Exercise - 7.2

General section

1. If $\cos 45^\circ = \frac{1}{\sqrt{2}}$, prove the followings:

a. $\sin 22\frac{1}{2}^\circ = \frac{1}{2}\sqrt{2-\sqrt{2}}$

we have,

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

or, ~~$\cos 45^\circ$~~

or, $\cos 2\cdot\left(22\frac{1}{2}\right) = \frac{1}{\sqrt{2}}$

or, $1 - 2\sin^2 22\frac{1}{2} = \frac{1}{\sqrt{2}}$

or, $1 - \frac{1}{\sqrt{2}} = 2\sin^2 22\cdot\frac{1}{2}$

or, $\frac{\sqrt{2}-1}{\sqrt{2}} = 2\sin^2 22\cdot\frac{1}{2}$

or, $\cancel{\frac{\sqrt{2}-1}{2\sqrt{2}}}$ $\frac{\sqrt{2}-1}{2\sqrt{2}} = \sin^2 22\frac{1}{2}$

or, $\sin^2 22\frac{1}{2} = \frac{\sqrt{2}(\sqrt{2}-1)}{\sqrt{2}\cdot 2\sqrt{2}}$

or, $\sin^2 22\frac{1}{2} = \frac{2-\sqrt{2}}{4}$

or, $\left(\sin 22\frac{1}{2}\right)^2 = \left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^2$

or, $\sin 22\frac{1}{2} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

$\therefore$ LHS = RHS proved

b. $\cos 22\frac{1}{2}^\circ = \frac{1}{2}\sqrt{2+\sqrt{2}}$

Soln,

we have, $\cos 45^\circ = \frac{1}{\sqrt{2}}$

or, $\cos 2.\left(22\frac{1}{2}\right) = \frac{1}{\sqrt{2}}$

or, $2\cos^2 22.\frac{1}{2} - 1 = \frac{1}{\sqrt{2}}$

or, $2\cos^2 22.\frac{1}{2} = 1 + \frac{1}{\sqrt{2}}$

or, $\cos^2 22.\frac{1}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$

or, $\cos^2 22.\frac{1}{2} = \frac{\sqrt{2}.(\sqrt{2}+1)}{\sqrt{2}.2\sqrt{2}}$

or, $\cos^2 22.\frac{1}{2} = \frac{2+\sqrt{2}}{4}$

or, $\left(\cos 22.\frac{1}{2}\right)^2 = \left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)^2$

or, $\cos 22.\frac{1}{2} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\therefore$ RHS proved,

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c. $\tan 22\frac{1}{2}^\circ = \sqrt{3-2\sqrt{2}}$

Soln,

we have,

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

or, $\cos 2.22\frac{1}{2} = \frac{1}{\sqrt{2}}$

$$\text{or, } \frac{1-\tan^2 22\tfrac{1}{2}}{1+\tan^2 22\tfrac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\text{or, } \sqrt{2} - \sqrt{2}\tan^2 22\tfrac{1}{2} = 1 + \tan^2 22\tfrac{1}{2}$$

$$\text{or, } \sqrt{2} - 1 = \sqrt{2}\tan^2 22\tfrac{1}{2} + \tan^2 22\tfrac{1}{2}$$

$$\text{or, } \sqrt{2} - 1 = \tan^2 22\tfrac{1}{2}(\sqrt{2}+1)$$

$$\text{or } \frac{\sqrt{2}-1}{(\sqrt{2}+1)} = \tan^2 22\tfrac{1}{2}$$

$$\text{or, } \tan^2 22\tfrac{1}{2} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$\text{or, } \tan^2 22\tfrac{1}{2} = \frac{(\sqrt{2})^2 - 2\sqrt{2} + 1}{2-1}$$

$$\text{or, } \tan^2 22\tfrac{1}{2} = 3 - 2\sqrt{2}$$

$$\text{or, } \left(\tan 22\tfrac{1}{2}\right)^2 = \left(\sqrt{3-2\sqrt{2}}\right)^2$$

$$\therefore \tan 22\tfrac{1}{2} = \sqrt{3-2\sqrt{2}}$$

Proved//

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2. If $\cos 30^\circ = \frac{\sqrt{3}}{2}$, show that

a. $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Soln,

we have, $\cos 30^\circ = \frac{\sqrt{3}}{2}$

Now,

$\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

LHS

$\sin 15^\circ$

$= \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3}}{2\sqrt{2}} - \dfrac{1}{2\sqrt{2}}$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

