Chapter 15

# Differential Equations

## Exercise 15.1

1. **Determine the order and the degree of following differential equations:**

**a)** $2x\frac{dy}{dx} + 3y = 0$

**Sol$^n$:** Since the equation consists highest order derivative $\frac{dy}{dx}$ and its power 1

$\therefore$ order of the differential equation = 1

degree of the differential equation = 1

**b)** $\left(\frac{dy}{dx}\right)^2 = 4x$

**Sol$^n$:** Since the equation consists highest order derivative $\frac{dy}{dx}$ and its power 2

$\therefore$ order of the differential equation = 1

degree of the differential equation = 2

**c)** $\frac{d^2y}{dx^2} + 2x = 0$

**Sol$^n$:** Since the equation consists highest order derivative $\frac{d^2y}{dx^2}$ and its power 1

$\therefore$ order of the differential equation = 2

degree of the differential equation = 1

**d)** $x\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + 5y = 0$

**Sol$^n$:** Since the equation consists highest order derivative $\frac{d^3y}{dx^3}$ and its power 2

$\therefore$ order of the differential equation = 3

degree of the differential equation = 2

**e)** $\left(\frac{d^3y}{dx^3}\right)^3 + \left(\frac{dy}{dx}\right)^5 + 8x = 0$

**Sol$^n$:** Since the equation consists highest order derivative $\frac{d^3y}{dx^3}$ and its power 3

$\therefore$ order of the differential equation = 3

degree of the differential equation = 3

**2. Solve the differential equations of the following curves.**

**a) y = mx**

$\Rightarrow \quad y = \frac{dy}{dx}x \qquad\qquad \Rightarrow x\frac{dy}{dx} - y = 0$

**b) $x^2 + y^2 = a^2$**

Differentiating both side w.r.to x

$2x + 2y\frac{dy}{dx} = 0 \qquad\qquad \Rightarrow x + y\frac{dy}{dx} = 0$

**c) $y = 2x + ax^2$**

Differentiating both side w.r.to x

$\frac{dy}{dx} = 2 + 2ax \qquad \Rightarrow \frac{dy}{dx} - 2 = 2ax \qquad \Rightarrow a = \frac{1}{2x}\left(\frac{dy}{dx} - 1\right)$

$\therefore \quad y = 2x + ax^2$

$\Rightarrow \quad y = 2x + \frac{1}{2x}\left(\frac{dy}{dx} - 1\right)x^2 \quad \Rightarrow y = 2x + \frac{1}{2}\frac{dy}{dx}x - x$

$\Rightarrow \quad y = x + \frac{x}{2}\frac{dy}{dx} \qquad\qquad \Rightarrow x\frac{dy}{dx} - 2y + 2x = 0$

**d) $y = ae^x + be^{-x}$**

Differentiating both side w.r.to x

$\frac{dy}{dx} = ae^x - be^{-x}$

Again, differentiating both sides w.r.to x

$\frac{d^2y}{dx^2} = ae^x + be^{-x} \qquad \Rightarrow \frac{d^2y}{dx^2} = y \qquad \Rightarrow \frac{d^2y}{dx^2} - y = 0$

**3. Verify that**

**a) $y = ax^n$ is the solution of $x\frac{dy}{dx} = ny$**

Here, $y = ax^n$ .... (i)

$\Rightarrow \quad \frac{dy}{dx} = nax^{n-1} \qquad \Rightarrow x\frac{dy}{dx} = nax^n \qquad \Rightarrow x\frac{dy}{dx} = ny$ (using i)

$\therefore \quad y = ax^n$ is the solution of $x\frac{dy}{dx} = ny$

**b) $y = ae^x$ is the solution of $\frac{dy}{dx} - y = 0$**

Here, $y = ae^x$ .... (i)

$\Rightarrow \quad \frac{dy}{dx} = ae^x \qquad \Rightarrow \frac{dy}{dx} = y$ (using i) $\quad \Rightarrow \frac{dy}{dx} - y = 0$

$\therefore \quad y = ae^x$ is the solution of $\frac{dy}{dx} - y = 0$

c) **y = ax + b is the solution of $\frac{d^2y}{dx^2} = 0$**

Here, $y = ax + b$

$\Rightarrow \quad \frac{dy}{dx} = a$

$\Rightarrow \quad \frac{d}{dx}\frac{dy}{dx} = 0 \qquad \Rightarrow \frac{d^2y}{dx^2} = 0$

$\therefore \quad y = ax + b$ is the solution of $\frac{d^2y}{dx^2} = 0$

d) **y -x + 1 = 0 is the solution of $(x + y)\, dy - (x^2 - y^2)dx = 0$**

Here, $y - x + 1 = 0$

$\Rightarrow \quad \frac{dy}{dx} - 1 = 0 \qquad \Rightarrow \frac{dy}{dx} = 1$

$\Rightarrow \quad \frac{dy}{dx} = x - y \qquad \Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{x + y}$

$\Rightarrow \quad (x + y)\, dy = (x^2 - y^2)dx \qquad \Rightarrow (x + y)\, dy - (x^2 - y^2)dx = 0$

$\therefore \quad y - x + 1 = 0$ is the solution of $(x + y)\, dy - (x^2 - y^2)dx = 0$