Aucoustics

The branch of physics that deals with the process of production, transmission and reception of sound is called aucoustics.

Pressure Amplitude:- (ΔP_{max})

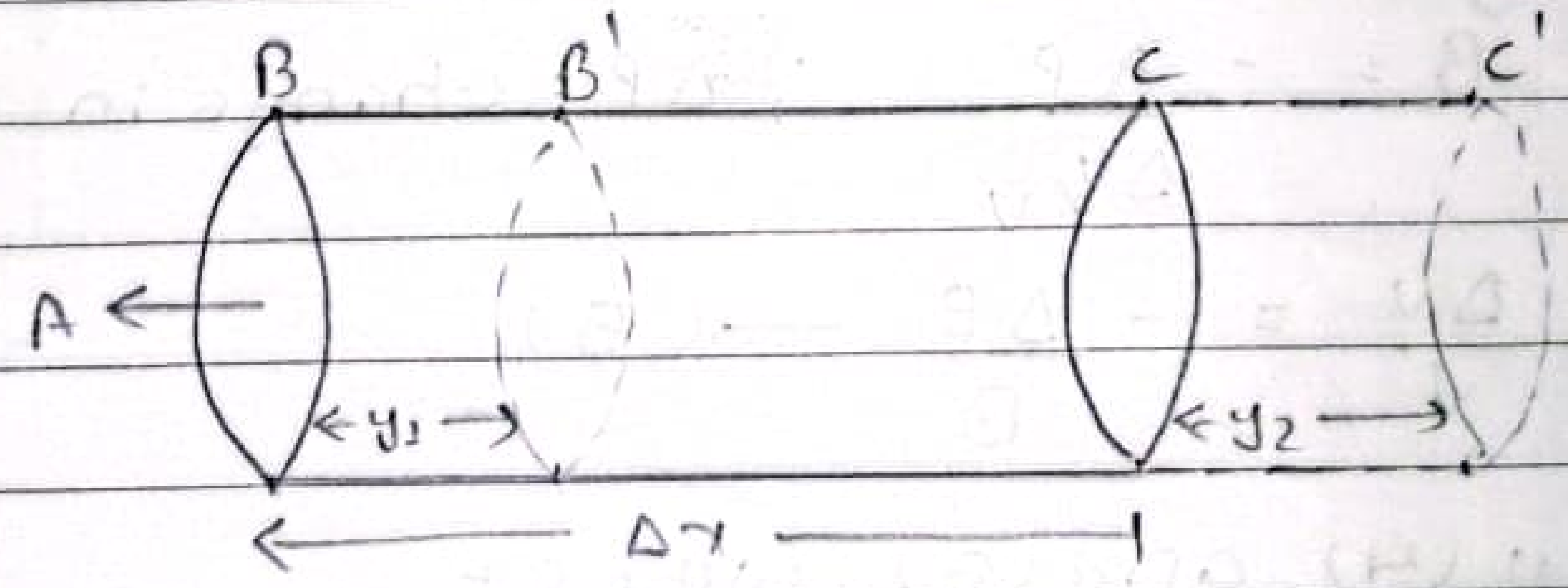
The maximum vibration (increase or decrease) of pressure as sound propagates in a medium is known as ~~sound propagat~~ pressure amplitude. Greater the pressure amplitude, louder will be the sound.

Let us consider a sound wave travelling in a medium whose displacement is:

$$y = a\sin(\omega t - kx) \quad \text{---(1)}$$

As the sound wave propagates, the particles move in the direction of propagation so there is variation in pressure at different positions of the medium.

In order to see the variation of pressure, let us consider an imaginary cylinder of air of cross-sectional area 'A' and length $BC = \Delta x$ as shown in figure.