RHS proved

b $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

soln

we have, $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$

Now, $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

LHS

$\cos 15^\circ$

$= \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ$

$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2}$

$= \dfrac{\sqrt{3}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ RHS proved

c. $\tan 15^\circ = 2-\sqrt{3}$

Soln,

We have, ~~$\cos 45^\circ = \frac{1}{\sqrt{2}}$~~ $\cos 30^\circ = \frac{\sqrt{3}}{2}$

or, $\cos 2\cdot 15^\circ = \frac{\sqrt{3}}{2}$

or $\frac{1-\tan^2 15}{1+\tan^2 15} = \frac{\sqrt{3}}{2}$

or $2 - 2\tan^2 15^\circ = \sqrt{3} + \sqrt{3}\tan^2 15^\circ$

or, $2-\sqrt{3} = 2\tan^2 15^\circ + \sqrt{3}\tan^2 15^\circ$

or $2-\sqrt{3} = \tan^2 15^\circ (2+\sqrt{3})$

or $\tan^2 15^\circ = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

or $\tan^2 15^\circ = \cancel{\frac{2^2 - 2\sqrt{3} + (\sqrt{3})^2}{4-3}} \frac{(2-\sqrt{3})^2}{4-3}$

or $\tan^2 15^\circ = \cancel{4 - 2\sqrt{3} + 3} \frac{(2-\sqrt{3})^2}{1}$

or $\tan 15^\circ = 2-\sqrt{3}$

$\therefore \tan 15^\circ = 2-\sqrt{3}$

Proved

3. If $\cos 330^\circ = \frac{\sqrt{3}}{2}$ show that :

a. $\sin 165^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Soln,

We have, $\cos 330^\circ = \frac{\sqrt{3}}{2}$

~~or, $\sin\theta = 1 - 2\sin^2\theta$~~

~~or~~

Also,

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\text{or, } \sin\theta = \sqrt{\frac{1-\cos^2\theta}{2}}$$

$$\text{So, } \sin 165 = \sqrt{\frac{1-\cos 330^\circ}{2}}$$

$$= \frac{\frac{1-\sqrt{3}}{2}}{2}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4} \times \frac{2}{2}}$$

$$= \sqrt{\frac{4-2\sqrt{3}}{8}}$$

$$= \sqrt{\frac{3-2\sqrt{3}+1}{8}}$$

$$= \sqrt{\frac{(\sqrt{3})^2-2\sqrt{3}.1+1^2}{8}}$$

$$= \sqrt{\frac{(\sqrt{3}-1)^2}{8}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Ans proved,

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b. $\cos 165 = -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

Soln,

we have, ~~cos 165~~ $\cos 330^\circ = \frac{\sqrt{3}}{2}$

$$\text{or, } \cos 2.165 = \frac{\sqrt{3}}{2}$$

or, $2\cos^2 165 - 1 = \frac{\sqrt{3}}{2}$

or, $2\cos^2 165 = 1 + \frac{\sqrt{3}}{2}$

or, $2\cos^2 165 = \frac{2+\sqrt{3}}{2}$

or, $\cos^2 165 = \frac{2+\sqrt{3}}{4} \times \frac{2}{2}$

or, $\cos^2 165 = \frac{4+2\sqrt{3}}{8}$

or, $\cos^2 165 = \frac{3+2\sqrt{3}+1}{8}$

or, $\cos^2 165 = \frac{(\sqrt{3})^2 + 2\cdot\sqrt{3}\cdot 1 + 1^2}{8}$

or, $(\cos 165)^2 = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2$

or, $\cos 165 = -+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

$\therefore \cos 165^\circ = -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

Proved.

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c. $\tan 165^\circ = \sqrt{3} - 2$

soln,

we have, $\cos 330^\circ = \frac{\sqrt{3}}{2}$

or, $\cos 2\cdot 165^\circ = \frac{\sqrt{3}}{2}$

or, $\frac{1-\tan^2 165}{1+\tan^2 165} = \frac{\sqrt{3}}{2}$

or, $2-2\,Tan^2 165 = \sqrt{3}+\sqrt{3}\,Tan^2 165$

or, $2-\sqrt{3} = 2Tan^2 165 + \sqrt{3}\,Tan^2 165$

or, $2-\sqrt{3} = Tan^2 165\,(2+\sqrt{3})$

or, $\frac{2-\sqrt{3}}{2+\sqrt{3}} = Tan^2 165$

or, $Tan^2 165 = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

or, $Tan^2 165 = -\frac{(\sqrt{3}-2)^2}{2-3}$

or, $(Tan 165)^{\cancel{2}} = \cancel{+}\frac{(\sqrt{3}-2)^{\cancel{2}}}{\cancel{-1}}$

or, $Tan 165 = \sqrt{3}-2$

$\therefore Tan 165 = \sqrt{3}-2$

Proved.

4a. If $\sin\frac{a}{3} = \frac{3}{5}$, Prove that $\sin a = \frac{117}{125}$

Soln,

We know,

$\sin a = \sin 3\cdot\frac{a}{3}$

$= 3\sin\frac{a}{3} - 4\sin^3\frac{a}{3}$

$= 3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3$

$\cancel{or}\ \frac{9}{5} - 4\times\frac{27}{125}$

$= \frac{9}{5} - \frac{108}{125}$

$= \frac{9\times25-108}{125}$

$= \frac{225 - 108}{225}$

$= \frac{117}{225}$

RHS proved.

