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Chapter-10
Thermal Expansion

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1. Thermal Expansion

Thermal expansion is the tendency of matter to change its shape, area, volume and density in response to a change in temperature, usually not including phase transitions.

Examples:
- Railways tracks are provided with gaps
- Electric wires kept ~~sak~~ sagging between poles during summer
- Tight lid of bottle can be loosening by running hot water over it

2. Laws of ~~Thermal~~ Expansion

There are mainly 3 laws of expansion. They are;

i. ~~Linear~~ Law of linear expansion

If increase in size is considered in one dimension, expansion is called linear expansion.

ii. Law of Superficial Expansion

Law of superficial expansion state that change in area is directly proportional to;

i) Original Area (A_1)

i.e. $\Delta A \propto A_1$ — (i)

(ii) Change in temperature

i.e. $\Delta A \propto \Delta\theta$, $\Delta\theta = \theta_2 - \theta_1$ — (ii)

Now, combining equation (i) and (ii), we get,

$$\Delta A \propto A_1 \Delta\theta$$

$$\text{or, } \Delta A = \beta A_1 \Delta\theta \quad \text{— (iii)}$$

when β = Coefficient of superficial expansion

$$\beta = \frac{\Delta A}{A_1 \Delta\theta}$$

from equation (iii)

$$\Delta A = A_2 - A_1$$

$$\therefore A_2 - A_1 = \beta A_1 \Delta\theta$$

$$\text{or, } A_2 = A_1 + \beta A_1 \Delta\theta$$

or, $A_2 = A_1 (1 + \beta \Delta \theta)$

iii. Law of cubical Expansion

Law of cubical expansion state that change in volume is directly proportional to ;

i) Original Volume (V_1)

i.e. $\Delta V \propto V_1$ —— (i)

ii) Change in temperature

i.e. $\Delta V \propto (\theta_2 - \theta_1)$ —— (ii)

Combining equation (i) and (ii)

$\Delta V \propto V_1 (\theta_2 - \theta_1)$

or, $\Delta V = \gamma V_1 (\theta_2 - \theta_1)$ —— (iii)

When,

γ = coefficient of cubical expansion

$$\gamma = \frac{\Delta V}{V_1 \Delta \theta}$$

From equation (iii), we get

$\Delta V = V_2 - V_1$

$\therefore V_2 - V_1 = \gamma A_1 \Delta \theta$

or, $V_2 = V_1 + \gamma V_1 \Delta \theta$

or, $V_2 = V_1 (1 + \gamma \Delta \theta)$

Q. An aluminium square plate of length 1 m at 10°c is heated to temperature 50°c. Find its final length and area. Given coefficient of linear expansion $\alpha_a = 2.4 \times 10^{-5} °c^{-1}$

Solution:

Given, Initial length $(l_1) = 1$ m

Initial temperature $(\theta_1) = 10°c$

Final temperature $(\theta_2) = 50°c$

Linear expansion of aluminium $(\alpha_a) = 2.4 \times 10^{-5} °c^{-1}$

Final length $(L_2) = ?$

Area $(A) = ?$

We know,

$$\alpha = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$$

or, $$2.4 \times 10^{-5} = \frac{l_2 - 1}{1(50-10)}$$

or, $$2.4 \times 10^{-5} = \frac{l_2 - 1}{40}$$

or, $9.6 \times 10^{-4} + 1 = l_2$

$\therefore l_2 = 1.00096$ m

Also,

Area (A) = $l^2 = (1.00096)^2\ m^2$

$= 1.0019209216\ m^2$

Relation between α and γ

Consider a cube of side l_1 and volume V_1 at θ_1°c. Let 'l_2' and 'V_2' be the length and volume at θ_2°c.

