Chapter - 12

Vectors

C/W

Date:

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Modulus of Vector

If $P(x,y)$ be any point on the plane.

$$\vec{OP} = (x,y) = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$|\vec{OP}| = \sqrt{x^2+y^2}$$

If $P(x,y,z)$ be any point on Space then,

$$|\vec{OP}| = \sqrt{x^2+y^2+z^2}$$

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are any two points on plane.

$$\vec{AB} = (x_2 - x_1, y_2 - y_1)$$

$$|\vec{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

If $A(x_1, y_2, z_1)$ and $B(x_2, y_2, z_2)$ are any two points on the Space then,

$$\vec{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$|\vec{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Unit Vector

A vector with magnitude 1 is called unit vector.

If $\vec{OP} = (x,y)$

Unit Vector of $\vec{OP} = \hat{OP} = \frac{\vec{OP}}{|\vec{OP}|} = \frac{(x,y)}{\sqrt{x^2+y^2}} = \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right)$

If $\vec{OP} = (x,y,z)$ on space.

$$\hat{OP} = \frac{\vec{OP}}{|\vec{OP}|} = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

Null Vector

Vector having magnitude 0 is called null Vector.

e.g. $\vec{a} = (0,0)$ then, $|\vec{a}| = 0$

Negative Vector

If $\vec{OP}$ is any vector the negative of OP is a vector having same magnitude but opposite direction.

Eg: $\vec{OP} = -\vec{PO}$

Like Vector

Two vector having same direction are called like vector

Unlike Vector

Two Vector having opposite direction are called unlike Vector.

Equal Vector

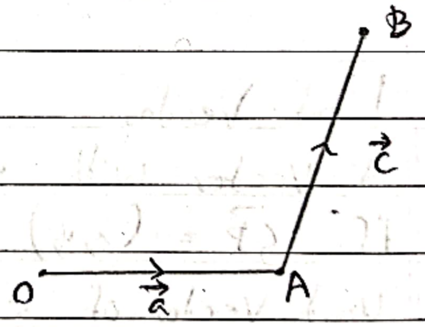
Two vector having same direction and equal magnitude are called equal vector.

Triangular Law of addition

If $\vec{OA} = \vec{a}$

$\vec{OB} = \vec{b}$

$\vec{AB} = \vec{c}$



Then, $\vec{OA} + \vec{AB} = \vec{OB}$

or, $\vec{AB} = \vec{OB} - \vec{OA}$

Collinear and Non-collinear Vector

Any number of vector are said to be collinear when all of them are parallel to the same line.

Otherwise, vector are said to be non-collinear.

Note:

i. If $\vec{a}$ and $\vec{b}$ are two non-zero non-collinear vector and ~~$x\vec{a}$~~ $x\vec{a} + y\vec{b} = 0$ then $x=0$ and $y=0$.

ii. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are two non-zero and collinear vector then one can be written as the linear combination of remaining two.

Eg: $\vec{a} = x\vec{b} + y\vec{c}$

Unit Vector along Co-ordinate X-axis:

$\vec{i}$ = Unit Vector along x-axis

$\vec{i} = (1,0)$

$\vec{j}$ = Unit vector along y-axis

$\vec{j} = (0,1)$

In space

$\vec{i} = (1,0,0)$

$\vec{j} = (0,1,0)$

$\vec{k} = (0,0,1)$

Coplanar and Non-Coplanar Vector

A system of Vector are said to coplanar if a plane can be drawn parallel to all of them.

Otherwise, Non-Coplanar.

Note:

i. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-zero and non-coplanar Vector and if (x, y, z) are three scalars such that $x\vec{a} + y\vec{b} + z\vec{c} = 0$, then $x=0$, $y=0$, $z=0$

ii. If 3 vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are co-planar then one vector can be written as linear combination of remaining two.

Eg: $\vec{a} = x\vec{b} + y\vec{c}$

Where, x and y are some scalar.

