Chapter - 11
Coordinates in space

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If 'd' be the distance between any two points (x_1, y_1, z_1) and $(x_2, y_2, ~~and~~ z_2)$ in the space then,

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

If $P(x, y, z)$ divides the line joining the any two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m_1 : m_2$ then

$$(x, y, z) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}, \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2} \right)$$

If P divides externally then,

$$(x, y, z) = \left(\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}, \frac{m_1 z_2 - m_2 z_1}{m_1 - m_2} \right)$$

If $P(x, y, z)$ is the midpoint of line joining $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ then,

$$(x, y, z) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Exercise - 11.1

1. Find the distance between the points.

a. $(-1, 4, 3)$ and $(2, 2, -3)$

Solution:

Using the distance formula,

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$= \sqrt{(2+1)^2 + (2-4)^2 + (-3-3)^2}$

$= \sqrt{9 + 4 + 36}$

$= 7$

b. $(4, -1, 5)$ and $(-4, 3, 6)$

Solution:

Using the distance formula,

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$= \sqrt{(-4-4)^2 + (3+1)^2 + (6-5)^2}$

$= \sqrt{64 + 16 + 1}$

$= 9$

2.a. Show that the points $(2, 0, -4)$, $(4, 2, 4)$ and $(10, 2, -2)$ are the vertices of an equilateral triangle.

Solution:

Let the given points be $A(2, 0, -4)$, $B(4, 2, 4)$ and $C(10, 2, -2)$

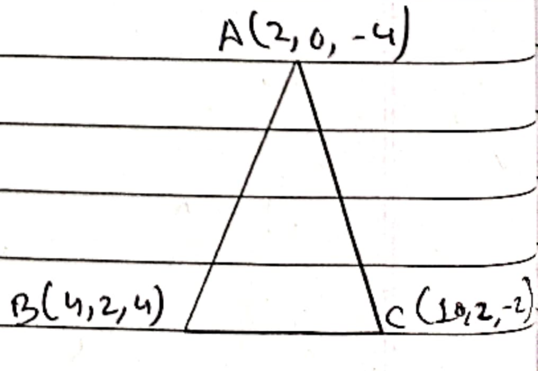
Now, distance between AB is;

$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$= \sqrt{(4-2)^2 + (2-0)^2 + (4+4)^2}$

$= \sqrt{4 + 4 + 64}$

$= \sqrt{72}$



Again,

Distance between BC is;

$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$= \sqrt{(10-4)^2 + (2-2)^2 + (-2-4)^2}$

$= \sqrt{36 + 36} = \sqrt{72}$

Distance between AC is;

$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$= \sqrt{(10-2)^2 + (2-0)^2 + (-2+4)^2}$

$= \sqrt{64+4+4}$

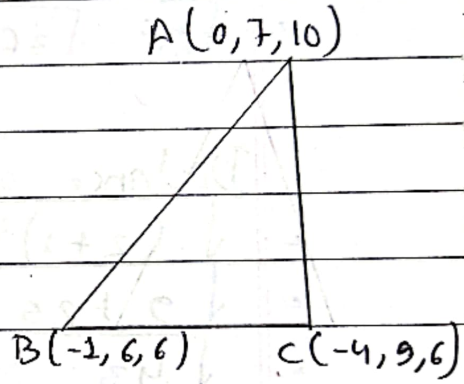
$= \sqrt{72}$

Since, AB = BC = AC. So, the given points are the vertices of an equilateral triangle.

b. Show that the point $(0,7,10)$, $(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled isosceles triangle.

Solution:

Let the given points be $A(0,7,10)$, $B(-1,6,6)$ and $C(-4,9,6)$



Now,

Distance of AB is

$= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

Distance of BC is

$= \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$

$= \sqrt{9+9+0}$

$= \sqrt{18}$

Distance of AC is

$= \sqrt{\cancel{(-4+1)}\,(-4-0)^2 + (9-7)^2 + (6-10)^2}$

$= \sqrt{16+4+16}$

$= 6$

Here, $AB = BC = \sqrt{18}$ and $AC = 6$. Hence it is an isosceles triangle.

