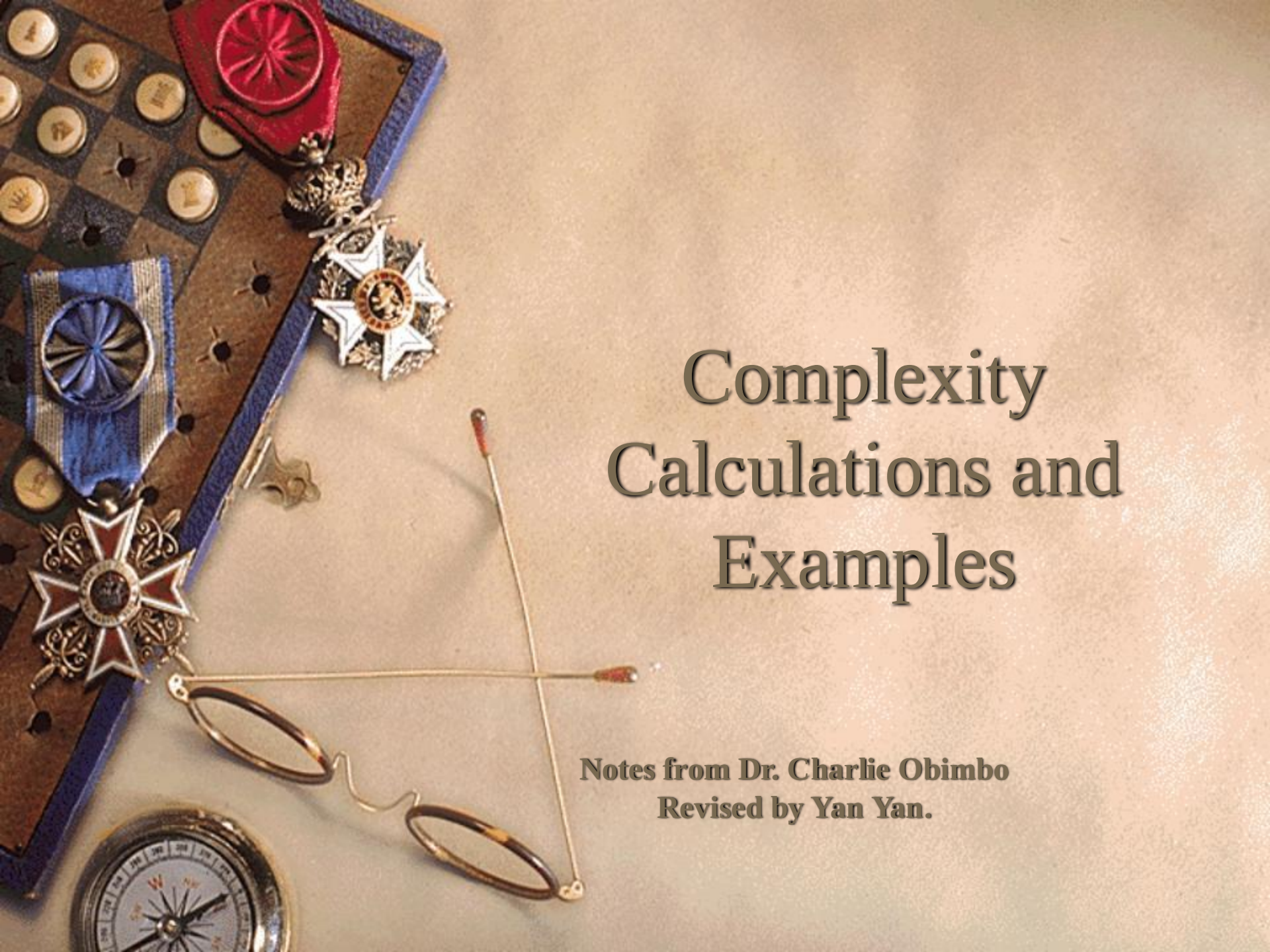


# Complexity Calculations and Examples

**Notes from Dr. Charlie Obimbo**
**Revised by Yan Yan.**

# Contents

1. Example on input size and complexity
2. Sorting algorithms and their complexities
    1. Big-O, Big Omega, & Theta

# Question

Computer solves problem of complexity $T(n) \in \Theta(n^3)$ & size $n = 1000$ in 1 minute. What size would be solved in 1 hour?

