H/W

Chapter-8
Coordinate Geometry

Date:

Page:

Exercise-8.1

General Section

1. Find the actual angle between the following pairs of straight lines.

a. $2x + 4y = 4$ and $x + 2y = 3$

Solution:

We have, $2x + 4y = 4$ ——— (i)

$x + 2y = 3$ ——— (ii)

From (i), Slope $(m_1) = -\frac{2}{4} = -\frac{1}{2}$

From (ii), Slope $(m_2) = -\frac{1}{2}$

Angle between them,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

$$= \frac{-\frac{1}{2} + \frac{1}{2}}{1 - \frac{1}{2} \cdot -\frac{1}{2}}$$

$\tan\theta = 0$

or, $\tan\theta = \tan 0^\circ$

$\therefore \theta = 0^\circ$

b. $x - \sqrt{3}y = a$ and $\sqrt{3}x - y = b$

Solution:

We have, $x - \sqrt{3}y = a$ —— (i)

$\sqrt{3}x - y = b$ —— (ii)

From (i), Slope $(m_1) = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

From (ii), Slope $(m) = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

Angle between them is given by;

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

$$= \frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{2}$$

$$= \frac{-2}{\sqrt{3}} \times \frac{1}{2}$$

$$= \frac{-1}{\sqrt{3}}$$

Taking positive 'value;

$$\tan\theta = \frac{1}{\sqrt{3}}$$

or, $\tan\theta = \tan 30°$

$\therefore \theta = 30°$

---

c. $y = (2-\sqrt{3})x + 4$ and, $y = (2+\sqrt{3})x - 7$

Solution:

we have, $y = (2-\sqrt{3})x + 4$ —(i)

$y = (2+\sqrt{3})x - 7$ —(ii)

Comparing equation (i) and (ii) with, $y = mx + c$

$m_1 = (2-\sqrt{3})$

$m_2 = (2+\sqrt{3})$

Now,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

$$= \frac{(2-\sqrt{3})-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{2-\sqrt{3}-2-\sqrt{3}}{1+4-3}$$

$$= \frac{-2\sqrt{3}}{2}$$

$$= -\sqrt{3}$$

Taking positive value

$\tan\theta = \sqrt{3}$

or $\tan\theta = \tan 60^\circ$

$\therefore \theta = 60^\circ$

---

d. $x\cos\alpha + y\sin\alpha = p$ and $x\sin\alpha - y\cos\alpha = q$

Solution:

we have,

$x\cos\alpha + y\sin\alpha = p$ —— (i)

$x\sin\alpha - y\cos\alpha = q$ —— (ii)

From (i), slope $(m_1) = -\frac{\cos\alpha}{\sin\alpha} = -\cot\alpha$

From (ii), slope $(m_2) = \frac{-\sin\alpha}{-\cos\alpha} = \tan\alpha$

Now, $\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$

$$= \frac{-\cot\alpha - \tan\alpha}{1 + -\cot\alpha \cdot \tan\alpha}$$

$$= \frac{-\cot\alpha - \tan\alpha}{1 - 1}$$

$$= \frac{-\cot\alpha - \tan\alpha}{0}$$

$$= \infty$$

Page:

$\tan\theta = \tan\infty$

or $\tan\theta = \tan 90°$

$\therefore \theta = 90°$

2. Find the obtuse angle between the following pairs of straight line.

a. $2x - y + 3 = 0$ and $3x + y + 3 = 0$

Solution:

We have, $2x - y + 3 = 0$ —— (i)

$3x + y + 3 = 0$ —— (ii)

From (i), slope $(m_1) = \frac{-2}{-1} = 2$

From (ii), slope $(m_2) = \frac{-3}{1} = -3$

Now, $\tan\theta = \frac{2+3}{1+2\times-3}$

$= \frac{5}{-5}$

or, $\tan\theta = -1$

~~Taking positive~~

or $\tan\theta = -\tan 45°$

or $\tan\theta = \tan(180° - 45°)$

$\therefore \theta = 135°$

b. $x + y + a = 0$ and $3x - 2y + b = 0$

Solution:

We have,

$x + y + a = 0$ —(i)

$3x - 2y + b = 0$ —(ii)

from (i), $m_1 = -1$

from (ii), $m_2 = \frac{-3}{-2} = \frac{3}{2}$

$$\tan\theta = \frac{-1 - \frac{3}{2}}{1 - 1 \cdot \frac{3}{2}}$$

or, $\tan\theta = \frac{\frac{-5}{2}}{\frac{-1}{2}}$

or, $\tan\theta = 5$

$\therefore \theta = \tan^{-1}(5)$

---

c. $y = (2-\sqrt{3})x + 5$ and $y = (2+\sqrt{3})x - 7$

Solution:

$y = (2-\sqrt{3})x + 5$ —— (i)

$y = (2+\sqrt{3})x - 7$ —— (ii)

from (i), $m_1 = (2-\sqrt{3})$

$m_2 = (2+\sqrt{3})$

Now, $\tan\theta = \frac{(2-\sqrt{3}) - (2+\sqrt{3})}{1 + (2-\sqrt{3})(2+\sqrt{3})}$

$= \frac{2-\sqrt{3}-2-\sqrt{3}}{1+4-3}$

$= \frac{-2\sqrt{3}}{2}$

or, $\tan\theta = -\sqrt{3}$

or, $\tan\theta = -\tan 60°$

or, $\tan\theta = \tan(180 - 60°)$

or, $\tan\theta = \tan 120°$

$\therefore \theta = 120°$

d. $y = -2$ and $y = \sqrt{3}x - 1$

Solution:

$y = -2 \qquad \cancel{y+2=0}$ ——— (i)

$y - \sqrt{3}x + 1 = 0$ ——— (ii)

from (i), $m_1 = 0$

from (ii), $m_2 = \sqrt{3}$

Now, $\tan\theta = \dfrac{0 - \sqrt{3}}{1 + 0\cdot\sqrt{3}}$

or, $\tan\theta = \dfrac{-\sqrt{3}}{1}$

or, $\tan\theta = -\tan 60°$

or, $\tan\theta = \tan(180 - 60°)$

$\therefore \theta = 120°$

3.a. Show that the line $3x + 5 - 4y = 0$ and $6x - 8y + 7 = 0$ are parallel to each other.

Solution:

$3x + 5 - 4y = 0$ ——— (i)

$6x - 8y + 7 = 0$ ——— (ii)

from (i), $m_1 = \dfrac{-3}{-4} = \dfrac{3}{4}$

from (ii), $m_2 = \dfrac{-6}{-8} = \dfrac{3}{4}$

Here, $m_1 = m_2$,

So, they are parallel to each other.

---

b. Prove that the following pairs of line $4x + 3y = 7$ and $3x - 4y + 5 = 0$ is perpendicular to each other.

Solution:

$4x + 3y = 7$ ——— (i)

$3x - 4y + 5 = 0$ ——— (ii)

from (i), $m_1 = \frac{-4}{3}$

from (ii), $m_2 = \frac{-3}{-4} = \frac{3}{4}$

Here, $m_1 \times m_2 = \frac{-4}{3} \times \frac{3}{4}$

$\therefore m_1 \cdot m_2 = -1$

So, they are perpendicular to each other.

---

c. Show that the lines $3x - y - 2 = 0$ and the line joining $(-2, 5)$ and $(-3, 2)$ are parallel to each other.

Solution :

$3x - y - 2 = 0$ —(i)

from(i) $= (m_1) = \frac{-3}{-1} = 3$

Also, $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{-3 + 2} = \frac{-3}{-1} = 3$

Here, $m_1 = m_2$.

So, they are parallel to each other.

---

d. Show that the line joining the points $(7, -5)$ and $(3, 4)$ is perpendicular to ~~ea~~ the line $4x - 9y + 7 = 0$.

Solution:

Let the given points be $A(7, -5)$ and $B(3, 4)$

Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4 + 5}{3 - 7} = \frac{9}{-4}$

Also, $4x - 9y + 7 = 0$ —(i)

from (i), $m_2 = \frac{-4}{-9} = \frac{4}{9}$

Here, $m_1 \cdot m_2 = \frac{9}{-4} \times \frac{4}{9}$

$\therefore m_1 \cdot m_2 = -1$

So, they are perpendicular to each other.

---

4. Find the value of k when

a. The pair of straight lines ~~are~~ $kx + 3y - 8 = 0$ and $2y - 3x = 1$ are parallel.