Now, $(AC)^2 = (AB)^2 + (BC)^2$

or, $6^2 = (\sqrt{18})^2 + (\sqrt{18})^2$

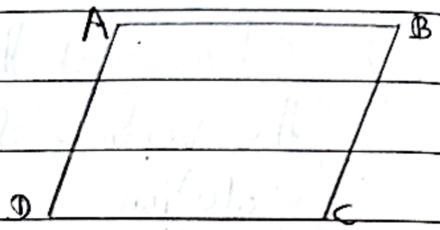
or, $36 = 36$.

Hence, it is ~~an isos~~ right angled triangle.

3.a. Show that the points $(1,2,3)$, $(-1,-2,-1)$, $(2,3,2)$ ~~and~~ are the vertices of a parallelogram.

Solution:

Let the points be $A(1,2,3)$, $B(-1,-2,-1)$, $C(2,3,2)$ and $D(4,7,6)$



Now,

Distance of the side AB is

$= \sqrt{(-1-1)^2 + (-2-2)^2 + (-1-3)^2}$

$= \sqrt{4 + \cancel{0} + 16 + 16}$

$= \cancel{\sqrt{26}}\ \sqrt{36}$

Distance of the ~~side~~ point BC is

$= \sqrt{(2+1)^2 + (3+2)^2 + (2+1)^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$

Distance of the points CD is

$= \sqrt{(4-2)^2 + (7-3)^2 + (6-2)^2}$

$= \sqrt{4 + 16 + 16}$

$= \sqrt{36}$

Distance between the point AD is

$= \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$

Here,

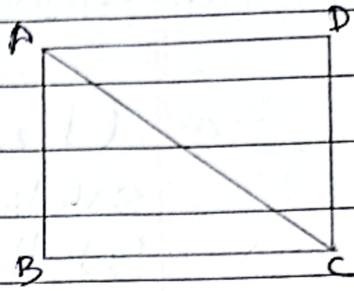
Length of AB = Length of CD and Length of BC = Length of AD. So, the given points are the vertices of a llgm.

6. Show that the points $(1,1,1)$, $(-2,4,1)$, $(-1,5,5)$ and $(2,2,5)$ are the vertices of a square.

Solution:

Let the given points be $A(1,1,1)$, $B(-2,4,1)$, $C(-1,5,5)$ and $D(2,2,5)$.

Now,



Distance between the points AB is

$= \sqrt{(-2-1)^2+(4-1)^2+(1-1)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

Distance between the points BC is

$= \sqrt{(-1+2)^2+(5-4)^2+(5-1)^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

Distance of between the points CD is

$= \sqrt{(2+1)^2+(2-5)^2+(5-5)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

Distance between the points AD is

$= \sqrt{(2-1)^2+(2-1)^2+(5-1)^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

Distance of the diagonal AC is

$= \sqrt{(-1-1)^2+(5-1)^2+(5-1)^2}$

$= \sqrt{4+16+16}$

$= 6$

Here, $AB = BC = CD = AD = \sqrt{18}$

Now, $(AC)^2 = (AB)^2 + (BC)^2$

or, $6^2 = (\sqrt{18})^2 + (\sqrt{18})^2$

or, $36 = 36$ [right angle proved]

Hence, the given points are the vertices of a square

4. Show that the following points are collinear.

a. $(1,2,3)$, $(-2,3,4)$ and $(7,0,1)$

Solution:

Let the given points be $A(1,2,3)$, $B(-2,3,4)$ and $C(7,0,1)$

Then,

$$AB = \sqrt{(-2-1)^2 + (3-2)^2 + (4-3)^2}$$
$$= \sqrt{9+1+1}$$
$$= \sqrt{11}$$

$$BC = \sqrt{(7+2)^2 + (0-3)^2 + (1-4)^2}$$
$$= \sqrt{81+9+9}$$
$$= \sqrt{99}$$

$$AC = \sqrt{(7-1)^2 + (0-2)^2 + (1-3)^2}$$
$$= \sqrt{36+4+4}$$
$$= \sqrt{44}$$

Now,

$$AB + AC = \sqrt{11} + \sqrt{44}$$
$$= \sqrt{99}$$
$$= BC \text{ proved}$$

Hence, A, B, C are collinear.

b. $(-2,3,5)$, $(1,2,3)$ and $(7,0,-1)$

Solution:

Let the given points be $A(-2,3,5)$, $B(1,2,3)$ and $C(7,0,-1)$

Then,

$$AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$

$$AC = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$

So,

$$AB + BC = \sqrt{14} + \sqrt{56}$$
$$= \sqrt{126}$$
$$= AC$$

Hence, A, B and C are collinear.

