# Chapter 1

# Properties of Stars

The word “astronomy” is derived from the Greek roots “astron”, meaning “star”, and “nomos”, meaning “law”. Although astronomers now study a wide range of objects, from dust grains to superclusters of galaxies, the study of stars, and the laws dictating their behavior, is still a key part of astronomy. A star is best defined as a luminous ball of gas powered by nuclear fusion in its interior. This definition distinguishes stars from smaller objects like planets and brown dwarfs, that are too cool inside for fusion to take place. It also distinguishes stars from stellar remnants like white dwarfs and neutron stars, that were stars once but which no longer host a fusion reactor in their interiors.[1]

It was not apparent, in the pre-Copernicus era, that the Sun and the stars in the night sky were members of the same class of objects. Consider Genesis 1:16 – “And God made two great lights: the greater light to rule the day, and the lesser light to rule the night; he made the stars also.” The Sun and Moon, from this viewpoint, are both “great lights”, bright and large (about thirty arcminutes across). The stars are mere afterthoughts, dim and small (much less than an arcminute across). However, once astronomers were aware of the immense distance to stars other than the Sun, they realized that the stars were all glowing spheres like the Sun. The Sun appears much brighter than the other stars we see simply because it is much closer to Earth than the other stars.

[1] Despite their name, “neutron stars” are not gaseous and not fusion-powered; thus, they are not stars by the strict definition we have adopted.

## 1.1 How Far is a Star?

A recurring problem in astronomy is the measurement of distances. When you look up at the night sky, you have no sense of depth. In Figure 1.1, for instance, it is not immediately clear that the fuzzy blob on the left is a galaxy two million light-years away and the fuzzy blob on the right is a comet a few light-minutes away. Simply gawking up at the sky with your

Figure 1.1: Left to right – the Andromeda galaxy, Comet Hale-Bopp. [Image credit: J. C. Casado (Astronomy Picture of the Day, 1999 March 14)]

naked eyes gives you little sense of how far away astronomical objects are. There are a few clues you can pick up – for instance, the Moon occults stars, so the stars must lie beyond the Moon – but quantitative measurement of distances is difficult.

Astronomers have developed many methods of estimating distances. I will only review a few of the more useful techniques. Within the Solar System, the distances between planets can be accurately measured using **radar**. A brief, powerful burst of radio waves is sent toward a planet, using the dish of a large radio telescope to collimate the radiation. After a time $\delta t$, a radio "echo" is detected; the time $\delta t$, which can be measured with great accuracy, is the round-trip travel time for a photon. The one-way distance from the radio telescope to the planet is then $d = c(\delta t)/2$. Thanks to careful radar measurements, distances within the Solar System are known with great accuracy. For instance, the length of the astronomical unit is known to better than one part per billion ($1\,\mathrm{AU} = 149{,}597{,}870.6\,\mathrm{km}$). The radar technique for measuring distances is only useful within the Solar System. Even if you had the patience to wait more than 8 years for a reflected radio signal from Proxima Centauri (the nearest star other than the Sun), the signal would be far too faint to detect with current technology.

The most useful tool in the astronomer's kit for measuring the distance to nearby stars is **stellar parallax**. Stellar parallax was mentioned in Chapter 2 of *Basic Astrophysics* as a proof of the Earth's motion around the Sun.[2] The Earth's motion around the Sun creates an apparent motion of nearby stars in a tiny ellipse. The semimajor axis of the ellipse has an angular size $\pi''$, given by the formula (*BA*, eq. 2.12)

$$\pi'' = \frac{206265}{d[\mathrm{AU}]} \ , \tag{1.1}$$

when $\pi''$ is measured in arcseconds. Thus, if the parallax $\pi''$ of a star is large enough to be measured with the equipment at hand, the distance $d$ to the star can be calculated:

$$d = \frac{206265\,\mathrm{AU}}{\pi''[\mathrm{arcsec}]} \ . \tag{1.2}$$

Since we know the length of the astronomical unit very well, the accuracy with which we know the distances to nearby stars is determined by the accuracy with which we can measure $\pi''$. The astronomical unit is an inconveniently small unit for measuring stellar distances. Thus, astronomers tend to measure stellar distances in **parsecs**, where

$$d = \frac{1\,\mathrm{parsec}}{\pi''[\mathrm{arcsec}]} \ . \tag{1.3}$$

Thus, the parsec (abbreviated 'pc') is the distance at which a star has a parallax angle $\pi''$ equal to one arcsecond.[3] In metric units, $1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m}$.

The angle $\pi''$, even for the Sun's nearest neighbors among the stars, is less than one arcsecond. When Friedrich Wilhelm Bessel first measured stellar parallax in 1838, he found that the star 61 Cygni had $\pi'' = 0.3\,\mathrm{arcsec}$; that's the size of a one-cent coin (U.S. or Canadian) as seen 14 kilometers away. Currently, the best measurements of parallax angles at visible wavelengths were those made by the *Hipparcos* satellite, which measured $\pi''$ for over 100,000 stars in our galaxy, with a typical accuracy of a milliarcsecond (0.001 arcsec).

The Sun's nearest neighbors are the three stars of the Alpha Centauri system (Figure 1.2). The nearest of the three is Alpha Centauri C, also

[2] I will frequently cite Professor Peterson's lecture notes for Astronomy 291; I'll refer to them by his chosen title *Basic Astrophysics*, or *BA* for short.

[3] Because of this definition, there are 206,265 astronomical units in one parsec, just as there are 206,265 arcseconds in a radian.

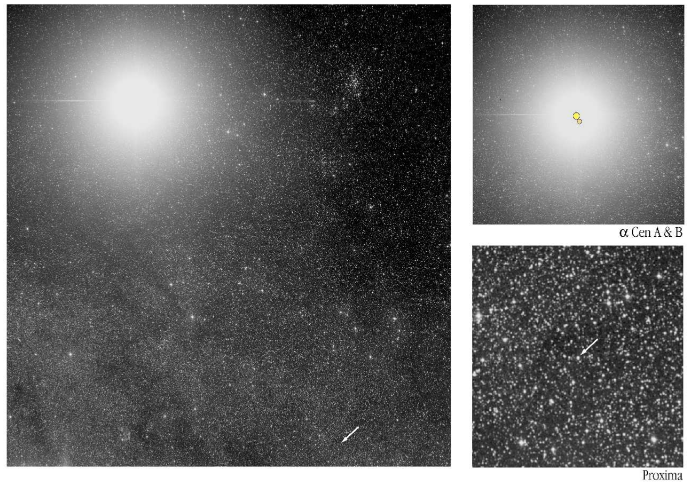


Figure 1.2: The Alpha Centauri system. [Image credit: European Southern Observatory (Astronomy Picture of the Day, 2003 Mar 23)]

known, from its proximity to us, as **Proxima Centauri**. From the *Hipparcos* measurements, we know the parallax, and thus the distance, to each star in the system.

