Chapter 9

Clusters and Superclusters

If the universe is a loaf of raisin bread, as Figure 7.17 suggests, then the raisins are not uniformly distributed through the loaf. Speaking less metaphorically, galaxies are not uniformly distributed through space. Figure 9.1 is a plot of the distribution of galaxies with $m_B < 19$) in the northern sky. The plot

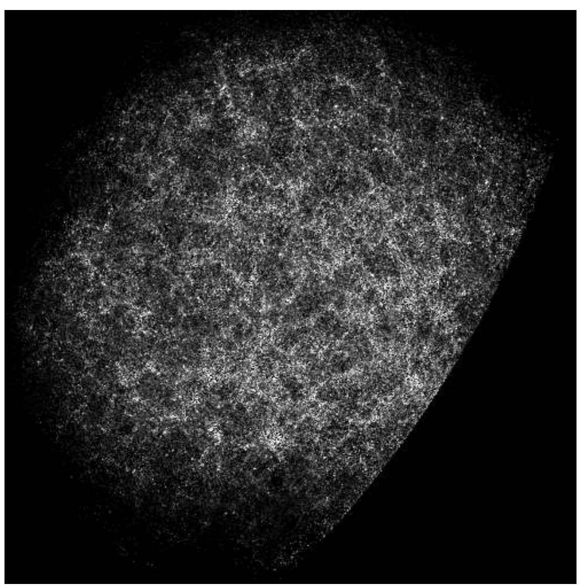

Figure 9.1: Distribution of galaxies in the Northern Galactic Hemisphere; the missing slice at lower right is inaccessible from Mount Hamilton, CA, where Lick Observatory is located. [Image credit: Seldner, et al. 1977, AJ, 82, 249]

is based on the Shane-Wirtanen catalog of galaxies. Shane and Wirtanen photographed the portion of the sky visible from Lick Observatory, then spent long hours counting the $\sim 10^6$ galaxies detected on the photographic plates. In the plot of the Shane-Wirtanen counts, we definitely see a non-uniform distribution. People tend to use the words "bubbly" or "spongy" when they describe the large-scale distribution of galaxies. The galaxies lie along walls or filaments, with particularly strong concentrations in the clusters where filaments meet.

The universe shows **hierarchical structure**; that is, it contains structure on a very wide range of length scales.

- **Stars** (typical diameter $d \sim 10^6\,\mathrm{km}$) are found largely in gravitationally bound objects called galaxies, containing $10^6 \rightarrow 10^{12}$ stars, plus gas, dust, and dark matter.

- **Galaxies** (typical diameter $d \sim 10\,\mathrm{kpc}$) are found largely in gravitationally bound objects called clusters, containing $10 \rightarrow 10^4$ galaxies, plus gas and dark matter.

- **Clusters** (typical diameter $d \sim 1\,\mathrm{Mpc}$) are found largely in currently collapsing objects called **superclusters**.

The superclusters are the largest structures you can see in Figure 9.1, and have a maximum length $d \sim 100\,\mathrm{Mpc}$.

9.1 Clusters of Galaxies

To start at home, our own galaxy is part of a relatively small cluster called the **Local Group**. The Local Group, mapped in Figure 9.2, contains at least 40 galaxies. The exact number of galaxies in the Local Group is not known, since most of the galaxies are inconspicuous dwarf spheroidal and dwarf irregular galaxies. The discovery of a new Local Group galaxy – a dwarf spheroidal given the name of Andromeda X – was announced as recently as January 2006. Like most small clusters, the Local Group is irregularly shaped. Most of the galaxies cluster around our own galaxy (labeled "Milky Way" in Figure 9.2) and M31 (labeled "Andromeda" in Figure 9.2). Our own galaxy and M31 contain most of the mass and luminosity in the Local Group. Third and fourth place go to M33 (an Sc galaxy) and the Large

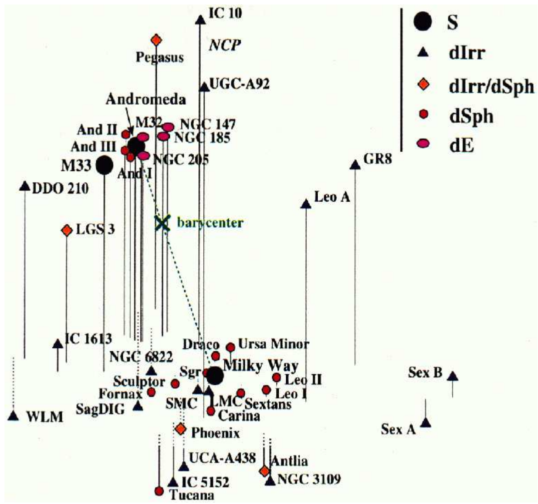

Figure 9.2: The Local Group, with galaxy types indicated; note the prevalence of dwarf galaxies.

Magellanic Cloud. Most of the remaining galaxies are low luminosity, low surface brightness “fluff muffins”.

The Local Group is a typical small cluster.[1] The nearest cluster much richer in galaxies than the Local Group is the Virgo Cluster. The distance to the center of the Virgo Cluster is $d = 16\,\mathrm{Mpc}$, found using Cepheid stars are standard candles. The diameter of the Virgo Cluster is $D \approx 2\,\mathrm{Mpc}$; the caveat should be added that the Virgo Cluster is not particularly close to being spherical. Since the Virgo Cluster is both large and nearby (as clusters go), it subtends a large angle on the sky. It sprawls over a region $\sim 7^\circ$ across, covering much of the constellation Virgo and stretching in Coma Berenices (Figure 9.3). Sixteen of the 2500 known galaxies in the Virgo Cluster are Messier objects. The four brightest galaxies in the cluster (M49, M60, M86, and M87) are all giant elliptical galaxies, swollen to a large size by cannibalizing smaller galaxies.[2] These four giant ellipticals each have an apparent magnitude $m_V \sim 9$, so even in a nearby cluster like Virgo, distance has rendered highly luminous galaxies invisible to the naked eye. There are several other Virgo-sized clusters within 60 Mpc of us (Abell 3655, Hydra,

[1]Clusters with fewer than fifty members are sometimes called “groups”, but the terminology is flexible.

