Trees

Notes by Yan Yan

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1. Tree
2. Binary Tree
3. Binary Tree Traversal

Learning Objectives

1. Describe the abstract structure of trees, and binary trees
2. Understand the tree terminology and apply them to describe trees and binary trees
3. Describe different traversal algorithms and apply the algorithms to perform traversals on binary trees

What is a Tree

- In computer science, a tree is an abstract model of a hierarchical structure
- A tree consists of nodes and edges with a parent-child relation
- Applications:
  - Organization charts
  - File systems
  - Decision process

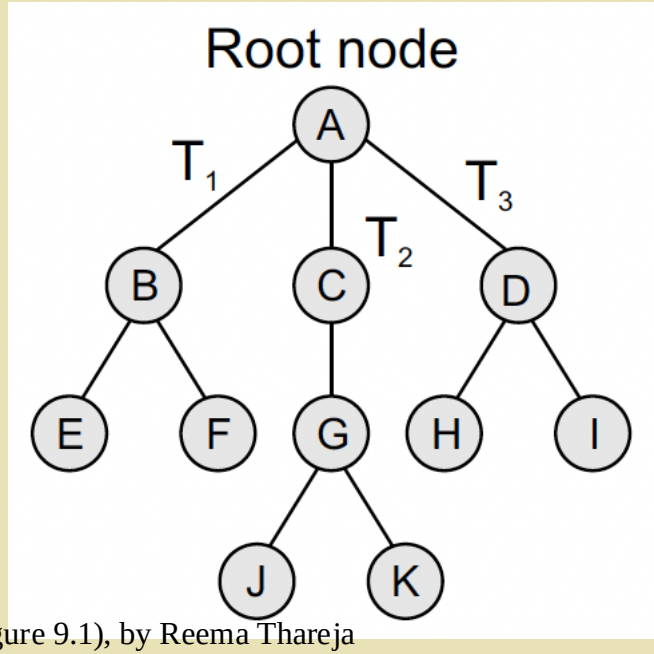


Figure from book Data Structures Using C, 2nd edition (Chapter 9, Figure 9.1), by Reema Thareja

