H/W

Chapter - 1

Function

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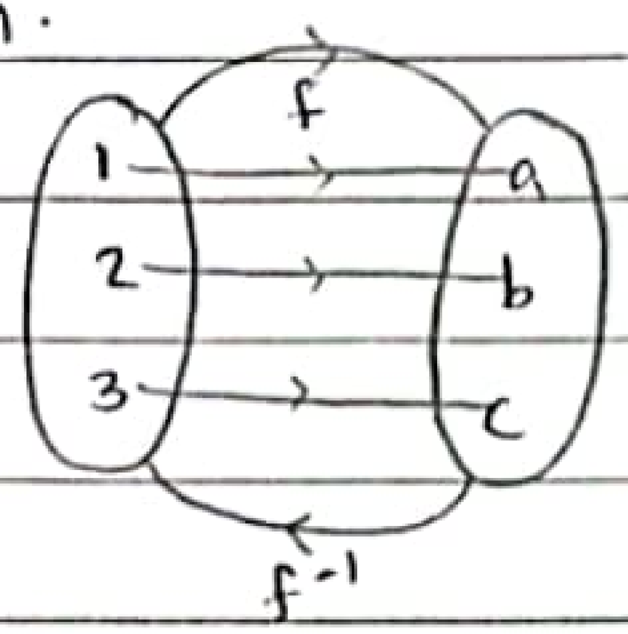
Exercise - 1.1

General section

1. Define inverse of a function with an arrow diagram.

Ans: Inverse of a function is a function which can reverse into another function.



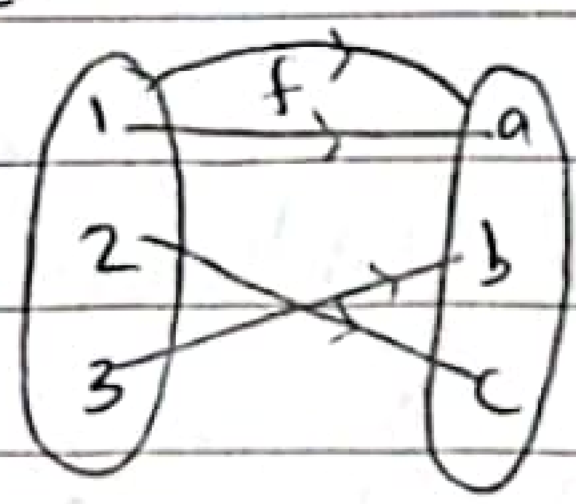
2a. Write the range and inverse of a function $f = \{(2, \frac{1}{2}), (3, \frac{1}{3}), (4, \frac{1}{4})\}$

Solution:

Range = $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$

$f^{-1} = \{(\frac{1}{2}, 2), (\frac{1}{3}, 3), (\frac{1}{4}, 4)\}$

b. Find the inverse of a function f from the arrow diagram given aside.



Solution:

$f^{-1} = \{(a, 1) (b, 3) (c, 2)\}$

3. Find the inverse of the function.

a. $f(x) = 5x - 1$

or, $y = 5x - 1$

Interchanging the role of x and y

$x = 5y - 1$

or, $\frac{x+1}{5} = y$

$\therefore f^{-1}(x) = \frac{x+1}{5}$

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b. $g : x \rightarrow 5x + 2$

solution:

$g(x) = 5x + 2$

or, $y = 5x + 2$

Interchanging the role of x and y

$x = 5y + 2$

or, $\frac{x-2}{5} = y$

$\therefore g^{-1}(x) = \frac{x-2}{5}$

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c. $f(x) = \frac{3x+7}{2}$

Solution:

$f(x) = y = \frac{3x+7}{2}$

Interchanging role of x and y.

