

Unit - 11 Surds

Laws of Surds

i) $\sqrt[n]{a} = a^{\frac{1}{n}}$ So, $(\sqrt[n]{a})^n = a$.

ii) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

iii) $\sqrt[n]{a} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

iv) $\sqrt[n]{a} + \sqrt[n]{a} = 2\sqrt[n]{a}$

v) $\sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$

vi) $2\sqrt[n]{a} - \sqrt[n]{a} = \sqrt[n]{a}$

Ex - 11.1

1. Simplify:

a) $\sqrt{2} + 5\sqrt{2} + 2\sqrt{2}$

b) $5\sqrt{3} + 6\sqrt{3} - 7\sqrt{3}$

$$= (1+5+2)\sqrt{2}$$

$$= (5+6-1)\sqrt{3}$$

$$= 8\sqrt{2}$$

$$= 10\sqrt{3}$$

c) $\sqrt{5} - 4\sqrt{5} + 6\sqrt{5}$

d) $8\sqrt[3]{4} - \sqrt[3]{4} - 3\sqrt[3]{4}$

$$= (1-4+6)\sqrt{5}$$

$$= (8-1-3)\sqrt[3]{4}$$

$$= 9\sqrt{5}$$

$$= 4\sqrt[3]{4}$$

$$e) \sqrt[3]{7} + 4\sqrt[3]{7} - 9\sqrt[3]{7} = (1+4-9)\sqrt[3]{7} = -4\sqrt[3]{7}$$

$$f) 2\sqrt[4]{6} - \sqrt[4]{6} - 3\sqrt[4]{6} = (2-1-3)\sqrt[4]{6} = -2\sqrt[4]{6}$$

2. Simplify:

$$a) \sqrt{2} \times \sqrt{3} \times \sqrt{5} \quad b) \sqrt{6} \times \sqrt{3} \times 2\sqrt{2}$$

$$= \sqrt{30} = \cancel{2} \cdot 2\sqrt{6} \times 3 \times 2 = 2\sqrt{36} = 12$$

$$c) \sqrt[3]{9} \times \sqrt[3]{3} \times \sqrt[3]{2} \quad d) \sqrt[5]{8} \div 2\sqrt{2}$$

$$= \sqrt[3]{9 \times 3 \times 2} = \frac{\sqrt[3]{54}}{2\sqrt{2}}$$

$$= \sqrt[3]{54} = \frac{10\sqrt{2}}{2\sqrt{2}}$$

$$= 9\sqrt{6} = 5.$$

$$e) \sqrt[5]{108} \div \sqrt[3]{2} \quad f) \sqrt[4]{360} \div 3\sqrt{20}$$

$$= \frac{\sqrt[5]{108}}{\sqrt[3]{2}} = \frac{4\sqrt[4]{360}}{3\sqrt{120}}$$

$$= \frac{24\sqrt{10}}{6\sqrt{300}}$$

$$= \frac{4\sqrt{3}}{3} \cdot 4\sqrt{5}.$$

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Simplify:

a. $\sqrt{32} + \sqrt{8} - \sqrt{72}$

$$\begin{aligned}
 &= \sqrt{2^2 \times 2^2 \times 2} + \sqrt{2^2 \times 2} - \sqrt{2^2 \times 2 \times 3^2} \\
 &= 2 \times 2\sqrt{2} + 2\sqrt{2} - 2 \times 3\sqrt{2} \\
 &= 4\sqrt{2} + 2\sqrt{2} - 6\sqrt{2} \\
 &= 6\sqrt{2} - 6\sqrt{2} \\
 &= 0
 \end{aligned}$$

b. $\sqrt{27} + \sqrt{75} - 8\sqrt{3}$

$$\begin{aligned}
 &= \sqrt{3^2 \times 3} + \\
 &= 3\sqrt{3} + 5\sqrt{3} - 8\sqrt{3} \\
 &= (3+5-8)\sqrt{3} \\
 &= 0\sqrt{3} \\
 &= 0
 \end{aligned}$$

c. $4\sqrt{45} - 3\sqrt{20} + 8\sqrt{5}$

$$\begin{aligned}
 &= 20(2\sqrt{5}) - 6\sqrt{5} + 8\sqrt{5} \\
 &= (12-6+8)\sqrt{5} \\
 &= 14\sqrt{5}
 \end{aligned}$$

d) $\sqrt{12} - \sqrt{75} + \sqrt{48}$

e) $\sqrt[3]{16} + \sqrt[3]{54} - \sqrt[3]{50}$

$$\begin{aligned} &= 2\sqrt{3} - 5\sqrt{3} + 4\sqrt{3} \\ &= (2-5+4)\sqrt{3} \\ &= 1\sqrt{3} \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} &= 2\sqrt[3]{2} + 3\sqrt[3]{2} - 5\sqrt[3]{2} \\ &= 6\sqrt[3]{2} - 5\sqrt[3]{2} \\ &= \sqrt[3]{2} \end{aligned}$$

f) $5\sqrt[3]{81} - 2\sqrt[3]{64} + \sqrt{375}$

$$\begin{aligned} &= 5\sqrt[3]{3 \times 3 \times 3 \times 3} - 2\sqrt[3]{2 \times 2 \times 2 \times 3} + \sqrt[3]{3 \times 5 \times 5 \times 5} \\ &= 15\sqrt[3]{3} - 4\sqrt[3]{3} + 5\sqrt[3]{5} \\ &= 16\sqrt[3]{3}, \end{aligned}$$

g) $4\sqrt[4]{405} - 3\sqrt[4]{80} - 2\sqrt[4]{5}$

$$\begin{aligned} &\approx 4\sqrt[4]{3^4} \\ &= 12\sqrt[4]{5} - 6\sqrt[4]{5} - 2\sqrt[4]{5} \\ &= 4\sqrt[4]{5} \end{aligned}$$

h) $3\sqrt{2} + \sqrt[4]{2500} - 4\sqrt{64} + 6\sqrt{8}$

$$\begin{aligned} &= 3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2} + 12\sqrt{2} \\ &= 18\sqrt{2} \end{aligned}$$

4a) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$

$$= \sqrt{3} \times \sqrt{3} - \sqrt{3} \times \sqrt{2} + \sqrt{2} \times \sqrt{3} - \sqrt{2} \times \sqrt{2}$$

$$\therefore 3 - \cancel{\sqrt{6}} + \sqrt{6} + \cancel{2}$$

$$= 1.$$

b) $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

$$= \cancel{\sqrt{5} \times \sqrt{5}} + \cancel{\sqrt{3} \times \sqrt{5}} - \sqrt{3} \times \sqrt{5} - \cancel{\sqrt{5} \times \sqrt{3}}$$

$$= 5 - \sqrt{15} - \sqrt{15} - 3$$

$$= 2$$

c) $(2\sqrt{5} + 3\sqrt{2})(2\sqrt{5} - 3\sqrt{2})$

$$= \cancel{2\sqrt{5}} 2 \times 2\sqrt{5} \cdot \sqrt{5} - 2 \times 3\sqrt{2} \cdot \sqrt{5} + 3 \times 2\sqrt{5} \cdot \sqrt{2} - \cancel{3\sqrt{2}}$$

