
Meeting Notes

THEORY407, Fall 2022

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2022-09-23

Humdrum Ch.1 It seems like a tool used for analyzing music similar to music21. With multiple functionalities, Humdrum can be used for counting specific notes, showing certain voice in a piece, and sometime even some specific analysis on harmonic/tonal functions. It is more like a library of tools on my idea of searching how similar two pieces are. We can conduct research with the same approach, but with different methods. Starting with these simple metrics:

- **Note Sequence:** How many notes are the same, how many are different?
 - Might be useful for comparing two pieces in the same work, such as *Gymnopédie*. But will yield a very low score if the key changed.
- **Note Sequence with Key:** How many notes are the same, given pieces in different key but represented in a \mathbb{Z}_7 format.
 - Can address some very basic issues such as exact transposition, but not effective if the phrase changes.
 - Can use the technique mentioned in the proposal to solve the above problem. But what if the phrase was changed a lot?

We then shall do the analysis with more and more complex methods such as counting leading tones, and chords using current available tools. In this way I believe we can come up with various ways to compare two pieces, but not being too impractical to do. The ultimate goal is also simple: output a "similarity score" for two pieces.

Will Read:

- Temperley, *A Bayesian Approach to Key Finding*
- Lerdahl, *Tonal Pitch Space* **Chapters ***
- Tymoczko, *Geometry of Musical Chords* **Chapters ***

2022-11-04

Test on modal music The Krumhansl-Schmuckler was tested on Bartok's *14 Bagatelles Op.47 No.1*. The algorithm recognized the right hand part as in E major/C# minor, but ignored the entire left hand in C Locrian/G Lydian. This is expected, since the algorithm only outputs one key given all notes in the piece and it seems like notes in the left hand sums up to a less duration than the right hand. So the KS algorithm put more weights in right hand than the left.

Link to a playable score: <https://musescore.com/user/4887176/scores/6403822>

Potential Solution We might do twice for the treble and bass clef separately (limited to piano), and then see if the two keys are the same. If they are, then we can say the piece is in one key.

Question How do we determine whether we should talk about keys separately?

Set-Class Similarity and Fourier Transform by Tymoczko Fourier transform assign two-dimensional vector whose components are:

$$V_{p,n} = (\cos(2\pi pn/12), \sin(2\pi pn/12))$$

Where for integer n from 0 to 6 and p in $\{0...11\}$ is the pitches in a chord. Each fourier component is the sum of all such component:

$$nth \text{ Fourier Component} = \sum_{p \in v} V_{p,n}$$

- Voice leading and set-class similarity. Steps to find the minimal Euclidean voice leading between two n -note multiset-classes A and B :
 - Choose a representative (prime form?) of A calculate the sum of its pitch classes.

- Find the n ($12/n$ semitones for each) transpositions of B with the same sum.
- For each of the transposition, calculate the L_2 norm of A and the vector. Do the same for inversions.
- Take the minimum of these $2n^2$ numbers and output the result.

- Fourier Magnitude

- In a set class space constructed by pitches of some perfectly even n -note chord, $n \in \{1...6\}$. Note the n -note chord means the chord even separate the 12 tone equal temperament pitches.
- Given a x -note set-class space, the n th Fourier component of a chord will decrease as pitches move away from the subset of pitches in n notes chord. Illustrated below:

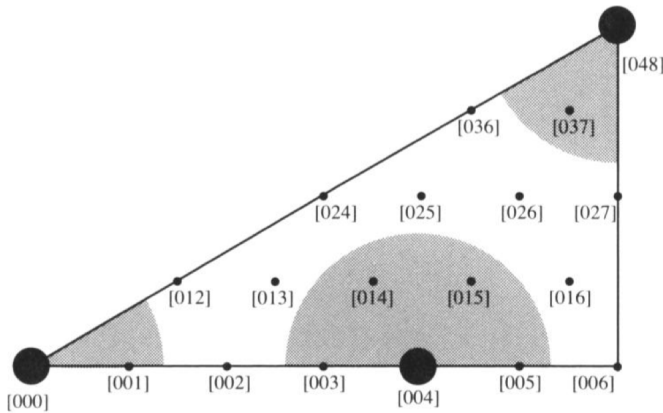


Figure 2. Set-classes in the shaded region will have a large third Fourier component, since they are near doubled subsets of $\{0, 4, 8\}$. Those in the unshaded region will have a smaller third Fourier component.

- We can use a linear equation to estimate the n -th Fourier component of chord given the *minimal voice leading* to the nearest doubled subset(VL). This means there is a close relationship between FC and the minimal Euclidean distance between two chords.
- 2 analysis procedure below:
- The analogy of minimal voice leading on a pitch class circle is: as the Fourier Component increases (which is the sum of all vectors), the minimal voice leading decreases.
- FC6 is the difference absolute value of difference between the number of a chord's notes in one whole tone scale and the number of its notes in the other ($[0, 2, 4, 6, 8, 10]$, $[1, 3, 5, 7, 9, 11]$).

a) Calculating the third Fourier component (FC_3) of $\{0, 2, 5\}$.

