F71PT/F71AH Portfolio Theory/Financial Economics 1
Tutorial 6

For Questions 5 and 6, you will need to use a suitable computer package to invert a large matrix.

1. Practice with Lagrangians. Part (b) gives you practice in solving a simple problem using Lagrangians. Part (a) gives you an alternative way to solve the problem so that you can check that at least in this case you do get the same optimal value.

   Suppose that $f(x) = f(x_1, x_2) = x_1^2 + 2x_2^2$.

   You wish to minimise $f(x)$ subject to $x_1 + x_2 = 1$.

   (a) The constraint implies that $x_2 = 1 - x_2$. Substitute this into $f(x)$ to minimise over x_1 and show that the optimal solution is $x = (2/3, 1/3)$.

   (b) Write down the Lagrangian function $L(x, \lambda)$ for this constrained optimisation problem.

   Minimise this function over (x, λ) and show that the optimal value of x is $x = (2/3, 1/3)$.

2. An investment market consists of portfolios constructed from three basic assets, A, B and C. The means, standard deviations and covariances between the returns on these assets are as shown in the following table:

Asset X	$\mathrm{E}[R_X]$ %	σ_X %	Covariance %%
A	10	14	$\sigma_{AB} = 25$
B	15	8	$\sigma_{AC} = 121$
C	20	25	$\sigma_{BC} = 49$

For any portfolio, P, we write:

$$P = \pi_A A + \pi_B B + \pi_C C, \quad \pi_A + \pi_B + \pi_C = 1$$

Consider the problem $(*)$, where:

$$(*) = \begin{cases} \text{minimise} & \sigma_P^2 \\ \text{subject to} & \pi_A + \pi_B + \pi_C = 1 \\ \text{and} & 10\pi_A + 15\pi_B + 20\pi_C = E_p \end{cases}$$

for a given constant E_p.
You are given that $P_1^* = (1.427,\ 0.147, -0.574)$ and $P_2^* = (0.742,\ 0.517, -0.259)$ are solutions to the problem $(*)$ for different values of E_P.

   (a) Calculate the expected returns on the portfolios P_1^* and P_2^*.

(b) Use the Two Fund Theorem to find the solution to the problem $(*)$ for $E_P = 8\%$.

(c) Use the Two Fund Theorem to find the solution to the problem $(**)$, where:

$$(**) = \begin{cases} \text{minimise} & \sigma_P^2 \\ \text{subject to} & \pi_A + \pi_B + \pi_C = 1 \\ \text{and} & 10\pi_A + 15\pi_B + 20\pi_C = 15 \\ \text{and} & 0 \le \pi_A, \pi_B, \pi_C \le 1 \end{cases}$$

that is, with no short selling allowed.
(*Hint:* First solve the problem (*) with $E_p = 15$.)

3. Suppose assets A, B and C have expected return 4%, 5% and 6%. The covariance matrix of the return of these three risky assets is

$$\begin{pmatrix} 4 & 3 & 3 \\ 3 & 9 & 3 \\ 3 & 3 & 4 \end{pmatrix}.$$

Find the composition of the minimum variance portfolio and its expected return.

4. There are three risky assets A, B and C in the market. Their expected value and covariance matrix of their returns are

$$\mu = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \qquad \text{V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

(a) Find the composition of minimum variance portfolio. Calculate the variance of the return of this portfolio.

(b) Find the composition of the efficient portfolio with 6% expected return. Calculate the variance of the return of this portfolio.

5. Consider four assets with expected rates of return $\mu_1 = 6\%$, $\mu_2 = 7\%$, $\mu_3 = 8\%$ and $\mu_4 = 10\%$, and variance-covariance matrix (units are %%)

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$$\begin{pmatrix} 10 & 8 & 0 & 15 \\ 8 & 15 & 0 & 16 \\ 0 & 0 & 40 & 0 \\ 15 & 16 & 0 & 35 \end{pmatrix}$$

(a) An investor wants to determine the minimum variance portfolio for a given expected rate of return E_P. Short selling is allowed. She expresses this problem in matrix notation as $Ay = b$. Write down the matrices A, y and b.

(b) Calculate A^{-1}.

(c) Let $\pi_1(E_P)$ denote the proportion of wealth invested in asset 1 to achieve the minimum variance for a given expected return, E_P. Show that $\pi_1(E_P)$ is a linear function of E_P.

6. Two risky assets, A and B, have expected rates of return 6% and 9%. The variances of their returns are 4%% and 10%%, respectively, and the covariance between them is 3%%. A risk-free asset has a rate of return of 4%. The investor can short sell both the risky assets and the risk-free asset. In what ratio will the investor hold assets A and B in an efficient portfolio?
(*Hint:* Use the Separation Theorem. It does not matter which rate of return we use for the portfolio (almost!)).