Wave Motion

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A wave is a continuous transfer of disturbance from one part of a medium to another through successive vibrations of the particles of the medium about their mean positions.

The wave carries energy and transport momentum but not the matter when it propagates in a medium.

Mechanical Wave

The waves which require a material medium for their propagation are called mechanical waves.

↳ Transverse Wave

The wave in which the particles of the medium vibrate about their mean position perpendicularly to the direction of propagation of the wave is called the transverse wave.

eg. waves produced in plucked string, waves on surface of water, radio waves, etc

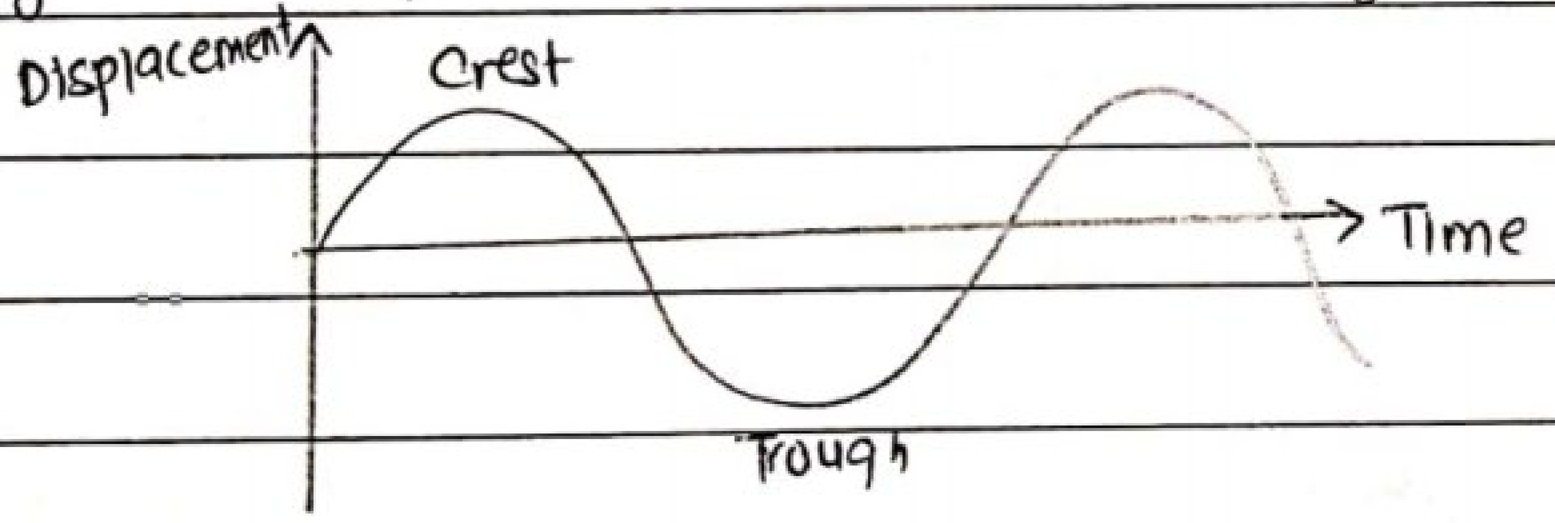


Fig: Transverse wave

↳ Longitudinal wave

The wave in which the particles of the medium vibrate along the direction of propagation of the wave is called longitudinal wave.

e.g. sound wave in air, wave in a spring which is suddenly compressed and released, sea waves, seismic P waves, etc

