Chapter 4

Formation and Evolution of Stars

The equations of stellar structure, as given in the previous chapter (equations 3.39 through 3.43), do not contain any time dependence. However, although the properties of stars usually change slowly, they must necessarily change. Stars have a non-zero luminosity and a finite fuel supply. This implies that they began nuclear fusion at some time in the past, and will exhaust their fuel supply at some time in the future. In this chapter, we will address some of the issues of stellar evolution, starting with the interstellar medium that supplies the raw material of star formation, and ending when a star wrings the last possible joule from nuclear fusion. (Chapter 5 will discuss stellar remnants – the dense "corpses" left over when a star no longer is powered by fusion.)

4.1 Interstellar Dust

Stars are formed by gravitational compression of the **interstellar medium**; that is, the low density mix of dust and gas that lies in the space between stars. As noted in section 2.5, the dust between stars can be detected because it causes extinction and reddening of starlight. Careful observations of dust yield clues that help us to determine the size, shape, and composition of dust grains.

Clue one: Dust causes reddening at both visible and ultraviolet wavelengths. Figure 4.1 shows the extinction A_λ as a function of wavelength λ, plotted as a function of $1/\lambda$. At all wavelengths $\lambda > 2200$ Å, correspond-

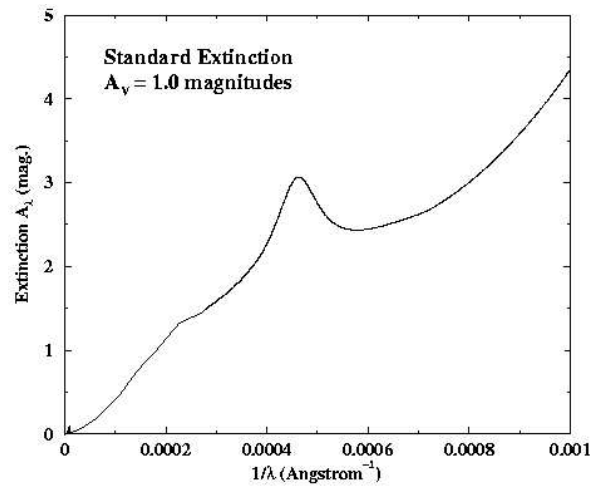

Figure 4.1: Dust extinction as a function of wavelength, normalized so that $A_V = 1$ magnitude.

ing to $1/\lambda < 0.00045$, the amount of extinction decreases with increasing wavelength. This differential extinction can only happen if the individual dust grains are smaller than the wavelength of light that they are scattering ($d \leq \lambda$, where d is the length of a dust grain). If interstellar matter were made of pebbles or boulders rather than dust grains, it would absorb all wavelengths of visible and near ultraviolet light equally. Detailed studies of the extinction as a function of wavelength give an estimate of $d \approx 50 \rightarrow 2000$ Å for the size of the dust particles. These minuscule grains are much smaller than the dust particles that you sweep from under the bed; they are more like the particles in cigarette smoke.

Clue two: Starlight is polarized by dust grains. A Polaroid filter polarizes light because it contains long polymer molecules that are aligned preferentially in one direction. Dust grains polarize light because they are nonspherical, and are aligned preferentially in one direction.[1] If dust grains were perfect spheres, they wouldn't cause polarization of light.

Clue three: The plot of extinction versus wavelength (Figure 4.1) has a "bump" at $\lambda \sim 2000$ Å $\sim 0.2\,\mu$m. What causes the excess extinction at this wavelength? It is known experimentally, that in graphite the bonds

[1]Dust grains tend to line up perpendicular to the interstellar magnetic field.

between carbon atoms absorb and emit light at wavelengths $\lambda \sim 0.2\,\mu$m. In the laboratory, and excellent fit to the extinction bump is given by graphite grains with $d \sim 0.02\,\mu$m $\sim 200\,$Å. (On the Earth, you find such tiny graphite particles in soot.)

Clue four: If you look at the infrared spectrum from isolated dusty clouds, you find that at long wavelengths ($\lambda > 100\,\mu$m), it's a blackbody spectrum with a typical temperature of $T \sim 200\,$K. At shorter wavelengths (Figure 4.2), there are absorption bands at $\lambda \sim 5\,\mu$m due to ices (frozen water,

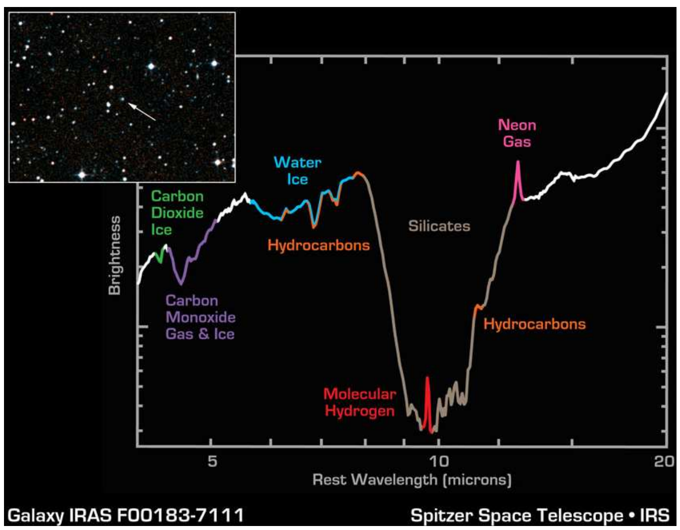

Figure 4.2: Infrared emission from a dust-rich galaxy. [Image credit: Spitzer Space Telescope]

carbon dioxide, carbon monoxide, and so forth) and at $\lambda \sim 10\,\mu$m due to silicates (a.k.a. "rock"). Thus, dust seems to contain a mix of volatile material, like ices, and refractory material, like silicates and graphite.

Models of dust featuring ice-covered flecks of silicate, mixed with small graphite grains, provide a good match with observations. Where does the dust come from? Cool supergiant stars, like Betelgeuse, have very strong stellar winds, a million times stronger than the solar wind. As the wind expands, it cools, and refractory material like graphite and silicates can condense out into tiny grains. A thin layer of frost can later form on these grains in the coolest, densest regions of the interstellar medium.

4.2 Interstellar Gas

At visible wavelengths, dust is the most prominent component of the interstellar medium. However, it only contributes a minority of the material between stars. By mass, the interstellar medium is only 1% dust; the remaining 99% is contributed by **interstellar gas**.[2] Although the bulk of the gas is low-density hydrogen and helium, which is not necessarily easy to detect, there are many methods we can use to detect interstellar gas.

