Chapter - 3
Fluid Statics

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Surface Tension
Surface Tension is a property of liquid by virtue of which liquid surface tries to occupy minimum surface area and behaves like a stretched membrane under tension.

$$\text{Surface Tension } (T) = \frac{\text{Force}}{\text{Length}}$$

SI unit is Nm^{-1}.

Cohesive force.
The force of attraction among the same types of ~~force~~ molecules is called cohesive force. Eg: Force of attraction between water molecules.

Adhesive Force
The force of attraction among the different types of molecules is called Adhesive force. Eg: force of attraction between ink molecules and molecules of white board, force of attraction between oil molecules and molecules of hair etc.

Surface Energy of liquid

Consider a rectangular wire frame ABCD in which the section AB of the wire is moveable. When the frame is dipped into a soap solution, a film is formed in the frame.

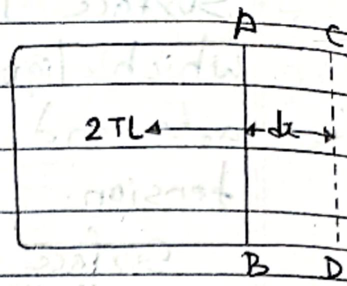


Fig: stretching of soap solution film

Let T be the surface Tension of the film and L be the length of wire AB then,

$$T = \frac{F}{2L}$$

$$\therefore F = T(2L)$$

Now,

$$\text{Work done } (dW) = F\,dx$$
$$= T\,2L.dx$$
$$= 2T\,dA$$

Where dA = Surface area of one side of free surface

Here,

Total amount of work done to change the area from A_1 to A_2 is;

$$W = \int_{A_1}^{A_2} 2T\,dA$$

$$\text{or, } W = 2T\int_{A_1}^{A_2} dA$$

$$\text{or, } W = T\left(2\int_{A_1}^{A_2} dA\right)$$

$$\text{or } W = \text{Surface Tension} \times \text{Increase in area}$$

$$\therefore \frac{W}{\text{Increase in area}} = \text{Surface Tension}$$

$\therefore$ Surface Tension (T) = Surface Energy (σ)

SI Unit of σ is Joule/metre square or Nm^{-1}

Angle of Contact

The angle θ which the tangent to the liquid surface at the point of contact with the solid surface inside the liquid is called angle of contact or capillary angle.

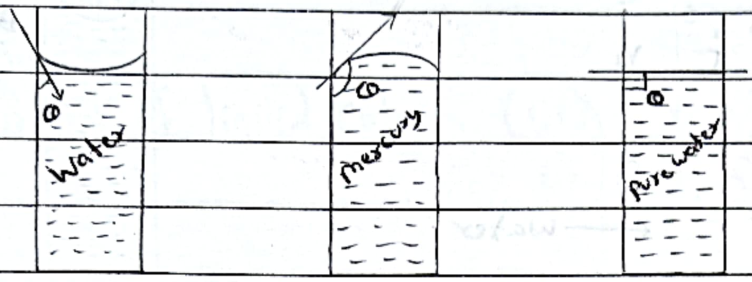


Fig: Angle of Contact

i. If adhesive force > cohesive force, the free surface of the liquid is concave in shape. This liquid wets the glass.

ii. If cohesive force > adhesive force, the free surface of the liquid is convex in shape. This liquid doesn't wet the glass.

iii. If adhesive force = cohesive force, the free surface of the liquid is plane.

Capillarity

A tube whose bore is comparable with the diameter of hair is called a capillary tube.

The phenomeon of rise or fall of a liquid in a capillary tube in comparison to the surrounding ~~is~~ is called capillary action or capillarity.

