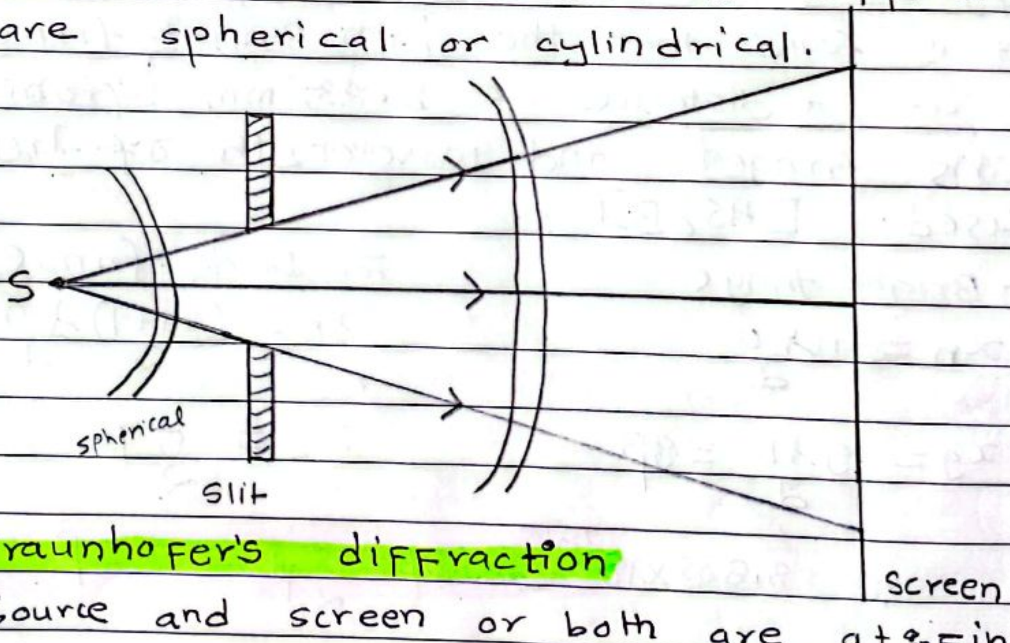
Diffraction

The phenomena of bending of light wave around the edge of an obstacle is called diffraction.
There are two types of diffraction.

① Freshell's diffraction
② Fraunhofer's diffraction.

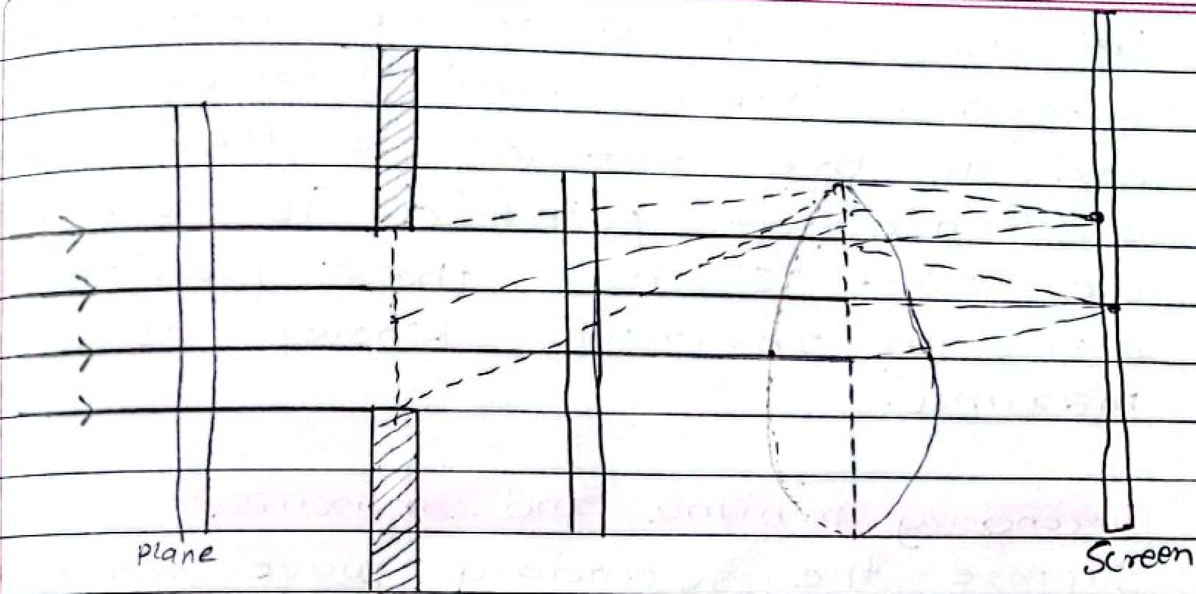
① Fresnell's diffraction

- The type of diffraction in which source and screen are at finite distance from obstacle is called Fresnell's diffraction.
- Generally, lens are not used to make light beam parallel.
- Incident wave as well as diffracted wave are spherical or cylindrical.

