$

or, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{1}{\sqrt{1+x+\Delta x} + \sqrt{1+x}}$

or, $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1+x} + \sqrt{1+x}}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1+x}}$

xiv. $(2x+3)^{1/2}$

Solution:

Let $y = (2x+3)^{1/2} = \sqrt{2x+3}$

Let Δx and Δy be small increment in x and y respectively then,

$y + \Delta y = \sqrt{2(x+\Delta x)+3}$

or, $\Delta y = \sqrt{2x+2\Delta x+3} - \sqrt{2x+3}$

or, $\Delta y = \sqrt{2x+2\cdot\Delta x+3} - \sqrt{2x+3} \times \dfrac{\sqrt{2x+2\cdot\Delta x+3} + \sqrt{2x+3}}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}$

or, $\Delta y = \dfrac{2x+2\Delta x+3-2x-3}{(\sqrt{2x+2\Delta x+3} + \sqrt{2x+3})}$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{2\Delta x}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}$

or, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \dfrac{2\Delta x}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}} \times \dfrac{1}{\Delta x}$

or, $\dfrac{dy}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{2}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}$

or, $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{2x+3} + \sqrt{2x+3}}$

$\therefore \dfrac{dy}{dx} = \dfrac{2}{2\sqrt{2x+3}} = \dfrac{1}{\sqrt{2x+3}}$

xv. $(1+x^2)^{1/2}$

Solution:

Let $y = (1+x^2)^{1/2} = \sqrt{1+x^2}$

Let Δx and Δy be the small increment in x and y respectively.

Then,

$$y + \Delta y = \sqrt{1+(x+\Delta x)^2}$$

or, $$\Delta y = \sqrt{1+(x+\Delta x)^2} - \sqrt{1+x^2}$$

or, $$\Delta y = \sqrt{1+(x+\Delta x)^2} - \sqrt{1+x^2} \times \frac{\sqrt{1+(x+\Delta x)^2} + \sqrt{1+x^2}}{\sqrt{1+(x+\Delta x)^2} + \sqrt{1+x^2}}$$

or, $$\Delta y = \frac{1 + x^2 + 2x.\Delta x + \Delta x^2 - 1 - x^2}{\sqrt{1+(x+\Delta x)^2} + \sqrt{1+x^2}}$$

or, $$\frac{\Delta y}{\Delta x} = \frac{2x.\Delta x + \Delta x^2}{\sqrt{1+(x+\Delta x)^2} + \sqrt{1+x^2}} \times \frac{1}{\Delta x}$$

or, $$\frac{\Delta y}{\Delta x} = \frac{\Delta x(2x + \Delta x)}{\sqrt{1+(x+\Delta x)^2} + \sqrt{1+x^2}} \times \frac{1}{\Delta x}$$

or, $$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{(2x + \Delta x)}{\sqrt{1+(x+\Delta x)^2} + \sqrt{1+x^2}}$$

or, $$\frac{dy}{dx} = \frac{2x}{\sqrt{1+x^2} + \sqrt{1+x^2}}$$

or, $$\frac{dy}{dx} = \frac{2x}{2\sqrt{1+x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$$

xvi. $\frac{1}{x^{1/2}}$

Solution:

Let $y = \frac{1}{x^{1/2}} = x^{-1/2}$

Let Δx and Δy be the small increment in x and y respectively.

Then,

or, $\Delta y = \dfrac{1}{\sqrt{x+\Delta x}} - \dfrac{1}{\sqrt{x}}$

or, $\Delta y = \sqrt{x} - \sqrt{x+\Delta x}$

$y + \Delta y = (x+\Delta x)^{-1/2}$

or, $\Delta y = (x+\Delta x)^{-1/2} - x^{-1/2}$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{(x+\Delta x)^{-1/2} - x^{-1/2}}{(x+\Delta x) - x}$

or, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{(x+\Delta x)^{-1/2} - x^{-1/2}}{(x+\Delta x) - x}$

or, $\dfrac{dy}{dx} = -\dfrac{1}{2} \times x^{(-1/2-1)}$ $\left[\dfrac{x^n - a^n}{x-a} = na^{n-1} \right]$

or, $\dfrac{dy}{dx} = -\dfrac{1}{2} x^{-3/2}$

$\therefore \dfrac{dy}{dx} = \dfrac{-1}{2x^{3/2}}$

xvii. $\dfrac{1}{(1-x)^{1/2}}$

Solution:

Let $y = \dfrac{1}{(1-x)^{1/2}} = (1-x)^{-1/2}$

Let Δx and Δy be the small increment in x and y respectiv-
then,

$y + \Delta y = \{1-(x+\Delta x)\}^{-1/2}$

or, $\Delta y = \{1-(x+\Delta x)\}^{-1/2} - (1-x)^{-1/2}$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{(1-x-\Delta x)^{-1/2} - (1-x)^{-1/2}}{(1-x-\Delta x) - (1-x)} \times (-1)$

or, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{(1-x-\Delta x)^{-1/2} - (1-x)^{-1/2}}{(1-x-\Delta x) - (1-x)} \times (-1)$

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or, $\frac{dy}{dx} = -\frac{1}{2} \times (1-x)^{-\frac{1}{2}-1} \times (-1)$

or, $\frac{dy}{dx} = \frac{1}{2} (1-x)^{-3/2}$

$\therefore \frac{dy}{dx} = \frac{1}{2(1-x)^{3/2}}$

xviii. $\frac{1}{(3x+4)^{1/2}}$

Solution:

Let $y = (3x+4)^{-1/2}$

Let Δx and Δy be the small increment in x and y respectively

Then,

$y + \Delta y = \{3(x+\Delta x)+4\}^{-1/2}$

or, $\Delta y = (3x+3\Delta x+4)^{-1/2} - (3x+4)^{-1/2}$

or, $\frac{\Delta y}{\Delta x} = \frac{(3x+3\Delta x+4)^{-1/2} - (3x+4)^{-1/2}}{(3x+3\Delta x+4)-(3x+4)} \times 3$

or, $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{(3x+3\Delta x+4)^{-1/2} - (3x+4)^{-1/2}}{(3x+3\Delta x+4)-(3x+4)} \times 3$

or, $\frac{dy}{dx} = -\frac{1}{2} (3x+4)^{-1/2-1} \times 3$

or, $\frac{dy}{dx} = -\frac{3}{2} (3x+4)^{-3/2}$

$\therefore \frac{dy}{dx} = \frac{-3}{2(3x+4)^{3/2}}$

xix. $\dfrac{ax+b}{\sqrt{x}}$

Solution

Let $y = \dfrac{ax+b}{\sqrt{x}} = \dfrac{ax}{\sqrt{x}} + \dfrac{b}{\sqrt{x}}$

$= \dfrac{a \cdot \sqrt{x} \cdot \sqrt{x}}{\sqrt{x}} + \dfrac{b}{\sqrt{x}}$

$= a \cdot x^{1/2} + bx^{-1/2}$

Let Δy and Δx be the increment in y and x respectively.