[2]As we’ve seen in the previous chapter, M87 is also a radio galaxy.

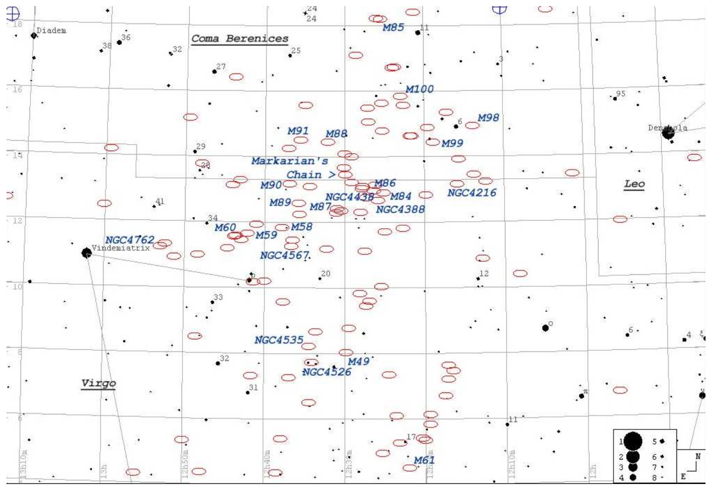

Figure 9.3: Finding chart for the Virgo Cluster.

and Centaurus, for instance) as well as numerous small Local-Group-sized clusters.

The nearest **extremely** rich cluster of galaxies is the Coma Cluster, so called because it is in the constellation Coma Berenices, slightly north of the Virgo Cluster. A snapshot of the Coma Cluster (seen in the left panel of Figure 9.4) shows only the most luminous galaxies in the cluster. The two brightest galaxies in the cluster, NGC 4889 and NGC 4874, have an apparent magnitude $m_V \sim 13$ (too faint for them to have made it into Messier's catalog). Deeper images of the Coma Cluster reveal as many as 10,000 galaxies.

The distance to the Coma Cluster can be estimated from the Hubble law. The 100 brightest galaxies in the cluster have an average redshift of $\langle z \rangle = 0.0232$. This corresponds to a radial velocity

$$v_r = c\langle z \rangle = 6960\,\mathrm{km\,s^{-1}}\ . \tag{9.1}$$

The distance to the Coma Cluster is, from the Hubble law,

$$d_{\mathrm{Coma}} = \frac{c}{H_0}\langle z \rangle = (4300\,\mathrm{Mpc})(0.0232) = 100\,\mathrm{Mpc}\ , \tag{9.2}$$

about six times the distance to the Virgo Cluster.

Knowing the distance, we can now estimate the absolute magnitude of the bright Coma galaxies, NGC 4889 and NGC 4874:

$$M_V = m_v - 5\log(d/10\,\mathrm{pc}) \approx 13 - 5\log(10^8/10)5A \approx 13 - 35 \approx -22\ . \tag{9.3}$$

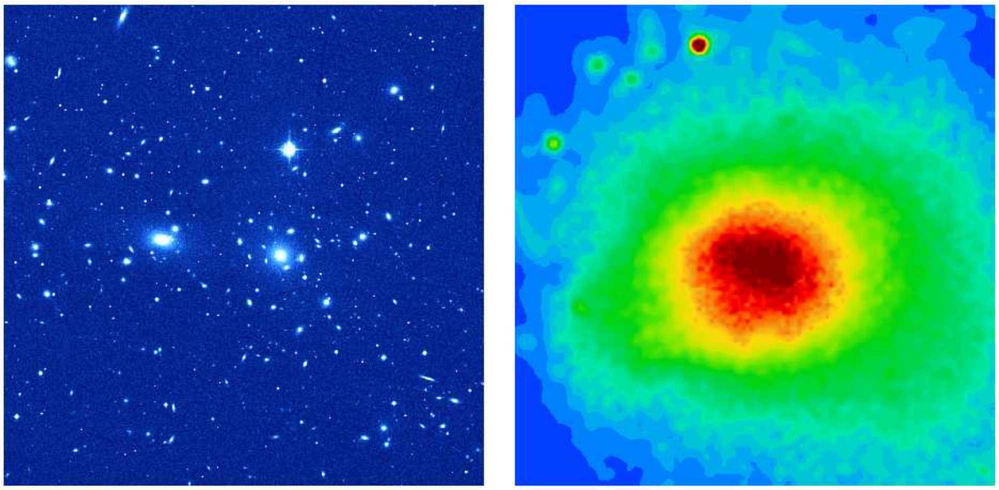

Figure 9.4: Left: Coma Cluster at visible wavelengths. Right: Coma Cluster at X-ray wavelengths (false color image).

This is a magnitude brighter than M31 or the Milky Way, and right up there with the very brightest galaxies known. The luminosity of the Coma Cluster as a whole is estimated to be

$$L_B \approx 8 \times 10^{12} L_{B,\odot} \ . \tag{9.4}$$

Our own galaxy has $L_{B,\mathrm{MW}} \approx 2 \times 10^{10} L_{B,\odot}$, so the luminosity of the Coma Cluster in the B band is equal to 400 times the luminosity of the Milky Way galaxy. Just as most of the visible light in a galaxy comes from a relatively few luminous stars, most of the visible light in a cluster comes from a relatively few luminous galaxies.