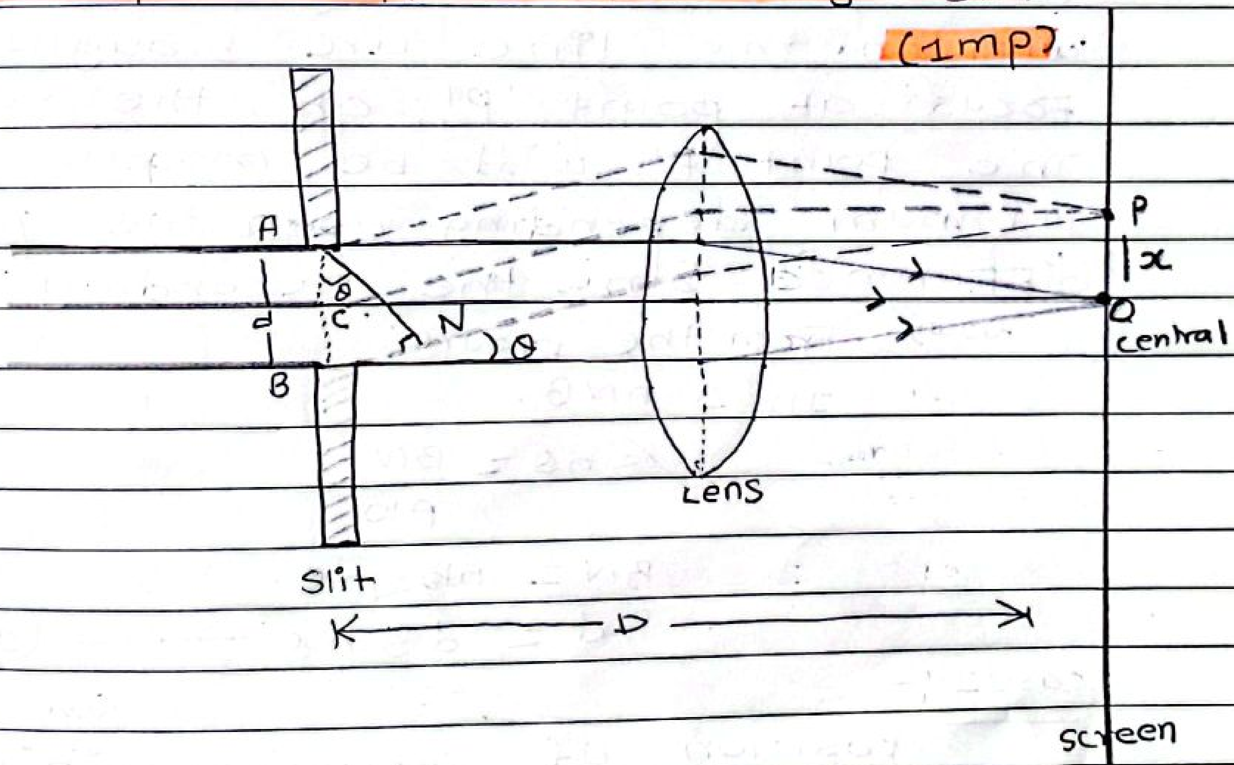


② Fraunhofer's diffraction

- Source and screen or both are at infinite distance from the obstacle.
- lens are used to make the light beam parallel.
- Incident wave as well as diffracted wave are plane.



Fraunhofer's diffraction at single slit. (1mp)



Let us consider a parallel beam of light incident normally on a slit AB of width 'd'. According to hygen's principle, every point in the plane wavefront AB acts as a source of secondary wavelet.

a) Central maxima

Let us consider O at the center of the screen which is equidistance from the end of the slit AB so the path difference at point O is zero. Thus the point 'O' is the position of maximum intensity known as central maxima.

b) Secondary minima and maxima.