- Proxima Centauri:
  - $\pi'' = 0.7723 \pm 0.0024\,\mathrm{arcsec}$
  - $d = 1\,\mathrm{pc}/\pi'' = 1.295 \pm 0.004\,\mathrm{pc} = 267{,}100 \pm 800\,\mathrm{AU}$
- Alpha Centauri A & B:
  - $\pi'' = 0.7421 \pm 0.0014\,\mathrm{arcsec}$
  - $d = 1.348 \pm 0.003\,\mathrm{pc} = 277{,}900 \pm 500\,\mathrm{AU}$

Alpha Centauri A & B form a closely bound binary pair, with an average separation of $a = 24\,\mathrm{AU}$ (slightly larger than the distance between the Sun and Uranus). Proxima Centauri, however, is $0.053\,\mathrm{pc} = 10{,}800\,\mathrm{AU}$ closer to us than Alpha Centauri A & B are. In addition, Proxima Centauri is separated from Alpha Centauri A & B by 7849 arcseconds (more than 2 degrees) as seen from Earth. This angular separation corresponds to a physical distance

$$D = \left(\frac{7849\,\mathrm{arcsec}}{206{,}265\,\mathrm{arcsec\,rad^{-1}}}\right) 267{,}100\,\mathrm{AU} = 10{,}200\,\mathrm{AU} \qquad (1.4)$$

at the distance of Proxima Centauri. Thus, the three-dimensional distance between Proxima Centauri and the Alpha Centauri A & B pair is

$$D = \left[(10{,}800\,\mathrm{AU})^2 + (10{,}200\,\mathrm{AU})^2\right]^{1/2} = 14{,}900\,\mathrm{AU}\ . \tag{1.5}$$

This separation is so large that some astronomers doubt whether Proxima Centauri is really bound to the other two stars; perhaps it is just passing through the neighborhood.

Even with the accurate angular measurements provided by *Hipparcos*, stellar parallax is useful only for stars within two hundred parsecs of us. That's less than 3% of the distance from here to the center of our galaxy. *Hipparcos* measured the parallax of $\sim 42{,}000$ stars with an error of less than 20%; compared to the estimated 200 billion stars in our galaxy, the number with distances estimated by stellar parallax is small. For the overwhelming majority of stars in our galaxy, we must develop techniques other than stellar parallax to find their distances. We will return to the problem of distance determination later in the text. For the moment, let's divert our attention to the problem of determining the **brightness** of a star.

## 1.2 How Bright is a Star?

In casual conversion, the word "brightness" is often used loosely. In astronomy, it is very useful to distinguish between **intrinsic brightness** and **apparent brightness**. The intrinsic brightness of a star is a measure of how much light the star emits in a given time. The apparent brightness of a star is a measure of how much starlight enters our pupils (or the aperture of our telescope) in a given time. The apparent brightness of a star, or any other luminous object, depends on both its intrinsic brightness and its distance; the further away a star is, the lower its apparent brightness.

The **intrinsic brightness** of a star is also known as its **luminosity** (symbolized by the letter $L$). The luminosity of a star is the rate at which it emits energy in the form of electromagnetic radiation. Luminosity is commonly measured in watts (abbreviated 'W'). For example, the Sun has a luminosity of

$$L_\odot = 3.86 \times 10^{26}\,\mathrm{W}\ . \tag{1.6}$$

This luminosity includes all electromagnetic radiation emitted by the Sun, from radio waves to gamma rays. It is also an average over time, since the

Sun's luminosity is slightly variable. The solar luminosity varies by about 0.1% over the course of a sunspot cycle.[4]

The **apparent brightness** of a star is also known as its **flux** (symbolized by the letter $f$). The flux of a star is the rate per unit area at which its energy strikes a surface held perpendicular to the star's rays. Flux is commonly measured in watts per square meter. The light emitted by stars is usually isotropic; that is, it's pretty much the same in all directions. Consider a transparent sphere of radius $d$ centered on a star of luminosity $L$. The flux of light energy through the sphere is the luminosity of the star divided by the sphere's area:

$$f = \frac{L}{4\pi d^2} \ . \tag{1.7}$$

The observed flux of a star falls off as the inverse square of its distance $d$. For example, the Sun's flux at the Earth's location is

$$f = \frac{L_\odot}{4\pi(1\,\mathrm{AU})^2} = \frac{3.86\times10^{26}\,\mathrm{W}}{4\pi(1.496\times10^{11}\,\mathrm{m})^2} = 1370\,\mathrm{W\,m^{-2}} \ . \tag{1.8}$$

Sunlight is a potentially potent power source on Earth; unfortunately, solar panels are inefficient, our atmosphere is not transparent, clouds are frequent, and half the Earth is in shadow at any given time.

In equation (1.8), we computed the Sun's flux, given its luminosity and distance. In practice, astronomers more commonly work in the other direction: after measuring the flux and distance of a star, they compute its luminosity. Consider, for example, the star Sirius, also known as Alpha Canis Majoris, the apparently brightest star in our night sky. From the Earth's northern hemisphere, Sirius can be seen in the winter sky, "dogging the heels" of the constellation Orion (Figure 1.3). The flux of Sirius is

$$f_{\mathrm{S}} = 1.2\times10^{-7}\,\mathrm{W\,m^{-2}} \ . \tag{1.9}$$

To intercept 1370 watts of sunlight, you'd need a panel one meter on a side; to intercept 1370 watts of Siriuslight, you'd need a panel roughly the size of Connecticut. The distance to Sirius, computed from its parallax, is

$$d_{\mathrm{S}} = 2.637\,\mathrm{pc} = 8.14\times10^{16}\,\mathrm{m} \ . \tag{1.10}$$

[4]Strange though it may seem, the Sun's luminosity is greatest when the number of sunspots is largest. This is because increased numbers of dark sunspots are correlated with increased numbers of bright plages. The increase in light from the plages more than compensates for the decrease in light from the sunspots.

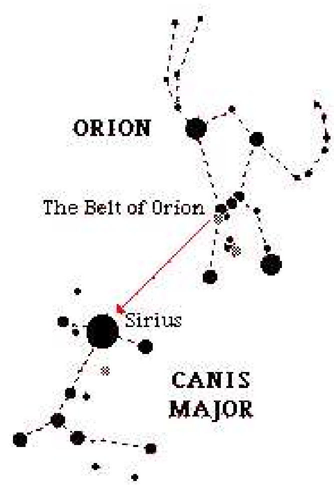


Figure 1.3: Sirius (Alpha Canis Majoris) and friends.

Thus, we can compute the luminosity of Sirius:

$$L_{\rm S} = 4\pi d_{\rm S}^2 f_{\rm S} = 1.0 \times 10^{28}\,{\rm W} = 26 L_\odot\ . \tag{1.11}$$

This tells us that stars don't all have the same luminosity.

Measuring the complete flux of a star, integrated over all wavelengths of light, is a difficult task. The history of stellar flux measurement began when a prehistoric human looked up at Sirius and said the prehistoric equivalent of "Gosh, that's a bright star!" As a flux measurement, this has two drawbacks. First, the human eye can only detect light in the wavelength range $400\,{\rm nm} < \lambda < 700\,{\rm nm}$ (or equivalently, 4000 to 7000 Angstroms). Second, the exclamation "Gosh, that's a bright star!" is not quantitative.

The first recorded attempt to quantify stellar flux at visible wavelengths was made by the Greek astronomer Hipparchus in the second century BC. After noting that stars differed in their apparent brightness, he classified them in six categories. The stars with the greatest flux were stars of the 1st magnitude. The stars with the next highest flux were stars of the 2nd magnitude, and so on down to stars of the 6th magnitude, which are the faintest stars visible to the human eye. After the invention of the telescope, the **apparent magnitude** scheme of Hipparchus was extended to fainter stars (7th magnitude, 8th magnitude, and so forth). A six-inch amateur telescope at a dark site can reach to 13th magnitude or so; the faintest objects in the Hubble Ultra Deep Field are about 30th magnitude. Improvements in measuring flux led to the introduction of fractional magnitudes; careful photometry can measure a star's flux to within 0.01 magnitudes.

