Exercise - 8.2

General Section

1. Find a single equation represented by the following line pair.

a. $x = 2y$ and $2x = y$

Solution:

Given, $x - 2y = 0$ —— (i)

$2x - y = 0$ —— (ii)

Now,

The single ~~represen~~ equation represented by (i) and (ii) is;

$(x-2y)(2x-y) = 0$

or, $2x^2 - xy - 4xy + 2y^2 = 0$

or, $2x^2 - 5xy + 2y^2 = 0$

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b. $x + y = 0$ and $x + 2y = 0$

Solution:

Given, $x + y = 0$ —— (i)

$x + 2y = 0$ —— (ii)

Now,

The single equation represented by (i) and (ii) is;

$(x+y)(x+2y) = 0$

or $x^2 + 2xy + xy + 2y^2 = 0$

or, $x^2 + 3xy + 2y^2 = 0$

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c. $x + y + 2 = 0$ and $x + 2y + 1 = 0$

Solution:

Given; $x + y + 2 = 0$ —— (i)

$x + 2y + 1 = 0$ —— (ii)

Now equation represented by (i) and (ii) is

$(x+y+2)(x+2y+1) = 0$

or, $x^2 + 2xy + x + xy + 2y^2 + y + 2x + 4y + 2 = 0$

or $x^2 + 3xy + 3x + 5y + 3x + 2y^2 + 2 = 0$

d. $2x - y = -3$ and $x - 3y = -4$

Solution

Given; $2x + 3 - y = 0$ — (i)

$x + 4 - 3y = 0$ — (ii)

Now, single equation represented by (i) and (ii) is given by,

$(2x+3-y)(x+4-3y) = 0$

or $2x^2 + 8x - 6xy + 3x + 12 - 9y - xy - 4y + 3y^2 = 0$

or $2x^2 + 3y^2 + 11x - 7xy - 13y + 12 = 0$

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e. $ax - by = 0$ and $bx + ay = 0$

Solution:

Given, $ax - by = 0$ — (i)

$bx + ay = 0$ — (ii)

Now, ~~of~~ single equation ~~of~~ represented by (i) and (ii) is given by:

$(ax - by)(bx + ay) = 0$

or $abx^2 + a^2xy - b^2xy - aby^2 = 0$

or $abx^2 - aby^2 - b^2xy + a^2xy = 0$

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2. Find the separate equations represented by the following equation

a. $2x^2 + 7xy + 3y^2 = 0$

Here,

$2x^2 + 7xy + 3y^2 = 0$

or $2x^2 + (6+1)xy + 3y^2 = 0$

or $2x^2 + 6xy + xy + 3y^2 = 0$

or $2x(x+3y) + y(x+3y) = 0$

or $(x+3y)(2x+y) = 0$

So,

$x + 3y = 0$ — (i)

$2x + y = 0$ — (ii) are the two required equation.

b. $x^2 - 5xy + 4y^2 = 0$

Here,

$x^2 - 5xy + 4y^2 = 0$

or $x^2 - (4+1)xy + 4y^2 = 0$

or $x^2 - 4xy - xy + 4y^2 = 0$

or $x(x-4y) - y(x-4y) = 0$

or $(x-4y)(x-y) = 0$

So,

$(x-4y) = 0$ — (i)

$(x-y) = 0$ — (ii) are the two required equations.

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c. $6x^2 - 5xy - 6y^2 = 0$

Here,

$6x^2 - 5xy - 6y^2 = 0$

or $6x^2 - (9-4)xy - 6y^2 = 0$

or $6x^2 - 9xy + 4xy - 6y^2 = 0$

or $3x(2x-3y) + 2y(2x-3y)$

or $(2x-3y)(3x+2y)$

So,

$(2x-3y)\ \cancel{(3x+2y)} = 0$ — (i)

$(3x+2y) = 0$ — (ii) are the required equation.

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d. $3x^2 - 4xy - 4y^2 = 0$

Here,

$3x^2 - 4xy - 4y^2 = 0$

or $3x^2 - (6-2)xy - 4y^2 = 0$

or $3x^2 - 6xy + 2xy - 4y^2 = 0$

or $(3x+2y)\ \cancel{3x+}\ (x-2y) = 0$

So,

$(3x+2y) = 0$

$(x-2y) = 0$ are the required equation.

e. $2x^2 + 5xy + 2y^2 = 0$

Here,

$2x^2 + 5xy + 2y^2 = 0$

or $2x^2 + (4+1)xy + 2y^2 = 0$

or $2x^2 + 4xy + xy + 2y^2 = 0$

or $2x(x+2y) + y(x+2y)$

or $(2x+y)(x+2y) = 0$

So,

$(2x+y) = 0$ — (i)

$(x+2y) = 0$ — (ii) are the required equation.