Measure of Surface Tension by Capillary Rise method:

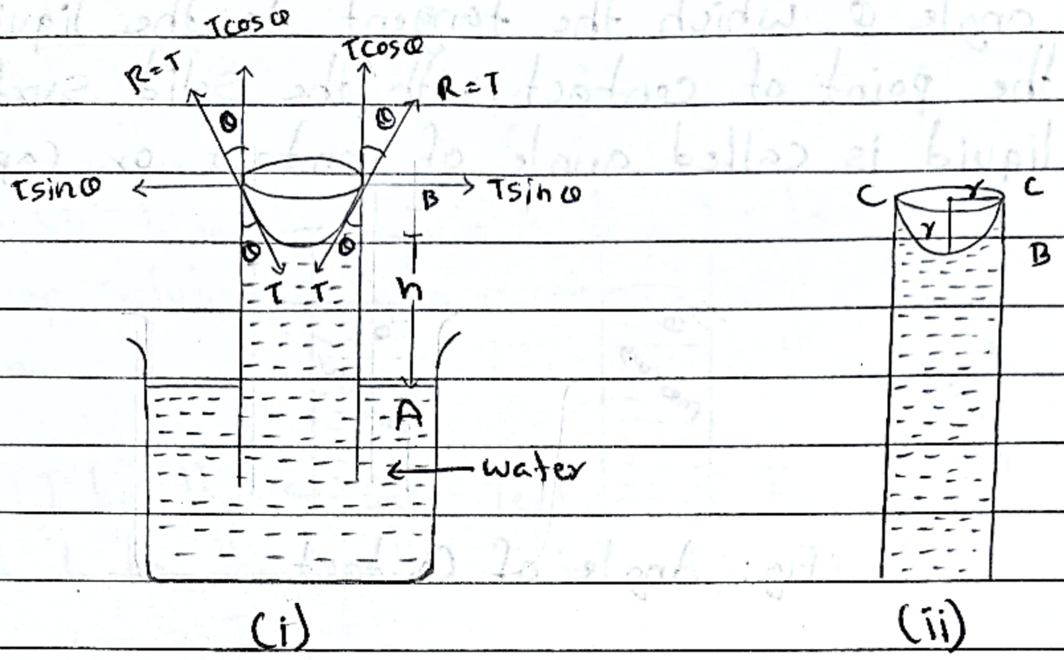


Fig: Surface Tension by Capillary rise

Let 'θ' be the angle of contact and 'r' be the internal radius of the capillary tube.

Now,

Total vertical force acting upward through-out the circumference is given by;

$$F = T\cos\theta \cdot 2\pi r \quad \text{—— (1)}$$

This force pulls the liquid upward until the weight of the liquid equals to upward force.

Also,

Volume of liquid in ~~min~~ meniscus is given by;

V_1 = [volume of cylinder of height 'r' and radius 'r'] − volume of hemisphere

$$\text{or, } V_1 = \pi r^2 \cdot r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \pi r^3 - \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^3$$

∴ Total volume of liquid raised is given by:

$$V = \pi r^2 h + \frac{1}{3}\pi r^3$$

$$\therefore V = \pi r^2 \left(h + \frac{r}{3}\right)$$

Again,

Weight of liquid column (W) = mg

$$\text{or, } W = \rho \times V \times g$$

$$= \rho \times \pi r^2 \left(h + \frac{r}{3}\right) g \quad \text{--- (11)}$$

AT Equilibrium:

Total upward force = wt. of liquid displace

$$\text{or, } T\cos\theta \, 2\pi r = \rho \times \pi r^2 \left(h + \frac{r}{3}\right) g$$

$$\text{or, } T = \frac{r\rho g\,(h + r/3)}{2\cos\theta}$$

For very small bore, $h \gg \frac{r}{3}$. So, $\frac{r}{3}$ can be neglected

$$\text{i.e. } T = \frac{rh\rho g}{2\cos\theta}$$

Excess pressure on a curved liquid Surface.

