Exercise - 7.1

General section

1.a. If $\sin A = \frac{3}{5}$, find the value of $\cos 2A$

Solution:

Here, $\sin A = \frac{3}{5}$

We know,

$$\cos 2A = 1 - 2\sin^2 A$$
$$= 1 - 2\left(\frac{3}{5}\right)^2$$
$$= 1 - 2 \times \frac{9}{25}$$
$$= \frac{25 - 18}{25}$$
$$= \frac{7}{25}$$

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b. If $\cos A = \frac{3}{5}$, find the value of $\sin 2A$.

Soln,

We know, $\cos A = \frac{3}{5}$

$$\cancel{\sin 2A = \cos A = \sqrt{1 - \sin^2 A}}$$
$$\therefore \sin A = \sqrt{1 - \cos^2 A}$$
$$= \sqrt{1 - (3/4)^2}$$
$$= \sqrt{1 - \cancel{\frac{9}{16}}\ \frac{9}{25}}$$
$$= \sqrt{\cancel{\frac{7}{16}}}\ \sqrt{16/25}$$
$$= \frac{4}{5}$$

we know,

~~CosA Cos2A =~~

$$\sin 2A = 2\sin A \cdot \cos A$$

$$= 2 \times \frac{4}{5} \times \frac{3}{5}$$

$$= \frac{24}{25}$$

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c. If $\tan A = \frac{3}{4}$, find the value of $\sin 2A$ and $\cos 2A$.

Soln.,

$$\tan A = \frac{3}{4}$$

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$= \frac{2 \times \left(\frac{3}{4}\right)}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{\cancel{3}}{\cancel{2}}}{\cancel{4}+\frac{9}{16}} \qquad 2 \times \frac{3}{4} \times \frac{16}{25}$$

$$= \frac{3}{2} \times \frac{16}{25}$$

$$= \frac{3}{\cancel{2}} \times \frac{\cancel{4}2}{13} \qquad = \frac{48}{50}$$

$$= \frac{6}{13} \qquad = \frac{24}{25}$$

Again,

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$= \frac{16-9}{16} \times \frac{16}{16+9}$$

$$= \frac{7}{25}$$

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d. If $\sin A = \frac{3}{4}$, find $\sin 3A$.

Soln,

$$\sin A = \frac{3}{4}$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$= 3 \times \frac{3}{4} - 4\left(\frac{3}{4}\right)^3$$

$$= \frac{9}{4} - 4 \times \frac{27}{64}$$

$$= \frac{9}{4} - \frac{108}{64}$$

$$= \frac{9 \times 16 - 108}{64}$$

2a. If $\cos 2A = \frac{7}{8}$, show that $\sin A = \frac{1}{4}$

Soln,

we know

$$\cos 2A = \frac{7}{8}$$

$$\text{or, } 1-2\sin^2 A = \frac{7}{8}$$

$$\text{or, } 1 - \frac{7}{8} = 2\sin^2 A$$

$$\text{or, } \frac{1}{8} = 2\sin^2 A$$

$$\text{or, } \sin^2 A = \frac{1}{16}$$

$$\text{or, } \sin A = \sqrt{\frac{1}{16}}$$

$$\therefore \sin A = \frac{1}{4}$$

Proved//

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b. If $\cos 2A = \frac{11}{25}$, show that $\cos A = \frac{6}{5\sqrt{2}}$

Soln,

we know,

$$\cos 2A = \frac{11}{25}$$

$$\text{or, } \cancel{1-2\sin^2 A = 1}$$

$$\text{or, } 2\cos^2 A - 1 = \frac{11}{25}$$

$$\text{or, } 1 + \frac{11}{25} = 2\cos^2 A$$

$$\text{or, } 2\cos^2 A = 36/25$$

$$\text{or, } \cos^2 A = \frac{36}{50}$$

$$\text{or } \cos A = \sqrt{36/50}$$

$$\therefore \cos A = \frac{6}{5\sqrt{2}}$$

Proved//

3. Prove the following identities:

a. $\frac{1-\cos 2A}{\sin 2A} = \tan A$

LHS

$$\frac{1-\cos 2A}{\sin 2A}$$

$$= \frac{1-(1-2\sin^2 A)}{2\sin A \cdot \cos A}$$

$$= \frac{1-1+2\sin^2 A}{2\sin A \cdot \cos A}$$

$$= \frac{\cancel{2}\sin^{\cancel{2}} A}{\cancel{2}\cancel{\sin A}\cdot \cos A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

RHS proved//

b. $\frac{\sin 2A}{1+\cos 2A} = \tan A$

LHS

$$\frac{\sin 2A}{1+\cos 2A}$$

$$= \frac{\sin 2A}{2\cos^2 A}$$

$$= \frac{2\sin A \cdot \cos A}{2\cos^2 A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

RHS proved//

c. $\frac{4\tan A}{1-\tan^4 A} = \tan 2A + \sin 2A$

RHS

$$\tan 2A + \sin 2A$$

$$= \frac{2\tan A}{1-\tan^2 A} + \frac{2\tan A}{1+\tan^2 A}$$

$$= \frac{2\tan A(1+\tan^2 A) + 2\tan A(1-\tan^2 A)}{1-\tan^4 A}$$

$$= \frac{2\tan A + \cancel{2\tan^3 A} + 2\tan A - \cancel{2\tan^3 A}}{1-\tan^4 A}$$

$$= \frac{2\tan A + 2\tan A}{1-\tan^4 A}$$

$$= \frac{4\tan A}{1-\tan^4 A}$$

LHS proved//

d. $\cos^4 A - \sin^4 A = \cos 2A$

~~RHS~~ LHS

$\cancel{\cos 2A}$ $\cos^4 A - \sin^4 A$

$= (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$

$= 1(\cos^2 A - \sin^2 A)$

$= \cos 2A$

RHS proved//

e. $\frac{1-\tan A}{1+\tan A} = \frac{1-\sin 2A}{\cos 2A}$

LHS

$$\frac{1-\tan A}{1+\tan A}$$

$$= \frac{1-\frac{\sin A}{\cos A}}{1+\frac{\sin A}{\cos A}}$$

$\overline{\cos^2 A - \sin^2 A}$

RHS

$$\frac{1 - \sin 2A}{\cos 2A}$$

$$= \frac{\sin^2 A + \cos^2 A - \cancel{\sin 2A} \; 2\sin A \cdot \cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{(\cos A - \sin A)^{\cancel{2}}}{(\cos A + \sin A)\,\cancel{(\cos A - \sin A)}}$$

$$= \frac{\dfrac{\cos A - \sin A}{\cos A}}{\dfrac{\cos A + \sin A}{\cos A}}$$

$$= \frac{1 - \tan A}{1 + \tan A}$$

LHS proved"

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f. $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A} = 2$

LHS

$$\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A}$$

$$= \frac{\cos A \cdot \sin 3A - \sin A \cdot \cos 3A}{\sin A \cdot \cos A}$$

$$= \frac{\cos A\,(3\sin A - 4\sin^3 A) - \sin A\,(4\cos^3 A - 3\cos A)}{\sin A \cdot \cos A}$$

$$= \frac{3\sin A \cdot \cos A - 4\sin^3 A \cdot \cos A - 4\cos^3 A \cdot \sin A + 3\cos A \cdot \sin A}{\sin A \cdot \cos A}$$

$$= \frac{6\sin A \cdot \cos A - 4\sin A \cdot \cos A\,(\sin^2 A + \cos^2 A)}{\sin A \cdot \cos A}$$

$$= \frac{2\sin A \cdot \cos A}{\sin A \cdot \cos A} = 2$$

RHS proved"

g. $\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} = 1 - \dfrac{1}{2}\sin 2A$

LHS

$\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A}$

$= \dfrac{\cancel{(\sin A + \cos A)}(\sin^2 A - \sin A \cos A + \cos^2 A)}{\cancel{(\sin A + \cos A)}}$

$= 1 - \sin A \cdot \cos A$

$= 1 - \dfrac{2 \sin A \cdot \cos A}{2}$ [Putting 2 both side]

$= 1 - \dfrac{\sin 2A}{2}$

$= 1 - \dfrac{1}{2}\sin 2A$

RHS proved.

i. $(1 + \sin 2\theta + \cos 2\theta)^2 = 4\cos^2\theta\,(1 + \sin 2\theta)$

LHS

$(1 + \sin 2\theta + \cos 2\theta)^2$

$= (\cancel{1} + 2\sin\theta \cdot \cos\theta + 2\cos^2\theta - \cancel{1})^2$

~~$= \{2\sin\theta(\cos\theta +$~~

$= \{2\cos\theta(\sin\theta + \cos\theta)\}^2$

$= (2\cos\theta)^2(\cos\theta + \sin\theta)^2$

$= 4\cos^2\theta\,(1 + \sin 2\theta)$

j. $\frac{1 - \cos 2A + \sin 2A}{1 + \sin 2A + \cos 2A} = \tan A$

LHS.

