

Exercise - - -

General Section:

2. Write down the first five terms of an A.P whose first term and common difference are given below:

- a. First term (a) = 1, common difference = 3

Solution:

$$\text{Here, first term } (a) = 1$$

$$\text{Common difference } (d) = 3$$

first five terms are,

$$t_1 = a = 1$$

$$t_2 = a + d = 1 + 3 = 4$$

$$t_3 = a + 2d = 1 + 2 \cdot 3 = 7$$

$$t_4 = a + 3d = 1 + 3 \cdot 3 = 10$$

$$t_5 = a + 4d = 1 + 4 \cdot 3 = 13$$

- b. First term (a) = -2, common difference = -2

Solution:

$$\text{First term } (a) = -2$$

$$\text{Common difference } (d) = -2$$

first five terms are;

$$t_1 = a = -2$$

$$t_2 = a + d = -2 - 2 = -4$$

$$t_3 = a + 2d = -2 - 2 \cdot 2 = -6$$

$$t_4 = a + 3d = -2 - 3 \cdot 2 = -8$$

$$t_5 = a + 4d = -2 - 4 \cdot 2 = -10$$

- c. First term (a) = $\frac{3}{5}$, common difference = $\frac{1}{2}$

Solution:

$$\text{First term } (a) = \frac{3}{5}$$

$$\text{Common difference } (d) = \frac{1}{2}$$

first five terms are;

$$t_1 = a = \frac{3}{15}$$

$$t_2 = a + d = \frac{3}{15} + \frac{1}{2} = \frac{11}{10}$$

$$t_3 = a + 2d = \frac{3}{15} + 2 \cdot \frac{1}{2} = \frac{8}{5}$$

$$t_4 = a + 3d = \frac{3}{15} + 3 \cdot \frac{1}{2} = \frac{21}{10}$$

$$t_5 = a + 4d = \frac{3}{15} + 4 \cdot \frac{1}{2} = \frac{13}{5}$$

2. find the 7th term and 9th term of an A.P where

a. first term (a) = 9, common difference = 5

Solution:

first term (a) = 9

common difference (d) = 5

7th term (t_7) = ?

9th term (t_9) = ?

We know,

$$t_n = a + (n-1)d$$

$$t_7 = 9 + (7-1)5$$

$$= 9 + 30$$

$$= 39$$

Also,

$$t_9 = 9 + (9-1)5$$

$$= 9 + 40$$

$$= 49$$

b. first term (a) = $\frac{1}{3}$, common difference = $-\frac{1}{3}$

Solution:

first term (a) = $\frac{1}{3}$

common difference (d) = $-\frac{1}{3}$

7th term (t_7) = ?

9th term (t_9) = ?

We know,

$$t_n = a + (n-1)d$$

$$t_7 = \frac{1}{3} + (7-1) \times -\frac{1}{3}$$

$$= \frac{1}{3} - \frac{6}{3}$$

$$= -\frac{5}{3}$$

Also,

$$t_9 = \frac{1}{3} + (9-1) \times -\frac{1}{3}$$

$$= \frac{1}{3} - \frac{8}{3}$$

$$= -\frac{7}{3}$$

3. Find the first term for each of the following A.P's

a. $d = -3$, 8^{th} term = 30

Solution:

Common difference (d) = -3

8^{th} term (t_8) = 30

first term (a) = ?

We know,

$$t_n = a + (n-1)d$$

$$\text{on } t_8 = a + (8-1) \cdot -3$$

$$\text{on } 30 = a + 7 \cdot -3$$

$$\text{on } a = 30 + 21$$

$$\therefore a = 51$$

b. 4th term = 27, common difference (d) = 5

Solution:

$$4^{\text{th}} \text{ term } (t_4) = 27$$

Common difference (d) = 5

first term (a) = ?

We know,

$$t_n = a + (n-1)d$$

$$\text{or } t_4 = a + (4-1)d$$

$$\text{or } 27 = a + 3d$$

$$\text{or } a = 27 - 3d$$

$$\therefore a = 12$$

4. Find the common difference of the following arithmetic sequences.

a. first term = 3, 7th term = 27

Solution:

first term (a) = 3

7th term (t_7) = 27

Common difference (d) = ?

We know,

$$t_n = a + (n-1)d$$

$$\text{or } t_7 = 3 + (7-1)d$$

$$\text{or } 27 = 3 + 6d$$

$$\text{or } \frac{27-3}{6} = d$$

$$\therefore d = 4$$

b. $a = 13$ and $t_{31} = -77$

Solution:

first term (a) = 13

31^{th} term (t_{31}) = -77

Common difference (d) = ?

We know,

$$t_n = a + (n-1)d$$

$$\text{or, } t_{31} = 13 + (31-1)d$$

$$\text{or, } -77 = 13 + 30d$$

$$\text{or, } \frac{-77 - 13}{30} = d$$

$$\therefore d = -3$$

- s. Determine the number of terms for each of the following A.P.'s.

a. First term = 1, common difference = 6, Last term = 49

Solution:

First term (a) = 1

Last term = 49

Common difference (d) = 6

No. of term (n) = ?

We know,

Let n be n^{th} term,

i.e. $t_n = n$

$$\text{or, } a + (n-1)d = n$$

$$\text{or, } 1 + (n-1)6 = 49$$

$$\text{or, } 1 + 6n - 6 = 49$$

$$\text{or, } n = \frac{49+5}{6}$$

$$\therefore n = 9$$

- b. $t_n = 1$, $a = 15$, $d = -2$

Solution:

First term (a) = 15

Last term (b) = 1

Common difference (d) = -2

No. of terms (n) = ?

Let t_n be n^{th} term

$$\text{i.e. } t_n = \cancel{49} - 1$$

$$\text{or } a + (n-1)d = \cancel{49} - 1$$

$$\text{or } 15 + (n-1) - 2 = \cancel{49} - 1$$

$$\text{or } 15 + (-2n + 2) = \cancel{49} - 1$$

$$\text{or } 15 - 2n + 2 = \cancel{49} - 1$$

$$\text{or } \frac{17 - 1}{2} = n$$

$$\therefore n = 8$$

• 44, 36, 28, ..., -380

Solution:

First term (a) = 44

Last term (b) = -380

Common difference (d) = $36 - 44$

$$= -8$$

No. of terms (t_n) = ?

Let -380 be n^{th} term

$$\text{i.e. } t_n = -380$$

$$\text{or } a + (n-1)d = -380$$

$$\text{or } 44 + (n-1) - 8 = -380$$

$$\text{or } 44 - 8n + 8 = -380$$

$$\text{or } 52 - 8n = -380$$

6.e. Is -42 a term of the series $7 + 2 - 3 + \dots$? Give reason.

