Ontology Languages
Description Logics

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Examples in this section are based on:

- D. Calvanese and D. Lembo (tutorial on DL @ISCW'07)
- F. Baader et al. The Description Logic Handbook

DESCRIPTION LOGICS

HOW TO MODEL KNOWLEDGE AND ASSERT INSTANCES

Is not completely built on top of RDFS
Takes some part of RDFS but all part of RDF

Logics-Based Knowledge Representation

First-Order Logic (FOL)

problem - computational complexity

- Suitable for knowledge representation
  - Classes as unary predicates
  - Properties / relationships as binary predicates
  - Constraints as logical formulas using those predicates
- Undecidability
  - In the general case, there is no algorithm that determines if a FOL formula implies another

undecidable
for some query takes a lot of time
it might not finish
so, first order logic is not used much in real life

Decidable Fragments of FOL

- Description Logics (binary predicates bounded number of variables)
- Datalog (Horn-clauses)

person identifies classes as unary predicate
person(x)
course(y)

binary predicate for relationship
teaches(x,y)

OWL choose FOL
OWL build on top of Description logic

Decidable Subsets of FOL

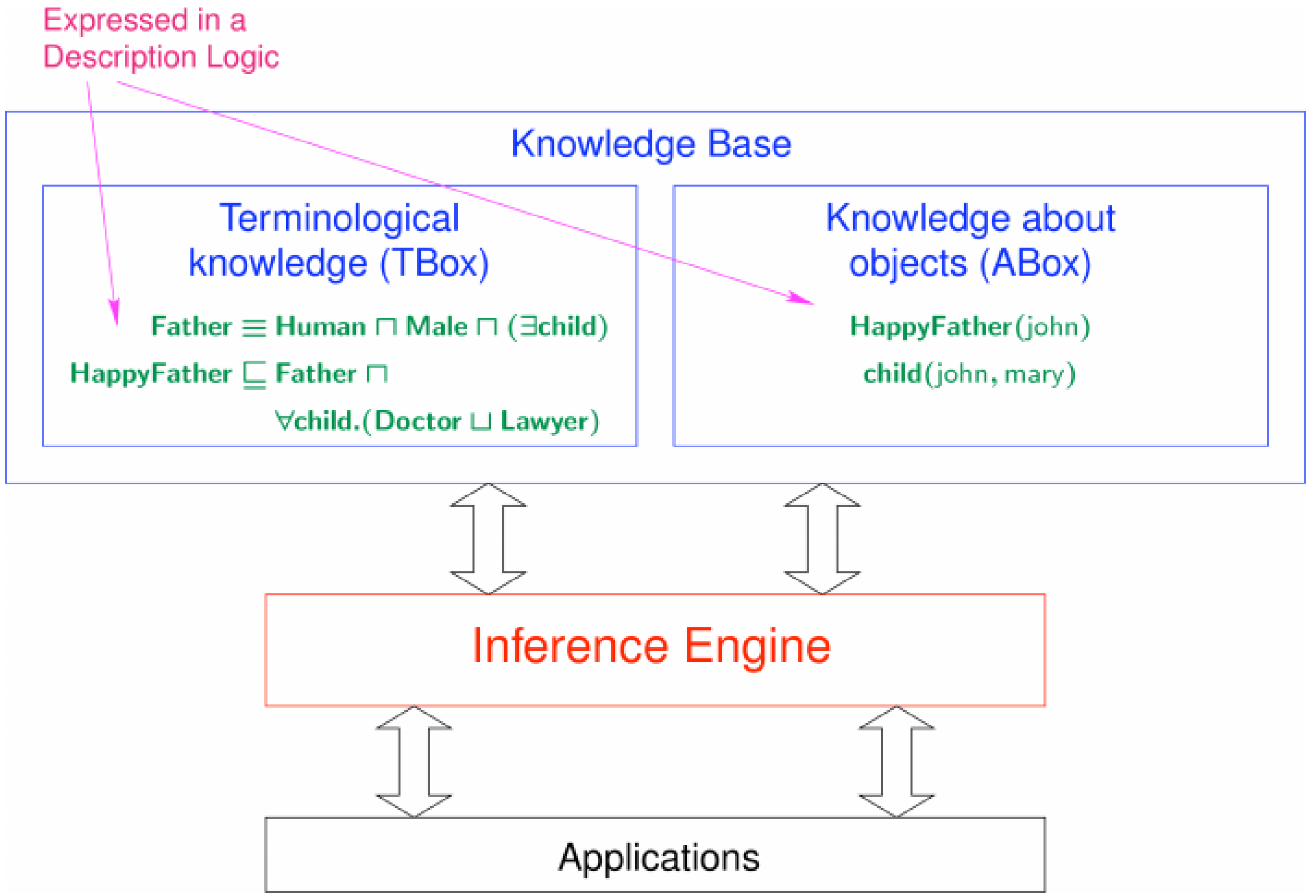
IN DATABASE

	Datalog	Description Logics
Focus	Instances	Knowledge
Approach	Centralized	Decentralized
Reasoning	Closed-world assumption	Open-world assumption
Unique name	Unique name assumption	Non-unique name assumption you can have two different variable for same thing

here, inference engine is always activated
in rdfs, it was upto us to enable/disable inference engine

in description logic, it is mandatory to define TBOX an abox
in rdfs, it was not mandatory

Description Logic (DL) Knowledge Base



to have ontology we must assert
1. there should be knowledge and instances
2. there will be answer for sure (though might take time)
3.

Description Logics and Ontologies

Description Logics are used to assert **knowledge** and **instances**

- The knowledge is asserted in the TBOX (DL terminology)
- The instances are asserted in the ABOX (DL assertions)

A DL TBOX and ABOX is a decidable subset of FOL. DL defines accordingly reasoning services for DL KBs

We say a *knowledge base* is an ontology if:

- It defines the ontology terminology (TBOX)