The apparent magnitude system was placed on a firm mathematical basis in the nineteenth century, where it was realized that a difference of 5 magnitudes represents a multiplicative factor of 100 in flux. To illustrate, consider two stars; star #1 has an apparent magnitude $m_1$ and star #2 has an apparent magnitude $m_2$. If $m_2 - m_1 = 5$, we say that star #1 is 5 magnitudes brighter than star #2.[5] With $m_2 - m_1 = 5$, the ratio of the star's fluxes is

$$\frac{f_1}{f_2} = 100 \ . \tag{1.12}$$

If $m_2 - m_1 = 1$ (that is, if star #1 is only 1 magnitude brighter than star #2), the ratio of fluxes is

$$\frac{f_1}{f_2} = 100^{1/5} = 10^{0.4} \approx 2.512 \ . \tag{1.13}$$

In general, the relation between apparent magnitude and flux is

$$\frac{f_1}{f_2} = 100^{(m_2 - m_1)/5} = 10^{0.4(m_2 - m_1)} \ , \tag{1.14}$$

or

$$m_2 - m_1 = 2.5 \log(f_1/f_2) \ , \tag{1.15}$$

where "log" represents a common logarithm, with base 10.

The apparent magnitude $m$ can be thought of as a logarithmic measure of the flux, with

$$m = C - 2.5 \log f \ . \tag{1.16}$$

The constant $C$ has historically chosen so that the star Vega, in the constellation Lyra, has an apparent magnitude of exactly zero.[6] Thus, $C = 2.5 \log f_{\rm Vega}$. Here are a few other apparent magnitudes of stars, taking into

[5] Please note: the apparent magnitude system is "bass-ackwards", in that *smaller* values of $m$ correspond to *larger* flux.

[6] Why Vega? It's an apparently bright star, which makes its flux easier to measure. It doesn't have a binary companion providing contaminating light. Finally, its luminosity doesn't vary significantly.

account only the flux at visible wavelengths:

| | |
|---|---|
| Sirius | $m = -1.5$ |
| Alpha Centauri A | $m = 0.0$ |
| Alpha Centauri B | $m = 1.4$ |
| Proxima Centauri | $m = 10.7$ |
| Sun | $m = -26.75$ |

This tells us that we receive the same flux of visible light from Alpha Centauri A as we do from Vega. However, the flux from Vega is greater than that from Proxima Centauri by a factor

$$\frac{f_{\rm Vega}}{f_{\rm Prox}} = 10^{0.4(10.7-0.0)} \approx 19{,}000 \; . \qquad (1.17)$$

Since a star's flux depends on both luminosity and distance, so does the star's apparent magnitudes. If we want a logarithmic measure of the luminosity alone, we use the **absolute magnitude** of a star, designated by the symbol $M$. The absolute magnitude $M$ of a star is defined as the apparent magnitude it would have if it were at a distance $d = 10\,{\rm pc}$.[7] Since the apparent magnitude of a star is

$$\begin{aligned} m &= C - 2.5\log f && (1.18)\\ &= C - 2.5\log L + 2.5\log(4\pi) + 5\log d \; , && (1.19) \end{aligned}$$

the absolute magnitude of the star is

$$\begin{aligned} M &= C - 2.5\log L + 2.5\log(4\pi) + 5\log(10\,{\rm pc}) && (1.20)\\ &= C - 2.5\log L + 2.5\log(4\pi) + 5 \; . && (1.21) \end{aligned}$$

If we measure the apparent magnitude of a star (by comparing its flux to that of Vega or some other standard star) and then measure the distance to the star (by finding its parallax), the absolute magnitude can then be computed:

$$M = m - 5\log d + 5 \; , \qquad (1.22)$$

where $d$ is measured in parsecs, or

$$M = m - 5\log(d/10\,{\rm pc}) \; . \qquad (1.23)$$

[7]Why 10 parsecs? When the absolute magnitude was first defined in the early 20th century, the only stars whose distances were accurately known were those within 10 parsecs of the Sun (and hence with parallaxes of more than 0.1 arcsec).

For instance, consider Proxima Centauri. In section 1.1, we found that its distance from us is $d = 1.295$; its apparent magnitude at visible wavelengths, as tabulated above, is $m = 10.7$. The absolute magnitude of Proxima Centauri is then

$$M = 10.7 - 5\log(1.295/10) = 15.1 \ . \tag{1.24}$$

If Proxima Centauri were 10 parsecs away from us, it would be farther away than it actually is, and hence would have a lower flux.

The difference between a star's apparent magnitude $m$ and its absolute magnitude $M$ is known as its **distance modulus**. From equation (1.23), the distance modulus is

$$m - M = 5\log(d/10\,\mathrm{pc}) \ . \tag{1.25}$$

Thus, the distance modulus is a logarithmic measure of the distance to a star. If you hear an astronomer say "That star has a distance modulus of 10", you know that the star in question is 100 parsecs away. Let's look at the apparent magnitude, absolute magnitude and distance modulus of our example stars:

| *star name* | $m$ | $M$ | $m-M$ |
|---|---|---|---|
| Sirius | $-1.5$ | 1.4 | $-2.9$ |
| Alpha Centauri A | 0.0 | 4.4 | $-4.4$ |
| Alpha Centauri B | 1.4 | 5.8 | $-4.4$ |
| Proxima Centauri | 10.7 | 15.1 | $-4.4$ |
| Sun | $-26.75$ | 4.83 | $-31.58$ |

Note the wide range of absolute magnitudes for these stars. The most luminous (Sirius) is 13.7 magnitudes brighter than the least luminous. That's a factor of $10^{0.4\times13.7} \approx 300{,}000$ in luminosity.

## 1.3 How Hot is a Star?

Stars are not monochromatic; they emit light with a wide range of wavelengths. Let $f_\lambda$ be the differential flux of a star, defined so that $f_\lambda d\lambda$ is the star's flux at wavelengths in the range $\lambda \to \lambda + d\lambda$. Figure 1.4 shows the differential flux of the star Vega at visible and near infrared wavelengths. The spectrum of Vega, like most stellar spectra, consists of a continuum (approximately a blackbody) with absorption lines superimposed. Since our

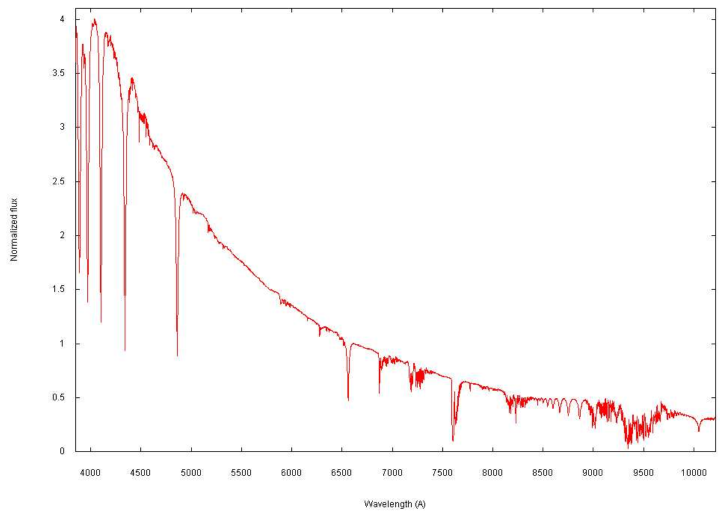


Figure 1.4: Spectrum (flux per unit wavelength) of the star Vega. The flux is in arbitrary units.

eyes can only detect radiation from wavelengths from 4000 angstroms to 7000 angstroms, they can give a misleading impression of the total flux of a star.[8] The total flux of a star,

$$f = \int_0^\infty f_\lambda d\lambda \ , \tag{1.26}$$

is also known as the **bolometric flux**. (A bolometer is a very sensitive thermometer that absorbs all the photons that strike it; it was invented by Samuel Langley in the 19th century, when he was attempting to measure the Sun's total flux.) Unfortunately, measuring the bolometric flux of apparently faint stars is difficult. For one thing, the Earth's atmosphere is opaque at many wavelengths, requiring you to place your detectors in orbit.