What is a Tree

- Applications Example:
  - File systems

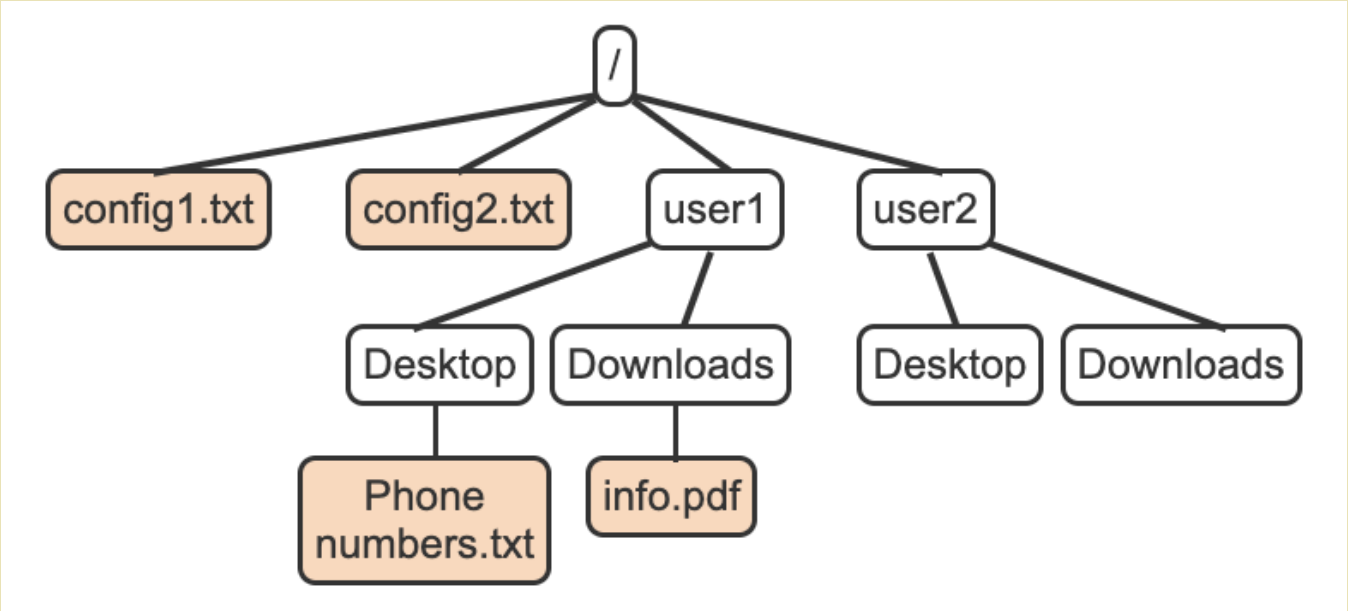


Figure 7.2 Applications of trees

Binary Tree

- In a **binary** tree, each node has **up to** two children, known as a **left** child and a **right** child.
  - One node has up to two successors

- "Binary" means two, referring to the two children

Binary Tree Terminology

- **Leaf**: A tree node with no children.
- **Internal** node: A node with at least one child.
- **Parent**: A node with a child is said to be that child's parent. A node's ancestors include the node's parent, the parent's parent, etc., up to the tree's root.
- **Degree** of a node: number of children that a node has
- **Root**: The one tree node with no parent (the "top" node).

Binary Tree Terminology

- **Edge**: the line connecting a node to any of its successors.
- **Path:** A sequence of consecutive edges.
- **Depth:** the length of the path from the root to a node *n*.
  - The depth of the root node is zero.
- **Height** of a tree: total number of nodes on the path from the root node to the deepest node in the tree.
  - A tree with only a root node has a height of 1

