UNIT 6: CALCULUS

Chapter 12

Derivatives

Exercise 12.1

1. **Find the derivatives of**

a) sinh 3x

Sol^n: Let y = sinh 3x

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(\sinh 3x) = \frac{d(\sinh 3x)}{d(3x)} \times \frac{d(3x)}{dx} = 3\cosh 3x$$

b) $\tanh \frac{x}{2}$

Sol^n: Let $y = \tanh \frac{x}{2}$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(\tanh \frac{x}{2}) = \frac{d(\tanh \frac{x}{2})}{d\left(\frac{x}{2}\right)} \times \frac{d\left(\frac{x}{2}\right)}{dx} = \sec h^2 \frac{x}{2} \times \frac{1}{2} = \frac{1}{2}\sec h^2 \frac{x}{2}$$

c) coth x^2

Sol^n: Let y = coth x^2

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(\coth x^2) = \frac{d(\coth x^2)}{d(x^2)} \times \frac{d(x^2)}{dx} = -\operatorname{cos\,ec}^2 hx^2 \times 2x = -2x \operatorname{cos\,ech}^2 x^2$$

d) sech^2 3x

Sol^n: Let y = sech^{2}3x

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(\sec h^2 3x) = \frac{d(\sec h3x)^2}{d(\sec h3x)} \times \frac{d(\sec h3x)}{d(3x)} \times \frac{d(3x)}{dx}$$

$$= 2\sec h3x. - \sec h3x. \tanh 3x \times 3 = -6\sec h^2 3x \tanh 3x$$

e) $\cosh^3 \frac{2x}{3}$

Sol^n: Let $y = \cosh^3 \frac{2x}{3}$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cosh^3 \frac{2x}{3}\right) = \frac{d\left(\cosh \frac{2x}{3}\right)^3}{d\left(\cosh \frac{2x}{3}\right)} \times \frac{d\left(\cosh \frac{2x}{3}\right)}{d\left(\frac{2x}{3}\right)} \times \frac{d\left(\frac{2x}{3}\right)}{dx}$$

$$= 3\cosh^2 \frac{2x}{3} \times \sinh \frac{2x}{3} \times \frac{2}{3} = 2\cosh^2 \frac{2x}{3} \sinh \frac{2x}{3}$$

f) $\cosh^{-1} \frac{x}{3}$

Sol^n: Let $y = \cosh^{-1}x$

$\Rightarrow \quad \cosh y = \frac{x}{3}$

Differentiating both sides w. r. to 'x'

$$\frac{d(\cosh y)}{dy} \times \frac{dy}{dx} = \frac{1}{3}$$

$$\Rightarrow \quad \sinh y \times \frac{dy}{dx} = \frac{1}{3} \Rightarrow \frac{dy}{dx} = \frac{1}{3}\frac{1}{\sinh y} = \frac{1}{3}\frac{1}{\sqrt{\cosh^2 y - 1}} = \frac{1}{3}\frac{1}{\sqrt{\left(\frac{x}{3}\right)^2 - 1}} = \frac{1}{\sqrt{x^2 - 9}}$$

2. Find the derivative of

a) $\sinh^2 x + \cosh^2 x$

Sol^n: Let $y = \sinh^2 x + \cosh^2 x$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sinh^2 x + \cos^2 x\right) = 2\sinh x.\cosh x + 2\cosh x \sinh x = \sinh 2x + \sinh 2x = 2\sinh 2x$$

b) $\coth x - \frac{1}{3}\coth^3 x$

Sol^n: Let $y = \coth x - \frac{1}{3}\coth^3 x$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\coth x - \frac{1}{3}\coth^3 x\right) = -\operatorname{cosech}^2 x - \frac{1}{3}(3\coth^2 x \times -\operatorname{cosech}^2 x)$$

$$= -\operatorname{cosech}^2 x + \coth x \operatorname{cosech}^2 x$$

$$= \operatorname{cosech}^2 x(\coth^2 x - 1) = \operatorname{cosech}^4 x$$

c) tanh 5x – sech 2x

Sol^n: Let y = tanh 5x – sech 2x

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(\tanh 5x - \operatorname{sec}h2x) = 5\ \text{sech}^2\ 5x + 2\text{sech}\ 2x\ \tanh 2x$$

