Magnetic Field

Magnetic Field

The space or region around a magnet (moving charge or current carrying conductor) up to which its magnetic influence can be experienced is called magnetic field.

The strength of magnetic field at any point in a field is called magnetic field intensity. It is denoted by $\vec{B}$ and its SI unit is Tesla (T).

Magnetic Field Lines

If an isolated N-pole of a bar magnet is moved in the direction of repelling force acting on it, the isolated pole will trace out a line called magnetic field lines.

Properties:

- The tangent draw to magnetic field lines gives the direction of the magnetic field.
- The closeness or density of field lines is directly proportional to the strength of the field.
- Magnetic field lines appear to emerge from the N-pole and terminate at the S-pole.
- Inside the magnet, the direction of magnetic lines is from S-Pole to N-Pole
- Magnetic field lines never intersect with each other.
- Magnetic field lines form a closed loop.
- Field lines have both direction and magnitude at any point in the field, so, magnetic field lines are represented by a vector.
- The magnetic field is stronger at the poles because the field lines are denser near the poles.

Force on Moving Charge Inside Magnetic Field (Lorentz Force)

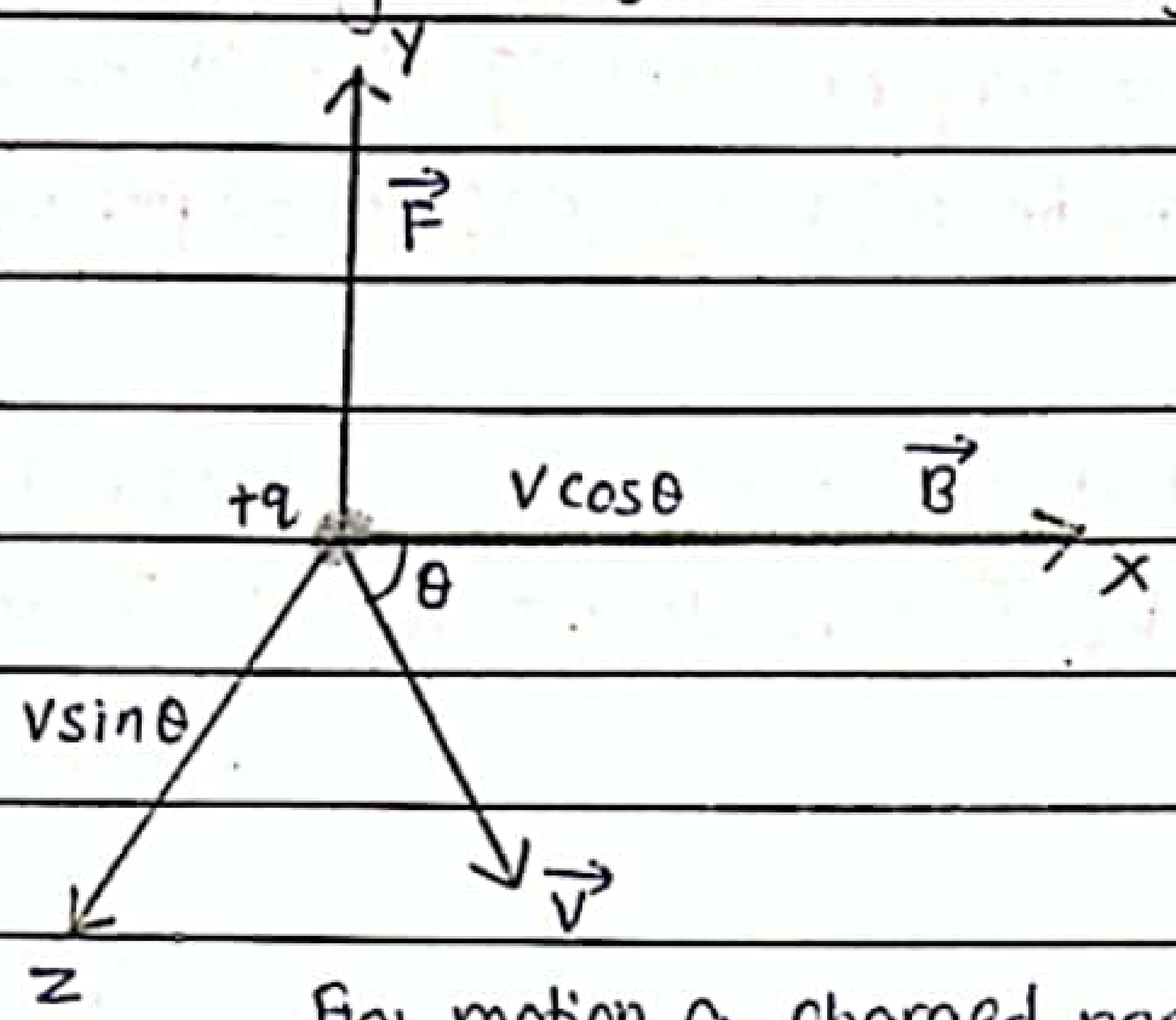

Fig: motion of charged particle in magnetic field

Consider a charge particle having charge +q moving with velocity v is in magnetic field B. Let, magnetic field along x-axis while charge particle is moving making angle θ with field. Then the charged particle experiences a force (F) which is directly proportional to:

i. Magnitude of charge (q)
i.e. $F \propto q$ ——— (i)

ii. Speed of charged particle (v)
i.e. $F \propto v$ ——— (ii)

iii. Magnitude of magnetic field (B)
i.e. $F \propto B$ ——— (iii)

iv. Sine of angle (θ) between magnetic field (B) and velocity (v).
i.e. $F \propto \sin\theta$ ——— (iv)

Combining all, we get

$F \propto qvB\sin\theta$

$F = k\,qvB\sin\theta$

where k is proportionality constant. In SI, k = 1

$\therefore$ $F = qvB\sin\theta$

$\therefore$ $\vec{F} = q(\vec{v} \times \vec{B})$

Special cases

i) If $\theta = 90^\circ$, $F = qvB$ which is the maximum force experienced by the charged particle

ii) If $\theta = 0^\circ$ or 180°

$F = 0$

$\therefore$ No force acts if the particle moves parallel or anti-parallel to field.

iii) If $v = 0$, $F = 0$

$\therefore$ No forces acts on stationary charged particle in magnetic field.

Force on Current carrying Conductor

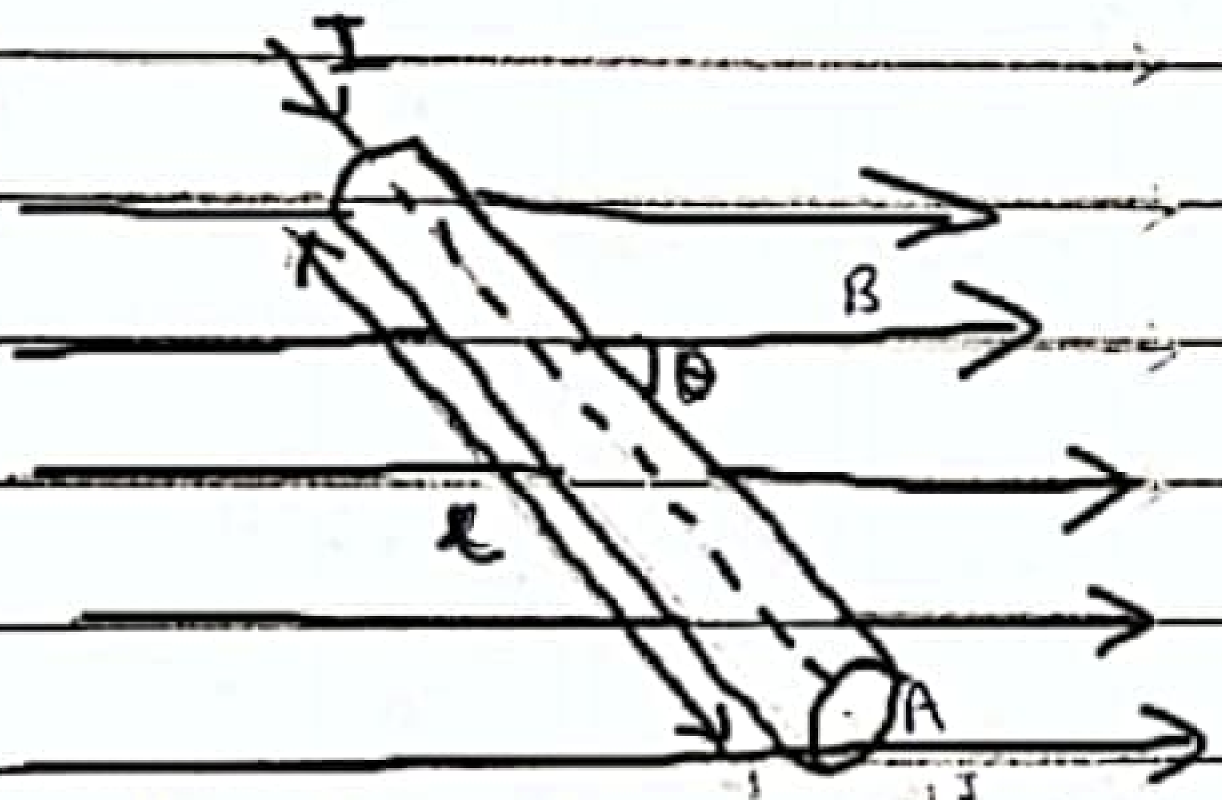

Fig: Current carrying conductor in magnetic field

Consider a conductor of length l and cross sectional area A placed at angle θ in a uniform magnetic field B. Let, I be the current passing through conductor, v_d be the drift velocity of free electrons, n be the electron density and e be the charge of an electron. Then,

Force on each electron, $\vec{F_e} = q(\vec{v_d} \times \vec{B})$

$\therefore F_e = qv_d B\sin\theta$

There are nAl free electrons in length l where Al is the volume of conductor.

So, total force F acting on the conductor is,

$F = (nAl) \cdot F_e = nAl \cdot q v_d B\sin\theta = (v_d q n A)(lB\sin\theta) = IlB\sin\theta$

$\therefore F = BIl\sin\theta$

$\therefore \vec{F} = I(\vec{l} \times \vec{B})$

Page : Date :

Special cases:

1. When $\theta = 0^\circ$ or 180°, $F = 0$

$\therefore$ No force is experienced by conductor when placed parallel or antiparallel to magnetic field.

2. When $\theta = 90^\circ$, $F = BIl$ i.e. maximum force.

Force and Torque on a Current Carrying Rectangular Coil in Uniform Magnetic Field.

Consider a rectangular coil PQRS carrying current I is suspended in horizontal uniform magnetic field B so it rotates freely about vertical axis XY as shown in the figure. Let, $PQ = RS = l$ and $QR = PS = b$.