Solution:

$kx + 3y - 8 = 0$ —— (i)

$2y - 3x = 11$ —— (ii)

From (i), $m_1 = \frac{-k}{3}$

From (ii), $m_2 = \frac{-3}{-2} = \frac{3}{2}$

Since, equation (i) and (ii) are parallel

$m_1 = m_2$

or, $\frac{-k}{3} = \frac{3}{2}$

or, $-2k = 9$

or $k = \frac{-9}{2}$

$\therefore k = -9/2$

---

b. The pair of straight lines $2x + 10y + 1 = 0$ and $x + ky + 1 = 0$ are parallel.

Solution:

$2x + 10y + 1 = 0$ —— (i)

$x + ky + 1 = 0$ —— (ii)

From (i), $m_1 = -2/10 = -1/5$

From (ii), $m_2 = -1/k$

Since, equation (i) and (ii) are parallel.

$m_1 = m_2$

or $\frac{-1}{5} = \frac{-1}{k}$

or $-k = -5$

$\therefore k = 5$

---

c. The lines $\sqrt{3}x + 2y = 9$ and $kx + \sqrt{3}y + 3 = 0$ are perpendicular to each other.

Solution:

$\sqrt{3}x + 2y = 9$ —(i)

$kx + \sqrt{3}y + 3 = 0$ —(ii)

from (i), $m_1 = \frac{-\sqrt{3}}{2}$

from (ii), $m_2 = \frac{-k}{\sqrt{3}}$

Since, equation (i) and (ii) are ~~parallel~~ perpendicular to each other.

$m_1 . m_2 = -1$

or $\frac{-\sqrt{3}}{2} \times \frac{-k}{\sqrt{3}} = -1$

or $\frac{k}{2} = -1$

$\therefore k = -2$

---

d. The lines $5x - 4y + 16 = 0$ and $4x + ky - 29 = 0$ are perpendicular to each other.

Solution:

$5x - 4y + 16 = 0$ —(i)

$4x + ky - 29 = 0$ —(ii)

From (i), $m_1 = 5/4$

From (ii), $m_2 = -4/k$

By question:

$m_1 . m_2 = -1$

or $\frac{5}{4} \times -\frac{4}{k} = -1$

or, $-5 = -k$

$\therefore k = 5$

---

e. The line given by the equation $3x - 4y + 7 = 0$ and $kx + 3y + 5 = 0$ are perpendicular to each other.

Solution:

$3x - 4y + 7 = 0$ —— (i)

$kx + 3y + 5 = 0$ —— (ii)

From (i), $m_1 = 3/4$

From (ii), $m_2 = -k/3$

By question:

$m_1 . m_2 = -1$

or $\frac{3}{4} \times -\frac{k}{3} = -1$

or $-k = -4$

$\therefore k = 4$

---

f. The lines $kx - 3y + 6 = 0$ is perpendicular to line joining $(4,3)$ and $(5,-3)$.

Solution:

$kx - 3y + 6 = 0$ —(i)

From (i), $m_1 = k/3$

Also,

Slope of Point $(4,3)$ and $(5,-3)$

$m_2 = \frac{-3-3}{5-4} = \frac{-6}{1}$

By question:

$m_1 \cdot m_2 = -1$

or, $\cancel{\frac{k}{3} \times -\frac{1}{2} = -1}$ $\qquad$ $\frac{k}{\cancel{3}} \times -\cancel{6}^{2} = -1$

or, $\cancel{-2k = -1}$ $\qquad$ or, $-2k = -1$

or, $\cancel{2k = 1}$ $\qquad$ $\therefore k = \frac{1}{2}$ #

$\cancel{\therefore k = 1/2}$

---

9. The line joining the points $(-5,7)$ and $(0,-2)$ is perpendicular to the line joining the points $(1,-3)$ and $(4,k)$

Solution:

Let the given points be, $A(-5,7)$, $B(0,-2)$, $C(1,-3)$ and $D(4,k)$

Slope of ~~(AB)~~ AB $(m_1) = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-2-7}{0+5} = \cancel{\frac{-10}{5}} = \frac{-9}{5}$

Slope of CD $(m_2) = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{k+3}{4-1}$

$= \frac{k+3}{3}$

By question:

$m_1 \cdot m_2 = -1$

or, $\frac{-9}{5} \times \frac{(k+3)}{3} = -1$

or, $\cancel{-2k-6 = -3}$ $\qquad$ $-9k - 27 = -15$

or, $\cancel{-2k = 3}$ $\qquad$ or, $k = \frac{-15+27}{-9}$

$\cancel{\therefore k = -3/2}$

$\therefore k = \boxed{\cancel{-3/4}}\ -4/3$

4. The line joining the points (8,2) and (4,1) is parallel to the line joining the points (6,4) and (k,2)

Solution:

slope of (8,2) and (4,1) $= \frac{y_2-y_1}{x_2-x_1}$

$$= \frac{1-2}{4-8} = \frac{1}{4}$$

slope of (6,4) and k,2) $= \frac{y_2-y_1}{x_2-x_1}$

$$= \frac{2-4}{k-6}$$

$$= \frac{-2}{k-6}$$

By question:

$m_1 = m_2$

or, $\frac{1}{4} = \frac{-2}{k-6}$

or, $k-6 = -8$

$\therefore k = \cancel{\ } -2$

---

5. Find the equation of the straight line which.

a. Passing through (2,3) and parallel to the line having slope $\frac{-4}{5}$

Solution:

Since, the lines are parallel, $m_1 = m_2 = \frac{-4}{5}$

The equation of st. line is given by;

$y-y_1 = m(x-x_1)$

or, $y-3 = \frac{-4}{5}(x-2)$

or, $5y-15 = -4x+8$

or, $4x+5y-23 = 0$ is the required equation.

b. Passing through (-1,7) and perpendicular to the line having slope -3.

Solution:

Given point $(x_1, y_1) = (-1, 7)$

Let the slope be 'm', then By question;

$$m \times (-3) = -1$$

or $-3m = -1$

$\therefore m = 1/3$

Hence, equation of st. line passing through (-1,7) and having slope $1/3$ is;

$$y - y_1 = m(x - x_1)$$

or $y - 7 = \frac{1}{3}(x+1)$

or, $3y - 21 = x + 1$

or, $x - 3y + 22 = 0$ is the required equation.

---

c. Passes through the point (2,3) and is parallel to the line $5x - 4y + 3 = 0$.

Solution:

Given, Point $(x_1, y_1) = (2, 3)$

$$5x - 4y + 3 = 0 \quad \text{— (i)}$$

From (i), $m_1 = \frac{-5}{-5} = \frac{5}{4}$

Since, they are parallel, $(m_1 = m_2) = 5/4$

Equation of st. line is given by;

$$y - y_1 = m(x - x_1)$$

or $y - 3 = \frac{5}{4}(x - 2)$

or, $4y - 12 = 5x - 10$

or, $5x - 4y + 2 = 0$ is the required equation.

d. Passes through the point (3,4) and is parallel to the straight line $3x + 4y = 12$.

Solution:

Given, Point $(x_1, y_1) = (3,4)$

$3x + 4y = 12$ — (i)

from (i), $m_1 = \frac{-3}{4}$

Since, they are parallel the equation of st. line is given by;

$y - y_1 = m(x - x_1)$

or, $y - 4 = \frac{-3}{4}(x - 3)$

or, $4y - 16 = -3x + 9$

or, $3x + 4y - 25 = 0$ is the required equation.

e. Passes through the point (-2,-3) and is perpendicular to the line $5x + 7y = 14$.

Solution:

Given, $(x_1, y_1) = (-2, -3)$

$5x + 7y = 14$ — (i)

from (i), $m_1 = \frac{-5}{7}$

Let the slope be m_2. By question

$m_2 \times \frac{-5}{7} = -1$

or, $m_2 = 7/5$

According to question. The equation of st. line is given by

$y - y_1 = m(x - x_1)$

or, $y + 3 = \frac{7}{5}(x + 2)$

or, $5y + 15 = 7x + 14$

or, $7x - 5y - 1 = 0$ is the required equation.

8. Passes through the point of intersection of the lines $x+y=3$ and $2x+y-5=0$ and parallel to the line joining the points $(4,8)$ and $(1,2)$

Solution:

Given, $x+y=3$ —(i)

$2x+y-5=0$ —(ii)

from (i) and (ii)

$x+y=3$

$\underline{2x+y=5}$

$-\quad -\quad -$

or, $-x=-2$

$\therefore x=2$

putting the value of x in equation (i)

$x+y=3$

or, $2+y=3$

$\therefore y=1$

Hence, $(x_1,y_1)=(2,1)$

Also,

slope of $(4,8)$ and $(1,2)$ $=\dfrac{y_2-y_1}{x_2-x_1}$

$=\dfrac{2-8}{1-4}=2$

Equation of st. line is given by;

$y-y_1=m(x-x_1)$

or, $y-1=2(x-1)$

or, $y-1=2x-2$

or, $2x-y-1=0$ is the required equation.