The volume of the cylinder when the wave does not propagate is

$$V = A\Delta x \quad -(2)$$

Let the end B is displaced by y_1 and C is displaced by y_2 when the sound propagate as shown in figure.

$$\Delta V = A(y_2 - y_1)$$

$$\Delta V = A\,\Delta y. \quad -(3)$$

Now;

Fractional change in volume is given by

$$\frac{\Delta V}{V} = \frac{A\Delta y}{A\Delta x} = \frac{\Delta y}{\Delta x}$$

or, $$\frac{\Delta V}{V} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

or, $$\frac{\Delta V}{V} = \frac{d}{dx}\left\{a\sin(\omega t - kx)\right\}$$

or, $$\frac{\Delta V}{V} = -ak\cos(\omega t - kx) \quad -(4)$$

eq (4) ~~chang~~ gives Fractional change in volume.

Also;

we know, By the defination of bulk modulus of elasticity

$$B = -\frac{\Delta P}{\Delta V/V}$$; ΔP = change in pressure.

or, $$\frac{\Delta V}{V} = -\frac{\Delta P}{B} \quad -(5)$$

From equ (4) and (5), we get

$$\cancel{+}\frac{\Delta P}{B} = \cancel{+}ak\cos(\omega t - kx)$$

$$\therefore \Delta P = Bak\cos(\omega t - kx) \quad -(6)$$

classmate
Date
Page

To get the maximum pressure the value of $\cos(\omega t - kx)$ must be 1

$$\therefore \Delta P_{max} = Bak$$

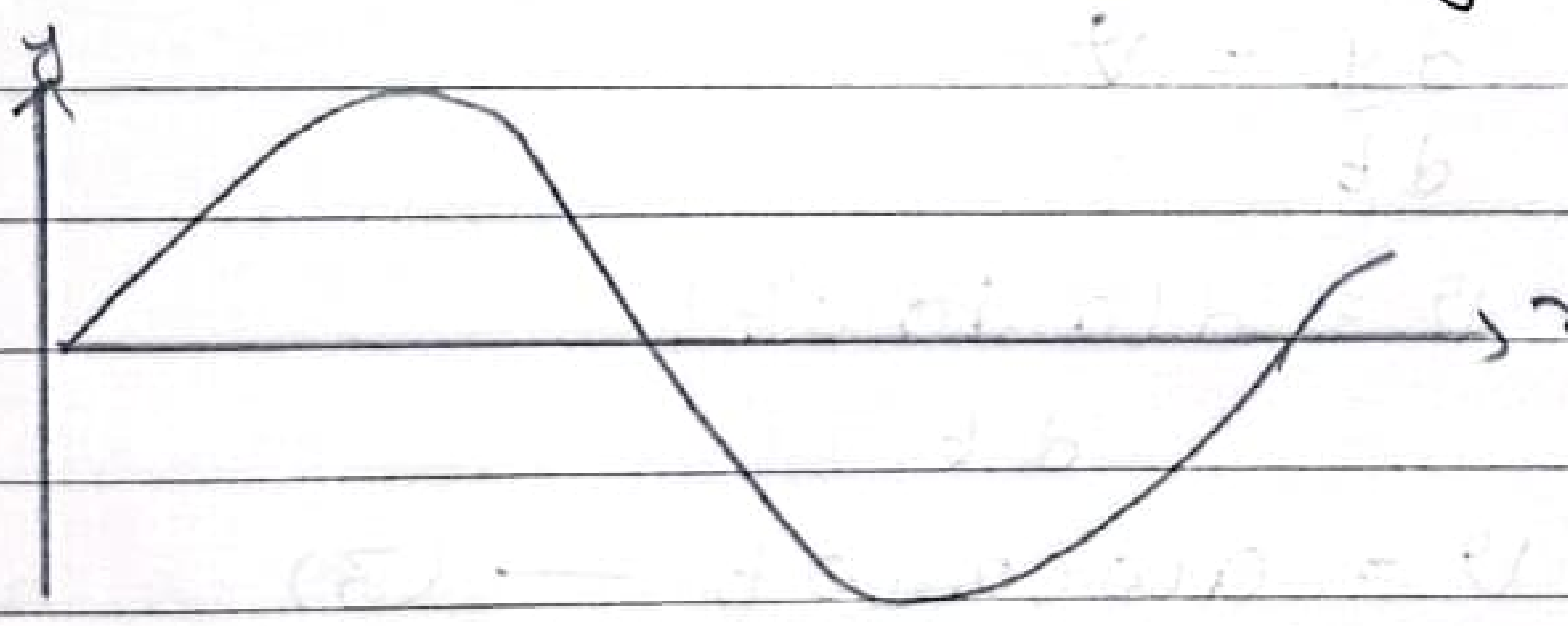
where B = Bulk modulus
a = amplitude
k = constant.