we have,

$$l_2 = l_1(1 + \alpha\Delta\theta) \text{ --- (i) } [\because \Delta\theta = \theta_2 - \theta_1]$$

$$V_2 = V_1(1 + \gamma\Delta\theta) \text{ --- (ii)}$$

Again,

we have, $V_2 = l_2^3 = \{l_1(1+\alpha\Delta\theta)\}^3$

$= l_1^3 (1+\alpha\Delta\theta)^3$

$= V_1\{1^3 + 3.1^2.\alpha\Delta\theta + 3.1.(\alpha\Delta\theta)^2 + (\alpha\Delta\theta)^3\}$

$= V_1\{1 + 3\alpha\Delta\theta + \cancel{3(\alpha\Delta\theta)^2} + \cancel{(\alpha\Delta\theta)^3}\}$

$\therefore V_2 = V_1(1 + 3\alpha\Delta\theta)$ --- (iii)

Now,

Comparing (ii) and (iii) we get

$V_2 = V_1(1 + 3\alpha\Delta\theta)$

$V_2 = 3\alpha$

Relation between α and β

Consider a square of length l_1 and area A_1 at θ_1°c and l_2 and A_2 be the length and area at θ_2°c.

We have,

$$l_2 = l_1 (1 + \alpha \Delta\theta) \quad \text{— (i)}$$

$$A_2 = A_1 (1 + \beta \Delta\theta) \quad \text{— (ii)}$$

Again we have,

$$A_2 = l_2^2 = \{l_1(1+\alpha\Delta\theta)\}^2$$

$$= l_1^2 (1 + \alpha\Delta\theta)^2$$

$$= A_1 (1^2 + 2 \cdot 1 \alpha\theta + \cancel{\alpha^2 \Delta\theta^2})$$

Neglating higher order term of α

$$\text{or, } A_2 = A_1 (1 + 2\alpha\Delta\theta) \quad \text{— (iii)}$$

Now,

Comparing equation (ii) and (iii)

$$\therefore \beta = 2\alpha$$

Q. A brass rod of length 0.40 metre and steel rod of length 0.60 metre. Both are initially at 0°c are heated to 85°c. If the increase in length is the same for both the rods. Calculate the linear expansivity of brass. The linear expansivity of steel is 12×10^{-6} °c^{-1}.

Solution:

Initial length of brass rod $(l_1^b) = 0.40$ m

Initial length of steel rod $(l_1^s) = 0.60$m

Initial temperature $(\theta_1) = 0$°c $= 273$ K

Final temperature $(\theta_2) = 85$°c $= 358$ K

Linear expansion of brass $(\alpha^b) = ?$

Linear expansion of steel $(\alpha^s) = 12 \times 10^{-6}$ °c^{-1}

$= 12 \times 10^{-6}$ k^{-1}

we know

$$\cancel{\frac{\alpha^b}{\alpha^s} = \frac{l^s}{l^b}}$$

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Change in length $(l_2^b - l_1^b = \Delta l^b)$ or $(l_2^s - l_1^s = \Delta l^s) = \Delta l$

i.e. $\Delta l^b = \Delta l^s = \Delta l$ (let)

We have,

$$\Delta l^b = \Delta l^s \quad [\because \Delta l = \alpha l_1 \Delta\theta]$$

or, $\alpha_b l_1^b \Delta\theta^b = \alpha_s l_1^s \Delta\theta^s$

or, $\alpha b = \dfrac{\alpha^s l_1^s \Delta\theta^s}{l_1^b \Delta\theta^b} \quad [\because \Delta\theta^s = \Delta\theta^b]$

$$= \frac{12\times10^{-6} \times 0.60}{0.40}$$

$$= 18\times10^{-6}\ k^{-1}$$

Hence, the co-efficient of linear expansivity of brass is $18\times10^{-6}\ k^{-1}$.