If you manage to measure the bolometric flux $f_{\rm bol}$ of a star, the apparent bolometric magnitude of the star is

$$m_{\rm bol} = C_{\rm bol} - 2.5 \log f_{\rm bol} \ . \tag{1.27}$$

Because of the difficulty of measuring bolometric fluxes, there was long debate about the appropriate value of the constant $C_{\rm bol}$. Finally, in 1997,

[8] Bees can detect light over a wavelength range of 3000 to 5500 angstroms; thus, a race of astronomically-minded bees would disagree with us over which stars have the highest flux at "visible" wavelengths.

Commission 25 of the International Astronomical Union (which deals with stellar photometry) requested that everyone use a scale on which the Sun's absolute bolometric magnitude is $M_{\mathrm{bol},\odot} = 4.74$. The Sun, rather than Vega, was chosen for the honor of normalizing the bolometric magnitude scale because the Sun is the only star whose bolometric flux has been measured with extremely high accuracy. With the IAU-approved normalization, the relation between absolute bolometric magnitude and luminosity is

$$M_{\mathrm{bol}} = 4.74 - 2.5 \log(L/L_\odot) \ , \tag{1.28}$$

or

$$L/L_\odot = 10^{0.4(4.74 - M_{\mathrm{bol}})} \ . \tag{1.29}$$

Given the difficulty of measuring bolometric fluxes, astronomers usually content themselves with measuring the flux over a strictly defined range of wavelengths. This is done, in practice, by putting a colored filter in the light path of a telescope, thus eliminating the unwanted wavelengths of light. Many different filter systems are in use. One of the most durably popular is the **Johnson system**, devised by Harold Johnson and his collaborators in the 1950s. There are five main filters in the Johnson system: $U$ (ultraviolet), $B$ (blue), $V$ (visual), $R$ (red), and $I$ (infrared). The spectral sensitivity, $S(\lambda)$, of the five basic Johnson filters is displayed in Figure 1.5. The spectral sensi-

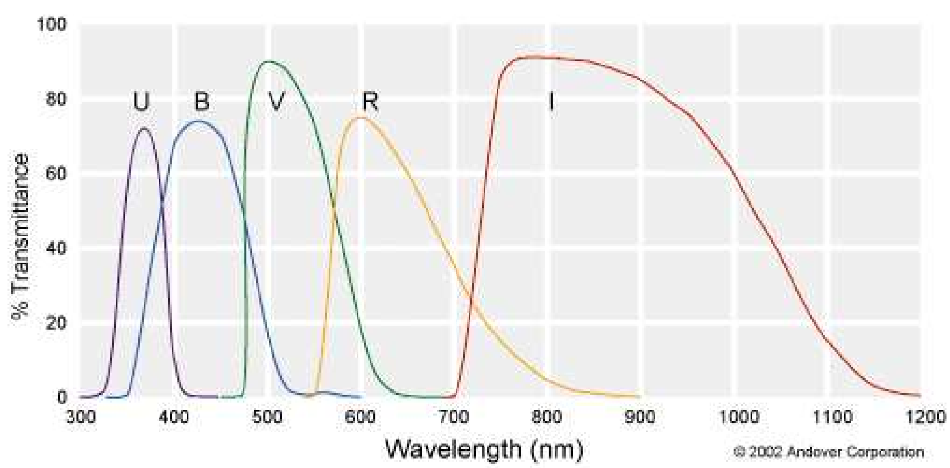


Figure 1.5: Spectral sensitivity $S(\lambda)$ of the Johnson filters. [Image credit: Andover Corporation]

tivity gives the fraction of the light at wavelength $\lambda$ that can pass through a filter. The Johnson filters were chosen to highlight different spectral regions. The $V$ filter, in particular, was chosen to approximate what our eyes see (although the $V$ filter has a narrower bandpass than the human eye). [9]

The flux of starlight seen through a particular filter depends on both the differential flux $f_\lambda$ of the star and the spectral sensitivity $S(\lambda)$ of the chosen filter. A star's flux through the Johnson $V$ filter, for instance, would be

$$f_V = \int_0^\infty f_\lambda S_V(\lambda) d\lambda \ , \tag{1.30}$$

where $f_\lambda$ is the star's flux and $S_V(\lambda)$ is the spectral sensitivity of the $V$ filter, as shown in Figure 1.5. The apparent magnitude of the star in the $V$ band is then

$$m_V = C_V - 2.5 \log f_V \ , \tag{1.31}$$

where $C_V$ is chosen so that the apparent magnitude of Vega is zero. Similarly, the $U$ band apparent magnitude is

$$m_U = C_U - 2.5 \log f_U \ , \tag{1.32}$$

the $B$ band apparent magnitude is

$$m_B = C_B - 2.5 \log f_B \ , \tag{1.33}$$

and so forth, through all the filters. The constant $C$, for each filter, is chosen so that Vega has an apparent magnitude of zero.

Multicolor photometry (that is, the practice of measuring a star's flux through multiple filters) is useful for two main reasons.

- The more different filters you look through, the more accurate your reconstruction of the star's spectrum and bolometric flux.
- Multicolor photometry yields information about the color, and hence the **temperature**, of a star.

The **color index** of a star is the difference of its apparent magnitude seen through two different filters. For instance, one popular color index is

$$B - V = m_B - m_V \ . \tag{1.34}$$

[9] The Johnson filters are popular in part because they are made from relatively inexpensive colored glass, available "off the shelf" from Schott Glaswerke, a German firm that specializes in coloring glass with organic dyes.

Other frequently used color indices are $U - B = m_U - m_B$ and $V - R = m_V - m_R$.[10] Color indices are useful because they depend on the temperature of the star's photosphere (usually called the star's "surface temperature", even though stars don't have a sharply defined surface). By definition, the star Vega has $B - V = 0$. The surface temperature of Vega is $T \approx 10{,}000\,\mathrm{K}$. If a star has $T > 10{,}000\,\mathrm{K}$, it will be bluer than Vega (its $B$ flux will be enhanced relative to its $V$ flux), and thus it will have $B - V < 0$, thanks to the bass-ackwards nature of the magnitude scale. By contrast, if a star has $T < 10{,}000\,\mathrm{K}$, it will be redder than Vega (its $V$ flux will be enhanced relative to its $B$ flux), and thus it will have $B - V > 0$. The relation between the surface temperature $T$ and the color index $B - V$ can be computed for a blackbody. However, since stars aren't exactly blackbodies, it's more useful to use the purely empirical relation

$$T = \frac{8540\,\mathrm{K}}{(B - V) + 0.865} , \tag{1.35}$$

which applies to stars with surface temperature in the range $4000\,\mathrm{K} < T < 10{,}000\,\mathrm{K}$. The bottom line is that measuring a color index such as $B - V$ gives you a quick and cheap way of estimating a star's surface temperature. It's usually much easier than taking a star's spectrum and finding the best-fitting blackbody curve.