Suppose the secondary wave travelling in the direction making an angle 'θ' with CO. This line are brought to focus at point 'P' on the screen. The point 'P' will be maximum or minimum depending upon the path difference btn. the secondary wave.

Now, from the figure:

In Δ ANB.

$$\sin\theta = \frac{BN}{AB}$$

$$BN = AB\sin\theta.$$

$$\therefore P.d = d\sin\theta \quad \text{———} \quad (1)$$

case I :-

Position of secondary minima:-

For secondary minima:-

$$P.d = n\lambda$$

$$\text{or, } d\sin\theta = n\lambda.$$

In general,

$$\boxed{d\sin\theta_n = n\lambda}$$

where, n = 1, 2, 3 ------

$\sin\theta_n \approx \theta_n$ (for small)

$$d\theta_n = n\lambda$$

$$\boxed{\theta_n = \frac{n\lambda}{d}}$$

classmate
Date
Page

* For 1st secondary minima:

$$n = 1$$

$$\boxed{\theta_1 = \frac{\lambda}{d}}$$

* For 2nd secondary minima:-

$$n = 2$$

$$\boxed{\theta_2 = \frac{2\lambda}{d}}$$

* For nth secondary minima:-

$$\boxed{\theta_n = \frac{n\lambda}{d}}$$

angular seperation.

<u>case II</u> :-

Position of secondary maxima.

$$pd = \frac{(2n+1)\lambda}{2}$$

$$d\sin\theta = \frac{(2n+1)\lambda}{2}$$

For small angle $\sin\theta \simeq \theta$.

$$d\theta = \frac{(2n+1)\lambda}{2}.$$

$$\theta = \frac{(2n+1)\lambda}{2d}.$$

In general,

$$\theta_n = \frac{(2n+1)\lambda}{2d}$$

where, $n = 0, 1, 2, 3$ - - - - - -

maxima	minima
$Pd = \left(\frac{2n+1}{2}\right)\lambda$	$Pd = n\lambda$
$\theta_n = \left(\frac{2n+1}{2}\right)\frac{\lambda}{d}$	$\theta_n = n\lambda/d$

* For first secondary maxima : n = 1

$$\therefore \theta_1 = \frac{3\lambda}{2d}$$

* For second secondary maxima : n = 2

$$\theta_2 = \frac{5\lambda}{2d}$$

$$\theta_n = \frac{(2n+1)\lambda}{2d}$$

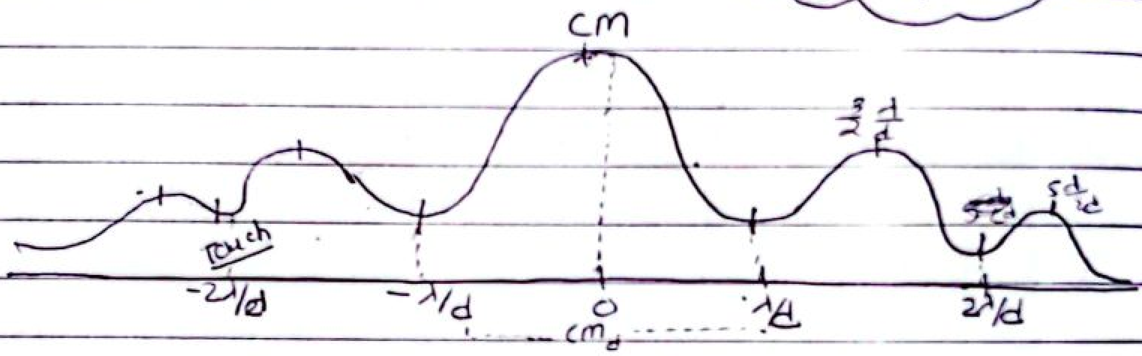
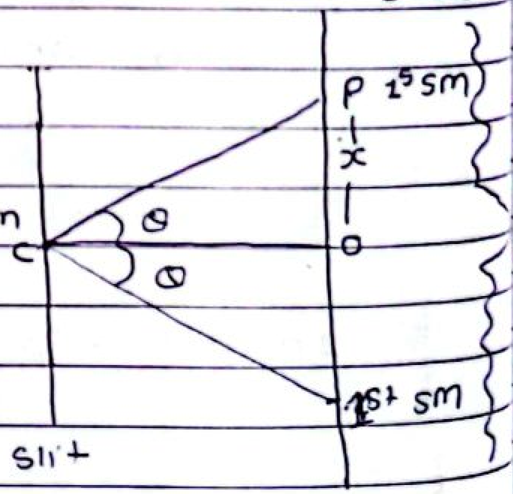


Fig:- Intensity distribution curve.
$\rightarrow \sin\theta_n$

Width of central maxima. IMP

The width of central maxima is defined as the distance between first minima on either side of central maxima.



