UNIT 7 : COMPUTATIONAL METHODS

Chapter 16

System of Linear Equations

Exercise 16.1

1. **Solve the following system of equations using Gauss Elimination method:**

a) $x + 2y = 5,\ 5x - 3y = -1$

Sol^n: $x + 2y = 5$ ……………. (i)

$5x - 3y = -1$ ……………. (ii)

Multiplying equation (i) by 5 and subtracting from (ii), we get

$-13y = -26$ ……… (iii)

Now, we have the following equations

$x + 2y = 5$ ……. (i)

$-13y = -26$ ……… (iii)

From eq. (iii), $-13y = -26$ $\Rightarrow$ $y = 2$

Substituting y = 2 in (i), we get

$x + 2 \times 2 = 5$ $\Rightarrow$ $x = 1$

$\therefore$ $x = 1, y = 2$

b) $3x - 2y = 5,\ 4x - y = 10$

Sol^n: $3x - 2y = 5$ …….. (i)

$4x - y = 10$ …….. (ii)

Multiplying eq. (i) by $\frac{4}{3}$ and subtracting from (ii), we get

$\frac{5}{3}y = \frac{10}{3}$ ……… (iii)

Now, we have the following equations,

$3x - 2y = 5$ …….. (i)

$\frac{5}{3}y = \frac{10}{3}$ …….. (iii)

From eq. (iii), $\frac{5}{3}y = \frac{10}{3}$ $\Rightarrow y = 2$

Substituting y = 2 in (i), we get

$3x - 2 \times 2 = 5$ $\Rightarrow$ $x = 3$

$\therefore$ $x = 3, y = 2$

c) $2x + 7y = 3,\ 6x - 5y = -17$

Sol^n: $2x + 7y = 3$ ……… (i)

$6x - 5y = -17$ ………. (ii)

Multiplying eq. (i) by 3 and subtracting from (ii), we get

$-26y = -26$ .... (iii)

Now, we have the following equations,

$2x + 7y = 3$ .... (i)

$-26y = -26$ .... (iii)

From equation (iii), $-26y = -26$ $\Rightarrow y = 1$

Substituting $y = 1$ in (i), we get

$2x + 7\times1 = 3$ $\Rightarrow x = -2$

$\therefore$ $x = -2, y = 1$

d) $5x - 8y = 28, 3x + 7y = 5$

Sol^n: $5x - 8y = 28$ .... (i)

$3x + 7y = 5$ .... (ii)

Multiplying eq. (i) by $\frac{3}{5}$ and subtracting from (ii),

$\frac{59y}{5} = -\frac{59}{5}$ .... (iii)

Now, we have the following equations

$5x - 8y = 28$ .... (i)

$\frac{59y}{5} = -\frac{59}{5}$ .... (iii)

From eq. (iii), $\frac{59y}{5} = -\frac{59}{5}$ $\Rightarrow y = -1$

Substituting $y = -1$ in (i), we get

$5x - 8 \times (-1) = 28$ $\Rightarrow x = 4$

$\therefore$ $x = 4, y = -1$

2. Solve the following system of equations using Gaussian elimination method

a) $x + 3y - 2z = 0, 2x - 3y + z = 1, 4x - 3y + z = 3$

Sol^n: $x + 3y - 2z = 0$ .... (i)

$2x - 3y + z = 1$ .... (ii)

$4x - 3y + z = 3$ .... (iii)

Multiplying equation (i) by 2 and subtracting from (ii), we get

$9y - 5z = -1$ .... (iv)

Again multiplying equation (i) by 4 and subtracting from (iii), we get

$5y - 3z = -1$ .... (v)

Multiplying equation (iv) by $\frac{5}{9}$ and subtracting from (v), we get

$-\frac{2}{9}z = -\frac{4}{9}$ .... (vi)

Now, we have the following equation

$x + 3y - 2z = 0$ .... (i)

