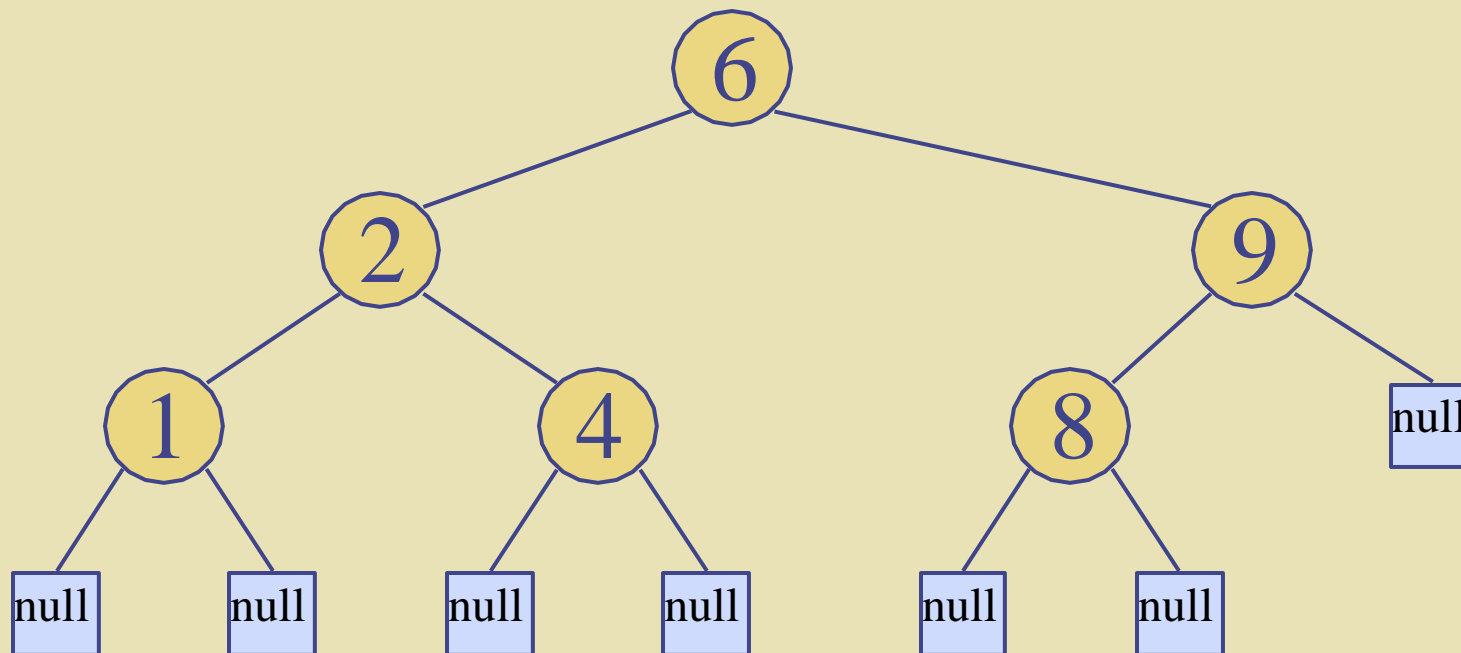# Binary Search Tree

Notes by Yan Yan

Contents

1. Binary Search Tree
2. Binary Search Tree Operations

Binary Search Tree

BST

◆ Nodes are arranged in an order.

◆ Assume no duplicates values allowed, then
  – Nodes in the left sub-tree have a value less than (<) the value of the root node.
  – Nodes in the right sub-tree have a value greater than (>) the value of root node.

Binary Search Tree -- Search

◆ To search for value (key = k), we trace a downward path starting at the root

◆ The next node visited depends on the outcome of the comparison of k with the key of the current node

◆ If we reach a leaf, the key is not found, we return NO_SUCH_KEY

```
BSTSearch(tree, key)
  cur = tree⋯→root
  while (cur is not null) do
    if (key == cur⋯→key) then
      return cur  // Found
    else if (key < cur⋯→key) then
      cur = cur⋯→left
    else
      cur = cur⋯→right
  end while
  return null  // Not found
```