$$\frac{c * (100)^3}{c * (x)^3} = \frac{1}{60}$$

$$(x)^3 = 60 * (100)^3$$

$$x = 3914$$

Note that $x \in N$

# Sorting

- **Sorting**: Rearranging the values in an array or collection into a specific order (usually into their "natural ordering").
  - one of the fundamental problems in computer science
- Input: A sequence of n objects
  - $s = \langle a_1, a_2, \ldots, a_n \rangle$
- Output: A permutation (reordering) $\langle a'_1, a'_2, \ldots, a'_n \rangle$ such that $a'_1 \leq a'_2 \leq \ldots \leq a'_n$.

# Sorting

- can be solved in many ways:
  - some are faster/slower than others
  - some use more/less memory than others
  - some work better with specific kinds of data

- comparison-based sorting: determining order by comparing pairs of elements:
  - `<`, `>`, `compareTo`, ...

# Sorting Algorithms

- **bubble sort**: swap adjacent pairs that are out of order
- **selection sort**: look for the smallest element, move to front
- **insertion sort**: build an increasingly large sorted front portion
- **merge sort**: recursively divide the array in half and sort it
- **quick sort**: recursively partition array based on a middle value

# Selection Sorting

- Orders a list of values by repeatedly putting the smallest or largest unplaced value into its final position.
  - Look through the list to find the smallest value.
  - Swap it so that it is at index 0.
  - Look through the list to find the second-smallest value.
  - Swap it so that it is at index 1.

    …
  - Repeat until all values are in their proper places.

# Selection sort example

- Initial array:

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | 22 | 18 | 12 | -4 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

- After 1st, 2nd, and 3rd passes:

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | **-4** | 18 | 12 | **22** | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

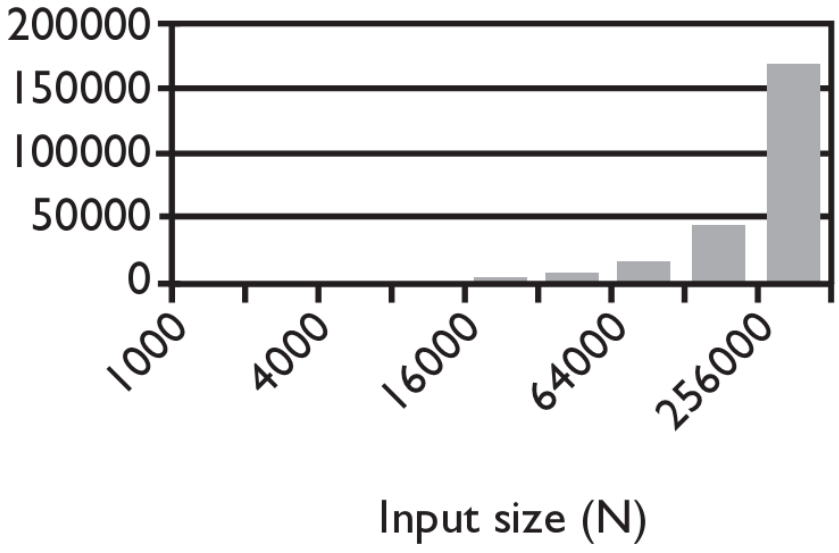
| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | **2** | 12 | 22 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | **18** | 85 | 42 | 98 | 25 |

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | 2 | **7** | 22 | 27 | 30 | 36 | 50 | **12** | 68 | 91 | 56 | 18 | 85 | 42 | 98 | 25 |

# Selection sort runtime

- What is the complexity class (Big-O) of selection sort?

| N | Runtime (ms) |
|---|---|
| 1000 | 0 |
| 2000 | 16 |
| 4000 | 47 |
| 8000 | 234 |
| 16000 | 657 |
| 32000 | 2562 |
| 64000 | 10265 |
| 128000 | 41141 |
| 256000 | 164985 |



# Similar algorithms

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | 22 | 18 | 12 | -4 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

