Chapter 2

Stellar Atmospheres

All that we know about stars other than the Sun comes from collecting photons (unless you count the 19 neutrinos that were detected from Supernova 1987a, a star in the process of collapsing). The problem with photons is that they tell us only what is happening in the *photosphere*, the relatively thin layer of a star from which the photons escape. When we compute the "radius of a star", for instance, we are really computing the radius of the star's photosphere. When astronomers talk about the "temperature of a star", they mean the temperature of the star's photosphere, unless they explicitly state otherwise.

A word of advice: studying the atmospheres of stars requires understanding how light interacts with matter. If you feel a bit rusty in your knowledge, you may want to review Chapter 6 of *BA* and/or Chapter 8 of Zeilik and Gregory.

2.1 Hydrostatic Equilibrium

To understand how a star's spectrum is produced, we must understand the basic physics of stellar atmospheres. In some ways, the atmosphere of a star is like the Earth's atmosphere; despite winds and storms, both types of atmosphere are in **hydrostatic equilibrium** (as described in section 10.1.2 of *BA*). In other ways, a star's atmosphere is unlike the Earth's. For one thing, the Earth's atmosphere rests upon a solid or liquid surface; since stars are completely gaseous, you can think of them as being nothing but atmosphere. Another difference between stellar atmospheres and the Earth's atmosphere is that the atmospheres of stars are relatively hot, and ionization becomes important.

For a spherical star in hydrostatic equilibrium (*BA*, equation 10.8),

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} \ , \tag{2.1}$$

where r is the distance from the star's center, P is the local pressure, M_r is the mass contained within a sphere of radius r, ρ is the local mass density, and G is the Newtonian gravitational constant. In other words, the upward force due to the pressure gradient (the left-hand side of equation 2.1) is exactly balanced by the inward force due to gravity (the right-hand side of equation 2.1).

The pressure at any point inside the star is well approximated by the perfect gas law:

$$P = nkT \ , \tag{2.2}$$

where n is the number density of particles (ions, free electrons, atoms, and molecules), k is the Boltzmann constant, and T is the temperature (in Kelvin).[1] When computing pressure forces, what counts is the total number density of particles, n; low-mass electrons and high-mass molecules contribute equally to the pressure. The number density n is simply related to the mass density ρ:

$$n = \frac{\rho}{\mu m_p} \ , \tag{2.3}$$

where μ is the mean molecular weight of the particles, measured in atomic mass units. Strictly defined, the atomic mass unit is the mass of a carbon-12 atom divided by 12. For practical purposes, as in equation (2.3), we can set it equal to the mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg.

Stars contain a mix of different elements. Most stars, however, contain mostly hydrogen and helium in their photospheres, with only small amounts of other elements. Thus, astronomers have drastically simplified their view of the periodic table. Like primitive tribesmen who count "one, two, many", astronomers often acknowledge the presence of only three elements: hydrogen (atomic number = 1), helium (atomic number = 2), and "metals" (atomic number > 2). Thus, in the jargon of astronomers, a "metal" is any element

[1] In some extremely luminous stars, the radiation pressure contributed by photons is large enough to be significant. However, in stars similar to the Sun, the radiation pressure is everywhere tiny compared to the gas pressure.

heavier than helium. The total mass density of a star's atmosphere can be broken down into three components:

$$\rho = \rho_{\rm H} + \rho_{\rm He} + \rho_{\rm metal} \ . \tag{2.4}$$

The chemical composition of the atmosphere can them be expressed in terms of the hydrogen mass fraction,

$$X \equiv \rho_{\rm H}/\rho \ , \tag{2.5}$$

the helium mass fraction,

$$Y \equiv \rho_{\rm He}/\rho \ , \tag{2.6}$$

and the "metal" mass fraction,

$$Z \equiv \rho_{\rm metal}/\rho = 1 - X - Y \ . \tag{2.7}$$

In section 8.1.1 of *BA*, for instance, you learned that the Sun's photosphere has $X_\odot = 0.734$, $Y_\odot = 0.250$, and $Z_\odot = 0.016$. Most of the mass of "metals" is contributed by oxygen and carbon, with neon, iron, and nitrogen rounding out the top five metals list.

The mean molecular weight μ of a gas depends both on its mass fractions (X, Y, and Z) and on its ionization state. Consider, for instance, a gas consisting of pure atomic hydrogen ($X = 1$). If it is neutral, it will have a mean molecular weight $\mu = 1$ and a number density $n = \rho/m_p$.[2] If the hydrogen is fully ionized, the number of particles is doubled, since one electron is freed from each atom. Thus, the number density of a hydrogen gas doubles to $n = 2\rho/m_p$ if it is fully ionized, and its mean molecular weight drops to $\mu = 1/2$. Now consider a gas made of pure helium ($Y = 1$). If it is neutral, it will have a mean molecular weight $\mu = 4$ and a number density $n = \rho/(4m_p)$, since each helium atom contains four nucleons – two protons and two neutrons.[3] If the helium is fully ionized, the number of particles is tripled, since two electrons are freed from each atom. Thus, the number density of a helium gas triples to $n = 3\rho/(4m_p)$ if it is fully ionized, and its mean molecular weight drops to $\mu = 4/3$. Finally, consider a gas made of "metals" ($Z = 1$). If the average number of nucleons per atom is A, the

[2] Our calculations will be sufficiently accurate if we assume that the mass of an electron is negligibly small compared to the mass of a proton.

[3] Our calculations will be sufficiently accurate if we assume that the mass of a neutron and the mass of a proton are the same.

number density of atoms for the gas in its neutral state is $n = \rho/(Am_p)$, and the mean molecular weight is A. If the number of protons and neutrons in each nucleus is roughly equal, then $\sim A/2$ electrons will be liberated from each atom when the gas is fully ionized. The number density of the gas will then be $n \sim \rho/(2m_p)$, assuming $A/2 \gg 1$, and its mean molecular weight will be $\mu \sim 2$.