$5y - 3z = -1$ .... (v)

$-\frac{2}{9}z = -\frac{4}{9}$ .... (vi)

From (vi), $-\frac{2}{9}z = -\frac{4}{9}$ $\Rightarrow z = 2$

Substituting the value of z in (v), we get

$5y - 3 \times 2 = -1$ $\Rightarrow y = 1$

Substituting the values of y and z in (i), we get

$x + 3 \times 1 - 2 \times 2 = 0$ $\Rightarrow x = 1$

$\therefore$ $x = 1, y = 1, z = 2.$

b) $\mathbf{x + 3y - z = -2,\ 3x + 2y - z = 3,\ -6x - 4y - 2z = 18}$

Sol^n: $x + 3y - z = -2$ .... (i)

$3x + 2y - z = 3$ ....(ii)

$-6x - 4y - 2z = 18$ .... (iii)

Multiplying eq (i) by 3 and subtracting from (ii), we get

$-7y + 2z = 9$ .... (iv)

Multiplying equation (ii) by 2 and adding (iii), we get

$-4z = 24$ .... (v)

Now, we have the following equations

$x + 3y - z = -2$ .... (i)

$-7y + 2z = 9$ .... (iv)

$-4z = 24$ .... (v)

From (v), $-4z = 24$ $\Rightarrow z = -6$

Substituting the value of z in (iv), we get

$-7y + 2 \times (-6) = 9$ $\Rightarrow y = -3$

Substituting the value of y and z in (v), we get

$x + 3 \times -3 + 6 = -2$ $\Rightarrow x = 1$

$\therefore$ $x = 1, y = -3, z = -6$

c) $\mathbf{x + 3y - 2z = 5,\ 3x + 5y + 6z = 7,\ 2x + 4y + 3z = 8}$

Sol^n: $x + 3y - 2z = 5$ .... (i)

$3x + 5y + 6z = 7$ .... (ii)

$2x + 4y + 3z = 8$ .... (iii)

Multiplying equation (i) by 3 and subtracting from (ii), we get

$-4y + 12z = -8$ .... (iv)

Again multiplying equation (i) by 2 and subtracting from (iii)

$-2y + 7z = -2$ .... (v)

Multiplying equation (iv) by 1/2 and subtracting from (v)

$z = 4$ .... (vi)

Now, we have the following equations

$x + 3y - 2z = 5$ .... (i)

$-2y + 7z = -2$ .... (v)

$z = -4$ .... (vi)

From (vi), $z = 2$

Substituting the value of z in (v), we get

$-2y + 7 \times 2 = -2$ $\Rightarrow y = 8$

Substituting the values of y and z in (i),

$x + 3 \times 8 - 2 \times 2 = 5 \qquad \Rightarrow x = -15$

$\therefore \quad x = -15, y = 8, z = 2.$

d) $2x - 3y + 3z = 27$, $4x + y - 2z = 0$, $-6x - 4y + 2z = 0$

Sol^n: $2x - 3y + 3z = 27$ .... (i)

$4x + y - 2z = 0$ .... (ii)

$-6x - 4y + 2z = 0$ .... (iii)

Multiplying equation (i) by 2 and subtracting from (ii), we get

$7y - 8z = -54$ .... (iv)

Multiplying equation (i) by 3 and adding with (iii), we get

$-13y + 11z = 81$ .... (v)

Multiplying equation (iv) by $\frac{13}{7}$ and adding (v), we get

$-\frac{27}{7}z = -\frac{135}{7}$ .... (vi)

Now, we have the following equation

$2x - 3y + 3z = 27$ .... (i)

$7y - 8z = -54$ .... (iv)

$-\frac{27}{7}z = -\frac{135}{7}$ .... (vi)

From (vi), $-\frac{27}{7}z = -\frac{135}{7} \qquad \Rightarrow z = \frac{135}{27} = 5$