5.a. Find the locus of a point which moves such that its distance from the fixed point $(-1,2,-2)$ is always 5.

Solution:

Let $P(x,y,z)$ be the point on the locus and it's distance from $A(1,2,-2)$ is always 5.

i.e. $AD = 5$

or, $\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2} = 5$

or, $\sqrt{x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + 4z + 1} = 5$

or, $x^2 \cancel{-2x} + y^2 + z^2 \cancel{-2xy} - 2x - 4y + 4z - 16 = 0$ which is the required equation:

b. Find the locus of a point which moves such that it is equidistant from two fixed point $(-1,2,3)$ and $(4,-1,5)$

Solution:

Let the points be $A(-1,2,3)$ and $B(4,-1,5)$

Let $P(a,b,c)$ be the point of locus.

Now,

$PA = \sqrt{(-1-a)^2 + (2-b)^2 + (3-c)^2}$

$= \sqrt{1+2a+a^2+4-4b+b^2+9-6c+c^2}$

Also

$PB = \sqrt{(4-a)^2 + (-1-b)^2 + (5-c)^2}$

$= \sqrt{16-8a+a^2+1+2b+b^2+25-10c+c^2}$

By question:

$(PA)^2 = (PB)^2$

or, $1+a^2+2a+4-4b+b^2+9-6c+c^2 = 16-8a+a^2+1+2b+b^2+25-10c+c^2$

or, $14+2a-4b+b^2-6c = 42-8a+2b+b^2-10c$

or, $10a - 6b + 4c = 28$

or, $5a-3b+2c-14=0$ is the required equation.

6. Find the coordinates of the points which is equidistant from the the four points O, A, B, and C where O is the origin and A, B and C are the points on the x-, y- and z-axis respectively at distance a, b and c from the origin.

Solution:

Let $P(x,y,z)$ be the coordinate of point which is equidistant from the four points $O(0,0,0)$, $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$

i.e. $OP = AP = BP = CP$

Now,

$(OP)^2 = (AP)^2$

or, $(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-a)^2 + (y-0)^2 + (z-0)^2$

or, $x^2+y^2+z^2 = x^2-2ax+a^2+y^2+z^2$

or, $0 = -2ax+a^2$

or, $0 = \cancel{a(-2x+a)}\ a(-2x+a)$

$\therefore x = \frac{a}{2}$

Also,

$(OP)^2 = (BP)^2$

or, $(x-0)^2+(y-0)^2+(z-0)^2 = (x-0)^2+(y-b)^2+(z-0)^2$

or, $x^2+y^2+z^2 = x^2+y^2-2yb+b^2+z^2$

or, $0 = -2yb+b^2$

or, $0 = b(-2y+b)$

$\therefore y = \frac{b}{2}$

Also,

$(OP)^2 = (CP)^2$

or, $(x-0)^2+(y-0)^2+(z-0)^2 = (x-0)^2+(y-0)^2+(z-c)^2$

or, $x^2+y^2+z^2 = x^2+y^2+z^2-2zc+c^2$

or, $0 = -2zc+c^2$

or, $0 = c(-2z+c)$

$\therefore z = \frac{c}{2}$

Hence, the required points is $P\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

7. Find the coordinates of ~~an~~ the point which divides the line segment joining each of the following pair of points internally in the ratio 1:2 and externally in the ratio 3:2

a. $(1,-2,3)$ and $(1,2,3)$

Solution:

Let the point $P(x,y,z)$ divides the line joining the point $A(1,-2,3)$ and $B(1,2,3)$ in the ratio 1:2 internally.

Here, $m_1 = 1$ and $m_2 = 2$

Now,

$$(x,y,z) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}, \frac{m_1z_2 + m_2z_1}{m_1+m_2}\right)$$

$$= \left(\frac{1.1 + 2.1}{1+2}, \frac{1.2 + 2.(-2)}{1+3}, \frac{1.3 + 2.3}{1+3}\right)$$

$$= \left(1, -\frac{2}{3}, 3\right)$$

Again,

For the external division, Let x', y', and z' divides the Point $A(2,-2,3)$ and $B(1,2,3)$. in the ratio 3:2.