Because bolometric fluxes are so difficult to measure, what astronomers generally do is measure the $V$ band apparent magnitude and then add a **bolometric correction**:

$$BC = m_{\mathrm{bol}} - m_V = M_{\mathrm{bol}} - M_V . \tag{1.36}$$

For instance, the Sun has $M_{\mathrm{bol}} = 4.74$ and $M_V - 4.83$, so its bolometric correction is $BC = 4.74 - 4.83 = -0.09$. The bolometric correction is smallest for stars with $T \approx 6700\,\mathrm{K}$, since these are the stars whose emission peaks in the $V$ band; for these stars, estimating the bolometric flux from the $V$ band flux yields a pretty good approximation. For hotter stars, most of the energy escapes at shorter wavelengths; for cooler stars, most of the energy escapes at longer wavelengths. The bolometric correction can be calculated by using model stellar atmospheres and computing how much of the total flux is emitted in the $V$ band. In practice, the bolometric correction is

[10] The universal convention is that color indices are $m$(shorter wavelength) - $m$(longer wavelength).

something that you can look up, because someone else has done the dirty work of calculating it for you. Appendix 4 of Zeilik & Gregory gives bolometric corrections for different classes of stars.

To review how magnitudes, color indices, and bolometric corrections work, let's do an example. The star Epsilon Eridani has $m_V = 3.73$ and $m_B = 4.61$; it's easily visible to the naked eye, but is not eye-catchingly bright.[11] Its color index is $B - V = 4.61 - 3.73 = 0.88$ (redder than Vega). The empirical relation given in equation (1.35) tells us that the surface temperature of Epsilon Eridani is $T = 8540\,\mathrm{K}/(0.88 + 0.865) = 4900\,\mathrm{K}$ (cooler than Vega). The bolometric correction for a normal star of this temperature is $BC = -0.40$. The apparent bolometric magnitude of Epsilon Eridani is $m_{\rm bol} = 3.73 - 0.40 = 3.33$. The distance to Epsilon Eridani, found by stellar parallax, is $d = 3.218\,\mathrm{pc}$, so the absolute bolometric magnitude is

$$M_{\rm bol} = m_{\rm bol} - 5\log(d/10\,\mathrm{pc}) = 5.80 \tag{1.37}$$

and its luminosity is

$$L/L_\odot = 10^{0.4(4.74 - M_{\rm bol})} = 10^{-0.424} = 0.38\ . \tag{1.38}$$

Thus, measuring the flux of a star through two filters and measuring its parallax is sufficient for us to compute its surface temperature and luminosity.

Warning: I have been making the assumption that the space between the star and our telescope is completely transparent. In the real universe, we must take the effect of **extinction** into account. **Atmospheric** extinction, due to the Earth's atmosphere, causes about 0.2 magnitudes of dimming when your telescope is pointing straight up (toward the zenith). Let $Z$ be the angle between the zenith direction and the direction in which the telescope is pointing; $Z = 0°$ when the telescope is pointing straight up, $Z = 90°$ when it's pointing toward the horizon. When $Z < 60°$, a useful approximation to the amount of dimming is

$$m_V(\text{above atmosphere}) = m_V(\text{observed}) - 0.2\sec Z\ . \tag{1.39}$$

(When $Z > 60°$, you should think twice about observing something so close to the horizon.) When you look up the apparent magnitude of a star in a reference work (Appendix 4 of the textbook, for instance), it will already be corrected for atmospheric extinction. **Interstellar** extinction, due to

[11] Epsilon Eridani is known to Star Trek buffs as the parent star of the planet Vulcan.

scattering by interstellar dust, can also be significant, especially within the disk of our galaxy, where most of the dust lies. We'll discuss interstellar extinction in more detail in Chapter 4, which deals with the interstellar medium.

## 1.4 How Big is a Star?

If you know the distance $d$ to a star, and its angular diameter $\alpha$, then its radius $r$ can be determined by a simple bit of trigonometry:

$$\frac{r}{d} = \tan\left(\frac{\alpha}{2}\right) . \tag{1.40}$$

Therefore,

$$r = d\tan\left(\frac{\alpha}{2}\right) \approx \frac{d\alpha}{2} , \tag{1.41}$$

in the small angle limit, where $\alpha \ll 1\,\mathrm{rad}$. Computing the Sun's radius is easy. The average distance to the Sun is $d = 1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$. The average angular diameter of the Sun is $\alpha = 1919\,\mathrm{arcsec} = 9.30 \times 10^{-3}\,\mathrm{rad}$, so

$$r_\odot = \frac{(1.496 \times 10^8\,\mathrm{km})(9.30 \times 10^{-3})}{2} = 696{,}000\,\mathrm{km} . \tag{1.42}$$

Stars other than the Sun have angular diameters that are very small, and hence difficult to measure. If we viewed the Sun from the location of Proxima Centauri ($d = 1.295\,\mathrm{pc} = 267{,}000\,\mathrm{AU}$), its angular size would be

$$\alpha = 1919\,\mathrm{arcsec}\left(\frac{1\,\mathrm{AU}}{267{,}000\,\mathrm{AU}}\right) = 7.2 \times 10^{-3}\,\mathrm{arcsec} . \tag{1.43}$$

Measuring an angular size of 7 milliarcseconds is difficult.

In fact, only one star other than the Sun has had its angular diameter resolved by direct imaging. The star Betelgeuse, in the constellation Orion, has been resolved by the Hubble Space Telescope (Figure 1.6). The distance to Betelgeuse, determined from its parallax, is

$$d_B = 131\,\mathrm{pc} = 2.70 \times 10^7\,\mathrm{AU} . \tag{1.44}$$

The angular diameter of Betelgeuse, measured by the Hubble Space Telescope, is

$$\alpha_B = 0.125\,\mathrm{arcsec} = 6.06 \times 10^{-7}\,\mathrm{rad} . \tag{1.45}$$

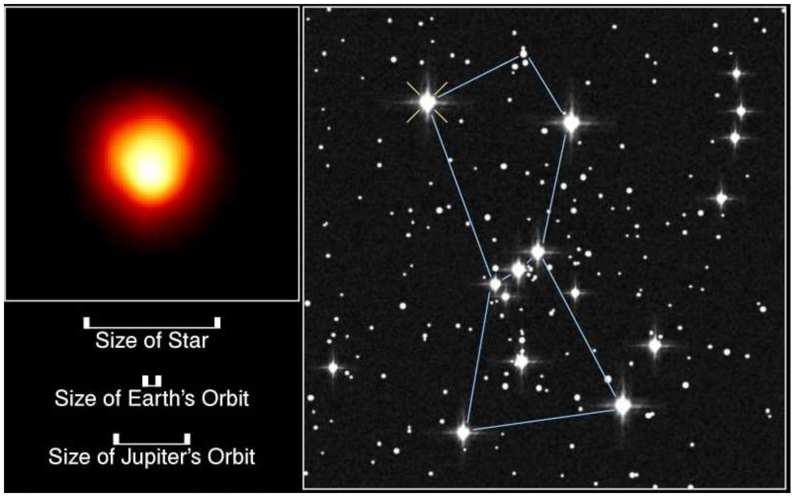


Figure 1.6: Hubble Space Telescope image of Betelgeuse, with finding chart. [Image credit: STScI]

Thus, its radius is

$$r_B = d_B \alpha_B / 2 = 8.2\,\mathrm{AU} = 1.2 \times 10^9\,\mathrm{km} = 1800 r_\odot \ . \qquad (1.46)$$

Betelgeuse is a supergiant star, swollen to an immense radius. If you placed its center at the Sun's location, it would extend far beyond Jupiter's orbit. Supergiant stars are rare; Betelgeuse is the nearest supergiant to the Sun. All the stars closer than Betelgeuse are smaller in radius, and all the stars larger than Betelgeuse are farther away.