---

f.9. Passes through the point $(4,6)$ and is perpendicular to the line $x-2y-2=0$.

Solution:

Given, $(x_1,y_1)=(4,6)$

$x - 2y - 2 = 0$ —(i)

from (i), slope $(m_1) = \frac{-1}{-2} = 1/2$

Also,

Let m_2 be the slope. Since, they are perpendicular;

$$\frac{1}{2} \times m_2 = -1$$

or $m_2 = -2$

Now, equation of line passing through (4,6) and having slope -2 is

$$y - y_1 = m(x - x_1)$$

or, $y - 6 = -2(x-4)$

or, $y - 6 = -2x + 8$

or $2x + y - 14 = 0$ is the required equation.

h. Divide the line joining the points A(-1,-4) and B(7,1) perpendicularly in the ratio 3:2.

Solution

Let P be the point that divides A(-1,-4) and B(7,1) in the ratio 3:2

Now, $P(x,y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$

$$= \left(\frac{3 \cdot 7 - 2 \cdot 1}{3+2}, \frac{3 \cdot 1 - 4 \cdot 2}{3+2} \right)$$

$$= \left(\frac{19}{5}, -1 \right)$$

Also, slope of line AB is;

$$AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1+4}{7+1} = \frac{5}{8}$$

Also,

Let m' be the slope of line perpendicular to AB

By question;

$m \cdot m_1 = -1$

or, $\frac{5}{8} \times m_1 = -\cancel{8}1$

$\therefore m_1 = -8/5$

Again,

The equation of st. line passing through A(-2,-4) and B(7,1) and has slope $-8/5$ is given by;

$y - y_1 = m(x - x_1)$

or, $y + 1 = \frac{-8}{5}\left(x - \frac{19}{5}\right)$

or, $y + 1 = \frac{-8}{5}\left(\frac{5x - 19}{5}\right)$

or, $y + 1 = \frac{-40x + 152}{25}$

or, $25y + 25 = -40x + 152$

or, $40x + 25y - 127 = 0$ is the required equation.

---

6. If the angle between the lines $(a^2 - b^2)x = (p+q)y$ and $(p^2 - q^2)x + (a+b)y = 0$ is the 90°, show that $(a-b)(p-q) = 1$

Solution:

Given;

$(a^2 - b^2)x = (p+q)y$

or, $(a^2 - b^2)x - (p+q)y = 0$ —(i)

Also,

$(p^2 - q^2)x + (a+b)y = 0$ —(ii)

from (i), $m_1 = \frac{a^2 - b^2}{(p+q)}$

from (ii), $m_2 = \frac{-(p^2 - q^2)}{(a+b)}$

we know,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

or, $$\tan 90^\circ = \frac{\frac{(a^2-b^2)}{(p+q)} + \frac{(p^2-q^2)}{(a+b)}}{1 + \frac{(a^2-b^2)}{(p+q)} \cdot -\frac{(p^2-q^2)}{(a+b)}}$$

or, $$\frac{1}{\cot 90^\circ} = \frac{\frac{(a^2-b^2)(a+b) + (p^2-q^2)(p+q)}{(p+q)(a+b)}}{1 + \frac{\cancel{(a+b)}(a-b) \times -\cancel{(p+q)}(p-q)}{\cancel{(p+q)}\cancel{(a+b)}}}$$

or, $$\frac{1}{0} = \frac{(a^2-b^2) + (p^2-q^2)}{1+\{(a-b)(p-q)\}}$$

or, $1 - (a-b)(p-q) = 0 \times \{(a^2-b^2) + (p^2-q^2)\}$

or, $1 - (a-b)(p-q) = 0$

or, $(a-b)(p-q) = 1$

proved,,

7.a. Find the equation of a straight line passing through the point $(2,3)$ and perpendicular to the line $4x-3y=10$.

Solution:

Given, $(x_1, y_1) = (2,3)$

$4x-3y=10$ — (i)

from (i), $m = \frac{4}{3}$

Let the slope of line be m'. Since they are perpendicular.

$m \cdot m' = -1$

or, $\frac{4}{3} \times m' = -1$

or, $m' = -\frac{3}{4}$

Equation of st. line passing through $(2,3)$ and has slope $-\frac{3}{4}$ is given by;

$y - y_1 = m(x - x_1)$

or $y - 3 = -\frac{3}{4}(x-2)$

or $4y - 12 = -3x + 6$

or $3x + 4y - 6 = 0$ is the required equation.

b. Find the equation of the straight line passing through the points $(-2,-3)$ and perpendicular to the line $5x+7y=14$

Solution:

Given; $(x_1, y_1) = (-2,-3)$

$5x+7y=14$ — (i)

from (i), $(m_1) = -\frac{5}{7}$

Let the slope of line be m_1.

By question:

$m \cdot m_1 = -1$

or $-\frac{5}{7} \cdot m_1 = -1$

$\therefore m_1 = \frac{7}{5}$

Now, equation of st. line passing through $(-2, -3)$ and has slope $\frac{7}{5}$ is given by;

$y - y_1 = m(x - x_1)$

or, $y + 3 = \frac{7}{5}(x + 2)$

or $5y + 15 = 7x + 14$

or, $7x - 5y - 1 = 0$ is the required equation.

---

c. Find the equation of the straight line passing through $(1,5)$ and parallel to line $x + y = 5$.

Solution:

Given, $(x_1, y_1) = (1, 5)$

$x + y = 5$ — (i)

from (i), $m = -1$

Since, they are parallel $m_1 = m_2 = -1$

Equation of line passing through $(1,5)$ and having slope -1 is given by;

$y - 5 = -1(x - 1)$

or $y - 5 = -x + 1$

or, $x + y - 6 = 0$ is the required equation:

---

8. Find the equation of the following lines:

a. Line passing through the point $(2,1)$ is parallel to the straight line joining the points $(2,3)$ and $(3,-2)$

Solution:

Given; $(x_1, y_1) = (2, 1)$

Slope of $(2,3)$ and $(3,-1)$ is $\dfrac{y_2-y_1}{x_2-x_1}$

$$= \frac{-1-3}{3-2}$$

$$= -4$$

Since they are parallel, $m_1 = m_2 = -4$

Equation of st. line passing through $(2,1)$ is given by;

$y - y_1 = m(x - x_1)$

or, $y - 1 = -4(x-2)$

or, $y - 1 = -4x + 8$

or, $4x + y - 9 = 0$ is the required equation.

b. Line passing through the point $(-4,1)$ and is parallel to the line joining the points $(-7,5)$ and $(-2,2)$

Solution:

Given; $(x_1, y_1) = (-4,1)$

Slope of $(-7,5)$ and $(-2,2)$ $= \dfrac{y_2-y_1}{x_2-x_1}$

$$= \frac{2-5}{-2+7}$$

$$= -\frac{3}{5}$$

Since, they are parallel, $m_1 = m_2 = -3/5$

Equation of st. line passing through $(-4,1)$ is given by;

$y - y_1 = m(x - x_1)$

or, $y - 1 = \dfrac{-3}{5}(x+4)$

or, $5y - 5 = -3x - 12$

or, $3x + 5y + 7 = 0$ is the required equation.

c. Line through the point $(2,-4)$ and perpendicular to the line through the points $(-3,-2)$ and $(4,3)$

Solution:

Given, $(x_1, y_1) = (2,-4)$

Slope of $(-3,-2)$ and $(4,3)$ $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{3+2}{4+3}$$

$$= \frac{5}{7}$$

Let the slope of st. line be (m_1). Since, they are perpendicul

$m \cdot m_1 = -1$

or $\frac{5}{7} m_1 = -1$

$\therefore m_1 = \frac{-7}{5}$

Equation of st. line passing through $(2,-4)$ and has slope $-7/5$ is given by;

$y - y_1 = m(x - x_1)$

or, $y + 4 = \frac{-7}{5}(x-2)$

or $5y + 20 = -7x + 14$

or, $7x + 5y + 6 = 0$ is the required equation.

d. Line through the points $(2,3)$ and perpendicular to the line joining the points $(-4,-7)$ and $(5,-2)$

Solution:

Given; $(x', y') = (2,3)$

Slope of $(-4,-7)$ and $(5,-2)$ $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2+7}{5+4}$

$= \frac{5}{9}$

Let the slope of st. line be m_1. since, they are perpendicular.