Thus, if a gas consists of a mix of hydrogen, helium, and metals, its number density of particles, in a fully ionized state, will be

$$n \approx X\left(\frac{2\rho}{m_p}\right) + Y\left(\frac{3\rho}{4m_p}\right) + Z\left(\frac{\rho}{2m_p}\right) \tag{2.8}$$

$$\approx \left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)\frac{\rho}{m_p} . \tag{2.9}$$

For a fully ionized gas, the mean molecular weight will then be

$$\mu(\text{ionized}) = \frac{\rho}{nm_p} = \left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)^{-1} . \tag{2.10}$$

For the Sun's photosphere, with $X_\odot = 0.734$, $Y_\odot = 0.250$, and $Z_\odot = 0.016$, the mean molecular weight, assuming total ionization, is

$$\mu_\odot = \frac{1}{1.468 + 0.188 + 0.008} = \frac{1}{1.664} = 0.60 . \tag{2.11}$$

By contrast, for a gas of neutral atoms, the mean molecular weight will be

$$\mu(\text{neutral}) = \left(X + \frac{Y}{4} + \frac{Z}{A}\right)^{-1} . \tag{2.12}$$

If the Sun's photosphere were neutral, its mean molecular weight would be $mu_\odot \sim 1.25$; since the rare metals contribute so little to the computation of μ, it doesn't really matter what their value of A is. We conclude that no matter how highly ionized the Sun's atmosphere is, its mean molecular weight will be the proverbial "factor of order unity".[4]

Ionized gas has a higher pressure than a gas of neutral atoms with the same mass density. This is for two reasons:

[4] A quick googling of the phrase "factor of order unity" yielded 17,000 hits. People seem to enjoy using this fancy phrase for a number close to one.

- Maintaining a gas in an ionized state requires a high temperature T; higher temperature implies a higher thermal velocity, and hence more pressure per particle.

- Ionization frees large quantities of electrons, thus increasing the number density n of particles at a given mass density.

Ionizing a gas increases both T and n in the relation $P = nkT$.

Stars are usually stable. The Sun has been shining away for 4.6 billion years without exploding or imploding. Some stars pulsate in and out perceptibly, and now and then a supernova explodes, but for the most part, stars are in sedate hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho = -g\rho \ , \tag{2.13}$$

where

$$g \equiv \frac{GM_r}{r^2} \tag{2.14}$$

is the gravitational acceleration (directed inward) at a distance r from the star's center. Since g and ρ are non-negative, the pressure gradient is $dP/dr \leq 0$. That is, the pressure becomes greater as you dive inward to the center of a star.

At the Sun's photosphere, $M_r = M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$; the mass of the chromosphere and corona above the photosphere are negligible. The radius of the photosphere is $r_\odot = 6.96 \times 10^8\,\mathrm{m}$. The gravitational acceleration at the Sun's photosphere is then

$$g_\odot = \frac{GM_\odot}{r_\odot^2} = 274\,\mathrm{m\,s^{-2}} \ , \tag{2.15}$$

about 28 times the gravitational acceleration at the Earth's surface. For a perfect gas, the mass density and pressure are related by the law

$$\rho = \frac{\mu m_p}{kT}P \ , \tag{2.16}$$

so the equation of hydrostatic equilibrium (equation 2.13) can be written in the form

$$\frac{dP}{dr} = -\frac{g\mu m_p}{kT}P \ . \tag{2.17}$$

If we are in a region of the star where g, μ, and T are roughly constant with radius, equation (2.17) has a solution of the form

$$P \propto \exp\left(-\frac{r}{H}\right) , \tag{2.18}$$

where the scale height H is

$$H = \frac{kT}{g\mu m_p} . \tag{2.19}$$

For the Sun's photosphere, with $T_\odot \approx 5800\,\mathrm{K}$ and $\mu_\odot \approx 0.60$, the scale height is

$$H_\odot = \frac{kT_\odot}{g_\odot \mu_\odot m_p} \approx 300\,\mathrm{km} . \tag{2.20}$$

This is much longer than the scale height $H \approx 8\,\mathrm{km}$ of the Earth's atmosphere, which is cooler and has a higher mean molecular weight. However, it is much shorter than the Sun's radius of $R_\odot \approx 700{,}000\,\mathrm{km}$. Thus, our assumption of constant $g_\odot$, $\mu_\odot$ and $T_\odot$ within the photosphere is a reasonable first approximation. Although the Sun doesn't have a sharply defined sur-

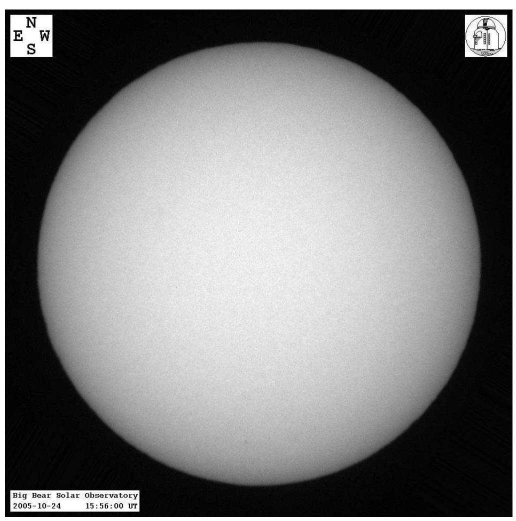

Figure 2.1: Image of the Sun at visible wavelengths. [Image credit: Big Bear Solar Observatory]

face, the pressure and density within the photosphere fall off exponentially with a short scale length: $H_\odot \ll R_\odot$. This gives an illusion of a sharp edge in images of the Sun at visible wavelengths (Figure 2.1).

2.2 Spectral Classification

A star's spectrum contains information about the photosphere's chemical composition and temperature. Every absorption line that you see in a star's spectrum represents the transition of an electron from a lower energy level to a higher level, in a particular element in a particular ionization state.[5] The solar spectrum (Figure 2.2) contains absorption lines from most of the