Method one of gas detection: Interstellar gas can be detected when it creates **absorption lines** in a star's spectrum. Kirchhoff's laws (*BA*, section 6.3) tell us that a relatively cool gas cloud seen against a hotter source of continuum emission (such as a star) will produce an absorption line spectrum. Figure 4.3 shows a small portion of spectrum of the star Zeta Ophiuchi, an O9 main sequence star that has $A_V \sim 1.5\,\mathrm{mag}$ from intervening dust. The

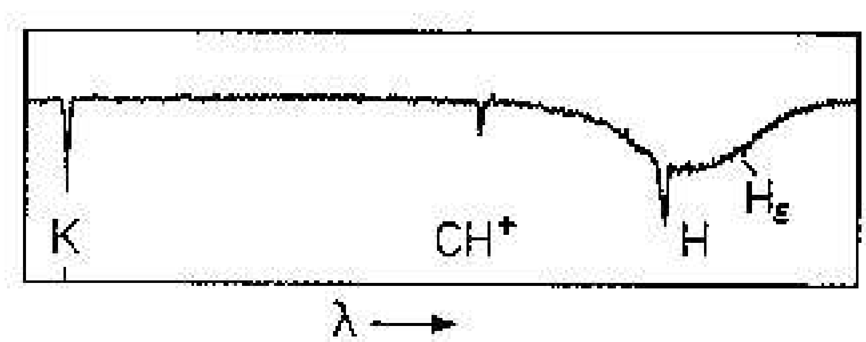

Figure 4.3: Narrow absorption lines of Ca II (labeled 'K' and 'H') and of CH^+, due to interstellar gas between us and ζ Ophiuchi.

narrow absorption lines are due to cool interstellar gas between us and the star. How do we know the absorption lines are from interstellar gas, and not from the outer layers of Zeta Ophiuchi itself?

- The lines are narrow, with very little thermal or pressure broadening. (Note how broad the Balmer ϵ line is in Figure 4.3; this line comes

[2]Parenthetic aside: the processes that cause gas to curdle up into stars are not ultra-efficient; in our galaxy, the total mass of interstellar gas is equal to 10 to 20% of the stellar mass.

Figure 4.4: The Orion Nebula; this Hubble Space Telescope image shows a region ~ 0.5 degrees across. [Image credit: NASA, ESA]

from the star's photosphere, which has both high temperature and high pressure, since Zeta Ophiuchi has a spectral type O9V.)

- The lines are of atoms in a low ionization state, or of molecules, which indicates a lower temperature than you would find the star's atmosphere.
- The lines show a different Doppler shift from the star's absorption lines.
- In general, interstellar absorption lines can be multiple, if there happen to be many individual gas clouds along the line of sight to the star.

Study of absorption lines from gas clouds indicates that interstellar clouds, like stellar photospheres, are mostly hydrogen and helium.

Method two of gas detection: Interstellar gas can be detected when it produces an **emission line spectrum**. Kirchhoff's laws tell us that a relatively warm gas cloud seen against a dark background will produce an emission line spectrum. An interstellar gas cloud that is hot enough to emit light at visible wavelengths is called an **emission nebula**. The Orion Nebula, shown in Figure 4.4, is a famous example of an emission nebula. To produce emission lines, electrons have to be lifted above the ground state (the $n = 1$ orbit) of their atom. Since interstellar gas is low in density, excitation of electrons is unlikely to occur by collisions between atoms. Instead, it occurs by the absorption of photons. In general, a nebula's gas absorbs ultraviolet light, which ionizes some of its atoms. Eventually, the ions and electrons recombine, but when they do, the electron usually is in a high energy level. As the electron cascades down to the lowest vacant energy level, it emits a photon with each downward quantum leap it makes. Thus, emission nebulae are an example of fluorescence, in which high-energy ultraviolet photons are converted to lower-energy visible photons. The source of ultraviolet photons varies between different types of emission nebulae:

- **HII region:** an emission nebula in which the source of ultraviolet light is one or more hot stars (spectral type O or B). The Orion Nebula and the Trifid Nebula are examples of HII regions.

- **Planetary nebula:** an emission nebula in which the source of ultraviolet light is a hot white dwarf. The Ring Nebula and the Spirograph Nebula are examples of planetary nebulae.[3]

The size of an emission nebula depends on the rate at which its central hot object can pour out ionizing photons. The more luminous the central object at ultraviolet wavelengths, the larger the ionized region around it.

Method three of gas detection: Interstellar gas can be detected when it produces continuum radio emission. When a hot gas is largely ionized, it can produce photons by the process known as **bremsstrahlung**, a German term that is literally translated as "braking radiation". Bremsstrahlung, also known as free-free emission, is produced when a free electron is accelerated by the electrostatic attraction of a positively charged ion. The electron emits a photon as it is accelerated. The initial kinetic energy of the electron, before it encounters the ion, will be of order the thermal energy, kT. Since the electron can't radiate away more energy than it has originally, the energy of the photon that it emits must be $hc/\lambda < kT$, implying

$$\lambda > \frac{hc}{kT} \sim 0.1\,\mathrm{mm}\left(\frac{T}{10^4\,\mathrm{K}}\right)^{-1} \tag{4.1}$$

for bremsstrahlung emission from a hot, ionized gas. Bremsstrahlung is thus of interest mainly to radio astronomers. If the ionized gas is in the presence of a magnetic field, it can also produce continuum radio emission via

[3] "Planetary nebulae" received their odd and highly inappropriate name from William Herschel in the 18th century. Seen through a small telescope, they look like a fuzzy disk, just as the planet Uranus did when Herschel discovered it.

synchrotron emission, as the electrons are accelerated along helical paths following the magnetic field lines. Synchrotron emission is produced copiously by young supernova remnants, in which ionized gas surrounds a highly magnetized neutron star.

Method four of gas detection: Interstellar gas can be detected when hydrogen atoms produce **21 centimeter** line emission. When a neutral hydrogen atom has its electron in the $n = 1$ energy level, the spin of the electron can be either parallel to the spin of the proton (left side of Figure 4.5) or antiparallel to the spin of the proton (right side of Figure 4.5). When the

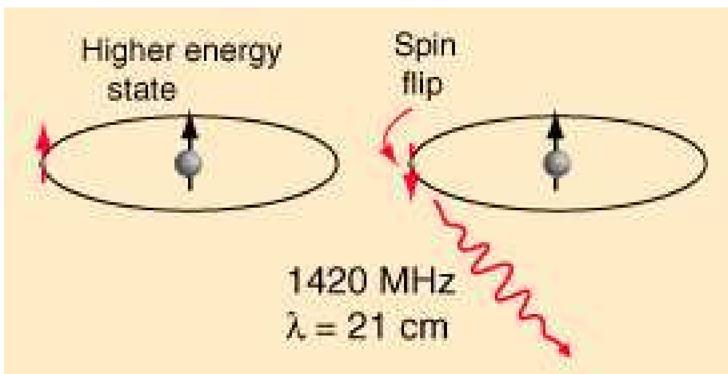

Figure 4.5: Spin flip transition in the hydrogen atom, leading to $\lambda = 21\,\mathrm{cm}$ emission.

spins are antiparallel, the energy of the atom is slightly smaller than when the spins are parallel. Because the antiparallel state is lower in energy, when the spins are parallel to each other, the electron can undergo a spontaneous "spin-flip" transition that inverts its spin. Because this transition violates a quantum mechanical selection rule, it is a forbidden transition. (Remember from section 6.1 of *BA* that a "forbidden" transition is not absolutely forbidden; it is merely far less probable than a "permitted" transition that doesn't violate any selection rules.) The half-life of the higher energy parallel state is roughly 10 million years. Thus, if you have a population of 3×10^{14} hydrogen atoms in the parallel state, only one atom per second will undergo the spontaneous spin-flip transition. As it does so, the small amount of energy lost is carried away by a photon with $\lambda = 21\,\mathrm{cm}$. An individual hydrogen atom will undergo a spin-flip transition only rarely, and the density of hydrogen atoms in interstellar space is low. However, space is big. Along a sufficiently long line of sight, the spin-flip transitions of hydrogen will produce a detectable signal at $\lambda = 21\,\mathrm{cm}$.