d) $\sqrt{\sinh x} + \frac{1}{\sqrt{\cosh x}}$

Sol^n: Let y = $\sqrt{\sinh x} + \frac{1}{\sqrt{\cosh x}}$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\sinh x} + \frac{1}{\sqrt{\cosh x}}\right) = \frac{d}{dx}(\sinh x)^{\frac{1}{2}} + \frac{d}{dx}(\sinh x)^{-\frac{1}{2}}$$

$$= \frac{1}{2}(\sinh x)^{\frac{1}{2}-1} - \frac{1}{2}(\cosh x)^{-\frac{1}{2}-1} = \frac{1}{2}\left(\frac{\cosh x}{\sqrt{\sinh x}} - \frac{\sinh x}{\sqrt{\cos^3 x}}\right)$$

e) tanh x coth x

Sol^n: Let y = tanhxcoth x

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \tanh x\frac{d}{dx}(\coth x) + \coth x\frac{d}{dx}(\tanh x)$$

$$= \tanh x. - \operatorname{cos}\text{ech}^2 x + \coth x \operatorname{sec}h^2 x = \coth x \operatorname{sec}h^2 x - \tanh x \operatorname{cos}\text{ech}^2 x$$

f) $\frac{\sinh 2x}{x + \cosh 2x}$

Sol^n: Let y = $\frac{\sinh 2x}{x + \cosh 2x}$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{(x + \cosh 2x)\frac{d}{dx}\sinh 2x - \sinh 2x\frac{d}{dx}(x + \cosh 2x)}{(x + \cosh 2x)^2}$$

$$= \frac{2\cosh 2x(x + \cosh 2x) - \sinh 2x(1 + 2\sinh 2x)}{(x + \cosh 2x)^2}$$

$$= \frac{2x\cosh 2x + 2\cosh^2 2x - \sinh 2x - 2\sinh^2 2x}{(x + \cosh 2x)^2}$$

$$= \frac{2x\cosh 2x + 2(\cosh^2 2x - \sinh^2 2x) - \sinh 2x}{(x + \cosh 2x)^2} = \frac{2x\cosh 2x - \sinh 2x + 2}{(x + \cosh 2x)^2}$$

f) $\dfrac{1}{\sqrt{\coth x+\tanh x}}$

Sol^n: Let $y = \dfrac{1}{\sqrt{\coth x+\tanh x}}$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx}=\frac{d}{dx}(\coth x+\tanh x)^{-\frac{1}{2}}$$

$$=\frac{d(\coth x+\tanh x)^{-\frac{1}{2}}}{d(\coth x+\tanh x)}\times\frac{d(\coth x+\tanh x)}{dx}$$

$$=-\frac{1}{2}(\coth x+\tanh x)^{-\frac{1}{2}-1}\times(-\operatorname{cosech}^2 x+\operatorname{sech}^2 x)$$

$$=\ -\frac{1}{2}(\coth x+\tanh x)^{-\frac{3}{2}}\times\frac{\sinh^2 x-\cosh^2 x}{\sinh^2 x\cosh^2 x}$$

$$=\frac{1}{2}(\coth x+\tanh x)^{-\frac{3}{2}}\times\frac{1}{4\sinh^2 x\cosh^2 x}\times 4=\frac{2(\coth x+\tanh x)^{-\frac{3}{2}}}{\sinh^2 2x}$$

f) $(\sinh^{-1}x+\cosh^{-1}x)^2$

Sol^n: Let $y = (\sinh^{-1}x+\cosh^{-1}x)^2$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx}=2(\sinh^{-1}x+\cosh^{-1}x)\frac{d}{dx}(\sinh^{-1}x+\cosh^{-1}x)$$

$$=2(\sinh^{-1}x+\cosh^{-1}x)\left(\frac{1}{\sqrt{1+x^2}}+\frac{1}{\sqrt{x^2-1}}\right).$$