Binary Tree Example

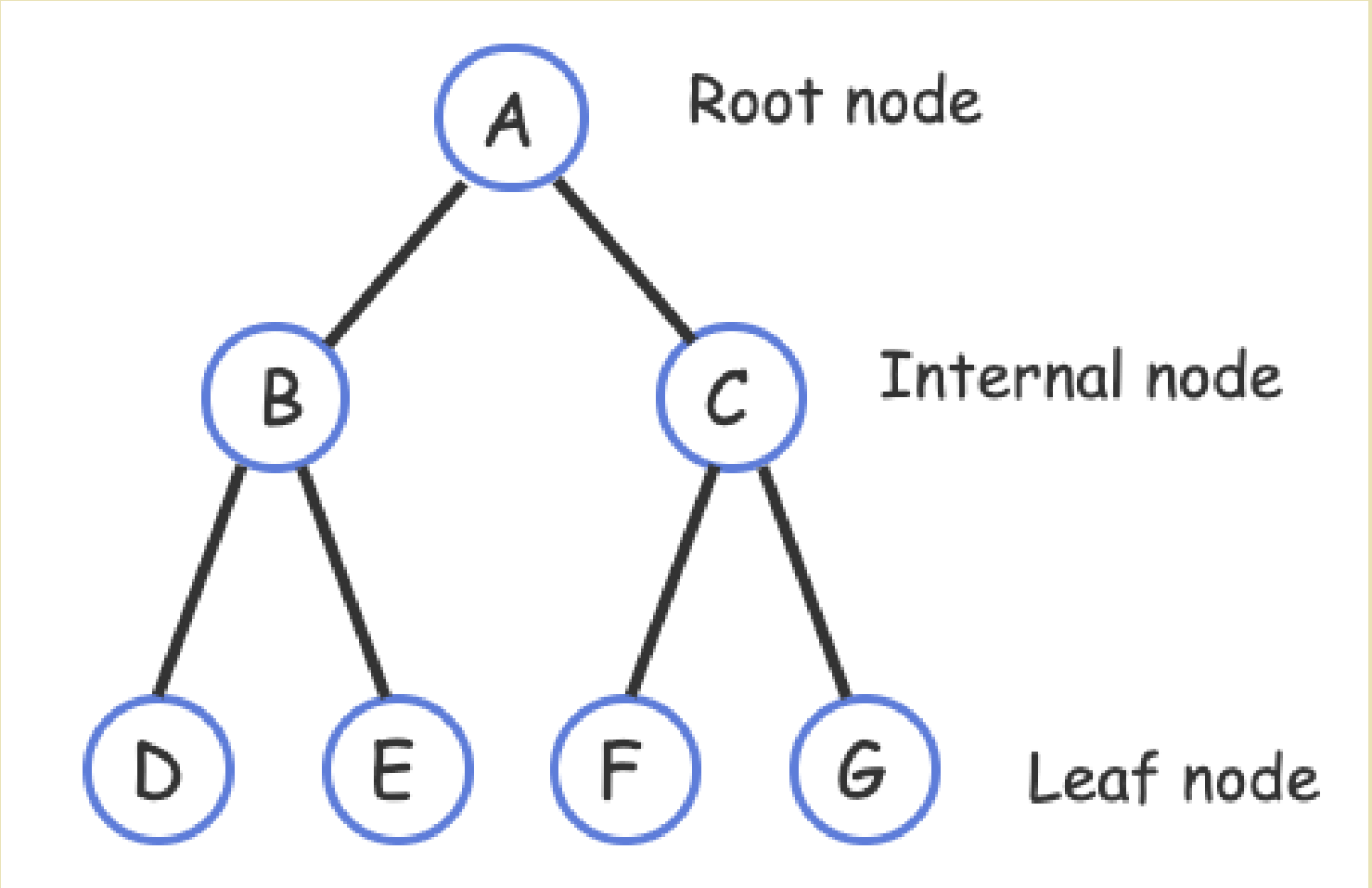
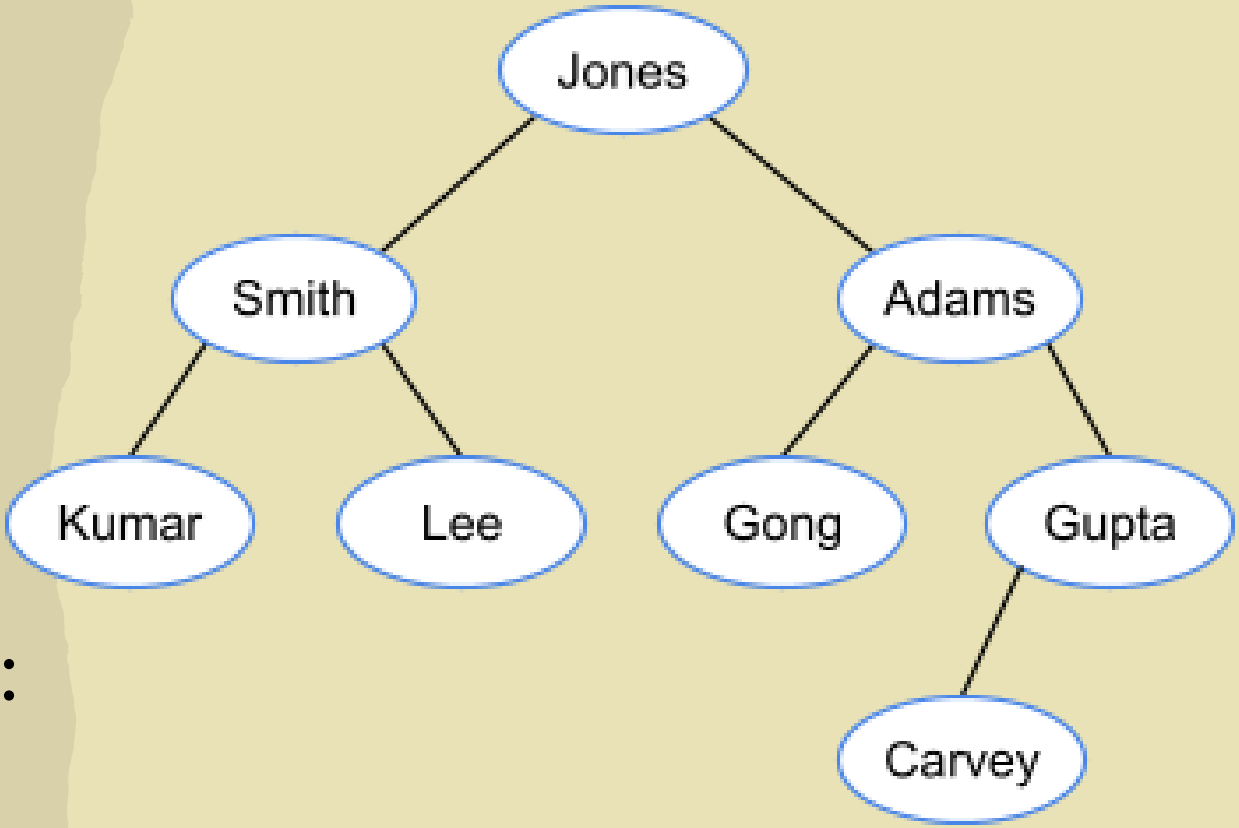