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f. $x^2 - 3xy - 4y^2 = 0$

Here,

$x^2 - 3xy - 4y^2 = 0$

or $x^2 - (4-1)xy - 4y^2 = 0$

or $x^2 - 4xy + xy - 4y^2 = 0$

or $x(x-4y) + y(x-4y) = 0$

or $(x-4y)(x+y) = 0$

So,

$(x-4y) = 0$ — (i)

$(x+y) = 0$ — (ii) are the required equation.

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g. $x^2 - 4y^2 = 0$

Here,

$x^2 - 4y^2 = 0$

or, $(x)^2 - (2y)^2 = 0$

or $(x+2y)(x-2y) = 0$

So,

$(x+2y) = 0$ — (i)

$(x-2y) = 0$ — (ii) are the required answer.

h. $x^2 + 2y - y^2 - 2x = 0$

Here,

$x^2 - 2x + 2y - y^2 = 0$

or $x^2 - y^2 - 2x + 2y = 0$

or $(x+y)(x-y) - 2(x-y) = 0$

or $(x-y)(x+y-2) = 0$

here,

$(x-y) = 0$ — (i)

$(x+y-2) = 0$ — (ii)

They are the required seperate equation.

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i. $\frac{1}{2}x^2 + 2xy + \frac{1}{2}y^2 = 0$

Here,

j. $xy - 4x - 5y + 20 = 0$

here,

$xy - 4x - 5y + 20 = 0$

or, $x(y-4) - 5(y-4) = 0$

or, $(y-4)(x-5) = 0$

So,

$(y-4) = 0$ — (i)

$(x-5) = 0$ — (ii) are the required equation.

3. Find the equation of two lines represented by the following equations.

a. $x^2 + 2xy\sec\theta + y^2 = 0$

or $x^2 + 2x.y\sec\theta + (y\sec\theta)^2 - (y^2\sec^2\theta) + y^2 = 0$

or $(x + y\sec\theta)^2 - y^2(\cancel{y}\sec^2\theta - 1) = 0$

or $(x + y\sec\theta)^2 - (y\tan\theta)^2 = 0$

or $(x + y\sec\theta + y\tan\theta)(x + y\sec\theta - y\tan\theta) = 0$

Hence,

$(x + y\sec\theta + y\tan\theta) = 0$ — (i)

$(x + y\sec\theta - y\tan\theta) = 0$ — (ii) are the required equation

b. $y^2 + 2xy\cot\theta - x^2 = 0$

or $y^2 + 2x.y\cot\theta + (x\cot\theta)^2 - (x\cot\theta)^2 - x^2 = 0$

or $(y + x\cot\theta)^2 - x^2\cot^2\theta + x^2$

or $(y + x\cot\theta)^2 - \cancel{x\cot\theta}(x\csc\theta)^2$

or $(y + x\cot\theta + x\csc\theta)(y + x\cot\theta - x\csc\theta) = 0$

Hence,

$(y + x\cot\theta + x\csc\theta) = 0$ — (i)

$(y + x\cot\theta - x\csc\theta) = 0$ — (ii) are required equation.

c. $y^2 - 2xy\cot\theta - x^2 = 0$

or, $y^2 - 2y.x\cot\theta + (x\cot\theta)^2 - (x\cot\theta)^2 - x^2 = 0$

or, $(y - x\cot\theta)^2 - x^2(\cot^2\theta - 1) = 0$

or, $(y - x\cot\theta)^2 - (x\operatorname{cosec}\theta)^2 = 0$

or, $(y - x\cot\theta + x\operatorname{cosec}\theta)(y - x\cot\theta - x\operatorname{cosec}\theta) = 0$

Hence,

$(y - x\cot\theta + x\operatorname{cosec}\theta) = 0$ — (i)

$(y - x\cot\theta - x\operatorname{cosec}\theta) = 0$ — (ii) are required equation.

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d. $x^2 + 2xy\operatorname{cosec}\theta + y^2 = 0$

or, $x^2 + 2xy\operatorname{cosec}\theta + (y\operatorname{cosec}\theta)^2 - (y\operatorname{cosec}\theta)^2 + y^2 = 0$

or, $(x + y\operatorname{cosec}\theta)^2 - y^2(\operatorname{cosec}^2\theta - 1) = 0$

or, $(x + y\operatorname{cosec}\theta)^2 - (y\cot\theta)^2 = 0$

or, $(x + y\operatorname{cosec}\theta + y\cot\theta)(x + y\operatorname{cosec}\theta - y\cot\theta) = 0$

Hence,

$(x + y\operatorname{cosec}\theta + y\cot\theta) = 0$ — (i)

$(x + y\operatorname{cosec}\theta - y\cot\theta) = 0$ — (ii) are required equatio