$$\therefore 20 - 6\sqrt{10} + 6\sqrt{10} - 18$$

$$= 2$$

d) $(\sqrt{2} + \sqrt{3})^2$

$$= (\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{3})$$

$$\therefore \sqrt{2} \times \sqrt{2} + \sqrt{2} \times \sqrt{3} + \sqrt{3} \times \sqrt{2} + \sqrt{3} \times \sqrt{3}$$

$$\therefore 2 + \sqrt{6} + \sqrt{6} + 3$$

$$= 5 + 2\sqrt{6}$$

e) $(\sqrt{5} - \sqrt{3})^2$

$$(\sqrt{5} - \sqrt{3})(\sqrt{5} - \sqrt{3})$$

$$(\sqrt{5} - \sqrt{3})(\sqrt{5} - \sqrt{3})$$

$$\sqrt{3} \cdot \sqrt{3} - \sqrt{5} \cdot \sqrt{3} = \sqrt{3} \times \sqrt{5} + \sqrt{3} \times \sqrt{5}$$

$$= 5 - \sqrt{15} - \sqrt{15} + 3$$

$$= 8 - 2\sqrt{15}$$

f) $(\sqrt{x+a} - \sqrt{x-a})^2$

$$(\sqrt{x+a} - \sqrt{x-a})(\sqrt{x+a} - \sqrt{x-a})$$

$$= \sqrt{x+a} \times \sqrt{x+a} - \sqrt{x+a} \times \sqrt{x-a} - \sqrt{x-a} \times \sqrt{x+a} + \sqrt{x-a} \times \sqrt{x-a}$$

$$= x+a - \cancel{\sqrt{x^2-a^2}} - \sqrt{x^2-a^2} + x-a$$

$$= 2(x - \sqrt{x^2-a^2})$$

g) $(2\sqrt{2} - \sqrt{3})(3\sqrt{2} + \sqrt{3})$

$$= 2 \times 3\sqrt{2} \cdot \sqrt{2} + 2\sqrt{2} \cdot \sqrt{3} - \cancel{8} - 3\sqrt{3} \cdot \sqrt{2} - \sqrt{3} \times \sqrt{3}$$

$$= 12 + 2\sqrt{8} - 9 - 3\sqrt{6} - 3$$

$$= 9 + 2\sqrt{8}$$

$$= 19\sqrt{8}$$

$$f) (3\sqrt{5} - 4\sqrt{2})(2\sqrt{5} + 2\sqrt{3})$$

$$\begin{aligned}
 &= 3 \cdot 2\sqrt{5} \cdot \sqrt{5} + 3 \cdot 2\sqrt{5} \cdot \sqrt{3} - 4 \cdot 2\sqrt{5} \cdot \sqrt{2} - 4 \cdot 2\sqrt{2} \cdot \sqrt{3} \\
 &= 30 + 6\sqrt{15} - 8\sqrt{15} - 8\sqrt{16} \\
 &= 30 + 6 - 8 - 8\sqrt{15} \\
 &= 30 + 6 - 8\sqrt{15} - 8\sqrt{16} \\
 &= 28\sqrt{15} - 8\sqrt{16}
 \end{aligned}$$

5. Simplify:

$$a) \frac{\sqrt{a^2 - b^2}}{\sqrt{a-b}}$$

$$b) \frac{\sqrt{x^2 - 9}}{\sqrt{x-3}}$$

$$2) (\sqrt{a+b}) \cdot (\sqrt{a-b})$$

$$= \frac{\sqrt{x^2 - 3^2}}{\sqrt{x-3}}$$

$$= (\sqrt{x+3}) (\sqrt{x-3})$$

$$= \sqrt{a+b}$$

$$= \sqrt{x+3}$$

$$c) \frac{\sqrt[3]{25x - x^2}}{\sqrt[3]{x+5}}$$

$$2) \frac{\sqrt[3]{5^2 - x^2}}{\sqrt[3]{x+5}}$$

$$\frac{(\sqrt[3]{5x}) (\sqrt[3]{5x})}{(\sqrt[3]{5x})}$$

$$\sqrt[3]{25x}$$

$$d) \frac{x-4}{\sqrt{x}+2}$$

$$= \frac{(\sqrt{x})^2 - (2)^2}{\sqrt{x}+2}$$

$$e) \frac{3x-16}{4+\sqrt{3x}}$$

$$= \frac{(\sqrt{3x})^2 - (4)^2}{4+\sqrt{3x}}$$

$$= \frac{(\cancel{\sqrt{x}+2})(\sqrt{x}-2)}{\cancel{(\sqrt{x}+2)}} = \frac{(\sqrt{3x}+4)(\sqrt{3x}-4)}{(4+\sqrt{3x})}$$

$$= \sqrt{x}-2 = \sqrt{3x}-4.$$

$$f) \frac{49-5x}{7-\sqrt{5x}}$$

$$\frac{(7)^2 - (\sqrt{5x})^2}{7-\sqrt{5x}}$$

$$\frac{(7+\sqrt{5x})(7-\sqrt{5x})}{(7-\sqrt{5x})}$$

$$7+\sqrt{5x}$$

6

Simplify

$$a. \frac{\sqrt{24} + \sqrt{54}}{10\sqrt{6}}$$

$$b. \frac{4\sqrt[3]{54} - 2\sqrt[3]{216}}{6\sqrt[3]{128}}$$

$$\frac{2\sqrt{6} + 3\sqrt{6}}{10\sqrt{6}} = \frac{12\sqrt[3]{6} - 16\sqrt[3]{10}}{48\sqrt[3]{2}}$$

$$\frac{5\sqrt{6}}{10\sqrt{6}} = \frac{36\sqrt[3]{2} - 50\sqrt[3]{2}}{48\sqrt[3]{2}}$$

$$\frac{\frac{1}{2}}{2} = \frac{-14\sqrt[3]{2}}{48\sqrt[3]{2}} = \frac{-7}{24}$$

$$c) \frac{3\sqrt[3]{81} - 3\sqrt[3]{243} + 2\sqrt[3]{375}}{18\sqrt[3]{192}}$$

$$\frac{3\sqrt[3]{3^3 \times 3} - 3\sqrt[3]{2^3 \times 3} + 2\sqrt[3]{5^3 \times 3}}{18\sqrt[3]{2^3 \times 2^3 \times 3}}$$

$$\frac{9\sqrt[3]{3} - 6\sqrt[3]{3} + 10\sqrt[3]{3}}{52\sqrt[3]{3}}$$

$$\frac{13\sqrt[3]{3}}{52\sqrt[3]{3}}$$

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$$= \frac{1}{\sqrt{2} + 1}$$