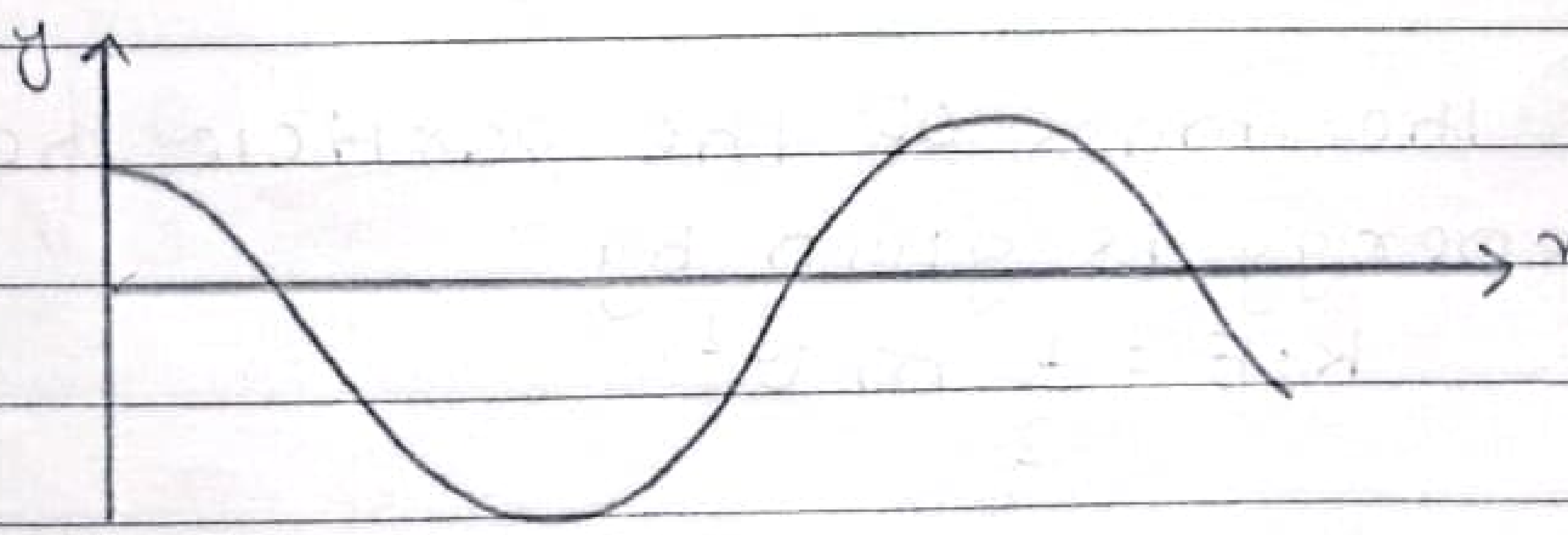
eq(6) can also be written as:

$$P = P_0 \cos(\omega t - kx) \quad — (3)$$

where $P_0 = Bak$ is the pressure amplitude.

A closer look at equ (4) and (3) shows that the pressure amplitude and displacement are 90^0 out of phase. This means pressure vibration is maximum when displacement is minimum and vice versa ~~aa~~ which is shown in graph below:-





Intensity of sound:-

Intensity of sound at a point is defined as the energy flowing per unit time per unit area of a surface held normally at the point to the direction of propagation of wave.

