Electromagnetic Induction

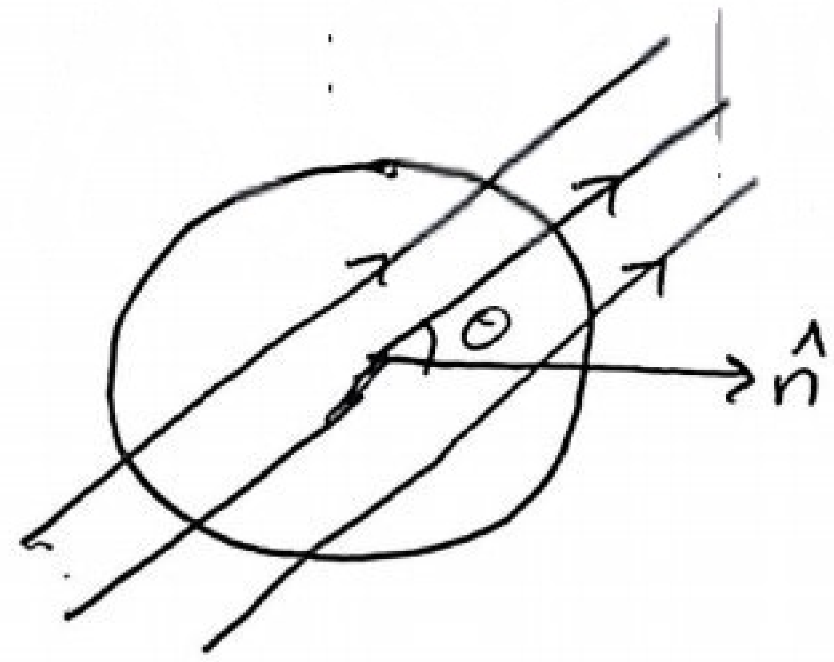
Topics to be covered

1)- Electromagnetic Induction
2)- Faraday's Law
3)- Induced EMF and current
4)- Lenz's Law
5)- Self and mutual induction

Magnetic Flux :-

→ The magnetic flux through any closed surface placed in a magnetic field is the total no. of magnetic lines of force crossing this surface normally.

$$\boxed{\begin{aligned}\phi &= BA\cos\theta \\ \phi &= \vec{B}\cdot\vec{A}\end{aligned}}$$



→ Magnetic flux is a scalar quantity and its dimension is $[ML^2T^{-2}A^{-1}]$

→ SI unit of magnetic flux is weber.
$[1 \text{ weber} = 1\ Tm^2]$

→ CGS unit of magnetic flux is Maxwell (Mx).
$[1 \text{ weber} = 10^8\ Mx]$

Faraday's Law of Electromagnetic Induction :-

First Law :-

Whenever the magnetic flux link with a closed circuit changes, an EMF (and hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomena is called "electromagnetic induction".

Second Law :-

The magnitude of the induced EMF is equal to the rate of change of magnetic flux link with the closed circuit. This is also known as "newmann law"

$$\left[|e| = \frac{d\phi}{dt} \right]$$

- According to Lenz rule for dirn of induced emf then,

$$\boxed{e = -\frac{d\phi}{dt}}$$

- The -ve sign indicates that the dirn of induced emf is such that it opposes the change in magnetic flux.

- If the coil consist of N turns, then the total emf will be

$$\boxed{e = \frac{-N d\phi}{dt}}$$

- This can be written as-

$$\boxed{e = -\frac{d(N\phi)}{dt}}$$

Here, the factor $N\phi$ is called the effective magnetic flux or no. of flux linkage in the coil. Its unit is weber-turn.

Induced Current in the Circuit :-

If the resistance of the closed circuit or coil is 'R' then the current induced in the coil or circuit will be

$$I = \frac{e}{R}$$

$$I = \frac{N\frac{d\phi}{dt}}{R}$$

$$\boxed{I = \frac{N}{R} \cdot \frac{d\phi}{dt}}$$

$$\left[\begin{array}{c} V = IR \\ I \alpha V \\ I = \frac{V}{R} \end{array} \right]$$

Induced Charge in the Circuit :-

Charge induced in the circuit in time interval (dt) is given by :

$$q = I dt$$

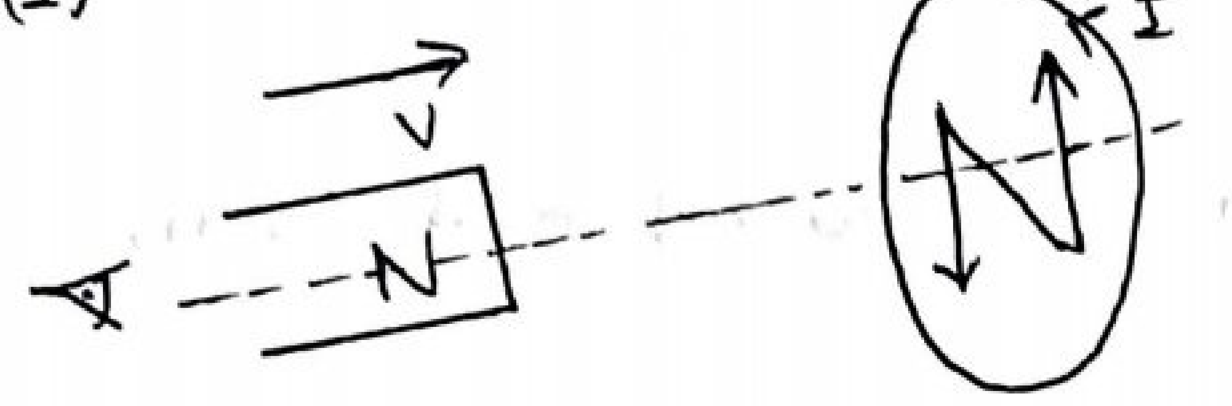
$$q = \left(\frac{N}{R} \cdot \frac{d\phi}{dt}\right) dt \Rightarrow \boxed{q = \frac{N}{R} \cdot d\phi}$$

