Chapter 4

Sequence and Series

Exercise 4.1

1. Given the nth term of the series, find the sum to n terms.

a) $n + n^2$ **b)** $n^2 - 3n$ **c)** $n(n + 1) - 2$ **d)** $3^n - 4n^3$

Sol^n: a) Here, the nth term of series, $t_n = n + n^2$

$\therefore$ $S_n = \Sigma t_n = \Sigma n + \Sigma n^2$

$$= \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)(3+2n+1)}{6}$$

$$= \frac{n(n+1)(n+2)}{3}.$$

b) Here, the nth term of series, $t_n = n^2 - 3n$

$\therefore$ $S_n = \Sigma t_n = \Sigma n^2 - \Sigma 3n$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{3n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1-9)}{6}$$

$$= \frac{n(n+1)(n-4)}{3}$$

c) Here, the nth term of series, $t_n = n(n + 1) - 2 = n^2 + n - 2$

$\therefore$ $S_n = \Sigma t_n = \Sigma n^2 + \Sigma n^{-2} + \Sigma 2$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - 2n$$

$$= \frac{n(n+1)(2n+1+3)}{6} - 2n$$

$$= \frac{n(n+1)(n+2)}{3} - 2n$$

$$= \frac{n(n^2+3n+2-6)}{3}$$

$$= \frac{n(n^2+3n-4)}{3}$$

$$= \frac{n(n^2+4n-n-4)}{3}$$

$$= \frac{n(n-1)(n+4)}{3}$$

d) Here, the nth term of the series, $t_n = 3^n - 4n^3$

$\therefore \quad S_n = \Sigma t_n = \Sigma 3^n - \Sigma 4n^3 = \dfrac{3(3^n - 1)}{3 - 1} - \dfrac{n^2(n + 1)^2}{4} = \dfrac{3(3^n - 1)}{2} - \dfrac{n^2(n + 1)^2}{4}$

2. Find the n^{th} term and then the sum of the first n terms of each of the following series:

a) 1.1 + 2.2 + 3.3 + 4.4 + ……

Sol^n: Here, thenthterm of given series, $t_n = n \times n = n^2$.

$\therefore \quad S_n = \Sigma t_n = \Sigma n^2 = \dfrac{n(n + 1)(2n + 1)}{6}$

b) 1·3 + 2·4 + 3·5 + ………

Sol^n: Here, the nth term of series, t_n $= (n^{th} \text{ term of } 1, 2, 3, \ldots) \times (n^{th} \text{ term of } 3, 4, 5, \ldots)$

$= [1 + (n - 1).1] \times [3 + (n - 1).1]$

$= n \times (n + 2) = n(n + 2)$

$\therefore \quad S_n = \Sigma t_n = \Sigma n(n + 2)$

$= \Sigma(n^2 + 2n)$

$= \dfrac{n(n + 1)(2n + 1)}{6} + 2\dfrac{n(n + 1)}{2} = \dfrac{n(n + 1)}{6}(2n + 1 + 6) = \dfrac{n(n + 1)(2n + 7)}{6}$

c) 1·3 + 3·5 + 5·7 + ……….

Sol^n: Here, the nth term of series, t_n $= (n^{th} \text{ term of } 1, 3, 5, \ldots) \times (n^{th} \text{ terms of } 3, 5, 7 \ldots)$

$= (2n - 1)(2n + 1) = 4n^2 - 1$

$\therefore \quad S_n = \Sigma t_n = \Sigma(4n^2 - 1)$

$= 4\left[\dfrac{n(n + 1)(2n + 1)}{6}\right] - n$

$= n\left[\dfrac{2(n + 1)(2n + 1)}{3} - 1\right] = \dfrac{n}{3}(4n^2 + 6n + 2 - 3) = \dfrac{n}{3}(4n^2 + 6n - 1)$.

d) 2.5 + 4.8 + 6.11 + ……

Sol^n: Here, the nth term series, t_n $= (n^{th} \text{ term of } 2, 4, 6, \ldots) \times (n^{th} \text{ term of } 5, 8, 11, \ldots)$

$= 2n \times [5 + (n - 1)3] = 2n(3n + 2) = 6n^2 + 4n$.