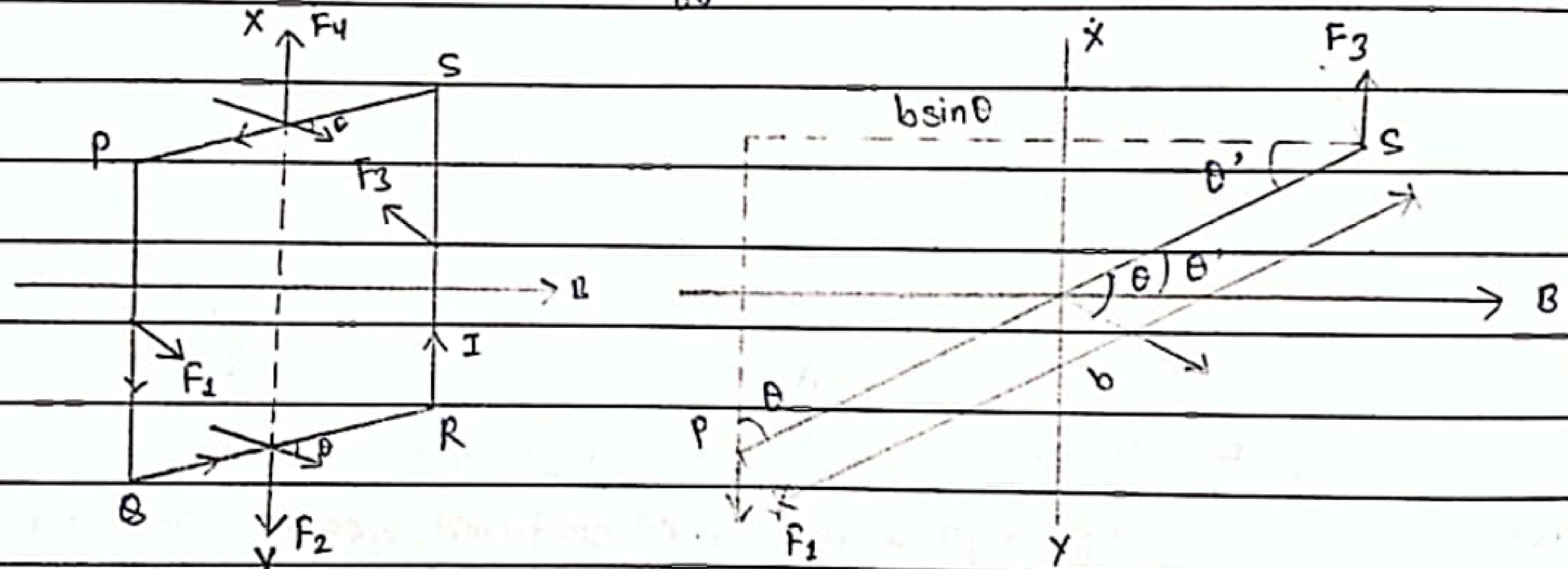

Fig: Torque on a current carrying rectangular coil in uniform magnetic field.

Let at any instant, the normal to the plane of coil makes an angle θ with the direction of magnetic field. The force on various arms of the coil is given by:

i) The inward force acting on the arm PQ.

$F_1 = BIl\sin\theta = BIl \quad [\because \theta = 90^\circ]$

ii) The downward force acting on arm QR.

$F_2 = BIb\sin\theta$

iii) The outword force acting on arm RS

$F_3 = BIl\sin\theta = BIl \quad [\because \theta = 90^\circ]$

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Page: Date:

iv) The upward force acting on arm SP:

$F_4 = BIb\sin\theta$

It is seen that forces F_2 and F_4 are equal in magnitude but opp. in direction and pass through same line of action. So, they cancel each other.

Forces F_1 and F_3 are also equal in magnitude and opposite in direction but do not passes through same line of action and cause to rotate the coil about a vertical axis.

Moment of couple or torque, τ is given by

τ = magnitude of either force × perpendicular distance between forces

$= F_1 \times b\sin\theta$

$= BIl \times b\sin\theta$

$= BIA\sin\theta$

where $A = l \times b$ is the area of the coil. Therefore, torque

$\tau = BIA\sin\theta$

If the coil has N turns, then

$\tau_N = BINA\sin\theta$

If the plane of the coil makes an angle θ' with the magnetic field, then the angle made by the side PS or QR with B is $\theta + \theta' = 90°$ or $\theta = 90 - \theta'$. Then,

$\tau_N = BINA\sin(90 - \theta')$

$\therefore \tau_N = BINA\cos\theta'$

Page: Date:

Moving Coil Galvanometer

It is an instrument which is used to measure the electric current. It is a sensitive electromagnetic device which can measure low current even of the few microamperes.

It is mainly divided into two types:

- Suspended coil galvanometer.
- Pivoted coil or Wetson galvanometer.

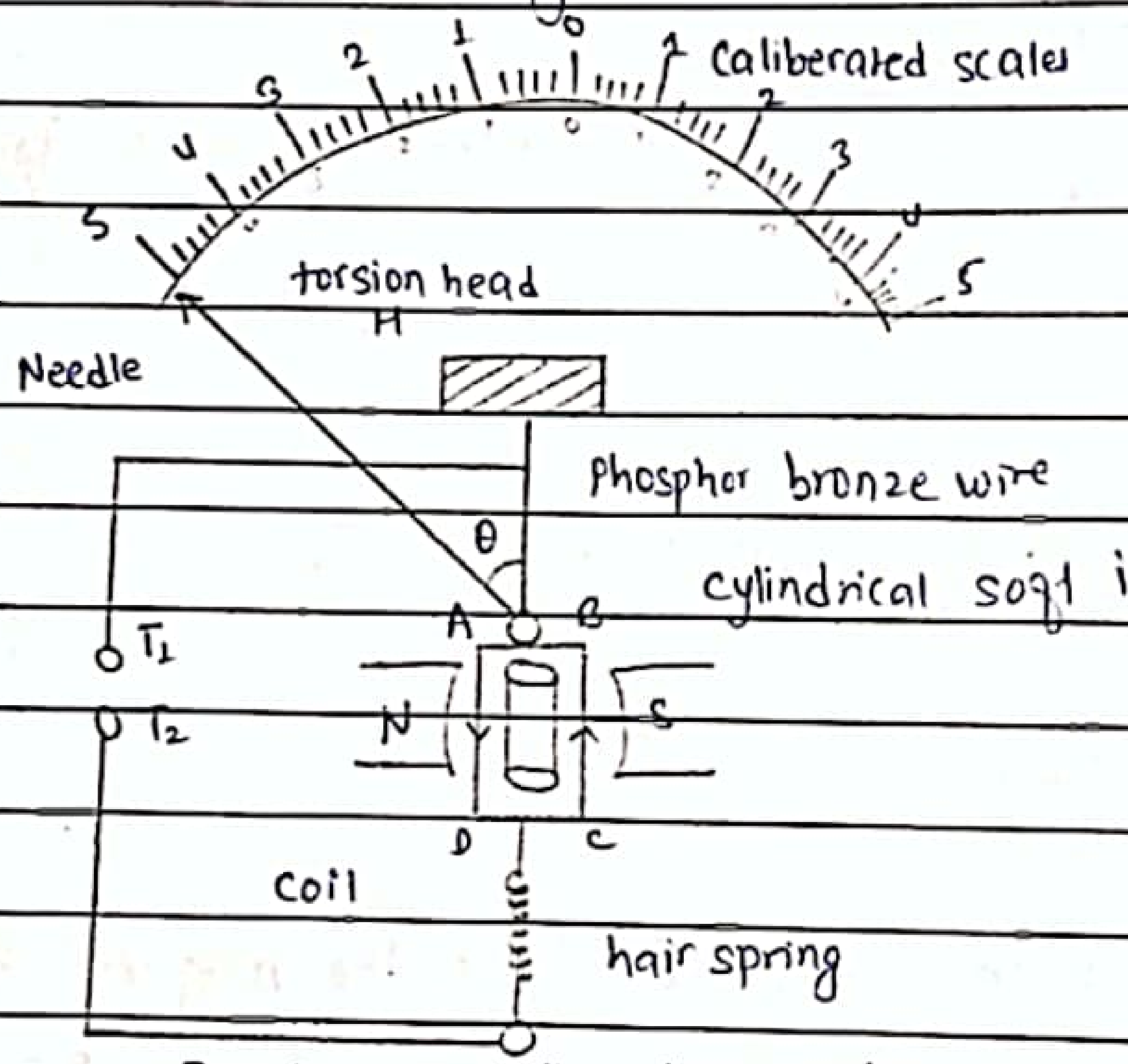

Fig: Moving coil galvanometer.

Principle: When a current carrying loop or coil is placed in a uniform magnetic field, the coil experiences a torque.

Construction:

The instrument contains a rectangular coil having large no. of turns wound on a non metallic frame. The coil is suspended between two poles of a permanent magnet which are cylindrical in shape. The coil is suspended by a phosphor bronze strip which acts as path for the current to the coil which is finally connected to terminal T_1 of the galvanometer. The other end of the coil is connected to a light spring which is finally connected to the terminal T_2 as shown in figure. The spring exerts a very small restoring torque on the coil. A piece of soft iron is placed within the frame of the coil.

Theory:

let, B = magnetic field intensity, I = Current passed through the coil.
l = length of the coil, b = breadth of the coil, $l \times b = A$ = area of the coil,
N = no. of turns of the coil.

When current flows through the coil, it experiences a torque given by

$\tau_N = BINA\sin\theta$

where θ is the angle made by the normal to the plane of the coil with the direction of the magnetic field. If θ is $90°$, then $\sin\theta = 1$. It is possible only when cylindrical poles of permanent magnet are used which produce a radial magnetic field.

Then, $\tau_N = BINA$

This torque is known as deflecting torque.

As the coil gets deflected, the suspension wire is twisted and a restoring torque is developed in it. If k is the restoring torque per unit twist of the suspension wire, then the restoring torque for deflection ϕ is

$\tau_R = k\phi$

For equilibrium of the coil, $\tau_N = \tau_R$

or, $BINA = k\phi$

or $I = \dfrac{k\phi}{BNA}$

or $I = G\phi$

where $G = \dfrac{k}{BNA}$ is called galvanometer constant.

$\therefore$ $I \propto \phi$

Thus deflection of the coil is directly proportional to the current flowing through it. Hence, linear scale is used in galvanometer to detect the current.

Page : Date :

Sensitivity of Galvanometer.

A galvanometer is said to be sensitive if small amount of current flowing through its coil produces a large deflection in it.

→ Current Sensitivity

The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$\text{Current sensitivity} = \frac{\phi}{I} = \frac{BNA}{K}$$

Current sensitivity can be increased by i) Increasing B ii) Increasing A iii) Increasing N . iv) Decreasing K

→ Voltage sensitivity

Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

$$\text{Voltage sensitivity} = \frac{\phi}{V} = \frac{\phi}{IR} = \frac{BNA}{KR}$$

Voltage sensitivity can be increased by i) Increasing B ii) Increasing A iii) Increasing N iv) Decreasing K v) Decreasing R.