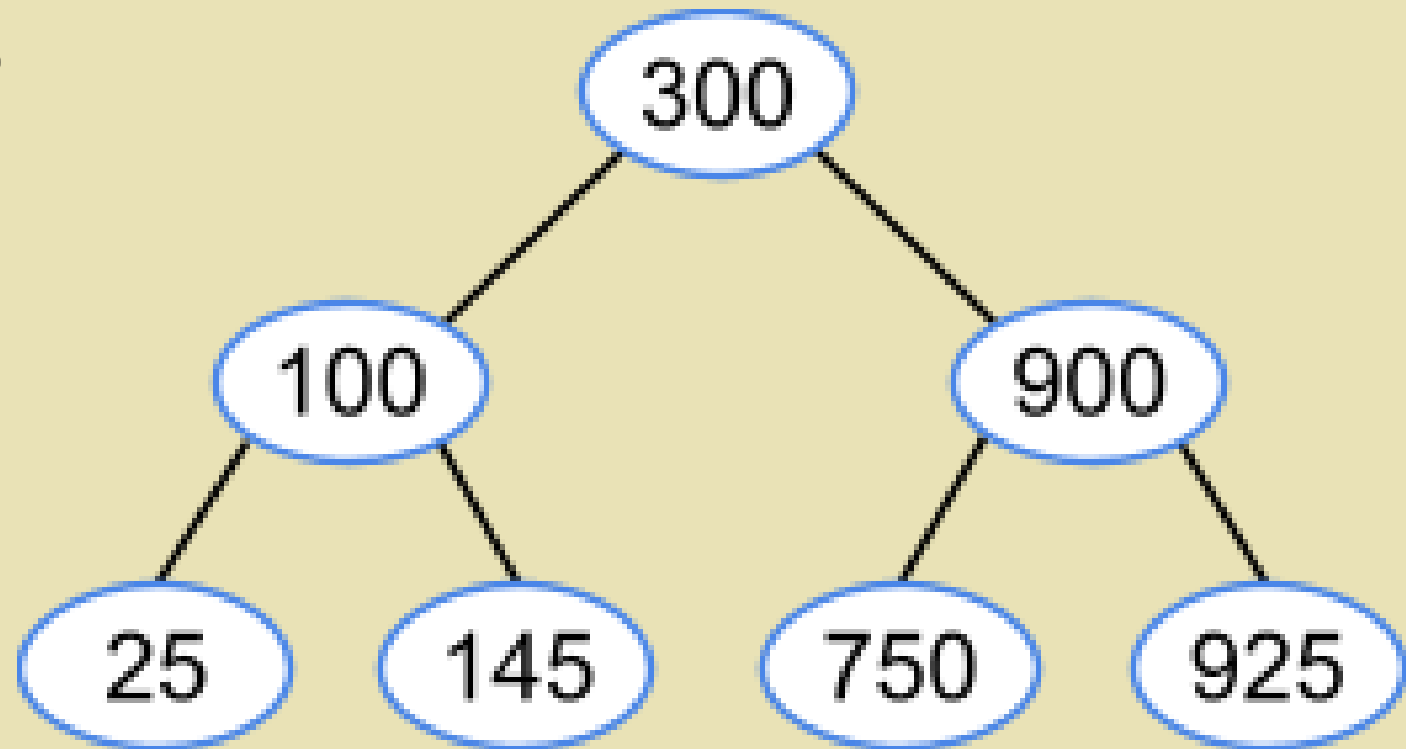# Binary Search Tree -- Search

◆ Example: search k=4 in this tree. BSTSearch(tree, 4)
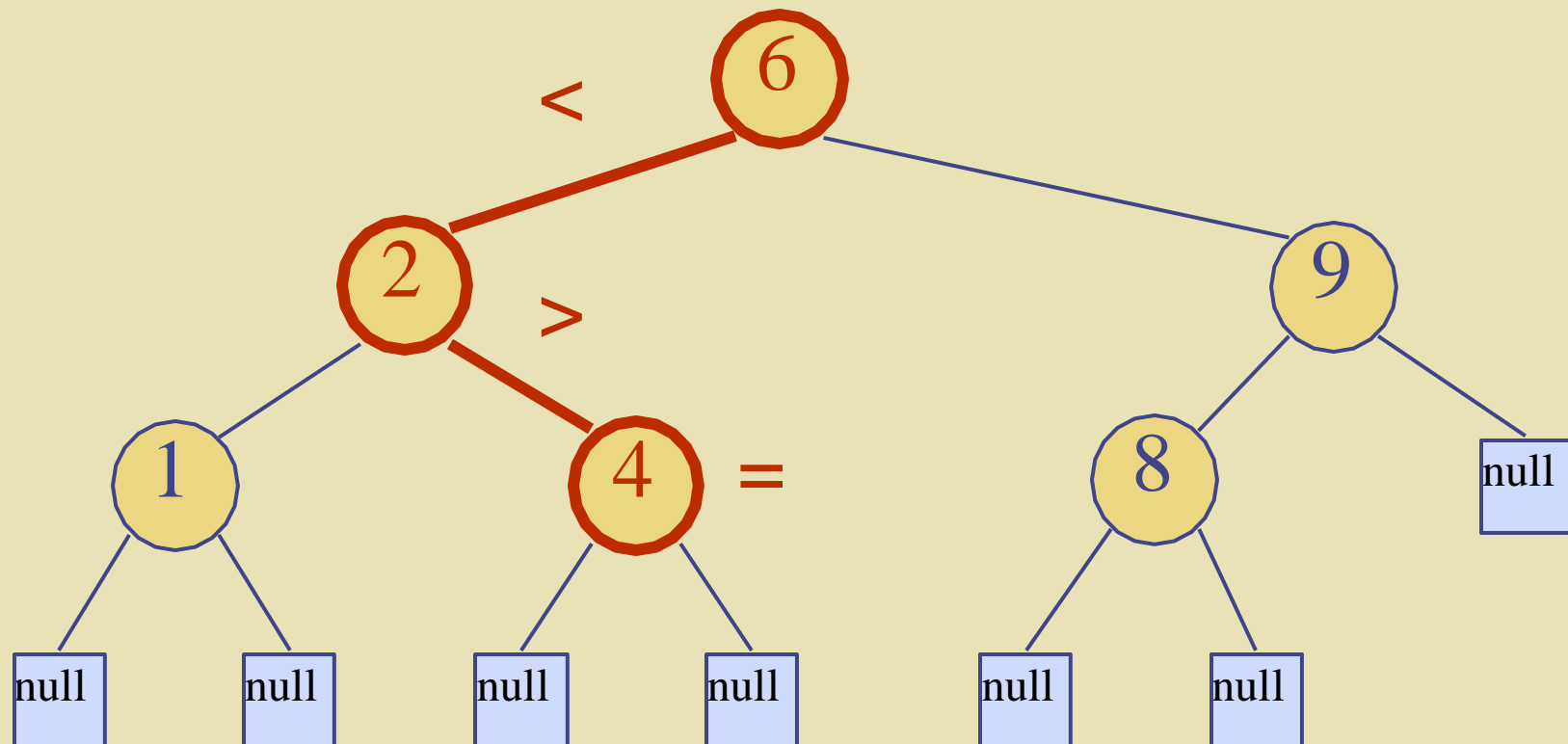
Binary Search Tree -- Search

◆ Exercise: What are the orders of nodes visited, if you search for

    A. 145

    B. 900

Binary Search Tree -- Insertion

- To perform insertion, we search for key k, and find a suitable location to insert as the leaf.

