20/8/24

COMPUTER GRAPHICS

Definition :-

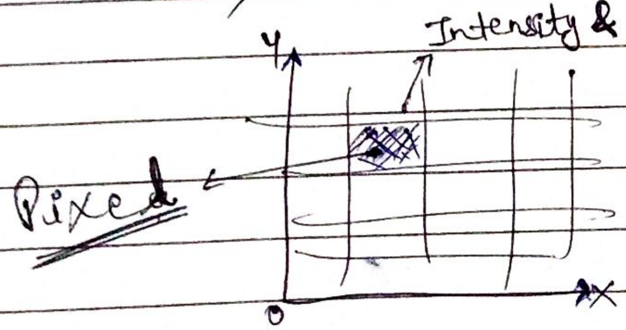
⇒ Computer Graphics is the field of visual computing where one utilises computers both to generate visual Images and to alter visual and spatial information from the real world.
(In space how it will be visible to us)

⇒ It is a pictorial representation, manipulation of data by a computer.

Eg:- Using in business graphics as like as ppts.

⇒ It is one of the most effective and commonly used way to communicate the processed information to the user.

• Pixel = Picture Element
⇒ (Smallest thing which is presented on the screen.)
⇒ (Smallest addressable thing Screen element.)



Applications-

1) Presentation Graphics.
2) Painting & Drawing
3) Web designing
4) Photo Editing
5) Scientific Visualization
6) Image Processing
7) Simulation
8) Education, Training, Entertainment
9) CAD (Computer Aided Design)
   CAM (Computer Aided Manufacturing)
10.) Animation & Games.

Types of Computer Graphics :-

1) Passive (Offline) CG
   Eg:- Movie

2) Interactive (Online) CG
   Eg :- Game

Components of CG :-

CPU	Update Process	Frame Buffer	Refresh Process	Display Controller	Pixel Information	Video Monitor or Display Device

Frame Buffer: Digital memory buffer where we store the picture information in an array.
VRAM
(Video RAM)

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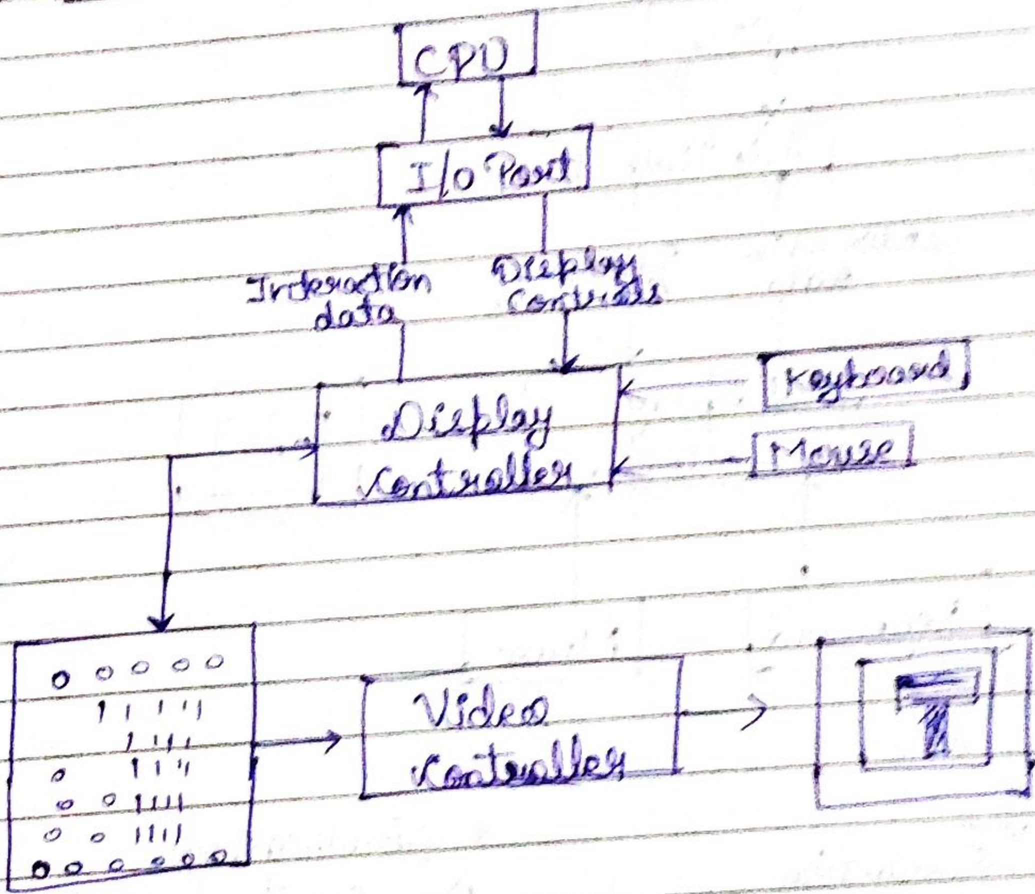
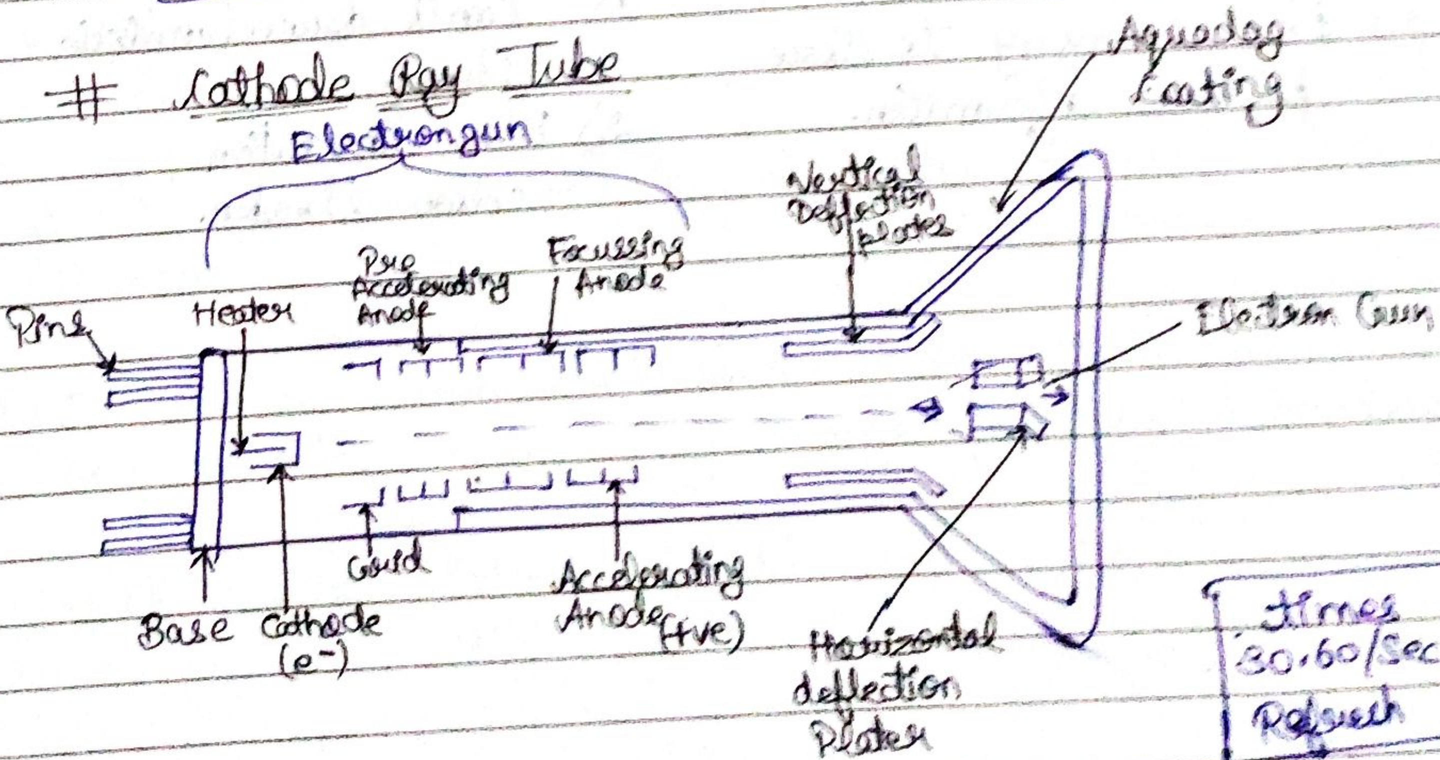
Raster Display

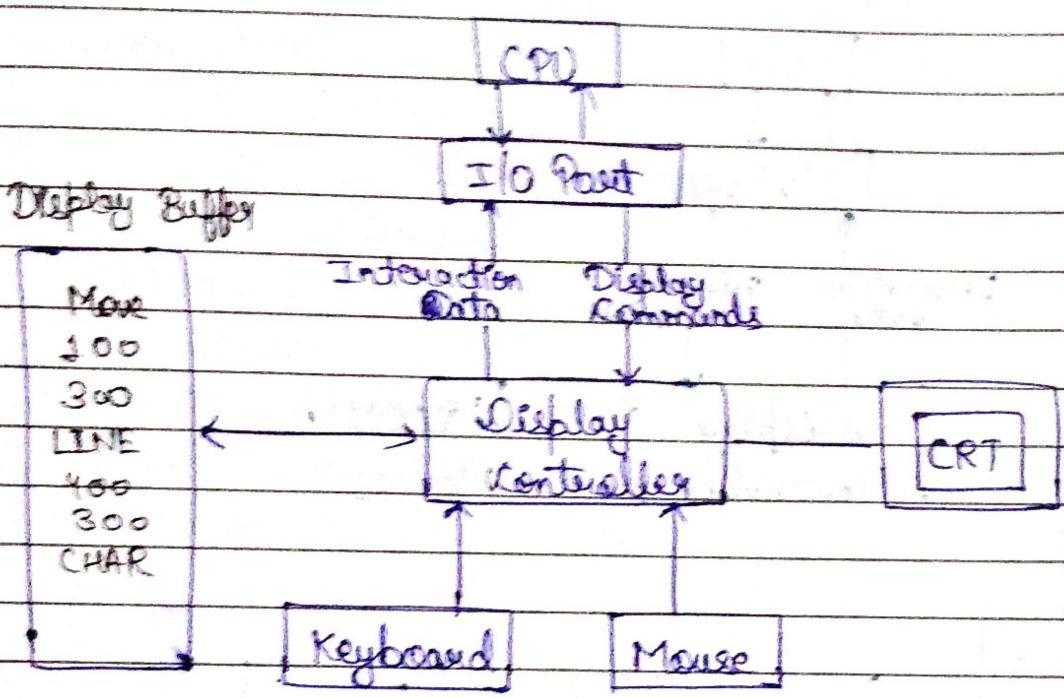


Cathode Ray Tube



times 30, 60/Sec Refresh

~~Rand~~ Architecture of Vector Display / Calligraphy Display
Random Scan Display



* **Advantages**
1) Higher Resolution
2) Less Memory to store picture Information.

* **Disadvantages**
1) Can't draw realistic items.
2) B/W limitation color stroke.

(* Resolution:- No. of pixels present on the screen.)

Dt.:
Pg.:

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CG

Difference between Random & Raster:-

Vector/Random Scan Display	Raster Scan Display
1.) Vector display only draws lines, points and characters.	1.) Raster display has the ability to display areas with solid colors or patterns.
2.) Higher Resolution.	2.) Less Resolution.
3.) More Expensive.	3.) Less Expensive.
4.) It uses monochrome (Produces black & white) or beam penetration type.	4.) It uses monochrome or shadow mask (R G B) type.
5.) Scan Conversion is not required.	5.) ~~Graphics Primitives~~ Scan conversion is required uses Interlacing (Whole picture scanning)
6.) Refresh Rate depends directly on picture complexity.	6.) Refresh Rate Independent of Picture complexity.
7.) It takes more space to store.	7.) Raster Images takes less space to store.
8.) It can only draw continuous and smooth lines.	8.) It can draw mathematical curves, polygons, & boundaries.
9.) .pdf, .svg. (Extension of file)	9.) .bmp, .jpg.

Delta

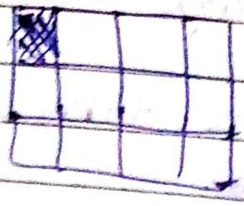
Q:- Characteristics of Display Device (DD) :-

1.) Display Size
2.) Refresh Rate - 60-120 Hz/sec (Normally)
3.) Resolution - dpi
4.) Aspect Ratio - 3 (4:3)
5.) Persistance. (How long the picture present on our screen.)
6.) Pixel Density
7.) Image Technology

Q:- Consider a Raster Scan System with the resolution of 1280 × 1024. What size of frame buffer is needed (in kilobytes) if 12 bits per pixel are to be stored.

