Chapter 6

Our Galaxy

Our study of stars began with a definition of the word "star". A sense of symmetry compels us to begin our study of galaxies with a definition of the word "galaxy". A galaxy is a collection of stars (between a million and a trillion of them, in round numbers), plus gas, dust, and dark matter, held together by gravity. A galaxy is bigger than a star cluster, such as the Pleiades, and smaller than a cluster of galaxies, such as the Virgo cluster. Although there are hundreds of billions of galaxies within the volume accessible to our telescopes, we will start by looking in depth at the galaxy in which we live: the Milky Way galaxy.

6.1 Overview: Morphology of our galaxy

On a dark night, far from city lights, you can see a luminous band of light across the sky, forming a great circle on the celestial sphere (Figure 6.1). In English, this band of light is called the **Milky Way**, because it looks, to the naked eye, like a luminous white fluid. In ancient Greece, it was called the "galaktikos kuklos", which literally translates as "milky circle". The Greek word "galaktikos" is the origin of the English word "galaxy".[1]

Although the Milky Way looks as if someone spilled glow-in-the-dark milk across the celestial sphere, when Galileo examined it with his telescope, he found that it is actually composed of a very large number of stars, individually very faint. A hypothesis that explains the existence of the Milky Way is that

[1] The Milky Way is tilted by 60.2° relative to the ecliptic, and by 62.6° relative to the celestial equator.

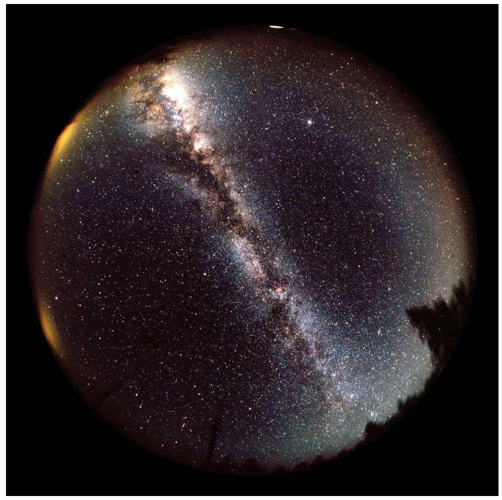

Figure 6.1: The Milky Way seen from Mount Graham, Arizona

the Sun is embedded in a thin disk of stars. When we look perpendicular to the disk, we see few stars, and the sky is dark. When we look in the plane of the disk, we see the many stars that make up the Milky Way. This disk of stars is a major component of the galaxy in which we live, which is therefore called the **Milky Way galaxy**. It is also called the Galaxy (with a capital 'G') or, if we're feeling particularly possessive, *our* galaxy.

The first method used to determine the size and shape of our galaxy was the method of **star counts**. To demonstrate how star counts work, let's start with some simplifying assumptions.

- All stars have the same absolute magnitude M. This is not true in general, but we can choose to look only at main sequence stars of a particular spectral type.
- The number density of stars, n, is constant within our galaxy.
- There is no absorption due to dust. (This is perhaps our most dubious assumption.)

A star of absolute magnitude M, will have an apparent magnitude m when it is at a distance

$$d = 10^{0.2(m-M+5)}\,\mathrm{pc}\ . \tag{6.1}$$

Every star closer than a distance d will be brighter than m. Thus, the total number of stars brighter than m will be

$$N(< m) = \frac{4\pi}{3} d^3 n = \frac{4\pi}{3} 10^{0.6(m-M+5)} n \ , \tag{6.2}$$

or, taking the logarithm,

$$\log_{10} N = 0.6m + \text{constant} \ . \tag{6.3}$$

By going one magnitude fainter, you should increase the number of stars you see in a given patch of sky by a factor $10^{0.6} \approx 4$.

If the galaxy were infinitely large, then as we counted stars as a function of their apparent magnitude, we would find that $\log N$ just kept increasing to arbitrarily large values of m. However, if there are no stars beyond a distance $d_{\rm max}$, then there will be no stars fainter than $m_{\rm max}$, where

$$m_{\rm max} = M + 5 \log d_{\rm max} - 5 \ . \tag{6.4}$$

Thus, if we find $m_{\rm max}$ for a particular patch of sky, we can find

$$d_{\rm max} = 10^{0.2(m_{\rm max} - M + 5)} \, \text{pc} \tag{6.5}$$

in that direction. For example, suppose you are counting G0V stars in a particular small patch of sky. G0 main sequence stars have $M_v = 4.4$. The faintest G0V stars that you can find have $m_{v,\rm max} = 16.4$. Thus, you compute that the most distance stars in the patch are at a distance

$$d_{\rm max} = 10^{0.2(16.4-4.4+5)} \, \text{pc} = 10^{3.4} \, \text{pc} = 2500 \, \text{pc} \ , \tag{6.6}$$

if you ignore the effects of dust.

The pioneers in using star counts to determine the shape of our galaxy were William and Caroline Herschel, the great sibling act of astronomy. In the late 18th century, they did star counts in various regions of the sky and came to the conclusion that our galaxy is shaped like a grindstone (Figure 6.2). (For those of you who haven't sharpened any knives the old-fashioned way recently, a grindstone is a thick disk made of coarse, abrasive stone.) Unfortunately, the Herschels were unaware of the existence of interstellar dust, and didn't take into account extinction by dust. As a result, they came to the erroneous conclusion that we are near the center of a relatively small galaxy. The "notch" in the grindstone, shown on the right side

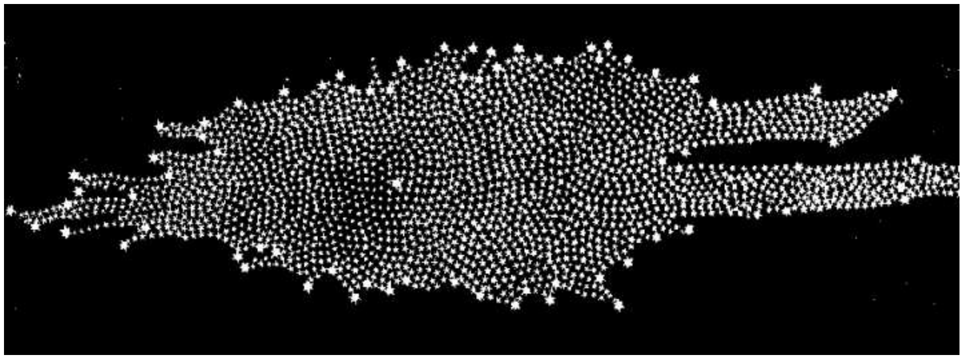

Figure 6.2: A cross-section through the Herschel's "grindstone" model of our galaxy. The large star to left of center marks the Sun's position.

of Figure 6.2, is actually the result of a dust lane down the center of the Milky Way.

A more accurate determination of our place in the Milky Way galaxy was provided by Harlow Shapley in the early 20th century. Shapley did it by looking away from the dust-laden Milky Way, and examining the distribution of **globular clusters**. A globular cluster is a very compact cluster of stars, containing as many as $\sim 10^6$ stars within a spherical region $\sim 30\,\mathrm{pc}$ in diameter. The H-R diagrams of globular clusters show an absence of hot main sequence stars, indicating that there are no young stars present. Typical ages estimated for globular clusters are $\sim 12\,\mathrm{Gyr}$. Our galaxy has about 150 globular clusters associated with it. The nearest globular cluster is M4, about $\sim 2\,\mathrm{kpc}$ away from us (Figure 6.3); the most distant of our galaxy's globular clusters is $\sim 100\,\mathrm{kpc}$ away. Shapley noted that the globular clusters are not uniformly distributed across the sky. Instead, they are concentrated in one half of the celestial sphere, centered on the constellation Sagittarius. Shapley concluded that the globular clusters are all orbiting the center of our galaxy, which lies in the direction of Sagittarius. In addition, he measured the distances to globular clusters, using RR Lyrae variable stars (which all have $M_V \approx 0.5$) as standard candles. Shapley's distances were, as it turns out, too large; he thought that RR Lyrae stars were more luminous than they actually are. However, he had the right order of magnitude. Modern distance measurements tell us that the distance to the galactic center is $R_0 = 8 \pm 1\,\mathrm{kpc}$, or about 26,000 light years.

Shapley's discovery that we are not at the center of our galaxy has been

Figure 6.3: The globular cluster M4 [Image credit: Kitt Peak]

called an extension of the Copernican Revolution. Copernicus said that the Earth is not at the center of the Solar System. Shapley said that the Solar System is not at the center of the Galaxy. (And before you ask, the Galaxy is not at the center of the universe. In fact, as far as we can tell, the universe doesn't have a center.)

In describing the shape of our galaxy, it is useful to break it down into three different components. The most luminous component of our galaxy is the **disk**. Defining the size of the disk is a bit tricky, since it doesn't have sharp edges. However, stars can be seen out to $R \sim 15\,\mathrm{kpc}$ from the galactic center. Thus, if you buy a "You are here" galactic t-shirt (Figure 6.4), it should show you roughly halfway to the edge of the visible disk.[2] However, the disk of the galaxy, when viewed in the 21-cm emission of atomic hydrogen, stretches out further, to $R \sim 25\,\mathrm{km}$ from the galactic center. The thickness of the disk is small compared to its radius. The vast majority of disk stars are less than 0.5 kpc from the midplane of the disk. Thus, the galaxy is shaped less than a grindstone than it is like a more modern artifact – a CD or DVD.