$$\left[\begin{array}{c} I = \frac{dq}{dt} \\ dq = I dt \end{array} \right]$$

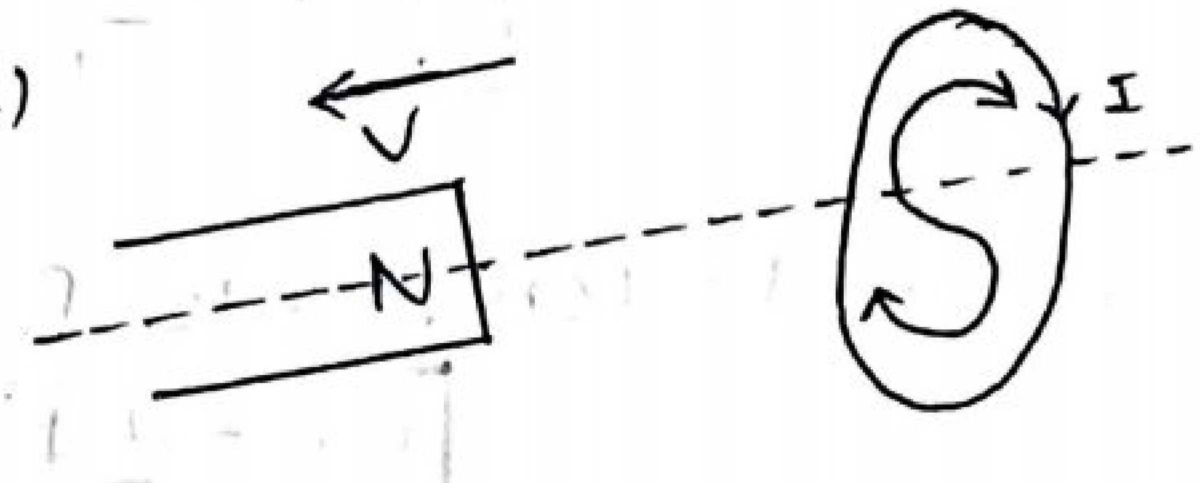
Lenz's Law :-

This law states that the dirn of induced current in a closed circuit is such that it opposes the cause or the change which produces it.

(1)



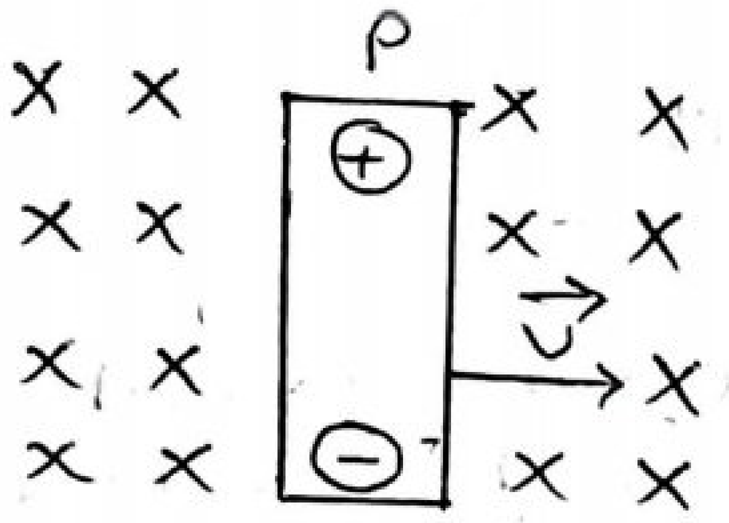
(2)



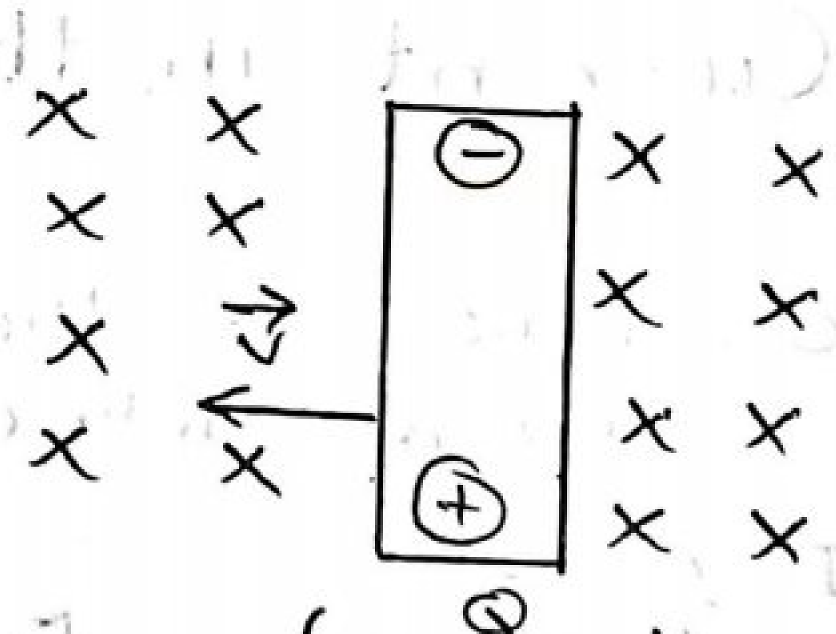
Lenz's Law & Law of Conservation of Energy :-