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e. $x^2 - 2xy\sec\theta + y^2 = 0$

or, $x^2 - 2xy\sec\theta + (y\sec\theta)^2 - (y\sec\theta)^2 + y^2 = 0$

or, $(x - y\sec\theta)^2 - y^2(\sec^2\theta - 1) = 0$

or, $(x - y\sec\theta)^2 - (y\tan\theta)^2 = 0$

or, $(x - y\sec\theta + y\tan\theta)(x - y\sec\theta - y\tan\theta) = 0$

Hence,

$(x - y\sec\theta + y\tan\theta) = 0$ — (i)

$(x - y\sec\theta - y\tan\theta) = 0$ — (ii) are required equation.

f. $x^2 - 2xy\cot 2\alpha + y^2 = 0$

or $x^2 - 2x.y\cot 2\alpha + (y\cot 2\alpha)^2 - (y\cot 2\alpha)^2 + y^2 = 0$

or, $(x - y\cot 2\alpha)^2 - y^2(\cot^2 2\alpha - 1)$

or $(x - y\cot 2\alpha)^2 - (y\operatorname{cosec} 2\alpha)^2$

or $(x - y\cot 2\alpha + y\operatorname{cosec} 2\alpha)(x - y\cot 2\alpha - y\operatorname{cosec} 2\alpha) = 0$

Hence,

$(x - y\cot 2\alpha + y\operatorname{cosec} 2\alpha) = 0$ — (i)

$(x - y\cot 2\alpha - y\operatorname{cosec} 2\alpha) = 0$ — (ii) are required equation

4.a. Determine the lines represented by each of the following equat

i. $xy - 3x + 2y - 6 = 0$

or $x(y-3) + 2(y-3) = 0$

or $(y-3)(y+3) = 0$

Hence,

$(y-3) = 0$ — (i)

$(y+3) = 0$ — (ii) are the required equation.

ii. $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$

or, $x^2 + 2x.y + y^2 - 2x - 2y - 15 = 0$

or $(x+y)^2 - 2(x+y) - 15 = 0$

or, $(x+y)^2 - (5-3)(x+y) - 15 = 0$

or $(x+y)^2 - 5(x+y) + 3(x+y) - 15 = 0$

or $x+y(x+y-5) + 3(x+y-5) = 0$

or $(x+y-5)(x+y+3) = 0$

Hence,

$(x+y-5) = 0$ — (i)

$(x+y+3) = 0$ — (ii) are the required equations.

5. Prove that the two lines represented by the following equation are perpendicular to each other.

a. $6x^2 - 5xy - 6y^2 = 0$

Here,

$6x^2 - 5xy - 6y^2 = 0$ —(i)

The homogeneous of second degree is

$ax^2 + \cancel{2hby}\ 2hxy + by^2 = 0$ —(ii)

Now,

Comparing equation (i) and (ii)

$a = 6 \qquad 2h = -5 \qquad b = -6$

or $h = -5/2$

Let,

θ be the angle between the lines represented by equation

$\tan\theta = \pm \dfrac{2\sqrt{h^2 - ab}}{a+b}$

or, $\tan\theta = \pm \dfrac{2\sqrt{(-5/2)^2 - (6 \times -6)}}{6-6}$

or, $\tan\theta = \pm \dfrac{2\sqrt{(-5/2)^2 - (-36)}}{0}$

or, $\tan\theta = \infty$

or, $\tan\theta = \tan 90°$

$\therefore \theta = 90°$

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c. $3x^2 + 4xy - 3y^2 = 0$

Here,

$3x^2 + 4xy - 3y^2 = 0$ —(i)

Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$

So, $a = 3 \qquad 2h = 4 \qquad b = -3$

$\therefore h = 2$

Let the angle represented by $3x^2 + 4xy - 3y^2 = 0$ be 'θ'

Now,

$$\tan\theta = \pm \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\text{or, } \tan\theta = \pm \frac{2\sqrt{(2)^2 - (3\times -3)}}{3-3}$$

$$\text{or, } \tan\theta = \infty$$

$$\text{or, } \tan\theta = \tan 90^\circ$$

$$\therefore \theta = 90^\circ$$

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b. $x^2 - y^2 = 0$

here,

$x^2 - y^2 = 0$ —(i)

Comparing equation (i) with $ax^2 + \cancel{2hby} + 2hxy + by^2 = 0$

So, $a = 1$ $\quad$ $2h = 0$ $\quad$ $b = -1$

$\therefore h = 0$

Let θ be the angle between perpendicular lines represented by $x^2 - y^2 = 0$

$$\tan\theta = \pm \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\text{or, } \tan\theta = \pm \frac{2\sqrt{0^2 - (1\times -1)}}{1-1}$$

$$\text{or, } \tan\theta = \infty$$

$$\text{or, } \tan\theta = \tan 90^\circ$$

$$\therefore \theta = 90^\circ$$

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d. $x^2 - y^2 + x + y = 0$

here,

$x^2 - y^2 \cancel{+ x + y} = 0$ —(i)

Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$

So, $a = 1$, $h = 0$ $\quad$ $b = -1$

Let θ be the angle between the perpendicular lines represented by $x^2-y^2=0$

$$\tan\theta = \pm\frac{2\sqrt{h^2-ab}}{a+b}$$

or, $$\tan\theta = \pm\frac{2\sqrt{0^2-1\times-1}}{1-1}$$

or, $\tan\theta = \infty$

or, $\tan\theta = \tan 90^\circ$

$\therefore \theta = 90^\circ$

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e. $x^2-y^2+x-y=0$

Here,

The homogeneous term of the equation is, $x^2-y^2=0$ — (i)

Also,

Comparing equation (i) with $ax^2+2hxy+by^2=0$, we get

$a=1$, $h=0$, $b=-1$

Now,

$a+b = 1-1$

$= 0$

$\therefore a+b=0$

$\therefore$ The line represented by the given equation are perpendicular to each other.

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f. $9x^2-13xy-9y^2+2x-3y+7=0$

Here, homogeneous term is, $9x^2-13xy-9y^2$ — (i)

Comparing it with, $ax^2+2hxy+by^2=0$, we get

$a=9$ $\quad h=-\frac{13}{2}$ $\quad b=-9$

Now,

$a+b = 9-9$

$\therefore a+b=0$

So, the line represented by the given equation is perpendicular.

6. Show that the two ~~tin~~ straight lines represented by the followi equation are coincident.

a. $16x^2 + 16xy + 4y^2 = 0$

Here,

$16x^2 + 16xy + 4y^2 = 0$ — (i)

Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$

$a = 16 \quad h = 8 \quad b = 4$

Since,

They are coincident;

$h^2 = ab$

or, $(8)^2 = 16 \times 4$

or, $64 = 64$

Proved.

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b. $2x^2 + 12xy + 18y^2 + 8x + 24y + 8 = 0$

Here,

homogeneous term of given equation is;

$2x^2 + 12xy + 18y^2$ — (i)

Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$

So, $a = 2 \quad h = 6 \quad b = 18$

Since,

They are coincident;

$h^2 = ab$

or, $(6)^2 = 18 \times 2$

or, $36 = 36$

Proved.

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c. $x^2 - 6xy + 9y^2 = 0$

Here,

$x^2 - 6xy + 9y^2 = 0$ — (i)

Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$

So, $a = 1$ $\quad h = \frac{-6}{2}$ $\quad b = 9$

$= -3$

since, They are coincident;

$h^2 = ab$

or $(-3)^2 = 9 \times 1$

or, $9 = 9$

proved.

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d. $9x^2 - 6xy + y^2 = 0$

Here,

$9x^2 - 6xy + y^2 = 0$ — (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$

So, $a = 9$ $\quad h = -3$ $\quad y = 1$

since,

they are coincient

$h^2 = ab$

or $(-3)^2 = 9 \times 1$

or $9 = 9$

Proved.

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e. $4x^2 - 12xy + 9y^2 = 0$

Here,

$4x^2 - 12xy + 9y^2 = 0$ — (i)

comparing, it with $ax^2 + 2hxy + by^2 = 0$

So, $a = 4$, $h = -6$ $\quad b = 9$

since, They are coincident;

$h^2 = ab$

or, $(-6)^2 = (9 \times 4)$

or, $36 = 36$ proved.

7. Find the value of k when the following equations are perpendicular to each other.

a. $kx^2 + xy - 3y^2 = 0$

Here,

$kx^2 + xy - 3y^2 = 0$ —(i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$

we, get,

$a = k \quad h = \frac{1}{2} \quad b = -3$

Since,

they are perpendicular to each other.

$a + b = 0$

or $k + (-3) = 0$

$\therefore k = 3$

b. $(k-25)x^2 + 5xy = 0$

Here,

$(k-25)x^2 + 5xy = 0$ —(i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$

So, $a = k-25 \quad 2h = 5 \quad b = 0$

$h = 5/2$

Since,

They are perpendicular to each other;

$a + b = 0$

or $k - 25 + 0 = 0$

$\therefore k = 25$

c. $(3k+4)x^2 - 48xy - k^2y^2 = 0$

Here,

$(3k+4)x^2 - 48xy - k^2y^2 = 0$ —(i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 3k+4$ $\quad 2h = -48$ $\quad b = -k^2$

$\therefore h = -24$

Since,

they are perpendicular to each other ;

$a + b = 0$

or $3k + 4 + (-k^2) = 0$

or $3k + 4 - k^2 = 0$

or, $\cancel{k^2 - 3k + 4}$ $k^2 - 3k - 4 = 0$

or, $k^2 - (4-1)k - 4 = 0$

or, $k^2 - 4k + k - 4 = 0$

or, $k(k-4) + 1(k-4) = 0$

or $(k+1)(k-4) = 0$

Either $\quad$ OR

$k + 1 = 0$ $\quad k - 4 = 0$

$\therefore k = -1$ $\quad \therefore k = 4$

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8. Find the value of k when the pair of lines represented by the following equations are coincident.