$$\text{i.e } I = \frac{Q}{A \times t} \quad -(1)$$

The displacement of the particle when the wave propagates is given by

$$y = a\sin\omega t \quad -(2)$$

The instantaneous displacement of the air layer due to the passage of the wave is given by

$$\text{i.e } \frac{dy}{dt} = v$$

$$\text{or, } v = \frac{d(a\sin\omega t)}{dt}$$

$$\therefore v = a\omega\cos\omega t \quad -(3)$$

If 'm' is the mass of the particle then, kinetic energy is given by

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m a^2 \omega^2 \cos^2\omega t \quad -(4)$$

We know;

Total energy is equal to maximum K.E

$$\text{i.e } Q = K.E_{max} \quad -(5)$$

For the value of K.E to be K.E max, $\cos \omega t = 1$

i.e $K.E_{max} = \frac{1}{2} m a^2 \omega^2$ — (6)

From equ (5) and (6)

$$\boxed{\rho = \frac{1}{2} m a^2 \omega^2} \quad — (7)$$

Then;

Substituting the value of (7) in equ(1)

$$I = \frac{\frac{1}{2} m a^2 \omega^2}{A \times t}$$

$$= \frac{\frac{1}{2} \rho V a^2 \omega^2}{A \cdot t} \quad [\because \rho = \frac{m}{V} \Rightarrow m = \rho V]$$

$$= \frac{1/2\ \rho . \cancel{A} . x . a^2 \omega^2}{\cancel{A} . t} \quad [\because V = A . x,\ x = \text{length}]$$

$$= \frac{1}{2} \rho . \left(\frac{x}{t}\right) a^2 \omega^2$$

$$\boxed{= \frac{1}{2} \rho v a^2 \omega^2} \quad — (8)$$

equation (8) can also be written in other form.

we know

$$v = \sqrt{\frac{B}{\rho}} \Rightarrow v^2 = \frac{B}{\rho} \Rightarrow B = \rho v^2$$

Then equ (8) becomes

$$I = \frac{1}{2} \frac{B}{v} a^2 \omega^2 \quad — (9)$$

Also;

$\omega = 2\pi f$

So, $I = \frac{1}{2} \rho v a^2 (2\pi f)^2 = \boxed{2 \rho \pi^2 a^2 f^2}$ — (10)

$\Rightarrow I = a^2$

Thus for given frequency, intensity is directly proportional to square of amplitude.

Note:-

At a distance r from a point source of sound, the energy Q will pass through the ~~source~~ surface of surrounding sphere of area $4\pi r^2$. so intensity

$$I = \frac{Q}{A \cdot t} = \frac{P}{A} = \frac{P}{4\pi r^2}$$

As P is constant

$$I \propto \frac{1}{r^2}$$

i.e intensity 'I' follows inverse square law.

Beats.

When two sound waves of slightly different frequencies but similar amplitude are produced simultaneously, the loudness increases and decreases periodically.

The rise and fall in the intensity of sound produced by the superposition of the two waves of slightly different frequencies are known as beats.

The number of beats heard per second is known as beat frequency.

$$f_b = f_1 - f_2 \text{ or } f_2 - f_1$$

Let us consider, two waves having same amplitude and slightly different frequencies f_1 and f_2 such that $f_1 > f_2$ and displacement of these waves is represented by

$$y_1 = a \sin \omega_1 t = a \sin 2\pi f_1 \cdot t \quad - (1)$$

$$y_2 = a \sin \omega_2 t = a \sin 2\pi f_2 \cdot t \quad - (2)$$

The resultant wave due to the superposition of these two waves is given by,

$$y = y_1 + y_2$$

or, $y = a \sin 2\pi f_1 t + a \sin 2\pi f_2 \cdot t$

or, $y = a\left[\sin 2\pi f_1 t + \sin 2\pi f_2 t\right]$

or, $y = 2a\left[\sin\left(\frac{2\pi(f_1+f_2)t}{2}\right) \cdot \cos\left(\frac{2\pi(f_1-f_2)t}{2}\right)\right]$

or, $y = 2a \cos \frac{2\pi(f_1-f_2)t}{2} \cdot \sin \frac{2\pi(f_1+f_2)t}{2}$

or, $y = A \sin 2\pi \frac{(f_1+f_2)}{2} t \quad - (3)$

equ(3) is resultant displacement of beat.

where, $A = 2a \cos \frac{2\pi(f_1-f_2)t}{2}$

which is the amplitude of beat.