$\therefore \quad S_n = \Sigma t_n = \Sigma(6n^2 + 4n)$

$= 6\left[\dfrac{n(n + 1)(2n + 1)}{6}\right] + 4\left[\dfrac{n(n + 1)}{2}\right]$

$= n(n + 1)(2n + 1 + 2) = n(n + 1)(2n + 3)$.

e) 5.3 + 7.5 + 9.7 + ……

Sol^n: Here, thenthterm of given series, t_n $= (n^{th} \text{ term of } 5, 7, 9, \ldots) \times (n^{th} \text{ term of } 3, 5, 7, \ldots)$

$= [5 + (n - 1)2] \times [3 + (n - 1)2]$

$= (2n + 3)(2n + 1) = 4n^2 + 8n + 3$.

$\therefore \quad S_n = \Sigma t_n = \Sigma(4n^2 + 8n + 3)$

$= 4\Sigma n^2 + 8\Sigma n + \Sigma 3$

$= 4\left[\dfrac{n(n + 1)(2n + 1)}{6}\right] + 8\left[\dfrac{n(n + 1)}{2}\right] + 3n$

$$= n(n+1)\left[\frac{2(2n+1)}{3}+4\right]+3n$$

$$= n(n+1)\left[\frac{4n+2+12}{3}\right]+3n$$

$$= \frac{n(n+1)(4n+14)}{3}+3n$$

$$= \frac{n(4n^2+18n+14+9)}{3}$$

$$= \frac{n(4n^2+18n+23)}{3}.$$

f) 2 + 6 + 12 + 20 + ...

Sol^n: Since $2+6+12+20+......=1.2+2.3+3.4+4.5+........$

The nth term of series, t_n = (n^{th} term of 1, 2, 3, 4........) × (n^{th} term of 2, 3, 4, 5 ....)

$$= [1+(n-1)1]\times[2+(n-1)1]$$

$$= n\times(n+1) = n(n+1)$$

$\therefore \quad S_n = \sum t_n \quad = \sum n(n+1)$

$$= \sum(n^2+n)$$

$$= \frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1}{3}+1\right]$$

$$= \frac{n(n+1)}{2}\left(\frac{2n+4}{3}\right)$$

$$= \frac{n(n+1)(n+2)}{3}.$$

Alternately,

Let $S_n = 2+6+12+20+.......+t_n$

$S_n = 2+6+12+................+t_{n-1}+t_n$

Subtracting

$0 = 2+4+6+8+.............+(t_n-t_{n-1})-t_n$

or, $t_n = 2+4+6+8+.............+n^{th}$ term.

$$= \frac{n}{2}\{2.2+(n-1).2\}$$

$$= \frac{n}{2}\{4+2n-2\} = n(n+1) = n^2+n$$

$\therefore \quad S_n = \sum t_n \quad = \sum(n^2+n)$

$$= \frac{n}{6}(n+1)(2n+1)+\frac{n}{2}(n+1)$$

$$= \frac{n(n+1)}{2}\left\{\frac{2n+1}{3}+1\right\} = \frac{n(n+1)}{2}\frac{(2n+1+3)}{3} = \frac{n}{3}(n+1)(n+2)$$

g) 2 + (2 + 3) + (2 + 3 + 4) + ……

Sol^n: Here, the nth term of the series is, $t_n = 2 + 3 + 4 + \dots\dots + (n + 1)$

$$= \frac{n}{2}[2.2 + (n - 1)1] \quad \text{[series consists n-terms]}$$

$$= \frac{n(n+3)}{2} = \frac{1}{2}(n^2 + 3n)$$

$\therefore \quad S_n = \Sigma t_n = \frac{1}{2}\Sigma(n^2 + 3n)$

$$= \frac{1}{2}(\Sigma n^2 + 3\Sigma n)$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1+9}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+10}{6}\right]$$

$$= \frac{n(n+1)(n+5)}{6}.$$

h) 1 + (1 + 3) + (1 + 3 + 5) + ...

