Chapter 7

Galaxies

The Hubble Ultra Deep Field (Figure 7.1) is the result of 800 exposures of a single field in Fornax, summing to a total exposure of over 11 days. The limiting magnitude in the V band is $m_V = 29$. Within the $3\,\text{arcmin} \times 3\,\text{arcmin}$ field of view of the Ultra Deep Field, there are $\sim 10{,}000$ galaxies. If you
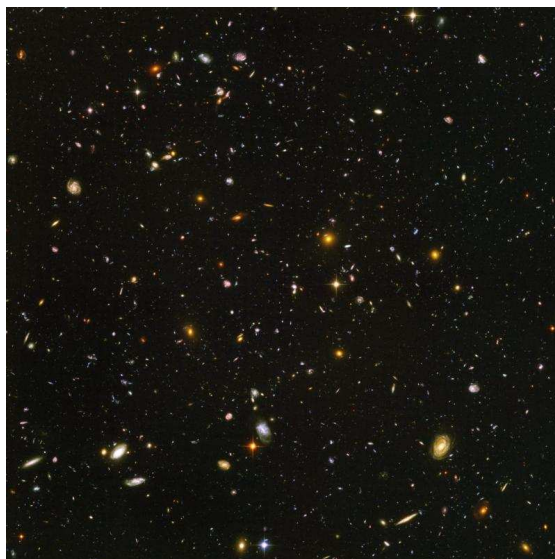

Figure 7.1: Hubble Ultra Deep Field.

multiply the ~ 1100 galaxies per square arcminute within the Hubble Ultra Deep Field by the 150 million square arcminutes on the celestial sphere, that implies that there are ~ 170 billion galaxies potentially observable by our

telescopes.

The universe is as full of galaxies as a pomegranate is of pips. Despite this fact, it wasn't until the 1920s that astronomers were convinced that large galaxies other than the Milky Way galaxy definitely existed. Edwin Hubble discovered Cepheid stars in the Andromeda Nebula (M31), and showed that the Andromeda Nebula is actually the Andromeda **Galaxy**, comparable in size to our own galaxy. Modern distance measures put the Andromeda Galaxy at a distance of 700 kpc from our own galaxy.

7.1 Galaxy Classification

Edwin Hubble, in addition to determining the true nature of the Andromeda Galaxy, also devised the classification scheme for galaxies that we use today. (Classification is an important first step in understanding. The purely empirical classification of stellar spectra, for instance, led to the physical understanding that the OBAFGKM sequence of spectral types is a temperature sequence.) Galaxies, unlike stars, are not customarily classified by their spectra. In practice, Hubble found it was most useful to classify galaxies by their shapes. The Hubble classification scheme for galaxies is thus a **morphological** classification.[1]

The Hubble scheme divides galaxies into three main classes: **elliptical**, **spiral**, and **irregular** galaxies. Our galaxy is an example of a spiral galaxy. As a useful mnemonic device, the different types of galaxies are laid out in what's generally called a "tuning fork" diagram, as shown in Figure 7.2. In the tuning fork diagram, elliptical galaxies are on the fork's handle, the two types of spiral galaxies (with and without central bars) provide the two tines of the fork, and irregular galaxies are dumped off to one side. Hubble erroneously though that the sequence shown in the tuning fork diagram was an evolutionary sequence, with galaxies moving from left to right on the diagram as they evolved. We now know that Hubble was wrong on this point: elliptical galaxies do *not* evolve into spiral galaxies. Nevertheless, the tuning fork is still appears in astronomy textbooks, as a convenient visual aid to remembering the different classes of galaxies.

Elliptical galaxies derive their name from the fact that they look like smooth, glowing, elliptical blobs, with no dark dust lanes, no spiral arms, and no bright patches of star formation. If you approximate the shape of an

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[1]The term "morphological" comes from the Greek root "morphos", meaning "shape".
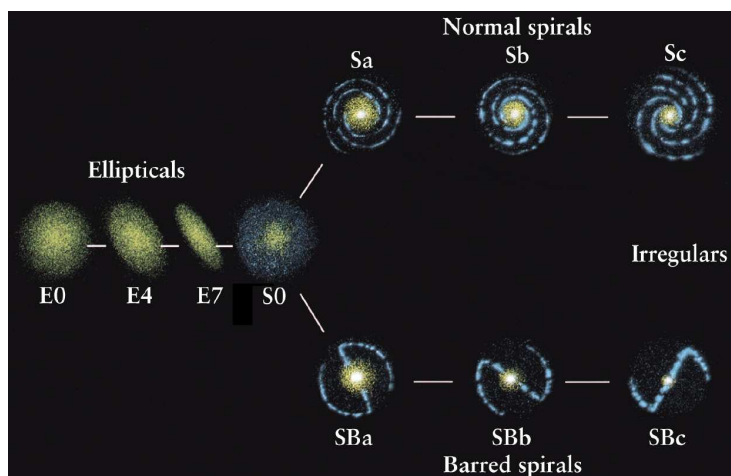
Figure 7.2: The tuning fork diagram of galaxies.

elliptical galaxy as a perfect ellipse, the size and shape of the ellipse are given by the semimajor axis a and the semiminor axis b, where $b \leq a$. The shape of the ellipse can be described by a single number. It might be the axis ratio $q \equiv b/a$, or the ellipticity $\varepsilon = 1 - q$, or the eccentricity $e = (1 - q^2)^{1/2}$. The Hubble classification scheme assigns to each elliptical galaxy a label "E**n**", where **n** is equal to ten times the ellipticity, rounded to the nearest integer. Thus, an E0 galaxy is nearly circular, while the flattest elliptical galaxies seen are around E6 or E7 (Figure 7.3). One unavoidable drawback to Hubble's
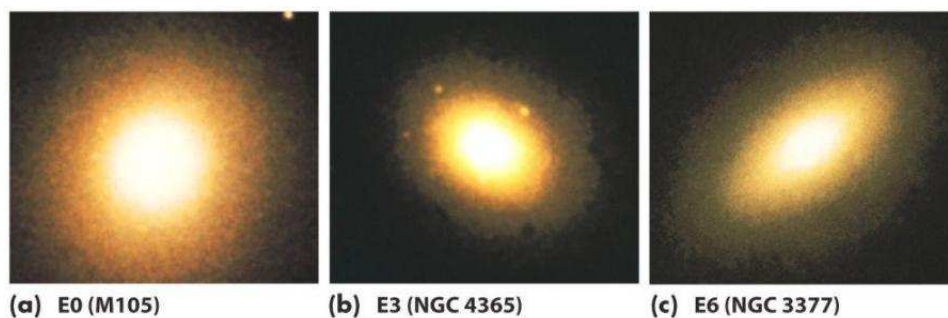

Figure 7.3: Elliptical galaxies, from circular (E0) to flattened (E6).

method for classifying elliptical galaxies is that they rely on the projected, two-dimensional shape of ellipticals, not on their intrinsic, three-dimensional

shape. Unfortunately, we can't nip 'round to the side for an alternate view, so we don't know whether an E0 galaxy is spherical, or an oblate spheroid seen face-on, or a prolate spheroid seen end-on.