Substituting the value of z in (iv), we get

$7y - 8 \times 5 = -54 \qquad \Rightarrow y = -2$

Substituting the value of y and z in (i), we get

$2x - 3 \times (-2) + 3 \times 5 = 27 \qquad \Rightarrow x = 3$

$\therefore \quad x = 3, y = -2, z = 5$

e) $3x + 2y - z = 1$, $2x - 2y + 4z = -2$, $-x + \frac{1}{2}y - z = 0$

Sol^n: $3x + 2y - z = 1$ ....(i)

$2x - 2y + 4z = -2$ ....(ii)

$-2x + y - 2z = 0$ .... (iii)

Multiplying equation (i) by $\frac{2}{3}$ and subtracting from (ii), we get

$-\frac{10}{3}y + \frac{14}{3}z = -\frac{8}{3}$ ....(iv)

Multiplying equation (i) by $\frac{1}{3}$ and adding(iii), we get

$\frac{7}{6}y - \frac{4}{3}z = -\frac{1}{3}$ ....(v)

Multiplying equation (iv) by $\frac{7}{20}$ and adding (v), we get

$$\frac{9}{30}z = -\frac{9}{15} \qquad \text{......... (vi)}$$

Now, we have the following three equation

$$3x + 2y - z = 1 \qquad \text{.......(i)}$$

$$-\frac{10}{3}y + \frac{14}{3}z = -\frac{8}{3} \qquad \text{....... (iv)}$$

$$\frac{9}{30}z = -\frac{9}{15} \qquad \text{......... (vi)}$$

From (vi), $\frac{9}{30}z = -\frac{9}{15}$ $\qquad \Rightarrow z = -2$

Substituting the value of z in (iv), we get

$$-\frac{10}{3}y + \frac{14}{3} \times (-2) = -\frac{8}{3} \qquad \Rightarrow y = -2$$

Substituting the values of y and z in (i)

$$3x + 2 \times (-2) - (-2) = 1 \qquad \Rightarrow x = 1$$

$\therefore$ $x = 1, y = -2, z = -2$

f) $\mathbf{3x_1 + 6x_2 + x_3 = 16, 2x_1 + 4x_2 + 3x_3 = 13, x_1 + 3x_2 + 2x_3 = 9}$

Sol^n:

$$3x_1 + 6x_2 + x_3 = 16 \qquad \text{........ (i)}$$

$$2x_1 + 4x_2 + 3x_3 = 13 \qquad \text{........ (ii)}$$

$$x_1 + 3x_2 + 2x_3 = 9 \qquad \text{........ (iii)}$$

Multiplying eq. (i) $\frac{2}{3}$ and subtracting from (ii)

$$\frac{7}{3}x_3 = \frac{7}{3} \qquad \text{....... (iv)}$$

Again multiplying eq. (i) by $\frac{1}{3}$ and subtracting from (iii)

$$x_2 + \frac{5}{3}x_3 = \frac{11}{3} \qquad \text{......... (v)}$$

Now, we have the following equations

$$3x_1 + 6x_2 + x_3 = 16 \qquad \text{........ (i)}$$

$$x_2 + \frac{5}{3}x_3 = \frac{11}{3} \qquad \text{......... (v)}$$

$$\frac{7}{3}x_3 = \frac{7}{3} \qquad \text{....... (iv)}$$

From equation (iv), $\frac{7}{3}x_3 = \frac{7}{3}$ $\qquad \Rightarrow x_3 = 1$

Substituting $x_3 = 1$ in (v), we get

$$x_2 + \frac{5}{3} = \frac{11}{3} \qquad \Rightarrow x_2 = \frac{11}{3} - \frac{5}{3} = 2$$

Substituting $x_2 = 2$ and $x_3 = 1$ in (i), we get

$$3x_1 + 6 \times 2 + 1 = 16 \qquad \Rightarrow x_1 = 1$$

$\therefore$ $x_1 = 1, x_2 = 2, x_3 = 1$

g) $\mathbf{x_1 - 2x_2 + 3x_3 = 10, 2x_1 + 3x_2 - 2x_3 = 1, -x_1 - 2x_2 + 4x_3 = 13}$