Solution:

$$\text{First term (a)} = 7$$

$$\text{Common difference (d)} = 2 - 7 \\ = -5$$

Let -42 be n^{th} term

$$t_n = -42$$

$$\text{i.e. } a + (n-1)d = -42$$

$$\text{or, } 7 + (n-1) \cdot -5 = -42$$

$$\text{or, } 7 + (-5n + 5) = -42$$

$$\text{or, } \frac{12 + 42}{5} = n$$

$$\therefore n = 10.8$$

Hence, n is not a true whole number.

a. Which term of the sequences $2, 4, 6, \dots, 512$ is 256 ?

Solution:

$$\text{First term (a)} = 2$$

$$\text{Last term} = 256$$

$$\text{Common difference (d)} = 4 - 2 \\ = 2$$

No. of terms (n) = ?

Let 256 be n^{th} term

$$\text{i.e. } t_n = 256$$

$$\text{or, } a + (n-1)d = 256$$

$$\text{or, } 2 + (n-1)2 = 256$$

$$\text{or, } 2 + 2n - 2 = 256$$

$$\text{or, } n = \frac{256}{2}$$

$$\therefore n = 128$$

b. Which term of the series $3 + 6 + 12 + \dots$ is 96?

Solution:

$$\text{First term (a)} = 3$$

$$\text{Last term} = 96$$

$$\text{Common difference (d)} = 6 - 3$$

$$= 3$$

No. of terms (n) = ?

Let 96 be n^{th} term

$$\text{i.e. } t_n = 96$$

$$\text{or } a + (n-1)d = 96$$

$$\text{or } 3 + (n-1)3 = 96$$

$$\text{or } 3 + 3n - 3 = 96$$

$$\text{or } n = \frac{96}{3}$$

$$\therefore n = 32$$

c. How many terms are there in the series $5 + 9 + 13 + \dots + 77$?

Solution:

$$\text{First term (a)} = 5$$

$$\text{Common difference (d)} = 9 - 5$$

$$= 4$$

$$\text{Last term} = 77$$

No. of terms (n) = ?

Let 77 be n^{th} term

$$\text{i.e. } t_n = 77$$

$$\text{or } a + (n-1)d = 77$$

$$\text{or } 5 + (n-1)4 = 77$$

$$\text{or } 5 + 4n - 4 = 77$$

$$\text{or } \frac{78}{4} = n$$

$$\therefore n = 19$$

d. Is 65 a term of the series $9 + 21 + 23 + 15 + \dots$?

Solution:

$$\text{First term}(a) = 9$$

$$\text{Last term} = 65$$

$$\text{Common Difference}(d) = 11 - 9 \\ = 2$$

No. of terms(n) = ?

Let 65 be n^{th} term

$$\text{i.e. } t_n = 65$$

$$\text{or, } a + (n-1)d = 65$$

$$\text{or, } 9 + (n-1)2 = 65$$

$$\text{or, } \frac{58}{2} = n$$

$$\therefore n = 29$$

It belongs to 29th term.

Creative Section

7.a. If fourth and the 10th term of an A.P. are 75 and 117 respectively. find the first term and common difference.

Solution:

$$4^{\text{th}} \text{ term } (t_4) = 75$$

$$10^{\text{th}} \text{ term } (t_{10}) = 117$$

$$\text{first term } (a) = ?$$

$$\text{common difference } (d) = ?$$

We know,

$$t_n = a + (n-1)d$$

$$\text{or, } t_4 = a + (4-1)d$$

$$\text{or, } 75 = a + 3d \quad \text{---(1)}$$

Also,

$$t_{10} = a + (10-1)d$$

$$\text{or } 117 = a + 9d \quad \text{--- (1)}$$

Now,

Subtracting equation (1) from (11)

$$\begin{array}{r} 117 = a + 9d \\ - 75 = a + 3d \\ \hline 42 = 6d \end{array}$$

$$\therefore d = 7$$

From (1)

$$75 = a + 3 \times 7$$

$$\text{or, } 75 - 21 = a$$

$$\therefore a = 54.$$

- b. The 3rd term and the 13th term of an arithmetic sequence are -40 and 0 respectively. What is the 28th term of the sequence.

Solution:

$$3^{\text{rd}} \text{ term } (t_3) = -40$$

$$13^{\text{th}} \text{ term } (t_{13}) = 0$$

$$28^{\text{th}} \text{ term } (t_{28}) = ?$$

We know,

$$t_n = a + (n-1)d$$

$$\text{or, } t_3 = a + (3-1)d$$

$$\text{or, } -40 = a + 2d \quad \text{--- (1)}$$

Also,

$$t_{13} = a + (13-1)d$$

$$\text{or, } 0 = a + 12d \quad \text{--- (11)}$$

Now,

Subtracting equation (1) from (11)

$$\begin{array}{rcl} a + 12d & = 0 \\ a + 2d & = -40 \\ \hline (-) & (-) & (-) \end{array}$$

$$\text{or } 10d = 40$$

$\therefore d = 4$

From ①

$$a + 2d = -40$$

$$\text{or } a + 2 \times 4 = -40$$

$$\therefore a = -48$$

Again,

28th term is

$$\begin{aligned} t_{28} &= a + (28-1)d \\ &= -48 + 27 \times 4 \\ &= 60 \end{aligned}$$

- c. If the 5th and 10th terms of an Arithmetic Sequences are 17 and 42 respectively. Find its 20th term.

Solution:

$$5^{\text{th}} \text{ term } (t_5) = 17$$

$$10^{\text{th}} \text{ term } (t_{10}) = 42$$

$$20^{\text{th}} \text{ term } (t_{20}) = ?$$

Now we know

$$t_n = a + (n-1)d$$

$$\text{or } t_5 = a + (5-1)d$$

$$\text{or } 17 = a + 4d \quad \text{--- (i)}$$

Also,

$$t_{10} = a + (10-1)d$$

$$\text{or } 42 = a + 9d \quad \text{--- (ii)}$$

Now,

Subtraction equation (i) from (ii)

$$\begin{array}{rcl} a + 9d & = 42 \\ a + 4d & = 17 \end{array}$$

(1) (2) (3)

$$5d = 25$$

$$\therefore d = 5$$

From (1)

$$17 = a + 4 \times 5$$

$$\text{or } 17 - 20 = a$$

$$\therefore a = -3$$

Again,

$$\begin{aligned} \text{28 } 20^{\text{th}} \text{ term } (t_{20}) &= a + (n-1)d \\ &= -3 + (20-1)5 \\ &= -3 + 95 \\ &= 92 \end{aligned}$$

8.a. If 6th term and 9th term of an A.P. are 23 and 35 which term is 67?