Stars less bloated than Betelgeuse must have their radii measured using **interferometry**. To see how the principle of interference can tell you the angular size of a star, consider the classic "two-slit" interference experiment illustrated in Figure 1.7. Light of wavelength $\lambda$ comes from a point source far to the left of the image, and strikes a wall in which two narrow slits have been cut. The light passing through one slit interferes with the light passing through the other slit, causing **constructive** interference where the wavecrests add together, and **destructive** interference where wavecrests from one slit encounter wave troughs from the other slit. If a screen is placed to the right of the slits, parallel to the wall, a pattern of bright and dark bands is seen (bright = constructive interference, dark = destructive interference). The distance between the bright bands on the screen is $x = d\lambda/b$, where $d$ is

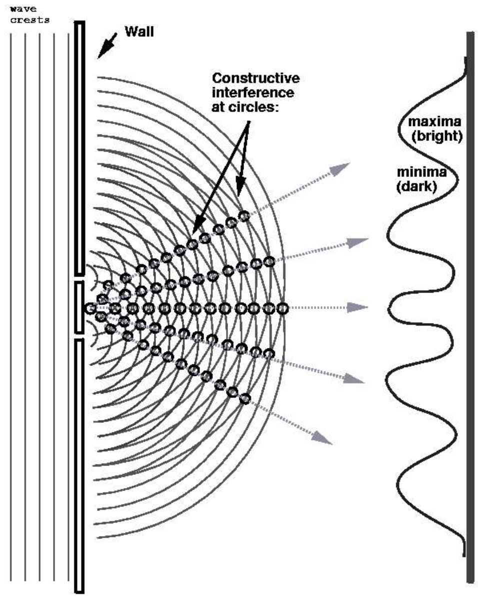


Figure 1.7: Two-slit interference.

the distance between the wall and the screen, and $b$ is the distance between the slits.

If stars were perfect point sources, they would produce interference bands in exactly this way. However, stars have a finite (although small) angular size $\alpha$. Thus, light from the upper limb of the star approaches the slits from a slightly different angle than light from the lower limb of the star. This means that when light from the *upper* limb of the star falls on the screen, its bright and dark bands are slightly displaced from the bright and dark bands produced by light from the *lower* limb. The interference pattern produced by a star of angular diameter $\alpha$ will be smeared out when

$$\alpha[\text{radians}] > \lambda/b \; . \tag{1.47}$$

Thus, if you observed a star of angular diameter $\alpha$ at a wavelength $\lambda$ through a pair of movable slits, the interference pattern would be smeared out once the baseline between the slits increased to

$$b > \lambda/\alpha \; . \tag{1.48}$$

Scaling to a plausible angular diameter for a nearby star,

$$b > 10\,\mathrm{m}\left(\frac{\lambda}{5000\,\mathrm{A}}\right)\left(\frac{\alpha}{0.01\,\mathrm{arcsec}}\right)^{-1} . \tag{1.49}$$

In practice, the technique of stellar interferometry doesn't use two slits in a wall; it uses two telescopes separated by a distance $b$.

The interferometer at the Very Large Telescope (VLT) in Chile uses telescopes separated by a distance as large as 140 meters; thus, it can measure angles smaller than a milliarcsecond. The VLT interferometer has been used, for instance, to find the angular diameters of the stars in the Alpha Centauri system:

- Alpha Centauri A: $\alpha = 8.5 \times 10^{-3}\,\mathrm{arcsec}$, $r = 1.23 r_\odot$.
- Alpha Centauri B: $\alpha = 6.0 \times 10^{-3}\,\mathrm{arcsec}$, $r = 0.87 r_\odot$.
- Proxima Centauri: $\alpha = 1.0 \times 10^{-3}\,\mathrm{arcsec}$, $r = 0.14 r_\odot$.

Note that the most luminous star in the system ($\alpha$ Cen A) is also the largest in size. It is particularly interesting to discover that Proxima Centauri isn't much larger than Jupiter, which has $r = 0.10 r_\odot$. We tend to think of stars as "big-ass" balls of gas, but the smallest stars aren't much larger than the biggest planets. At the moment, there are roughly a thousand stars whose radii have been measured using interferometric techniques. They span the range from supergiants like Betelgeuse to dwarfs like Proxima Centauri.

When a star's radius and luminosity are both known, we have a new way of estimating its temperature. For a spherical blackbody,

$$L = 4\pi r^2 \sigma T^4 , \tag{1.50}$$

where $\sigma$ is the Stefan-Boltzmann constant. Whether a star is really a blackbody or not, we can assign it an **effective temperature**, defined as

$$T_{\mathrm{eff}} = \left(\frac{L}{4\pi r^2 \sigma}\right)^{1/4} . \tag{1.51}$$

If this temperature doesn't agree closely with the temperature as estimated from the star's spectrum or color index, then the star must be very far indeed from being a blackbody.

## 1.5 How Massive is a Star?

The radius of Betelgeuse is 1800 times the radius of the Sun, meaning that the volume of Betelgeuse is 5.8 billion times the Sun's volume. This leads inevitably to the question, "Betelgeuse – is it fat, or is it just fluffy?" If the mass of Betelgeuse is similar to that of the Sun, it must be very fluffy, with a density billions of times lower than the Sun's. If, by contrast, the density of Betelgeuse is similar to that of the Sun, it must be very fat, with a mass billions of times the Sun's mass.

The mass of a star can be determined using Kepler's Third Law, as modified by Isaac Newton (*BA*, section 3.2):

$$M_1 + M_2 = \frac{4\pi^2}{G}\frac{a^3}{P^2} , \tag{1.52}$$

where $M_1$ and $M_2$ are the masses of two objects orbiting their mutual center of mass, $P$ is their orbital period, and $a$ is the semimajor axis of their relative orbit.[12] Kepler's Law restricts us to measuring the total mass ($M_1 + M_2$) of a binary system. Finding the mass of an isolated star is like recording the sound of one hand clapping. Fortunately, the majority of stars in the solar neighborhood have companions: either a star, or a substellar object (a brown dwarf or planet), or a formerly stellar object (a white dwarf).

Binary stellar systems are usually classified by the way in which they are detected. There are three main classes.

- **Visual binary:** The two stars in the binary system are individually resolved in your telescope. You can tell they are not a chance superposition of stars at different distances because one star moves on an elliptical orbit relative to the other. Visual binaries tend to have large separations, and hence long periods.

- **Spectroscopic binary:** The two stars are unresolved, appearing as a single blob. However, the spectrum of the binary system shows absorption lines that oscillate in wavelength as the stars' radial velocities change. Spectroscopic binaries tend to have high orbital speeds, and hence small orbits and short periods.

[12]Unfortunately, the letter $M$ does double duty, indicating both a star's mass and its absolute magnitude. The context should make it clear which meaning is appropriate.