Exercise - 12.1

1. Show that the following points are collinear.

a. $(1,-2,3)$, $(2,3,-4)$ and $(0,-7,10)$

Solution:

Let O be the origin. Let A, B and C be three points with position vectors $(1,-2,3)$, $(2,3,-4)$ and $(0,-7,10)$ respectively. Then,

$$\vec{OA} = \vec{i} - 2\vec{j} + 3\vec{k}$$
$$\vec{OB} = 2\vec{i} + 3\vec{j} - 4\vec{k}$$
$$\vec{OC} = -7\vec{j} + 10\vec{k}$$

Now,

$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= \vec{i} + 5\vec{j} - 7\vec{k}$$

Also,

$$\vec{AC} = \vec{OC} - \vec{OA}$$
$$= -\vec{i} - 5\vec{j} + 7\vec{k}$$
$$= -(\vec{i} + 5\vec{j} - 7\vec{k})$$

Here, $\vec{AC} = -\vec{AB}$, which shows that $\vec{AB}$ and $\vec{AC}$ are parallel. But they start from the same point A. So, A, B, C are collinear.

b. $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$, $3\vec{i} + 8\vec{j} - 6\vec{k}$

Solution:

Let the given vectors be $A(1,2,4)$, $B(2,5,-1)$, $C(3,8,-6)$

Then,

$$\vec{OA} = \vec{i} + 2\vec{j} + 4\vec{k}$$
$$\vec{OB} = 2\vec{i} + 5\vec{j} - \vec{k}$$
$$\vec{OC} = 3\vec{i} + 8\vec{j} - 6\vec{k}$$

Now,

$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= 2\vec{i} + 5\vec{j} - \vec{k} - \vec{i} - 2\vec{j} - 4\vec{k}$$

$= \vec{i} + 3\vec{j} - 5\vec{k}$

Also,

$\vec{AC} = \vec{OC} - \vec{OA}$

$= 3\vec{i} + 8\vec{j} - 6\vec{k} - \vec{i} - 2\vec{j} - 4\vec{k}$

$= 2\vec{i} + 6\vec{j} - 10\vec{k}$

$= 2(\vec{i} + 3\vec{j} - 5\vec{k})$

Here, $\vec{AC} = 2\vec{AB}$, which shows $\vec{AB}$ and $\vec{AC}$ are parallel.

$\therefore$ The given three vectors are collinear.

c. $2\vec{i} + 5\vec{j} - 8\vec{k}$, $3\vec{i} + 2\vec{j} - \vec{k}$, $4\vec{i} - \vec{j} + 6\vec{k}$

Solution:

Let the given vectors be $A(2, 5, -8)$, $B(3, 2, -1)$ and $C(4, -1, 6)$ then,

$\vec{OA} = 2\vec{i} + 5\vec{j} - 8\vec{k}$

$\vec{OB} = 3\vec{i} + 2\vec{j} - \vec{k}$

$\vec{OC} = 4\vec{i} - \vec{j} + 6\vec{k}$

Now,

$\vec{AB} = \vec{OB} - \vec{OA}$

$= 3\vec{i} + 2\vec{j} - \vec{k} - 2\vec{i} - 5\vec{j} + 8\vec{k}$

$= \vec{i} - 3\vec{j} + 7\vec{k}$

Also,

$\vec{AC} = \vec{OC} - \vec{OA}$

$= 4\vec{i} - \vec{j} + 6\vec{k} - 2\vec{i} - 5\vec{j} + 8\vec{k}$

$= 2\vec{i} - 6\vec{j} + 14\vec{k}$

$= 2(\vec{i} - 3\vec{j} + 7\vec{k})$

Here, $\vec{AC} = 2\vec{AB}$, which shows $\vec{AB}$ and $\vec{AC}$ are parallel.

$\therefore$ The given three vectors are collinear.

d. $\vec{i} - 4\vec{j} + \vec{k}$, $2\vec{i} + 3\vec{j} - 5\vec{k}$, $-11\vec{j} + 7\vec{k}$

Solution:

Let the given vectors be $A(1,-4,1)$, $B(2,3,-5)$ and $C(0,-11,7)$

Then,

$\vec{OA} = \vec{i} - 4\vec{j} + \vec{k}$

$\vec{OB} = 2\vec{i} + 3\vec{j} - 5\vec{k}$

$\vec{OC} = -11\vec{j} + 7\vec{k}$

Now,

$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= 2\vec{i} + 3\vec{j} - 5\vec{k} - \vec{i} + 4\vec{j} - \vec{k}$$
$$= \vec{i} + 7\vec{j} - 6\vec{k}$$

Also,

$$\vec{AC} = \vec{OC} - \vec{OA}$$
$$= -11\vec{j} + 7\vec{k} - \vec{i} + 4\vec{j} - \vec{k}$$
$$= -\vec{i} - 7\vec{j} + 6\vec{k}$$
$$= -(\vec{i} + 7\vec{j} - 6\vec{k})$$

Here, $\vec{AC} = -\vec{AB}$, which shows that $\vec{AB}$ and $\vec{AC}$ are parallel.