Solution:

$$6^{\text{th}} \text{ term } (t_6) = 23$$

$$9^{\text{th}} \text{ term } (t_9) = 35$$

Now,

$$t_n = a + (n-1)d$$

$$\text{or } t_6 = a + (6-1)d$$

$$\text{or } 23 = a + 5d \quad \text{--- (i)}$$

Also,

$$t_9 = a + (9-1)d$$

$$\text{or } 35 = a + 8d \quad \text{--- (ii)}$$

Subtracting equation (i) from (ii)

$$a + 8d = 35$$

$$a + 5d = 23$$

$$\underline{- \quad - \quad - \quad -}$$

$$3d = 12$$

$$\therefore d = 4$$

from ①

$$23 = a + 5d$$

$$\text{or } 23 = a + 5 \times 4$$

$$\text{or } 23 - 20 = a$$

$$\therefore a = 3$$

Let t_n be n^{th} term;

$$t_n = \{a + (n-1)d\}$$

$$t_n = 3 + (n-1)4$$

$$\text{or } t_n = 3 + 4n - 4$$

$$\text{or } \frac{t_n + 4 - 3}{4} = n$$

c. $n = 17^{\text{th}}$ term

b. The 5^{th} term of an A.P is 28 and the 9^{th} term is 48. Is 93 a term of the sequence? Give reason:

Solution:

$$5^{\text{th}} \text{ term } (t_5) = 28$$

$$9^{\text{th}} \text{ term } (t_9) = 48$$

Now,

$$t_n = a + (n-1)d$$

$$\text{or } t_5 = a + (5-1)d$$

$$\text{or } 28 = a + 4d \quad \text{--- (i)}$$

Also,

$$t_9 = a + (9-1)d$$

$$\text{or } 48 = a + 8d \quad \text{--- (ii)}$$

Subtracting equation (i) from (ii)

$$a + 8d = 48$$

$$\begin{array}{r} a + 4d = 28 \\ (-) \quad (-) \quad (-) \\ \hline 4d = 20 \end{array}$$

$$\text{or } 4d = 20$$

$$\therefore d = 5$$

from (i)

$$28 = a + 4d$$

$$\text{or } 28 = a + 4 \times 5$$

$$\text{or } 28 - 20 = a$$

$$\therefore a = 8$$

Let a_n be n^{th} term then

$$t_n = a + (n-1)d$$

$$\text{or } a_n = a + (n-1)5$$

$$\text{or } a_n = 8 + 5n - 5$$

$$\text{or } a_n + 8 = 5n$$

s

$$\therefore n = 18$$

c. Is 40 a term of an arithmetic sequence whose 11th term and 15th terms are 23 and 35 respectively.

Solution:

$$11^{\text{th}} \text{ term } (t_{11}) = 23$$

$$15^{\text{th}} \text{ term } (t_{15}) = 35$$

We know,

$$t_n = a + (n-1)d$$

$$t_{11} = a + (11-1)d$$

$$\text{or } 23 = a + 10d \quad \text{--- (i)}$$

Also,

$$t_{15} = a + (15-1)d$$

$$\text{or } 35 = a + 14d \quad \text{--- (ii)}$$

Subtracting (i) from (ii)

$$\cancel{a} + 14d = 35$$

$$\cancel{a} + 10d = 23$$

$$(-) \quad (-) \quad (-)$$

$$\therefore 4d = 12$$

$$\therefore d = 3$$

from ①

$$23 = a + 10d$$

$$\text{or } 23 = a + 10 \times 3$$

$$\text{or } 23 - 30 = a$$

$$\therefore a = -7$$

Let u_0 be the n^{th} term

$$t_n = a + (n-1)d$$

$$\text{or } u_0 = -7 + (n-1)3$$

$$\text{or } u_0 = -7 + 3n - 3$$

$$\text{or } \frac{u_0 + 7 + 3}{3} = n$$

$$\therefore n = \frac{50}{3}$$

No, u_0 is not a arithmetic sequence.

g.a. If the n^{th} term of the A.P $7, 12, 17, 22, \dots$ is equal to the n^{th} term of another A.P $27, 30, 33, 36, \dots$ find the value of n .

Solution:

$$7, 12, 17, 22, \dots$$

$$\text{first term (a)} = 7$$

$$\text{Common difference (d)} = 12 - 7 \\ = 5$$

We know,

$$t_n = a + (n-1)d$$

$$= 7 + (n-1)5$$

$$= 7 + 5n - 5 \quad \text{--- ①}$$

Again,

$$\text{for } 27, 30, 33, 36, \dots$$

$$\text{first term (a)} = 27$$

Common difference (d) = 3

Now,

$$t_n = a + (n-1)d$$

$$\therefore 27 = a + (n-1)3$$

$$\therefore 27 = a + 3n - 3 \quad \text{--- (i)}$$

By question:

$$7 + 5n - 5 = 27 + 3n - 3$$

$$\text{or, } 2 + 5n = 2n + 3n$$

$$\text{or, } 5n - 3n = 24 - 2$$

$$\text{or, } 2n = 22$$

$$\therefore n = 11$$

- b. If the 12^{th} , 85^{th} and the last term of an arithmetic sequence are 38, 257 and 395 respectively. calculate how many terms are there in the sequence.

Solution

$$12^{\text{th}} \text{ term } (t_{12}) = 38$$

$$85^{\text{th}} \text{ term } (t_{85}) = 257$$

$$\text{Last term} = 395$$

We know,

$$t_n = a + (n-1)d$$

$$\text{or, } t_{12} = a + (12-1)d$$

$$\text{or, } 38 = a + 11d \quad \text{--- (i)}$$

Also,

$$t_{85} = a + (85-1)d$$

$$\text{or, } 257 = a + 84d \quad \text{--- (ii)}$$

Subtracting (i) from (ii)

$$0 + 84d = 257$$

$$\begin{array}{r} a + 11d = 38 \\ (-) \quad (-) \\ \hline 73d = 219 \end{array}$$

$$\text{or, } 73d = 219$$

$$\therefore d = 3$$

from ①

$$38 = a + 11 \times 3$$

$$\therefore 38 - 33 = a$$

$$\therefore a = 5$$

Let 395 be n th term

$$t_n = a + (n-1)d$$

$$\therefore 395 = 5 + (n-1)3$$

$$\therefore 395 = 5 + 3n - 3$$

$$\therefore \frac{395 - 5 + 3}{3} = n$$

$$\therefore n = 131$$

∴ There are 131 terms

10.a. Ram earns a monthly salary of Rs 20,000 per month. This salary increases by Rs. 5000 per year. How much will his salary be in the 10th year?