Sol Resolution = 1280 × 1024
When 1 Pixel = 12 bits
Size of raster = 1280 × 1024 × 12 bits
= 15,728,640

12 bits



Convert Bit into Bytes
$= \frac{15728640}{8}$

= 1966080 Ans

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Assignment 1

Q. Explain LED, OLED and curved LED Displays.

Q. Consider 3 different raster systems with resolutions of 1) 640 x 480 2) 1280 X 1024 3) 2560 X 2048
What size of frame buffer (in bytes) is needed for each of these systems to store 12 bits per pixel.
How much storage is required for each system if 24 bits per pixel are to be stored?

(For 12 bits)

Sol. (1) Resolution = 640 x 480
1 Pixel = 12 bits
Size = 640 x 480 x 12 bits
= 3686400

Convert into bytes = $\frac{3686400}{8}$

= 4,60,800 bytes.
≃ 450 KB

For (24 bits)

Size = 640 x 480 x 24
= 7372800

In bytes = $\frac{7372800}{8}$ = 921600

(For 12 bits)

(ii) Resolution = 1280×1024

Size = $1280 \times 1024 \times 12$ bits

= 15728 640 bits

Convert bit into bytes = $\frac{15728640}{8}$

= 1966080 byte

(For 24 bits)

Size = $1280 \times 1024 \times$

(iii) Resolution = 2560×2048

Size = $2560 \times 2048 \times 12$ bits

= 62,914,560

Convert bit into bytes = $\frac{62,914,560}{8}$

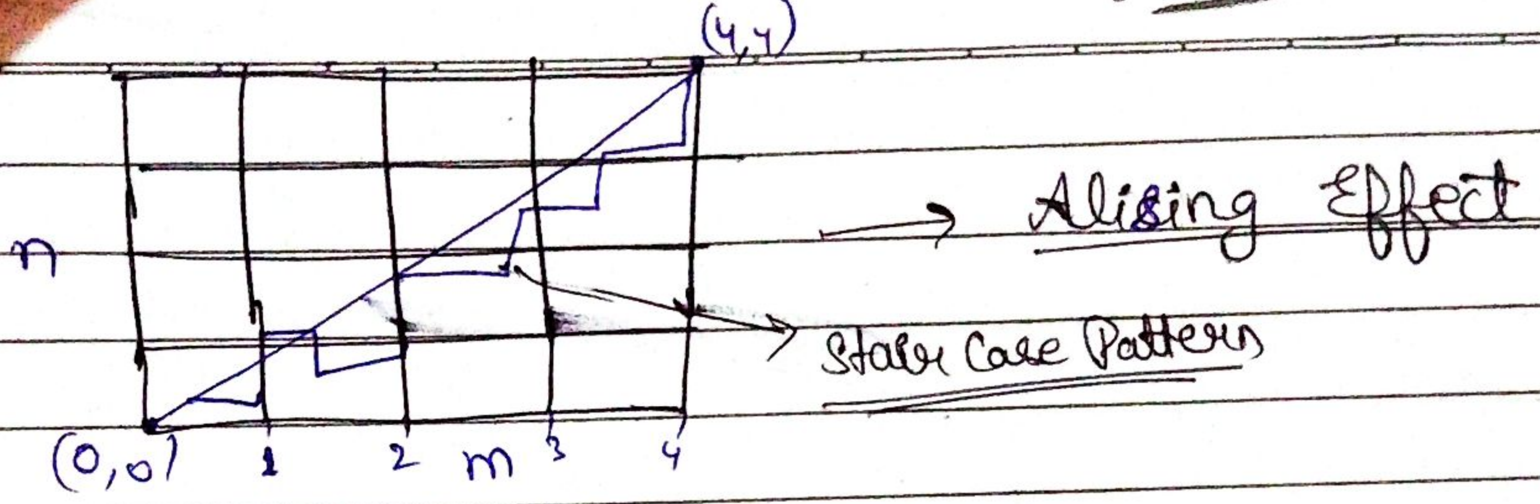
= 7864320

≃ 786 KB

Q- Find the aspect ratio of raster system using 8 × 10 inches screen and 100 pixel / inch.

Sol. Aspect Ratio = $\frac{\text{Width}}{\text{Height}} = \frac{\cancel{8}^{4} \times 100}{\cancel{10}_{5} \times 100} = 4:5$ Ans

#



⇒ Alising in CG refers to visual distortion that occur when representing a continuous signal or function. Jagged Edges or Stair Case Patterns in Images are seen.

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Equation of Line - (Cartesian Form)

$y = mx + c$

m = slope of line
c = constant or y-intercept.

If the line has 2 endpoints (x_0, y_0) & (x_n, y_n)

Slope of the line $= m = \frac{y_n - y_0}{x_n - x_0} = \frac{\Delta y}{\Delta x}$

$c = y_0 - m.x_n$

$\Delta y = y_2 - y_1$, $\Delta x = x_2 - x_1$

or $\Delta y = m.\Delta x$ or $m = \frac{\Delta y}{\Delta x}$

$\Delta x = \frac{\Delta y}{m}$

Putpixel (x, y, COLOR);
putpixel (x, y, RED);

DELTA Notebook

Antialising Techniques

1) High Resolution
2) Super Sampling
3) Area Sampling or Pre Filtering.

* Cases:-

$$m = \frac{\Delta y}{\Delta x}$$

1.) Case 1 - when $m < 1$

$$x_2 = x_1 + 1$$
$$y_2 = y_1 + m$$

2.) Case 2 - when $m > 1$

$$y_2 = y_1 + 1$$
$$x_2 = x_1 + \frac{1}{m}$$

Q:- Consider a line frome (0,0), 2,4,6
Use the simple DDA algorithm to rasterize this line.

Sol:- $A(0,0)$, $B(4,6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{4-0} = \frac{3}{2} > 1$$

$$y_2 = y_1 + 1$$
$$6 = 0 + 1$$
$$6 \neq 1$$

Dt.:
Pg.:

1st Iteration

$$x_2 = x_1 + \frac{1}{m}$$

$$4 = 0 + \frac{1}{m}$$

$$4 = \frac{0+1}{m}$$

$$4 = \frac{1}{m}$$

$$m = \frac{1}{4}.$$

$$x_2 = x_1 + \frac{1}{m}$$

$$= 0 + \frac{1}{3/2}$$

$$= 0 + \frac{2}{3}$$

$$\boxed{x_2 = \frac{2}{3}}$$

$$y_2 = y_1 + 1$$

$$= 0 + 1$$

$$\boxed{y_2 = 1}$$

∴ Points are $(x, y) = \left(\frac{2}{3}, 1\right) \approx (1, 1)$

IInd Iteration

$$x_3 = x_2 + \frac{1}{m} = 1 + \frac{1}{3/2} = \frac{1}{1} + \frac{2}{3}$$

$$= \frac{3+2}{3} = \frac{5}{3}$$

$$\boxed{x_3 = \frac{5}{3}} \approx 2$$

$$y_3 = y_2 + 1$$

$$= 1 + 1$$

$$\boxed{y_3 = 2}$$

∴ Points are $(x_3, y_3) = (2, 2)$ Ans

3rd Iteration:-

$$x_4 = x_3 + \frac{1}{m} = 2 + \frac{1}{3/2} = \frac{2}{1} + \frac{2}{3}$$
$$= \frac{6+2}{3} = \frac{8}{3} \approx 2$$

$$y_4 = y_3 + 1$$
$$= 2 + 1$$
$$\boxed{y_4 = 3}$$

$\therefore$ Points are $(x_4, y_4) = (2, 3)$.

4 th Iteration

$$x_5 = x_4 + \frac{1}{m} = 2 + \frac{1}{3/2} = \frac{2}{1} + \frac{2}{3} = \frac{6+2}{3}$$
$$= \frac{8}{3} \approx 3$$

$$y_5 = \cancel{y_4} + 1$$
$$= 3 + 1$$
$$\boxed{y_5 = 4}$$

$\therefore$ Points are $(x_5, y_5) = (3, 4)$.

5 th Iteration

$$x_6 = x_5 + \frac{1}{m} = 3 + \frac{1}{3/2} = \frac{3}{1} + \frac{2}{3} = \frac{9+2}{3}$$
$$= \frac{11}{3} \approx 4$$

$$y_6 = y_5 + 1$$
$$= 4 + 1 = 5.$$

$\therefore$ Points are $(x_6, y_6) = (4, 5)$.

7 th Iteration:-

$$x_7 = x_6 + \frac{1}{m} = 4 + \frac{2}{3} = \frac{12+2}{3} = \frac{14}{3} \approx 4$$

$$y_7 = y_6 + 1 = 5 + 1 = 6$$

$\therefore$ Points are $(x_7, y_7) = (4, 6)$

Q:- $A(\overset{x_1}{2}, \overset{y_1}{4})$ $B(\overset{x_2}{9}, \overset{y_2}{9})$

Sol:- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-4}{9-2} = \frac{5}{7} < 1$ (Case 1)

Ist Iteration:-

$$x_2 = x_1 + 1 = 2 + 1 = 3$$
$$y_2 = y_1 + m = \frac{4}{1} + \frac{5}{7} = \frac{28+5}{7} = \frac{33}{7} \approx 4$$

$\therefore$ Points are $(x_2, y_2) = (3, 4)$.

IInd Iteration:-

$$x_3 = x_2 + 1 = 3 + 1 = 4$$
$$y_3 = y_2 + m = 5 + \frac{5}{7} = \frac{35+5}{7} = \frac{40}{7} \approx 5$$

$\therefore$ Points are $(x_3, y_3) = (4, 5)$

III rd Iteration :-

$x_4 = x_3 + 1 = 4 + 1 = 5$

$y_4 = y_3 + m = \frac{5}{1} + \frac{5}{7} = \frac{35+5}{7} = \frac{40}{7} \approx 6$

∴ Points are $(x_4, y_4) = (5, 6)$

IV th Iteration :-

$x_5 = x_4 + 1 = 5 + 1 = 6$

$y_5 = y_4 + m = \frac{6}{1} + \frac{5}{7} = \frac{42+5}{7} = \frac{47}{7} \approx 7$

∴ Points are $(x_5, y_5) = (6, 7)$

V th Iteration :-

$x_6 = x_5 + 1 = 6 + 1 = 7$

$y_6 = y_5 + m = \frac{7}{1} + \frac{5}{7} = \frac{49+5}{7} = \frac{54}{7} = 8$

∴ Points are $(x_6, y_6) = (7, 8)$

VI th Iteration :-

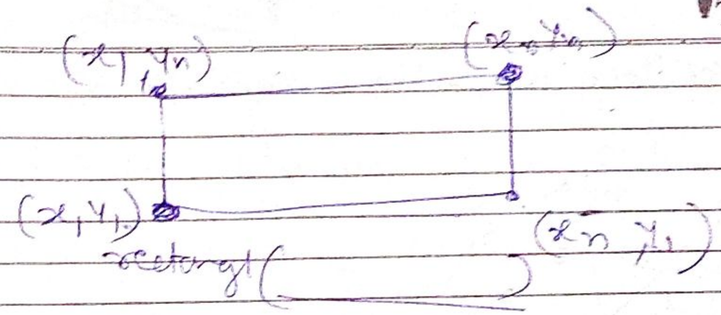
$x_7 = x_6 + 1 = 7 + 1 = 8$

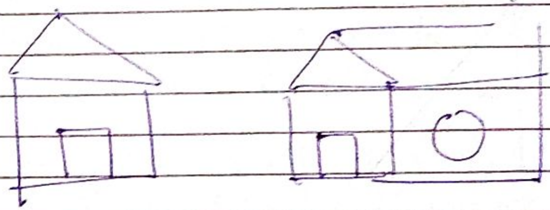
$y_7 = y_6 + m = 8 + \frac{5}{7} = \frac{56+5}{7} = \frac{61}{7} = 8$

VII th Iteration :-

$x_8 = x_7 + 1 = 8 + 1 = 9$

$y_8 = y_7 + m = 8 + \frac{5}{7} =$





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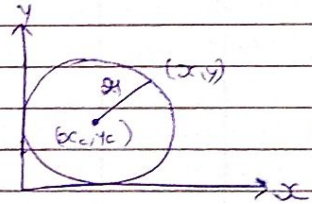
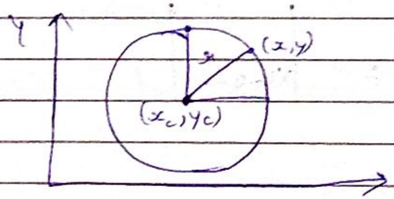
$x^2 + y^2 = r^2$