- Special case: A node inserted into an empty tree will become the tree's root.

```
BSTInsert(tree, node)
   if (tree⋯⇢root is null) then
      tree⋯⇢root = node
   else
      currentNode = tree⋯⇢root
      while (currentNode is not null) do
         if (node⋯⇢key < currentNode⋯⇢key) then
            if (currentNode⋯⇢left is null) then
               currentNode⋯⇢left = node
               currentNode = null
            else
               currentNode = currentNode⋯⇢left
            end if
         else
            if (currentNode⋯⇢right is null) then
               currentNode⋯⇢right = node
               currentNode = null
            else
               currentNode = currentNode⋯⇢right
            end if
         end if
      end while
   end if
```
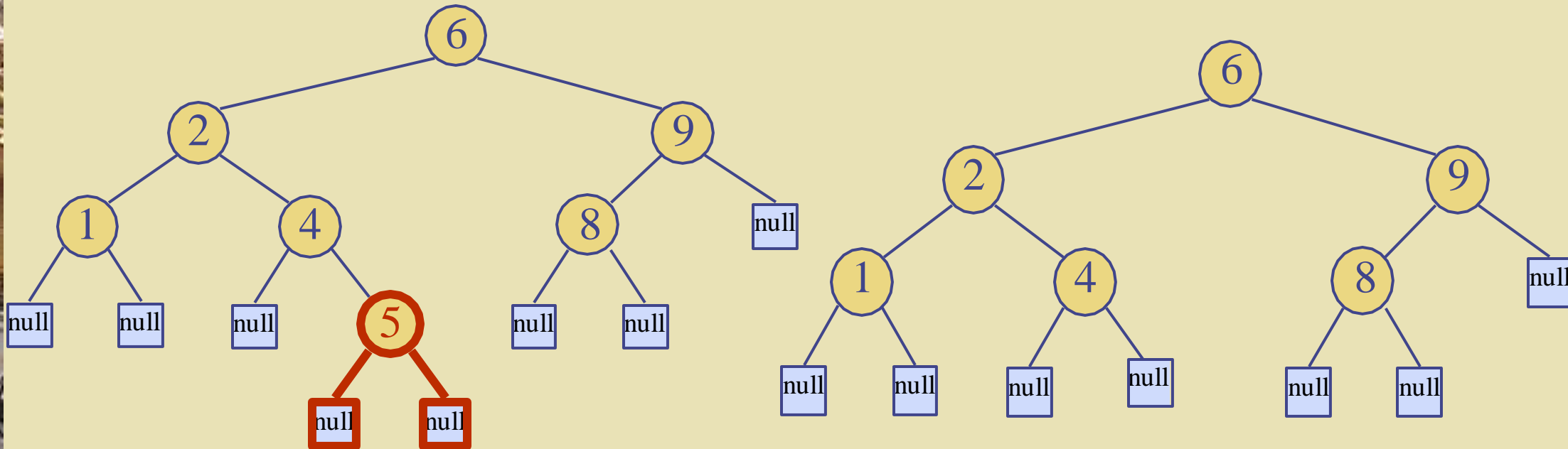
Binary Search Tree -- Insertion

◆ Example: insert 5 in this tree. BSTInsert(tree, 5)
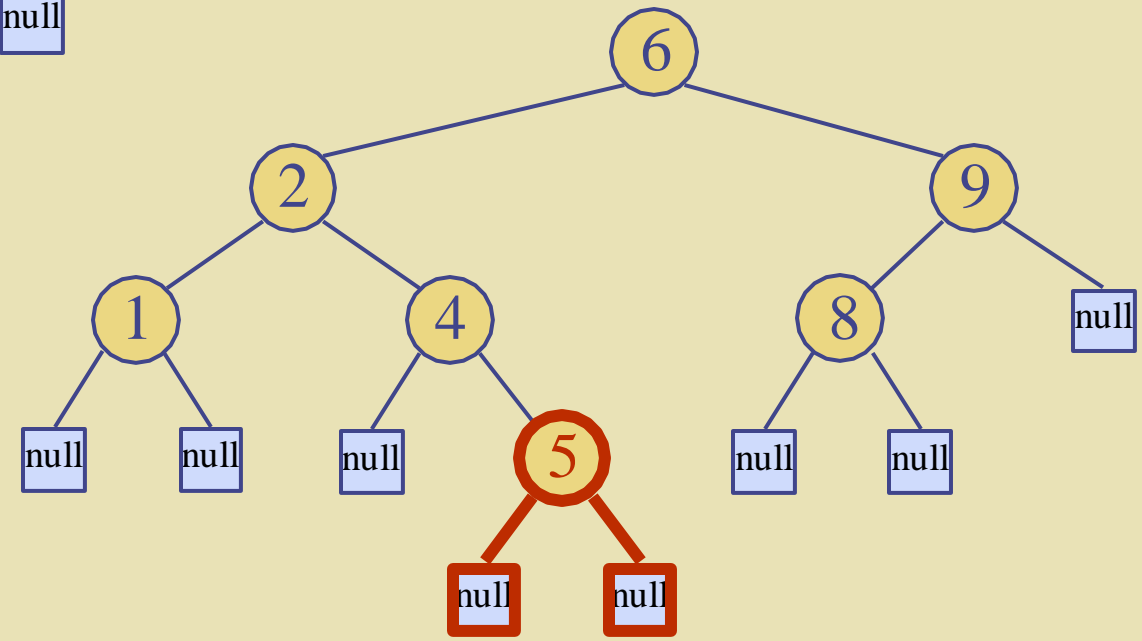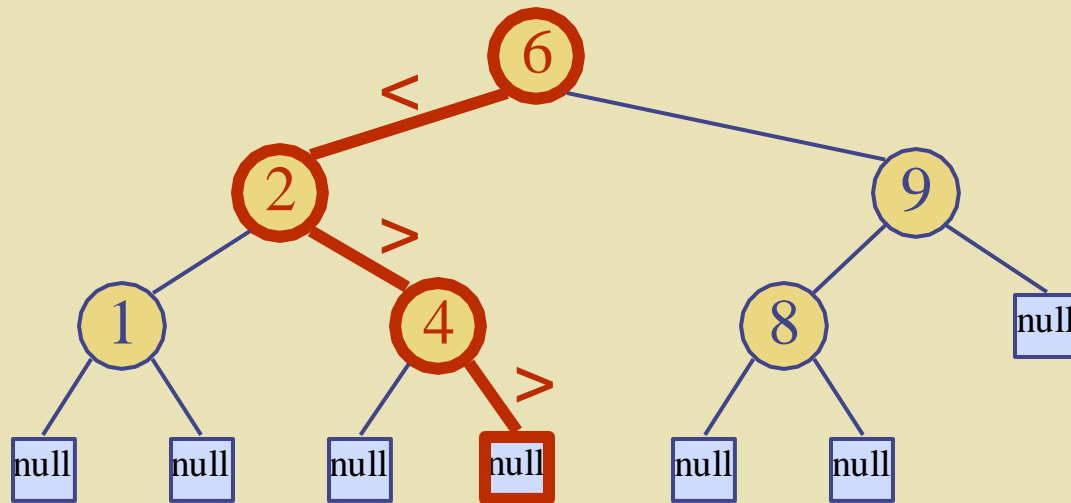
Binary Search Tree -- Removal