Condition for maxima.

For $A = A_{max}$ i.e $A = 2a$,

$\cos \frac{2\pi(f_1-f_2)}{2} \cdot t = \pm 1$

or, $\cos \frac{2\pi(f_1-f_2)t}{2} = \cos n\pi$

where $n = 0, 1, 2, 3, \ldots$

classmate

Date

Page

or, $\frac{\cancel{2}\pi(f_1-f_2)t}{\cancel{2}} = n\pi$

$\therefore \quad t = \frac{n}{(f_1-f_2)}$

which is time for beat maxima.

Thus, time period for maxima are

$0, \frac{1}{f_1-f_2}, \frac{2}{f_1-f_2}, \ldots$

The time interval betwo consecutive maxima is the ~~tim~~ period and it is given by

$T = \frac{2}{f_1-f_2} - \frac{1}{f_1-f_2} = \frac{1}{f_1-f_2}$

$\therefore \quad T = \frac{1}{f_1-f_2}$

Condition for minima

Resultant amplitude is minimum when $\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$ is minimum i.e.

i.e $\quad \cos 2\pi\left(\frac{f_1-f_2}{2}\right)t = 0$

or, $\quad \cos 2\pi\left(\frac{f_1-f_2}{2}\right)t = \cos(2n+1)\frac{\pi}{2}$

where $n = 0, 1, 2, \ldots$

or, $\quad \cancel{2}\cancel{\pi}\left(\frac{f_1-f_2}{\cancel{2}}\right)t = (2n+1)\frac{\pi}{2}$

$\therefore \quad t = \frac{(2n+1)}{2(f_1-f_2)}$

Thus, time period for minima are

$\frac{1}{2(f_1-f_2)}, \frac{3}{2(f_1-f_2)}, \ldots$

So, the time interval between two consecutive minima i.e. beat period is given as.

$$T = \frac{3}{2(f_1-f_2)} - \frac{1}{2(f_1-f_2)}$$

$$T = \frac{1}{(f_1-f_2)}$$

Musical sound and Noise.

A desirable sound that produces pleasing effect on the listeners is called musical sound. Eg sound of piano, flute etc.

The sound that produces an unpleasant effect on the listener is called noise. All sounds other than musical ~~so~~ notes are noises.

Characteristics of musical sound.

1) Pitch

Pitch is a property that helps us to differentiate between a shrill sound and a flat sound. Shrill sound has high pitch and high frequency and flat one has low pitch and low frequency. However, frequency and pitch are not the same thing. frequency is measurable quantity whereas pitch is a sensation that corresponds to or is related to the frequency.

classmate
Date
Page

2. Loudness

Loudness is a subjective sensation that depends on listener. Loudness (or intensity) depends upon the following factors.

a) Amplitude of vibration of ~~sound~~ source:-
Greater is the amplitude of the vibration of source, larger is the intensity and hence, loudness of the sound also increases.

b) Direction of motion of wind:-
If the wind is blowing in the direction of propagation of sound, the loudness of the sound increases and on the other hand, if the wind is blowing in the opposite direction of travel of sound, the loudness decreases.

c) Presence of other bodies:- The loudness of sound is increased due to the presence of other bodies near the source of sound.

d) Surface area of vibrating body:-
The greater the surface area of vibrating body, the larger is the loudness of the sound.

e) Frequency of sound:- The loudness of a vibrating body is directly proportional to the square of the frequency of the vibrating body

3. Quality or Timbre
It is the property of sound that helps us to distinguish between two sounds of same pitch and loudness. It depends on the intensity and number of harmonics present in the sound.

Threshold of Hearing and Pain.

The minimum intensity of sound that can be heared by an ear is called the threshold of hearing. It is represented by I_0 and is given as

$$I_0 = 10^{-12} \, W/m^2$$ for pure tone of frequency 1kHz.