Method five of gas detection: Interstellar gas can be detected when molecules produce **radio line emission**. In the denser regions of interstellar space, individual atoms can join together to form molecules. When a small molecule such as CH or CO spins about one of its axes, it can undergo quantum transitions from one spin state to another. If the transition is from a rapidly rotating state to a less rapidly rotating state, the lost kinetic energy of rotation is carried away by a photon. For instance, when a carbon monoxide (CO) molecule makes a transition from the $J = 1$ rotational state to the $J = 0$ ground rotational state, it emits a photon with $\lambda = 2.6\,\mathrm{mm}$, a wavelength at which the Earth's atmosphere is conveniently transparent. A map of the Milky Way's emission at 2.6 mm (Figure 4.6) reveals that the CO – and by inference, the other molecular gas – in our galaxy is confined to a thin layer near the midplane of the galaxy. At higher resolution, it is found

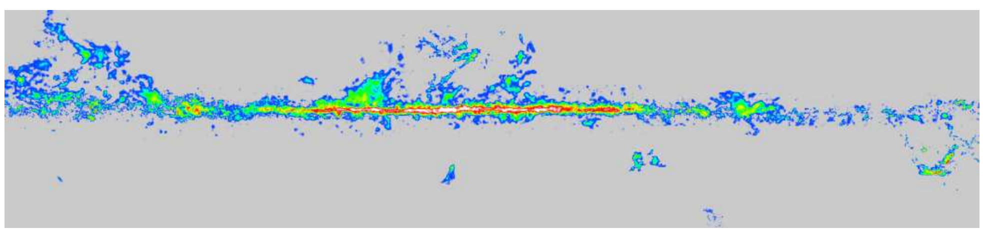

Figure 4.6: False color map of the CO emission from the Milky Way. [Image credit: Astronomy Picture of the Day, 30 April 1997]

that most of the molecules are in relatively small, dense, and cold molecular clouds.

The interstellar medium is by no means homogeneous. Some consists of very hot ionized gas, while some consists of much cooler gas containing H_2 and other molecules. Astronomers have labeled different components of the interstellar gas:

- Cold Molecular Clouds:
  $T \sim 10\,\mathrm{K}$, $n > 10^9\,\mathrm{m}^{-3}$

- Cool Atomic Clouds ('HI regions'):
  $T \sim 100\,\mathrm{K}$, $n \sim 10^8\,\mathrm{m}^{-3}$

- Warm Partially Ionized Gas ('intercloud medium'):
  $T \sim 7000\,\mathrm{K}$, $n \sim 4 \times 10^5\,\mathrm{m}^{-3}$
- Hot Ionized Gas ('HII regions'):
  $T \sim 10^4\,\mathrm{K}$, $n \sim 10^6\,\mathrm{m}^{-3}$
- Very Hot Ionized Gas ('coronal gas'):
  $T \sim 10^6\,\mathrm{K}$, $n < 10^4\,\mathrm{m}^{-3}$

Which component is the most common? That depends on how you look at it: cold molecular clouds contain $\sim 50\%$ of the *mass* of the interstellar medium, while coronal gas comprises $\sim 70\%$ of the *volume*. The coronal gas takes the form of "bubbles" blown by supernovae (exploding stars). The Sun is near the middle of such a bubble, about 100 parsecs in diameter, shown in Figure 4.7. When most of the gas was cleared from this "Local Bubble", most

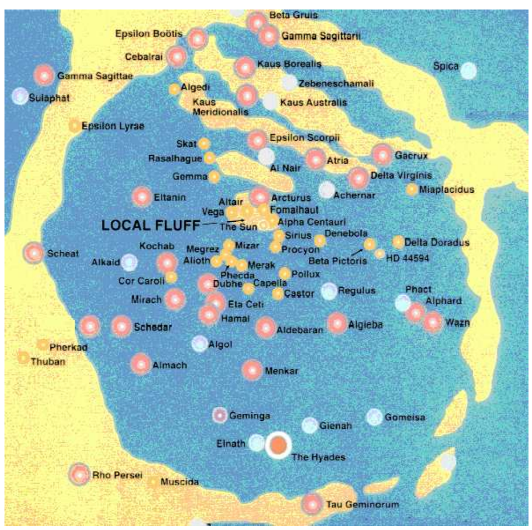

Figure 4.7: Schematic drawing of the Local Bubble; the region pictured is $\sim 100\,\mathrm{pc}$ across. Blue = lower density gas, yellow = higher density gas.

of the dust was as well. Thus, stars less than $\sim 50\,\mathrm{pc}$ from us suffer little extinction from dust. Although a coronal bubble contains gas at a very high temperature, it contains very little thermal energy, thanks to its low density. If you were tossed out the airlock of your spaceship without a spacesuit, you wouldn't have to worry about freezing (or roasting) to death. Heat losses by convection and conduction would be negligible, and it would take about 5 minutes for your temperature to drop by one degree Celsius due to radiative heat losses. By that time, you would have asphyxiated.[4]

Stars form by the gravitational collapse of the densest, coolest regions of the interstellar medium. These regions are the relatively dense cores of molecular clouds, which have densities as high as

$$n_{\rm mc} \sim 10^{12}\,\text{molecules/}\,\text{m}^3\ . \tag{4.2}$$

(To put things in perspective, the Earth's atmosphere at sea level has $n \sim 10^{25}$ molecules per cubic meter; even the densest part of the interstellar medium would qualify as a high-grade vacuum in a terrestrial laboratory.) If the gas is pure molecular hydrogen, the mass density will be

$$\rho_{\rm mc} \approx 2m_p n_{\rm mc} \approx 3\times10^{-15}\,\text{kg}\,\text{m}^{-3} \approx 5\times10^{-12} M_\odot\,\text{AU}^{-3}\ . \tag{4.3}$$

Mixing in heavier particles, like He atoms and CO molecules, will raise the mass density, but this is an adequate order-of-magnitude estimate.

4.3 Star Formation

In order to become a star, even the densest regions of the interstellar gas must obviously be greatly compressed. The average density of the Sun is $\rho_\odot \approx 1400\,\text{kg}\,\text{m}^{-3}$. For a spherical molecular cloud core to become as dense as the Sun, it must decrease its radius by a factor

$$\frac{r_\odot}{r_{\rm mc}} = \left(\frac{\rho_{\rm mc}}{\rho_\odot}\right)^{1/3} \approx \left(\frac{3\times10^{-15}\,\text{kg}\,\text{m}^{-3}}{1400\,\text{kg}\,\text{m}^{-3}}\right)^{1/3} \approx 10^{-6}\ . \tag{4.4}$$

If a star ends with a radius of $1r_\odot$, it must have started with a radius $r \sim 10^6 r_\odot \sim 4000\,\text{AU}$.