Figure from the zybook 7.1 Binary trees

Binary Tree Example



- The root node is _____.
- Smith's left child: ____

How many internal nodes does the tree have?
How many leaf nodes does the tree have?

Figure from the zybook 7.1 Binary trees

Complete Binary Tree

- Two properties
  - every level, except possibly the last, is completely filled.
  - all nodes appear as far left as possible
- Example

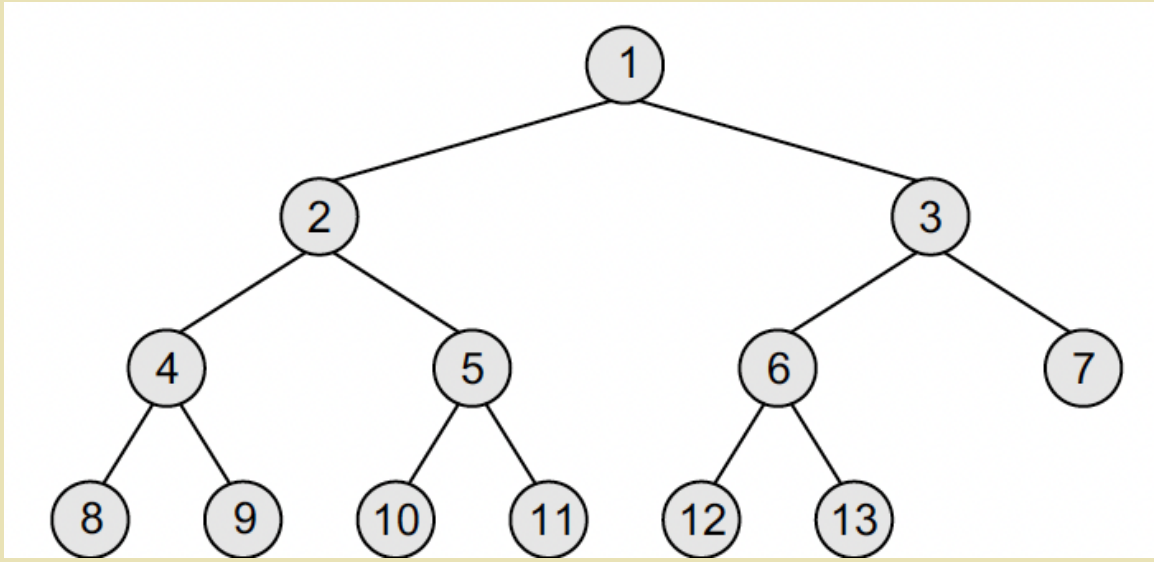


Figure from book Data Structures Using C, 2nd edition (Chapter 9, Figure 9.7), by Reema Thareja

Binary Tree Representation

- Linked representation of Binary Trees

```
struct node {
struct node *left;
int data;
struct node *right;
};
```

- Every binary tree has a pointer ROOT, which points to the root element (topmost element) of the tree.
  - If ROOT = NULL, then the tree is empty.