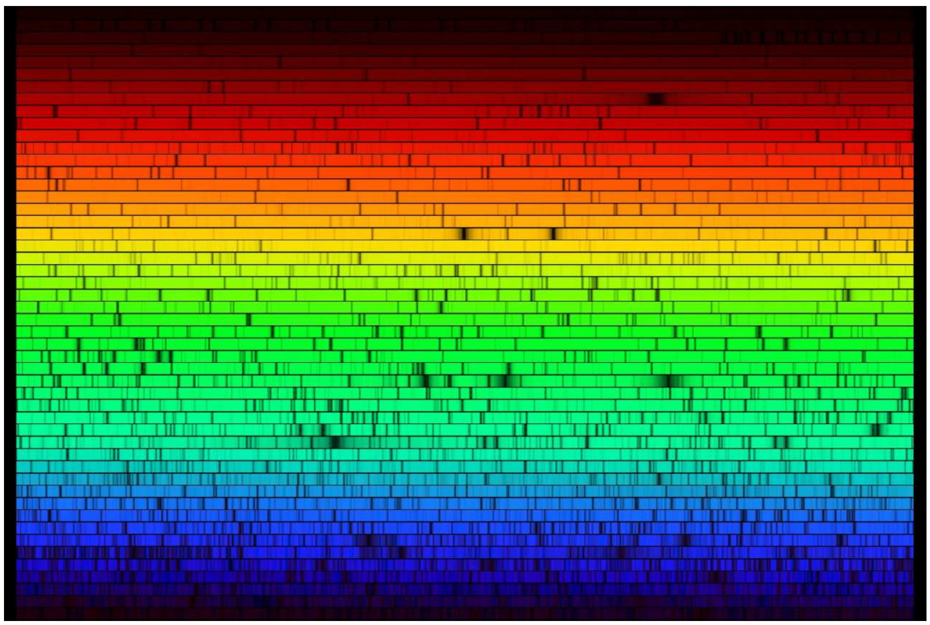

Figure 2.2: Spectrum of the Sun at visible wavelengths [Image credit: NOAO/AURA/NSF]

elements found in nature. Interestingly, the element helium, the second most abundant element in the universe (after hydrogen), reveals its presence only by emission lines from the Sun's chromosphere. During a solar eclipse in the year 1868, the astronomer Norman Lockyer discovered a previously unknown emission line at $\lambda = 5876\,\text{Å}$ in the chromosphere's spectrum. He attributed the emission line to a new element, which he named "helium", after the Greek sun god Helios.

The strength of absorption lines, as measured by their equivalent width W, depends on the temperature of the photosphere. For example, the ab-

[5] In some cooler stars, you also see absorption bands due to molecules.

sence of helium absorption lines in the Sun's photospheric spectrum doesn't mean that all the helium has run off to the chromosphere; rather, it indicates that the photosphere isn't hot enough to excite helium atoms above their ground state. As a relatively simple example of how line strength depends on temperature, let's consider the **Balmer lines** of hydrogen. Balmer absorption lines are created when electrons in the $n = 2$ energy level absorb photons of the correct energy to lift them to a higher ($n > 2$) energy level (Figure 2.3). For instance, the Balmer alpha line (Hα) is produced when an

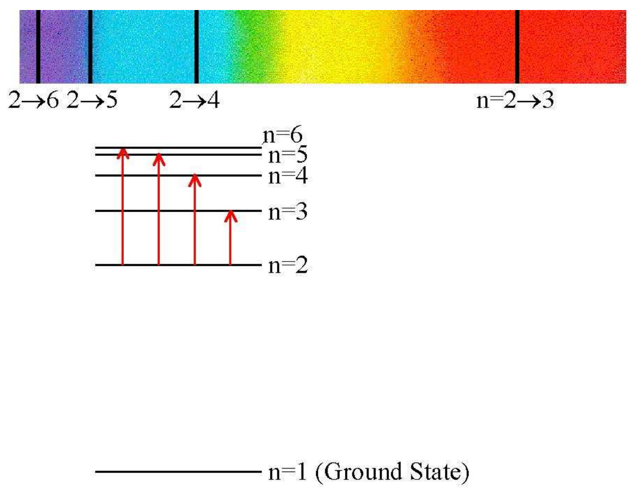

Figure 2.3: Electron energy levels of the hydrogen atom, with Balmer absorption line transitions. [Image credit: Richard Pogge]

electron jumps from the $n = 2$ level to the $n = 3$ level. To make this leap, the electron must absorb a photon of energy $\Delta E = 1.890\,\mathrm{eV}$ (*BA*, Table 6.1). This corresponds to a wavelength

$$\lambda = \frac{hc}{\Delta E} = 6563\,\text{Å} \ , \tag{2.21}$$

in the red region of the visible spectrum. Similarly, the Hβ caused by a jump from $n = 2$ to $n = 4$, requires a photon of energy $\Delta E = 2.552\,\mathrm{eV}$ and wavelength $\lambda = 4861\,\text{Å}$ – and so forth, through the entire Balmer series.

To be capable of producing a Balmer absorption line, a hydrogen atom must fulfill two conditions:

- The H atom must be neutral.

- The atom's electron must start in the $n = 2$ energy level.

The **Saha equation** tells us the ratio of ionized atoms (naked protons) to neutral atoms (*BA*, equation 6.62):

$$\frac{n^+}{n^0} \propto \frac{(kT)^{3/2}}{n_e} \exp\left(-\frac{\chi}{kT}\right) \ , \tag{2.22}$$

where n^+ is the number density of positive ions, n^0 is the number density of neutral atoms, n_e is the number density of free electrons, T is the gas temperature, and $\chi = 13.6\,\mathrm{eV}$ is the ionization potential of a hydrogen atom (the minimum energy required to rip an electron from the ground state and convert it to a free electron). As $T \to 0$, $n^+/n^0 \to 0$; all the atoms are neutral at low temperatures. As $T \to \infty$, $n^+/n^0 \to \infty$; all the atoms are ionized at high temperatures.

The **Boltzmann equation** tells us the ratio of atoms with electrons in the $n = 2$ level to atoms with electrons in the ground ($n = 1$) level (*BA*, equation 6.61):

$$\frac{n_2^0}{n_1^0} \propto \exp\left(-\frac{E_2 - E_1}{kT}\right) \ , \tag{2.23}$$

where n_2^0 is the number density of neutral atoms with electrons at the $n = 2$ level, n_1^0 is the number density of neutral atoms with electrons at the $n = 1$ level, and $E_2 - E_1 = 10.19\,\mathrm{eV}$ is the energy required to lift an electron from the $n = 1$ state to the $n = 2$ state. As $T \to 0$, $n_2^0/n_1^0 \to 0$; the ground state is strongly favored over the excited state at low temperatures. As $T \to \infty$, $n_2^0/n_1^0 \to$ constant.

As the temperature T of gaseous hydrogen increases, the fraction of neutral atoms that have their electron in the $n = 2$ level *increases*. However, as T increases, the fraction of atoms that are neutral *decreases*. These two competing effects maximize the total number of electrons in the $n = 2$ level at $T \approx 10{,}000\,\mathrm{K}$, as shown in Figure (6.12) of *BA*. This is roughly the temperature of the photospheres of Vega and Sirius A. The Balmer lines can be used to estimate the temperature of a star; the stars with the strongest Balmer lines are those with $T \approx 10{,}000\,\mathrm{K}$. What about stars with weak Balmer lines? There are three possible reasons why the Balmer lines might be weak or nonexistent:

- There's little or no hydrogen present in the photosphere. (Given the ubiquity of hydrogen in the universe, this is a highly implausible explanation.)