Although the shape of a single elliptical galaxy can't be unambiguously determined from its two-dimensional image, statistical statements can be made about the intrinsic shapes of elliptical galaxies, after looking at large data sets.[2] The typical elliptical galaxy must be a triaxial ellipsoid, with principal axes of three different lengths. (Zeilik and Gregory say that elliptical galaxies are oblate spheroids, but this is wrong.) The surface brightness of bright elliptical galaxies, $I(r)$, usually follows the law

$$\log I \propto -r^{1/4} \ , \tag{7.1}$$

where r is the distance from the galaxy's center. (Zeilik and Gregory say that $\log I \propto r^{-1/4}$, but this is wrongedy-wrong-wrong.) The luminosities of ellipticals cover a very wide range. The most luminous giant ellipticals have $M_V \sim -23$, or $L_V \sim 10^{11} L_{V,\odot}$.[3]

Spiral galaxies derive their name from their spiral arms, most easily seen when we view the galaxy face-on. The spiral structure of these galaxies was first noted by Lord Rosse in 1845, when he viewed M51 (the Whirlpool Galaxy) with the 72-inch telescope, the Leviathan of Parsonstown.[4] Every spiral galaxy has a central bulge, a rotating disk, and spiral arms within the disk, containing gas, dust, and star-forming regions. There are three main subdivisions of spiral galaxies, as illustrated in Figure 7.4. The main classes are:

- **Sa**: Big bulge, tightly wound spiral arms, little gas and dust.

- **Sb**: Medium bulge, moderately wound spiral arms, middling amounts of gas and dust.

- **Sc**: Small bulge, loosely wound spiral arms, lots of gas and dust.

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[2]As a simple example, globular clusters must all be nearly spherical. Why? Because they all look nearly circular in projection. The only shape that always looks circular, from any angle, is a sphere.

[3]Cautionary footnote: all absolute magnitudes and V-band luminosities in this section are approximate.

[4]He wasn't certain, however, whether he was looking at a galaxy, a smaller cluster of stars in our own galaxy, or perhaps a nearby planetary system in the process of formation.

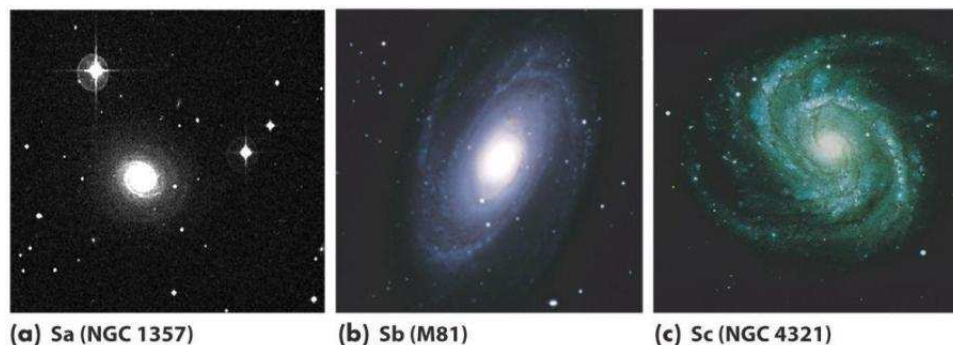(a) Sa (NGC 1357)   (b) Sb (M81)   (c) Sc (NGC 4321)

Figure 7.4: Spiral galaxies, from Sa to Sc.

Many spiral galaxies, perhaps even the majority of them, have an elongated central bar of stars. Barred spirals, like "ordinary" spirals, can be further subdivided into SBa, SBb, and SBc, with the capital B standing for Barred. Examples of barred spiral galaxies are shown in Figure 7.5. Our own galaxy
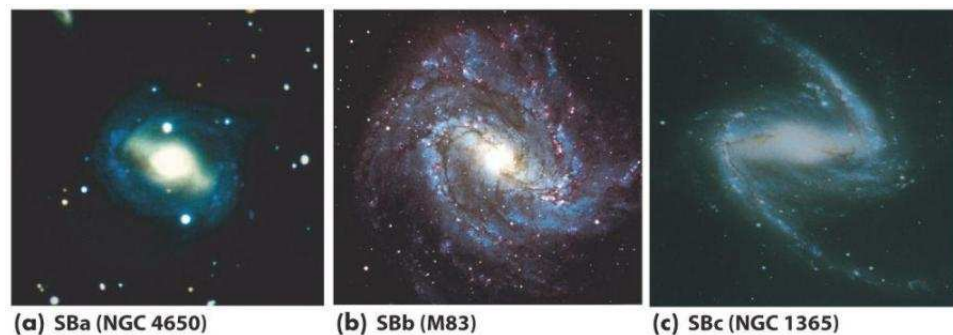

(a) SBa (NGC 4650)   (b) SBb (M83)   (c) SBc (NGC 1365)

Figure 7.5: Barred spiral galaxies, from SBa to SBc.

has a bar, though its degree of "barrishness" is hard to tell from our location inside the disk. At a guess, the Hubble classification of our galaxy would be SBb. The surface brightness of the disks of spiral galaxies falls off exponentially with distance from the galaxy's center:

$$\log I \propto -r \ . \tag{7.2}$$

The bulges can frequently be fit with the $\log I \propto -r^{1/4}$ law that applies to elliptical galaxies. Our galaxy and M31 (the Andromeda Galaxy) are both bright spiral galaxies, with $M_V \sim -21$, or $L_V \sim 2 \times 10^{10} L_{V,\odot}$.

The Hubble classification for spiral galaxies has been extended to type Sd, which represents spirals with minuscule bulges and huge amounts of gas and dust. A final type of spiral galaxy is the "Magellanic spiral", also referred to as type Sm. Magellanic spirals are systems similar to the Large Magellanic Cloud, which has a very prominent bar, rudimentary spiral arms, and lots of active star formation. Since the Large Magellanic Cloud has many young stars, some of them massive, it is host to the occasional core-collapse (type II) supernova, like Supernova 1987a.

There exist galaxies intermediate between spiral and elliptical galaxies; these are called **S0** galaxies.[5] S0 galaxies have flat rotating disks, like spiral galaxies. However, like elliptical galaxies, they have very little gas and dust, and no spiral arms at all. In tribute to their hybrid nature, they are placed at the Y-junction of the "tuning fork", between the ellipticals and spirals (Figure 7.2). S0 galaxies are sometimes also referred to as lenticular galaxies, since they have big central bulges, which give them a shape like a convex lens (Figure 7.6).

Figure 7.6: NGC 3115, an edge-on S0 galaxy ($d \approx 10\,\mathrm{Mpc}$).

Irregular galaxies are the last of Hubble's main classes. As their name implies, they are amorphous, lacking any regular shape. Irregular galaxies are rich in gas and dust, and have copious star formation. A nearby example of an irregular galaxy is the Small Magellanic Cloud, in which there is not even a hint of spiral structure. The Small Magellanic Cloud looks like an

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[5]That's "S zero", not "S oh".

egg-shaped smear of stars punctuated with emission nebulae. See Figure 7.7 to compare the Large and Small Magellanic Clouds.