The second component of our galaxy is the **bulge**. If you look at the Milky Way at infrared wavelengths, to minimize the effects of dust, you see a bulge in the direction of Sagittarius, near the galactic center (Figure 6.5). The central bulge of our galaxy is about 1 kpc in radius, so you can see it stick out "above" and "below" the disk. At the very center of the bulge,

[2] If it doesn't, you've fallen victim to an unscrupulous t-shirt vendor from the edge of the galactic disk.

Figure 6.4: T-shirt cartography.

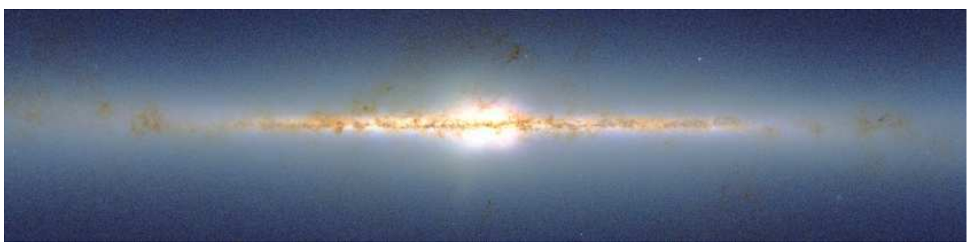

Figure 6.5: Milky Way in the infrared; note the central bulge. [Image credit: 2MASS]

there is a tiny nucleus, very bright at radio wavelengths.

The third component of our galaxy is the **halo**. The halo is a roughly spherical distribution of stars, about 100 kpc in radius; thus, it extends far beyond the disk. The halo has the same luminosity as the bulge, but is stars are spread over a volume $\sim 100^3 \sim 10^6$ times larger. The three stellar components of our galaxy – disk, bulge, and halo – differ in their stellar populations as well as in their size and shape. The disk contains relatively young stars, for the most part, and is where most star formation is occurring today. Disk stars tend to be rich in "metals", with a mass fraction in metals of $Z \geq 0.01$. In the jargon of astronomers, stars that are young and metal-rich are called **population I** stars. By contrast, the halo contains stars that are relatively old and low in metals ($Z \leq 0.001$). The astronomical term for old, metal-poor stars is **population II** stars. The bulge contains a mixture of old and young stars, so it is a place where population I and population II coexist.[3]

Making a complete census of the stars in our galaxy is difficult, because of all the dust. However, a estimate of the luminosity of different components can be made:

- Disk: $L_B = 19 \times 10^9 L_\odot$
- Bulge: $L_B = 2 \times 10^9 L_\odot$
- Halo: $L_B = 2 \times 10^9 L_\odot$
- Grand Total: $L_B = 23 \times 10^9 L_\odot$

Our galaxy's total luminosity of 23 billion solar luminosities is fairly bright, as galaxies go. If all the stars in the Milky Way galaxy were identical to the Sun, then we'd conclude that our galaxy contains 23 billion stars. However, most stars are dim little M dwarfs that contribute little to the total luminosity (particularly in the B band). The best current estimate is that our galaxy contains 200 billion stars.

If you could see the Milky Way galaxy from outside, oriented so that the disk was edge-on, it would probably look like the galaxy NGC 891 (Figure 6.6). Notice the prominent dustlane running down the middle of the galaxy. It is generally true, for disk-dominated galaxies like the Milky Way and NGC 891, that the gas and dust is confined to a much thinner disk than the stars.

If you could see the Milky Way galaxy oriented so that the disk was face-on, it would probably look like the galaxy M83 (Figure 6.7). In this orientation, you can clearly see the most striking feature of galaxies similar to the Milky Way galaxy; their **spiral arms**. Disk-dominated galaxies like M83 and the Milky Way galaxy are thus referred to as "spiral galaxies".[4] Images taken at radio wavelengths indicate that the atomic and molecular gas tend to be concentrated along spiral arms, with lower-density ionized

[3] Exasperating fact of the day: Core collapse supernovae occur in massive, short-lived (and thus, of necessity, young) stars. This means that type II supernovae only occur among population I stars. This is the sort of jargon confusion that drives astronomers to drink.

[4] The edge-on galaxy NGC 891 is part of the "spiral galaxy" class, as well; we just can't see its spiral arms because it is edge-on with respect to us.

Figure 6.6: NGC 891 ($d = 9.8\,\mathrm{Mpc}$), shown in the Large Binocular Telescope "first light" image.

gas filling in the regions between arms. Images taken at ultraviolet and blue wavelengths indicate that the luminous, hot, and short-lived O & B stars also tend to lie along spiral arms. Thus, spiral arms are star-forming factories; they are where dense molecular clouds are converted into stars. The Sun is located in a short stub of a spiral arm, usually called the Orion Arm, or Local Spur, nestled between the longer Perseus and Sagittarius Arms (Figure 6.8). The Orion Nebula, the local hot-bed of star formation, is situated within the Orion Arm.

6.2 Overview: Kinematics and dynamics of our galaxy

An image of a spiral galaxy, like that of Figure 6.7, looks very dynamic, like a snapshot of a hurricane. In fact, the stars in the disk of our galaxy are all rotating in the sense indicated by Figure 6.8); that is, the disk is rotating such that the spiral arms are *trailing.* Stars in the disk are on nearly circular orbits, close to the midplane of the disk, all orbiting in the same direction around the center of the Galaxy. By contrast, stars in the halo are on elongated orbits, at random orientations relative to the disk, with some moving in the same sense as the disk stars and others moving in the opposite sense. (On much different length scales, the disk of our galaxy can be compared to the planets in the Solar System, while the halo of our galaxy can be compared

Figure 6.7: The spiral galaxy M83 ($d = 4.5\,\mathrm{Mpc}$) [Image credit: ESO VLT]

to the Oort Cloud.)

The Sun's orbital speed around the Galactic center is estimated to be[5]

$$v_0 = 220\,\mathrm{km\,s^{-1}} = 225\,\mathrm{kpc\,Gyr^{-1}}\ . \tag{6.7}$$

Later on, we'll discuss how the orbital speeds of the Sun and other stars are actually determined. For now, let's just consider the implications of this orbital motion. The distance of the Sun from the Galactic center is

$$R_0 = 8\,\mathrm{kpc}\ . \tag{6.8}$$

If we make the approximation that the Sun is on a perfectly circular orbit, we find that its orbital period is

$$P_0 = \frac{2\pi R_0}{v_0} = \frac{50.3\,\mathrm{kpc}}{225\,\mathrm{kpc\,Gyr^{-1}}} = 0.22\,\mathrm{Gyr}\ . \tag{6.9}$$

During the Sun's 4.6 Gyr lifetime, it has gone around the Galactic center just over 20 times.[6]

To find out how much mass is inside the Sun's orbit, we may use Kepler's Third Law:

$$M_\odot + M_G = \frac{a^3}{P^2}\ , \tag{6.10}$$

[5]Note the useful coincidence that $1\,\mathrm{km\,s^{-1}} \approx 1\,\mathrm{pc\,Myr^{-1}} \approx 1\,\mathrm{kpc\,Gyr^{-1}}$.

[6]In some sense, the Sun is no longer a teenager.

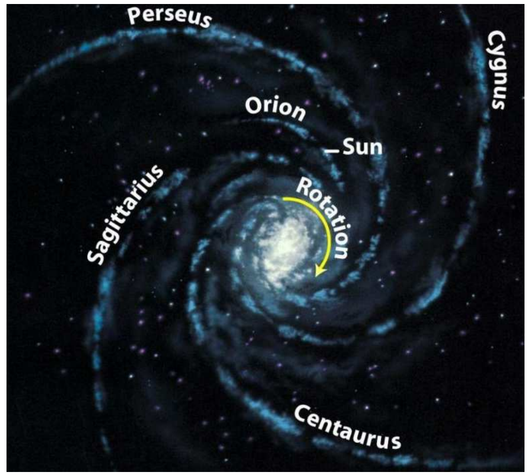

Figure 6.8: Spiral arms in our galaxy, and the direction of galactic rotation.

where M_G is the mass (measured in solar masses) inside a sphere of radius $R_0 = 8\,\mathrm{kpc}$ centered on the Galactic center, $a = 8\,\mathrm{kpc} = 1.65 \times 10^9\,\mathrm{AU}$, and $P = 2.2 \times 10^8\,\mathrm{yr}$. Strictly speaking, it must be confessed, we can only use Kepler's Third Law if the two bodies involved are spherical. Since the matter distribution of our galaxy is flattened, the mass M_G computed from equation 6.10 will be slightly inaccurate. Still, it will be good enough for a first estimate. Since the mass of the Galaxy is much greater than the mass of just one star, we may assume $M_G \gg M_\odot$ and

$$M_G \approx \frac{a^3}{P^2} \approx \frac{(1.65 \times 10^9)^3}{(2.2 \times 10^8)^2} M_\odot \approx 9.3 \times 10^{10} M_\odot \ . \tag{6.11}$$

Ninety-three billion solar masses is a lot of matter, especially when you consider that the total luminosity of our galaxy is only 23 billion solar luminosities. The material of which our galaxy is made must have, on average, a higher mass-to-light ratio than the Sun. Moreover, the mass $M_G = 9.3 \times 10^{10} M_\odot$ contains only the mass *inside* the Sun's orbit. The mass outside the Sun's orbit has no net effect on the Sun's orbital motion (assuming that it's spherically distributed).