3. Find the derivatives of

a) sech ($\tan^{-1}$x)

Sol^n: Let y = sech ($\tan^{-1}$x)

Differentiating both sides w.r. to 'x'

$$\frac{dy}{dx}=\frac{d(\operatorname{sech}(\tan^{-1}x))}{dx}=\frac{d(\operatorname{sech}(\tan^{-1}x))}{d(\tan^{-1}x)}\cdot\frac{d(\tan^{-1}x)}{dx}=-\operatorname{sech}(\tan^{-1}x)\tanh(\tan^{-1}x)\frac{1}{1+x^2}.$$

b) $\operatorname{sech}^{-1}x - \cosh^{-1}x$

Sol^n: Let $y = \operatorname{sech}^{-1}x - \cosh^{-1}x$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx}=\frac{d}{dx}(\operatorname{sech}^{-1}x-\cosh^{-1}x)=\frac{d}{dx}\operatorname{sech}^{-1}x-\frac{d}{dx}\cosh^{-1}x$$

$$=-\frac{1}{x\sqrt{1-x^2}}-\frac{1}{\sqrt{x^2-1}}.$$

c) Arc tan sinh x

Sol^n: Let y = Arc tan sinh x

i.e. $y = \tan^{-1} \sinh x$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}\sinh x)$$

$$= \frac{d(\tan^{-1}\sinh x)}{d(\sinh x)} \cdot \frac{d(\sinh x)}{dx} = \frac{1}{1+\sinh^2 x}\cosh x = \frac{\cosh x}{\cosh^2 x} = \text{sech} x.$$

d) $2\tanh^{-1}\left(\tan\frac{1}{2}x\right)$

Sol^n: Let $y = 2\tanh^{-1}\left(\tan\frac{1}{2}x\right)$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left\{(\tanh^{-1})\left(\tan\frac{1}{2}x\right)\right\}$$

$$= 2\frac{d(\tanh^{-1}(\tan\frac{1}{2}x))}{d(\tan\frac{1}{2}x)} \cdot \frac{d(\tan\frac{1}{2}x)}{d(\frac{1}{2}x)} \cdot \frac{d(\frac{1}{2}x)}{dx}$$

$$= 2\frac{1}{1-\tan^2\frac{1}{2}x}\sec^2\frac{x}{2}\times\frac{1}{2}$$

$$= \frac{1+\tan^2\frac{1}{2}x}{1-\tan^2\frac{1}{2}x} = \frac{1}{\cos x} = \sec x$$

e) $\sinh^{-1}(\cos 2x)$

Sol^n: Let $y = \sinh^{-1}(\cos 2x)$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d\left[\sinh^{-1}(\cos 2x)\right]}{d(\cos 2x)}\times\frac{d(\cos 2x)}{d(2x)}\times\frac{d(2x)}{dx}$$

$$= \frac{1}{\sqrt{1+\cos^2 2x}}\times -\sin 2x\times 2 = -\frac{2\sin 2x}{\sqrt{1+\cos^2 2x}}$$

f) coth (Arc sin x)

Sol^n: Let y = coth (Arc sin x) = $\coth(\sin^{-1}x)$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d\left[\coth(\sin^{-1}x)\right]}{d(\sin^{-1}x)}\times\frac{d(\sin^{-1}x)}{dx}$$

$$= -\text{cosec}^2(\sin^{-1}x)\frac{1}{\sqrt{1-x^2}} = -\frac{\text{cosec}^2(\sin^{-1}x)}{\sqrt{1-x^2}}$$

4. Find the derivative of

a) $e^{\sinh x}$

Sol^n: Let $y = e^{\sinh x}$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(e^{\sinh x}) = \frac{d(e^{\sinh x})}{d(\sinh x)} \cdot \frac{d(\sinh x)}{dx}$$

$$= e^{\sinh x}\cosh x$$

b) $e^{\tanh\frac{x}{2}}$

Sol^n Let $y = e^{\tanh\frac{x}{2}}$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d\left[e^{\tanh\frac{x}{2}}\right]}{d\left(\tan\frac{x}{2}\right)} \times \frac{d\left(\tan\frac{x}{2}\right)}{d\left(\frac{x}{2}\right)} \times \frac{d\left(\frac{x}{2}\right)}{dx} = e^{\tanh\frac{x}{2}} \times \sec^2\frac{x}{2} \times \frac{1}{2} = \frac{1}{2}e^{\tanh\frac{x}{2}}\sec^2\frac{x}{2}$$

c) log(tanhx)