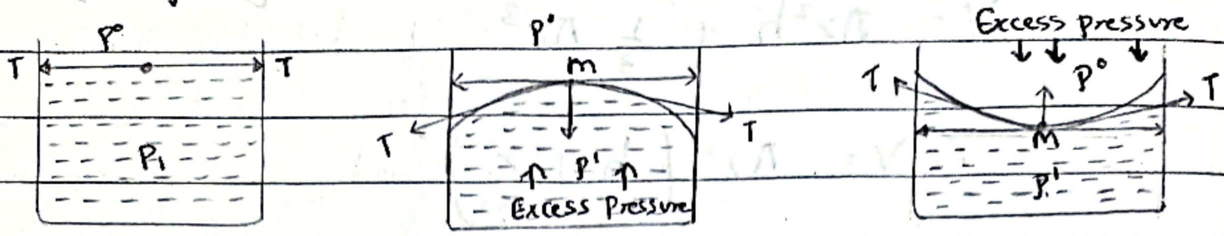
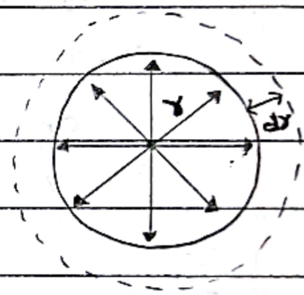


Fig (1) $P_o = P_i$ | Fig (2) $P_i > P_o$ | Fig (3) $P_o > P_i$

Here,

P_o = Outside pressure and P_i = Inside pressure

Determination of Excess Pressure



Consider a spherical liquid drop of radius 'r' and surface Tension 'T'.

P_i = pressure inside drop

P_o = outside pressure

Now,

$P = P_i - P_o$ [As, $P_i > P_o$]

When the size of drop gets enlarged, the radius increases from r to $(r + dr)$

Also,

Work done to increase surface area is given by;

dW = Excess force x distance

= Pressure x Area x distance [$\because F = P \times A$]

$= (P_i - P_o) \times 4\pi r^2 \times dr$ —— ①

Similarily, Increase in PE of the liquid is given by:

dE = Surface Tension (T) x increase in area

$= T \times [4\pi(r+dr)^2 - 4\pi r^2]$

$= T \times 4\pi [r^2 + 2r \cdot dr + dr^2 - r^2]$

$= 4\pi T [2r dr + dr^2]$

Since, dr is very small, its higher power i.e. dr^2 can be neglected. Then,

$$dE = 4\pi T \times 2r \cdot dr \quad \text{——— (ii)}$$

from (i) and (ii)

$$(P_i - P_o) \times 4\pi r^2 \times dr = 4\pi T \times 2r \cdot dr$$

$$P_i - P_o = \frac{2T}{R} \quad \text{——— (iii)}$$

Viscosity

Viscosity is the property of the liquid or gas by virtue of which it opposes the relative motion between its consecutive layers.

Cause of Viscosity

Internal friction between the molecules in a fluid is the main cause of viscosity. Consider a flowing fluid to be made up of layers that moves in relation to another. These layers scrape against each other, and the more the friction there is, the slower the flow (the more force required to achieve flow)

Some causes of viscosity:

a. Temperature

Viscosity decreases as temperature increases. Warmer fluids have more kinetic energy, which reduces molecular attraction and makes them flow more easily.

2. Pressure

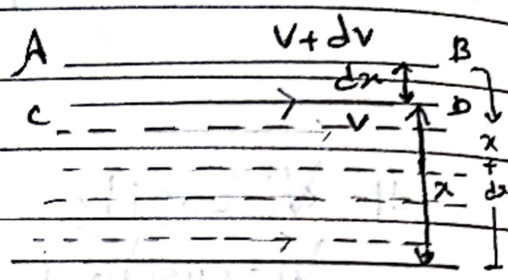
Except water, the viscosity of the liquid increases with the increase in pressure. In case of water at normal temperature, the viscosity decreases with the increase in pressure for few hundred atm. pressure.

Newton's Law of Viscosity

Consider two layers of liquid x and y at a distance x and $x + dx$ respectively from the fixed layers. Let v and v + dv be their respective velocities.

