

∴ Its value after 3 years is Rs 1,749,20.

Chapter - 8 H.C.F and L.C.M

Exercise - 8.1

General Section

1. Find the H.C.F of the following expression.

a. $2x^3(x+2)(x-2)$ and $ux(x+2)(x+3)$

Solution

1st expression

$$2x^3(x+2)(x-2)$$

2nd expression

$$ux(x+2)(x+3)$$

$$= 2 \times x \times x \times x (x+2)(x-2) = 2 \times x \times x (x+2)(x+3)$$

$$\therefore \text{H.C.F} = 2x(x+2)$$

b. $ux^2(x-1)(x+2)$ and $6x^2y(x-1)(x-4)$

Solution:

1st expression

$$ux^2(x-1)(x+2)$$

2nd expression

$$6x^2y(x-1)(x-4)$$

$$= 2 \times x \times x \times y (x-1)(x+2)$$

~~= 2 \times 3 \cdot x \cdot x \cdot y (x-1)(x-4)~~

$$\therefore \text{H.C.F} = 2xy(x-1)$$

c. $6a^2b^2(a-b)(2a+3b)$ and $9a^3b^3(2a+3b)(a+b)$

Soln,

1st expression

$$6a^2b^2(a-b)(2a+3b)$$

2nd expression

$$9a^3b^3(2a+3b)(a+b)$$

$$= 2 \times 3 \cdot a \cdot a \cdot b \cdot b (a-b)(2a+3b)$$

$$= 3 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b (2a+3b)(a+b)$$

$$\therefore \text{H.C.F} = 3a^2b^2(2a+3b)$$

d. $12a^3b^3(a-3b)(a+b-2)$ and $16a^3b^2(a+3b)(a+b-2)$

Soln.,

1st expression

$$12a^3b^3(a-3b)(a+b-2)$$

$$= 4 \times 3 \cdot a^2 \cdot b^3 (a-3b)(a+b-2)$$

$$\therefore \text{H.C.F} = 4a^2b^2(a+b-2)$$

2nd expression

$$16a^3b^2(a+3b)(a+b-2)$$

$$= 4 \cdot 4 \cdot a^3b^2(a+3b)(a+b-2)$$

e. $(p+q)(p-q)$, $(p-q)(p^2+pq+q^2)$ and $(p-q)(p+q)(p^2+q^2)$

Soln.,

1st expression

$$(p+q)(p-q)$$

2nd expression

$$(p-q)(p^2+pq+q^2)$$

3rd expression

$$(p-q)(p+q)(p^2+q^2)$$

$$\therefore \text{H.C.F} = (p-q)$$

2. Find the H.C.F of the following expressions.

a. $a^2 - b^2$ and $a^3 + b^3$

Soln.,

1st expression

$$a^2 - b^2$$

$$(a+b)(a-b)$$

$$\therefore \text{H.C.F} = (a+b)$$

2nd expression

$$a^3 + b^3$$

$$= (a+b)(a^2 - ab + b^2)$$

b. $a^2 - 2a$ and $a^4 - 8a$

Solution:

1st expression

$$a^2 - 2a$$

$$a(a-2)$$

$$\therefore \text{H.C.F} = a(a-2)$$

2nd expression

$$a^4 - 8a$$

$$= a(a^3 - 2^3)$$

$$= a(a-2)(a^2 + 2a + 4)$$

c. $x^2 - 9$ and $3x + 9$

Solution:

1st expression

$$x^2 - 9$$

2nd expression

$$3x + 9$$

∴ $x^2 - 9$

∴ $(x+3)(x-3)$

$$\therefore \text{H.C.F} = (x+3)$$

d. $x^2 - y^2$ and $x^2 + 2xy + y^2$

Solution:

1st expression

$$x^2 - y^2$$

2nd expression

$$x^2 + 2xy + y^2$$

∴ $(x+y)(x-y)$

$$= x^2 + (1+1)xy + y^2$$

$$= x^2 + 2xy + y^2$$

$$= x(x+y) + y(x+y)$$

$$= (x+y)(x+y)$$

$$\therefore \text{H.C.F} = (x+y)$$

e. $4x^2 - 100$ and $4x + 20$

Solution:

1st expression

$$4x^2 - 100$$

2nd expression

$$4x + 20$$

∴ $4(x^2 - 25)$

$$= 4(x+5)$$

∴ $4(x+5)(x-5)$

$$\therefore \text{H.C.F} = 4(x+5)$$

f. $4x^3 + 8x^2$ and $5x^3 - 20x$

Solution:

1st expression

$$4x^2(x+2)$$

2nd expression

$$5x^3 - 20x$$

$$\therefore \text{H.C.F} = x(x+2)$$

$$= 5x(x^2 - 4)$$

$$= 5x(x+2)(x-2)$$

g. $a^2 - b^2 - 2a + 2$ and $a^2 - ab - a$

Solution

1st expression

$$a^2 - b^2 - 2a + 1$$

$$a^2 - 2a + 1 - b^2$$

$$\therefore (a-1)^2 - b^2$$

$$\therefore (a-1+b)(a-1-b)$$

$$\therefore \text{H.C.F} = (a-b-1)$$

2nd expression

$$a^2 - ab - a$$

$$\therefore a(a-b-1)$$

h. $x^3 - 8y^3$ and $x^2 + 2xy + 4y^2$

Solution:

