Chapter-14
Probability

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Permutation

Permutation means number of arrangement of object or number. on certain order.
Arrangement of 'n' object ~~are~~ taken 'r' at a time. is denoted by ^nP_r or $P(n,r)$

$$P(n,r) = \frac{n!}{(n-r)!}$$

Number of permutation of n object taken n at a time is $P(n,r) = n!$

Combination

Combination means ~~se~~ number of selection of object or number without regarding order.
The number of selection of 'n' object taken 'r' at a time is given by ^nc_r or $c(n,r) = \frac{n!}{(n-r)! \times r!}$

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Exercise - 14.1

1. A die is thrown once. Determine the probability of getting—

a. an even number

Solution:

Here, No. of favourable cases (m) = 3

Total possible cases (n) = 6

We know,

$$P(E) = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$$

b. a number ≥ 3

Solution:

$$P(E) = \frac{m}{n} = \frac{4}{6} = \frac{2}{3}$$

c. a number ≤ 4

Solution:

$$P(E) = \frac{m}{n} = \frac{4}{6} = \frac{2}{3}$$

2. A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is;

a. A spade

Solution:

Here, $m = 13$ and $n = 52$

Now, $P(S) = \frac{m}{n} = \frac{13}{52}$

$$= \frac{1}{4}$$

b. a red 8, a red 9 or a red 10

Solution:

Here, $M = 6$ and $n = 52$

Now, $P(E) = \frac{m}{n} = \frac{6}{52}$

$= \frac{3}{26}$

c. a king or a diamond

Solution:

Here, $m = 16$ $n = 52$

We know,

$P(E) = \frac{m}{n} = \frac{16}{52}$

$= \frac{4}{13}$

d. a black or an ace

Here, $m = 28$ $n = 52$

Now, $P(B \cup A) = \frac{m}{n} = \frac{28}{52}$

$= \frac{7}{13}$

e. Neither a heart nor a queen

Solution:

Here, $m = 16$ $n = 52$

Now, $P(\overline{H \cup Q}) = \frac{m}{n} = 1 - \frac{16}{52}$

$= \frac{9}{13}$

3.a. From 20 tickets marked from 1 to 20, one is drawn at random. Find the probability that,

i. It is a multiple of 4 and 5

Solution:

No. of favourable cases, $(m) = 1$

No. of possible cases, $(n) = 20$

We know, $P(E) = \frac{m}{n} = \frac{1}{20}$

ii. It is a multiple of 4 or 5.

Solution:

Here, $m = 8$, $n = 20$

We know, $P(M) = \frac{m}{n} = \frac{8}{20}$

$= \frac{2}{5}$

b. Find the probability that there will be 5 Sundays in the month of June?

Solution:

Here, $m = 2$ $n = 7$

Now, $P(S) = \frac{m}{n} = \frac{2}{7}$

4.a. If $P(A) = 0.4$, $P(B) = 0.35$ and $P(A \cup B) = 0.55$, find $P(A \cap B)$. Are the events A and B independent?

Solution:

Here, $P(A) = 0.4$

$P(B) = 0.35$

$P(A \cup B) = 0.55$

$P(A \cap B) = ?$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or, $0.55 = 0.4 + 0.35 - P(A \cap B)$

$\therefore P(A \cap B) = 0.2$

No, the given events A and B are not independent.

b. If A and B are two independent events with $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{5}$, find $P(A \cup B)$ and $P(A \cup B)$

Solution:

Given, $P(A) = 2/3$

$P(B) = 3/5$

$P(A \cup B) = ?$

$\overline{P(A \cup B)} = ?$

We know, $P(A \cap B) = P(A) \cdot P(B)$

$$= \frac{2}{3} \times \frac{3}{5}$$

$$= 0.4 = \frac{6}{15}$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{3} + \frac{3}{5} - \frac{2}{5}$$

$$\therefore P(A \cup B) = \frac{13}{15}$$

Also, $P(\overline{A \cup B}) = 1 - P(\overline{A \cup B})$

$$= 1 - \frac{13}{15}$$

$$= \frac{2}{15}$$

c. If $P(\overline{A\cup B}) = 0.48$ and $P(A) = 0.4$, find $P(B)$ if

i. A and B are mutually exclusive events.