$m \cdot m_1 = -1$

or, $\frac{5}{9} \cdot m_1 = -1$

$\therefore m_1 = -9/5$

Equation of st. line passing through (2,3) and having slope $\frac{-9}{5}$ is given by;

$y - y_1 = m(x - x_1)$

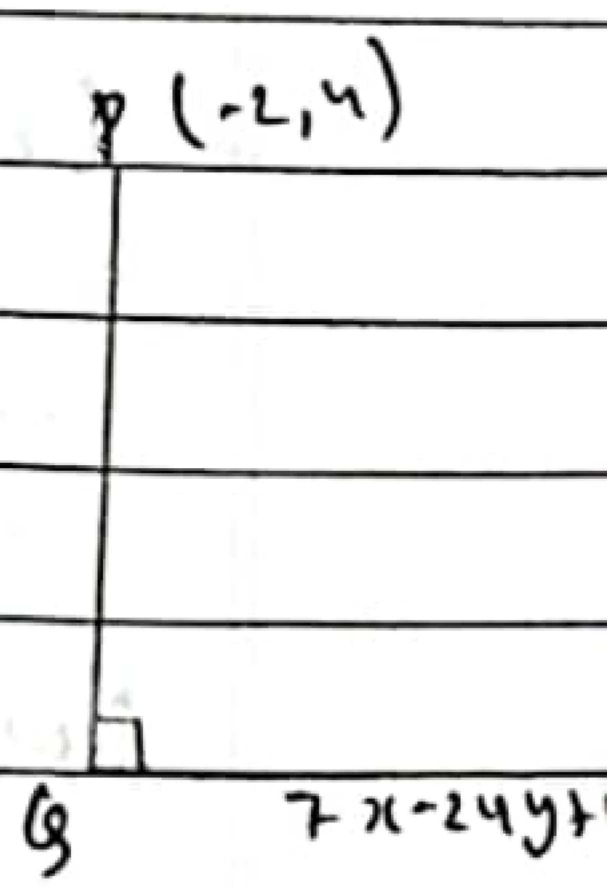
or, $y - 3 = \frac{-9}{5}(x - 2)$

or, $5y - 15 = -9x + 18$

or, $9x + 5y - 33 = 0$ is the required equation.

9.a. From the point P(-2,4) if PQ is drawn perpendicular to the line $7x - 24y + 10 = 0$. find the equation of line PQ.

Solution:



Slope of $7x - 24y + 10 = 0$ is,

$m = \frac{-7}{-24}$

$= \frac{7}{24}$

Let the slope of PQ be (m_1), then

$\frac{7}{24} \times m_1 = -1$

$\therefore m_1 = \frac{-24}{7}$

Now equation of straight line is given by;

$$y - y_1 = m(x - x_1)$$

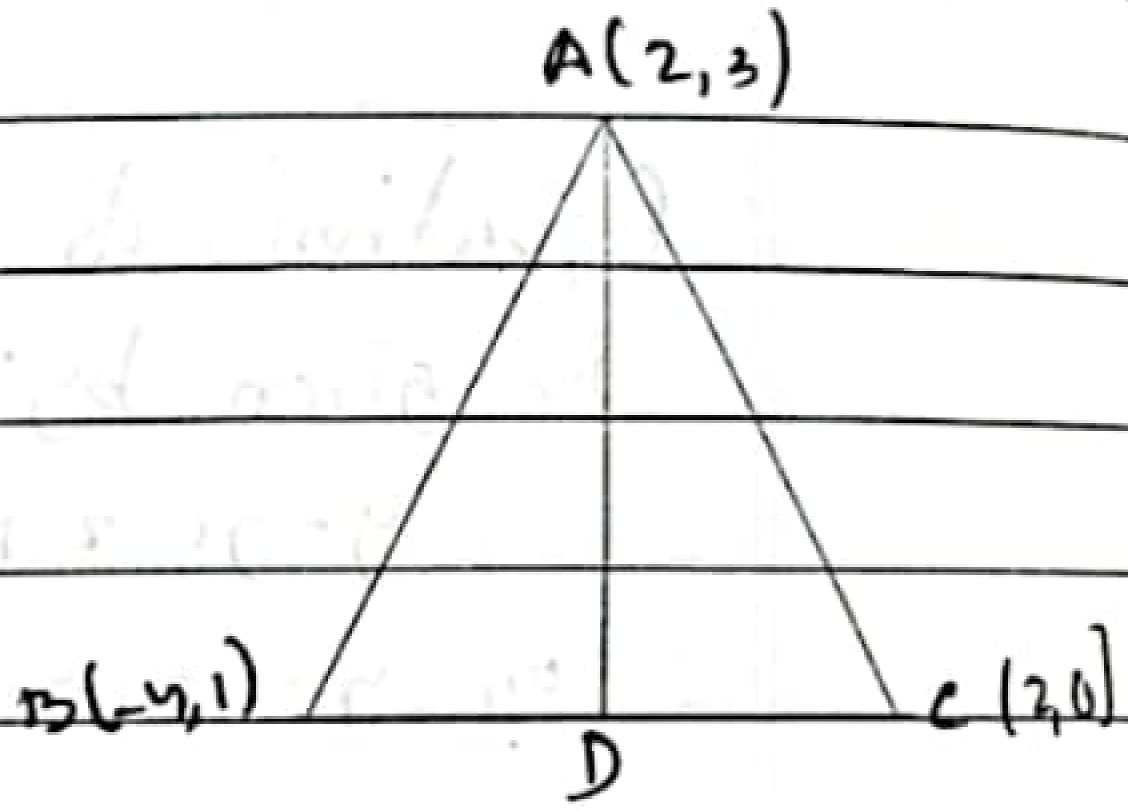
or, $y - 4 = \frac{-24}{7}(x+2)$

or, $7y - 28 = -24x - 48$

or, $24x + 7y + 20 = 0$ is the required equation.

---

b. Find the equation of the altitude of triangle ABC ~~which~~ with vertices A(2,3), B(-4,1) and C(2,0) drawn from the first vertex A(2,3)



Solution:

Let AD be the vertex drawn from vertex A(2,3).

here,

Slope of BC = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-1}{2+4}$

$= \frac{-1}{6}$

Let the slope of AD be m', then

$\frac{-1}{6} \times m' = -1$

or, $m' = 6$

Now, Equation of line is given by;

$$y - y_1 = m(x - x_1)$$

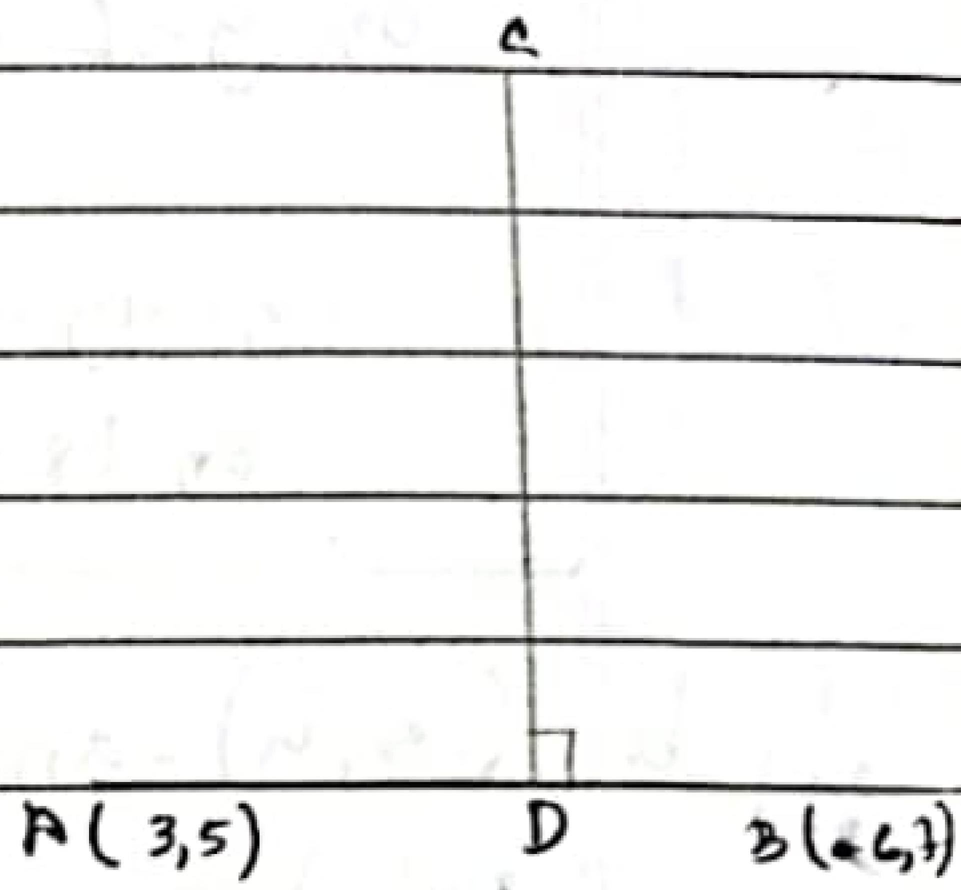
or, $y - 3 = 6(x - 2)$

or, $y - 3 = 6x - 12$

or, $6x - y - 9 = 0$ is the required equation.