Solution:

$$\text{first term } (a) = 20,000$$

$$\text{common difference } (d) = 5000$$

$$10^{\text{th}} \text{ term } (t_{10}) = ?$$

We know,

$$t_n = a + (n-1)d$$

$$\therefore t_{10} = 20,000 + (10-1)5000$$

$$= 20,000 + 45000$$

$$< \text{Rs } 65000$$

∴ His salary will be Rs 65000 in 10 years.

b. A man repays a loan of Rs 3250 by first paying Rs. 20 in the first day then increases the payment by Rs. 15 every day. How long will it take to clear his debt?

By question:

$$S_n = \frac{n}{2} [2 \times 20 + (n-1) 15]$$

$$\text{or } 3250 \times 2 = n (40 + 15n - 15)$$

$$\text{or } 6500 = 25n + 15n^2$$

$$\text{or } 15n^2 + 25n - 6500 = 0$$

$$\text{or } 5 (3n^2 + 5n - 1300) = 0$$

$$\text{or } 3n^2 + 65n - 1300 = 0$$

$$\text{or } (3n + 65)(n - 20) = 0$$

$$\therefore n = \frac{-65}{3} \text{ or } n = 20$$

Exercise - 3.2

General Section

2. Find the sum of the following series.

a. $4 + 7 + 10 + \dots$ up to 12 terms.

Solution:

We know $a = 4$, $d = 7 - 4 = 3$, $n = 12$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{12}{2} [2 \times 4 + (12-1) \times 3]$$

$$= 6 (8 + 33)$$

$$= 6 \times 41$$

$$= 246$$

e. $2 + \frac{1}{2} + 0 + \dots - \frac{17}{2}$

Solution:

Let $-\frac{17}{2}$ be n th term, then

$$t_n = a + (n-1)d$$

$$0_7 - \frac{17}{2} = 1 + (n-1) \times -\frac{1}{2}$$

$$0_7 - \frac{17}{2} - 1 = -\frac{(n-1)}{2}$$

$$0_7 - \frac{17-2}{2} = -\frac{(n-1)}{2}$$

$$0_7 - 15 = -n+1$$

$$0_7 - 15 - 1 = -n$$

$$\therefore n = 20$$

Now,

$$S_n = \frac{n}{2} [a + l]$$

$$= \frac{20}{2} \left[1 - \frac{17}{2} \right]$$

$$= \frac{20}{2} \times \frac{(2-17)}{2}$$

$$= 20 \times -15$$

$$= -75$$

b. $4n + 3n + 3n + \dots$ up to 50 terms

Solution:

First term (a) = $4n$

Common difference (d) = -5

No. of terms (N) = 50

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{50}{2} [2 \times 4n + (50-1)(-5)]$$

$$= 25 [88 - 245]$$

$$= -3925$$

$$\text{c. } 8 + 13 + 18 + \dots + 63$$

Solution:

$$\text{First term (a)} = 8$$

$$\text{Common difference (d)} = 5$$

$$\text{Last term (l)} = 63$$

We know,

$$S_n = \frac{n}{2} (a + l)$$

Let 63 be n^{th} term,

$$t_n = a + (n-1)d$$

$$\text{or } 63 = 8 + (n-1)5$$

$$\text{or } 63 = 8 + 5n - 5$$

$$\text{or } 63 - 8 + 5 = n$$

$$\therefore n = 12$$

Now,

$$S_n = \frac{n}{2} (a + l)$$

$$= 12 (8 + 63)$$

$$= 6 \times 71$$

$$= 426$$

$$\text{d. } 2 - 9 - 20 - \dots - 130$$

Solution:

$$\text{First term (a)} = 2$$

$$\text{Common difference (d)} = -9 - 2 - (-9 - 2)$$

$$= -11$$

Last term (l) = -130

Let -130 be n^{th} term;

$$t_n = a + (n-1)d$$

$$a_7 - 130 = 2 + (n-1) \cdot -11$$

$$a_7 - 130 = 2 - 11n + 11$$

$$a_7 - 130 = 13 - 11n$$

$$a_7 = 13 - 11n$$

$$\therefore n = 13$$

Now,

$$S_n = \frac{n}{2} (a + l)$$

$$= \frac{13}{2} (2 + -130)$$

$$= -832$$

f. $\frac{13}{2} + \frac{21}{4} + 4 + \dots$ up to 8th term

Solution:

First term (a) = $\frac{13}{2}$

Common difference (d) = $-\frac{5}{4}$

No. of terms (n) = 8

We know,

$$S_n = \frac{n}{2} \left\{ 2a + (n-1)d \right\}$$

$$= \frac{8}{2} \left[2 \times \frac{13}{2} + (8-1) \cdot -\frac{5}{4} \right]$$

$$= 4 \left(12 \frac{1}{4} \right)$$

$$= 17$$

2.a. The first term of an arithmetic series is 32 with common difference 8. Find the sum of first 8 terms.

Solution:

$$\text{First term} (a) = 32$$

$$\text{Common difference} (d) = 8$$

$$\text{No. of terms} (n) = 8$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{8}{2} [2 \times 32 + (8-1)8]$$

$$= 4(64 + 56)$$

$$= 480$$

b. If the fourth term of an A.P is 4 and common difference 6. Find the sum of first 16 terms.

Solution:

$$4^{\text{th}} \text{ term} (t_4) = 4$$

$$\text{Common difference } (d) = 6$$

$$\text{16}^{\text{th}} \text{ No. of term} (n) = 16$$

We know,

$$t_n = a + (n-1)d$$

$$\therefore t_4 = a + (4-1)6$$

$$\therefore 4 = a + 3 \times 6$$

$$\therefore 4 = a + 18$$

$$\therefore a = -14$$

Now,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{16}{2} [2 \times -14 + (16-1)6]$$

$$2 \quad 8 \cdot (-28 + 90)$$

$$= 496$$

3.a. The sum of 15 terms of an A.P is 435 and the common difference is 4. find the first term.

Solution:

$$\text{Sum of 15 terms} (S_n) = 435$$

$$\text{Common difference} (d) = 4$$

$$\text{No. of terms} (n) = 15$$

$$\text{first term} (a) = ?$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or } 435 = \frac{15}{2} [2a + (15-1)4]$$

$$\text{or } \frac{435 \times 2}{15} = 2a + 56$$

$$\text{or } \frac{870}{15} = 2a + 56$$

$$\therefore a = 1$$

b. Find the first term of an arithmetic series whose sum of its first seven terms is 0 and the common difference is -3.

