# Mechanical Waves



The waves which require a material medium for their propagation are called mechanical waves. Sound waves in air, waves in stretched string are examples of mechanical waves.

## Speed of Wave Motion.

The speed of wave motion is the distance travelled by the wave in a given interval of time.

## Velocity of Sound wave in Solid and liquid

When a sound wave propagates through a medium, its different regions undergo varying stress and strain. So, the velocity of sound waves depends on the modulus of elasticity (E), inertia and density ($\rho$) of the medium. It can be shown that the velocity of sound, $v = \sqrt{\frac{E}{\rho}}$

i. Speed of sound in solid: When a wave travels along the rod, Young's Modulus Y is relevant for modulus of elasticity. Thus, $v = \sqrt{\frac{Y}{\rho}}$

ii. Speed of sound wave in liquid: The wave propagates in all directions, bulk modulus B is relevant for modulus of elasticity. Thus, $v = \sqrt{\frac{B}{\rho}}$

iii. Speed of Transverse wave through a stretched string:

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension of the string and $\mu$ is the mass per unit length of the string.

iv. Speed of electromagnetic wave:

$$v = \sqrt{\frac{1}{\epsilon\mu}}$$

where, $\epsilon$ and $\mu$ are the permittivity & permeability of a medium through which EM wave is propagated.

v. Speed of sound in extended solid:

$$v = \sqrt{\frac{B + 4\eta/3}{\rho}}$$ where B = Bulk modulus of elasticity

$\eta$ = modulus of rigidity.

## Newton's Formula For Velocity of Sound in Gas.

Newton assumed that the propagation of sound wave is very slow. Hence, it obeys the thermodynamic process called the isothermal process. In such process temperature variation is negligible. There is no any temperature difference between the region of compression and rarefaction.

Under isothermal process,

$$PV = \text{constant}$$

Differentiating,

$$PdV + VdP = 0$$

$$PdV = -VdP$$

or $P = -\frac{VdP}{dV} = -\frac{dP}{\frac{dV}{V}}$

Here, dp and dv refer the change of pressure and volume in gas.

Also, Bulk modus of elasticity $(B) = -\frac{dP}{\frac{-dV}{V}}$

So, $P = B$.

From the expression of speed of sound in a medium,

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}}$$

This is Newton's Formula for velocity of sound in Gas.

For air at NTP

$P = 760$ mm of Hg $= 1.01 \times 10^5\ N/m^2$

$\rho = 1.29\ kg/m^3$.

$\therefore v = \sqrt{\frac{1.01 \times 10^5}{1.29}} \approx 280\ m/s.$

Experimentally observed value of speed of sound in air at NTP is 332 m/s. Due to this difference, it was thought that Newtons formula needed to be modified with a necessary correction. Later on, Laplace corrected the above relation with necessary modification.

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## laplace's Correction

Unlike Newton's assumption, laplace assumed that the propagation of sound wave is very fast so that heat cannot be shared by compressions and rarefaction is such very short time. So, the temperature of gas changes i.e. the process is adiabatic rather than isothermal. Thus, under adiabatic process

$$PV^{\gamma} = \text{constant}$$

where $\gamma = \frac{C_P}{C_V}$ is the specific heat ration.

Differentiating,

$$d(PV^{\gamma}) = 0$$

$$P\gamma V^{\gamma-1}\, dV + V^{\gamma}\, dP = 0$$

Dividing by $V^{\gamma-1}$,

$$P\gamma\, dV + V\, dP = 0$$

or, $$\gamma P = -\frac{dP}{dV/V} = B$$

where B is the Bulk modulus of elasticity,

The speed of sound in air is given by,

$$V = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

This is laplace's formula for the speed of sound in a gas.

For air, $\gamma = 1.41$ and at NTP,

$$V = \sqrt{\frac{1.41 \times 1.01 \times 10^5}{1.41}} = 331.6\, m/s.$$

This result closely agrees with the experimental value. Thus, Laplace's formula gives the correct value for velocity of sound in air.