The mass of the Coma Cluster can be estimated from the virial theorem (equation 7.20):

$$M \approx 7.5 \frac{\sigma^2 r_h}{G} \ . \tag{9.5}$$

Now *galaxies*, not stars, are the individual point masses whose line of sight velocity we measure. For the 100 brightest galaxies in the Coma Cluster, the dispersion in the radial velocities is

$$\sigma = 880\,\mathrm{km\,s^{-1}} = 8.8 \times 10^5\,\mathrm{m\,s^{-1}} \ . \tag{9.6}$$

The half-light radius of the cluster is

$$r_h = 1.5\,\mathrm{Mpc} = 4.6 \times 10^{22}\,\mathrm{m} \ . \tag{9.7}$$

The virial theorem estimate of the mass is then

$$\begin{aligned} M &\approx 7.5\frac{(8.8\times10^5\,\mathrm{m\,s^{-1}})^2(4.6\times10^{22}\,\mathrm{m})}{6.67\times10^{-11}\,\mathrm{m^3\,s^{-2}\,kg^{-1}}} \\ &\approx 4\times10^{45}\,\mathrm{kg}\approx 2\times10^{15}M_\odot\ . \end{aligned} \tag{9.8}$$

With a mass-to-light ratio $M/L_B \approx 250 M_\odot/L_{B,\odot}$, the Coma Cluster must be amply supplied with dark matter.

The mass of the Coma Cluster can also be estimated by looking at its X-ray emission (right panel of Figure 9.4). The X-ray emission, produced by hot intergalactic gas, looks smoothly distributed, which suggests that the hot gas is in hydrostatic equilibrium. From the X-ray spectrum, it is estimated that the average temperature of the hot gas is

$$\langle T_{\rm gas}\rangle \approx 1\times10^8\,\mathrm{K}\ , \tag{9.9}$$

and that the total amount of gas radiating X-rays is

$$M_{\rm gas} \approx 2\times10^{14}M_\odot\ , \tag{9.10}$$

about 10% of the total mass of the cluster. Back when we were discussing stellar interiors (section 3.1), we noted that if a sphere of gas is in hydrostatic equilibrium its central temperature must be

$$T_c \approx \frac{2GM\mu m_p}{rk}\ , \tag{9.11}$$

if the pressure is described by a perfect gas law. If we substitute $r_h \sim r/2$ and $T_c \sim \langle T\rangle$, we can estimate the mass of the Coma Cluster:

$$M \sim \frac{r_h k\langle T\rangle}{G\mu m_p}\ . \tag{9.12}$$

With $\mu \approx 0.6$ (assuming a mix of ionized hydrogen and helium with a few metals), $r_h \approx 1.5\,\mathrm{Mpc}$, and $\langle T\rangle \approx 10^8\,\mathrm{K}$, the mass estimated from X-ray emission is

$$M \sim 10^{45}\,\mathrm{kg} \sim 0.5\times10^{15}M_\odot\ . \tag{9.13}$$

Despite the crudity of our estimate, we get a mass in the same ballpark as the our earlier mass estimate $M \approx 2\times10^{15}M_\odot$ from the virial theorem.[3]

[3] More sophisticated models of the X-ray emission yield $M = (1.3\pm0.5)\times10^{15}M_\odot$ within 3 Mpc of the cluster's center.

9.2 When Galaxies Collide!

The novel *When Worlds Collide*, by Wylie and Balmer, is a classic of science fiction. But how often do objects actually collide in the real universe? Consider a population of stars, each with radius r. One particular star is moving with a speed v relative to the average velocity of the stars in its vicinity. When its center comes within a distance $2r$ of the center of any other star, the two stars will collide. During a time t, the star sweeps out a cylindrical volume of length vt, and radius $2r$; any star whose center lies within this cylinder will collide with the moving star. The volume of the cylinder is

$$V(t) = vt(4\pi r^2) \ . \tag{9.14}$$

If the number density of stars is n, the average number of stars colliding with our moving star will be

$$N_\star(t) = nV(t) = nvt(4\pi r^2) \ . \tag{9.15}$$

The mean time between collisions, $t_\star$, will be roughly the time required to make $N_\star = 1$. That is,

$$t_\star \approx \frac{1}{nv(4\pi r^2)} \ . \tag{9.16}$$

Scaled to the properties of stars in the solar neighborhood,

$$t_\star \approx 5 \times 10^{10}\,\mathrm{Gyr} \left(\frac{r}{1 r_\odot}\right)^{-2} \left(\frac{v}{30\,\mathrm{km\,s^{-1}}}\right)^{-1} \left(\frac{n}{0.1\,\mathrm{pc^{-3}}}\right)^{-1} \ . \tag{9.17}$$

During the Sun's lifetime of 4.6 Gyr, its probability of slamming into another star was only 1 in 10 billion; no wonder such a collision hasn't occurred. In the central parsec of our galaxy, where $n \sim 10^7\,\mathrm{pc^{-3}}$ and $v \sim 200\,\mathrm{km\,s^{-1}}$, the time between collisions is only $t_\star \sim 80\,\mathrm{Gyr}$, nearly a billion times shorter than in the solar neighborhood. Since $t_\star$ near the Galactic center is only 8 times the age of the Galaxy, the probability that a low-mass star undergoes a collision during its lifetime is no longer negligible.

If two main sequence stars collide with a relative velocity smaller than their escape velocity, they will form a more massive main sequence star. There is evidence for such stellar merger remnants in the dense cores of globular clusters. Since globular clusters have ages of $t \sim 10\,\mathrm{Gyr}$ or more, all stars with initial mass $M > 1 M_\odot$ should have evolved off the main sequence.