Hall Effect

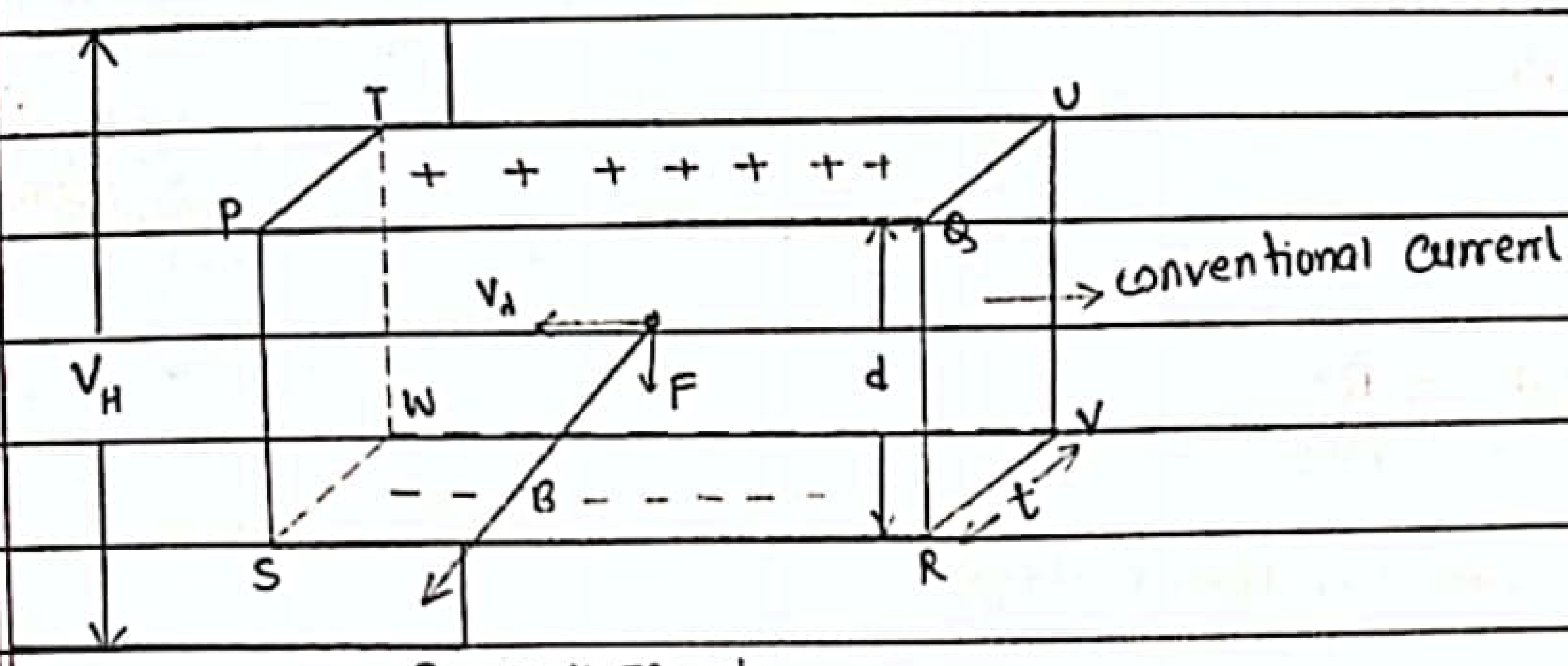

Fig: Hall Effect

When a magnetic field is applied to a current carrying conductor, a voltage is set up in the direction perpendicular to both the direction of current and magnetic field. This effect is called Hall effect.

Let us consider a slab of width d and thickness t so that the area of cross section is txd. Suppose conventional current flows towards right so that free electrons move to left with drift velocity V_d as shown. When a uniform magnetic field is applied from face TUVW to front face PQRS, each electron experiences a Lorentz force, $F = qV_dB$ in vertically downward direction. So, electrons accumulate to lower face VWSR which becomes negatively charged and upper plate positively charged. So, pd or emf is developed across lower and upper face of slab which passes electron flow towards downward direction. The flow ceases when emf reaches to critical value V_H called Hall voltage.

Then, Electrical field Intensity, $E = \frac{V_H}{d}$

Electric force, $F = eE = \frac{eV_H}{d}$

If this force is balanced by Lorentz force, the electrons are in equilibrium. Thus,

$F = F_{lorentz}$

Page : Date :

or $\frac{eV_H}{d} = BeV_d$

$\therefore V_H = BV_d d$ --- (i)

As, $V_d = \frac{I}{enA}$,

$V_H = \frac{BId}{enA} = \frac{BId}{entd} = \frac{BI}{net}$

This is the expression for Hall voltage.

Hall constant,

$R_H = \frac{E}{JB} = \frac{V_H}{d \cdot I/A \cdot B} = \frac{V_H \cdot A}{dIB} = \frac{V_H \cdot d \times t}{dIB} = \frac{B \times I \times d \times t}{net \; IBd}$

$\therefore R_H = \frac{1}{ne}$

Biot and Savart law

Consider a conductor xy through which current I is flowing an a magnetic field is produced around it. To find the magnetic field at point P, let us consider an element AB of length dl which makes angle θ with line joining to P. Let the distance of P from dl be r.

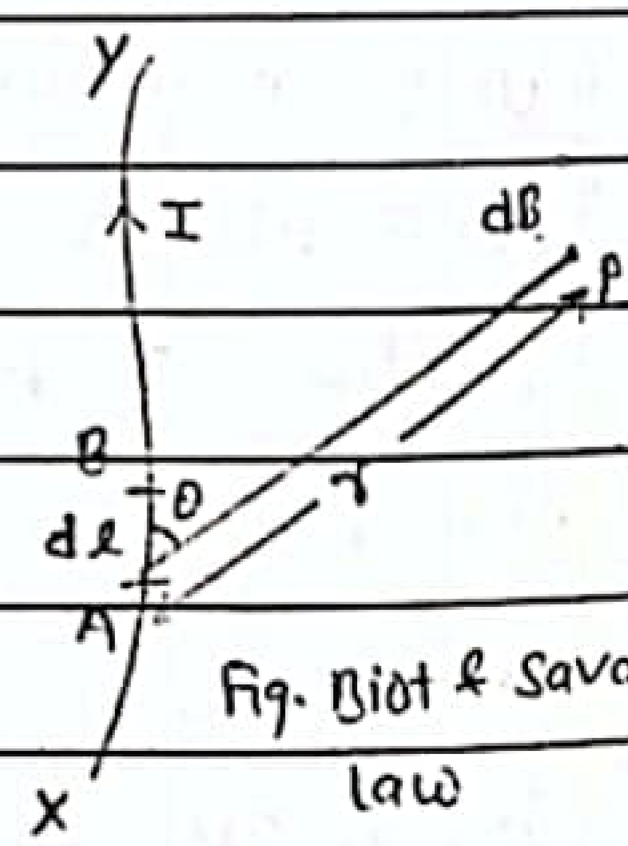

Fig. Biot & Savart law

From Biot and Savart law, magnetic field dB produced by the element at that point P is

i. directly proportional to magnitude of current passed

$dB \propto I$

ii. directly proportional to length of the element

$dB \propto dl$

iii. directly proportional to sine of the angle between the conductor and line joining P.

$dB \propto \sin\theta$

iv. inversely proportional to square of the distance between the element and point P.

$dB \propto \frac{1}{r^2}$

Combining above equations,

$dB \propto \frac{I dl \sin\theta}{r^2}$

or $dB = \frac{k I dl \sin\theta}{r^2}$

where k is proportionality constant. In SI units, $k = \frac{\mu_0}{4\pi}$ where μ_0 is the permeability of free space and its value is $4\pi \times 10^{-7}$ H/m.

$\therefore dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$

In vector form,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3}$$

Direction of $\vec{dB}$: The direction of dB at P is perpendicularly inward to plane containing $\vec{dl}$ and $\vec{r}$.

Applications of Biot and Savart law:

1. Magnetic field at the centre of current carrying circular coil.

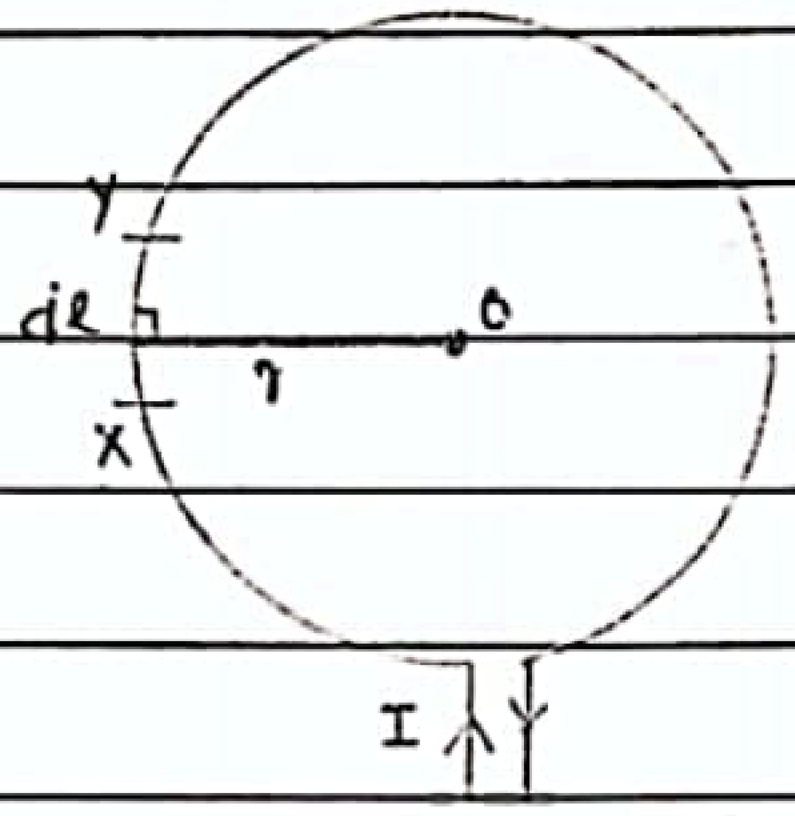

Fig: magnetic field at the centre of circular coil.

Consider a circular coil of radius r carrying current I which produces a magnetic field around it. To find magnetic field at O, consider an element XY of length dl such that the radius $CO = r$ makes angle $\theta = 90^\circ$ with the element. From Biot and savart law, dB produced by this element at O is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2} \quad [\because \sin 90^\circ = 1]$$

To find out magnetic field produced by whole coil,

$$B = \int_0^{2\pi r} dB = \int_0^{2\pi r} \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\left[l\right]_0^{2\pi r} = \frac{\mu_0}{4\pi}\frac{2\pi r I}{r^2} = \frac{\mu_0 I}{2r}$$

If the coil has N turns, then

$$B = \frac{\mu_0 N I}{2r}$$

The direction of magnetic field B is perpendicular to the plane of the coil.