Step 1: assign the vector $(\cos 2\pi pn/12, \sin 2\pi pn/12)$ to each pitch class p .

$$0 \rightarrow (\cos(2\pi(0 \times 3)/12), \sin(2\pi(0 \times 3)/12)) = (1, 0)$$

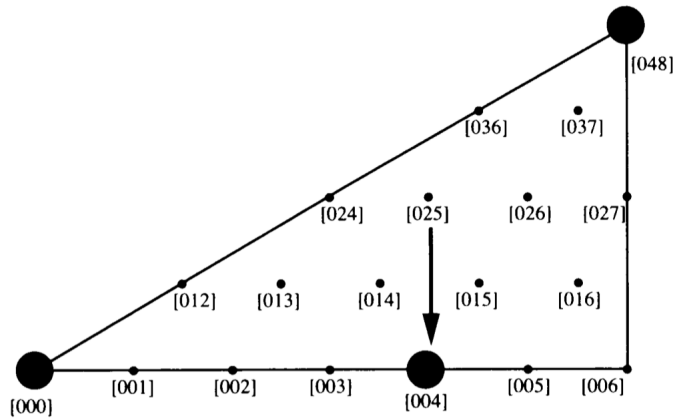
$$2 \rightarrow (\cos(2\pi(2 \times 3)/12), \sin(2\pi(2 \times 3)/12)) = (-1, 0)$$

$$5 \rightarrow (\cos(2\pi(5 \times 3)/12), \sin(2\pi(5 \times 3)/12)) = (0, 1)$$

Step 2: add these vectors: $(1, 0) + (-1, 0) + (0, 1) = (0, 1)$

Step 3: determine the length of this sum: $\|(0, 1)\| = \sqrt{0^2 + 1^2} = 1$

b) Determining the distance to the nearest doubled subset of $\{0, 4, 8\}$.



Following the procedure in Section I, we learn that $(0, 2, 5) \rightarrow (1, 1, 5)$ is the minimal voice leading, moving two voices by one semitone each, and with a Euclidean size of $\sqrt{1^2 + 1^2} \cong 1.41$

Reflection and Questions

1. Both Temperley and Tymoczko are trying to find the similarity in terms of "fitness" to a certain pitch set class. In Temperley's case, it is the fitness to major and minor scale, while in Tymoczko's case, it is the fitness to a certain evenly distributed $12/n$ -notes chord.
2. What is the purpose of doing this? For Tymoczko, maybe it is demonstrating certain piece confirms such minimal voice leading motion?
<http://dmitri.mycpanel.princeton.edu/ChordGeometries.html>
3. So it seems like the notion of "motion" (no puns intended) here is important, how should we show such motion and make analysis upon it?

A proposed struture to show such motions This is only a speculative model to show such motions in terms of intervals through time. Fix certain time untis and a pitch class space (k -euqal temperament) we can represent a piece of music as intervals between notes in the same voice through time. For each voice, if the piece alst for m time units, there would be a 1D array of size $m - 1$ where the each entry of the array is the interval between the pitches n_i, n_{i+1} on time $i, i + 1$. We can also counts the intervals between all voices on the same time (Lewin's IFUNC) and utilize his Fourier test for further analysis. Given a proper time unit, we can also represent a "segment" of notes by finding the

closest "evenly distributed pitch class set"(Tymoczko's) and then do analysis in terms of these summarized segments in correspondence with the notion of "notes, phrase, section, passage".

Questions

1. How do we determine a proper time unit?
2. What kind of analysis shall we use for the horizontal interval vectors?
3. This is most suitable for piano/counterpoint, what about other instruments?

Will Read:

- Tymoczko's Geometry of Chord Spaces
- Issacson's The Interval Angle
- Yust's Set Theory

2022-11-27

Yust's Set Theory Introduced Fourier componenets and phase spaces.

- **n Phase Space:** A circle evenly divided by n interval(in the context of 12-equal temperament).
- **Fourier Component:** See summarization for Tymoczko's paper.

The process is similar to DFT, which can be seen as a matrix multiplication written as:

$$F_k = \frac{1}{n} \sum_{j=0}^{n-1} e^{-2\pi i \frac{jk}{n}}$$

Where F_n is the n -th Fourier component, k is the pitch class, and $n = 12$. Or, more generally: $\hat{f}_k = [[\text{DFT}]] \cdot f_k$. Think of the process as matrix multiplication. Different phase space reveals different properties of music. Ph_5 indicates the a set's affinity toward diatonic collection. Each 6 phase space approximates:

1. chromaticism
2. quartal harmony?(not sure what this means)

3. hexatonicity
4. octatonicity
5. diatonicity
6. whole-tone quality.

Scale Theory? Evenness? Might need more explanation. We can also use DFT to calculate the common tones of two sets.

$$\frac{1}{12} \sum_{n=0}^{11} |f_n(A)| |f_n(B)| \cos(\phi_n(A) - \phi_n(B))$$

.Intuitively, common tones will show harmonic closeness. Another aspect in the common tone analysis is the "wormhole" effect. Which is simply apply analysis in two phase spaces.

Questions

- What can I do for DFT in the interval space?
- Since DFT can be seen as a matrix operation, what does the eigen value of a DFT matrix show?
- Structure for my writing toward the end of calss?

Will Read: Isaacson's The Interval Angle