However, globular clusters are observed to have a few stars on the main sequence with $M > 1 M_\odot$. These stars are called "blue stragglers", since they are hotter, and hence bluer, than expected for main sequence stars in an old globular cluster. Figure 9.5 shows the Hertzsprung-Russell diagram for the globular cluster M55, with the blue straggler stars indicated. Blue

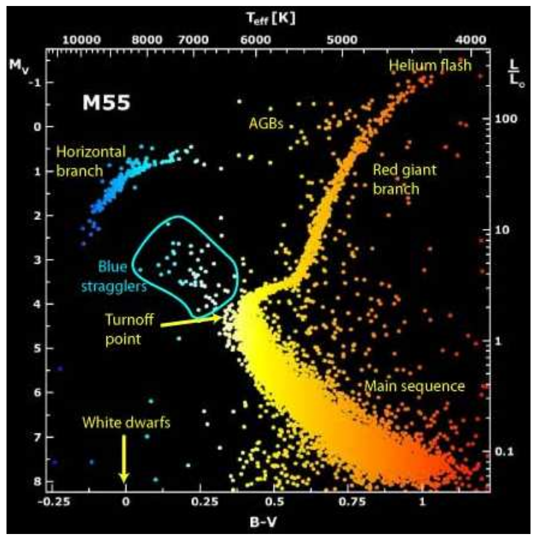

Figure 9.5: HR diagram of the globular cluster M55 ($d \approx 5\,\text{kpc}$, in the constellation Sagittarius). The blue straggler stars are circled.

stragglers can also form by the merger of a close binary star; in that case, the blue straggler that forms will be rotating very rapidly, since it contains all the orbital angular momentum of the initial binary.

We can also ask how often *galaxies* collide with each other. Galaxies are much larger than stars, but also more widely spaced. Let's take, as an example, the Coma Cluster, which contains about 10,000 detectable galaxies. The typical size of these galaxies is

$$r \approx 3\,\text{kpc} \approx 1.3 \times 10^{11} r_\odot\ . \tag{9.18}$$

The typical relative speed, assuming an isotropic velocity dispersion, is

$$v \approx \sqrt{3}\sigma = \sqrt{3}(880\,\text{km}\,\text{s}^{-1}) \approx 1500\,\text{km}\,\text{s}^{-1}\ . \tag{9.19}$$

The average number density of galaxies, assuming that half the galaxies are inside the half-light radius, is

$$n \approx \frac{N/2}{(4\pi/3)r_h^3} \approx \frac{5000}{(4\pi/3)(1.5\,\mathrm{Mpc})^3} \approx 350\,\mathrm{Mpc}^{-3} \approx 3.5 \times 10^{-16}\,\mathrm{pc}^{-3} \; . \tag{9.20}$$

By plugging the above values for r, v, and n into equation (9.17), we find that the average time between collisions for a galaxy in the Coma Cluster is

$$t_\star \approx 17\,\mathrm{Gyr} \approx 1.2 H_0^{-1} \; . \tag{9.21}$$

Thus, the collision time is comparable to the Hubble time, and a typical galaxy in the Coma Cluster is as likely as not to undergo a collision. In a rich cluster, collisions between galaxies are the rule rather than the exception. In poor clusters and groups, the velocities and number densities are lower, but there is still a significant probability of collisions.[4]

Naturally, for every head-on collision, where the galaxies actually interpenetrate, there are many close encounters, where there is no overlap, but the tidal distortions of each galaxy are significant. Suppose two galaxies are on hyperbolic orbits about their mutual center of mass. Near the time of closest approach, each galaxy raises tidal bulges on the other galaxy. If the differential tidal force is strong enough to liberate loosely bound stars from a galaxy, then the stars from the tidal bulge closer to the center of mass will have a faster orbital speed (and thus will *lead* the main body of the galaxy), while stars from the tidal bulge farther from the center of mass will have a slower orbital speed (and thus will *lag* the main body of the galaxy). Galaxy interactions can be modeled using computer simulations. In a typical n-body simulation, the galaxies are approximated as a distribution of n point masses, interacting according to an inverse square law gravitational force. The grandmother of all n-body simulations was performed by Erik Holmberg in 1941; he used an analog computer consisting of $n = 74$ light bulbs being pushed across a $3\,\mathrm{m} \times 4\,\mathrm{m}$ patch of floor covered with black paper.[5] Despite the sim-

[4]Note that when two galaxies collide, the individual stars within the galaxy do not collide with each other. Their cross-sections are just too tiny, even when you jack up the relative velocities to $v \sim 1500\,\mathrm{km\,s}^{-1}$ and double the number density n of stars.

[5]Since the flux from each bulb was an inverse square law, just like gravity, the computed acceleration for each bulb was taken to be equal to the net flux of light at the bulb's position. At each time step, the velocity of each bulb was recomputed, and the bulb was moved the appropriate distance across the black paper.

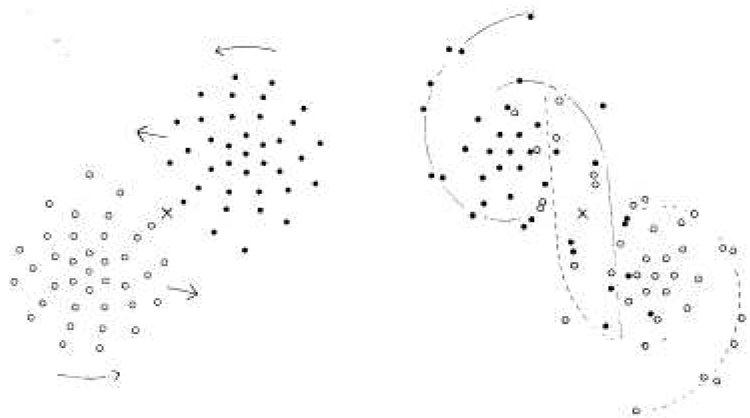

Figure 9.6: One of Holmberg's simulated galaxy encounters. Left: before closest approach. Right: after closest approach, with tidal tails sketched in. [Image credit: Holmberg 1941, ApJ, 94, 385]

plicity of Holmberg's "computer", he was able to reproduce the tidal tails seen in interacting galaxies (Figure 9.6). More recent numerical simulations (see Figure 9.7 for an example) can contain $n > 10^8$ particles. It's the same physics, though. If you've seen one inverse square law, you've seen them all.