$\therefore$ The given three vectors are collinear.

2.a. If $3\vec{i} + \vec{j} - \vec{k}$ and $\lambda\vec{i} - 4\vec{j} + 4\vec{k}$ are collinear, find the value of λ.

Solution:

Let $\vec{a} = 3\vec{i} + \vec{j} - \vec{k}$

$\vec{b} = \lambda\vec{i} - 4\vec{j} + 4\vec{k}$

Suppose,

$$\vec{a} = x\vec{b}$$

or, $3\vec{i} + \vec{j} - \vec{k} = x(\lambda\vec{i} - 4\vec{j} + 4\vec{k})$

or, $3\vec{i} + \vec{j} - \vec{k} = \lambda x\vec{i} - 4x\vec{j} + 4x\vec{k}$

or, $-4x = 1$

$\therefore x = \frac{-1}{4}$

Also,

$\lambda x = 3$

or, $\lambda \cdot \left(-\frac{1}{4}\right) = 3$

$\therefore \lambda = -12$

b. Find the value of λ so that the three points with position vectors $-2\vec{i} + 3\vec{j} + \lambda\vec{k}$, $\vec{i} + 2\vec{j} + 3\vec{k}$ and $7\vec{i} - \vec{k}$ are collinear.

Solution:

Let $\vec{a} = -2\vec{i} + 3\vec{j} + \lambda\vec{k}$

$\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$

$\vec{c} = 7\vec{i} - \vec{k}$

Suppose,

$\vec{a} = x\vec{b} + y\vec{c}$

or, $-2\vec{i} + 3\vec{j} + \lambda\vec{k} = x(\vec{i} + 2\vec{j} + 3\vec{k}) + y(7\vec{i} - \vec{k})$

or, $-2\vec{i} + 3\vec{j} + \lambda\vec{k} = x\vec{i} + 2x\vec{j} + 3x\vec{k} + 7y\vec{i} - y\vec{k}$

or, $-2\vec{i} + 3\vec{j} + \lambda\vec{k} = (x + 7y)\vec{i} + (2x - 0.y)\vec{j} + (3x - y)\vec{k}$

Now, Equating the corresponding vectors

$-2 = x + 7y$ —— (i)

$3 = 2x$

or, $x = \frac{3}{2}$ —— (ii)

Also, $\lambda = 3x - y$ —— (iii)

from (i) and (ii)

$-2 = \frac{3}{2} + 7y$

$\therefore y = -\frac{1}{2}$

Using the value in equation (iii), we get

$\lambda = 5$

c. Find the value of λ so that the three points with position vectors $-\vec{i}-\vec{j}-\vec{k}$, $\vec{i}+3\vec{j}+2\vec{k}$ and $5\vec{i}+\lambda\vec{j}+8\vec{k}$ are collinear.

Solution:

Let $\vec{a} = -\vec{i}-\vec{j}-\vec{k}$

$\vec{b} = \vec{i}+3\vec{j}+2\vec{k}$

$\vec{c} = 5\vec{i}+\lambda\vec{j}+8\vec{k}$

Suppose,

$$\vec{c} = x\vec{a} + y\vec{b}$$

or, $5\vec{i}+\lambda\vec{j}+8\vec{k} = x(-\vec{i}-\vec{j}-\vec{k}) + y(\vec{i}+3\vec{j}+2\vec{k})$

or, $5\vec{i}+\lambda\vec{j}+8\vec{k} = -x\vec{i}-x\vec{j}-x\vec{k} + y\vec{i}+3y\vec{j}+2y\vec{k}$

or, $5\vec{i}+\lambda\vec{j}+8\vec{k} = (y-x)\vec{i}+(3y-x)\vec{j}+(2y-x)\vec{k}$