Ans

Ex-11.2

1. Rationalise the denominators and simplify.

$$a) \frac{1}{\sqrt{2} + 1}$$

$$b) \frac{1}{\sqrt{3} - 2}$$

$$= \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{1}{\sqrt{3} - 2} \times \frac{\sqrt{3} + 2}{\sqrt{3} + 2}$$

$$= \frac{\sqrt{2} - 1}{\sqrt{2}^2 - 1^2}$$

$$= \frac{\sqrt{3} + 2}{\sqrt{3}^2 - 2^2}$$

$$2) \frac{\sqrt{2} - 1}{2 - 1}$$

$$= \frac{\sqrt{3} + 2}{3 - 4}$$

$$= \sqrt{2} - 1,$$

$$= \sqrt{3} + 2$$

$$= -\sqrt{3} + 2$$

c) $\frac{3}{\sqrt{3}-\sqrt{2}}$

$$= \frac{3}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{3(\sqrt{3}+\sqrt{2})}{\sqrt{3}^2 - \sqrt{2}^2}$$

$$= 3(\sqrt{3}+\sqrt{2})$$

$$= \frac{3(\sqrt{3}+\sqrt{2})}{3}$$

$$= \sqrt{3} + \sqrt{2}$$

d) $\frac{4}{2\sqrt{3}-\sqrt{2}}$

$$= \frac{4}{2\sqrt{3}-\sqrt{2}} \times \frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$$

$$= \frac{8\sqrt{3}+\sqrt{2}}{4(\sqrt{3}^2 - \sqrt{2}^2)}$$

$$= \frac{8\sqrt{3}+\sqrt{2}}{4(3-2)}$$

$$= \frac{8\sqrt{3}+\sqrt{2}}{4}$$

$$= 2\sqrt{3} + \sqrt{2}$$

e) $\frac{8}{3\sqrt{7}+2\sqrt{3}}$

$$= \frac{8}{3\sqrt{7}+2\sqrt{3}} \times \frac{3\sqrt{7}-2\sqrt{3}}{3\sqrt{7}-2\sqrt{3}}$$

$$= \frac{15\sqrt{7}-2\sqrt{3}}{9\cancel{6\sqrt{7}^2} \quad 9\sqrt{7}^2 - 4\sqrt{3}^2}$$

$$\frac{15\sqrt{7}-2\sqrt{3}}{63-12}$$

$$= \frac{15\sqrt{7}-2\sqrt{3}}{51} = \frac{5(3\sqrt{7}-2\sqrt{3})}{17}$$

$$f) \frac{2\sqrt{2}}{\sqrt{6} + \sqrt{3}}$$

$$= \frac{2\sqrt{2}}{\sqrt{6} + \sqrt{3}} \times \frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} - \sqrt{3}}$$

$$g) \frac{5\sqrt{3}}{2\sqrt{3} - \sqrt{2}}$$

$$= \frac{5\sqrt{3}}{2\sqrt{3} - \sqrt{2}} \times \frac{2\sqrt{3} + \sqrt{2}}{2\sqrt{3} + \sqrt{2}}$$

$$= \frac{4\sqrt{3} - 2\sqrt{6}}{\sqrt{6} - \sqrt{3}^2}$$

$$= \frac{4\sqrt{3} - 2\sqrt{6}}{6 - 3}$$

$$= \frac{2}{3} (2\sqrt{3} - \sqrt{6})$$

$$= \frac{30 + 5\sqrt{6}}{4\sqrt{3}^2 - \sqrt{2}^2}$$

$$= \frac{30 + 35\sqrt{6}}{12 - 2} = \frac{30 + 35\sqrt{6}}{10}$$

$$= \frac{7\sqrt{6} - \sqrt{21}}{2}$$

h)

$$\frac{4\sqrt{5}}{2\sqrt{3} + \sqrt{5}}$$

$$= \frac{4\sqrt{5}}{2\sqrt{3} + \sqrt{5}} \times \frac{2\sqrt{3} - \sqrt{5}}{2\sqrt{3} - \sqrt{5}}$$

$$= \frac{8\sqrt{15} - 20}{4\sqrt{3}^2 - \sqrt{5}^2}$$

$$= \frac{8\sqrt{15} - 20}{8}$$

$$= \frac{4(2\sqrt{15} - 5)}{7}$$

2. Rationalise the denominators and simplify.

$$a) \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$b) \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\Rightarrow \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= 3 + 2\sqrt{2}$$

$$= 2 - \sqrt{3} \quad (d+1)$$

$$c) \frac{-\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$= \underline{5 - 2\sqrt{6}}$$

$$e) \frac{2\sqrt{3} - 3\sqrt{2}}{2\sqrt{3} + 3\sqrt{2}}$$

$$= \frac{2\sqrt{3} - 3\sqrt{2}}{2\sqrt{3} + 3\sqrt{2}}$$

$$= 2\sqrt{6} - 5$$

$$d) \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= 4 + \sqrt{15}$$

$$f) \frac{3\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= \frac{3\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= 6 + \sqrt{15}$$

$$g) \frac{\sqrt{a+b} - \sqrt{a-b}}{\sqrt{a+b} + \sqrt{a-b}}$$

$$= \frac{\sqrt{a+b} - \sqrt{a-b}}{\sqrt{a+b} + \sqrt{a-b}} \times \frac{\sqrt{a+b} + \sqrt{a-b}}{\sqrt{a+b} + \sqrt{a-b}}$$

$$= \frac{(\sqrt{a+b} - \sqrt{a-b})^2}{(\sqrt{a+b})^2 - (\sqrt{a-b})^2}$$

$$= \frac{(\sqrt{a+b})^2 - 2\sqrt{a+b} \cdot \sqrt{a-b} + (\sqrt{a-b})^2}{a+b - a+b}$$

$$\frac{a+b-2\sqrt{a^2-b^2}}{2b} + a-b$$

$$\frac{2a-2\sqrt{a^2-b^2}}{2b} = \frac{(a+\sqrt{a^2-b^2})}{(a-\sqrt{a^2-b^2})}$$

b) $\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}$

$$\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} \times \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}$$

$$\frac{(\sqrt{x+1}-\sqrt{x-1})^2}{(\sqrt{x+1})^2 - (\sqrt{x-1})^2}$$

$$= \frac{(\sqrt{x+1})^2 - 2\sqrt{x+1}\sqrt{x-1} + (\sqrt{x-1})^2}{x+1-x+1}$$

$$\frac{x+1-\sqrt{x^2-1}+x-1}{2}$$

$$2x-\frac{\sqrt{x^2-1}}{2}$$

$$\text{R) } \frac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$$

$$\frac{\sqrt{2}}{(\sqrt{2} + \sqrt{3}) - \sqrt{5}} \times \frac{(\sqrt{2} + \sqrt{3}) + \sqrt{5}}{(\sqrt{2} + \sqrt{3}) + \sqrt{5}}$$

$$\frac{2 + \sqrt{6} + \sqrt{10}}{(\sqrt{2} + \sqrt{5})^2 - (\sqrt{5})^2}$$

$$\frac{2 + \sqrt{6} + \sqrt{10}}{(\sqrt{2})^2 + 2 \cdot \sqrt{2} \cdot \sqrt{3} + \sqrt{3}^2 - 5}$$

$$\frac{2 + \sqrt{6} + \sqrt{10}}{2 + 2\sqrt{6} + 3 - 5}$$

$$\frac{2 + \sqrt{6} + \sqrt{10}}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{2\sqrt{6} + 6 + \sqrt{60}}{2 \times 6}$$

$$6 + 2\sqrt{6} + \sqrt{6} \times 10$$

12

$$6 + 2\sqrt{6} + 2\sqrt{10}$$

12

$$\frac{2(3 + \sqrt{6} + \sqrt{10})}{10\sqrt{5}}$$

$$3 + \sqrt{6} + \sqrt{10}$$

3. Simplify:

$$a) 3\sqrt{5} - \frac{1}{\sqrt{5}}$$

Soln,

$$= 3\sqrt{5} - \frac{1}{\sqrt{5}}$$

$$= 3\sqrt{5} \times \frac{\sqrt{5}}{\sqrt{5}} - \frac{1}{\sqrt{5}}$$

$$= \frac{(15-1) \times \sqrt{5}}{\sqrt{5} \sqrt{5}}$$

$$= \frac{14\sqrt{5}}{5}$$

$$b) \frac{7}{\sqrt{3}} + 2\sqrt{3}$$

$$= 7 + 2\sqrt{3} \times \sqrt{3}$$

$$= \frac{7+6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{13\sqrt{3}}{3}$$