Many interacting galaxies can be seen in the real universe, particularly in rich clusters, where close encounters are more frequent. The Coma Cluster, for instance, contains a pair of galaxies known as "the Mice" because of their long tidal tails (Figure 9.8). The Mice, known also as NGC 4676 A & B, are a favorite system of astronomers doing computer models of galaxy encounters. The Mice can be modeled as a pair of identical spiral galaxies that have just passed pericenter in their encounter. ("Just", in this case, means that closest approach was one or two hundred million years ago.)

The Cartwheel Galaxy, shown in Figure 9.9, is another interesting system involving interacting galaxies. It's located in the constellation Sculptor, about 130 Mpc away. The Cartwheel is a galaxy that recently had a smaller but denser galaxy zip through it at high speed; higher than the escape velocity from the Cartwheel Galaxy. The impulse provided by the high-speed intruder caused a circular "ripple" to run outward from the Cartwheel's center, similar to the ripple caused when a rock is dropped into a pool of water. The circular ripple causes the Cartwheel Galaxy's ring-like appearance.[6] The

[6]The intruding galaxy is not necessarily either of the two galaxies on the right side of

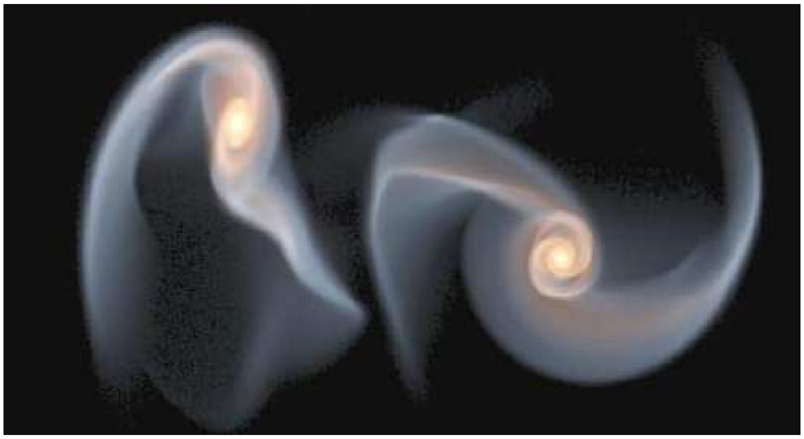

Figure 9.7: A recent simulated galaxy encounter. [Image credit: John Dubinski]

outer ring of the Cartwheel is very blue because the ripple running through the interstellar gas triggered star formation, creating hot, short-lived O & B stars.

If the relative speed of the galaxies is comparable to or less than the escape velocity of the galaxies, then a nearly head-on encounter will result in the **merger** of the galaxies to form a single larger galaxy. Things to remember when galaxies collide:

- Individual stars do not collide with each other. There will not be a sudden bonanza of blue stragglers.

- Galaxy collisions are *inelastic*. That is, during the collision, some of the orbital kinetic energy of the galaxies is converted to internal energy, in the form of random motions of stars. Thus, a pair of galaxies that start out on hyperbolic orbits relative to each other can still end up as a bound system.

- Although stars don't collide, gas clouds do. A giant molecular cloud has $r_{\rm gmc} \sim 10\,{\rm pc} \sim 4 \times 10^8 r_\odot$. The large cross-section for molecular clouds means that the gas clouds will collide when two gas-rich galaxies pass through each other.

Figure 9.9. The high-speed culprit may already have fled the scene of the crime.

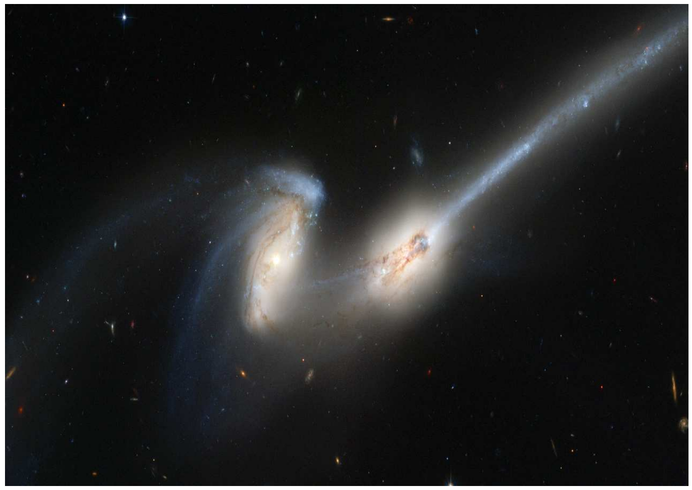

Figure 9.8: The Mice ($d \approx 100\,\mathrm{Mpc}$). [Image credit: HST]

Thanks to the colliding gas clouds, merging galaxies are hotbeds of star formation. The class of galaxies known as ULIRGs (**U**ltra**L**uminous **I**nfra**R**ed **G**alaxies) are frequently found to be merging galaxies, in which large numbers of dust-enshrouded protostars produce copious amounts of far infrared light. Merging galaxies also produce lots of blue and ultraviolet light, thanks to the large numbers of O & B stars produced. The merging galaxies known as "the Antennae" (a tribute to their long tidal tails) are 20 Mpc away in the constellation Corvus. The Antennae are very luminous in their central regions, both at blue wavelengths and at infrared wavelengths (Figure 9.10).