2. Magnetic field on the Axis of Current carrying circular coil

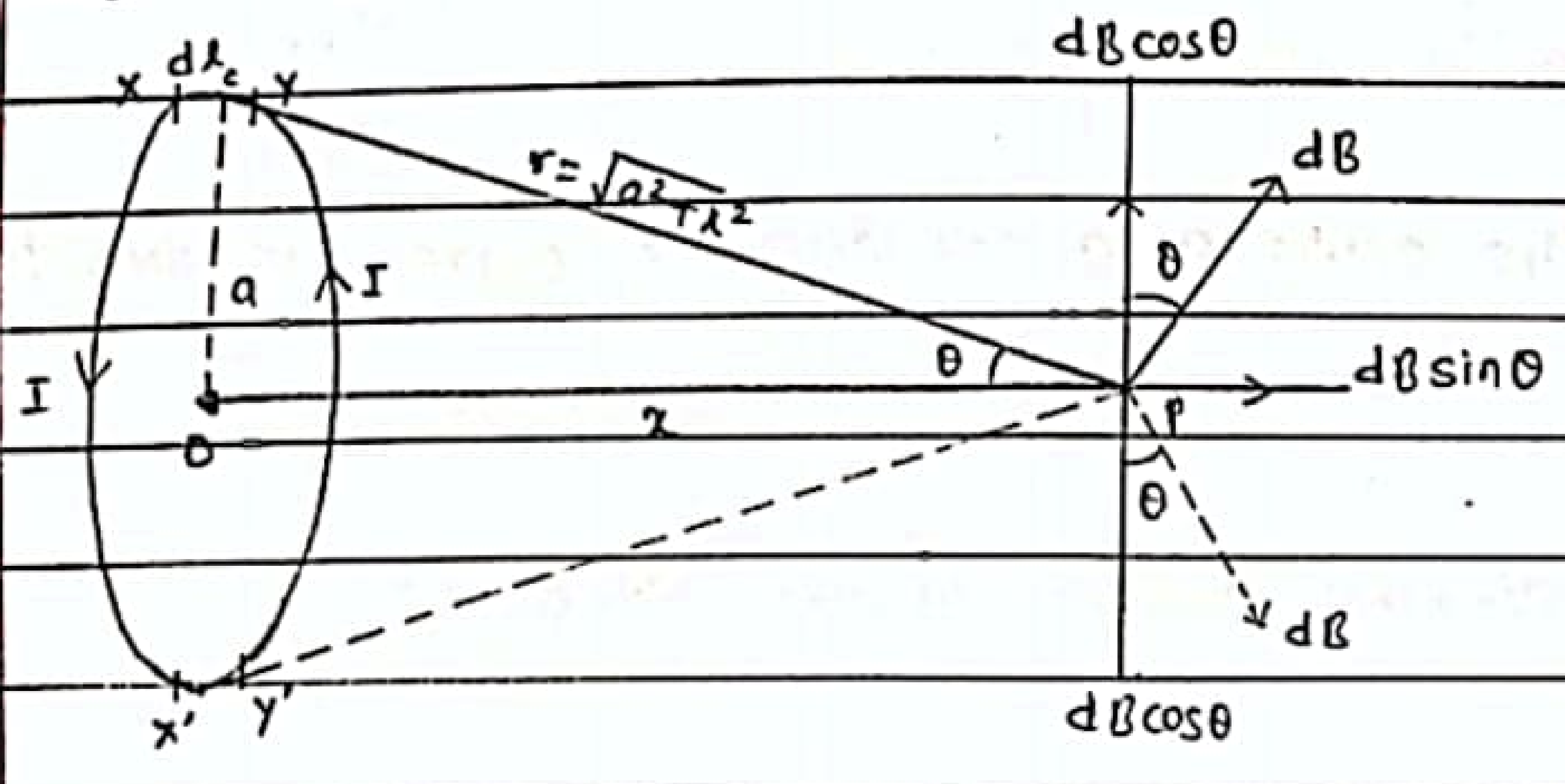

Fig: Magnetic field on the axis of circular coil

Consider a circular coil of radius a carrying current I. let, the plane of the coil be perpendicular to the plane of the paper. To find a magnetic field at a point P on the axis of the coil at a distance x from a centre O of the coil, take a small element XY of length dl of the coil which is at a distance $r = \sqrt{a^2+x^2}$ from P where $x = OP$.

From Biot and Savart law, magnetic field at P due to element XY is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2} \quad [\because \theta = 90^\circ]$$

dB is perpendicular to the plane formed by dl and r. As the coil is symmetrical about its axis, every element of length dl has equal and opposite element at the end of a diameter of the coil. The perpendicular components cancel each other but horizontal components remain all along the axis of the coil. So, magnetic field at P due to whole coil is:

$$B = \int_0^{2\pi a} dB\sin\theta = \int_0^{2\pi a}\frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin\theta = \frac{\mu_0 I\sin\theta}{4\pi r^2}\int_0^{2\pi a} dl = \frac{\mu_0 I\sin\theta}{4\pi r^2}[l]_0^{2\pi a} = \frac{\mu_0 I a\sin\theta}{2r^2}$$

In $\triangle COP$, $\sin\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$

$$\therefore B = \frac{\mu_0 I}{2(x^2+a^2)} \cdot \frac{a}{(x^2+a^2)^{1/2}} \cdot a = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

If the coil has N turns,

$$B = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$$

The magnetic field at the centre of a coil carrying current is along the axis of the coil.

3. Magnetic Field due to straight current carrying conductor.

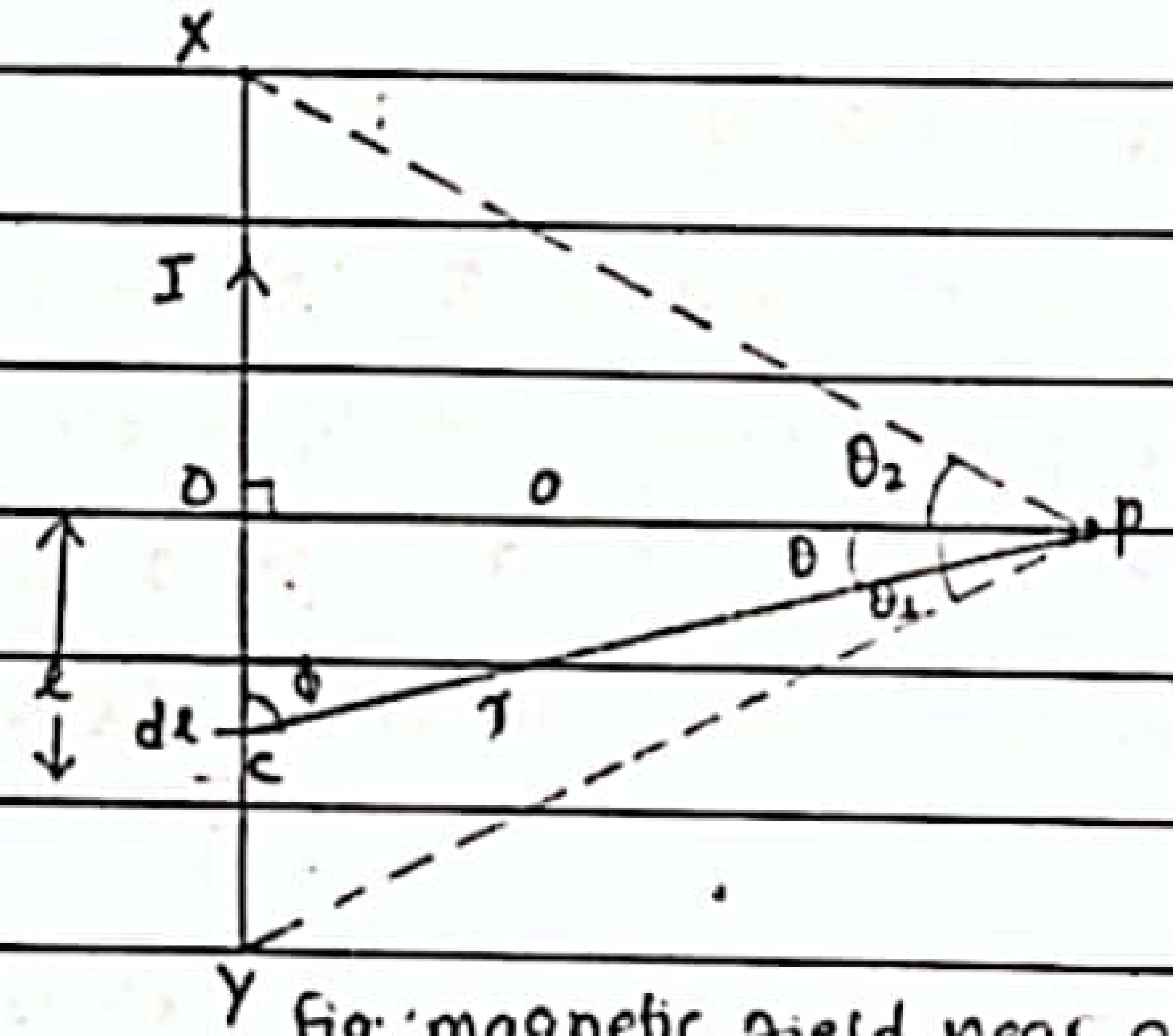

Fig: magnetic field near a long straight conductor

Consider a long straight conductor XY carrying current I. Let, P be the point at distance a from the conductor where magnetic field is to be calculated. Take a small element dl of conductor at a distance r from point P. According to Biot and Savart law, the magnetic field at point P due to small element dl is:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2} \quad \text{---- (i)}$$

Draw PO normal on conductor XY. In Δ POC,

$$\sin\phi = \frac{a}{r} = \cos\theta \Rightarrow r = \frac{a}{\cos\theta}$$

Also, $\tan\theta = \frac{l}{a}$

or, $l = a\tan\theta$

Diff w.r.t. θ,

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$dl = a\sec^2\theta \, d\theta.$

Then, from (i),

$$dB = \frac{\mu_0}{4\pi} \frac{I\,(a\sec^2\theta\, d\theta)}{\frac{a^2}{\cos^2\theta}} . \cos\theta$$

$$= \frac{\mu_0}{4\pi} \frac{I\, a \sec^2\theta\, d\theta\, \cos^2\theta \cdot \cos\theta}{a^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I\cos\theta\, d\theta}{a}$$

Magnetic field due to whole conductor XY can be calculated by integrating within θ_2 and $-\theta_1$.

$$\therefore B = \int_{-\theta_1}^{\theta_2} dB = \frac{\mu_0 I}{4\pi a}\int_{-\theta_1}^{\theta_2} \cos\theta\, d\theta = \frac{\mu_0 I}{4\pi a}\left[\sin\theta\right]_{-\theta_1}^{\theta_2} = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

For infinitely long conductor, $\theta_1 = \theta_2 = \pi/2$

$$\therefore B = \frac{\mu_0 I}{4\pi a}(\sin\pi/2 + \sin\pi/2)$$

$$\therefore B = \frac{\mu_0 I}{2\pi a}$$

The direction of the field can be found by using right hand rule which is directed inward to the plane of paper at P.

