c/w

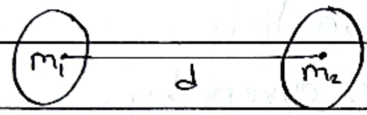
Chapter-7
Gravitation

Page No.
Date: / /

Forces in Nature

i. Gravitation (mass dependent)
ii. Electromagnetic (charge dependent)
iii. Short range nuclear force (distance dependent active in 10^{-15} m)
iv. Weak force (Active in 10^{-17} m)

Newton's Law of Gravitation



Let us consider the force of attraction between two bodies of mass 'm_1' and (m_2) separated by distance 'd' from their centre is directly proportional to the product of their mass and inversely proportional to the square of their separation from their centre.

i.e. $F \propto m_1 . m_2$ —— (i)

Also, $F \propto \dfrac{1}{d^2}$ —— (ii)

Combining equation (i) and (ii)

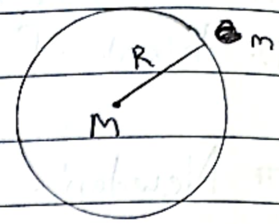
$$F \propto \frac{m_1 . m_2}{d_2}$$

or, $F = \dfrac{G m_1 . m_2}{d_2}$ —— (iii)

Where G is proportionality constant called universal Gravitational constant and it's value is $6.67 \times 10^{-11}\ Nm^2 kg^{-2}$

Acceleration due to gravity

Let us consider a body of ~~'m'~~ mass 'm' lies on a surface of the earth having mass 'M' and radius 'R'.



Then,
According to the Newton's Law of gravitation, the force of attraction between earth and body is given by:

$$F = \frac{GMm}{R^2} \quad \text{—— (I)}$$

If 'g' be the acceleration due to gravity than according to the Newton's second law of motion, the force ~~of~~ on the body is given by:

$$F = mg \quad \text{—— (II)}$$

From equation (I) and (II), we get:

$$mg = \frac{GMm}{R^2}$$

$$\text{or, } g = \frac{GM}{R^2} \quad \text{—— (III)}$$

This is the req. expression for acceleration due to gravity and it shows that 'g' is independent on the mass of the body.

Variation of acceleration due to gravity.

1. Due to shape of earth:

We know,

$$g = \frac{GM}{R^2}$$

$$\Rightarrow g \propto \frac{1}{R^2}$$

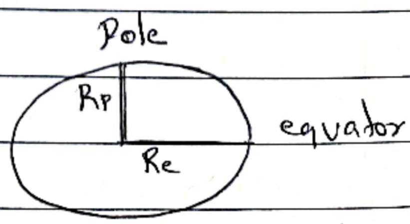


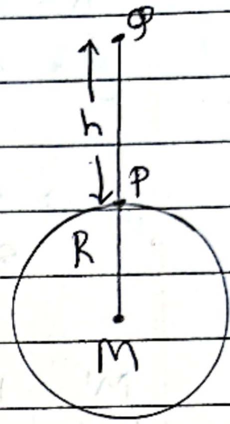
Fig: Variation of 'g' due to the shape of earth.

h/w

Date: / /

Since, Polar radius (R_p) is smaller than equatorial radius (R_e). So, acceleration due to gravity at pole is greater than that of a equator. i.e $g_p > g_e$ when $R_e > R_p$

2. Due to height (Altitude)



Let 'M' be the mass of earth and 'R' be its radius. Also, let P be any point on the earth surface. So acceleration due to gravity at point 'P' is given by;

$$g = \frac{GM}{R^2} \quad \text{—— (i)}$$

Again, let Q be any point at height 'h' from the surface of the earth. So, acceleration due to gravity at point Q is given by:

$$g' = \frac{GM}{(R+h)^2} \quad \text{—— (ii)}$$

Dividing eq. (ii) by (i)

$$\frac{g'}{g} = \frac{R^2}{\left[R\left(1+\frac{h}{R}\right)\right]^2}$$

$$\text{or, } \frac{g'}{g} = \frac{1}{\left(1+h/R\right)^2}$$

Using binomial expansion and neglecting higher power terms,

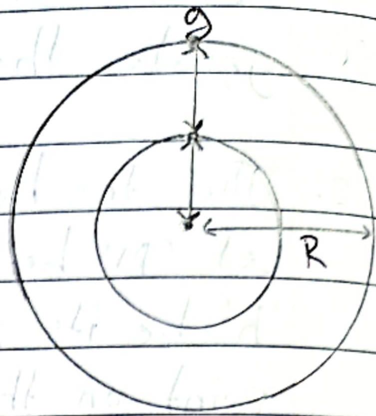
$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

$$\text{or, } g' = g\left(1 - \frac{2h}{R}\right) \quad \text{—— (iii)}$$

Equation (iii) shows that acceleration due to gravity goes on decreasing when height increases.