Sol^n: $x_1 - 2x_2 + 3x_3 = 10$ .... (i)

$2x_1 + 3x_2 - 2x_3 = 1$ .... (ii)

$-x_1 - 2x_2 + 4x_3 = 13$ .... (iii)

Multiplying equation (i) by 2 and then subtracting from (ii), we get

$7x_2 - 8x_3 = -19$ .... (iv)

Adding (i) and (iii), we get

$-4x_2 + 7x_3 = 23$ .... (v)

Multiplying equation (iv) by $\frac{4}{7}$ and adding (v), we get

$\frac{17}{7}x_3 = \frac{85}{7}$ .... (vi)

Now, we have the following equation,

$x_1 - 2x_2 + 3x_3 = 10$ .... (i)

$-4x_2 + 7x_3 = 23$ .... (v)

$\frac{17}{7}x_3 = \frac{85}{7}$ .... (vi)

From (iii), $\frac{17}{7}x_3 = \frac{85}{7}$ $\Rightarrow x_3 = 5$

Substituting the value of x_3 in (ii), we get

$-4 \times x_2 + 7 \times 5 = 23$ $\Rightarrow x_2 = 3$

Substituting the value of x_2 and x_3 in (i), we get

$x_1 - 2 \times 3 + 3 \times 5 = 10$ $\Rightarrow x_3 = 5$

$\therefore$ $x_1 = 1, x_2 = 3, x_3 = 5.$

h) $\mathbf{3x_1 - x_2 + x_3 = 2, -15x_1 + 6x_2 - 5x_3 = 5, 5x_1 - 2x_2 + 2x_3 = 1}$

Sol^n: $3x_1 - x_2 + x_3 = 2$ .... (i)

$-15x_1 + 6x_2 - 5x_3 = 5$ .... (ii)

$5x_1 - 2x_2 + 2x_3 = 1$ .... (iii)

Multiplying equation (i) by 5 and adding with (ii), we get

$x_2 = 15$ ...... (iv)

Multiplying equation (i) by $\frac{5}{3}$ and adding with (ii), we get

$-\frac{1}{3}x_2 + \frac{1}{3}x_3 = -\frac{7}{3}$ .... (v)

Multiplying equation (iv) by $\frac{1}{3}$ and adding (v), we get

$\frac{1}{3}x_3 = \frac{8}{3}$ ……. (vi)

Now, we have following system of equations

$3x_1 - x_2 + x_3 = 2$ .... (i)

$x_2 = 15$ ...... (iv)

$\frac{1}{3}x_3 = \frac{8}{3}$ ……. (vi)

From (vi), $x_3 = 8$

From (iv), $x_2 = 15$

Substituting the values of x_2 and x_3 in (i), we get

$3x_1 - 15 + 8 = 2 \qquad \Rightarrow x_1 = 3$

$\therefore \quad x_1 = 3, x_2 = 15, x_3 = 8$

3. Solve, using Gaussian elimination method with partial pivot, the following equations:

a) $5x + 4y = 9, x - 5y = 25$

Sol^n: Given, $5x + 4y = 9$ .... (i)

$x - 5y = 25$ .... (ii)

"***Gaussian elimination method of partial pivoting***"

Multiplying eq^n (i) by $\frac{1}{5}$ and then subtracting from eq^n (ii), we get

$$-\frac{29}{5}y = \frac{116}{5}$$

Now, we have the following equations

$5x + 4y = 9$ .... (i)

$-\frac{29}{5}y = \frac{116}{5}$ .... (iii)