- The photospheric temperature is $T \gg 10{,}000\,\mathrm{K}$.

- The photospheric temperature is $T \ll 10{,}000\,\mathrm{K}$.

We can distinguish between the high temperature case and the low temperature case by looking at the absorption lines of elements other than hydrogen.

Consider a neutral helium atom. Its ionization potential is $\chi = 24.5\,\mathrm{eV}$, roughly twice the ionization potential of hydrogen. The energy required to lift an electron from the ground level to the first excited level is $E_2 - E_1 = 20.9\,\mathrm{eV}$, roughly twice the equivalent energy in a hydrogen atom. Given the relative energy scales, we expect absorption by helium to occur at energies (and hence temperatures) roughly twice that of hydrogen. If a star has weak Balmer lines and strong helium lines, we conclude that it has $T > 10{,}000\,\mathrm{K}$.

Now consider an atom with a single electron in its outer shell (lithium, sodium, and so forth). These atoms have low ionization potentials (for sodium, $\chi = 5.1\,\mathrm{eV}$). Thus, lines of neutral sodium are seen only in relatively cool stars; in hotter stars, the sodium is nearly all ionized. If a star has weak Balmer lines and strong sodium lines, we conclude that it has $T < 10{,}000\,\mathrm{K}$. We can also conclude that a star is cool if it has molecular absorption bands. The dissociation energy for molecules tends to be small compared to the ionization energy of hydrogen. Titanium oxide is a relatively tough molecule; its dissociation energy is about $6.9\,\mathrm{eV}$. Strong TiO absorption bands are seen in stars with $T \approx 3000\,\mathrm{K}$.

Detailed study of the strength of different absorption lines provides a good estimate of the photospheric temperature. Given the vagaries of history, however, it shouldn't astonish you greatly that the spectral classification of stars long predates the realization that the *spectral sequence* of stars is a *temperature sequence*. In the 1860's, the priest/astronomer Angelo Secchi was the first person to make an empirical classification of stellar spectra. He noted that some stars have many absorption lines in their visible spectra, while others have relatively few. He therefore divided stars into five classes, based on the number of absorption lines he could detect. The small number of classes made Secchi's scheme of limited usefulness.

The spectral classification scheme we use today had its origin around the year 1890, when Edward Pickering, at the Harvard College Observatory, undertook the task of sorting out thousands of stellar spectra. Needing a collaborator who was experienced at making order out of chaos, he hired his housekeeper, Williamina Fleming, as his assistant. Pickering and Fleming proposed a scheme in which each spectrum was assigned a letter, from 'A' through 'Q'. The letter 'J' was not used (apparently because it looks like 'I' when scribbled quickly). This meant that Pickering and Fleming had 16 classes in all. The letter 'P' was assigned to planetary nebulae (a type of compact gaseous nebula) and the letter 'Q' was their wastebasket class – if a spectrum was too bizarre to fit any other class, it went there. The other letters in their scheme were assigned in order of decreasing strength of the Balmer lines, with A stars having the strongest Balmer lines, and O stars the weakest.

The classification of Pickering and Fleming started out as a purely empirical scheme, like sorting olives by size. At the beginning of the 20th century, however, astronomers began to have a clearer idea of how the strength of the Balmer lines depended on temperature. Another Harvard astronomer, Annie J. Cannon, tossed out the redundant classes in the Pickering-Fleming scheme and reordered the remaining classes according to temperature. The order, from hot to cold, was **OBAFGKM**.[6] With higher resolution spectra, Cannon was able to refine the classification further. A single letter was subdivided into numbered subtypes. For instance, G stars are subdivided into G0 stars (the hottest), G1, G2, G3, G4, G5, G6, G7, G8, and G9 stars (the coolest). The Sun, for instance, is a G2 star.

The coolest stars in the standard classification scheme were M9 stars, with $T \approx 2400\,\mathrm{K}$. For many decades, astronomers toiled to discover star-like objects cooler than 2400 K. Such relatively cool bodies are extremely difficult to detect at visible wavelengths, since most of their luminosity emerges in the infrared. A blackbody with $T \approx 1500\,\mathrm{K}$, for instance, has a spectrum peaking at $\lambda_{\max} \approx 2\,\mu\mathrm{m}$. Not coincidentally, an all-sky infrared survey called 2MASS[7] examined the sky at wavelengths of $\sim 2\,\mu\mathrm{m}$, with one of its major goals being the discovery of objects cooler than M9 stars. This goal was attained. The 2MASS survey found a number of cool dwarfs, comparable in size to an ordinary "main sequence" M9 star, but with temperatures $T < 2400\,\mathrm{K}$. Cool dwarfs with a temperature of $T \sim 2000\,\mathrm{K}$ have been given the name of "L dwarfs".[8] L dwarfs have distinctive spectral characteristics:

[6] The traditional mnemonic for this sequence is "Oh Be A Fine Girl; Kiss Me. However, if you prefer kissing guys (or goats or gorillas), feel free to make the appropriate substitution.

[7] 2 Micron All-Sky Survey.

[8] Why L? It was available, since it was one of the classes discarded by Cannon.

- Disappearance of TiO (and other oxide) absorption bands common in M dwarfs.
- Appearance of metal hydride absorption bands (FeH, CrH, and so forth).
- Greater flux in the infrared than in the visible.

According to `dwarfarchives.org`, there are 412 L dwarfs known as of January 2006. The nearest L dwarf is at a distance $d = 7.3\,\mathrm{pc}$; the crudely estimated number density of L dwarfs in our vicinity is $n \sim 0.01\,\mathrm{pc}^{-3}$, about a tenth the number density of M stars.

Encouraged by their success, astronomers looked for still cooler dwarfs. With considerable effort, they found a few dim cool dwarfs that display methane absorption bands, similar to those seen in the spectrum of Jupiter. Methane is a relatively frail molecule, compared to metal oxides and metal hydrides; it dissociates at $T \geq 1300\,\mathrm{K}$. The ultracool dwarfs that have $T < 1300\,\mathrm{K}$ have been given the name of 'T' dwarfs.[9] The distinctive spectral characteristic of T dwarfs is their methane absorption bands. The dwarf archive currently lists 67 T dwarfs; the small number of T dwarfs compared to L dwarfs is due to the lower luminosity of T dwarfs. The estimated number density of T dwarfs is $n \sim 0.01\,\mathrm{pc}^{-3}$, comparable to the number density of L dwarfs. The nearest T dwarfs are a pair of companions to the star ϵ Indi, at a distance $d = 3.6\,\mathrm{pc}$ from the Sun.