Solution:

$$\text{Sum of series} (S_n) = 0$$

$$\text{Common difference} (d) = -3$$

$$\text{No. of terms} (n) = 7$$

We know

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or } 0 = \frac{7}{2} [2a + 6 \times (-3)]$$

$$0_1 \quad 0 = 2a - 18$$

$$0_2 \quad \frac{18}{2} = a$$

$$\therefore a = 9$$

c. An arithmetic series has 10 terms. If its last term is 50 and the sum of its ten terms is 275, find the first term.

Solution:

$$\text{No. of terms (n)} = 10$$

$$\text{Last term (l)} = 50$$

$$\text{Sum of series (S}_n\text{)} = 275$$

$$\text{first term (a)} = ?$$

We know,

$$S_n = \frac{n}{2} (a + l)$$

$$0_1 \quad 275 = \frac{10}{2} (a + 50)$$

$$0_1 \quad 55 = a + 50$$

$$0_2 \quad 55 - 50 = a$$

$$\therefore a = 5$$

d. An arithmetic series with 9 terms has the sum 135. If the first term is 3, find the last term.

Solution:

$$\text{No. of terms (n)} = 9$$

$$\text{Sum of terms (S}_n\text{)} = 135$$

$$\text{First term (a)} = 3$$

$$\text{Last term (l)} = ?$$

We know,

$$S_n = \frac{n}{2} (a + l)$$

$$\text{Q7. } 135 = \frac{9}{2} [3+1]$$

$$\text{Q7. } \frac{135 \times 2}{9} = 3+1$$

$$\therefore 30 - 3 = 1$$
$$\therefore d = 27$$

Q8. Find the number of terms in an arithmetic progression which has its first term 16, common difference 4 and the sum 120.

Solution:

$$\text{First term (a)} = 16$$

$$\text{Common difference (d)} = 4$$

$$\text{Sum of number (S}_n\text{)} = 120$$

$$\text{No. of terms (N)} = ?$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{Q7. } 120 = \frac{N}{2} [2 \times 16 + (n-1)4]$$

$$\text{Q7. } 240 = N [32 + 4n - 4]$$

$$\text{Q7. } 240 = N (28 + 4n)$$

$$\text{Q7. } 240 = 28n + 4n^2$$

$$\text{Q7. } 4n^2 + 28n - 240 = 0$$

$$\text{Q7. } 4(n^2 + 7n - 60) = 0$$

$$\text{Q7. } n^2 + 12n - 60 = 0$$

$$\text{Q7. } n(n+12) - 5(n+12) = 0$$

$$\text{Q7. } (n+12)(n-5) = 0$$

Either

or

$$n+12 = 0$$

$$n-5 = 0$$

$$\therefore n = -12$$

$$\therefore n = 5$$

- b. Find the number of terms in an arithmetic series which has its first term 7, common difference 4 and the sum 900.

Solution:

First term(a) = 7

Common difference(d) = 4

Sum (S_n) = 900

No. of terms(n) = ?

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or, } 900 = 900 \cdot \frac{n}{2} [2 \times 7 + (n-1)4]$$

$$\text{or, } 1800 = n [14 + 4n - 4]$$

$$\text{or, } 1800 = 14n + 4n^2 - 4n$$

$$\text{or, } 4n^2 + 10n - 1800 = 0$$

$$\text{or, } 2(2n^2 + 5n - 900) = 0$$

$$\text{or, } 2n^2 + 5n - 900 = 0$$

$$\text{or, } n(2n + 45) - 20(2n + 45) = 0$$

$$\text{or, } (2n + 45)(n - 20) = 0$$

Either

$$2n + 45 = 0$$

OR

$$n - 20 = 0$$

$$\therefore n = -\frac{45}{2}$$

$$\therefore n = 20$$

- c. Find the number of terms in an arithmetic series which has its first term 49, last term -31 and the sum 153.

Solution:

First term(a) = 49

Last term(l) = -31

Sum(S_n) = 153

No. of terms(N) = ?

We know,

$$S_n = \frac{n}{2} (a + l)$$

$$\text{or, } 153 = \frac{n}{2} [49 - 31]$$

$$\text{or, } 153 = n [18]$$

$$\text{or, } n = \frac{306}{18}$$

$$\therefore n = 17$$

d. The first and the last term of an A.P are 7 and 23 respectively. Find the no. of terms if its sum is 90.

Solution:

$$\text{First term}(a) = 7$$

$$\text{Last term}(l) = 23$$

$$\text{Sum}(S_n) = 90$$

$$\text{No. of terms}(n) = ?$$

We know,

$$S_n = \frac{n}{2} (a + l)$$

$$\text{or, } 90 = \frac{n}{2} [7 + 23]$$

$$\text{or, } 180 = 30n$$

$$\text{or, } n = \frac{180}{30}$$

$$\therefore n = 6$$

s.d. What is the common difference of an A.P when the first term is 1, the last term 50 and the sum 204.

Solution:

$$\text{First term}(a) = 1$$

$$\text{Last term}(l) = 50$$

$$\text{Sum}(S_n) = 204$$

Common difference (d) ?

We know,

$$S_n = \frac{n}{2} (a + l)$$

$$\therefore 204 = \frac{n}{2} [1 + 50]$$

$$\therefore \frac{n(51)}{2} = 204$$

$$\therefore n = 8$$

Again,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_8 = \frac{8}{2} [2a + (8-1)d]$$

$$\therefore 204 = 4 [2 + 7d]$$

$$\therefore \frac{204}{4} = 2 + 7d$$

$$\therefore 51 - 2 = 7d$$

$$\therefore d = \frac{49}{7}$$

$$\therefore d = 7$$

S.Q. The first term of an A.P is 5, number of terms is 30 and sum of terms is 1455. Find the common difference.

Solution:

$$\text{First term } (a) = 5$$

$$\text{Number of terms } (n) = 30$$

$$\text{Sum of terms } (S_n) = 1455$$

Common difference (d) ?

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{Q1. } 1n55 = \frac{30}{2} [2 \times 5 + (30-1)d]$$

$$\text{Q2. } 1455 = 15 [10 + 29d]$$

$$\text{Q3. } \frac{1455}{15} = 10 + 29d$$

$$\text{Q4. } \frac{97 - 10}{25} = d$$

$$\therefore d = 3$$

S.b. The sum of an AP of 10 terms is 210. Find the common difference if the first term is 3.