- The asserted instances (ABOX) are complaint with the terminology (i.e., TBOX)
- It provides **sound** reasoning services

Thus:

- Any Description Logic KB is always an ontology
- A RDFS KB is an ontology if:
  - You define a TBOX
  - The RDFS ABOX is compliant with the TBOX
  - You use sound inference rules (i.e., those defined by the SPARQL community)
- Strictly speaking, although many people say the opposite, a RDF knowledge base is not an ontology if we follow the definition above

RDF alone is not enough to define ontology

complex concept and role can be created from atomic concept and roles
course(y), person(x), teaches(x,y) is atomic roles and concept
there exists teaches(x,y) is complex

Description Logic: TBOX

A DL TBOX is characterized by a set of constructs for building **complex concepts and roles** from **atomic concepts and roles**:

- Concepts correspond to classes
- Roles correspond to relationships

Atomic concepts / roles:

- Must be explicitly defined by the user (e.g., the person concept or the lectures role)

Complex concepts / roles:

- They are derived from atomic concepts or roles (e.g., a lecturer is a person who lectures)
- They must be derived using the pre-defined **concept and role constructs** provided by the description logic

It is called TBOX because it defines the **terminology** (of the domain)

- It is equivalent to the metadata / schema layer we have used for RDFS

Description Logic: TBOX

A DL TBOX is characterized by a set of constructs for building **complex concepts and roles** from **atomic concepts and roles**:

- Concepts correspond to classes
- Roles correspond to relationships

<u>**A DL TBOX formal semantics**</u> are given in terms of interpretations:

An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of:

- a nonempty set $\Delta^{\mathcal{I}}$, the domain of $\mathcal{I}$
- an interpretation function $\cdot^{\mathcal{I}}$, which maps
  - each individual a to an element $a^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$
  - each atomic concept A to a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$
  - each atomic role P to a subset $P^{\mathcal{I}}$ of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

Concept Constructs

Atomic concepts and roles are defined explicitly by the user!

