Exercise - 7.3

General section

1. Express as the sum or difference of sine or cosine of the following product of sine and cosine.

a. $\cos 81° \cdot \sin 9°$

Soln,

$\frac{1}{2} \times 2 \cos 81° \cdot \sin 9°$

$= \frac{1}{2} [\sin(81+9) - \sin(81-9)]$

$= \frac{1}{2} [\sin 90° - \sin 72°]$

Ans,

b. $\sin 72° \cdot \cos 81°$

Soln,

$= \frac{1}{2} \times 2 \sin 72° \cdot \cos 81°$

$= \frac{1}{2} [\sin(72+81) + \sin(72-81)]$

$= \frac{1}{2} [\sin 153° - \sin 9]$

c. $\cos 27° \cdot \cos 54$

Soln,

$\frac{1}{2} \times 2 \cos 27° \cdot \cos 54$

$= \frac{1}{2} [\cos(27°+54) + \cancel{\sin} \cos(27-54)]$

$= \frac{1}{2} [\cos 81 + \cos 27°]$

d. $\sin 30° \cdot \cos 2\theta$

Soln.

$\frac{1}{2} \times 2 \sin 30° \cdot \cos 2\theta$

$= \frac{1}{2} [\sin(30+2\theta) + \sin(30-2\theta)]$

$= \frac{1}{2} [\sin 5\theta + \sin 1\theta]$

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e. $2\cos 3\theta \cdot \cancel{\cos 7\theta}\ \cos 7\theta$

Soln.

$2\cos 3\theta \cdot \cancel{\cos 7\theta\theta}\ \cos 7\theta$

$= \cancel{\cos(3\theta+7\theta) + \cos(3\theta-7\theta)}$

$= \cos(3\theta+7\theta) + \cos(3\theta-7\theta)$

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2. Express the sum or the difference of sine or cosine into the product form.

a. $\sin 6\theta + \sin 4\theta$

$= 2\sin\left(\frac{6\theta+4\theta}{2}\right) \cdot \cos\left(\frac{6\theta-4\theta}{2}\right)$

$= 2\sin\frac{10\theta}{2} \cdot \cos\frac{2\theta}{2}$

$= 2\sin 5\theta \cdot \cos\theta$

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b. $\cos 75° + \cos 15°$

$= 2\cos\left(\frac{75+15}{2}\right) \cdot \cos\left(\frac{75-15}{2}\right)$

$= 2\cos\frac{90}{2} \cdot \cos\frac{60}{2}$

$= 2\cos 45° \cdot \cos 30°$

$= \cancel{2} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{\cancel{2}}$

$= \frac{\sqrt{3}}{\sqrt{2}}$

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c. $\sin 70^\circ - \sin 140^\circ$

$= 2\cos\left(\frac{70^\circ + 140}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$= 2\cos \frac{210}{2} \cdot \sin\left(\frac{70^\circ - 140}{2}\right)$

$= 2\cos \frac{210^\circ}{2} \cdot \sin \frac{-70}{2}$

$= 2\cos 105^\circ \cdot \sin -35^\circ$

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d. $\sin 75^\circ - \sin 15^\circ$

$= \cancel{\sin}\ 2\cos\left(\frac{75 + 15}{2}\right) \cdot \sin\left(\frac{75^\circ - 15^\circ}{2}\right)$

$= 2\cos \frac{90}{2} \cdot \sin \frac{60}{2}$

$= 2\cos 45^\circ \cdot \sin 30^\circ$

$= \cancel{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{\cancel{2}}$

$= \frac{1}{\sqrt{2}}$

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e. $\cos 25^\circ - \cos 37^\circ$

$= \cancel{2\sin(C+} - 2\sin\left(\frac{25^\circ + 37^\circ}{2}\right) \cdot \sin\left(\frac{37 - 25}{2}\right)$

$= -2\sin \frac{62}{2} \cdot \sin \frac{12}{2}$

$= -2\sin 31^\circ \cdot \sin 6^\circ$

3. Prove the following:

a. $\sin 75^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$

LHS

$\sin 75^\circ - \sin 15^\circ$

$= 2\cos\left(\frac{75^\circ + 15^\circ}{2}\right) \cdot \sin\left(\frac{75^\circ - 15^\circ}{2}\right)$

$= 2\cos\frac{90^\circ}{2} \cdot \cos\frac{60^\circ}{2}$

$= 2\cos 45^\circ \cdot \cos 30^\circ$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{1}{\sqrt{2}}$

Proved.

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b. $\sin 18^\circ + \cos 18^\circ = \sqrt{2}\cos 27^\circ$

LHS

$\sin 18^\circ + \cos 18^\circ$

$\sin(90^\circ - 72^\circ) + \cos 18^\circ$

$= \cos 72^\circ + \cos 18^\circ$

$= 2\cos\left(\frac{72+18}{2}\right) \cdot \cos\left(\frac{72-18}{2}\right)$

$= 2\cos\frac{90}{2} \cdot \cos\frac{54}{2}$

$= 2\cos 45 \cdot \cos 27^\circ$

$= \overset{\sqrt{2}}{\cancel{2}} \times \frac{1}{\cancel{\sqrt{2}}} \cdot \cos 27$

$= \sqrt{2}\cos 27^\circ$

Proved.

c. $\sin 50^\circ + \sin 70^\circ = \sqrt{3}\cos 10^\circ$

Soln,

$\sin 50^\circ + \sin 70^\circ$

$= 2\sin\left(\frac{50^\circ + 70^\circ}{2}\right) \cdot \cos\left(\frac{50 - 70^\circ}{2}\right)$

$= 2\sin\frac{120}{2} \cdot \cos\frac{20}{2}$

$= 2\sin 60^\circ \cdot \cos 10^\circ$

$= \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} \times \cos 10^\circ$

$= \sqrt{3}\cos 10^\circ$

Proved,

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d. $\cos 52^\circ + \cos 68^\circ + \cos 172^\circ = 0$

LHS

$\cos 52^\circ + \cos 68^\circ + \cos 172^\circ$

$= \cos 52^\circ + (\cos 68^\circ + \cancel{\sin}\cos 172^\circ)$

$= \cos 52^\circ + 2\cos\left(\frac{68+172}{2}\right) \cdot \cos\left(\frac{68-172}{2}\right)$