Let 'x' be the distance of first minima from O. Then we know that for first minima.

$$\theta = \lambda/d \quad \text{--- (1)}$$

Also from figure in Δ CPO

$$\tan\theta = x/D$$

for small angle $\tan\theta \approx \theta$

$$\therefore \theta = x/D \quad \text{--- (2)}$$

from (1) and (2)

$$\frac{\lambda}{d} = \frac{x}{D}.$$

width of

$$\boxed{x = \frac{\lambda D}{d}}$$

$$x_n = \frac{n\lambda D}{d}$$

center maxima देखि nth minima samma ko distance

width of central maxima $= 2x$

$$= \frac{2\lambda D}{d}$$ #

* width of central maxima is double than that of first secondary minima or maxima.
* Intensity goes on decreasing.

For first central maxima

$$\theta = \frac{3\lambda}{2d}$$

from Δ CPO $\theta = x/D$.

$$\therefore \frac{x}{D} = \frac{3\lambda}{2d}$$

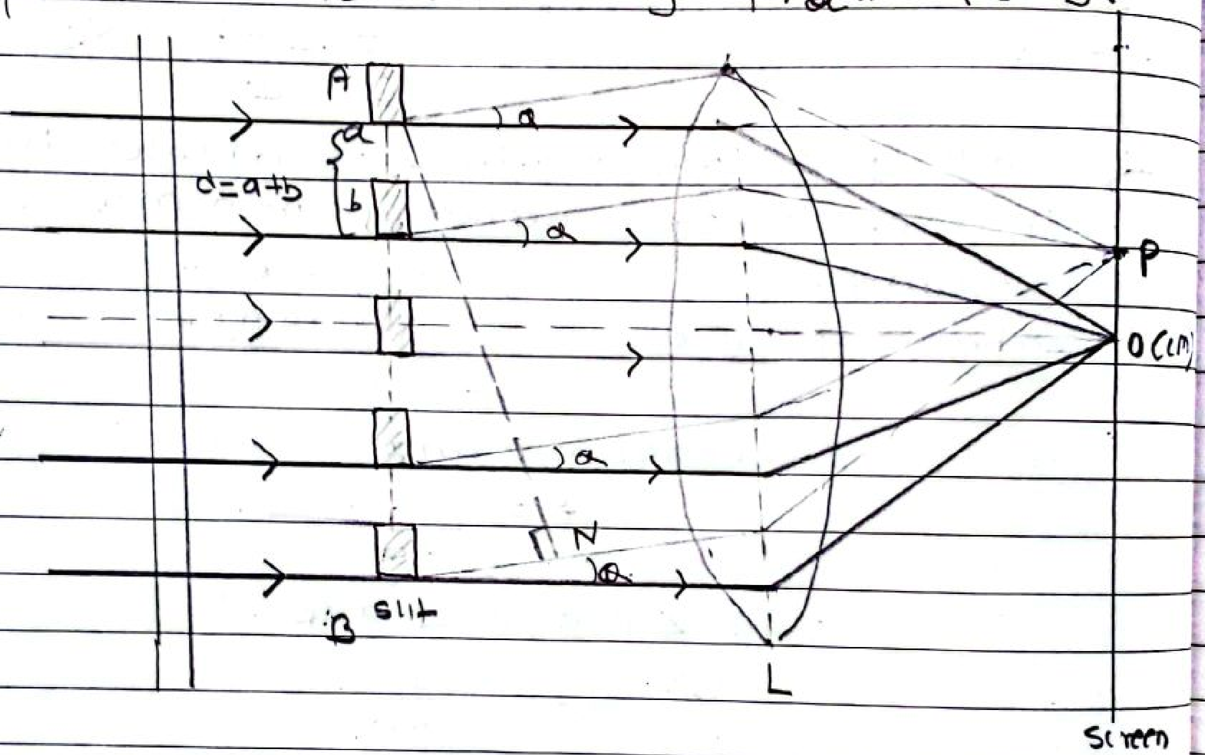
$$\text{or, } x = \frac{3\lambda D}{2d} = \frac{3}{2}\left(\frac{\lambda D}{d}\right)$$ #.

$$x_n = \frac{3n}{2}\left(\frac{\lambda D}{d}\right)$$ which gives the distance of nth maxima from central maxima.