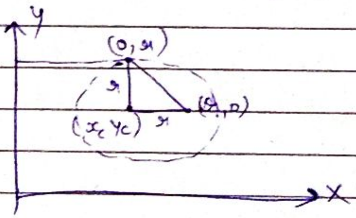
#



$(x - x_c)^2 + (y - y_c)^2 = r^2$



$-x, y$	I x, y +ve
$-x, -y$	$+x, -y$

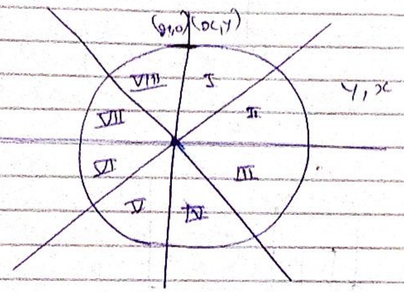


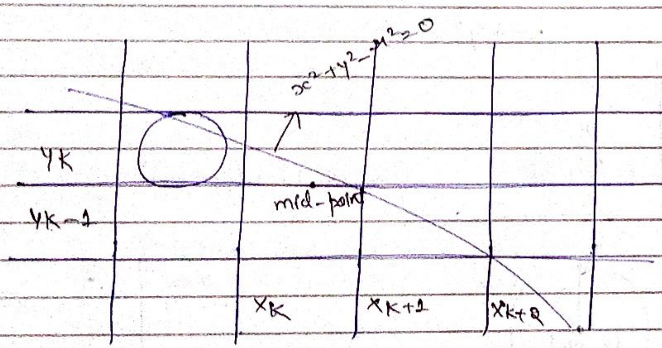
$m = \frac{\sin\theta}{\cos\theta}$

$x = x_c + r\cos\theta$

$y = y_c + r\sin\theta$

Bresenham Mid Point Circle Generation:-





$f_c(x,y) = x^2 + y^2 - r^2$

$f_c(x,y) < 0$

decision parameter $= 0$

> 0

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DDA Algorithm → Line Generation Algorithm

1.) Starting Point (x_0, y_0) Ending Point (x_n, y_n)

Calculate :- Δx, Δy & Slope m

$\Delta x = x_n - x_0$, $\Delta y = y_n - y_0$, $\boxed{m = \frac{\Delta y}{\Delta x}}$

2.) Case I - When slope (m) of line < 1

Suppose current point is (x_p, y_p) then next point (x_{p+1}, y_{p+1})

x_{p+1} = round off $(x_p + 1)$

y_{p+1} = round off $(y_p + m)$

3.) Case II - When slope $m = 1$

x_{p+1} = roundoff $(1 + x_p)$

y_{p+1} = roundoff $(y_p + 1)$

4.) Case III - When slope $m > 1$

x_{p+1} = roundoff $(x_p + \frac{1}{m})$

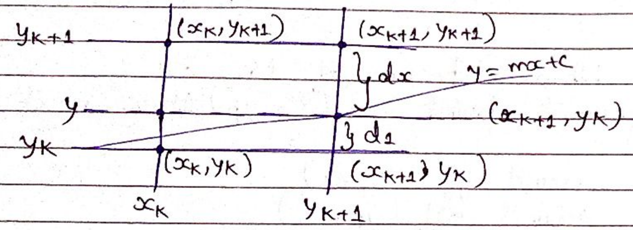
y_{p+1} = roundoff $(1 + y_p)$

5.) keep repeating untill end point is reached.

* Bresenham line generation algorithm :-

1.) More effective   2.) No roundoff value

CASE I :- When slope $m<1$



$$y = m(x+1)+c \quad \text{—①}$$
$$d_1 = y - y_n \quad \text{—②}$$
$$d_1 = m(x_k+1) + c - y_k \quad \text{—③} \quad \text{using ①}$$
$$d_2 = (y_{k+1}) - y = y_{k+1} - m(x_{k+1}) - c \quad \text{—④}$$
$$[\because y_{k+1} = y_k + 1]$$

Using ③ & ④ find $d_1 - d_2$

$$d_1 - d_2 = [m(x_k+1) + c - y_k] - [y_{k+1} - m(x_k+1) - c]$$
$$d_1 - d_2 = mx_k + m + c - y_k - y_k - 1 - mx_k + m - c$$
$$= 2m(x_k+1) - 2y_k + 2c - 1 \quad \text{—⑤}$$

Decision Parameter P for k^{th} step is P_k

$$P_k = \Delta x \, (d_1 - d_2) \quad \text{—⑥}$$
$$P_k = \Delta x (d_1 - d_2)$$
$$= \Delta x [2m(x_k+1) - 2y_k + 2c - 1$$
$$\left\{ \Delta x, \; m = \frac{\Delta y}{\Delta x} \right\}$$

$$P_k = \Delta x \left[2\left(\frac{\Delta y}{\Delta x} (x_k+1) - 2y_k + 2c - 1 \right] \right.$$
$$P_k = 2\Delta y x_k - 2\Delta x y_k + b \quad \text{—⑦}$$
$$b = 2\Delta y + \Delta x(2c - 1) \quad \text{—⑧}$$
$$P_{k+1} = 2\Delta y x_{k+1} - 2\Delta x y_{k+1} + b \quad \text{—⑨}$$

Subtract ⑦ from ⑨

$$P_{k+1} - P_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$
$$= 2\Delta y - 2\Delta x [y_{k+1} - y_k]$$

$P_k < 0 \cdot \; y_{k+1} = y_k$

$P_k > 0 \quad y_{k+1} = y_k + 1$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

At $(x_0 y_0)$

$$\boxed{P_0 = 2\Delta y - \Delta x}$$

Algorithm $m<1$

1.) Input 2 Endpoints & store left point (x_0, y_0).
2.) Load $(x_0 \; y_0)$ frame buffer & plot first point
3.) Calculate Δx, Δy, $2\Delta y$, $2\Delta y - 2\Delta x$ & obtain decision parameter $\boxed{P_0 = 2\Delta y - \Delta x}$.
4.) At each x_k along the line, Starting at $k=0$
if $P_k < 0$, next plot (x_{k+1}, y_k)
& $P_{k+1} = P_k + 2\Delta y$
else plot (x_{k+1}, y_{k+1})
& $P_{k+1} = P_k + 2(\Delta y - \Delta x)$
5.) Repeat step 4 Δx times.

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Bresenham Line Generation Algorithm :-

~~case~~ CASE I – $m < 1$

$\Delta x = x_1 - x_0$, $\Delta y = y_1 - y_0$

P_0 = Initial decision parameter = $2\Delta y - \Delta x$

```
while (x0 <= x1)
do
{
    putpixel (x0 y0)
    if (Pi > 0) then {
        x0 = x0 + 1;
        y0 = y0 + 1;
        Pi+1 = Pi + 2(Δy - Δx);
    }
    if (Pi < 0) then {
        x0 = x0 + 1;
        y0 = y0;
        Pi = Pi + 2Δy
    }
}
```

* x_0, y_0 = Starting Points
* P_0 = Initial decision parameter

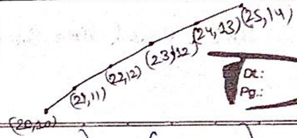
PROBLEM :- Draw line segment joining (20, 10) & (25, 14) using Bresenham line generation algorithm.

Solution :- $(x_0\ y_0) = (20, 10)$ $(x_1\ y_1) = (25, 14)$

m = slope = $\frac{y_1 - y_0}{x_1 - x_0} = \frac{14 - 10}{25 - 20} = \frac{4}{5} < 1$

~~As, $\Delta y = 5$, & $\Delta x = 4$~~

∴ Case I , ~~$\frac{\Delta x}{\Delta y} = \frac{5}{4}$~~ $\frac{\Delta Y}{\Delta X} = \frac{4}{5}$



Plot first point $(x_0, y_0) = (20, 10)$

$P_i = 2\Delta y - \Delta x$

$2(4) - 5 = 8 - 5 = 3$

Iteration $i = 1$, $P_1 = 2\Delta y - \Delta x = \cancel{2(5) - 4 = 10 - 4 = 6}$

as , $P_1 = 3 > 0$ ∴, $x_0 = x_0 + 1 = 20 + 1 = 21$

$y_0 = y_0 + 1 = 10 + 1 = 11$

∴, $(x_1, y_1) = 21, 11$

plot (21, 11)

Iteration $i = 2$ as $P_1 > 0$

$P_2 = P_1 + 2(\Delta y - \Delta x)$

$= 3 + 2(4 - 5) = 3 - 2 = 1$

$P_2 > 0$ ∴ $(x_2\ y_2) = (22, 12)$ plot (22, 12)

Iteration $i = 3$ as $P_2 > 0$

$P_3 = P_2 + 2(\Delta y - \Delta x) = 1 + 2(4 - 5) = -1$

$P_3 < 0$ ∴ $x_0 = x_0 + 1 = 22 + 1 = 23$

$y_0 = y_0 = 12$

∴, $(x_3\ y_3) = (23, 12)$

plot (23, 12)

Iteration $i = 4$ as $P_3 < 0$

$P_4 = P_3 + 2\Delta y = -1 + 2(4) = 7$

∴, $x_0 = x_0 + 1 = 23 + 1 = 24$

$y_0 = y_0 + 1 = 12 + 1 = 13$ ∴, plot (24, 13)

Iteration $i = 5$ as $P_4 > 0$

$P_5 = P_4 + 2(\Delta y - \Delta x) = 7 + 2(4 - 5) = 5$

∴ $x_0 = x_0 + 1 = 24 + 1 = 25$ ∴, plot (25, 14)

$y_0 = y_0 + 1 = 13 + 1 = 14$

for $i = 6$ $x_0 > x_1$ ∴, Stop Algo. terminate.

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Bresenham Line Generation Algorithm :-

Q:- Draw a line segment joining (20,10) & (30,18) using Bresenham line Generation.

Sol. $(x_0, y_0) = (20, 10)$
$(x_1, y_1) = (30, 18)$

$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{18-10}{30-20} = \frac{8}{10} < 1$

∴, Case I , $\frac{\Delta y}{\Delta x} = \frac{8}{10}$

I = 0 , plot $(x_0, y_0) = (20, 10)$

I = 1 $P_1 = 2\Delta y - \Delta x$
$= 2(8) - 10 = 16 - 10 = 6$

∴, $P_1 > 0$, plot $(x_1, y_1) = (21, 11)$

$x_1 = x_0 + 1$ & $y_1 = y_0 + 1$
$x_1 = 20 + 1$ $y_1 = 10 + 1$
$= 21$ $y_1 = 11$

6 − 4

6 + 2(−2)
~~6~~ − 8
= −2

I = 2 $P_2 = P_1 + 2(\Delta y - \Delta x)$
$= 6 + 2(8 - 10)$
$= 6 + 2(2) = 6 + 4 = \cancel{2}$.

∴, $P_2 > 0$

$x_2 = x_1 + 1$ & $y_2 = y_1 + 1$
$x_2 = 21 + 1$ $y_2 = 11 + 1$
$= 22$ $= 12$

∴, Plot $(x_2, y_2) = (22, 12)$.

I = 3 $P_3 = P_2 + 2(\Delta y - \Delta x)$
$= 2 + 2(8 - 10) = 2 + 2(-2) = 2 - 4$
$= -2$

∴, $P_3 < 0$

$x_3 = x_2 + 1$ & $y_3 = y_2 + 1$
$x_3 = 22 + 1$ $y_3 = 12 + 1$
$x_3 = 23$ $y_3 = 13$

Plot $(x_3, y_3) = (23, 13)$.

I = 4 $P_4 = P_3 + 2(\Delta y - \Delta x)$
$= -2 + 2(8 - 10) = -2 + 2(-2)$
$= -2 - 4 = -6$

∴, $P_4 < 0$

$x_4 = x_3 + 1$ & $y_4 = y_3 + 1$
$x_4 = 23 + 1 = 24$ $y_4 = 13 + 1 = 14$

Plot $(x_4, y_4) = (24, 14)$.

I = 5 $P_5 = P_4 + 2(\Delta y - \Delta x)$
$= -6 + 2(8 - 10) = -6 + 2(-2)$
$= -6 - 4 = -10$

∴, $P_5 < 0$

$x_5 = x_4 + 1$ & $y_5 = y_4 + 1$
$x_5 = 24 + 1 = 25$ $y_5 = 14 + 1 = 15$

Plot $(x_5, y_5) = (25, 15)$

I = 6 $P_6 = P_5 + 2(\Delta y - \Delta x)$
$= -10 + 2(8 - 10) = -10 + 2(-2)$
$= -10 - 4 = -14$

∴, $P_6 < 0$, $x_6 = x_5 + 1 = 25 + 1 = 26$
$y_6 = y_5 + 1 = 15 + 1 = 16$

Plot $(x_6, y_6) = (26, 16)$.

$I=7$, $P_7 = P_6 + 2(\Delta y - \Delta x)$
$= -14 + 2(8-10) = -14 + 2(-2)$
$= -14 - 4$
$= -18$

$\therefore P_7 < 0$

$x_7 = x_6 + 1$	$y_7 = y_7 + 1$
$x_7 = 26 + 1$	$y_7 = 16 + 1$
$x_7 = 27$	$y_7 = 17$

Plot $(x_7, y_7) = (27, 17)$

$I=8$, $P_8 = P_7 + 2(\Delta y - \Delta x)$
$= -18 + 2(8-10)$
$= -18 + 2(-2) = -18 - 4 = -22$

$\therefore P_8 < 0$

$x_8 = x_7 + 1$	$y_8 = y_8 + 1$
$x_8 = 27 + 1$	$y_8 = 17 + 1$
$x_8 = 28$	$y_8 = 18$

Plot $(28, 18)$

$I=9$, $P_9 = P_8 + 2(\Delta y - \Delta x)$
$= -22 + 2(8-10) = -22 + 2(-2)$
$= -22 - 4 = 26$

$\therefore P_9 < 0$

$x_9 = x_8 + 1$	$y_9 = y_8 + 1$
$x_9 = 28 + 1$	$= 18 + 1 = 19$
$x_9 = 29$	

Plot $(29, 19)$

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Bresenham Circle Drawing Algorithm:-

1) Assign the starting point coordinate (x_0, y_0)
Here Radius = R.
$\therefore$ Initial point is $(0, R) = (x_0, y_0)$.