4. Magnetic field on the Axis of a Solenoid.

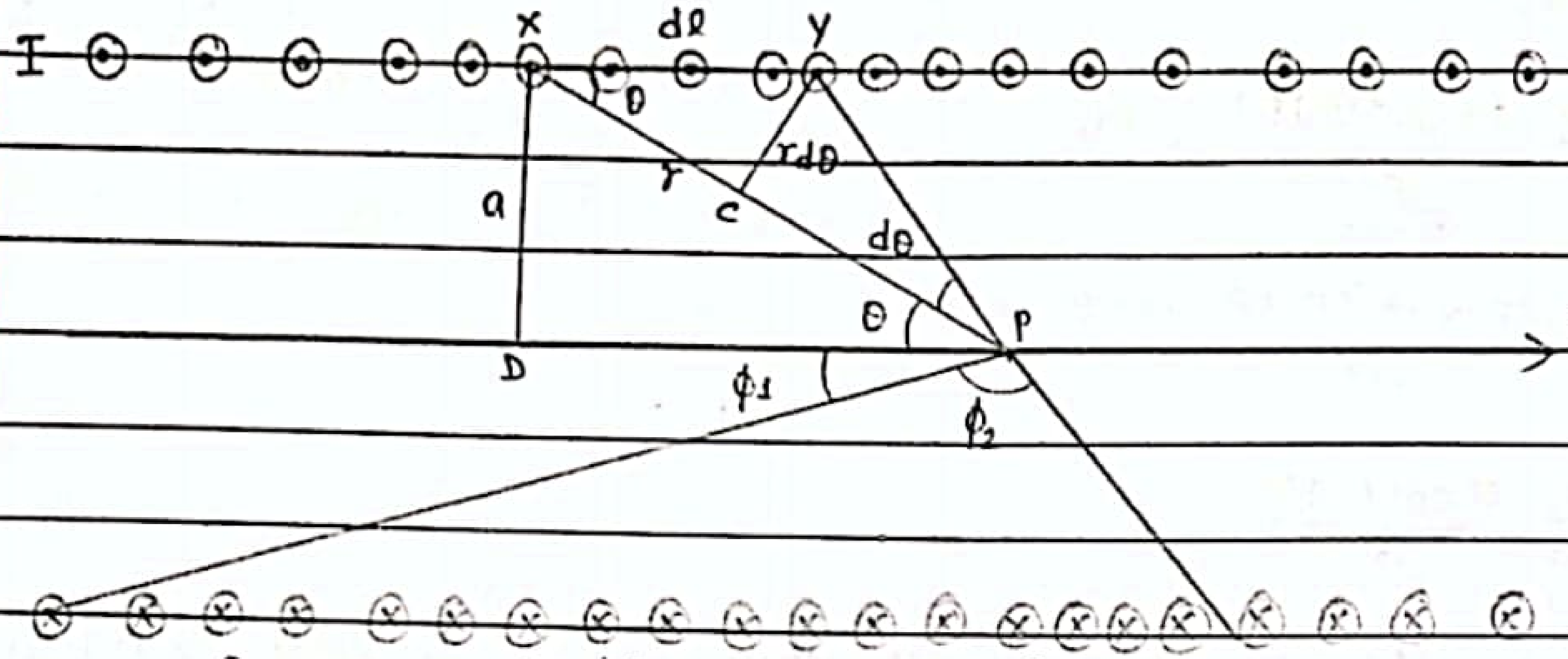

fig: Magnetic field on the axis of long solenoid

Consider a solenoid of radius a having n no. of turns per unit length. When current I flows through it, magnetic field is produced inside the solenoid. To find out magnetic field at a point P on the axis of the solenoid, let us consider an element XY of length dl that makes angle dθ at P. let, XP = r, XD = a and ∠XPD = θ.

Since magnetic field produced by a single coil is $\frac{\mu_0 I a \sin\theta}{2r^2}$, so a small magnetic field produced by dl having ndl turns at P is:

$$dB = \frac{\mu_0 I a \sin\theta}{2r^2} ndl \quad \text{--- (i)}$$

In △YCP, for small angle dθ, YC = rdθ

In △XYC, $\sin\theta = \frac{YC}{dl} \Rightarrow YC = dl\sin\theta$

Then, $rd\theta = dl\sin\theta$

$$dl = \frac{rd\theta}{\sin\theta}$$

From (i),

$$dB = \frac{\mu_0 I a \sin\theta}{2r^2} \cdot n \cdot \frac{rd\theta}{\sin\theta} = \frac{\mu_0 I a n}{2r} d\theta$$

In ΔXDP, $\sin\theta = \frac{a}{r} \Rightarrow r = \frac{a}{\sin\theta}$

Then,

$$dB = \frac{\mu_0 I a n d\theta}{2} \cdot \frac{\sin\theta}{a}$$

or, $$dB = \frac{\mu_0 n I \sin\theta \, d\theta}{2}$$

If the radius r makes angles ϕ_1 and ϕ_2 at P from two ends of coil, total magnetic field at point P can be obtained by integrating from ϕ_1 to ϕ_2. so,

$$B = \int_{\phi_1}^{\phi_2} \frac{\mu_0 n I \sin\theta \, d\theta}{2} = \frac{\mu_0 n I}{2}[-\cos\theta]_{\phi_1}^{\phi_2} = \frac{\mu_0 n I}{2}(\cos\phi_1 - \cos\phi_2)$$

If the solenoid is very long then $\phi_1 = 0°$ and $\phi_2 = \pi$. so,

$$B = \frac{\mu_0 n I}{2}(\cos 0° - \cos\pi) = \mu_0 n I$$

$\therefore$ $B = \mu_0 n I$

The direction of the field is along the axis DP of the solenoid.

Ampere's law

Ampere's law states that the line integral of the magnetic field around any closed path in free space is equal to μ_0 times the total current enclosed by the path.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

where μ_0 is permeability of free space, $\vec{B}$ is magnetic field, dl is small element and I is current. The closed path is called Amperian loop.

Proof.

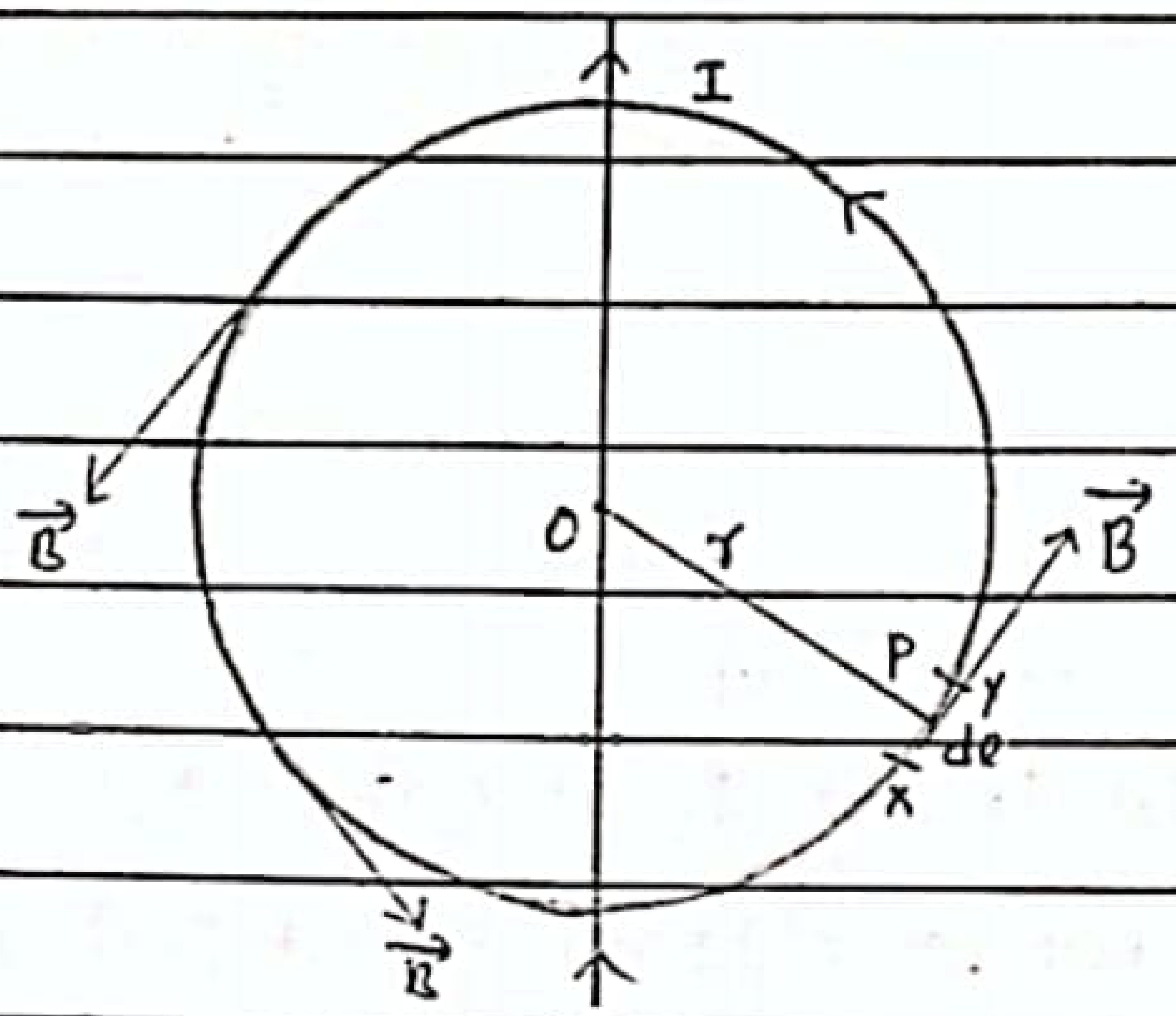

Fig: Ampere's law for straight conductor

Consider a straight conductor carrying current I which produces magnetic field in which magnetic lines of force are concentric circles with centre at the conductor. Magnetic field at P at a distance r from the conductor is

$$B = \frac{\mu_0 I}{2\pi r}$$

The direction of B is along the tangent to the circle. The closed loop is the circle of radius r. let, XY be a small element of length dl. Since, field B is tangent at every point on the loop, so $\vec{dl}$ and $\vec{B}$ are in same direction. The magnitude of $\vec{B}$ is same at every point of the loop. Therefore, the line integral of $\vec{B}$ over this closed loop is

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos 0^\circ = \oint B\,dl$$

Now,

$$\oint B\,dl = \oint \frac{\mu_0 I}{2\pi r}\cdot dl = \frac{\mu_0 I}{2\pi r}\cdot 2\pi r = \mu_0 I$$

This proves Ampere's law.