Diffraction grating

→ It is an arrangement consisting large no. of parallel slits of equal width and seperated from each other by equal opaque portion is called diffraction grating.

If the slits are parallel to eachother then the diffraction grating is called plane transmission grating. It was first constructed by Fraunhofer's.



→ Let us consider a plane transmission grating AB whose slits are ⊥r to the plane of paper. If 'a' be the width of slit 'b' be the thickness of opaque portion. Then the distance (a+b) is called the grating elements.

→ If 'N' be the no. of lines per unit length of the grating. Then the grating element is given by $(a+b) = \frac{1}{N}$

→ Let us consider a parallel beam of monochromatic light is incident on the grating surface AB. The light diffract through N slight is focused by the Lens 'L' on the screen.

- The diffraction formed on the screen is called Fraunhofer's diffraction. The central maxima is formed at O.
- For the point 'P' to be maximum. The p.d of two waves should be integral multiple of λ. i.e.

$$pd = n\lambda.$$

$$\text{or, } d\sin\theta = n\lambda.$$

$$\text{or, } (a+b)\sin\theta = n\lambda$$

$$\text{or, } \frac{1}{N}\sin\theta = n\lambda. \quad \text{——— (1)}$$

- If θ_1 be the the diffraction for first secondary maxima. Then we can write.

$$(a+b)\sin\theta_1 = \lambda.$$

$$\frac{1}{N}\sin\theta_1 = \lambda$$

If θ_2 be the diffraction for secondary maxima Then we can write.

$$\frac{1}{N}\sin\theta_2 = 2\lambda.$$

In general, $\boxed{\frac{1}{N}\sin\theta_n = n\lambda.}$

Application of Diffraction grating.

- It is used for accurate estimation of wavelength.
- The velocity of ultrasonic can be measured with diffraction technique.
- The location, size and shape of ulcer, tumours etc can be found by the diffraction technique [ultrasound scanning]

Resolving Power of optical instruments.

→ The ability of the optical instrument to resolve the image of two nearby point object is called Resolving Power.

Application of Resolving power of optical instruments.

1) To see two object close together as seperate.
2) To see two spectral line as distinct from each other.

Resolving power of microscope.

→ It is defined as the reciprocal of the smallest distance between two point object at which they can be just resolve when seen through the microscope.

$$RP\left(\frac{1}{d\theta}\right) = \frac{2\mu \sin\theta}{\lambda}$$

where

$d\theta = \frac{\lambda}{2\mu \sin\theta}$ is the limit of resolution

λ = wavelength of light

μ = Refractive index of the medium enclosed btn. object & lens

θ = half angle of cone from each point object.

Resolving power of Telescope.

→ It is defined as the reciprocal of the smallest angular seperation btn two distant object whose image can be just resolve by it.

$$RP\left(\frac{1}{d\theta}\right) = \frac{D}{1.22\lambda}$$

① How wide is the central diffraction break on a screen 3.5 cm behind a 0.01 mm slit illuminated by 500 nm light?

→ width $(x) = \frac{2\lambda D}{d}$

$$= \frac{2 \times 500 \times 10^{-9} \times 3.5 \times 10^{-2}}{0.01 \times 10^{-3}}$$

$$= 3.5 \times 10^{-3}\ m.$$

② A parallel beam of monochromatic light is allowed to incident normally on a plane transmission grating having 500 lines/mm and second order spectral lines is found at 30°. calculate the wavelength of light.

→ we have, $\frac{1}{N} \sin\theta_n = n\lambda$.

$N = 500/mm$
$= 500 \times 10 \times 100$
$= 5 \times 10^5\ /m$.

$$\frac{1}{5 \times 10^5} \sin 30 = 2\lambda.$$

$$\therefore \lambda = 5 \times 10^{-7}\ m$$

$\frac{1}{N} \sin \theta_n = n\lambda$

Q) A parallel beam of light incident normally on a diffraction grating. The angle between 1st order spectra on either side of the normal is 27° 42 min (27° 42'). Assuming that wavelength of light is 5.893×10^{-7} m. Find no. of lines per mm in grating.