2) Calculate the Initial decision parameter
$\boxed{P_0 = 3 - 2R}$

3) Suppose the current point is (x_k, y_k) and the next point (x_{k+1}, y_{k+1}), Find the next point of the first octant, depending on the value of the decision parameter $= P_k$.

CASE 1:- $P_k < 0$

$x_{k+1} = x_k + 1$
$y_{k+1} = y_k$

$\boxed{P_{k+1} = P_k + 4x_{k+1} + 6}$

CASE 2:- $P_k >= 0$

$x_{k+1} = x_k + 1$
$y_{k+1} = y_k - 1$

$\boxed{P_{k+1} = P_k + 4(x_{k+1} - y_{k+1}) + 10}$

Q) Given Center Coordinate $(0,0)$ and $R = 8$, using the Bresenham circle ~~Direct~~ Drawing Algorithm.

Sol. $P_0 = 3 - 2R$
$= 3 - 2(8) = 3 - 16 = -13$

1.) $(x_0, y_0) = (0, R) = (0, 8)$
2) $P_0 = 3 - 2R = 3 - 2(8) = -13$
3) As, $P_0 < 0$
$\therefore x_1 = x_0 + 1$
$y_1 = y_0$.
$\therefore (x_1, y_1) = (1, 8)$.

$P_1 = P_0 + 4x_1 + 6 = -13+4+6 = -13+10$

$\boxed{P_1 = -3}$

4.) $P_1 < 0$

$\therefore (x_2, y_2) = (2,8)$

$x_2 = x_1 + 1$
$= 1+1$
$= 2$

$y_2 = y_1$
$= 8$

$P_2 = P_1 + 4x_2 + 6 = -3 + 4(2) + 6$
$= -3+8+6 = -3+14$
$= 11.$

5.) As, $P_2 > 0$

$x_3 = x_2 + 1 = 2+1 = 3$
$y_3 = y_2 - 1 = 8-1 = 7$
$\therefore (x_3, y_3) = (3,7)$

~~$\therefore P_3 = P_2 + 4x_2 + 6$ $= 11 + 4(3) + 6$~~
$= 11 + 12 + 6 =$

$\therefore P_3 = P_2 + 4(x_3 - y_3) + 10$
$= 11 + 4(3-7) + 10$
$= 11 + 4(-4) + 10 = 11 - 16 + 10$
$= 11 - 6 = 5$

6.) As, $P_3 > 0$

$x_4 = x_3 + 1 = 3+1 = 4$
$y_4 = y_3 - 1 = 7-1 = 6$
$\Big\} (x_4, y_4) = (4,6)$

$\therefore P_4 = P_3 + 4(x_4 - y_4) + 10$
$= 5 + 4(4-6) + 10$
$= 5 + 4(-2) + 10 = 5 - 8 + 10 = 7$

7.) As, $P_4 > 0$

$x_5 = 5, \quad y_5 = 5$

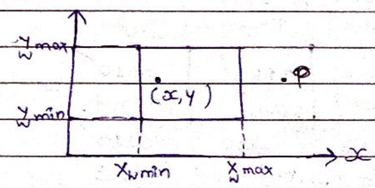
Hence, $x_5 = y_5$ $\therefore$ STOP.

Octant I P_k	P_{k+1}	(x_{k+1}, y_{k+1})
		(0,8)
-13	-3	(1,8)
-3	11	(2,8)
11	5	(3,7)
5	7	(4,6)
7		5,5

5/9/24 UNIT-2

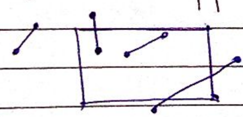
CLIPPING :-

1.) Point Clipping :-



$X_{wmin} \le x < X_{wmax}$
$Y_{wmin} \le y < Y_{wmax}$

2.) Line Clipping :-



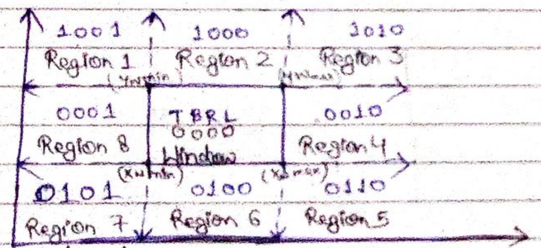
1.) Cohen Sutherland
2.) Cyrus beck
3.) Midpoint subdivision.

3) Polygon clipping
4.) Text clipping
5) Curve clipping

Windowing Clipping :-

⇒ When drawings are to complex it is difficult to read. In such situations we will display only those portions of the drawing that we are interested.
Method for selecting and enlarging portions of drawing is called 'Windowing'. The technique for not showing that part of the drawing, which one is not Interested is called "clipping."

1.) Cohen Sutterland line Clipping :-

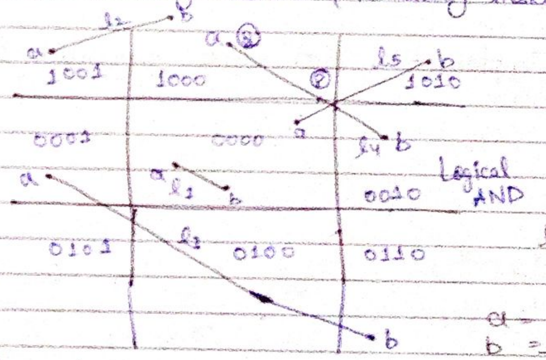


1 → T B R L = 1   Top Bottom Right left
Or
A B R L
Above   Or → Below
ABLR

Case 1 – Totally visible or Trivial Acceptance

Case 2 – Totally Invisible or Trivial Rejection

Case 3 – Partially visible (Partially inside the window).



logical A

Logical AND

l_2 – 1001
1000
1000

l_2 – 0001
0110
0000

a – 0000
b = 0000
0000 $\boxed{l_1}$

l_4 = 1000
0010
0000

l_5 = 0000
1010
0000

1.) Top Case / Above :-
$y - y_1 = m(x - x_1)$.   Eqⁿ of line PQ

Slope
$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$

Here, $y = y_{wmax}$

Coordinates of line of intersection are (x, y_{wmax})
$\therefore$ Eqⁿ of line between point a & intersection point is

$y_{wmax} - y_1 = m(x - x_1)$

$\therefore x = x_1 + \frac{1}{m}(y_{wmax} - y_1)$ —— ①

2) Bottom / Below Edge — $y = y_{wmin}$

1. $x = x_2 + \frac{1}{m}(y_{wmin} - y_2)$

9/9/24 LAB

Q1- WAP to implement DDA line generation algo

$dx = x_2 - x_1$

$dy = y_2 - y_1$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{7}$

$dx > dy$

m

(x_0, y_0)

(x, y)

```
if (m<1)
{
    x = x0 + 1;
    y = y0 + m;
else
    x = x0 + 1/m;
    y = y0 + 1;
```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Cyrus-Beck line clipping Algorithm :-

* Dot Product.

P_0 P_1

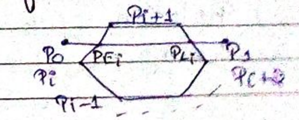
$P(t) = P_0 + t(P_1 - P_0)$

$0 \le t \le 1$ —— (1)

$P(0) = P_0$

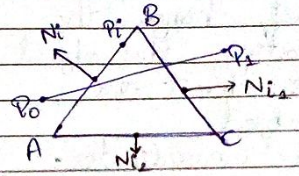
$P(1) = P_1$

Parametric Equation of a line



PEi = Potentially entering point

PLi = Potentially leaving point



1.) P_0P_1 − Line N_i − Normal vector to the edge / i^{th} edge $i = 1, 2, 3, ...$

2.) Any point P_{Ei}

3.) $N_i \cdot (P(t) - P_{Ei}) = 0$ —— (2)

$< = 0$ inside the polygon

$> = 0$ outside the polygon

$= 0$ on the polygon edge

$N_i \cdot (P_0 + t(P_1 - P_0) - P_{Ei}) = 0$

$N_i \cdot P_0 + N_i \cdot tP_1 - N_i t P_0 - N_i P_{Ei} = 0$

$N_i \cdot tP_1 - N_i \cdot tP_0 = N_i \cdot P_{Ei} - N_i \cdot P_0$

$t(N_i \cdot P_1 - N_i \cdot P_0) = N_i \cdot (P_{Ei} - P_0)$

$\therefore\ t = \frac{N_i \cdot (P_{Ei} - P_0)}{N_i \cdot (P_1 - P_0)}$ —— (3)

Denominator $N_i \cdot (P_1 - P_0) = D$

If $D > 0$ leaving point

$D < 0$ Entering point

This is for i^{th} edge

t_1, t_2, t_3

P_E = Highest value } $P(t)$
P_L = Lowest value }

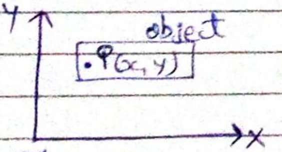
Transformation :-

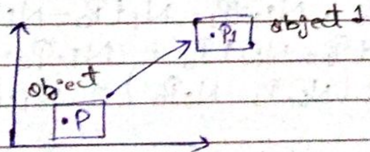
1.) Translation
2.) Rotation
3.) Scaling < $\frac{1}{2}$, 2

⇒ Transformation is a process of changing the position of the object or rotating the object or changing the size of the object or changing the shape of the object.

⇒ The objects are refer by their coordinates. Changes in orientation, size & shape are accomplished with geometric transformations that allow us to calculate the new ~~co~~ coordinates. The Basic Geometric transformations are translation, rotation and scaling.

1) Translation Transformation (t_v = translation Vector)





After translation

$P^1 = P \cdot T_V$

$x' = x + t_x \qquad y^1 = y + t_y$

$P(x, y, 1) \qquad P1(x', y', 1)$

T_V is the translation in matrix form

Homogeneous Coordinate

$(x, y) \Rightarrow (x, y, H)$

$H = 1$

DELTA Notebook

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Homogeneous Coordinate Systems (HCS) :-

(X, Y, H) $\boxed{H=1}$

2D or Euclidian System ⇒ ~~Homo~~ HCS

Any point $(x, y) \Rightarrow (x, y, H)$

$(x, y) \Rightarrow (x, y, 1)$

$P' = P \cdot T_V$

$$(x', y', 1) = (x, y, 1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

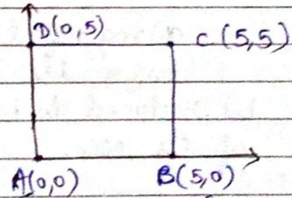
$$= (x + 0 + t_x,\ 0 + y + t_y)$$

$$= (x + t_x,\ y + t_y)$$

Q:- $A(0,0)$, $B(5,0)$, $C(5,5)$, $D(0,5)$

$t_x = 2$, $t_y = 3$ translation ~~Vecto~~ Factor

Sol:-



$$\begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 5 & 1 \\ 0 & 5 & 1 \end{bmatrix}$$

$$[A'B'C'D'] = [A\ B\ C\ D] \cdot T_V$$

$$[A'B'C'D'] = \begin{bmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 5 & 1 \\ 0 & 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+2 & 0+0+3 & 0+0+1 \\ 5+0+2 & 0+0+3 & 0+0+1 \\ 5+0+2 & 0+5+3 & 0+0+1 \\ 0+0+2 & 0+5+3 & 0+0+1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 7 & 3 & 1 \\ 7 & 8 & 1 \\ 2 & 8 & 1 \end{bmatrix}$$

DELTA Notebook

2.) Scaling :-

⇒ Let $P(x, y)$ any point. S_x & S_y are Scaling Factors

$\therefore\ P'(x'y') = S.P = (x, y) \cdot \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

$x' = x.S_x$ & $y' = y.S_y$

~~1) When S_x~~

3D Matric =

$$P'(x'y'1) = (x\ y\ 1) \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.) When $S_x = S_y = 2$ or > 1

2.) When $S_x = S_y = \frac{1}{2}$ or < 1

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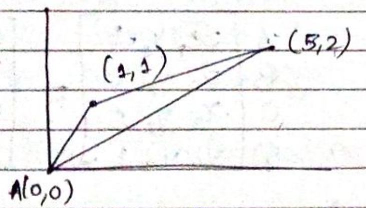
Q:- $A(0, 0)$ $B(1, 1)$ $C(5, 2)$

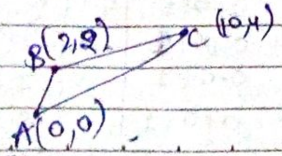


a.) magnified to twice its size.

b.) reduced to half of its size.