1st expression

$$x^3 - 8y^3$$

$$\therefore x^3 - (2y)^3$$

$$\therefore (x-2y)(x^2+2xy+4y^2)$$

2nd expression

$$x^2 + 2xy + 4y^2$$

$$\therefore x^2 + (2+2)xy + 4y^2$$

$$\therefore x(x+2y) + 2y(x+2y)$$

$$\therefore (x+2y)(x+2y)$$

$$\therefore \text{H.C.F} = (x^2 + 2xy + 4y^2)$$

i. $p^4 - 16$ and $p^2 - p - 6$

Solution:

1st expression

$$p^4 - 16$$

$$(p^2)^2 - (4)^2$$

$$(p^2+4)(p^2-4)$$

$$(p^2+4)(p+2)(p-2)$$

2nd expression

$$p^2 - p - 6$$

$$\therefore p^2 - (3-2)p - 6$$

$$\therefore p^2 - 3p + 2p - 6$$

$$\therefore p(p-3) + 2(p-3)$$

$$\therefore (p+2)(p-3)$$

$$\therefore \text{H.C.F} = (p+2)$$

j. $x^4 + 4y^4$ and $2x^3y + 4xy^3 + 4x^2y^2$

Solution

1st expression

$$x^4 + 4y^4$$

$$\therefore (x^2)^2 + (2y^2)^2$$

$$= (x^2 + 2y^2)^2 - 4x^2y^2$$

$$= (x^2 + 2y^2)^2 - (2xy)^2$$

$$= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$$

$$\therefore \text{H.C.F} = (x^2 + 2y^2 + 2xy)$$

2nd expression

$$2x^3y + 4xy^3 + 4x^2y^2$$

$$= 2xy(x^2 + 2y^2 + 2xy)$$

k. $a^3 - 1$ and $a^4 + a^2 + 1$

Solution:

1st expression

$$a^3 - 1$$

$$\therefore (a-1)(a^2 + a + 1)$$

2nd expression

$$a^4 + a^2 + 1$$

$$= (a^2) + a^2 + 1 + (a^2)^2 + 1^2 + a^2$$

$$= (a^2 + 1)^2 - (a^2 + 1)a^2$$

$$= (a^2 + 1)^2 - (a)^2$$

$$= (a^2 + a + 1)(a^2 - a + 1)$$

$$\therefore \text{H.C.F} = (a^2 + a + 1)$$

l. $16x^4 - 4x^2 - 4x - 1$, $8x^3 - 1$

Solution,

1st expression

$$16x^4 - 4x^2 - 4x - 1$$

$$= 16x^4 - 2(4x^2 + 2x + 1)^2$$

$$= 16x^4 - 2((2x)^2 + 2 \cdot 2 \cdot 1 + 1^2)^2$$

$$= 16x^4 - 2(2x+1)^4$$

$$= (4x^2)^2 - (2x+1)^4$$

$$= (4x^2 + 2x + 1)(4x^2 - 2x - 1)$$

2nd expression:

$$8x^3 - 1$$

$$= (2x)^3 - 1^3$$

$$= (2x-1)(4x^2 + 2x + 1)$$

$$= (2x-1)(4x^2 + 2x + 1)$$

$$\therefore \text{H.C.F} = (4x^2 + 2x + 1)$$

Creative Section.

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3. Find the H.C.F of the following expression.

a. $a^2 - 4, a^3 + 8, a^2 + 5a + 6$

Solution

1st expression

$$a^2 - 4$$

$$\therefore (a+2)(a-2)$$

2nd expression

$$a^3 + 8$$

$$\therefore (a+2)(a^2 + 2a + 4)$$

3rd expression

$$a^2 + 5a + 6$$

$$\therefore a^2 + (3+2)a + 6$$

$$\begin{aligned} &= a^2 + 3a + 2a + 6 \\ &= a(a+3) + 2(a+3) \end{aligned}$$

$$\therefore \text{H.C.F} = (a+2)$$

b. $x^2 - 9, x^3 - 27, x^2 + x - 12$

Solution:

1st expression

$$x^2 - 9$$

$$(x^2 - (3)^2)$$

$$\therefore (x+3)(x-3)$$

2nd expression

$$x^3 - 27$$

$$(x^3 - (3)^3)$$

$$\therefore (x-3)(x^2 + 3x + 9)$$

3rd expression

$$x^2 + x - 12$$

$$\therefore x^2 + (4-3)x - 12$$

$$\therefore x^2 + 4x - 3x - 12$$

$$\therefore x(x+4) - 3(x+4)$$

$$\therefore (x-3)(x+4)$$

$$\therefore \text{H.C.F} = (x+4)$$

c. $4x^2 - 9, 2x^2 + x - 3, 8x^3 + 27$

Solution

1st expression

$$4x^2 - 9$$

$$(2x)^2 - (3)^2$$

2nd expression

$$2x^2 + x - 3$$

$$\therefore 2x^2 + (3-2)x - 3$$

$$\begin{aligned}
 & \text{1. } (2x+3)(2x-3) \\
 & \quad = 2x^2 + 3x - 2x - 3 \\
 & \quad = x(2x+3) - 1(2x+3) \\
 & \quad = (x-1)(2x+3)
 \end{aligned}$$

3^{rd} expression

$$8x^3 + 27$$

$$(2x)^3 + (3)^3$$

$$(2x+3)(4x^2 - 6x + 9)$$

$$\therefore \text{H.C.F} = (2x+3)$$

$$\text{d. } 3x^2 - 8x + 4, 2x^2 - 5x + 2, x^2 - 8x$$

Solution:

1st expression

$$3x^2 - 8x + 4$$

$$= 3x^2 + (6+2)x + 4$$

$$= 3x^2 - 6x - 2x + 4$$

$$= 3x(x-2) - 2(x-2)$$

$$= (3x-2)(x-2)$$

2nd expression

$$2x^2 - 5x + 2$$

$$= 2x^2 - (4+1)x + 2$$

$$= 2x^2 - 4x - x + 2$$

$$= 2x(x-2) - 1(x-2)$$

$$= (x-2)(2x-1)$$

3rd expression

$$x^4 - 8x$$

$$x(x^3 - 2^3)$$

$$= x(x-2)(x^2 + 2x + 4)$$

$$\therefore \text{H.C.F} = (x-2)$$

$$\text{e. } 5a^3 - 20a, a^3 - 3a^2 - 10a, a^3 - a^2 - 2a + 8$$

Solution:

1st expression

$$5a^3 - 20a$$

$$= 5a(a^2 - 2^2)$$

$$= 5a(a+2)(a-2)$$

2nd expression

$$a^3 - 3a^2 - 10a$$

$$= a(a^2 - 3a - 10)$$

$$= a\{a^2 - (5-2)a - 10\}$$

$$= a(a+2)(a-5)$$

3^{rd} expression

$$a^3 - a^2 - 2a + 8$$

$$= a^3 + 2^3 - a^2 - 2a$$

$$= (a+2)(a^2 - 2a + 4) - a(a+2)$$

$$= (a+2)(a^2 - 2a + 4 - a)$$

$$= (a+2)(a^2 - 3a + 4)$$

$$\therefore \text{H.C.F} = (a+2)$$

b. $m^3 - m^2 - m + 1$, $2m^4 - 2m$, $3m^2 + 3m - 6$

Solution:

1st expression

$$m^3 - m^2 - m + 1$$

$$= m^3 + 1^3 - m^2 - m$$

$$= (m+1)(m^2 - m + 1) - m(m+1)$$

$$= (m+1)(m^2 - m - m + 1)$$

$$= (m+1)(m^2 - 2m + 1)$$

$$= (m+1)\{(m)^2 - 2 \cdot m \cdot 1 + 1^2\}$$

$$= (m+1)\{m-1\}^2$$

2nd expression

$$2m^4 - 2m$$

$$= 2m(m^3 - 1^3)$$

$$= 2m(m-1)(m^2 + m + 1)$$

3rd expression

$$3m^2 + 3m - 6$$

$$= 3(m^2 + m - 2)$$

$$= 3\{m^2 + (2-1)m - 2\}$$

$$= 3\{m^2 + 2m - m - 2\}$$

$$= 3\{m(m+2) - 1(m+2)\}$$

$$= 3(m+2)(m-1)$$

$$\therefore \text{H.C.F} = (m-1)$$

$$g. (a+b)^2 - 4ab, a^3 - b^3, a^2 + ab - 2b^2$$

Solution:

1st expression

$$(a+b)^2 - 4ab$$

$$= a^2 + 2ab + b^2 - 4ab$$

$$= a^2 - 2ab + b^2$$

$$= (a)^2 - 2 \cdot a \cdot b + b^2$$

$$= (a-b)^2$$

$$\therefore (a-b)(a-b)$$

2nd expression

$$a^3 - b^3$$

$$= (a-b)(a^2 + ab + b^2)$$

3rd expression

$$a^2 + ab - 2b^2$$

$$= a^2 + (2-1)ab - 2b^2$$

$$= a^2 + 2ab - ab - 2b^2$$

$$= a(a+2b) - b(a+2b)$$

$$= (a-b)(a+2b)$$

$$\therefore \text{H.C.F} = a-b$$

$$h. x^3 - 6xy^3, x^2 - 6xy + 8y^2, x^2 - 16y^2$$

Solution

1st expression

$$x^3 - 6xy^3$$

$$= (x)^3 - (xy)^3$$

$$= (x-xy)(x^2 + xxy + 16y^2)$$

2nd expression

$$x^2 - 6xy + 8y^2$$

$$= x^2 - (4+2)xy + 8y^2$$

$$= x^2 - 4xy - 2xy + 8y^2$$

$$= x(x-4y) - 2y(x-4y)$$

$$= (x-4y)(x-2y)$$

3rd expression

$$x^2 - 16y^2$$

$$= (x)^2 - (4y)^2$$

$$= (x+4y)(x-4y)$$

$$\therefore \text{H.C.F} = (x-4y)$$

$$i. \quad 4x^4 + 16x^3 - 20x^2, \quad 3x^3 + 14x^2 - 5x, \quad x^4 + 12x^2$$

Solution:

1st expression

$$4x^4 + 16x^3 - 20x^2$$

$$\therefore 4x^2(x^2 + 4x - 5)$$

$$\therefore 4x^2\{x^2 + (5-1)x - 5\}$$

$$\therefore 4x^2\{x^2 + 5x - x - 5\}$$

$$\therefore 4x^2\{x(x+5) - 1(x+5)\}$$

$$\therefore 4x^2(x+5)(x-1)$$

2nd expression

$$3x^3 + 14x^2 - 5x$$

$$\therefore x(3x^2 + 14x - 5)$$

$$\therefore x\{3x^2 + (15-1)x - 5\}$$

$$\therefore x\{3x^2 + 15x - x - 5\}$$

$$\therefore x\{3x^2 + 3x(x+5) - 1(x+5)\}$$

$$\therefore x(x+5)(3x-1)$$

3rd expression

$$x^4 + 12x^2$$

$$\therefore x(x^3 + 5^3)$$

$$\therefore x(x+5)(x^2 + 5x + 25)$$

$$\therefore \text{H.C.F} = x(x+5)$$

$$j. \quad x^3 - 1, \quad x^4 + x^2 + 1, \quad x^3 + 1 + 2x^2 + 2x$$

Solution:

1st expression

$$x^3 - 1$$

$$(x)^3 - (1)^3$$

$$\therefore (x-1)(x^2+x+1)$$

2nd expression

$$x^4 + x^2 + 1$$

$$\therefore (x^2)^2 + (1)^2 + x^2$$

$$\therefore (x^2+1)^2 - 2x^2 + x^2$$

$$\therefore (x^2+1)^2 - (x)^2$$

$$\therefore (x^2+x+1)(x^2-x+1)$$

3rd expression

$$x^3 + 1 + 2x^2 + 2x$$

$$\therefore (x)^3 + (1)^3 + 2x^2 + 2x$$

$$\therefore (x+1)(x^2 - x + 1) + 2x(x+1)$$

$$\therefore (x+1)(x^2 - x + 1 + 2x)$$

$$\therefore (x+1)(x^2 + x + 1)$$

$$\therefore \text{H.C.F} = (x^2 + x + 1)$$

k. $8x^3 + 27y^3$, $16x^4 + 36x^2y^2 + 81y^4$, $4x^3 - 6x^2y + 9xy^2$

Solution:

1st expression:

$= 8x^3 + 27y^3$

$= (2x)^3 + (3y)^3$

$= (2x+3y)(4x^2 - 6xy + 9y^2)$

2nd expression:

$= 16x^4 + 36x^2y^2 + 81y^4$

$= (4x^2)^2 + (9y^2)^2 + 36x^2y^2$

$= (4x^2 + 9y^2)^2 - 2 \cdot 4x^2 \cdot 9y^2 + 36x^2y^2$

$= (4x^2 + 9y^2)^2 - (6xy)^2$

$= (4x^2 + 6xy + 9y^2)(4x^2 - 6xy + 9y^2)$

3rd expression:

$= 4x^3 - 6x^2y + 9xy^2$

$\therefore H.C.F = 4x^2 - 6xy + 9y^2$

$\therefore H.C.F = (4x^2 - 6xy + 9y^2)$

l. $x^3y + y^4$, $x^4 + x^2y^2 + y^4$, $2ax^3 - 2ax^2y + 2axy^2$

Solution:

1st expression:

$= x^3y + y^4$

$= y(x^3 + y^3)$

$= y(x+y)(x^2 - xy + y^2)$

2nd expression:

$= x^4 + x^2y^2 + y^4$

$= (x^2)^2 + (y^2)^2 + (xy)^2$

$= (x^2 + y^2)^2 - 2x^2y^2 + x^2y^2$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2 + xy + y^2)(x^2 - xy + y^2)$

3rd expression:

$= 2ax^3 - 2ax^2y + 2axy^2$

$= 2ax(x^2 - xy + y^2)$

$\therefore H.C.F = (x^2 - xy + y^2)$

m. $a^2 + 2ab + b^2 - c^2$, $b^2 + 2bc + c^2 - a^2$, $c^2 + 2ca + a^2 - b^2$

Solution:

1st expression:

$= a^2 + 2ab + b^2 - c^2$

$= (a+b)^2 - c^2 = (a+b+c)(a+b-c)$

2^{nd} expression

$$b^2 + 2bc + c^2 - a^2$$

$$(b+c)^2 - (a)^2$$

$$(b+c+a)(b+c-a)$$

$$(a+b+c)(b+c-a)$$

$$\therefore \text{H.C.F} = (a+b+c)$$

3^{rd} expression

$$c^2 + 2(a+c-b)^2$$

$$(c+a)^2 - (b)^2$$

$$(c+a+b)(c+a-b)$$

$$(a+b+c)(a+c-b)$$

n. $9x^2 - 4y^2 - 8yz - 4z^2$, $4z^2 - 4y^2 - 9x^2 - 12xy$, $9x^2 + 12xz + 4z^2 - 4y^2$

Solution

1st expression

$$9x^2 - 4y^2 - 8yz - 4z^2$$

$$= 9x^2 - (4y^2 + 8yz + 4z^2)$$

$$= 9x^2 - [(2y)^2 + 2 \cdot 2yz + (2z)^2]$$

$$= 9x^2 - (2y+2z)^2$$

$$= (3x)^2 - (2y+2z)^2$$

$$= (3x+2y+2z)(3x-2y-2z)$$

2^{nd} expression

$$4z^2 - 4y^2 - 9x^2 - 12xy + 4y^2$$

$$= 4z^2 - (4y^2 + 9x^2 + 12xy)$$

$$= 4z^2 - [(2y)^2 + 2 \cdot 2y \cdot 3x + (3x)^2]$$

$$= 4z^2 - (2y+3x)^2$$

$$= (2z+2y+3x)(2z-2y-3x)$$

$$= (3x+2y+2z)(2z-2y-3x)$$

3rd expression

$$9x^2 + 12xz + 4z^2 - 4y^2$$

$$= 9x^2 + (12x(3x)^2 + 2 \cdot 3x \cdot 2 + (2z)^2) - 4y^2$$

$$= (3x+2z)^2 - (2y)^2$$

$$= (3x+2y+2z)(3x-2y+2z)$$

$$\therefore \text{H.C.F} = (3x+2y+2z)$$

o. $x^2 - 18x - 19 + 20y - y^2$, $x^2 + x - y^2 + y$, $x^2 - y^2 + 2y - 1$

Soln,

1st expression

$$x^2 - 18x - 19 + 20y - y^2$$

$$= x^2 - 2 \cdot x \cdot 9 + 9^2 - 9^2 - 19 + 20y - y^2$$

$$= (x-9)^2 - 100 + 20y - y^2$$

$$= (x-9)^2 - (10^2 - 20y + y^2)$$

$$\begin{aligned}
 &= (x-y)^2 - (10-y)^2 \\
 &= (x-y+10-y)(x-y-10+y) \\
 &= (x-y+1)(x+y-19)
 \end{aligned}$$