Solution:

Given, $P(\overline{A\cup B}) = 0.48$

$P(A) = 0.4$

$P(B) = ?$

Since, the events are mutually exclusive

$$P(A\cup B) = P(A) + P(B)$$
$$= 0.4 + P(B)$$

Also,

$$P(\overline{A\cup B}) = 1 - P(A\cup B)$$
$$\text{or, } 0.48 = 1 - [0.4 + P(B)]$$
$$\text{or, } 0.48 + P(B) + 0.4 = 1$$
$$\therefore P(B) = 0.12$$

ii. A and B are independent events

Solution:

Since, the events are independent

$$P(\overline{A\cup B}) = 1 - [P(A\cup B)]$$
$$\text{or, } 0.48 = 1 - [P(A) + P(B) - P(A\cap B)]$$
$$\text{or, } 0.48 = 1 - [0.4 + P(B) - \{P(A).P(B)\}]$$
$$\text{or, } 0.48 = 1 - [0.4 + P(B) - 0.4.P(B)]$$
$$\text{or, } 0.48 + 0.4 + P(B) + 0.4(P(B) = 1$$
$$\text{or, } P(B) + 0.4P(B) = 1 - 0.48 - 0.4$$

$$P(\overline{A\cup B}) = 1 - P(A\cup B)$$
$$\therefore P(A\cup B) = 0.52$$

Also,

$$P(A\cap B) = P(A) * P(B) \cancel{- P(A\cap B)}$$
$$= 0.4 \times P(B)$$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or, $0.52 = 0.4 + P(B) - 0.4\,P(B)$

or, $\frac{0.52 - 0.4}{} = P(B)$

$\therefore P(B) = 0.2$

5.a. A coin and a die are thrown simultaneously. Find the probability of getting

i. Tail on the coin and 6 on the die.

Solution:

Here, $P(T) = \frac{1}{2}$

$P(S) = \frac{1}{6}$

Now, $P(T \cap S) = P(T) \cdot P(S)$

$= \frac{1}{2} \cdot \frac{1}{6}$

$= \frac{1}{12}$

ii. head on the coin and even number in the die.

Solution:

Here, $P(H) = \frac{1}{2}$

$P(E) = \frac{3}{6}$

Now, $P(H \cap E) = \frac{1}{2} \cdot \frac{3}{6}$

$= \frac{3}{12} = \frac{1}{4}$

b. A card is drawn from a well shuffled pack of 52 cards and at the same time a die is rolled. Find the probability of getting a spade on the pack of card and a number ~~&~~ greater than 2 on the die.

Solution:

Here, $P(S) = \frac{13}{52}$

Also, $P(T) = \frac{4}{6}$

Now, $P(S \cap T) = P(S) . P(T)$

$= \frac{13}{52} \times \frac{4}{6}$

$= \frac{1}{6}$

c. A card is drawn from a pack of 52 cards and noted. Then the card is replaced in the same pack and the second card is drawn. Find the probability that

i. Both are kings

Solution:

Here, probability of getting king on 1^{st} drawn, $P(k) = \cancel{\frac{1}{13}} \; \frac{4}{52}$

probability of getting another king on 2^{nd} drawn, $P(k') = \cancel{\frac{1}{13}} \; \frac{4}{52}$

Now,

$P(k \cap k') = \cancel{\frac{1}{13} . \frac{1}{13}} \; \frac{4}{52} . \frac{4}{52}$

$= \frac{1}{169}$

ii. first card is a face card and the second is an ace.