$= \cos 52^\circ + 2\cos 120^\circ \cdot \cos 52^\circ$

$= \cos 52^\circ + 2 \times \frac{-1}{2} \cdot \cos 52^\circ$

$= \cos 52^\circ - \cos 52^\circ$

$= 0$

RHS proved,

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e. $\frac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ - \sin 8^\circ} = \tan 53^\circ$

LHS

$\frac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ - \sin 8^\circ}$

$$= \frac{\frac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ}}{\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ}}$$

$$= \frac{1 + \tan 8^\circ}{1 - \tan 8^\circ}$$

$$= \frac{\tan 45^\circ + \tan 8^\circ}{1 - \tan 45^\circ . \tan 8^\circ}$$

$= \tan(45^\circ + 8^\circ)$

$= \tan 53^\circ$

proved

Creative Section

4. Prove the following.

a. $\frac{\cos 10^\circ - \sin 10^\circ}{\cos 10^\circ + \sin 10^\circ} = \cot 55^\circ$

LHS

$$\frac{\cos 10^\circ - \sin 10^\circ}{\cos 10^\circ + \sin 10^\circ}$$

$$= \frac{\frac{\cos 10^\circ - \sin 10^\circ}{\sin 10^\circ}}{\frac{\cos 10^\circ + \sin 10^\circ}{\sin 10^\circ}}$$

$$= \frac{\cot 10^\circ - 1}{\cot 10^\circ + 1}$$

$$= \frac{\cot 10^\circ . \cot 45^\circ - 1}{\cot 10^\circ + \cot 45^\circ}$$

$= \cot(10^\circ + 45^\circ)$

$= \cot 55^\circ$

proved

b. $\dfrac{\sin 7A - \sin 5A}{\cos 7A + \cos 5A} = \tan A$

LHS

$\dfrac{\sin 7A - \sin 5A}{\cos 7A + \cos 5A}$

$= \dfrac{\cancel{2\sin\left(7A-5\right.} \quad 2\cos\left(\frac{7A+5A}{2}\right)\cdot\sin\left(\frac{7A-5A}{2}\right)}{\cancel{2\cos 6A} \quad 2\cos\left(\frac{7A+5A}{2}\right)\cdot\cos\left(\frac{7A-5A}{2}\right)}$

$= \dfrac{2\cancel{\cos 6A}\cdot\sin A}{2\cancel{\cos 6A}\cdot\cos A}$

$= \dfrac{\sin A}{\cos A}$

$= \tan A$

c. $\dfrac{\sin A.\sin 2A + \sin 3A.\sin 6A}{\sin A.\cos 2A + \sin 3A.\cos 6A} = \tan 5A$

LHS

$\dfrac{\sin A\cdot\sin 2A + \sin 3A\cdot\sin 6A}{\sin A\cdot\cos 2A + \sin 3A\cdot\cos 6A}$

$= \dfrac{2\sin A.\sin 2A + 2\sin 3A\cdot\sin 6A}{2\sin A.\cos 2A + 2\sin 3A.\cos 6A}$

$= \dfrac{\cos(A-2A) - \cos(A+2A) + \cos(3A-6A) - \cos(3A+6A)}{\sin(A+2A) + \sin(A-2A) + \sin(3A+6A) + \sin(3A-6A)}$

$= \dfrac{\cos(-A) - \cos 3A + \cos(-3A) - \cos 9A}{\sin 3A + \sin(-A) + \sin 9A + \sin(-3A)}$

$= \dfrac{\cos A - \cancel{\cos 3A} + \cancel{\cos 3A} - \cos 9A}{\cancel{\sin 3A} - \sin A + \sin 9A - \cancel{\sin 3A}}$

$= \dfrac{\cos A - \cos 9A}{\sin 9A - \sin A}$

$= \dfrac{\cancel{-2\sin(A+9A)}}{}$

$= \dfrac{2\sin\left(\frac{A+9A}{2}\right)\cdot\sin\left(\frac{9A-A}{2}\right)}{2\cos\left(\frac{9A+A}{2}\right)\cdot\sin\left(\frac{9A-A}{2}\right)}$

$= \dfrac{\cancel{2}\sin 5A\cdot\cancel{\sin 4A}}{\cancel{2}\cos 5A\cdot\cancel{\sin 4A}}$

$= \dfrac{\sin 5A}{\cos 5A}$

$= \tan 5A$

RHS proved.

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d. $\dfrac{\cos 2A\cdot\cos 3A - \cos 2A\cdot\cos 7A}{\sin 4A\cdot\sin 3A - \sin 2A\cdot\sin 5A} = \dfrac{\sin 7A + \sin 3A}{\sin A}$

LHS

$\dfrac{\cos 2A\cdot\cos 3A - \cos 2A\cdot\cos 7A}{\sin 4A\cdot\sin 3A - \sin 2A\cdot\sin 5A}$

$= \dfrac{\cos 2A(\cos 3A - \cos 7A)}{\frac{1}{2}\left[\left\{\cos(4A-3A) - \cos(4A+3A)\right\} - \left\{\cos(2A-5A) - \cos(2A+5A)\right\}\right]}$

$= \dfrac{\frac{2\cos 2A\cdot 2\sin\left(\frac{3A+7A}{2}\right)\cdot\sin\left(\frac{7A-3A}{2}\right)}{1}}{\cos A - \cancel{\cos 7A} - \cos(-3A) + \cancel{\cos 7A}}$