Construct	Syntax	Example	Semantics
atomic concept	A	Doctor	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
atomic role	P	hasChild	$P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
atomic negation	$\neg A$	$\neg$Doctor	$\Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$
conjunction	$C \sqcap D$	Hum $\sqcap$ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
(unqual.) exist. res.	$\exists R$	$\exists$hasChild	$\{\ a \mid \exists b.\, (a, b) \in R^{\mathcal{I}}\ \}$
value restriction	$\forall R.C$	$\forall$hasChild.Male	$\{a \mid \forall b.\, (a, b) \in R^{\mathcal{I}} \to b \in C^{\mathcal{I}}\}$
bottom	$\bot$		$\emptyset$

(C, D denote arbitrary concepts and R an arbitrary role)

The above constructs form the basic language $\mathcal{AL}$ of the family of $\mathcal{AL}$ languages.

Additional Concept and Role Constructs

Construct	$\mathcal{AL}\cdot$	Syntax	Semantics
disjunction	$\mathcal{U}$	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
top		$\top$	$\Delta^{\mathcal{I}}$
qual. exist. res.	$\mathcal{E}$	$\exists R.C$	$\{ a \mid \exists b. (a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}} \}$
(full) negation	$\mathcal{C}$	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
number restrictions	$\mathcal{N}$	$(\geq k\, R)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}}\} \geq k \}$
		$(\leq k\, R)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}}\} \leq k \}$
qual. number restrictions	$\mathcal{Q}$	$(\geq k\, R.C)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \geq k \}$
		$(\leq k\, R.C)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \leq k \}$
inverse role	$\mathcal{I}$	R^{-}	$\{ (a,b) \mid (b,a) \in R^{\mathcal{I}} \}$
role closure	reg	$\mathcal{R}^{*}$	$(R^{\mathcal{I}})^{*}$

negation of concept

negation of complex concept

is role

is role

Understanding DL Axioms

What is the meaning of these axioms? Write the **interpretation** corresponding to each axiom

$$\forall \mathsf{hasChild}.(\mathsf{Doctor} \sqcup \mathsf{Lawyer})$$

$$\exists \mathsf{hasChild}.\mathsf{Doctor}$$

$$\neg(\mathsf{Doctor} \sqcup \mathsf{Lawyer})$$

$$(\geq 2\, \mathsf{hasChild}) \sqcap (\leq 1\, \mathsf{sibling})$$

$$(\geq 2\, \mathsf{hasChild}.\, \mathsf{Doctor})$$

$$\forall \mathsf{hasChild}^{-}.\mathsf{Doctor}$$

$$\exists \mathsf{hasChild}^{*}.\mathsf{Doctor}$$

TBOX Definition

A DL TBOX only includes terminological axioms of the following form

- Inclusion (*subsumption*) $C_1 \sqsubseteq C_2$ is satisfied by $\mathcal{I}$ if $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$

  $R_1 \sqsubseteq R_2$ is satisfied by $\mathcal{I}$ if $R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$

  Example: $\mathrm{PhDStudent} \sqsubseteq \mathrm{Student} \sqcap \mathrm{Researcher}$

- Equivalence $C_1 \sqsubseteq C_2,\ C_2 \sqsubseteq C_1$

  Example: $\mathrm{PhDStudent} \equiv \mathrm{Student} \sqcap \mathrm{Researcher}$

Description Logics: ABOX

Defines instances in terms of the terminological axioms defined in the TBOX

- Concept assertions
  - Student(Pere)
- Role assertions
  - Teaches(Oscar, Pere)

We **cannot** assert instances for a concept not defined previously in the TBOX

We can assert instances of both atomic and complex concepts / roles

It is called ABOX because it defines **assertions** on the TBOX concepts and roles

- It is equivalent to the instance layer we have used for RDFS

Example of DL Knowledge Base

TBox assertions:

- Inclusion assertions on concepts:

$$\begin{array}{rcl} \mathsf{Father} & \equiv & \mathsf{Human} \sqcap \mathsf{Male} \sqcap \exists \mathsf{hasChild} \\ \mathsf{HappyFather} & \sqsubseteq & \mathsf{Father} \sqcap \forall \mathsf{hasChild}.(\mathsf{Doctor} \sqcup \mathsf{Lawyer} \sqcup \mathsf{HappyPerson}) \\ \mathsf{HappyAnc} & \sqsubseteq & \forall \mathsf{descendant}.\mathsf{HappyFather} \\ \mathsf{Teacher} & \sqsubseteq & \neg \mathsf{Doctor} \sqcap \neg \mathsf{Lawyer} \end{array}$$

- Inclusion assertions on roles:

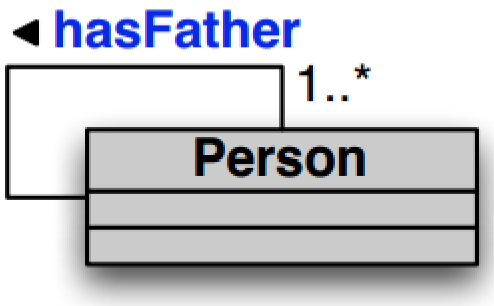
$$\mathsf{hasChild} \sqsubseteq \mathsf{descendant} \qquad \mathsf{hasFather} \sqsubseteq \mathsf{hasChild}^-$$

ABox membership assertions:

- $\mathsf{Teacher}(\mathtt{mary}),\quad \mathsf{hasFather}(\mathtt{mary}, \mathtt{john}),\quad \mathsf{HappyAnc}(\mathtt{john})$