a. $4x^2 + 2kxy + 9y^2 = 0$

Here,

$4x^2 + 2kxy + 9y^2 = 0$ — (1)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 4$ $\quad 2h = 2k$ $\quad b = 9$

$\therefore h = \frac{2k}{2} = k$

Since, they are coincident ;

$h^2 = ab$

or, $k^2 = 4 \times 9$

or $k = \sqrt{36}$

$\therefore k = \pm 6$

b. $(k+2)x^2 + 3xy + 4y^2 = 0$

Here,

$(k+2)x^2 + 3xy + 4y^2 = 0$ —(i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = k+2$, $h = \frac{3}{2}$ $b = 4$

Since, they are coincident;

$h^2 = ab$

or, $\left(\frac{3}{2}\right)^2 = (k+2)4$

or, $\frac{9}{4} = 4k + 8$

or, $9 = 16k + 32$

or, $9 - 32 = 16k$

or, $-23 = 16k$

$\therefore k = \frac{-23}{16}$

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c. $(10k-1)x^2 + (5k+3)xy + (k-1)y^2 = 0$

Here,

$(10k-1)x^2 + (5k+3)xy + (k-1)y^2 = 0$ —(i)

Comparing it with, $ax^2 + 2hxy + by^2 = 0$, we get,

$a = 10k - 1$ $h = \frac{5k+3}{2}$ $b = k-1$

Since, they are coincident;

~~at~~ $h^2 = ab$

or, $\left(\frac{5k+3}{2}\right)^2 = (10k-1)(k-1)$

or, $\frac{25k + 30k + 9}{4} = \cancel{10k^2 - 10k +}\ 10k^2 - 10k - k + 1$

or, $25k^2 + 30k + 9 = 4(10k^2 - 11k + 1)$

or $25k^2 + 30k + 9 = 40k^2 - 44k + 4$

or $25k^2 - 40k^2 + 30k + 44k + 9 - 4 = 0$

or $-15k^2 + 74k + 5 = 0$

or $-(15k^2 - 74k - 5) = 0$

or $15k^2 - 74k - 5 = 0$

or $15k^2 - (75 - 1)k - 5 = 0$

or $15k^2 - 75k + k - 5 = 0$

or $15k(k-5) + 1(k-5) = 0$

or, $(k-5)(15k+1) = 0$

Now,

Either	OR
$k - 5 = 0$	$15k + 1 = 0$
$\therefore k = 5$	$\therefore k = \frac{-1}{15}$

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9. Find the angle between the pair of lines.

a. $x^2 + 9xy + 14y^2 = 0$

Here,

$x^2 + 9xy + 14y^2 = 0$ — (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 1$, $h = \frac{9}{2}$, $b = 14$

If θ be the angle between the lines then,

$$\tan\theta = \frac{\pm 2\sqrt{h^2 - ab}}{a+b}$$

or $$\tan\theta = \frac{\pm 2\sqrt{81/4 - 1.14}}{1+14}$$

or $$\tan\theta = \frac{\pm 2\sqrt{\frac{81-56}{2}}}{15}$$

$\pm \; 2 \times 14$

or, $\tan\theta = \pm \dfrac{\cancel{2} \times \cancel{3}5}{15}$

or, $\tan\theta = \pm \dfrac{1}{3}$

or, $\tan\theta = \pm \cancel{1}/3$

or, $\theta = \tan^{-1} \cancel{(\pm 1/3)} \left(\frac{1}{3}\right)$

b. $2x^2 + 7xy + 3y^2 = 0$

here,

$2x^2 + 7xy + 3y^2 = 0$ — (i)

Comparing it with ~~$2x^2$~~ $ax^2 + 2hxy + by^2 = 0$, we get

$a = 2 \quad h = \frac{7}{2} \quad b = 3$

Let θ, be the angle between the lines, then

$\tan\theta = \pm \dfrac{2\sqrt{h^2 - ab}}{a+b}$

or $\tan\theta = \pm \dfrac{2\sqrt{\frac{49}{4} - 6}}{5}$

or $\tan\theta = \pm \dfrac{2\sqrt{\frac{49-24}{4}}}{5}$

or $\tan\theta = \pm \dfrac{2 \times \frac{5}{2}}{5}$

or $\tan\theta = \pm 1$

or $\tan\theta = \pm \tan 45^\circ$

$\therefore \theta = 45^\circ$ or, $\theta = 135^\circ$

c. $x^2 - 7xy + y^2 = 0$

Here,

$x^2 - 7xy + y^2 = 0 \quad -(1)$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, then we get