The enormous compression required to create a star from interstellar gas is provided by gravity, the universal trash compactor. Consider a spherical gas cloud with an initial radius $r_0 \approx 4000\,\text{AU}$ and mass $M \sim 1M_\odot$. If the

[4]Other unpleasant things happen when you undergo explosive decompression. You rapidly dehydrate as your water content evaporates into the near-vacuum of space. Any gas trapped within your body expands rapidly, so you're likely to burst your eardrums and experience excruciating sinus pains (not to mention the fact that your intestinal tract contains about a liter of gas at any given time...) Still, brief exposure to vacuum is not necessarily fatal.

cloud is not rotating, and is very cold, the molecules of which it is made will fall inward on radial orbits, with eccentricity $e = 1$. An orbiting molecule must satisfy Kepler's Third Law:

$$P^2 = \frac{4\pi^2}{G}\frac{a^3}{M} , \tag{4.5}$$

where P is the orbital period of the molecule, a is the semimajor axis of its highly eccentric orbit, and M is the mass contained in a sphere whose radius is equal to the molecule's distance from the cloud's center. As the molecule plummets inward, M should be constant, since the molecules closer to the cloud's center will fall inward with an equal or shorter period. If the molecule starts at a radius r_0 and falls on a radial orbit, $a = r_0/2$. The freefall time $t_{\rm ff}$ is defined as the time it takes the molecule to reach the center; thus, $t_{\rm ff} = P/2$. From Kepler's Third Law,

$$4t_{\rm ff}^2 = \frac{4\pi^2}{G}\frac{r_0^3}{8M} = \frac{4\pi^2}{G}\frac{r_0^3}{8}\frac{3}{4\pi R_0^3\rho_0} , \tag{4.6}$$

where ρ_0 is the initial average density of the cloud. Thus,

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} \approx 4\times 10^4\,{\rm yr}\left(\frac{3\times 10^{-15}\,{\rm kg\,m^{-3}}}{\rho_0}\right)^{1/2} , \tag{4.7}$$

scaling to the typical density of a molecular cloud core. Note that the freefall collapse time depends only on the average density ρ_0 of a cloud, and not on its initial radius r_0.

It takes a mere 40 millennia for a dense molecular cloud core to collapse under its own gravity. Nevertheless, our galaxy, which has been around for 10 million millennia, is still cluttered with molecular cloud cores that have not collapsed. This is because the clouds are in hydrostatic equilibrium; the inward force due to gravity is balanced by the outward force provided by a pressure gradient. However, not every equilibrium is a *stable* equilibrium.[5] Consider a spherical gas cloud of radius r_0 and mass M, initially in hydrostatic equilibrium. We give it a slight squeeze, so that its radius goes from r_0 to $r_0(1-\epsilon)$. The gravitational force at its surface increases, and the pressure gradient between its center and surface must therefore increase if hydrostatic equilibrium is to be maintained. Consider what happens at the molecular level as the cloud is squeezed:

[5]Consider a pencil balanced on its point; it's in equilibrium, but a tiny disturbance will topple it.

- Molecules near the surface are scrunched closer together, increasing their density and pressure.
- These molecules jostle against molecules a littler further into the cloud, increasing their density and pressure.
- These in turn jostle against molecules still further in, and so on, until the center of the cloud is reached.

In short, the pressure within the sphere is altered by a pressure wave – otherwise known as a **sound wave** – traveling through the cloud. The change in pressure required to restore hydrostatic equilibrium thus travels at the speed of sound. If the inward-traveling sound wave reaches the center in a time less than the freefall time, the cloud is saved. If not, the cloud collapses.

If you're a fan of science fiction, you may be familiar with the tagline of the movie *Alien* (Figure 4.8): "In space, no one can hear you scream." If

Figure 4.8: Why can't they hear you??

this is true, talking about sound in outer space may seem absurd. However, sound waves can travel through space, if their wavelength is long enough. The propagation of sound depends on molecules and atoms bumping into each other. In the low density of an interstellar cloud, a molecule will travel

$\sim 10^4$ km before bumping into another molecule. As a consequence, only sound waves with $\lambda \geq 10^4$ km can travel through a molecular cloud. If you had vocal cords $\sim 10^4$ km long, someone could hear you scream in space – if their eardrums were sensitive to frequencies of $\sim 10^{-5}$ Hz. But we digress.

The collapse time for a cloud is (equation 4.7)

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} \tag{4.8}$$

The time required to build up a pressure gradient within a cloud is

$$t_{\rm press} = \frac{r_0}{c_s} , \tag{4.9}$$

where c_s is the sound speed within the cloud:

$$c_s = \left(\frac{\gamma kT}{\mu m_p}\right)^{1/2} . \tag{4.10}$$

In equation (4.10), γ is the adiabatic index, equal to $\gamma = 7/5$ for a gas of diatomic molecules like H_2, T is the gas temperature, and μ is the mean molecular weight. The cloud is unstable if

$$t_{\rm ff} < t_{\rm press} , \tag{4.11}$$

or

$$\left(\frac{3\pi}{32G\rho_0}\right)^{1/2} < r_0 \left(\frac{\mu m_p}{\gamma kT}\right)^{1/2} . \tag{4.12}$$

The freefall collapse time $t_{\rm ff}$ is independent of the cloud's initial radius r_0 and the sound travel time is linearly proportional to r_0. This implies that for a given density ρ_0 and temperature T there is a *maximum* radius r_J for which a cloud is stable against collapse. The critical radius r_J is known as the **Jeans length**, after the astronomer James Jeans, who first realized its importance.

From equation (4.12), the Jeans length is

$$r_J = \left(\frac{3\pi\gamma kT}{32G\rho_0\mu m_p}\right)^{1/2} . \tag{4.13}$$

For pure molecular hydrogen, with $\mu = 2$ and $\gamma = 7/5$, the numerical value of the Jeans length is

$$r_J \approx 2000\,\mathrm{AU} \left(\frac{T}{10\,\mathrm{K}}\right)^{1/2} \left(\frac{\rho_0}{3 \times 10^{-15}\,\mathrm{kg\,m^{-3}}}\right)^{-1/2} \tag{4.14}$$

The mass within a sphere of radius r_J is called the Jeans mass:

$$M_J \approx 0.2\,\mathrm{M}_\odot \left(\frac{T}{10\,\mathrm{K}}\right)^{3/2} \left(\frac{\rho_0}{3 \times 10^{-15}\,\mathrm{kg\,m^{-3}}}\right)^{-1/2} . \tag{4.15}$$

This hints why stars form only in the coolest, densest regions of the interstellar gas. In hotter, lower density regions, the Jeans mass is very much bigger than a star's mass.

If a dense core inside a molecular cloud is bigger than its Jeans length, then squeezing it (with a supernova shock wave, for instance), will trigger a gravitational collapse. What stops the collapse? Usually, the collapse is stopped by conservation of angular momentum. When we look at dense interstellar clouds, they are usually rotating slowly. The Horsehead Nebula, for instance, has a rotation speed of $\sim 1\,\mathrm{km\,s^{-1}}$. If a cloud starts with rotation speed v_0 and radius r_0, when it collapses to a final radius r_f, its rotation speed will be given by the law of conservation of angular momentum: $v_0 r_0 = v_f r_f$, or

$$v_f = \left(\frac{r_0}{r_f}\right) v_0 . \tag{4.16}$$

If the radius of a molecular cloud core decreases by a factor $\sim 10^{-6}$, as suggested earlier, its final rotation velocity will be $v_f \sim 10^6 v_0$. For an initial rotation speed of $v_0 \sim 1\,\mathrm{km\,s^{-1}}$, like that of the Horsehead Nebula, the final rotation speed will be faster than light. Oops.