Comparing the corresponding like vectors,

$y - x = 5$

or, $y = x + 5$ —— (i)

Also,

$2y - x = 8$

or, $2(x+5) - x = 8$

or, $2x + 10 - x = 8$

or, $x = -2$ —— (ii)

Also,

$3y - x = \lambda$

or, $3(x+5) - (-2) = \lambda$

or, $3(-2+5) + 2 = \lambda$

$\therefore \lambda = 11$

3. Show that the following vectors are Coplanar.

a. $(3,-7,8)$, $(3,-2,1)$, $(1,1,-2)$

Solution:

Let $\vec{a} = 3\vec{i} - 7\vec{j} + 8\vec{k}$

$\vec{b} = 3\vec{i} - 2\vec{j} + \vec{k}$

$\vec{c} = \vec{i} + \vec{j} - 2\vec{k}$

Suppose,

$\vec{a} = x\vec{b} + y\vec{c}$

or, $3\vec{i} - 7\vec{j} + 8\vec{k} = 3x\vec{i} - 2x\vec{j} + x\vec{k} + y\vec{i} + y\vec{j} - 2y\vec{k}$

or, $3\vec{i} - 7\vec{j} + 8\vec{k} = (3x+y)\vec{i} + (y-2x)\vec{j} + (x-2y)\vec{k}$

Equating the coefficient of like vectors;

$3x + y = 3$ —— (i)

$-2x + y = -7$ —— (ii)

$x - 2y = 8$ —— (iii)

Solving equation (i) and (ii), we get

$x = 2$ and $y = -3$

putting the value of x and y in $\vec{a} = x\vec{b} + y\vec{c}$, we get

$\vec{a} = 2\vec{b} - 3\vec{c}$

$\therefore$ The given vectors are coplanar.

b. $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$, $-\vec{b} + 2\vec{c}$

Solution:

Let $\vec{r_1} = \vec{a} - 2\vec{b} + 3\vec{c}$

$\vec{r_2} = -2\vec{a} + 3\vec{b} - 4\vec{c}$

$\vec{r_3} = -\vec{b} + 2\vec{c}$

Suppose,

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$ —— (i)

or, $-\vec{b} + 2\vec{c} = x\vec{a} - 2x\vec{b} + 3x\vec{c} - 2y\vec{a} + 3y\vec{b} - 4y\vec{c}$

or, $-\vec{b} + 2\vec{c} = (x-2y)\vec{a} + (3y-2x)\vec{b} + (3x-4y)\vec{c}$

Equating the coefficient of like vectors;

$x - 2y = 0$ —— (ii)

$3y - 2x = -1$ —— (iii)

$3x - 4y = 2$ —— (iv)

Solving equation (ii) and (iii), we get

$x = 2$ and $y = 1$

putting the value of x and y in equation (i), we get

$\vec{r_3} = 2\vec{r_1} + \vec{r_2}$

$\therefore$ The given vectors are coplanar.

c. $\vec{a} - 3\vec{b} + 5\vec{c}$, $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$

Solution:

Let $\vec{r_1} = \vec{a} - 3\vec{b} + 5\vec{c}$

$\vec{r_2} = \vec{a} - 2\vec{b} + 3\vec{c}$

$\vec{r_3} = -2\vec{a} + 3\vec{b} - 4\vec{c}$

Suppose,

$\vec{r_1} = x\vec{r_2} + y\vec{r_3}$ —— (i)

or, $\vec{a} - 3\vec{b} + 5\vec{c} = x\vec{a} - 2x\vec{b} + 3x\vec{c} - 2y\vec{a} + 3y\vec{b} - 4y\vec{c}$

or, $\vec{a} - 3\vec{b} + 5\vec{c} = (x - 2y)\vec{a} + (3y - 2x)\vec{b} + (3x - 4y)\vec{c}$

Equating the corresponding like vectors

$x - 2y = 1$ —— (ii)

$3y - 2x = -3$ —— (iii)

$3x - 4y = 5$ —— (iv)

Solving (ii) and (iii) we get;

$x = 3$ and $y = 1$

Using the values in equation (i), we get

$\vec{r_1} = 3\vec{r_2} + \vec{r_3}$

$\therefore$ The given vectors are coplanar.