Now,

According to Newton's law of Viscous flow, the viscous force 'f' acting tangentially on the liquid layer is;



a. Directly proportional to Area

i.e. $F \propto A$ —— ①

b. Directly proportional to velocity gradient

i.e. $F \propto \frac{dv}{dx}$ —— ⑪

from ① and ⑪, we get;

$$F \propto A.\frac{dv}{dx}$$

$$\therefore F = -\eta A.\frac{dv}{dx}$$ [∵ η = Coefficient of viscosity]

Also,

$$\eta = \frac{F}{A.\frac{dv}{dx}}$$

SI unit of coefficient of viscosity = NSm^{-2}

Stroke's Law

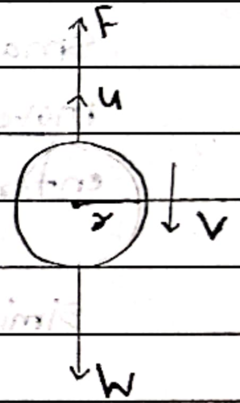
Stroke found that the viscous force, f acting on a spherical body of radius 'r' moving through a viscous fluid is directly proportional to sphere's velocity, ~~and~~ its radius as well as the viscosity of the fluid.

Mathematically,

$$F = 6\pi\eta r v_1$$

Expression for Terminal Velocity

Terminal velocity is the constant maximum velocity acquired by a body while falling through a viscous fluid.



Let 'r' be the radius of spherical ball falling through the viscous fluid of density 'σ' and coeff. of viscosity. Let 's' be the density of the material of the ball.

The force acting on a body is;

i. It's weight, W downward

ii. Upward thrust, u equal to wt. of displaced fluid.

iii. Viscous force, f in the direction opposite to the direction of motion of a body.

When the body is falling with terminal velocity 'V'

$$W = F + U$$

$$\text{or, } mg = 6\pi\eta r V_t + V\sigma g$$

$$\text{or, } \frac{4}{3}\pi r^3 \times s g = 6\pi\eta r V_t + \frac{4}{3}\pi r^3 \sigma g$$

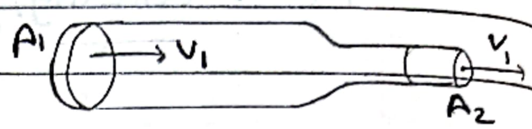
$$\text{or, } \frac{4}{3}\pi r^3 (s - \sigma) g = 6\pi\eta r V_t$$

$$\text{or, } \eta = \frac{2}{9} \frac{r^2 (s - \sigma) g}{V_t}$$

$$\therefore V = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Equation of Continuity

Let A_1 and A_2 be CSA of two ends of a portion of a tube. Let V_1 and V_2 be the Velocity of the liquid at CSA A_1 and A_2 resp.



Assuming no fluid enters or leaves out the tube. Let small time interval 'dt' be the time when the fluid moves a distance $V_1 dt$ from A_1. So volume of the fluid entering from A_1 during interval dt is

$$V_1 = A_1 V_1 dt$$

Similarily, $V_2 = A_2 V_2 dt$

Let dm_1 and dm_2 be the mass of fluid entering into and flowing out from CSA A_1 and A_2 of the tube resp. Then

$$dm_1 = V_1 \rho = A_1 \rho V_1 dt$$

and $$dm_2 = V_2 \rho = A_2 \rho V_2 dt$$

In steady flow, the total mass in the tube is constant. So,

$$dm_1 = dm_2$$

or, $$A_1 \rho V_1 dt = A_2 \rho V_2 dt$$

$$\therefore A_1 V_1 = A_2 V_2 = \text{Constant}$$

This means,

$$AV = \text{Constant}$$

i.e. $$A \propto \frac{1}{V}$$

Bernoullis Theorem

It states that for the streamline flow of an ideal liquid (incompressible and non-viscous liquid), the sum of pressure energy, K.E and PE per unit mass remains constant at every section throughout the flow.

Mathematically,

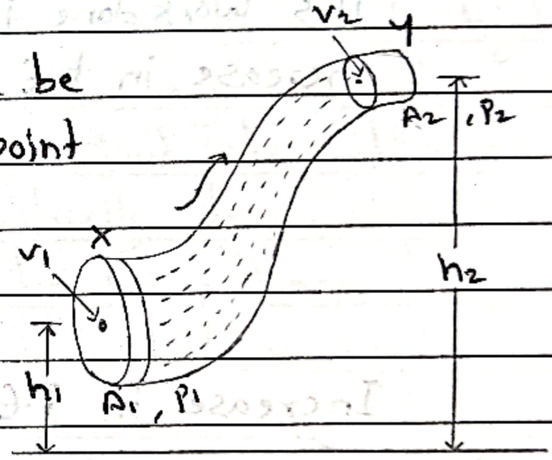
$$\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}$$

Proof :

Consider the streamlined flow of an ideal fluid through a pipe ~~of~~ XY of varying CSA.