In fact, if angular momentum is conserved, the cloud will stop falling inward when it forms a rotationally supported disk, in which the gravitational acceleration is just sufficient to keep material on a circular orbit:

$$\frac{GM}{r_f^2} = \frac{v_f^2}{r_f} . \tag{4.17}$$

Combining equations (4.16) and (4.17), we find that the disk's radius will be

$$r_f = \frac{v_0^2 r_0^2}{GM} \approx 200\,\mathrm{AU} \left(\frac{v_0}{0.1\,\mathrm{km\,s^{-1}}}\right)^2 \left(\frac{r_0}{4000\,\mathrm{AU}}\right)^2 \left(\frac{M}{1 M_\odot}\right)^{-1} . \tag{4.18}$$

Even when the initial rotation speed is small, the cloud will collapse to a disk much larger than a star. Such protoplanetary disks, sometimes called

Figure 4.9: Edge-on protoplanetary disk (or "proplyd") in the Orion Nebula. [Image credit: HST]

"proplyds" for short, can be seen in the denser regions of the Orion Nebula (Figure 4.9).

A protoplanetary disk contains far more angular momentum than a star does. create a star from a rotationally supported planetary disk, some of the material must lose angular momentum and fall to the center, where it forms a **protostar**. (A protostar is the slowly rotating ball of gas that will eventually become a star, but which has not yet started fusion in its center.) One hint of where a protostar dumps its angular momentum is provided by the present Solar System. The orbital angular momentum of Jupiter is

$$J_{\rm Jup} \approx 2 \times 10^{43}\,{\rm kg\,m^2\,s^{-1}}\ . \tag{4.19}$$

The rotational angular momentum of the Sun is only

$$J_\odot \approx 1 \times 10^{42}\,{\rm kg\,m^2\,s^{-1}}\ . \tag{4.20}$$

Although the Sun contains 99.8% of the mass of the Solar System, it contains less than 5% of the angular momentum. A relatively small amount of mass on a very large orbit can act as a "scapegoat" for angular momentum, carrying most of the initial angular momentum of the collapsing cloud. Part of the protoSun's angular momentum was carried by viscous torques to the outer part of the protoplanetary disk.[6] Another part of the protoSun's angular momentum is carried away by a magnetized protosolar wind; this is much the same mechanism described in section 8.3 of *BA*.

The brief cartoon version of star formation is shown in the four images of Figure 4.10. The steps can be summarized as follows:

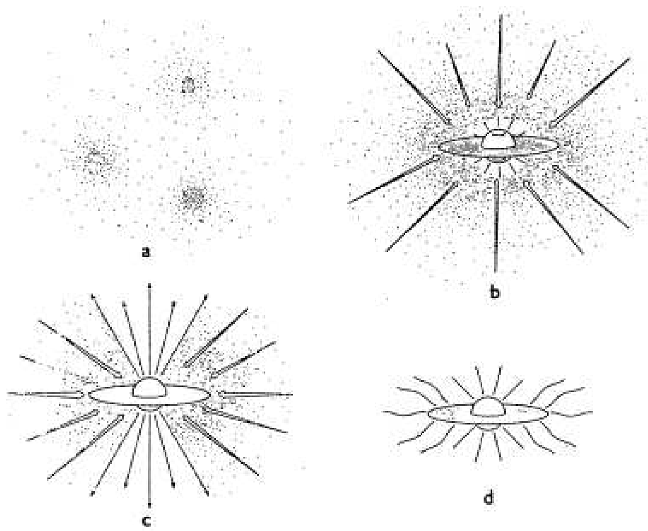

Figure 4.10: The four stages of star formation [Image credit: Shu, Adams, & Lizano]

- (a) The dense core of a molecular cloud is perturbed by a shock wave, and starts to collapse.
- (b) The core collapses to form a rotationally supported disk. The gas in the disk is threaded by magnetic field lines. The central dense region of the disk is the protostar.
- (c) Hot gas from the disk moves out along the magnetic field lines, forming a strong stellar wind. Since the magnetic field lines rotate

[6]To visualize viscous torques, think of a disk made of extremely sticky, viscous material. If the inner regions are moving more rapidly than the outer regions, they will tend to drag the outer regions along with them, speeding up the outer regions and increasing their angular momentum.

along with the disk, the gas traveling along the field lines carries much angular momentum.

- (d) Within the disk, dust clumps to form planetesimals; planetesimals collide to form planets. The remaining gas in the disk is blown away by the stellar wind.

The protostar slowly contracts, on the Kelvin-Helmholtz timescale, as it radiates away its gravitational potential energy. When its central regions become hot and dense enough for hydrogen fusion to ignite, the protostar becomes a star.

4.4 Evolution of Sun-like Stars

Once the Sun started fusing hydrogen in its core, its time evolution became very gradual. Solving the time-dependent equations of stellar evolution indicates that the Sun's fusion-powered life began 4.6 billion years ago. Since the Sun's main sequence lifetime is 10 billion years, the Sun has exhausted nearly half the hydrogen in its core. During the Sun's 10 billion years on the main sequence, not much changes in its global structure. The most significant changes are due to the changing composition of the Sun's core. Fusing hydrogen into helium increases the mean molecular weight μ of the gas. (As shown in section 2.1, fully ionized hydrogen has $\mu = 1/2$, while fully ionized helium has $\mu = 4/3$.) In order to maintain the central pressure P_c required for the Sun to remain in hydrostatic equilibrium, either the central temperature T_c or the central mass density ρ_c, or both, will have to increase. This in turn increases the energy generation rate ϵ and drives up the luminosity of the Sun. When the Sun began fusion, 4.6 billion years ago, its luminosity was $\sim 0.7L_\odot$; about 6 billion years from now, right before it runs out of hydrogen in its core, the Sun's luminosity will be $\sim 2.2L_\odot$.[7] A minor side effect of the Sun's luminosity increase is that $\sim 3.5\,\mathrm{Gyr}$ from now, the increase flux at the Earth's location will trigger a runaway greenhouse effect; the Earth's oceans will evaporate, and the Earth's climate will resemble that of Venus today.

What happens when the Sun (or a Sun-like main sequence star) runs out of hydrogen in its core? The Sun is still losing energy from its photosphere

[7] Tripling the Sun's luminosity sounds impressive, but when it occurs over 10 billion years, it only requires an increase of 0.01% every million years.

when the central hydrogen runs out. (The photosphere doesn't have a "gas gauge" to watch, and thus is unaware that the core has run out of fuel; it just keeps on radiating because it keeps on being hotter than its surroundings.) The energy lost to interstellar space has to come from somewhere; the source on which the star falls back is the old standby, gravitational potential energy. The helium core of the Sun slowly contracts inward, converting its gravitational potential energy into thermal energy. The layer above the core is heated to the point where it starts fusing its hydrogen into helium. During this phase of the Sun's life, the main energy source is much closer to the surface than it was before. The outer layers of the Sun absorb the energy emitted by the hydrogen-fusing shell; their temperature and pressure increase, and they expand outward. The radius of the photosphere increases from $\sim 1r_\odot$ to $\sim 170r_\odot$.[8] The swollen photosphere drops in temperature from 6000 K to 3000 K, but the large increase in surface area means that the Sun's luminosity climbs from $2.2L_\odot$ to $2000L_\odot$. During this stage of the Sun's evolution, its location on a Hertzsprung-Russell diagram (Figure 4.11) moves upward and to the right. When the Sun fuses hydrogen into helium

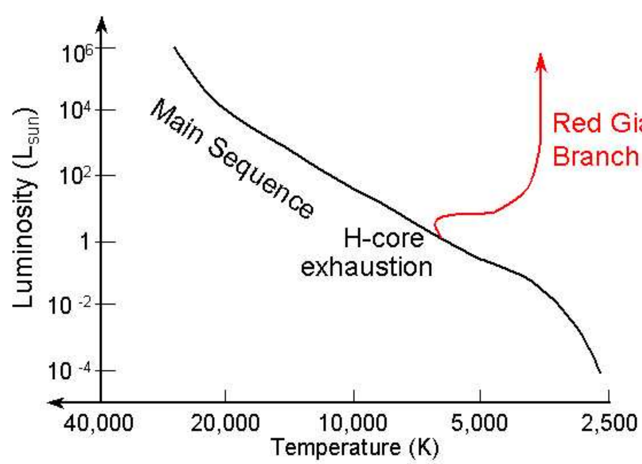

Figure 4.11: Climbing the Red Giant Branch. [Image credit: R. Pogge]

in a shell outside its core, it is very big and very red. Hence it is called a **red giant**, and its evolution away from the main sequence is referred to as "climbing the red giant branch".