$x = \frac{3y+7}{2}$

or, $\frac{2x-7}{3} = y$

$\therefore f^{-1}(x) = \frac{2x-7}{3}$

d. $g(x) = \frac{2x+5}{2}, x \in R$

Solution:

$g(x) = \frac{2x+5}{2}$

or, $y = \frac{2x+5}{2}$

Interchanging role of x and y

$x = \frac{2y+5}{2}$

or, $\frac{2x-5}{2} = y$

$\therefore g^{-1}(x) = \frac{2x-5}{2}$

e. $h(x) = \frac{x-5}{2x}$

or, $h(x) = y = \frac{x-5}{2x}$

Interchanging role of x and y

or, $x = \frac{y-5}{2y}$

or, $2xy = y-5$

or, $5 = y-2xy$

or, $5 = y(1-2x)$

or, $\frac{5}{1-2x} = y$

$\therefore h^{-1}(x) = \frac{5}{1-2x}$

f. $f = \left\{\left(x, \frac{5x-3}{2x+1}\right), x \in R\right\}$

Solution:

$$f(x) = \frac{5x-3}{2x+1}$$

or, $y = \frac{5x-3}{2x+1}$

Interchanging the role of x and y.

$$x = \frac{5y-3}{2y+1}$$

or, $2xy + x \cancel{+3} = 5y - 3$

or, $\cancel{x(2y+1) = 5y-3}$

or, $x + 3 = 5y - 2xy$

or, $y = \frac{x+3}{5-2x}$

$\therefore f^{-1}(x) = \frac{x+3}{5-2x}$

5a. If $f(x) = \frac{3x-4}{5}$, find $f^{-1}(x)$ and $f^{-1}(3)$

Solution:

$\cancel{f(x)}$ $f(x) = y = \frac{3x-4}{5}$

Interchanging the role of x and y

$$x = \frac{3y-4}{5}$$

or, $\frac{5x+4}{3} = y$

$\therefore f^{-1}(x) = \frac{5x+4}{3}$

Also

$$f^{-1}(3) = \frac{5\times3+4}{3}$$

$$= 19/3$$

b. If $g(x) = \frac{3x-2}{4}$, find $g^{-1}(-2)$

Solution:

$$g(x) = y = \frac{3x-2}{4}$$

Interchanging the role of x and y

$$x = \frac{3y-2}{4}$$

or $\frac{4x+2}{3} = y$

or $g^{-1}(x) = \frac{4x+2}{3}$

or $g^{-1}(-2) = \frac{4\times(-2)+2}{3}$

$$= \frac{-8+2}{3}$$

$$= -2 \#$$

c. If $h(x) = \frac{1}{5x+4}$, find $h^{-1}\left(\frac{-2}{3}\right)$

Solution:

$$h(x) = y = \frac{1}{5x+4}$$

or Interchanging role of x and y.

$$x = \frac{1}{5y+4}$$

or $5xy + 4x = 1$

or ~~$4x = 1 - 5xy$~~

or $x(5y + 4) = 1$

or $5y + 4 = \frac{1}{x}$

or $5y = \frac{1}{x} - 4$

or $y = \frac{1 - 4x}{5x}$

or $h^{-1}(x) = \frac{1 - 4x}{5x}$

~~or, $h^{-1} = +$~~

or $h^{-1}\left(\frac{-2}{3}\right) = \frac{1 - 4 \times \left(\frac{-2}{3}\right)}{5 \times \left(-\frac{2}{3}\right)}$

$= \frac{3 + 8}{3} \times \frac{3}{-10}$

$= \cancel{\frac{8}{12}} \; \frac{-11}{10}$

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6. Define composite function with example.

Ans: Composite Function is a function whose value are found from two given functions by applying one function to an independent variable and

A composite function is generally a function that is written inside another function. for example: $f[g(x)]$ is the composite function of $f(x)$ and $g(x)$.