c) $\frac{3}{\sqrt{5}} + \frac{\sqrt{5}}{2}$

(d) $\frac{7}{\sqrt{3}} - \frac{\sqrt{3}}{4}$

$$\frac{3}{\sqrt{5}} \cdot \frac{x\sqrt{5}}{\sqrt{5}} + \cancel{\frac{\sqrt{5}}{2}} \Rightarrow \frac{7}{\sqrt{3}} \cdot \frac{x\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{3}}{4}$$

$$\frac{3\sqrt{5}}{5} + \frac{\sqrt{5}}{2}$$

$$\Rightarrow \frac{7\sqrt{3}}{3} - \frac{\sqrt{3}}{4}$$

$$\frac{3\sqrt{5} \cdot 2 + \sqrt{5} \cdot 5}{10}$$

$$\Rightarrow \frac{4 \cdot 7\sqrt{3} - \sqrt{3} \cdot 3}{12}$$

$$\frac{6\sqrt{5} + 5\sqrt{5}}{10}$$

$$\Rightarrow 28\sqrt{3} - 3\sqrt{3}$$

$$\frac{11\sqrt{5}}{10}$$

$$\Rightarrow \frac{28\sqrt{3} - 3\sqrt{3}}{12}$$

$$\Rightarrow \frac{25\sqrt{3}}{12}$$

e) $\frac{\sqrt{2}}{5} + \frac{3}{\sqrt{2}}$

f) $\frac{\sqrt{7}}{3} + \frac{5}{2\sqrt{7}}$

$$\Rightarrow \frac{\sqrt{2}}{5} + \frac{3}{\sqrt{2}} \cdot \frac{x\sqrt{2}}{\sqrt{2}} = \frac{7}{\sqrt{3}} + \frac{5}{2\sqrt{7}} \cdot \frac{x\sqrt{7}}{\sqrt{7}}$$

$$\Rightarrow \frac{\sqrt{2}}{5} + \frac{3\sqrt{2}}{2}$$

$$\Rightarrow \frac{7}{\sqrt{3}} + \frac{5\sqrt{7}}{14}$$

$$= \frac{\sqrt{2} \times 2 + 3\sqrt{2} \times 5}{10} \Rightarrow \frac{\sqrt{7} \times 14 + 5\sqrt{7} \times 3}{42}$$

$$\Rightarrow \frac{17\sqrt{2}}{10}$$

$$= \frac{29\sqrt{1}}{42}$$

4. Simplify.

$$a. \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$b) \frac{5+3\sqrt{5}}{\sqrt{5}+2} - \frac{5-3\sqrt{5}}{\sqrt{5}-2}$$

$$= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$\Rightarrow (\sqrt{5}-2)(5+3\sqrt{5}) - (5-3\sqrt{5})(5-4)$$

$$= \frac{3+2\sqrt{8}+1+3-2\sqrt{8}+1}{\sqrt{3}^2 - 1^2} \Rightarrow \frac{5-\sqrt{5}-(5-\sqrt{5})}{1} = 10$$

$$= \frac{8-1}{3-1} = 10$$

$$= 4.$$

$$c) \frac{\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{(\sqrt{3} + \sqrt{2})^2 + (\sqrt{3} - \sqrt{2})^2}{3 - 2}$$

$$= \frac{10}{1} = 10$$

$$d) \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} - \sqrt{a}} - \frac{\sqrt{x} - \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

$$= \frac{(\sqrt{x} + \sqrt{a})^2 - (\sqrt{x} - \sqrt{a})^2}{x - a}$$

$$= \frac{x + 2\sqrt{ax} + a - x + 2\sqrt{ax} - a}{x - a}$$

$$\Rightarrow \frac{4\sqrt{ax}}{x - a}$$

$$e) \frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}} - \frac{x - \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}}$$

$$= \frac{(x + \sqrt{x^2 - 1})^2 - (x - \sqrt{x^2 - 1})^2}{x^2 - x^2 + 1}$$

$$e) \frac{x^2 + 2x\sqrt{x^2-1} + x^2 - 1 - x^2 + 2x\cdot \sqrt{x^2-1 - x^2+1}}{1}$$

$$\Rightarrow 4x\sqrt{x^2-1}$$

$$f) \frac{a-\sqrt{a^2-1}}{a+\sqrt{a^2-1}} + \frac{a+\sqrt{a^2-1}}{a-\sqrt{a^2-1}}$$

$$= \frac{(a-\sqrt{a^2-1})^2 + (a+\sqrt{a^2-1})^2}{a^2-a^2+1}$$

$$\rightarrow \frac{a^2 - 2a \cdot \sqrt{a^2-1} + a^2-1 + a^2 + 2a \cdot \sqrt{a^2-1} \cdot a^2-1}{1}$$

$$\Rightarrow 4a^2 + 2$$

$$\Rightarrow 2(2a^2 + 1),$$

5. Simplify.

$$a) 3\sqrt{20} + \frac{4}{\sqrt{5}} + \frac{\sqrt{5} + 3}{\sqrt{5} - 3}$$

$$= 3\sqrt{2^2 \cdot 5} + \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} + \frac{\sqrt{5} + 3}{\sqrt{5} - 3} \cdot \frac{\sqrt{5} + 3}{\sqrt{5} + 3}$$

$$= 3 \cdot 2\sqrt{5} + \frac{4\sqrt{5}}{5} + \frac{(\sqrt{5} + 3)^2}{(\sqrt{5})^2 - (3)^2}$$

$$\frac{6\sqrt{5} + 4\sqrt{5}}{5} + \frac{14 + 6\sqrt{5}}{5 - 9}$$

$$\frac{30\sqrt{5} + 4\sqrt{5}}{5} + \frac{14 + 6\sqrt{5}}{-4}$$

$$\frac{34\sqrt{5}}{5} - \frac{14 + 6\sqrt{5}}{4}$$

$$136\sqrt{5} - 70 - 30\sqrt{5}$$

$$\frac{66\sqrt{5} - 70}{20}$$

$$2(53\sqrt{5} - 35)$$

$$\frac{10}{10}$$

$$2(53\sqrt{5} - 35),$$

$$6) \frac{\sqrt{72} - 48}{\sqrt{50}} - \frac{45}{\sqrt{128}} + 2\sqrt{98}$$

$$= \frac{6\sqrt{2}}{5\sqrt{2}} - \frac{48}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} - \frac{45}{8\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}$$