$\frac{1 - \cos 2A + \sin 2A}{1 + \sin 2A + \cos 2A}$

$= \frac{1 - (1 - 2\sin^2 A) + \sin 2A}{\cancel{1} + \sin 2A + 2\cos^2 A - \cancel{1}}$

$= \frac{1 - 1 + 2\sin^2 A + 2\sin A \cdot \cos A}{2\sin A \cdot \cos A + 2\cos^2 A}$

$= \frac{2\sin^2 A + 2\sin A \cdot \cos A}{2\sin A \cdot \cos A + 2\cos^2 A}$

$= \frac{\cancel{2}\sin A \cancel{(\sin A + \cos A)}}{\cancel{2}\cos A \cancel{(\sin A + \cos A)}}$

$= \frac{\sin A}{\cos A}$

$= \tan A$

RHS proved.

Creative section

4. Prove the following:

a. $\frac{1}{\sin 2A} + \frac{\cos 4A}{\sin 4A} = \cot A - \operatorname{cosec} 4A$

LHS

$= \frac{1}{\sin 2A} + \frac{\cos 4A}{\sin 4A}$

$= \frac{1}{\sin 2A} + \frac{\cos 2(2A)}{\sin 2(2A)}$

$= \frac{1}{\sin 2A} + \frac{2\cos^2 2A - 1}{2\sin 2A \cdot \cos 2A}$

$= \dfrac{2\cos 2A + 2\cos^2 2A - 1}{2\sin 2A \cdot \cos 2A}$

$= \dfrac{\cancel{2\cos 2A +} 2\cos 2A\,(1 + \cos 2A) - 1}{2\sin 2A \cdot \cos 2A}$

$= \dfrac{2\cos 2A \cdot 2\cos^2 A - 1}{2\sin 2A \cdot \cos 2A}$

$= \dfrac{2\cos 2A \cdot 2\cos^2 A}{2\sin 2A \cdot \cos 2A} - \dfrac{1}{2\sin 2A \cdot \cos 2A}$

$= \dfrac{2\cos^2 A}{\sin 2A} - \dfrac{1}{\sin 4A}$

$= \dfrac{2\cos^2 A}{2\sin A \cdot \cos A} - \dfrac{1}{\sin 4A}$

$= \dfrac{\cos A}{\sin A} - \dfrac{1}{\sin 4A}$

$= \cot A - \operatorname{cosec} 4A$

RHS proved,,

b. $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ} = 4$

LHS

$\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ} = 4$

$= \dfrac{1}{\sin 10} - \dfrac{\tan 60^\circ}{\cos 10^\circ}$

$= \dfrac{1}{\sin 10} - \dfrac{\sin 60^\circ}{\cos 60^\circ \cdot \cos 10^\circ}$

$= \dfrac{\cos 60^\circ \cdot \cos 10^\circ - \sin 60^\circ \cdot \sin 10^\circ}{\sin 10 \cdot \cos 10 \cdot \cos 60^\circ}$

$= \dfrac{\cos(60^\circ + 10^\circ)}{\sin 10 \cdot \cos 10 \cdot \cos 60}$

$$= \frac{2\cos 70}{2 \sin 10 \cdot \cos 10 \cdot \cos 60^\circ}$$

$$= \cancel{\frac{2\cos 70}{\cos 2(10^\circ)}} \quad \frac{2\cos 70^\circ}{\sin 2(10^\circ) \cdot \cos 60^\circ}$$

$$= \cancel{\frac{2\cos 70^\circ}{\cos 20^\circ}} \quad \frac{2\cos 70^\circ}{\sin 20^\circ \cdot \cos 60^\circ}$$

$$= \frac{2\cos(90-20)}{\sin 20^\circ \cdot \cos 60^\circ}$$

$$= \frac{2\cancel{\sin 20^\circ}}{\cancel{\sin 20^\circ} \cdot \cos 60^\circ}$$

$$= \frac{2}{\cos 60^\circ}$$

$$= 2 \times \frac{2}{1}$$

$$= 4$$

RHS proved//

C. $\cos^6\theta + \sin^6\theta = \frac{1}{4}(1+3\cos^2 2\theta) = \frac{1}{8}(5+3\cos 4\theta$

LHS.

$\cos^6\theta + \sin^6\theta$

$= (\cos^2\theta)^3 + (\sin^2\theta)^3$

$= (\cos^2\theta + \sin^2\theta)(\cos^4\theta - \cos^2\theta \cdot \sin^2\theta + \sin^4\theta)$

$= 1 \cdot (\cos^4\theta + \sin^4\theta - \cos^2\theta \cdot \sin^2\theta)$

$= (\cos^2\theta - \sin^2\theta)^2 + 2\cos^2\theta \cdot \sin^2\theta - \cos^2\theta \cdot \sin^2\theta$

$= (\cos 2\theta)^2 + \cos^2\theta \cdot \sin^2\theta$

$= \cos^2 2\theta + \frac{4}{4} \cdot \cos^2\theta \cdot \sin^2\theta$

$= \cos^2 2\theta + \frac{1}{4}(2\cos\theta \cdot \sin\theta)^2$

$$= \frac{4\cos^2 2\theta + \sin^2 2\theta}{4}$$

$$= \frac{4\cos^2 2\theta + 1 - \cos^2 2\theta}{4}$$

$$= \frac{3\cos^2 2\theta + 1}{4}$$

$$= \frac{1}{4} + \frac{3\cos^2 2\theta}{4}$$

$$= \frac{1}{4}\left(1 + 3\cos^2 2\theta\right)$$

$$= \frac{1}{4}\left\{1 + \frac{3}{2}(1 + \cos 4\theta)\right\}$$

$$= \frac{1}{4}\left(\frac{2 + 3 + 3\cos 4\theta}{2}\right)$$

$$= \frac{1}{8}\left(5 + 3\cos 4\theta\right) = \text{RHS}$$

Proved//

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d. $\cos^6\theta - \sin^6\theta = \cos 2\theta\left(1 - \frac{1}{4}\sin^2 2\theta\right)$

LHS

$\cos^6\theta - \sin^6\theta$

$= (\cos^2\theta)^3 - (\sin^2\theta)^3$

$= (\cos^2\theta - \sin^2\theta)(\cos^4\theta + \cos^2\theta \cdot \sin^2\theta + \sin^4\theta)$

$= \cos 2\theta \cdot (\cos^2\theta)^2 + 2\cos^2\theta \cdot \sin^2\theta + (\sin^2\theta)^2 - \cos^2\theta \cdot \sin^2\theta$

$= \cos 2\theta\ (\cos^2\theta + \sin^2\theta)^2 - \cos^2\theta \cdot \sin^2\theta$

$= \cos 2\theta\ (1 - \cos^2\theta \cdot \sin^2\theta)$

$$= \cos 2\theta\left(1 - \frac{4}{4} \times \cos^2\theta \cdot \sin^2\theta\right)$$

$$= \cos 2\theta\left\{1 - \frac{(2\cos\theta \cdot \sin\theta)^2}{4}\right\}$$

$= \cos 2\theta \left\{ 1 - \frac{1}{4} (\sin 2\theta)^2 \right\}$

$= \cos 2\theta \left(1 - \frac{1}{4} \sin^2 2\theta \right)$

Rhs proved,,

e. $\sin^4\theta = \frac{3}{8} - \frac{1}{2} \cos 2\theta + \frac{1}{8} \cos 4\theta$

Lhs

$\sin^4\theta$

$= \sin^2\theta \cdot \sin^2\theta$ $\qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$= \frac{1 - \cos 2\theta}{2} \cdot \frac{1 - \cos^2\theta}{2}$ $\qquad \cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$= \frac{1}{4} (1 - \cos 2\theta)^2$

$= \frac{1}{4} (1 - 2\cos 2\theta + \cos^2 2\theta)$

$= \frac{1}{4} \left(1 - 2\cos 2\theta + \frac{1 + \cos 4\theta}{2}\right)$

$= \frac{1}{4} - \frac{1}{2} \cos 2\theta + \frac{1}{8} + \frac{\cos 4\theta}{8}$

$= \frac{1}{4} + \frac{1}{8} - \frac{1}{2} \cos 2\theta + \frac{1}{8} \cos 4\theta$

$= \frac{3}{8} - \frac{1}{2} \cos 2\theta + \frac{1}{8} \cos 4\theta$

Rhs proved,,

f. $\cancel{\sin}\ \cos^4\theta = \frac{1}{8}(3+4\cos 2\theta + \cos 4\theta)$

LHS

$\cos^4\theta$

$= \cancel{(\cos^2\theta)^2}\ \cos^2\theta \cdot \cos^2\theta$

$= \frac{1+\cos 2\theta}{2} \cdot \frac{1+\cos 2\theta}{2}$

$= \frac{1}{4}(1+\cos 2\theta)^2$

$= \frac{1}{4}\{1+2\cos 2\theta + (\cos 2\theta)^2\}$

$= \frac{1}{4}(1+2\cos 2\theta + \cos^2 2\theta)$

$= \frac{1}{4}\left(1+2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right)$