3. Variation of 'g' due to depth

Let, m = mass of a body at 'x' depth from earth's sphere.



M = mass of earth

R = Radius of earth

g = acceleration due to gravity on surface of earth.

g' = acceleration due to gravity on ~~surface of earth~~ at 'x' depth from earth surface.

M' = mass of earth of radius (R−x)

Now,

At surface

$$g = \frac{GM}{R^2} \quad \text{——— (i)}$$

$$g' = \frac{GM'}{(R-x)^2} \quad \text{——— (ii)}$$

Now,

$$\frac{g'}{g} = \frac{R^2}{(R-x)^2} \cdot \frac{M'}{M}$$

$$= \frac{R^2}{(R-x)^2} \cdot \frac{\rho \frac{4\pi}{3}(R-x)^3}{\rho \frac{4\pi}{3} R^3}$$

or, $$\frac{g'}{g} = \frac{R-x}{R}$$

or, $$g' = g\left(1 - \frac{x}{R}\right)$$

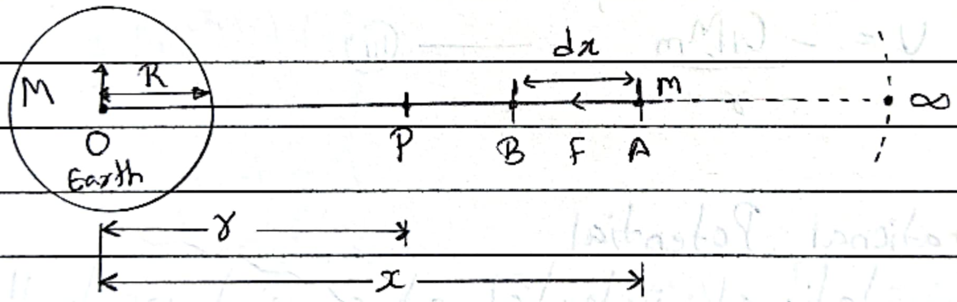
$$g' < g$$

Gravitational Field
The space around the earth where a body experiences its force of attraction is called its field. It is represented by F.

Gravitational Strength
The gravitational force experienced per unit mass of a body in its field is called gravitational field strength.

$$E = \frac{F}{m} = \frac{GMm}{R^2 \cdot m} = \frac{GM}{R^2} = g = \text{acc. due to gravity}$$

Gravitational Potential Energy
Gravitational Potential energy at a point is defined as "amount of work done on bringing mass 'm' of any body from infinity to that point.



Let M = mass of earth
R = radius of earth

Let P be a point at a distance 'r' from the centre O of the earth.

m = mass of body at infinity

Now,

$$F = \frac{GMm}{x^2} \quad \text{——} \ (\text{I})$$

Work done dW by this force on moving the body in small displacement dx from Point A to B is,

$$dW = \frac{GMm}{x^2}\,dx \quad \text{——} \ (\text{II}) \quad \text{Where, } \frac{GMm}{x^2} = F$$

Now, Integrating equation ② from ∞ to r. Therefore,

$$W = \int_{\infty}^{r} dW$$

$$= \int_{\infty}^{r} \frac{GMm}{x^2} dx$$

$$= GMm \int_{\infty}^{r} x^{-2} . dx$$

$$= GMm \left[\frac{-1}{x} \right]_{\infty}^{r}$$

$$= - GMm \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$\therefore W = - \frac{GMm}{r}$$

This work done is equal to gravitational potential energy 'U' of mass m at point P.

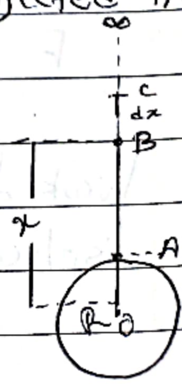
$$\therefore U = - \frac{GMm}{r} \quad \text{——— (iii)}$$

Gravitational Potential

The gravitational potential at a point inside the gravitational field is defined as the amount of work done in bringing a unit mass from infinity to that point without acceleration. It is denoted by V. It's SI unit is $J kg^{-1}$.

Escape Velocity

It is the velocity of a body with which if projected from earth surface goes into space.