From eq^n (iii), $-\frac{29}{5}y = \frac{116}{5} \qquad \Rightarrow y = -4$

Substituting $y = -4$ in eq^n (i), we get

$5x + 4 \times (-4) = 9 \qquad \Rightarrow \quad x = 5$

$\therefore \quad x = 5, y = -4$

"***Gaussian elimination method with partial pivot***"

The matrix form of the above equations is,

$$\begin{pmatrix} 5 & 4 & : & 9 \\ 1 & -5 & : & 25 \end{pmatrix}$$

Here, $a_{11} > a_{12}$

Applying $R_2 \rightarrow 5 \times R_2 - R_1$, we get

$$\begin{pmatrix} 5 & 4 & : & 9 \\ 0 & -29 & : & 116 \end{pmatrix}$$

Then, $5x + 4y = 9$ .... (iii)

$-29y = 116$ ……. (iv)

From eq^n (iv), $-29y = 116 \qquad \Rightarrow y = -4$

Substituting $y = -4$ in eq^n (iii), we get

$5x + 4 \times (-4) = 9 \qquad \Rightarrow \quad x = 5$

$\therefore \quad x = 5, y = -4$

b) $x - 8y = -26, 4x + 3y = 1$

Sol^n: Given, $x - 8y = -26$ .... (i)

$4x + 3y = 1$ .... (ii)

The matrix form of the above equations is,

$$\begin{pmatrix} 1 & -8 : -26 \\ 4 & 3 : \ 1 \end{pmatrix}$$

Here, $|a_{11}| < |a_{21}|$

Applying $R_1 \leftrightarrow R_2$ to make $|a_{11}| > |a_{21}|$, we get

$$\begin{pmatrix} 4 & 3 & : & 1 \\ 1 & -8 & : & -26 \end{pmatrix}$$

$R_2 \to 4 \times R_2 - R_1$, we get

$$\begin{pmatrix} 4 & 3 & : & 1 \\ 0 & -35 & : & -105 \end{pmatrix}$$

Then, $4x + 3y = 1$ .... (iii)

$-35y = -105$ .... (iv)

From eq^n (iv), $-35y = -105$ $\Rightarrow y = 3.$

Substituting $y = 3$ in eq^n (iii), we have

$4x + 3 \times 3 = 1$ $\Rightarrow$ $x = -2$

$\therefore$ $x = -2, y = 3$

c) $3x + y - 2z = 10, 2x + 4y - 5z = 24, x - 2y + 3z = -11$

Sol^n: Given, $3x + y - 2z = 10$ .... (i)

$2x + 4y - 5z = 24$ .... (ii)

$x - 2y + 3z = -11$ .... (iii)

The matrix form of the above equations is,

$$\begin{pmatrix} 3 & 1 & -2 : & 10 \\ 2 & 4 & -5 : & 24 \\ 1 & -2 & 3 : & -11 \end{pmatrix}$$

Here, $a_{11} > a_{21}$ and a_{31}

Applying $R_2 \to R_2 - \frac{1}{3} \times R_1$ and $R_3 \to R_3 - \frac{1}{3} \times R_1$, we get

$$\begin{pmatrix} 3 & 1 & -2 : & 10 \\ 0 & \frac{10}{3} & -\frac{11}{3} : & \frac{52}{3} \\ 0 & -\frac{7}{3} & \frac{11}{3} : & -\frac{43}{3} \end{pmatrix}$$

Here, $|a_{22}| > |a_{32}|$

Applying $R_3 \to R_3 + \frac{7}{10} R_2$, we get

$$\begin{pmatrix} 3 & 1 & -2 : & 10 \\ 0 & \frac{10}{3} & -\frac{11}{3} : & \frac{52}{3} \\ 0 & -\frac{7}{3} & \frac{11}{10} : & -\frac{33}{15} \end{pmatrix}$$