[8] $170r_\odot \approx 0.8$ AU; at this point, we can kiss Mercury and Venus goodbye.

As the hydrogen-fusing shell eats its way outward through the Sun, a larger and larger sphere of helium is left behind in the center. As the core of ionized helium becomes more massive, it is squeezed together by its own gravity until the free electrons become **degenerate**. (The term "degenerate" is not a value judgment about the electrons' lifestyle; to a physicist, electrons are degenerate when they are sufficiently close together that the Pauli exclusion principle comes into play, and prevents any two electrons from occupying the same state.) The degenerate electrons produce a pressure that is entirely independent of the temperature of the helium nuclei.

The **degenerate electron pressure** is a consequence of the Heisenberg uncertainty principle, which states that you can't simultaneously specify the position x and momentum p of a particle. There is always an uncertainty in each such that

$$\Delta x \Delta p = \hbar \ , \tag{4.21}$$

where $\hbar = h/(2\pi) = 1.05 \times 10^{-34}\,\mathrm{J\,s}$ is the reduced Planck constant. Suppose that the degenerate electrons have a number density n_e. In their cramped conditions, each electron is confined to a volume $V \sim n_e^{-1}$. Thus, the location of each electron is determined with an uncertainty $\Delta x \sim V^{1/3} \sim n_e^{-1/3}$. From the uncertainty principle, the uncertainty in the electron momentum is

$$\Delta p \sim \frac{\hbar}{\Delta x} \sim \hbar n_e^{1/3} \ . \tag{4.22}$$

If the electrons are nonrelativistic,

$$\Delta v = \frac{\Delta p}{m_e} \sim \frac{\hbar n_e^{1/3}}{m_e} \ , \tag{4.23}$$

where $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$ is the mass of the electron.

Thanks to the uncertainty principle, degenerate electrons are zipping around with a speed $v_e \propto n_e^{1/3}$ regardless of how low the temperature drops. These "Heisenberg speeds" contribute to the pressure, just as the thermal speeds do. For ordinary thermal motions, the electron speeds are

$$v_{\rm th} \sim \left(\frac{kT}{m_e}\right)^{1/2} \ , \tag{4.24}$$

and the pressure contributed by thermal motions of electrons is

$$P_{\rm th} = n_e kT \sim n_e m_e v_{\rm th}^2 \ . \tag{4.25}$$

By analogy, the "Heisenberg speeds" contribute a pressure

$$P_{\rm degen} \sim n_e m_e (\Delta v)^2 \sim n_e m_e \left(\frac{\hbar n_e^{1/3}}{m_e} \right)^2 \sim \hbar^2 \frac{n_e^{5/3}}{m_e} \ . \tag{4.26}$$

We label a population of electrons as "degenerate" when $P_{\rm degen} > P_{\rm th}$.

As the helium core continues to grow, it is supported by degenerate electron pressure even as the temperature of the helium nuclei increases. When the temperature of the helium nuclei reaches 10^8 K, fusion of helium into carbon by the triple alpha process begins (as detailed in section 3.3). As the energy released by fusion is dumped into the core, the temperature of the helium nuclei, and hence the rate of the triple alpha process, shoots up rapidly. Since the core pressure is provided by degenerate electrons, the pressure initially remains constant despite the rapid temperature rise. This means that the mass density of the core remains constant while the temperature increases; just the conditions needed for runaway fusion. The initiation of helium fusion in a degenerate core is called the **helium flash**. The fusion runaway continues until the temperature rises to 3.5×10^8 K. At this temperature the electrons finally become nondegenerate, and the dominant source of pressure is ordinary thermal motions.

After the helium flash, the Sun settles into a steady state; helium is fused into carbon (and a little oxygen) in the core, while hydrogen is fused into helium in a shell outside the core. At this stage of the evolution, the Sun is "on the horizontal branch" (Figure 4.12). This odd name is given because all stars have very nearly the same luminosity – about $100 L_\odot$ – when they are at this stage of their evolution. Thus, they fall along a horizontal line in the H-R diagram. As a horizontal branch star, the Sun is smaller ($r \sim 10 r_\odot$) and hotter ($T \sim 5000$ K) than during its red giant phase.

There's enough helium in the core to fuel the star by the triple alpha process for 100 million years. When helium runs out, the core contracts, the released gravitational potential energy heats the shell above, and fusion of helium into carbon begins in the shell above the carbon core. During this phase, the Sun is a red giant again, only with *two* fusion shells; the inner converts helium to carbon (and a little oxygen) and the outer converts hydrogen to helium. At this stage, the Sun is an "asymptotic giant branch" star, or AGB star. This name is given because the star's path on the H-R diagram (Figure 4.13) is asymptotic to the path it followed on its first climb up the giant branch.

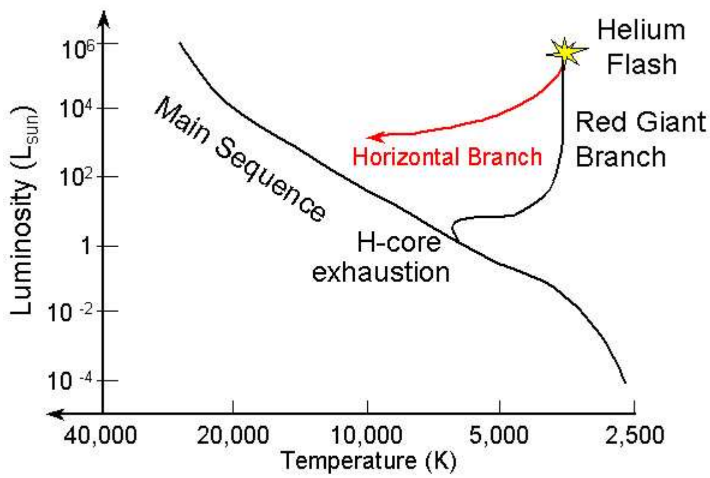

Figure 4.12: On the horizontal branch. [Image credit: R. Pogge]

The "layered look" of an AGB star is shown schematically in Figure 4.14. The relative sizes of the different layers are not shown to scale. The outer cool envelope is actually much larger than the central fusion shells and the carbon/oxygen core. The life of the Sun as an AGB star is relatively brief; this is the most luminous stage of the Sun's existence, with a predicted maximum luminosity of $\sim 3000 L_\odot$. An AGB star is unstable, and pulsates in and out. The outward pulsations eject the outer layers of the star in huge gusts of stellar wind. The final pulses blow away the hydrogen-fusing and helium-fusing shells. Only the inert carbon/oxygen core remains.