Binary Tree Representation

- Example

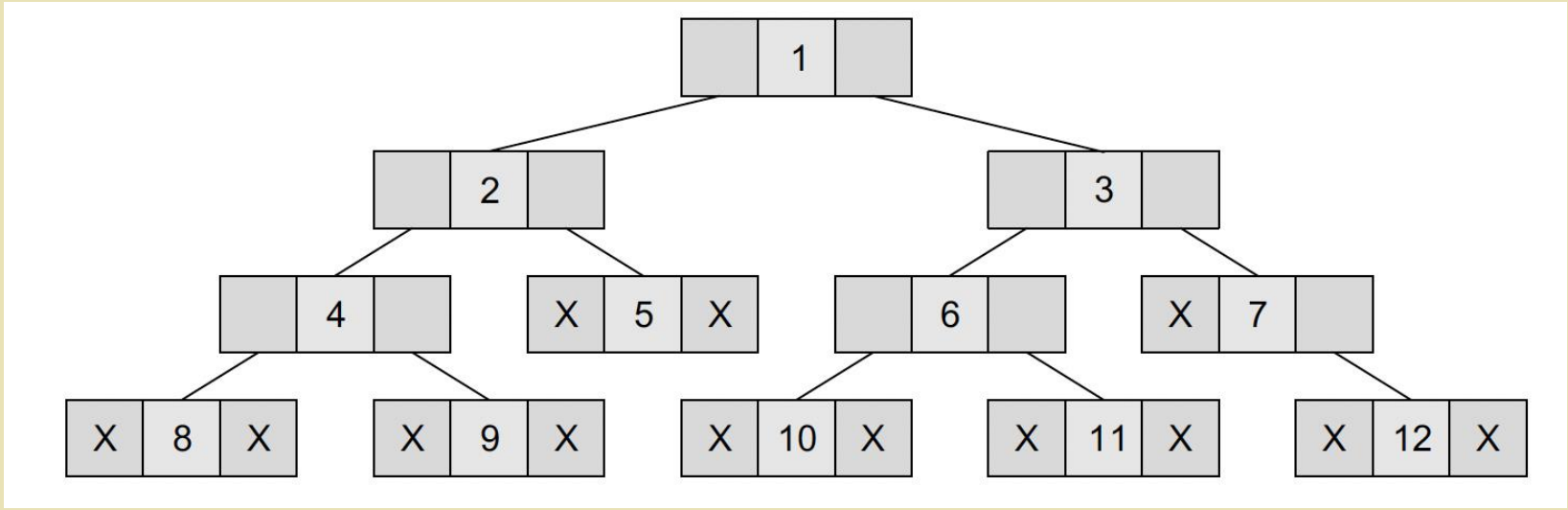


Figure from book Data Structures Using C, 2nd edition (Chapter 9, Figure 9.9), by Reema Thareja

Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
- internal nodes: operators
- external nodes: operands