Figure 7.7: Left to right: Large Magellanic Cloud ($d \approx 55\,\mathrm{kpc}$), Small Magellanic Cloud ($d \approx 65\,\mathrm{kpc}$).

Although Hubble's classification scheme is useful, it has some restrictions. First of all, due to the technical limitations of Hubble's day, it applies only to luminous galaxies with high surface brightness. The lowest luminosity galaxies are called **dwarf galaxies**. These dwarfs tend to be low in surface brightness as well as low in total luminosity; that is, the stars they contain are spread over a relatively wide area on the sky. Thus, dwarf galaxies are hard to detect. Some dwarf galaxies are elliptical in shape, and contain little gas and dust; these are called **dwarf ellipticals**. A dwarf elliptical generally has $M_V > -18$, or $L_V < 10^9 L_{V,\odot}$. The very dimmest dwarf ellipticals, with $M_V > -14$, or $L_V < 3 \times 10^7 L_{V,\odot}$, are often called "dwarf spheroidals". The dwarf spheroidal galaxy Leo I is shown in Figure 7.8. Note that individual stars can be resolved in this dwarf spheroidal, which is only 250 kpc away. Some dwarf galaxies do contain lots of gas and dust; these are called **dwarf irregulars**.[6] Just as inconspicuous M dwarfs are the most common type of star, inconspicuous dwarf galaxies are the most common type of galaxy. An estimated 90% of the galaxies in our immediate neighborhood (less than a megaparsec away) are dwarfs.

Finally, some galaxies don't fit into Hubble's classification scheme because they are just plain weird. The galaxy Centaurus A (Figure 7.9) has

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[6]Apparently, there are no dwarf spiral galaxies; spiral structure apparently only appears in massive, luminous galaxies.

Figure 7.8: The dwarf spheroidal Leo I ($d \approx 250 \, \text{kpc}$

been called "a pathological object". It resembles an elliptical galaxy, but it has a prominent dust lane slashing across its middle – something that ordinary ellipticals just don't have. Centaurus A is also a strong radio source; its name indicates that it is the brightest radio source in the constellation Centaurus. It is probable that Centaurus A has recently cannibalized a dust-rich companion galaxy. The galaxy NGC 7252 (Figure 7.10) has been called "a train wreck". Although NGC 7252 bears a single catalog number in the New General Catalog, it is actually a pair of galaxies that have not yet finished the process of merging together. The long tails extending away from the train wreck have been stretched out by tidal forces. Roughly 0.5% of nearby bright galaxies are estimated to be members of merging pairs.

7.2 Galaxy Spectra

Although a galaxy's shape contains interesting information, it doesn't tell the whole story. Useful information can also be derived from the spectrum of a galaxy. **Visible** light emitted by a galaxy is primarily produced by stars (which have an absorption line spectrum) and by hot gas (which has an emission line spectrum). Most of the starlight comes from a small number of very luminous stars, not from the huge number of dim M dwarfs. A single main sequence O star, with $M_V \approx -5$, produces as much visible light as

Figure 7.9: Centaurus A, a peculiar galaxy ($d \approx 3.5\,\mathrm{Mpc}$).

100 million main sequence M stars, with $M_V \approx +15$. In spiral and irregular galaxies, where stars are currently forming, the brightest stars are young, hot main sequence stars of spectral type O and B. In elliptical galaxies, where star formation has usually ceased long ago, the brightest stars are red giants. Thus, elliptical galaxies, whose bright stars are relatively cool, are redder in color than spiral and irregular galaxies, whose bright stars are very hot.

The spectra of different types of galaxies are similar in appearance. Elliptical galaxies have strong absorption lines and no emission lines; the integrated light of all the stars in an elliptical galaxy produces a spectrum similar to a star of spectral type K. Spiral galaxies typically have both strong absorption lines and moderately strong emission lines; the absorption lines are similar to those of a star of spectral type F or G. (Irregular galaxies have spectra similar to those of spiral galaxies.) About 1 or 2 percent of bright galaxies are **Active Galaxies**, in which a very large fraction of the light is nonstellar in origin. The nonstellar light in an active galaxy comes from a small but luminous central nucleus; thus, active galaxies are also referred to as Active Galactic Nuclei, or **AGNs**, for short. The spectrum of an AGN has extremely strong emission lines, indicating the presence of large quantities of hot gas.

Infrared light at 10 to 100 microns comes primarily from warm dust, with temperature $T \sim 100\,\mathrm{K}$. Infrared emission is stronger from spiral and irregular galaxies than from elliptical galaxies. Within spiral galaxies, the
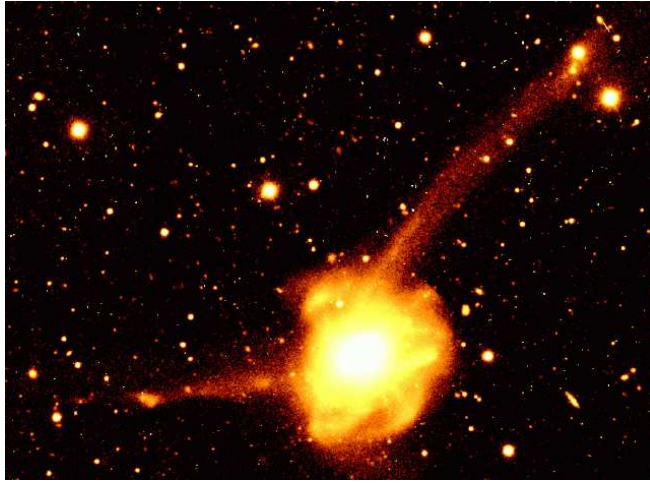
Figure 7.10: NGC 7252, the remnant of a galaxy merger ($d \approx 60\,\mathrm{Mpc}$).

infrared emission is greatest from the spiral arms, where the dust is concentrated.

Radio emission is usually stronger in spirals than in ellipticals – although there are many exceptions to the rule. In addition, many radio-loud galaxies, like Centaurus A (Figure 7.9), are peculiar in their morphology. Active galactic nuclei are strong sources of synchrotron emission. Gas clouds in spiral galaxies are strong sources of line emission. Seen at $\lambda = 21\,\mathrm{cm}$, the disk of a spiral galaxies appears larger than at visible wavelengths. This is what tells us that the gaseous disk is larger than the stellar disk.

Ultraviolet light comes primarily from hot, short-lived stars. Thus, ultraviolet light traces the arms of spiral galaxies, where most of the star formation occurs. The small amount of ultraviolet light from elliptical galaxies comes from relatively hot helium-fusing stars.

X-rays from galaxies come primarily from a relatively small number of X-ray binaries. In an X-ray binary, a black hole or neutron star accretes gas from a stellar companion.[7] In addition, some X-rays come from the hot coronal gas in a galaxy.

Because photons of different energy are created by different physical phenomena, a galaxy can change its appearance dramatically when viewed at different wavelengths. Consider, for example, the irregular galaxy M82, shown

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[7]The X-ray source V404 Cygni, discussed in section 5.3, is an example of an X-ray binary consisting of a black hole and a star.

