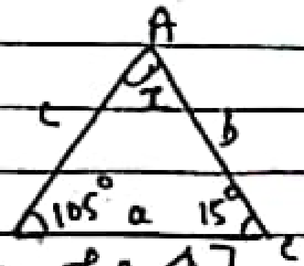
Solution of Triangle

EXERCISE 7.1



I) Solⁿ, Given,

we know that

$\angle BAC + \angle ACB + \angle CBA = 180°$ [∵ sum of a Δ]

or, $x + 15° + 105° = 180°$

or, $x + 120° = 180°$

or, $x = 60°$ //

Now, $a : b : c = \sin A : \sin B : \sin C$

$= \sin 105° : \sin 15° : \sin 60°$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} : \frac{\sqrt{3}-1}{2\sqrt{2}} : \frac{\sqrt{3}}{2}$

$= \sqrt{3}+1 : \sqrt{3}-1 : \sqrt{6}$ //

(ii) Solⁿ, $A = 45°$, $B = 60°$, $c = ?$

we know that

$A + B + C = 180°$

or, $45° + 60° + C = 180°$

$C = 75°$ //

Now, fine sine law

$\frac{a}{\sin A} = \frac{c}{\sin C}$

or, $\frac{a}{c} = \frac{\sin A}{\sin C}$

$= \frac{\sin 45}{\sin 75} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

or, $\frac{a}{c} = \frac{2}{\sqrt{3}+1}$

$a : c = 2 : \sqrt{3}+1$

(iii) Soln, Given,

$A = 60°$

$B : C = 1 : 3$

$a : b : c = ?$

Now, we know that, $B = k, C = 3k$

we have,

$A + B + C = 180°$

or, $60° + k + 3k = 180°$

or, $4k = 120°$

So, $k = 30°$

$\therefore B = 30°, C = 90°$

Also,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or, $$\frac{a}{\frac{\sqrt{3}}{2}} = \frac{b}{\frac{1}{2}} = \frac{c}{1}$$

or, $$\frac{a}{\sqrt{3}} = \frac{b}{1} = \frac{c}{2}$$

Hence, $a : b : c = \sqrt{3} : 1 : 2$ and $A = 60°, B = 30°,$
$C = 90°$ //

(iv) Soln, Given:-
$\Rightarrow$ $A : B : C = 2 : 3 : 7$

To prove:- $a : b : c = \sqrt{2} : 2 : (\sqrt{3}+1)$

Now, Let $A = 2k$,
$B = 3k$
$C = 7k$

$\therefore A + B + C = 180^\circ$
$2k + 3k + 7k = 180^\circ$
or, $12k = 180^\circ$
$k = 15^\circ$

$\therefore A = 2 \times 15^\circ = 30^\circ$
$B = 3 \times 15^\circ = 45^\circ$
$C = 7 \times 15^\circ = 105^\circ$

Now, Using sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or, $$\frac{a}{\frac{1}{2}} = \frac{b}{\frac{1}{\sqrt{2}}} = \frac{c}{\cancel{\frac{\sqrt{6}+\sqrt{2}}{4}}\ \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

or, $$\frac{a}{\sqrt{2}} = \frac{b}{2\sqrt{2}} = \frac{c}{\sqrt{3}+1}$$

Hence, $a : b : c = \sqrt{2} : 2\sqrt{2} : \sqrt{3}+1$ proved //

$\Rightarrow$ (v) Soln, Given:- $\cos A = \frac{4}{5}$, $\cos B = \frac{3}{5}$

To find:- $a : b : c$

Now,

Since, $\cos A = \frac{b}{h} = \frac{4}{5}$, so $\cos B = \frac{3}{5} = \frac{b}{h}$