$= \dfrac{2\cdot 2\cos 2A\cdot\sin 5A\cdot\sin 2A}{\cos 5A - \cos 3A}$

$= \cancel{4\cos 2A\cdot\sin 5}$

$\dfrac{4\cos 2A\cdot\sin 5A\cdot\sin 2A}{\cos A - \cos 3A}$

$$= \frac{2 \cdot 2\cos 2A \cdot \sin 2A \cdot \sin 5A}{2 \sin \frac{A+3A}{2} \cdot \sin \frac{3A-A}{2}}$$

$$= \frac{2 \sin 4A \cdot \sin 5A}{2 \sin 2A \cdot \sin A}$$

$$= \frac{2 \cdot \sin 2 \cdot 2A \cdot \sin 5A}{2 \sin 2A \cdot \sin A}$$

$$= \frac{2 \cdot \cancel{2 \sin 2A \cdot \cos A}\, \cos 2A \cdot \sin 5A}{\cancel{2 \sin 2A} \cdot \sin A}$$

$$= \frac{2 \cos 2A \cdot \sin 5A}{\sin A}$$

$$= \frac{\sin(2A+5A) - \sin(2A-5A)}{\sin A}$$

$$= \frac{\sin 7A - \sin(-3A)}{\sin A}$$

$$= \frac{\sin 7A + \sin 3A}{\sin A}$$

RHS proved.

e. $\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A$

LHS

$$\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A}$$

$$= \frac{\sin 5A + \sin 3A + \sin 7A + \sin A}{\cos 5A + \cos 3A + \cos 7A + \cos A}$$

$$= \frac{2 \cancel{\cos} \sin\left(\frac{5A+3A}{2}\right) \cdot \cos\left(\frac{5A-3A}{2}\right) + 2\sin\left(\frac{7A+A}{2}\right) \cdot \cos\left(\frac{7A-A}{2}\right)}{2\cos\left(\frac{5A+3A}{2}\right) \cdot \cos\left(\frac{5A-3A}{2}\right) + 2\cos\left(\frac{7A+A}{2}\right) \cdot \cos\left(\frac{7A-A}{2}\right)}$$

$$= \frac{2 \sin 4A \cdot \cos A + 2 \sin 4A \cdot \cos 3A}{2 \cos 4A \cdot \cos A + 2 \cos 4A \cdot \cos 3A}$$

$$= \frac{2 \sin 4A \, \cancel{(\cos A + \cos 3A)}}{2 \cos 4A \, \cancel{(\cos A + \cos 3A)}}$$

$$= \frac{\cancel{2} \sin 4A}{\cancel{2} \cos 4A}$$

$$= \tan 4A$$

RHS proved.

f. $\sin\alpha + \sin\left(\alpha + \frac{2\pi}{3}\right) + \sin\left(\alpha + \frac{4\pi}{3}\right) = 0$

LHS

$\sin\alpha + \sin\left(\alpha + \frac{2\pi}{3}\right) + \sin\left(\alpha + \frac{4\pi}{3}\right)$

$= \cancel{\sin\alpha + \sin\left\{\alpha + \left(\pi - \frac{\pi}{3}\right)\right\} + \sin\left\{\alpha + \left(\pi + \frac{\pi}{3}\right)\right\}}$

$= \sin\alpha + 2\sin\left(\frac{\alpha + \frac{2\pi}{3} + \alpha + \frac{4\pi}{3}}{2}\right) \cdot \cos\left(\frac{\alpha + \frac{2\pi}{3} - \alpha - \frac{4\pi}{3}}{2}\right)$

$= \sin\alpha + 2\sin\left(\frac{2\alpha + \cancel{2}\frac{\cancel{6}\pi}{\cancel{3}}}{2}\right) \cdot \cos\left(\frac{\frac{2\pi}{3}}{2}\right)$

$= \sin\alpha + 2\sin \frac{\cancel{2}(\alpha + \pi)}{\cancel{2}} \cdot \cos\frac{\pi}{3}$

$= \sin\alpha + 2\sin(\pi + \alpha) \cdot \cos 60^\circ$

$= \sin\alpha + (-2\sin\alpha) \cdot \frac{1}{2}$

$= \sin\alpha - \cancel{2}\sin\alpha \cdot \frac{1}{\cancel{2}}$

$= \sin\alpha - \sin\alpha$

$= 0$

RHS Proved

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g. $\frac{\sin 50 - \sin 70 - \sin 40 + \sin 80}{\cos 40 - \cos 50 - \cos 80 + \cos 70} = \cot 60$

LHS

$\frac{\sin 50 - \sin 70 + \sin 80 - \sin 40}{\cos 40 - \cos 80 + \cos 70 - \cancel{\cos 40}\cos 5A}$

$= 2\cos\left(\frac{5\theta+7\theta}{2}\right)\cdot\sin\left(\frac{5\theta-7\theta}{2}\right) + 2\cos\left(\frac{8\theta+4\theta}{2}\right)\cdot\sin\left(\frac{8\theta-4\theta}{2}\right)$

$- 2\sin\left(\frac{4\theta+8\theta}{2}\right)\cdot\sin\left(\frac{4\theta-8\theta}{2}\right) + \left(-2\sin\frac{7\theta+5\theta}{2}\right)\cdot\sin\left(\frac{7\theta-5\theta}{2}\right)$

$= \dfrac{2\cos 6\theta\cdot\sin(-\theta) + 2\cos 6\theta\cdot\sin 2\theta}{-2\sin 6\theta\cdot\sin(-2\theta) + (-2\sin 6\theta)\cdot\sin\theta}$

$= \dfrac{2\cos 6\theta\cdot\sin 2\theta - 2\cos 6\theta\cdot\sin 2\theta}{2\sin 6\theta\cdot\sin 2\theta - 2\sin 6\theta\cdot\sin\theta}$

$= \dfrac{2\cos 6\theta\,(\sin 2\theta - \sin 2\theta)}{2\sin 6\theta\,(\sin 2\theta - \sin\theta)}$

$= \dfrac{2\cos 6\theta}{2\sin 6\theta}$

$= \cot 6\theta$

RHS proved.