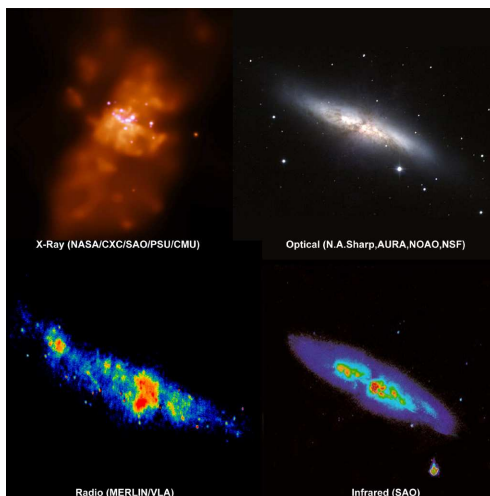in Figure 7.11 at four different wavelengths. At infrared wavelengths, M82

Figure 7.11: The galaxy M82, at wavelengths ranging from X-rays to radio.

has the highest flux of any galaxy in the sky. Although it's at a distance $d \approx 3.5\,\mathrm{Mpc}$, about five times the distance to M31, it is more than 25 times as luminous in the infrared. The excess infrared emission from M82 is caused by recent star formation inside dusty clouds. The dust absorbs the light from luminous young stars and reradiates it at longer wavelengths. Note also that the hot, X-ray emitting gas stretches out beyond the star-inhabited region. It's thought that the gas in M82 has been heated by supernova explosions to the point where it is expanding outward in a "galactic wind". M82 is an example of a **starburst galaxy**, a galaxy which has recently experienced a major episode of star formation.[8]

The visible spectra of galaxies generally contain absorption or emission lines that can be used to compute a radial velocity for the galaxy. We measure a redshift, $z \equiv \Delta\lambda/\lambda$, and compute a radial velocity $v_r = cz$, assuming we are in the nonrelativistic limit, where $z \ll 1$ and $v_r \ll c$. Because galaxies are resolved, extended objects, we can measure the radial velocity as a function of position on the galaxy's image. From this measurement, we can deduce how fast the stars in the galaxy are orbiting, and hence how massive the galaxy is.

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[8]In the case of M82, the star formation may have been triggered by a tidal encounter with its neighboring galaxy, M81.

As an example, suppose you are looking at a spiral galaxy in which the stars in the disk are on perfectly circular orbits about the galaxy's center. You see the disk at an inclination i, where $i = 0°$ if the disk is face-on and $i = 90°$ if the disk is edge-on. You see the intrinsically circular disk thanks to the effects of perspective, as an ellipse of axis ratio $q = b/a$, where a is the semimajor axis and b is the semiminor axis. If the disk is infinitesimally thin and perfectly circular, then the apparent axis ratio of the ellipse will be

$$q = \cos i \ . \tag{7.3}$$

Thus, a face-on disk ($i = 0°$) appears circular ($q = 1$), while an edge-on disk ($i = 90°$) appears as a line segment ($q = 0$). After you measure the apparent axis ratio q of a spiral galaxy, you can compute the inclination $i = \cos^{-1} q$. (There will be an error in your calculation, of course, since disks are neither infinitesimally thin nor perfectly circular, but in most circumstances, the error will be acceptably small.) For instance, the nearby spiral galaxy M31, as seen in Figure 7.12, has an apparent axis ratio $q = 0.3$. This implies that

Figure 7.12: The spiral galaxy M31, a.k.a. the Andromeda Galaxy ($d = 700\,\mathrm{kpc}$).

we are viewing M31 at a relatively high inclination of $i = 73°$.[9]

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[9]Since M31 is at a low galactic latitude ($b \approx -22°$), inhabitants of M31 see the Milky Way at a high inclination, as well.

Now suppose we measure the radial velocity $v_r = cz$ along the apparent long axis of the disk of M31. The observed radial velocity v_r will be related to the orbital speed v_c by the relation

$$v_r(R) = v_c(R) \sin i + v_{r,0} \ , \qquad\qquad (7.4)$$

where $v_{r,0}$ is the radial velocity of the center of M31, and R is the distance measured from the center of M31. If we want to know the orbital speed as a function of radial distance, we must compute

$$v_c(R) = \frac{v_r(R) - v_{r,0}}{\sin i} = \frac{v_r(R) - v_{r,0}}{\sqrt{1 - \cos^2 i}} = \frac{v_r(R) - v_{r,0}}{\sqrt{1 - q^2}} \ . \qquad (7.5)$$

Although this equation applies to any rotationally supported disk, let's continue to use M31 as our example. M31 is at a distance $d = 700\,\text{kpc}$ from us; at this distance, 1 arcsec corresponds to a length $r = 700{,}000\,\text{AU} = 3.4\,\text{pc}$. The center of M31 is moving toward us, with a radial velocity $v_{r,0} = -270\,\text{km s}^{-1}$ relative to the Sun. At an angular distance $R'' = 600$ arcsec from the center of M31, along its apparent long axis, we measure a Doppler shift $z = -0.00010$, corresponding to a radial velocity $v_r = cz = -30\,\text{km s}^{-1}$. We can compute that

$$R = (600\,\text{arcsec})(3.4\,\text{pc arcsec}^{-1}) = 2040\,\text{pc} = 2.04\,\text{kpc} \ . \qquad (7.6)$$

The orbital speed at this distance from M31 is (from equation 7.5)

$$v_c(R) = \frac{v_r(R) - v_{r,0}}{\sqrt{1 - q^2}} = \frac{-30\,\text{km s}^{-1} + 270\,\text{km s}^{-1}}{\sqrt{1 - (0.3)^2}} = 250\,\text{km s}^{-1} \ . \qquad (7.7)$$

In fact, the rotation curve of M31 is observed to be flat out to nearly 3 degrees ($R'' \approx 10{,}800$ arcsec) from the center of the galaxy, corresponding to a physical distance $R \approx 36\,\text{kpc}$ from the center. At $R \approx 36\,\text{kpc} \approx 1.1 \times 10^{21}\,\text{m}$, the calculated orbital speed is $v_c \approx 230\,\text{km s}^{-1} \approx 2.3 \times 10^5\,\text{m s}^{-1}$. The deduced mass of M31 is then

$$
\begin{aligned}
M(R) \ &\approx \ \frac{v_c^2 R}{G} \approx \frac{(2.3 \times 10^5\,\text{m s}^{-1})^2 (1.1 \times 10^{21}\,\text{m})}{6.67 \times 10^{-11}\,\text{kg m}^3\,\text{s}^{-2}} \\
&\approx \ 9 \times 10^{41}\,\text{kg} \approx 4 \times 10^{11} M_\odot \ .
\end{aligned}
\qquad (7.8)
$$

Nearly half a trillion solar masses, with no sign of a Keplerian fall-off, which would indicate the edge of a massive dark halo.