- **bubble sort**: Make repeated passes, swapping adjacent values

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | 18 | 12 | -4 | 22 | 27 | 30 | 36 | 7 | 50 | 68 | 56 | 2 | 85 | 42 | 91 | 25 | 98 |

22 → 50 → 91 → 98 →

- **insertion sort**: Shift each element into a sorted sub-array

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | 12 | 18 | 22 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

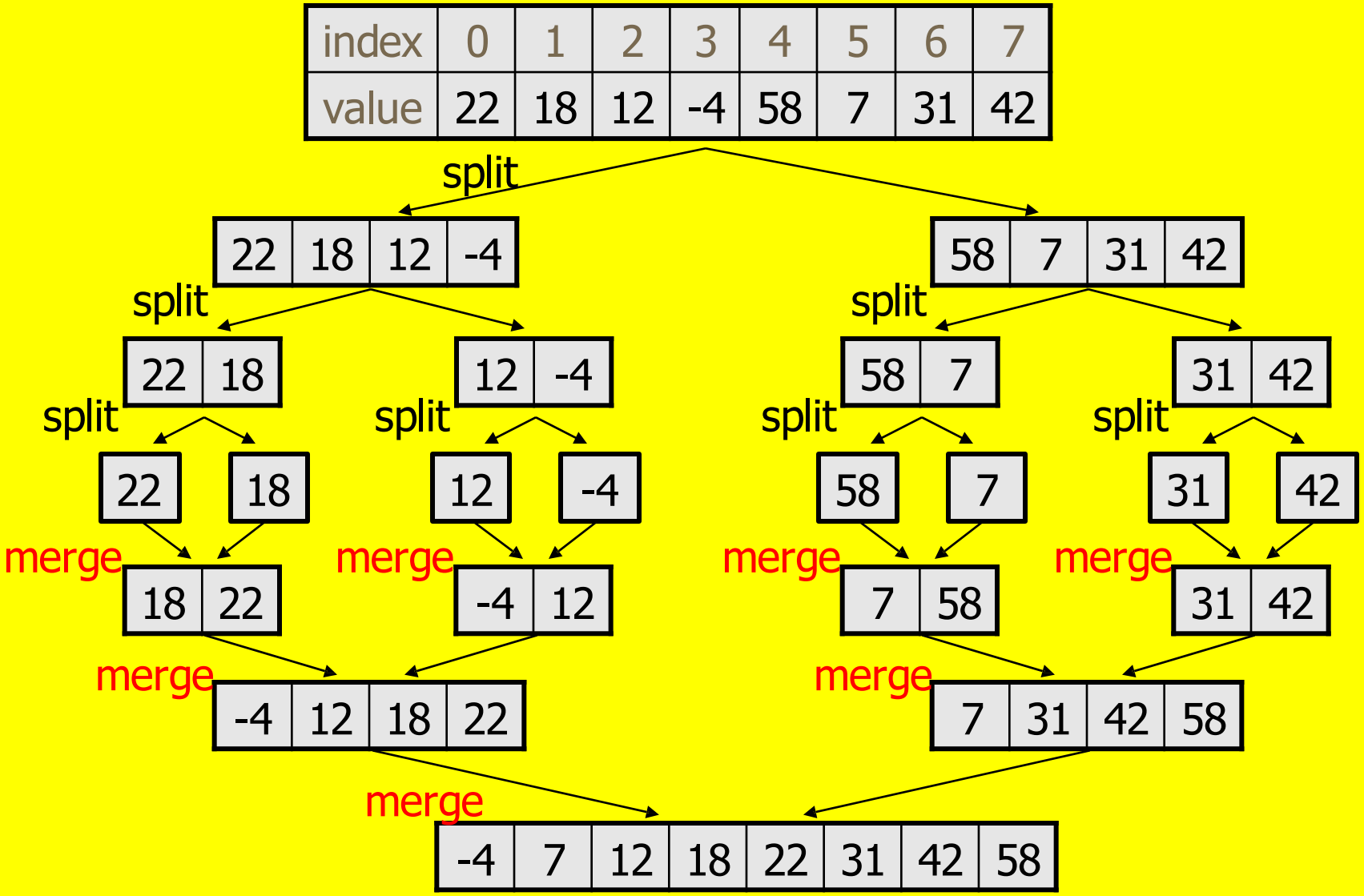
sorted sub-array (indexes 0-7)