Let P_1, a_1, h_1, v_1 and P_2, a_2, h_2, v_2 be the pressure, CSA and velocity at point X and Y resp.

Also, let in time 'Δt' the fluid reach from point X to Y.



Now,

Force acting on a fluid at end X,

$$F_1 = P_1 a_1$$

$\therefore$ Work done on the fluid at end 'X'

$$W_1 = F_1 S_1 \quad [\because S_1 = \text{displacement at } 1^{st} \text{ end}]$$

$$= P_1 a_1 v_1 \Delta t$$

Similarily,

Work done by fluid at end 'Y'

$$W_2 = F_2 S_2$$

$$= P_2 a_2 v_2 \Delta t$$

Thus,

Net work done on the fluid from X to Y is

$$W = W_1 - W_2$$

i.e. $W = P_1 a_1 v_1 \Delta t - P_2 a_2 v_2 \Delta t$ —— (I)

from Equation of continuity:

$$a_1 v_1 = a_2 v_2$$

Since the liquid is incompressible (assume)

$$a_1 v_1 \Delta t = V_1 \text{ and } a_2 v_2 \Delta t = V_2$$ [$\because V$ = Volume of liquid displaced]

$$\text{i.e. } V_1 = V_2 = \frac{m}{\rho}$$

From equation (I)

$$W = P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho}$$

$$\therefore W = (P_1 - P_2)\frac{m}{\rho} \text{ —— (II)}$$

This work done is used in giving K.E and PE to the liquid

$$\text{Increase in K.E} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$= \frac{1}{2} m (v_2^2 - v_1^2) \text{ —— (III)}$$

$$\text{Increase in P.E} = mgh_2 - mgh_1$$

$$= mg(h_2 - h_1) \text{ —— (IV)}$$

According to work energy theorem

Work done by pressure energy = Increase in (KE + PE)

$$\text{i.e. } (P_1 - P_2)\frac{m}{\rho} = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

$$\text{or, } \frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + gh_2 - gh_1$$

$$\text{or, } \frac{P_1}{\rho} + gh_1 + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2} v_2^2$$

In general, $\frac{P}{\rho} + \frac{1}{2} v^2 + gh = \text{Constant}.$

Give reasons:

1. Does a ship sink more in river water or in sea water? Explain.

Ans: The density of sea water is greater than the density of river water, so the weight of the liquid displaced should be greater in river water than in sea water. Hence, ship sink more in river water than in sea water.

2. In still air a helium filled balloon rises up to a certain height and then stop rising. Why?

Ans: In the lower height, the atmospheric pressure is high so the balloon displaces the sufficient weight of the air to get the upthrust to lift it up. But when it rises up, the atmospheric pressure gradually decreases. So the amount of air displaced will be low. Hence, the upthrust is not sufficient to lift the balloon up.

3. Why is a soap solution a better cleansing agent than ordinary water?

Ans: The addition of soap helps to decrease the surface tension of water and the decrease in surface tension enhances the wetting and washing power. Hence, the soap solution is a better cleansing agent than ordinary water.

4. Antiseptics used in cuts and wounds of human flesh have low surface Tension. Why?

Ans: Antiseptics have low surface tension. so that it can

Spread over a large area over the wound and paralize the germs. It helps to grow the new tissue in place of the infected tissue.

5. Hot soup is more tasty than a cold one, why?

Ans: Hot soup is more tasty than a cold one because the surface tension of hot soup is less as the surface tension decrease with increase in temperature. The low surface tension helps to spread the soup all over the tongue so that the taste buds on tongue receive it.