Solution:

$$\text{No. of term (n)} = 10$$

$$\text{Sum (S}_n\text{)} = 210$$

$$\text{First term (a)} = 3$$

$$\text{Common difference (d)} = ?$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{Q5. } 210 = \frac{10}{2} [2 \times 3 + (10-1)d]$$

$$\text{Q6. } 210 = 5 [6 + 9d]$$

$$\text{Q7. } \frac{210}{5} = 5(6 + 9d)$$

$$\text{Q8. } \frac{42 - 6}{9} = d$$

$$\therefore d = 4$$

5.c. The first term of an A.P is 9, the number of terms 7 and their sum is 0. Find the common difference.

Solution:

$$\text{First term}(a) = 9$$

$$\text{No. of term}(n) = 7$$

$$\text{Sum}(S_n) = 0$$

$$\text{Common Difference}(d) = ?$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore 0 = \frac{7}{2} [2 \times 9 + (7-1)d]$$

$$\therefore 0 = 18 + 6d$$

$$\therefore d = -\frac{18}{6}$$

$$\therefore d = -3$$

Creative Section.

6.a. If the 9th term and 29th term of A.P are 40 and 60 respectively, find the sum of first 30 terms.

Solution:

$$9^{\text{th}} \text{ term}(t_9) = 40$$

$$29^{\text{th}} \text{ term}(t_{29}) = 60$$

$$\text{No. of term}(n) = 30$$

$$\text{Sum}(S_n) = ?$$

We know,

$$t_n = a + (n-1)d$$

$$\therefore t_9 = a + (9-1)d$$

$$\therefore 40 = a + 8d \quad \text{--- (i)}$$

Also,

$$t_{29} = a + (29-1)d$$

$$a_3 = 60 = a + 2d \quad \text{--- (i)}$$

Subtracting equation (i) from (ii)

$$a + 2d = 60$$

$$\begin{array}{r} a + 8d = 40 \\ (-) \quad (-) \quad (-) \end{array}$$

$$a_7 = 20d = 20$$

$$\therefore d = 1$$

from eqn. (i)

$$a_7 = a + 8d$$

$$a_7 = a - 8 = a$$

$$\therefore a = 32$$

Also,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{30}{2} [2 \times 32 + (30-1)d]$$

$$= 15 [64 + 29d]$$

$$= 15 [64 + 29 \times 1]$$

$$= 15 \times 93$$

$$= 1395$$

- b. If the fourth term of an A.P is 1 and the sum of its first eight term is 18, find the tenth term of the series.

Solution:

$$n^{\text{th}} \text{ term } (t_4) = 1$$

$$\text{Sum } (S_8) = 18$$

$$t_{10} = ?$$

We know,

$$t_n = a + (n-1)d$$

$$a_4 = a + (4-1)d$$

$$a_4 = 1 = a + 3d \quad \text{--- (i)}$$

Also,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or } 18 = \frac{8}{2} [2a + (8-1)d]$$

$$\text{or } 18 = 4 [2a + 7d]$$

$$\text{or } 18 = 8a + 28d \quad \text{--- (i)}$$

from subtracting equation (i) from (ii) after multiplying equation (i) by 4:

$$8a + 28d = 18$$

$$\cancel{8a} + \cancel{28d} = 8$$

(-)(-) (-)

$$4d = 10$$

$$\therefore d = 5/2$$

Now,

$$t_{10} = a \text{ from (i)}$$

$$18 = 8a + 28 \times \frac{5}{2}$$

$$\text{or } 18 = \frac{16a + 140}{2}$$

$$\text{or } \frac{36 - 140}{16} = a$$

$$\therefore a = \frac{-13}{2}$$

Now,

$$t_{10} = a + (n-1)d$$

$$= \frac{-13}{2} + \frac{(10-1) \cdot 5}{2}$$

$$= \frac{-13}{2} + \frac{45}{2}$$

$$= 16$$

c. If the third term of an arithmetic sequence is 1 and the fifth term is 7, find the sum of first ten term of sequence.

Solution:

$$\text{Third term } (t_3) = 1$$

$$5^{\text{th}} \text{ term } (t_5) = 7$$

$$S_{10} ?$$

We know,

$$t_n = a + (n-1)d$$

$$t_3 = a + (3-1)d$$

$$\text{or, } 1 = a + 2d \quad \text{(i)}$$

Also,

$$t_5 = a + (5-1)d$$

$$\text{or, } 7 = a + 4d \quad \text{(ii)}$$

Subtracting (i) from (ii)

$$a + 4d = 7$$

$$a + 2d = 1$$

$$(-) \quad (-) \quad (-)$$

$$2d = 6$$

$$\therefore d = 3$$

from (i)

$$1 = a + 2 \times 3$$

$$\text{or, } 1 - 6 = a$$

$$\therefore a = -5$$

Now,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or, } S_{10} = \frac{10}{2} [2 \times (-5) + (10-1)3]$$

$$= 5 [-10 + 27]$$

7.a. If the sum of the first three terms of an arithmetic series is 42 and that of the first 5 term is 80, find 20th term.

Solution:

$$\text{Sum}(S_3) = 42$$

$$\text{Sum}(S_5) = 80$$

~~Sum of 20 term (t_{20}) = ?~~

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_3 = \frac{3}{2} [2a + (3-1)d]$$

$$\text{or } \frac{42 \times 2}{3} = 2a + 2d$$

$$\text{or } 28 = 2(a+d)$$

$$\text{or } \frac{28}{2} = a+d$$

$$\text{or } a+d = 14 \quad \text{--- (i)}$$

Also,

$$S_5 = \frac{5}{2} [2a + (5-1)d]$$

$$\text{or } \frac{80 \times 2}{25} = 2a+4d$$

$$\text{or } 32 = 2(a+2d)$$

$$\text{or } a+2d = 16 \quad \text{--- (ii)}$$

Subtracting (i) from (ii)

$$a+2d = 16$$

$$a+d = 14$$

— — —

$$\therefore d = 2$$

Putting the value of d in equation (i).

$$14 = a+d$$

$$\text{or } 14 = a + 2$$

$$\therefore a = 12$$

Again,

$$t_n = a + (n-1)d$$

$$t_{20} = 12 + (20-1)2$$

$$= 12 + 19 \times 2$$

$$= 12 + 38$$

$$= 50$$

- b. If the sum of first seven terms of an A.P is 14 and the sum of first ten terms is 125, then find the fourth term of the series.