d. $-\vec{a}+4\vec{b}+3\vec{c}$, $2\vec{a}-3\vec{b}-5\vec{c}$, $2\vec{a}+7\vec{b}-3\vec{c}$

Solution:

Let $\vec{r_1} = -\vec{a}+4\vec{b}+3\vec{c}$

$\vec{r_2} = 2\vec{a}-3\vec{b}-5\vec{c}$

$\vec{r_3} = 2\vec{a}+7\vec{b}-3\vec{c}$

Suppose,

$\vec{r_1} = x\vec{r_2}+y\vec{r_3}$ —— (i)

or, $-\vec{a}+4\vec{b}+3\vec{c} = 2x\vec{a}-3x\vec{b}-5x\vec{c}+2y\vec{a}+7y\vec{b}-3y\vec{c}$

or, $-\vec{a}+4\vec{b}+3\vec{c} = (2x+2y)\vec{a}+(7y-3x)\vec{b}-(5x+3y)\vec{c}$

Equating the coefficient of like vectors

$2x+2y=-1$ —— (ii)

$-3x+7y=4$ —— (iii)

$-5x-3y=3$ —— (iv)

Solving (i) and (ii), we get;

$x = -3/4$ and $y = 1/4$

putting these value in equation (i), we get

$\vec{r_1} = -\frac{3}{4}\vec{r_2}+\frac{1}{4}\vec{r_3}$

4.a. Express $\vec{r} = (4,7)$ as the linear combination of $\vec{a} = (5,-4)$ and $\vec{b}(-2,5)$

Solution:

Let $\vec{r} = 4\vec{i}+7\vec{j}$

$\vec{a} = 5\vec{i}-4\vec{j}$

$\vec{b} = -2\vec{i}+5\vec{j}$

Suppose,

$\vec{r} = x\vec{a}+y\vec{b}$

or, $4\vec{i}+7\vec{j} = 5x\vec{i}-4x\vec{j}-2y\vec{i}+5y\vec{j}$

or, $4\vec{i}+7\vec{j} = (5x-2y)\vec{i}+(-4x+5y)\vec{j}$

Then,

$4 = 5x-2y$ —— (i)

$-4x + 5y = 7$ —— (ii)

Solving (i) and (ii), we get

$x = 2$ $\quad$ $y = 3$

Now,

$\vec{r} = 2\vec{a} + 3\vec{b}$

b. Express $\vec{r} = (8, -5)$ as the linear combination of $\vec{a} = (2, -3)$ and $\vec{b} = (-1, 2)$

Solution:

Let $\vec{r} = 8\vec{i} - 5\vec{j}$

$\vec{a} = 2\vec{i} - 3\vec{j}$

$\vec{b} = -\vec{i} + 2\vec{j}$

suppose;

$\vec{r} = x\vec{a} + y\vec{b}$

or, $8\vec{i} - 5\vec{j} = 2x\vec{i} - 3x\vec{j} - y\vec{i} + 2y\vec{j}$

or, $8\vec{i} - 5\vec{j} = (2x - y)\vec{i} - (-3x + 2y)\vec{j}$

Then,

$8 = 2x - y$ —— (i)

$-5 = -3x + 2y$ —— (ii)

Solving (i) and (ii) we get

$x = 3$ $\quad$ $y = -2$

Then,

$\vec{r} = 3\vec{a} - 2\vec{b}$

c. Express $\vec{r} = (-2, 16, 2)$ as the linear combination of $\vec{a} = (0, 3, 4)$, $\vec{b} = (0, 0, -2)$ and $\vec{c} = (1, -5, 0)$.

Solution:

Let, $\vec{r} = -2\vec{i} + 16\vec{j} + 2\vec{k}$

$\vec{a} = 3\vec{j} + 4\vec{k}$

$\vec{b} = -2\vec{k}$

$\vec{c} = \vec{i} - 5\vec{j}$

Suppose,

$\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$

or, $-2\vec{i} + 16\vec{j} + 2\vec{k} = 3x\vec{j} + 4x\vec{k} - 2y\vec{k} + z\vec{i} - 5z\vec{j}$

or, $-2\vec{i} + 16\vec{j} + 2\vec{k} = (3x - 5z)\vec{j} + z\vec{i} + (4x - 2y)\vec{k}$

Now,

$-2 = z$ —— (i)

$16 = 3x - 5z$ —— (ii)

$2 = 4x - 2y$ —— (iii)

On solving (i) and (ii), we get

$z = -2$ and $x = 2$

on solving (ii) and (iii), we get

$y = 3$

Now,

$\vec{r} = 2\vec{a} + 3\vec{b} - 2\vec{c}$

5.a. Let $\vec{a} = 2\vec{i} + \vec{j}$, $\vec{b} = \vec{i} + \vec{j}$ and $\vec{c} = \vec{i} - \vec{j}$. Find the scalars x and y such that $\vec{a} = x\vec{b} + y\vec{c}$.