For stars and gas in the disk of our galaxy to be on stable circular orbits, we require

$$\frac{v(R)^2}{R} = \frac{GM(R)}{R^2} \ , \tag{6.12}$$

where $v(R)$ is the orbital speed of a star on an orbit of radius R, and $M(R)$ is the mass inside a sphere of radius R centered on the Galactic center. We can thus determine the mass M inside a star's orbit from its orbital speed v:

$$M(R) = \frac{v(R)^2 R}{G} \ . \tag{6.13}$$

Most of the Galaxy's luminosity is provided by the disk, whose brightness falls off exponentially with a scale length $R_s \approx 3\,\text{kpc}$. Thus, most the Galaxy's luminosity lies inside the Sun's orbit. If most of the *mass* lies inside the Sun's orbit as well, we would expect $M \approx$ constant for $R > R_0$, and hence $v \propto R^{-1/2}$.[7]

Observations of stars and gas clouds in our galaxy reveal that the orbital speed v in the disk does not fall off with distance from the center (Figure 6.9). Instead, all the way to the outer fringes of the disk, the orbital speed is con-

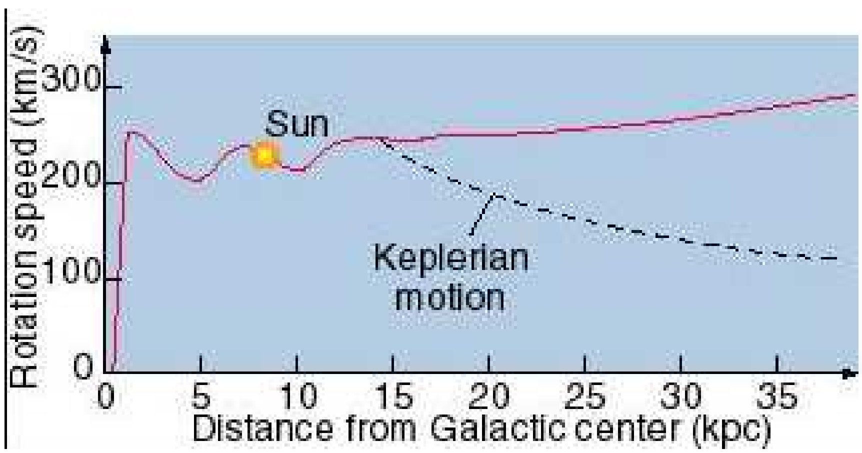

Figure 6.9: Rotation of our galaxy compared to a Keplerian system.

stant, or even slowly rising with radius. Thus, there is more mass outside the Sun's radius than inside. If we estimate from Figure 6.9 that $v = 270\,\text{km}\,\text{s}^{-1}$ at $R = 2R_0 = 16\,\text{kpc}$, we find that the mass inside that radius is

$$\begin{aligned} M(2R_0) &\approx \frac{(2.7 \times 10^5\,\text{m}\,\text{s}^{-1})^2 (4.94 \times 10^{20}\,\text{m})}{6.67 \times 10^{-11}\,\text{m}^3\,\text{s}^{-2}\,\text{kg}^{-1}} & (6.14) \\ &\approx 5.4 \times 10^{41}\,\text{kg} \approx 2.7 \times 10^{11} M_\odot \ . & (6.15) \end{aligned}$$

[7] A system which has $v \propto R^{-1/2}$ is called a **Keplerian** system, since it obeys Kepler's Third Law. The Solar System is a Keplerian system since nearly all its mass is concentrated in the Sun.

Since v is roughly constant outside the Sun's orbital radius, the mass must increase as $M \propto R$, implying an average mass density $\rho \propto R^{-2}$ in a region where the luminosity density is plummeting exponentially.

The most obvious explanation is that there must be **dark matter** in the outer regions of our galaxy. The phrase "dark matter" is the term used by astronomers to refer to matter which is too dim to be detected using current technology.[8] The obvious question to pose is "What's the (dark) matter?" It's hard to determine what something is made of when you can't see it. In recent years, there have been three major candidates to play the role of dark matter.

The first candidate is the **neutrino**. A neutrino can only interact with other particles via gravity or the weak nuclear force. The weak nuclear force is weak indeed; a solar neutrino zips through the Sun as if it weren't there – for that matter, it would zip through a slab of iron a light-year thick as if it weren't there. Since neutrinos snub photons just as they snub other particles, they are a possible candidate for the dark matter, if they have enough mass. As mentioned in section 3.4, recent experiments indicate that the three flavors of neutrino – electron, muon, and tau – have masses that differ from each other. Although the exact mass of each flavor has not been determined, there are fairly strict upper limits on their mass from various experiments. Even though neutrinos are very common particles, their low masses mean that they contribute at most a few percent of the dark matter present in the universe.

The second candidate is the **WIMP**. The term WIMP is an acronym for Weakly Interacting Massive Particle. Supersymmetric extensions to the Standard Model of particle physics predict massive particles that only interact through the weak nuclear force (and through gravity, of course). Think of them as the obese cousins of neutrinos. Particle physicists give these hypothetical particles names like photinos, gravitinos, axinos, sneutrinos, and gluinos. However, since they are massive and weakly interacting, astronomers lump them all together under the generic label of Weakly Interacting Massive Particles. Although WIMPs have been searched for in particle accelerator experiments, they have not yet been found. Of course, since their predicted rest mass is quite large (typical numbers are ~ 100 times the mass of the proton), you wouldn't expect to produce them in the current generation of acceler-

[8]Dark matter might also be called "invisible matter", or maybe "transparent matter", but dark matter is the name that has stuck.

ators. However, the Large Hadron Collider at CERN (currently scheduled to be switched on in 2007) is expected to search for supersymmetric particles with thousands of times the mass of the proton. Other experiments are searching for WIMPs in the same way you detect neutrinos; by building a really big detector and waiting for those very rare interactions mediated by the weak nuclear force. So far, there exist only upper limits on the WIMP interaction rate. But the search goes on...

The third candidate is something completely different: the **MACHO**. The term MACHO is a (slightly strained) acronym for MAssive Compact Halo Object.[9] MACHOs are dim, dense objects with masses comparable to, or somewhat less than, the mass of the Sun. Brown dwarfs, old cold white dwarfs, neutron stars, and black holes can all be MACHOS if they are located in the halo of our galaxy. MACHOs can be detected by acting as **gravitational lenses**. One of the predictions of the theory of General Relativity is that massive objects can bend the path of light. One of the early experimental supports for General Relativity came in the year 1919, when observations of stellar positions during a solar eclipse revealed that the Sun had bent the path of the starlight, by an angle consistent with the predictions of Einstein. Because massive compact objects can bend light, they can act as lenses, making distant stars appear higher in flux than they ordinarily would. Suppose a MACHO in the halo of our galaxy moves direction between us and a star in the Large Magellanic Cloud (see the left panel of Figure 6.10). As the MACHO moves toward the star, the lensed flux we receive grows larger; as the MACHO moves away again, the lensed flux decreases to its original value (as shown in the right panel of Figure 6.10). The net effect is that the lensed star looks brighter for a period of a few weeks, typically. Different research groups have carefully monitored the brightness of stars in the Magellanic Clouds, hoping to catch MACHOs in the act of lensing. Although some lensing events have been detected, there are fewer MACHOs than are needed to contribute all the dark matter. Only $\sim 20\%$ of the dark matter in the halo can consist of MACHOs. There's still plenty of room for WIMPs in the Galaxy.

[9] The term MACHO was first devised as a spoof of the term WIMP.

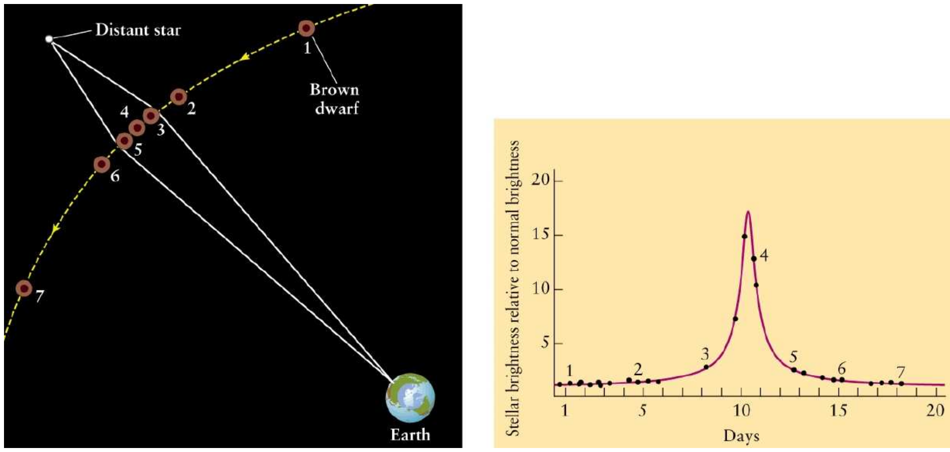

Figure 6.10: Left: gravitational lensing of a distant star by a MACHO. Right: the resulting light curve of the star.