- **Eclipsing binary:** The two stars are unresolved. However, the orbit of the system is seen nearly edge on, so the stars periodically eclipse each other, causing dips in the flux. Eclipsing binaries tend to have small separations, and hence short periods.

An example of a visual binary is the Sirius system. It was discovered in the 19th century that the star we know as "Sirius" (Figure 1.3) has a much fainter companion. The brighter component of the binary system is now known as Sirius A, while its dim companion is called Sirius B. At visible wavelengths, Sirius A is roughly 8000 times brighter than Sirius B; at X-ray wavelengths, however, they are more nearly equal in brightness, as shown in Figure 1.8. Sirius A has a surface temperature $T = 9900\,\mathrm{K}$ (estimated

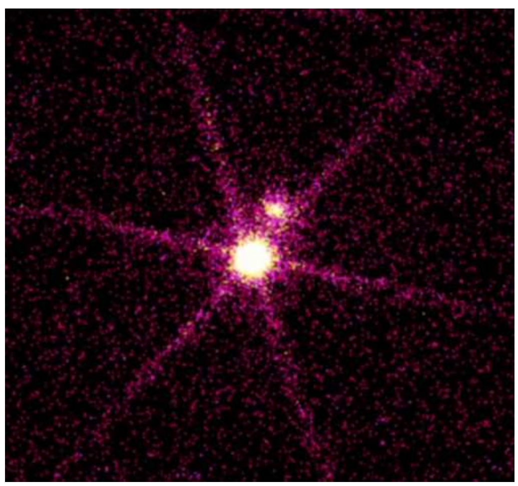

Figure 1.8: Sirius A (with diffraction spikes) and Sirius B (above and to the right) seen in X-rays [Image credit: Chandra X-ray Observatory]

from its spectrum) and a luminosity $L = 26L_\odot$. By contrast, Sirius B has a surface temperature $T = 24{,}800\,\mathrm{K}$ but a luminosity of only $L = 0.024L_\odot$, most of which emerges at wavelengths too short for the human eye to detect.

Sirius B is less luminous than Sirius A, despite having a much higher surface temperature. This means that Sirius B must have a much smaller surface area. From the relation among radius, temperature, and luminosity for a blackbody,

$$\frac{r_B}{r_A} = \left(\frac{L_B}{L_A}\right)^{1/2}\left(\frac{T_A}{T_B}\right)^2 = (0.00092)^{1/2}(0.40)^2 = 0.0048\ . \tag{1.53}$$

Since the radius of Sirius A is known to be $r_A = 1.71 r_\odot$, from stellar interferometry, the radius of Sirius B must be

$$r_B = 0.0048 r_A = 0.0084 r_\odot = 0.92 r_{\rm Earth} \ . \tag{1.54}$$

When Sirius B and similar objects were determined to have high temperatures but low luminosities, they were named "white dwarfs", since they are small in size (comparable in size to the Earth) but high enough in temperature to be white hot.

The motion of Sirius B relative to Sirius A has been traced for well over a century; thus, the relative orbit of the two objects, as projected onto the plane of the sky, is well known (see Figure 1.9). The orbital period of the

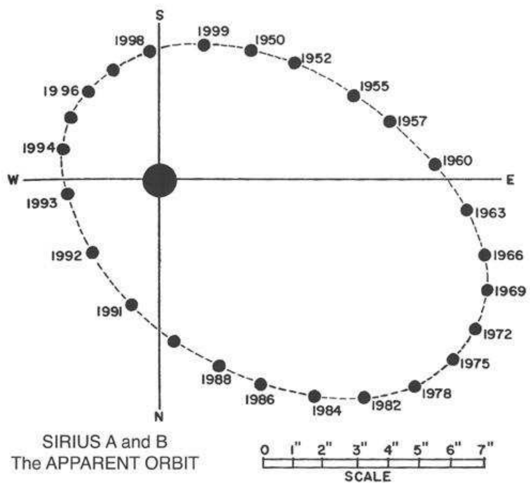


Figure 1.9: The projected orbit of Sirius B relative to Sirius A.

Sirius system is $P = 50.18\,$yr. To find the semimajor axis $a$ of the relative orbit, we must *deproject* the observed ellipse. That is, we must ask the question, "What ellipse, with Sirius A at one focus, appears (when viewed at an angle), like the ellipse in Figure 1.9?" This question has a unique answer. I won't go into the geometrical details,[13] but will simply quote the answer. The true three-dimensional orbit of Sirius B relative to Sirius A has a semimajor axis of angular length

$$a'' = 7.51\,\mathrm{arcsec} = 3.64 \times 10^{-5}\,\mathrm{rad} \tag{1.55}$$

[13] A good textbook on celestial mechanics will give you all the details you might want – and then some.

and is viewed at an inclination $i = 44°$.[14] Since the distance to the Sirius system is $d = 2.637\,\mathrm{pc} = 544{,}000\,\mathrm{AU}$, the semimajor axis of the orbit of Sirius B relative to Sirius A is

$$a = a''d = 19.8\,\mathrm{AU} \ , \tag{1.56}$$

comparable to the average distance from Uranus to the Sun, and slightly smaller than the average distance between Alpha Centauri A and B.

We now have the necessary information to compute the total mass of the Sirius system. If the masses $M_1$ and $M_2$ are in solar masses, $a$ is in AU, and $P$ is in years, Kepler's Third Law can be written in the form

$$M_1 + M_2 = \frac{a^3}{P^2} \ . \tag{1.57}$$

(For planets orbiting the Sun, with $M_1 = 1M_\odot$ and $M_2 \ll 1M_\odot$, this reduces to the familiar formula $P^2 = a^3$.) For the Sirius system,

$$M_1 + M_2 = \frac{(19.8)^3}{(50.18)^2} = 3.08M_\odot \ . \tag{1.58}$$

We know that Sirius A and Sirius B, taken together, have a mass more than three times that of the Sun; but how is the mass allocated between the two objects?

To determine how the total mass is divided between Sirius A and Sirius B, we must locate the **center of mass** of the binary system. Suppose that two spherical stars, with masses $M_1$ and $M_2$, are separated by a distance $a$, as shown in Figure 1.10. If $a_1$ is the distance from $M_1$ to the center of mass, and $a_2$ is the distance from $M_2$ to the center of mass, then

$$\frac{a_2}{a_1} = \frac{M_1}{M_2} \ . \tag{1.59}$$

The center of mass is always closer to the more massive star. If we can locate the center of mass of a binary system, we can find the ratio of the masses. Combined with the sum of the masses, found from Kepler's Third Law, this enables us to find the individual masses of the stars in the binary.

[14] An inclination $i = 0°$ means we are looking at the orbit face-on; an inclination $i = 90°$ means we are looking at the orbit edge-on.