Then, $3x + y - 2z = 10$ .... (iv)

$$\frac{10}{3}y - \frac{11}{3}z = \frac{52}{3} \qquad \text{......... (v)}$$

$$\frac{11}{10}z = -\frac{33}{15} \qquad \text{......... (vi)}$$

From (vi), $\frac{11}{10}z = -\frac{33}{15} \quad \Rightarrow \quad z = -2$

Substituting $z = -2$ in (v), we get

$$\frac{10}{3}y - \frac{11}{3} \times -2 = \frac{52}{3} \qquad \Rightarrow y = 3$$

Substituting $y = 3$ and $z = -2$ in eq^n (iv), we get

$$3x + 3 - 2(-2) = 10 \qquad \Rightarrow \quad x = 1$$

$\therefore \quad x = 1, y = 3, z = -2.$

d) $x - y + z = 10, 4x - 2y - 5z = 9, 3x + 4y - 2z = 1$

Sol^n: Given, $x - y + z = 10$ .... (i)

$4x - 2y - 5z = 9$ .... (ii)

$3x + 4y - 2z = 1$ .... (iii)

The matrix form of the above equations is,

$$\begin{pmatrix} 1 & -1 & 1 & : & 10 \\ 4 & -2 & -5 & : & 9 \\ 3 & 4 & -2 & : & 1 \end{pmatrix}$$

Here, $a_{11} < a_{21}$ and a_{31}

Applying $R_1 \leftrightarrow R_2$ to make $a_{11} > a_{21}$ and a_{31}, we get

$$\begin{pmatrix} 3 & -2 & -5 & : & 9 \\ 1 & -1 & 1 & : & 10 \\ 3 & 4 & -2 & : & 1 \end{pmatrix}$$

$R_2 \rightarrow R_2 - \frac{1}{4} \times R_1$ and $R_3 \rightarrow R_3 - \frac{3}{4} \times R_1$, we get

$$\begin{pmatrix} 4 & -2 & -5 & : & 9 \\ 0 & -\frac{1}{2} & \frac{9}{4} & : & \frac{31}{4} \\ 0 & \frac{11}{2} & \frac{7}{4} & : & -\frac{23}{4} \end{pmatrix}$$

Here, $|a_{22}| < |a_{23}|$

Applying $R_2 \leftrightarrow R_3$ to make $|a_{22}| > |a_{23}|$, we get

$$\begin{pmatrix} 4 & -2 & -5 & : & 9 \\ 0 & \frac{11}{2} & \frac{7}{4} & : & -\frac{23}{4} \\ 0 & -\frac{1}{2} & \frac{9}{4} & : & \frac{31}{4} \end{pmatrix}$$

Applying $R_3 \to R_3 + 11 \times R_2$, we get

$$\left(\begin{array}{ccc:c} 4 & -2 & -5 & 9 \\ 0 & \frac{11}{2} & \frac{7}{4} & -\frac{23}{4} \\ 0 & 0 & \frac{53}{22} & \frac{159}{22} \end{array}\right)$$

Then, $4x - 2y - 5z = 9$ .... (iv)

$\frac{11}{2}y - \frac{7}{4}z = -31$ .... (v)

$\frac{52}{22}z = \frac{159}{22}$ .... (vi)

From eq^n (vi), $\frac{52}{22}z = \frac{159}{22}$ $\Rightarrow z = 3$

Substituting $z = 3$ in eq^n (v), we get

$\frac{11}{2}y - \frac{7}{4} \times 3 = -31$ $\Rightarrow y = -2$

Substituting $z = 3$ and $y = -2$ in eq^n (iv), we get

$4x - 2(-2) - 5 \times 3 = 9$ $\Rightarrow x = 5$

$\therefore$ $x = 5, y = -2, z = 3$.