The normal human ear tolerable to maximum intensity is $1 \, W/m^2$. This is called threshold of ~~be~~ pain.

Relation between Intensity and Loudness of sound.

The weber Fechner law states that the loudness of sound L is proportional to the logarithm of the intensity of sound. i.e

$$L \propto \log_{10} I$$

$$\text{or, } L = k \log_{10} I \quad \text{— (i)}$$

where k = constant. This is a weber-Fechner relation.

Let I_0 be the threshold of hearing. The loudness is L_0 for the corresponding threshold of hearing is

$$L_0 = k \log_{10} I_0 \quad \text{—(ii)}$$

Then

difference in Loudness

$$L - L_0 = k \log_{10} I - k \log_{10} I_0$$

$$\text{or, } L - L_0 = k \log\left(\frac{I}{I_0}\right) \quad \text{— (iii)}$$

The difference $L - L_0$ indicate as to how much loudness of a given sound is above the minimum value of hearing. Thus, this is called intensity level (β)

$$\therefore \beta = k \log_{10}\left(\frac{I}{I_0}\right) \quad -(iv)$$

If $k = 1$, β is measured in a unit called bel.
If $I = 10 I_0$, then $\beta = \log_{10} \frac{10 I_0}{I_0} = 1$ bel.

So, intensity level of sound is said to be one bel if its intensity is 10 times of threshold of hearing.

A decibel is one tenth of a bell

So, $$\beta = 10\,dB \log_{10}\left(\frac{I}{I_0}\right) \quad -(v)$$

classmate
Date
Page

Droppler's effect :-

Droppler's effect in sound is defined as the change in frequency of the sound heard by the observer due to relative motion between source and observer. The frequency (pitch) so heard by the observer is known as apparent frequency. The apparent frequency is given by

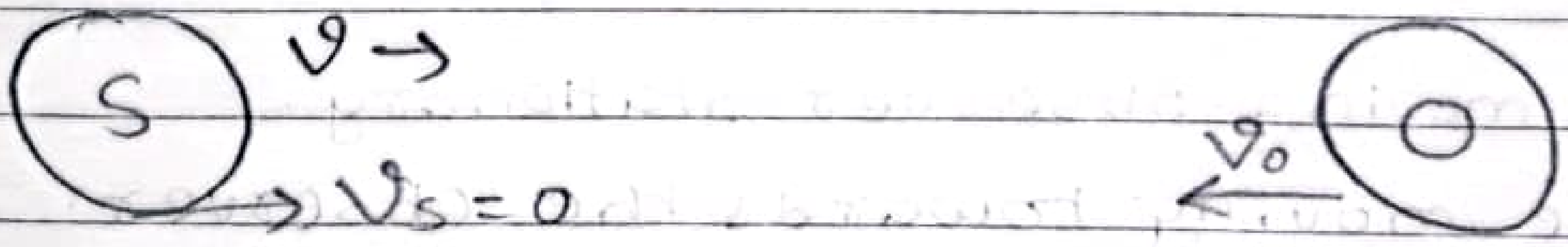
$$f' = \frac{\text{Relative velocity of sound w.r.t observer}}{\text{wavelength reaching to observer.}}$$

Different cases of the droppler's effect are discussed below:-

1. Let, v = velocity of sound
v_s = Velocity of source
v_o = velocity of observer.
f = frequency of sound (source)
f' = apparent frequency.

1. Source stationary - observer moving

a) Observer moving towards stationary source:-



For a sound source producing sound of velocity 'v', 'f' no. of waves emitted per second will be contained in a length of 'v'. So wavelength reaching to the observer is given by: $\lambda = \frac{v}{f}$ — (1)

classmate
Date
Page

The relative velocity of the sound with respect to observer is equal to $v + v_o$

So, the apparent frequency is:

$$f' = \frac{v + v_o}{v/f}$$

$$f' = \left(\frac{v + v_o}{v}\right) f$$

$$\frac{f'}{f} = \frac{v + v_o}{v} > 1$$

$$f' > f$$

Thus the apparent frequency is greater than the frequency of sound.