Sol:- ⓐ $[A'B'C'] = [ABC] \cdot T_{S_{x,y}}$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+1 \\ 2+0+0 & 0+2+0 & 0+0+1 \\ 10+0+0 & 0+4+0 & 0+0+1 \end{bmatrix}$$



$$= \begin{bmatrix} 0 & 0 & 1 \\ 2 & 2 & 1 \\ 10 & 4 & 1 \end{bmatrix}$$

Sol: ⓑ $S_x = S_y = \frac{1}{2}$

Transformation Matrix for Scaling =

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

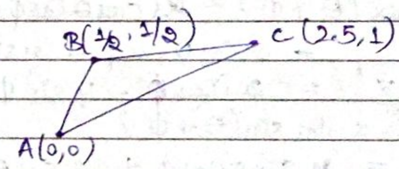
$(A'B'C') = (ABC) \cdot T_{S_{x,y}}$

$$(A'B'C') = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+1 \\ 1/2+0+0 & 0+1/2+0 & 0+0+1 \\ 5/2+0+0 & 0+1+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 1 \\ 5/2 & 1 & 1 \end{bmatrix}$$

After Scaling

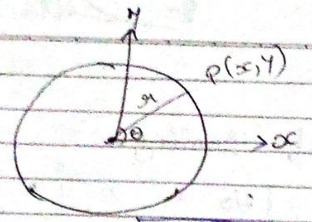


Case 1:- If S_x & S_y are less than 1 i.e, S_x & $S_y < 1$

Case 2:- S_x & $S_y > 1$

Case 3:- Same Scaling factor, $S_x = S_y$ = Uniform Scaling.

#



$x = r\cos\theta$

$y = r\sin\theta$

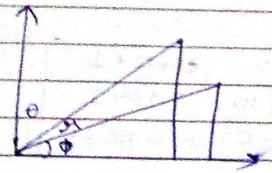
$r = \sqrt{x^2+y^2}$

$$\boxed{m = \tan\theta = \frac{\Delta y}{\Delta x}}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$

* Rotation about origin



$P(x,y) = P(r\cdot\cos\phi,\ r\cdot\sin\phi)$

$P'(x'y') = P[r\cdot\cos(\phi+\theta),\ r\sin(\phi+\theta)]$

$x' = r\cdot\cos(\theta+\phi) = r(\cos\theta\cos\phi - \sin\theta\sin\phi)$

$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$

$$\boxed{x' = x\cdot\cos\phi - y\sin\phi}$$

$y' = r\cdot\sin(\theta+\phi)$

$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$$\boxed{y' = y\cos\phi + x\sin\phi}$$

$\therefore R_\theta = x' = x\cos\phi - y\sin\phi$

$y' = y\cdot\cos\phi + x\sin\phi$

$$\therefore (x'y') = (xy)\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$

In

$\therefore$ HCS → Rotation Transformation matrix is In Anti Clockwise direction (i.e, Positive)

$$(x'y'1) = (xy1)\begin{bmatrix}\cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1\end{bmatrix}$$

For Clockwise Rotation

$$R(-\theta) = \begin{bmatrix}\cos(-\theta) & \sin(-\theta) & 0 \\ -\sin(-\theta) & \sin(-\theta) & 0 \\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1\end{bmatrix}$$

FORMULAS :-

1.) For translation :- $(x', y', 1) = (x, y, 1)\begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1\end{bmatrix}$

2.) For Scaling :- $(x'y'1) = (xy1)\begin{bmatrix}S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1\end{bmatrix}$

3.) For Rotation :- Positive or AntiClockwise

$$(x'y'1) = (xy1)\begin{bmatrix}\cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1\end{bmatrix}$$

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1) Rotation about the origin — Anti Clockwise Rotation

$$(x'\ y'\ 1) = (x\ y\ 1)\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \quad \theta \text{ is positive}$$

i.e, $\boxed{P'_n = P_n \cdot R_\theta}$

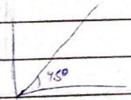
2.) Clockwise Rotation when $\theta = -\theta$

$$P'_n = P_n \cdot R_\theta$$

$$(x'\ y'\ 1) = (x\ y\ 1)\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

Q1 Perform 45° rotation of a triangle A(0,0), B(1,1), C(5,2) about the origin.

Sol- $(x'\ y'\ 1) = (x\ y\ 1)\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$



$$= \begin{bmatrix}0 & 0 & 1\\ 1 & 1 & 1\\ 5 & 2 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}0+0+0 & 0+0+0 & 0+0+1\\ \cos\theta - \sin\theta + 0 & \cos\theta - \sin\theta + 0 & 0+0+1\\ 5\cos\theta - 2\sin\theta + 0 & 5\sin\theta + 2\cos\theta + 0 & 0+0+1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 1\\ \cos\theta - \sin\theta & \cos\theta - \sin\theta & 1\\ 5\cos\theta - 2\sin\theta & 5\sin\theta + 2\cos\theta & 1\end{bmatrix}$$

$$R_{45} = \begin{bmatrix}\cos 45 & \sin 45 & 0\\ -\sin 45 & \cos 45 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2 & 0\\ -\sqrt{2}/2 & \sqrt{2}/2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

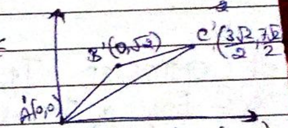
$$[A'B'C'] = [ABC] \cdot R_\theta$$

$$= \begin{bmatrix}0 & 0 & 1\\ 1 & 1 & 1\\ 5 & 2 & 1\end{bmatrix} \cdot \begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2 & 0\\ -\sqrt{2}/2 & \sqrt{2}/2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 1\\ \sqrt{2}/2 - \sqrt{2}/2 + 0 & \sqrt{2}/2 + \sqrt{2}/2 & 1\\ \frac{5\sqrt{2}}{2} - \sqrt{2} + 0 & \frac{5\sqrt{2}}{2} + \sqrt{2} & 1\end{bmatrix}$$

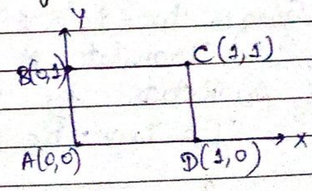
$$= \begin{bmatrix}0 & 0 & 1\\ 0 & \sqrt{2} & 1\\ \frac{3\sqrt{2}}{2} & \frac{7\sqrt{2}}{2} & 1\end{bmatrix} \text{ Ans}$$

$\frac{5\sqrt{2}}{2} - \sqrt{2}$, $\frac{5\sqrt{2} - 2\sqrt{2}}{2}$, $5 \times \frac{\sqrt{2}}{2}$, $2 \times \frac{\sqrt{2}}{2}$, $\sqrt{2} + \sqrt{2}$



Q2 A square consists of matrices A(0,0) B(0,1) C(1,1) D(1,0). Translation factors $t_x = 5$, $t_y = 7$. Determine new location of square.

Sol- $(A\ B\ C\ D) = \begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$



Transformation Matrix for translation

$t_x = 5$, $t_y = 7$

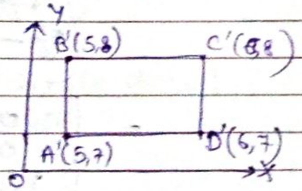
$$T_v = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ t_x & t_y & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 5 & 7 & 1\end{bmatrix}$$

$$[A'B'C'D'] = [A\ B\ C\ D] \cdot T_v$$

$$= \underset{(4\times 3)}{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}} \underset{(3\times 3)}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 7 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 0+0+5 & 0+0+7 & 0+0+1 \\ 0+0+5 & 0+1+7 & 0+0+1 \\ 1+0+5 & 0+1+7 & 0+0+1 \\ 1+0+5 & 0+0+7 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 & 1 \\ 5 & 8 & 1 \\ 6 & 8 & 1 \\ 6 & 7 & 1 \end{bmatrix}$$



COMPOSITE TRANSFORMATION

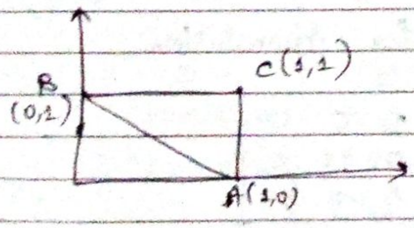
Q:- For a transformation of a triangle A(1,0), B(0,1), C(1,1) by translating 1 unit in x direction & y direction & then rotating 45° about the origin. Find new coordinates.

A:- Given:- $t_x = t_y = 1$ $\theta = 45°$ positive

1.) Translate it by $t_x = t_y = 1$

2.) Rotate by 45° about the origin

$T = T_{x,y} \cdot R_\theta$



$$[ABC] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[A'B'C'] = [A\ B\ C] \cdot T$$

$$= \underset{(ABC)}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}} \cdot \underset{(T_{x,y})}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}} \cdot \underset{(R_\theta)}{\begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 1+0+1 & 0+0+1 & 0+0+1 \\ 0+0+1 & 0+1+1 & 0+0+1 \\ 1+0+1 & 0+1+1 & 0+0+1 \end{bmatrix} \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} - \frac{\sqrt{2}}{2} + 0 & \sqrt{2} + \frac{\sqrt{2}}{2} + 0 & 0+0+1 \\ \sqrt{2}/2 - \sqrt{2} + 0 & \sqrt{2}/2 + \sqrt{2} + 0 & 0+0+1 \\ \sqrt{2} - \sqrt{2} + 0 & \sqrt{2} + \sqrt{2} + 0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & \frac{3\sqrt{2}}{2} & 1 \\ -\sqrt{2}/2 & 3\sqrt{2}/2 & 1 \\ 0 & 2\sqrt{2} & 1 \end{bmatrix} \quad \text{Ans.}$$

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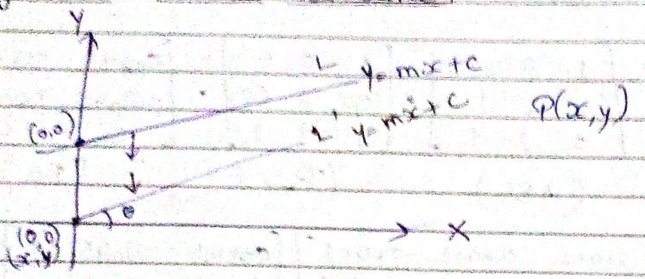
* Reflection about a line :-



Reflection about x-axis

⇒ Reflection is a transformation which generates the mirror of image of an object. Mirror Reflection helps in achieving 8-way symmetry for the circle to simplify the scan conversion process. For Reflection we need to know the reference axis or reference plane depending on whether the object is 2-D or 3-D.

⇒ Let a line 'L' represented by $\boxed{y = mx + c}$ where "m = slope" with respect to the x-axis and "C" is the intercept on y-axis.

⇒ Let, $P'(x' \ y')$ be the mirror reflection about the line "L" of point $P(x,y)$.

⇒ The transformation about mirror reflection about this line "L" consist of the following Basic Transformation.

STEPS :-

1.) Translate the intersection point $A(0, c)$ to the origin, this shifts the line L to L'.

2.) Rotate the shifted line L' by $-\theta$ degrees so that the line L' aligns with the x-axis.

3.) Mirror reflection about x-axis.

4.) Rotate the x-axis back by θ degrees.