It is only practical to reconstruct the rotation curve from measuring radial velocities v_r, and not from measuring proper motions μ. If we viewed a spiral galaxy face-on $(i = 0°)$, a star with orbital speed v_c would have a proper motion μ'' given by the relation (equation 6.22):

$$\mu'' = \frac{v_c}{4.74d} \text{ arcsec yr}^{-1} \, , \tag{7.9}$$

where v_c is in kilometers per second and d is in parsecs. If we saw M31 face-on, we'd expect proper motions of approximately

$$\mu'' \approx \frac{250}{4.74(700,000)} \text{ arcsec yr}^{-1} \approx 8 \times 10^{-5} \text{ arcsec yr}^{-1} \, . \tag{7.10}$$

Future space-based interferometry missions may be able to measure proper motions of this magnitude, but at the moment, it's too small to measure.

The spectra of elliptical galaxies reveal different kinematics from spiral galaxies. In ellipticals, the orbital speed v_c is found to be small compared to that of comparably sized spiral galaxies. However, the width of the absorption lines in ellipticals is much greater than you expect from the temperature of their stars. The added width is due to the velocity dispersion σ of the stars. In an elliptical galaxy, stars are not on orderly, near-circular orbits, like stars in the disk of a spiral galaxy. Instead, they are on eccentric, randomly oriented orbits, like the stars in the halo of a spiral galaxy. Thus, along any line of sight through an elliptical, you will see stars with a wide range of radial velocities, and hence a wide range of Doppler shifts.

An interesting nearby elliptical galaxy is NGC 4365, a bright galaxy in the Virgo Cluster, about 16 Mpc away from us. In the Hubble classification scheme, it is labeled an 'E3' galaxy'; its apparent diameter is about 6 arminutes, so it is well resolved from Earth. Its surface brightness, shown in false color in the left panel of Figure 7.13, is smooth and featureless, with no bright patches of star formation and no dark dust lanes. Along the semi-major axis, the surface brightness follows the usual law for bright elliptical galaxies: $\log I \propto -r^{1/4}$. The average radial velocity, $v_r - v_{r,0}$, is shown in the central panel of Figure 7.13. The main body of the galaxy rotates about the apparent major axis, with a maximum velocity of $v \sim 50 \text{ km s}^{-1}$, much lower than the rotation speed in spiral galaxies of similar luminosity. Note also that the central core in NGC 4365 is rotation in a different direction from the rest of the galaxy! This is actually fairly common in elliptical galaxies; the "kinematically decoupled core", as the jargon goes, may be the remnants
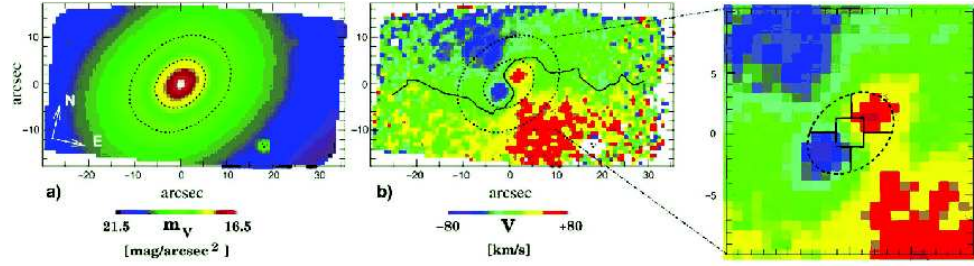
Figure 7.13: Surface brightness (left) and mean radial velocity (center) of NGC 4365; note that the core (right inset) is rotating in a different direction. [Image credit: Davies, et al. 2001, ApJL, 548, 33]

of a small but dense galaxy that has been cannibalized by the larger, fluffier galaxy.

The dispersion in radial velocity, σ, is quite large in NGC 4365, particularly when compared to the average radial velocity. The dispersion is as large as $\sigma \approx 275\,\mathrm{km\,s^{-1}}$ in the central regions of NGC 4365 (Figure 7.14), and remains as high as $\sigma \approx 200\,\mathrm{km\,s^{-1}}$ farther from the center. If we think
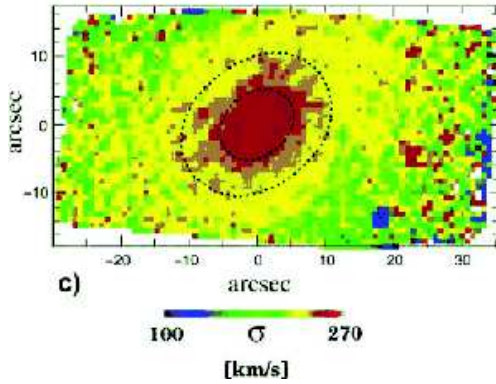

Figure 7.14: Line-of-sight velocity dispersion σ in NGC 4365. [Image credit: Davies, et al. 2001, ApJL, 548, 33]

of the individual stars in NGC 4365 as point masses in a gas, we can think of NGC 4365 as a system that is *pressure supported* rather than *rotationally supported*. The mean square velocity of the stars is then a measure of the "temperature" of the gas of stars.

The observed velocity dispersion σ along the line of sight can be used

to estimate the mass of an elliptical galaxy, or any other system dominated by random stellar motions rather than ordered orbital motions. The mass estimate involved the use of the **virial theorem**, first set forth by Rudolf Clausius in the nineteenth century. The virial theorem states that if a self-gravitating system of stars, such as a galaxy or star cluster, is in equilibrium (neither expanding nor contracting), there is a simple relation between the total kinetic energy K of all the stars and the gravitational potential energy U of the system:

$$2K = -U \ . \tag{7.11}$$

A derivation of the virial theorem is straightforward but tedious, so I won't work through it here.[10]

The total kinetic energy of a system of N stars is

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 \ , \tag{7.12}$$

where m_i is the mass of the ith star, and $\vec{v}_i$ is its velocity with respect to the center of mass of the system. The kinetic energy can also be written in the form

$$K = \frac{1}{2} M \langle v^2 \rangle \ , \tag{7.13}$$

where M is the total mass of the stars and $\langle v^2 \rangle$ is the mass-weighted mean square velocity of the stars.

The potential energy of a system of stars will be

$$U \sim -\frac{GM^2}{r} \ , \tag{7.14}$$

where r is an appropriately defined radius for the system. Finding an "appropriate" radius for a galaxy might be difficult; remember that galaxies don't have sharp, clearly defined edges. For an elliptical galaxy, it's found that a good approximation is

$$U \approx -0.4 \frac{GM^2}{r_h} \ , \tag{7.15}$$

where r_h is the half-mass radius of the system; that is, the radius of a sphere (centered on the galaxy's center) large enough to contain half the mass of the

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[10]You can find a rather sketchy derivation in section P2-5 of Zeilik and Gregory.

galaxy. With this approximation, the virial theorem (equation 7.11) becomes

$$M\langle v^2 \rangle = 0.4 \frac{GM^2}{r_h} \ , \tag{7.16}$$

or

$$M \approx 2.5 \frac{\langle v^2 \rangle r_h}{G} \ . \tag{7.17}$$

Note the similarity of equation (7.17) to the equation we used to determine the mass of a spiral galaxy:

$$M = \frac{v_c^2 R}{G} \ . \tag{7.18}$$

In each case, we square a velocity, multiply it by a radius, and divide by Newton's gravitational constant G.