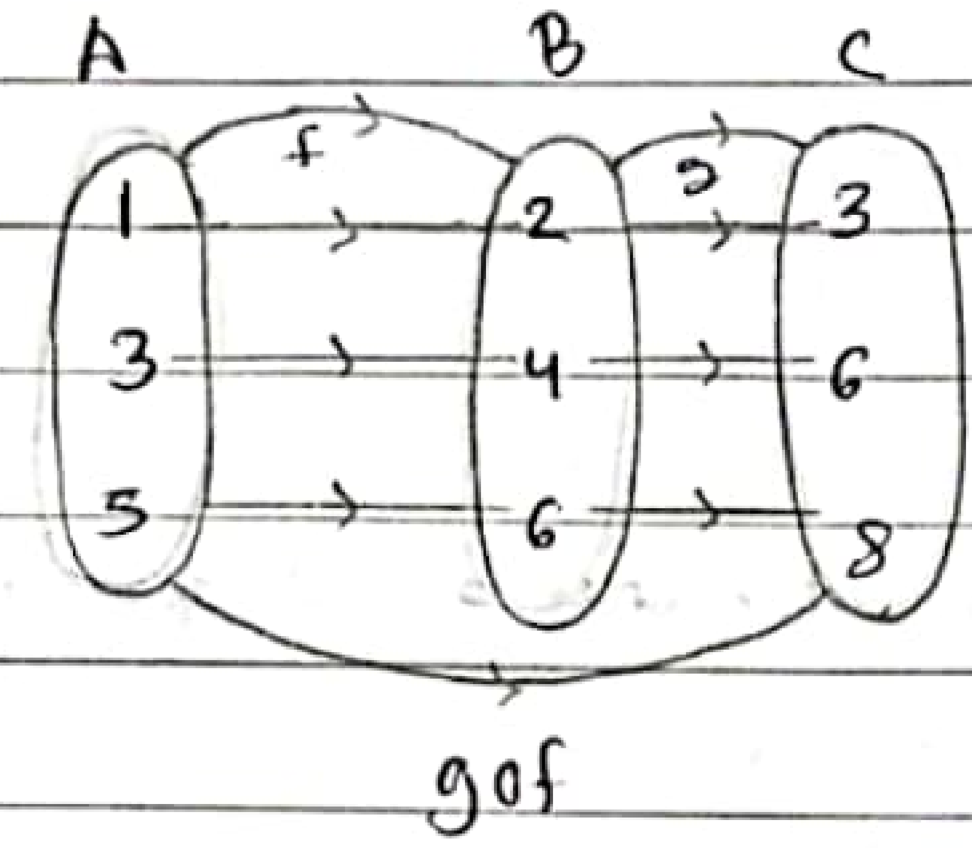
7.a. If $f = \{(1,2), (3,4), (5,6)\}$ and $g = \{(2,3), (4,6), (6,8)\}$ find the composite of gof. Does fog exist.

Solution:

Here, $f = \{(1,2), (3,4), (5,6)\}$

$g = \{(2,3), (4,6), (6,8)\}$

Now,



Also,

$gof = \{(1,3), (3,6) (5,8)$

Yes, fog exist.