5.) Translate the origin back to the intercept point (0,c).

$$M_L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -c & 1 \end{bmatrix} \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Clockwise Rotation}(-\theta)} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Reflection Matrix}} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

~~Tes~~

3-D Transformation

1) 3D Translation

$$P'(x' \ y' \ z' \ 1) = P(x \ y \ z \ 1) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$x' = x + t_x$

$y' = y + t_y$

$z' = z + t_z$

$$\boxed{\therefore P'_n = P_n \cdot T_r}$$

2.) Transformation for 3D Rotation

$Z = 0$ plane i.e. XY plane.

$$(x'\ y'\ z'\ 1) = (x\ y\ z\ 1)\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.) Transformation for 3D Scaling

if $S_x = S_y = S_z$

$$(x'\ y'\ z'\ 1) = (x\ y\ z\ 1)\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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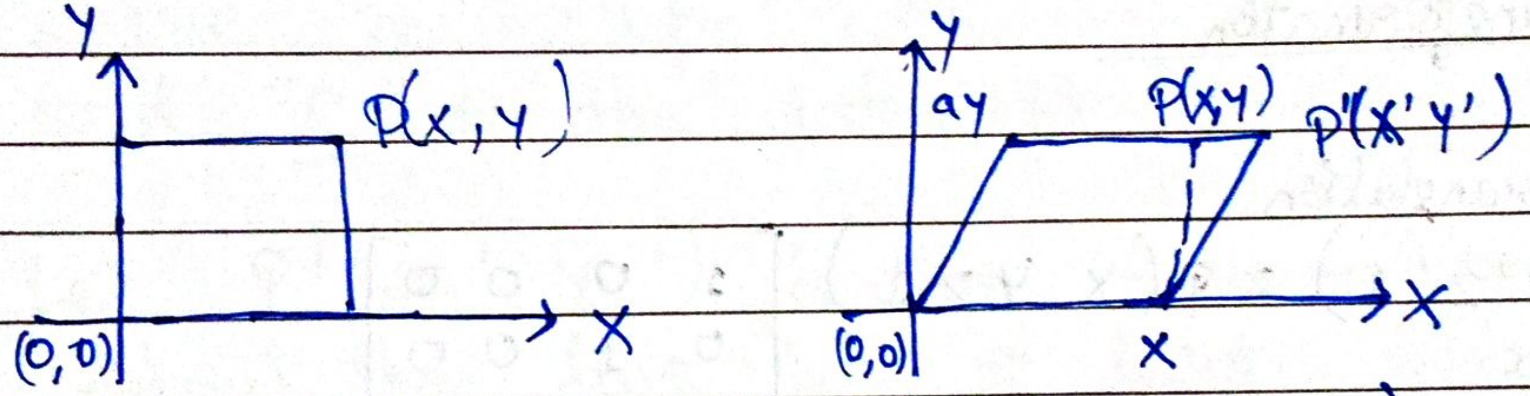
Shearing Transformation :-

1.) X-shear about the origin -



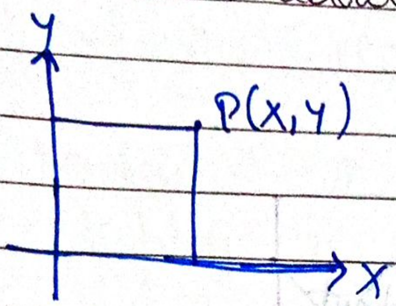
P(x, y) P'(x'y')

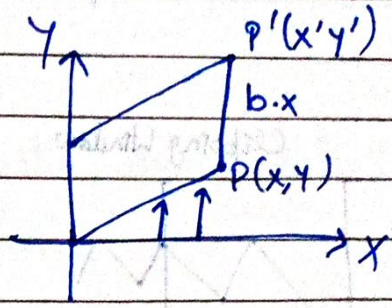
$\therefore$, $\left.\begin{aligned} x' &= x + ay \\ y' &= y \end{aligned}\right\} = Sh_x(a)$ where, a = constant or shear parameter

$$(x' \; y' \; 1) = (x \; y \; 1)\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $\boxed{P'_n = P_n \cdot Sh_x(a)}$

2) Y- Shear about the origin





$$\left.\begin{aligned} X' &= X \\ Y' &= Y + bx \end{aligned}\right\} = Sh_y(b)$$

b = constant or shear parameter

$$(x'\ y'\ 1) = (x\ y\ 1)\begin{bmatrix} 1 & b & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

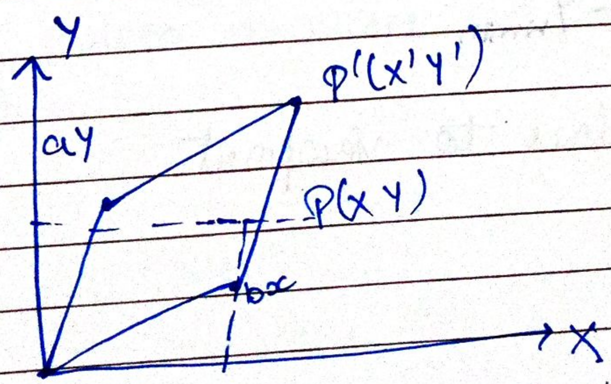
$$\therefore \boxed{P'_n = P_n \cdot Sh_y(b)}$$

3) XY shear about origin

$$X' = Y + aY$$
$$Y' = Y + bx$$

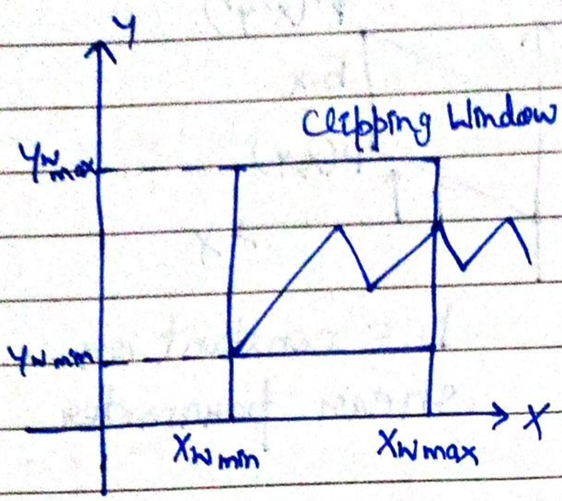
$$(x'\ y'\ 1) = (x\ y\ 1)\begin{bmatrix} 1 & b & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

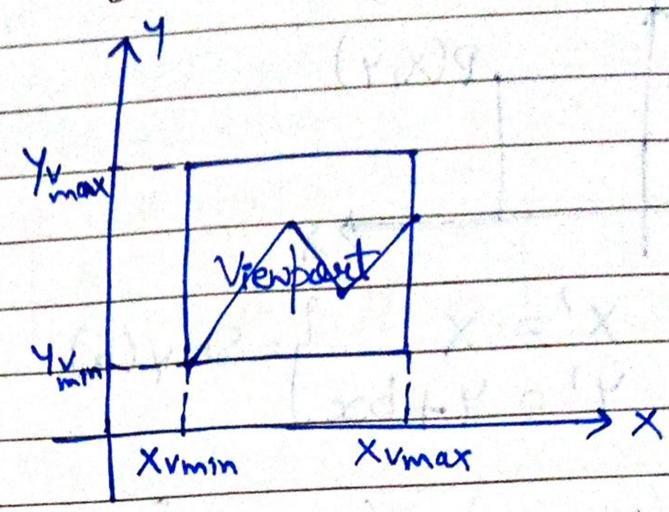
$$\therefore \boxed{P'_n = P_n \cdot Sh_{xy}(a,b)}$$



DELTA Notebook

Window to Viewport Transformation :-





a.) X_v & Y_v = ?

$$\frac{X_v - X_{vmin}}{X_{vmax} - X_{vmin}} = \frac{X_w - X_{wmin}}{X_{wmax} - X_{wmin}}$$

$$\frac{Y_v - Y_{vmin}}{Y_{vmax} - Y_{vmin}} = \frac{Y_w - Y_{wmin}}{Y_{wmax} - Y_{wmin}}$$

$$X_v = X_{vmin} + (X_w - X_{wmin}) S_x$$
$$Y_v = Y_{vmin} + (Y_w - Y_{wmin}) S_y$$

$$S_x = \frac{X_{vmax} - X_{vmin}}{X_{wmax} - X_{wmin}}$$

$$S_y = \frac{Y_{vmax} - Y_{vmin}}{Y_{wmax} - Y_{wmin}}$$

Mapping of window to viewport

1) Scaling
2) Translation

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Windowing & clipping :-

=> Window - A world coordinate area selected for display is called a window.

=> Viewport - A area on a display device to which a window is mapped is called a viewport

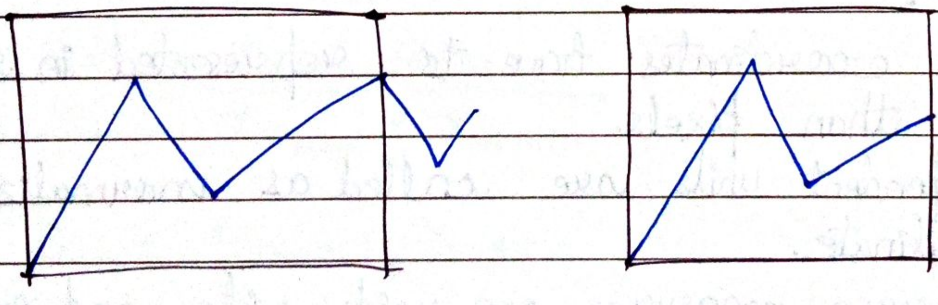
1) Diff.

Window Transformation

1) Different display devices have diff. screen sizes and resolutions which is measured in pixels.
2) Pictures on a screen with high resolutions appears small in size, whereas same picture on screen with less resolution appears ~~with~~ big in size.
3) To avoid this we make our program device independent.
4) So, picture coordinates have to represented in some units other than pixels.
5) Device Independent units are called as normalized device coordinate.
6) In this, screen measures one unit wide and one unit high.

Viewing Pipe line - 2D Viewing Transformations

Modelling Coordinates → Construct World coordinates scene from modelling coordinate Transformations → World Coordinates → Convert World coordinates to viewing Coordinates → Viewing Coordinates → Project Transformation → Projection Coordinates → Transform Projection to Normalized Coordinates → Normalized Coordinates → Map Normalized Coordinating to Device Coordinates → Device Coordinates



Window Viewport

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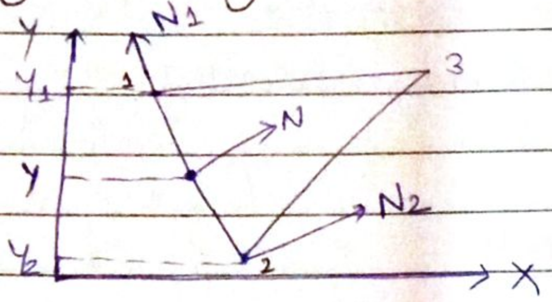
Phong Shading :-



$$N = \frac{Y - Y_2}{Y_1 - Y_2} N_1 + \frac{Y_1 - Y}{Y_1 - Y_2} N_2$$

Gouraud Shading

Intensity Interpolation.

1.) Avg. Unit Normal vector
2.) Apply Illumination model
3.) Linearly interpolate the vertex Intensity

• Disadvantage

→ Shading Machbands.

Phong Shading :-

1.) Determine average unit normal vector at each polygon vertex.
2.) Linearly interpolate the vertex normals over the surface of polygon.
3.) Apply illumination model along each scan line to find pixel Intensities.

• Advantages :-

a.) It reduces the machband effect.
b.) It displays more ~~rel~~ realistic highlights on the surface.
c.) It gives more accurate results.
~~d.) It requires~~

DELTA Notebook

• Disadvantages :-
a) It requires more calculation.
b) More Extensive.
c) It is slower than Gouraud Shading.

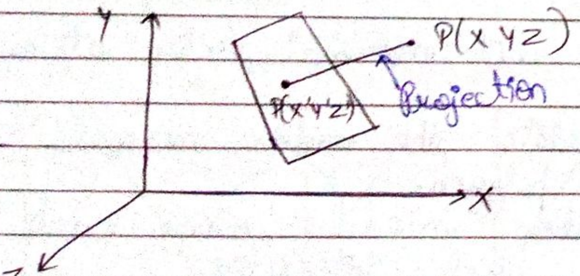
UNIT-4

Projections

Parallel Projection | Perspective Projection

→ Parallel Projection is the projection when the lines of projections are parallel to each other.

→ Lines of projections are not parallel to each other and converge (Meet) at COP.



1.) View Plane or Plane of projection (POP).
2.) Projectors.
3.) Centre of Projection (COP).

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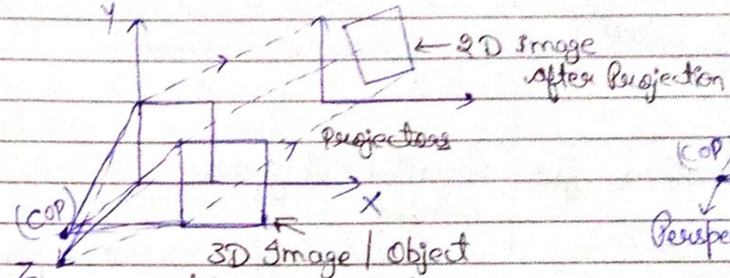
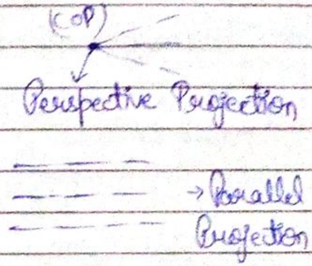
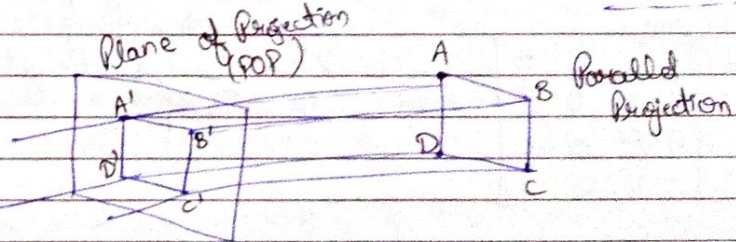
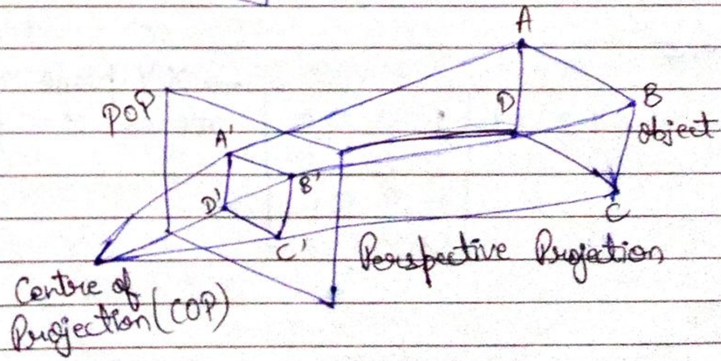
Projections :- It is the process of converting a 3D object into a 2D object. ~~Into~~

n	n-1
3D	2D
(X Y Z)	(X Y)
	z=0 plane









Delta

Projection
- Parallel
  - Orthographic (Perpendicular to the projection line.)
  - Oblique (Not Perpendicular)
- Perspective
  - 1 point
  - 2 point
  - 3 point