Solution:

Probability of getting face card, $P(F) = \frac{12}{52}$

Probability of getting an ace, $P(A) = \frac{4}{52}$

Now, $P(F \cap A) = P(F) \cdot P(A)$

$= \frac{12}{52} \times \frac{4}{52}$

$= \frac{3}{169}$

6.a. A bag contains 5 white and 3 black balls. Another bag contains 4 white and 6 black balls. One ball is drawn from each bag, find the probability that

i. Both are white

Solution:

Probability of getting white ball from bag A, $P(A) = \frac{5}{8}$

Probability of getting white ball from bag B, $P(B) = \frac{4}{10}$

Now, $P(A \cap B) = P(A) \cdot P(B)$

$= \frac{5}{8} \cdot \frac{4}{10}$

$= \frac{1}{4}$

ii. Both are black

Solution:

From bag A, ~~prop~~ probability of getting black ball, $P(A) = \frac{3}{8}$

From bag B, probability of getting black ball, $P(B) = \frac{6}{10}$

Now,

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{3}{8} \cdot \frac{6}{10}$$

$$= \frac{9}{40}$$

iii. One is white and another is black balls

Solution:

From bag A, probability of getting white ball, $P(W_1) = \frac{5}{8}$

" " " " " " Black ", $P(B_1) = 3/8$

From bag B, Probability of getting black ball, $P(B_2) = \frac{6}{10}$

from bag B, Probability of getting white ball, $P(W_2) = 4/10$

Now, $P(W_1B_2 \text{ or } W_2B_1) = P(W_1B_2) + P(W_2B_1)$

$$= \frac{5}{8} \cdot \frac{6}{10} + \frac{4}{10} \cdot \frac{3}{8}$$

$$= \frac{21}{40}$$

b. Two coins are tossed simultaneously. Find the sample space. Find the probability of getting;

i. both heads

Solutions:

Here, sample space = {HH, HT, TT, TH}

Here, Probability of getting both heads = $\frac{1}{4}$

ii. At least one head.

Solution:

From the above sample space,

Probability of getting at least one head = $\frac{3}{4}$

c. A bag contains 20 tickets numbered from 1 to 20. Two tickets are drawn one by one with replacement. Find the probability of getting;

i. An even number in first and odd number in the second ticket.

Solution:

Here, Probability of getting even no. $P(E) = \frac{10}{20}$

Probability of getting odd no. $P(O) = \frac{10}{20}$

Now,

$$P(E \cap O) = P(E) \cdot P(O)$$

$$= \frac{10}{20} \cdot \frac{10}{20}$$

$$= \frac{1}{4}$$

ii. a multiple of 4 in the first and the multiple of 5 in second ticket.

Solution:

Probability of getting multiple of 4, $P(F) = \frac{5}{20}$

Probability of getting multiple of 5, $P(M) = \frac{4}{20}$

Now,

$$P(F \cap M) = P(F) \cdot P(M)$$

$$= \frac{5}{20} \cdot \frac{4}{20}$$

$$= \frac{1}{20}$$

d. Two dice are rolled together. Find the probability of getting;

i. Odd number in both dice

Solution:

Probability of getting odd no. in 1st roll, $P(A) = \frac{3}{6}$

Probability of getting odd no. in 2nd roll, $P(B) = \frac{3}{6}$

Now,

$$P(A \cap B) = \frac{3}{6} \times \frac{3}{6}$$

$$= \frac{1}{4}$$

ii. A number divisible by 3 in first die and a prime number in the second die.

Solution:

In 1st die, probability of getting no. divisible by 5, $P(F) = \frac{2}{6}$

In 2nd die, Probability of getting prime number, $P(P) = \frac{3}{6}$

Now,

$$P(F \cap P) = \frac{2}{6} \times \frac{3}{6}$$

$$= \frac{1}{6}$$

iii. A total of 8

Solution:

Here, total possible outcome $(n) = 36$

~~Possibl~~ favourable cases $(m) = 5$

We know, $P(F) = \cancel{8}\ \frac{m}{n} = \frac{5}{36}$,

iv. a total of 9 or 6

Solution:

Here, Total Possible cases (n) = 36

Total possible case = { (6,3) (5,4) (3,6) (4,5) (4,2) (2,4)
(3,3) (5,1) (1,5)

No. of favourable cases (m) = 9

Now, $P(9 \text{ or } 6) = \frac{9}{36}$

$= \frac{1}{4}$

7.a. The chance that A can solve the problem is $\frac{3}{5}$ and the chance that B can solve the same problem is $\frac{2}{3}$. Find the probability that;

i. The problem is solved by A and B

Solution:

Here, $P(A) = 3/5$

$P(B) = 2/3$

Now, $P(A \cap B) = P(A) \cdot P(B)$

$= \frac{3}{5} \cdot \frac{2}{3}$

$= \frac{2}{5}$

ii. The problem is solved

Solution:

We know, $P(A \cap B) = 2/5$

$P(A) = 3/5$ and $P(B) = 2/3$

Then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 3/5 + 2/3 - 2/5$

$= 13/15$

iii. None of them can solve the problem

Solution:

We know, $P(A \cup B) = 13/15$

Now, $P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - \frac{13}{15}$

$= \frac{2}{15}$

b. A problem in statistics is given to three students A, B and C whose chance of solving it are 1/3, 1/4, 2/5 respectively. Find the probability that i) the problem is solved.

Solution:

Here, $P(A) = 1/3$

$P(B) = 1/4$

$P(C) = 1/5$

$P(A \cup B \cup C) = ?$

We know, $P(A \cup B \cup C) = \cancel{P(A) + P(B) + P(C) - P(A \cap B \cap C)}$

$= \cancel{\frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}}$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$

$- P(A \cap B \cap C)$

$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{3} \cdot \frac{1}{4} - \frac{1}{3} \cdot \frac{1}{5} - \frac{1}{4} \cdot \frac{1}{5}$

$- \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}$

$=$

$$P(A \text{ or } B \text{ or } C) = 1 - P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$$

$$= 1 - \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}$$

$$= \frac{3}{5}$$

ii. The problem is solved by only one of them.

solution:

$$P(\text{solved by only one}) = P(A) \cdot P(\bar{B}) \cdot P(\bar{C}) + P(B) \cdot P(\bar{A}) \cdot P(\bar{C}) + P(C) \cdot P(\bar{A}) \cdot P(\bar{B})$$

$$= \frac{1}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} + \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{4}{5} + \frac{1}{5} \cdot \frac{2}{3} \cdot \frac{3}{4}$$

$$= \frac{13}{30}$$

c. Mita and sita appear in an interview for two post; the probability of Mita's selection in the interview is 80% and that of sita is 75%. find the probability that;

i. both of them will be selected

Solution:

Here, $P(M) = \frac{80}{100} = \frac{8}{10}$

$P(S) = \frac{75}{100} = \frac{3}{4}$

Now, $P(M \cap S) = \cancel{8}\ P(M) \cdot P(S)$

$$= \frac{8}{10} \cdot \frac{3}{4}$$

$$= \frac{3}{5}$$

ii. At least one of them will be selected

Solution:

We know, $P(M) = 8/10$

$P(S) = 3/4$

$P(M \cap S) = 3/5$

Now, $P(M \cup S) = P(M) + P(S) - P(M \cap S)$

$= \frac{8}{10} + \frac{3}{4} - \frac{3}{5}$

$= \frac{19}{20}$

iii. none of them will be selected

Solution:

$P(M \cup S) = 19/20$

$P(\overline{M \cup S}) = ?$

We know, $P(\overline{M \cup S}) = 1 - P(M \cup S)$

$= 1 - \frac{19}{20}$

$= \frac{1}{20}$

iv. Only one of them will be selected.

Solution:

$P(\text{only one}) = P(M) \cdot P(\overline{S}) + P(S) \cdot P(\overline{M})$

$= \frac{8}{10} \cdot \frac{1}{4} + \frac{3}{4} \cdot \frac{2}{10}$

$= \frac{7}{20}$

8.a. Two students are selected from section A and B of grade XI, one from each group. Section A consists of 12 girls and 20 boys and section B consists of 16 girls and 16 boys. Find the probability of getting;

i. both boys

Solution:

Probability of getting boys from A, $P(B) = \frac{20}{32}$

Probability of getting boys from B, $P(B') = \frac{16}{32}$

Now,

$P(B \cap B') = \frac{20}{32} \cdot \frac{16}{32}$

$= \frac{5}{16}$

ii. Boths girls

Solution:

$P(G) = \frac{12}{32}$

$P(G') = \frac{16}{32}$

Now, $P(G \cap G') = \frac{12}{32} \cdot \frac{16}{32}$

$= \frac{3}{16}$

iii. One is girl and other is boy

Solution:

$P(\text{one girl and one boy}) = P(B) \cdot P(G') + P(G) \cdot P(B')$

$= \frac{20}{32} \cdot \frac{16}{32} + \frac{12}{32} \cdot \frac{16}{32} \Rightarrow \frac{1}{2}$

b. Two cards are drawn from a well-shuffled packs of 52 cards one by one with replacement. Find the probability ~~of getting~~ that

i. Both are red cards

Solution:

Here, $P(R) = \frac{\cancel{13}}{\cancel{52}} \; \frac{26}{52}$

$P(R') = \frac{26}{52}$

Now, $P(R \cap R') = \frac{26}{52} \cdot \frac{26}{52} = \frac{1}{4}$

ii. Both are aces

Solution:

$P(A) = \frac{4}{52}$

$P(A') = \frac{4}{52}$

$P(A \cap A') = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$

iii. first is black card and second is red king.

Solution:

$P(B) = \frac{26}{52}$

$P(R) = \frac{2}{52}$

$P(B \cap R) = \frac{26}{52} \cdot \frac{2}{52}$

$= \frac{1}{52}$

iv. One is heart and other is black card.

Solution:

$p(\text{one heart and another black}) = P(H) \cdot P(B)$

$= \frac{13}{52} \cdot \frac{26}{52}$

$= \frac{1}{8}$

c. A bag contains 6 white and 3 blue balls. Two balls are drawn one after another with replacement. Find the probability that (i) both are white

Solution:

$P(W) = \frac{6}{9}$

$P(B) = \frac{3}{9}$

Now, $P(\text{both white}) = P(W) \cdot P(W)$

$= \frac{6}{9} \cdot \frac{6}{9}$

$= \frac{4}{9}$

ii. Both are blue

Solution:

$P(B) = \frac{3}{9}$

$\cancel{P(B \cap B')}$ $P(\text{both blue}) = P(B) \cdot P(B)$

$= \frac{3}{9} \cdot \frac{3}{9}$

$= \frac{1}{9}$

iii. they are different colour

Solution:

$$P(\text{different color}) = P(W)\cdot P(B) + P(B)\cdot P(W)$$

$$= \frac{6}{9}\cdot\frac{3}{9} + \cancel{P(3}\ \frac{3}{9}\cdot\frac{6}{9}$$

$$= 4/9$$

d. A box contains 4 red, 6 white and 2 blue balls. from this ~~Box~~ box 3 balls are drawn in succession (i.e. one by one). Find the probability that they are in order red, white and blue if each ball is replaced before the other ball is drawn.

Solution:

$$P(R) = \frac{4}{12}$$

$$P(W) = \frac{6}{12}$$

$$P(B) = \frac{2}{12}$$

Now, $$P(R\cap W\cap B) = P(R)\cdot P(W)\cdot P(B)$$

$$= \frac{4}{12}\cdot\frac{6}{12}\cdot\frac{2}{12}$$

$$= \frac{1}{36}$$

e. A lot contains 9 items of which 3 are defectives. Three item are choosen from the lot at random one after another with replacement. Find the probability that

i. All three are defectives

Solution:

$$P(D) = \frac{3}{9}$$

$$P(ND) = \frac{6}{9}$$

$$P(\text{All defective}) = \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9}$$

$$= \frac{1}{27}$$

ii. All three are non-defective

Solution:

$$P(\text{All non defective}) = P(ND) \cdot \cancel{P} \; P(ND') \cdot P(ND'')$$

$$= \frac{6}{9} \times \frac{6}{9} \times \frac{6}{9}$$

$$= \frac{8}{27}$$

iii. Only the first one is defective.

Solution:

$$P(\text{only } 1^{st} \text{ defective}) = \cancel{P(ND)} \; P(D) \cdot P(ND) \cdot P(ND')$$

$$= \frac{3}{9} \cdot \frac{6}{9} \cdot \frac{6}{9}$$

$$= \frac{4}{27}$$