Portfolio Theory
Tutorial 2 – 2022

1. The rate of return on investments A and B can be 2%, 4%, 5%, 6%, 8% or 9%. The probability of the return taking each of these values is shown in the table below:

Return	Probability for A	Probability for B
2%	0.25	0
4%	0.25	0.5
5%	0	0.25
6%	0.25	0
8%	0.25	0
9%	0	0.25

Let $F_A(x)$ and $F_B(x)$ denote the distribution functions of the rates of return on investments A and B, respectively.

(a) By drawing graphs of $F_A(x)$ and $F_B(x)$, or otherwise, determine whether one of the investments has first order stochastic dominance over the other, and, if so, which one.

(b) By calculating;

$$\int_{-\infty}^{6\%} F_A(y)\,dy \quad \text{and} \quad \int_{-\infty}^{6\%} F_B(y)\,dy$$

or otherwise, determine whether one of the investments has second order stochastic dominance over the other, and, if so, which one.

2. Investors can choose between four portfolios, A, B, C and D. The returns from these portfolios depend on which of four scenarios occurs; the relevant information is shown in the following table.

Scenario	*Prob*	A	B	C	D
1	$\frac{1}{4}$	80	40	50	90
2	$\frac{1}{4}$	100	80	60	90
3	$\frac{1}{4}$	120	120	130	140
4	$\frac{1}{4}$	140	200	140	140

(a) Calculate the expected return from each of these portfolios.

(b) For each of the portfolios determine which of the other portfolios it dominates in the sense of:

   (i) Absolute dominance.

   (ii) First order stochastic dominance.

   (iii) Second order stochastic dominance.

3. Consider two investment portfolios A and B. Each has a return which is normally distributed with variance σ^2. The expected return from A is μ_A, and the expected return from B is μ_B. Assume that $\mu_A > \mu_B$. Show that portfolio A has first order stochastic dominance over B.

4. Consider two investment portfolios A and B. The return from each portfolio is normally distributed with expected value μ. The variance of the return from A is σ_A^2; the variance of the return from B is σ_B^2. Assume that $\sigma_A > \sigma_B$.

   (a) Show that neither of the two investments has first order stochastic dominance over the other one.

   (b) Show that B has second order stochastic dominance over A.
   Hint: For $x > \mu$ show that:

$$\int_{-\infty}^{x} (F_A(y) - F_B(y))\, dy > \int_{-\infty}^{\infty} (F_A(y) - F_B(y))\, dy = 0$$