Sol^n Let $y = \log(\tanh x)$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d[\log(\tanh x)]}{d(\tanh x)} \times \frac{d(\tanh x)}{dx}$$

$$= \frac{1}{\tanh x} \times -\operatorname{sech}^2 x$$

$$= -\frac{1}{\sinh x \cosh x} = -\frac{2}{\sinh 2x} = -2\operatorname{cosech} 2x$$

d) $\log\sinh\left(\frac{x}{a}\right)$

Sol^n: Let $y = \log\sinh\left(\frac{x}{a}\right)$

Differentiating both sides w. r .to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log\sinh\frac{x}{a}\right)$$

$$= \log\sinh\frac{x}{a}\frac{d}{d\left(\sinh\frac{x}{a}\right)} \times \frac{d\left(\sinh\frac{x}{a}\right)}{d\left(\frac{x}{a}\right)} \times \frac{d\left(\frac{x}{a}\right)}{dx}.$$

$$= \frac{1}{\sinh\frac{x}{a}}\cosh\frac{x}{a} \cdot \frac{1}{a} = \frac{1}{a}\coth\frac{x}{a}$$

e) $\log(\cosh x^2)$

Sol^n: Let $y = \log(\cosh x^2)$

Differentiating both sides w. r. to 'x'

$$\frac{dy}{dx} = \frac{d[\log(\cosh x^2)]}{d(\cosh x^2)} \times \frac{d(\cosh x^2)}{d(x^2)} \times \frac{d(x^2)}{dx}$$

$$= \frac{1}{\cosh x^2} \times \sinh x^2 \times 2x = 2x\tanh x^2$$

5. Find the derivatives of

a) $x^{\cosh x/a}$

Sol^n: Let $y = x^{\cosh x/a}$

Taking log on both sides, we get

$$\log y = \cosh\frac{x}{a}\log x$$

Differentiating both sides w. r. to 'x'

$$\frac{d}{dx}(\log y) = \frac{d}{dx}(\cosh\frac{x}{a}\log x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cosh\frac{x}{a}\cdot\frac{1}{x} + \log x \sinh\frac{x}{a}\cdot\frac{1}{a}$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{1}{x}\cosh\frac{x}{a} + \frac{1}{a}\log x \sinh\frac{x}{a}\right)$$

$$\therefore \frac{dy}{dx} = x^{\cosh x/a}\left(\frac{1}{x}\cosh\frac{x}{a} + \frac{1}{a}\log x\sinh\frac{x}{a}\right).$$

b) $x^{\sinh x^2/a}$

Sol^n: Let $y = x^{\sinh x^2/a}$

Taking log on both sides, we get

$$\log y = \sin h\frac{x^2}{a}\ \log x$$

Differentiating both sides w. r. to 'x' We get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \cosh\frac{x^2}{a}\frac{2x}{a}\log x + \sinh\frac{x^2}{a}\cdot\frac{1}{x}$$

$$\therefore \frac{dy}{dx} = y\left(\frac{\sinh\frac{x^2}{a}}{x} + \frac{2x\log x}{a}\cdot\cosh\frac{x^2}{a}\right)$$

$$= x^{\sinh x^2/a}\left(\frac{\sinh\frac{x^2}{a}}{x} + \frac{2x\log x}{a}\cdot\cosh\frac{x^2}{a}\right)$$