The extended spectral classification scheme is now **OBAFGKMLT**.[10] An approximate (but handy) translation of spectral class into temperature is:

O	40,000 K
B	20,000 K
A	9000 K
F	7000 K
G	5500 K
K	4500 K
M	3000 K
L	2000 K
T	< 1300 K

[9] Why T? It was available, since it wasn't part of the original Pickering/Fleming classification scheme.

[10] One proposed mnemonic is "Oh Be A Fine Guy; Kiss My Left Toe". Feel free to substitute any other bit of your anatomy starting with T.

You may have noticed that we've been using the terms "L dwarfs" and "T dwarfs" rather than "L stars" and "T stars". This is because L and T dwarfs are not hot and dense enough to fuse hydrogen into helium in their cores. Thus, they are not stars by the strict definition of the term. Rather, L and T dwarfs are examples of **brown dwarfs**, object that fall into the gap between stars and planets. Brown dwarfs are balls of gas, but they are balls of gas without central fusion reactors. Thus, they cool down with time. A brown dwarf that starts as an L dwarf will end as a T dwarf. Main sequence stars, by contrast, maintain a roughly constant surface temperature as long as they have a supply of hydrogen to fuse into helium at their centers.

As seen in Figure 2.4, M, L, and T dwarfs are all comparable in size to Jupiter. Because of the difference in temperatures, $L_M > L_L > L_T > L_{\rm Jup}$.

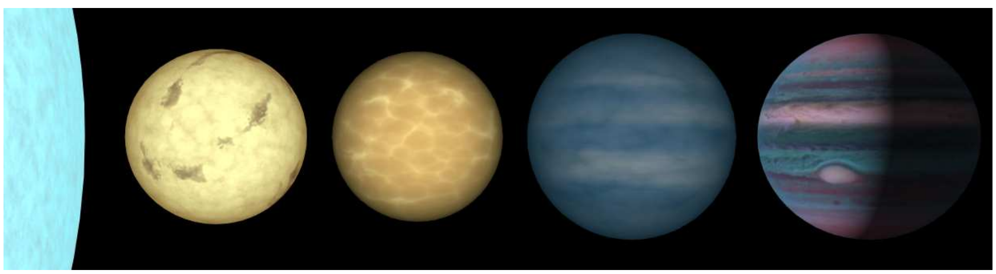

Figure 2.4: False color image of Sun, M dwarf, L dwarf, T dwarf, and Jupiter, as seen at near infrared wavelengths. [Image credit: IPAC]

Figure 2.4 is astronomically correct in that it shows "clouds" and "weather" on the T dwarf, similar to the clouds and weather on Jupiter.

2.3 Luminosity Classes

The spectral classes O through T are a temperature sequence. Although the temperature is the most important parameter determining a star's spectrum, it is not the only one. In the 1930s, W. W. Morgan and Philip Keenan added an extension to the old OBAFGKM scheme by introducing the concept of **luminosity classes**. Empirically, the six luminosity classes, I, II, III, IV, V, and VI, correspond to different absorption line widths, with luminosity class I having the narrowest lines at a given temperature and luminosity class VI having the broadest. As an example, Figure 2.5 shows the spectra of two stars, each with spectral class O9. The upper spectrum, which has

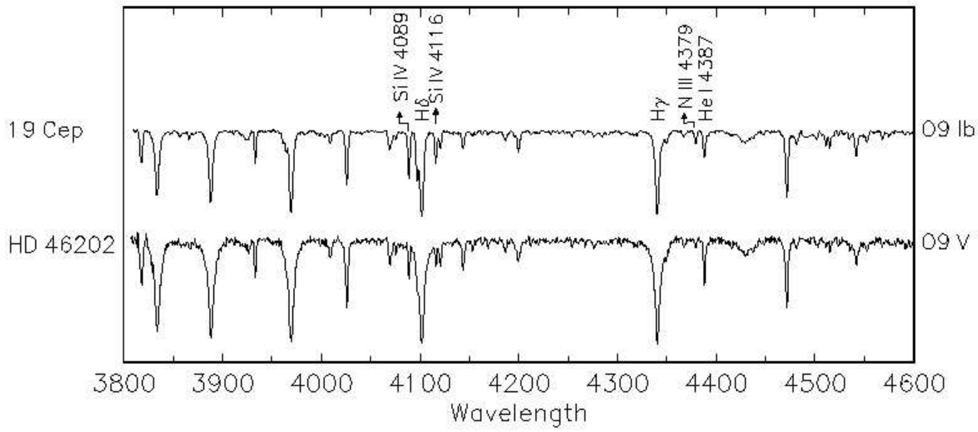

Figure 2.5: Spectra of two O9 stars. Upper: luminosity class I. Lower: luminosity class V.

luminosity class I, has narrower absorption lines than the lower spectrum, which has luminosity class V.

In practice, it is found that the six luminosity classes correspond to stars of different radii:

I	supergiant
II	bright giant
III	giant
IV	subgiant
V	dwarf (main sequence)
VI	subdwarf

The majority of stars (like the Sun, Sirius A, Alpha Centauri A, Alpha Centauri B, Proxima Centauri, and Vega) are of luminosity class V. Betelgeuse is a supergiant, of luminosity class I. Arcturus and Capella are examples of giants, with luminosity class III.[11]

Why should supergiants (luminosity class I) have narrower absorption lines than ordinary main sequence stars (luminosity class V) of the same surface temperature? To understand, let's first think about what it means to look at the photosphere of a star. If you look at a star from a distance, the optical depth τ increases as you look farther into the star's interior. For

[11] White dwarfs are sometimes classified as luminosity class VII, and supergiants are often subdivided into class Ia (superdupergiants) and class Ib (ordinary supergiants).

a thin spherical shell of radius r and thickness dr, the relation between the shell's optical thickness $d\tau$ and physical thickness dr is

$$d\tau = -\rho(r)\kappa(r)dr \ , \tag{2.24}$$

where ρ is the mass density of the shell and κ is the **opacity**. The opacity κ is found by adding together the cross sections of all the absorbers and scatterers in the shell, and dividing by the total mass of the shell. Thus, the units of κ are $\mathrm{m^2\,kg^{-1}}$. At the Sun's photosphere, $\kappa \approx 3\,\mathrm{m^2\,kg^{-1}}$, but in general, the opacity is a function of temperature, density, and chemical composition. The negative sign in equation (2.24) is a reminder that τ increases as r decreases. For a thin shell, $d\tau$ is just the probability that a photon is absorbed or scattered as it passes through the shell. As you dive inward into a star, the photosphere is where the optical depth reaches a value $\tau = 1$.