Solution:

Given, $\vec{a} = 2\vec{i} + \vec{j}$

$\vec{b} = \vec{i} + \vec{j}$

$\vec{c} = \vec{i} - \vec{j}$

We know,

$\vec{a} = x\vec{b} + y\vec{c}$

or, $2\vec{i} + \vec{j} = x\vec{i} + x\vec{j} + y\vec{i} - y\vec{j}$

or, $2\vec{i} + \vec{j} = (x+y)\vec{i} + (x-y)\vec{j}$

Then,

$x + y = 2$ —— (i)

$x - y = 1$ —— (ii)

On solving (i) and (ii), we get

$x = \frac{3}{2}$ and $y = \frac{1}{2}$

b. Find the value of λ so that the vectors $4\vec{i} + 5\vec{j} + \vec{k}$, $5\vec{i} + \lambda\vec{j} + 4\vec{k}$ and $-\vec{j} - \vec{k}$ are coplanar.

Solution:

Let $\vec{r_1} = 4\vec{i} + 5\vec{j} + \vec{k}$

$\vec{r_2} = 5\vec{i} + \lambda\vec{j} + 4\vec{k}$

$\vec{r_3} = -\vec{j} - \vec{k}$

By questions; the given vectors are coplanar.

i.e. $\vec{r_2} = x\vec{r_1} + \cancel{y r_1}\ y\vec{r_3}$

or, $5\vec{i} + \lambda\vec{j} + 4\vec{k} = 4x\vec{i} + 5x\vec{j} + x\vec{k} - y\vec{j} - y\vec{k}$

~~Then,~~ or, $5\vec{i} + \lambda\vec{j} + 4\vec{k} = 4x\vec{i} + (5x - y)\vec{j} + (x - y)\vec{k}$

Then,

$4x = 5$

or, $x = 5/4$ —— (i)

$5x - y = \lambda$ —— (ii)

$x - y = 4$ —— (iii)

On solving equation (i) and (iii) we get

$y = -11/4$

Using this values in equation (ii), we get

$\lambda = 9$

Linearly dependent and Independent

A system of vectors $\vec{a}, \vec{b}, \vec{c}, \ldots \vec{v}$ are said to be linearly dependent if there exist a relation $x\vec{a} + y\vec{b} + z\vec{c} + \ldots + t\vec{v} = 0$, where $x, y, z, \ldots t$ are scalar such that one of them is not zero otherwise linearly independent.

Note:

i. The two vectors $\vec{a} = (a_1, a_2)$, $\vec{b} = (b_1, b_2)$ will be linearly dependent or independent according as

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0 \text{ or } \neq 0$$

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ii. If three vectors $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$ and $\vec{c} = (c_1, c_2, c_3)$ are linearly dependent or independent according as;

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0 \text{ or } \neq 0$$

6. Examine whether the following vectors in each case are linearly dependent or independent.

a. $\vec{i} + \vec{k}$, $\vec{i} + \vec{j}$ and $-\vec{i} - \vec{k}$

Solution:

Let $\vec{a} = \vec{i} + \vec{k}$

$\vec{b} = \vec{i} + \vec{j}$

$\vec{c} = -\vec{i} - \vec{k}$

Now,

$$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} - 0\begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix}$$

$= 1(-1-0) - 0 + 1(0+1)$

$= -1 + 1$

$= 0$

$\therefore$ The given vectors are linearly dependent.

