



$$\frac{\partial \mu}{\partial x_i} = \frac{1}{N}$$

$$\begin{aligned} \frac{\partial v}{\partial x_i} &= \frac{2}{N} \sum_{k=1}^N (x_k - \mu) \left(\delta_{ik} - \frac{1}{N} \right) \\ &= \frac{2}{N} (x_i - \mu) - \frac{2}{N^2} \sum_{k=1}^N (x_k - \mu) \\ &= \frac{2}{N} (x_i - \mu) \end{aligned}$$

$N\mu - N\mu = 0$

μ is a function
that takes x_i
as input

$$\begin{aligned} \frac{\partial \sigma}{\partial x_i} &= \frac{\partial \sigma}{\partial v} \cdot \frac{\partial v}{\partial x_i} \\ &= \frac{1}{2\sqrt{v+e}} \cdot \frac{2}{N} (x_i - \mu) \\ &= \frac{x_i - \mu}{N\sigma} \end{aligned}$$

$$\begin{aligned} \frac{\partial Y_j}{\partial x_i} &= \frac{1}{\sigma^2} \cdot \left\{ \frac{\partial (x_j - \mu)}{\partial x_i} \cdot \sigma - (x_j - \mu) \frac{\partial \sigma}{\partial x_i} \right\} \\ &= \frac{1}{\sigma^2} \left\{ \left(\delta_{ij} - \frac{1}{N} \right) \cdot \sigma - \frac{1}{N\sigma} \cdot (x_j - \mu)(x_i - \mu) \right\} \\ &= \frac{1}{\sigma} \left\{ \delta_{ij} - \frac{1}{N} - \frac{1}{N} \cdot \frac{(x_j - \mu)}{\sigma} \cdot \frac{(x_i - \mu)}{\sigma} \right\} \\ &= \frac{1}{\sigma} \left\{ \delta_{ij} - \frac{1}{N} (1 + y_i y_j) \right\} \end{aligned}$$

$$\frac{\partial L}{\partial Y_j} = \gamma \cdot \frac{\partial L}{\partial z_j}$$

$$\frac{\partial L}{\partial x_i} = \sum_{j=1}^N \frac{\partial L}{\partial Y_j} \cdot \frac{\partial Y_j}{\partial x_i}$$

$$= \frac{\gamma}{\sigma} \sum_{j=1}^N \frac{\partial L}{\partial z_j} \cdot \left\{ \delta_{ij} - \frac{1}{N} (1 + y_i y_j) \right\}$$

$$= \frac{\gamma}{\sigma} \left\{ \frac{\partial L}{\partial z_i} - \frac{1}{N} \sum_{j=1}^N (1 + y_i y_j) \frac{\partial L}{\partial z_j} \right\} = \frac{\gamma}{\sigma} \left(\frac{\partial L}{\partial z_i} - \frac{1}{N} \sum_{j=1}^N \frac{\partial L}{\partial z_j} - \frac{y_i}{N} \sum_{j=1}^N y_j \frac{\partial L}{\partial z_j} \right)$$

$\frac{dL}{dz}$