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h. $\sin^2\left(\frac{\pi}{8}+\frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\sin\theta$

LHS

$\sin^2\left(\frac{\pi}{8}+\frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{\theta}{2}\right)$

$= \dfrac{1-\cos 2\left(\frac{\pi}{8}+\frac{\theta}{2}\right)}{2} - \dfrac{1-\cos 2\left(\frac{\pi}{8}-\frac{\theta}{2}\right)}{2}$

$= \frac{1}{2}\left[\cancel{1} - \cos\left(\frac{\pi}{4}+\theta\right) - \cancel{1} + \cos\left(\frac{\pi}{4}-\theta\right)\right]$

$= \frac{1}{2}\left[\cancel{-\cos 45^\circ\cdot\cos\theta} + ~~\cos 45\theta\cdot\cos\theta~~ \sin\frac{\pi}{4}\cdot\sin\theta + \cancel{\cos\frac{\pi}{4}\cdot\cos\theta} + \sin 45^\circ\cdot\sin\theta\right]$

$= \frac{1}{2}\times\sin 45^\circ\cdot\sin\theta + \sin 45^\circ\cdot\sin\theta$

$= \frac{1}{\cancel{2}} \times \cancel{2} \sin 45^\circ \cdot \sin\theta$

$= \frac{1}{\sqrt{2}} \sin\theta$

RHS proved

i. $\frac{1 - \cos\theta + \cos B - \cos(\theta + B)}{1 + \cos\theta - \cos B - \cos(\theta + B)} = \tan\frac{\theta}{2} \cdot \cot\frac{B}{2}$

LHS

$\frac{1 - \cos\theta + \cos B - \cos(\theta + B)}{1 + \cos\theta - \cos B - \cos(\theta + B)}$

$= \frac{1 - (\cos\theta - \cos B) - \cos 2\left(\frac{\theta + B}{2}\right)}{1 + (\cos\theta - \cos B) - \cos 2\left(\frac{\theta + B}{2}\right)}$

$= \frac{1 - 2\sin\frac{\theta + B}{2} \cdot \sin\left\{\frac{B - \theta}{2}\right. - \left\{1 - 2\sin^2\left(\frac{\theta + B}{2}\right)\right\}}{1 + 2\sin\frac{\theta + B}{2} \cdot \sin\frac{B - \theta}{2} - \left\{1 - 2\sin^2\left(\frac{\theta + B}{2}\right)\right\}}$

$= \frac{\cancel{1} - 2\sin\frac{\theta + B}{2} \cdot \sin\frac{B - \theta}{2} - \cancel{1} + 2\sin^2\left(\frac{\theta + B}{2}\right)}{\cancel{1} + 2\sin\frac{\theta + B}{2} \cdot \sin\frac{B - \theta}{2} - \cancel{1} + 2\sin^2\left(\frac{\theta + B}{2}\right)}$

$= \frac{\cancel{2\sin^2\left(\frac{\theta + B}{2}\right)}\left[\sin\frac{\theta + B}{2} - \sin\frac{B - \theta}{2}\right]}{\cancel{2\sin\frac{\theta + B}{2}}\left[\sin\frac{B - \theta}{2} + \sin\frac{\theta + B}{2}\right]}$

$= \frac{2\cos\left(\frac{\frac{\theta + B}{2} + \frac{B - \theta}{2}}{2}\right) \cdot \sin\left(\frac{\frac{\theta + B}{2} - \frac{B - \theta}{2}}{2}\right)}{2\sin\left(\frac{\sin\frac{B - \theta}{2} + \sin\frac{\theta + B}{2}}{2}\right) \cdot \cos\left(\frac{B - \theta}{2} - \frac{\theta + B}{2}\right)}$

$$= \frac{\cos\left(\frac{B/2 + B/2}{2}\right) \cdot \sin\left(\frac{\theta/2 + \theta/2}{2}\right)}{\sin\left(\frac{B/2 + B/2}{2}\right) \cdot \cos\left\{-\left(\frac{\theta/2 + \theta/2}{2}\right)\right\}}$$

$$= \frac{\cos B/2 \cdot \sin \theta/2}{\sin B/2 \cdot \cos \theta/2}$$

$$= \tan \theta/2 \cdot \cot B/2$$

RHS proved.

5. Prove the following identities :

a. $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = 3/16$

LHS

$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ$

$= \sin 60^\circ \cdot \frac{1}{2} \cdot 2\sin 80 \cdot \sin 40 \cdot \sin 20$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2}\left[2 \sin 80^\circ \cdot \sin 40^\circ\right] \cdot \sin 20^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2}\left[\cancel{\sin(80^\circ} \cos(80^\circ - 40^\circ) - \cos(80^\circ + 40^\circ)\right] \cdot \sin 20^\circ$

$= \frac{\sqrt{3}}{4}\left[\cancel{\sin 40^\circ} \cos 40^\circ - \cos 120^\circ\right] \cdot \sin 20^\circ$

$= \frac{\sqrt{3}}{4}\left(\cos 40^\circ + \frac{1}{2}\right) \cdot \sin 20^\circ$

$= \frac{\sqrt{3}}{4}\left(\cos 40^\circ \cdot \sin 20^\circ + \frac{1}{2} \sin 20^\circ\right)$

$= \frac{\sqrt{3}}{4} \times \frac{1}{2} \times 2 \cos 40^\circ \cdot \sin 20^\circ + \frac{1}{2} \sin 20^\circ$

$= \cancel{\frac{\sqrt{3}}{4} \times \frac{1}{2}\left(\sin(40^\circ + 20^\circ)\right.} - \sin(40^\circ - 20^\circ) + \frac{1}{2} \sin 20^\circ$

$= \cancel{\frac{\sqrt{3}}{8} \times \sin 60^\circ - \sin 20^\circ}$

$= \frac{\sqrt{3}}{4} \times \left[\frac{1}{2} \{ \sin(40^\circ + 20^\circ) - \sin(40^\circ - 20^\circ) \} + \frac{1}{2} \sin 20^\circ \right]$

$= \frac{\sqrt{3}}{4} \left[\frac{1}{2} \sin 60^\circ - \frac{1}{2} \sin 20^\circ \right] + \frac{1}{2} \sin 20^\circ$

$= \frac{\sqrt{3}}{4} \times \frac{1}{2} \sin 60^\circ - \cancel{\frac{1}{2} \sin 20^\circ} + \cancel{\frac{1}{2} \sin 20^\circ}$