b. Observer moving away from the stationary source:-

so, $$f' = \left(\frac{v - v_o}{v}\right) f$$

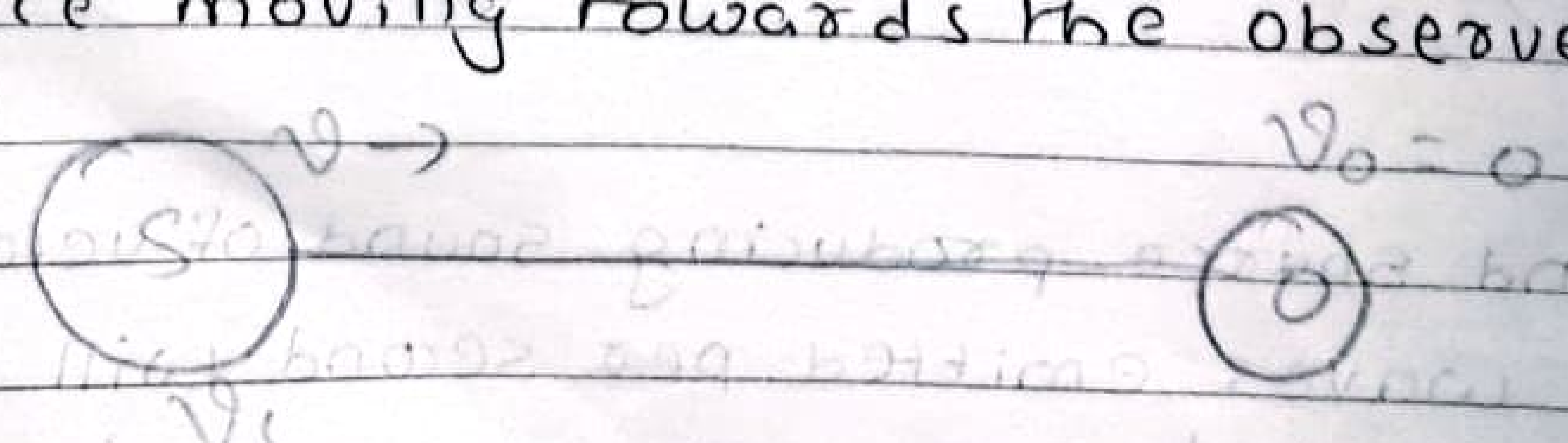
$$\frac{f'}{f} = \frac{v - v_o}{v} < 1$$

$$\frac{f'}{f} < 1$$

$$f' < f$$

2. Source moving - observer stationary.

a) Source moving towards the observer.



The 'f' waves emitted by source in one second are occupied within a distance of 'v' but if the source is also moving, it covers a distance 'v_s' in the same direction in which sound is propagating. So, the 'f' waves emitted in one second will be occupied by within $v - v_s$ distance. So, the wavelength reaching to the observer is given by $\lambda' = \frac{v - v_s}{f}$

So, the apparent frequency is:

$$f' = \frac{v}{(v - v_s)} \cdot f$$

$$\frac{f'}{f} = \frac{v}{v - v_s} > 1$$

$$f' > f$$

b) Source moving away from the observer.

In this case, the relative velocity of sound with respect to source is $v + v_s$. So, the apparent frequency is:

$$f' = \frac{v}{v + v_s} \cdot f$$

$$\frac{f'}{f} = \frac{v}{v + v_s}$$

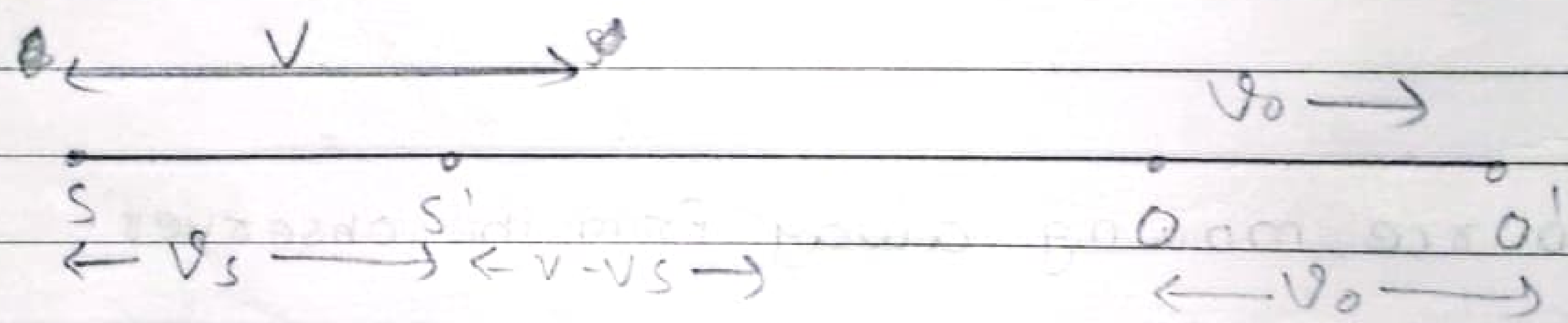
$$\frac{f'}{f} < 1$$

$$f' < f$$

3. Both source and observer moving.

Let us consider a source of sound 'S', moving with velocity 'v_s', emits sound of frequency 'f' and velocity 'v'. Also, 'v_o' be the velocity of observer. Let us consider both source and observer moving along the same line towards the positive x-axis such that 'S' reaches to 'S'' and O reaches to 'O'' as shown in figure:-
If the source emits 'f' waves in one second then these waves occupy a distance of $v - v_s$. This means the wavelength gets shorter. So, the wavelength is given by

$$\lambda' = \frac{v - v_s}{f} \quad \text{--- (1)}$$



Because of the relative velocity between source and observer. There will be change in frequency heard by obserber which is given by:

$$f' = \frac{\text{Relative velocity of sound w.r.to observer } (v')}{\text{the wavelength } (\lambda')}$$

Here,

$$v' = v - v_o$$

So,

$$f' = \frac{v - v_o}{\lambda'} = \left(\frac{v - v_o}{v - v_s}\right) f.$$

This is the general equation for apparent frequency.

Case I:

Source moving towards observer - observer moving towards source.

We know;

$$f' = \left(\frac{v - v_o}{v - v_s}\right) f$$

~~when~~ $v_s = v_s$, $v_o = -v_o$

$\therefore$ $f' = \left(\frac{v + v_o}{v - v_s}\right) f$.   Since $v + v_o > v - v_s$

$f' > f$.

Case II: ~~So~~

Source and observer moving away from each other.

We know;

$$\cancel{f' = \left(\frac{v - v_o}{v - v_s}\right) \times f}$$

$$v' = v - v_o$$

$$\lambda' = \frac{v + v_s}{f}$$

Then

$$f' = \frac{v'}{\lambda'} = \frac{v - v_o}{(v + v_s)/f} = \frac{v - v_o}{v + v_s} \times f$$

Since $v + v_s > v - v_o$

$f' < f$

Case III :

Source and observer moving in same direction such that source leading the observer.

$v' = v + v_o$

$\lambda' = \frac{v + v_s}{f}$

$\therefore f' = \frac{v'}{\lambda'}$

$f' = \frac{v + v_o}{(v + v_s)/f}$

$\therefore f' = \frac{v + v_o}{v + v_s} \times f$

If $v_o > v_s$, $f' > f$

If $v_o < v_s$, $f' < f$

Case IV:

Source and observer moving in same direction such that observer leading the source.

$v' = v - v_o$

$\lambda' = \frac{v - v_s}{f}$

$\therefore f' = \frac{v'}{\lambda'} = \frac{v - v_o}{(v - v_s)/f} = \frac{v - v_o}{v - v_s} \times f$

If $v_o > v_s$; $f' < f$

$v_o < v_s$ $f' > f$.

Limitation of Droppler Effect.

The Droppler's principle can only be applied in the case where the relative velocity between the source and observer is less than the velocity of sound. So, the peinciple is not applicable if the source moves towards the observer with supersonic velocity.