The naked core is extremely hot, and emits copious ultraviolet light. It ionizes the surrounding gas and produces the type of emission nebula that we call a "planetary nebula". The carbon/oxygen core is small, dense, and supported by electron degeneracy pressure; it is the type of stellar remnant that we call a "white dwarf". It is expected that the Sun will be able to blow away $\sim 40\%$ of its mass, leaving behind a white dwarf with mass $M_{\rm wd} \sim 0.6 M_\odot$. The white dwarf is too cool to fuse its carbon and oxygen to heavier elements, and too stiff to collapse to a smaller volume and decrease its gravitational potential. With no energy source available other than thermal energy, the white dwarf gradually cools down, until it becomes a "black dwarf" in the distant future.[9]

[9] This may be disappointing to you, but the Sun will never become a supernova or a black hole. Sorry.

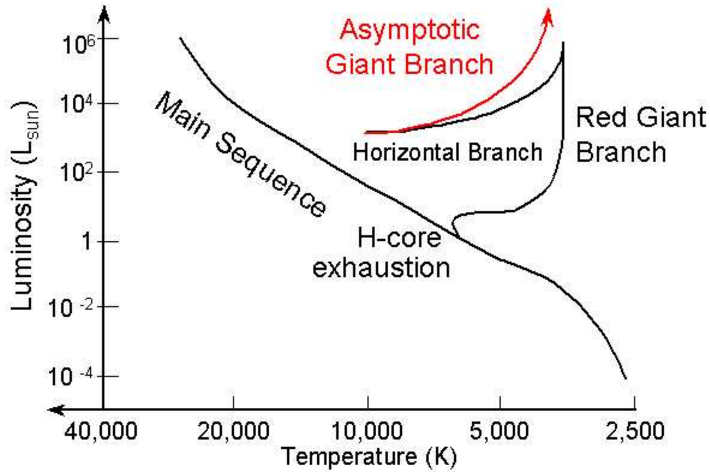

Figure 4.13: Climbing the asymptotic giant branch. [Image credit: R. Pogge]

The life of the Sun is a play in five acts, plus a prologue and epilogue:

Prologue: Protostar ($t \sim 50\,\mathrm{Myr}$)
No fusion; powered by gravity
Act I: Main sequence ($t \sim 10\,\mathrm{Gyr}$)
fusion of H to He in core
Act II: Red giant branch ($t \sim 1\,\mathrm{Gyr}$)
fusion of H to He in shell
Act III: Horizontal branch ($t \sim 100\,\mathrm{Myr}$)
fusion of H to He in shell, He to C in core
Act IV: Asymptotic giant branch ($t \sim 20\,\mathrm{Myr}$)
fusion of H to He in outer shell, He to C in inner shell
Act V: Planetary nebula ($t \sim 50\,\mathrm{kyr}$)
no fusion; hot core emits UV, gas fluoresces
Epilogue: White dwarf ($t \to \infty$)
no fusion; white dwarf cools down

4.5 Pulsating Variable Stars

During the Sun's main sequence lifetime, its properties change only slowly. However, there do exist stars whose luminosities, and other properties, vary periodically on time scales of less than a year. These stars are known as

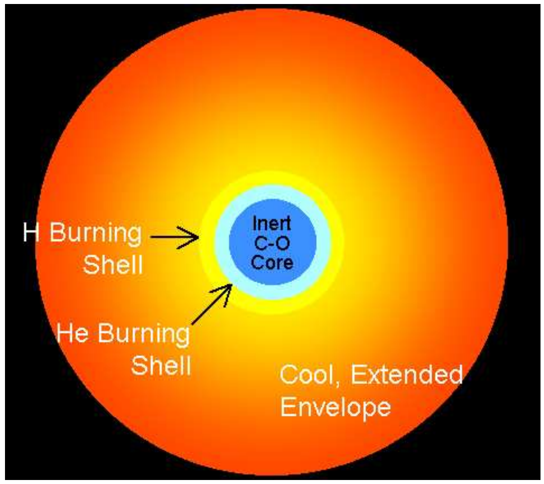

Figure 4.14: Layers of an asymptotic giant branch star (not to scale). [Image credit: R. Pogge]

pulsating variable stars. Zeilik & Gregory (in their Table 18-1) list several types of pulsating variables. I'll focus on the two most important types: **Cepheid stars** (called "classical Cepheids" by Zeilik & Gregory) and **RR Lyrae** stars. Each of these classes is named after its prototype; that is, the first star of that class to be recognized. This was Delta Cephei in the case of the Cepheids, and RR Lyrae itself in the case of the RR Lyrae stars.

We can see the difference between Cepheids and RR Lyrae stars by tabulating some of their properties:

	Cepheid	RR Lyrae
M_v (average)	-0.5 to -6	0.5 to 1
spectral type	F, G, K	A, F
pulsation period	1 to 50 days	1.5 to 24 hours
mass	3 to 18 $M_\odot$	0.5 to 0.7 $M_\odot$
evolutionary stage	supergiant	horizontal branch
metallicity	high	low

Cepheid and RR Lyrae variables are different in many properties. However, if you plot their locations on an H-R diagram (Figure 4.15), you find they are adjacent to each other, on a diagonal stripe called the **instability strip**. Unlike the Sun and other stable main sequence stars, Cepheid and

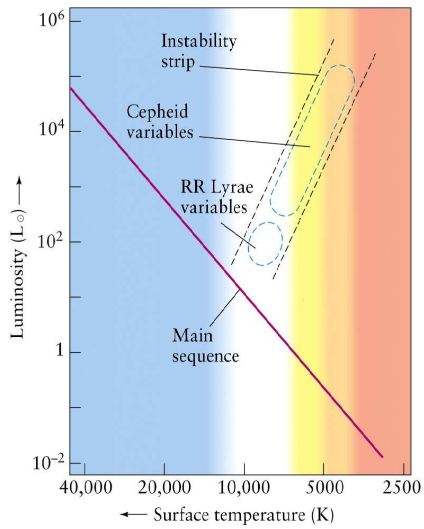

Figure 4.15: Location of Cepheids and RR Lyrae stars on a Hertzsprung-Russell diagram.

RR Lyrae stars pulsate in and out: they actually grow larger and smaller in radius.

As an example of a pulsating variable star, consider Delta Cephei. Over a period of $P = 5.6\,\mathrm{days}$, its apparent magnitude varies by $\Delta m \approx 1$; this corresponds to $f_{\max}/f_{\min} \approx 10^{0.4} \approx 2.5$. A steep rise in flux (Figure 4.16) is followed by a slow decline. The radial velocity fluctuates from $v_r \approx -20\,\mathrm{km\,s^{-1}}$ when the photosphere expands toward us to $v_r \approx +20\,\mathrm{km\,s^{-1}}$ when the photosphere contracts away from us. The effective temperature changes from $T_{\rm eff} \approx 5600\,\mathrm{K}$ when the star is near its minimum luminosity to $T_{\rm eff} \approx 6600\,\mathrm{K}$ when the star is near its maximum luminosity. The variations in radius can be deduced from the star's changes in luminosity and effective temperature: $r \propto L^{1/2}/T_{\rm eff}^2$. The radius of Delta Cephei varies by nearly 15% over the course of one cycle; this means that the maximum volume is 50% greater than the minimum volume.