5. If $\tan\frac{A}{3} = \frac{1}{5}$, prove that $\tan A = \frac{37}{55}$

Soln.,

we know,

$$\tan A = \tan 3 \cdot \frac{A}{3}$$

$$= \frac{3\tan\frac{A}{3} - \tan^3\frac{A}{3}}{1 - 3\tan^2\frac{A}{3}}$$

$$= \frac{3 \times \frac{1}{5} - \left(\frac{1}{5}\right)^3}{1 - 3\left(\frac{1}{5}\right)^2}$$

$$= \frac{\frac{3}{5} - \frac{1}{125}}{1 - \frac{3}{25}}$$

$$= \frac{\frac{75 - 1}{125}}{\frac{25 - 3}{25}}$$

$$= \frac{\cancel{74}^{37}}{\cancel{125}_{\cancel{25}_{5}}} \times \frac{\cancel{25}^{\cancel{5}\,1}}{\cancel{22}_{11}} = \frac{37}{55}$$

6. If $\cos\frac{\theta}{3} = \frac{1}{2}\left(p + \frac{1}{p}\right)$, prove that $\cos\theta = \frac{1}{2}\left(p^3 + \frac{1}{p^3}\right)$

Soln:

we know,

$\cos\theta = \cos 3\frac{\theta}{3}$

$= 4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}$

$= 4\left\{\frac{1}{2}\left(p + \frac{1}{p}\right)\right\}^3 - 3 \times \frac{1}{2}\left(p + \frac{1}{p}\right)$

$= \frac{4}{8}\left\{p^3 + 3p^{\cancel{2}}.\frac{1}{\cancel{p}} + 3\cancel{p}.\frac{1}{p^{\cancel{2}}} + \frac{1}{p^3}\right\} - \frac{3}{2}\left(p + \frac{1}{p}\right)$

$= \frac{\cancel{4}2}{\cancel{8}4}\left\{p^3 + \frac{1}{p^3} + 3\left(p + \frac{1}{p}\right)\right\} - \frac{3}{2}\left(p + \frac{1}{p}\right)$

$= \frac{1}{2}\left(p^3 + \frac{1}{p^3}\right) + \cancel{\frac{3}{2}\left(p + \frac{1}{p}\right)} - \cancel{\frac{3}{2}\left(p + \frac{1}{p}\right)}$

$= \frac{1}{2}\left(p^3 + \frac{1}{p^3}\right)$

Proved.

2. If $\cos 30° = \frac{\sqrt{3}}{2}$, show that:

a. $\sin 15° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

We have, $\cos 30° = \frac{\sqrt{3}}{2}$

or, $\cos 2.15° = \frac{\sqrt{3}}{2}$

or, $1 - 2\sin^2 15 = \frac{\sqrt{3}}{2}$

or, $1 - \frac{\sqrt{3}}{2} = 2\sin^2 15$

or, $\frac{2-\sqrt{3}}{2} = 2\sin^2 15$

or, $\sin^2 15 = \frac{(2-\sqrt{3})}{4} \times \frac{2}{2}$

or, $\sin^2 15 = \frac{4-2\sqrt{3}}{8}$

or, $\sin^2 15 = \frac{3-2\sqrt{3}+1}{8}$

or, $\sin^2 15 = \frac{(\sqrt{3})^2 - 2\sqrt{3}.1 + 1^2}{8}$

or, $\sin^2 15 = \left(\frac{\sqrt{3}-1}{\cancel{4}\ 8\ \cancel{8}}\right)^2$

or, $\sin 15 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\therefore \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Proved

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b. $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

we have, $\cos 30^\circ = \frac{\sqrt{3}}{2}$

or, $\cos 2.15 = \frac{\sqrt{3}}{2}$

or, $2\cos^2 15 - 1 = \frac{\sqrt{3}}{2}$

or, $2\cos^2 15 = 1 + \frac{\sqrt{3}}{2}$

or, $2\cos^2 15 = \frac{2+\sqrt{3}}{2}$

$$\text{or, } \cos^2 15^\circ = \frac{(2+\sqrt{3})}{4} \times \frac{2}{2}$$

$$\text{or, } \cos^2 15^\circ = \frac{4+2\sqrt{3}}{8}$$

$$\text{or, } \cos^2 15^\circ = \frac{3+2\sqrt{3}+1}{2\sqrt{2}}$$

$$\text{or, } \cos^2 15^\circ = \frac{(\sqrt{3})^2+2\sqrt{3}\cdot 1+1^2}{2\sqrt{2}}$$

$$\text{or, } \cos^2 15 = \frac{(\sqrt{3}+1)^2}{2\sqrt{2}}$$

$$\text{or, } \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\therefore \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Creative section