c) $x^{\cosh^2 x/a}$

Sol^n: Let $y = x^{\cosh^2 x/a}$

Taking log on both sides, we get

$$\log y = \cosh^2\frac{x}{a}\cdot\log x$$

Differentiating both sides w. r. to 'x' we get

$$\frac{dy}{dx}(\log y) = \frac{d}{dx}\left(\cosh^2\frac{x}{a}\log x\right)$$

$$\Rightarrow \frac{d(\log y)}{dy}\frac{dy}{dx} = \cosh^2\frac{x}{a}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\cosh^2\frac{x}{a}\right)$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \cosh^2\frac{x}{a}\cdot\frac{1}{x} + \log x\frac{d\left(\cosh^2\frac{x}{a}\right)}{d\left(\cosh\frac{x}{a}\right)}\times\frac{d\left(\cosh\frac{x}{a}\right)}{d\left(\frac{x}{a}\right)}\times\frac{d\left(\frac{x}{a}\right)}{dx}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} \cosh^2\frac{x}{a} + \log x.2 \cosh\frac{x}{a} .\sinh\frac{x}{a} .\frac{1}{a}$$

$$\therefore \frac{dy}{dx} = y\left(\frac{1}{x}\cos^2\frac{x}{a} + \frac{1}{a}\log x \sinh\frac{2x}{a}\right) = x^{\cosh^2 x/a}\left(\frac{1}{x}\cos^2\frac{x}{a} + \frac{1}{a}\log x \sinh\frac{2x}{a}\right).$$

d) $x^{\tanh^{-1}\frac{x}{3}}$

Sol^n: Let $y = x^{\tanh^{-1}\frac{x}{3}}$

Taking log on both sides we get,

$$\log y = \log x^{\tanh^{-1}\frac{x}{3}} = \tanh^{-1}\frac{x}{3}\ \log x$$

Differentiating both sides w. r. to 'x'

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1}\frac{x}{3}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\tanh^{-1}\frac{x}{3}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1}\frac{x}{3}.\frac{1}{x} + \log x\frac{1}{1-\left(\frac{x}{3}\right)^2}.\frac{1}{3}$$

$$\frac{dy}{dx} = y\left[\frac{1}{x}\tan^{-1}\frac{x}{3} + \frac{3\log x}{9-x^2}\right] = x^{\tanh^{-1}\frac{x}{3}}\left[\frac{1}{x}\tan^{-1}\frac{x}{3} + \frac{3\log x}{9-x^2}\right]$$

6. Find the derivatives of

a) $\left(\sinh\frac{x}{a}\right)^{x^2}$

Sol^n: Let, $y = \left(\sinh\frac{x}{a}\right)^{x2}$

Taking log on both sides we get

$$\log y = x^2 \log \sinh\frac{x}{a}$$

Differentiating both sides w. r to 'x' we get

$$\frac{d}{dx}(\log y) = \frac{d}{dx}(x^2 \log \sinh\frac{x}{a})$$

$$\Rightarrow \frac{d(\log y)}{dy} \times \frac{dy}{dx} = x^2\frac{d}{dx}\ (\log \sinh\frac{x}{a}\ ) + \log \sinh\frac{x}{a}\frac{d}{dx}\ (x^2)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x^2}{\sinh\frac{x}{a}}\cosh\frac{x}{a}\frac{1}{a} + 2x\log \sinh\frac{x}{a}$$

$$\therefore \frac{dy}{dx} = y\left(\frac{x^2}{a}\coth\frac{x}{a} + 2x\log x \sinh\frac{x}{a}\right)$$

$$= \left(\sinh\frac{x}{a}\right)^{x2}\left(\frac{x^2}{a}\coth\frac{x}{a} + 2x\log x \sinh\frac{x}{a}\right)$$