$= \frac{\sqrt{3}}{4} \times \frac{1}{2} \times \sin 60^\circ$

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$

$= \frac{3}{16}$

RHS proved$_{11}$

b. $\cos 40^\circ \cdot \cos 80^\circ \cdot \cos 160^\circ = -\frac{1}{8}$

LHS

$\cos 40^\circ \cdot \cos 80^\circ \cdot \cos 160^\circ$

$\frac{1}{2} \cdot 2 \cos 40^\circ \cdot \cos 80^\circ \cdot \cos 160^\circ$

$= \frac{1}{2} \{ \cos(40^\circ + 80^\circ) + \cos(40^\circ - 80^\circ) \} \cdot \cos 160^\circ$

$= \frac{1}{2} (\cos 120^\circ + \cos 40^\circ) \cdot \cos 160^\circ$

$= \frac{1}{2} \left\{ -\frac{1}{2} \cos 160^\circ + \cos 40^\circ \cdot \cos 160^\circ \right\}$

$= \frac{1}{2} \left\{ \frac{-\cos 160^\circ + 2 \cos 40^\circ \cdot \cos 160^\circ}{2} \right\}$

$= \frac{1}{4} \{ -\cos 160^\circ + \cos(40^\circ + 160^\circ) + \cos(40 - 160^\circ) \}$

$= \frac{1}{4}\left\{-\cos 160^\circ + \cos 200^\circ + \cos 120^\circ\right\}$

$= \frac{1}{4}\left\{\cos 200^\circ - \cos 160^\circ - \frac{1}{2}\right\}$

$= \frac{1}{4}\left\{\left(2\sin\frac{200+160}{2} \cdot \sin\frac{160^\circ-200}{2}\right) - \frac{1}{2}\right\}$

$= \frac{1}{4}\left\{2\sin 180^\circ \cdot \sin(-20^\circ) - \frac{1}{2}\right\}$

$= \frac{1}{4}\left\{2 \times 0 \cdot \sin(-20) - \frac{1}{2}\right\}$

$= \frac{1}{4}\left\{0 - \frac{1}{2}\right\}$

$= \frac{-1}{8}$

RHS proved.

C. $\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 80^\circ = \sqrt{3}$

LHS

$\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 80^\circ$

$= \frac{2\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ}{2\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ}$

$= \frac{\{\cos(40^\circ-80^\circ) - \cos(40^\circ+80^\circ)\} \cdot \cancel{\cos}\, \sin 20^\circ}{\{\cos(40^\circ+80^\circ) + \cos(40^\circ-80^\circ)\} \cdot \cos 20^\circ}$

$= \frac{\{\cos 40^\circ - \cos 120^\circ\} \cdot \sin 20^\circ}{\{\cos 120^\circ + \cos 40^\circ\} \cdot \cos 20^\circ}$

$= \frac{\left\{\cos 40^\circ \cdot \sin 20^\circ + \frac{1}{2} \cdot \sin 20^\circ\right\}}{\left\{-\frac{1}{2} \cdot \cos 20^\circ + \cos 40^\circ \cdot \cos 20^\circ\right\}}$

$= \frac{2\cos 40^\circ \cdot \sin 20^\circ + \sin 20^\circ}{2} \times \frac{2}{-\cos 20^\circ + 2\cos 40^\circ \cdot \cos 20^\circ}$

$$= \frac{\sin(40°+20°) - \sin(40°-20°) + \sin 20°}{-\cos 20° + \cos(40°+20°) + \cos(40°-20°)}$$

$$= \frac{\sin 60° - \cancel{\sin 20°} + \cancel{\sin 20°}}{-\cancel{\cos 20°} + \cos 60° + \cancel{\cos 20°}}$$

$$= \frac{\sin 60°}{\cos 60°}$$

$$= \tan 60°$$

$$= \sqrt{3}$$

RHS proved

d. $16 \sin 10° \cdot \sin 30° \cdot \sin 50° \cdot \sin 70° = 1$

LHS

$16 \sin 10° \cdot \sin 30° \cdot \sin 50° \cdot \sin 70°$

$$= 16 \times \sin 30 \left\{ \frac{1}{2} \times 2 \sin 10° \cdot \sin 70° \right\} \cdot \sin 50°$$

$$= 16 \times \frac{1}{2} \times \frac{1}{2} \left\{ \cos(10-70) - \cos(10+70) \right\} \cdot \sin 50°$$

$$= \frac{16}{4} \left\{ \cos 60 - \cos 80 \right\} \cdot \sin 50$$

$$= 4 \left\{ \frac{1}{2} \cdot \sin 50 - \cos 80 \cdot \sin 50 \right\}$$

$$= 4 \left\{ \frac{\sin 50 - 2 \cos 80 \cdot \sin 50}{2} \right\}$$

$$= \frac{4}{2} \left\{ \sin 50° - \left(\sin(80+50) - \sin(80-50) \right) \right\}$$

$$= 2 \left\{ \sin 50 - \sin 130 + \sin 30 \right\}$$

$$= 2 \left\{ \sin 50 - \sin(180-50) + \sin 30 \right\}$$

$$= 2 \left\{ \sin 50 - \sin 50 + \frac{1}{2} \right\}$$

$$= \cancel{2 \sin 50} - \cancel{2 \sin 50} + \frac{\cancel{2}^{1}}{\cancel{2}}$$

$= 1$ RHS proved

e. $2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

LHS

$2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=-\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=0$

RHS proved.