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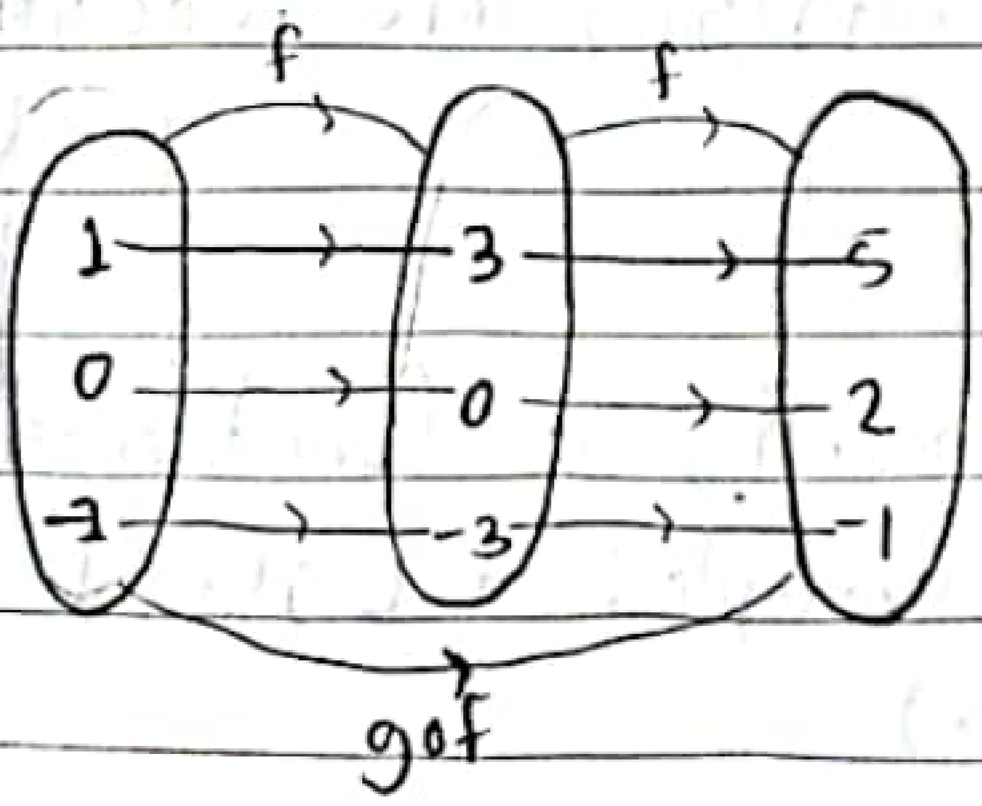
b. If $f = \{(1,3), (0,0), (-1,-3)\}$ and $g = \{(0,2), (-3,-1), (3,5)\}$ then find gof in an arrow diagram and also present it in an order pair.

Solution:

Here, $f = \{(1,3), (0,0), (-1,-3)\}$

$g = \{(0,2), ~~(-3,-1)~~ (-3,-1), (3,5)\}$

Now,



Here,

$gof = \{(1,5), (0,2), (-1,-1)\}$

c. If $g = \{(1,2),(2,3),(3,4)\}$ and $h = \{(2,3),(3,4),(4,5)\}$ then calculate $hog(1)$ and $hog(2)$.

Solution:

Here, $g = \{(1,2)(2,3)(3,4)\}$

$h = \{(2,3),(3,4),(4,5)\}$

Now,

$hog = \{(1,3),(2,4),(3,5)\}$

Here, $hog(1) = 3$

$hog(2) = 4$

8.a. If $f(x) = x$ and $g(x) = x+1$ then find

i. $fog(x)$

Now,

$fog(x) = f\{g(x)\}$

$= f(x+1)$

$= x+1$

ii. $(gof)(x)$

Now,

$(gof)(x) = g\{f(x)\}$

$= g(x)$

$= x+1$

iii. $(fof)(x)$

Now,

$(fof)(x) = f\{f(x)\}$

$= f(x)$

$= x$

iv. $(gog)(x)$

Now,

$(gog)(x) = g\{g(x)\}$

$= g(x+1)$

$= g(x+1)+1$

$= g(x+2)$

$= x+2$

v. $(fog)(1)$

Now,

$(fog) = \cancel{f\{g(y)\}}\ f\{g(x)\}$

$= f(x+1)$

$= x+1$

or, $fog(1) = 1+1 = 2$

vi. $(gof)(-1/2)$

Now, $gof = \cancel{g\{g(x)}\ g\{\cancel{g}\,f(x)\}$

$= g(x)$

$= x+1$

or, $gof(-1/2) = -\frac{1}{2}+1 = \frac{1}{2}$

b. If $f(x) = x-5$ and $g(x) = 5-x$, check if $(fog)(x) = (gof)(x)$.