6.3 Local Stellar Motions

In the previous sections of this chapter, we have examined the global picture of our galaxy, outlining its general size, shape, mass, and composition. In this section, however, we will be thinking locally rather than globally, focusing on the motion of stars in the solar neighborhood, within 5 parsecs of the Sun. The location of these stars, relative to the Sun, is fairly easy to determine, since their parallaxes can be measured accurately. It is also relatively easy to determine their *velocity* relative to the Sun. Consider, for instance, the radial velocity v_r of a star relative to the Sun – that's just the rate at which the distance between the star and the Sun is changing (Figure 6.11). The radial velocity relative to your telescope can be found from the Doppler shift of the star's absorption lines:

$$v_r = \frac{\Delta\lambda}{\lambda} c \ . \qquad (6.16)$$

It is important to correct the measured radial velocity for the Earth's orbital motion around the Sun ($v_{\rm orb} \approx 30\,{\rm km\,s^{-1}}$). If you are striving for high accuracy, you must also correct for the Earth's rotation speed at the location of your telescope ($v_{\rm rot} \leq 0.5\,{\rm km\,s^{-1}}$). If the star you are observing is part of a spectroscopic binary system, you can separate the radial velocity of the star relative to the center of mass and the radial velocity of the center of mass

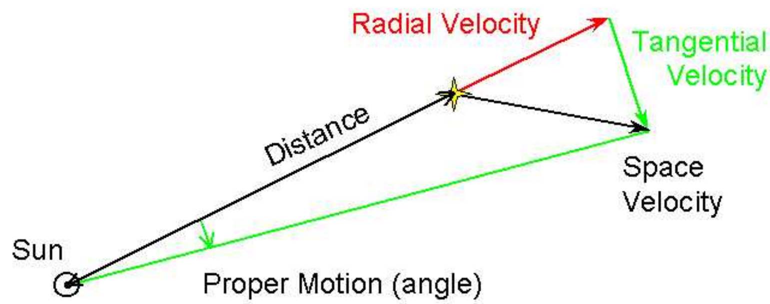

Figure 6.11: Components of a star's velocity relative to the Sun.

itself. This can be done by averaging the radial velocity of the star over an entire orbital period.

The radial velocity of forty stellar systems within 5 parsecs of the Sun is plotted in Figure 6.12. Notice a few interesting results:

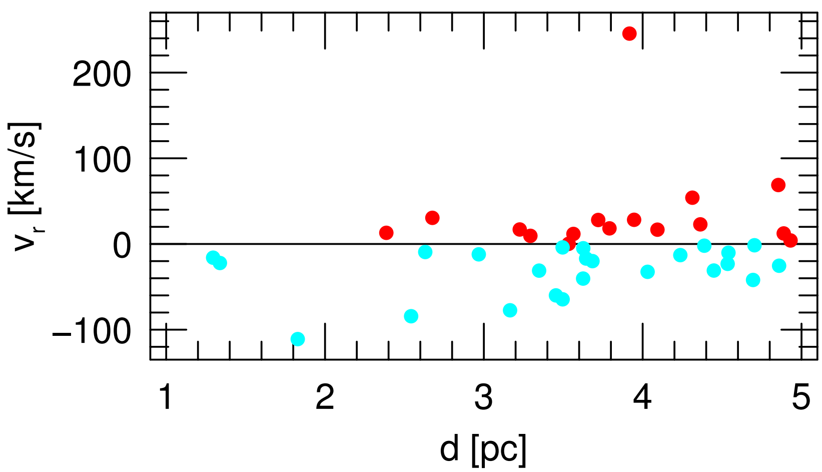

Figure 6.12: Radial velocity of stars within 5 pc of the Sun.

- The radial velocities show approximately equal numbers of blueshifts ($v_r < 0$) and redshifts ($v_r > 0$).
- There is one notable outlier on the plot: Kapteyn's star, alias CD-45°1841, alias Gliese 191, which is moving away from the Sun with

$v_r \approx 250\,\mathrm{km\,s^{-1}}$.

- If Kapteyn's star is left out of the sample, the root mean square radial velocity of nearby stars (relative to the Sun) is $v_r \sim 35\,\mathrm{km\,s^{-1}}$.

Kapteyn's star is distinctly different in its kinematic properties from the other neighborhood stars because is belongs to the *halo*, not the disk. The Sun, and its neighbors in the disk, are orbiting the Galactic center at $\sim 220\,\mathrm{km\,s^{-1}}$, passing by Kapteyn's star, which is on a nearly radial orbit. Halo stars can be recognized by their high velocity relative to the Sun and by their low metallicity.[10]

The radial velocity v_r only gives you one component of the star's three-dimensional velocity. To know completely the star's velocity through space, you must also determine the star's tangential velocity v_t, the component of the velocity perpendicular to the Sun – star line (Figure 6.11). In the nonrelativistic limit, the tangential velocity doesn't produce a Doppler shift. However, the tangential velocity can be determined indirectly, because it produces a **proper motion** μ, which is the rate of change of the star's angular position on the celestial sphere. In the small angle limit,

$$\mu = \frac{v_t}{d} \,, \tag{6.17}$$

where μ is in radians per year, v_t is in parsecs per year, and d is in parsecs. In measuring μ, you must correct for the elliptical motion due to parallax, and for the orbital motion of the star, if it's in a binary system. For instance, Barnard's Star, an M dwarf with $m_V = 9.6$, has the largest proper motion of any star in the night sky (Figure 6.13). Barnard's Star has a parallax of $\pi'' = 0.547\,\mathrm{arcsec}$, implying a distance of only $d = 1.83\,\mathrm{pc}$.[11] The proper motion of Barnard's Star is $\mu = 10.358\,\mathrm{arcsec\,yr^{-1}}$. This is a huge proper motion by stellar standards; it would take Barnard's Star only 170 years to cross the width of the full Moon.

Knowing the distance d and proper motion μ, we can compute the tangential velocity from equation (6.17):

$$\frac{v_t}{1\,\mathrm{pc\,yr^{-1}}} = \left(\frac{d}{1\,\mathrm{pc}}\right)\left(\frac{\mu}{1\,\mathrm{rad\,yr^{-1}}}\right) \,. \tag{6.18}$$

[10] Kapteyn's star is an M subdwarf with a metallicity 1/7 that of the Sun.

[11] Barnard's Star is the fifth closest star to the Earth, after the Sun, Proxima Centauri, and Alpha Centauri A & B.

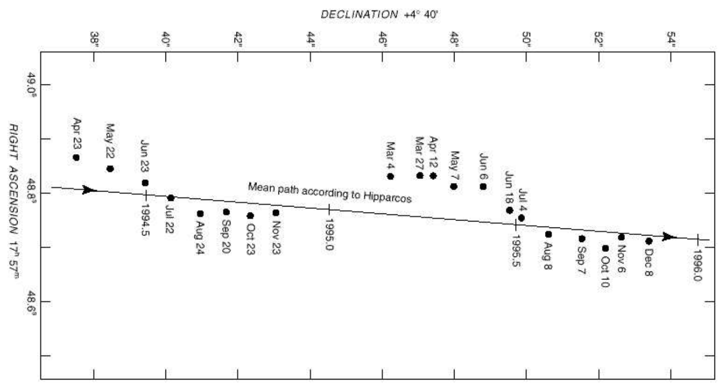

Figure 6.13: Proper motion and parallactic motion of Barnard's Star.

Among astronomers, however, the preferred unit of speed is not the "parsec per year", but the "kilometer per second". The preferred unit of proper motion is not the "radian per year" but the "arcsecond per year". We can make the translation to astronomer-approved units by noting that

$$1\,\mathrm{rad\,yr^{-1}} = 206{,}265\,\mathrm{arcsec\,yr^{-1}} \tag{6.19}$$

and that

$$1\,\mathrm{pc\,yr^{-1}} = \frac{3.086\times10^{13}\,\mathrm{km}}{3.16\times10^{7}\,\mathrm{s}} = 9.77\times10^{5}\,\mathrm{km\,s^{-1}}\ . \tag{6.20}$$

Thus,

$$\begin{aligned} &\left(\frac{1\,\mathrm{pc\,yr^{-1}}}{9.77\times10^{5}\,\mathrm{km\,s^{-1}}}\right)\left(\frac{v_t}{1\,\mathrm{pc\,yr^{-1}}}\right) \\ &= \left(\frac{1\,\mathrm{rad\,yr^{-1}}}{2.063\times10^{5}\,\mathrm{arcsec\,yr^{-1}}}\right)\left(\frac{\mu}{1\,\mathrm{rad\,yr^{-1}}}\right)\left(\frac{d}{1\,\mathrm{pc}}\right)\ . \end{aligned} \tag{6.21}$$

This means that in the preferred units,

$$\frac{v_t}{1\,\mathrm{km\,s^{-1}}} = 4.74\left(\frac{d}{1\,\mathrm{pc}}\right)\left(\frac{\mu}{1\,\mathrm{arcsec\,yr^{-1}}}\right)\ . \tag{6.22}$$

If the parallax π'' is measured in arcseconds, and the proper motion μ'' is measured in arcseconds per year, the tangential velocity of a star is

$$v_t = 4.74\left(\frac{\mu''}{\pi''}\right)\,\mathrm{km\,s^{-1}}\ . \tag{6.23}$$

For Barnard's Star, as an example,

$$v_t = 4.74 \left(\frac{10.358}{0.547} \right) \text{km s}^{-1} = 89.8 \, \text{km s}^{-1} \; . \tag{6.24}$$

The tangential velocity of stellar systems within 5 parsecs of the Sun is shown in Figure 6.14. The high proper motion of Barnard's Star is due both to the

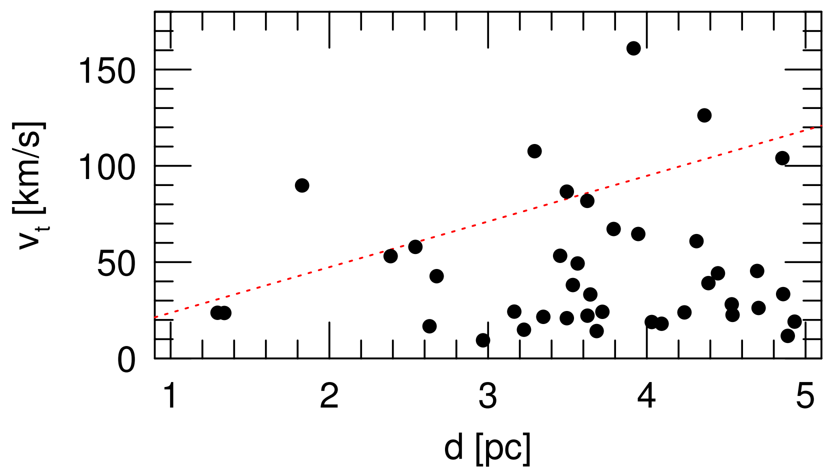

Figure 6.14: Tangential velocity of stars within 5 pc of the Sun. Stars above the dotted line have $\mu'' > 5 \, \text{arcsec yr}^{-1}$.

fact that it is very close to us and to the fact that its tangential velocity is higher than the average for nearby stars.