---

f. $(\cos A+\cos B)^2+(\sin A+\sin B)^2=4\cos^2\left(\frac{A-B}{2}\right)$

LHS

$(\cos A+\cos B)^2+(\sin A+\sin B)^2$

$=\cos^2A+2\cos A\cdot\cos B+\cos^2B+\sin^2A+2\sin A\cdot\sin B+\sin^2B$

$=1+1+2\cos A\cdot\cos B+2\sin A\cdot\sin B$

$=2+2(\cos A\cdot\cos B+\sin A\cdot\sin B)$

$=2+2\cos(A-B)$

$=2+2\cos2\left(\frac{A-B}{2}\right)$

$=2+2\left\{2\cos^2\left(\frac{A-B}{2}\right)-1\right\}$

$=\cancel{2}+4\cos^2\left(\frac{A-B}{2}\right)-\cancel{2}$

$=4\cos^2\left(\frac{A-B}{2}\right)$

proved.

g. $\sin A \cdot \sin(60° - A) \cdot \sin(60° + A) = \frac{1}{4} \sin 3A$

LHS

$\sin A \cdot \sin(60 - A) \cdot \sin(60° + A)$

$= \frac{1}{2} \sin A \cdot 2 \sin(60 - A) \cdot \sin(60 + A)$

$= \frac{1}{2} \sin A \{ \cos(60 - A - 60 - A) - \cos(60 - A + 60 + A) \}$

$= \frac{1}{2} \sin A \{ \cos 2A - \cos 120° \}$

$= \frac{1}{2} \sin A \{ \cos 2A + \frac{1}{2} \}$

$= \frac{1}{2} \sin A \cdot \cos 2A + \frac{1}{4} \sin A$

$= \frac{1}{2} \times \frac{2}{2} \sin A \cdot \cos 2A + \frac{1}{4} \sin A$

$= \frac{1}{4} \{ \sin(A + 2A) + \sin(A - 2A) \} + \frac{1}{4} \sin A$

$= \frac{1}{4} \{ \sin 3A + \sin(-A) \} + \frac{1}{4} \sin A$

$= \frac{1}{4} \sin 3A - \frac{1}{4} \sin A + \frac{1}{4} \sin A$

$= \frac{1}{4} \sin 3A$

RHS proved.

h. $\cos A \cdot \cos(60° - A) \cdot \cos(60° + A) = \frac{1}{4} \cos 3A$

LHS

$\cos A \cdot \cos(60° - A) \cdot \cos(60° + A)$

$= \frac{1}{2} \cos A \cdot 2 \cos(60° - A) \cdot \cos(60° + A)$

$= \frac{1}{2} \cos A \{ \cos(60 - A + 60 + A) + \cos(60 - A - 60 - A) \}$

$= \frac{1}{2} \cos A \{ \cos 120 + \cos 2A \}$

$= \frac{1}{2} \cos A \{ -\frac{1}{2} + \cos 2A \}$

$= -\frac{1}{4} \cos A + \frac{1}{2} \cos A \cdot \cos 2A$

$= \cancel{\frac{1}{2} \times \frac{2}{2} \cos A} \; \frac{1}{2} \times \frac{2}{2} \cos A \cdot \cos 2A - \frac{1}{4} \cos A$

$= \frac{1}{4} \{ \cos(A+2A) + \cos(A-2A) \} - \frac{1}{4} \cos A$

$= \frac{1}{4} \cos 3A + \cancel{\frac{1}{4} \cos A} - \cancel{\frac{1}{4} \cos A}$

$= \frac{1}{4} \cos 3A$

RHS Proved.

---

i. $\cos^3\theta \cdot \sin^2\theta = \frac{1}{16} (2\cos\theta - \cos 3\theta - \cos 5\theta)$

RHS

$\frac{1}{16} (2\cos\theta - \cos 3\theta - \cos 5\theta)$

$= \frac{1}{16} \{ 2\cos\theta - (\cos 3\theta + \cos 5\theta) \}$

$= \frac{1}{16} \left[2\cos\theta - \left\{ 2\cos\left(\frac{3\theta+5\theta}{2}\right) \cdot \cos\left(\frac{3\theta-5\theta}{2}\right) \right\} \right]$

$= \frac{1}{16} [2\cos\theta - 2\cos 4\theta \cdot \cos\theta]$

$= \frac{1}{16} [2\cos\theta (1 - \cos 4\theta)]$

$= \frac{1}{16} [2\cos\theta (1 - \cos 2 \cdot 2\theta)]$

$= \frac{1}{16}\left[2\cos\theta\left(2\sin^2 2\theta\right)\right]$

$= \frac{1}{\cancel{16}_4} \times \cancel{4}\cos\theta \cdot \sin^2 2\theta$

$= \frac{1}{4}\cos\theta \cdot \sin^2 2\theta$

$= \frac{1}{4}\cos\theta\,(2\sin\theta\cdot\cos\theta)^2$

$= \cancel{\frac{1}{4}\cos\theta\cdot(4\sin^2\theta\cdot\cos^2\theta)}$

$= \cancel{\frac{1}{4}\cos\theta\cdot\sin 4\sin^2\theta\cdot\cos^3\theta}$

$= \frac{1}{4}\cos\theta\cdot 4\sin^2\theta\cdot\cos^2\theta$

$= \frac{1}{\cancel{4}}\,\cancel{\sin}\,\cos^3\theta\cdot\cancel{4}\sin^2\theta$

$= \cos^3\theta\cdot\sin^2\theta$

LHS proved.

j. $\frac{\sin^2 A - \sin^2 B}{\sin A\cdot\cos A - \sin B\cdot\cos B} = \tan(A+B)$

LHS

$\frac{\sin^2 A - \sin^2 B}{\sin A\cdot\cos A - \sin B\cdot\cos B}$

$= \frac{2\sin^2 A - 2\sin^2 B}{2\sin A\cdot\cos A - 2\sin B\cdot\cos B}$ [multiplying by 2]

$= \frac{1-\cos 2A - (1-\cos 2B)}{\sin 2A - \sin 2B}$

$= \frac{\cancel{1}-\cos 2A - \cancel{1} + \cos 2B}{\sin 2A - \sin 2B}$

$= \dfrac{\cos 2B - \cos 2A}{\sin 2A - \sin 2B}$

$= \dfrac{\cancel{2} \sin\left(\frac{2B+2A}{2}\right) \cdot \sin\left(\frac{2B-2B}{2}\right)}{\cancel{2} \cos\left(\frac{2A+2B}{2}\right) \cdot \sin\left(\frac{2A-2B}{2}\right)}$

$= \dfrac{\sin(B+A)}{\cos(A+B)} \cdot \dfrac{\cancel{\sin\left(\frac{2A-2B}{2}\right)}}{\cancel{\sin\left(\frac{2A-2B}{2}\right)}}$

$= \dfrac{\sin(A+B)}{\cos(A+B)}$

$= \tan(A+B)$

Rhs proved.