7. Prove the following:

a. $$\sin A = \frac{2 \tan \frac{A}{2}}{1 + \tan \frac{A}{2}}$$

RHS

$$\frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$$

$$= \frac{2 \frac{\sin A/2}{\cos A/2}}{1 + \frac{\sin^2 A/2}{\cos^2 A/2}}$$

$$= \frac{\dfrac{2\,\cancel{\cos}\,\sin A/2}{\cos A/2}}{\dfrac{\cos^2 A/2 + \sin^2 A/2}{\cos^2 A/2}}$$

$$= \frac{\dfrac{2\sin A/2}{\cancel{\cos A/2}}}{\dfrac{1}{\cos A/2}}$$

$$= 2\sin A/2 \cdot \cos A/2$$

$$= \sin A$$

LHS proved//

b. $$\frac{1 - \tan^2 A/2}{1 + \tan^2 A/2} = \cos A$$

LHS

$$\frac{1 - \tan^2 A/2}{1 + \tan^2 A/2}$$

$$= \frac{1 - \dfrac{\sin^2 A/2}{\cos^2 A/2}}{1 + \dfrac{\sin^2 A/2}{\cos^2 A/2}}$$

$$= \frac{\dfrac{\cos^2 A/2 - \sin^2 A/2}{\cancel{\cos^2 A/2}}}{\dfrac{\cos^2 A/2 + \sin^2 A/2}{\cancel{\cos^2 A/2}}}$$

$$= \frac{\cos^2 A/2 - \sin^2 A/2}{1}$$

$$= \cos A$$

RHS proved//

c. $\dfrac{1 + \cos a}{\sin a} = \cot \dfrac{a}{2}$

LHS

$\dfrac{1 + \cos a}{\sin a}$

$= \dfrac{\cancel{1} + 2\cos^2 A/2 - \cancel{1}}{2\sin A/2 \cdot \cos A/2}$

$= \dfrac{\cancel{2}\cos^{\cancel{2}} A/2}{\cancel{2}\sin A/2 \cdot \cancel{\cos A/2}}$

$= \dfrac{\cos A/2}{\sin A/2}$

$= \cot \dfrac{a}{2}$

RHS proved

d. $\dfrac{1 + \sec a}{\tan \alpha} = \cancel{\dfrac{\cot \alpha}{2}} \quad \cot \dfrac{\alpha}{2}$

LHS

$\dfrac{1 + \sec \alpha}{\tan \alpha}$

$= \dfrac{1 + \dfrac{1}{\cos a}}{\dfrac{\sin a}{\cos a}}$

$= \dfrac{\dfrac{\cos a + 1}{\cancel{\cos \alpha}}}{\dfrac{\sin \alpha}{\cancel{\cos a}}}$

$= \dfrac{\cancel{2}\cos^{\cancel{2}} a/2 - \cancel{1} + \cancel{1}}{\cancel{2}\sin a/2 \cdot \cancel{\cos a/2}} = \cot \dfrac{a}{2}$

RHS proved

e. $$\frac{\sin^3\frac{\theta}{2} + \cos^3\frac{\theta}{2}}{\sin\frac{\theta}{2} + \cos\frac{\theta}{2}} = 1 - \frac{1}{2}\sin\theta$$

LHS

$$\frac{\sin^3\frac{\theta}{2} + \cos^3\frac{\theta}{2}}{\sin\frac{\theta}{2} + \cos\frac{\theta}{2}}$$

$$= \frac{\cancel{\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)}\left(\sin^2\frac{\theta}{2} - \sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right)}{\cancel{\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)}}$$

$$= \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} - \sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$$

$$= 1 - \sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$$

$$= 1 - \frac{2}{2}\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$$

$$= 1 - \frac{\sin\theta}{2}$$

$$= 1 - \frac{1}{2}\sin\theta$$

RHS proved.

f. $\dfrac{1+\sin\theta}{\cos\theta} = \dfrac{1+\tan\theta/2}{1-\tan\theta/2}$

RHS

$\dfrac{1+\tan\theta/2}{1-\tan\theta/2}$

$= \dfrac{1+\dfrac{\sin\theta/2}{\cos\theta/2}}{1-\cancel{\tan\theta/2}\dfrac{\sin\theta/2}{\cos\theta/2}}$

$= \dfrac{\dfrac{\cos\theta/2+\sin\theta/2}{\cancel{\cos\theta/2}}}{\dfrac{\cos\theta/2-\sin\theta/2}{\cancel{\cos\theta/2}}}$

$= \dfrac{\cos\theta/2+\sin\theta/2}{\cos\theta/2-\sin\theta/2} \times \dfrac{\cos\theta/2+\sin\theta/2}{\cos\theta/2+\sin\theta/2}$