Then,

$y + \Delta y = a(x+\Delta x)^{1/2} + b(x+\Delta x)^{-1/2}$

or, $\Delta y = a(x+\Delta x)^{1/2} + b(x+\Delta x)^{-1/2} - ax^{1/2} - bx^{-1/2}$

or, $\Delta y = a(x+\Delta x)^{1/2} - ax^{1/2} + b(x+\Delta x)^{-1/2} - bx^{-1/2}$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{a\{(x+\Delta x)^{1/2} - x^{1/2}\} + b\{(x+\Delta x)^{-1/2} - x^{-1/2}\}}{\Delta x}$

or, $\dfrac{\Delta y}{\Delta x} = \dfrac{a\{(x+\Delta x)^{1/2} - x^{1/2}\}}{\Delta x} + \dfrac{b\{(x+\Delta x)^{-1/2} - x^{-1/2}\}}{\Delta x}$

or, $\lim\limits_{\Delta x \to 0} = \lim\limits_{\Delta x \to 0} \left[\dfrac{a\{(x+\Delta x)^{1/2} - x^{1/2}\}}{\Delta x} + \dfrac{b\{(x+\Delta x)^{-1/2} - x^{-1/2}\}}{\Delta x} \right]$

or, $\dfrac{dy}{dx} = \lim\limits_{\Delta x \to 0} \left[a \cdot \dfrac{(x+\Delta x)^{1/2} - x^{1/2}}{(x+\Delta x) - x} + b \times \dfrac{(x+\Delta x)^{-1/2} - x^{-1/2}}{(x+\Delta x) - x} \right]$

or, $\dfrac{dy}{dx} = a \times \dfrac{1}{2} \cdot x^{\frac{1}{2}-1} + b\left(-\dfrac{1}{2}\right) x^{-\frac{1}{2}-1}$

or, $\dfrac{dy}{dx} = \dfrac{ax^{-1/2}}{2} - b\dfrac{x^{-3/2}}{2}$

or, $\dfrac{dy}{dx} = \dfrac{a}{2x^{1/2}} - \dfrac{b}{2x^{3/2}}$

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Rule of Differentation

1. The Sum Rule

$$\frac{d(u+v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

2. The Product Rule

$$\frac{d(u.v)}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx} \quad , \quad \frac{dk.u}{dx} = k.\frac{du}{dx}$$

Note: Derivative of constant number is zero.

3. The Power Rule

$$\frac{dx^n}{dx} = nx^{n-1}$$

4. The Quotient Rule

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v.\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Eg: $$\frac{d\left(\frac{2x+5}{3x+1}\right)}{dx}$$

$$= \frac{(3x+1)\frac{d(2x+5)}{dx} - (2x+5)\frac{d(3x+1)}{dx}}{(3x+1)^2}$$

5. The chain Rule.

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx}$$

2. Find the derivatives of the following:

i. x^5

Solution:

Let $y = x^5$

Then, $\dfrac{dy}{dx} = \dfrac{d}{dx}(x^5)$

$= 5 \cdot x^{5-1}$

$= 5x^4$

ii. $5x^7$

Solution :

Let $y = 5x^7$

Then, $\dfrac{dy}{dx} = \dfrac{d}{dx}(5x^7)$

$= 5 \dfrac{d}{dx}(x^7)$

$= 5 \cdot 7x^{7-1}$

$= 35x^6$

iii. $3x^2 - 5x + 7$

Solution:

Let $y = 3x^2 - 5x + 7$

Then, $\dfrac{dy}{dx} = \dfrac{d}{dx}(3x^2 - 5x + 7)$

$= \dfrac{d}{dx} 3x^2 - \dfrac{d}{dx} 5x + \dfrac{d}{dx}(7)$

$= 3 \cdot \dfrac{d}{dx}(x^2) - 5 \cdot \dfrac{d}{dx}(x) + 0$

$= 3 \cdot 2x^{2-1} - 5 \cdot 1$

$= 6x - 5$

iv. $\dfrac{3x^3+2x-1}{2x^2}$

Let $y = \dfrac{3x^3+2x-1}{2x^2}$

Then, $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{3x^3+2x-1}{2x^2}\right)$

$$= \frac{2x^2 \cdot \frac{d}{dx}(3x^3+2x-1) - (3x^3+2x-2)\cdot \frac{d}{dx} \times 2x^2}{(2x^2)^2}$$

$$= \frac{2x^2 \cdot \frac{d}{dx}3x^3 + \frac{d}{dx}2x - \frac{d}{dx}(1) - (3x^3+2x-1)\ 2\cdot\frac{d}{dx}x^2}{4x^4}$$

$$= \frac{2x^2\cdot(3\cdot 3x^2 + 2\cdot 1) - (3x^3+2x-1)\ 2\cdot 2x}{4x^4}$$

$$= \frac{18x^4 + 4x^2 - (3x^3+2x-1)\ 4x}{4x^4}$$

$$= \frac{18x^4 + 4x^2 - 12x^4 - 8x^2 + 4x}{4x^4}$$

$$= \frac{6x^4 - 4x^2 + 4x}{4x^4}$$

$$= \frac{2x(3x^3 - 2x + 2)}{2\cdot x\cdot 2x^3}$$

$$= \frac{3x^3 - 2x + 2}{2x^3}$$

v. $2x^{3/4} - 3x^{1/2} - 5x^{1/4}$

Solution:

Let $y = 2x^{3/4} - 3x^{1/2} - 5x^{1/4}$

Then,

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(2x^{3/4} - 3x^{1/2} - 5x^{1/4}\right)$

$= \frac{d}{dx}\left(2x^{3/4}\right) - \frac{d}{dx}\left(3x^{1/2}\right) - \frac{d}{dx}\left(5x^{1/4}\right)$

$= 2 \cdot \frac{d}{dx}\left(x^{3/4}\right) - 3 \cdot \frac{d}{dx}\left(x^{1/2}\right) - 5 \cdot \frac{d}{dx}\left(x^{1/4}\right)$

$= 2 \times \frac{3}{4} x^{3/4 - 1} - 3 \cdot \frac{1}{2} x^{1/2 - 1} - 5 \cdot \frac{1}{4} x^{1/4 - 1}$

$= \frac{6}{4} x^{-1/4} - \frac{3}{2} x^{-1/2} - \frac{5}{4} x^{-3/4}$

$= \frac{3}{2} x^{-1/4} - \frac{3x^{-1/2}}{2} - \frac{5}{4} x^{-3/4}$

$= \frac{3}{2x^{1/4}} - \frac{3}{2x^{1/2}} - \frac{5}{4x^{3/4}}$

$= \frac{3}{2x^{1/4}} \times \frac{2}{2} \times \frac{x^{1/2}}{x^{1/2}} - \frac{3}{2x^{1/2}} \times \frac{2}{2} \times \frac{x^{1/4}}{x^{1/4}} - \frac{5}{4x^{3/4}}$

$= \frac{6x^{1/2}}{4x^{3/4}} - \frac{6x^{1/4}}{4x^{3/4}} - \frac{5}{4x^{3/4}}$

$= \frac{6x^{1/2} - 6x^{1/4} - 5}{4x^{3/4}}$

vi. $\frac{2x + 3x^{3/4} + x^{1/2} + 1}{x^{1/4}}$

Solution:

Let $y = \frac{2x + 3x^{3/4} + x^{1/2} + 1}{x^{1/4}}$

$= \frac{2x}{x^{1/4}} + \frac{3x^{3/4}}{x^{1/4}} + \frac{x^{1/2}}{x^{1/4}} + \frac{1}{x^{1/4}}$

$= 2x^{3/4} + 3x^{1/2} + x^{1/4} + x^{-1/4}$

Then,

$\frac{dy}{dx} = \frac{d}{dx}\left(2x^{3/4} + 3x^{1/2} + x^{1/4} + x^{-1/4}\right)$

