ASTRONOMY 292
Dr. Ryden – Winter 2006

Problem Set 7
due Wednesday, March 8
at class time

Note: In solving these problems,
you'll probably find my lecture
notes more useful than the textbook.

1) Suppose you are in an infinitely large, infinitely old universe in which the average number density of stars is $n_\star = 10^9\,\mathrm{Mpc}^{-3}$ and the average stellar radius is equal to the Sun's radius: $r_\star = 1\,\mathrm{r}_\odot$. How far, on average, can you see in any direction before your line of sight hits a star? (Assume that standard Euclidean geometry holds true.) If the stars are clumped into galaxies with number density $n_{\rm gal} = 1\,\mathrm{Mpc}^{-3}$ and average radius $r_{\rm gal} = 2\,\mathrm{kpc}$, how far, on average, can you see in any direction before your line of sight hits a galaxy?

2) Imagine a universe full of regulation baseballs, each of mass $m_{\rm bb} = 0.145\,\mathrm{kg}$ and radius $r_{\rm bb} = 0.0369\,\mathrm{m}$. If the baseballs are uniformly distributed throughout the universe, what number density of baseballs is required to make the density equal to the current critical density, $\rho_{c,0} = (3H_0^2)/(8\pi G)$? Given this density of baseballs, how far, on average, would be able to see in any direction before your line of sight intersected a baseball? In fact, we can see galaxies at a distance $d \approx c/H_0 \approx 4300\,\mathrm{Mpc}$; does the transparency of the universe on this length scale place useful limits on the number density of intergalactic baseballs?

3) Just as the universe has a Cosmic Microwave Background dating back to the time when the universe was opaque to photons, it also has a Cosmic Neutrino Background dating back to the earlier time when the universe was opaque to neutrinos. The calculated number density of cosmic neutrinos is $n_\nu = 3.36 \times 10^8\,\mathrm{m}^{-3}$. How many cosmic neutrinos are inside your body right now? What average neutrino mass, m_ν, would be required to make the density of neutrinos equal to the critical density $\rho_{c,0}$?

4) Suppose you are in a Newtonian universe whose density is exactly equal to the critical density $\rho_{c,0}$. The scale factor $a(t)$ is implicitly given by the relation

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho_{c,0}}{3}\frac{1}{a^3} \ .$$

What is the functional form of $a(t)$, given the boundary condition $a = 1$ at $t = t_0$? What is t_0 in terms of the Hubble constant H_0? In our universe, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and the oldest globular clusters have an age of $t_{\mathrm{GC}} = 13\,\mathrm{Gyr}$; are these two observations consistent with a Newtonian universe with $\rho_0 = \rho_{c,0}$?

5) Given that the current scale factor is $a(t_0) = 1$, at what scale factor did the temperature of the cosmic background radiation equal the temperature of the Sun's photosphere? At what scale factor did it equal the temperature of the Sun's center? If the current mass density of the universe is equal to $0.3\rho_{c,0}$, what was the mass density of the universe when the temperature was equal to that of the Sun's center? Compare this mass density to the average density of the Sun.