

Portfolio Theory Tutorial 5



- 1 (a) Consider a portfolio consisting of two independent assets, A and B , with the following distribution

<i>Prob</i>	<i>A</i>	<i>B</i>
1/3	45	30
1/3	60	60
1/3	75	90

Short sales are disallowed. Find the opportunity set formed by these two assets and the efficient frontier.

Hint: We know that $E(A) = E(B) = 60$, any portfolio formed from these 2 assets must have the same mean of 60. Now consider the range of possible values of the standard deviation of such portfolios.

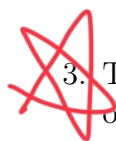
- (b) Now consider a risk free asset that has guaranteed proceeds of 50. Find the efficient frontier formed by the assets A , B and the risk free asset. (Short sales and risk-free borrowing are disallowed).
- (c) Describe how your answers in part (a) and in part (b) would change if short sales and risk-free borrowing are both allowed.
- (d) Comment on your results.
2. Prove that the efficient frontier is always a concave function of σ . (Recall that a function $h(x)$ is concave if for any $0 \leq c \leq 1$, and any 2 points y and y , $h(cx + (1 - c)y) \geq ch(x) + (1 - c)h(y)$.)

Hint: Consider 2 portfolios A and B on the efficient frontier, with expected returns and standard deviation μ_A , σ_A , μ_B , σ_B respectively. Show that for any $0 \leq \pi_A \leq 1$, the portfolio on the efficient frontier with expected return

$$\pi_A \mu_A + (1 - \pi_A) \mu_B$$

has standard deviation no greater than

$$\pi_A \sigma_A + (1 - \pi_A) \sigma_B.$$



3. There are three assets in the market, two risky assets A and B , and one risk free asset F . The expected return and variance of the return of these three assets are given below:

Return	Mean(%)	Variance(%%)
A	5	100
B	10	900
F	3	0

The correlation coefficient between A and B is 0.5. Consider a portfolio P formed by assets A and B only. Let π be the proportion of capital invested in asset A .

- (a) Find the expected return, r , of the portfolio P in terms of π .
- (b) Find the standard deviation of the return, σ , of the portfolio P in terms of π .
- (c) Use the results in (a) and (b), find σ in terms of r .
- (d) Let C be the efficient portfolio formed by assets A and B . By the result in part (c), calculate the slope of efficient frontier (the line passing through F and C) in terms of r . Hence, find the expected return and the composition of portfolio C .

4. Suppose there are n uncorrelated assets available in the market. The expected values r of their returns are the same but the variances are different. The return of asset i has a variance of σ_i for $i = 1, \dots, n$. Identify the composition of the minimum variance portfolio and its corresponding variance.