6. How is the rise of liquid affected, if the top ~~is~~ of the capillary tube is closed?

Ans: If the top of ~~liquid~~ the capillary tube is closed, the liquid will rise to lesser height as compared to the ordinary condition. This is because the air gets compressed and the pressure will increase. The compressed air will oppose the rise of liquid.

7. Why it is difficult to introduce mercury in a fine capillary tube?

Ans: The angle of contact between the mercury and glass is obtuse ($\theta > 90°$). Due to this, the mercury descends in the glass so that it is difficult to introduce mercury in the glass capillary tube.

8. Hotter liquid moves faster than colder one. Why?

Ans: The liquid having low coefficient of viscosity will move faster than that of high coefficient of viscosity. The coefficient of viscosity of colder liquid is high than that of hot one, hence the hotter liquid move faster than cold one.

9. Why are light roofs blown ~~up~~ off during cyclones or strong winds storms?

Ans: During cyclones, the velocity of wind suddenly decreases when enters into the houses, but ~~in~~ its velocity remains high above the roof. This makes high pressure below the roof and low pressure above the roof. The high pressure below the roof lifts it up and hence roofs gets blown off.

10. An aeroplane runs for some distance on the ~~ra~~ runway before taking off. Why?

Ans: Generally, air doesn't strike an aeroplane with a large velocity. To get the lift, the aeroplane runs for some distance on the runway before taking off. Due to it's special shape, the velocity of air above the plane increases and hence pressure decreases. The aeroplane gets an uplift.

11. A tiny liquid drop is spherical but a larger drop has oval shape. Why?

Ans:

Numerical Problems

1. What is the pressure on a swimmer 20 m below the surface of a lake?

Solution:

Depth (h) = 20 m

Density of water (ρ) = 1000 kgm^{-3}

Acceleration due to gravity (g) = 9.8 ms^{-2}

The pressure on a swimmer 20m below the surface of the lake is given by;

$$P = P_{atm} + h\rho g$$
$$= 1.01 \times 10^5 + 20 \times 1000 \times 9.8$$
$$= 1.21 \times 10^5 \ Nm^{-2}$$

2. Find the work done required to break up a drop of water of radius 5×10^{-3} m into eight drops of water, assuming isothermal condition.

Solution:

Radius of large drop (R) = 5×10^{-3} m

No. of drop (n) = 8

Work done (W) = ?

Surface Tension of water (T) = $72 \times 10^{-3} Nm^{-1}$

Since, the mass of water drop remains constant

$$\left(\frac{4}{3}\pi R^3\right)\rho = 8\left(\frac{4}{3}\pi r^3\right)\rho$$

or, $R^3 = 8r^3$

or, $R = 2r$

or, $r = \dfrac{5 \times 10^{-3}}{2}$

$\therefore r = 2.5 \times 10^{-3}$ m.

Now,

Surface Area of large drop $= 4\pi R^2$

$= 4\pi (5 \times 10^{-3})^2$

$= 3.14 \times 10^{-4} \, m^2$

Surface area of smaller drops $= 8 \times 4\pi r^2$

$= 8 \times 4\pi (2.5 \times 10^{-3})^2$

$= 6.28 \times 10^{-4} \, m^2$

Again,

Increase in Area $= (6.28 - 3.14) \times 10^{-4} \, m^2$

$= 3.14 \times 10^{-4} \, m^2$

Now,

Work done (W) = Increase in area $\times T$

$= 3.14 \times 10^{-4} \times 72 \times 10^{-3}$

$= 2.26 \times 10^{-5}$ Joule

3. Water flows steadily through a horizontal tube which consists of 100 parts joined end to end. One part is 21 cm long and has a diameter of 0.225 cm and the other is 7.0 cm long and has a diameter of 0.075 cm. If the pressure difference between the ends of tube is 14 cm of water. Find the pressure difference between the ends of each parts.

Solution:

Here, Length of one part $(l_1) = 21$ cm

Radius of one part $(r_1) = 0.1125$ cm

Length of other part $(l_2) = 7$ cm

Radius of other part $(r_2) = 0.0375$ cm

Let P_1 and P_2 be the pressure difference, and η be the viscosity of water.