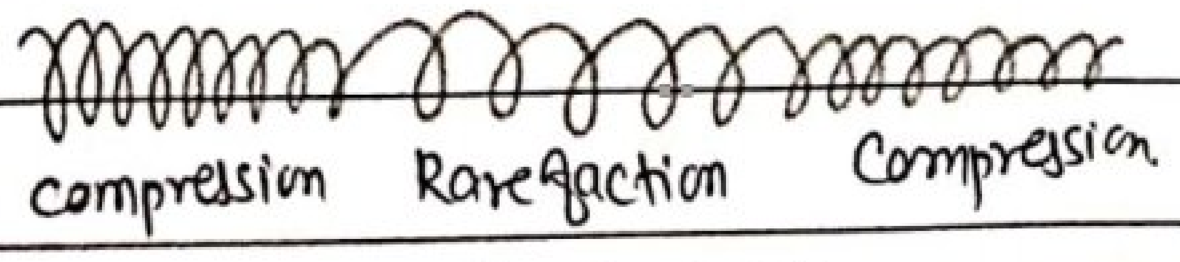


Fig: Longitudinal wave.

Progressive Wave

A progressive wave is one in which the disturbance is continuously transmitted along the direction of propagation of wave.

Wave Properties.

1. Wavelength

The distance between two nearest particles of the medium vibrating in same phase is called wavelength λ. It is equal to the distance travelled by the wave during the time at which any particles of the medium completes one complete vibration.

2. Frequency:

It is the no. of waves or crest passing through a given point per second. This is equal to no. of oscillations completed by the particles of the medium in one second.

3. Period

It is the time required for one complete wave to pass a given point. Since, f waves are produced in one second, then $f = 1/T$.

4. Amplitude

The maximum displacement of particles of medium from equilibrium positions when a wave passes through it is called the amplitude 'a' of wave.

5. Wave velocity

The velocity of a wave is the distance traveled by a crest in one second. A wave travels a distance of one wavelength λ in a time equal to one period T. Then, wave velocity, $v = \lambda/T = f\lambda$.

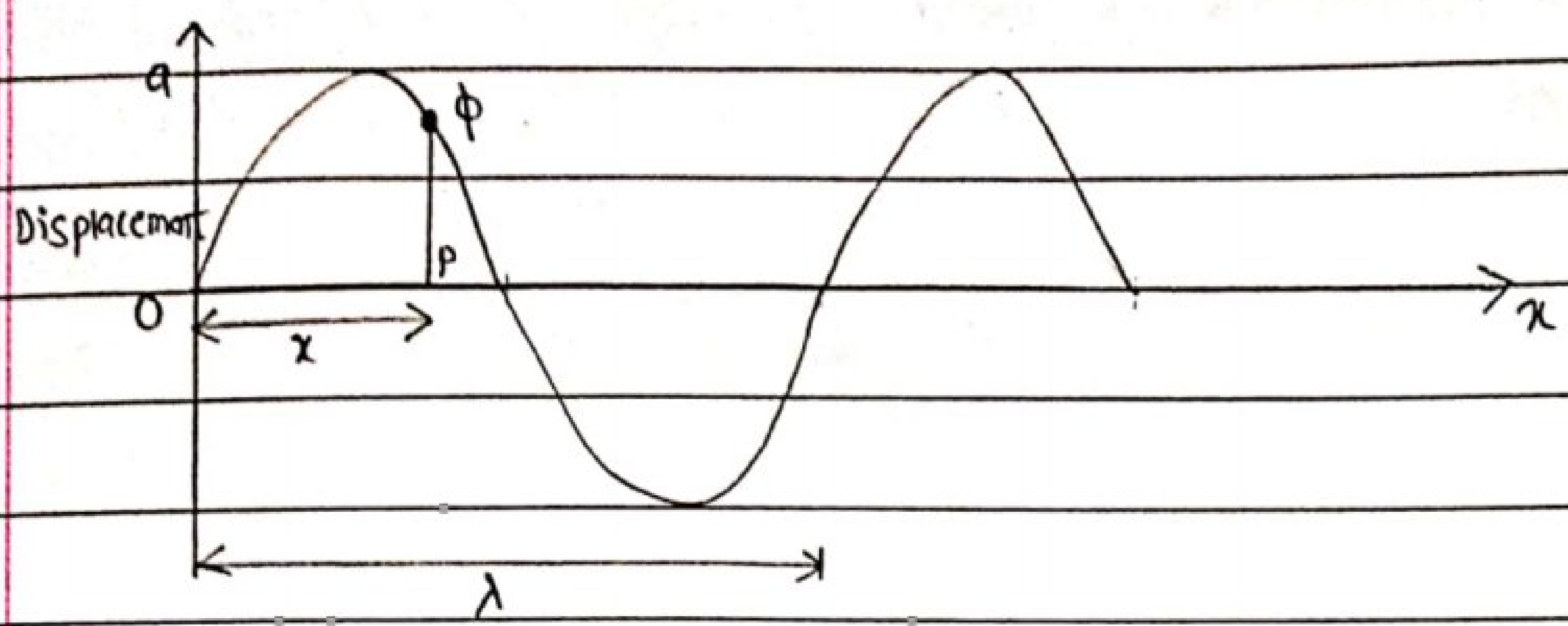
6. Phase.