Galaxy mergers are like car crashes, in that they are very effective at increasing entropy. Galaxies differ in the amount of entropy (or disorder) they contain. Spiral galaxies contain stars on orderly, one-way, nearly circular orbits; spirals are *low* in entropy. Elliptical galaxies contain stars on disorderly, randomly oriented orbits; ellipticals are *high* in entropy. When two neat orderly compact cars collide, you don't get a neat orderly SUV; you get a tangled heap of rubble. Similarly, when two neat orderly spiral galaxies collide, you don't get out a neat orderly giant spiral; you get a tangled heap of stars (otherwise known as an elliptical galaxy).

The giant elliptical galaxies in the centers of rich clusters, like NGC 4889 and NGC 4874 in the middle of the Coma cluster, have grown to their present

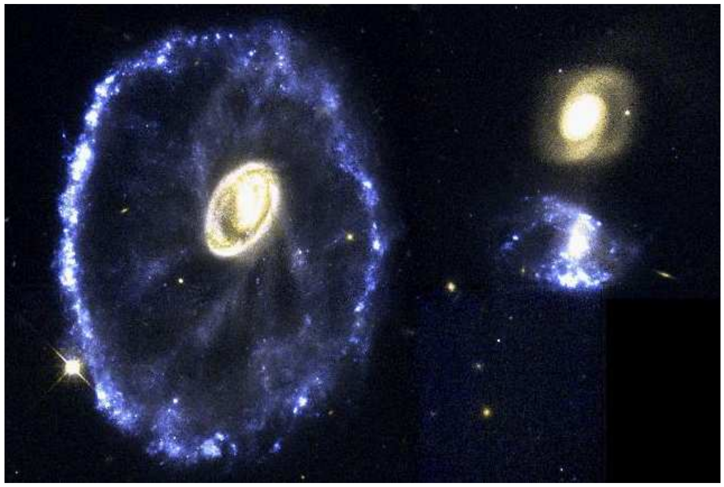

Figure 9.9: The Cartwheel and companions ($d \approx 130\,\mathrm{Mpc}$). [Image credit: HST]

large size by the process of **galactic cannibalism**.[7] The "cannibalism" refers to the fact that when a large galaxy merges with a small one, the large galaxy retains its identity, while the smaller one is disrupted. Sometimes "partially digested" cannibalized galaxies can be seen as multiple nuclei within a giant elliptical galaxy.

Our galaxy is a cannibal, as well, but only on a low level. It is currently tidally disrupting the Sagittarius dwarf galaxy and the Canis Majoris dwarf galaxy, which are disintegrating into long tidal streams that will eventually be mixed in with the halo of our galaxy. These minor encounters are trivial in comparison to our upcoming encounter with M31 (the Andromeda Galaxy). M31 is blueshifted with respect to our galactic center. The relative velocity of the centers of the two galaxies is $v_r = -123\,\mathrm{km\,s} = -126\,\mathrm{kpc\,Gyr^{-1}}$. The tangential velocity of M31 relative to our galaxy is unknown, thanks to the difficulty of measuring proper motions at large distances, but is likely to be small. Thus, in a time

$$t \sim \frac{d_{\mathrm{M31}}}{|v_r|} \sim \frac{700\,\mathrm{kpc}}{126\,\mathrm{kpc\,Gyr^{-1}}} \sim 6\,\mathrm{Gyr}\ , \tag{9.22}$$

M31 and our own galaxy will merge. Detailed computer simulations indi-

[7] Zeilik & Gregory refer to this process, rather pedantically, as "galaxian cannibalism", but everyone in the real world calls it "galactic cannibalism".

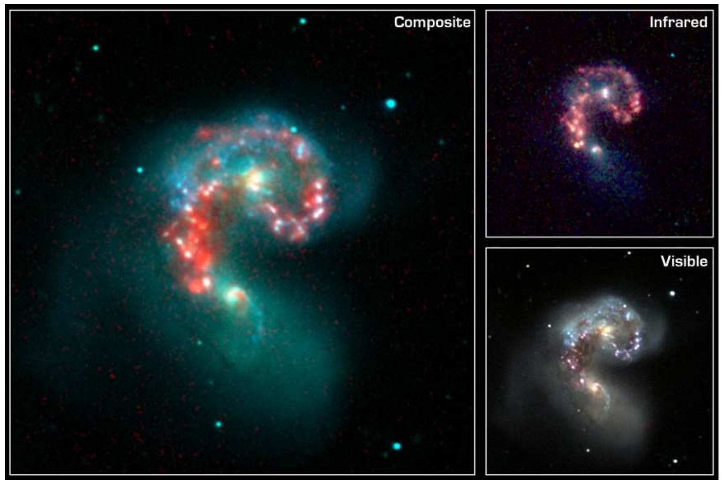

Figure 9.10: Infrared and visible images of the center of the Antennae. [Image credit: Spitzer & Hubble Space Telescopes]

cate that in $\sim 3.2\,\mathrm{Gyr}$, the two galaxies will have a close passage, inducing long tidal tails. The two distorted galaxies will fall back toward each other; about $\sim 4.6\,\mathrm{Gyr}$ from now, they will have formed a large elliptical galaxy, surrounded by tidal debris. When the Sun becomes a red giant, it will be a member of an elliptical galaxy, not a spiral galaxy.