← 7

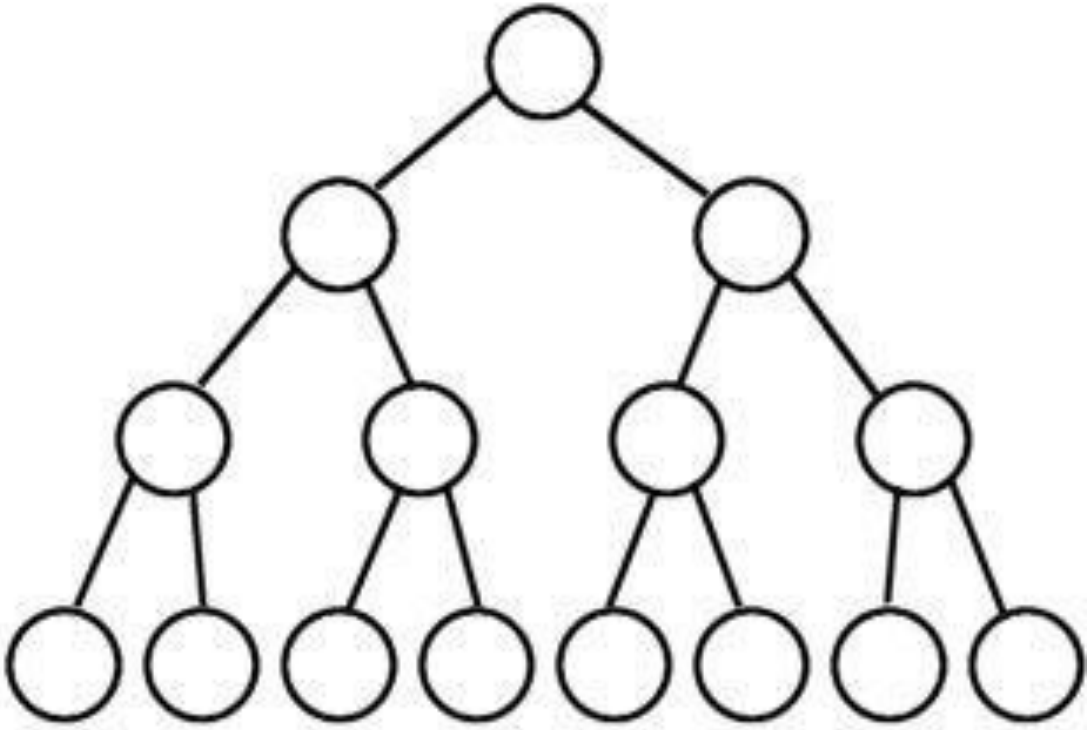
# Merge sort

- Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.
  - Often implemented recursively.
  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

# Merge sort example



**Full** Binary Tree



| Height | # of Nodes | **Tot # of Nodes** |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 2 | 3 |
| 2 | 4 | 7 |
| 3 | 8 | 15 |

# Sorting Algorithms Complexity Summary

| Algorithm | Time Complexity | | Space Complexity |
|---|---|---|---|
| | Best | Worst | Worst |
| Bubble Sort | $\Omega(n)$ | $O(n^2)$ | $O(1)$ |
| Selection Sort | $\Omega(n^2)$ | $O(n^2)$ | $O(1)$ |
| Insertion Sort | $\Omega(n)$ | $O(n^2)$ | $O(1)$ |
| Merge Sort | $\Omega(n \log(n))$ | $O(n \log(n))$ | $O(n)$ |
| Quick Sort | $\Omega(n \log(n))$ | $O(n^2)$ | $O(n)$ |

# References and Useful Resources

- Merge Sort
  - https://www.geeksforgeeks.org/time-and-space-complexity-analysis-of-merge-sort/
- Time complexity of sorting algorithms
  - https://www.geeksforgeeks.org/time-complexities-of-all-sorting-algorithms/
  - https://www.boardinfinity.com/blog/time-complexity-of-sorting-algorithms/

That’s about this lecture!