$= \frac{1}{4}\left(\frac{2+4\cos 2\theta+1+\cos 4\theta}{2}\right)$

$= \frac{1}{8}(3+4\cos 2\theta+\cos 4\theta)$

RHS proved//

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g. $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$

LHS

$\tan A + \cancel{2\tan 2A}\ 2\tan 2A + 4\tan 4A + 8\cot 8A$

$= \tan A + 2\tan 2A + 4\tan 4A + 8 \times \frac{1}{\tan 8A}$

$= \tan A + 2\tan 2A + 4\tan 4A + 8 \times \frac{1}{\frac{2\tan 4A}{1-\tan^2 4A}}$

$= \tan A + 2\tan 2A + 4\tan 4A + \cancel{8}^{4} \times \frac{1-\tan^2 4A}{\cancel{2}\tan 4A}$

$= \tan A + 2\tan 2A + \frac{\cancel{4\tan^2 4A} + 4 - \cancel{4\tan^2 4A}}{\cancel{2}\tan 4A}$

$= \tan A + 2\tan 2A + \frac{4}{\tan 4A}$

$= \tan A + 2\tan 2A + \cancel{4}^{2}\frac{(1-\tan^2 2A)}{\cancel{2}\tan 2A}$

~~$\tan A + 2\tan 2A + 2 - 2\tan^2 2A$~~

$= \tan A + 2\tan 2A + \frac{2-2\tan^2 2A}{\cancel{\tan A}\,\tan 2A}$

$= \tan A + \frac{\cancel{2\tan^2 2A} + 2 - \cancel{2\tan^2 2A}}{\tan 2A}$

$= \tan A + \frac{2}{\tan 2A}$

$= \tan A + \cancel{2}\frac{(1-\tan^2 A)}{\cancel{2}\tan A}$

$= \frac{\cancel{\tan^2 A} + 1 - \cancel{\tan^2 A}}{\tan A}$

$= \frac{1}{\tan A}$

$= \cot A$

RHS proved,,

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bi. $(2\cos\theta + 1)(2\cos\theta - 1)(2\cos 2\theta - 1) = 2\cos 4\theta + 1$

RHS

$2\cos 4\theta + 1$

$= 2\cos 2.2\theta + 1$

$= 2\,\cancel{\cos}\,(2\cos^2 2\theta - 1) + 1$

$= 4\cos^2 2\theta - 2 + 1$

$= (2\cos 2\theta)^2 - 1^2$

$= (2\cos 2\theta + 1)\ \cancel{fet}\ (2\cos 2\theta - 1)$

$= 2(2\cos^2\theta - 1) + 1 \quad (2\cos 2\theta - 1)$

$= (4\cos^2\theta - 2 + 1)\ (2\cos 2\theta - 1)$

$= (2\cos\theta)^2 - 1^2\ (2\cos 2\theta - 1)$

$= (2\cos\theta + 1)\ (2\cos\theta - 1)\ (2\cos 2\theta - 1)$

h. $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

LHS

$\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1}$

$= \dfrac{\dfrac{1}{\cos 8\theta} - 1}{\dfrac{1}{\cos 4\theta} - 1}$

$= \dfrac{\dfrac{1 - \cos 8\theta}{\cos 8\theta}}{\dfrac{1 - \cos 4\theta}{\cos 4\theta}}$

$= \dfrac{1 - \cos 8\theta}{\cos 8\theta} \times \dfrac{\cos 4\theta}{1 - \cos 4\theta}$

$= \dfrac{1 - (1 - 2\sin^2 4\theta)}{\cos 8\theta} \times \dfrac{\cos 4\theta}{\{1 - (1 - 2\sin^2 2\theta)}$

$= \dfrac{\cancel{1} - \cancel{1} + 2\sin^2 4\theta \cdot \cos 4\theta}{\cos 8\theta \cdot (\cancel{1} - \cancel{1} + 2\sin^2 2\theta)}$

$= \dfrac{2\sin^2 4\theta \cdot \cos 4\theta}{\cos 8\theta \cdot 2\sin^2 2\theta}$

$= \dfrac{\sin 8\theta \cdot \sin 4\theta}{\cos 8\theta \cdot 2\sin^2 2\theta}$

$= \tan 8\theta \times \dfrac{\cancel{2}\cancel{\sin 2\theta} \cdot \cos 2\theta}{\cancel{2}\sin^{\cancel{2}} 2\theta}$

$= \tan 8\theta \cdot \cot\theta$

$= \dfrac{\tan 8\theta}{\tan 2\theta}$ RHS proved

j. $\cos^8\theta + \sin^8\theta = 1 - \sin^2 2\theta + \frac{1}{8}\sin^4 2\theta$

LHS.

$\cos^8\theta + \sin^8\theta$

$= (\cos^4\theta)^2 + (\sin^4\theta)^2$

$= (\cos^4\theta - \sin^4\theta)^2 + 2\cos^4\theta.\sin^4\theta$

$= \{(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)\}^2 + \frac{8}{8} \times 2\cos^4\theta \cdot \sin^4\theta$

$= \cos^2 2\theta + \frac{1}{8}(2\cos\theta \cdot \sin\theta)^4$

$= \cos^2 2\theta + \frac{1}{8} \times (\sin 2\theta)^4$

$= \cos^2 2\theta + \frac{1}{8}\sin^4 2\theta$

$= 1 - \sin^2 2\theta + \frac{1}{8}\sin^4 2\theta$

Rhs proved,,

3.h. $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$

LHS

$\frac{\sin\theta + 2\sin\theta \cdot \cos\theta}{\cancel{1} + \cos\theta + 2\cos^2\theta - \cancel{1}}$

$= \frac{\sin\theta\cancel{(1+2\cos\theta)}}{\cos\theta\cancel{(1+2\cos\theta)}}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

Rhs proved,

k. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8A}}} = 2\cos A$

LHS

$\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8A}}}$

$= \sqrt{2+\sqrt{2+\sqrt{2+2(2\cos^2 4A-1)}}}$

$= \sqrt{2+\sqrt{2+\sqrt{\cancel{2}+4\cos^2 4A-\cancel{2}}}}$

$= \sqrt{2+\sqrt{2+\sqrt{(2\cos 4A)^2}}}$

$= \sqrt{2+\sqrt{2+2\cos 4A}}$

$= \sqrt{2+\sqrt{2+2(2\cos^2 2A-1)}}$

$= \sqrt{2+\sqrt{2+4\cos^2 2A-2}}$

$= \sqrt{2+\sqrt{(2\cos 2A)^2}}$

$= \sqrt{2+2\cos 2A}$

$=$

$= \sqrt{2+2(2\cos^2 A-1)}$

$= \sqrt{\cancel{2}+4\cos^2 A-\cancel{2}}$

$= \cancel{\sqrt{}}(2\cos A)^{\cancel{2}}$

$= 2\cos A$

RHS proved

1. $\frac{1}{\operatorname{Tan} 3A + \operatorname{Tan} A} - \frac{1}{\operatorname{Cot} 3A + \operatorname{Cot} A} = \operatorname{Cot} 4A$

L.H.S.

$\frac{1}{\operatorname{Tan} 3A + \operatorname{Tan} A} - \frac{1}{\operatorname{Cot} 3A + \operatorname{Cot} A}$

$= \frac{1}{\operatorname{Tan} 3A + \operatorname{Tan} A} - \frac{1}{\frac{1}{\operatorname{Tan} 3A} + \frac{1}{\cancel{\operatorname{Cot} A}\ \operatorname{Tan} A}}$

$= \frac{1}{\operatorname{Tan} 3A + \operatorname{Tan} A} - \frac{1}{\frac{\operatorname{Tan} A + \operatorname{Tan} 3A}{\operatorname{Tan} 3A \cdot \cancel{\operatorname{Cot} A}\ \operatorname{Tan} A}}$

$= \frac{1}{\operatorname{Tan} 3A + \operatorname{Tan} A} - \frac{\operatorname{Tan} 3A \cdot \operatorname{Tan} A}{\operatorname{Tan} A + \operatorname{Tan} 3A}$

$= \frac{1 - \operatorname{Tan} A \cdot \operatorname{Tan} 3A}{\operatorname{Tan} A + \operatorname{Tan} 3A}$

$= \frac{1}{\frac{\operatorname{Tan} A + \operatorname{Tan} 3A}{1 - \operatorname{Tan} A \cdot \operatorname{Tan} 3A}}$

$= \frac{1}{\operatorname{Tan}(A + 3A)}$

$= \frac{1}{\operatorname{Tan} 4A}$

$= \operatorname{Cot} 4A$

RHS proved,,

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m. $\frac{4(\cos^4 A + \sin^4 A)}{\cos^4 A - \sin^4 A} = (3 + \cos 4A) \sec 2A$