9.3 Superclusters and Voids

In the hierarchy of structures in our universe, the average density of a star is greater than that of a galaxy. The average density of a galaxy is greater than that of a cluster. Finally, the average density of a cluster is greater than that of a supercluster. The superclusters are the largest structures in the universe today, and they consist of regions that are just now collapsing under their own gravity. For superclusters to be collapsing today, they must have a freefall time $t_{\rm ff}$ that is comparable to the age of the universe, which in turn is comparable to the Hubble time, H_0^{-1}. Using the value of the freefall time in terms of the average density ρ_0 of a structure (equation 4.7), we find that for superclusters,

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} \approx H_0^{-1} \ . \tag{9.23}$$

Thus, the average density of a supercluster must be

$$\rho_0 \approx \frac{3\pi H_0^2}{32G} \approx 2 \times 10^{-26}\,\mathrm{kg\,m^{-3}} \approx 3 \times 10^{11} M_\odot\,\mathrm{Mpc^{-3}}\ . \tag{9.24}$$

This is equivalent to 14 hydrogen atoms per cubic meter; even compared to the low-density coronal gas of the interstellar medium, the average density of a supercluster is not great.

Superclusters are most easily seen in three-dimensional maps of the universe, rather than in two-dimensional projections on the sky (such as Figure 9.1). A "redshift map" of the universe can be made by using the redshift z of a galaxy as a surrogate for its distance, since $d \propto z$ for small z. To make such a redshift map, you can start by measuring redshifts for galaxies in a long, narrow strip of the sky. Among the earliest redshift maps of this kind were those produced in the 1980s by the CfA Redshift Survey.[8] A wedge of the universe from the CfA survey is shown in Figure 9.11. To make

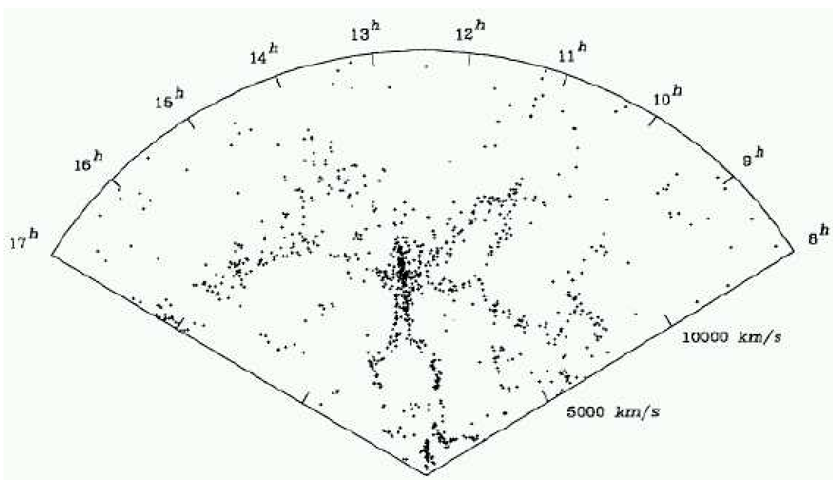

Figure 9.11: A slice of the universe: each dark dot represents a galaxy. [Image credit: ApJL, 1986, 302, L1]

this redshift map, redshifts were measured for galaxies with right ascension $8^h < \alpha < 17^h$ and declination $26.5° < \delta < 32.5°$, down to a limiting apparent magnitude $m_B = 15.5$. For the 1061 galaxies meeting these criteria, $v_r = cz$ was plotted versus right ascension. Note that the dense regions, or superclusters, in the redshift map tend to be elongated, while the underdense regions,

[8] "CfA" stands for the Harvard-Smithsonian **C**enter **f**or **A**strophysics.

or voids, are more nearly spherical (or more nearly circular, in the case of this thin slice).

A more recent redshift map, with a fainter apparent magnitude limit, is shown in Figure 9.12. This shows a slice from the Sloan Digital Sky Survey (SDSS). When the Sloan Digital Sky Survey is complete, it will have provided

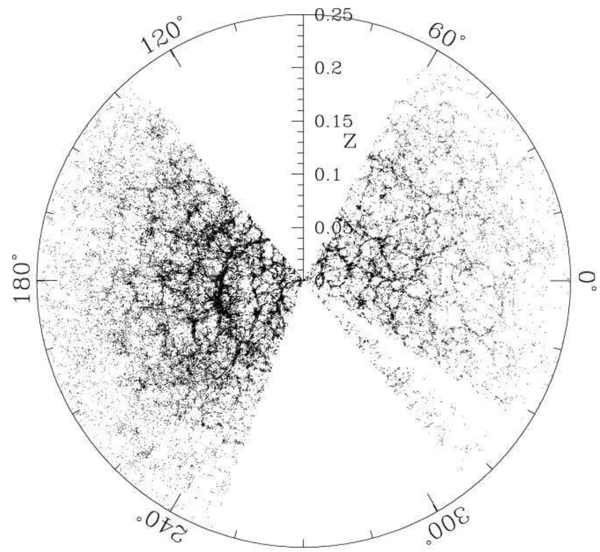

Figure 9.12: A bigger slice of the universe: each tiny dark dot represents a galaxy.

redshifts for over a million galaxies in the Northern Galactic Hemisphere, down to a limiting apparent magnitude $m_r = 17.8$.[9] This gives a detailed map of the galaxy distribution out to a redshift $z \sim 0.2$, corresponding to a distance $d \sim (c/H_0)z \sim 900\,\mathrm{Mpc}$. Thus the Sloan Digital Sky Survey probes the universe to four times the distance of the CfA Redshift Survey, which only reached to $z \sim 0.05$.

We learn something about the large scale structure of the universe just from looking at redshift maps. Superclusters are elongated filaments or walls, not spheres. Dense objects tend to collapse first along their long axis, forming a two-dimensional wall, then along their second-shortest axis, forming a one-dimensional filaments. Voids, by contrast, are roughly spherical. As

[9]The Sloan r band is centered at a wavelength $\lambda \approx 6160\,\text{Å}$, and is roughly comparable to the Johnson R band.

underdense regions expand to fill the space vacated by the collapsing superclusters, they tend to become more spherical with time. Voids, by definition, have a very low density of bright galaxies. The density of non-stellar matter in voids is not always well known. The transparency of intergalactic space places an upper limit on the density of dust: $\rho_{\rm dust} < 4 \times 10^{-30}\,{\rm kg\,m^{-3}}$. The limits on Lyman alpha absorption by neutral gas in voids places a stringent upper limit on the density of neutral hydrogen: $\rho_{\rm H} < 10^{-36}\,{\rm kg\,m^{-3}}$. However, the amount of ionized hydrogen and of dark matter in voids is not as well constrained.