- Work has to be done in moving the magnet closer to the coil against force of repulsion (Fig 1).
- Similarly, when the North pole of the magnet is moved away from the coil, work has to be done against force of attraction (Fig 2).
- Finally, this work appears as electric energy in the form of induced current. Thus, Lenz's Law is valid and consequence of law of conservation of energy.

Motional EMF :-



(Fig-1)



(Fig-2)

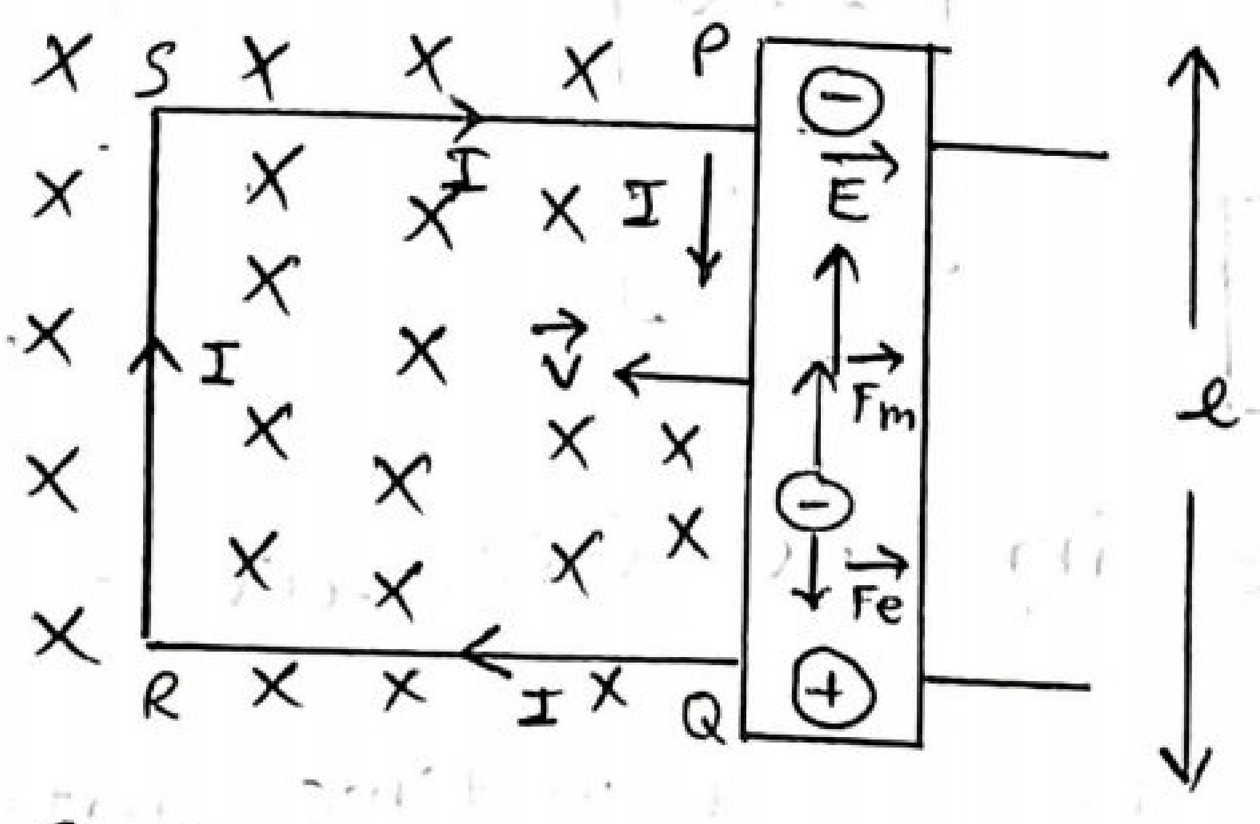
$F_m = F_e$

$qvB = qE$

$E = vB$

$\frac{e}{l} = vB$

$\boxed{e = vlB}$



- This emf induced across the ends of a conductor due to its motion in a magnetic field is called motional emf.