LHS

$\frac{4(\cos^4 A + \sin^4 A)}{\cos^4 A - \sin^4 A}$

$$= \frac{4\left((\cos^2A + \sin^2A)^2 - 2\cos^2A.\sin^2A\right)}{(\cos^2A + \sin^2A)\,(\cos^2A - \sin^2A)}$$

$$= \frac{4\left(1 - 2\cos^2A.\sin^2A\right)}{(\cos^2A - \sin^2A)}$$

$$= \frac{4\left(1 - \frac{2}{2}\times 2\cos^2A.\sin^2A\right)}{\cos 2A}$$

$$= \frac{4\left\{1 - \frac{1}{2}(2\cos A.\sin A)^2\right\}}{\cos 2A}$$

$$= \frac{4 - 2\sin^2 2A}{\cos 2A}$$

$$= \frac{3 + 1 - 2\sin^2 2A}{\cos 2A}$$

$$= \frac{3 + (1 - 2\sin^2 2A)}{\cos 2A}$$

$$= \frac{3 + \cancel{\cos 2A}\cos 2.2A}{\cos 2A}$$

$$= 3 + \cos 4A \times \frac{1}{\cos 2A}$$

$$= (3 + \cos 4A)\sec 2A$$

RHS proved,,

n. $\frac{1}{\tan 3A + \tan A} + \frac{1}{\cot 3A + \cot A} = \frac{1}{2}\operatorname{cosec} 2A$

LHS

$$\frac{1}{\tan 3A + \tan A} + \frac{1}{\cot 3A + \cot A}$$

$$= \cancel{\frac{1}{\tan 3A + \tan A}} + \frac{1}{\dots}$$

$$= \frac{1}{\tan 3A + \tan A} + \frac{\tan A \cdot \tan 3A}{\tan 3A + \tan A}$$

$$= \frac{1 + \tan A \cdot \tan 3A}{\cancel{\tan 3A + \tan A + \tan A}}$$

$$= \frac{1}{\frac{\sin 3A}{\cos 3A} + \frac{\sin A}{\cos A}} + \frac{1}{\frac{\cos 3A}{\sin 3A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin 3A \cdot \cos A + \sin A \cdot \cos 3A}{\cos 3A \cdot \cos A}} + \frac{1}{\frac{\cos 3A \cdot \sin A + \sin 3A \cdot \cos A}{\cancel{\cos 3A} \; \sin 3A \cdot \sin A}}$$

$$= \frac{\cos 3A \cdot \cos A}{\sin(3A + A)} + \frac{\sin 3A \cdot \sin A}{\sin(3A + A)}$$

$$= \frac{\cos 3A \cdot \cos A + \sin 3A \cdot \sin A}{\sin(3A + A)}$$

$$= \frac{\cos(3A - A)}{\sin 4A} = \frac{\cos 2A}{\sin 2 \cdot 2A} = \frac{\cancel{\cos 2A}}{2 \sin 2A \cdot \cancel{\cos 2A}} = \frac{1}{2} \operatorname{cosec} 2A$$

Q. $\sin^2 A \cdot \cos 2B - \sin^2 B \cdot \cos 2A = \cos^2 B - \cos^2 A$

LHS

$\sin^2 A \cdot \cos 2B - \sin^2 B \cdot \cos 2A$

$= \sin^2 A \cdot (1 - 2\sin^2 B) - \sin^2 B \cdot (1 - 2\sin^2 A)$

$= \sin^2 A - 2\cancel{\sin^2 A \cdot \sin^2 B} - \sin^2 B + 2\cancel{\sin^2 A \cdot \sin^2 B}$

$= \sin^2 A - \sin^2 B$

$= \cancel{1 - \cos^2 B - \{1 - \sin \cos^2 B\}}$

$= \cancel{1 - \cos^2 B - 1 +}$

$= 1 - \cos^2 A - (1 - \cos^2 B)$

$= \cancel{1} - \cos^2 A - \cancel{1} + \cos^2 B$

$= \cos^2 B - \cos^2 A$ proved

p. $4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$

Soln,

$4\cos^3 10^\circ + 4\sin^3 20^\circ = 3\cos 10^\circ + 3\sin 20^\circ$

$= 4\cos^3 10^\circ - 3\cos 10^\circ = 3\sin 20^\circ - 4\sin^3 20^\circ$

$= \cos 3\times 10^\circ = \sin 3\times 20^\circ$

$= \cos 30^\circ = \sin 60^\circ$

$= \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$\therefore$ LHS = RHS

---

q. $\operatorname{cosec} 2A + \operatorname{cosec} 4A + \operatorname{cosec} 8A = \cot A - \cot 8A$

or, $\operatorname{cosec} 2A + \operatorname{cosec} 4A + \operatorname{cosec} 8A + \cot 8A = \cot A$

LHS

$\operatorname{cosec} 2A + \operatorname{cosec} 4A + \operatorname{cosec} 8A + \cot 8A$

$= \operatorname{cosec} 2A + \operatorname{cosec} 4A + \frac{1}{\sin 8A} + \frac{\cos 8A}{\sin 8A}$

$= \operatorname{cosec} 2A + \operatorname{cosec} 4A + \frac{1 + \cos 8A}{\sin 8A}$

$= \operatorname{cosec} 2A + \operatorname{cosec} 4A + \frac{\cancel{1} + \cancel{2}\cos^{\cancel{2}} 4A - \cancel{1}}{\cancel{2}\sin 4A \cdot \cancel{\cos A}}$

$= \operatorname{cosec} 2A + \frac{1}{\sin 4A} + \frac{\cos 4A}{\sin 4A}$

$= \operatorname{cosec} 2A + \frac{1 + \cos 4A}{\sin 4A}$

$= \operatorname{cosec} 2A + \frac{\cancel{1} + 2\cos^{\cancel{2}} 2A - \cancel{1}}{\cancel{2}\sin 2A \cdot \cancel{\cos 2A}}$

$= \operatorname{cosec} 2A + \frac{\cos 2A}{\sin 2A}$

$= \frac{1}{\sin 2A} + \frac{\cos 2A}{\sin 2A}$

$= \dfrac{1 + \cos 2A}{\sin 2A}$

$= \dfrac{\cancel{1} + \cancel{2}\cos^2 A - \cancel{1}}{\cancel{2}\sin A \cdot \cancel{\cos A}}$

$= \dfrac{\cos A}{\sin A}$

$= \cot A$

RHS proved//

---

r. $\sin^2 A + \sin^2 B \cdot \cos 2A = \sin^2 B + \sin^2 A \cdot \cos 2B$

~~or, $\sin^2 B \cdot \cos 2A - \sin^2 B + \sin^2$~~

or, $\sin^2 B \cdot \cos 2A - \sin^2 A \cdot \cos 2B = \sin^2 B - \sin^2 A$

LHS

$\sin^2 B \cdot \cos 2A - \sin^2 A \cdot \cos 2B$

$= \sin^2 B \cdot (1 - 2\sin^2 A) - \sin^2 A \cdot (1 - 2\sin^2 B)$

$= \sin^2 B - \cancel{2\sin^2 A \cdot \sin^2 B} - \sin^2 A + \cancel{2\sin^2 A \cdot \sin^2 B}$

$= \sin^2 B - \sin^2 A$

$=$ RHS proved//

---

5. Prove the following:

a. $1 + \cos^2 2\theta = 2(\cos^4\theta + \sin^4\theta)$

RHS

$= 2\cos^4\theta + \sin^4\theta$

$= \cancel{2\{(\cos^2\theta)^2 + (\sin^2\theta}$

$= 2\{(\cos^2\theta)^2 + (\sin^2\theta)^2$

$= 2\{(\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta \cdot \sin^2\theta$

$= 2 - 4\cos^2\theta \cdot \sin^2\theta$

$= 2 - (2\cos\theta \cdot \sin\theta)^2$

$= \cancel{2 \quad \cos^2 2\theta}$

$= \cancel{2 - (1 - 2\sin^2\theta)}$

$= \cancel{2 - \sin^2 2\theta}$

$= \cancel{2 - (1 - \cos^2 2\theta)}$

$= \cancel{2 - 2 + 2 + 2\cos^2 2A}$

$= \cancel{2 - 1 + 2\cos^2 2\theta}$

$= 2 - \sin^2 2\theta$

$= 2 - (1 - \cos^2 2\theta)$

$= 2 - 1 + \cos^2 2\theta$

$= 1 + \cos^2 2\theta$

~~RHS~~ LHS proved//

b. $2\cos^2\theta + \cos^2 2\theta - 2\cos^2\theta \cdot \cos 2\theta = 1$

LHS

$2\cos^2\theta + \cos^2 2\theta - 2\cos^2\theta \cdot \cos 2\theta$

$= 2\cos^2\theta + (\cos^2\theta - \sin^2\theta)^2 - 2\cos^2\theta \cdot (1 - 2\sin^2\theta)$

$= \cancel{2\cos^2\theta} + \cos^4\theta - 2\cos^2\theta \cdot \sin^2\theta + \sin^4\theta - \cancel{2\cos^2\theta} + 4\cos^2\theta \cdot \sin^2\theta$

