Portfolio Theory
Tutorial 8

1. The expected return on a unit invested in a risky security, A, is 1.13 and the variance of this return 0.01. The return on a unit invested in the risk-free asset is 1.04.

   (a) Given that A is an efficient portfolio derive the equation for the capital market line.

   (b) The expected return on a unit invested in the market, M, is 1.09. Using the result in part (a), calculate the variance of the return on the market.

2. In a market that satisfies the assumptions of CAPM, units invested in assets A and B have expected returns of 1.07 and 1.10, respectively. Assets A and B have β values 0.8 and 1.2, respectively. Use the security market line to calculate the risk-free rate and the expected return on the market portfolio.

3. Suppose there are only three assets, A, B and C, in a market. Their expected returns are known to be 10%, 15% and 10%, respectively.

   Portfolios X and Y are known to lie on the minimum variance frontier (but they are not necessarily efficient) and their compositions are:

   $$X = 0.6A + 0.2B + 0.2C, \qquad Y = 0.8A - 0.2B + 0.4C.$$

   [* The minimum variance frontier is defined by portfolios that achieve the minimum variance for given level of expected return. This includes the lower side of the opportunity set as well as the upper, north-west facing side (the efficient frontier).]

   Note that the market portfolio is efficient.

   (a) Using the above information, find the minimum and maximum possible expected return of the market portfolio.

   (b) Suppose X is the (global) minimum variance portfolio. Does this change the answers to part (a)?

4. Consider a market in which the assumptions of the CAPM hold.

Suppose there are n mutually independent assets. The return on asset i has variance σ_i^2. The market capitalization of asset i is W_i. Therefore, the total market capitalization $M = \sum_{i=1}^{n} W_i$ and let $w_i = W_i/M$ for $i = 1, 2, \ldots, n$. Hence the market portfolio is $(w_1, w_2, \ldots, w_n)$.

(a) Find the expression for β_j in terms of the $w_i's$ and $\sigma_i's$.

(b) With the result in part (a), verify that $\beta_M = 1$.

5. A long term investor can choose one of two strategies (1 and 2).

If she follows strategy i then the total return in year t is given by $R(i,t)$ per unit invested at the start of the period.

For each i, the $R(i,t)$ for $t = 1, 2, \ldots$ are assumed to be i.i.d..

Under Strategy 1: $R(1,t)$ can take each of the values 0.9, 1.1 and 1.3 with equal probability.

Under Strategy 2: $R(2,t)$ can take each of the values 0.85, 1.05, 1.25 and 1.45 with equal probability.

(a) Write down a formula for the total return up to time T, $X(T)$, of an initial investment of 1 at time 0.

(b) State the Kelly Criterion and identify which of strategies 1 and 2 should be selected on the basis of the Kelly Criterion.

(c) An investor has a power utility $u(X(T)) = \gamma^{-1} X(T)^{\gamma}$ for a fixed time horizon T and will use the expected utility criterion to choose between strategies 1 and 2.

Show that the optimal decision does not depend on T.

Show numerically that Strategy 2 is optimal when $\gamma = 0.5$ and Strategy 1 is optimal when $\gamma = -5$.

Why is it intuitively reasonable that the choice should switch from Strategy 2 to Strategy 1 as γ becomes more and more negative?