$= \dfrac{(\cos\theta/2+\sin\theta/2)^2}{\cos^2\theta/2-\sin^2\theta/2}$

$= \dfrac{\cos^2\theta/2+2\cos\theta/2.\sin\theta/2+\sin^2\theta/2}{\cos\theta}$

$= \dfrac{1+2\cos\theta/2\cdot\sin\theta/2}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$

LHS proved//

g. $\dfrac{\sin\alpha+\sin\alpha/2}{1+\cos\alpha/2+\cos\alpha} = \tan\dfrac{\alpha}{2}$

LHS

$\dfrac{\sin\alpha+\sin\alpha/2}{1+\cos\alpha/2+\cos\alpha}$

$= \dfrac{2\sin a/2 \cdot \cos a/2 + \sin a/2}{\cancel{1} + \cos a/2 + 2\cos^2 a/2 - \cancel{1}}$

$= \dfrac{\sin a/2 \,(2\cos a/2 + 1)}{\cos a/2\,(1 + 2\cos a/2)}$

$= \tan a/2 \,\dfrac{\cancel{(2\cos a/2 + 1)}}{\cancel{(2\cos a/2 + 1)}}$

$= \tan \dfrac{a}{2}$

RHS proved.

h. $\cos^4 \theta/2 - \sin^4 \theta/2 = \cos\theta$

LHS

$\cos^4 \theta/2 - \sin^4 \theta/2$

$(\cos^2 \theta/2)^2 - (\sin^2 \theta/2)^2$

$= (\cos^2 \theta/2 + \sin^2 \theta/2)(\cos^2 \theta/2 - \sin^2 \theta/2)$

$= 1\,(\cos^2 \theta/2 - \sin^2 \theta/2)$

$= \cos\theta$

RHS proved.

i. $\dfrac{\sin 2a}{1 + \cos 2a} \cdot \dfrac{\cos a}{1 + \cos a} = \tan \dfrac{a}{2}$

LHS

$\dfrac{\sin 2\alpha}{1 + \cos 2\alpha} \cdot \dfrac{\cos a}{1 + \cos a}$

$= \dfrac{2\sin a \cdot \cancel{\cos a}}{\cancel{1} + 2\cos^2 a - \cancel{1}} \cdot \dfrac{\cos a}{\cancel{1} + 2\cos^2 a/2 - \cancel{1}}$

$= \dfrac{\cancel{2}\sin\alpha}{\cancel{2}\cancel{\cos\alpha}} \cdot \dfrac{\cancel{\cos a}}{2\cos^2 a}$

$= \dfrac{\cancel{2}\sin a/2 \cdot \cancel{\cos a/2}}{\cancel{2}\cos^2 a/2}$

$= \dfrac{\cancel{\cos a/2}}{5}$

$= \frac{\sin \alpha/2}{\cos \alpha/2}$

$= \tan \frac{\alpha}{2}$

RHS proved//

8. Prove the following:

a. $\tan\left(45^\circ + \frac{A}{2}\right) = \sec A + \tan A$

LHS

$\tan\left(45^\circ + A/2\right)$

$= \frac{\tan 45^\circ + \tan A/2}{1 - \tan 45^\circ \cdot \tan A/2}$

$= \frac{1 + \tan A/2}{1 - \tan A/2}$

$= \frac{1 + \frac{\sin A/2}{\cos A/2}}{1 - \frac{\sin A/2}{\cos A/2}}$

$= \frac{\frac{\cos A/2 + \sin A/2}{\cos A/2}}{\frac{\cos A/2 - \sin A/2}{\cos A/2}}$

$= \frac{\cos A/2 + \sin A/2}{\cos A/2 - \sin A/2} \times \frac{\cos A/2 + \sin A/2}{\cos A/2 + \sin A/2}$

$= \frac{(\cos A/2 + \sin A/2)^2}{\cos^2 A/2 - \sin^2 A/2}$

$= \frac{\cos^2 A/2 + \cancel{2\sin A} 2\cos A/2 . \sin A/2 + \sin^2 A/2}{\cos A}$

$$= \frac{1 + 2 \cancel{\sin A} \cos A/2 \cdot \sin A/2}{\cos A}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \sec A + \tan A$$

RHS proved.

b. $\tan\left(45^\circ - \frac{A}{2}\right) = \sqrt{\frac{1 - \sin A}{1 + \cos A}}$