If the average tangential velocity v_t doesn't vary with distance from the Sun, then *on average*, nearby stars will have a higher proper motion than more distant stars. One way to search for nearby stars, if you don't have the time or patience to measure parallaxes for every star in the sky, is to start by looking at stars with high proper motion. The history books state that in the year 1838, Friedrich Wilhelm Bessel measured the parallax of 61 Cygni. What they sometimes don't tell you is why Bessel chose that star to observe: it is, after all, a humble fifth magnitude star. Bessel, in fact, chose 61 Cygni because it has the highest proper motion of any star visible to the naked eye, $\mu'' = 5.2 \, \text{arcsec yr}^{-1}$.[12] Of course, any star, no matter its distance, will have

[12]The unusual nature of Kapteyn's Star was first recognized in the year 1897, when this otherwise boring M dwarf was found to have a proper motion of nearly 9 arcseconds per year. At the time, this was the largest proper motion known, defeated only when E. E. Barnard measured the proper motion of Barnard's Star in 1916.

zero proper motion if it's moving straight toward us or straight away from us. An example of a nearby star with small proper motion is Gliese 710, which has $v_r = -13.9\,\mathrm{km\,s^{-1}}$ and $\mu'' = 0.014\,\mathrm{arcsec\,yr^{-1}}$. At the moment, Gliese 710 is over 19 parsecs away from us, and has $m_V = 9.7$. In 1.4 Myr, however, is will be only 0.34 parsecs away (about a fourth the present distance to Proxima Centauri) and have $m_V = 0.9$.

Putting it all together, the total speed relative to the Sun,

$$v = (v_r^2 + v_t^2)^{1/2} \;, \tag{6.25}$$

is called the "space motion" or "space velocity". A plot of the space motion for nearby stars (Figure 6.15) reveals that Kapteyn's Star again stands out

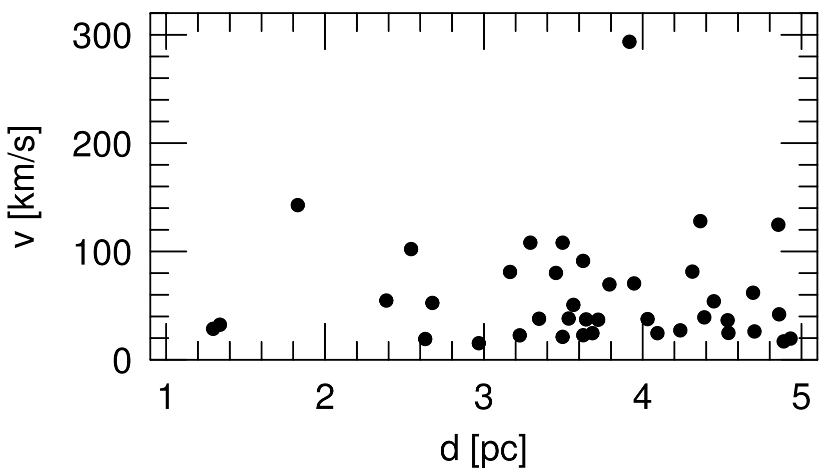

Figure 6.15: Space velocity of stars within 5 pc of the Sun. The outlier at $d \approx 3.9\,\mathrm{pc}$, $v \approx 300\,\mathrm{km\,s^{-1}}$ is Kapteyn's star.

like the proverbial sore thumb. Its space motion of $v \approx 300\,\mathrm{km\,s^{-1}}$ is twice that of Barnard's Star, the next speediest star in the solar neighborhood. The average space motion of stars within 5 parsecs of the Sun is $v \sim 50\,\mathrm{km\,s^{-1}} \sim 50\,\mathrm{pc\,Myr^{-1}}$. This indicates that the list of stars within 5 parsecs of us will be thoroughly revised on timescales $t \sim 5\,\mathrm{pc}/50\,\mathrm{pc\,Myr^{-1}} \sim 0.1\,\mathrm{Myr}$.

6.4 The Local Standard of Rest

If the disk of our galaxy were perfectly orderly, with all the stars on exactly circular orbits in the same plane, it would be simple to compute the expected velocity of stars relative to the Sun. However, the Sun and the other disk stars are not on perfectly circular orbits. A typical orbit is neither circular nor elliptical, but forms a complicated "rosette" pattern, as shown in Figure 6.16. Since the Sun is on a complicated noncircular orbit, using it as the origin for

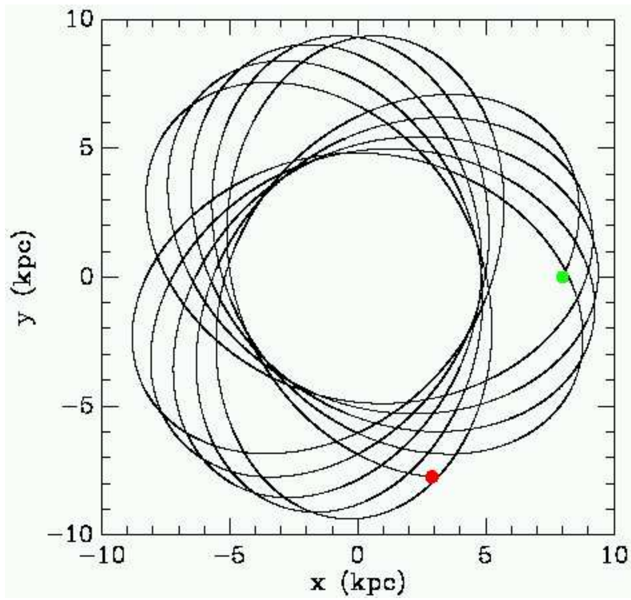

Figure 6.16: Typical orbit for a disk star in the solar neighborhood.

our reference frame makes the mathematics unnecessarily complicated.[13]

It is mathematically convenient to use an idealized reference frame for our study of motion within the Galaxy. This reference frame is called the **Local Standard of Rest**, or LSR. The Local Standard of Rest has its origin at the Sun's location ($R_0 = 8\,\mathrm{kpc}$), and is moving in a circular orbit with $v_0 = 220\,\mathrm{km\,s^{-1}}$. In other words, the LSR is doing what the Sun would be doing *if* it were on a perfectly circular orbit.

The Sun is moving with respect to the LSR at a speed of $\sim 20\,\mathrm{km\,s^{-1}}$. How can we tell? In the solar neighborhood, the circular speed

$$v_c(R) \equiv \left(\frac{GM(R)}{R}\right)^{1/2} \approx 220\,\mathrm{km\,s^{-1}} \tag{6.26}$$

[13]Poor Ptolemy...since the Earth is on a noncircular orbit around the Sun, using it as the origin for his reference frame made the Ptolemaic model for the Solar System unnecessarily complicated. Think of all those equants, epicycles, and deferents!

doesn't depend strongly on R, as shown in Figure 6.9. Thus, if the Sun were moving at the same velocity as the LSR, the average radial velocity of nearby stars ($d \ll R_0$) would be zero in all directions. Now suppose that the Sun is moving at a velocity $\Delta\vec{v}$ relative to the LSR. Stars in the direction of the Sun's motion will be blueshifted, with $v_r = -\Delta\vec{v}$ on average (the Sun will be overtaking them). Stars opposite the direction of the Sun's motion with be redshifted, with $v_r = +\Delta\vec{v}$ on average (the Sun will be pulling away from them). The point toward which the Sun is moving, *relative to the LSR*, is called the **apex**; the opposite point on the celestial sphere is called the **antapex**. Statistical analysis of the radial velocity of nearby stars reveals that the apex of the Sun's motion is in the constellation Hercules, and the antapex is in the constellation Columba.

In the disk of our galaxy, it's convenient to use cylindrical coordinates, rather than cartesian or spherical (Figure 6.17). The cylindrical coordinates

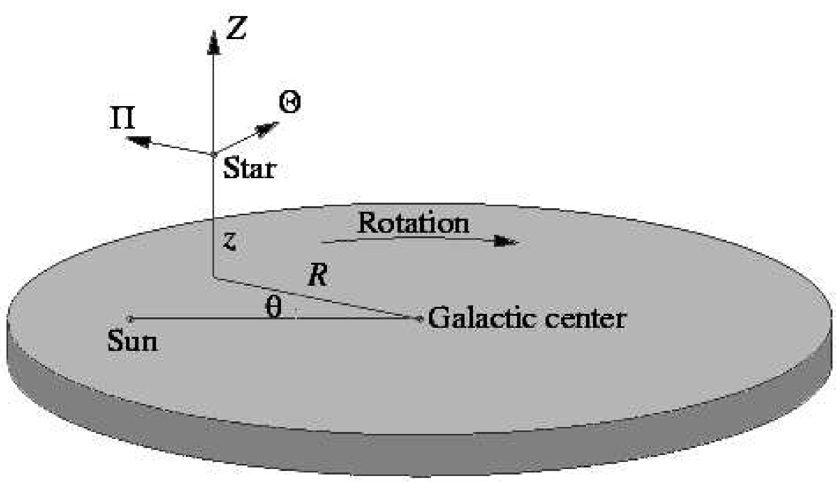

Figure 6.17: Cylindrical coordinates for position and velocity.