b) $\left(\cosh\frac{x}{a}\right)^{\log x}$

Sol^n: Let $y = \left(\cosh\frac{x}{a}\right)^{\log x}$

Taking log on both sides we get

$$\log y = \log x.\ \log\cosh\frac{x}{a}$$

Differentiating both sides w,r to 'x' we get

$$\frac{d}{dx}(\log y) = \frac{d}{dx}\left(\log x \log\cosh\frac{x}{a}\right)$$

$$\Rightarrow \quad \frac{d(\log y)}{dx}\frac{dy}{dx} = \frac{\log x\, d\left(\log\cosh\frac{x}{a}\right)}{d\left(\cosh\frac{x}{a}\right)} \times \frac{d\left(\cosh\frac{x}{a}\right)}{d\left(\frac{x}{a}\right)} \times \frac{d\left(\frac{x}{a}\right)}{dx} + \log\cosh\frac{x}{a}\frac{d}{dx}(\log x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log x\frac{1}{\cosh\frac{x}{a}}\sinh\frac{x}{a}\frac{1}{a} + \log\cosh\frac{x}{a}\frac{1}{x}$$

$$\therefore \quad \frac{dy}{dx} = y\left(\frac{1}{a}\log x\ .\ \tanh\frac{x}{a} + \frac{1}{x}\log\cosh\frac{x}{a}\right) = \left(\cosh\frac{x}{a}\right)^{\log x}\left(\frac{1}{a}\log x\ .\ \tanh\frac{x}{a} + \frac{1}{x}\log\cosh\frac{x}{a}\right).$$

c) $(\sinh x)^{\tanh x}$

Sol^n: Let $y = (\sinh x)^{\tanh x}$

Taking log on both sides we get,

$$\log y = \log(\sinh x)^{\tanh x} = \tanh x \log(\sinh x)$$

Differentiating both sides w. r. to 'x'

$$\frac{1}{y}\frac{dy}{dx} = \tanh x\frac{d}{dx}\log(\sinh x) + \log(\sinh x)\frac{d}{dx}(\tanh x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \tanh x\ .\ \frac{1}{\sinh x}.\cosh x + \log(\sinh x)\ \mathrm{sech}^2 x$$

$$\therefore \quad \frac{dy}{dx} = y(\ 1 + \log(\sinh x)\ \mathrm{sech}^2 x) = (\sinh x)^{\tanh x}(\ 1 + \log(\sinh x)\ \mathrm{sech}^2 x)$$

d) $(\log x)^{\sinh x}$

Sol^n: Let $y = (\log x)^{\sinh x}$

Taking log on both sides we get,

$$\log y = \log(\log x)^{\sinh x} = \sinh x\ .\ \log(\log x)$$

Differentiating both sides w. r. to 'x'

$$\frac{1}{y}\frac{dy}{dx} = \sinh x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(\sinh x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \sinh x\ .\ \frac{1}{\log x}.\frac{1}{x} + \log(\log x)\cosh x$$

$$\therefore \quad \frac{dy}{dx} = y\left[\frac{\sinh x}{x\log x} + \cosh x.\log(\log x)\right] = (\log x)^{\sinh x}\left[\frac{\sinh x}{x\log x} + \cosh x.\log(\log x)\right]$$

e) $(\cosh x)^{\sinh^{-1}x}$

Sol^n: Let $y = (\cosh x)^{\sinh^{-1}x}$

Taking log on both sides

$$\log y = \sinh^{-1}x . \log\cosh x$$

Differentiating both sides w,r to 'x' we get

$$\frac{d}{dx}(\log y) = \frac{d}{dx}(\sinh^{-1} x \log\cosh x)$$

$$\Rightarrow \frac{d(\log y)}{dx}\frac{dy}{dx} = \sinh^{-1} x \frac{d(\log \cosh x)}{d(\cosh x)}\frac{d(\cosh x)}{dx} + \log.\cosh x\frac{d}{dx}(\sinh^{-1}x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(\sinh^{-1} x \frac{1}{\cosh x}\sinh x + \log\ \cosh x \frac{1}{\sqrt{1+x^2}}\right)$$

$$\therefore \frac{dy}{dx} = y\left(\sinh^{-1} x \frac{1}{\cosh x}\sinh x + \log\ \cosh x \frac{1}{\sqrt{1+x^2}}\right)$$

$$= (\cosh x)^{\sinh^{-1}x}\left(\sinh^{-1} x \tanh x + \frac{x}{\sqrt{1+x^2}}\log\ \cosh x\right)$$