Unfortunately, we can't measure the mass-weighted mean square velocity $\langle v^2 \rangle$ for an elliptical galaxy. The practical difficulties of measuring proper motions for stars in external galaxies means that we only have information about the velocity along the line of sight. Moreover, the dispersion σ that we measure is luminosity-weighted, not mass-weighted. If we assume that the red giants that provide the bulk of an elliptical galaxy's weight have the same dispersion as the rest of the galaxy's stars, we can ignore the difference between mass-weighting and luminosity-weighting. If, in addition, we assume that the velocity dispersion is *isotropic* (the same in all three dimensions), we may write

$$\langle v^2 \rangle = 3\sigma^2 \tag{7.19}$$

if the galaxy's net rotation speed is small compared to its velocity dispersion. This leads to a mass estimate

$$M \approx 7.5 \frac{\sigma^2 r_h}{G} \ . \tag{7.20}$$

Unfortunately, we can't measure the half-mass radius r_h, only the half-light radius (and in projection, at that!). In estimating the total mass of a galaxy, we sometimes have to sigh with resignation, and make the additional assumption that the half-mass radius equals the half-light radius.

As an example of the virial theorem in action, consider the dwarf spheroidal galaxy Leo I (Figure 7.8), a member of the Local Group of galaxies, at a distance $d \approx 250\,\text{kpc}$ from us. The half-light radius of Leo I is $r_h'' \approx 4.0\,\text{arcmin} \approx$

240 arcsec; at a distance of $d \approx 250 \, \text{kpc}$, this corresponds to a physical distance

$$r_h \approx 290 \, \text{pc} \approx 8.9 \times 10^{18} \, \text{m} \ . \tag{7.21}$$

Because the stars in Leo I are individually resolved, a radial velocity can be measured for each bright star in the galaxy. The dispersion in the radial velocities is

$$\sigma = 8.8 \, \text{km s}^{-1} \approx 8.8 \times 10^3 \, \text{m s}^{-1} \ . \tag{7.22}$$

The estimated mass of Leo I is then

$$
\begin{aligned}
M &\approx 7.5 \frac{(8.8 \times 10^3 \, \text{m s}^{-1})^2 (8.9 \times 10^{18} \, \text{m})}{6.67 \times 10^{-11} \, \text{m}^3 \, \text{s}^{-1} \, \text{kg}^{-1}} \\
&\approx 8 \times 10^{37} \, \text{kg} \approx 4 \times 10^7 M_\odot \ .
\end{aligned}
\tag{7.23}
$$

Since the V band luminosity of Leo I is $L_V = 4.9 \times 10^6 L_{V,\odot}$, this implies a mass-to-light ratio for Leo I of $M/L_V \approx 8 M_\odot / L_{V,\odot}$. Such a high mass-to-light ratio suggests that Leo I may contain significant amounts of dark matter.

7.3 Distances to Galaxies

Knowing the distance to galaxies is of great interest to astronomers, just as it was of great interest to know the distance to stars within our galaxy. In practice, we work our way outward to larger distances by using a **distance ladder**, with each "rung" in distance depending on a lower rung. We have already encountered the lowest rungs of the distance ladder:

- **Radar** determines distances out to $\sim 10 \, \text{AU}$. Radar distances depend on knowing the speed of light.

- **Stellar parallax** determines distances out to $\sim 100 \, \text{pc}$. Stellar parallax distances depend on knowing the length of the astronomical unit (determined using radar).

- **Spectroscopic parallax** determines distances out to $\sim 10 \, \text{kpc}$. Spectroscopic parallax distances depend on knowing the distance to nearby main sequence stars (determined using stellar parallax).

The technique of spectroscopic parallax is an example of a **standard candle** technique. To astronomers, a "standard candle" is an object whose luminosity L you know and whose flux f you can measure. The distance d can then be computed from the formula

$$d = \left(\frac{L}{4\pi f}\right)^{1/2} \; , \tag{7.24}$$

or expressed in terms of magnitudes,

$$d = 10^{0.2(m-M+5)} \, \text{pc} \; , \tag{7.25}$$

assuming no dust extinction.

To measure the distance to external galaxies (as opposed to the distance to stars within our own galaxy), a very luminous standard candle is required. Cepheid pulsating stars are a favorite standard candle for nearby galaxies. The distances to nearby Cepheids can be determined by stellar parallax, for the very nearest Cepheids, and by spectroscopic parallax, in the case of Cepheids in clusters with main sequence stars. The period - luminosity relation for Cepheids depends on the filter you are using. In the V band, a useful fit is found to be

$$\overline{M}_V = -2.76 \log(p/10 \, \text{days}) - 4.16 \; , \tag{7.26}$$

where $\overline{M}_V$ is the absolute V magnitude averaged over a complete period p of the Cepheid. Using ground-based telescopes, the apparent magnitudes of Cepheids can be measured to $d \sim 4 \, \text{Mpc}$. Using the Hubble Space Telescope, the apparent magnitudes can be measured to $d \sim 25 \, \text{Mpc}$.

Most of the hundreds of billions of galaxies in the visible universe are at distances greater than 25 Mpc. In order to measure their distances, we need a brighter standard candle. Such a standard candle is provided by type Ia supernovae. Type II (core collapse) supernovae, although very bright, are not very standardized; they have a variety of progenitor masses, and hence a variety of luminosities. Type Ia supernovae, however, are more standardized, since they all result from white dwarfs pushed over the Chandrasekhar limit.

In any given galaxy, type Ia supernova are rare. In galaxies similar to the Milky Way Galaxy, there are roughly one per century, on average. However, galaxies are sufficiently numerous that we expect an average of three type Ia supernovae to go off each year within 25 Mpc of our galaxy. Since these

supernovae occur in galaxies which contain observable Cepheid stars, we can tie the supernova length scale to the Cepheid length scale, and add another run to the distance ladder. Careful study of nearby type Ia supernovae reveals that they don't all have exactly the same luminosity. This is bad news for a standard candle. However, the peak luminosity is found to be correlated with the rate of decline of the luminosity after the peak; brighter supernovae have a slower decline (Figure 7.15). This means that we can
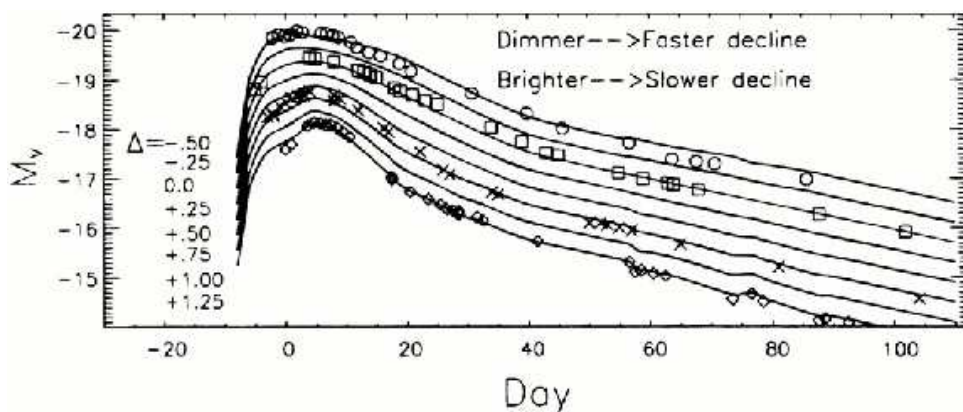