We know,

$$F = \frac{GMm}{x^2}$$

$$= GMm x^{-2}$$

Also, $dW = \cdot F dx$

or, $dW = GMmx^{-2}\,dx$

Integrating

$$W = \int_R^{\infty} GMmx^{-2}\,dx$$

$$= GMm\left[\frac{x^{-2+1}}{-2+1}\right]_R^{\infty}$$

$$= \left[-\frac{1}{x}\right]_R^{\infty} GMm$$

$$= -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$W_{A\infty} = \frac{GMm}{R} \quad \text{——— (i)}$$

$$K.E = \frac{1}{2}MV_e^2 \quad \text{——— (ii)}$$

But, $K.E = W_{\infty A}$

or, $\frac{1}{2}MV_e^2 = \frac{GMm}{R}$

or, $V_e^2 = \frac{2GM}{R}$

or, $V_e = \sqrt{2GM/R}$

or, $V_e = \sqrt{2gR}$

Note: $\sqrt{\frac{2GM}{R} \cdot \frac{R}{R}}$

$= \sqrt{2\left(\frac{GM}{R^2}\right)R} = \sqrt{2gR}$

Orbital Velocity of Satellite

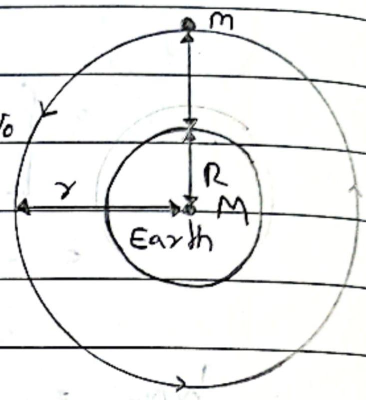
The velocity required to keep a satellite into its orbit is called the orbital velocity of the satellite. It is denoted by V_o.

Let m = mass of satellite

M = mass of planet

r = radius of orbit of satellite

V_o = Orbital Velocity of satellite



Now, Centripetal force on satellite

$$F = \frac{MV_o^2}{r} \quad \text{—— (i)}$$

Gravitational force between planet and satellite

$$F = \frac{GMm}{r^2} \quad \text{—— (ii)}$$

From (i) and (ii)

$$V_o = \sqrt{\frac{GM}{r}}$$

or, $$V_o = \sqrt{\frac{GM}{R+h}} \qquad [\because r = R+h]$$

or, $$V_o = \sqrt{\frac{gR^2}{R+h}} \quad \text{—— (iii)} \qquad \left[\because g = \frac{GM}{R^2}\right]$$

Now,

When satellite is ~~cot~~ close to earth surface, then $R+h \to R$ i.e. $h \to 0$

from equation (iii), we get

$$V_o = \sqrt{\frac{gR^2}{R+0}}$$

$$\therefore V_o = \sqrt{gR} = 7.2 \text{ km/s}$$

Time Period of Satellite (T)

It is the time taken by the satellite to make one complete revolution around the Earth.

$$\therefore T = \frac{\text{Circumference of orbit}}{\text{orbital Velocity}}$$

$$\text{or, } T = \frac{2\pi r}{V_0}$$

$$\text{or, } T = 2\pi r\sqrt{\frac{r}{GM}} \qquad \left[\because V_0 = \sqrt{\frac{GM}{r}}\right]$$

$$\text{or, } T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$\text{or, } T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \qquad \left[\because r = R+h \ \& \ GM = gR^2\right]$$

When $h = 0$

$$T = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ minutes}$$

Height of Satellite (h)

We have,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$\text{or, } \frac{4\pi(R+h)^3}{GM} = \cancel{T^2}\ \cancel{4\pi^2}\ T^2$$

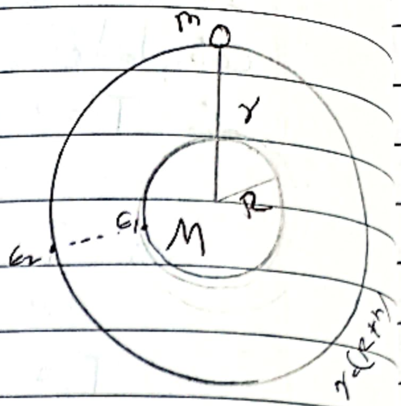
$$\text{or, } (R+h)^{3/\times 1/3} = \left[\frac{T^2 GM.R^2}{4\pi^2 R^2}\right]^{1/3} = \left[\frac{T^2 g R^2}{4\pi^2}\right]^{1/3}$$

$$\text{or, } (R+h) = \left(\frac{T^2 g R^2}{4\pi^2}\right)^{1/3}$$

$$\text{or, } h = \left(\frac{T^2 g R^2}{4\pi^2}\right)^{1/3} - R$$

Energy of Satellite

Let us consider a satellite of mass 'm' is revolving around the earth of mass 'M' and radius 'R' on it's orbit of radius 'r'



Now,

Gravitational force ~~of earth~~ between earth and satellite is given by;

$$F = \frac{GMm}{r^2} \quad \text{—— (i)}$$

The gravitational force provides the necessary centripetal force

i.e. $F = F_r$

$$\text{or, } \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\text{or, } mv^2 = \frac{GMm}{r}$$

$$\text{or, } \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$\therefore K.E = \frac{GMm}{2r} \quad \text{—— (ii)}$$

Again, P.E at a point on it's orbit is given by

$$P.E = -\frac{GMm}{r} \quad \text{—— (iii)}$$

Then, Total energy of satellite is given by;

$$E = K.E + P.E$$

$$= \frac{GMm}{2r} - \frac{GMm}{r}$$

$$= \frac{GMm - 2GMm}{2r}$$

$$\therefore E = -\frac{GMm}{2r}$$

[$\because$ Here, -ve sign shows that the satellite is attracted towards earth.]