◆ The algorithm first searches for a matching node just like the search algorithm. If found (call this node X), the algorithm performs one of the following sub-algorithms:

1. Remove a leaf node: If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with null. Else, if X was the root, the root pointer is assigned with null, and the BST is now empty.

Binary Search Tree -- Removal

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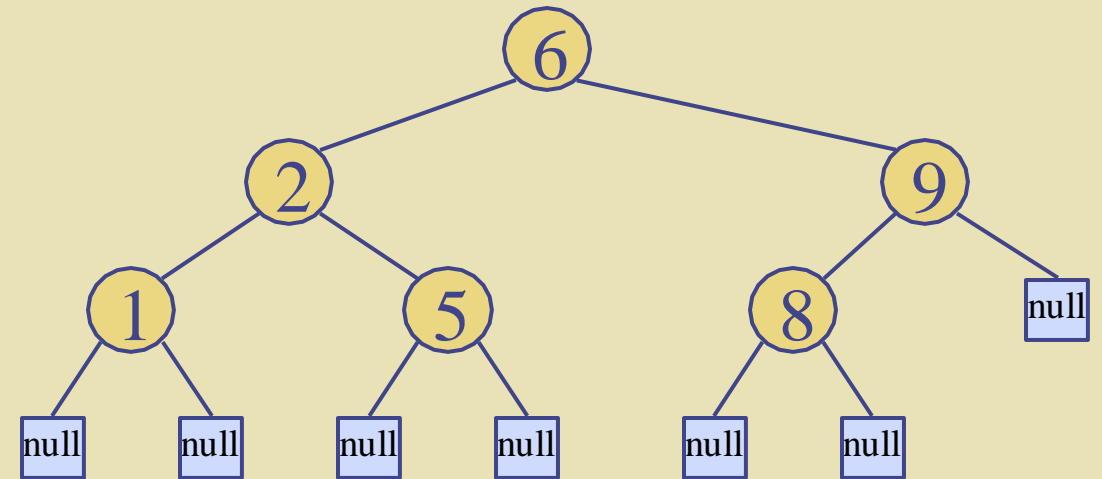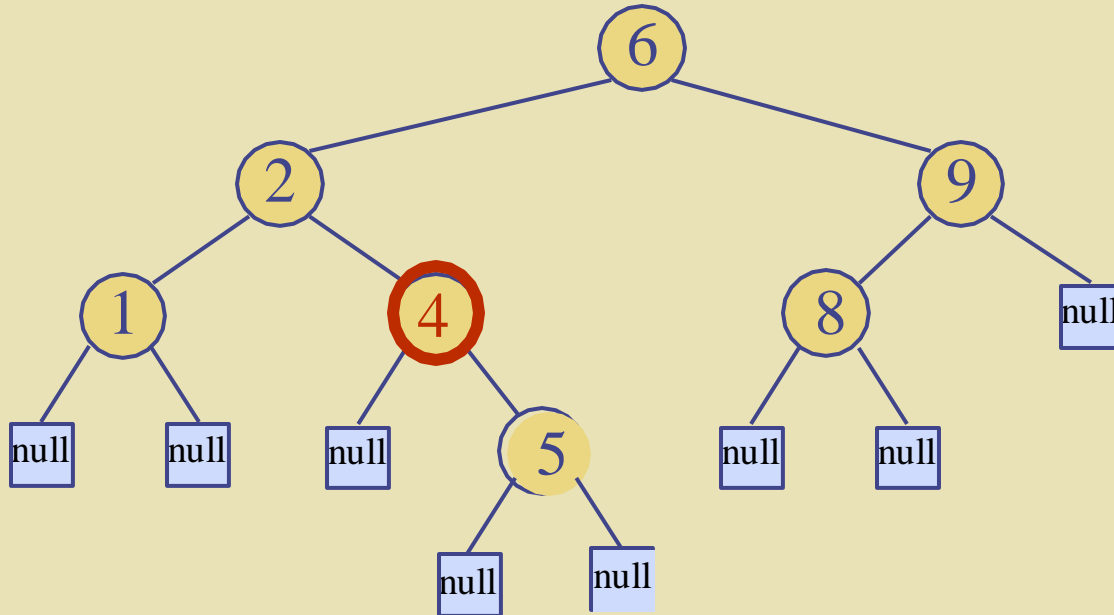
2. Remove an internal node with single child: If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with X's single child. Else, if X was the root, the root pointer is assigned with X's single child.

Binary Search Tree -- Removal

- Remove an internal node with single child: If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with X's single child. Else, if X was the root, the root pointer is assigned with X's single child.

Binary Search Tree -- Removal

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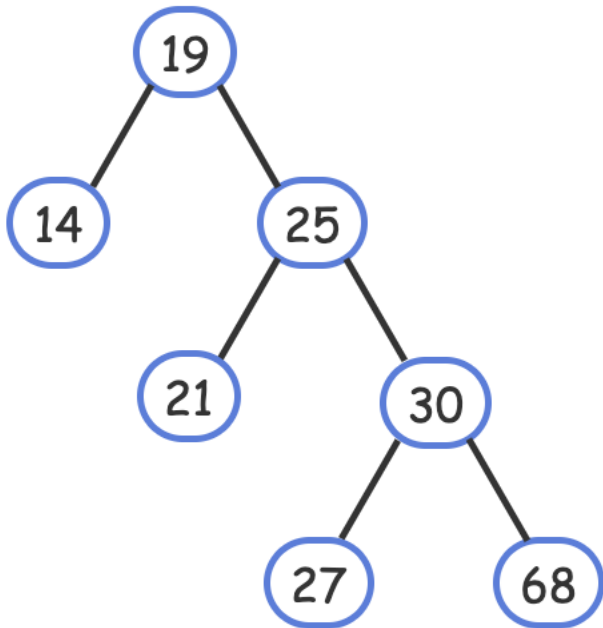
3. Remove an internal node with two children: This case is the hardest. First, the algorithm locates X's successor (the leftmost child of X's right subtree), and copies the successor to X. Then, the algorithm recursively removes the successor from the right subtree.