$= (\cos^2\theta)^2 + (\sin^2\theta)^2 + 2\cos^2\theta \cdot \sin^2\theta$

$= (\cos^2\theta + \sin^2\theta)^2 - \cancel{2\cos^2\theta \cdot \sin^2\theta} + \cancel{2\cos^2\theta \cdot \sin^2\theta}$

$= 1^2$

$= 1$

RHS proved,

c. $\cos^3 A \cdot \sin 3A + \sin^3 A \cdot \cos 3A = \frac{3}{4} \sin 4A$

LHS

$\cos^3 A \cdot \sin 3A + \sin^3 A \cdot \cos 3A$

$= \cos^3 A \cdot (3\sin A - 4\sin^3 A) + \sin^3 A (4\cos^3 A - 3\cos A)$

$= 3\sin A \cdot \cos^3 A - \cancel{4\sin^3 A \cos^3 A} + \cancel{4\cos^3 A \cdot \sin^3 A} - 3\sin^3 A \cdot \cos A$

$= 3\sin A \cdot \cos^3 A - 3\sin^3 A \cdot \cos A$

$= 3\sin A \cdot \cos A (\cos^2 A - \sin^2 A)$

$= 3\sin A \cdot \cos A \cdot \cos 2A$

$= \frac{3}{2} \times 2\sin A \cdot \cos A \cdot \cos 2A$

$= \frac{3}{2} \sin 2A \cdot \cos 2A$

$= \frac{3}{2} \times \frac{2}{2} \sin 2A \cdot \cos 2A$

$= \frac{3}{4} \times 2\sin 2A \cdot \cos 2A$

$= \frac{3}{4} \sin 4A$

RHS proved//

d. $\cos^3 2\theta + 3\cos 2\theta = 4(\cos^6\theta - \sin^6\theta)$

LHS

$\cos^3 2\theta + 3\cos 2\theta$

$= (2\cos^2\theta - 1)^3 + 3(2\cos^2\theta - 1)$

$= 8\cos^6\theta - 3\cdot 2\cos^2\theta\cdot 1(2\cos^2\theta - 1) - 1^3 + 3(2\cos^2\theta - 1)$

$= 8\cos^6\theta - 1 - 6\cos^2\theta(2\cos^2\theta - 1) + 6\cos^2\theta - 3$

$= 8\cos^6\theta - 1 - 12\cos^4\theta + 6\cos^2\theta + 6\cos^2\theta - 3$

$= 8\cos^6\theta - 12\cos^4\theta + 12\cos^2\theta - 4$

$= 4[2\cos^6\theta - 3\cos^2\theta(\cos^2\theta - 1) - 1]$

$= 4[2\cos^6\theta + 3\cos^2\theta(1 - \cos^2\theta) - (\sin^2\theta + \cos^2\theta)^3]$

$= 4[2\cos^6\theta + 3\cos^2\theta\cdot\sin^2\theta - \{\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cdot\cos^2\theta(\cos^2\theta + \sin^2\theta)\}]$

$= 4[2\cos^6\theta + 3\cos^2\theta\cdot\sin^2\theta - \sin^6\theta - \cos^6\theta - 3\sin^2\theta\cdot\cos^2\theta]$

$= 4[2\cos^6\theta - \cos^6\theta - \sin^6\theta]$

$= 4[\cos^6\theta - \sin^6\theta]$

RHS proved//

e. $\cos^3 A \cdot \cos 3A + \sin^3 A \cdot \sin 3A = \cos^3 2A$

LHS

$\cos^3 A \cdot \cos 3A + \sin^3 A \cdot \sin 3A$

$= \cos^3 A \cdot (4\cos^3 A - 3\cos A) + \sin^3 A\,(3\sin A - 4\sin^3 A)$

$= 4\cos^6 A - 3\cos^4 A + 3\sin^4 A - 4\sin^6 A$

$= 4(\cos^6 A - \sin^6 A) - 3(\cos^4 A - \sin^4 A)$

$= 4[(\cos^2 A - \sin^2 A)(\cos^4 A + \sin^2 A \cdot \cos A + \sin^4 A)] - 3(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$

$= 4[\cos 2A\,(\cos^2 A + \sin^2 A)^2 - 2\cos^2 A \cdot \sin^2 A + \sin^2 A \cdot \cos^2 A] - 3\cos 2A$

$= \cancel{4[\cos 2A}$

$= 4\cos 2A\,[1 - \sin^2 A \cdot \cos^2 A] - 3\cos 2A$

$= \cos 2A\,[4 - 4\sin^2 A \cdot \cos^2 A] - 3\cos 2A$

$= \cos 2A \cdot [4 - (2\sin A \cdot \cos A)^2] - 3\cos 2A$

$= \cos 2A\,[4 - \overset{\sin^2 2A}{\cancel{\sin \cos 2A}}] - 3\cos 2A$

$= \cos 2A\,\cancel{[(4 - \cos^2 2A - 3)]}\,[4 - \sin^2 2A] - 3\cos 2A$

$= \cos 2A\,\cancel{[(1 - \cos^2 2A)]}\,(4 - \sin^2 2A - 3)$

$= \cos 2A \cdot (1 - \sin^2 2A)$

$= \cos 2A \cdot \cos^2 2A$

$= \cos^3 2A$ proved,,

f. $\tan 20° + 4\sin 20° = \sqrt{3}$

LHS

$\tan 20° + 4\sin 20°$

$= \dfrac{\sin 20°}{\cos 20°} + 4\sin 20°$

$= \dfrac{\sin 20° + 4\sin 20° \cdot \cos 20°}{\cos 20°}$

$= \dfrac{\sin 20° + 2 \cdot 2\sin 20° \cdot \cos 20°}{\cos 20°}$

$= \dfrac{\sin 20° + 2 \cdot \sin 40°}{\cos 20°}$

$$= \frac{\sin(60^\circ - 40^\circ) + 2\cdot\sin 40^\circ}{\cos 20^\circ}$$

$$= \frac{\sin 60^\circ\cdot\cos 40 - \cos 60^\circ\cdot\sin 40^\circ + 2\cdot\sin 40^\circ}{\cos 20^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}\cos 40^\circ - \frac{\cancel{4}\,1}{2}\sin 40^\circ + 2\sin 40^\circ}{\cos 20^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}\cdot\cos 40^\circ + \frac{3}{2}\sin 40^\circ}{\cos 20^\circ}$$

$$= \frac{\frac{\sqrt{3}}{\cancel{2}}\left(\frac{\cos 40^\circ}{2} + \frac{\sqrt{3}}{2}\sin 40^\circ\right)}{\cos 20^\circ}$$

$$= \frac{\sqrt{3}\left(\frac{1}{2}\times\cos 40^\circ + \frac{\sqrt{3}}{2}\sin 40^\circ\right)}{\cos 20^\circ}$$

$$= \sqrt{3}\left(\frac{\cos 60^\circ\cdot\cos 40^\circ + \sin 60^\circ\cdot\sin 40^\circ}{\cos 20^\circ}\right)$$

$$= \frac{\sqrt{3}\{\cos(60^\circ - 40^\circ)\}}{\cos 20^\circ}$$

$$= \sqrt{3}\,\frac{\cos 20^\circ}{\cos 20^\circ}$$

$$= \sqrt{3}$$

RHS proved//

9. $\sec 2\theta + \tan 2\theta = \tan(45^\circ + \theta)$

LHS

$\sec 2\theta + \tan 2\theta$

$= \dfrac{1}{\cos 2\theta} + \dfrac{\sin 2\theta}{\cos 2\theta}$

$= \dfrac{1 + \sin 2\theta}{\cos 2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta}{\cos 2\theta}$

$= \dfrac{(\sin\theta + \cos\theta)^2}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{(\sin\theta + \cos\theta)\cdot\cancel{(\sin\theta + \cos\theta)}}{\cancel{(\cos\theta + \sin\theta)}(\cos\theta - \sin\theta)}$

$= \dfrac{(\sin\theta + \cos\theta)}{(\cos\theta - \sin\theta)}$

$= \dfrac{\dfrac{(\sin\theta + \cos\theta)}{\cos\theta}}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}$ [dividing by 'cosθ']

$= \dfrac{\tan\theta + 1}{1 - \tan\theta}$

$= \dfrac{1 + \tan\theta}{1 - \tan\theta}$

$= \dfrac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ\cdot\tan\theta}$

$= \tan(45^\circ + \theta)$

RHS proved$_{//}$

h. $$\frac{1+\tan^2\left(\frac{\pi}{4}-A\right)}{1-\tan^2\left(\frac{\pi}{4}-A\right)} = \operatorname{cosec}2A$$

LHS

$$\frac{1+\tan^2(\pi/4-A)}{1-\tan^2(\pi/4-A)}$$

$$= \frac{1}{\frac{1-\tan^2(\pi/4-A)}{1+\tan^2(\pi/4-A)}}$$

$$= \frac{1}{\cos 2(\pi/4-A)}$$

$$= \frac{1}{\cos\left(\frac{2\pi}{4}-2A\right)}$$

$$= \frac{1}{\cos(90^\circ-2A)}$$

$$= \frac{1}{\sin 2A}$$

$= \operatorname{cosec}2A$

RHS proved.