1)- Current induced in the loop -

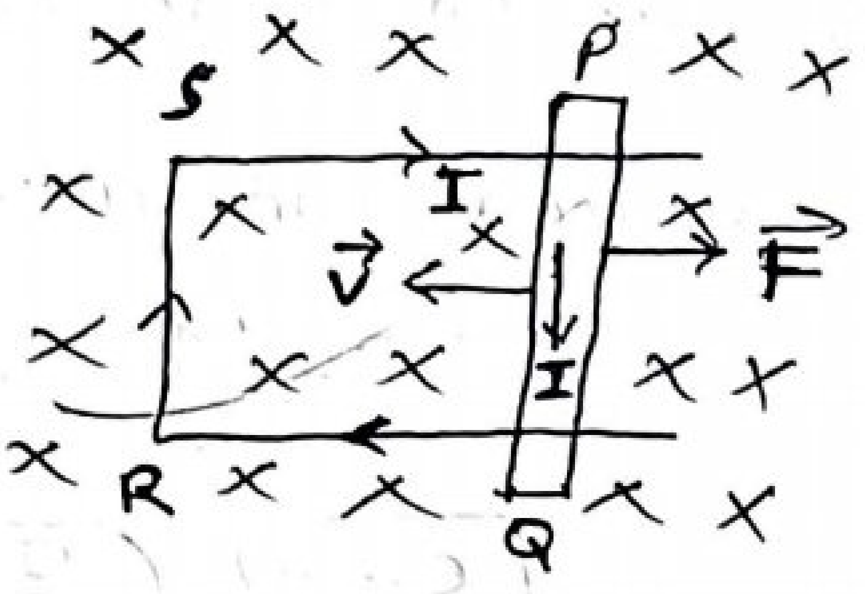
Let R be the resistance of the moving arm PQ of the rectangular loop PQRS, then-

$$\boxed{I = \frac{e}{R} = \frac{vlB}{R}}$$

2)- Force on the moving arm -

The moving arm PQ of length l and carrying current I experiences a force in the perpendicular magnetic field.

$$F = IlB$$
$$F = \left(\frac{vlB}{R}\right) lB$$
$$\boxed{F = \frac{v\, l^2 B^2}{R}}$$



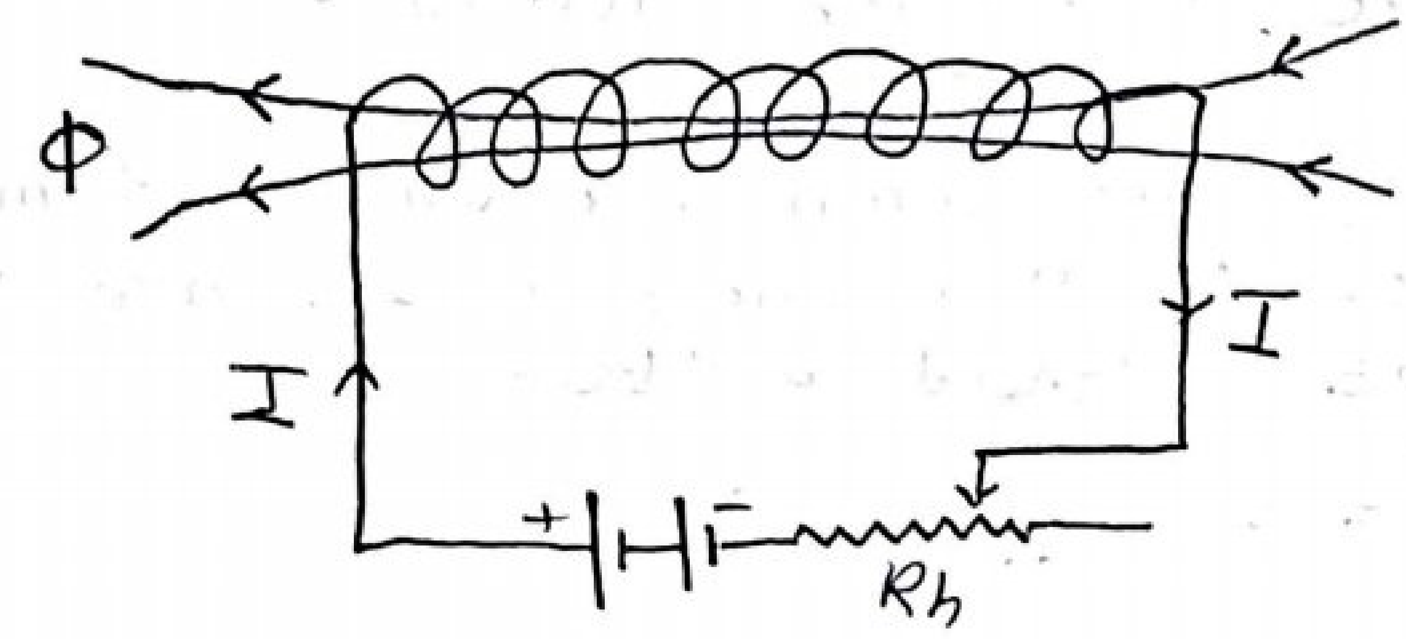
3)- Power delivered by external force -

The power supplied by external force to maintain the motion of the moveable arm.

$$P = Fv$$
$$P = \left(\frac{vl^2B^2}{R}\right)v$$
$$\boxed{P = \frac{v^2 l^2 B^2}{R}}$$

Self Induction :-

When a changing current flows through a coil, the magnetic ~~flux~~ flux linked with the coil changes and opposing emf is induced in the coil. This emf is called self induced emf or back emf and the phenomena is known as self induction.