$5^2 = p^2 + 4^2$ $\qquad$ $5^2 = p^2 + 3^2$

or, $3 = p$ $\qquad$ $p = 4$

$\sin A = \frac{3}{5}$ $\qquad$ $\sin B = \frac{4}{5}$

Now, $\sin C = \sin(A+B)$ $\quad [\because A+B+C=180^\circ]$

$= \sin A \cos B + \cos A \sin B$

$= \frac{3}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{4}{5}$

$= \frac{9}{25} + \frac{16}{25}$

$= \frac{25}{25}$

$\sin C = 1$

$\therefore C = 90^\circ$

We have, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$= \frac{a}{3/5} \times \frac{1}{5} + \frac{b}{4/5} \times \frac{1}{5} = \frac{c}{1} \times \frac{1}{5}$

$= \frac{a}{3} = \frac{b}{4} = \frac{c}{5}$

$\therefore a:b:c = 3:4:5$

2i) Solⁿ, Given

$a = 2$, $b = \sqrt{2}$, $c = \sqrt{3} + 1$

Now, $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(\sqrt{2})^2 + (\sqrt{3}+1)^2 - (2)^2}{2 \times \sqrt{2} \times (\sqrt{3}+1)}$

$$= \frac{2+3+2\sqrt{3}+1-4}{2\cdot\sqrt{2}\cdot(\sqrt{3}+1)}$$

$$= \frac{1}{\sqrt{2}}$$

$\cos A = \cos 45^\circ$

$\therefore A = 45^\circ$

Again, $\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{(\sqrt{3}+1)^2+4-2}{2\cdot(\sqrt{3}+1)\cdot 2}$

$$= \frac{3+2\sqrt{3}+1+4-2}{4(\sqrt{3}+1)}$$

$$= \frac{6+2\sqrt{3}}{4(\sqrt{3}+1)}$$

$$= \cos 30^\circ$$

$\therefore B = 30^\circ$

$\therefore A+B+C = 180^\circ$

$45^\circ + 30^\circ + C = 180^\circ$

$\therefore C = 105^\circ$ //

(ii) Sol:- Given,

$a = 3+\sqrt{3}$, $b = 2\sqrt{3}$, $c = \sqrt{6}$,

To find:- A, B, C = ?

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{(2\sqrt{3})^2+(\sqrt{6})^2-(3+\sqrt{3})^2}{2\times 2\sqrt{3}\times\sqrt{6}}$

$$= \frac{12+6-(9+2\cdot 6\sqrt{3}+3)}{4\sqrt{3}\times\sqrt{6}}$$

$$= \frac{\cancel{12}+6-\cancel{12}+6\sqrt{3}}{12\sqrt{2}}$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}} \qquad \therefore A = 105^\circ //$$

$$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{6+(9+6\sqrt{3}+3)-12}{2\sqrt{6}(3+\sqrt{3})}$$

$$= \frac{6+6\sqrt{3}}{2\sqrt{6}(3+\sqrt{3})}$$

$$= \frac{3}{\sqrt{18}} = \frac{1}{\sqrt{2}}$$

$$= \cos 45^\circ$$

$\therefore B = 45^\circ$

$\therefore A+B+C = 180^\circ$

$105^\circ + 45^\circ + C = 180^\circ$

$C = 30^\circ$ //

(iii) Sol^n, In ΔABC, we have

$a:b:c = 2:\sqrt{6}:\sqrt{3}+1$

So, $a = 2K$, $b = \sqrt{6}K$, $c = (\sqrt{3}+1)K$

To find :- A, B, C

Now, $\cos A = \frac{6k^2+(3+2\sqrt{3}+1)k^2 - 4k^2}{2\cdot\sqrt{6}K\cdot(\sqrt{3}+1)K}$

$$= \frac{6+2\sqrt{3}}{2\sqrt{6}(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{3}(\sqrt{3}+1)}{2\sqrt{6}(\sqrt{3}+1)} = \frac{1}{\sqrt{2}}$$