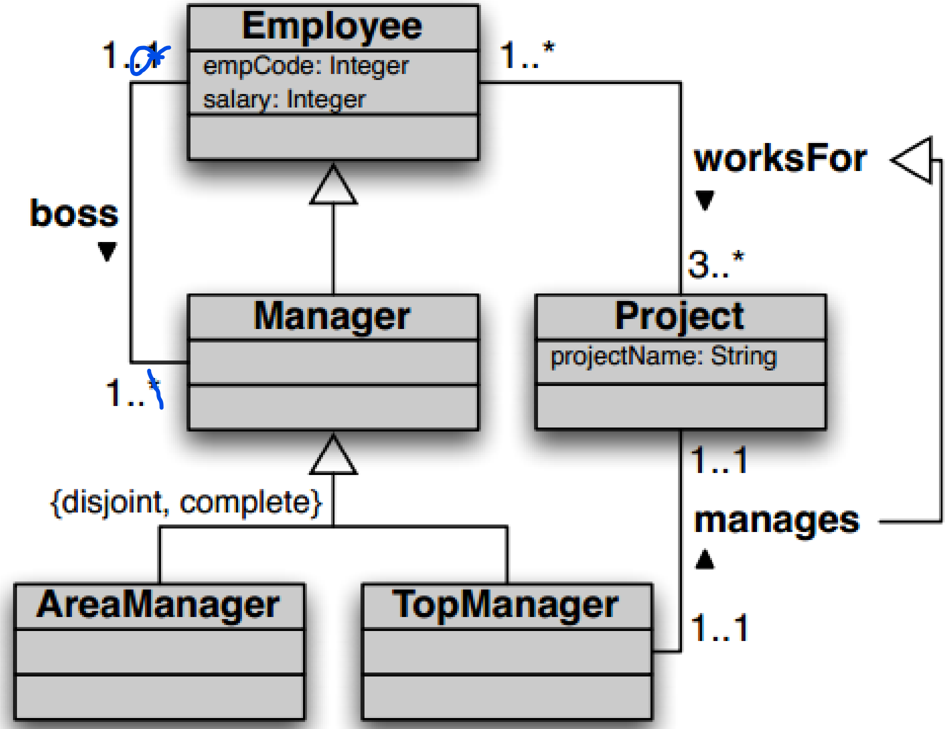
Exercise

Represent as concept expressions the following UML diagram



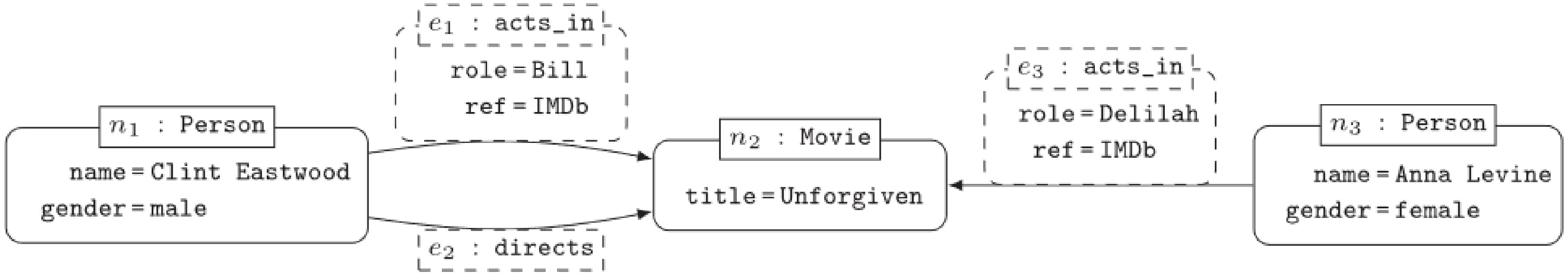
Exercise II

Represent as concept expressions the following UML diagram



Exercise III

Create a DL KB capturing as much constraints as possible from the following graph:



How to represent relationship between two entities when there is blank node in between? Ex: to show a person can act in multiple movies

Examples in this section are based on:

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- F. Baader et al. The Description Logic Handbook

DESCRIPTION LOGICS

REASONING

Model of a DL Ontology

Model of a DL knowledge base

An interpretation $\mathcal{I}$ is a model of $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ if it satisfies all assertions in $\mathcal{T}$ and all assertions in $\mathcal{A}$.

model of ontology that satisfies ...

$\mathcal{O}$ is said to be satisfiable if it admits a model.

The fundamental reasoning service from which all other ones can be easily derived is . . .

Logical implication

$\mathcal{O}$ logically implies and assertion α, written $\mathcal{O} \models \alpha$, if α is satisfied by all models of $\mathcal{O}$.

TBOX Reasoning

concept tbox model

- Concept Satisfiability: C is satisfiable wrt $\mathcal{T}$, if there is a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}}$ is not empty, i.e., $\mathcal{T} \not\models C \equiv \bot$.
- Subsumption: C_1 is subsumed by C_2 wrt $\mathcal{T}$, if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \sqsubseteq C_2$.
- Equivalence: C_1 and C_2 are equivalent wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} = C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \equiv C_2$.
- Disjointness: C_1 and C_2 are disjoint wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}} = \emptyset$, i.e., $\mathcal{T} \models C_1 \sqcap C_2 \equiv \bot$.
- Functionality implication: A functionality assertion $(\textbf{funct } R)$ is logically implied by $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$, we have that $(o, o_1) \in R^{\mathcal{I}}$ and $(o, o_2) \in R^{\mathcal{I}}$ implies $o_1 = o_2$, i.e., $\mathcal{T} \models (\textbf{funct } R)$

Reasoning Complexity

Complexity of concept satisfiability: [DLNN97]

union or
existential quantification

$\mathcal{AL}$, $\mathcal{ALN}$	N- cardinality	PTIME
$\mathcal{ALU}$, $\mathcal{ALUN}$		NP-complete
$\mathcal{ALE}$		coNP-complete
$\mathcal{ALC}$, $\mathcal{ALCN}$, $\mathcal{ALCI}$, $\mathcal{ALCQI}$		PSPACE-complete

Observations:

- Two sources of complexity:
  - union ($\mathcal{U}$) of type NP,
  - existential quantification ($\mathcal{E}$) of type coNP.