Sol^n: Since $1 + (1 + 3) + (1 + 3 + 5) + \dots = 1^2 + 2^2 + 3^2 + \dots,$

The nth term of series $(t_n) = n^2$

$\therefore \quad S_n = \Sigma t_n = \frac{n(n+1)(2n+1)}{6}.$

3. a) Find the general term and then the sum of first n terms: $1 \cdot n + 2 \cdot (n - 1) + 3 \cdot (n - 2) + \dots$

Sol^n: Here, the general term of series, $t_r = (r^{th} \text{ term of } 1, 2, 3 \dots) \times (r^{th} \text{ terms of } n, n - 1, n - 2)$

$$= [1 + (r - 1).1] \times [n + (r - 1).(-1)]$$

$$= r(n - r + 1) = nr - r^2 + r$$

$\therefore \quad S_n = \sum_{r=1}^{n} t_r = n\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$

$$= n(1+2+3+\dots+n) - (1^2+2^2+3^2+\dots+n^2) + (1 + 2 + 3 + \dots + n)$$

$$= n.\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{3n - 2n - 1 + 3}{3}\right] = \frac{n(n+1)}{2}.\frac{n+2}{3} = \frac{n(n+1)(n+2)}{6}.$$

b) The natural numbers are grouped as follows:

(1), (2, 3), (4, 5, 6), (7, 8, 9, 10), ……

Find the first term of the nth group and its sum.

Sol^n: Here, the groups of the natural numbers as (1), (2, 3), (4, 5, 6), (7, 8, 9, 10), …… .

Let, the series of first element of each group be

$S_n = 1 + 2 + 4 + 7 + \dots\dots$

Let, t_n be nth term of series such that t_n will be first term of nth group, then

$$S_n = 1 + 2 + 4 + 7 + \ldots\ldots + t_{n-1} + t_n$$

$$S_n = 1 + 2 + 4 + \ldots\ldots\ldots\ldots + t_{n-1} + t_n$$

Subtracting, we get

$$0 = 1 + 1 + 2 + 3 + \ldots\ldots\ldots\ldots \text{ upto n terms - } t_n$$

$$t_n = 1 + (1 + 2 + 3 + \ldots\ldots\ldots \text{ upto } (n-1) \text{ terms})$$

$$t_n = 1 + \frac{n-1}{2}[2.1 + (n - 1 - 1)1\,] = 1 + \frac{n(n-1)}{2} = \frac{n^2 - n + 2}{2}$$

The first term of n group is $\frac{n^2 - n + 2}{2}$

Since the nth group consists n counting number with first term(a) = $\frac{n^2 - n + 2}{2}$,

The sum of numbers in the n^{th} group = $\frac{n}{2}\left[2.\frac{n^2 - n + 2}{2} + (n-1).1\right] = \frac{n(n^2+1)}{2}$.

4. Sum to n terms the following series:

a) $1^2 \cdot 1 + 2^2 \cdot 3 + 3^2 \cdot 5 + \ldots\ldots\ldots$

Sol^n: The nth term of the given series, t_n = (n^{th}terms of 1, 2, 3,.....)^2 × (n^{th} term of 1, 3, 5,....)