Applications of Ampere's law

1. Magnetic field due to a straight current carrying conductor.

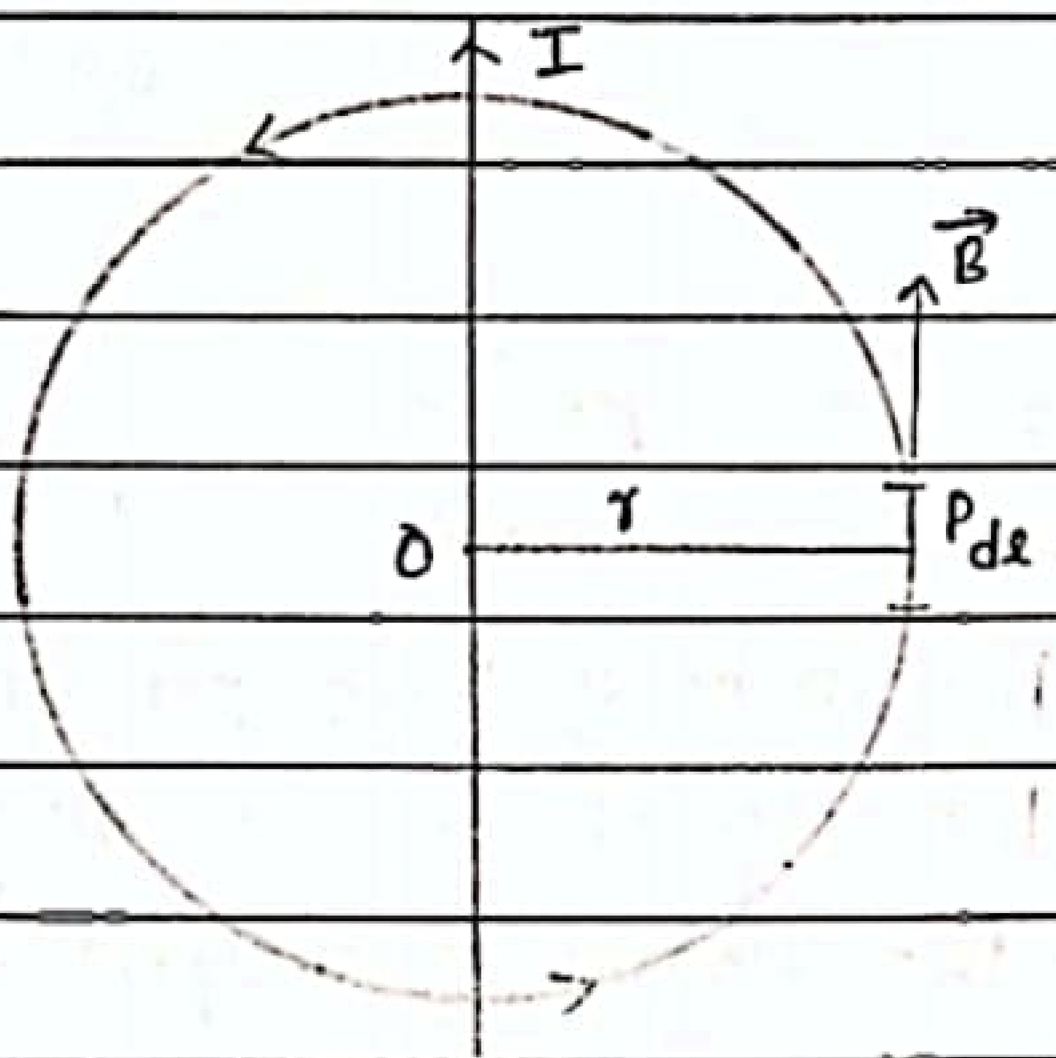

Fig: magnetic field due to straight conductor.

Consider a long straight conductor carrying current I. We have to find the magnetic field at point P at a perpendicular distance r from the conductor. The magnetic lines of force are concentric circles centred at the conductor. If we choose a circle of radius r as the closed path, the magnetic of $\vec{B}$ at every point on the path is same. The field $\vec{B}$ is tangent to $\vec{dl}$ and at every point on the circle. Thus applying Ampere's law,

$$\oint \vec{B}\,\vec{dl} = \mu_0 . I$$

or $$\oint B\, dl \cos 0^\circ = \mu_0 I$$

or, $$B . 2\pi r = \mu_0 I$$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$

2. Magnetic field due to a current carrying solenoid.

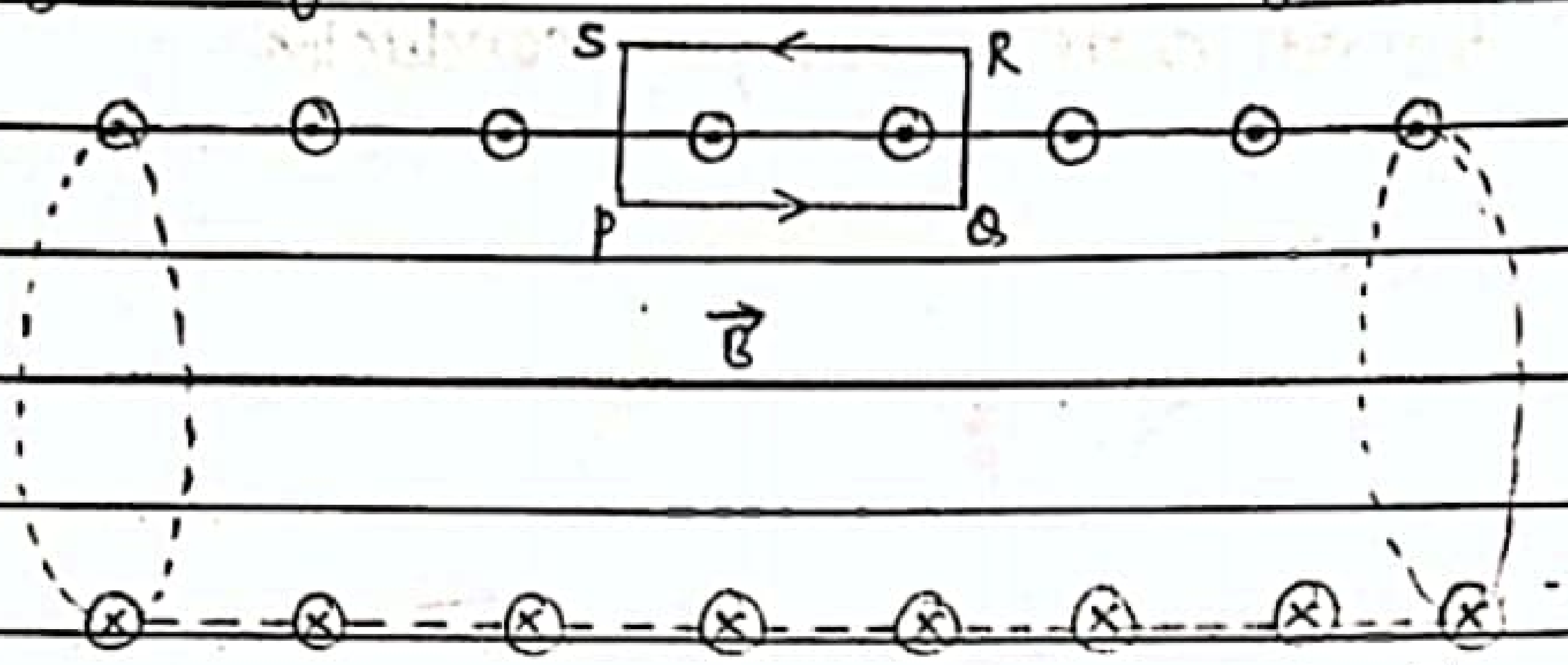

Fig: magnetic field due to solenoid

Consider a very long solenoid have n no. of turns per unit length. Let, I be the current flowing through the solenoid so that magnetic field is produced. The magnetic field (B) inside the solenoid is uniform, strong and directed along the axis of the solenoid. The magnetic field outside the solenoid is very weak and can be neglected.

In order to use Ampere's law, let us draw a closed path PQRS as shown in figures, where $PQ = l$. The line integral of $\vec{B}$ over the closed path PQRS is

$$\oint \vec{B} dl = \int_P^Q \vec{B}\cdot\vec{dl} + \int_Q^R \vec{B}\cdot\vec{dl} + \int_R^S \vec{B}\cdot\vec{dl} + \int_S^P \vec{B}\cdot\vec{dl}$$