Then,

$$(x',y',z') = \left(\frac{m_1x_2 - m_2x_1}{m_1-m_2}, \frac{m_1y_2 - m_2y_1}{m_1-m_2}, \frac{m_1z_2 - m_2z_1}{m_1-m_2}\right)$$

$$= \left(\frac{3.1 - 2.1}{3-2}, \frac{3.2 + 2.2}{3-2}, \frac{3.3 - 2.3}{3-2}\right)$$

$$\therefore (x',y',z') = (1, 10, 3)$$

b. $(2,0,1)$ and $(4,-2,5)$

Solution:

Let the point $P(x,y,z)$ divides the line $A(2,0,1)$ and $B(4,-2,5)$ in the ratio 1:2 internally and $P'(x',y',z')$ divides the line in the ratio 3:2 externally.

Now, For internal division:

$$(x,y,z) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}, \frac{m_1z_2 + m_2z_1}{m_1+m_2}\right)$$

$$= \left(\frac{1.4 + 2.2}{1+2}, \frac{1.(-2) + 2.0}{1+2}, \frac{1.5 + 2.1}{1+3}\right)$$

$$= \left(\frac{8}{3}, \frac{-2}{3}, \frac{7}{3}\right)$$

For external division:

$$P'(x', y', z') = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2} \right)$$

$$= \left(\frac{3.4 - 2.2}{3-2}, \frac{3.(-2) - 2.0}{3-2}, \frac{3.5 - 2.1}{3-2} \right)$$

$$\therefore (x', y', z') = (8, -6, 13)$$

8. Find the Coordinates of the mid-point of the join of each of the following pair of points.

a. $(-4, 3, 6)$ and $(2, 1, -3)$

Solution:

Let (x, y, z) be the mid-points of the line joining the lines $A(-4, 3, 6)$ and $B(2, 1, -3)$

Then,

$$(x, y, z) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

$$= \left(\frac{-4+2}{2}, \frac{3+1}{2}, \frac{6-3}{2} \right)$$

$$= (-1, 2, 3/2)$$

$$\therefore (x, y, z) = \left(-1, 2, \frac{3}{2} \right)$$

b. $(2, 5, -8)$ and $(4, -1, 6)$

Solution:

Let the point (x, y, z) be the mid points of the line joining the points $A(2, 5, -8)$ and $B(4, -1, 6)$.

Then,

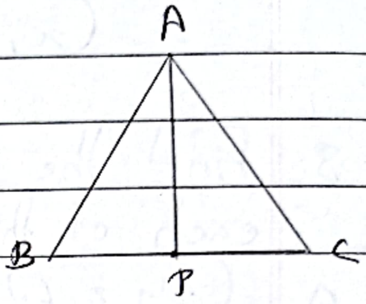
$$(x, y, z) = \left(\frac{2+4}{2}, \frac{5-1}{2}, \frac{-8+6}{2} \right)$$

$$= (3, 2, -1)$$

9. If $A(3,4,5)$, $B(-1,2,0)$ and $C(-3,4,-2)$ are the vertices of the triangle ABC, find the length of the median joining the vertex $A(3,4,5)$ and the middle point of its opposite side.

Solution:

Let P be the mid-point of the line BC



then,

$$P(x,y,z) = \left(\frac{-3-1}{2}, \frac{4+2}{2}, \frac{-2+0}{2}\right)$$

$$= (-2, 3, -1)$$

Here

AP is the median and the distance between them is given by;

$$\sqrt{(-2-3)^2+(3-4)^2+(-1-5)^2}$$

$$= \sqrt{25+1+36}$$

$$= \sqrt{62}$$

Hence, the distance or length of the median is $\sqrt{62}$ units.

10.a. Find the ratio in which the line joining the points $(-2,4,7)$ and $(3,-5,-8)$ is divided by the xy-plane.