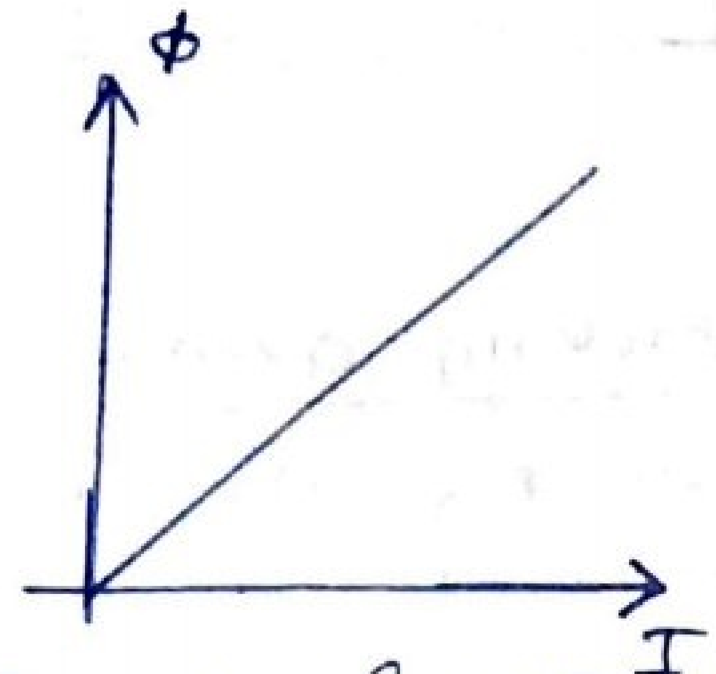


Self Inductance (coefficient of self induction) :-

→ At any instant, the magnetic flux link with the coil proprotion to the current through it.

$$\phi \propto I$$

$$\boxed{\phi = LI}$$



→ Where, the proportionality constant 'L' is called self inductance of the coil.
It is also known as coefficient of self induction.

→ The induced emf set-up in the coil will be

$$e = -\frac{d\phi}{dt}$$

$$\boxed{e = -L\left(\frac{dI}{dt}\right)}$$

→ Unit of self inductance is "Henry".

$$\cancel{1 = H} \left[1H = 1V sec/A\right]$$

→ Dimension of self inductance : $[ML^2T^{-2}A^{-2}]$

Physical Significance of Self Inductance :-
(Inertia of Electricity)

→ Self induction of a coil is the property by virtue of which it tend to maintain the magnetic flux link with it and opposes any change in the flux by inducing current in it. This property of a coil is analogous to mechanical inertia. That's why self induction is called inertia of electricity.

Calculation of Self Inductance :-

1)- Self Inductance of a Long Solenoid :-

- Consider a long air solenoid having n turns per unit length. If current in solenoid is I, then the magnetic field inside the solenoid will be -

$$[B = \mu_0 n I] \ldots (1)$$

- If A is cross sectional area of solenoid, then effective flux link with solenoid -

$$\left[\phi = NBA\right] \quad (N \rightarrow \text{no. of turns in the solenoid})$$

$$\phi = (nl)(\mu_0 n I)A \quad \left(n = \frac{N}{l}\right)$$

$$\left[\phi = \mu_0 n^2 I (Al)\right] \cdots (2)$$

But, $\phi = LI \cdots (3)$

From eq^n 2 and eq^n 3 -

$$\boxed{L = \mu_0 n^2 Al}$$

This is the self inductance of long solenoid

2)- Self Inductance of a Plane Circular Coil :-

- Consider a plane circular coil of radius r and no. of turns N. If current flowing in the coil is I, then magnetic field at the centre of the coil -

$$\left[B = \frac{\mu_0 N I}{2r}\right]$$

- Effective magnetic flux link with the coil -

$$\left[\phi = NBA\right]$$

$$\phi = N\left(\frac{\mu_0 N I}{2r}\right)(\pi r^2)$$

$$\frac{\phi}{I} = \frac{1}{2}\mu_0 \pi N^2 r$$

$$\boxed{L = \frac{1}{2}\mu_0 \pi N^2 r}$$

Energy Stored in an Inductor :-

$$\boxed{U = \frac{1}{2} L I^2}$$

"The energy resides in the inductor in the form of magnetic field."