$a = 1, \quad h = \frac{-7}{2}, \quad b = 1$

Let θ be the angle between the lines, then

$$\tan\theta = \frac{\pm 2\sqrt{h^2 - ab}}{a+b}$$

or, $$\tan\theta = \frac{\pm 2\sqrt{\frac{49}{4} - 1}}{2}$$

or $$\tan\theta = \frac{\pm \cancel{2}\sqrt{\frac{45}{4}}}{\cancel{2}}$$

or, $$\tan\theta = \pm\frac{3\sqrt{5}}{2}$$

or $$\theta = \frac{\pm 3\sqrt{5}}{2}$$

$$\therefore \theta = \pm \tan^{-1}\left(\frac{3\sqrt{5}}{2}\right)$$

---

d. $x^2 + 3xy + 2y^2 = 0$

Here,

$x^2 + 3xy + 2y^2 = 0 \quad -(1)$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 1 \quad h = \frac{3}{2} \quad b = 2$

Let θ be the angle between the lines, then

$$\tan\theta = \frac{\pm 2\sqrt{h^2 - ab}}{a+b}$$

or, $$\tan\theta = \frac{\pm 2\sqrt{\frac{9}{4} - 2}}{3}$$

or, $$\tan\theta = \frac{\pm 2\sqrt{\frac{1}{4}}}{3}$$

or, $$\tan\theta = \frac{\pm 2 \times \frac{1}{2}}{3}$$

or, $\tan\theta = \pm\frac{1}{3}$

$\therefore \theta = \tan^{-1}(\pm 1/3)$

e. $4x^2 + 5xy - 6y^2 = 0$

here,

$4x^2 + 5xy - 6y^2 = 0$ — (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 4$ $h = \frac{5}{2}$ $b = -6$

Let θ be the angle between the lines, then

$$\tan\theta = \pm\frac{2\sqrt{h^2-ab}}{a+b}$$

or, $\tan\theta = \pm\frac{2\sqrt{\frac{25}{4}+24}}{-2}$

or, $\tan\theta = \pm\frac{2\sqrt{\frac{121}{4}}}{-2}$

or, $\tan\theta = \pm\frac{2\times 11/2}{-2}$

or, $\tan\theta = \pm\frac{11}{-2}$

or, $\theta = \tan^{-1}(\pm 11/2)$

f. $x^2 - 2xy\sec\theta + y^2 = 0$

Here,

$x^2 - 2xy\sec\theta + y^2 = 0$ — (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 1$ $h = -\sec\theta$ $b = 1$

Let α be the angle between the lines represented by the given equation;

$$\tan\alpha = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$$

or, $\tan\alpha = \frac{\pm 2\sqrt{\sec^2\theta - 1}}{2}$

or, $\tan\alpha = \pm\sqrt{\tan^2\theta}$

~~or, $\theta = \pm\tan^{-1}(\pm\tan\theta)$~~

or, $\tan\alpha = \tan\theta$

$\therefore$ ~~$\alpha = \theta$~~ $\therefore \alpha = \theta$

---

g. $x^2 - 2xy\cot\theta - y^2 = 0$

Here,

$x^2 - 2xy\cot\theta - y^2 = 0$ — (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 1$ $h = -\cot$ $b = -1$

Let, θ be the angle between the pair of lines, then

$$\tan\theta = \frac{\mp 2\sqrt{h^2-ab}}{a+b}$$

or, $\tan\theta = \frac{\pm 2\sqrt{\cot^2\theta - 1}}{0}$

or, $\tan\theta = \infty$

or, $\tan\theta = \tan 90°$

$\therefore \theta = 90°$

---

h. $x^2 + 2xy\ \mathrm{cosec}\,\alpha + y^2 = 0$

Here,

$x^2 + 2xy\ \mathrm{cosec}\,\alpha + y^2 = 0$ — (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 1$ $h = \mathrm{cosec}\,\alpha$ $b = 1$

Also,

Let α be the angle between the pair of line.

we know,

$$\tan\alpha = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$$

or, $\tan\alpha = \frac{\pm 2\sqrt{\operatorname{cosec}^2\theta - 1}}{2}$

or $\tan\alpha = \pm\sqrt{\cot^2\theta}$

or, $\tan\alpha = \pm\cot\theta$

or $\alpha = \tan^{-1}(\pm\cot\theta)$

Taking (+ve) sign

$\tan\alpha = \tan(90° - \alpha)$

$\therefore \alpha = 90° - \alpha$

Taking (-ve) sign

$\tan\alpha = -\tan(90-\alpha)$

$\therefore \alpha = 90 + \alpha$

---

i. $9x^2 - 13xy - 9y^2 + 2x - 3y + 7 = 0$

here,

Homogeneous term is $9x^2 - 13xy - 9y^2$ —(i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 9$ $\quad h = \frac{-13}{2}$ $\quad a = -9$

Let α be the angle between the lines, we know,

$$\tan\alpha = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$$

or $\tan\alpha = \frac{\pm 2\sqrt{\frac{169}{4} + 81}}{0}$

or, $\tan\alpha = \infty$

or, $\tan\alpha = \pm\tan 90°$

$\therefore \alpha = \pm 90°$

---

j. $x^2 + 6xy + 9y^2 + 4x + 12y - 15 = 0$

soln

Homogeneous term is $x^2 + 6xy + 9y^2 = 0$ —(i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$