Q. The length of an iron rod is measured by a brass scale. When both of them are at 10°c, the measured length is 50 cm. What is the length of the rod at 40°c when measured by the brass scale at 40°c? (α for brass $= 24\times10^{-6}\ °c^{-1}$, α for iron $= 16\times10^{-6}\ °c^{-1}$)

Solution:

Initial temperature $(\theta_1) = 10°c$

Final temperature $(\theta_2) = 40°c$

Linear expansivity of brass $(\alpha_b) = 24\times10^{-6}\ °c^{-1}$

Linear expansivity of iron $(\alpha_i) = 16\times10^{-6}\ °c^{-1}$

We have,

Initial length of brass = Initial length of iron (l_1) = 50 cm

Final length of iron rod $(l_2) = ?$

We know,

$$l_2 = l_1[1+\alpha_i(\theta_2-\theta_1)]$$

$$= 50[1+16\times10^{-6}(40-10)]$$

$$= 50.024\text{ cm}$$

~~∴ The l~~

Again,

Length of 1 cm division of brass scale at 40°c

$= 1[1+24\times10^{-6}(40-10)]$

$= 1.00072$ cm

Now,

length of the rod measured by brass scale at 40°c

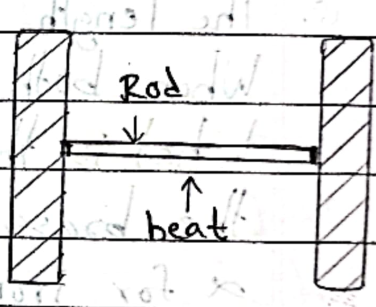
$= \dfrac{50.024}{1.00072}$

$= 49.988$ cm

∴ The length of iron rod at 40°c is 49.988 cm.

Force set up due to Expansion or Contraction and Thermal Stress.

Consider a metallic rod of length l_1 and CSA 'A' is fixed between two rigid supports.



Let,

l_1 = Length of rod at θ_1°c

l_2 = Length of rod at θ_2°c

$\alpha = \dfrac{\Delta l}{l_1 \Delta\theta}$

or, $\Delta l = \alpha l_1 \Delta\theta$ —— (i)

Also,

$Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{\Delta l / l_1}$ [∵ Y = Young's Modulus]

$Y = \dfrac{F}{A} \times \dfrac{l_1}{\Delta l}$

or $F = \dfrac{YA\Delta l}{l_1}$ —— (ii)

from equation (i) and (ii)

$F = \dfrac{YA\alpha l_1 \Delta\theta}{l_1}$

$F = YA\alpha\Delta\theta$

Q. A wire that is 1.5 m long at 20°c is found to increase in length by 1.9 cm when warmed to 420°c (a) Compute its average coefficient of linear expansion for this temperature range. (b) The wire is stretched just taut (zero tension) at 420°c. Find the stress in the wire if it is cooled to 20°c without being allowed to contract. (Young's modulus for the wire is 2×10^{11} Pa).

Solution:

Initial length $(l_1) = 1.5$ m

Change in length $(\Delta l) = 1.9$ cm

$= 1.9\times10^{-2}$ m

Change in temperature $(\Delta\theta) = (420 - 20°c)$

$= 400°c$

Coefficient of linear expansion $(\alpha) = ?$

Thermal stress $= ?$

We know,

$$\alpha = \frac{\Delta l}{l_1 \Delta\theta}$$

$$= \frac{1.9\times10^{-2}}{1.5\times400}$$

$$= 3.17\times10^{-5}\ k^{-1}$$

Also,

Young's Modulus $(Y) = 2\times10^{11}$ Pascal

Then,

$$\text{Stress} = \frac{F}{A} = \cancel{\frac{Y A \alpha \Delta\theta}{A}}\ Y\alpha\Delta\theta$$

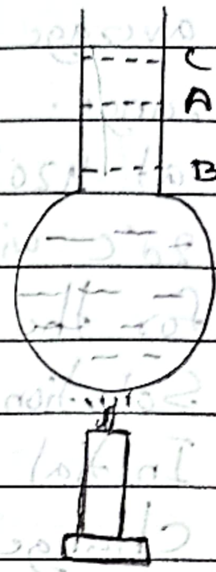
$$= 2\times10^{11}\times3.17\times10^{-5}\times400$$

$$= 2.54\times10^{9}\ N/m^2$$

Hence, the coefficient of linear expansion and thermal stress are $3.17\times10^{-5}\ k^{-1}$ and $2.54\times10^{9}\ N/m^2$ respectively.