$= \frac{d}{dx}\left(2x^{3/4}\right) + \frac{d}{dx}\left(3x^{1/2}\right) + \frac{d}{dx}\left(x^{1/4}\right) + \frac{d}{dx}\left(x^{-1/4}\right)$

$= 2 \cdot \frac{d}{dx}(x^{3/4}) + 3 \cdot \frac{d}{dx}(x^{1/2}) + \frac{d}{dx}(x^{1/4}) + \frac{d}{dx}(x^{-1/4})$

$= 2 \times \frac{3}{4} x^{3/4 - 1} + 3 \times \frac{1}{2} x^{1/2 - 1} + \frac{1}{4} x^{1/4 - 1} + \left(-\frac{1}{4}\right) x^{-1/4 - 1}$

$= \frac{3}{2} x^{-1/4} + \frac{3}{2} x^{-1/2} + \frac{1}{4} x^{-\frac{3}{4}} - \frac{1}{4} x^{-5/4}$

$= \frac{3}{2x^{1/4}} + \frac{3}{2x^{1/2}} + \frac{1}{4x^{3/4}} - \frac{1}{4x^{5/4}}$

---

3. Use the product rule to calculate the derivatives of :

i. $3x^2(2x-1)$

Solution:

Let $y = 3x^2(2x-1)$

Then, $\frac{dy}{dx} = \frac{d}{dx}\{3x^2(2x-1)\}$

$= 3x^2 \frac{d}{dx}(2x-1) + (2x-1)\frac{d}{dx} 3x^2$

$= 3x^2 \times \left(\frac{d}{dx} 2x - \frac{d}{dx} 1\right) + (2x-1)\ 3 \cdot \frac{d}{dx} \cdot x^2$

$= 3x^2 \cdot 2 \cdot 1 - 0 + (2x-1) \cdot 3 \cdot 2x$

$= 6x^2 + (2x-1)\,6x$

$= 6x^2 + 12x^2 - 6x$

$= 18x^2 - 6x$

$= 6x(3x-1)$

ii. $(2x^2+1)(3x^2-2)$

Solution:

Let $y = (2x^2+1)(3x^2-2)$

Then, $\frac{dy}{dx} = \frac{d}{dx}\{(2x^2+1)(3x^2-2)\}$

$= (2x^2+1) \cdot \frac{d}{dx}(3x^2-2) + (3x^2-2) \cdot \frac{d}{dx}(2x^2+1)$

$= (2x^2+1)\left(\frac{d}{dx}3x^2 - \frac{d}{dx}(2)\right) + (3x^2-2)\left(\frac{d}{dx}(2x^2) + \frac{d}{dx}1\right)$

$= (2x^2+1)\left(3 \cdot \frac{d}{dx} \cdot x^2 - 0\right) + (3x^2-2)\left(2 \cdot \frac{d}{dx} x^2 + 0\right)$

$= (2x^2+1)\ 3 \cdot 2 \cdot x + (3x^2-2)\ 2 \cdot 2 \cdot x$

$= (2x^2+1)\ 6x + (3x^2-2)\ 4x$

$= 12x^3 + 6x + 12x^3 - 8x$

$= 24x^3 - 2x$

$= 2x(12x^2-1)$

iii. $(3x^4+5)(4x^5-3)$

Solution:

Let $y = (3x^4+5)(4x^5-3)$

Then, $\frac{dy}{dx} = (3x^4+5)\frac{d}{dx}(4x^5-3) + (4x^5-3) \cdot \frac{d}{dx}(3x^4+5)$

$= 3x^4+5\left(\frac{d}{dx}4x^5 - \frac{d}{dx}(3)\right) + (4x^5-3)\left(\frac{d}{dx}3x^4 + \frac{d}{dx}5\right)$

$= (3x^4+5)\left(4 \cdot \frac{d}{dx}x^5 - 0\right) + (4x^5-3)\left(3\frac{d}{dx}x^4 + 0\right)$

$= (3x^4+5)\ 4 \cdot 5 \cdot x^4 + (4x^5-3)\ 3 \cdot 4 \cdot x^3$

$= (3x^4+5)\ 20x^4 + (4x^5-3)\ 12x^3$

$= 60x^8 + 100x^4 + 48x^8 - 36x^3$

$= 108x^8 + \cancel{48x}\ 100x^4 - 36x^3$

$= 4x^3(27x^5 + 25x - 9)$

iv. $(3x^2+5x-1)(x^2+3)$

Let $y = (3x^2+5x-1)(x^2+3)$

Then, $\frac{dy}{dx} = \frac{d}{dx}(3x^2+5x-1)(x^2+3)$

$$= (3x^2+5x-1) \cdot \frac{d}{dx}(x^2+3) + (x^2+3) \cdot \frac{d}{dx}(3x^2+5x-1)$$

$$= (3x^2+5x-1)\left(\frac{d}{dx}x^2 + \frac{d}{dx}(3)\right) + (x^2+3)\left(\frac{d}{dx}3x^2 + \frac{d}{dx}5x - \frac{d}{dx}\times 1\right)$$

$$= (3x^2+5x-1) \times 2x + (x^2+3)\left(3 \cdot 2x + 5 - 0\right)$$

$$= 6x^3 + 10x^2 - 2x + (x^2+3)(6x+5)$$

$$= 6x^3 + 10x^2 - 2x + 6x^3 \cancel{+ x^2} + 18x + 15 + 5x^2$$

$$= 12x^3 + 15x^2 + 16x + 15$$

v. $(a + \sqrt{x})(a - \sqrt{x})$

Solution:

Let $y = (a + x^{1/2})(a - x^{1/2})$

Then,

$$\frac{dy}{dx} = (a + x^{1/2})\frac{d}{dx}(a - x^{1/2}) + (a - x^{1/2})\frac{d}{dx}(a + x^{1/2})$$

$$= (a + x^{1/2})\left[\frac{d}{dx}(a) - \frac{d}{dx}x^{1/2}\right] + (a - x^{1/2})\left[\frac{d}{dx}\times a + \frac{d}{dx}x^{1/2}\right]$$

$$= (a + x^{1/2})\left[0 - \frac{1}{2}x^{1/2-1}\right] + (a - x^{1/2})\left[0 + \frac{1}{2}x^{1/2-1}\right]$$

$$= (a + x^{1/2})\left(-\frac{1}{2}x^{-1/2}\right) + (a - x^{1/2}) \times \frac{1}{2}x^{-1/2}$$

$$= -\frac{a}{2}x^{-1/2} - \frac{1}{2}x^{1/2-1/2} + \frac{a}{2}x^{-1/2} - \frac{1}{2}x^{1/2-1/2}$$

$$= \frac{-a}{2x^{1/2}} + \frac{a}{2x^{1/2}} - \frac{1}{2}\cdot 1 - \frac{1}{2}\cdot 1$$

$$= -\frac{1}{2} - \frac{1}{2}$$

$$= \frac{-2}{2}$$

$$= -1$$

4. Use the quotient rule to find the derivatives of:

i. $\frac{x}{1+x}$

Solution:

Let $y = \frac{x}{1+x}$

Differentiating both sides w.r.t. 'x'

or, $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{1+x}\right)$

$$= \frac{(1+x)\cdot\frac{d}{dx}(x) - x\cdot\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$= \frac{(1+x)\cdot 1 - x\cdot 1}{(1+x)^2}$$

$$= \frac{1}{(1+x)^2}$$

ii. $\frac{x^2}{1-x^2}$

Solution:

Let $y = \frac{x^2}{1-x^2}$

Differentiating both sides w.r.t 'x'

or, $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2}{1-x^2}\right)$

$$= \frac{(1-x^2)\cdot\frac{d}{dx}(x^2) - x^2\cdot\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$= \frac{(1-x^2)\cdot 2x - x^2\cdot(-2x)}{(1-x^2)^2}$$

$$= \frac{2x - 2x^3 + 2x^3}{(1-x^2)^2}$$

$$= \frac{2x}{(1-x^2)^2}$$

iii. $\dfrac{x^2 - a^2}{x^2 + a^2}$

Solution:

Let $y = \dfrac{x^2 - a^2}{x^2 + a^2}$

Differentiating both sides w.r.t 'x'

or, $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{x^2 - a^2}{x^2 + a^2}\right)$

$$= \frac{(x^2 + a^2) \cdot \frac{d}{dx}(x^2 - a^2) - (x^2 - a^2) \cdot \frac{d}{dx}(x^2 + a^2)}{(x^2 + a^2)^2}$$

$$= \frac{(x^2 + a^2) \cdot 2x - (x^2 - a^2) \cdot 2x}{(x^2 + a^2)^2}$$

$$= \frac{2x^3 + 2a^2x - 2x^3 + 2a^2x}{(x^2 + a^2)^2}$$

$$= \frac{4a^2x}{(x^2 + a^2)^2}$$

iv. $\dfrac{3}{x^2}$

Solution:

Let $y = \dfrac{3}{x^2}$

Differentiating both sides w.r.t 'x'

Then, $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{3}{x^2}\right)$

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$$= \frac{x^2 \cdot \frac{d}{dx} \times 3 - 3 \cdot \frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2 \times 0 - 3 \cdot 2x}{x^4}$$

$$= \frac{-6}{x^3}$$

v. $\frac{x^2 - 2x}{x+1}$

Solution:

Let $y = \frac{x^2 - 2x}{x+1}$

Differentiating both sides w.r.t. 'x'

or, $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2 - 2x}{x+1}\right)$

$$= \frac{(x+1) \cdot \frac{d}{dx}(x^2 - 2x) - (x^2 - 2x) \cdot \frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1) \cdot (2x-2) - (x^2 - 2x) \cdot 1}{(x+1)^2}$$

$$= \frac{2x^2 - 2x + 2x - 2 - x^2 + 2x}{(x+1)^2}$$

$$= \frac{x^2 + 2x - 2}{(x+1)^2}$$

5. Use the general power rule to calculate the derivative of :

i. $(2x+3)^2$

Solution:

Let $y = (2x+3)^2$

Differentiating both sides w.r.t 'x'

or, $\frac{dy}{dx} = \frac{d}{dx}(2x+3)^2$

$= \frac{d(2x+3)^2}{d(2x+3)} \times \frac{d(2x+3)}{dx}$

$= 2(2x+3) \cdot 2 \cdot 1$

$= 4(2x+3)$

ii. $(3-2x)^3$

Solution:

Let $y = (3-2x)^3$

Differentiating both sides w.r.t 'x'

or, $\frac{dy}{dx} = \frac{d(3-2x)^3}{d(3-2x)} \cdot \frac{d(3-2x)}{dx}$

$= 3(3-2x)^2 \times (-2)$

$= -6(3-2x)^2$

iii. $(3x^2+2x-1)^4$

Solution:

Let $y = (3x^2+2x-1)^4$

Differentiating both sides w.r.t 'x'

or, $\frac{dy}{dx} = \frac{d(3x^2+2x-1)^4}{d(3x^2+2x-1)} \cdot \frac{d(3x^2+2x-1)}{dx}$

$= 4(3x^2+2x-1)^3 \cdot (2 \cdot 3x+2)$

$= 4(3x^2+2x-1)^3 \cdot 2(3x+1)$

iv. $(2x^2 + 3x - 3)^{-6}$

Solution:

Let $y = (2x^2 + 3x - 3)^{-6}$

Differentiating both sides w.r.t 'x'

or, $\frac{dy}{dx} = \frac{d(2x^2+3x-3)^{-6}}{d(2x^2+3x-3)} \cdot \frac{d(2x^2+3x-3)}{dx}$

$= -6(2x^2+3x-3)^{-7} \cdot (2 \cdot 2x + 3)$

$= -6(4x+3)(2x^2+3x-3)^{-7}$

v. $\sqrt{(8-5x)}$

Solution:

Let $y = \sqrt{(8-5x)} = (8-5x)^{1/2}$

Differentiating both sides w.r.t 'x'

or, $\frac{dy}{dx} = \frac{d(8-5x)^{1/2}}{d(8-5x)} \cdot \frac{d(8-5x)}{dx}$

$= \frac{1}{2}(8-5x)^{-1/2} \cdot (-5)$

$= \frac{-5}{2(8-5x)^{1/2}}$

$= \frac{-5}{2\sqrt{8-5x}}$

vi. $(2x^2 - 3x + 1)^{3/4}$

Solution:

Let $y = (2x^2 - 3x + 1)$

Differentiating both side w.r.t 'x'

or, $\frac{dy}{dx} = \frac{d(2x^2-3x+1)^{3/4}}{d(2x^2-3x+1)} \cdot \frac{d(2x^2-3x+1)}{dx}$

$= \frac{3}{4}(2x^2-3x+1)^{-1/4} \cdot (2 \cdot 2x - 3)$

$= \frac{3(4x-3)}{4}(2x^2-3x+1)^{-1/4}$

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vii. $\dfrac{1}{\sqrt{(ax^2+bx+c)}}$

Solution:

Let $y = (ax^2+bx+c)^{-1/2}$

Differentiating both sides w.r.t. 'x'

or, $\dfrac{dy}{dx} = \dfrac{d(ax^2+bx+c)^{-1/2}}{d(ax^2+bx+c)} \cdot \dfrac{d(ax^2+bx+c)}{dx}$

$= -\dfrac{1}{2}(ax^2+bx+c)^{-3/2}(a \cdot 2 \cdot x + b \cdot 1)$

$= -\dfrac{1}{2}(2ax+b)(ax^2+bx+c)^{-3/2}$

viii. $\dfrac{1}{\sqrt[3]{3x^2-4x-1}}$

Solution:

Let $y = (3x^2-4x-1)^{-1/3}$

Differentiating both sides w.r.t 'x'

or, $\dfrac{dy}{dx} = \dfrac{d(3x^2-4x-1)^{-1/3}}{d(3x^2-4x-1)} \cdot \dfrac{d(3x^2-4x-1)}{dx}$

$= -\dfrac{1}{3}(3x^2-4x-1)^{-4/3}(2 \cdot 3 \cdot x - 4)$

$= -\dfrac{1}{3}(3x^2-4x-1)^{-4/3}\, 2(3x-2)$

$= -\dfrac{2}{3}(3x^2-4x-1)^{-4/3}(3x-2)$

$= \cancel{-\dfrac{2}{3}(3x^2 - 4x - 1}$

$= -\dfrac{2}{3}(3x-2)(3x^2-4x-1)^{-4/3}$

ix. $\frac{1}{\sqrt{a^n - x^n}}$

Solution:

Let $y = (a^n - x^n)^{-1/2}$

Differentiating both sides w.r.t 'x'

$$\text{or, } \frac{dy}{dx} = \frac{d(a^n - x^n)^{-1/2}}{d(a^n - x^n)} \cdot \frac{d(a^n - x^n)}{dx}$$

$$= -\frac{1}{2}(a^n - x^n)^{-3/2} \cdot \left(\frac{d a^n}{dx} - \frac{d x^n}{dx}\right)$$

$$= -\frac{1}{2}(a^n - x^n)^{-3/2}\left(0 - n x^{n-1}\right)$$

$$= \frac{1}{2}(n x^{n-1})(a^n - x^n)^{-3/2}$$

x. $\frac{1}{\sqrt{(x+a)} - \sqrt{x}}$

Solution:

$$\text{Let } y = \frac{1}{\sqrt{x+a} - \sqrt{x}} \times \frac{\sqrt{x+a} + \sqrt{x}}{\sqrt{x+a} + \sqrt{x}}$$

$$= \frac{\sqrt{x+a} + \sqrt{x}}{x+a-x}$$

$$= \frac{1}{a}(\sqrt{x+a} + \sqrt{x})$$

Differentiating both side w.r.t 'x'

$$\text{or, } \frac{dy}{dx} = \frac{1}{a}\left[\frac{d}{dx}(\sqrt{x+a} + \sqrt{x})\right]$$

$$= \frac{1}{a}\left[\frac{d(x+a)^{1/2}}{d(x+a)} \times \frac{d(x+a)}{dx} + \frac{d(x)^{1/2}}{dx}\right]$$

$$= \frac{1}{a}\left[\frac{1}{2}(x+a)^{-1/2} \cdot 1 + \frac{1}{2}x^{-1/2}\right]$$

$$= \frac{1}{a} \times \frac{1}{2}\left[\frac{1}{(x+a)^{1/2}} + \frac{1}{x^{1/2}}\right]$$

$$= \frac{1}{2a}\left(\frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x}}\right)$$

xi. $\dfrac{1}{x-\sqrt{a^2+x^2}}$

Solution:

Let $y = \dfrac{1}{x-\sqrt{a^2-x^2}} \times \dfrac{x+\sqrt{a^2+x^2}}{x+\sqrt{a^2+x^2}}$

$$= \frac{x+\sqrt{a^2-x^2}}{x^2-a^2-x^2}$$

$$= -\frac{1}{a^2}\left(x+\sqrt{a^2+x^2}\right)$$

Differentiating both sides w.r.t 'x'

or, $\dfrac{dy}{dx} = -\dfrac{1}{a^2}\left[\dfrac{d\{x+\sqrt{a^2+x^2}\}}{dx}\right]$

$$= -\frac{1}{a^2}\left[\frac{d}{dx}(x) + \frac{d(a^2+x^2)^{1/2}}{dx}\right]$$

$$= -\frac{1}{a^2}\left[1 + \frac{d(a^2+x^2)^{1/2}}{d(a^2+x^2)} \cdot \frac{d(a^2+x^2)}{dx}\right]$$

$$= -\frac{1}{a^2}\left[1 + \frac{1}{2}(a^2+x^2)^{-1/2} \cdot 2x\right]$$

$$= -\frac{1}{a^2}\left[1 + \frac{2x}{2}(a^2+x^2)^{-1/2}\right]$$

$$= \frac{-1}{a^2}\left(1 + \frac{x}{\sqrt{a^2+x^2}}\right)$$

xii. $\dfrac{1}{\sqrt{x+a}+\sqrt{x-a}}$

Solution:

Let $y = \dfrac{1}{\sqrt{x+a}+\sqrt{x-a}} \times \dfrac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}$

$$= \frac{\sqrt{x+a} - \sqrt{x-a}}{x+a-x+a}$$

$$= \frac{1}{2a}\left[(x+a)^{1/2} - (x-a)^{1/2}\right]$$

Differentiating both sides w.r.t 'x'

or, $$\frac{dy}{dx} = \frac{1}{2a}\left[\frac{d}{dx}\left\{(x+a)^{1/2} - (x-a)^{1/2}\right\}\right]$$

$$= \frac{1}{2a}\left[\frac{d(x+a)^{1/2}}{d(x+a)} \cdot \frac{d(x+a)}{dx} - \frac{d(x-a)^{1/2}}{d(x-a)} \cdot \frac{d(x-a)}{dx}\right]$$

$$= \frac{1}{2a}\left[\frac{1}{2}(x+a)^{-1/2} \cdot 1 - \frac{1}{2}(x-a)^{-1/2} \cdot 1\right]$$

$$= \frac{1}{2a}\left(\frac{1}{2\sqrt{x+a}} - \frac{1}{2\sqrt{x-a}}\right)$$

$$= \frac{1}{2a} \times \frac{1}{2}\left(\frac{1}{\sqrt{x+a}} - \frac{1}{\sqrt{x-a}}\right)$$

$$= \frac{1}{4a}\left(\frac{1}{\sqrt{x+a}} - \frac{1}{\sqrt{x-a}}\right)$$

---

6. Use the chain rule to calculate $\frac{dy}{dx}$, if

i. $y = 2u^2 - 3u + 1$ and $u = 2x^2$

Solution:

$y = 2u^2 - 3u + 1$

Differentiating both sides w.r.t 'u'.

$$\frac{dy}{du} = \frac{d(2u^2 - 3u + 1)}{du}$$

$$= 4u - 3$$

Also,

$u = 2x^2$

Differentiating both sides w.r.t 'x'

or, $$\frac{du}{dx} = \frac{d}{dx}(2x^2)$$

$$= 4x$$

Now,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (4u-3) \cdot 4x$$

$$= (4 \cdot 2x^2 - 3) \cdot 4x$$

$$= 4x(8x^2-3)$$

ii. $y = 2u^2 + 3$ and $u = 3x^2 - 1$

Solution:

$y = 2u^2 + 3$

Differentiating both sides w.r.t 'u'

or, $\frac{dy}{du} = \frac{d(2u^2+3)}{du}$

$= 4u$

Also,

$u = 3x^2 - 1$

Differentiating both sides w.r.t 'x'

or, $\frac{du}{dx} = \frac{d(3x^2-1)}{dx}$

$= 6x$

Now, $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{du}{dx}$

$= 4u \cdot 6x$

$= 4(3x^2-1)6x$

$= 24(3x^2-1)$

iii. $y = 5t^2 + 6t - 7$ and $t = x^3 - 2$

Solution:

$y = 5t^2 + 6t - 7$

Differentating both sides w.r.t 't'

or, $\frac{dy}{dt} = \frac{d(5t^2+6t-7)}{dt}$

$= 10t + 6$

Also,

$t = x^3 - 2$

Differentiating both side w.r.t 'x'

or, $\frac{dt}{dx} = \frac{d}{dx}(x^3 - 2)$

$= 3x^2$

Now,

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= (10t + 6) \cdot 3x^2$

$= \{10(x^3 - 2) + 6\} \cdot 3x^2$

$= 3x^2(10x^3 - 14)$

$= 6x^2(5x^3 - 7)$

iv. $y = \frac{t-1}{2t}$ and $t = \sqrt{x+1}$

Solution:

Let $y = \frac{t-1}{2t} = \frac{1}{2} - \frac{1}{2}t^{-1}$

Differentiating both sides w.r.t. 't'

or, $\frac{dy}{dt} = \frac{d}{dt}\left(\frac{1}{2} - \frac{1}{2}t^{-1}\right)$

$= -\frac{1}{2}(-1)t^{-2}$

$= \frac{1}{2t^2}$

Again,

$t = \sqrt{x+1} = (x+1)^{1/2}$

Differentiating both sides w.r.t 'x'

or, $\frac{dt}{dx} = \frac{d}{dx}(x+1)^{1/2}$

$= \frac{d(x+1)^{1/2}}{d(x+1)} \cdot \frac{d(x+1)}{dx}$

$= \frac{1}{2}(x+1)^{-1/2} \cdot 1$

$= \frac{1}{2\sqrt{x+1}}$

Now,

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{1}{2t^2} \cdot \frac{1}{2\sqrt{x+1}}$