$a = 1$ $\quad h = 3$ $\quad b = 9$

Let θ be the angle between the lines.

$$\tan\theta = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$$

or $$\tan\theta = \frac{\pm 2\sqrt{9-9}}{9+1}$$

or $$\tan\theta = \frac{0}{10}$$

or $\tan\theta = \pm 0$

or $\tan\theta = \pm \tan 0^\circ$

$\therefore \theta = 0^\circ$

---

k. $y^2 + 2xy\cot\theta - x^2 = 0$

Here,

$y^2 + 2xy\cot\theta - x^2 = 0$ —(i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 1 \quad h = \cot\theta \quad b = -1$

Let α be the angle between the lines.

$$\tan\alpha = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$$

or, $$\tan\alpha = \frac{\pm 2\sqrt{\cot^2+1}}{-1+1}$$

or $$\tan\alpha = \frac{\pm 2\operatorname{cosec}\theta}{0}$$

or $\tan\alpha = \infty$

or $\tan\alpha = \tan 90^\circ$

$\therefore \alpha = 90^\circ$

Creative Section

20a) Find the equation of two lines represented by the equation $2x^2 +$ $7xy + 3y^2 = 0$. Also, find the angle between them.

Solution:

Here, $2x^2 + 7xy + 3y^2 = 0$ — (i)

or $2x^2 + (6+1)xy + 3y^2 = 0$

or $2x^2 + 6xy + xy + 3y^2 = 0$

or $2x(x+3y) + y(x+3y) = 0$

or $(x+3y)(2x+y) = 0$

So,

$(x+3y) = 0$ — (ii)

$(2x+y) = 0$ — (iii)

Also,

Comparing equation (i) with $ax^2 + 2hxy + by^2$, we get

$a = 2$ $\quad h = \frac{7}{2}$ $\quad b = 3$

Let θ be the angle between them;

$$\tan\theta = \pm \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\text{or } \tan\theta = \pm \frac{2\sqrt{\frac{49}{4} - 6}}{5}$$

$$\text{or } \tan\theta = \pm \frac{2\sqrt{\frac{25}{4}}}{5}$$

$$\text{or } \tan\theta = \pm \frac{2 \times \frac{5}{2}}{5}$$

or $\tan\theta = \pm 1$

or $\tan\theta = \pm \tan 45^\circ$

When,

$\tan\theta = 1$ $\qquad\qquad$ $\tan\theta = -1$

$\theta = 45^\circ$ $\qquad\qquad$ $\theta = 135^\circ$

b. Find the equation of pair of lines represented by the equation $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$. Also, find the angle between them.

Here,

$$x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0 \quad \text{—(i)}$$

or $x^2 - 2x.y\operatorname{cosec}\theta + (y\operatorname{cosec}\theta)^2 + y^2 - y^2\operatorname{cosec}^2\theta = 0$

or $(x - y\operatorname{cosec}\theta)^2 - y^2\operatorname{cosec}^2\theta + y^2 = 0$

or $(x - y\operatorname{cosec}\theta)^2 - y^2(\operatorname{cosec}^2\theta - 1) = 0$

or $(x - y\operatorname{cosec}\theta)^2 - y^2\cot^2\theta = 0$

or $(x - y\operatorname{cosec}\theta)^2 - (y\cot\theta)^2 = 0$

or $(x - y\operatorname{cosec}\theta + y\cot\theta)(x - y\operatorname{cosec}\theta - y\cot\theta) = 0$

So,

$$(x - y\operatorname{cosec}\theta + y\cot\theta) = 0 \quad \text{—(ii)}$$

$$(x - y\operatorname{cosec}\theta - y\cot\theta) = 0 \quad \text{—(iii)}$$

Also,

Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 1 \quad h = -\operatorname{cosec}\theta \quad b = 1$

Let α be the angle between the lines:

$$\tan\alpha = \frac{\pm \cancel{2}\sqrt{\operatorname{cosec}^2\theta - 1}}{\cancel{2}}$$

or $\tan\alpha = \cot\theta$

or $\alpha = \tan^{-1}(\pm\cot\theta)$

Taking (+ve)

$\tan\alpha = \tan(90 - \theta)$

$\therefore \alpha = 90° - \theta$

Taking (−ve)

$\tan\alpha = \tan(90 + \theta)$

$\therefore \alpha = 90 + \theta$

11.a. What is the single equation of straight lines through the origin and perpendicular to the lines represented by $x^2 - 5xy + 4y^2 = 0$

Solution:

Here,

$x^2 - 5xy + 4y^2 = 0$.

or $x^2 - (4+1)xy + 4y^2 = 0$

or $x^2 - 4xy - xy + 4y^2 = 0$

or $x(x-4y) - y(x-4y) = 0$

or, $(x-4y)(x-y) = 0$

So,

$(x-4y) = 0$ — (i)