To see why pulsating variable stars pulsate (while the Sun is content to be stable), start by taking a star of radius r and squeezing slightly, so that its radius decreases to $r - dr$. As we saw in section 4.3, squeezing a gas sphere makes a sound wave travel toward its center. Since a star in hydrostatic equilibrium is smaller than its Jeans length, the sound wave will reach the

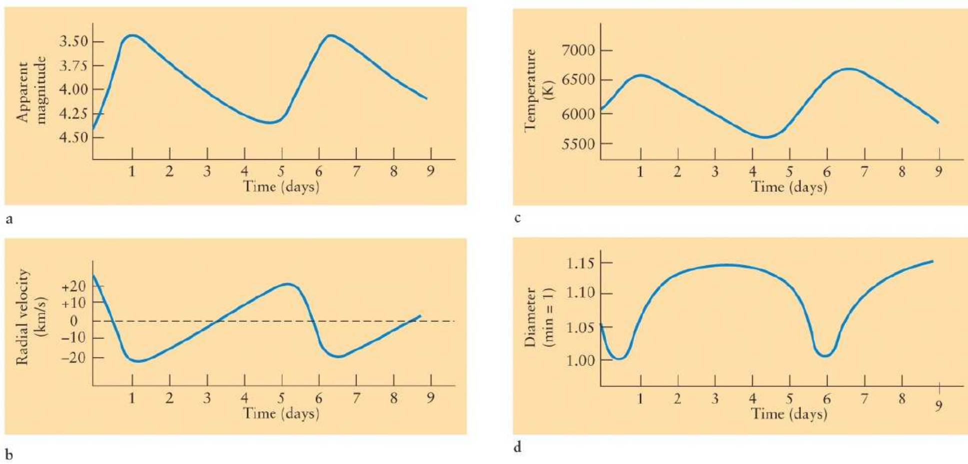

Figure 4.16: Varying properties of Delta Cephei.

center in a time less than the collapse time. When the sound wave reaches the center, it expands back to the surface; when it reaches the surface it reflects back to the center. Thus, squeezing a star in a spherically symmetric manner results in a standing sound wave, or **acoustic oscillation**.

The period p of the acoustic oscillation is the time required for a sound wave to make a round trip from photosphere to center to photosphere. This time is

$$p = \frac{2r}{c_s} = 2r\left(\frac{\mu m_p}{\gamma k \langle T \rangle}\right)^{1/2} , \tag{4.27}$$

where c_s is the average sound speed in the star; the relation between c_s and the average temperature $\langle T \rangle$ is taken from equation (4.10). If we take the average temperature to be roughly half the central temperature (as approximated in equation 3.8), we find a period of

$$p \approx 2r\left(\frac{2\mu m_p}{\gamma k T_c}\right)^{1/2} \approx 2r\left(\frac{r}{\gamma G M}\right)^{1/2} . \tag{4.28}$$

Note that the period for acoustic oscillations is

$$p \approx \left(\frac{4r^3}{\gamma G M}\right)^{1/2} \approx \left(\frac{3}{\pi \gamma G \langle \rho \rangle}\right)^{1/2} , \tag{4.29}$$

so stars of greater density have shorter periods for acoustic oscillations.[10] This tells that that the short-period RR Lyrae stars are higher in density than the longer-period Cepheids.

Any star will undergo acoustic oscillations when it is squeezed or otherwise perturbed. In the Sun, helioseismology reveals that the oscillations are of small amplitude; this is true in most other stars as well. The acoustic oscillations are damped by the viscosity of the gas. Cepheids and RR Lyraes have high amplitude pulsations because their acoustic oscillations are **driven**. Like a little girl on a swing who gets repeated pushes from Mom at the same point of her swing, a pulsating variable star is driven by a periodic force that has the same period p as the acoustic oscillations.

The driving force behind the pulsations of Cepheids and RR Lyrae stars is related to changes in opacity. To see how opacity changes can drive oscillations, let's consider the process step by step.

- A layer near the surface is heated to a temperature of $T \sim 40{,}000\,\mathrm{K}$. At this temperature, He^+ is ionized to He^{++}.
- The opacity shoots upward, due to scattering by the newly freed electrons.
- Light is absorbed at the bottom of the opaque layer, increasing the temperature and pressure gradient across the layer.
- Driven by the increased pressure gradient, the opaque layer expands outward and cools.
- He^{++} recombines with free electrons to form He^+.
- The opacity plummets, and previously trapped photons rush outward through the newly transparent layer.
- The layer contracts and reheats to $40{,}000\,\mathrm{K}$, where the whole cycle begins again.

A layer with $T \sim 40{,}000\,\mathrm{K}$ is near the photosphere of the star. The central regions, where the fusion reactions occur at much higher temperatures, are unaffected by the opacity changes far above them. The fusion generates energy at a steady rate; however, the changes in opacity cause the energy to

[10]You can verify that $p/2 < t_{\mathrm{ff}}$, ensuring that the star is in stable equilibrium.

be released in periodic outbursts.[11] Stars in the instability strip (Figure 4.15) have a helium ionization layer in which the natural period for the transition from opaque to transparent to opaque just matches the period for acoustic oscillations in the star.

Within the instability strip, the most luminous stars are large, low-density supergiants. Since their density ρ is low, their acoustic oscillation period, $p \propto \rho^{-1/2}$, is long. As a result, there is a **period - luminosity relation** for Cepheid stars, with the more luminous stars having longer periods. In fact, the period - luminosity relation for Cepheids was discovered empirically long before anyone knew the precise mechanism driving their luminosity fluctuations. In 1912, Henrietta Swan Leavitt was studying variable stars in the Large and Small Magellanic Clouds. When she plotted the apparent magnitude of each Cepheid as a function of its oscillation period p, she found $m \propto \log p$. Since the Clouds are small compared to their distance from us, the stars in each Cloud are all at roughly the same distance from us, implying

$$M \propto \log p \ . \tag{4.30}$$

To calibrate the period - luminosity relation, and find exactly which absolute magnitude M corresponds to a given period p, we need to know the distance d to at least some of the Cepheids out there. Unfortunately, Cepheid variables, like all highly luminous stars, are rare; even the closest Cepheids are a little too far away to have their parallax measured accurately.[12] The calibration for the Cepheid period - luminosity relationship given by Zeilik & Gregory is

$$\overline{M}_V = -2.76 \log(p/10\,\mathrm{days}) - 4.16 \ , \tag{4.31}$$

where $\overline{M}_V$ is the absolute magnitude in the V band, averaged over a complete period. The scatter in absolute magnitude at a given period is about 0.3 magnitudes.

If you measure the period p of a Cepheid, you can compute its absolute magnitude from equation (4.31). After measuring its average apparent magnitude $\overline{m}_V$, you can then compute its distance using the relation

$$5 \log(d/10\,\mathrm{pc}) = \overline{m}_V - \overline{M}_V \ , \tag{4.32}$$

[11]Think of a pot of boiling water with a heavy lid. The water evaporates at a steady rate; however, the water vapor only escapes in periodic "puffs" when the pressure grows high enough to lift the lid.

[12]The nearest Cepheid is Polaris, at $d = 130 \pm 10\,\mathrm{pc}$.

assuming there's no extinction. Thus, Cepheids make excellent **standard candles**; this is the term used by astronomers for an object whose luminosity is known, and whose distance can thus be calculated once its flux is measured. Cepheids are bright enough to be seen in the Virgo cluster of galaxies, whose distance (as measured using Cepheids) is approximately 18 million parsecs. RR Lyrae stars can also be used as standard candles, but since they are less luminous, they can't be seen as far away as Cepheids.