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$$e) \frac{20\sqrt{2} - 48\sqrt{2}}{10} - \frac{45\sqrt{2}}{16}$$

$$\Rightarrow \frac{80\sqrt{2} - 8 \cdot 48\sqrt{2}}{80} - \frac{8 \cdot 45\sqrt{2}}{16}$$

$$\Rightarrow \frac{1600\sqrt{2} - 384\sqrt{2}}{160} - \frac{228\sqrt{2}}{16}$$

$$= \frac{1992\sqrt{2}}{160} = 12.45\sqrt{2}$$

$$c) \frac{2\sqrt{10}}{\sqrt{3}+1} - \frac{2\sqrt{5}}{\sqrt{6}+2} - \frac{\sqrt{10}}{\sqrt{2}+1}$$

$$2 \quad \frac{2\sqrt{10}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} - \frac{2\sqrt{5}}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2} - \frac{\sqrt{10}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$2 \quad \frac{2\sqrt{30}-2\sqrt{10}}{\sqrt{3}^2-1^2} - \frac{2\sqrt{80}-4\sqrt{5}}{\sqrt{6}^2-2^2} - \frac{\sqrt{20}-\sqrt{10}}{\sqrt{2}^2-1^2}$$

$$\Rightarrow \frac{2(\sqrt{30}-\sqrt{10})}{2} - \frac{2(\sqrt{80}-2\sqrt{5})}{2} - \frac{2\sqrt{5}-\sqrt{10}}{1}$$

$$\Rightarrow \sqrt{30}-\sqrt{10}-\sqrt{30}+2\sqrt{5}-2\sqrt{5}+\sqrt{10}$$

$$\Rightarrow 0.$$

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6 Simplify.

$$2) \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} + \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}}$$

$$\Rightarrow \frac{(\sqrt{x+1} + \sqrt{x-1})^2 + (\sqrt{x+1} - \sqrt{x-1})^2}{(\sqrt{x+1} + \sqrt{x-1})(\sqrt{x+1} - \sqrt{x-1})}$$

$$= \frac{x+1 + 2\sqrt{x+1} \cdot \sqrt{x-1} + x-1 + 2\sqrt{x+1} \cdot \sqrt{x-1} + (x+1) - (x-1)}{(\sqrt{x+1})^2 - (\sqrt{x-1})^2}$$

$$\Rightarrow \frac{4x}{x+1-x+1}$$

$$= 2x$$

$$= 2x$$

$$b) \frac{\sqrt{a+b} + \sqrt{a-b}}{\sqrt{a+b} - \sqrt{a-b}} + \frac{\sqrt{a+b} - \sqrt{a-b}}{\sqrt{a+b} + \sqrt{a-b}}$$

$$\Rightarrow \frac{(\sqrt{a+b} + \sqrt{a-b})^2 + (\sqrt{a+b} - \sqrt{a-b})^2}{a+b - a+b}$$

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$$= a+b+2\sqrt{ab} - \sqrt{a+b+a-b+a+b-2\sqrt{ab}} \cdot \sqrt{a-b+a-b}$$

$$= \frac{4a}{2b} \quad \text{Simplifying}$$

$$2) \frac{2x}{b}$$

$$\text{c)} \frac{\sqrt{2x+3} + \sqrt{2x-3}}{\sqrt{2x+3} - \sqrt{2x-3}} + \frac{\sqrt{2x+3} - \sqrt{2x-3}}{\sqrt{2x+3} + \sqrt{2x-3}}$$

$$\Rightarrow \cancel{(2x+3)}$$

$$\Rightarrow (\sqrt{2x+3} + \sqrt{2x-3})^2 + (\sqrt{2x+3} - \sqrt{2x-3})^2$$

$$2x+3 - 2x+3$$

$$8x = 12$$

$$\Rightarrow 2x+3 + 2 \cdot \sqrt{2x+3} \cdot \sqrt{2x-3} + 2x-3 + 2x+3 - 2 \cdot \sqrt{2x+3} \cdot \sqrt{2x-3}$$

$$2x+3 - 2x+3$$

$$2x = 12 \rightarrow x = 6$$

$$= \frac{8x}{6}$$

$$\text{Solved with parametric: } 57 = 5(1+2x)$$

$$= \frac{4x}{3}$$

$$28 = 1 + 2x \\ 27 = 2x \\ x = 13.5$$

$$\begin{aligned}
 d) & \frac{\sqrt{x^2+2} + \sqrt{x^2-2}}{\sqrt{x^2+2} - \sqrt{x^2-2}} + \frac{\sqrt{x^2+2} - \sqrt{x^2-2}}{\sqrt{x^2+2} + \sqrt{x^2-2}} \\
 & = \frac{(\sqrt{x^2+2} + \sqrt{x^2-2})^2 + (\sqrt{x^2+2} - \sqrt{x^2-2})^2}{x^2+2-x^2+2} \\
 & = \frac{x^2+2+2\cdot\sqrt{x^2+2}\cdot\sqrt{x^2-2}+x^2-2+x^2+2-2\cdot\sqrt{x^2+2}\cdot\sqrt{x^2-2}}{x^2-2+x^2-2} \\
 & = \frac{4x^2}{4} \\
 & = x^2
 \end{aligned}$$

Ex-11.3

1. Solve $\sqrt{x+1} = 5$

$$a) \sqrt{x+1} = 5$$

Squaring both sides.
 $(\sqrt{x+1})^2 = 5^2$

$$x+1 = 25$$

$$\therefore x = 24$$

~~a)~~- checking for $x=24$

$$\sqrt{24+5} = 5$$

$$\sqrt{25} = 5$$

$$5 = 5 \text{ (True)}$$

b) $\sqrt{2x-5} = 7$

Squaring both sides.

$$(\sqrt{2x-5})^2 = 7^2$$

$$2x-5 = 49$$

$$2x = 54$$

Checking for $x=27$

on $\sqrt{2x-5} = 7$

on $\sqrt{2 \times 27 - 5} = 7$

on $\sqrt{49} = 7$

$\therefore 7 = 7 \text{ (True)}$

d) $\sqrt{2x+3} - 3 = 0$

$$\sqrt{2x+3} = 3$$

Squaring both sides.

$$(\sqrt{2x+3})^2 = 3^2$$

$$2x+3 = 9$$

$$2x = 6$$

$$x = 3$$

Checking for $x = 3$

$$\text{Or, } \sqrt[3]{2x+5} = 3$$

$$\text{or, } \sqrt[3]{9} = 3$$

$$\therefore 3 = 3 \text{ (True)}$$

g) $\sqrt[3]{x+5} - 1 = 2$

$$\text{or, } \sqrt[3]{x+5} = 3$$

Cubing both sides

$$(\sqrt[3]{x+5})^3 = 3^3$$

$$x+5 = 27$$

$$x = 22$$

Checking for $x = 22$

$$\text{or, } \sqrt[3]{22+5} - 1 = 2$$

$$\text{or, } \sqrt[3]{27} - 1 = 2$$

$$\text{or, } 3 - 1 = 2$$

$$\therefore 2 = 2 \text{ (True)}$$

g) $\sqrt[4]{2x-1} - 2 = 1$

$$\text{Or, } \sqrt[4]{2x-1} = 3$$

Raising 4^{th} on both sides

$$(\sqrt[4]{2x-1})^4 = 3^4$$

$$\begin{aligned}2x - 1 &= 81 \\2x &= 82 \\\therefore x &= 41,\end{aligned}$$

h) $\sqrt{x+5} = \sqrt{2x-3}$

Squaring both sides.