$(x-4y) = 0$ — (ii)

from (i), $m_1 = \frac{1}{4}$

Slope of line perpendicular to equation (i) is

$m.m_1 = -1$

or $\frac{1}{4}m_1 = -1$

$\therefore m' = -4$

Also,

equation of line passing through origin and have slope -4 is

$y - y_1 = m(x - x_1)$

or $y - 0 = -4(x-0)$

or $y = -4x$

or $4x + y = 0$ — (iii)

Also

Taking equation (ii), $m_2 = 1$

Slope of line ~~perde~~ perpendicular to equation (ii) is given by (m_2')

~~$y - y_1 = m_2(x - x_1)$~~

~~or $y - 0$~~ $m.m_2 = -1$

or $1.m_2 = -1$

$\therefore m_2 = -1$

Again,

Equation of line passing through origin and have slope -1 is

$y - y_1 = m(x - x_1)$

or, $y - 0 = -1(x - 0)$

or, $y = -x$

or, $x + y = 0$ —— (iv)

Also,

Combining equation (iii) and (iv), we get

$(x+y)(4x+y) = 0$

or $4x^2 + 5xy + y^2 = 0$ is the required equation.

---

b. Find the equation of two straight lines which passes through the point (2,3) and parallel to the line $x^2 - 6xy + 8y^2 = 0$

Solution:

Here,

$x^2 - 6xy + 8y^2 = 0$ —— (i)

or $x^2 - (4+2)xy + 8y^2 = 0$

or $x^2 - 4xy - 2xy + 8y^2 = 0$

or $x(x - 4y) - 2y(x - 4y) = 0$

or $(x - 4y)(x - 2y) = 0$

So,

$(x - 4y) = 0$ —— (ii)

$(x - 2y) = 0$ —— (iii)

from (ii), $m_1 = \frac{1}{4}$

from (iii), $m_2 = \frac{1}{2}$

Also,

Equation of line parallel to (ii) is, $m_1' = \frac{1}{4}$

Equation of line parallel to (iii) is $m_2' = \frac{1}{2}$

Now,

Equation of line passing through $(2,3)$ and have slope $\frac{1}{4}$ is

$$y - y_1 = m(x - x_1)$$

or, $y - 3 = \frac{1}{4}(x - 2)$

or, $4y - 12 = x - 2$

or, $x - 4y + 10 = 0$ —— (iv)

Also,

Equation of line passing through $(2,3)$ and have slope $\frac{1}{2}$ is

$$y - y_1 = m(x - x_1)$$

or, $y - 3 = \frac{1}{2}(x - 2)$

or, $2y - 6 = x - 2$

or, $x - 2y + 4 = 0$ —— (v)

Hence,

required equation is

$$(x - 4y + 10)(x - 2y + 4) = 0$$

or, $x^2 - 2xy + 4x - 4xy + 8y^2 - 16y + 10x - 20y + 40 = 0$

or, $x^2 - 6xy + 14x - 36y + 40 = 0$ #

---

12 a) Find the separate equations when lines represented by $kx^2 + 8xy - 3y^2 = 0$ are orthogonal to each other.

Here,

$kx^2 + 8xy - 3y^2 = 0$ —— (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = k \quad h = 4 \quad b = -3$

Since,

they are orthogonal

$a + b = 0$

or, $k - 3 = 0$

$\therefore k = 3$

Putting the value of 'k' in equation (i)

$3x^2 + 8xy - 3y^2 = 0$

or $3x^2 + (9-1)xy - 3y^2 = 0$

or, $3x^2 + 9xy - xy - 3y^2 = 0$

or, $3x(x+3y) - y(x+3y) = 0$

or $(x+3y)(3x-y) = 0$

Hence,

$(x+3y) = 0$ — (ii)

$(3x-y) = 0$ — (iii) are the required equations.

---

b. Find the two equation when the lines represented by $9x^2 + 12xy + my^2 - 3x - 2y = 0$ are coincident.

Here,

The homogeneous term is $9x^2 + 12xy + my^2$ — (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 9 \quad h = 6 \quad b = m$

since,

They are coincident

$h^2 = ab$

or $36 = 9m$

$m = \frac{36}{9}$

$\therefore m = 4$

Putting the value of m in equation (i)

$9x^2 + 12xy + 4y^2 \cancel{=0} - 3x - 2y = 0$

~~or $9x^2 + (6+6)xy + 4y^2 = 0$~~

~~or $9x^2 + 6xy + 6xy + 4y^2 = 0$~~

~~or $3x(x+2y) + 2y + 3$~~

or $(3x)^2 + 2.3x.2y + (2y)^2 - (3x+2y) = 0$

or, $(3x+2y)^2 - (3x+2y) = 0$

or $(3x+2y)(3x+2y-1) = 0$