Solution:

$$\text{Sum of 7 terms } (S_7) = 14$$

$$\text{Sum of 10 terms } (S_{10}) = 125$$

$$\text{Fourth term } (t_4) = ?$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or } S_7 = \frac{7}{2} [2a + (7-1)d]$$

$$\text{or } 14 \times 2 = 2a + 6d$$

$$\text{or } 4 = 2(a + 3d)$$

$$\text{or } \frac{4}{2} = a + 3d$$

$$\text{or } a + 3d = 2 \quad \text{--- (i)}$$

Also,

$$S_{10} = \frac{10}{2} [2a + (10-1)d]$$

$$\text{or } 125 = 5 [2a + 9d]$$

$$\text{or } 25 = 2a + 9d \quad \text{--- (ii)}$$

Multiplying equation by 2 and subtracting ① from ②

$$2a + 9d = 25$$

$$\begin{array}{r} 2a + 6d = 4 \\ (-) \quad (-) \quad (-) \\ \hline 3d = 21 \end{array}$$

$$\therefore d = 7$$

Putting the value of 'd' in equation ①

$$2a + 6x7 = 4$$

$$\text{on } a = \frac{-38}{2}$$

$$\therefore a = -19$$

Now,

$$\begin{aligned} t_n &= a + (n-1)d \\ &= -19 + (n-1)7 \\ &= -19 + 21 \\ &= 2 \end{aligned}$$

- c. The sum of first n terms of an AP is 26 and the sum of first 8 terms is 100. Find the sum of first 12 terms.

Solution:

$$\text{Sum of } n \text{ terms } (S_n) = 26$$

$$\text{Sum of 8 terms } (S_8) = 100$$

$$\text{Sum of 12 terms } (S_{12}) = ?$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{26}{2} [2a + (n-1)d]$$

$$\text{on } 26 = 2 [2a + 3d]$$

$$\text{or } 13 = 2a + 3d \quad \text{--- ①}$$

Also,

$$S_8 = \frac{8}{2} [2a + (8-1)d]$$

$$\therefore 100 = 4 [2a + 7d]$$

$$\therefore 25 = 2a + 7d \quad \text{--- (i)}$$

Subtracting (i) from (ii)

$$2a + 7d = 225$$

$$2a + 3d = 13$$

$$\begin{array}{r} (-) \quad (-) \\ \hline \end{array}$$

$$4d = 12$$

$$\therefore d = 3$$

Putting the value of d in equation (i)

$$13 = 2a + 3 \times 3$$

$$\therefore \frac{13 - 9}{2} = a$$

$$\therefore a = 2$$

Now,

$$S_{12} = \frac{12}{2} [2 \times 2 + (12-1)3]$$

$$= 6 [4 + 33]$$

$$= 6 \times 37$$

$$= 222$$

- 8.a. Find three numbers in A.P such that their sum is 36 and their product is 1140.

Solution:

Let 'a' be the first term and 'd' be the common difference then,

$a-d, a, a+d$ are three terms of A.P

By question,

$$(a-d) + a + (a+d) = 36$$

$$\text{or } a - d + a + d = 36$$

$$\text{or } 3a = 36$$

$$\therefore a = 12$$

Again,

$$(a-d) \times (a) \times (a+d) = 1140$$

$$\text{or } (a^2 - d^2) a = 1140$$

$$\text{or } (12^2 - d^2) 12 = 1140$$

$$\text{or } 144 - d^2 = \frac{1140}{12}$$

$$\text{or } 144 - 95 = d^2$$

$$\text{or } 1728 - 12d^2 = 1140$$

$$\text{or } 1728 - 1140 = 12d^2$$

$$\text{or } \frac{588}{12} = d^2$$

$$\text{or } 7^2 = d^2$$

$$\therefore d = 7 \text{ cm}$$

when,

$$a = 12 \text{ and } d = 7$$

The terms are:

$$a - d = 12 - 7 = 5$$

$$a = 12 = 12$$

$$a + d = 12 + 7 = 19$$

When, $a = 12$ and $d = 7$

The terms are:

$$a + d = 12 + 7 = 19$$

$$a = 12$$

$$a - d = 12 - 7 = 5$$

b. Find three numbers in A.P such that their sum is 21 and their product is 280.

Solution:

If 'a' is the first term and 'd' is the common difference then
 $a-d$, a and $a+d$ are three terms of A.P

By question:

$$a-d + a + a+d = 21$$

$$\text{or, } 3a = 21$$

$$\therefore a = 7$$

Again,

$$(a-d) \times a \times (a+d) = 280$$

$$\text{or, } (a^2 - d^2) a = 280$$

$$\text{or, } (7^2 - d^2) 7 = 280$$

$$\text{or, } 49 - d^2 = 280 / 7$$

$$\text{or, } 49 - d^2 = 40$$

$$\text{or, } d^2 = 9$$

$$\therefore d = \pm 3$$

When, $a=7$ and $d=3$, the terms are;

$$a+b = 7+3 = 10$$

$$a = 7$$

$$a-b = 7-3 = 4$$

Also,

When, $a=7$ and $d=-3$, the terms are

$$a+b = 7-3 = 4$$

$$a = 7$$

$$a-b = 7+3 = 10$$

c. Find the three numbers in A.P such that their sum is 12 and the sum of their squares is 50.

Solution:

If 'a' is the first term and 'd' is the difference then
 $a-d$, a , $a+d$ are the three terms.

By question:

$$a-d + a + a+d = 12$$

$$\text{or } 3a = 12$$

$$\therefore a = 4$$

Also,

$$(a-d)^2 + a^2 + (a+d)^2 = 50$$

$$\text{or } a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 50$$

$$\text{or } 3a^2 + 2d^2 = 50$$

$$\text{or } 3 \times 4^2 + 2d^2 = 50$$

$$\text{or } 2d^2 = 50 - 32$$

$$\text{or } d^2 = \frac{18}{2}$$

$$\text{or } d = \pm \sqrt{9}$$

$$\therefore d = \pm 3$$

when $a = 4$ and $d = 3$, the terms are

$$a+b = 4+3 = 7$$

$$a = 4$$

$$a-b = 4-3 = 1$$

Also

when $a = 4$ and $d = -3$, the terms are

$$a+b = 4-3 = 1$$

$$a = 4$$

$$a-b = 4+3 = 7$$

Ques. If the 5^{th} term of AP is 46, find the sum of first 11 terms.

Solution:

Given,

$$5^{\text{th}} \text{ term} (t_5) = 46$$

Sum of 11 term (S_{11}) = ?