Figure 7.15: Empirical light curves of type Ia supernovae. [Image credit: Riess et al. 1996, ApJ, 473, 88]

predict a supernova's peak luminosity from its rate of decline, just as we can predict a Cepheid's average luminosity from its pulsation rate. This makes type Ia supernovae a much more attractive standard candle. The average peak luminosity of nearby type Ia supernovae is $M_V = -19.2$. This is about 1/3 the luminosity of the Milky Way Galaxy or M31, and about 3 times the luminosity of the Large Magellanic Cloud. Thus, a type Ia supernova is briefly equal in luminosity to a mid-sized galaxy. Basically, if a galaxy is bright enough for you to detect, a supernova in that galaxy will be bright enough to detect as well (unless it is very deeply buried in dust).

Type Ia supernovae have the drawback of being rare. If you want to find the distance to any particular galaxy, you might have to wait decades – or even centuries – for a type Ia supernova to occur. If you are impatient, and want to estimate a galaxy's distance *right now*, there is an alternative. You can use the entire galaxy as a standard candle. For nearby ellipticals whose distance is known, the velocity dispersion σ is discovered to be correlated

with the total V band luminosity, L_V. The relation between σ and L_V, known as the Faber-Jackson relation after its discoverers, is

$$\frac{L_V}{2 \times 10^{10} L_{V,\odot}} = \left(\frac{\sigma}{200 \,\mathrm{km\,s^{-1}}}\right)^4 \,, \tag{7.27}$$

or in terms of absolute magnitudes,

$$M_V = -21.4 - 10 \log \left(\frac{\sigma}{200 \,\mathrm{km\,s^{-1}}}\right) \,. \tag{7.28}$$

For spiral galaxies, there is a similar relation, called the Tully-Fisher relation, which relates the peak rotation speed v_c to the absolute magnitude M of the galaxy. In the B band,

$$M_B = -20.8 - 10.2 \log \left(\frac{v_c}{200 \,\mathrm{km\,s^{-1}}}\right) \,. \tag{7.29}$$

There is plenty of scatter in the Faber-Jackson and Tully-Fisher relation, but sometimes its the best you can do. The scatter can be reduced in the Faber-Jackson relation by including information about the galaxies' central surface brightness. The scatter can be reduced in the Tully-Fisher relation by making observations in the infrared, where the dimming effects of dust aren't as strong.

7.4   The Hubble Law

In addition to determining the distance to a galaxy from a standard candle, we can determine its radial velocity from its redshift. In the nonrelativistic limit, $v_r = cz = c(\Delta\lambda/\lambda)$. As noted in section 6.3, nearby stars within our own galaxy show a mixture of redshifts ($z > 0$) and blueshifts ($z < 0$). This is no surprise; our galaxy is neither expanding nor contracting. Early in the 20th century, Vesto Slipher began what was then the non-trivial task of measuring the wavelength shifts of nearby galaxies. By the year 1917, he had measured z for a sample of 25 nearby spiral galaxies (or "spiral nebulae", as Slipher called them back then). He was surprised to find that 21 out of the 25 galaxies were *redshifted*, and only 4 were blueshifted. This proportion of redshifts was unlikely to have occurred by chance.[11]   Slipher was also

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[11]If you flipped a coin 25 times, the probability of having 21 or more heads is less than 1 in 2000.

surprised by the size of the deduced radial velocities; some of the spiral galaxies had $cz > 1000\,\mathrm{km\,s^{-1}}$.

Another surprise came in 1929, when Edwin Hubble examined how the wavelength shift z depended on d, the measured distance to a galaxy. Hubble had z for about 50 galaxies but had distance estimates for only 25 of them. When he plotted cz versus d, he found an approximate linear relation, as shown in Figure 7.16. The linear relation between redshift and distance can

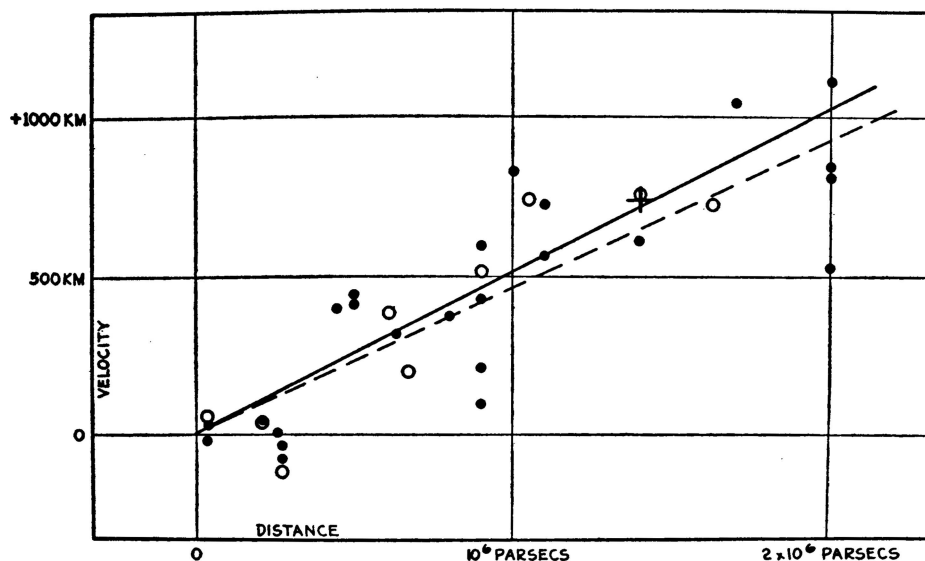

Figure 7.16: The Hubble diagram, according to Hubble. [Image credit: Hubble 1929, Proc. Nat. Acad. Sci., 15, 168]

be written in the form

$$cz = H_0 d \,, \tag{7.30}$$

where the constant H_0, pronounced "H naught", is now called the **Hubble constant**. As a further tribute to Hubble, the plot of cz versus d is called a Hubble diagram, and the equation $cz = H_0 d$ is called the Hubble law.