Arithmetic Expression Tree

- Example: arithmetic expression tree for the expression *(a – b) + (c * d)*

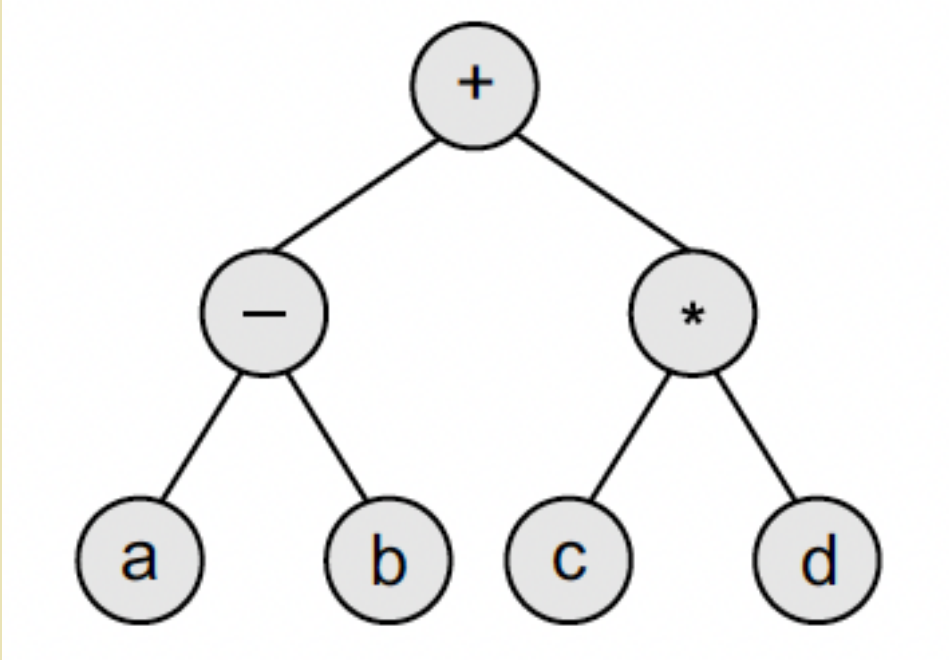
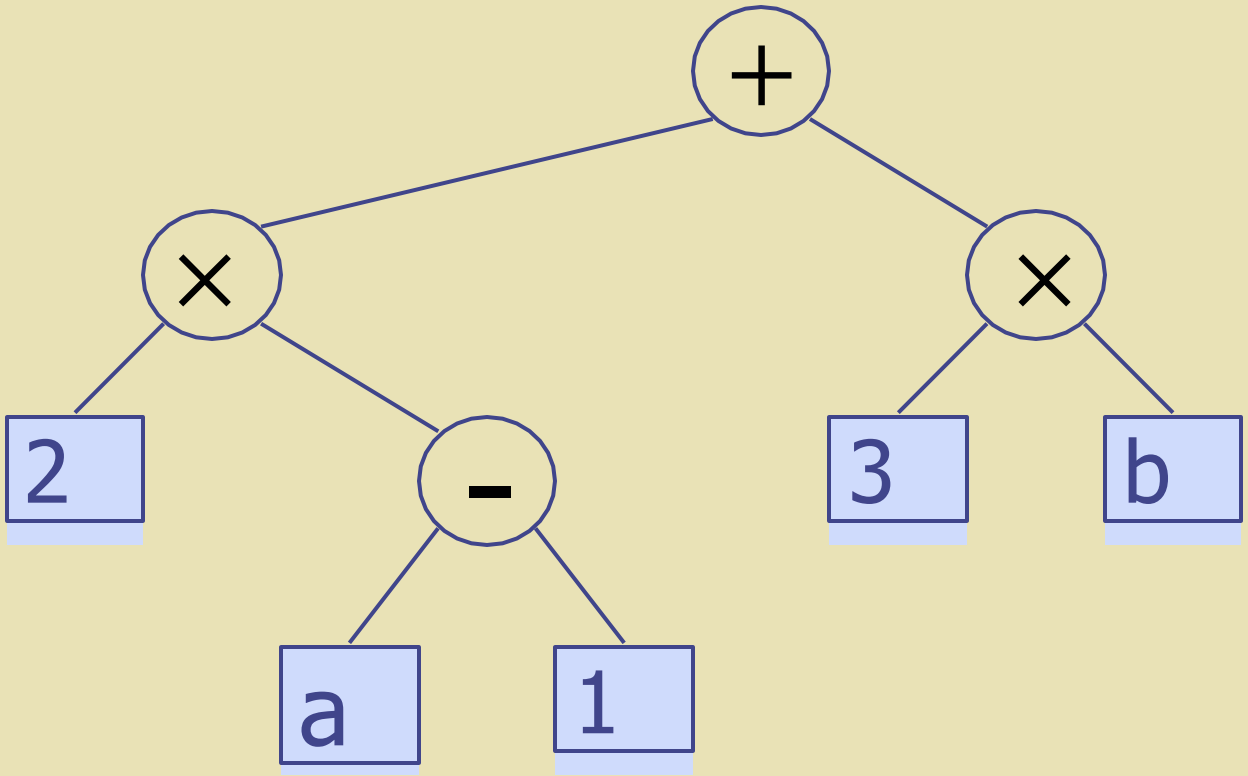