Solution:

$$(fog)(x) = f\{g(x)\}$$
$$= f\{\cancel{57}\ 5-x\}$$
$$= \cancel{f}\{5-x \cancel{+10} -5\}$$
$$= -x$$

Also,

$$(gof)(x) = g\{f(x)\}$$
$$= g\ (x-5)$$
$$= 5-(x-5)$$
$$= \cancel{\alpha}\ 5-x+5$$
$$= 10-x$$

9a. If $g:x = 5x-2$ and $h:x = \cancel{5x}\ \frac{5+x}{2}$, find $(hog)^{-1}(x)$

Solution:

Here, $g(x) = 5x-2$

$h(x) = \cancel{5x}\ \frac{5+x}{2}$

Now,

$$hog(x) = h\{g(x)\}$$
$$= h\{5x-2\}$$
$$= \frac{5+5x-2}{2}$$

$$\therefore hog(x) = \frac{5x+3}{2}$$

Also,

Let $hog(x) = y = \frac{5x+3}{2}$

Interchanging the role of x and y.

$$x = \frac{5y+3}{2}$$

or $\frac{2x-3}{5} = y$

$\therefore (hog)^{-1}(x) = \frac{2x-3}{5}$

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6. If $f(x) = \frac{x+3}{2}$ and $g(x) = 3x+4$, find $fof^{-1}(x)$ and $fog^{-1}(x)$. Also find out $f^{-1}og^{-1}(1)$.

Solution:

Given, $f(x) = \frac{x+3}{2}$

$g(x) = 3x+4$

Now,

$$fof(x) = f\{f(x)\}$$
$$= f\left(\frac{x+3}{2}\right)$$
$$= \frac{\frac{x+3}{2}+3}{2}$$
$$= \frac{x+3+6}{2} \times \frac{1}{2}$$
$$= \frac{x+9}{4}$$

Also,

$fof(x) = y = \frac{x+9}{4}$

Interchanging role of x and y

$x = \frac{y+9}{4}$

or $\frac{4x-9}{1} = y$

$\therefore fof^{-1}(x) = 4x-9$

Also,

$$Fog(x) = f\{g(x)\}$$
$$= f\{3x+4\}$$
$$= \frac{3x+4+3}{2}$$
$$= \frac{3x+7}{2}$$

Again, let $fog(x) = y = \frac{3x+7}{2}$

Interchanging role of x and y

$$x = \frac{3y+7}{2}$$

or, $\frac{2x-7}{3} = y$

$$\therefore fog^{-1}(x) = \frac{2x-7}{3}$$

At last,

$\cancel{f^{-1}og^{-1}} = f^{-1}(x) = y = \frac{x+3}{2}$

or, $2x-3 = y$

$\therefore f^{-1}(x) = 2x-3$

Also,

$g(x) = y = 3x+4$

or, $x = 3y+4$

or, $\frac{x-4}{3} = y$

$\therefore g^{-1}(x) = \frac{x-4}{3}$

Now,

$f^{-1}og^{-1}(x) = f^{-1}\{g^{-1}(x)\}$

$= f^{-1}\left\{\frac{x-4}{3}\right\}$

$= 2\left(\frac{x-4}{3}\right) - 3$

$= \frac{2x-8-9}{3} = \frac{2x-17}{3}$

Also,

$f^{-1}og^{-1}(1) = \frac{2\times1-17}{3}$

$\therefore f^{-1}og^{-1}(1) = -5$

Creative Section:

10a. If $f(x) = 2x+3$ and $f^{-1}(7) = k$, then find out value of k.

Solution:

Here, $f(x) = 2x+3$

$y = 2x+3$

or, $x = 2y+3$

or, $\frac{x-3}{2} = y$

$\therefore f^{-1}(x) = \frac{x-3}{2}$

or, $f^{-1}(7) = \frac{7-3}{2}$

or, $k = \frac{4}{2}$

$\therefore k = 2$

b. If $f: x \rightarrow 4x+3$ and if $f(x) = f^{-1}(x)$, find the value of x

Solution:

$f(x) = 4x+3$

or, $y = 4x+3$

or, $x = 4y+3$

or, $\frac{x-3}{4} = y$

$\therefore f^{-1}(x) = \frac{x-3}{4}$

we know, $f(x) = f^{-1}(x)$

or, $4x+3 = \frac{x-3}{4}$

or, $16x+12 = x-3$

or, $15x = -15$

$\therefore x = -1$

c. If $g = \{(x, 5x-k)\}$ and if $g^{-1}(2) = \frac{4}{5}$, find the value of k.