Then,

Volume of water flowing per sec in 1st part $\left(\frac{V}{t}\right)_1$

$$= \frac{\pi}{8} \times \frac{P_1 r_1^4}{\eta l_1}$$

Volume of water flowing per sec in 2^{nd} part $\left(\frac{V}{t}\right)_2$

$$= \frac{\pi}{8} \times \frac{P_2 r_2^4}{\eta L_2}$$

Since,

$$\left(\frac{V}{t}\right)_1 = \left(\frac{V}{t}\right)_2$$

or, $\frac{\pi}{8} \times \frac{P_1 r_1^4}{\eta L_1} = \frac{\pi}{8} \frac{P_2 r_2^4}{\eta L_2}$

or, $\frac{P_1 r_1^4}{\eta L_1} = \frac{P_2 r_2^4}{\eta L_2}$

or, $\frac{P_2 (0.0375)^4}{7} = \frac{P_1 (0.1125)^4}{21}$

or, $P_1 = 0.037 P_2$

Again,

Pressure difference between two ends = 14 cm

Then, $P_1 + P_2 = 14$

or, $0.037 P_2 + P_2 = 14$

or, $1.037 P_2 = 14$

$\therefore P_2 = 13.5$ cm

$\therefore P_1 = 0.5$ cm

4. Water flows steadily through a horizontal pipe of non-uniform cross section. If the pressure of water is 4×10^4 Nm^{-2} at a point where the velocity of flow is $2 ms^{-1}$ and cross section is 0.02 m^2, what is the pressure at a point where cross section reduces to 0.01 m^2?

Solution:

For one end of pipe,

Pressure $(P_1) = 4 \times 10^4$ Nm^{-2}

Velocity $(v_1) = 2 ms^{-1}$

Area $(A_1) = 0.02 m^2$

For another end of pipe:
Pressure (P_2) = ?
Area $(A_2) = 0.01\ m^2$
Velocity (V_2) = ?
We know, from continuity equation

$$A_1 V_1 = A_2 V_2$$

$$\text{or } V_2 = \frac{2 \times 0.02}{0.01}$$

$$= 4ms^{-1}$$

Now, using Bernoulli's equation,

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho V_2^2 \qquad [\because \rho = 1000\ kgm^{-3}]$$

$$\text{or, } 4 \times 10^4 + \frac{1}{2} \times 1000 \times 2^2 = P_2 + \frac{1}{2} \times 1000 \times 4^2$$

$$\text{or, } 42000 = P_2 + 8000$$

$$\text{or, } P_2 = 42000 - 8000$$

$$= 34000\ Nm^{-2}$$

$$= 3.4 \times 10^4\ Nm^{-2}$$

5. Two spherical rain drops of equal size are falling vertically through air with a terminal velocity of 0.15ms What would the terminal velocity be if these two drops were to coalesce to from a larger spherical drop?

Solution:
Terminal velocity of each small drop (v) = $0.15\ ms^{-1}$
The terminal velocity of each small drop is;

$$V = \frac{2r^2(\rho - \sigma)g}{9\eta} \quad \text{—— (i)}$$

Let R be the radius of combined drop. The terminal velocity of large drop is,

$$\therefore V' = \frac{2R^2(\rho - \sigma)g}{9\eta} \quad \text{—— (ii)}$$

Dividing equation (ii) by (i), we get;

$$\therefore \frac{V'}{V} = \frac{R^2}{r^2} \quad \text{—— (iii)}$$

Volume of large drop = Volume of two small drop

$$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3$$

$$\therefore R = 2^{1/3} r \quad \text{—— (iv)}$$

from (iii) and (iv), we have,

$$V' = V \times \frac{R^2}{r^2}$$

$$\text{or, } V' = V \times \frac{2^{1/3} r^2}{r^2}$$

$$\text{or, } V' = V \times 2^{1/3}$$

$$\text{or, } V' = \cancel{0 \times}\ 0.15 \times 2^{1/3}$$

$$\therefore V' = 0.238\ ms^{-1}$$

Hence,

The terminal velocity of large drop is $0.238\ ms^{-1}$