As it turned out, Hubble severely underestimated the distance to nearby galaxies, since his standard candles were actually much more luminous than he thought.[12] The farthest galaxies in Figure 7.16 are in the Virgo Cluster of galaxies. Hubble thought they were $\sim 2\,\mathrm{Mpc}$ away; current dis-

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[12]He was using the brightest star in each galaxy as his standard candle. However, in

tance estimates put the Virgo cluster at a distance $d \sim 16\,\mathrm{Mpc}$. Thus, although Hubble thought the Hubble constant was $H_0 \sim 1000\,\mathrm{km\,s^{-1}}/2\,\mathrm{Mpc} \sim 500\,\mathrm{km\,s^{-1}}\,\mathrm{Mpc}$, the current best estimate of the Hubble constant is much smaller:

$$H_0 = 70 \pm 5\,\mathrm{km\,s^{-1}}\,\mathrm{Mpc^{-1}} \ . \tag{7.31}$$

After Hubble's erroneously high value of H_0 was rejected, there was a decades-long debate over whether the true value of the Hubble constant was $H_0 \approx 50\,\mathrm{km\,s^{-1}}\,\mathrm{Mpc^{-1}}$ or $100\,\mathrm{km\,s^{-1}}\,\mathrm{Mpc^{-1}}$. As a relic of that long debate, you sometimes see the Hubble constant written in the form

$$H_0 = 100h\ \mathrm{km\,s^{-1}}\,\mathrm{Mpc^{-1}} \ , \tag{7.32}$$

where $0.5 < h < 1$. Since the value of the Hubble constant is pinned down much more accurately now, please feel free to substitute $h = 0.7$ whenever you encounter it.

When cosmologically naïve individuals first encounter the Hubble law, they are likely to say, "Why are all the galaxies moving away from *us*? Was it something we said? Do we have the galactic equivalent of bad breath?" In fact, there is nothing particularly special about us. We are *not* the center of the expansion; in fact, there *is* no center of expansion. The Hubble law is the natural result of the homogeneous, isotropic expansion of the universe. Saying that the expansion is **homogeneous** means that it is the same at all locations. Saying that it is **isotropic** means that it is the same in all directions. For homogeneous expansion, H_0 is the same at all positions; for isotropic expansion, H_0 is the same in all directions at a given location. The usual analogy compares the universe to a loaf of raisin bread expanding homogeneously and isotropically, so that its shape remains the same as the loaf expands (Figure 7.17). In such an expansion, each raisin sees every other raisin moving away with a velocity proportional to its distance. Note that the raisins themselves don't expand, since they are held firmly together by intermolecular forces. Similarly, in the globally expanding universe, galaxies themselves don't expand, since they are held firmly together by gravity. The Hubble expansion is not the result of some mysterious force that is prying apart the entire universe down to the tiniest scales; it's simply an empirical statement about kinematics on large scales. Widely separated galaxies are

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more distant galaxies, what he thought was the brightest star was actually a compact, ultraluminous HII region, which can be 50 times more luminous than even the brightest stars.
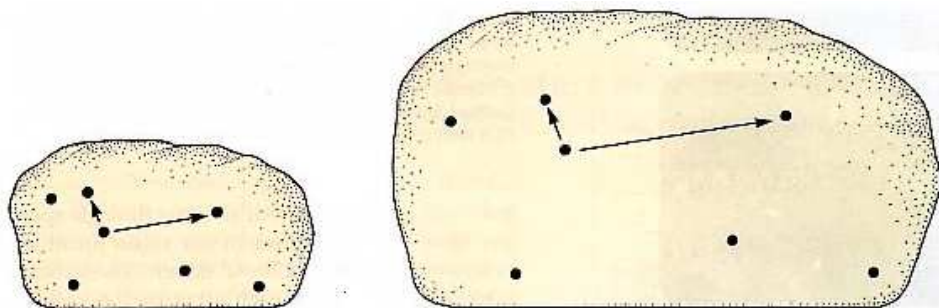
Figure 7.17: To see the universe in a loaf of raisin bread...

moving away from each other, with a speed proportional to the distance between them.

If galaxies are moving apart from each other, they must have been closer together in the past. Thus, the discovery by Hubble of the expansion of the universe led naturally to the **Big Bang** model for the universe. A Big Bang model can be defined as a universe that starts in an extremely dense, compressed state, and then expands to increasingly low densities. Consider a pair of galaxies that are currently separated by a distance d, and thus have a relative speed $v_r = H_0 d$. If the relative speed of the galaxies has been constant, then the time it took to reach their current separation is

$$t = \frac{d}{v_r} = \frac{d}{H_0 d} = \frac{1}{H_0} \; , \tag{7.33}$$

assuming that the galaxies started out very close to each other $(d_{\text{init}} \ll d)$. Notice that t is independent of the present separation d. This means that at a time t before the present, all pairs of galaxies were very close to each other.

The Hubble law thus gives us a natural time scale for the expansion of the universe: the Hubble time, which is simply

$$H_0^{-1} = 14 \pm 1 \, \text{Gyr} = (4.4 \pm 0.3) \times 10^{17} \, \text{s} \; . \tag{7.34}$$

In calculating that the universe has been expanding for one Hubble time, we assumed that the relative speed of any pair of galaxies has been constant. This is not necessarily true. The gravitational attraction between galaxies tends to decelerate the expansion; in this case, the expansion was faster in the past than it is now, and the universe is younger than H_0^{-1}. On the other hand, it has been theorized that the universe contains "dark energy", which

causes the expansion to accelerate outward; in such a case, the expansion was slower in the past than it is now, and the universe is older than H_0^{-1}. It is reassuring to note, in any case, that the estimated age of the oldest globular clusters is close to 14 gigayears. Hubble's initial (erroneously large) value for the Hubble constant implied a Hubble time of only $2\,\text{Gyr}$, which was embarrassingly short compared to the known age of the Earth.[13]

The expansion of the universe has profound cosmological and philosophical implications. At the moment, let's postpone the profound philosophy, and be practical and down-to-earth. The Hubble law, from a practical viewpoint, gives us another way of estimating the distance to a galaxy. We simply measure the redshift z, and then compute

$$d = \frac{c}{H_0}z = (4300 \pm 300\,\text{Mpc})z \ . \tag{7.35}$$

Notice that just as the Hubble time, $1/H_0$, gives a natural time scale in an expanding universe, the Hubble distance, c/H_0, gives a natural length scale. If a galaxy is moving away from us at 1% the speed of light, its distance from us is 1% of the Hubble distance, or $43\,\text{Mpc}$. We should keep in mind, however, the sources of error involved when we use equation (7.35) to estimate distances. First of all, we don't know the Hubble distance exactly. Second, there is a significant amount of scatter in the Hubble diagram (Figure 7.16). This is because in addition to the perfectly homogeneous Hubble expansion, galaxies have *peculiar velocities* of order $\sim 10^{-3}c$ caused by the gravitational attraction of their neighboring galaxies. These peculiar velocities cause errors of order $\sim 10^{-3}(c/H_0) \sim 4\,\text{Mpc}$ in the distance estimates.[14] Finally, we must keep in mind that the linear relation between d and z only holds true in the limit $z \ll 1$. At higher redshifts, nonlinear relativistic corrections must be taken into account.

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[13]The universe can't be younger than the objects it contains. In the words of a lady who was asked her age, "I am older than my teeth, and the same age as my tongue."

[14]For example, the radial velocity of M31 relative to the Milky Way Galaxy is negative, since the two galaxies form a gravitationally bound system in which they are on strongly radial orbits about their center of mass. If you stared straight at M31 while computing its distance from equation (7.35), you would conclude it was behind you!