Since the equation of hydrostatic equilibrium tells us that

$$\frac{dP}{dr} = -g\rho \ , \tag{2.25}$$

we can write the dependence of pressure on optical depth is

$$\frac{dP}{d\tau} = \frac{dP}{dr}\frac{dr}{d\tau} = \frac{g}{\kappa} \ . \tag{2.26}$$

If we assume that g/κ is roughly constant in the star's atmosphere,

$$P = \frac{g}{\kappa}\tau \ . \tag{2.27}$$

As you dive into a star's atmosphere, the increase in pressure and increase in optical depth exponentially increase with the same scale height. Since $\tau = 1$, by definition, at the photosphere, we can compute the pressure at the location of the photosphere as

$$P_{\mathrm{phot}} = \frac{g_{\mathrm{phot}}}{\kappa_{\mathrm{phot}}} \ . \tag{2.28}$$

For the Sun, for instance, the pressure in the photosphere is $P_\odot \sim 100\,\mathrm{N\,m^{-2}} \sim 10^{-3}\,\mathrm{atm}$. In combination with the photospheric temperature of $T_\odot = 5800\,\mathrm{K}$ and a mean molecular weight of order unity, this implies a density in the photosphere of $\rho_\odot \sim 10^{-5}\,\mathrm{kg\,m^{-3}}$.

In general, then, stars with a higher gravitational acceleration g will have higher pressures P in their photosphere, since the opacity κ doesn't vary greatly from one star to another. This means that stars with high acceleration will have more *pressure broadening* (also known as "collisional broadening") of their absorption lines.[12] As a specific example, let's compare Betelgeuse and Proxima Centauri. Betelgeuse is an M2I star, and has an absolute magnitude $M_V \approx -5.5$. Proxima Centauri is an M5V star, and has an absolute magnitude $M_V \approx 15$. Betelgeuse and Proxima Centauri are similar in surface temperature ($T \sim 3000\,\mathrm{K}$), but differ by over 20 magnitudes in M_V; that's a difference of $\sim 10^8$ in luminosity. Betelgeuse is more luminous than Proxima Centauri by a factor of $\sim 10^8$ because it's larger in radius by a factor $\sim 10^4$.[13] Because Betelgeuse is much larger than Proxima Centauri, the gravitational acceleration in its photosphere is much smaller:

$$\frac{g_{\rm Betel}}{g_{\rm Prox}} = \frac{M_{\rm Betel}}{M_{\rm Prox}} \left(\frac{r_{\rm Betel}}{r_{\rm Prox}} \right)^{-2} \approx (200)(13{,}000)^{-2} \approx 10^{-6} \; . \tag{2.29}$$

Thus, the pressure at Proxima Centauri's photosphere will be greater by a factor $\sim 10^6$, leading to a larger amount of pressure broadening of its absorption lines.

The Sun's complete spectral classification is **G2V**. This grouping of three symbols contains a wealth of information. Empirically, the spectral class 'G2' indicates that the Sun's spectrum has weak Balmer lines and very strong Ca II lines. (Ca II is calcium with one electron stripped away; given the low ionization potential of calcium, this does not imply a high temperature.) By deduction from the relative strength of Balmer lines and Ca II lines, the spectral class G2 corresponds to a surface temperature $T = 5800\,\mathrm{K}$. Empirically, the luminosity class 'V' means that the Sun's absorption lines are broad. By deduction, the luminosity class V corresponds to an ordinary main sequence star, with relatively high pressure in its photosphere.

2.4 Hertzsprung-Russell Diagrams

In the early twentieth century, when the OBAFGKM spectral classification system was being sorted out, it occurred independently to a pair of

[12] Pressure broadening is described in section 6.3.3 of *BA*.

[13] Remember from section 1.4 that $r_{\rm Bet} \approx 1800 r_\odot$, whereas $r_{\rm Prox} \approx 0.14 r_\odot$.

astronomers that it might be interesting to plot the absolute visual magnitude of stars versus their spectral type. (This is the approximate equivalent of plotting luminosity versus surface temperature.) The two scientists who had this idea were the Danish astronomer Ejnar Hertzsprung and the American astronomer Henry Norris Russell. Their joint invention – the plot of absolute magnitude versus spectral class – is called the "Hertzsprung-Russell diagram", or **H-R diagram** for short.

Russell's first published H-R diagram is shown as the left panel in Figure 2.6. (Hertzsprung's diagram looked very similar.) To generate his plot,

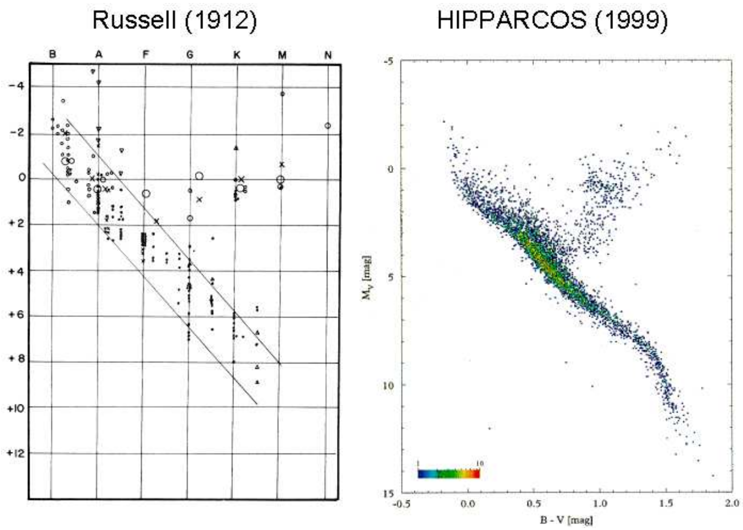

Figure 2.6: Left: Henry Russell's original H-R diagram. Right: H-R diagram for stars observed by *Hipparchos*.