i. $\cos^2(120^\circ-A)+\cos^2A+\cos^2(120^\circ+A) = \frac{3}{2}$

LHS

$\cos^2(120^\circ-A)+\cos^2A+\cos^2(120^\circ+A)$

$= (\cos 120^\circ\cdot\cos A+\sin 120^\circ\cdot\sin A)^2+\cos^2A+(\cos 120^\circ\cdot\cos A-\sin 120^\circ\cdot\sin A)^2$

$= \left(-\frac{1}{2}\cdot\cos A+\frac{\sqrt{3}}{2}\cdot\sin A\right)^2+\cos^2A+\left(-\frac{1}{2}\cdot\cos A-\frac{\sqrt{3}}{2}\cdot\sin A\right)^2$

$= \frac{3}{4}\sin^2A - \cancel{\frac{2\sqrt{3}}{2}\sin A\cos A} - \frac{1}{2}\cos A + \left(\frac{1}{2}\cos A\right)^2 + \cos^2A$

$+ \left(\frac{1}{2}\cos A\right)^2 + 2\cdot\frac{1}{2}\cos A\cdot\frac{\sqrt{3}}{2}\sin A + \left(\frac{\sqrt{3}}{2}\sin A\right)^2$

$= \frac{3}{4}\sin^2A - \cancel{\frac{\sqrt{3}}{2}\sin A\cdot\cos A} + \frac{1}{4}\cos^2A + \cos^2A + \frac{1}{4}\cos^2A + \cancel{\frac{\sqrt{3}}{2}}$

$\cancel{\sin A\cdot\cos A} + \frac{3}{4}\sin^2A$

$= \frac{3}{4}\sin^2A + \frac{3}{4}\sin^2A + \frac{1}{4}\cos^2A + \frac{1}{4}\cos^2A + \cos^2A$

$= \cancel{\frac{3}{4}\sin^2A + 3}$

$= \frac{3}{2}\sin^2A + \frac{3}{2}\cos^2A$

$= \frac{3}{2}(\sin^2A + \cos^2A)$

$= \frac{3}{2}\times 1$

$= \frac{3}{2}$

Ans proved

j. $\frac{\cos\theta - \sqrt{1+\sin2\theta}}{\sin\theta - \sqrt{1+\sin2\theta}} = \tan\theta$

LHS

$\frac{\cos\theta - \sqrt{1+\sin2\theta}}{\sin\theta - \sqrt{1+\sin2\theta}}$

$= \frac{\cos\theta - \sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta}}{\sin\theta - \sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta}}$

$= \frac{\cos\theta - \sqrt{(\sin\theta + \cos\theta)^2}}{\sin\theta - \sqrt{(\sin\theta + \cos\theta)^2}}$

$$= \frac{\cos\theta - \sin\theta - \cos\theta}{\sin\theta - \sin\theta - \cos\theta}$$

$$= \frac{+\sin\theta}{+\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

RHS proved.

6. Prove the following:

a. If $2\tan a = 3\tan B$, then show that $\tan(a-B) = \frac{\sin 2B}{5-\cos 2B}$

Soln,

Given; $2\tan a = 3\tan B$

or, $\tan a = \frac{3}{2}\tan B$

To prove: $\tan(a-B) = \frac{\sin 2B}{5-\cos 2B}$

LHS

$\tan(a-B)$

$$= \frac{\tan\alpha - \tan B}{1+\tan\alpha \cdot \tan B}$$

$$= \frac{\frac{3}{2}\tan B - \tan B}{1+\frac{3}{2}\tan B \cdot \tan B}$$

$$= \frac{\frac{3\tan B - 2\tan B}{2}}{\frac{2+3\tan^2 B}{2}}$$

$$= \frac{\tan B}{2 + 3\tan^2 B}$$

$$= \frac{\frac{\sin B}{\cos B}}{2 + 3\frac{\sin^2 B}{\cos^2 B}}$$

$$= \frac{\frac{\sin B}{\cos B}}{\frac{2\cos^2 B + 3\sin^2 B}{\cos^2 B}}$$

$$= \frac{\sin B}{\cancel{\cos B}} \times \frac{\cos^{\cancel{2}} B}{2\cos^2 B + 3\sin^2 B}$$

$$= \frac{\sin B \cdot \cos B}{2\cos^2 B + 3(1 - \cos^2 B)}$$

$$= \frac{\sin B \cdot \cos B}{2\cos^2 B + 3 - 3\cos^2 B}$$

$$= \frac{2\sin B \cdot \cos B}{2(3 - \cos^2 B)}$$

$$= \frac{\sin 2B}{6 - 2\cos^2 B}$$

$$= \frac{\sin 2B}{\cancel{6}\,5 - (2\cos^2 B - 1)}$$

$$= \frac{\sin 2B}{5 - \cos 2B}$$

RHS proved//

6. If $2\,\text{Tan}\alpha = 3\tan B$, then show that $\text{Tan}(\alpha+B) = \dfrac{5\sin 2B}{5\cos 2B - 1}$

Soln.

Given; $2\,\text{Tan}\,\alpha = 3\,\text{Tan}B$

or $\text{Tan}\,\alpha = \dfrac{3}{2}\text{Tan}B$

To prove: $\text{Tan}(\alpha+B) = \dfrac{5\sin 2B}{5\cos 2B - 1}$

L.H.S

$$\text{Tan}(\alpha+B) = \frac{\text{Tan}\alpha + \text{Tan}B}{1 - \text{Tan}\alpha . \text{Tan}B}$$

$$= \frac{3/2\,\text{Tan}B + \text{Tan}B}{1 - \frac{3}{2}\text{Tan}B . \text{Tan}B}$$

$$= \frac{\frac{3\,\text{Tan}B + 2\,\text{Tan}B}{2}}{\frac{2 - 3\,\text{Tan}^2 B}{2}}$$

$$= \frac{5\,\text{Tan}B}{2 - 3\,\text{Tan}^2 B}$$

$$= \frac{\frac{5\sin B}{\cos B}}{2 - \frac{3\sin B}{\cos^2 B}}$$

$$= \frac{\frac{5\sin B}{\cos B}}{\frac{2\cos^2 B - 3\sin^2 B}{\cos^2 B}}$$

$$= \frac{5\sin B}{\cancel{\cos B}} \times \frac{\cos^2 B}{2\cos^2 B - 3\sin^2 B}$$

$$\frac{5 \times 2\sin B \cdot \cos B}{2 \times 2\cos^2 B - 3 \times 2\sin^2 B}$$

$$= \frac{5\sin 2B}{4\cos^2 B - 6\sin^2 B}$$

$$= \frac{5\sin 2B}{4\cos^2 B - 4\sin^2 B - 2\sin^2 B}$$

$$= \frac{5\sin 2B}{4\cos 2B + 1 - 2\sin^2 B - 1}$$

$$= \frac{5\sin 2B}{4\cos 2B + \cos 2B - 1}$$

$$= \frac{5\sin 2B}{5\cos 2B - 1}$$

Proved.

c. If $\tan\theta = \frac{b}{a}$ then show that $a.\cos 2\theta + b.\sin 2\theta = a$

Soln,

Given: $\tan\theta = \frac{b}{a}$

To show: $a.\cos 2\theta + b.\sin 2\theta = a$

LHS

$a.\cos 2\theta + b.\sin 2\theta$

$= a.\frac{1-\tan^2\theta}{1+\tan^2\theta} + b.\frac{2\tan\theta}{1+\tan^2\theta}$

$= \dfrac{a - a\left(\frac{b}{a}\right)^2 + b.2.\frac{b}{a}}{1+\tan^2\theta}$

$= \dfrac{a - \cancel{a}.\frac{b^2}{a^{\cancel{2}}} + 2b \times \frac{b}{a}}{1 + \frac{b^2}{a^2}}$

$= \dfrac{a - \frac{b^2}{a} + \frac{2b^2}{a}}{\frac{a^2+b^2}{a^2}}$

$= \dfrac{\frac{a^2-b^2+2b^2}{a}}{\frac{a^2+b^2}{a^2}}$

$= \frac{\cancel{a^2+b^2}}{\cancel{a}} \times \frac{a^{\cancel{2}}}{\cancel{a^2+b^2}}$

$= a$

RHS proved,

H/W

d. If $\cos A = \frac{1}{2}\left(a + \frac{1}{a}\right)$ then show that $\cos 2A = \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right)$

and $\cos 3A = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

Soln,

Given, $\cos A = \frac{1}{2}$

To show: $\cos 2A = \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right)$

LHS.