2^{nd} expression

$$x^2 + x - y^2 + ty$$

$$x^2 - y^2 + tx + ty$$

$$(x+ty)(x-y) + t(x+ty)$$

$$(x+ty)(x-y+1)$$

$$\therefore \text{H.C.F} = (x-y+1)$$

3^{rd} expression

$$x^2 - y^2 + 2y - 1$$

$$= x^2 - (y^2 - 2y + 1)$$

$$= x^2 - (y-1)^2$$

$$= (x+y-1)(x-y+1)$$

Exercise-8.2

General Section

1. Find the L.C.M of the following expressions.

a. $3x(x+1)(x-1)$ and $2x^2(x-1)(x+3)$

Solution:

1st expression

$$3x(x+1)(x-1)$$

2nd expression

$$2x^2(x-1)(x+3)$$

$$\therefore \text{L.C.M} = 6x^2(x^2-1)(x+3)$$

b. $4x^3(x-3)(x+2)$ and $6x^2(x+2)(x+3)$

Solution:

1st expression

$$4x^3(x-3)(x+2)$$

2nd expression

$$6x^2(x+2)(x+3)$$

$$\therefore \text{L.C.M} = 12x^3(x+2)(x^2-9)$$

c. $8a^2b(a-b)(a^2+ab+b^2)$ and $12ab^2(a-b)(a+ab)$

Solution:

1st expression

$$8a^2b(a-b)(a^2+ab+b^2)$$

2nd expression

$$12ab^2(a-b)(a+ab)$$

$$\therefore \text{L.C.M} = 4ab(a-b)(a+b)(a^2+ab+b^2) \cdot 2a \cdot 3b$$

$$= 24a^2b^2(a+b)(a^3-b^3)$$

d. $(x+2)(x+3), (x+3)(x-2), (x-2)(x-3)$

Solution:

1st expression

$$(x+2)(x+3)$$

2nd expression

$$(x+3)(x-2)$$

3rd expression

$$(x-2)(x-3)$$

$$\therefore \text{L.C.M} = (x-2)(x+3)(x-3)(x+2)$$

$$= (x^2-4)(x^2-9)$$

e. $(a-3)(a-4), (a-4)(a-5), (a-5)(a-3)$

Solution:

1st expression

$$(a-3)(a-4)$$

2nd expression

$$(a-4)(a-5)$$

3rd expression

$$(a-5)(a-3)$$

$$\therefore \text{L.C.M} = (a-3)(a-4)(a-5)$$

2. Find the LCM of the following expressions.

a. $3x^2 + 6x, 2x^3 + 4x^2$

Solution:

1st expression

$$3x^2 + 6x$$

2nd expression

$$2x^3 + 4x^2$$

2. $3x(x+2)$

$$= 2x^2(x+2)$$

$$\therefore \text{L.C.M} = 6x^2(x+2)$$

b. $ax^2 + ax, ax^2 - a$

Solution:

1st expression

$$ax^2 + ax$$

2nd expression

$$ax^2 - a$$

2. $ax(x^2 + 1)$

$$= a(x^2 - 1)$$

$$\therefore \text{L.C.M} = ax(x^2 - 1)$$

$$= a(x+1)(x-1)$$

$$c. 4x^5y^4 + 2x^4y^5, 10x^4y^3 + 5x^3y^4$$

Solution:

1st expression

$$4x^5y^4 + 2x^4y^5$$

2nd expression

$$10x^4y^3 + 5x^3y^4$$

$$= 2x^4y^4(2xy + ty)$$

$$= 5x^3y^3(2x + ty)$$

$$\therefore \text{L.C.M} = 10x^4y^4(2x + ty)$$

$$d. x^2 - xy, x^3y - xy^3$$

Solution:

~~x~~ (x - ^{1st} expression)

$$x^2 - xy$$

2nd expression

$$x^3y - xy^3$$

$$= x(x-y)$$

$$= xy(x^2 - y^2)$$

$$= xy(xy)(x-y)$$

$$\therefore \text{L.C.M} = xy(x^2 - y^2)$$

$$e. 4x^2 - 2x, 8x^3 - 2x$$

Solution

1st expression

$$4x^2 - 2x$$

2nd expression

$$8x^3 - 2x$$

$$= 2x(2x-1)$$

$$= 2x(4x^2 - 1^2)$$

$$= 2x(2x+1)(2x-1)$$

$$\therefore \text{L.C.M} = 2x^2(4x^2 - 1)$$

$$f. a^3 - b^3, a^2 + ab + b^2$$

Solution.