Creative Section

20. Find the equation to the perpendicular bisector of the line join of the two points:



a. (3,5) and (-6,7)

Solution:

Let the given points be A(3,5) and B(-6,7) and Let CD be the perpendicular to AB.

Coordinate of D is;

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{3-6}{2}, \frac{5+7}{2}\right)$$

$$= \left(\frac{-3}{2}, 6\right)$$

Now, slope of line AB is given by;

$$\left(\frac{y_2-y_1}{x_2-x_1}\right)$$

$$= \left(\frac{7-5}{-6-3}\right)$$

$$= \left(\frac{2}{-9}\right)$$

Also, Let the slope of line CD be m'.

$$m.m' = -1$$

$$\text{or, } \frac{2}{-9} \times m' = -1$$

$$\therefore m' = \frac{9}{2}$$

Now, Slope of line passing through (-3/2, 6) and having slope 9/2 is given by;

$$y-y_1 = m(x-x_1)$$

$$y - 6 = \frac{9}{2}\left(x + \frac{3}{2}\right)$$

or, $y - 6 = \frac{9}{2}\left(\frac{2x+3}{2}\right)$

or, $4y - 24 = 18x + 27$

or, $18x - 4y + 51 = 0$ is the required equation.

b. $(5,4)$ and $(7,12)$

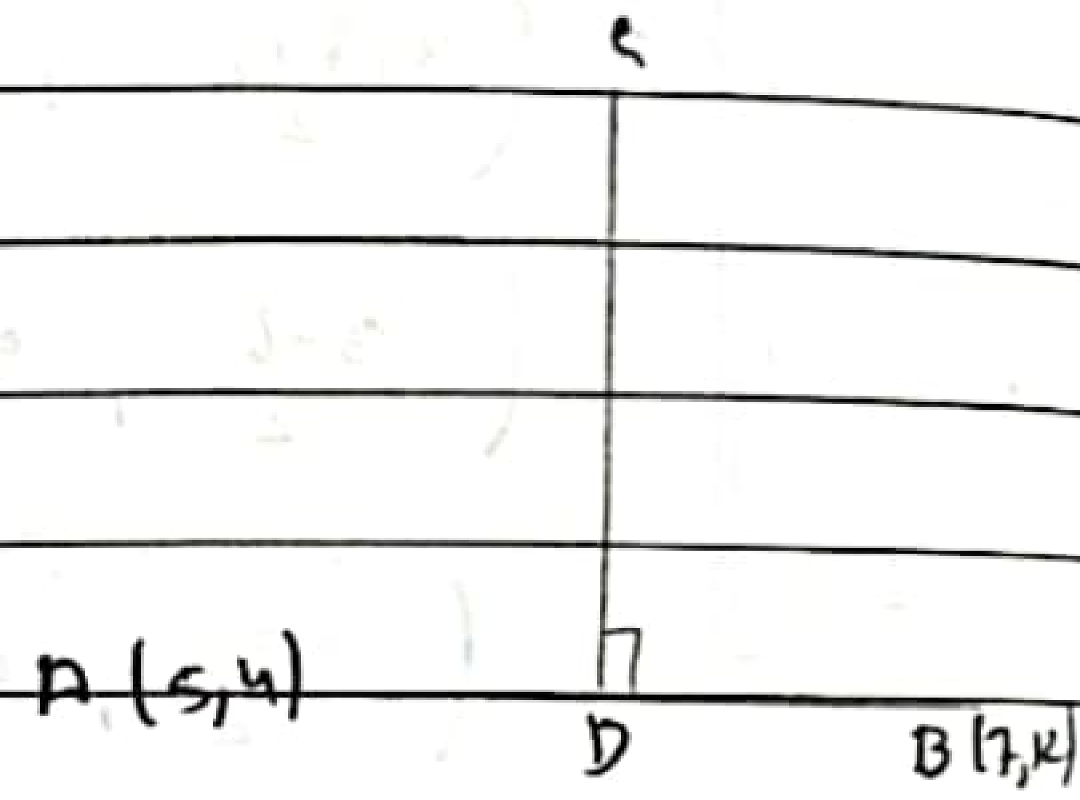
Solution:

Let the points be $A(5,4)$ and $B(7,12)$

Let CD be the perpendicular of AB

having vertices $= \left(\frac{5+7}{2}, \frac{4+12}{2}\right)$

$= (6,8)$



Also,

Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{12 - 4}{7 - 5}$

$= \frac{8}{2} = 4$

Also, $m \cdot m_1 = -1$

or, $4 \cdot m_1 = -1$

$\therefore m_1 = \cancel{-4}\ -\frac{1}{4}$

Also,

Slope of line passing through $(6,8)$ and having slope $-\frac{1}{4}$ is given by;

$y - y_1 = m(x - x_1)$

or, $y - 8 = -\frac{1}{4}(x - 6)$

$\cancel{\text{or } y - 8 = -4x + 24}$

$\cancel{\text{or } 4x + y - 32 = 0}$

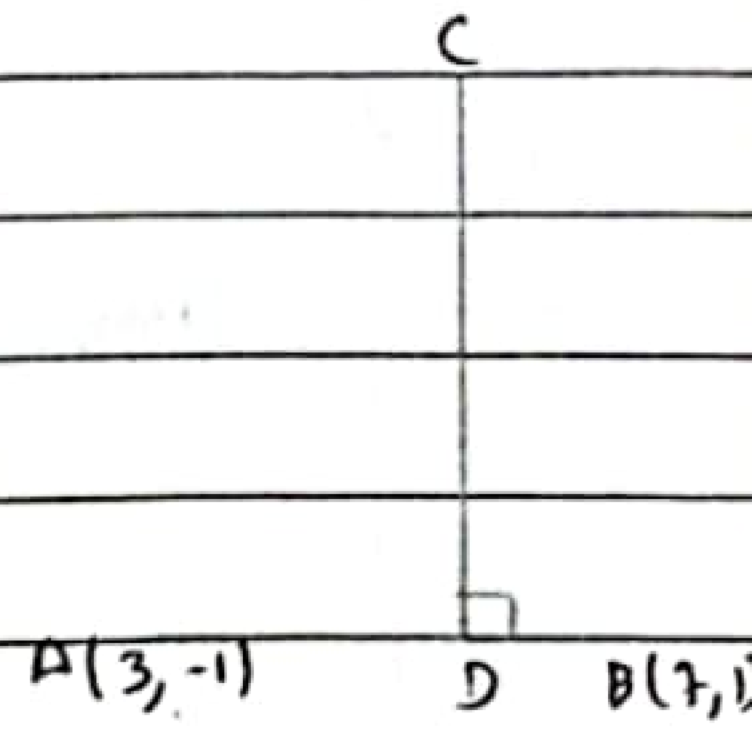
or, $4y - 32 = -x + 6$

or, $x + 4y - 38 = 0$ is the required equation

c. (3,-1) and (7,1)

Solution:

Let A(3,-1) and B(7,1) be the given points.