It represents the current position of the wave relative to some reference position. It is measured in angles.

7. Particle velocity:

The velocity of the vibrating particles of a medium when a wave is passing through it is called particle velocity v_p. Relation between particle velocity and wave velocity is $v_p = -v\,(dy/dx)$. Particle velocity (dy/dt) changes with time but wave velocity $(v = \lambda f)$ is constant. So, acceleration of wave is zero but that of particle is not zero.

Equation of a Progressive Wave



Suppose a wave travelling from left to right along x-axis as shown in figure. Consider a particle at the origin O vibrating simple harmonically and its displacement at any instant t is:

$$y = a\sin\omega t \quad \text{—— (i)}$$

where a is amplitude of the particle and ω, its angular velocity. Let, another particle at P at a distance x from O. Since, the disturbance will reach later to the particles to right of O, the phase of motion of these particles lags to that of O and it goes on increasing. Let, ϕ be the phase difference of the particle at P, then its displacement is:

$$y = a\sin(\omega t - \phi) \quad \text{—— (ii)}$$

For a distance of λ, phase difference is 2π. So, the phase difference ϕ at P at a distance x from O is $(x/\lambda)2\pi$. From (ii),

$$y = a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

The quantity $2\pi/\lambda = k$ is called the wave number or propagation constant. Then,

$$y = a\sin(\omega t - kx) \quad \text{—— (iii)}$$

$$y = a\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{—— (iv)}$$

Again, $\omega = 2\pi f = 2\pi v/\lambda$. So,

$$y = a\sin\left(\frac{2\pi v t}{\lambda} - \frac{2\pi x}{\lambda}\right) = a\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{—— (v)}$$

If the wave travels from right to left, equation of wave is

$$y = a\sin\frac{2\pi}{\lambda}(vt + x).$$

Differential Equation of Wave Motion

The general wave ~~motion~~ equation is

$$y = a\sin(\omega t - kx). \quad \text{---} (1)$$

Diff. (i) w.r.t. t

$$\frac{dy}{dt} = a\omega\cos(\omega t - kx)$$

Again differentiating,

$$\frac{d^2y}{dt^2} = -a\omega^2\sin(\omega t - kx)$$
$$= -\omega^2 y$$

$$y = -\frac{1}{\omega^2}\frac{d^2y}{dt^2} \quad \text{---} (II)$$

Diff (1) w.r.t. x.

$$\frac{dy}{dx} = -ak\cos(\omega t - kx)$$

Again differentiating,

$$\frac{d^2y}{dx^2} = -k^2 a\sin(\omega t - kx) = -k^2 y$$

$$y = -\frac{1}{k^2}\frac{d^2y}{dx^2} \quad \text{---} (III)$$

Equating (II) and (III)

$$-\frac{1}{\omega^2}\frac{d^2y}{dt^2} = -\frac{1}{k^2}\frac{d^2y}{dx^2}$$

$$\frac{k^2}{\omega^2}\frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$$

$$\frac{1}{v^2}\frac{d^2y}{dt^2} = \frac{d^2y}{dx^2} \quad \left[\because \frac{k^2}{\omega^2} = \left(\frac{2\pi/\lambda}{2\pi f}\right)^2 = \frac{1}{\lambda^2 f^2} = \frac{1}{v^2}\right]$$

$$\therefore \frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

This is the differential wave equation.