Solution:

Given, $g(x) = 5x - k$

or $y = 5x - k$

or $x = 5y - k$

or $\frac{x+k}{5} = y$

$\therefore g^{-1}(x) = \frac{x+k}{5}$

Also, $g^{-1}(2) = \frac{2+k}{5}$

or $\frac{4}{5} = \frac{2+k}{5}$

or $4 - 2 = k$

$\therefore k = 2$

11a. If $f(x) = \frac{x-2}{2x+1}$ and $g(x) = \frac{1}{x}$ and if $f^{-1}(x) = gof(x)$, find the value of x.

Solution:

Let $f(x) = y = \frac{x-2}{2x+1}$

or $x = \frac{y-2}{2y+1}$

or $2xy + x = y - 2$

or $x + 2 = y - 2xy$

or $x + 2 = y(1 - 2x)$

or $\frac{x+2}{1-2x} = y$

$\therefore f^{-1}(x) = \frac{x+2}{}$

Again,

$$gof = g\{f(x)\}$$
$$= g\left\{\frac{x-2}{2x+1}\right\}$$
$$= \frac{1}{\frac{x-2}{2x+1}}$$
$$= \frac{2x+1}{x-2}$$

$$\therefore gof = \frac{2x+1}{x-2}$$

By question:

$$f^{-1}(x) = gof(x)$$

or $\dfrac{x+2}{1-2x} = \dfrac{2x+1}{x-2}$

or, $(x+2)(x-2) = (1-2x)(2x+1)$

or $x^2-4 = 2x+1-4x^2-2x$

or $x^2+4x^2 = 1+4+2x-2x$

or $5x^2 = 5$

or $x^2 = \dfrac{5}{5}$

or $x = \sqrt{1}$

$\therefore x = \pm 1$

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a. If $f(x) = 4x-7$ and $g(x) = 3x-5$. If ~~fg~~ $fog^{-1}(x) = 15$, find the value of x.

Solution:

$$fog(x) = f\{g(x)\}$$
$$= f\{3x-5\}$$
$$= 4(3x-5)-7$$
$$= 12x-20-7 = 12x-27$$

Let, $fog(x) = y = 12x - 27$

or $x = 12y - 27$

or $\frac{x+27}{12} = y$

$\therefore fog^{-1}(x) = \frac{x+27}{12}$

~~Also, $fog^{-1}($~~

or $15 = \frac{x+27}{12}$

or $180 - 27 = x$

$\therefore x = 153$

12a. If the functions $f(x) = \frac{x}{x-2}$ and $g(x) = bx - 2$. Also, $gof(4) = -8$, find the values of $f^{-1}(-2)$ and b.

Solution:

$gof(x) = g\{f(x)\}$

$= g\left\{\frac{x}{x-2}\right\}$

$= \frac{b.x}{x-2} - 2$

$= \frac{bx - 2x + 4}{x-2}$

we know,

$gof(4) = \frac{b.4 - 2.4 + 4}{4-2}$

or $-8 = \frac{4b - 4}{2}$

or

or $\frac{-16+4}{4} = b$

$\therefore b = -3$

Again,

Let $y = \frac{x}{x-2}$

Interchanging position of x and y

$$x = \frac{y}{y-2}$$

or, $xy - 2x = y$

or, $xy - y = 2x$

or, $y(x-1) = 2x$

or, $y = \frac{2x}{x-1}$

$\therefore$ ~~f-1~~ $f^{-1}(x) = \frac{2x}{x-1}$

Also, $f^{-1}(2) = \frac{2\times 2}{2-1}$

$= \frac{4}{1} = 4$

---

b. If $f(x) = 3x + a$ and if $fof(6) = 10$, find the value of a and $f^{-1}(4)$.