We know

$$t_n = a + (n-1)d$$

$$t_1 = a + 5d$$

$$\text{or } 46 = a + 5d \quad \text{--- (1)}$$

Also,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{11} = \frac{11}{2} [2a + (11-1)d]$$

$$S_{11} = \frac{11}{2} [2a + 10d]$$

$$S_{11} = \frac{11}{2} \times 2(a + 5d)$$

$$S_{11} = 11 \times 46 \quad (\because a + 5d = 46)$$

$$S_{11} = 506$$

$$\therefore S_{11} = 506$$

- b. In an arithmetic sequence, the sixth term is equal to 3 times the fourth term and the sum of the first three term is -12. Find AP
Solution:

By question:

$$t_6 = 3 \times t_4$$

$$\text{or } a + (n-1)d = 3(a + (n-4)d)$$

$$\text{or } a + (6-1)d = 3[a + (4-1)d]$$

$$\text{or } a + 5d = 3a + 9d$$

$$\text{or } -2a = 4d$$

$$\therefore a = -2d \quad \text{--- (1)}$$

Also,

$$S_3 = -12$$

$$\text{or } \frac{n}{2} [2a + (n-1)d] = -12$$

$$o_1 \frac{3}{2} [2a + (3-1)d] = -12$$

$$o_1 \frac{3}{2} [2a + 2d] = -12$$

$$o_1 \frac{3}{2} [(2x-2d) + 2d] = -12$$

$$o_1 \frac{3}{2} (-4d + 2d) = -12$$

$$o_1 \frac{3}{2} x - 2d = -12$$

$$o_1 73d = +12$$

$$\therefore d = 4$$

from ①

$$a = -2d$$

$$o_1 a = -2 \times 4$$

$$\therefore a = -8$$

Now AP is given below,

$$a, a+d, a+2d \quad a = -8$$

$$-8, -8+4, -8+2 \times 4 = -4$$

$$a+2d = -8+2 \times 4 = 0$$

c. If the 8th term of an AP is 45, find the sum of first 15 terms.

Solution:

$$8^{\text{th}} \text{ term } (t_8) = 45$$

We know,

$$t_n = a + (n-1)d$$

$$o_1 t_8 = a + (8-1)d$$

$$o_1 45 = a + 7d \quad \text{--- ①}$$

Also,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{Q1. } S_{15} = \frac{15}{2} [2a + (15-1)d]$$

$$= \frac{15}{2} [2a + 14d]$$

$$= 15 \times \frac{a+7d}{2}$$

$$= 15 \times 45$$

$$= 675$$

Ans,

a. The first and last term of an A.S are -24 and 72 respectively. If the sum of all terms of series is 600. find the number of terms and common difference.

Solution:

$$\text{First term}(a) = -24$$

$$\text{Last term }(l) = 72$$

$$\text{Sum}(S_n) = 600$$

$$\text{No. of terms}(n) = ?$$

$$\text{Common difference}(d) = ?$$

We know,

$$S_n = \frac{n}{2} [a+l]$$

$$\text{Q1. } 600 = \frac{n}{2} [-24+72]$$

$$\text{Again, Q1. } 1200 = n \times 48$$

$$\text{Q1. } n = \frac{1200}{48}$$

$$\therefore n = 25$$

Again,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$a) 660 = \frac{25}{2} [2x_2 + (25-1)d]$$

$$a) 2n = \frac{1}{2} [48 + 24d]$$

$$a) 48 = -48 + 24d$$

$$a) \frac{96}{24} = d$$

$$\therefore d = 4$$

- b. In an A.S., the sum of first ten terms is 520. If the seventh term is double of third term, calculate the first term and common difference.

Solution:

$$\text{Sum of ten terms } (S_{10}) = 520$$

By question:

$$t_7 = 2t_3$$

$$a + (n-1)d = 2(a + (n-1)d)$$

$$a + (7-1)d = a + (3-1)d \quad \{ \rightarrow 2(a + (3-1)d) \}$$

$$a + 6d = 2a + 4d$$

$$6d - 4d = 2a - a$$

$$2d = a \quad \leftarrow \textcircled{1}$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$a) S_{10} = \frac{10}{2} [2x_2 + 9d]$$

$$a) \frac{10n}{10} = 4d + 9d$$

$$a) \frac{10n}{13} = d$$

$$\therefore d = 8$$

Putting the value of d in equation ①

$$a = 2d$$

$$\text{or } a = 2 \times 8$$

$$\therefore a = 16$$

11a. Find the sum of the first 50 natural numbers.

Solution:

$$\text{First term}(a) = 50$$

$$\text{Last term}(l) = 50$$

$$\text{No. of terms}(n) = 50$$

We know,

$$S_n = \frac{n}{2} [a + l]$$

$$\text{or } S_{50} = \frac{50}{2} (1 + 50)$$

$$\text{or } S_{50} = 25 \times 51$$

$$\therefore S_{50} = 1275$$

b. Find the sum of the first 100 odd numbers.

Solution:

The sequence of odd numbers is 1, 3, 5, ...

$$\text{First term}(a) = 1$$

$$\text{Common difference}(d) = 2$$

$$\text{No. of term}(n) = 100$$

$$\text{Sum}(S_n) = ?$$

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or } S_{100} = \frac{100}{2} [2 \times 1 + 99 \times 2]$$

$$\therefore S_{100} = 10,000$$

c. Find the sum of first 50 even numbers.

Solution:

Series The sequence of first 50 even no. is 2, 4, 6, 8 ...

first term (a) = 2

Common difference (d) = 2

No. of term (n) = 50

Sum of term (s_n) = ?

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or } S_{50} = \frac{50}{2} [2 \times 2 + 49 \times 2]$$

$$\text{or } S_{50} = 25 \times 102$$

$$\therefore S_{50} = 2550$$

12a. Find the sum of all numbers between 200 and 400 divisible by 7.

Solution:

The sequence of no. between 200 and 400 which are divisible by 7 is 203, 210, 217, 224, ..., ~~289~~ 399

first term (a) = 203

Common difference (d) = $210 - 203$

$$= 7$$

Last term (a_n) = 399

We know,

$$t_n = a + (n-1)d$$

$$\text{or } 399 = 203 + (n-1)7$$

$$\text{or } 399 = 203 + 7n - 7$$

$$\text{or } \frac{399 - 203 + 7}{7} = n$$

$$\therefore n = 29$$

Now,

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$= \frac{29}{2} [203 + 399]$$

$$\therefore S_n = 8729$$

- b. Find the sum of all numbers between 1 and 100 which are divisible by 3.

Solution:

The sequence of no. between 1 and 100 which are divisible by 3 are 3, 6, 9, 12, ..., 99

Here,

$$\text{first term}(a) = 3$$

$$\text{Last term}(l) = 99$$

$$\text{Common difference}(d) = 12 - 9$$

$$= 3$$

We know,

$$t_n = a + (n-1)d$$

$$\text{or } 99 = 3 + (n-1)3$$

$$\text{or } 99 = 3 + 3n - 3$$

$$\text{or } \frac{99-3}{3} = n$$

$$\therefore n = 33$$

Now,

$$S_n = \frac{n}{2} (a+d)$$

$$= \frac{33}{2} (3+99)$$

$$= 1683$$