f) $\left(\cosh^{-1} x\right)^{\sinh x}$

Sol^n: Let $y = (\cosh^{-1} x)^{\sinh x}$

Taking log on both sides we get,

$$\log y = \log(\cosh^{-1} x)^{\sinh x} = \sinh x . \log(\cosh^{-1} x)$$

Differentiating both sides w. r. to 'x'

$$\frac{1}{y}\frac{dy}{dx} = \sinh x \frac{d}{dx}\log(\cosh^{-1} x) + \log(\cosh^{-1} x)\frac{d}{dx}(\sinh x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sinh x . \frac{1}{\cosh^{-1} x}.\frac{1}{\sqrt{x^2-1}} + \log(\cosh^{-1} x)\cosh x$$

$$\therefore \frac{dy}{dx} = y\left[\frac{\sinh x}{\cosh^{-1} x\sqrt{x^2-1}} + \cosh x.\log(\cosh^{-1} x)\right]$$

$$= (\cosh^{-1} x)^{\sinh x}\left[\frac{\sinh x}{\cosh^{-1} x\sqrt{x^2-1}} + \cosh x.\log(\cosh^{-1} x)\right]$$

g) $(\tanh x)^{\cosh^{-1}3x}$

Sol^n: Let $y = (\tanh x)^{\cosh^{-1}3x}$

Taking log on both sides we get,

$$\log y = \log(\tanh x)^{\cosh^{-1}3x} = \cosh^{-1}3x . \log(\tanh x)$$

Differentiating both sides w. r. to 'x'

$$\frac{1}{y}\frac{dy}{dx} = \cosh^{-1} 3x \frac{d}{dx}\log(\text{tahh } x) + \log(\tanh x)\frac{d}{dx}\left(\cosh^{-1} 3x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cosh^{-1} 3x \frac{1}{\tanh x}.\text{sec}\,h^2 x + \log(\tanh x)\frac{1}{\sqrt{(3x)^2-1}}.3$$

$$\therefore \quad \frac{dy}{dx} = y\left[\frac{\cosh^{-1}3x}{\sinh x\cosh x} + \frac{3\log(\tanh x)}{\sqrt{(3x)^2-1}}\right]$$

$$= y\left[\frac{2\cosh^{-1}3x}{\sinh 2x} + \frac{3\log(\tanh x)}{\sqrt{(3x)^2-1}}\right]$$

$$= (\tanh x)^{\cosh^{-1}3x}\left[2\,\mathrm{cosech}2x\cosh^{-1}3x + \frac{3\log(\tanh x)}{\sqrt{9x^2-1}}\right]$$

h. $\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}$

Sol^n: Let, $y = \left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}$

Taking log on both sides we get

$$\log y = \log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx} = nx.\log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)$$

Differentiating both sides w,r to 'x' we get

$$\frac{d}{dx}(\log y) = \frac{d}{dx}\left\{nx\log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)\right\}$$

$$\Rightarrow \frac{d(\log y)}{dx}\frac{dy}{dx} = n\left\{x\frac{d}{dx}\log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right) + \log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)\times\frac{d}{dx}(x)\right\}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = n\left\{x\frac{d\left(\log\left(\sinh\frac{x}{a}\cos\frac{x}{a}\right)\right)}{d\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)}\right\}\frac{d\left(\sin\frac{x}{a} + \cosh\frac{x}{a}\right)}{d\left(\frac{x}{a}\right)}\times\frac{d\left(\frac{x}{a}\right)}{dx} + \log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right).\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = nx\left(\frac{1}{\sinh\frac{x}{a} + \cosh\frac{x}{a}}\right)\left(\cosh\frac{x}{a} + \sinh\frac{x}{a}\right)\frac{1}{a} + n\log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{nx}{a} + n\log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)\right) = n\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}\left(\frac{x}{a} + \log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)\right).$$

Hints and solutions of MCQ's

1. Differentiability of a function $\Rightarrow$ continuity of a function but converse may not be true.
2. The function $f(x) = |x|$ is continuous as such that

$$\lim_{x\to 0^-}|x| = \lim_{x\to 0^-} -x = 0,$$

$$\lim_{x\to 0^+}|x| = \lim_{x\to 0^+} x = 0 \text{ and}$$

$$\boxed{|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0\end{cases}}$$

and $f(0) = |0| = 0$ are equal

But not differentiable such that

$$Lf'(0) = \lim_{h\to 0^-}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0^-}\frac{-(-h)}{-h} = -1$$

and $$Rf'(0) = \lim_{h\to 0^+}\frac{f(0+h)-f(0)}{-h} = \lim_{h\to 0}\frac{h-0}{h} = 1$$

are not equal.