Principle of Superposition of Waves

It states that the resultant displacement of the particle is equal to the vector sum of individual displacements due to different waves. If y be the resultant displacement of a particle and $y_1, y_2, --$ are displacements due to individual waves, then according to the principle of superposition of waves, we have

$y = y_1 \pm y_2 \pm ---$

Stationary wave:

When two progressive waves of same amplitude and frequency travel in a medium in exactly the opposite direction, a resultant wave is formed. This resultant wave is called stationary wave or standing wave.

The position of particle at zero displacement is called node (N) and the position of particle at which the maximum displacement takes place is called antinode (AN).

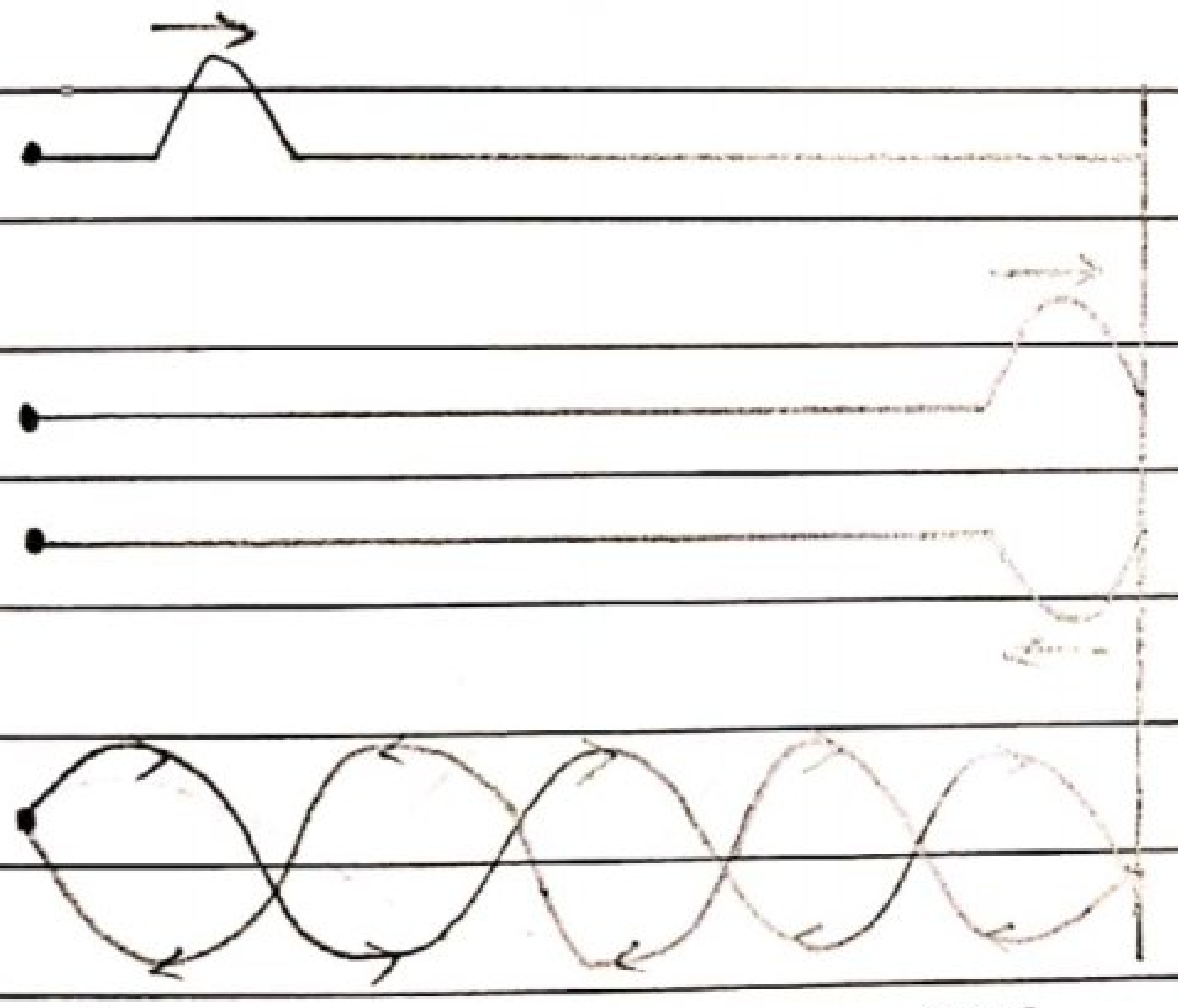


Fig: Formation of stationary wave.

Equation of stationary wave

Let, y_1 and y_2 be the displacements of two progressive waves of same amplitude a and wavelength λ travelling in opposite direction simultaneously with same velocity v. The equations of these waves may be expressed as follows,

$y_1 = a\sin(\omega t - kx)$ —— (i)

$y_2 = a\sin(\omega t + kx)$ —— (ii)

Thus, the resultant displacement of the particle of medium due to both the waves will be determined from the principle of superposition as

$y = y_1 + y_2$

$= a\sin(\omega t - kx) + a\sin(\omega t + kx)$

$= a[\sin(\omega t - kx) + \sin(\omega t + kx)]$

$= 2a\cos kx . \sin\omega t.$

$= A\sin\omega t$

$\therefore\ y = A\sin\omega t$

This represent a SHM whose amplitude is $A = 2a\cos kx$ and $\sin\omega t$ gives the nature of oscillation.

Condition for Maximum Amplitude