Russell first computed the absolute magnitude of stars with known distances. He then plotted the absolute magnitude of each star versus its spectral class, determined from its spectrum. Russell found that the stars were *not* splattered randomly around the plot. Instead, they fell along a broad band from the upper left (hot, luminous stars) to the lower right (cool, dim stars). The general results of Russell are confirmed by the more data-rich plot seen in the right panel of Figure 2.6. This panel plots M_V versus $B-V$ color index for a sample of nearly 5000 stars whose parallax was measured with an error $< 5\%$ by the *Hipparcos* satellite. Since the $B-V$ color index and the OBAFGKM spectral class are both related to surface temperature, an H-R diagram can use either quantity along its x-axis.

The diagonal band from upper left to lower right on the H-R diagram is called the **main sequence**, and is the reason for the term "main sequence stars". All the stars on the main sequence have luminosity class V, and are relatively dense, small dwarf stars. The small number of stars above and to the right of the main sequence (cool but luminous stars) are of luminosity class III. The scattered points below and to the left of the main sequence are white dwarfs, those dense stellar remnants of which Sirius B is an example.

It is instructive to look at the H-R diagrams for different populations of stars. As an example, let's start by looking at all stars within a few parsecs of the Sun. These are stars whose distance, and hence absolute magnitude, is well known. They should also constitute a "fair sample" of the stars in our galaxy, since we don't think there's anything particularly special about our location. What do we find when we look at the H-R diagram for local stars? (Figure 2.7). To paraphrase Abraham Lincoln, God must have loved

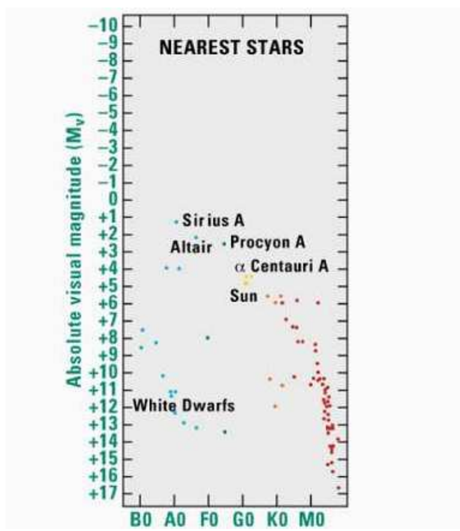

Figure 2.7: H-R diagram for nearby stars.

M main sequence stars, because he made so many of them. There are no giants or supergiants within 5 parsecs of the Sun. On the main sequence, there is nothing hotter or more luminous than Sirius A (spectral class A1, $T \sim 10{,}000\,\mathrm{K}$). However, over half the stars in our neighborhood are type MV: small, cool, main sequence stars.

As another example, let's look at the apparently brightest stars (that is, the stars with the highest flux as seen from Earth). The H-R diagram of the highest-flux stars is shown in Figure 2.8. The apparently brightest stars are

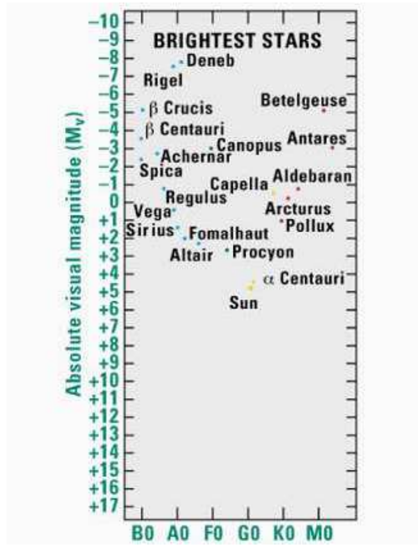

Figure 2.8: H-R diagram for apparently bright stars.

an unrepresentative sample of stars. Most of them are hot, luminous main sequence stars, giants (luminosity class III), and supergiants (luminosity class I). Although supergiants are extremely rare, their high luminosity makes them visible over long distances.[14] The M dwarfs that make up more than half the population of stars are all invisible to the naked eye. Even Proxima Centauri would have to be moved to one-tenth its present distance to be visible.

The fact that the main sequence is a relatively narrow band on the H-R diagram (Figure 2.6) give us a new way of estimating the distances to stars; this new method is called **spectroscopic parallax**. To see how spectroscopic parallax wroks, suppose that you take the spectrum of a star and find that it has luminosity class V; that is, it's on the main sequence. For any main sequence star, its absolute magnitude (or equivalently, its luminosity) is determined by its spectral type. For instance, an O5 main sequence star has $M_V = -6.0$; an M5 main sequence star has $M_V = 12.3$.[15] If you measure

[14]Supergiants are the "celebrities" of the galaxy; although there aren't many of them, they have a talent for publicity, so everyone knows about them.

[15]The absolute magnitudes of main sequence stars can be read from an H-R diagram, or taken from a table such as the one given in Appendix 4 of Zeilik and Gregory.

the apparent magnitude m_V of a star, you can find its distance from the relation

$$\log(d/10\,\mathrm{pc}) = m_V - M_V \ . \tag{2.30}$$

There will be some error in this calculation. Not all stars of the same spectral classification have *exactly* the same absolute magnitude; the main sequence is a band, not an infinitesimally thin line. In addition, we have ignored any dimming by **interstellar dust** along the line of sight to the star.

2.5 Extinction and Reddening by Dust

The life of stellar astronomers would be simpler if our galaxy were perfectly transparent. It isn't. The interstellar medium – a fancy term for "the stuff between stars" consists mostly of gas, and partly of dust (small solid particles). By terrestrial standards, the density of dust is very low. On average, there's one tiny dust mote per million cubic meters of space.[16] On relatively short scales, the dimming due to dust is not significant. We can see Betelgeuse very well, thank you, despite the fact that it is four quadrillion kilometers away. However, over long distances, the effects of dust in our galaxy become significant. One very striking manifestation of interstellar dust is the ragged-looking dark lane down the middle of the Milky Way (Figure 2.9). This is not due to a lack of stars in the dark regions; instead it's because dust in that direction hides distant stars from our sight.