LHS

$\tan(45^\circ - A/2)$

$$= \frac{\tan 45^\circ - \tan A/2}{1 + \tan 45^\circ \cdot \tan A/2}$$

$$= \frac{1 - \frac{\sin A/2}{\cos A/2}}{1 + \frac{\sin A/2}{\cos A/2}}$$

$$= \frac{\frac{\cos A/2 - \sin A/2}{\cancel{\cos A/2}}}{\frac{\cos A/2 + \sin A/2}{\cancel{\cos A/2}}}$$

$$= \frac{\cos A/2 - \sin A/2}{\cos A/2 + \sin A/2} \times \frac{\cos A/2 - \sin A/2}{\cos A/2 - \sin A/2}$$

$$= \frac{\cos^2 A/2 - 2\cos A/2 \cdot \sin A/2 + \sin^2 A/2}{\cos^2 A/2 - \sin^2 A/2}$$

$$= \frac{1 - \sin A}{\cos A}$$

$$= \sec A - \tan A$$

RHS

$$\sqrt{\frac{1-\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{1-\sin A}{1+\sin A} \times \frac{1-\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\left(\frac{1-\sin A}{\cos A}\right)^2}$$

$$= \frac{1-\sin A}{\cos A}$$

$$= \sec A - \tan A$$

$\therefore$ LHS = RHS proved.

c. $1 - 2\sin^2\left(45^\circ - \frac{A}{2}\right) = \sin A$

LHS

$$1 - 2\sin^2\left(45^\circ - \frac{A}{2}\right)$$

$$= \cos 2\left(45^\circ - \frac{A}{2}\right)$$

$$= \cos(90^\circ - A)$$

$$= \sin A$$

RHS proved.

d. $\tan\left(45^\circ + \frac{\theta}{2}\right) + \tan\left(45^\circ - \frac{\theta}{2}\right) = 2\sec\theta$

LHS

$$\tan\left(45^\circ + \frac{\theta}{2}\right) + \tan\left(45^\circ - \frac{\theta}{2}\right)$$

$= \frac{\tan 45^\circ + \tan \theta/2}{1 - \tan 45^\circ \cdot \tan \theta/2} + \frac{\tan 45^\circ - \tan \theta/2}{1 + \tan 45^\circ \cdot \tan \theta/2}$

$= \frac{1 + \frac{\sin \theta/2}{\cos \theta/2}}{1 - \frac{\sin \theta/2}{\cos \theta/2}} + \frac{1 - \frac{\sin \theta/2}{\cos \theta/2}}{1 + \frac{\sin \theta/2}{\cos \theta/2}}$

$= \frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 - \sin \theta/2} + \frac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2 + \sin \theta/2}$

$= \frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 - \sin \theta/2} \times \frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 + \sin \theta/2} + \frac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2 + \sin \theta/2} \times \frac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2 - \sin \theta/2}$

$= \frac{\cos^2 \theta/2 + 2\cos \theta/2 \cdot \cos \theta/2 + \sin^2 \theta/2}{\cos^2 \theta/2 - \sin^2 \theta} + \frac{\cos^2 \theta/2 - 2\sin^2 \theta \cdot \cancel{\sin^2\theta} \cos^2\theta/2 + \sin}{\cos^2 \theta/2 - \sin^2 \theta}$

$= \frac{1 + \sin A}{\cos A} + \frac{1 - \sin A}{\cos A}$

$= \frac{1 + \cancel{\sin A} + 1 - \cancel{\sin A}}{\cos A}$

$= 2 \sec A$

RHS proved.

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e. $(\cos A - \cos B)^2 + (\sin A - \sin B)^2 = 4 \sin^2 \left(\frac{A - B}{2}\right)$

LHS

$(\cos A - \cos B)^2 + (\sin A - \sin B)^2$

$= \cos^2 A - 2\cos A \cdot \cos B + \cos^2 B + \sin^2 A - 2\sin A \cdot \sin B + \sin^2 B$

$= 1 + 1 - 2\cos A \cos B \cancel{+ \sin} - 2\sin A \cdot \sin B$

$= 2 - 2(\cos A \cdot \cos B + \sin A \cdot \sin B)$

$= 2 - 2\cos(A - B)$

$= 2 - 2\cos 2\left(\frac{A - B}{2}\right)$

$= 2 - 2\left\{1 - 2\sin^2\left(\frac{A - B}{2}\right)\right\}$

$= 2 - 2 + 4\sin^2\left(\frac{A - B}{2}\right)$

$= 4\sin^2\left(\frac{A-B}{2}\right)$

RHS proved,,

f. $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2\left(\frac{A-B}{2}\right)$

LHS

$(\cos A + \cos B)^2 + (\sin A - \sin B)^2$

$= \cos^2 A + 2\cos A.\cos B + \cos^2 B + \sin^2 A + 2\sin A.\sin B + \sin^2 B$

$= 1 + 1 + 2\cos A\cdot\cos B + 2\sin A.\sin B$

$= 2 + 2(\cos A.\cos B + \sin A.\sin B)$

$= 2 + 2\cos(A-B)$

$= 2 + 2\cos 2\left(\frac{A-B}{2}\right)$

$= 2 + 2\left\{2\cos^2\left(\frac{A-B}{2}\right) - 1\right\}$

$= \cancel{2} + 4\cos^2\left(\frac{A-B}{2}\right) - \cancel{2}$

$= 4\cos^2\left(\frac{A-B}{2}\right)$

RHS proved,,

g. Find the value of $\sin 18°$. Hence find ~~tos~~ $\cos 18°$, $\cos 36°$, $\sin 36°$ $\sin 72°$, $\cos 72°$, $\sin 54°$ and $\cos 54°$.