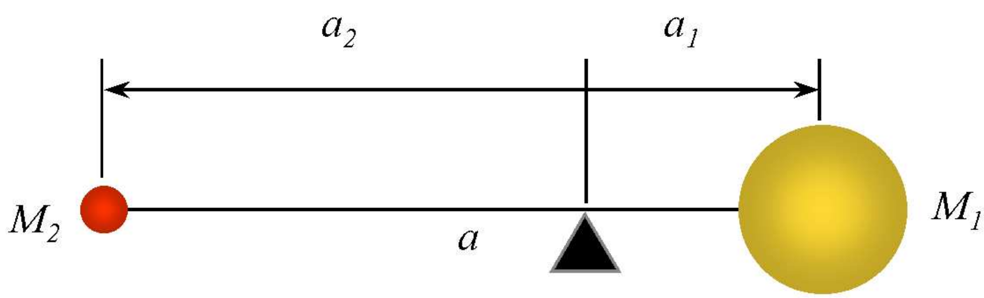


Figure 1.10: The center of mass of a binary system. [Image credit: Richard Pogge]

The center of mass of the Sirius system is orbiting the center of our galaxy with an orbital period of roughly 240 million years; so is the center of mass of the Solar System. During half a century (the orbital period of Sirius B relative to Sirius A), the motion of the center of mass of the Sirius system can be very well approximated as a straight line. When the positions of Sirius B and Sirius A are plotted relative to background objects (such as distant quasars), they show the "wobbly" motion displayed in Figure 1.11. The small annual wiggles due to stellar parallax are not shown in this figure;

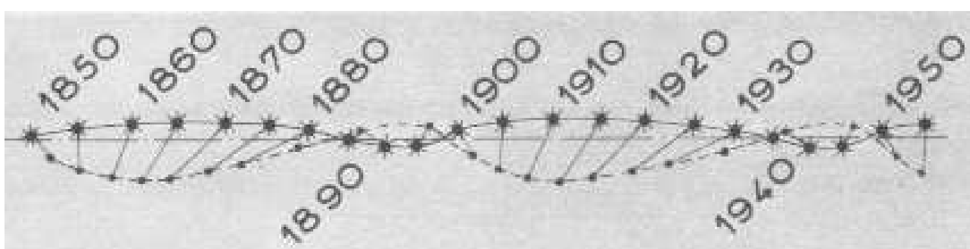


Figure 1.11: The long-term motion of the Sirius system. Asterisk = Sirius A, dot = Sirius B.

only the larger wiggles with period $P = 50.18\,\mathrm{yr}$ due to the relative motion of Sirius A and Sirius B. If Sirius B had a negligibly small mass – as you might expect from its tiny volume – then the motion of Sirius A would be a straight line. Since Sirius A does show wobbles in its motion, the mass of Sirius B must be significant.[15] The motion of the center of mass is a straight

[15]In fact, the wobbles in the motion of Sirius A were first noticed by Friedrich Bessel (the

line if we assume

$$\frac{a_B}{a_A} = 2.2 = \frac{M_A}{M_B} \ . \tag{1.60}$$

That is, Sirius A is just over twice the mass of Sirius B. When we combine our two bits of information, $M_A + M_B = 3.08 M_\odot$ and $M_A = 2.2 M_B$, we find the solution

$$M_B = 0.96 M_\odot \ , \qquad M_A = 2.12 M_\odot \ . \tag{1.61}$$

When astronomers first learned that white dwarfs like Sirius B had masses comparable to the Sun, but volumes comparable to the Earth, they were gobsmacked and/or flabbergasted. The average density of Sirius B is more than two tons per cubic centimeter.

If we plot radius ($r$) versus mass ($M$) for stars whose mass we know (Figure 1.12), we find that most stars lie along a well-defined mass – radius relation. The stars that obey the mass – radius relation are called "main

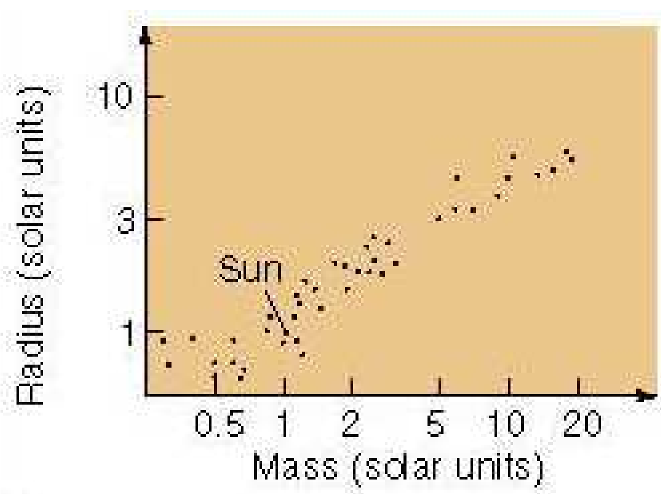


Figure 1.12: Stellar radius versus mass (logarithmic scale).

sequence" stars. The Sun, Sirius A, Epsilon Eridani, and the stars of the Alpha Centauri system are all main sequence stars. As you can see, more massive main sequence stars are larger in radius, but not with the $M \propto r^3$ relation that you would expect if all stars had the same density. Stars that are large in radius are lower in density. An empirical fit to the mass – radius

parallax guy) in 1844, nearly twenty years before Sirius B was seen through a telescope.

relation is

$$M \propto r^{1.7} \quad M < 1.3M_\odot \tag{1.62}$$
$$M \propto r \quad M \geq 1.3M_\odot\ . \tag{1.63}$$

Among the stars that don't fall on the usual mass – radius relation are Betelgeuse (which has an overly large radius for its mass) and Sirius B (which has an overly small radius for its mass). Thus, supergiants like Betelgeuse and white dwarfs like Sirius B are special cases, that don't follow the same relations as ordinary "main sequence" stars.

If we plot stellar luminosity versus mass (Figure 1.13), we find that most stars lie along a well-defined mass – luminosity relation. More massive stars

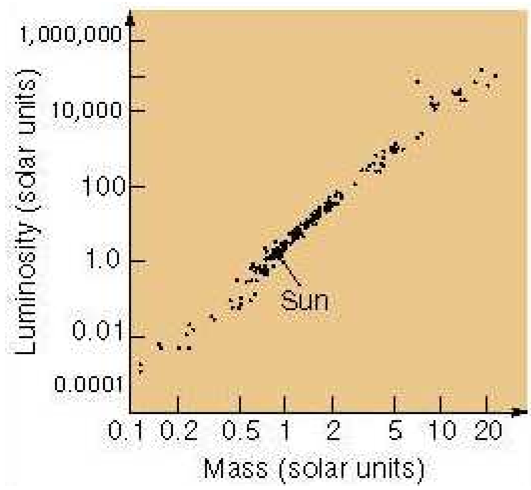


Figure 1.13: Stellar luminosity versus mass (logarithmic scale).

are higher in luminosity. A reasonably good empirical fit to the mass – luminosity relation is

$$L/L_\odot = 0.23(M/M_\odot)^{2.3} \quad M < 0.43M_\odot \tag{1.64}$$
$$L/L_\odot = (M/M_\odot)^4 \quad M \geq 0.43M_\odot\ . \tag{1.65}$$

Note the very steep dependence of luminosity upon mass, particularly for high-mass stars; this has important implications for stellar evolution. A star is its own fuel tank; that is, it powers itself by fusion of the material that it contains. Thus, the total fuel supply of a star is proportional to its mass $M$.

The rate at which it uses fuel is proportional to its luminosity. The lifetime $\tau$ of a star before it exhausts its fuel is then $\tau \propto M/L$. For the observed mass – luminosity relation,

$$\tau \propto M/L \propto M^{-1.3} \quad M < 0.43 M_\odot \tag{1.66}$$
$$\tau \propto M/L \propto M^{-3} \quad M > 0.43 M_\odot \ . \tag{1.67}$$

For example, Sirius A, which has a mass more than twice that of the Sun, will have a lifetime less than 1/8 as long, before it runs out of fuel for its fusion “engine”.