Solution:

$fof = \cancel{f\{g(x}\ f\{f(x)\}$

$= f\{3x+a\}$

$= 3(3x+a) + a$

$= 9x + 3a + a$

$= 9x + 4a$

Also,

$fof(6) = 9\times 6 + 4a$

or, $10 = 54 + 4a$

$\therefore a = -11$

Also,

Let $y = 3x + a$

Interchanging the position of x and y.

$x = 3y + a$

or $\frac{x-a}{3} = y$

$\therefore f^{-1}(x) = \frac{x-a}{3}$

or $f^{-1}(4) = \frac{4-(-11)}{3}$

or $f^{-1}(4) = \frac{4+11}{3}$

$\therefore f^{-1}(4) = 5$

---

c. If $f(x) = (2x+k)$ and $fof(4) = 10$, find the value of k.

Solution:

$fof(x) = f\{f(x)\}$

$= f\{2x+k\}$

$= 2\{2x+k\} + k$

$= 4x + 2k + k$

$= 4x + 3k$

Also,

$fof(4) = 4\times4 + 3k$

or $10 = 16 + 3k$

or $\frac{10-16}{3} = k$

$\therefore k = -2$

13a. If $f(x) = \frac{2x+5}{x+2}$; $x \neq -2$, find $f^{-1}(x)$ and show that $fof^{-1}(x)$ is an identity function.

12.

Solution:

Let $f(x) = y = \frac{2x+5}{x+2}$

Interchanging x and y.

or, $x = \frac{2y+5}{y+2}$

or, $xy + 2x = 2y + 5$

or, $2x - 5 = 2y - xy$

or, $2x - 5 = y(2-x)$

or, $y = \frac{2x-5}{2-x}$

$\therefore f^{-1}(x) = \frac{2x-5}{2-x}$

Also,

$fof^{-1} = f\{(f^{-1}x)\}$

$= f\left\{\frac{2x-5}{2-x}\right\}$

$= \dfrac{2 \times \frac{2x-5}{2-x} + 5}{\frac{2x-5}{2-x} + 2}$

$= \frac{4x - 10 + 10 - 5x}{(2-x)} \times \frac{(2-x)}{2x-5+4-2x}$

$= \frac{4x-5x}{-5+4}$

$= \frac{-x}{-1}$

$= x$

$\therefore fof^{-1}(x)$ is an identity matrix.

b. If the function $f(x) = \frac{2x+1}{4}$, prove that $f^{-1} o f(x)$ is an identity function.

Solution:

Let $f(x) = y = \frac{2x+1}{4}$

Interchanging x and y

$$x = \frac{2y+1}{4}$$

or $\frac{4x-1}{2} = y$

$$\therefore f^{-1}(x) = \frac{4x-1}{2}$$

Again,

$$f^{-1} o f(x) = f^{-1}\{f(x)\}$$

$$= f^{-1}\left\{\frac{2x+1}{4}\right\}$$

$$= \frac{4 \times \frac{(2x+1)}{4} - 1}{2}$$

$$= \frac{8x+4-4}{4} \times \frac{1}{2}$$

$$= \frac{8x}{8} = x$$

$\therefore f^{-1} o f(x)$ is an identity ~~matrix~~ function.

14a. If $f(x+3) = x+6$, find $f^{-1}(x)$

Solution:

Here,

$$f(x+3) = x+6$$

or, $f(x+3) = (x+3)+6-3$

or, $f(x+3) = (x+3)+3$

This can be written as;

$f(x) = x+3$

For inverse

Let $f(x) = y = x+3$

or $x = y+3$

or $x-3 = y$

$\therefore f^{-1}(x) = x-3$

---

b. If ~~f(x)~~ $f(2x+5) = 4x+13$, find $f^{-1}(x)$.

Solution:

Here, $f(2x+5) = 4x+13$

or $f(2x+5) = (2x+5)\times 2+3$

This can be written as;

$f(x) = 2x+3$

Let $f(x) = 2x+3 = y$

Interchanging x and y

$x = 2y+3$

or $\frac{x-3}{2} = y$

$\therefore f^{-1} = \frac{x-3}{2}$

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c. If $f(3x+4) = 5x+8$, find $f^{-1}(x)$.