→ Soln: Given $2\theta = 27° 42''$ $\qquad 2\theta = 27.7$

$n\lambda = d \sin\theta$ $\qquad \theta = 27.7/2$

$1 \times 5.893 \times 10^{-7} = d \sin(27.7)$

$d = 1.26 \times 10^{-6}$ m

$d = \frac{1}{N} \Rightarrow N = 788037.43$ per m

Q) A diffraction grating has 400 lines per mm is illuminated normally by monochromatic light of wavelength 600 nm calculate the grating spacing, angle at which the first order maxima, and the no. of diffraction maxima obtained.

→ $N = 400$ lines / mm $= 400 \times 10^3$ lines per metre

$\lambda = 600$ nm $= 600 \times 10^{-6}$ m

$d = ?$ $\qquad \theta_1 = ?$ $\qquad n = ?$

$\frac{1}{N} \sin \theta_n = n\lambda$

$\frac{1}{400 \times 10^3} \sin \theta_1 = 1 \times 600 \times 10^{-6} \Rightarrow \sin\theta_1 =$

$N = 400$ lines/mm $= 400 \times 10^3$ lines/m

$\lambda = 600\,nm = 600 \times 10^{-9} m$ $\quad d = ? = \frac{1}{N} = \frac{1}{4 \times 10^5} = 0.25 \times 10^{-5} m$

Also

$d \sin\theta_1 = \lambda$ (for first order)

$\sin\theta_1 = \frac{\lambda}{d} = \frac{600 \times 10^{-9}}{0.25 \times 10^{-5}} = 0.24$

$\theta_1 = \sin^{-1}(0.24)$

$= 13.88^0$ Ans

For no of maxima obtained

$\theta = 90 \quad \sin\theta = 1$

$\therefore d \sin\theta = n\lambda$

$n = d/\lambda = \frac{0.25 \times 10^{-5}}{600 \times 10^{-9}}$

$= 4$ Ans

Q) Two spectral lines of sodium D1 and D2 have wavelength of approximately 5890 A° and 5896 A°. A sodium lamp send incident plane wave on to a slit of width 2 μm. A screen is located 2m from the slit. Find the spacing btn. on the first maxima of two sodium lines as measured on the screen.

[$9 \times 10^{-4} m$].

→ For D1

$\lambda_1 = 5890\,A^0$

$= 5.890 \times 10^{-7} m$

For D2

$\lambda_2 = 5896\,A^0$

$= 5.896 \times 10^{-7} m$

$d = 2\mu m = 2 \times 10^{-6} m$

$D = 2m$

For first maxima

$\frac{(2n+1)\lambda}{2} = d\,\theta_1$

$\theta_1 = \frac{3\lambda_1}{2d}$

$= 0.44175\,m$

For first maxima

$\frac{(2n+1)\lambda_2}{2} = d\,\theta_2$

$\theta_2 = \frac{3\lambda_2}{2d}$

$= 0.8844$

$d = \theta_2 - \theta_1$

$= 0.44265\,m$ Ans

classmate

Date
Page

SO)

1) Why the intensity of secondary maximum does became less as compared to the central maximum.

→ The central maxima is due to the constructive interference of wavelets from all parts of the slit. With the increase in the value of 'n', the wavelets from lesser and lesser parts of the slit produces constructive interference to form secondary maxima.

2) can diffraction be without interference?

→ As diffraction is basically spreading of light when passes through obstacle so it can take place without interference.

3) can interference be without diffraction?

→ As superposition of light from different source, causes interference. so it cannot be without diffraction.

$y = \frac{2\lambda D}{d}$ $\boxed{\frac{y}{\alpha} = \frac{\lambda D}{d}}$

classmate

Date ________
Page ________

a) What will happen in diffraction pattern of single slit diffraction experiment if slit is replaced by circular aparture?

→ circular bright and dark ~~fringe~~ rings will appear in diffraction pattern.

Diffraction Important Points

$d\sin\theta = n\lambda$ (minima)

$d\sin\theta = \left(\frac{2n+1}{2}\right)\lambda$ (maxima)

Diffraction grating

$$\left\{\begin{aligned} d\sin\theta &= n\lambda \\ (a+b)\sin\theta &= n\lambda \\ \frac{1}{N}\sin\theta &= n\lambda \end{aligned}\right\}$$

n_{max} $(\sin\theta = \sin 90 = 1)$

$n_{max} = \frac{d}{\lambda}$