1st expression

$$a^3 - b^3$$

2nd expression

$$a^2 + ab + b^2$$

$$= (a-b)(a^2 + ab + b^2)$$

$$\therefore \text{L.C.M} = (a-b)(a^2 + ab + b^2)$$

$$(a-b) = a^3 - b^3$$

g) $x^2 + 5x + 6$, $x^2 - 4$

Solution:

1st expression

$x^2 + 5x + 6$

$x^2 + (3+2)x + 6$

$x^2 + 3x + 2x + 6$

$x(x+3) + 2(x+3)$

$(x+3)(x+2)$

2nd expression

$x^2 - 4$

$(x+2)(x-2)$

$\therefore \text{L.C.M} = (x^2 - 4)(x+3)$

h) $x^2 - 9$, $3x^3 + 81$

Solution:

1st expression :

$x^2 - 9$

2nd expression

$3x^3 + 81$

$(x+3)(x-3)$

$3(x^3 + 3^3)$

$3(x+3)(x^2 - 3x + 9)$

$\therefore \text{L.C.M} = 3(x-3)(x+3)(x^2 - 3x + 9)$

$= 3(x+3)(x^3 + 27)(x^2 - 9)(x^2 - 3x + 9)$

i) $a^4b - ab^4$, $a^4b^2 - a^2b^4$

Solution:

1st expression

$a^4b^4 - ab^4$

2nd expression

$a^4b^2 - a^2b^4$

$ab(a^3 - b^3)$

$= a^2b^2(a^2 - b^2)$

$ab(a-b)(a^2 + ab + b^2)$

$= a^2b^2(a+b)(a-b)$

$\therefore \text{L.C.M} = a^2b^2(a-b)(a+b)(a^2 + ab + b^2)$

$= a^2b^2(a^2 - (a^2 - b^2))(a^2 + ab + b^2)$

$$j. x^u + x^2y^2 + y^u , x^3 - y^3$$

Solution:

1st expression

$$\therefore x^u + x^2y^2 + y^u$$

$$= (x^2)^{u/2} + (x^2)^2 + (y^2)^{u/2} + x^2y^2$$

$$= (x^2 + y^2)^2 - 2x^2y^2 + x^2y^2$$

$$= (x^2 + y^2)^2 - (xy)^2$$

$$= (x^2 + xy + y^2)(x^2 - xy + y^2)$$

$$\therefore \text{L.C.M} = (x-y)(x^2 - xy + y^2)(x^2 + xy + y^2)$$

$$= (x^2 - xy + y^2)(x^3 - y^3)$$

$$k. a^u + a^2b^2 + b^u , a^3 + b^3$$

Solution:

1st expression

$$a^u + a^2b^2 + b^u$$

$$= (a^2)^{u/2} + (b^2)^2 + a^2b^2$$

$$= (a^2 + b^2)^2 - 2a^2b^2 + a^2b^2$$

$$= (a^2 + b^2)^2 - (ab)^2$$

$$= (a^2 + ab + b^2)(a^2 - ab + b^2)$$

2nd expression

$$a^3 + b^3$$

$$= (a+b)(a^2 - ab + b^2)$$

$$\therefore \text{L.C.M} = (a^2 - ab + b^2)(a+b)(a^2 + ab + b^2)$$

$$= (a^3 + b^3)(a^2 + ab + b^2)$$

$$l. 6x^2 - x - 1 , 5x^4 + 2x$$

Solution

1st expression

$$6x^2 - x - 1$$

$$= 6x^2 - (3-2)x - 1$$

$$= 6x^2 - 3x + 2x - 1$$

$$= 3x(2x-1) + 1(2x-1)$$

$$= (2x-1)(3x+1)$$

2nd expression

$$5x^4 + 2x$$

$$= 2x(27x^3 + 1^3)$$

$$= 2x(3x+1)(9x^2 - 3x + 1)$$

$$\therefore \text{L.C.M} = 2x(27x^3 + 1)(2x-1)$$

Creative section

3. Find the L.C.M of the following:

a. $a^2 - 4, a^3 - 8, (a+2)^2$

Solution

1st expression

$$a^2 - 4$$

$$= (a+2)(a-2)$$

2nd expression

$$a^3 - 8$$

$$= (a-2)(a^2 + 2a + 4)$$

3rd expression

$$(a+2)^2$$

$$(a+2)(a+2)$$

$$\therefore \text{L.C.M} = (a-2)(a+2)(a+2)(a^2 + 2a + 4)$$

$$= (a+2)^2(a^3 - 8)$$

b. $(a-3)^2, a^2 - 9, a^3 + 27$

Solution:

1st expression

$$(a-3)^2$$

$$= (a-3)(a-3)$$

2nd expression

$$a^2 - 9$$

$$= (a+3)(a-3)$$

3rd expression

$$a^3 + 27$$

$$\therefore \text{L.C.M} = (a-3)(a+3)(a-3)(a^2 - 3a + 9)$$

$$= (a-3)^2(a^3 - 27)$$

c. $a^3 - 2a^2 + a, a^3 + a^2 - 2a, a^3 - 4a$

Solution:

1st expression

$$a^3 - 2a^2 + a$$

$$= a(a^2 - 2a + 1)$$

$$= a(a-1)^2$$

2nd expression

$$a^3 + a^2 - 2a$$

$$= a(a^2 + a - 2)$$

$$= a(a+2)(a-1)$$

3rd expression

$$a^3 - 4a$$

$$= a(a^2 - 2^2)$$

$$= a(a+2)(a-2)$$

$$\therefore \text{L.C.M} = a(a-1)(a+2)(a-2)(a-1)$$

$$= a(a-1)^2(a^2 - 4)$$

$$d \quad x^4 - y^4, \quad x^2 - y^2, \quad x^3 - y^3$$

Solution:

1st expression

$$x^4 - y^4$$

$$= (x^2 + y^2)(x^2 - y^2)$$

$$= (x^2 + y^2)(x+y)(x-y)$$

2nd expression

$$x^2 - y^2$$

$$= (x+y)(x-y)$$

3rd expression

$$x^3 - y^3$$

$$= (x-y)(x^2 + xy + y^2)$$

$$\therefore \text{L.C.M} = (x-y)(x+y)(x^2 + y^2)(x^2 + xy + y^2)$$

$$= (x^4 - y^4)(x^2 + xy + y^2)$$

$$e. \quad 4x^3 - 10x^2 + 4x, \quad 3x^4 - 8x^3 + 4x^2, \quad x^4 - 8x$$

Solution:

1st expression

$$4x^3 - 10x^2 + 4x$$

$$= 2x(2x^2 - 5x + 2)$$

$$= 2x(2x^2 - 4x - x + 2)$$

$$= 2x[2x(x-2) - 1(x-2)]$$

$$= 2x(x-2)(2x-1)$$

2nd expression

$$3x^4 - 8x^3 + 4x^2$$

$$= x^2(3x^2 - 8x + 4)$$

$$= x^2(3x^2 - 6x - 2x + 4)$$

$$= x^2\{3x(x-2) - 2(x-2)\}$$

$$= x^2(x-2)(3x-2)$$

3rd expression

$$x^4 - 8x$$

$$= x(x^3 - 2^3)$$

$$= x(x-2)(x^2 + 2x + 4)$$

$$\therefore \text{L.C.M} = x(x-2) \cdot 2(2x-1) \cdot x(3x-2) (x^2 + 2x + 4)$$

$$= 2x^2(x-2)(2x-1)(3x-2)(x^2 + 2x + 4)$$

f. $x^3 - 9x, x^4 - 2x^3 - 3x^2, x^3 - 27$

Solution

1st expression

$$x^3 - 9x$$

$$x(x^2 - 9)$$

$$x(x+3)(x-3)$$

2nd expression

$$x^4 - 2x^3 - 3x^2$$

$$= x^2(x^2 - 2x - 3)$$

$$= x^2(x^2 - 3x + x - 3)$$

$$= x^2(x-3)(x-1)$$

3rd expression

$$x^3 - 27$$

$$x^3 - 3^3$$

$$(x-3)(x^2 - 3x + 9)$$

$$\therefore \text{L.C.M} = x^2(x-3)(x+3)(x-1)(x^2 - 3x + 9)$$

$$= x^2(x^3 - 27)(x+3)(x-1)$$

g. $a^3 - 4a, a^4 - a^3 - 2a^2, a^3 - 8$

Solution

1st expression

$$a^3 - 4a$$

2nd expression

$$a^4 - a^3 - 2a^2$$

$$= a(a^2 - 2^2)$$

$$= a^2(a^2 - a - 2)$$

$$= a(a+2)(a-2)$$

$$= a^2(a^2 - 2a + a - 2)$$

$$(a-2)(a+2)(a-1)$$

$$= a^2(a-2)(a+1)$$

3rd expression

$$a^3 - 8$$

$$a^3 - 2^3$$

$$(a-2)(a^2 - 2a + 4)$$

$$\therefore \text{L.C.M} = a^2(a-2)(a+1)(a+2)(a^2 - 2a + 4)$$

$$= a^2(a^3 - 8)(a-2)(a+1)$$

h. $a^6 + a^2 + 1$, $a^3 - 1$, $a^3 - a^2 + a$

Solution:

1st expression

$$a^6 + a^2 + 1$$

2nd expression

$$a^3 - 1$$

$$(a^2)^2 + (1)^2 + a^2$$

$$= (a-1)(a^2+a+1)$$

$$= (a^2+1)^2 - 2a^2 + a^2$$

$$= (a^2+1)^2 - (a)^2$$

$$= (a^2+a+1)(a^2-a+1)$$

3rd expression

$$a^3 - a^2 + a$$

$$a(a^2 - a + 1)$$

$$\therefore \text{L.C.M} = a(a-1)(a^2+a+1)(a^2-a+1)$$

$$= a(a^3-1)(a^2-a+1)$$

i. $a^2 + 2ab + b^2 - c^2$, $b^2 + 2bc + c^2 - a^2$, $c^2 + 2ca + a^2 - b^2$

Solution:

1st expression

$$a^2 + 2ab + b^2 - c^2$$

2nd expression

$$b^2 + 2bc + c^2 - a^2$$

$$= (b+c)^2 - a^2$$

$$= (b+c+a)(b+c-a)$$

3rd expression

$$c^2 + 2ca + a^2 - b^2$$

$$(c+a)^2 - b^2$$

$$= (a+b+c)(a+c-b)$$

$$\therefore \text{L.C.M} = (a+b+c)(a+b-c)(b+c-a)(a+c-b)$$

$$j. \quad x^3 - 1, \quad x^4 + x^2 + 1, \quad x^3 + 1 + 2x^2 + 2x$$

Solution:

1st expression

$$x^3 - 1$$

$$\therefore (x-1)(x^2-x+1)$$

2nd expression

$$x^4 + x^2 + 1$$

$$= (x^2)^2 + 1^2 + x^2$$

$$= (x^2+1)^2 - 2x^2 + x^2$$

$$= (x^2+x+1)(x^2-x+1)$$

3rd expression

$$x^3 + 1 + 2x^2 + 2x$$

$$\therefore (x+1)(x^2-x+1) + 2x(x+1)$$

$$\therefore (x+1)(x^2+x+1)$$

$$\therefore \text{L.C.M.} = (x^2-1)(x^2-x+1)(x^2+x+1)$$

$$k. \quad 8x^3 + 27y^3, \quad 16x^4 + 36x^2y^2 + 81y^4, \quad 4x^3 - 6x^2y + 9xy^2$$

Solution:

1st expression

$$8x^3 + 27y^3$$

$$\therefore (2x)^3 + (3y)^3$$

$$\therefore (2x+3y)(4x^2 - 6xy + 9y^2)$$

2nd expression

$$16x^4 + 36x^2y^2 + 81y^4$$

$$= (4x^2)^2 + (9y^2)^2 + 36x^2y^2$$

$$= (4x^2+9y^2)^2 - 2 \cdot 4x^2 \cdot 9y^2 + 36x^2y^2$$

$$= (4x^2+9y^2)^2 - (6xy)^2$$

$$= (4x^2 + 6xy + 9y^2)(4x^2 - 6xy + 9y^2)$$

$$(4x^2 - 6xy + 9y^2)$$

3rd expression

$$4x^3 - 6x^2y + 9xy^2$$

$$\therefore x(4x^2 - 6xy + 9y^2)$$

$$\therefore \text{L.C.M.} = x(2x+3y)(4x^2 - 6xy + 9y^2)$$

$$1. \quad x^3y + y^4, x^4 + x^2y^2 - ty^4, 2ax^3 - 2axy^2 + 2axy^2$$

Solution:

1st expression

$$x^3y + y^4$$

$$\therefore y(x^3 + y^3)$$

$$\therefore y(xy)(x^2 - xy + y^2)$$

2nd expression

$$x^4 + x^2y^2 - ty^4$$

$$\therefore (x^2)^2 + (y^2)^2 + x^2y^2$$

$$\therefore (x^2 + y^2)^2 - 2x^2y^2 + x^2y^2$$

$$\therefore (x^2 + xy + y^2)(x^2 - xy + y^2)$$

3rd expression

$$2ax^3 - 2axy^2 + 2axy^2$$

$$\therefore 2ax(x^2 - xy + y^2)$$

$$\therefore \text{L.C.M} = 2axy(x^2 + xy + y^2)(x^2 - xy + y^2)$$

$$2. \quad a^2 + 2ab \quad x^3 - 2x^2y + 2xy^2 - y^3, x^4 - y^4, x^3 + y^3$$

Solution:

1st expression

$$x^3 - 2x^2y + 2xy^2 - y^3$$

$$\therefore x(x^2 - 2xy) + y^2 +$$

$$\therefore x^3 - y^3 - 2x^2y + 2xy^2$$

$$\therefore (x-y)(x^2 + xy + y^2) - 2xy(x-y)$$

$$\therefore (x-y)(x^2 - xy + y^2)$$

2nd expression

$$x^4 - y^4$$

$$\therefore (x^2)^2 - (y^2)^2$$

$$\therefore (x^2 + y^2)(x^2 - y^2)$$

$$\therefore (x^2 + y^2)(x+y)(x-y)$$

3rd expression

$$x^3 + y^3$$

$$\therefore (x+y)(x^2 - xy + y^2)$$

$$\therefore \text{L.C.M} = (x^2 + y^2)(x+y)(x-y)(x^2 - xy + y^2)$$

$$\therefore (x^4 - y^4)(x^2 - xy + y^2)$$

k. $x^2 + 3x + 2$, $x^2 + 5x + 6$, $x^2 + 4x + 3$

Solution:

1st expression

$$x^2 + 3x + 2$$

$$\therefore x^2 + (2+1)x + 2$$

$$= x^2 + 2x + x + 2$$

$$= (x+2)(x+1)$$

2nd expression

$$x^2 + 5x + 6$$

$$= x^2 + 3x + 2x + 6$$

$$= x(x+3) + 2(x+3)$$

$$= (x+2)(x+3)$$

3rd expression

$$x^2 + 4x + 3$$

$$= x^2 + 3x + x + 3$$

$$= (x+3)(x+1)$$

$$\therefore \text{L.C.M} = (x+1)(x+2)(x+3)$$

l. $2x^3 + 2x^2 - 12x$, $6x^3 - 6x^2 - 72x$, $4x^3 - 24x^2 + 32x$

Solution:

1st expression

$$2x^3 + 2x^2 - 12x$$

$$= 2x(x^2 + x - 6)$$

$$= 2x(x^2 + 3x - 2x - 6)$$

$$= 2x(x+3)(x-2)$$

2nd expression

$$6x^3 - 6x^2 - 72x$$

$$= 6x(x^2 - x - 12)$$

$$= 6x(x^2 - 4x + 3x - 12)$$

$$= 6x(x-4) + (x+3)$$

3rd expression

$$4x^3 - 24x^2 + 32x$$

$$= 4x(x^2 - 6x + 8)$$

$$= 4x(x^2 - 8x + 4x - 2x + 8)$$

$$= 4x(x-4)(x-2)$$

$$\therefore \text{L.C.M} = 2x \cdot 3x \cdot 2x (x-2) \cancel{(x+3)} (x+3)(x-4)$$

$$= 12x(x-2)(x+3)(x-4)$$