Solution:

Here, $f(3x+4) = 5x+8$

or $f(3x+4) = (3x+4)\frac{5}{3} + \frac{4}{3}$

This can be written as;

$f(x) = \frac{5x+4}{3}$

Let $f(x) = y = \frac{5x+4}{3}$

Interchanging x and y.

$$x = \frac{5y+4}{3}$$

or $\frac{3x-4}{5} = y$

$\therefore f^{-1}(x) = \frac{3x-4}{5}$

15a. If $f^{-1}(x) = \frac{x-1}{3}$, find the function $f(x)$

Let $f^{-1}(x) = y = \frac{x-1}{3}$

Interchanging x and y

$$x = \frac{y-1}{3}$$

or $3x+1 = y$

$\therefore f(x) = 3x+1$

---

b. If $f^{-1}(x) = x-2$, find $f(x)$ and $f(3)$

Solution:

Let $f^{-1}(x) = y = x-2$

Interchanging x and y

$$x = y-2$$

or $x+2 = y$

$\therefore f(x) = x+2$

Also,

$f(3) = 3+2$

$= 5$

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16a. If $(f o g)(x) = \frac{7x-1}{3}$ and $f(x) = 3x+5$, find $g(x)$, where $g(x)$ is linear.

Solution:

$(f o g)(x) = \frac{7x-1}{3}$

or, $f\{g(x)\} = \frac{7x-1}{3}$

or, $f(y) = \frac{7x-1}{3}$

or, $3y+5 = \frac{7x-1}{3}$

or, $9y+15 = 7x-1$

or, $y = \frac{7x-16}{9}$

$\therefore g(x) = \frac{7x-16}{9}$

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b. If $g(x) = 2x$ and $(f o g)(x) = 6x-2$, find $f(x)$, where $f(x)$ is a linear function.

Solution:

$(f o g)(x) = 6x-2$

or, $f\{g(x)\} = 6x-2$

or, $f\{2x\} = 6x-2$

or, $f(2y) = 6x-2$

or, $2y = 6x-2$

or, $y = \frac{6x-2}{2}$

or, $y = 3x-1$

$\therefore f(x) = 3x-1$

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4. If $f(x) = x^2 - 5$, whose range is 4. find the show f(x) in an arrow diagram and find the inverse of f(x) if possible.

Solution:

Here, range of $f(x) = x^2 - 5$ is 4

or, $x^2 - 5 = 4$

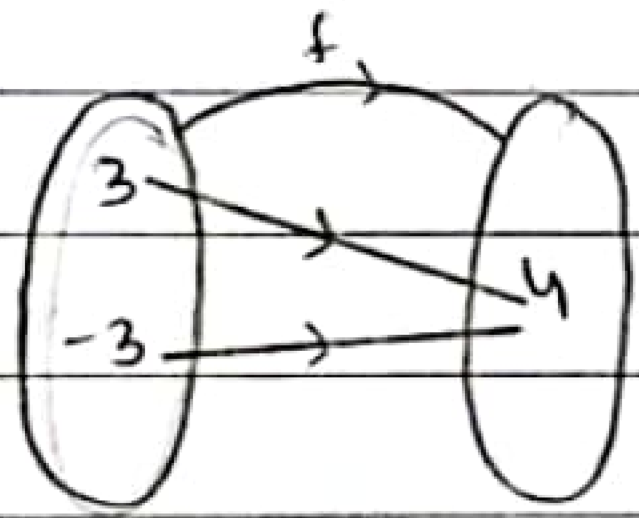
or, $x^2 = 9$

or, $x = \sqrt{9}$

$\therefore x = \pm 3$

Here, domain $= 3, -3$

Range $= 4$



Mapping diagram.

Here,

The inverse of f(x) is not possible as it does not have unique elements.