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b. $2\vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{i} - \vec{j} + 2\vec{k}$ and $5\vec{i} + 6\vec{j} + 8\vec{k}$

Solution:

Let, $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$

$\vec{b} = \vec{i} - \vec{j} + 2\vec{k}$

$\vec{c} = 5\vec{i} + 6\vec{j} + 8\vec{k}$

Now,

$$\begin{vmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 5 & 6 & 8 \end{vmatrix} = 2\begin{vmatrix} -1 & 2 \\ 6 & 8 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 \\ 5 & 8 \end{vmatrix} + 4\begin{vmatrix} 1 & -1 \\ 5 & 6 \end{vmatrix}$$

$= 2(-8-12) - 3(8-10) + 4(6+5)$

$= -40 + 6 + 44$

$= 10 \neq 0$

$\therefore$ The given vectors are linearly ~~dep~~ independent.

c. $(1, -2, 1)$, $(2, 1, 1)$ and $(7, -4, 1)$

Solution:

Let, $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$

$\vec{b} = 2\vec{i} + \vec{j} + \vec{k}$

$\vec{c} = 7\vec{i} - 4\vec{j} + \vec{k}$

Now,

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 7 & -4 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ -4 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 7 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 7 & -4 \end{vmatrix}$$

$= 1(1+4) + 2(2-7) + 1(-8-7)$

$= 5 - 10 - 15$

$= -20 \neq 0$

$\therefore$ The given vector is linearly ~~depe~~ independent.

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d. $2\vec{i} + \vec{j} - 3\vec{k}$, $-3\vec{i} + 2\vec{j} + 3\vec{k}$ and $2\vec{i} - 2\vec{j} + 4\vec{k}$

Solution:

Let, $\vec{a} = 2\vec{i} + \vec{j} - 3\vec{k}$

$\vec{b} = -3\vec{i} + 2\vec{j} + 3\vec{k}$

$\vec{c} = 2\vec{i} - 2\vec{j} + 4\vec{k}$

Now,

$$\begin{vmatrix} 2 & 1 & -3 \\ -3 & 2 & 3 \\ 2 & -2 & 4 \end{vmatrix} = 2\begin{vmatrix} 2 & 3 \\ -2 & 4 \end{vmatrix} - 1\begin{vmatrix} -3 & 3 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} -3 & 2 \\ 2 & -2 \end{vmatrix}$$

$$= 2(8+6) - 1(-12-6) - 3(6-4)$$

$$= 28 + 6 - 6$$

$$= 28 \neq 0$$

$\therefore$ The given vectors are linearly independent.

e. $3\vec{i} - 2\vec{j} + 2\vec{k}$, $6\vec{i} + \vec{j} - \vec{k}$ and $-2\vec{i} - 3\vec{j} + 2\vec{k}$

Solution:

Let, $\vec{a} = 3\vec{i} - 2\vec{j} + 2\vec{k}$

$\vec{b} = 6\vec{i} + \vec{j} - \vec{k}$

$\vec{c} = -2\vec{i} - 3\vec{j} + 2\vec{k}$

Now,

$$\begin{vmatrix} 3 & -2 & 2 \\ 6 & 1 & -1 \\ -2 & -3 & 2 \end{vmatrix} = 3\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} + 2\begin{vmatrix} 6 & -1 \\ -2 & 2 \end{vmatrix} + 2\begin{vmatrix} 6 & 1 \\ -2 & -3 \end{vmatrix}$$

$$= 3(2+3) + 2(12+2) + 2(-18+2)$$

$$= 15 + 28 - 32$$

$$= 11 \neq 0$$

$\therefore$ The given vectors are linearly independent.

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f. $\vec{i} + \vec{j} - k$, $2\vec{i} + 3\vec{j} - 3\vec{k}$ and $\vec{i} - 3\vec{j} + 3\vec{k}$

Solution:

Let $\vec{a} = \vec{i} + \vec{j} - \vec{k}$

$\vec{b} = 2\vec{i} + 3\vec{j} - 3\vec{k}$

$\vec{c} = \vec{i} - 3\vec{j} + 3\vec{k}$

Now,

$$\begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & -3 \\ 1 & -3 & 3 \end{vmatrix} = 1 \begin{vmatrix} 3 & -3 \\ -3 & 3 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 1 & 3 \end{vmatrix} - 1 \begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= 1(9 - 9) - 1(6+3) - 1(-6-3)$$

$$= 0 - 9 + 9$$

$$= 0$$

$\therefore$ The given vectors are linearly dependent.