(R, θ, z) are chosen such that $R = 0$ and $z = 0$ at the Galactic center. The azimuthal coordinate is $\theta = 0$ at the Sun's location and increases in the direction of the LSR's direction of motion. The z coordinate increases as you go north.[14] The location of the Sun in this cylindrical coordinate system is $(R_0, \theta_0, z_0) = (8\,\text{kpc}, 0, 0\,\text{kpc})$. The velocities in the three directions are

[14]The Milky Way divides the celestial sphere into two hemispheres. The hemisphere that happens to contain the North Celestial Pole is called the northern hemisphere; its central point is the North Galactic Pole.

- Π = speed in the R direction (positive away from the Galactic center).
- Θ = speed in the θ direction (positive in the direction of motion of the LSR).
- Z = speed in the z direction (positive toward the North Galactic Pole).

The velocity of the Local Standard of Rest is then

$$(\Pi_0, \Theta_0, Z_0) = (0, 220\,\mathrm{km\,s^{-1}}, 0)\ . \tag{6.27}$$

The velocity of the Sun relative to the LSR is

$$\begin{aligned}(\Pi_\odot - \Pi_0, \Theta_\odot - \Theta_0, Z_\odot - Z_0) = \\ (-10.4\,\mathrm{km\,s^{-1}}, 14.8\,\mathrm{km\,s^{-1}}, 7.3\,\mathrm{km\,s^{-1}})\ .\end{aligned} \tag{6.28}$$

The Sun is currently moving *inward* (toward the Galactic center). The Sun is moving *forward* (faster than the LSR in the azimuthal direction). The Sun is moving *northward* (toward the North Galactic Pole). The net motion is $\Delta v = 19.5\,\mathrm{km\,s^{-1}}$ in the direction of Hercules, about 23° north of the Milky Way.

The Local Standard of Rest is itself moving at $\Theta_0 = 220\,\mathrm{km\,s^{-1}}$ in the direction of Cygnus, 90° away from the Galactic center in Sagittarius. Sometimes the Sun's extra velocity Δv, which is $< 9\%$ of the speed of the LSR, is small enough to be ignored, and we can pretend the Sun is on a circular orbit. At other times, it must be taken into account.[15]

6.5 Differential Rotation of Our Galaxy

Since we know the Sun's velocity relative to the LSR, if we measure the velocity of a star relative to the Sun, it is a straightforward piece of vector algebra to convert it into a velocity relative to the LSR. This is useful because the analysis of stellar velocities relative to the Local Standard of Rest tells us about the rotation of our galaxy. The orbital speed of a star on a circular orbit is

$$\Theta(R) = \left(\frac{GM(R)}{R}\right)^{1/2}\ . \tag{6.29}$$

[15]Similarly, sometimes we can pretend the Earth's orbit is circular; sometimes we must take its eccentricity into account. It's all a matter of how accurate you need to be for a given problem.

The orbital speed Θ can be converted to an **angular velocity** ω:

$$\omega(R) \equiv \frac{\Theta(R)}{R} . \qquad (6.30)$$

At the Sun's location, the angular velocity is

$$\omega_0 = \frac{\Theta_0}{R_0} = \frac{220\,\mathrm{km\,s^{-1}}}{8\,\mathrm{kpc}} = 27.5\,\mathrm{km\,s^{-1}\,kpc^{-1}} . \qquad (6.31)$$

In other units, this becomes $0.028\,\mathrm{rad\,Myr^{-1}}$, or $5.8 \times 10^{-3}\,\mathrm{arcsec\,yr^{-1}}$.

There are different types of rotation, some of which we have already encountered.

- **Keplerian rotation**, in which all the mass is concentrated at the center of a system. $M =$ constant, $\Theta \propto R^{-1/2}$, $\omega \propto R^{-3/2}$.
- **Constant orbital speed**, a fair approximation for most of our galaxy. $\Theta =$ constant, $M \propto R$, $\omega \propto R^{-1}$.
- **Rigid-body rotation**, seen, for instance, in a rotating wheel. $\omega =$ constant, $\Theta \propto R$, $M \propto R^3$

If the disk of our galaxy were in rigid-body rotation, then stars would have the same orbital period, $P = 2\pi/\omega$, regardless of distance from the Galactic center. However, the disk is actually in **differential rotation**, with ω decreasing with radius. We expect stars closer to the Galactic center to be passing us, while stars farther from the Galactic center fall behind.

Let's now reconstruct the analysis of the Galaxy's differential rotation performed by Jan Oort in the 1920s.[16] This analysis is an exercise in trigonometry, so we should start by examining the diagram in Figure 6.18. This diagram – let's call it the Oort Diagram – is central to an understanding of galactic rotation. The Sun's location, which defines the origin of the Local Standard of Rest, is at a distance $R_0 = 8\,\mathrm{kpc}$ from the Galactic center. The LSR is moving on a circular orbit with speed $\Theta_0 = 220\,\mathrm{km\,s^{-1}}$. We observe a star in the disk of the Galaxy at a galactic longitude ℓ; the galactic longitude is just the angle between the star and the Galactic center, assuming that the

[16]This is the same Oort after whom the Oort Cloud is named. The dynamics of stellar systems, such as the Galaxy, aren't that different from the dynamics of a swarm of comets, such as the Oort Cloud.

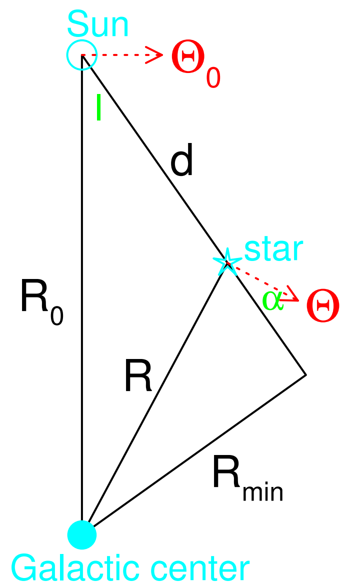

Figure 6.18: Oort Diagram, giving the geometry of Galactic rotation.

star is at $z = 0$, in the midplane of the disk.[17] The star is at a distance d from us. The quantities ℓ and d are things that we can measure. The star is at a distance R from the Galactic center, and is moving on a circular orbit with speed Θ.[18]

How can we determine the orbital speed $\Theta(R)$ from observations of a star? One thing we can determine from observations of the star is v_r, its radial velocity relative to the LSR. From the Oort Diagram (Figure 6.18), we see that

$$v_r = \Theta \cos\alpha - \Theta_0 \cos(90^\circ - \ell) = \Theta \cos\alpha - \Theta_0 \sin\ell \ , \tag{6.32}$$

where α is the angle between the star's velocity vector and the line from the star to the Sun. We cannot measure α directly, so we must eliminate α from equation (6.32) by using trigonometry. The lines from the Sun to the star to the Galactic center define a triangle whose vertex angles equal ℓ at the Sun,

[17] The value of ℓ runs, by convention, from 0° to 360°, with $\ell = 90^\circ$ in the direction of motion of the LSR.

[18] In general, of course, stars in the disk aren't on *perfectly* circular orbits. However, unless we goof up and accidentally look at a halo star, the approximation of a circular orbit is close enough to give useful results.

$90° + \alpha$ at the star, and thus $180° - \alpha - \ell$ at the Galactic center. From the Law of Sines,

$$\frac{\sin \ell}{R} = \frac{\sin(90° + \alpha)}{R_0} \ , \tag{6.33}$$

and thus

$$\frac{\sin \ell}{R} = \frac{\cos \alpha}{R_0} \ . \tag{6.34}$$

By substituting equation (6.34) into equation (6.32), we find

$$v_r = \Theta \frac{R_0}{R} \sin \ell - \Theta_0 \sin \ell = \left(\frac{\Theta}{R} - \frac{\Theta_0}{R_0} \right) R_0 \sin \ell \ , \tag{6.35}$$

or

$$v_r = (\omega - \omega_0) R_0 \sin \ell \ . \tag{6.36}$$

Equation (6.36) is the **first Oort equation**, which permits you to compute ω in terms of the observables v_r and ℓ and the known values of ω_0 and R_0.[19]

Another thing we can determine from observations of the star is v_t, its tangential velocity relative to the LSR. From the Oort Diagram (Figure 6.18), we see that

$$v_t = \Theta \sin \alpha - \Theta_0 \cos \ell \ . \tag{6.37}$$

Once again, we need to eliminate the unmeasurable angle α. From inspection of the Oort Diagram, we can deduce that

$$R_0 \cos \ell = d + R \sin \alpha \tag{6.38}$$

and thus

$$\sin \alpha = \frac{1}{R}(R_0 \cos \ell - d) \ . \tag{6.39}$$

By substituting equation (6.39) into equation (6.37), we find

$$v_t = \frac{\Theta}{R} R_0 \cos \ell - \frac{\Theta}{R} d - \Theta_0 \cos \ell = \left(\frac{\Theta}{R} - \frac{\Theta_0}{R_0} \right) R_0 \cos \ell - \frac{\Theta}{R} d \ , \tag{6.40}$$

or

$$v_t = (\omega - \omega_0) R_0 \cos \ell - \omega d \ . \tag{6.41}$$

[19]Sanity check: for rigid-body rotation, $\omega = \omega_0$ and thus $v_r = 0$. In other words, the distance between points on a rigid wheel remains constant as the wheel spins.