Solution:

Let $P(a,b,0)$ on xy-plane divides the line joining $A(-2,4,7)$ and $B(3,-5,-8)$ in the ratio $m_1:m_2$

We know,

$$(a,b,0) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$$\text{or, } (a,b,0) = \left(\frac{m_1.3-2m_2}{m_1+m_2}, \frac{m_1\times -5+m_2.4}{m_1+m_2}, \frac{m_1(-8)+m_2.7}{m_1+m_2}\right)$$

$$\text{or, } (a,b,0) = \left(\frac{3m_1-2m_2}{m_1+m_2}, \frac{-5m_1+4m_2}{m_1+m_2}, \frac{-8m_1+7m_2}{m_1+m_2}\right)$$

or, $\frac{-8m_1 + 7m_2}{m_1 + m_2} = 0$

or, $-8m_1 + 7m_2 = 0$

or, $7m_1 = 8m_1$

or, $\frac{m_1}{m_2} = \frac{7}{8}$

Hence, the required ratio is 7:8.

b. Find the ratio in which the yz-plane divides the line joining the points (4, 6, 7) and (-1, 2, 5). Find also the coordinate of the points on the yz-plane.

Solution:

Let P(0, a, b) be the point on yz-plane which divides the line joining the points A(4, 6, 7) and B(-1, 2, 5) in the ratio $m_1 : m_2$.

Then,

$$(0, a, b) = \left(\frac{m.(-1) + m_2.4}{m_1 + m_2}, \frac{m_1.2 + m_2.6}{m_1 + m_2}, \frac{m_1.5 + m_2.7}{m_1 + m_2} \right)$$

~~So~~ or, $(0, a, b) = \left(\frac{-m_1 + 4m_2}{m_1 + m_2}, \frac{2m_1 + 6m_2}{m_1 + m_2}, \frac{5m_1 + 7m_2}{m_1 + m_2} \right)$

So,

$0 = \frac{-m_1 + 4m_2}{m_1 + m_2}$

or, $0 = -m_1 + 4m_2$

or, $m_1 = 4m_2$

or, $\frac{m_1}{m_2} = \frac{4}{1}$

Hence, $m_1 = 4$ and $m_2 = 1$.

Now, putting the value of m_1 and m_2 in the above equation to find the value of a and b.

We know,

$$a = \frac{2m_1 + 6m_2}{m_1 + m_2}$$

$$\text{or, } a = \frac{2 \times 4 + 6 \times 1}{4 + 1}$$

$$\therefore a = \frac{14}{5}$$

Also,

$$b = \frac{5m_1 + 7m_2}{m_1 + m_2}$$

$$\text{or, } b = \frac{5 \cdot 4 + 7 \cdot 1}{4 + 1}$$

$$= \frac{27}{5}$$

Hence, the required coordinates are $\left(0, \frac{14}{5}, \frac{27}{5}\right)$

c. Given three collinear points $A(3, 2, -4)$, $B(5, 4, -6)$ and $C(9, 8, -10)$, find the ratio in which B divides AC.

Solution:

Let $B(5, 4, -6)$ divides the line AC joining $A(3, 2, -4)$ and $C(9, 8, -10)$ in the ratio $m_1 : m_2$.

Now,

$$B(5, 4, -6) = \left(\frac{m_1 \cdot 9 + m_2 \cdot 3}{m_1 + m_2}, \frac{m_1 \cdot 8 + m_2 \cdot 2}{m_1 + m_2}, \frac{m_1(-10) + m_2 \cdot (-4)}{m_1 + m_2}\right)$$

$$= \left(\frac{9m_1 + 3m_2}{m_1 + m_2}, \frac{8m_1 + 2m_2}{m_1 + m_2}, \frac{-10m_1 - 4m_2}{m_1 + m_2}\right)$$

Here,

$$5 = \frac{9m_1 + 3m_2}{m_1 + m_2}$$

$$\text{or, } 5m_1 + 5m_2 = 9m_1 + 3m_2$$

$$\text{or, } 2m_2 = 4m_1$$

$$\text{or, } \frac{m_1}{m_2} = \frac{1}{2}$$

$$\therefore m_1 : m_2 = 1:2$$

11.a. Find the point where the line through the point (1,2,3) and (4,-4,9) meets the zx plane

Solution:

Let P(a, 0, b) be the point on zx plane where A(1,2,3) and B(4,-4,9) meet. Let P divides AB in the ratio $m_1 : m_2$

Then,

$$(a,0,b) = \left(\frac{m_1 4 + m_2 . 1}{m_1 + m_2}, \frac{m_1(-4) + m_2 . 2}{m_1 + m_2}, \frac{m_1 . 9 + m_2 . 3}{m_1 + m_2}\right)$$

$$= \left(\frac{4m_1 + m_2}{m_1 + m_2}, \frac{-4m_1 + 2m_2}{m_1 + m_2}, \frac{9m_1 + 3m_2}{m_1 + m_2}\right)$$

Here,

$$0 = \frac{-4m_1 + 2m_2}{m_1 + m_2}$$

$$\text{or, } 4m_1 = 2m_2$$

$$\text{or, } \frac{m_1}{m_2} = \frac{1}{2}$$

Now, putting the value of m_1 and m_2 above to get the value of a and b.

$$\text{So, } a = \left(\frac{4m_1 + m_2}{m_1 + m_2}\right), \quad b = \frac{m_1 . 9 + m_2 . 3}{m_1 + m_2}$$

$$\text{or, } a = \left(\frac{4 . 1 + 2}{3}\right) \quad = \frac{1 . 9 + 2 . 3}{3}$$

$$\therefore a = 2 \quad = 5$$

Hence,

The required points on zx plane is (2,0,5).

b. Find the points where the line joining the points $(2,-3,1)$ and $(3,-4,-5)$ ~~meet~~ cuts the plane $2x+y+z=7$.

Solution:

Given plane is $2x+y+z=7$

Let the plane divides the line AB in the ratio $m_1:m_2$ at the point $P(a,b,c)$

Then,

$$(a,b,c) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$$= \left(\frac{m_1.3+m_2.2}{m_1+m_2}, \frac{m_1(-4)+m_2.(-3)}{m_1+m_2}, \frac{m_1(-5)+m_2.1}{m_1+m_2}\right)$$

$$= \left(\frac{3m_1+2m_2}{m_1+m_2}, \frac{-4m_1-3m_2}{m_1+m_2}, \frac{-5m_1+m_2}{m_1+m_2}\right)$$

Here,

(a,b,c) lies on the plane. So, it must satisfy the equation $2x+y+z=7$

or, $2\left(\frac{3m_1+2m_2}{m_1+m_2}\right)+\left(\frac{-4m_1-3m_2}{m_1+m_2}\right)+\left(\frac{-5m_1+m_2}{m_1+m_2}\right)=7$

or, $6m_1+4m_2-4m_1-3m_2-5m_1+m_2=7m_1+7m_2$

or, $-3m_1+2m_2=7m_1+7m_2$

or, $-7m_2+2m_2=7m_1+3m_1$

or, $-5m_2=10m_1$

or, $\frac{-5}{10}=\frac{m_1}{m_2}$

$\therefore \frac{m_1}{m_2}=\frac{-1}{2}$

Now, putting the value of m_1 and m_2 in the above calculations to get the value of a, b and c.

$$(a,b,c) = \left(\frac{3.(-1)+2.2}{-1+2}, \frac{+4.1-3.2}{-1+2}, \frac{+5.1+2}{-1+2}\right)$$

or, $(a,b,c) = \left(\frac{-3+4}{1}, \frac{4-6}{1}, \frac{5+2}{1}\right)$

$= (1,-2,7)$

Hence, the required points are $(1,-2,7)$

12. Show that the following represent the vertices of a llgm.

a. $(3,0,1)$, $(2,2,2)$, $(-1,3,3)$ and $(0,1,2)$

Solution:

Let the vertices of a llgm be $A(3,0,1)$, $B(2,2,2)$, $C(-1,3,3)$ and $D(0,1,2)$

Now,

$AB = \sqrt{(2-3)^2+(2-0)^2+(2-1)^2}$

$= \sqrt{1+4+1}$

$= \sqrt{6}$

$BC = \sqrt{(-1-2)^2+(3-2)^2+(3-2)^2}$

$= \sqrt{9+1+1}$

$= \sqrt{11}$

$CD = \sqrt{(0+1)^2+(1-3)^2+(2-3)^2}$

$= \sqrt{1+4+1}$

$= \sqrt{6}$

$AD = \sqrt{(0-3)^2+(1-0)^2+(2-1)^2}$

$= \sqrt{9+1+1}$

$= \sqrt{11}$

Here, $AB = CD$ and $BC = AD$. So, the given points are the points of a parallelogram.

b. (1,3,4), (-1,6,10), (-7,4,7) and (-5,1,1)

Solution:

Let the given points be A (1,3,4), B(-1,6,10), C(-7,4,7) and D(-5,1,1).