$$(\sqrt{x+5})^2 = (\sqrt{2x-3})^2$$

$$\text{or } x+5 = 2x-3$$

$$x = 8$$

Checking for $x=8$

$$\sqrt{x+5} = \sqrt{2x-3}$$

$$\sqrt{8+5} = \sqrt{2 \times 8 - 3}$$

$$\sqrt{13} = \sqrt{13} \quad (\text{True})$$

$$\therefore x = 8.$$

Ex-11.3

3. Solve:

$$\text{i) } \frac{\sqrt{x} + \sqrt{7}}{\sqrt{x} - \sqrt{7}} = 3$$

$$E - \cancel{KL} = 2 + \cancel{KL}$$

$$\sqrt{x} + \sqrt{7} = 3\sqrt{x} - 3\sqrt{7}$$

$$\sqrt{x} - 3\sqrt{x} = -3\sqrt{7} - \sqrt{7}$$

$$-2\sqrt{x} = -4\sqrt{7} \quad : \sqrt{x} = 2\sqrt{7} \quad (\text{div.})$$

sq both sides

$$(\sqrt{x})^2 = (2\sqrt{7})^2$$

$$x = 4 \times 7$$

$$\therefore x = 28$$

Checking for $x = 28$

$$\frac{\sqrt{28} + \sqrt{7}}{\sqrt{28} - \sqrt{7}} = 3$$

$$\frac{\sqrt{28} + \sqrt{7}}{\sqrt{28} - \sqrt{7}} = 3$$

$$\frac{2\sqrt{7} + \sqrt{7}}{2\sqrt{7} - \sqrt{7}} = 3$$

$$\frac{3\sqrt{7}}{\sqrt{7}} = 3$$

$$3 = 3 \quad \text{True}$$

$$x = 28,$$

h) $\frac{\sqrt{y} + \sqrt{5}}{\sqrt{y} - \sqrt{5}} = 3$

$$\sqrt{y} + \sqrt{5} = 3\sqrt{y} - 3\sqrt{5}$$

$$\sqrt{y} - 3\sqrt{y} = -3\sqrt{5} - \sqrt{5}$$

$$-2\sqrt{y} = -4\sqrt{5}$$

$$\sqrt{y} = 2\sqrt{5}$$

Sq both sides

$$(\sqrt{y})^2 = (2\sqrt{5})^2$$

$$y = 4 \times 5$$

$$\therefore y = 20$$

Checking for $y = 20$

$$\frac{\sqrt{20} + \sqrt{5}}{\sqrt{20} - \sqrt{5}} = 3$$

$$\sqrt{20} - \sqrt{5}$$

$$2\sqrt{5} + \sqrt{5} = 3$$

$$2\sqrt{5} - \sqrt{5}$$

$$3\sqrt{5} = 3$$

$$\sqrt{5}$$

$3 = 3$ True.

$$y = 20$$

$$4c) 2x+1 = \sqrt{4x^2+3x+6}$$

Sq. both sides.

$$(2x+1)^2 = (\sqrt{4x^2+3x+6})^2$$

$$\text{or } (2x)^2 + 2 \cdot 2x \cdot 1 + 1^2 = 4x^2 + 3x + 6$$

$$\text{or } 4x^2 + 4x + 1 - 4x^2 - 3x - 6 = 0$$

$$\text{or, } x - 5 = 0$$

$$\therefore x = 5$$

Checking for $x = 5$

$$2x+1 = \sqrt{4x^2+3x+6}$$

$$10+1 = \sqrt{100+15+6}$$

$$11 = \sqrt{121}$$

$$11 = 11 \text{ true}$$

$$\therefore x = 5$$

$$5f) \sqrt{x} + \sqrt{x-\sqrt{1-x}} = 1$$

$$\sqrt{x-\sqrt{1-x}} = 1 - \sqrt{x}$$

Sq. both sides:

$$x - \sqrt{1-x} = 1 - 2\sqrt{x} + x$$

$$-\sqrt{1-x} = 1 - 2\sqrt{x}$$

Sq. both sides

$$1-x = 1^2 - 2 \cdot 1 \cdot 2\sqrt{x} + (2\sqrt{x})^2$$

$$1-x = 1 - 4\sqrt{x} + 4x$$

$$1-x - 1 + 4\sqrt{x} - 4x = 0$$

$$4\sqrt{x} - 5x = 0$$

$$4\sqrt{x} = 5x$$

Sq. both sides

$$(4\sqrt{x})^2 = (5x)^2$$

$$16x = 25x^2$$

$$16x = 25x^2$$

$$25x^2 - 16x = 0$$

$$x(25x - 16) = 0$$

Either

OR

$$x = 0$$

$$25x - 16 = 0$$

$$25x = 16$$

$$\therefore x = \frac{16}{25}$$

Checking for $x = 0$

$$\sqrt{0} + \sqrt{0 - \sqrt{1-0}} = 1$$

$$0 + \sqrt{0 - \sqrt{1}} = 1$$

$$0 + \sqrt{0 - 1} = 1$$

$$\sqrt{-1} = \pm 1$$

Checking for $x = \frac{16}{25}$

$$\sqrt{\frac{16}{25}} + \sqrt{\frac{16}{25} - \sqrt{1 - \frac{16}{25}}} = 1$$

$$\frac{4}{5} + \sqrt{\frac{16}{5} - \frac{16}{25}} = 1$$

$$\frac{4}{5} + \sqrt{\frac{16}{5} - \frac{9}{25}} = 1$$

$$\frac{4}{5} + \sqrt{\frac{16}{25} - \frac{3}{5}} = 1$$

$$\frac{4}{5} + \sqrt{\frac{16-15}{25}} = 1$$

$$\frac{4}{5} + \frac{1}{5} = 1$$

$$\frac{5}{5} = 1$$

$$1 = 1 \text{ True.}$$

$$x = \frac{16}{25}$$

$$6f \frac{7x-36}{6+5\sqrt{2x}} = 9 - 5\sqrt{7x-11}$$

$$\frac{(\sqrt{7x})^2 - 6^2}{6+5\sqrt{2x}} = \frac{27 - 5\sqrt{7x-11}}{3}$$

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$$(\sqrt{7x+6})(\sqrt{7x}-6) = 38 - 5\sqrt{7x}$$

$$3\sqrt{7x} - 18 = 38 - 5\sqrt{7x}$$

$$3\sqrt{7x} + 5\sqrt{7x} = 38 + 18$$

$$8\sqrt{7x} = 56$$

$$\sqrt{7x} = 7$$

Sq. both sides.

$$7x = 49$$

$$x = 7$$

Checking for $x = 7$

$$\frac{7x7-36}{6+\sqrt{7x7}} = \frac{9-5\sqrt{7x7}+11}{3}$$

$$\frac{19}{6+7} = \frac{9-5x7-11}{3}$$

$$\frac{13}{13} = \frac{9-24}{3}$$

$$1 = 1 \quad \text{True}$$

$$\therefore x = 7$$

$$78) \frac{\sqrt{2} + \sqrt{a}}{\sqrt{a} - \sqrt{a}} + \frac{\sqrt{2} - \sqrt{a}}{\sqrt{a} + \sqrt{a}} = 6$$

$$\frac{(\sqrt{2} + \sqrt{a})^2}{(\sqrt{a} - \sqrt{a})(\sqrt{2} + \sqrt{a})} + \frac{(\sqrt{2} - \sqrt{a})^2}{(\sqrt{a} + \sqrt{a})(\sqrt{2} - \sqrt{a})} = 6$$

$$\frac{2 + 2\sqrt{2}\sqrt{a} + \sqrt{a} + 2 - 2\sqrt{2} - \sqrt{a} + 2}{2 - a} = 6$$