k. $\cos \frac{2\pi^c}{7} + \cos \frac{4\pi^c}{7} + \cos \frac{8\pi^c}{7} = -\frac{1}{2}$

Lhs

$\cos \frac{2\pi^c}{7} + \cos \frac{4\pi^c}{7} + \cos \frac{8\pi^c}{7}$

$= \cos \frac{2\pi^c}{7} + \cos \frac{4\pi^c}{7} + \cos\left(2\pi - \frac{6\pi}{7}\right)$

$= \cos \frac{2\pi^c}{7} + \cos \frac{4\pi^c}{7} + \cos \frac{6\pi^c}{7}$

$= \dfrac{2 \sin \frac{\pi^c}{7} \left(\cos \frac{2\pi^c}{7} + \cos \frac{4\pi^c}{7} + \cos \frac{6\pi^c}{7} \right)}{2 \sin \pi^c/7}$

$= \dfrac{2 \sin \frac{\pi^c}{7} \cdot \cos \frac{2\pi^c}{7} + 2 \sin \frac{\pi^c}{7} \cdot \cos \frac{4\pi^c}{7} + 2 \sin \frac{\pi^c}{7} \cdot \cos \frac{6\pi^c}{7}}{2 \sin \pi^c/7}$

$$= \frac{\sin\left(\frac{\pi^c}{7}+\frac{2\pi^c}{7}\right)+\sin\left(\frac{\pi^c}{7}-\frac{2\pi^c}{7}\right)+\sin\left(\frac{\pi^c}{7}+\frac{4\pi^c}{7}\right)+\sin\left(\frac{\pi^c}{7}-\frac{4\pi^c}{7}\right)+\sin\left(\frac{\pi^c}{7}+\frac{6\pi^c}{7}\right)+\sin\left(\frac{\pi^c}{7}-\frac{6\pi^c}{7}\right)}{\cancel{2\pi^c/7}\ 2\sin \pi^c/7}$$

$$= \frac{\sin\left(\frac{3\pi^c}{7}\right)+\sin\left(\frac{-\pi^c}{7}\right)+\sin\left(\frac{5\pi^c}{7}\right)+\sin\left(\frac{-3\pi^c}{7}\right)+\sin\left(\frac{7\pi^c}{7}\right)+\sin\left(\frac{-5\pi^c}{7}\right)}{2\sin \cancel{8}\pi^c/7}$$

$$= \frac{\cancel{\sin\frac{3\pi^c}{7}}-\sin\frac{\pi^c}{7}+\cancel{\sin\frac{5\pi^c}{7}}-\cancel{\sin\frac{3\pi^c}{7}}+\sin\frac{7\pi^c}{7}-\cancel{\sin\frac{5\pi^c}{7}}}{2\sin \cancel{8}\pi^c/7}$$

$$= \frac{-\sin\frac{\pi^c}{7}+\sin\frac{\cancel{7}\pi^c}{\cancel{7}}}{2\sin\frac{\pi^c}{7}}$$

$$= \frac{-\cancel{\sin\frac{\pi^c}{7}}+\sin 180^\circ}{2\cancel{\sin\frac{\pi^c}{7}}}$$

$$= \frac{-1}{2}+0$$

$$= \frac{-1}{2}$$

RHS proved.

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1. $\tan(A)+\tan(60^\circ+A)+\tan(120^\circ+A) = 3\tan 3A$

LHS

$\tan(A)+\tan(60^\circ+A)+\tan(120^\circ+A)$

$$= \tan(A)+\frac{\tan 60+\tan A}{1-\tan 60\cdot\tan A}+\frac{\tan 120+\tan A}{1-\tan 120\cdot\tan 20}$$

$$= \tan(A)+\frac{\sqrt{3}+\tan A}{1-\sqrt{3}\tan A}+\frac{(-\sqrt{3})+\tan A}{1-(-\sqrt{3})\tan A}$$

$$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} + \frac{\tan A - \sqrt{3}}{1 + \sqrt{3}\tan A}$$

$$= \tan A + \frac{(\sqrt{3} + \tan A)(1 + \sqrt{3}\tan A) + (\tan A - \sqrt{3})(1 - \sqrt{3}\tan A)}{(1 - \sqrt{3}\tan A)(1 + \sqrt{3}\tan A)}$$

$$= \tan A + \frac{\cancel{\sqrt{3}} + 3\tan A + \tan A + \cancel{\sqrt{3}\tan^2 A} + \tan A - \cancel{\sqrt{3}\tan^2 A} - \cancel{\sqrt{3}} + 3\tan A}{1 - 3\tan^2 A}$$

$$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A}$$

$$= \frac{3(3\tan A - \tan^3 A)}{1 - 3\tan^2 A}$$

$$= 3 \cdot \tan 3A$$

$$= 3\tan 3A$$

RHS proved.

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m. $\tan(60° + A) \cdot \tan(60° - A) = \frac{2\cos 2A + 1}{2\cos 2A - 1}$

LHS

$\tan(60° + A) \cdot \tan(60° - A)$

$$= \frac{\sin(60° + A)}{\cos(60° + A)} \cdot \frac{\sin(60° - A)}{\cos(60° - A)}$$

$$= \frac{2\sin(60° + A) \cdot \sin(60° - A)}{2\cos(60° + A) \cos(60° - A)}$$

$$= \frac{\cos(60° + A - 60° + A) - \cos(60° + A + 60 - A)}{\cos(60° + A + 60° - A) + \cos(60 + A - 60 + A)}$$

$$= \frac{\cos 2A - \cos 120}{\cos 120 + \cos 2A}$$

$$= \frac{\cos 2A + \frac{1}{2}}{-\frac{1}{2} + \cos 2A}$$

$$= \frac{\frac{2\cos 2A + 1}{\cancel{2}}}{\frac{-1 + 2\cos 2A}{\cancel{2}}}$$

$$= \frac{2\cos 2A + 1}{2\cos 2A - 1}$$

Ans proved,,

6. If $\sin\alpha = k\sin B$, prove that $\tan\left(\frac{\alpha - B}{2}\right) = \left(\frac{k-1}{k+1}\right) \cdot \tan\left(\frac{\alpha + B}{2}\right)$