$$= [1 + (n-1).1]^2 \times [1 + (n-1).2] = n^2(2n-1) = 2n^3 - n^2$$

$$\therefore \quad S_n = \Sigma t_n = \Sigma(2n^3 - n^2)$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 - \left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{n^2(n+1)^2}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)^2}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{6}\,[3n\,(n+1) - (2n+1)]$$

$$= \frac{n(n+1)}{6}\,(3n^2+3n-2n-1)$$

b) $3.1^2 + 4.2^2 + 5.3^2 + \ldots\ldots$

Sol^n: The nth tern of the series is, t_n = (nth term of 3,4,5.....) × (nth term of $1^2, 2^2, 3^2, \ldots$)

$$= [3 + (n-1)1] \times n^2 = (3 + n - 1)\,n^2 = n^3 + 2n^2.$$

$$\therefore \quad S_n = \Sigma\, n^3 + 2\Sigma\, n^2$$

$$= \frac{n^2(n+1)^2}{4} + 2\left\{\frac{n(n+1)(2n+1)}{6}\right\}$$

$$= \frac{1}{12}[3n^2(n+1)^2 + 4n(n+1)(2n+1)]$$

$$= \frac{1}{12}[n(n+1)\{3n(n+1) + 4(2n+1)\}]$$

$$= \frac{1}{12}[n(n+1)(3n^2 + 3n + 8n + 4)]$$

$$= \frac{1}{12}[n(n+1)(3n^2+11n+4)]$$

$$= \frac{1}{12}n(n+1)(3n^2+11n+4) .$$

c) $1^2.2 + 2^2.3 + 3^2.4 + ...$

Sol^n: The nth term of the series is t_n $= [n^{th}$ term of $1^2, 2^2, 3^2, ...] \times [n^{th}$ term of $2, 3, 4, ...]$

$= n^2 . [2 + (n - 1)] = n^2(n+1) = n^3 + n^2$

Let S_n denotes the sum of the given series.

Then,

$$\therefore \quad S_n = \Sigma t_n$$

$$= \Sigma n^3 + \Sigma n^2$$

$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{12}[3n^2(n+1)^2 + 2n(n+1)(2n+1)]$$

$$= \frac{1}{12}[n(n+1)\{3n(n+1) + 2(2n+1)\}]$$

$$= \frac{1}{12}n(n+1)(3n^2+3n+4n+2)$$

$$= \frac{1}{12}n(n+1)(3n^2+7n+2)$$

$$= \frac{1}{12}n(n+1)(3n^2+6n+n+2)$$

$$= \frac{1}{12}n(n+1)\{(3n(n+2) + 1(n+2)\}$$

$$= \frac{1}{12}n(n+1)(n+2)(3n+1) .$$

d) $1.2^2 + 3.4^2 + 5.6^2 + ...$

Sol^n: The nth term of series is, $t_n = [$nth term of $1, 3, 5, ...] \times [$nth term $2^2, 4^2, 6^2, ...]$

$= (2n - 1) \times 4n^2 = 8n^3 - 4n^2$

$$\therefore \quad S_n = \Sigma t_n$$

$$= \Sigma(8n^3 - 4n^2)$$

$$= 8\Sigma n^3 - 4\Sigma n^2$$

$$= 8 \times \frac{n^2(n+1)^2}{4} - 4 \times \frac{n(n+1)(2n+1)}{6}$$

$$= 2n^2(n+1)^2 - \frac{2n(n+1)(2n+1)}{3}$$

$$= 2n(n+1)\left[n(n+1) - \frac{2n+1}{3}\right]$$

$$= 2n(n+1)\left[\frac{3n^2+3n-2n-1}{3}\right]$$

$$= \frac{2n(n+1)(3n^2+n-1)}{3} .$$

e) $\mathbf{1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots}$

Sol^n: The nth term of given series is, $t_n = [1 + (n-1)2]^2 = (2n-1)^2$

$\therefore \quad S_n = \Sigma t_n$

$= \Sigma(2n-1)^2$

$= \Sigma(4n^2 - 4n + 1)$

$= \dfrac{4n(n+1)(2n+1)}{6} - 4\dfrac{n(n+1)}{2} + n$

$= n\left[\dfrac{2(n+1)(2n+1)}{3} - 2(n+1) + 1\right]$

$= \dfrac{n}{3}\ [2(n+1)(2n+1) - 6(n+1) + 3]$

$= \dfrac{n}{3}\ (4n^2 + 2n + 4n + 2 - 6n - 6 + 3)$

$= \dfrac{1}{3}n(4n^2 - 1)$.