The amplitude $2a\cos kx$ will be maximum if $A = \pm 2a$

So, $2a\cos kx = \pm 2a$

$\cos kx = \pm 1$

$\cos \frac{2\pi}{\lambda} x = \cos n\pi$

$\frac{2\pi}{\lambda} x = n\pi$

$x = \frac{n\lambda}{2}$

This is the condition for antinode formation.

For $n=0$, $x_0 = 0$, For $n=1$, $x_1 = \lambda/2$, For $n=2$, $x_2 = 2\lambda/2 = \lambda$, For $n=3$, $x_3 = 3\lambda/2$

Hence antinodes occur at the point where

Plase Difference $(\phi) = 0, \pi, 2\pi, 3\pi, \ldots\ n\pi$

Path Difference $(x) = 0, \lambda/2, \lambda, 3\lambda/2, \ldots\ n\lambda/2$

$\therefore$ The condition of antinode is

$x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots\ n\lambda/2$

The distance between two consequtive antinodes $= \frac{n\lambda}{2} - \frac{(n-1)\lambda}{2} = \lambda/2.$

Condition for Minimum Amplitude

The amplitude $A = 2a\cos kx$ will be minimum if $A = 0$

So, $2a\cos kx = 0$

or, $\cos kx = 0$

or $\cos \frac{2\pi}{\lambda} x = \frac{(2n+1)\pi}{2}$

or $\frac{2\pi x}{\lambda} = (2n+1)\frac{\pi}{2}$

or $x = (2n+1)\frac{\lambda}{4}$

This is the condition for node formation.

For $n=0$, $x_0 = \frac{\lambda}{4}$, For $n=1$, $x_0 = \frac{3\lambda}{4}$, For $n=2$, $x_2 = \frac{5\lambda}{4}$

Hence, the nodes occur at the positions where,

Phase Difference $(\phi) = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots (2n+1)\frac{\pi}{2}$

Path Difference $(x) = \lambda/4, 3\lambda/4, 5\lambda/4 \ldots (2n+1)\lambda/4$.

The distance between two consecutive node $= (2n+1)\lambda/4 - (2\{n-1\}+1)\lambda/4 = \lambda/2$

Distance between any consecutive node and antinode $= (2n+1)\lambda/4 - n\lambda/2 = \lambda/4$

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