Solution:

Let $\theta = \cancel{18f}\ 18°$

So, $5\theta = 90°$

$\therefore 2\theta + 3\theta = \cancel{90}\ 90°$

or, $2\theta = 90° - 3\theta$

Taking sin on both sides, we get

$\sin 2\theta = \sin(90° - 3\theta) = \cancel{\sin}\ \cos 3\theta$

or, $2\sin\theta.\cos\theta = 4\cos^3\theta - 3\cos\theta$

$$\text{or, } 2\sin\theta\cdot\cancel{\cos\theta} = \cancel{\cos\theta}\,(4\cos^2\theta - 3)$$

$$\text{or, } 2\sin\theta = 4\cos^2\theta - 3$$

$$\text{or, } 2\sin\theta = 4(1-\sin^2\theta) - 3$$

$$\text{or, } 2\sin\theta = 4 - 4\sin^2\theta - 3$$

$$\text{or, } 2\sin\theta + 4\sin^2\theta - 1 = 0$$

$$\text{or, } 4\sin^2\theta + 2\sin\theta - 1 = 0$$

Now,

$$\sin\theta = \frac{-2 \pm \sqrt{2^2 - 4\cdot4(-1)}}{2\cdot4}$$

$$\text{or, } \sin\theta = \frac{-2 \pm \sqrt{20}}{8}$$

$$\text{or, } \sin\theta = \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\text{or, } \sin\theta = \frac{\cancel{2}(\sqrt{5}-1)}{\cancel{8}\,4}$$

$$\therefore \sin\theta = \frac{\sqrt{5}-1}{4}$$

Again,

$$\cos 18^\circ = \sqrt{1-\sin^2 18^\circ}$$

$$= \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= \sqrt{1 - \frac{[(\sqrt{5})^2 - 2\sqrt{5} + 1]}{16}}$$

$$= \sqrt{1 - \frac{(5 - 2\sqrt{5} + 1)}{16}}$$

$$= \sqrt{\frac{16 - 5 + 2\sqrt{5} - 1}{16}}$$

$$= \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

Again,

~~Sin 36° = Sin18° x2~~

$\cos 36° = ?$

We know,

$\cos 2(18°) = \cos 36°$

Now,

$\cos 2(18°) = 1 - 2\sin^2 18°$

$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$

$= 1 - \cancel{2}\left(\frac{5 - 2\sqrt{5} + 1}{\cancel{16}\,8}\right)$

$= \cancel{\frac{8 - 5 + 2\sqrt{5} - 1}{8}}$

$= \frac{8 - 6 + 2\sqrt{5}}{8}$

$= \frac{2 + 2\sqrt{5}}{8}$

$= \frac{\cancel{2}(1+\sqrt{5})}{\cancel{8}\,4}$

$= \frac{\sqrt{5}+1}{4}$

Again,

$\cancel{\cos}\ \sin 36° = \sqrt{1 - \cos^2 36°}$

$= \sqrt{1 - \left(\frac{\sqrt{5}+1}{4}\right)^2}$

$= \sqrt{\frac{16 - (5 + 2\sqrt{5} + 1)}{16}}$

$= \sqrt{\frac{16 - 6 - 2\sqrt{5}}{16}}$

$= \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

Again,

$$\sin 72^\circ = \cancel{\sin 2(36^\circ)}.\ \sin(90^\circ - 18^\circ)$$
$$= \cos 18^\circ$$
$$= \sqrt{\frac{10+2\sqrt{5}}{4}}$$

Also,

$$\cos 72^\circ = \cos(90^\circ - 18^\circ)$$
$$= \sin 18^\circ$$
$$= \frac{\sqrt{5}-1}{4}$$

Again,

$$\sin 54^\circ = \sin(90^\circ - 36^\circ)$$
$$= \cos 36^\circ$$
$$= \frac{\sqrt{5}+1}{4}$$

Also,

$$\cos 54^\circ = \cos(90^\circ - 36^\circ)$$
$$= \sin 36^\circ$$
$$= \sqrt{\frac{10-2\sqrt{5}}{4}}$$

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10. Show that $\tan 7\frac{1}{2} = \sqrt{6} - \sqrt{3} + 2\sqrt{2} - 2$. (Use $\tan\frac{A}{2} = \frac{1-\cos}{\sin}$