→ Since, for a solenoid : $\left[L = \mu_0 n^2 Al\right]$

$$B = \mu_0 n I \Rightarrow I = \frac{B}{\mu_0 n}$$

$$\therefore U = \frac{1}{2}(\mu_0 n^2 Al)\left(\frac{B^2}{\mu_0^2 n^2}\right)$$

$$\left[U = \frac{1}{2}\frac{B^2 Al}{\mu_0}\right]$$

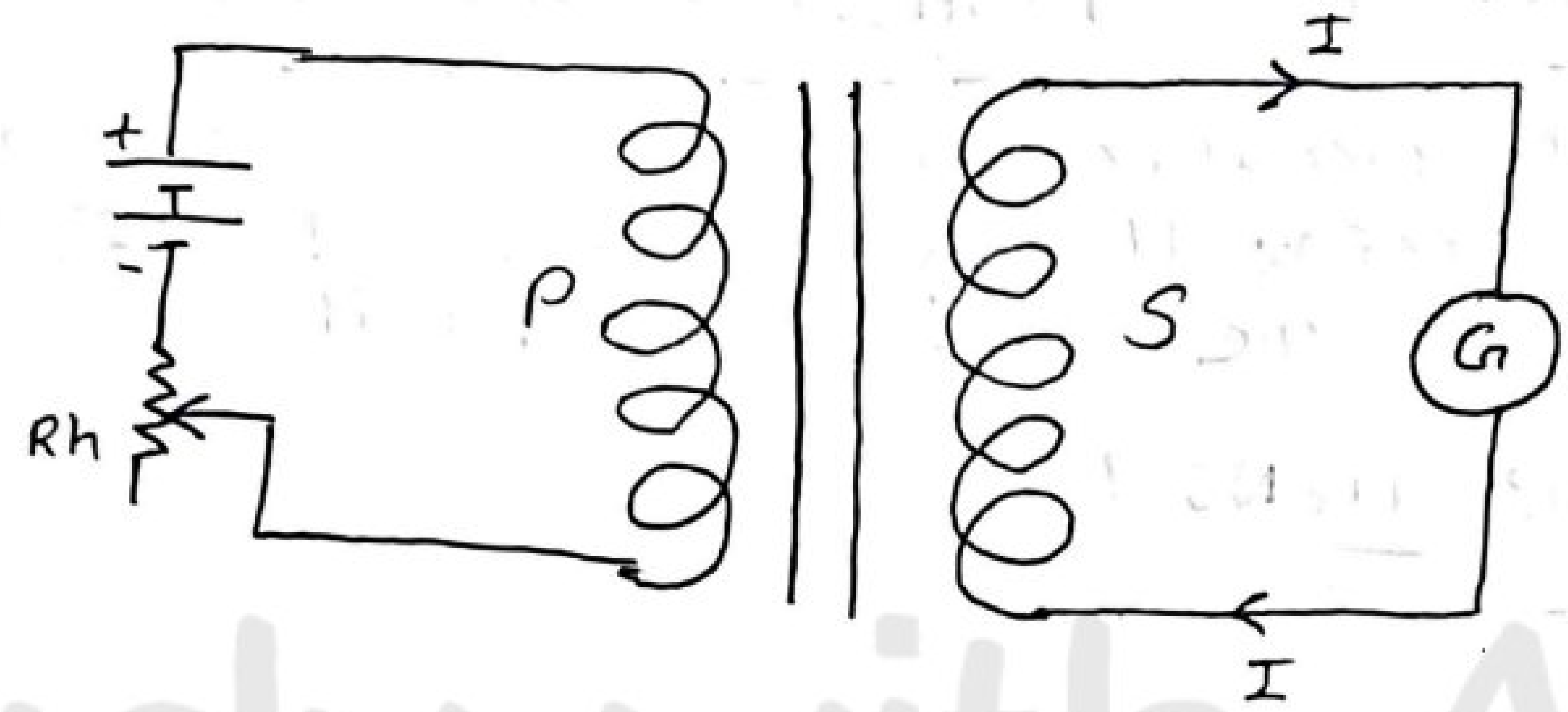
→ The magnetic energy stored per unit volume in the solenoid -

$$u = \frac{U}{Al}$$

$$\left[u = \frac{1}{2}\frac{B^2}{\mu_0}\right]$$

Mutual Induction :-

→ It is the phenomena of production of induced emf in one coil due to change of current in the neighbouring coil.



Mutual Inductance or Coefficient of Mutual Induction:-

→ At any instant the magnetic flux link with the secondary coil proportional to the current in the primary coil.

$$\phi \propto I$$

$$\boxed{\phi = MI}$$

→ Where, M is the coefficient of mutual induction or mutual inductance of the two coil.