$$2x + 2a = 6x - 6a$$

$$2x - 6x = -6a - 2a$$

$$-4x = -8a$$

$$x = 2a$$

Checking for $x = 2a$

$$\frac{\sqrt{2a} + \sqrt{a}}{\sqrt{2a} - \sqrt{a}} + \frac{\sqrt{2a} - \sqrt{a}}{\sqrt{2a} + \sqrt{a}} = 6$$

$$\frac{(\sqrt{2a} + \sqrt{a})^2}{(\sqrt{2a} - \sqrt{a})(\sqrt{2a} + \sqrt{a})} + \frac{(\sqrt{2a} - \sqrt{a})^2}{(\sqrt{2a} + \sqrt{a})(\sqrt{2a} - \sqrt{a})} = 6$$

$$\frac{2a + 2\sqrt{2a}\cdot\sqrt{a} + \sqrt{a} + 2a - 2\sqrt{2a}\cdot\sqrt{a} + 2a}{2a - a} = 6$$

$$\frac{6a}{a} = 6$$

$G = G'$ June

$$\chi = 2a$$

Checking for $x = 16$

$$\sqrt{x-7} = \sqrt{x} - 1$$

$$\sqrt{16-7} = \sqrt{16} - 1$$

$$3 = 3 \text{ (True)}$$

c) $\sqrt{x+8} - 2 = \sqrt{x}$

$$\sqrt{x+8} - 2 = \sqrt{x}$$

$$\text{or } \sqrt{x+8} = 2 + \sqrt{x}$$

Squaring both sides of the equation, we get,

$$(\sqrt{x+8})^2 = (2 + \sqrt{x})^2$$

$$x+8 = 2^2 + 2 \cdot 2\sqrt{x} + (\sqrt{x})^2$$

$$x+8 = 4 + 4\sqrt{x} + x$$

$$4 - \sqrt{x} = 4$$

$$\sqrt{x} = 1$$

$$x = 1$$

Checking for $x = 1$

$$\sqrt{x+8} - 2 = \sqrt{x}$$

$$\sqrt{1+8} - 2 = \sqrt{1}$$

$$3 - 2 = 1$$

$$1 = 1 \text{ (True)}$$

e) $\sqrt{x} = 6 - \sqrt{x-24}$

Sln,

$$\sqrt{x-24} = 6 - \sqrt{x}$$

Squaring both sides of the equation, we get

$$(\sqrt{x-24})^2 = (6 - \sqrt{x})^2$$

$$x-24 = 6^2 + 2 \cdot 6\sqrt{x} - (\sqrt{x})^2$$

$$x-24 = 36 + 12\sqrt{x} - x$$

$$12\sqrt{x} = 60$$

$$\sqrt{x} = 5$$

$$x = 25$$

Checking for $x = 25$

$$\sqrt{25-24} = 6 - \sqrt{25}$$

$$\sqrt{1} = 1$$

$$1 = 1 \text{ (True)}$$

g) $x + \sqrt{x^2 - 20} = 10$

Sln,

$$(x + \sqrt{x^2 - 20}) + (-x) = 10 + (-x)$$

$$x + \sqrt{x^2 - 20} - x = 10 - x$$

$$x - x + \sqrt{x^2 - 20} = -x + 10$$

$$\sqrt{x^2 - 20} = -x + 10$$

Squaring both sides

$$(\sqrt{x^2 - 20})^2 = (-x + 10)^2$$

$$x^2 - 20 = (-x)^2 + 2(-x)10 + 10^2$$

$$x^2 - 20 = x^2 - 2x \cdot 10 + 100$$

$$x^2 - 20 = x^2 - 20x + 100$$

$$-20 = -20x + 100$$

$$-120 = -20x$$

$$x = 6$$

Checking for $x = 6$

$$6 + \sqrt{6^2 - 20} = 60$$

$$(0 \text{ (True)})$$

3. Solve.

$$\frac{x-1}{\sqrt{x+1}} = 1$$

Sln,

$$\frac{(x-1)(\sqrt{x+1})}{(\sqrt{x+1})(\sqrt{x-1})} = 1$$

$$\frac{(x-1)(\sqrt{x-1})}{(x-1)} = 1$$

$$(x-1)$$

$$\therefore \sqrt{x}-1 = 1$$

$$\therefore \sqrt{x} = 2$$

$$\therefore x = 4$$

$$Q) \frac{2x-9}{\sqrt{x}+3}$$

$$\therefore \frac{(\sqrt{x})^2 - 3^2}{\sqrt{x}+3} = 1$$

$$\therefore (\sqrt{x}+3)(\sqrt{x}-3) = 1$$

$$\therefore \sqrt{x}-3 = 1$$

$$\therefore \sqrt{x} = 4$$

$$\therefore x = 16$$

$$L = \frac{2 + K_{SP}}{2} + n - K_{SP} C$$

$$Z = \frac{2 + K_{SP}}{2} + Z - K_{SP} N$$

$$\frac{2}{2 + K_{SP}} - Z = \frac{(2 + K_{SP})}{(6 - K_{SP})(6 + K_{SP})}$$

$$\frac{2}{2 + K_{SP}} - Z = Z - (K_{SP})$$

$$3\sqrt{5}x = 6$$

$$\sqrt{5}x = 2$$

$$x = 4/5$$

$$f) \quad \sqrt{x-3} = 2$$

$$\sqrt{x}$$

$$\begin{aligned}
 & \Rightarrow (\sqrt{x}-3)^2 = 2^2 \\
 & \Rightarrow \sqrt{x} - 15 = 4 \\
 & \Rightarrow \sqrt{x} = 19 \\
 & \Rightarrow x = 361
 \end{aligned}$$

$$= \sqrt{x} = 5$$

$$x = 25$$

$$x = 25 \quad \text{is a solution.}$$

4. Solve:

$$a) \quad \sqrt{2x+7} = x+2$$

\Rightarrow Squaring both sides of the equation, we get

$$(\sqrt{2x+7})^2 = (x+2)^2$$

$$2x+7 = x^2+4x+4$$

$$x^2-2x = 11$$

$$x(x-2) = 11$$

$$\text{Either } x = 11 \text{ or } x-2 = 11$$

$$x = 13$$

6) $\sqrt{2x+9} = 13 - x$.

Soln,

Separating both sides.

$$(\sqrt{2x+9})^2 = (13-x)^2$$

$$2x+9 = 13^2 - 2 \cdot 13 \cdot x + x^2$$

$$2x+9 = 169 - 26x + x^2$$

$$x^2 + 160 - 28x = 0$$

$$x^2 - 20x - 8x + 160 = 0$$

$$(x-20)(x-8) = 0$$

∴ $x = 20$

Either. OR. -

$$x-20 = 0 \quad x-8 = 0$$

$$x = 20 \quad x = 8$$

Substituting $x = 20$ in the original equation, we get

$$\sqrt{2 \cdot 20 + 9} = 13 - 20$$

$$\sqrt{49} = -7 \text{ which is false.}$$

Substituting $x = 8$ in the given equation, we get

$$\sqrt{2 \cdot 8 + 9} = 13 - 8$$

$$\sqrt{25} = 5 \text{ which is true.}$$

So, the required value of x is 5.

d) $3x - \sqrt{7x+2} = 2$

Ans:

$$-\sqrt{7x+2} = -3x+2$$

Squaring both sides.

$$(\sqrt{7x+2})^2 = (-3x+2)^2$$

$$7x+2 = (3x)^2 + 2(-3x)2 + 2^2$$

$$7x+2 = 9x^2 - 12x + 4$$

$$7x+2 - (9x^2 - 12x + 4) = 0$$

~~$$7x - 9x^2 - 2 = 0$$~~

$$9x^2 - 19x + 2 = 0$$

$$9x^2 - 18x - x + 2 = 0$$

$$9x(x-2) - 1(x-2) = 0$$

$$(9x-1)(x-2) = 0$$

Either:

$$9x-1 = 0$$

$$x = 1/9$$

$$x = 1$$

Substituting $x=2$ in the original eqn, we get:

$$3x^2 - \sqrt{7x^2 + 2} = 2$$

$$6 - \sqrt{14+2} = 2$$

$$2 = 2 \text{ which is true.}$$

∴ Ans. The required value of x is 2.