$= \dfrac{1}{6}n(n+1)(3n^2 + n - 1)$.

f) $\mathbf{1^3 + 3^3 + 5^3 + \ldots\ldots\ldots\ldots}$

Sol^n: The nth term of series is, $t_n = \{1 + (n-1)2\}^3 = (2n-1)^3 \qquad = 8n^3 - 12n^2 + 6n - 1$

$\therefore \quad S_n = \Sigma t_n$

$= \Sigma(8n^3 - 12n^2 + 6n - 1)$

$= 8\Sigma n^3 - 12\Sigma n^2 + 6\Sigma n - \Sigma 1$

$= 8\left[\dfrac{n(n+1)}{2}\right]^2 - 12\left[\dfrac{n(n+1)(2n+1)}{6}\right] + \left[\dfrac{6n(n+1)}{2}\right] - n$

$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3n(n+1) - n$

$= n[2n(n^2 + 2n + 1) - 2(n+1)(2n+1) + 3(n+1) - 1]$

$= n(2n^3 + 4n^2 + 2n - 4n^2 - 2n - 4n - 2 - 3n + 3 - 1)$

$= n(2n^2 - 4n + 3n) = n(2n^2 - n)$

$\therefore \quad S_n = n^2(2n^2 - 1)$

5. Find the nth term and then sum of n terms of the following series:

a) $\mathbf{2 + 4 + 8 + 14 + 22 + \ldots\ldots\ldots}$

Sol^n: Let, $S_n = 2 + 4 + 8 + 14 + 22 + \ldots\ldots\ldots\ldots + t_{n-1} + t_n$

$S_n = 2 + 4 + 8 + 14 + 22 + \ldots\ldots + t_{n-2} + t_{n-1} + t_n$

Subtracting,

$0 = 2 + 2 + 4 + 6 + 8 + 10 + \ldots\ldots + (t_n - t_{n-1}) - t_n$

$\therefore \quad t_n = 2 + 2(1 + 2 + 3 + 4 + 5 + \ldots\ldots$ to $(n-1)$ terms$)$

$= 2 + 2 \times \dfrac{(n-1)}{2}[2 \times 1 + (n-1-1)1] \qquad \left[\because S_n = \dfrac{n}{2}[2a + (n-1)d]\right]$

$= 2 + n(n-1) = n^2 - n + 2$.

$\therefore \quad S_n = \Sigma t_n$

$= \Sigma(n^2 - n + 2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 2n$

$= n \times \frac{(2n^2 + 3n + 1 - 3n - 3 + 12)}{6}$

$= n \times \frac{(2n^2 + 10)}{6} = \frac{1}{3}n(n^2 + 5)$

b) 3 + 7 + 13 + 21 + 31 + ...

Sol^n: Let, $S_n = 3 + 7 + 13 + 21 + 31 + \ldots\ldots\ldots\ldots + t_{n-1} + t_n$

$S_n = 3 + 7 + 13 + 21 + 31 + \ldots\ldots + t_{n-2} + t_{n-1} + t_n$

Subtracting,

$0 = 3 + 4 + 6 + 8 + 10 + \ldots\ldots + (t_n - t_{n-1}) - t_n$

$\therefore \quad t_n = 1 + \{2(1 + 2 + 3 + 4 + 5 + \ldots\ldots \text{ to n terms})\}$

$= 1 + 2 \times \frac{n(n+1)}{2} = n^2 + n + 1.$

$\therefore \quad S_n = \Sigma t_n$

$= \Sigma(n^2 + n + 1)$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$

$= n \times \frac{(2n^2 + 3n + 1 + 3n + 3 + 6)}{6}$

$= n \times \frac{(2n^2 + 6n + 10)}{6} = \frac{1}{3}n(n^2 + 3n + 5)$

c) 2 + 5 + 10 + 17 + 26 + ...