Soln,,

$\sin\alpha = k\sin B$

or $\frac{\sin\alpha}{\sin B} = \frac{k}{1}$

or $\frac{\sin\alpha + \sin B}{\sin\alpha - \sin B} = \frac{k+1}{k-1}$

or $\frac{\cancel{2}\sin\frac{\alpha + B}{2} \cdot \cos\frac{\alpha - B}{2}}{\cancel{2}\cos\frac{\alpha + B}{2} \cdot \sin\frac{\alpha - B}{2}} = \frac{k+1}{k-1}$

or, $\tan\frac{\alpha + B}{2} \cdot \cot\frac{\alpha - B}{2} = \frac{k+1}{k-1}$

or, $\tan\frac{\alpha + B}{2} \cdot \frac{1}{\tan\frac{\alpha - B}{2}}$

$= \cancel{\Phi}$ or, $\tan\left(\frac{\alpha+\beta}{2}\right) = \tan\left(\frac{\alpha-\beta}{2}\right)\cdot\left(\frac{k+1}{k-1}\right)$

or, $\left(\frac{k-1}{k+1}\right)\cdot\tan\left(\frac{\alpha+\beta}{2}\right) = \tan\left(\frac{\alpha-\beta}{2}\right)$

or, $\tan\left(\frac{\alpha-\beta}{2}\right) = \left(\frac{k-1}{k+1}\right)\cdot\tan\left(\frac{\alpha+\beta}{2}\right)$

$\therefore \tan\left(\frac{\alpha+\beta}{2}\right) = \left(\frac{k-1}{k+1}\right)\cdot\tan\left(\frac{\alpha+\beta}{2}\right)$

Proved,,

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7. If $\sin\alpha + \sin\beta = x$ and $\cos\alpha + \cos\beta = y$. Prove that $\tan\left(\frac{\alpha-\beta}{2}\right) = \pm\sqrt{\frac{4-x^2-y^2}{x^2+y^2}}$

Solution:

$\sin\alpha + \sin\beta = x$

on squaring both side

$(\sin\alpha + \sin\beta)^2 = x^2$ —— (i)

Also,

$\cos\alpha + \cos\beta = y$

Squaring both side

$(\cos\alpha + \cos\beta)^2 = y^2$ —— (ii)

Adding equation (i) and (ii)

$(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 = x^2+y^2$

or, $\sin^2\alpha + 2\sin\alpha.\sin\beta + \sin^2\beta + \cos^2\alpha + 2\cos\alpha.\cos\beta + \cos^2\beta = x^2+y^2$

or, $1+1+2\sin\alpha\cdot\sin\beta + 2\cos\alpha.\cos\beta = x^2+y^2$

or, $\cancel{2+2\,\sin+2}\ 2+2\sin\alpha.\sin\beta + 2\cos\alpha.\cos\beta = x^2+y^2$

or, $2 + 2(\sin\alpha.\sin\beta + \cos\alpha.\cos\beta) = x^2+y^2$

~~or, $2+2\sin(\alpha+\beta) = x^2+y^2$~~

or, $2+2\cos(\alpha-\beta) = x^2+y^2$

or, $2(1+\cos(\alpha-\beta)) = x^2+y^2$

or, $2\{2\cos^2\left(\frac{\alpha-\beta}{2}\right)\} = x^2+y^2$

or, $4\cos^2\left(\frac{\alpha-\beta}{2}\right) = x^2+y^2$

or, $\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4}\, x^2+y^2$

or, $\frac{1}{\sec^2\left(\frac{\alpha-\beta}{2}\right)} = \frac{1}{4}\, x^2+y^2$

or, $\frac{1}{1+\tan^2\left(\frac{\alpha-\beta}{2}\right)} = \frac{1}{4}\, x^2+y^2$

or, $4 = x^2+y^2 + \tan^2\left(\frac{\alpha-\beta}{2}\right).x^2+y^2$

or, $\frac{4-x^2-y^2}{x^2+y^2} = \tan^2\left(\frac{\alpha-\beta}{2}\right)$

or, $\tan\left(\frac{\alpha-\beta}{2}\right) = \pm\sqrt{\frac{4-x^2-y^2}{x^2+y^2}}$

~~Pr~~ proved//

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8. If $x\cos\alpha + y\sin\alpha = k$ and $x\cos\beta + y\sin\beta = k$ then prove that $x = y\cot\left(\frac{\alpha+\beta}{2}\right)$

Solution

$x\cos\alpha + y\sin\alpha = k$ — (i)

$x\cos\beta + y\sin\beta = k$ — (ii)

From equation (i) and (ii)

$x\cos\alpha + y\sin\alpha = x\cos\beta + y\sin\beta$

or, $x\cos\alpha - x\cos\beta = y\sin\beta - y\sin\alpha$

or, $x(\cos\alpha - \cos\beta) = y(\sin\beta - \sin\alpha)$

or, $x = y\frac{(\sin\beta-\sin\alpha)}{(\cos\alpha-\cos\beta)}$

or, $x = y\,\frac{2\cos\left(\frac{\beta+\alpha}{2}\right).\sin\left(\frac{\beta-\alpha}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right).\sin\left(\frac{\beta-\alpha}{2}\right)}$

or, $x = y \cdot \dfrac{2\cos\left(\frac{\beta+\alpha}{2}\right)}{2\sin\left(\frac{\beta+\alpha}{2}\right)}$

or, $x = y\cot\left(\frac{\beta+\alpha}{2}\right)$

$\therefore x = y\cot\left(\frac{\alpha+\beta}{2}\right)$

Proved.

Exercise - 7.4

General Section

1. If $A+B+C = \pi^c$, prove the following

a. $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

Solution:

We have,

$A+B+C = \pi$

or, $A+B = \pi - C$

or, $\tan(A+B) = \tan(\pi - C)$

or, $\dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = -\tan C$

or, $\tan A + \tan B = -\tan C + \tan A \cdot \tan B \cdot \tan C$

or, $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

proved.

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b. $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A = 1$

Solution:

We have,

$A+B+C = \pi$

or, $A+B = \pi - C$