$\Delta E = GMm\left(\frac{1}{R} - \frac{1}{2(R+h)}\right)$, ΔE = energy req. to launch satellite

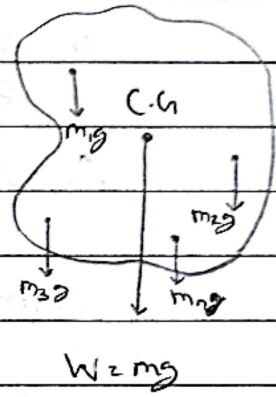
Geostationary Satellite

The satellite which seems to be stationary when viewed from a point on the earth's surface is called geostationary satellite.

Centre of Gravity (C.G)

The centre of gravity of a body is defined as a point at which the algebraic sum of the moments of weights of all the particles constituting the body is zero.

Let us consider a body of Mass 'M' consist of n particles of masses $m_1, m_2, m_3, \dots m_n$ resp. The force due to gravity on these masses are $m_1g, m_2g, m_3g, \dots m_ng$ resp. all acting vertically downwards. Then,



$$W = m_1g + m_2g + m_3g \dots + m_ng$$

$$\therefore W = Mg$$

Where ~~ea~~ M is the sum of all ~~forces~~ masses

Centre of Mass (C.M)

A point with respect to a body at which the whole mass of the body is supposed to be concentrated, is called centre of mass.

Let us consider a system of n particles of masses $m_1, m_2, m_3 \dots m_n$ with coordinate position $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots (x_n, y_n)$, respectively.

Let (x, y) be the coordinate of C.M.

The various ~~force~~ masses force acting are $F_1 = m_1a_1$, $f_2 = m_2a_2, \dots f_n = m_na_n$

Where $a_1, a_2, a_3 \dots a_n$ are acceleration produced on masses $m_1, m_2, m_3 \dots m_n$ resp.

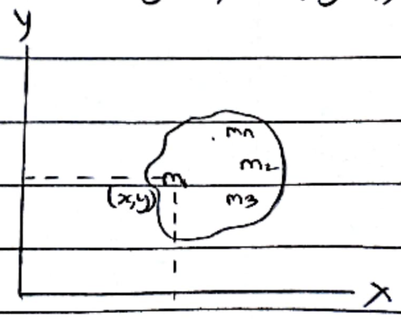


Fig: coordinate of $m_1, m_2, \dots m_n$ particles of x-y plane

Total force acting on a body,

$F_1 + f_2 + f_3 + \dots + f_n$

$= m_1a_1 + m_2a_2 + m_3a_3 + \dots + m_na_n$

or, $\Sigma f = m_1\frac{d^2x_1}{dt^2} + m_2\frac{d^2x_2}{dt^2} + m_3\frac{d^2x_3}{dt^3} + \dots + m_n\frac{d^2x_n}{dt^2}$

or, $\Sigma f = \frac{d^2}{dt^2}(m_1x_1 + m_2x_2 + \dots + m_nx_n)$

Total mass $\Sigma m = m_1 + m_2 + \dots + m_n$

Now, $\frac{\Sigma f}{\Sigma m} = \frac{d^2}{dt^2}\left(\frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{\Sigma m}\right)$

or, $\frac{\Sigma f}{\Sigma m} = \frac{d^2}{dt^2}\left(\frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1 + m_2 + \dots + m_n}\right)$

$\therefore x = \frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1 + m_2 + \dots m_n}$

Similarily,

$y = \frac{m_1y_1 + m_2y_2 + \dots + m_ny_n}{m_1 + m_2 + m_3 \dots + m_n}$

So, $(x,y) = \left(\frac{m_1x_1 + m_2x_2 + m_nx_n}{m_1 + m_2 + \dots + m_n}, \frac{m_1y_1 + m_2y_2 + \dots m_ny_n}{m_1 + m_2 + m_3 + \dots + m_n}\right)$

Condition for a body in stable Equilibrium

i. The C.G of the body should lie as low as possible

ii. The base of the body should be as large as possible.

iii. C.G should lie with in the base of the body on displaced position.