$\cos 2A$

$= 2\cos^2 A - 1$

$= 2\left\{\frac{1}{2}\left(a + \frac{1}{a}\right)\right\}^2 - 1$

$= 2 \times \frac{1}{4}\left(a + \frac{1}{a}\right)^2 - 1$

$= \frac{1}{2}\left(a^2 + 2.a.\frac{1}{a} + \frac{1}{a^2}\right) - 1$

$= \frac{1}{2}\left(a^2 + \frac{1}{a^2} + 2\right) - 1$

$= \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) + 2 \times \frac{1}{2} - 1$

$= \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) + \cancel{1} - \cancel{1}$

$= \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right)$

RHS proved,,

$\cos 3A = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

Lhs

$\cos 3A$

$= 4\cos^3 A - 3\cos A$

$= 4\left\{\frac{1}{2}\left(a + \frac{1}{a}\right)\right\}^3 - 3\left\{\frac{1}{2}\left(a + \frac{1}{a}\right)\right\}$

$= 4 \times \frac{1}{8}\left(a + \frac{1}{a}\right)^3 - \frac{3}{2}\left(a + \frac{1}{a}\right)$

$= \frac{1}{2}\left(a^3 + 3a^2 \cdot \frac{1}{a} + 3a \cdot \frac{1}{a^2} + \frac{1}{a^3}\right) - \frac{3}{2}\left(a + \frac{1}{a}\right)$

$= \frac{1}{2}\left\{a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)\right\} - \frac{3}{2}\left(a + \frac{1}{a}\right)$

$= \frac{1}{2}\left\{a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) - 3\left(a + \frac{1}{a}\right)\right\}$

$= \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

Rhs proved

e. If $\tan\theta = \frac{1}{7}$ and $\tan\phi = \frac{1}{3}$, prove that $\cos 2\theta = \sin 4\phi$

Soln

Lhs

~~Tanθ~~ $\cos 2\theta$

$= \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= \frac{1 + \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2}$

$$= \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}}$$

$$= \frac{\frac{49-1}{\cancel{49}}}{\frac{49+1}{\cancel{49}}}$$

$$= \frac{48}{50}$$

$$= \frac{24}{25}$$

Again, RHS

$\sin 4\phi$

$= \cancel{8.2\,\tan}\ 2\sin 2\phi \cdot \cos 2\phi$

$$= 2 \cdot \frac{2\tan\phi}{1+\tan^2\phi} \times \frac{1-\tan^2\phi}{1+\tan^2\phi}$$

$$= \frac{2 \cdot 2 \cdot \frac{1}{3}}{1+\left(\frac{1}{3}\right)^2} \times \frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2}$$

$$= \frac{\frac{4}{3}}{\frac{10}{9}} \times \frac{\frac{8}{9}}{\frac{10}{9}}$$

$$= \frac{\cancel{4}2}{\cancel{3}} \times \frac{\cancel{9}3}{\cancel{10}5} \times \frac{8}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{10}5}$$

$$= \frac{24}{25}$$

$\therefore$ LHS = RHS

Proved

7. Prove the following:

a. $8\cos^3 20^\circ - 8\sin^3 10^\circ - 6\cos 20^\circ + 6\sin 10^\circ = 2$

Soln.

LHS

$8\cos^3 20^\circ - 8\sin^3 10^\circ - 6\cos 20^\circ + 6\sin 10^\circ$

$= 8\cos^3 20^\circ - 6\cos 20^\circ + 6\sin 10^\circ - 8\sin^3 10^\circ$

$= 2(4\cos^3 20^\circ - 3\cos 20^\circ) + 2(3\sin 10^\circ - 4\sin^3 10^\circ)$

$= \cancel{2 \cdot \sin 3 \cdot 20^\circ + 2 \cdot 5}$

$= 2 \cdot \cos 3 \cdot 20^\circ + 2 \cdot \sin 3 \cdot 10^\circ$

$= 2 \cdot \cos 60^\circ + 2\sin 30^\circ$

$= \cancel{2} \times \frac{1}{\cancel{2}} + \cancel{2} \times \frac{1}{\cancel{2}}$

$= 1 + 1$

$= 2$

RHS proved.

b. $16\cos\frac{2\pi}{15} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15} \cdot \cos\frac{16\pi}{15} = 1$

LHS

$16\cos\frac{2\pi}{16} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15} \cdot \cos\frac{16\pi}{15}$

$= \frac{8 \cdot 2\cos\frac{2\pi}{16}}{\sin\frac{2\pi}{16}} \times \sin\frac{2\pi}{15} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15} \cdot \cos\frac{16\pi}{15}$

$= \frac{8 \cdot \sin\frac{4\pi}{15}}{\sin\frac{2\pi}{15}} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15} \cdot \cos\frac{16\pi}{15}$

$$= \frac{4 \cdot 2 \sin \frac{4\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}}{\sin \frac{2\pi}{16}}$$

$$= \frac{4 \sin \frac{8\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}}{\sin \frac{2\pi}{15}}$$

$$= \frac{2 \sin \frac{16\pi}{15} \cdot \cos \frac{16\pi}{15}}{\sin \frac{2\pi}{15}}$$

$$= \frac{\sin \frac{32\pi}{15}}{\sin \frac{2\pi}{15}}$$

$$= \frac{\sin \left(2\pi + \frac{2\pi}{15} \right)}{\sin \frac{2\pi}{15}}$$

$$= \frac{\sin \frac{2\pi}{15}}{\sin \frac{2\pi}{15}}$$

$$= 1$$

RHS proved

c. $\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{8\pi}{7}=\frac{1}{8}$

LHS

$\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{8\pi}{7}$

$$=\frac{2\cdot\cos\frac{2\pi}{7}\times\sin\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{8\pi}{7}}{2\cdot\sin\frac{2\pi}{7}}$$

$$=\frac{2\cdot\sin\frac{4\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{8\pi}{7}}{2\cdot2\sin\frac{2\pi}{7}}$$

$$=\frac{2\cdot\sin\frac{8\pi}{7}\cdot\cos\frac{8\pi}{7}}{2\cdot4\sin\frac{2\pi}{7}}$$

$$=\frac{\sin\frac{16\pi}{7}}{8\sin\frac{2\pi}{7}}$$

$$=\frac{\sin\left(2\pi+\frac{2\pi}{7}\right)}{8\sin\frac{2\pi}{7}}$$

$$=\frac{\cancel{\sin\frac{2\pi}{7}}}{8\cancel{\sin\frac{2\pi}{7}}}$$

$$=\frac{1}{8}$$

RHS proved.

d. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=\frac{1}{8}$

LHS

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left\{1+\cos\left(\pi-\frac{\pi}{8}\right)\right\}\left(1+\cos\frac{3\pi}{8}\right)\left\{1+\cos\left(\pi-\frac{3\pi}{8}\right)\right\}$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)$

$=\left(1^2-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$=\left(1-\cos 2.\frac{\pi}{8}+1\right)\left(1-\cos 2.\frac{3\pi}{8}+1\right)$

$=\left(\frac{2-\cos\frac{\pi}{4}-1}{2}\right)\left(\frac{2-\cos\frac{3\pi}{4}-1}{2}\right)$

$=\left(\frac{(1-\cos 45^\circ)(1-\cos 135^\circ)}{4}\right)$

$=\frac{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}{4}$

$=\frac{1^2-\left(\frac{1}{\sqrt{2}}\right)^2}{4}$

$=\frac{1-\frac{1}{2}}{4}$

$=\cancel{\frac{1}{2}\times 4}\ \frac{\frac{1}{2}}{4}$

$=\frac{1}{8}$ RHS proved.

e. $\cos^4\left(\frac{\pi}{8}\right) + \cancel{\sin^4}\cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right) = \frac{3}{2}$

LHS

$\cos^4\left(\frac{\pi}{8}\right) + \cancel{\sin^4}\cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5x}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$

$= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{\pi}{8}\right)$

$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos\frac{3\pi}{8} + \cos^4\frac{\pi}{8}$

$= 2\cos^4\frac{\pi}{8} + 2\cos\frac{3\pi}{8}$

$= 2\left[\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right]$

$= 2\left[\left(\cos^2\frac{\pi}{8}\right)^2 + \left(\cos^2\frac{3\pi}{8}\right)^2\right]$

$= \cancel{2\left[\left(\frac{1+\cos 2\frac{\pi}{8}}{2}\right)^2 + \left(\frac{1+\cos 2\cdot 3\pi/8}{2}\right)^2\right]}$

$= 2\left(\frac{\cos 45^\circ + 1}{2}\right)^2 + 2\left(\cos^2\frac{3\pi}{8}\right)^2$

$= \frac{2}{4}\left(\frac{1}{\sqrt{2}} + 1\right)^2 + 2\left(\frac{\cos 3\times 45 + 1}{2}\right)^2$

$= \frac{2}{4}\left(\frac{1}{\sqrt{2}} + 1\right)^2 + \frac{2}{4}\left(-\frac{1}{\sqrt{2}} + 1\right)^2$

$= \frac{1}{2}\left(\frac{1}{2} + 2\cdot\frac{1}{\sqrt{2}} + 1\right) + \frac{1}{2}\left(1 - 2\frac{1}{\sqrt{2}} + \frac{1}{2}\right)$

$= \frac{1}{4} + \cancel{\frac{1}{\sqrt{2}}} + \frac{1}{2} + \frac{1}{2} - \cancel{\frac{1}{\sqrt{2}}} + \frac{1}{4}$