$$e = -\frac{d\phi}{dt}$$

$$e = -\frac{d(MI)}{dt}$$

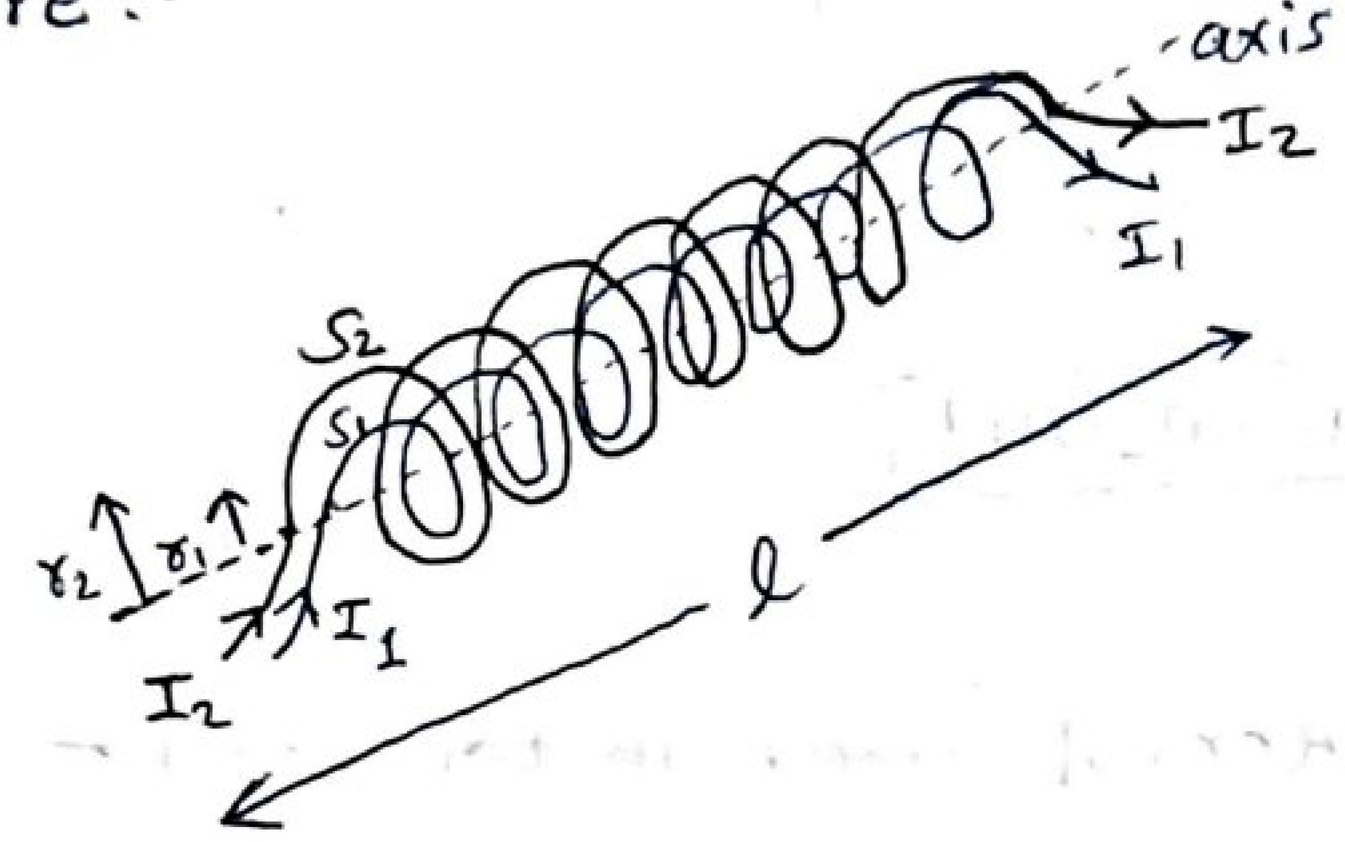
$$\boxed{e = -M\frac{dI}{dt}}$$

→ SI unit of mutual inductance is Henry (H).

$$[1H = 1\,Vs/A = 1\,Wb/A]$$

Mutual Inductance of two Long Solenoid:-

- Consider two long solenoid co-axially placed as shown in the figure:-



- At first a time varying current I_2 through S_2, then the total magnetic flux link with the inner solenoid S_1 is given by—

$$\phi_1 = B_2 A N_1$$

$$\phi_1 = (\mu_0 n_2 I_2) A N_1 \quad (\because B = \mu_0 nI)$$

$$\frac{\phi_1}{I_2} = \mu_0 n_2 A N_1$$

Then, the mutual inductance of the coil S_1 with respect to coil S_2 —

$$M_{12} = \frac{\phi_1}{I_2}$$

$$\boxed{M_{12} = \mu_0 n_2 A N_1}$$

But, $n_2 = \frac{N_2}{l}$

$$\left[M_{12} = \frac{\mu_0 N_1 N_2 A}{l}\right] \cdots (1)$$

- Now, the flux link with the outer solenoid S_2 due to current I_1 in the inner solenoid S_1, then the total flux link with the outer solenoid S_2 is given by—

$$\phi_2 = B_1 A N_2$$

$$\phi_2 = (\mu_0 n_1 I_1) A N_2$$

$$\frac{\phi_2}{I_1} = \mu_0 n_1 A N_2$$

$$M_{21} = \frac{\phi_2}{I_1}$$

$$\left[M_{21} = \mu_0 n_1 A N_2\right]$$

But $n_1 = \frac{N_1}{l}$

$$\left[M_{21} = \frac{\mu_0 N_1 N_2 A}{l}\right] \cdots (2)$$

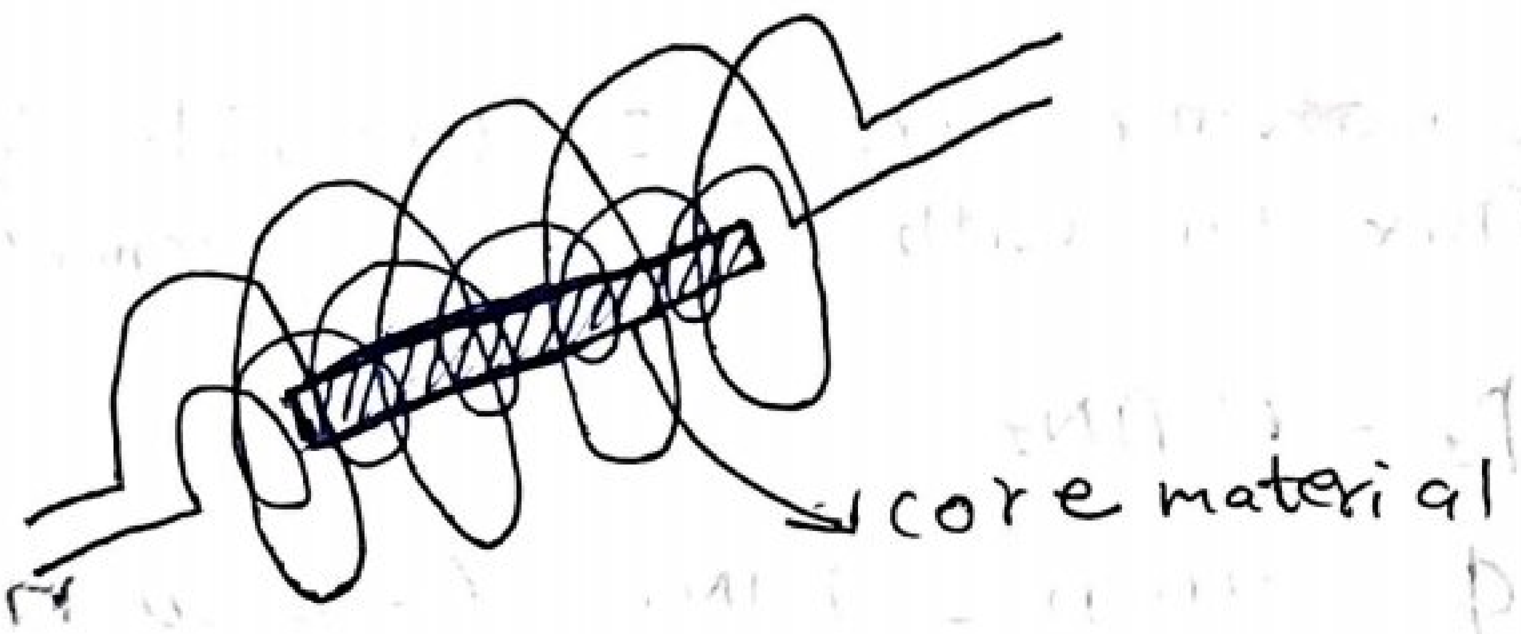
- Therefore, from equation 1 and 2, we can write in general -