Redshift maps should be used with the caveat that not all redshifts are due to the expansion of the universe. For instance, rich clusters in redshift maps show the "finger of God" effect, which means that they are elongated in the radial direction. Look at Figure 9.11 to see an example of a "finger of God". At the center of Figure 9.11 is a structure that looks vaguely like a bow-legged stick figure. The elongated torso of the stick figure is actually the Coma Cluster. Note how the Coma Cluster is stretched out in the radial direction. The "finger of God" effect received its name because the grossly elongated cluster is jokingly compared to God's finger pointing accusingly at the observer, while the Voice of God booms out, "You are WRONG!"

What the miserable sinning observer has done wrong, in this case, is to assume that the redshift z measured for each galaxy in the cluster can be converted directly to a distance using the simple formula $d = (c/H_0)z$. In fact, if a cluster has a velocity dispersion σ along the line of sight, then some galaxies will have a radial velocity of approximately $+\sigma$ relative to the cluster's center of mass, while other galaxies will have a radial velocity of approximately $-\sigma$. Thus, two galaxies that are actually close to each other physically can have redshifts that differ by

$$\Delta z \sim \frac{2\sigma}{c} \ . \tag{9.25}$$

If we naïvely convert observed redshifts to distances using the Hubble law, we will conclude that the find of God has a length

$$\Delta d = \frac{c}{H_0}\Delta z \sim \frac{2\sigma}{H_0} \ . \tag{9.26}$$

The Coma Cluster, which has $\sigma = 880\,{\rm km\,s^{-1}}$, appears to be stretched to a length of $\Delta d \sim 25\,{\rm Mpc}$ in a redshift map, when its actual diameter is $d \sim 3\,{\rm Mpc}$, an order of magnitude smaller.

Since the largest structures in the universe are superclusters and voids about 100 Mpc across, we expect that the region withing a few hundred megaparsecs of us, containing several superclusters and several voids, should be a fair sample of the universe at the present day. When we perform a census of galaxies in this region (out to $z \sim 0.05$), we can compute the number density of galaxies as a function of their luminosity. This function, known as the **luminosity function** of galaxies, is found to be well fitted by a power-law with an exponential cutoff:

$$\Phi(L) = \Phi_* \left(\frac{L}{L_*}\right)^{\alpha} \exp\left(-\frac{L}{L_*}\right) \frac{dL}{L_*} , \tag{9.27}$$

where $\Phi(L)dL$ is the number density of galaxies with luminosity in the range L to $L + dL$. The luminosity function of equation (9.27) is often called a "Schechter function", after the astronomer who first applied it to the galaxy luminosity function. Schechter's own plot of the galaxy luminosity function is given in Figure 9.13. The exponential cutoff in the luminosity function

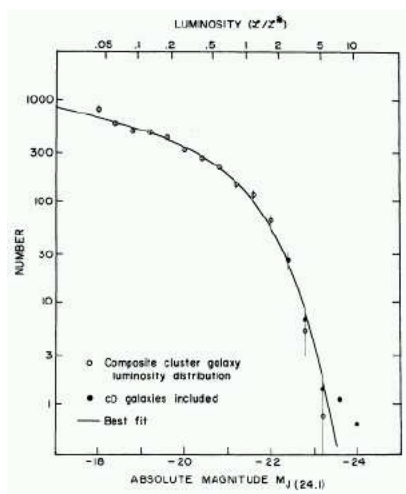

Figure 9.13: The number density of galaxies as a function of luminosity. [Image credit: Schechter, 1976, ApJ, 203, 297]

occurs at

$$L_* \approx 2 \times 10^{10} L_\odot \approx L_{\rm MW} . \tag{9.28}$$

There are relatively few galaxies with luminosities greater than that of the Milky Way Galaxy, $L_{\rm MW}$. Most of the scarce ultraluminous galaxies are fat "cannibals" in rich clusters of galaxies. At luminosities below the cutoff, the luminosity function is a power-law with index

$$\alpha \approx -1.2 \ . \tag{9.29}$$

Thus, the luminosity function is weighted toward dim galaxies. Presumably, there is a cutoff at low luminosities as well as at high luminosities; otherwise, the total number density of galaxies,

$$n_{\rm gal} = \int_0^\infty \Phi(L) dL \ , \tag{9.30}$$

would go to infinity as $L \to 0$. However, given the extreme difficulty of counting dwarf galaxies (we don't even know how many are in the Local Group, after all), how the luminosity function cuts off at low luminosities is not well determined.

The normalization of the luminosity function is

$$\Phi_* \approx 0.01 \, {\rm Mpc}^{-3} \ . \tag{9.31}$$

The total luminosity density of galaxies (in $L_\odot \, {\rm Mpc}^{-3}$) does not diverge. By integrating the luminosity function of equation (9.27) weighted by the luminosity, we find a total luminosity density[10]

$$\rho_L = \int_0^\infty L\Phi(L) dL = \Phi_* L_* \Gamma(2+\alpha) \approx 2.3 \times 10^8 L_\odot \, {\rm Mpc}^{-3} \ . \tag{9.32}$$

This is equivalent to having a single 40 watt light bulb inside a sphere 1 AU in radius. By terrestrial standards, the universe is a poorly lit place.

[10]Since most of the light is contributed by the bright galaxies with $L \sim L_*$, the uncertainty in the faint end of the luminosity function doesn't greatly affect our estimate of the luminosity density of the universe.