$= \frac{1}{2} + 1$

$= \frac{3}{2}$ RHS

$\cos 2\cdot\frac{\pi}{8} = 2\cos^2\frac{\pi}{8} - 1$

$\cos\frac{\pi}{4} + 1 = 2\cos^2\frac{\pi}{8}$

$\frac{\cos 45^\circ + 1}{2} = \cos^2\frac{\pi}{8}$

f. $\sin^6\left(\frac{\pi}{8}\right) + \sin^6\left(\frac{3\pi}{8}\right) + \sin^6\left(\frac{5\pi}{8}\right) + \sin^6\left(\frac{7\pi}{8}\right) = \frac{5}{4}$

Soln,,

LHS

$\sin^6\left(\frac{\pi}{8}\right) + \sin^6\left(\frac{3\pi}{8}\right) + \sin^6\left(\frac{5\pi}{8}\right) + \sin^6\left(\frac{7\pi}{8}\right)$

$= \sin^6\left(\frac{\pi}{8}\right) + \sin^6\left(\frac{3\pi}{8}\right) + \left\{\sin\left(\frac{\pi}{2} + \frac{\pi}{8}\right)\right\}^6 + \left\{\sin\left(\frac{\pi}{2} + \frac{3\pi}{8}\right)\right\}$

$= \sin^6\left(\frac{\pi}{8}\right) + \sin^6\left(\frac{3\pi}{8}\right) + \cos^6\frac{\pi}{8} + \cos^6\frac{3\pi}{8}$

$= \left(\sin^2\frac{\pi}{8}\right)^3 + \left(\cos^2\frac{\pi}{8}\right)^3 + \left(\sin^2\frac{3\pi}{8}\right)^3 + \left(\cos^2\frac{3\pi}{8}\right)^3$

$= \left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right)\left(\sin^4\frac{\pi}{8} - \sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right)$

$+ \left(\sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8}\right)\left(\sin^4\frac{3\pi}{8} - \sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right)$

$= 1\cdot\left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right)^2 - 2\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8} + 1\left(\sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8}\right)^2$

$- 2\sin^2\frac{3\pi}{8}\cdot\cos^2\frac{3\pi}{8} - \sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8} - \sin^2\frac{3\pi}{8}\cdot\cos^2\frac{3\pi}{8}$

$= 1 - \frac{3}{4}\left(2\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}\right)^2 + 1 - \frac{3}{4}\left(2\sin\frac{3\pi}{8}\cdot\cos\frac{3\pi}{8}\right)^2$

$= 2 - \frac{3}{4}\sin^2 2\cdot\frac{\pi}{8} - \frac{3}{4}\sin^2 2\cdot\frac{3\pi}{8}$

$= 2 - \frac{3}{4}\left(\sin^2 45^\circ + \sin^2 135^\circ\right)$

$= 2 - \frac{3}{4}\left\{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2\right\}$

$= 2 - \frac{3}{4}\left(\frac{1}{2} + \frac{1}{2}\right)$

$= \frac{8-3}{4} = \frac{5}{4}$ RHS Proved,,

g. $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

LHS

$\cos 5\theta$

$= \cos(3\theta + 2\theta)$

$= \cos 3\theta \cdot \cos 2\theta - \sin 3\theta \cdot \sin 2\theta$

$= (4\cos^3\theta - 3\cos\theta)(\cos^2\theta - \sin^2\theta) - (3\sin\theta - 4\sin^3\theta)(2\sin\theta\cdot\cos\theta)$

$= 4\cos^5\theta - 4\cos^3\theta\cdot\sin^2\theta - 3\cos^3\theta + 3\cos\theta\cdot\sin^2\theta - 6\sin^2\theta\cdot\cos\theta + 8\sin^4\theta\cdot\cos\theta$

$= 4\cos^5\theta - 4\cos^3\theta(1-\cos^2\theta) - 3\cos^3\theta + 3\cos\theta\cdot(1-\cos^2\theta) \cancel{- 6(1-\cos}$
$- 6(1-\cos^2\theta)\cdot\cos\theta + 8(1-\cos^2\theta)^2\cdot\cos\theta$

$= 4\cos^5\theta - 4\cos^3\theta + 4\cos^5\theta - 3\cos^3\theta + 3\cos\theta - 3\cos^3\theta$
$- 6\cos\theta + 6\cos^3\theta + 8\cos\theta \cancel{- 8\cos^3\theta} - 16\cos^3\theta + 8\cos^5\theta$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

RHS proved

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h. $\frac{2\sin A}{\cos 3A} + \frac{2\sin 3A}{\cos 9A} + \frac{2\sin 9A}{\cos 27A} = \tan 27A - \tan A$

LHS

$\frac{2\sin A}{\cos 3A} + \frac{2\sin 3A}{\cos 9A} + \frac{2\sin 9A}{\cos 27A}$

$= \frac{2\sin A\cdot\cos A}{\cos 3A\cdot\cos A} + \frac{2\sin 3A\cdot\cos 3A}{\cos 9A\cdot\cos 3A} + \frac{2\sin 9A\cdot\cos 9A}{\cos 27A\cdot\cos 9A}$

$= \frac{\sin 2A}{\cos 3A\cdot\cos A} + \frac{\sin 6A}{\cos 9A\cdot\cos 3A} + \frac{\sin 18A}{\cos 27A\cdot\cos 9A}$

$= \frac{\sin(3A - A)}{\cos 3A\cdot\cos A} + \frac{\sin(9A - 3A)}{\cos 9A\cdot\cos 3A} + \frac{\sin(27A - 9A)}{\cos 27A\cdot\cos 9A}$

$= \frac{\sin 3A\cdot\cos A - \cos 3A\cdot\sin A}{\cos 3A\cdot\cos A} + \frac{\sin 9A\cdot\cos 3A - \cos 9A\cdot\sin 3A}{\cos 9A\cdot\cos 3A}$
$+ \frac{\sin 27A\cdot\cos 9A - \cos 27A\cdot\sin 9A}{\cos 27A\cdot\cos 9A}$

$$= \frac{\sin 3A \cdot \cancel{\cos A}}{\cos 3A \cdot \cancel{\cos A}} - \frac{\cancel{\cos 3A} \cdot \sin A}{\cancel{\cos 3A} \cdot \cos A} + \frac{\sin 9A \cdot \cancel{\cos 3A}}{\cos 9A \cdot \cancel{\cos 3A}}$$

$$- \frac{\cancel{\cos 9A} \cdot \sin 3A}{\cancel{\cos 9A} \cdot \cos 3A} + \frac{\sin 27A \cdot \cancel{\cos 9A}}{\cos 27A \cdot \cancel{\cos 9A}} - \frac{\cancel{\cos 27A} \cdot \sin 9A}{\cancel{\cos 27A} \cdot \cos 9A}$$

$$= \cancel{\tan 3A} - \tan A + \cancel{\tan 9A} - \cancel{\tan 3A} + \tan 27A - \cancel{\tan 9A}$$

$$= \tan 27A - \tan A$$

RHS proved

j. $(1 + \sec 2A)(1 + \sec 4A)(1 + \sec 8A) = \tan 8A \cdot \cot A$

LHS

$(1 + \sec 2A)(1 + \sec 4A)(1 + \sec 8A)$

$$= \left(1 + \frac{1}{\cos 2A}\right)\left(1 + \frac{1}{\cos 4A}\right)\left(1 + \frac{1}{\cos 8A}\right)$$

$$= \frac{\cos 2A + 1}{\cos 2A} \cdot \frac{\cos 4A + 1}{\cos 4A} \cdot \frac{\cos 8A + 1}{\cos 8A}$$

$$= \frac{2\cos^2 A - 1 + 1}{\cos 2A} \cdot \frac{2\cos^2 2A - 1 + 1}{\cos 4A} \cdot \frac{2\cos^2 4A - 1 + 1}{\cos 8A}$$

$$= \frac{8\cos^2 A \cdot \cos^{\cancel{2}} 2A \cdot \cos^{\cancel{2}} 4A}{\cancel{\cos 2A} \cdot \cancel{\cos 4A} \cdot \cos 8A}$$

$$= \frac{8\cos^2 A \cdot \cos 2A \cdot \cos 4A}{\cos 8A} \cdot \frac{\sin A}{\sin A}$$

$$= \frac{4(2\cos A \cdot \sin A) \cdot \cancel{\sin A} \cdot \cos 2A \cdot \cos 4A \cdot \cos A}{\cos 8A \cdot \sin A}$$

$$= \frac{4\sin 2A \cdot \cos 2A \cdot \cos 4A}{\cos 8A} \cdot \frac{\cancel{\sin A}\,\cos A}{\sin A}$$

$$= \frac{2 \cdot \sin 4A \cdot \cos 4A}{\cos 8A} \cdot \frac{\cos A}{\sin A}$$

$$= \frac{\sin 8A}{\cos 8A} \cdot \cot A$$

$= \tan 8A \cdot \cot A$ RHS proved