  When they are combined, the complexity jumps to PSPACE.
- Number restrictions ($\mathcal{N}$) do not add to the complexity.

Ontology Reasoning

- Ontology Satisfiability: Verify whether an ontology $\mathcal{O}$ is satisfiable, i.e., whether $\mathcal{O}$ admits at least one model.
- Concept Instance Checking: Verify whether an individual c is an instance of a concept C in $\mathcal{O}$, i.e., whether $\mathcal{O} \models C(c)$.
- Role Instance Checking: Verify whether a pair (c_1, c_2) of individuals is an instance of a role R in $\mathcal{O}$, i.e., whether $\mathcal{O} \models R(c_1, c_2)$.
- Query Answering:

The certain answers to $q(\vec{x})$ over $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, denoted $cert(q, \mathcal{O})$, ...

are the tuples $\vec{c}$ of constants of $\mathcal{A}$ such that $\vec{c} \in q^{\mathcal{I}}$, for every model $\mathcal{I}$ of $\mathcal{O}$.

Example

TBOX:

$Researcher \sqsubseteq \neg Professor$
$Researcher \sqsubseteq \neg Lecturer$
$\exists TeachesTo^{-} \sqsubseteq Student$

$Student \sqcap \neg Undergrad \sqsubseteq GraduateStudent$
$\exists TeachesTo.Undergrad \sqsubseteq Professor \sqcup Lecturer$ whoever is teaching undergrad should be either professor or lecturer

TBOX Inferences: in copy

ABOX:

$TeachesTo(dupond, pierre)$
$\neg GraduateStudent(pierre)$
$\neg Professor(dupond)$

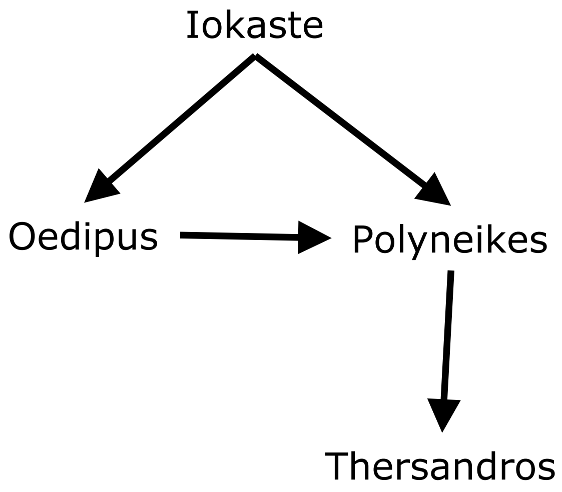
Ontology Inferences:

in copy

Open-World Assumption

Something evaluates false **only if** it contradicts other information in the ontology

hasSon(Iokaste,Oedipus)
hasSon(Iokaste,Polyneikes)
hasSon(Oedipus,Polyneikes)
hasSon(Polyneikes,Thersandros)
patricide(Oedipus)
¬patricide(Thersandros)



$\text{Query} \equiv \exists \text{hasSon}.(\text{patricide} \sqcap \exists \text{hasSon}.\neg \text{patricide})$

$\text{ABox} \models \text{Query}(\text{Iokaste})$?

Modeling with Description Logics

It is hard to build good ontologies with DL

- The names of the classes are irrelevant
- Classes are overlapping by default
- Domain and range definitions are axioms, not constraints
- Open world assumption
  - Anything might be true unless explicit asserted knowledge contradicts it (negation)
- Non-unique name assumption
  - Although families such as the DL-Lite family assume the unique name assumption

In this course, we aim at modeling usual data models and we will solely focus on modeling UML-like TBOXes (like the examples we have seen during this lecture)

Summary

Description Logics

- TBOX
  - Constructs
  - Formal Semantics
- ABOX
- Reasoning
  - Open-World Assumption