Now,

$$AB = \sqrt{(-1-1)^2+(6-3)^2+(10-4)^2}$$
$$= \sqrt{4+9+36}$$
$$= \sqrt{49}$$
$$= 7 \text{ units.}$$

$$BC = \sqrt{(-7+1)^2+(4-6)^2+(7-10)^2}$$
$$= \sqrt{36+4+9}$$
$$= \sqrt{49}$$
$$= 7 \text{ units}$$

$$CD = \sqrt{(-5+7)^2+(1-4)^2+(1-7)^2}$$
$$= \sqrt{4+9+36}$$
$$= \sqrt{49}$$
$$= 7 \text{ units}$$

$$AD = \sqrt{(-5-1)^2+(1-3)^2+(1-4)^2}$$
$$= \sqrt{36+4+9}$$
$$= \sqrt{49}$$
$$= 7 \text{ units}$$

Here,

All the opposite sides are equal in length. Hence, the given points represents the parallelogram.

13. Two vertices of a triangle ABC are A(2, −4, 3) and B(3, −1, −2) and its centroid is (1, 0, 3). Find the third vertex C.

Solution:

Let the vertices of a triangle be A(2, −4, 3), B(3, −1, −2) and C(x, y, z). Let P(1, 0, 3) be the centroid of the triangle ABC.

Now, using centroid formula;

$$P(1,0,3) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$\text{or } (1,0,3) = \left(\frac{2+3+x}{3}, \frac{-4-1+y}{3}, \frac{3-2+z}{3}\right)$$

$$\text{or } (1,0,3) = \left(\frac{5+x}{3}, \frac{-5+y}{3}, \frac{1+z}{3}\right)$$

Comparing the corresponding elements, we get;

$$1 = \frac{5+x}{3} \qquad 0 = \frac{-5+y}{3} \qquad 3 = \frac{1+z}{3}$$

$$\therefore x = -2 \qquad \therefore y = 5 \qquad \therefore z = 8$$

Hence,

The required ~~co~~ points of the third vertex is ~~(-2,~~ (−2, 5, 8)

14. Three vertices of a parallelogram ABCD are A(−5, 5, 2), B(−9, −1, 2) and C(−3, −3, 0). Find the coordinates of the fourth vertex.

Solution:

Let the vertices of a ||gm be A(−5, 5, 2), B(−9, −1, 2), C(−3, −3, 0) and D(x, y, z).

Now,

~~Distance~~ Mid-point of diagonal AC is given by;

$= \sqrt{\cancel{\quad}}$

$$= \left(\frac{-5-3}{2}, \frac{-3+5}{2}, \frac{2+0}{2} \right)$$

$$= (-4, 1, 1)$$

Again,

Mid-point of diagonal BD is given by;

$$= \left(\frac{-9+x}{2}, \frac{-1+y}{2}, \frac{2+z}{2} \right)$$

Since, the given points represent the parallelogram, their diagonal should bisect each other. It ~~meas~~ means both the ~~mid~~ diagonal should have same mid-point.

So, $(-4, 1, 1) = \left(\frac{-9+x}{2}, \frac{-1+y}{2}, \frac{2+z}{2} \right)$

Comparing the corresponding elements, we get;

$-4 = \frac{-9+x}{2}$ $\quad$ $1 = \frac{-1+y}{2}$ $\quad$ $1 = \frac{2+z}{2}$

or $-8+9 = x$ $\quad$ or, $2+1 = y$ $\quad$ or $2-2 = z$

$\therefore x = 1$ $\quad$ $\therefore y = 3$ $\quad$ $\therefore z = 0$

Hence,

The points of the fourth vertex is $D(1, 3, 0)$