3. y = f(x) is differentiable at x = a, if $Lf'(a) = Rf'(a)$

4. $\frac{d}{dx}\sinh 2x = \frac{d\sinh 2x}{d(2x)} \times \frac{d(2x)}{dx} = \cosh 2x \,.\, 2 = 2\cosh 2x$

5. $\frac{d}{dx}\cosh\frac{x}{2} = \frac{d\cosh\frac{x}{2}}{d\left(\frac{x}{2}\right)} \times \frac{d\left(\frac{x}{2}\right)}{dx} = \frac{1}{2}\sinh\frac{x}{2}$

6. $\frac{d}{dx}e^{\tanh 3x} = \frac{de^{\tanh 3x}}{d(\tanh 3x)} \times \frac{d\tanh 3x}{d(3x)} \times \frac{d(3x)}{dx} = -3\operatorname{sech}^2 3x\, e^{\tanh 3x}$

7. $\frac{d\sinh^{-1} e^{-x}}{d\left(e^{-x}\right)} \times \frac{d(e^{-x})}{dx} = \frac{1}{\sqrt{1+(e^{-x})^2}} \,.\, -e^{-x} = -\frac{e^{-x}}{\sqrt{1+\frac{1}{e^{2x}}}} = -\frac{e^{-x}\, e^{x}}{\sqrt{1+e^{2x}}} = -\frac{1}{\sqrt{1+e^{2x}}}$

8. $\frac{dy}{dx} = \frac{d\ln\left(\sinh\frac{x}{a}\right)}{d\left(\sinh\frac{x}{a}\right)} \times \frac{d\left(\sinh\frac{x}{a}\right)}{d\left(\frac{x}{a}\right)} \times \frac{d\left(\frac{x}{a}\right)}{dx} = \frac{1}{\sinh\frac{x}{a}} \times \cosh\frac{x}{a} \times \frac{1}{a} = \frac{1}{a}\coth\frac{x}{a}$

9. $\frac{dy}{dx} = \frac{d\, e^{\cosh^{-1}\frac{x}{a}}}{d\left(\cosh^{-1}\frac{x}{a}\right)} \times \frac{d\left(\cosh^{-1}\frac{x}{a}\right)}{d\left(\frac{x}{a}\right)} \times \frac{d\left(\frac{x}{a}\right)}{dx} = e^{\cosh^{-1}\frac{x}{a}} \times -\frac{1}{\sqrt{\frac{x^2}{a^2}-1}} \times \frac{1}{a} = \frac{-e^{\cosh^{-1}\frac{x}{a}}}{\sqrt{x^2-a^2}}$

10. $\frac{dy}{dx} = \frac{d\tan^{-1}(\cosh x)}{d(\cosh x)} \times \frac{d\cosh x}{dx} = \frac{1}{1+\cosh^2 x} \times \sinh x = \frac{\sinh x}{1+\cosh^2 x}$

11. $\frac{dy}{dx} = \frac{2\coth^{-1}(\sin 2x)}{d(\sin 2x)} \times \frac{d(\sin 2x)}{dx} = \frac{1}{1-\sin^2 2x} \times 2\cos 2x = \frac{2\cos 2x}{\cos^2 2x} = 2\sec 2x$

12. $\frac{dy}{dx} = \frac{d\, e^{\tanh^{-1}x}}{d(\tanh^{-1}x)} \times \frac{d(\tanh^{-1}x)}{dx} = \frac{e^{\tanh^{-1}x}}{1-x^2}$