Now, $\int_P^Q \vec{B}\vec{dl} = \int_P^Q B\,dl\cos 0^\circ = Bl$

Also, $\int_Q^R \vec{B}\cdot\vec{dl} = \int_S^P \vec{B}\,\vec{dl} = 0$

And, $\int_R^S \vec{B}\cdot\vec{dl} = 0$

$\therefore$ $\oint \vec{B}\cdot\vec{dl} = Bl$

From Ampere's law,

$\oint \vec{B}\cdot\vec{dl} = \mu_0 \times$ no. of turns in PQRSP $\times I$

$\oint \vec{B}\cdot\vec{dl} = \mu_0 n l I$

So, $Bl = \mu_0 n l I$

$\therefore B = \mu_0 n I$

3. Magnetic Field due to a Current carrying Toroid.

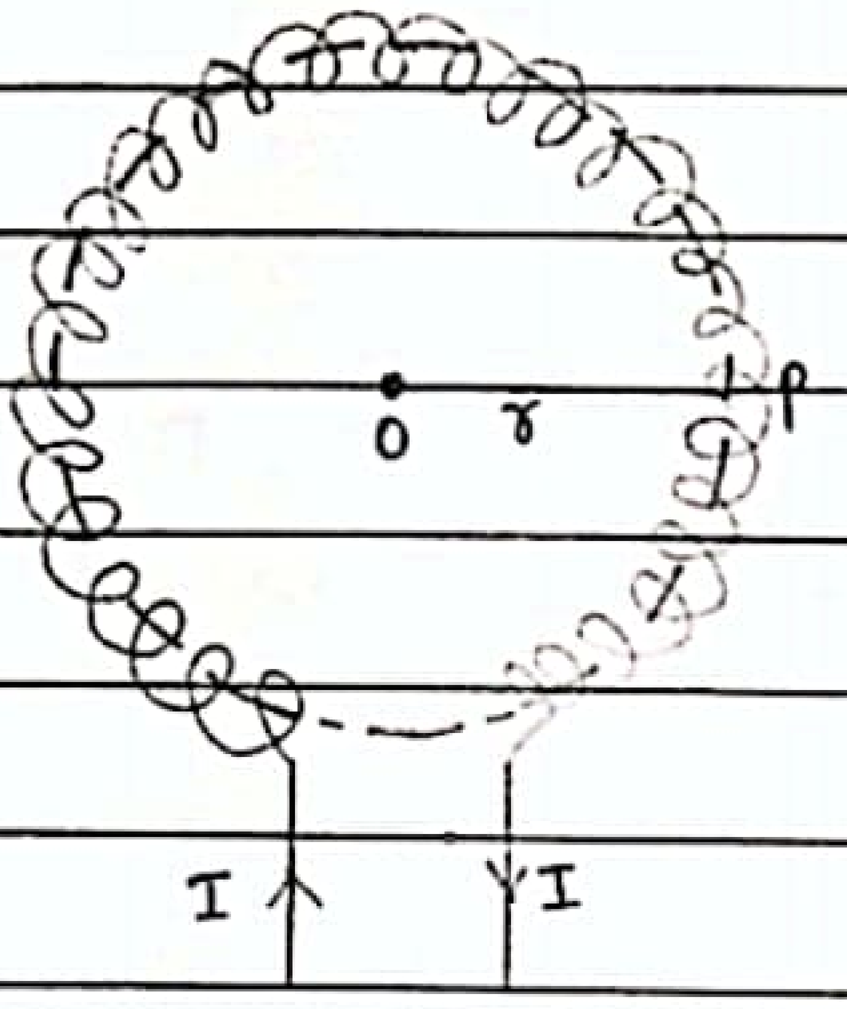

Fig: A toroid carrying current I

Consider a toroid having n no. of turns per unit length. When a current I flows through the toroid, a magnetic field is produced in it. Magnetic lines of force inside the toroid are uniform circular, concentric with the centre of the toroid.

In order to apply Ampere's circuital law, let us draw a closed path inside the turns as a circle of radius r as shown in fig. The magnitude of $\vec{B}$ is same everywhere on the closed path. The field is tangent everywhere on the circular path and the angle between $\vec{dl}$ and $\vec{B}$ is zero. Thus, line integral of $\vec{B}$ over this closed path is:

$\oint \vec{B}\cdot\vec{dl} = \int B\, dl \cos 0° = B \times 2\pi r$

$\therefore$ $\oint \vec{B}\cdot\vec{dl} = B(2\pi r)$.

According to Ampere's law,

$\oint \vec{B}\cdot\vec{dl} = \mu_0 \times$ net current enclosed by the closed path

Let, the toroid has N turns. Then the total current enclosed by the Amperian loop is NI. So,

$\oint \vec{B}\cdot\vec{dl} = \mu_0 (NI)$

or $B \times 2\pi r = \mu_0 NI$

or $B = \mu_0 \left(\frac{N}{2\pi r}\right)\cdot I = \mu_0 n I$

Where $\frac{N}{2\pi r} = n =$ no. of turns per unit length.

$\therefore B = \mu_0 n I$

Helmholtz Coil

Consider a circular coil of radius a carrying current I. The magnetic field produced by the coil at distance x from centre of the coil on its axis is given by

$$B = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}$$

It means magnetic field along the axis of a single coil varies with distance x from the centre of coil as shown in fig (a). So, a single coil can't produce a uniform magnetic field.

In order to obtain uniform magnetic field, Helmholtz used two coaxial parallel coils of equal radius a separated by same distance a. In this case, when same current flows around each coil in the same direction, the resultant magnetic field B is uniform for some distance on either side of the point on their axis midway between the coils. At distance a/2 from centre of the coil, the rate of variation of the field remains constant on either side of the point. As a result, net magnetic field remains the same. Such arrangement of coil is called Helmholtz coil.

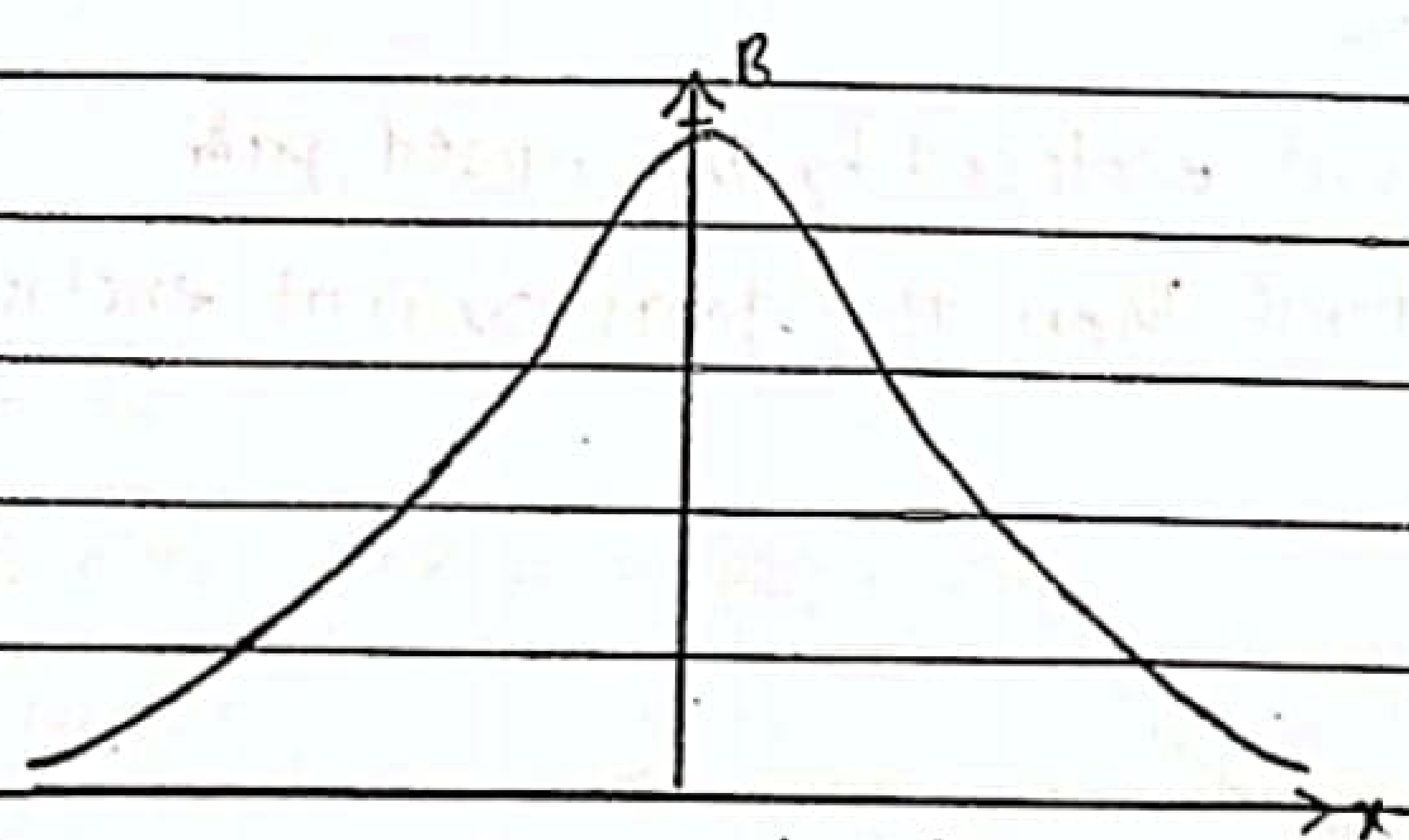

Fig: magnetic field produced in a single circular coil.

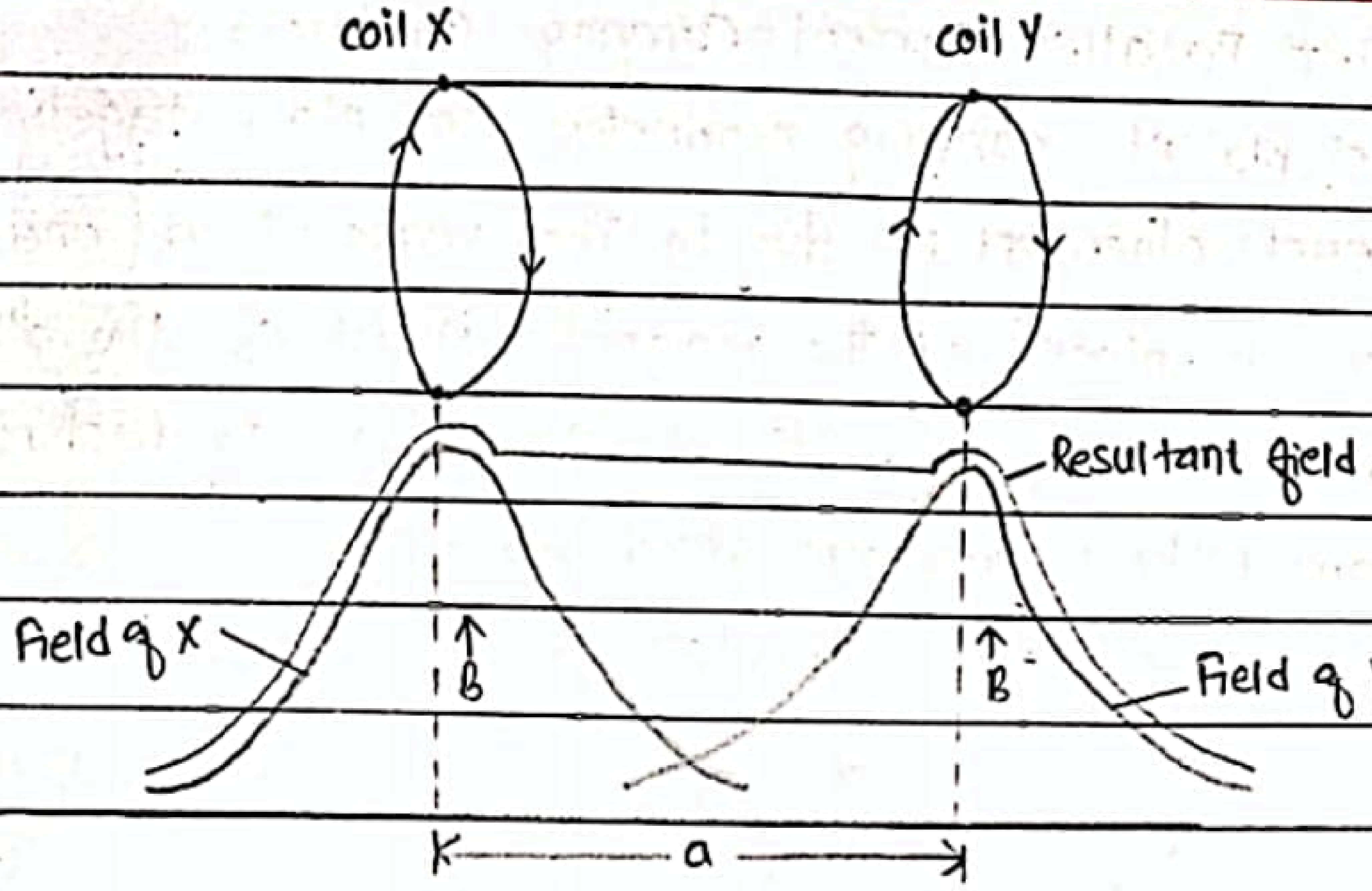

Fig: Helmholtz coil

If the coil has N no. of turns, then,

$$B = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$$

Magnetic field at $x = a/2$ due to coil X is

$$B_x = \frac{\mu_0 N I a^2}{2\left[a^2 + \left(\frac{a}{2}\right)^2\right]^{3/2}} = \frac{\mu_0 N I a^2}{2(5/4\, a^2)^{3/2}} = \frac{\mu_0 N I a^2}{2(5/4)^{3/2} a^3} = \frac{\mu_0 N I}{2(5/4)^{3/2} a}$$