Transformations for Parallel Projections

$$P_{Parallel,Z} = \begin{cases} x' = x \\ y' = y \\ z' = 0 \end{cases}$$

X Y Plane or Z=0 plane

P(X Y Z) is any point in space then projected point P'(x'y'z') can be obtained as

$P' = P \cdot P_{par,Z}$

In Matrix Form,

$$P_{par,Z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P'_h = P_h \cdot P_{ar,Z}$

→ XY plane means Z=0 plane

$$(x'\ y'\ z'\ 1) = (x\ y\ z\ 1)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Delta

DDA Line Drawing Algorithm :-

⇒ DDA :- It stands for Digital Differential Analyzer. It is an incremental method of scan conversion of line. In this method calculation is performed at each step but by using results of previous step:-

Step 1 :- Start Algo.
Step 2 :- Declare x, y, x_2, y_2 dx dy x.y
Step 3 :- Enter Value of $x, y, x_2\ y_2$
Step 4 :- $dx = x_2 - x_1$
Step 5 :- $dy = y_2 - y_1$
Step 6 :- if ABS(dx) > ABS(dy)
then, step = abs(dx)
else step = abs(dy)
Step 7 :- $x_{inc} = dx / step$
$y_{inc} = dy / step$
assign $x = x_1$ & $y = y_1$.
Step 8 :- Set pixel(x, y)
Step 9 :- $x = x + x_{inc}$
$y = y + y_{inc}$
Set Pixels(Round(x), Round(y))

↳ ABS = Modulus
If (−ve) value occurs then we will considered is (+ve)

Step 10 :- Repeat step 9 until $\boxed{x = x_2}$

Slope (m) $= \frac{\Delta dy}{\Delta dx}$ or $\frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1}$

Q1 :- $P_1(5,5)$ $P_2(10,9)$

⇒ $x_1 = 5$, $y_1 = 5$ & $x_2 = 10$, $y_2 = 9$

$dx = x_2 - x_1 = 10 - 5 = 5$
$dy = y_2 - y_1 = 9 - 5 = 4$
⇒ if $5 > 4$, step = 5

$x_{inc} = \frac{5}{5} = 1$ $y_{inc} = \frac{4}{5} = 0.8$

1) $x = 5$, $y = 5$
Point (5, 5)

2.) $x = x + x_{inc} = 5 + 1 = 6$
$y = y + y_{inc} = 5 + 0.8 = 5.8$
Point (6, 6)

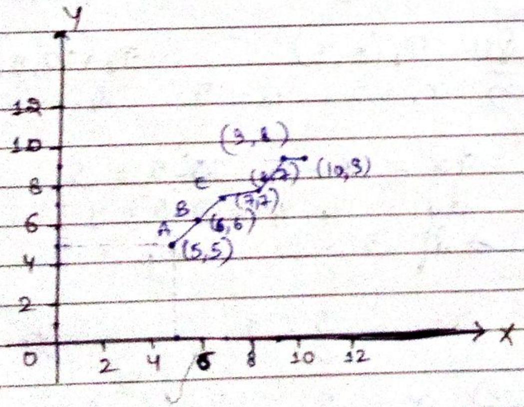
3.) $x = x + x_{inc} = 6 + 1 = 7$
$y = y + y_{inc} = 5.8 + 0.8 = 6.6$
$\therefore$ Point $(7, 7)$

4.) $x = x + x_{inc} = 7 + 1 = 8$
$y = y + y_{inc} = 6.6 + 0.8 = 7.4$
$\therefore$ Point $(8, 7)$

5.) $x = x + x_{inc} = 8 + 1 = 9$
$y = y + y_{inc} = 7.4 + 0.8 = 8.2$
$\therefore$ Points $(9, 8)$

6.) $x = x + x_{inc} = 9 + 1 = 10$
$y = y + y_{inc} = 8.2 + 0.8 = 9.2$
$\therefore$ Points $(10, 9)$

x	y
5	5
6	6
7	7
8	7
9	8
10	9



DDA

Q2:- $P_1 = (2, 3)$, $P_2 = (6, 15)$

Sol:- $x_1 = 2$ & $y_1 = 3$, $x_2 = 6$ & $y_2 = 15$

$dx = x_2 - x_1 = 6 - 2 = 4$
$dy = y_2 - y_1 = 15 - 3 = 12$

$\therefore$ if $ABS(dx) > ABS(dy)$
$4 > 12$ = Not Satisfied
else
step = abs(dy) = 12

$\Rightarrow x_{inc} = \frac{dx}{step} = \frac{4}{12} = \frac{1}{3}$

$\Rightarrow y_{inc} = \frac{dy}{step} = \frac{12}{12} = 1$

1.) $x_1, y_1 = 2, 3$

2.) $x_2 = x_1 + x_{inc} = 2 + 1/3 = 7/3 = 2$
$y_2 = y_1 + y_{inc} = 3 + 1 = 4$

3.) $x_3 = x_2 + x_{inc} = 2 + \frac{1}{3} = 3$
$y_3 = y_2 + y_{inc} = 4 + 1 = 5$

4.) $x_4 = x_3 + x_{inc} = 3 + \frac{1}{3} = \frac{9+1}{3} = \frac{10}{3} = 3$
$y_4 = y_3 + y_{inc} = 5 + 1 = 6$

Delta

5.) $x_5 = x_4 + x_{inc} = 3 + \frac{1}{3} = 3$

$y_5 = y_4 + y_{inc} = 6 + 1 = 7$

6.) $x_6 = x_5 + x_{inc} = 3 + \frac{1}{3} = 4$

$y_6 = y_5 + y_{inc} = 7 + 1 = 8$

7.) $x_7 = x_6 + x_{inc} = \frac{4}{1} + \frac{1}{3} = \frac{12+1}{3} = \frac{13}{3} = 4$

$y_7 = y_6 + y_{inc} = 8 + 1 = 9$

8.) $x_8 = x_7 + x_{inc} = 4 + \frac{1}{3} = 4$

$y_8 = y_7 + y_{inc} = 9 + 1 = 10$

9.) $x_9 = x_8 + x_{inc} = 4 + \frac{1}{3} = 5$

$y_9 = y_8 + y_{inc} = 10 + 1 = 11$

10.) $x_{10} = x_9 + x_{inc} = \frac{5}{1} + \frac{1}{3} = \frac{15+1}{3} = \frac{16}{3} = 5$

$y_{10} = y_9 + y_{inc} = 11 + 1 = 12$

11.) $x_{11} = x_{10} + x_{inc} = 5 + \frac{1}{3} = 5$

$y_{11} = y_{10} + y_{inc} = 12 + 1 = 13$

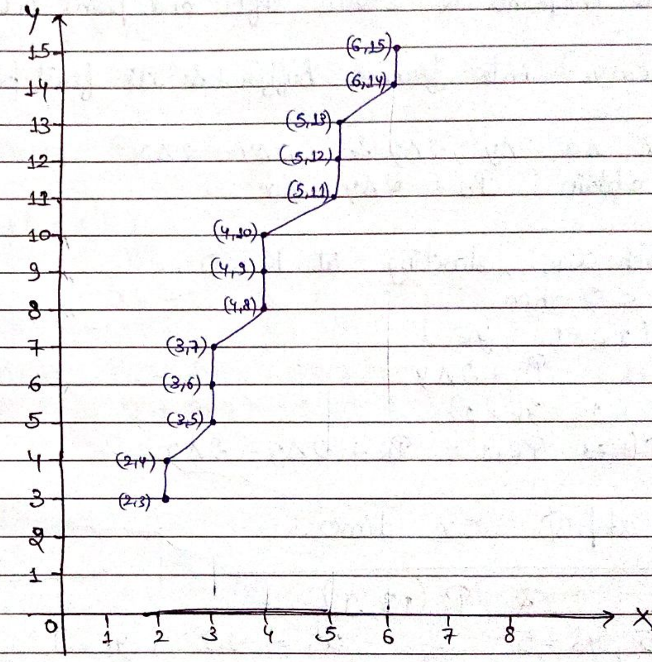
Delta

12.) $x_{12} = x_{11} + x_{inc} = 5 + \frac{1}{3} = 6$

$y_{12} = y_{11} + y_{inc} = 13 + 1 = 14$

13.) $x_{13} = x_{12} + x_{inc} = 6 + \frac{1}{3} = 6$

$y_{13} = y_{12} + y_{inc} = 14 + 1 = 15$



x	y
2	3
2	4
3	5
3	6
3	7
4	8
4	9
4	10
5	11
5	12
5	13
6	14
6	15

Bresenham's Line Algorithm :-

→ It is used for scan converting a line.
→ It Involves only integer addition, ~~subst~~ subtraction & multiplication operations.
→ Next pixel selected is that one who has the least distance from true line.

Step 1:- Input 2 endpoints & store left end point in (x_0, y_0)

Step 2:- Load (x_0, y_0) into frame buffer & its first point

Step 3:- Calculate Δx, Δy, $2\Delta y$ & $2\Delta y - 2\Delta x$ and obtain $P_0 = 2\Delta y - \Delta x$

Step 4:- At each x_k, starting at $k = 0$
if, $P_k < 0$ then
$(x_k + 1, y_k)$
and $P_{k+1} = P_k + 2\Delta y$
else $(x_k + 1, y_k + 1)$
and ~~Pk~~ $P_{k+1} = P_k + 2\Delta y - 2\Delta x$

Step 5:- Repeat step ④ Δx times.

Q1:- $P_1(5, 5)$ & $P_2(13, 9)$
Sol:- $x_0 = 5, y_0 = 5$ & $x_1 = 13$, $y_1 = 9$

$dx = x_1 - x_0 = 13 - 5 = 8$
$dy = y_1 - y_0 = 9 - 5 = 4$

$P_0 = 2\Delta y - \Delta y = 2(4) - 8 = 8 - 8 = 0$

1.) $P_1 = P_0 + 2\Delta y - 2\Delta x$
$= 0 + 2(4) - 2(8) = 0 + 8 - 16 = -8$
$\therefore (x_k + 1, y_k + 1) = (5+1), (5+1)$
Points $= (6, 6)$

~~2.) $P_2 = P_1 + 2\Delta y - 2\Delta x$~~
~~$= -8 + 2(4) - 2(8) = -8 + 8 - 16 =$~~

At $(6, 6)$:-
$P_k < 0$ → $(7, 6)$ = New Points

$P_k = P_1 + 2\Delta y = -8 + 2(4) = 0$

At $(7, 6)$:-
$P_k = 0$
$x_k + 1, y_{k+1} = 7+1, 6+1 = (8, 7)$

$P_{k+1} = P_k + 2\Delta y - 2\Delta x$
$= 0 + 2(4) - 2(8) = 0 + 8 - 16 = -8$

At $(8, 7)$:-
$P_k < 0$
$\therefore (x_{k+1}, y) = (8+1, 7) = (9, 7)$

$P_k = P_{k+1} + 2\Delta y = -8 + 2(4) = -8 + 8 = 0$

At $(9, 7)$:-
$P_k = 0$, $(10, 8)$
$\therefore P_{k+1} = 0 + 2(4) - 2(8)$
$= 0 + 8 - 16 = -8$

At (10,8) :-

$P_K < 0$, (11,8)

$P_K = -8 + 2(4) = 0$

At (11,8) :-

$P_K = 0$, (12,9)

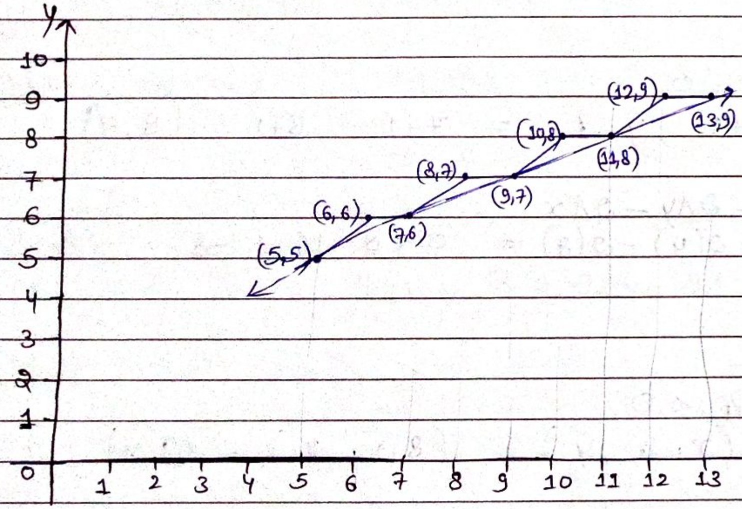
$P_K = 0 + 2(4) - 2(8) = 8 - 16 = -8$

At (12,9) :-

$P_K < 0$, (13,9)