In general, dust causes both **extinction** (it makes distant stars look fainter than they otherwise would) as well as **reddening** (it makes distant stars look redder than they otherwise would). Here on Earth, you can see extinction and reddening of the Sun during a sunset. The Sun looks dimmer as well as redder just before it sets (you can look straight at it without major discomfort). Sunsets are particularly red shortly after a volcanic eruption tosses dust into the upper atmosphere.[17]

If you aren't careful, extinction by dust can cause you to overestimate the distance to a star when you use spectroscopic parallax. In the absence of dust,

$$m_V - M_V = 5\log(d/10\,\mathrm{pc}) \ . \tag{2.31}$$

[16] Think of a single speck of dust in the middle of Ohio Stadium, and you get the general idea.

[17] The eruption of Krakatoa in August 1883 caused several months of dramatically red sunsets, commemorated by Edvard Munch in his famous painting "The Scream".

Figure 2.9: The Milky Way at visible wavelengths; note the dark patches due to interstellar dust.

In the presence of dust extinction,

$$m_V - M_V = 5\log(d/10\,\mathrm{pc}) + A_V\ , \tag{2.32}$$

where A_V is the number of magnitudes of extinction in the V band. The value of A_V is always non-negative; dust makes stars appear dimmer, not brighter. The difficulty in determining the true distance to a star lies in determining a value for A_V. Since dust is clumpy, we can't simply assume that A_V is proportional to distance. We might, by chance, be peering through either a dusty region or through a region that is dust-free.

The value of A_V can be determined indirectly, from the fact that dust causes reddening as well as extinction. As sketched in Figure 2.10, long wavelengths of visible light (red and orange) penetrate dust clouds more readily than short wavelengths (blue and violet), which are scattered in all directions. An observer at position A in Figure 2.10 will observe a reddened star. An observer at position B will observe a bluish dust cloud.[18]

Stated mathematically, in the V band,

$$m_V - M_V = 5\log(d/10\,\mathrm{pc}) + A_V \tag{2.33}$$

and in the B band,

$$m_B - M_B = 5\log(d/10\,\mathrm{pc}) + A_B\ , \tag{2.34}$$

[18]The sky is blue because sunsets are red, and vice versa.

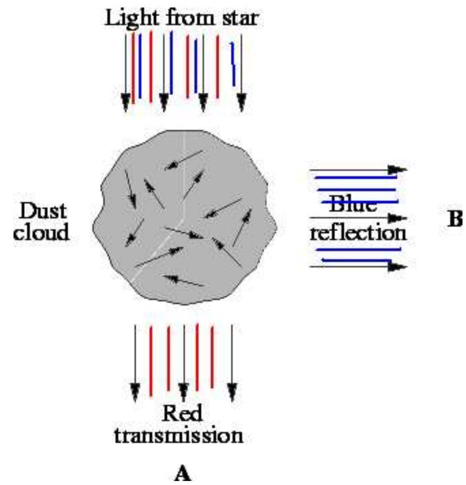

Figure 2.10: Preferential scattering of short wavelength light by dust.

with $A_B > A_V$. There is more extinction in the B band than in the V band because the B band is at shorter wavelengths (Figure 1.5). Subtracting equation (2.33) from equation (2.34), we find

$$[m_B - m_V] - [M_B - M_V] = A_B - A_V \ . \tag{2.35}$$

In equation (2.35), the apparent magnitudes m_B and m_V are what we observe with our telescope, peering through an unknown amount of dust. The absolute magnitudes M_B and M_V are intrinsic to the star; they are what we would observe if the stars were 10 parsecs away in a completely transparent, dust-free environment. By looking at the spectrum of each star, we can determine its spectral type, and look up M_V and M_B for a star of that particular type. Thus,

$$(B - V)_{\text{observed}} - (B - V)_{\text{intrinsic}} = A_B - A_V \ . \tag{2.36}$$

We know $(B - V)_{\text{observed}}$ from our B and V band images. We can look up $(B-V)_{\text{intrinsic}}$ once we determine the spectral type of the star. Thus, we can compute the difference between the B band and V band extinction, $A_B - A_V$, even if we don't know A_B and A_V individually. This difference in extinctions is known as the **color excess** of the star, and is symbolized as

$$E(B - V) \equiv A_B - A_V \ . \tag{2.37}$$

Note that $E(B-V) \geq 0$, indicating that stars are always reddened, and never "bluened", by dust. For stars whose distance is known independently (from stellar parallax, for instance), it is found that

$$A_V \approx 3 \times E(B-V) \ . \tag{2.38}$$

In other words, for every 4 magnitudes of extinction in the B band, there are only 3 magnitudes of extinction in the V band. Equation (2.38) is very useful, since it expresses the hard-to-measure extinction A_V in terms of the easier-to-measure color excess $E(B-V)$.

Let's do an example of how the color excess works in practice. We observe a star whose spectral type is A0V, the same as Vega. Such a star has an intrinsic color of

$$(B-V)_{\rm intrinsic} = 0.0 \tag{2.39}$$

and an absolute V magnitude of $M_V = 0.6$. Observing the star through a B filter then a V filter, we find

$$(B-V)_{\rm observed} = 0.5 \tag{2.40}$$

and an apparent V magnitude of $m_V = 12.1$. The color excess of this star is

$$E(B-V) = (B-V)_{\rm observed} - (B-V)_{\rm intrinsic} = 0.5 \ , \tag{2.41}$$

and its extinction in the V band, from equation (2.38), is

$$A_V \approx 3E(B-V) \approx 1.5 \ . \tag{2.42}$$

Thus, from equation (2.33),

$$5 \log(d/10\,{\rm pc}) = m_V - M_V - A_V \approx 12.1 - 0.6 - 1.5 \approx 10.0 \ , \tag{2.43}$$

and thus $d \approx 1000\,$pc. Note that without correcting for extinction, we would have found $d/10\,{\rm pc} = 10^{2.3}$, and thus $d \approx 2000\,$pc.[19] Correcting for extinction is crucial for computing spectroscopic parallaxes of distant stars. Otherwise, you may badly overestimate the distance. If you're off by a factor of 2 at $d \sim 1\,$kpc, it will be a factor of 4 at $d \sim 2\,$kpc and 8 at $d \sim 3\,$kpc.

In this section, we have treated dust as a damnable nuisance that dims the light from the stars that we are studying. Some astronomers, though, love dust for its own sake. We'll return to the question of dust in Chapter 4, in which we study the properties of the interstellar medium. In the meantime, however, it's time to delve into the mysterious opaque interiors of stars.

[19] The numbers for this example were chosen with care; 1.5 magnitudes of extinction in the V band for a star 1 kiloparsec away is not unusual.