Equation (6.41) is the **second Oort equation**, which permits you to compute ω in terms of the observables v_t, ℓ, and d, and the known values of ω_0 and R_0.[20]

The Oort equations in their full glory (equations 6.36 and 6.41) are a bit daunting. However, we can simplify them by considering only nearby stars, with $d \ll R_0$ (stars within one or two hundred parsecs of us, for instance). For these nearby stars, we can expand the angular velocity $\omega(R)$ in a Taylor series around $R = R_0$:

$$\omega(R) \approx \omega(R_0) + \frac{d\omega}{dR}\Big|_{R=R_0}(R - R_0) \ . \tag{6.42}$$

Thus,

$$\omega - \omega_0 \approx \frac{d\omega}{dR}\Big|_{R=R_0}(R - R_0) \ , \tag{6.43}$$

and the first Oort equation becomes

$$v_r \approx R_0 \left(\frac{d\omega}{dR}\right)_{R=R_0}(R - R_0)\sin\ell \ . \tag{6.44}$$

For stars with $d \ll R_0$, $R - R_0 \approx -d\cos\ell$, meaning that we can write

$$v_r \approx -R_0 \left(\frac{d\omega}{dR}\right)_{R=R_0} d\cos\ell\sin\ell \ . \tag{6.45}$$

Using the trigonometric identity $2\cos\ell\sin\ell = \sin 2\ell$, the first Oort equation can be written in the simplified form

$$v_r \approx Ad\sin 2\ell \ , \tag{6.46}$$

where

$$A \equiv -\frac{R_0}{2}\left(\frac{d\omega}{dR}\right)_{R=R_0} \ . \tag{6.47}$$

The **Oort constant** A is a measurement of the shear; that is, the degree to which the disk of our galaxy does not rotate like a rigid body. Equation (6.46) implies that the radial velocity of nearby stars on circular orbits will be zero

[20]Sanity check: for rigid-body rotation, $\omega = \omega_0$ and thus $v_t = -\omega_0 d$. Imagine you are on a merry-go-round rotating counterclockwise as seen from above; if you look at someone else standing on the merry-go-round with you, he will appear to move right-to-left relative to distant background objects, with a proper motion $\mu = \omega_0$ and hence $v_t = d\mu = d\omega_0$.

when $\ell = 0°$ and $180°$ (toward and away from the Galactic center) but also when $\ell = 90°$ and $270°$ (toward and away from the LSR's direction of motion).

A similar analysis (details left for the reader) yields a similar simplification of the second Oort equation when $d \ll R_0$:

$$v_t \approx d(A \cos 2\ell + B) \ , \tag{6.48}$$

where

$$B \equiv A - \omega_0 \tag{6.49}$$

is the second Oort constant.

A plot of v_r versus galactic longitude ℓ for stars all at the same distance d from the Sun shows a sinusoidal pattern with amplitude Ad; see, for instance, the results of Figure 6.19. A recent fit to the radial velocities of Cepheids[21]

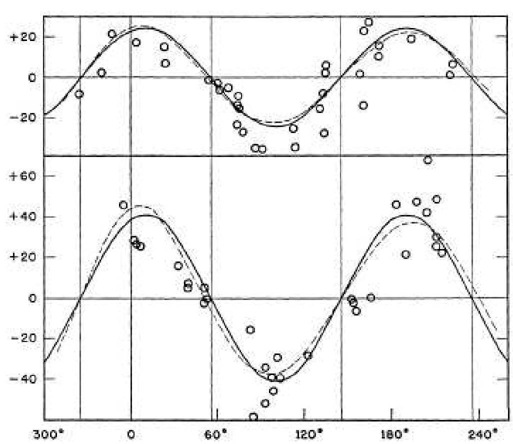

Figure 6.19: Radial velocity (corrected to LSR) versus ℓ for Cepheid stars. Top panel: Cepheids with $d \sim 1.5$ kpc. Bottom panel: Cepheids with $d \sim 3$ kpc. [Image credit: Joy, A. H. 1939, ApJ, 89, 356]

yields $A = 14.8 \pm 0.8 \, \mathrm{km\,s^{-1}\,kpc^{-1}}$. A plot of v_t/d – that is, the proper motion – versus ℓ also shows a sinusoidal pattern, offset in the vertical direction by a value B (Figure 6.20). The best fit to the proper motion of Cepheids yields $B = -12.4 \pm 0.6 \, \mathrm{km\,s^{-1}\,kpc^{-1}}$. By the way, Oort's original value

[21]Feast & Whitelock, 1997, MNRAS, 291, 683

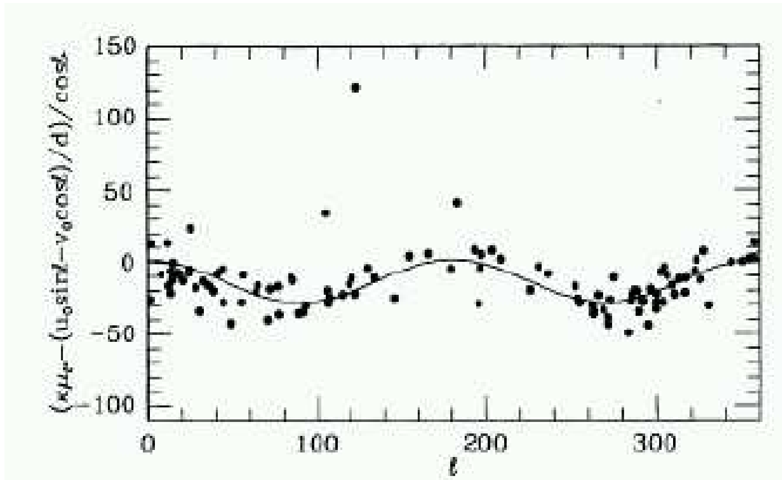

Figure 6.20: Proper motion of Cepheid stars within 2 kpc of the Sun. [Image credit: Feast & Whitelock, 1997, MNRAS, 291, 683]

for the Oort constants, found in 1927, were $A = 19 \pm 3\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ and $B = -24 \pm 5\,\mathrm{km\,s^{-1}\,kpc^{-1}}$. His non-zero value for A was the first definitive evidence that our galaxy is in differential rotation. Note that our best current values for the Oort constants yield a local angular velocity

$$\omega_0 = A - B = 27.2 \pm 1.0\,\mathrm{km\,s^{-1}\,kpc^{-1}} \,, \tag{6.50}$$

in agreement with the value of $\omega_0 = 27.5\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ that we have been assuming.

6.6 Determining the Rotation Curve

The Oort constants A and B tells us the value of the angular velocity ω and its radial derivative $d\omega/dR$ at the Sun's location in the disk. This enables us to recreate the rotation curve of the disk in the Sun's immediate vicinity. To determine the full rotation curve for the Galaxy, we must use the Oort equations in their complete form. Suppose we observe a star at galactic longitude ℓ, and measure its distance d and its radial velocity v_r relative to the Local Standard of Rest (Figure 6.18). From the Law of Cosines, we can determine the distance R of the star from the Galactic center:

$$R = \left(R_0^2 + d^2 - 2dR_0 \cos\ell\right)^{1/2} \,. \tag{6.51}$$

From the first Oort equation (eq. 6.36),

$$v_r = (\omega - \omega_0) R_0 \sin \ell \ , \tag{6.52}$$

we can determine the angular velocity at R:

$$\omega(R) = \omega_0 + \frac{v_r}{R_0 \sin \ell} \ . \tag{6.53}$$

We can then compute the orbital velocity $\Theta(R) = R\omega(R)$.

The problem with this method of determine the Galactic rotation curve (Θ as a function of R) is that if you want to find Θ for a wide range of R, you must observe stars several kiloparsecs away, near the midplane of the disk where the dust is thickest. Thus, the extinction corrections will be large, and the resulting errors in distance will be sizable. One way to pierce through the dust is to look at radio emission from gas clouds instead of visible light from stars. The 21 cm emission from atomic hydrogen and the 2.6 mm emission from carbon monoxide is largely unaffected by dust. The problem with using gas clouds as your source of emission is that determining the distance to a gas cloud is difficult. Measuring their parallax is impractical, since they are distant fuzzy blobs instead of nearby unresolved sources.[22]

Despite the difficulty in determining the distance to gas clouds, you can still derive some information from a gas cloud of known galactic longitude ℓ and radial velocity v_r (but unknown distance d). We can start by rewriting the Law of Cosines (equation 6.51) to give the distance d in terms of R, R_0, and ℓ.

$$d = R_0 \cos \ell \pm \sqrt{R^2 - R_0^2 \sin^2 \ell} \ . \tag{6.54}$$

Along a line of sight with fixed galactic longitude ℓ, every gas cloud will have $R \geq R_{\rm min}$, where $R_{\rm min} \equiv d \sin \ell$ (see Figure 6.18); this assumes, implicitly, that $\sin \ell \geq 0$, which is true in the quadrants $0^\circ < \ell < 90^\circ$ and $270^\circ < \ell < 360^\circ$. The point along the line where $R = R_{\rm min}$ is called the **tangent point**.[23] For values of R in the range $R_0 > R > R_{\rm min}$, a single value of R corresponds to *two* values of d, one on the near side of the tangent point, as seen from the Sun, and the other on the far side. In general, even if we knew

[22] If you knew the clouds' physical size in AU, you could measure their angular size in arcseconds, and thus compute their distance. Unfortunately, gas clouds aren't of uniform size.

[23] The tangent point is where the line of sight is *tangent* to a circle of radius $R_{\rm min}$ centered on the Galactic center.

R for a particular gas cloud, its distance d from the Sun is still ambiguous. Is it on the near side of the tangent point, or the far side?