Sol^n: Let $S_n = 2 + 5 + 10 + 17 + 26 + \ldots\ldots\ldots\ldots + t_{n-1} + t_n$

$S_n = 2 + 5 + 10 + 17 + 26 + \ldots\ldots + t_{n-2} + t_{n-1} + t_n$

subtracting,

$0 = 2 + 3 + 5 + 7 + 9 + \ldots\ldots + (t_n - t_{n-1}) - t_n$

$\therefore \quad t_n = 1 + (1 + 3 + 5 + 7 + 9 + \ldots\ldots \text{ to n terms})$

$= 1 + n^2 = n^2 + 1.$

$\therefore \quad S_n = \Sigma t_n$

$= \Sigma(n^2 + 1)$

$= \frac{n(n+1)(2n+1)}{6} + n$

$= n \times \frac{(2n^2 + 3n + 1 + 6)}{6}$

$= \frac{1}{6}n(2n^2 + 3n + 7).$

d) $5 + 11 + 19 + 29 + 41 + \ldots\ldots$

Sol^n: Let, $S_n = 5 + 11 + 19 + 29 + 41 + \ldots\ldots\ \ldots\ldots + t_{n-1} + t_n$

$S_n = 5 + 11 + 19 + 29 + 41 + \ldots\ldots + t_{n-2} + t_{n-1} + t_n$

Subtracting,

$0 = 5 + 6 + 8 + 10 + 12 + \ldots\ldots + (t_n - t_{n-1}) - t_n$

$\therefore\ t_n = 5 + (6 + 8 + 10 + 12 + \ldots\ldots \text{ to } (n-1) \text{ terms})$

$= 5 + \frac{n-1}{2}[2 \times 6 + (n - 1 - 1)2]$ $\qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 5 + \frac{(n-1)(2n+8)}{2}$

$= 5 + (n-1)(n+4) = 5 + n^2 + 3n - 4 = n^2 + 3n + 1.$

$\therefore\ S_n = \Sigma t_n$

$= \Sigma(n^2 + 3n + 1)$

$= \Sigma n^2 + 3\Sigma n + \Sigma 1$

$= \frac{n(n+1)(2n+1)}{6} + 3 \times \frac{n(n+1)}{2} + n$

$= n \times \frac{(2n^2 + 3n + 1 + 9n + 9 + 6)}{6}$

$= \frac{1}{6}n(2n^2 + 12n + 16)$

$= \frac{1}{3}n(n^2 + 6n + 8)$

$= \frac{1}{3}n(n+2)(n+4).$

e) $5 + 7 + 13 + 31 + 85 + \ldots\ldots$

Sol^n: Let, $S_n = 5 + 7 + 13 + 31 + 85 + \ldots\ldots\ \ldots\ldots + t_{n-1} + t_n$

$S_n = 5 + 7 + 13 + 31 + 85 + \ldots\ldots + t_{n-2} + t_{n-1} + t_n$

Subtracting,

$0 = 5 + 2 + 6 + 18 + 54 + \ldots\ldots + (t_n - t_{n-1}) - t_n$

$\therefore\ t_n = 5 + (2 + 6 + 18 + 54 + \ldots\ldots \text{ to } (n-1) \text{ terms})$

$= 5 + \frac{2(3^{n-1} - 1)}{3 - 1}$ $\qquad \left[\because S_n = \frac{a(r^n - 1)}{r - 1}\right]$

$= 5 + 3^{n-1} - 1 = 3^{n-1} + 4.$

$\therefore\ S_n = \Sigma t_n$

$= \Sigma 3^{n-1} + \Sigma 4$

$= (1 + 3 + 9 + 27 + \ldots\ldots \text{ to n terms}) + 4n$

$= \frac{1(3^n - 1)}{3 - 1} + 4n$

$= \frac{3^n - 1}{2} + 4n$

$= \frac{1}{2}(3^n + 8n - 1).$

f) 2 + 5 + 14 + 41 + ………….