Let CD be the perpendicular to AB having coordinate;

$$\left(\frac{3+7}{2}, \frac{-1+1}{2}\right)$$

$$(5,0)$$

Slope of line AB is given by;

$$\left(\frac{y_2-y_1}{x_2-x_1}\right)$$

$$= \left(\frac{1+1}{7-3}\right)$$

$$= \left(\frac{2}{4}\right) = \frac{1}{2}$$

~~As~~ Let slope of CD be m'. Now

$$m.m' = -1$$

or $\frac{1}{2} \cdot m' = -1$

$\therefore m' = -2$

Also, Slope of line passing through (5,0) and having slope -2 is given by;

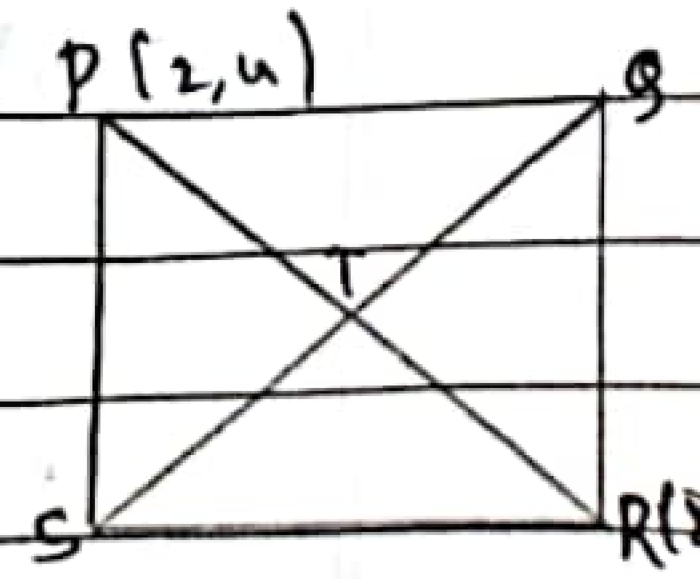
$$y-y_1 = m(x-x_1)$$

or $y-0 = -2(x-5)$

or $y-0 = -2x+10$

or $2x+y-10 = 0$ is the required equation.

11.a. Let P(2,4) and R(8,10) ~~be~~ are the opposite vertices of a rhombus PQRS. Find the equation of diagonal QS.



Solution:

Here, PQRS is a rhombus PR and QS are diagonals which are intersected at T

Coordinate of T is;

$$\left(\frac{2+8}{2}, \frac{4+10}{2}\right)$$

$$= (5,7)$$

Now slope of PR is $= \frac{10-4}{8-2}$

$$= \frac{6}{6} = 1$$

Let the slope of QS be m', then

$$1 \cdot m' = -1$$

$$\therefore m' = -1$$

Now, ~~line of eq~~ equation of line passing through (5,7) and having slope -1 is;

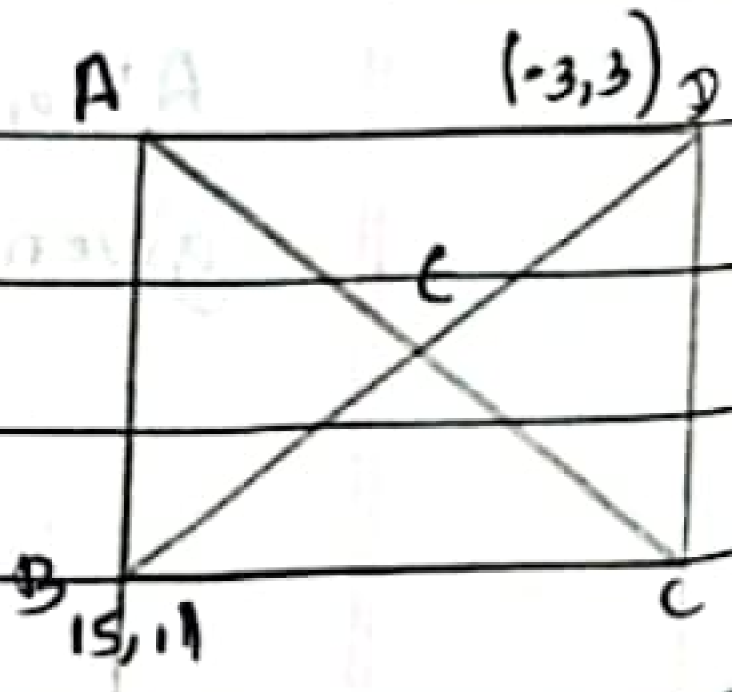
$$y - y_1 = m(x - x_1)$$

$$\text{or } y - 7 = -1(x-5)$$

or $x + y - 12 = 0$ is the required equation.

b. Let B(5,1) and D(-3,3) are two opposite vertices of a square ABCD. find the equation of the diagonal AC.

Solution.



~~Let~~ Here, ABCD are the ~~vertices~~ sides of square having diagonal AC and BD which meet at E

Coordinate of E is given by;

$$\left(\frac{5-3}{2}, \frac{1+3}{2}\right)$$

$$(1, 2)$$

Also,

slope of BD is given by;

$$= \left(\frac{3-1}{-3-5}\right)$$

$$= -2/8$$

since, AC and BD are perpendicular to each other.

$$-\frac{2}{8} \times m' = -1$$

or, $m' = \frac{8}{2} = 4$

Also,
Slope of AC is given by;

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

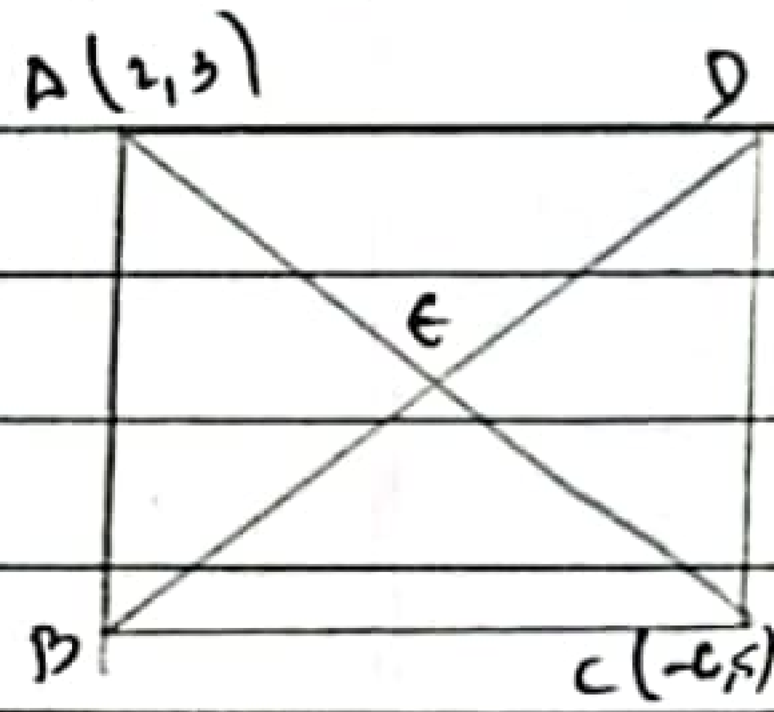
$y - 2 = 4x - 4$

$4x - y - 2 = 0$ is the required equation.

---

c. If (2,3) and (-6,5) are the end points of the diagonals of a square, find the equation of other diagonal.

Solution:



Let the given point be A(2,3) and C(-6,5).
Here, ABCD is a square with diagonal AC and BD meeting at E.

~~Diago~~ Coordinate of E is given by;

$$\left(\frac{2-6}{2}, \frac{3+5}{2}\right)$$

$(-2, 4)$

Also, slope of AC is $= \frac{5-3}{-6-2}$

$$= -\frac{1}{4}$$

Since, AD and BD are perpendicular. Let slope of BD be m'

$$-\frac{1}{4} \times m' = -1$$

$\therefore m' = 4$

Slope of diagonal BD is given by;

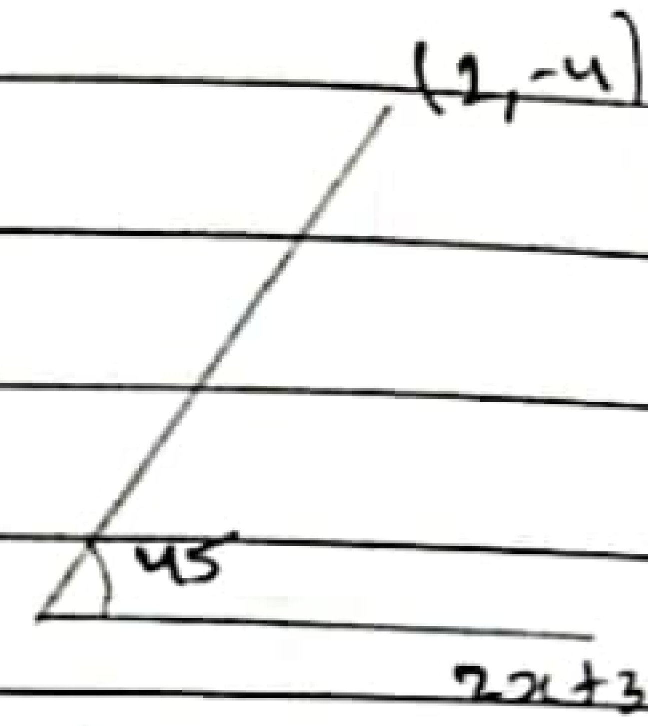
$y - y_1 = m(x - x_1)$

or $y - 4 = 4(x + 2)$

or $4x - y - 12 = 0$ is the required equation.