Binary Search Tree -- Removal

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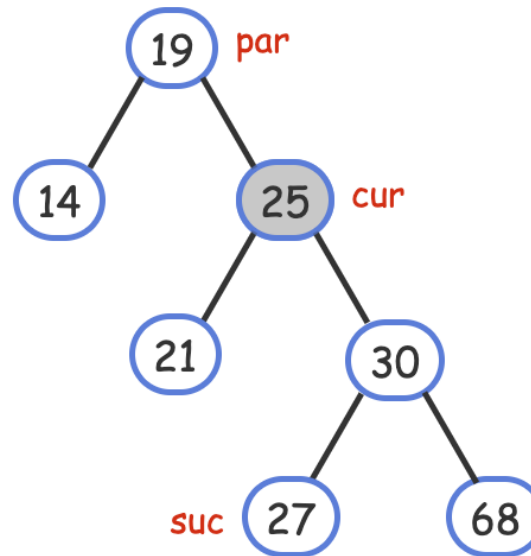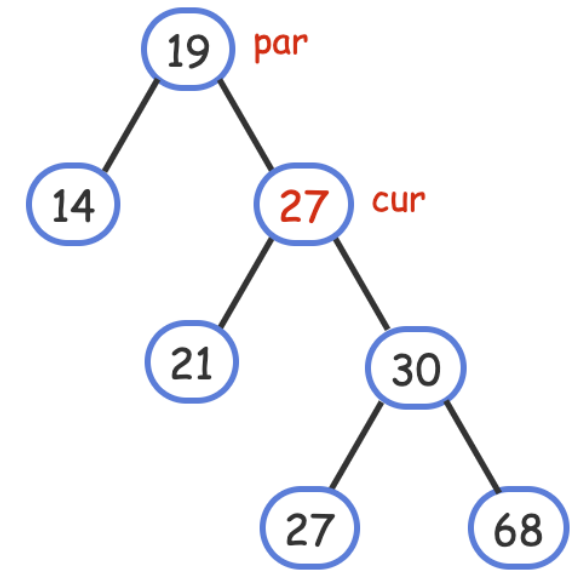
◆ Remove 25

BSTRemove(tree, 25)

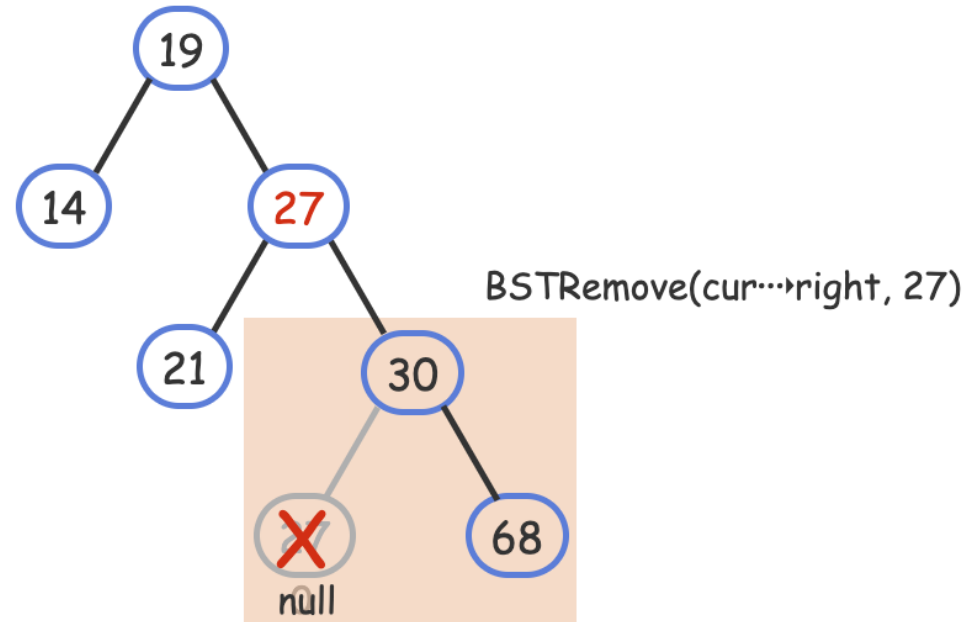Remove internal node with two children

BSTRemove(tree, 25)

Remove internal node with two children

Binary Search Tree -- Removal

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♦ Remove 25

BSTRemove(tree, 25)

BSTRemove(cur···right, 27)

Remove internal node
with two children

Binary Search Tree -- Removal

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– Remove a leaf node: If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with null. Else, if X was the root, the root pointer is assigned with null, and the BST is now empty.

– Remove an internal node with single child: If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with X's single child. Else, if X was the root, the root pointer is assigned with X's single child.

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That's about this lecture!