Similarly, magnetic field at $x = a/2$ due to coil Y is

$$B_y = \frac{\mu_0 N I}{2(5/4)^{3/2} a}$$

As both B_x and B_y are acting in same direction, their resultant magnetic field at the mid point is

$$B = B_x + B_y = \frac{2\,\mu_0 N I}{2(5/4)^{3/2} a} = 0.72\,\frac{\mu_0 N I}{a}$$

Force between Two Parallel Current Carrying Conductor

When two parallel current carrying conductors are close together, they exert forces to each other. It is due to the reason that one current carrying conductor is placed in the magnetic field of the other and magnetic field produced by them interact each other. Conductors with like current attract and unlike currents repel each other.

i. When currents pass in same direction

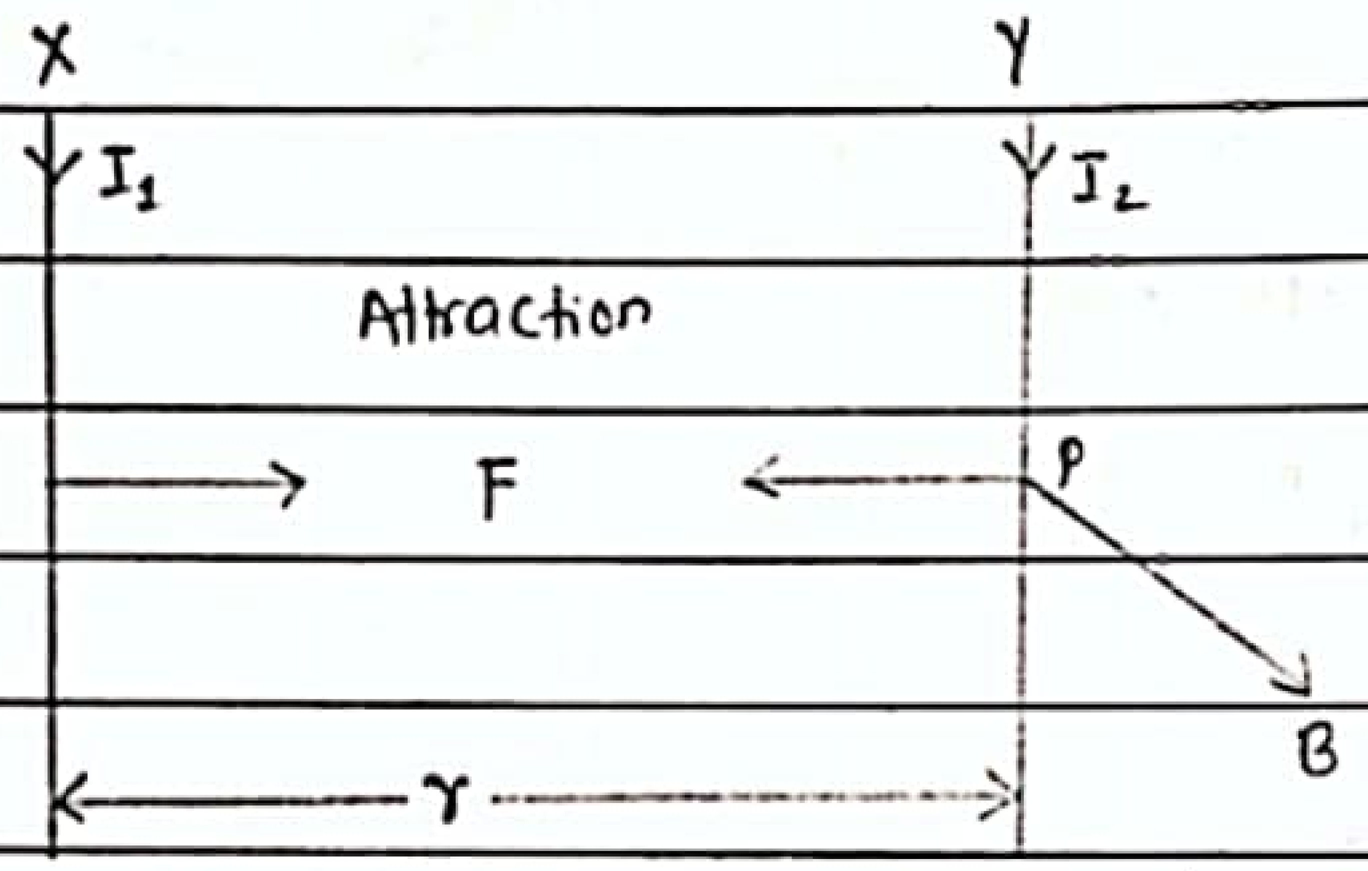

Fig: Attractive force between two conductors

Consider two infinitely long parallel conductors X and Y carrying currents I_1 and I_2 respectively in the same direction. Let they are separated at distance r in the plane of paper.

The magnetic field produced by X at P through where Y is placed is

$$B_x = \frac{\mu_0 I_1}{2\pi r} \quad \text{--- (i)}$$

The direction of the field is perpendicular to the plane of the paper and is directed outward. Since, conductor Y with current I_2 is in the magnetic field B_x, the force experienced by a length dl of conductor Y is:

$$F_y = B_x I_2 dl \quad \text{--- (ii)}$$

From (i) & (ii)

$$F_y = \frac{\mu_0 I_1 I_2 dl}{2\pi r}$$

$$\frac{F_y}{dl} = F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

The force per unit length, F is in the plane of the paper and is directed towards the conductor x.

Similarly, force per unit length on conductor y is

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

and is directed towards conductor y.

ii. When currents pass in opposite direction.

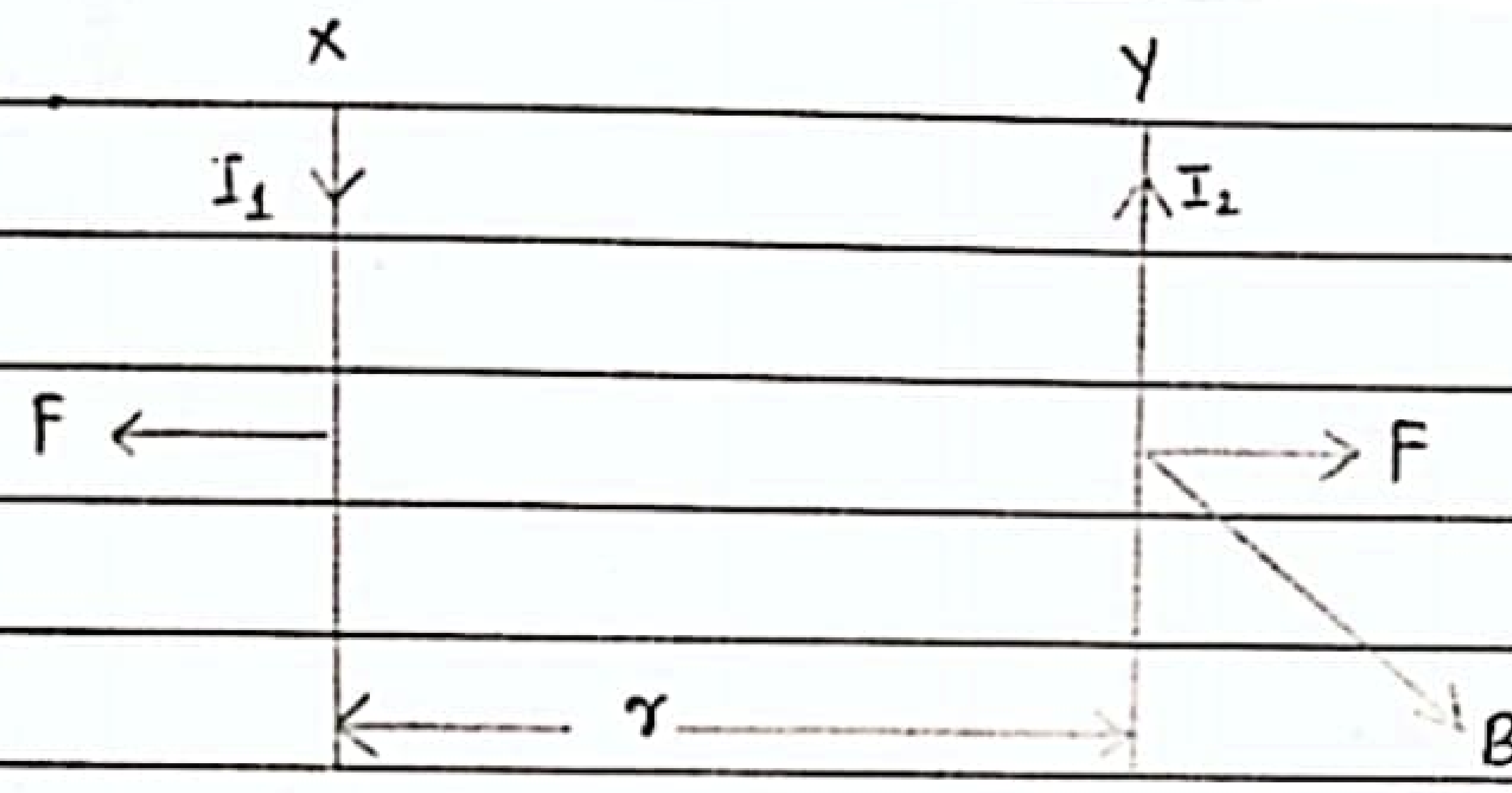

Fig: Repulsive force between two conductors

When currents I_1 and I_2 pass through two conductors x and y respectively in opposite direction to each other, it can be found that, $F_x = -F_y$

It means that the forces produced in the conductors have same magnitude but repel each other.

Ampere

Force between two current carrying conducts per unit length is:

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

If $I_1 = I_2 = 1A$ and $r = 1m$, then

$$F = \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 1} = 2 \times 10^{-7}\,N$$

Thus, one ampere is the current flowing in each of two infinitely long parallel conductors 1m apart resulting the force of 2×10^{-7} N per meter on each conductor.