12.a. Determine the equation of the straight lines through $(1, -4)$ that makes an angle of $45°$ with the straight line $2x + 3y + 7 = 0$.

Solution:



Here,

slope of the line $2x + 3y + 7 = 0$ is $-\frac{2}{3}$

Let,

slope of required line be m

Then, $\tan 45° = \dfrac{m - \left(-\frac{2}{3}\right)}{1 + m\left(-\frac{2}{3}\right)}$

or $1 = \dfrac{\frac{3m + 2}{3}}{\frac{3 - 2m}{3}}$

or $1 = \pm \dfrac{3m + 2}{3 - 2m}$

Taking positive (+ve), $3 - 2m = 3m + 2$

or $-5m = -1$

$\therefore m = \frac{1}{5}$

Taking negative (-ve): $3 - 2m = -3m - 2$

or, $5 = -m$

$\therefore m = -5$

Now,

The equation of line passing through $(1, -4)$ and having slope $\frac{1}{5}$ is

$y - y_1 = m(x - x_1)$

or, $y + 4 = \frac{1}{5}(x - 1)$

or, $5y + 20 = x - 1$

or, $x - 5y - 21 = 0$

Again,

Equation of line passing through and having slope 5 is given by:

$y - y_1 = m(x - x_1)$

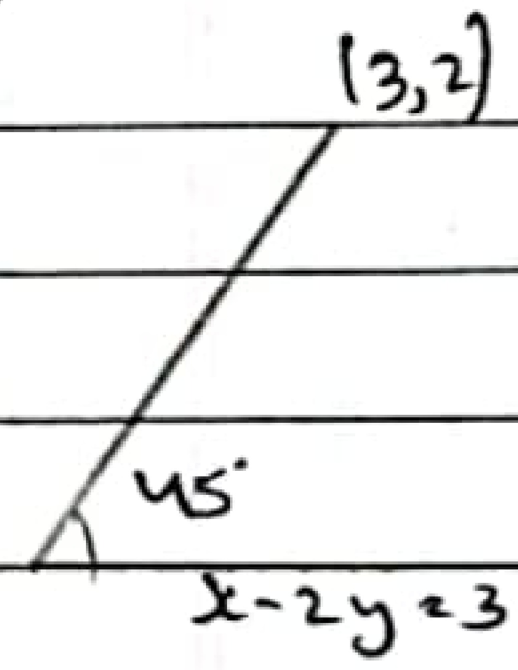
or, $y + 4 = -5(x - 1)$

or, $y + 4 = -5x + 5$

$5x + y - 1 = 0$ is the required equation.

---

b. Find the equation of straight line passing through the point (3,2) and making an angle of 45° with the line $x - 2y = 3$



Solution:

here, slope of line $x - 2y = 3$ is $\frac{1}{2}$

Also, slope of required line be m.

Now,

$$\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

$$\text{or, } \tan 45 = \pm \frac{m - \frac{1}{2}}{1 + m \cdot \frac{1}{2}}$$

$$\text{or, } 1 = \pm \frac{\frac{2m - 1}{2}}{\frac{2 + m}{2}}$$

$$\text{or, } 1 = \pm \frac{2m - 1}{2 + m}$$

Taking positive (+ve)

$2 + m = 2m - 1$

or, $m = 3$

Taking negative (-ve)

$2 + m = -2m + 1$

or, $m + 2m = 1 - 2$

or, $3m = -1$

$\therefore m = -1/3$

Now,

slope of line passing through (3,2) and having slope 3 is

$y - 2 = 3(x - 3)$

or, $y - 2 = 3x - 9$

$3x - y - 7 = 0$

Also,

slope of line passing through (3,2) and having slope $-\frac{1}{3}$ is

$y - 2 = -\frac{1}{3}(x - 3)$

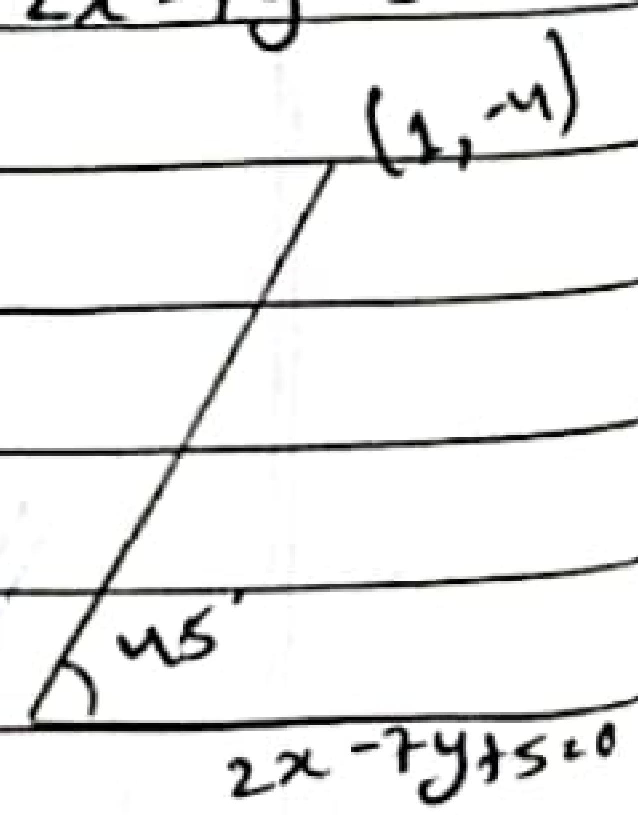
or, $3y - 6 = -x + 3$

or, $x + 3y - 3 = 0$

---

c. Find the equation of ~~the~~ two lines passing through the point (1,-4) and making an angle of 45° with the line $2x - 7y + 5 = 0$.

Solution:



Here, slope of line $2x - 7y + 5 = 0$ is $\frac{2}{7}$

Let the slope of required line be m

Now,

$$\tan 45^\circ = \pm \frac{m - \frac{2}{7}}{1 + m\cdot\frac{2}{7}}$$

or, $1 = \pm \dfrac{\frac{7m-2}{7}}{\frac{7+2m}{7}}$

or $1 = \pm \dfrac{7m-2}{7+2m}$

Taking positive (+ve)

$7+2m = 7m-2$

$\therefore m = \frac{9}{5}$

Taking negative

$7+2m = -7m+2$

$\therefore m = \frac{-5}{9}$

Now, equation of line passing through $(1,-4)$ and have ~~the~~ slope $\frac{9}{5}$ is given by;

$y+4 = \frac{9}{5}(x-1)$

or, $5y+20 = 9x-9$

or, $9x-5y-29 = 0$

Also,

~~Slope of~~ equation of line passing through $(1,-4)$ and have slope $\frac{-5}{9}$ is given by;

$y+4 = \frac{-5}{9}(x-1)$

or, $9y+36 = -5x+5$

or $5x+9y+31 = 0$

d. Find the equation of the straight line passing through the point $(3,-2)$ and making an angle of $60°$ with the line $\sqrt{3}x+y-1=0$.

Solution:

Slope of line $\sqrt{3}x+y-1=0$ is $-\sqrt{3}$

Let the slope of required line be m

Now, $\tan 60 = \pm \frac{m+\sqrt{3}}{1+m\sqrt{3}}$

or, $\sqrt{3} = \pm \frac{m+\sqrt{3}}{1-m\sqrt{3}}$

Taking positive (+ve)

$\sqrt{3}(1-m\sqrt{3}) = m+\sqrt{3}$

or, $\sqrt{3} - 3m = m+\sqrt{3}$

or, $\sqrt{3}-\sqrt{3} = m+3m$

or, $4m = 0$

$\therefore m = 0$

Taking negative (-ve)

$\sqrt{3}(1-m\sqrt{3}) = -m-\sqrt{3}$

or, $\sqrt{3} - 3m = -m-\sqrt{3}$

or, $\sqrt{3}+\sqrt{3} = -m+3m$

or, $2\sqrt{3} = 2m$

$\therefore m = \sqrt{3}$

Now,

Equation of line passing through $(3,-2)$ and having slope 0 is given by;

$y+2 = 0(x-3)$

$y+2 = 0$

Also,

equation of line passing through $(3,-2)$ and having slope $\sqrt{3}$ is given by;

$y+2 = \sqrt{3}(x-3)$

$y+2 = \sqrt{3}x - 3\sqrt{3}$

or, $\sqrt{3}x - y = 2+3\sqrt{3}$

---

e. Determine k so that $3x-ky-8=0$ shall make an angle of 45° with the line $3x+5y-17=0$.

