Portfolio Theory
Tutorial 1

1. An investor has current wealth 1, all of which must be invested in one of two assets. Asset A has a return that is uniformly distributed on [1.02, 1.08]. The return from asset B will be 1.03, 1.04, 1.06 or 1.07, each with probability $\frac{1}{4}$.

   The investor has the quadratic utility function

   $$u(x) = x - \alpha x^2,$$

   where $0 < \alpha < 0.4629$. Determine which asset the investor will prefer.

2. An agent's utility function is

   $$u(x) = 2\sqrt{x}.$$

   (a) Calculate the certainty equivalent of the following uncertain increase in wealth:

   £2000 with probability 0.5 and £0 with probability 0.5, assuming:

   (i) The agent's current wealth is £0.

   (ii) The agent's current wealth is £1,000.

   (b) Comment on your answers to parts (i) and (ii).

3. A risk averse person has an exponential utility function

   $$u(x) = 1 - \exp(-\alpha x).$$

   Three random payoffs are available for her to choose from. It is found that she prefers A to B and prefers B to C. Find the possible range of the value of α.

A	$Prob$	B	$Prob$	C	$Prob$
5	1/4	7	1	5	1/3
9	3/4			9	2/3

4. A homeowner is considering buying buildings insurance. His current total wealth is £200,000. Over one year there is a 1% chance of his house suffering £100,000 of damage, and a 5% chance of suffering £10,000 of damage.

(a) A large insurer sells many thousands of identical policies to homeowners like the one above. What premium should they charge per policy each year if they expect zero profit?

(b) The homeowner has a log utility function. What is the maximum price he is willing to pay for this insurance?

(c) Comment on the difference between your answers in parts (a) and (b).

5. Hyperbolic absolute risk aversion (HARA) utility function is defined as

$$u(x) = \frac{1-\gamma}{\gamma}\left(\frac{ax}{1-\gamma} + b\right)^{\gamma}, \qquad a, b, x > 0.$$

(a) Show that the absolute risk aversion function is of form $1/(cx+d)$. Hence, find c and d in terms of a, b and γ.

(b) By considering $\gamma \to 1$, show that risk neutral (linear) utility function is a special case of HARA utility function.