4. Solve the following system of equations using Gaussian elimination method if possible;

a) $x_1 - x_2 + x_3 = 1, 3x_1 + x_2 + 5x_3 = 11, 4x_1 + 2x_2 + 7x_3 = 16$

Sol^n: $x_1 - x_2 + x_3 = 1$ .... (i)

$3x_1 + x_2 + 5x_3 = 11$ .... (ii)

$4x_1 + 2x_2 + 7x_3 = 16$ .... (iii)

Multiplying equation (i) by 3 and then subtracting from (ii)

$4x_2 + 2x_3 = 8$ .... (iv)

Again multiplying equation (i) by 4 and then subtracting from (iii)

$6x_2 + 3x_3 = 12$ .... (v)

Multiplying equation (v) by $\frac{3}{2}$ and then subtracting from (v),

$0 = 0$

or, $0.x_3 = 0$ .... (vi)

Now, we have the following equations,

$x_1 - x_2 + x_3 = 1$ .... (i)

$4x_2 + 2x_3 = 8$ .... (iv)

$0.x_3 = 0$ .... (vi)

The equation (vi) is true for all values of x_3.

$\therefore$ The system of equation has infinitely many solutions hence consistent.

Thus, if $x_3 = k$

From (iv), $4x_2 + 2k = 8$ $\Rightarrow x_2 = \frac{4-k}{2}$

d) $\mathbf{x_1 + 2x_2 + 3x_3 = 4, 4x_2 + 5x_2 + 6x_3 = -7, 7x_1 + 8x_2 + 9x_3 = 10}$

Sol^n: $x_1 + 2x_2 + 3x_3 = 4$ .... (i)

$4x_2 + 5x_2 + 6x_3 = -7$ .... (ii)

$7x_1 + 8x_2 + 9x_3 = 10$ .... (iii)

Multiplying eq. (i) by 4 and then subtracting from (ii),

$-3x_2 - 6x_3 = -23$ .... (iv)

Again multiplying eq. (i) by 7 and then subtracting from (iii)

$-6x_2 - 12x_3 = -18$ .... (v)

Multiplying eq. (iv) by 2 and then subtracting from (v)

$0 = 28$ .... (vi)

Now, we have the following equations,

$x_1 + 2x_2 + 3x_3 = 4$ .... (i)

$-3x_2 - 6x_3 = -23$ .... (iv)

$0.x_3 = 28$ .... (vi)

There is no value for $x_3$that satisfies equation (vi)

$\therefore$ The system has no solution hence inconsistent.

Hint and Solution of MCQ's

1. To solve the system
   $a_1x + b_1y = c_1$ and
   $a_2x + b_2y = c_2$ using Gauss-elimination the co-efficient of first variable in first equation is non-zero ($a \neq 0$).
2. To solve the system
   $a_1x + b_1y + c_1z = d_1$
   $a_2x + b_2y + c_2z = d_2$
   $a_3x + b_3y + c_3z = d_3$, using Gauss-elimination if $a_1 = 0$, interchange either first and second equation or first and third equation.
3. After forward elimination by Gaussian method, the system of equation changed into upper triangular matrix.
4. To solve
   $2x - y = 5$ ……… (i) and
   $3x + y = 10$ ………. (ii)
   eliminate x, using the technique c) equation (1) - $\frac{2}{3}$ equation (2)
5. After elimination the system reduces the form
   $px + qy = m$
   $ry = n$ such that $r \neq 0$
   then the solution system has unique solution.
6. In the system using Gaussian elimination, the co-efficient a_{11}, a_{22} and a_{33} are known as pivot elements

7. The system of equations in x, y, z such that after the forward elimination z becomes free variable, i.e. $0.z = 0$ (true for any choices of z) then the system is consistent and has infinitely many solution.
8. If forward elimination of Gaussian method gives

   $0.z = k \neq 0$, then the system is inconsistent and has no solution.
9. To solve the system

   $2x + 2y + z = 6$... (i)

   $4x + 2y + 3z = 4$ ..... (ii)

   $x - y + z = 0$ .... (iii), using partial pivoting rearrange the system in order 2, 1, 3, so that the absolute value of co-efficient of x in equation (i) would be large.