Solution:

Here, $3x-ky-8=0$ — (i)

Slope, $m_1 = \frac{-3}{-k} = \frac{3}{k}$

Also, $3x+5y-17=0$ — (ii)

Slope, $m_2 = \frac{-3}{5}$

Angle between them is 45°

we know,

$$\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

$$\text{or, } \tan 45^\circ = \frac{\frac{3}{k} + \frac{3}{5}}{1 + \frac{3}{k} \cdot -\frac{3}{5}}$$

$$\text{or, } 1 = \pm \frac{\frac{15 + 3k}{5k}}{\frac{5k - 9}{5k}}$$

$$\text{or, } 1 = \pm \frac{15 + 3k}{5k - 9}$$

Taking (+ve)

$5k - 9 = 15 + 3k$

or, $5k - 3k = 15 + 9$

or, $k = \frac{24}{2}$

$\therefore k = 12$

Also,

Taking (-ve)

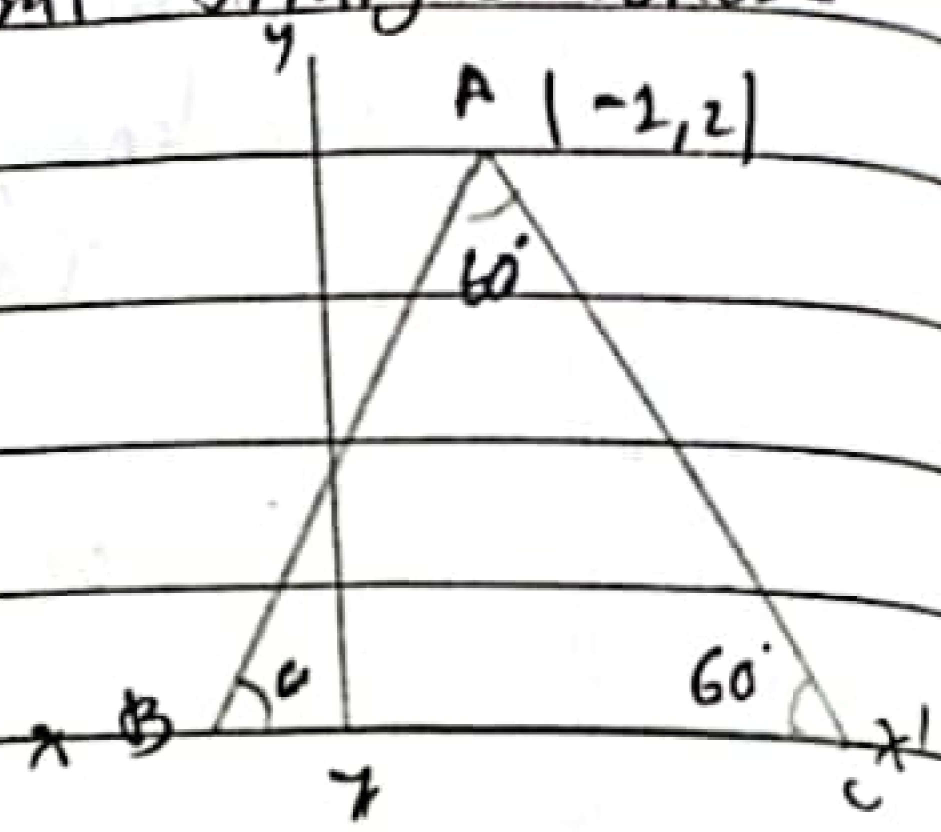
$5k - 9 = -15 - 3k$

or, $5k + 3k = -15 + 9$

or, $k = -\frac{6}{8}$

$\therefore k = \frac{-3}{4}$

13 a. Find the equation of the sides of an equilateral triangle whose vertex is $(-1,2)$ and base is $y=0$.



Solution:

Let ABC be an equilateral triangle with vertex $A(-1,2)$ and base $y=0$

Then, $\angle ABC$, $\angle BCA$ and $\angle CAB$ are equal i.e. $60°$ each.

Now

Slope of AB $= \tan\theta$

$= \tan 60°$

$= \sqrt{3}$

Also,

Slope of AC is $= \tan 120°$

$= -\sqrt{3}$

Now

Equation of AB is given by;

$y - y_1 = m(x - x_1)$

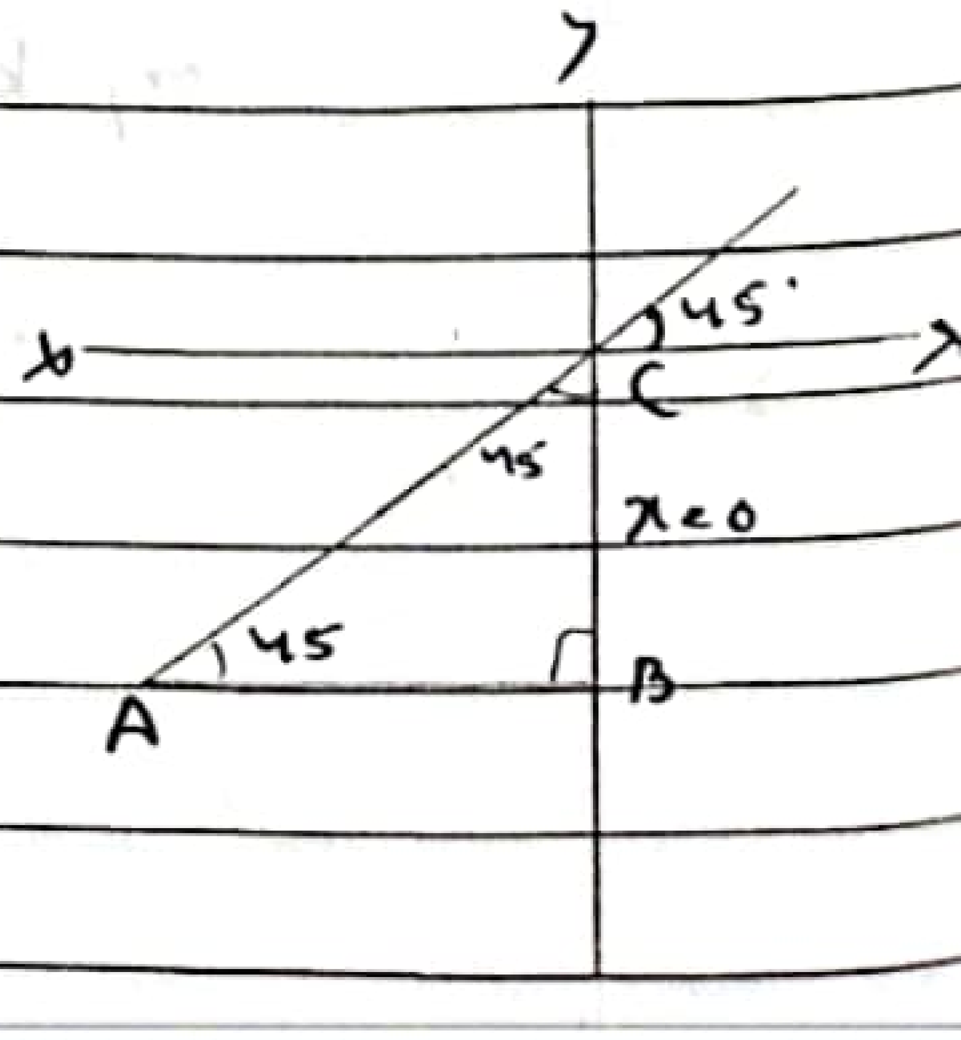
or, $y - 2 = \sqrt{3}(x+1)$

or, $y - 2 = \sqrt{3}x + \sqrt{3}$

$\sqrt{3}x - y + \sqrt{3} + 2 = 0$ is the required equation

b. Find the equation of the side of the right angled isosceles triangle whose vertex is $(-2,-3)$ and whose base is $x=0$.

Solution.



Let ABC be the right angled isosceles triangle with vertex $(-2,-3)$ and base $x=0$

Then,

$\angle ABC = 90°$

$\angle BAC = \angle BCA = 45°$

Slope of AC = $\tan 45^\circ$

$= 1$

Now,

equation of line passing through $(-2,-3)$ and having slope 1 is given by;

$y - y_1 = m(x - x_1)$

or, $y + 3 = 1(x+2)$

or, $y + 3 = x + 2$

or, $x - y - 1 = 0$

Again,

Equation of line BC is $x = 0$

Again,

Slope of AB = $\tan 0^\circ$

$= 0$

Now,

Equation is line passing through $(-2,-3)$ and have slope 0 is given by;

$y - y_1 = m(x - x_1)$

or, $y + 3 = 0(x+2)$

or, $y + 3 = 0$