Cool atomic gas clouds and cold molecular clouds are sufficiently common in our galaxy that a line of sight through the disk usually passes through several of them. Since ω decreases with R in the disk, the gas cloud closest to the tangent point will have the largest angular velocity ω, and hence the largest radial velocity,

$$v_r = (\omega - \omega_0) R_{\rm min} \ . \tag{6.55}$$

This leads to the **tangent point method** for determining the rotation curve of the Galaxy. Start by pointing your radio telescope at a given galactic longitude ℓ along the Milky Way. Along the chosen line of sight, the tangent point lies at a distance $R_{\rm min} = R_0 \sin \ell$ from the Galactic center.[24] In the molecular or atomic emission lines that you observe, you will typically find several peaks; see, for instance, the CO emission line displayed in Figure 6.21. Each peak in the emission line corresponds to a different gas cloud with a

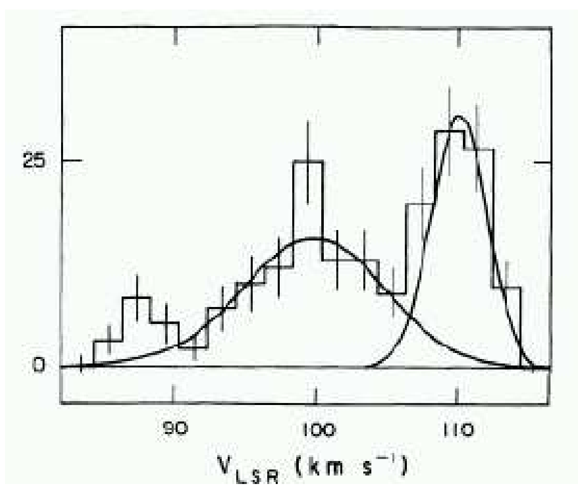

Figure 6.21: Carbon monoxide emission at Galactic longitude $\ell = 31.5°$. [Image credit: Clemens, D. P. 1985, ApJ, 295, 422]

different radial velocity v_r. The peak with the highest radial velocity tells you $v_{r,\rm max}$, the radial velocity of the most rapidly receding cloud along the

[24]Again, we are assuming that we are looking in the quadrants closest to the Galactic center, with $0° < \ell < 90°$, and $270° < \ell < 360°$.

line of sight. From the first Oort equation (eq. 6.36), this radial velocity corresponds to an angular velocity for the cloud of

$$\omega_{\rm max} = \omega_0 + \frac{v_{r,\rm max}}{R_{\rm min}} \ . \tag{6.56}$$

If we assume that the cloud is located *exactly* at the tangent point, then the angular velocity at the tangent point is

$$\omega(R_{\rm min}) = \omega_{\rm max} = \omega_0 + \frac{v_{r,\rm max}}{R_{\rm min}} \ , \tag{6.57}$$

and the orbital velocity at the tangent point is

$$\Theta(R_{\rm min}) = R_{\rm min}\omega(R_{\rm min}) = \omega_0 R_{\rm min} + v_{r,\rm max} \ . \tag{6.58}$$

Note the error built into this method; if the cloud isn't exactly at the tangent point, both R and Θ for the cloud will be *larger* than we have computed.

As an example, let's use the data presented in Figure 6.21, the result of observing CO emission at the Galactic longitude $\ell = 31.5°$, in the constellation Aquila. Along this line of sight, the tangent point is at a distance $R_{\rm min} = (8\,{\rm kpc})\sin 31.5° = 4.18\,{\rm kpc}$ from the Galactic center. The emission peak with the highest radial velocity is at $v_r = +109.9\,{\rm km\,s^{-1}}$, corresponding to an angular velocity (equation 6.56)

$$\omega_{\rm max} = 27.5\,{\rm km\,s^{-1}} + \frac{109.9\,{\rm km\,s^{-1}}}{4.18\,{\rm kpc}} = 53.8\,{\rm km\,s^{-1}\,kpc^{-1}} \ . \tag{6.59}$$

This angular velocity corresponds to an orbital velocity

$$\Theta(R_{\rm min}) = \omega_{\rm max} R_{\rm min} = 225\,{\rm km\,s^{-1}} \ , \tag{6.60}$$

if the molecular cloud is exactly at the tangent point.

By looking along many lines of sight at different Galactic longitudes, we can build up a plot of $\Theta(R)$ versus R; an example is shown in Figure 6.22. Note that the tangent point method only works for gas clouds with $R < R_0$. To find the rotation curve outside R_0, you really do need to observe objects whose distance is known unambiguously. Once you have an more-or-less accurate idea of $\omega(R)$, you can compute ω for each gas cloud along a line of sight and use the plot of ω versus R to determine the distance R of each gas cloud from the Galactic center. Finding d, the distance from the Sun, is

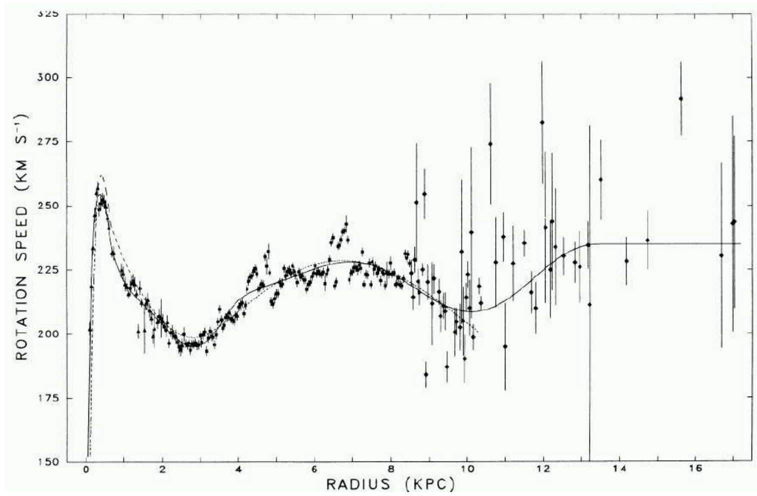

Figure 6.22: Rotation curve of our galaxy. [Image credit: Clemens, D. P. 1985, ApJ, 295, 422]

still fraught with ambiguity. Often you can use the angular size of the gas cloud to guess whether it's at the nearer value of d or the farther value of d. Translating from ω to d makes it possible to make maps of the gas distribution in our galaxy. For instance, Figure 6.23 shows the surface density of atomic hydrogen (HI) in our galaxy. Note the features in the map that seem to point directly away from the Sun. This is because errors in d tend to stretch out compact, nearly spherical structures into long smears along the line of sight. In addition, structures near the Galactic center, in the middle of the figure, are very muddled and noisy. This is because there are strongly noncircular motions near the center of our Galaxy.

Plots of the surface density of gas as a function of R are less noisy, since they don't require dealing with the ambiguity in d for a particular gas cloud. Figure 6.24 shows the surface density of atomic gas (dashed line) and molecular gas (solid line) as a function of R, the distance from the center of the Galaxy. Inside $R \approx 5\,\mathrm{kpc}$, most gas in our galaxy is molecular; outside $R \approx 5\,\mathrm{kpc}$, most of the gas is atomic. The key difference is density. Molecules can only for in relatively dense regions of interstellar space, where dust grains act as a catalyst for the formation of molecules.

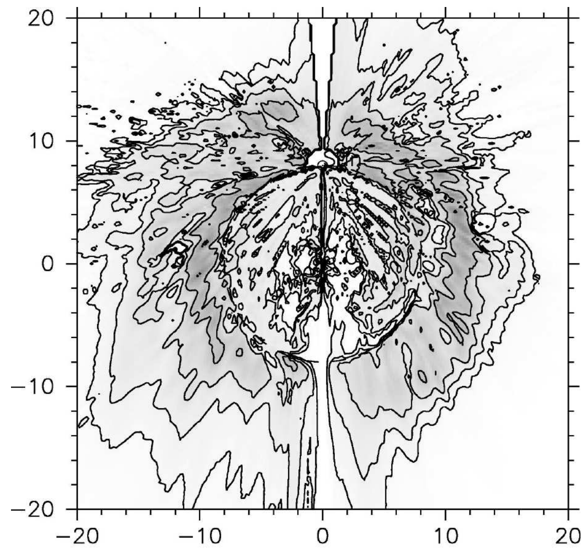

Figure 6.23: Surface density of atomic hydrogen in our galaxy. The small circle in the upper center marks the location of the Sun. [Image credit: Nakanishi et al. 2003, PASJ, 55, 191]

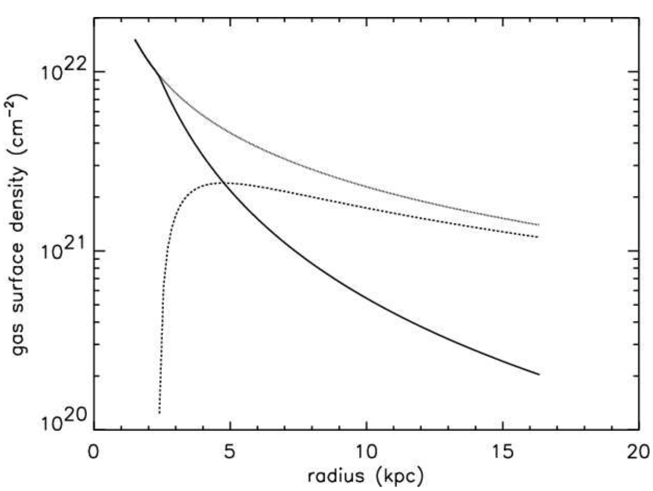

Figure 6.24: Surface density of atomic gas (dashed) and molecular gas (solid). The gray line gives the total gas density.