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$$8) \sqrt{3x+1} - \sqrt{x-1} = 2$$

$$\sqrt{3x+1} = \sqrt{x-1} + 2$$

Squaring both sides.

$$(3x+1)^2 = (\sqrt{x-1} + 2)^2$$

$$3x+1 = (\sqrt{x-1})^2 + 2 \cdot \sqrt{x-1} \cdot 2 + 2^2$$

$$3x+1 = x-1 + 4\sqrt{x-1} + 4$$

$$3x+1 = x+4 \cdot \sqrt{x-1} - 1 + 4$$

~~Both sides~~

$$2x-2 = 4 \cdot \sqrt{x-1}$$

A squ.

Squaring the both sides

$$(2x-2)^2 = (4\sqrt{x-1})^2$$

$$4x^2 - 8x + 4 = 16x - 16$$

$$4x^2 - 8x + 4 - 16x + 16 = 0$$

$$4x^2 - 24x + 20 = 0$$

$$2^2(x^2 - (6x) + 5) = 0$$

$$x^2 - 6x + 5 = 0$$

$$x^2 - 5x - x + 5 = 0$$

$$(x-5)(x-1) = 0$$

E: Here

$$x-5=0 \quad \text{or} \quad x-1=0$$

$$x=5$$

$$x=1$$

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Substituting $x = 5$ in original eqn, we get.

$$\sqrt{3x+1} - \sqrt{5-x} = 2$$

$\Rightarrow 2 = 2$ which is True.

Substituting $x = 1$ in original eqn we get

$$\sqrt{3x+1} - \sqrt{5-x} = 2$$

$1 \neq 2$ which is True.

\therefore So the required value is (C.S.U.)

5. Solve:

a) ~~$\sqrt{5+x}$~~ $\sqrt{x} + \sqrt{5+x} = \frac{15}{\sqrt{5+x}}$

Solve, $\sqrt{x} = 15 - \sqrt{5+x}$

on $\sqrt{x} = 15 - \frac{\sqrt{5+x}}{\sqrt{5+x}}$

on $\sqrt{x(5+x)} = 15 - \sqrt{(5+x)^2}$

on $\sqrt{x(5+x)} = 15 - 5-x$

on $x(5+x) = 10 - x$

Squaring both sides

$$\text{or } (\sqrt{x(5+x)})^2 = (10-x)^2$$

$$\text{or } x(5+x) = 10^2 - 2 \times 10 \times 4x + 4x^2$$

$$\text{or } 5x + x^2 = 100 - 20x + 4x^2$$

$$\text{or } x^2 - x^2 + 5x + 20x = 100$$

$$\text{or } x = \frac{100}{25}$$

$$x = 4$$

$$\text{c) } 2\sqrt{x} - \sqrt{4x-3} = \sqrt{4x-3}$$

$$\sqrt{4x-3}$$

Solve,

$$2\sqrt{x} = 1 + \sqrt{4x-3}$$

$$\sqrt{4x-3}$$

$$2\sqrt{x} = 1 + (\sqrt{4x-3}) (\sqrt{4x-3})$$

$$\sqrt{4x-3}$$

~~$$2\sqrt{x} = 1 + 4x - 3$$~~

$$2\sqrt{x}(4x-3) = 1 + \sqrt{(4x-3)(4x-3)}$$

$$2\sqrt{x}(4x-3) = 1 + (4x-3)^2$$

$$2\sqrt{x}(4x-3) = 1 + 4x - 3$$

$$2\sqrt{x}(4x-3) = 4x - 2$$

$$2\sqrt{x}(4x-3) = 2(2x-1)$$

$$\sqrt{x(4x-3)} = 2x-1$$

Squaring both sides

$$\begin{aligned} (\sqrt{x(4x-3)})^2 &= (2x+1)^2 \\ x(4x-3) &= (2x)^2 - 2 \cdot 2x \cdot 1 + 1^2 \\ x(4x-3) &= 4x^2 - 4x + 1 \\ 4x^2 - 3x &= 4x^2 + 4x - 1 = 0 \\ x &= 1 \end{aligned}$$

~~$$\frac{dx}{\sqrt{2x+5}} = \frac{3+5\sqrt{x}}{\sqrt{x+2}}$$~~

Q: solve :-

a) $\frac{x-1}{\sqrt{x+1}} = \frac{3+4\sqrt{x}-1}{2}$

solve,
 $(\sqrt{x})^2 - (1)^2 = 4 + \sqrt{x} - 1$

$$on \quad \sqrt{x} - 1 = 4 + \sqrt{x} - 1$$

$$on \quad \sqrt{x} - 1 - \sqrt{x} - 1 = 4$$

$$on \quad 0 \cdot x - 1 - (4x - 1) = 4$$

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$$x - 1 = 8$$

$$\text{on } \sqrt{x} = 8 + 1$$

squaring both sides.

$$(\sqrt{x})^2 = 9^2$$

$$x = 81$$

$$\text{c) } y - 25 = 4 + \sqrt{y - 5}$$

$$5 + \sqrt{y - 5}$$

• soln,

$$(\sqrt{y})^2 - 5^2 = 4 + \sqrt{y - 5}$$

$$\sqrt{y} + 5$$

$$(\sqrt{y} + 5)(\sqrt{y} - 5) = 4 + \sqrt{y - 5}$$

$$\sqrt{y} + 5$$

$$\sqrt{y} - 5 = 4 + \sqrt{y - 5}$$

$$5$$

$$\sqrt{y} - 5 - \sqrt{y - 5} = 4$$

$$5 - (\sqrt{y - 5}) - (\sqrt{y - 5}) = 4$$

$$5$$

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$$5\sqrt{y} - 25 - \sqrt{y} + 5 = 20$$

$$4\sqrt{y} - 20 = 20$$

$$4\sqrt{y} = 40$$

$$\sqrt{y} = 10$$

Squaring both sides.

$$(\sqrt{y})^2 = 10^2$$

$$\therefore y = 100$$

$$\text{e) } \frac{3x-4}{2+3x} - \frac{\sqrt{3x}-2}{2} = 2$$

$$(\sqrt{3x}-2)^2 - (\sqrt{3x}-2) \cdot 2 = 2 \\ \sqrt{3x}+2$$

$$(\sqrt{3x}-2) - \frac{3x-2}{2} = 2$$

$$2\sqrt{3x}-4-\sqrt{3x}+2=4$$

$$\sqrt{3x}-2=4$$

$$\sqrt{3x}=6$$

Squaring both sides

$$(\sqrt{3x})^2 = 6^2$$

$$3x = 36$$

$$x = 12$$