Expansion of liquid (Real and Apparent expansion).

Solution:



Let V_1 = Initial volume of liquid

$BC = AB + AC$

BC = Real change in volume

AB = change in volume of glass vessel.

AC = Apparent change in volume

Again,

Assume,

BC = ΔV_r = Real change in volume

i.e. $\Delta V_r = \gamma_r V_1 \Delta\theta$ —— (i)

γ_r = Coefficient of real expansion of volume of liquid.

AC = ΔV_a = Apparent change in volume

i.e. $\Delta V_a = \gamma_a V_1 \Delta\theta$ —— (ii)

AB = ΔV_g = Change in volume of glass vessel

i.e. $\Delta V_g = \gamma_g V_1 \Delta\theta = 3\alpha_g V_1 \Delta\theta$ —— (iii) $(\because \gamma_g = 3\alpha_g)$

We have,

$BC = AB + AC$

or $\Delta V_r = \Delta V_g + \Delta V_a$

or $\gamma_r V_1 \Delta\theta = \gamma_g V_1 \Delta\theta + \gamma_a V_1 \Delta\theta$

or $\gamma_r = \gamma_g + \gamma_a$

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Variation of density with temperature.

Solution:

Let us consider a substance of mass 'm'.

Let ~~us~~ V_1 be the volume of that given substance at temperature 'θ_1'. then it's density (ρ_1) is given by;

$$\rho_1 = \frac{m}{V_1} \quad \text{---} (i)$$

Let the given substance is heated to θ_2 temperature then it's corresponding volume will be V_2 and it's density ρ_2 is given by;

$$\rho_2 = \frac{m}{V_2} \quad \text{---} (ii)$$

from cubical Expansion,

$$V_2 = V_1 (1 + \gamma \Delta\theta) \quad \text{---} (iii)$$

Now,

Using equation (iii) in (ii) we get;

$$\rho_2 = \frac{m}{V_1 (1 + \gamma \Delta\theta)}$$

or, $$\rho_2 = \frac{\rho_1}{1 + \gamma \Delta\theta}$$

$\therefore$ Hence, density decreases with increase in temperature.

Thermal Expansion

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1. Linear Expansion

Let a metal rod having Length 'L_1' at temp. θ_1°c. When it is heated to θ_2°c then its corresponding length will be 'L_2'

here,

$$\Delta L = L_2 - L_1$$

$$\Delta\theta = \theta_2 - \theta_1$$

We know,

$$\Delta L \propto L_1 \quad \text{--- (i)}$$

$$\Delta L \propto \Delta\theta \quad \text{--- (ii)}$$

Combining equation (i) and (ii)

$$\Delta L \propto L_1 \Delta\theta$$

or, $\Delta L = \alpha L_1 \Delta\theta$

Where α is the coefficient of linear expansion.

$$\alpha = \frac{\Delta L}{L_1 \Delta\theta}$$

α is defined as the ratio of change in length per unit degre in rise in temperature.

$$\Delta L = \alpha L_1 \Delta\theta$$

or, $L_2 - L_1 = \alpha L_1 \Delta\theta$

$$L_2 = L_1 + \alpha L_1 \Delta\theta$$

$$\therefore L_2 = L_1(1 + \alpha\Delta\theta)$$

2. Superficial Expansion

Let us suppose a metallic sheet having area 'A_1' at θ_1°c.
When temperature reaches θ_2 its corresponding area will be 'A_2'.
Experiment shows that;

$(A_2 - A_1) \propto A_1$ —— (i)

$(A_2 - A_1) \propto \Delta\theta$ —— (ii)

Combining equation (i) and (ii)