$P_K = -8 + 8 = 0$

x	y
5	5
6	6
7	6
8	7
9	7
10	8
11	8
12	9
13	9

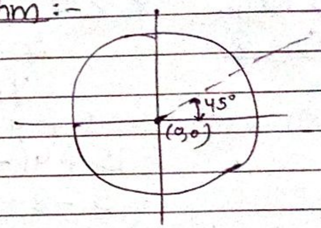


Bresenham circle Drawing Algorithm :-

(0,0) & $r = 8$

1) Start Point → $x_i = 0$

$y_i = r = 8$



2) Decision Parameter (P)

$P_i = 3 - 2R = 3 - 2(8) = 3 - 16 = -13$

Case 1:- $P < 0$:-

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i$

$P_{i+1} = P_i + 4(x_{i+1}) + 6$

Case 2:- $P \geq 0$:-

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i - 1$

$P_{i+1} = P_i + 4(x_{i+1} - y_{i+1}) + 10$

1.) Here, $P < 0$, (0,8)

Using Case 1:- $x_{i+1} = 0 + 1 = 1$

$y_{i+1} = 8$

$P_{i+1} = -13 + 4(1) + 6$

$= -13 + 4 + 6 = -13 + 10 = -3$

∴ New Points = (1,8)

2.) Now, $P < 0$, (1,8)

14
-3
11

Case 1:- $x_1 = 1 + 1 = 2$

$y_1 = 8$

$P_1 = -3 + 4(2) + 6 = -3 + 8 + 6 = -3 + 14 = 11$

∴ New Points = (2,8)

11 -6

11
+10
21
-16
5

3.) Now, $P \geq 0$, (2,8)

Case 2:- $x_1 = 2 + 1 = 3$

$y_1 = 8 - 1 = 7$

$P_1 = 11 + 4(3 - 7) + 10 = 11 + 4(-4) + 10 = 11 - 16 + 10 = 5$

∴ New Points = (3,7).

Delta

Dt:
Pg.:

13-8

4.) Now, $P \geq 0$, $(3,7)$

Case 2:- $x_1 = 3+1 = 4$

$y_1 = 7-1 = 6$

$P_1 = 5 + 4(4-6)+10$

$= 5+4(-2)+10$

$= 5-8+10 = 7$

$\therefore$, New Points $= (4,6)$.

5.) Now, $P \geq 0$, $(4,6)$

Case 2:- $x_1 = 4+1 = 5$

$y_1 = 6-1 = 5$

$P_1 = 7 + 4(5-5)+10$

$= 7+4(0)+10 = 7+10 = 17$

$\therefore$, New Points $= (5,5)$.

Delta

Dt:
Pg.:

23/10/24

Bezier Curve :-

$\Rightarrow$ Control Points - $n+1$

$\Rightarrow$ Position vector $P(u)$ - describes path of an approximating Bezier Polynomial function between P_0 & P_n.

Bezier Curve : $P(u) = \sum_{i=0}^{n} P_i B_{n,i}(u)$ —— (1)

where:- $B_{n,i} = nC_i u^i (1-u)^{n-i}$ —— (2)

Cubic Bezier Curve $n = 3$

$P(u) = \sum_{i=0}^{3} P_i B_{i,3}(u)$ —— (3)

$\Rightarrow P_0 B_{3,0}(u) + P_1 B_{3,1}(u) + P_2 B_{3,2}(u) + P_{3,3}(u)$

Now, find each Blending function

$B_{n,i}(u) = nC_i \cdot u^i (1-u)^{n-i}$

a.) $B_{3,0}(u) = 3C_0 u^0 (1-u)^{3-0}$

$= \frac{3!}{0!(3-0)!} \cdot 1(1-u)^3 = (1-u)^3$

b.) $B_{3,1}(u) = \frac{3!}{1!(3-1)!} \cdot u^1 (1-u)^{3-1}$

$= \frac{3!}{1!(2)!} u \cdot (1-u)^2 = 3u(1-u)^2$

c.) $B_{3,2}(u) = 3C_2 u^2 (1-u)^{3-2} = \frac{3!}{2!(3-2)!} u^2 (1-u)$

$= 3u^2(1-u)$

d.) $B_{33}(u) = 3C_3 u^3 (1-u)^{3-3} = \frac{3!}{3!(3-3)!} u^3$

Using a, b, c, d

$$P(u) = P_0(1-u)^3 + 3P_1 u(1-u)^2 + 3P_2 u^2(1-u) + P_3 u^3$$

Q. Perform 2D translation of a line.
Q:- Perform 2D scaling of a triangle in C.
Q:- Perform the following 2D transformation operation — translation, rotation, scaling.

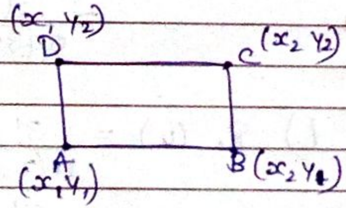
2D transformation :-

⇒ It is essential for graphics system to allow user to change the way objects appear.
⇒ The effects that changing size, position or its orientation is called Transformation.

$$[X'] = [X][T]$$

→ Rotation
→ Translation
→ Reflection
→ Scaling
→ Shearing

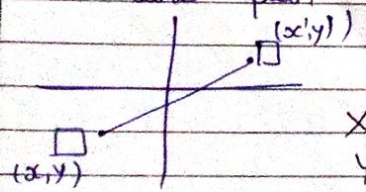
$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_1 \\ x_2 & y_2 \\ x_1 & y_2 \end{bmatrix}$$



Translation :- Repositioning an object along a straight line path.

Eg:- $P = (-3.7, -4.1)$ (x) (y) (t_x) (t_y)

$T = (7.1, 8.2)$



$x' = x + t_x$
$y' = y + t_y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Q:- $P = (8.6, -1)$
$T = (0.4, -0.2)$

$$\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 8.6 \\ -1 \end{bmatrix} + \begin{bmatrix} 0.4 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 9 \\ -1.2 \end{bmatrix}$$ Ans.

* Scale :- Alter the size of an object.

$P = (x, y)$
$S = (S_x, S_y)$
$x' = S_x \cdot x$
$y' = S_y \cdot y$

$$\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Case 1:- $S_x = S_y = 1$, then $x' = x$ & $y' = y$
Case 2:- When $S_x = S_y > 1$, then object ~~exa~~ expands
Case 3:- When $S_x = S_y$ & $0 < S_x, S_y < 1$, then object gets compressed.
Case 4:- When $S_x \neq S_y$. There is no uniform scaling

Q:- $P = (1.4, 2.2)$
$S = (3, 3)$

⇒ $x' = 3 \times 1.4 = 4.2$
$y' = 3 \times 2.2 = 6.6$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4.2 \\ 6.6 \end{bmatrix}$$

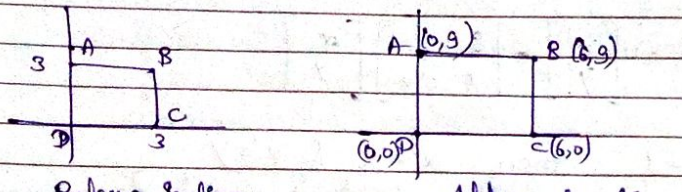
Q:- A(0,3) B(3,3) C(3,0) D(0,0)
Apply scaling 2 towards X-axis & 3 towards Y-axis.

Sol:- A(0,3) → $x' = S_x \cdot x = 2 \times 0 = 0$ ∴ $A' = (0, 9)$
$S_x = 2$ $y' = S_y \cdot y = 3 \times 3 = 9$
$S_y = 3$

B(3,3) → $x' = 2 \times 3 = 6$ ∴ $B' = (6, 9)$
$y' = 3 \times 3 = 9$

$C(3,0) \rightarrow x' = 2\times 3 = 6$, $y' = 3\times 0 = 0$ ∴ $C' = (6,0)$

$D(0,0) \rightarrow x' = 2\times 0 = 0$, $y' = 3\times 0 = 0$ ∴ $D' = (0,0)$



Before Scaling   After Scaling

* **Rotation:-** Repositions an object along a circular path.

⇒ Types:-

- **Anti-Clockwise:-** The +ve value of pivot (θ°) point rotates an object in counter clockwise.
- **Clockwise:-** The -ve value of pivot point rotates an object in clockwise.

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Clockwise

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Anti-Clockwise

Q:- $A(3,4)$ & $B(12,15)$ Rotate through 45° anticlockwise

Sol. $R = \begin{bmatrix} \cos 45° & \sin 45° \\ -\sin 45° & \cos 45° \end{bmatrix} = \begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 \\ -1/\sqrt2 & 1/\sqrt2 \end{bmatrix}$

$A(3,4) = [x'\ y'] = [3\ \ 4]\begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 \\ -1/\sqrt2 & 1/\sqrt2 \end{bmatrix}$

$= \left[\frac{3}{\sqrt2} - \frac{4}{\sqrt2} \quad \frac{3}{\sqrt2} + \frac{4}{\sqrt2}\right] = \left[-1/\sqrt2 \quad 7/\sqrt2\right]$

$B(12,15) = [x'\ y'] = [12\ \ 15]\begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 \\ -1/\sqrt2 & 1/\sqrt2 \end{bmatrix}$

$= \left[\frac{12}{\sqrt2} - \frac{15}{\sqrt2} \quad \frac{12}{\sqrt2} + \frac{15}{\sqrt2}\right]$

$= \left[-3/\sqrt2 \quad 27/\sqrt2\right]$

∴ So, line AB After rotation at 45° become

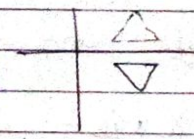
$\left[-1/\sqrt2 \quad 7/\sqrt2\right]$, $\left[-3/\sqrt2 \quad 27/\sqrt2\right]$

or

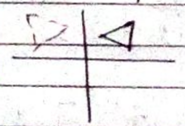
$[-0.707 \quad 4.94]$, $[-2.121 \quad 19.089]$

* **Reflection:-** It produces a mirror image of an object

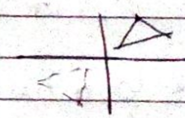
→ Reflection about x:- $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$



→ Reflection about y:- $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

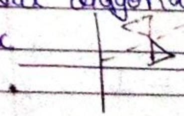


→ Reflection about origin:- $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$



→ Reflection about diagonal:-

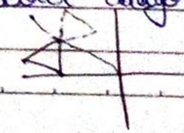
$y = x$



$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

→ Reflection about diagonal

$y = -x$



$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

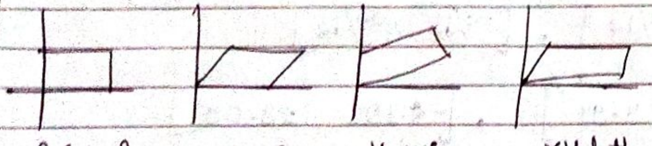
* **Shearing**:- 2D Shearing is an Ideal technique to change the shape of existing object in 2D plane.

X-axis ⇒ $X_{new} = X_{old} + Sh_x * Y_{old}$ $\quad \begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$

$Y_{new} = Y_{old}$

Y-axis ⇒ $X_{new} = X_{old}$ $\quad \begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$

$Y_{new} = Y_{old} + Sh_y * X_{old}$



XY both direction $= \begin{bmatrix} 1 & Sh_x \\ Sh_y & 1 \end{bmatrix}$

Q:- A(1,1) B(0,0) C(1,0). Apply 2 on X-axis & 2 on Y-axis

Sol:- $Sh_x = 2$ & $Sh_y = 2$

X-axis:- $Sh_x = 2$

A(1,1) = $X_{new} = 1 + 2 \times 1 = 1 + 2 = 3$

$Y_{new} = 1$

∴ A'(3,1)

B(0,0) = $X_{new} = 0 + 2 \times 0 = 0$

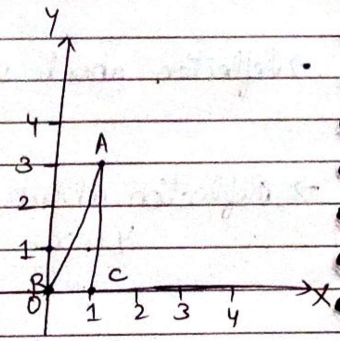
$Y_{new} = 0$

∴ B'(0,0)

C(1,0) = $X_{new} = 1 + 2 \times 0 = 1$

$Y_{new} = 0$

∴ C'(1,0)



Y-axis:- $Sh_y = 2$

A(1,1), $Y_{new} = 1 + 2 \times 1 = 3$

$X_{new} = 1$

A'(1,3)

B(0,0) = $Y_{new} = 0$, $X_{new} = 0$ B'(0,0)

C(1,0) = $Y_{new} = 0 + 2 \times 1 = 2$, $X_{new} = 1$, C'(1,2)