$= \frac{1}{2(x+1)} \cdot \frac{1}{2\sqrt{x+1}}$

$= \frac{1}{4(x+1)^{3/2}}$

v. $y = (2u^2+3)^{1/3}$ and $u = \sqrt{(2x+1)}$

solution:

$y = (2u^2+3)^{1/3}$

Differentiating both side w.r.t 'u'

or, $\frac{dy}{du} = \frac{d}{du}(2u^2+3)^{1/3}$

$= \frac{d(2u^2+3)^{1/3}}{d(2u^2+3)} \cdot \frac{d(2u^2+3)}{du}$

$= \frac{1}{3}(2u^2+3)^{-2/3} \cdot 4u$

Again, $u = \sqrt{(2x+1)} = (2x+1)^{1/2}$

Differentiating both side w.r.t 'x'

or, $\frac{du}{dx} = \frac{d(2x+1)^{1/2}}{d(2x+1)} \cdot \frac{d(2x+1)}{dx}$

$= \frac{1}{2}(2x+1)^{-1/2} \cdot 2$

$= \dfrac{1}{(2x+1)^{1/2}}$

Now,

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$= \dfrac{4u}{3(2u^2+3)^{2/3}} \cdot \dfrac{1}{(2x+1)^{1/2}}$

$= \dfrac{4u}{3\{2(2x+1)^{1/2}+3\}^{2/3}} \cdot \dfrac{1}{(2x+1)^{1/2}}$

$= \dfrac{4\sqrt{2x+1}}{3(}$

=

=

=

vi. $y = \dfrac{t}{t^2-1}$ and $t = 3x^2+1$

Solution:

$y = \dfrac{t}{t^2-1}$

Differentiating both side w.r.t. 't'.

or, $\dfrac{dy}{dt} = \dfrac{d}{dt}\left(\dfrac{t}{t^2-1}\right)$

$= (t^2-1)\dfrac{d}{dt}(t) - t \cdot \dfrac{d}{dt}(t^2-1) \Big/ (t^2-1)^2$

$= \dfrac{(t^2-1) \cdot 1 - t \cdot 2t}{(t^2-1)^2}$

$= \dfrac{-t^2-1}{(t^2-1)^2}$

Again,

$t = 3x^2 + 1$

Differentiating both side w.r.t 'x'

$\frac{dt}{dx} = \frac{d}{dx}(3x^2+1)$

$= 6x$

Now,

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{-t^2-1}{(t^2-1)^2} \cdot 6x$

$= -\frac{\{(3x^2+1)^2+1\} \cdot 6x}{\{(3x^2+1)^2-1\}^2}$

$= -\frac{(9x^4+6x^2+1+1) \cdot 6x}{\{9x^4+6x^2-1+1\}^2}$

$= -\frac{(9x^4+6x^2+2) \cdot 6x}{(9x^4+6x^2)^2}$

$=$

---

7. Use implicit differentiation to obtain $\frac{dy}{dx}$ in the following

i. $x^2 + y^2 = 16$

Solution:

Differentiating both sides w.r.t 'x'

or, $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(16)$

or, $\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$

or, $2x + \frac{dy^2}{dy} \cdot \frac{dy}{dx} = 0$

or, $2x + 2y \cdot \frac{dy}{dx} = 0$

or, $2y \frac{dy}{dx} = -2x$

or, $\frac{dy}{dx} = \frac{-2x}{2y}$

$\therefore \frac{dy}{dx} = \frac{-x}{y}$

ii. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Solution:

Differentiating both side w.r.t 'x'

or, $\frac{d}{dx}\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = \frac{d}{dx}(1)$

or, $\frac{1}{a^2} \cdot \frac{d}{dx}(x^2) - \frac{1}{b^2} \cdot \frac{d}{dx}(y^2) = 0$

or, $\frac{1}{a^2} \cdot 2x - \frac{1}{b^2} \cdot \frac{dy^2}{dy} \cdot \frac{dy}{dx} = 0$

or, $\frac{1}{a^2} \cdot 2x - \cancel{B}\,\frac{1}{b^2} \cdot 2y \cdot \frac{dy}{dx} = 0$

or, $\frac{2x}{a^2} - \cancel{\frac{2y}{b^2}}\,\frac{2y}{b^2} \cdot \frac{dy}{dx} = 0$

or $\frac{2x}{a^2} = \frac{2y}{b^2} \cdot \frac{dy}{dx}$

or, $2xb^2 = 2a^2y \cdot \frac{dy}{dx}$

or, $\cancel{\frac{dy}{dx} = \frac{2a^2y}{2xb^2}}$

or, $\dfrac{2xb^2}{2a^2y} = \dfrac{dy}{dx}$

$\therefore \dfrac{dy}{dx} = \dfrac{b^2x}{a^2y}$

iii. $y^2 = 4ax$

Solution:

Differentiating both sides w.r.t 'x'

$\dfrac{d}{dx}(y^2) = \dfrac{d}{dx}(4ax)$

or, $\dfrac{dy^2}{dy} \cdot \dfrac{dy}{dx} = 4 \cdot \left[a \cdot \dfrac{d}{dx}(x) + x \cdot \dfrac{d}{dx}(a) \right]$

or, $2y \cdot \dfrac{dy}{dx} = 4a \cdot 1$

or, $\dfrac{dy}{dx} = \dfrac{4a}{2y}$

$\therefore \dfrac{dy}{dx} = \dfrac{2a}{y}$

iv. $2x^2 + 3xy + 2y^2 = 0$

Solution:

~~Der~~ Differentiating both side w.r.t 'x'

or, $\dfrac{d}{dx}(2x^2 + 3xy + 2y^2) = 0$

or, $2\dfrac{d}{dx}(x^2) + 3\left(x \cdot \dfrac{d}{dx}(y) + y \cdot \dfrac{dx}{dx} \right) + 2 \cdot \dfrac{dy^2}{dy} \cdot \dfrac{dy}{dx} = 0$

or, $4x + 3x \cdot \dfrac{dy}{dx} + 3y \cdot 1 + 4y \cdot \dfrac{dy}{dx} = 0$

or, $\dfrac{dy}{dx}\left(3x + 4y \right) = -4x - 3y$

$\therefore \dfrac{dy}{dx} = \dfrac{-4x-3y}{3x+4y}$

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v. $x^2 + 2x^2y = y^3$

Solution:

Differentiating both sides w.r.t 'x'

$\frac{d}{dx}(x^2 + 2x^2y) = \frac{d}{dx}(y^3)$

or, $\frac{d(x^2)}{dx} + 2\frac{d}{dx}(x^2 \cdot y) = \frac{dy^3}{dy} \cdot \frac{dy}{dx}$

or, $2x + 2\left(x^2 \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x^2)\right) = 3y^2 \cdot \frac{dy}{dx}$

or, $2x + 2\left(x^2 \frac{dy}{dx} + 2xy\right) = 3y^2 \cdot \frac{dy}{dx}$

or, $2x + 2x^2 \cdot \frac{dy}{dx} + 4xy = 3y^2 \cdot \frac{dy}{dx}$

or, $2x(1 + 2xy) = 3y^2 \cdot \frac{dy}{dx} - 2x^2 \cdot \frac{dy}{dx}$

or, $2x(1 + 2xy) = \frac{dy}{dx}(3y^2 - 2x^2)$

$\therefore \frac{dy}{dx} = \frac{2x(1 + 2xy)}{(3y^2 - 2x^2)}$

vi. $x^3y^6 = (x+y)^9$

Solution:

Differentiating both sides w.r.t. 'x'

or, $\frac{d}{dx}(x^3y^6) = \frac{d}{dx}(x+y)^9$

or, $x^3 \cdot \frac{d}{dx}y^6 + y^6 \cdot \frac{d}{dx}x^3 = \frac{d(x+y)^9}{d(x+y)} \cdot \frac{d(x+y)}{dx}$

or, $x^3 \cdot \frac{dy^6}{dy} \cdot \frac{dy}{dx} + y^6 \cdot 3x^2 = 9(x+y)^8\left\{1 + \frac{dy}{dx}\right\}$

or, $x^3 \cdot 6y^5 \cdot \frac{dy}{dx} + 3x^2y^6 = 9(x+y)^8 + 9(x+y)^8 \frac{dy}{dx}$

or, $6x^3y^5\frac{dy}{dx} \cancel{8} - 9(x+y)^8\frac{dy}{dx} = 9(x+y)^8 - 3x^2y^6$

or, $\frac{dy}{dx} = \frac{9(x+y)^8 - 3x^2y^6}{6x^3y^5 - 9(x+y)^8}$

or, $\frac{dy}{dx} = \frac{9\frac{(x+y)^9}{(x+y)} - 3x^2y^6}{6x^3y^5 - 9\frac{(x+y)^9}{(x+y)}}$

or, $\frac{dy}{dx} = \frac{\frac{9x^3y^6 - 3x^2y^6(x+y)}{(x+y)}}{\frac{(x+y)6x^3y^5 - 9(x^3y^6)}{(x+y)}}$ $[\because x^3y^6 = (x+y)^9]$

or, $\frac{dy}{dx} = \frac{9x^3y^6 - 3x^3y^6 - 3x^2y^7}{6x^4y^5 + 6x^3y^6 - 9x^3y^6}$

or, $\frac{dy}{dx} = \frac{6x^3y^6 - 3x^2y^7}{6x^4y^5 - 3x^3y^6}$

or, $\frac{dy}{dx} = \frac{3x^2y^6(2x-y)}{3x^3y^5(2x-y)}$

or, $\frac{dy}{dx} = \frac{y^{6-5}}{x^{3-2}}$

$\therefore \frac{dy}{dx} = \frac{y}{x}$

vii. $x^2y + xy^2 = a^3$

Solution:

Differentiating both sides w.r.t 'x'

or, $\frac{d(x^2y + xy^2)}{dx} = \frac{d(a^3)}{dx}$

or, $\frac{d(x^2y)}{dx} + \frac{d(xy^2)}{dx} = 0$

or, $x^2 \cdot \frac{dy}{dx} + y \cdot \frac{dx^2}{dx} + x \cdot \frac{dy^2}{dx} + y^2 \cdot \frac{dx}{dx} = 0$

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or, $x^2 \cdot \frac{dy}{dx} + 2xy + x \cdot \frac{dy^2}{dy} \cdot \frac{dy}{dx} + y^2 \cdot 1 = 0$

or, $x^2 \cdot \frac{dy}{dx} + 2xy + 2xy \cdot \frac{dy}{dx} + y^2 = 0$

or, $x^2 \frac{dy}{dx} + 2xy \cdot \frac{dy}{dx} = -2xy - y^2$

or, $\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$

or, $\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$

$\therefore \frac{dy}{dx} = \frac{-y(2x+y)}{x(x+2y)}$

7.viii. $x^3 + y^3 = 3xy^2$

Solution:

Differentiating both sides w.r.t 'x'.

or, $\frac{d(x^3+y^3)}{dx} = \frac{d(3xy^2)}{dx}$

or, $\frac{d(x^3)}{dx} + \frac{d(y^3)}{dx} = 3 \cdot \frac{d(xy^2)}{dx}$

or, $3x^2 + \frac{dy^3}{dy} \cdot \frac{dy}{dx} = 3\left\{x \cdot \frac{dy^2}{dy} \cdot \frac{dy}{dx} + y^2 \cdot \frac{d(x)}{dx}\right.$

or, $3x^2 + 3y^2 \cdot \frac{dy}{dx} = 3\left\{2xy \cdot \frac{dy}{dx} + y^2 \cdot 1\right\}$

or, $3x^2 + 3y^2 \cdot \frac{dy}{dx} = 6xy \cdot \frac{dy}{dx} + 3y^2$

or, $3x^2 - 3y^2 = 6xy \cdot \frac{dy}{dx} - 3y^2 \cdot \frac{dy}{dx}$

or, $3(x^2-y^2) = \frac{dy}{dx}(6xy - 3y^2)$

or, $\frac{3(x^2-y^2)}{6xy-3y^2} = \frac{dy}{dx}$

or, $\frac{dy}{dx} = \frac{3(x^2-y^2)}{3y(2x-y)}$

$\therefore \frac{dy}{dx} = \frac{(x^2-y^2)}{y(2x-y)}$

ix. $x^2y^2 = x^2 + y^2$

Solution:

Differentiating both side ~~with~~ w.r.t 'x'

or, $\frac{d(x^2y^2)}{dx} = \frac{d(x^2+y^2)}{dx}$

or, $x^2 \cdot \frac{dy^2}{dy} \cdot \frac{dy}{dx} + y^2 \cdot \frac{dx^2}{dx} = \frac{d(x^2)}{dx} + \frac{dy^2}{dy} \cdot \frac{dy}{dx}$

or, $2x^2y \cdot \frac{dy}{dx} + 2xy^2 = 2x + 2y \cdot \frac{dy}{dx}$

or, $2x^2y \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} = 2x - 2xy^2$

or, $\frac{dy}{dx}(2x^2y - 2y) = 2x(1 - y^2)$

or, $\frac{dy}{dx} = \frac{2x(1-y^2)}{2y(x^2-1)}$

$\therefore \frac{dy}{dx} = \frac{x}{y} \cdot \frac{(1-y^2)}{(x^2-1)}$

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8. Find $\frac{dy}{dx}$ when,

a. $x = 2a(t^2+1)$, $y = 2at^3$

Solution:

Differentiating both side w.r.t 't'

or, $\frac{dx}{dt} = 2a \cdot \frac{d(t^2+1)}{dt}$

or, $\frac{dx}{dt} = 2a\left\{\frac{dt^2}{dt} + \frac{d}{dt}(1)\right\}$

or, $\frac{dx}{dt} = 2a\{2t + 0\}$

$= 4at$

$\therefore \frac{dt}{dx} = \frac{1}{4at}$

Also,

$y = 2at^3$

Differentiating both sides w.r.t 't'

$\frac{dy}{dt} = 2a \cdot \frac{d}{dt}(t^3)$

$= 2a \times 3t^2$

$= 6at^2$

Now,

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= 6at^2 \times \frac{1}{4at}$$

$$= \frac{3t^{2-1}}{2}$$

$$= \frac{3}{2}t$$

b. $x = 2t^2 - 1$, $y = t^4 - 3$

Solution:

$x = 2t^2 - 1$ and $y = t^4 - 3$

Differentiating both sides w.r.t 't'

$$\frac{dx}{dt} = 2 \cdot \frac{d}{dt}(t^2 - 1) \quad , \quad \frac{dy}{dt} = \frac{d(t^4 - 3)}{dt}$$

$$= 2 \cdot 2t \qquad \text{or, } \frac{dy}{dt} = 4t^3$$

$$\text{or, } \frac{dt}{dx} = \frac{1}{4t}$$

Now,

$$\frac{dy}{dx} = \cancel{dx}\,\frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= 4t^3 \cdot \frac{1}{4t}$$

$$= t^2$$

c. $x = u^3 + 3u + 1, \quad y = 3u^2 + 1$

Solution:

$x = u^3 + 3u + 1, \quad y = 3u^2 + 1$

Differentiating both side w.r.t 'u'

or, $\frac{dx}{du} = \frac{d}{du}(u^3 + 3u + 1), \qquad \frac{dy}{du} = \frac{d}{du}(3u^2+1)$

$= 3u^2 + 3 \qquad\qquad = 3 \cdot 2 \cdot u$

$\qquad\qquad\qquad\qquad = 6u$

or, $\frac{du}{dx} = \frac{1}{3(u^2+1)}$

Now,

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 6u \cdot \frac{1}{3(u^2+1)}$

$= \frac{2u}{u^2+1}$

d. $x = \frac{2a}{t}, \quad y = \frac{1}{t^2}$

Solution:

$x = 2at^{-1}, \quad y = t^{-2}$

Differentiating both sides w.r.t 't'

$\frac{dx}{dt} = 2a \cdot \frac{dt^{-1}}{dt}, \qquad \frac{dy}{dt} = \frac{dt^{-2}}{dt}$

$= -2at^{-2} \qquad\qquad = -2t^{-3}$

or, $\frac{dt}{dx} = \frac{1}{-2at^{-2}}$

Now,

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$$= -2t^{-3} \cdot \frac{1}{-2at^{-2}}$$

$$= \frac{t^{-3+2}}{a}$$

$$= \frac{1}{at}$$

e. $x = v^2 - 1$, $y = \frac{v-1}{2v}$

Solution:

$x = v^2 - 1$

Differentiating both sides w.r.t 'v'

$$\frac{dx}{dv} = \frac{d}{dv}(v^2-1)$$

$$= 2v$$

Also,

$y = \frac{v-1}{2v}$

Differentiating both side w.r.t 'v'

$$\frac{dy}{dv} = \frac{d}{dv}\left(\frac{v-1}{2v}\right)$$

$$= \frac{2v \cdot \frac{d(v-1)}{dv} - (v-1) \cdot \frac{d}{dv} \cdot 2v}{(2v)^2}$$

$$= \frac{2v - (v-1)2}{4v^2}$$

$$= \frac{2v - 2v + 2}{4v^2}$$

$$= \frac{2}{4v^2}$$

Now, $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$

$$= \frac{2}{4v^2} \cdot \frac{1}{2v}$$

$$= \frac{1}{4v^3}$$

f. $x = \frac{2av}{1+v}$, $y = \frac{2av}{1-v}$

Solution:

$x = \frac{2av}{1+v}$

Differentiating both side w.r.t 'v'

$$\frac{dx}{dv} = \frac{d}{dv}\left(\frac{2av}{1+v}\right)$$

$$= \frac{(1+v) \cdot \frac{d}{dv}(2av) - 2av \cdot \frac{d}{dv}(1+v)}{(1+v)^2}$$

$$= \frac{(1+v) \cdot 2a - 2av}{(1+v)^2}$$

$$= \frac{2a + 2av - 2av}{(1+v)^2}$$

$$= \frac{2a}{(1+v)^2}$$

Also,

$y = \frac{2av}{1-v}$

Differentiating both side w.r.t 'v'

$$\frac{dy}{dv} = \frac{d}{dv}\left(\frac{2av}{1-v}\right)$$

$$= \frac{(1-v)\cdot\frac{d}{dv}(2av) - (2av)\cdot\frac{d}{dv}(1-v)}{(1-v)^2}$$

$$= \frac{(1-v)\cdot 2a - 2av\cdot(-1)}{(1-v)^2}$$

$$= \frac{2a - 2av + 2av}{(1-v)^2}$$

$$= \frac{2a}{(1-v)^2}$$

Now,

$$\frac{dy}{dx} = \frac{dy}{dv}\cdot\frac{dv}{dx}$$

$$= \frac{2a}{(1-v)^2} \times \frac{(1+v)^2}{2a}$$

$$= \frac{(1+v)^2}{(1-v)^2}$$

9:- Differentiate x^6 w.r.t. x^2

Solution:

Let $y = x^6$

Then, $\frac{dy}{dx} = \frac{d}{dx}(x^6)$

$= 6x^5$

Again, Let $z = x^2$

Then, $\frac{dz}{dx} = \frac{d}{dx}(x^2)$

$= 2x$

Now, $\frac{d(x^6)}{d(x^2)} = \frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{6x^5}{2x}$

$= 3x^4$

ii. Differentiate $(3x+1)^4$ w.r.t $(3x+1)$

Solution:

Let $y = (3x+1)^4$

Then, $\frac{dy}{dx} = \frac{d}{dx}(3x+1)^4$

$$= \frac{d(3x+1)^4}{d(3x+1)} \cdot \frac{d(3x+1)}{dx}$$

$$= 4(3x+1)^3 \cdot (3)$$

$$= 12(3x+1)^3$$

Also,

Let $z = (3x+1)$

Then, $\frac{dz}{dx} = \frac{d}{dx}(3x+1)$

$$= 3$$

Now,

$$\frac{d(3x+1)^4}{d(3x+1)} = \frac{dy}{dz} = \frac{dy/dx}{dz/dx}$$

$$= \frac{12(3x+1)^3}{3}$$

$$= 4(3x+1)^3$$

iii. Differentiate $2x^6 - 3x^4 + x^2$ w.r.t. x^3

Solution:

Let $y = 2x^6 - 3x^4 + x^2$

Then, $\frac{dy}{dx} = \frac{d(2x^6 - 3x^4 + x^2)}{dx}$

$$= 12x^5 - 12x^3 + 2x$$

Also,

Let $z = x^3$

Then, $\frac{dz}{dx} = \frac{dx^3}{dx} = 3x^2$

Now,

$$\frac{d(2x^6 - 3x^4 + x^2)}{d(x^3)} = \frac{dy}{dz} = \frac{dy/dx}{dz/dx}$$

$$= \frac{12x^5 - 12x^3 + 2x}{3x^2}$$

$$= 4x^3 - 4x + \frac{2}{3x}$$

iv. Differentiate $(5x-1)^6$ w.r.t $(x+1)^2$

Solution:

Let $y = (5x-1)^6$

Then, $\frac{dy}{dx} = \frac{d(5x-1)^6}{dx}$

$$= \frac{d(5x-1)^6}{d(5x-1)} \cdot \frac{d(5x-1)}{dx}$$

$$= 6(5x-1)^5 \cancel{(-5)} (5)$$

Also,

let $z = (x+1)^2$

Then, $\frac{dz}{dx} = \frac{d(x+1)^2}{dx}$

$$= \frac{d(x+1)^2}{d(x+1)} \cdot \frac{d(x+1)}{dx}$$

$$= 2(x+1).$$

Now,

$$\frac{d(5x-1)^6}{d(x+1)^2} = \frac{dy}{dz} = \frac{dy/dx}{dz/dx}$$

$$= \frac{30(5x-1)^5}{2(x+1)}$$

$$= \frac{15(5x-1)^5}{x+1}$$