$$\boxed{M = \frac{\mu_0 N_1 N_2 A}{l}}$$

This can be written as -

$$\boxed{M = \mu_0 n_1 n_2 (Al)}$$

- If the core material used in this coil -



$$\boxed{M = \mu_0 \mu_r n_1 n_2 A l}$$

Ques: A small piece of metal wire is dragged across the gap betwen the pole pieces of a magnet in 0.5 sec. The magnetic flux b/w the pole pieces is known to be 8×10^{-4} weber. Estimate the emf induced in the wire.

$\Rightarrow$ $$|e| = \frac{8 \times 10^{-4} - 0}{0.5 - 0} \quad \left[\because |e| = \frac{d\phi}{dt} = \frac{\phi_2 - \phi_1}{t_2 - t_1}\right]$$

$|e| = 16 \times 10^{-4}$ volt Ans

Ques: The magnetic flux through a coil perpendicular to the plane is varying according to the relation $\phi = 5t^3 + 4t^2 + 2t - 5$ wb. Calculate the induced current through the coil at $t = 2$ sec. If the resistance of the coil 5Ω.

$\Rightarrow$

$\phi = 5t^3 + 4t^2 + 2t - 5$

$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(5t^3 + 4t^2 + 2t - 5)$

$|e| = (15t^2 + 8t + 2)$ v

At $t = 2$ sec.

$|e| = 15 \times (2)^2 + 8 \times 2 + 2$

$|e| = 60 + 16 + 2$

$|e| = 78$ v

⇒ Now, $V = IR$

$I = \frac{V}{R} = \frac{|e|}{R} = \frac{78}{5} = 15.6\ A$ Ans

Ques A 10 Ω resistance coil has 1000 turns and at a time time 5.5×10^{-4} wb of flux passes through it. If the flux falls to 0.5×10^{-4} wb in 0.1 sec. Find the emf generated in volt and the charge flows through the coil in coulomb.

⇒ $R = 10\,\Omega$

$N = 1000$

$\phi_1 = 5.5\times10^{-4}$

$\phi_f = 0.5\times10^{-4}$

$t_i = 0$ sec

$t_f = 0.1$ sec.

$\cancel{|e| = \frac{d\phi}{dt} = \frac{\phi_f - \phi_i}{t_f - t_i} = \frac{0.5\times10^{-4} - 5.5\times10^{-4}}{0.1 - 0}}$

$\cancel{|e| = \frac{-5\times10^{-4}}{0.1}}$

$\cancel{e = 5\times10^{-3}\ V}$

$$e = -N\frac{d\phi}{dt} = -N\frac{(\phi_2 - \phi_1)}{(t_2 - t_1)} = -1000\,\frac{(0.5\times10^{-4} - 5.5\times10^{-4})}{0.1 - 0}$$

$$e = \frac{-1000(-5\times10^{-4})}{0.1} = \frac{5}{0.1} = 5\ V$$

$\therefore [e = 5V]$ Ans

⇒ $I = \frac{e}{R} = \frac{5}{10} = 0.5A$

$q = It = 0.5\times0.1 = [0.05\,c]$ Ans

Ques: A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T. If the induced emf is 11 V when the magnetic field changes to 1000 T in 4 sec, then what is the no. of turn in the coil?

⇒ $B_1 = 6000T$

$e = 11$ volt

$B_2 = 1000\ T$

$t = 4$ sec.

$A = \pi r^2 = \pi\times(0.01)^2$

$N = ?$

$$e = \frac{d\phi}{dt} = \frac{\phi_2 - \phi_1}{t_2 - t_1} = \frac{NB_2A - NB_1A}{t_2 - t_1}$$

$$e = NA\frac{(B_2 - B_1)}{t_2 - t_1}$$

$$11 = \frac{N\times0.0001\times3.14\times5000}{4}$$

$$N = \frac{11\times4}{0.0001\times3.14\times5000} = \boxed{28.1}$$ Ans