Soln,

Here,

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$$

$$\tan\frac{30^\circ}{2} = \frac{1-\cos 30^\circ}{\sin 30^\circ}$$

$$\text{or, } \tan 15^\circ = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$\text{or, } \tan 15' = \frac{\frac{2-\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}}$$

$$\text{or, } \tan 2\left(\frac{15}{2}\right) = \frac{2-\sqrt{3}}{1}$$

$$\text{or, } \frac{2\tan 15/2}{1-\tan^2 15/2} = \frac{2-\sqrt{3}}{1}$$

$$\text{or, } 2\tan 15/2 = (2-\sqrt{3})(1-\tan^2 15/2)$$

$$\text{or, } 2\tan 15/2 = 2-\sqrt{3} - 2-\sqrt{3}\tan^2 15/2$$

$$\text{or, } (2-\sqrt{3})\tan^2 15/2 + 2\tan 15/2 - (2-\sqrt{3}) = 0$$

we know,

$$\tan 15/2 = \frac{-2 \pm \sqrt{(2)^2 - 4(2-\sqrt{3})\{-(2-\sqrt{3})\}}}{2(2-\sqrt{3})}$$

$$= -2 \pm \sqrt{4+4\{4-4\sqrt{3}+3\}}$$

$$= \frac{-2 \pm \sqrt{4+16\cancel{4}-16\sqrt{3}+12}}{\cancel{2\sqrt{3}}\ 2(2-\sqrt{3})}$$

$$= \frac{-2 \pm \sqrt{32-16\sqrt{3}}}{2(2-\sqrt{3})}$$

$$= \frac{-2 \pm 4\sqrt{2-\sqrt{3}}}{2(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{2+\sqrt{3}}$$

$$= \frac{-2(2+\sqrt{3}) \pm 4\sqrt{2-\sqrt{3}}\,(2+\sqrt{3})}{2(4-3)}$$

$$= \frac{\cancel{2}\{-(2+\sqrt{3}) \pm 2\sqrt{2-\sqrt{3}}\,(2+\sqrt{3})\}}{\cancel{2}}$$

$$= -2-\sqrt{3} \pm 2\sqrt{2-\sqrt{3}}\,(2+\sqrt{3})$$

$$= \cancel{-2-\sqrt{3} \pm 2\sqrt{2^2-(\sqrt{3})^2}}$$

$$= -2-\sqrt{3} \pm 2\sqrt{2-\sqrt{3}} \times \sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{3}}$$

$$= -2-\sqrt{3} \pm 2\sqrt{2^2-(\sqrt{3})^2}\cdot\sqrt{2+\sqrt{3}}$$

$$= -2-\sqrt{3} \pm 2\sqrt{2+\sqrt{3}}$$

$$= -2-\sqrt{3} \pm \sqrt{2}\sqrt{4+2\sqrt{3}}$$

$= -2-\sqrt{3} \pm \sqrt{2}\sqrt{(\sqrt{3})^2+2\sqrt{3}+1}$

$= -2-\sqrt{3} \pm \sqrt{2}\sqrt{(\sqrt{3}+1)^2}$

$= -2-\sqrt{3} \pm \sqrt{2}(\sqrt{3}+1)$

$= -2-\sqrt{3} \pm \sqrt{6}+\sqrt{2}$

proved,,

11. $\sin 6^\circ \cdot \sin 42^\circ \cdot \sin 66^\circ \cdot \sin 78^\circ = 1/16$

LHS

$\sin 6^\circ \cdot \sin 42^\circ \cdot \sin 66^\circ \cdot \sin 78^\circ$

$\cancel{2\sin}\ \frac{2\cos 6^\circ \cdot \sin 6^\circ \cdot \sin 42^\circ \cdot \sin 66^\circ \cdot \sin(90^\circ-12)}{2\cos 6^\circ}$

$= \cancel{\sin 24^\circ}\ \frac{\sin 12^\circ \cdot \cancel{\sin 6^\circ} \cdot \sin 42^\circ \cdot \sin 66^\circ \cdot \cancel{\sin}\cos 12^\circ}{2\cos 6^\circ}$

$= \frac{2\cos 12 \cdot \sin 12^\circ \cdot \cancel{\sin 6^\circ} \cdot \sin 42^\circ \cdot \sin(90^\circ-24)}{2\times 2\cos 6^\circ}$

$= \sin 24^\circ \cdot \cancel{\sin 6^\circ}\ \sin 42^\circ \cdot \cancel{\sin 24^\circ}\ \cos 24^\circ$

$= \frac{2\sin 24^\circ \cdot \cos 24^\circ \cdot \sin 42^\circ}{2\times 2\times 2\cos 6^\circ}$

$= \frac{2\sin 48^\circ \cdot \sin(90^\circ-48^\circ)}{16\cos 6^\circ}$

$= \frac{2\sin 48^\circ \cdot \cos 48^\circ}{16\cos 6^\circ}$

$= \frac{\sin 96^\circ}{16\cos 6^\circ}$

$= \frac{\sin(90^\circ+6^\circ)}{16\cos 6}$

$= \frac{\cancel{\cos 6^\circ}}{16\cancel{\cos 6^\circ}}$

$= \frac{1}{16}$

RHS proved,,