$\therefore A = 45^\circ$

Again, $\cos B = \frac{(3+2\sqrt{3}+1)k^2+4k^2-6k^2}{2(\sqrt{3}+1)k.2k}$

$$= \frac{2+2\sqrt{3}}{4(\sqrt{3}+1)}$$

$= \frac{1}{\sqrt{2}} = \cos 60°$

$\therefore B = 60°$ //

$\therefore C = 180-(45+60) = 75°$

Hence, $A = 45°$, $B = 60°$ & $C = 75°$ //

3.i) Solⁿ, $C = 30°$, $B = 45°$, $c = 6\sqrt{2}$

To find:- a, b, A

Now,

$$A = 180 - (B+C) = 105°$$

Again, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

or, $\frac{a}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{b}{\frac{1}{\sqrt{2}}} = \frac{c}{\frac{1}{2}}$

or, $\frac{a}{\sqrt{3}+1} = \frac{b}{2} = \frac{6\sqrt{2}}{\sqrt{2}}$ $[\because c = 6\sqrt{2}]$

Taking 1st & 3rd Taking 1st & 2nd.

$a = \frac{6\sqrt{2}}{\sqrt{2}}(\sqrt{3}+1) = 6(\sqrt{3}+1)$ & $b = 12$ //

(ii) Solⁿ, $A = 60°$, $B = 75°$, $c = 2\sqrt{2}$

To find: b, c, C

Now, $C = 180 - (A+B) = 180-(60+75) = 45°$

And, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

or, $\frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{b}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{c}{\frac{1}{\sqrt{2}}}$

or, $\sqrt{2} = \frac{b}{\sqrt{3}+1} = \frac{c}{2}$

So, $C = 45^\circ$; $b = \sqrt{6}+2$ & $c = 2\sqrt{2}$

iii) Sol^n, $A = 30^\circ$, $B = 45^\circ$, $b = 2$

To find :- a, c, C

Now, $C = 180 - (A+B) = 180 - (30+45) = 105^\circ$

Again, $\frac{a}{\sin 30} = \frac{b}{\sin 45^\circ} = \frac{c}{\sin 105^\circ}$

or, $\frac{a}{\sqrt{2}} = \frac{2}{\frac{1}{\sqrt{2}}} = \frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

or, $\frac{a}{\sqrt{2}} = 1 = \frac{c}{\sqrt{3}+1}$

$\therefore a = \sqrt{2}$, $c = \sqrt{3}+1$

$\therefore C = 105^\circ$, $a = \sqrt{2}$, $c = \sqrt{3}+1$ //

4 i) Sol^n, Here, $a = 2$, $b = 4$, $C = 60^\circ$

To find :- A and B

We have, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

or, $\frac{1}{2} = \frac{4+16-c^2}{2.2.4}$

or, $c^2 = 12$

$\therefore c = 2\sqrt{3}$

Now,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{2}{\sin A} = \frac{4}{\sin B} = 4$$

Now, $\sin A = \frac{1}{2}$ $\qquad \sin B = 1$

$A = 30^\circ$, $\qquad B = 90^\circ$

$\therefore$ Hence, $A = 30^\circ$, $B = 90^\circ$ //

(ii) Soln, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

or, $\frac{1}{2} = \frac{(3 - 2\sqrt{3} + 1) + (3 + 2\sqrt{3} + 1) - a^2}{2(\sqrt{3} - 1)(\sqrt{3} + 1)}$

or, $3 - 1 = 8 - a^2$

$\therefore a = \sqrt{6}$

Now, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

or, $\frac{\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3} - 1}{\sin B} = \frac{\sqrt{3} + 1}{\sin C}$

So, $\sin B = \frac{\sqrt{3} - 1}{2.\sqrt{2}} = \sin 15^\circ$, $\therefore B = 15^\circ$

And, $\sin C = \frac{\sqrt{3} + 1}{2\sqrt{2}} = \sin 75^\circ$ or $\sin 105^\circ$

So, $C = 105^\circ$ [$C = 75^\circ$ is not possible]

$\therefore a = \sqrt{6}$, $B = 15^\circ$, $C = 105^\circ$ //

(iii) Soln, Here

$a = \sqrt{57}$, $\Delta = 2\sqrt{3}$, $A = 60^\circ$

To find: b and c

Since, $\Delta = \frac{abc}{4R}$

or, $2\sqrt{3} = \frac{\sqrt{57} \times bc}{4 \times \sqrt{19}}$

$bc = \frac{2\sqrt{3} \times 4 \times \sqrt{19}}{\sqrt{57}}$

$\therefore bc = 8$ ---------- (i)