Sol^n: Let, $S_n = 2 + 5 + 14 + 41 + \ldots\ldots\ldots + t_{n-1} + t_n$

$S_n = 2 + 5 + 14 + 41 + \ldots\ldots + t_{n-2} + t_{n-1} + t_n$

Subtracting,

$0 = 2 + 3 + 9 + 27 + \ldots\ldots + (t_n - t_{n-1}) - t_n$

$\therefore\ t_n = 2 + (3 + 9 + 27 + \ldots\ldots \text{ to } (n-1) \text{ terms})$

$= 2 + \dfrac{3(3^{n-1} - 1)}{3 - 1}$

$= \dfrac{1}{2}(4 + 3 \times 3^{n-1} - 3)$ $\left[\because S_n = \dfrac{a(r^n - 1)}{r - 1}\right]$

$= \dfrac{1}{2}(3^n + 1).$

$\therefore\ S_n = \Sigma t_n$

$= \dfrac{1}{2}(\Sigma 3^n + \Sigma 1)$

$= \dfrac{1}{2}[(3 + 9 + 27 + \ldots\ldots \text{ to n terms}) + n]$

$= \dfrac{1}{2}\left[\dfrac{3(3^n - 1)}{3 - 1} + n\right]$

$= \dfrac{1}{2}\left[\dfrac{3^{n+1} - 3}{2} + n\right]$

$= \dfrac{1}{2}(3^{n+1} + n - 3).$

Hint and Solution of MCQ's

1. The sum of first n-natural numbers is $\dfrac{n(n+1)}{2}$
2. The sum of first n - odd numbers is n^2
3. The sum of squares of first n-natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$
4. $3 + 4 + 5 + 6 + \ldots\ldots + 20$
   $= (1 + 2 + 3 + 4 + 5 + 6 + \ldots\ldots + 20) - (1 + 2)$
   $= \dfrac{20(20+1)}{2} - 3$
   $= 207$
5. $6 + 8 + 10 + \ldots\ldots$ 10 terms
   $= (2 + 4 + 6 + 8 + 10 + \ldots\ldots \text{ 10 terms}) - (2 + 4)$
   $= 10(10 + 1) - 6 \quad = 104$ [sum of first n -even numbers is n (n + 1)]
6. $2^2 + 3^2 + 4^2 + \ldots\ldots + 6^2$
   $= (1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 6^2) - 1^2$
   $= \dfrac{6(6+1)(2.6+1)}{6} - 1$
   $= 90$

7. n^{th} term of 2, 3, 4, ... $= 2 + (n-1)1 = n + 1$

n^{th} term of 3, 4, 5, ... $= 3 + (n-1)1 = n + 2$

$\therefore$ n^{th} term of $2.3 + 3.4 + 4.5 + \ldots = (n+1)(n+2) = n^2 + 3n + 2$

8. The n^{th} term of given series $= t_n = 2 + 4 + 6 + \ldots\ldots$ to n terms

$= n(n+1)$ [Sum of first n – even number]

9. The n^{th} term of given series is equal to

$t_n = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots$ to n terms

$= \dfrac{n^2(n+1)^2}{4}$ [Sum of cubes of first n-natural numbers]

10. The sum of n terms of given series is equal to

$s_n = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2$

$= \dfrac{n(n+1)(2n+1)}{6}$ [Sum of squares of first n-natural numbers]

11. Let n^n term of given series be t_n and the sum is S_n. Then

$$
\begin{array}{l}
S_n = 1 + 3 + 7 + 15 + 31 + \ldots\ldots\ldots + t_n \\
S_n = 1 + 3 + 7 + 15 + \ldots\ldots\ldots + t_{n-1} + t_n \\
\hline
0 = 1 + 2 + 4 + 8 + 16 + \ldots\ldots\ldots \text{ to n terms} - t_n
\end{array}
$$

$\therefore$ $t_n = 1 + 2 + 4 + 8 + 16 + \ldots\ldots\ldots$ to n terms

$= \dfrac{1(2^n - 1)}{2-1}$ [Sum of G.S.]

$= 2^n - 1$