$\cancel{A_1 \propto \Delta\theta}$

~~or,~~ $\cancel{A_1 \propto (\theta_2 - \theta_1)}$

$(A_2 - A_1) \propto A_1 \Delta\theta$

or, $(A_2 - A_1) = \beta A_1 \Delta\theta$ [$\because \beta$ = Coefficient of superficial expansivity]

or, $A_2 = A_1 + \beta A_1 \Delta\theta$

or, $A_2 = A_1 (1 + \beta\Delta\theta)$

$\therefore A_2 = A_1 (1 + \beta\Delta\theta)$

3. Cubical Expansion

Let us suppose a metallic cube of volume 'V_1' at θ_1°c and when it is heated to θ_2°c its corresponding temperature is θ_2°c
Experiment shows that;

$(V_2 - V_1) \propto V_1$ —— (i)

$(V_2 - V_1) \propto (\theta_2 - \theta_1)$ —— (ii)

Combining (i) and (ii)

$(V_2 - V_1) \propto V_1 (\theta_2 - \theta_1)$

or, $(V_2 - V_1) = \gamma V_1 (\theta_2 - \theta_1)$ [$\because \gamma$ = Coefficient of cubical expansion

or, $V_2 = V_1 + \gamma V_1 (\theta_2 - \theta_1)$

$\therefore V_2 = V_1 (1 + \gamma\Delta\theta)$

Determination of Linear Expansivity of a Solid by Pullinger's Apparatus.

For this experiment a metallic rod is kept inside a hollow cylinder having three openings. One for keeping thermometer and other two for steam in and out. An electric circuit containing battery and ~~galvom~~ galvanometer is connected in the terminals along with a spherometer

Now,

Let, L_1 = Original length of rod

θ_1 = Initial temperature of rod

S_1 = Initial reading of spherometer

θ_2 = Final temperature of rod

S_2 = final reading of spherometer

We know,

Change in length of rod, $\Delta L = \cancel{L_2 - L_1}\ S_2 - S_1$

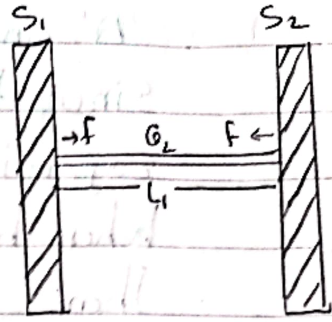
Change in temperature, $\Delta\theta = \theta_2 - \theta_1$

Since, L_1 is the initial/original length of rod then its linear expansitivity is given by;

$$\alpha = \frac{\Delta L}{L_1 \Delta\theta}$$

or, $$\alpha = \frac{S_2 - S_1}{L_1(\theta_2 - \theta_1)}$$

Force Set up due to Expansion or Contraction and Thermal stress



Let us consider a metallic rod 'L_1' at temperature θ_1°c which is fixed between two rigid substance S_1 and S_2 as shown in figure.

When it is heated to θ_2°c its final length will be L_2

from Linear Expansion

$$L_2 = L_1(1+\alpha\Delta\theta)$$

or, $L_2 = L_1 + L_1\alpha\Delta\theta$

or, $L_2 - L_1 = L_1\alpha\Delta\theta$ ——(1)

To prevent the linear Expansion of the rod, the rigid support apply certain force which is equal to force setup due to expansion of rod. The corresponding stress is called ~~Young~~ thermal stress.

We have,

from Young's Modulus;

$$Y = \frac{\text{Normal stress}}{\text{Logitudinal strain}}$$

or, $Y = \dfrac{F/A}{(L_2-L_1)/L_1}$

or, $Y = \dfrac{F}{A} \times \dfrac{L_1}{L_2-L_1}$

or, $Y = \dfrac{F}{A} \times \dfrac{L_1}{L_1\alpha\Delta\theta}$ $\quad [\because L_1 = L_1\alpha\Delta\theta]$

or, $Y.A.\alpha\Delta\theta = F$

$\therefore F = YA\alpha\Delta\theta$