Also,

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

or, $\frac{1}{\cancel{2}} = \frac{b^2 + c^2 - 57}{\cancel{2} \cdot 8}$

or, $8 = b^2 + c^2 - 57$

or, $b^2 + c^2 - 65 = 0$

or, $b^2 + \frac{64}{b^2} - 65 = 0$ $\left[c = \frac{8}{b}\right]$

or, $b^4 - 65b^2 + 64 = 0$

or, $b^2(b^2 - 64) - (b^2 - 64) = 0$

Either, or,

$b = 1, 8$ $c = 8, 1$ [from (i)]

Hence, $b = 1$ or 8 & $c = 8$ or 1

5 i) Sol^n, Here, $a = 3$, $b = 3\sqrt{3}$, $A = 30°$

To find: c, B, C

Now, $\frac{a}{\sin A} = \frac{b}{\sin B}$

or, $\frac{3}{\sin 30} = \frac{3\sqrt{3}}{\sin B}$

or, $\frac{\cancel{3}}{\frac{1}{2}} = \frac{\cancel{3}\sqrt{3}}{\sin B}$ or, $\sin B = \frac{\sqrt{3}}{2}$

$\therefore B = 60°$ or $120°$ (Ambiguous case)

Now, When $B = 60°$,

$C = 180 - (A + B)$

$= 180 - (30 + 60)$

$= 90°$

Again, $\frac{a}{\sin A} = \frac{c}{\sin C}$

or, $\frac{3}{1/2} = \frac{c}{\sin 90}$

$\therefore C = 90°, c = 6, B = 60°$

Again, When $B = 120°$

$\therefore C = 180 - (30 + 120°)$

$= 30°$

Again, $\frac{a}{\sin A} = \frac{c}{\sin C}$

or, $\frac{3}{1/\cancel{2}} = \frac{c}{1/\cancel{2}}$

$\therefore c = 3, B = 120°, C = 3$

Hence, the two solutions are, $B = 60°$, $C = 90°$, $c = 6$

and $B = 120°$, $C = 30°$ & $c = 3$

(11) Sol^n, Given,

$a = 2$, $b = \sqrt{3}+1$, $A = 45^\circ$

To find:- c, B, C

Now, $\frac{a}{\sin A} = \frac{b}{\sin B}$

or, $\frac{2}{1/\sqrt{2}} = \frac{\sqrt{3}+1}{\sin B}$

or, $\sin B = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore B = 75^\circ$ or 105° (Ambiguous case)

Now,

When $B = 75^\circ$

$C = 180 - (45+75) = 60^\circ$

Again, $\frac{a}{\sin A} = \frac{c}{\sin C}$

or, $\frac{2}{1/\sqrt{2}} = \frac{c}{\sqrt{3}/2}$

$\therefore c = \sqrt{6}$

$\therefore B = 75^\circ$, $C = 60^\circ$ & $c = \sqrt{6}$

Again, $B = 105^\circ$ Then,

$C = 180 - (45+105)$

$= 30^\circ$

Again, $\frac{a}{\sin A} = \frac{c}{\sin C}$

or, $\frac{2}{1/\sqrt{2}} = \frac{c}{1/2}$

$\therefore c = \sqrt{2}$ //

(iii) Sol^n, Here, $A = 30^\circ$, $a = 6$, $b = 4$

To find: B, C, c

Now, $\frac{a}{\sin A} = \frac{b}{\sin B}$

or, $\frac{6}{1/2} = \frac{4}{\sin B}$

$\sin B = \frac{1}{3}$

$\therefore B = 19.47^\circ$ (No two solutions)

Now,

$C = 180 - (30 + 19.47^\circ)$

$= 130.47^\circ$

Again, $\frac{a}{\sin A} = \frac{c}{\sin C}$

or, $\frac{6}{1/2} = \frac{c}{\sin(130.47)}$

$\therefore c = 9$

Hence, $B = 19.47^\circ$, $C = 130.47^\circ$ & $c = 9$ //