## Factors Affecting the speed of sound in a Gas.

1. Effect of Temperature

For a gas of mass m in a volume V.

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{\rho V}}$$

$$= \sqrt{\frac{\gamma nRT}{m}} = \sqrt{\frac{\gamma nRT}{nM}} \quad (\because m = nM \text{ \& } M = \text{molar mass of a gas}).$$

$$= \sqrt{\frac{\gamma RT}{M}}$$

$\therefore$ $v \propto \sqrt{T}$

Let, $v_1$ and $v_2$ be the speed of sound at temperatures $T_1$ and $T_2$ respectively in a gas.

Then,

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

2. Effect of pressure

For n-mole of gas, velocity of sound in gas is

$$v = \sqrt{\frac{\gamma RT}{M}}$$

For a gas, $\gamma$, R & M are constant. This shows that if the temperature of the gas is constant, the velocity of sound in gas is also constant. So, it is independent of the pressure of the gas.

3. Effect of Density:

Consider two gases at same pressure P having different density $\rho_1$ and $\rho_2$. Then,

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \text{ and } v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

Dividing (i) by (ii),

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

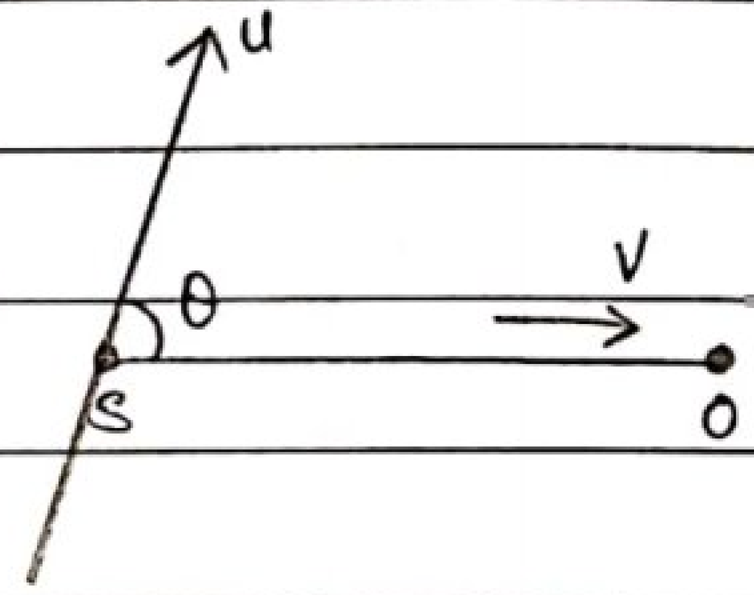
$\therefore$ $v \propto \frac{1}{\sqrt{\rho}}$

4. Effect of Humidity

The density of water vapour is smaller than that of dry air and the presence of moisture in air reduces the density of air in the atmosphere. So, $\rho_{moist} < \rho_{dry}$ and as the velocity of sound in gas is inversely proportional to the square root of its density, velocity of sound in moist air is greater than in dry air. Greater the humidity, higher is the velocity of sound.

5. Effect of Wind

Consider U and v be the speed of wind and speed of sound wave resp. Let the wind blows at an angle θ with the line joining the sound source (S) and sound observer (O) as shown.




The resultant speed of sound wave:

$V_R = v + u\cos\theta$

i. If the wind blow towards the observer. (i.e. $\theta = 0°$)

$V_R = v + U\cos 0° = v + u$ (maximum velocity).

ii. If the wind blow in perpendicular direction of source-observer position ($\theta = 90°$)

$V_R = v + U\cos 90° = v$ (no effect).

iii. If wind blows in opposite direction of observer ($\theta = 180°$)

$V_R = v + U\cos 180° = v - U$ (minimum velocity).

6. Effect of frequency, wavelength and Amplitude.

The velocity of sound in air is independent of both frequency and wavelength. To a large extent, velocity is independent of amplitude as well. If amplitude is very large, the compression and rarefaction may result in large temperature variations and it affects the velocity of sound.