Figure from book Data Structures Using C, 2nd edition (Chapter 9, Figure 9.13), by Reema Thareja

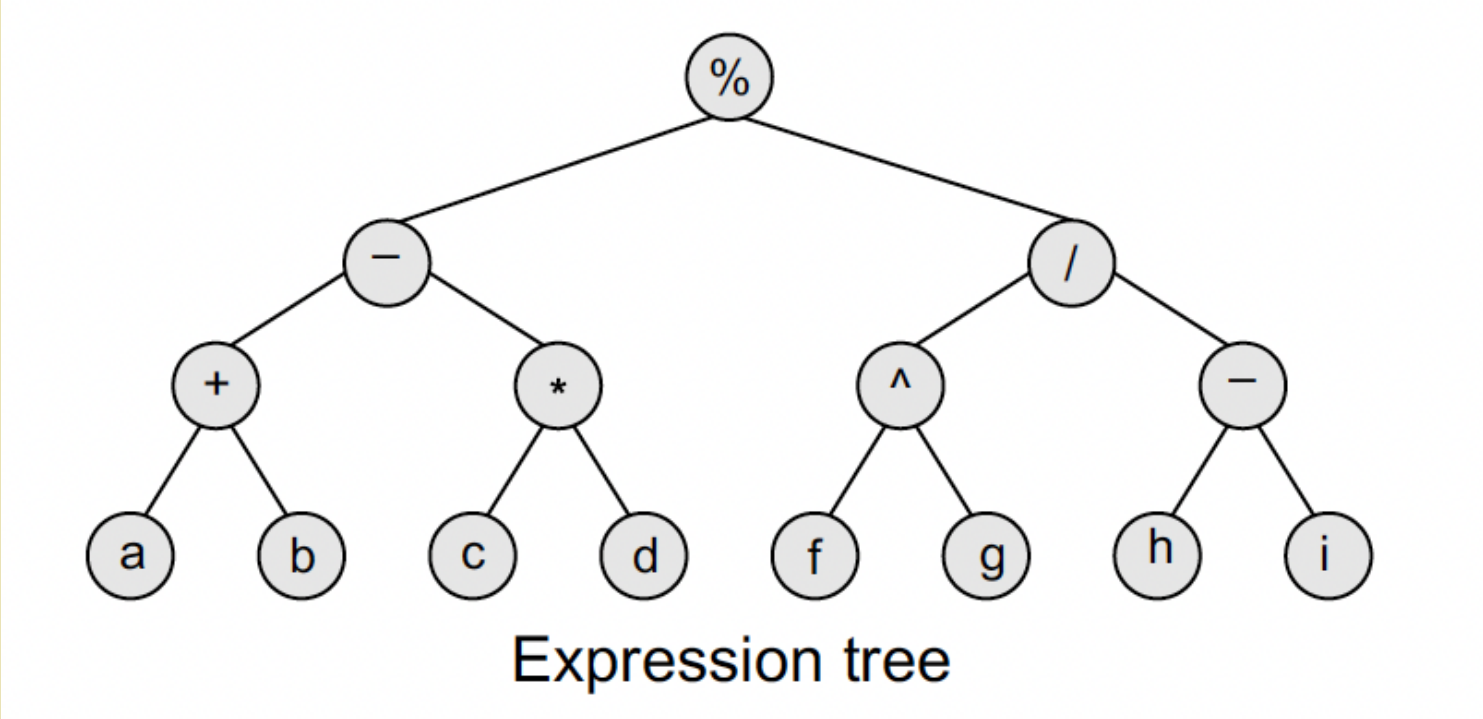
Arithmetic Expression Tree

- Exercise: What is the expression from the arithmetic expression tree?



Arithmetic Expression Tree

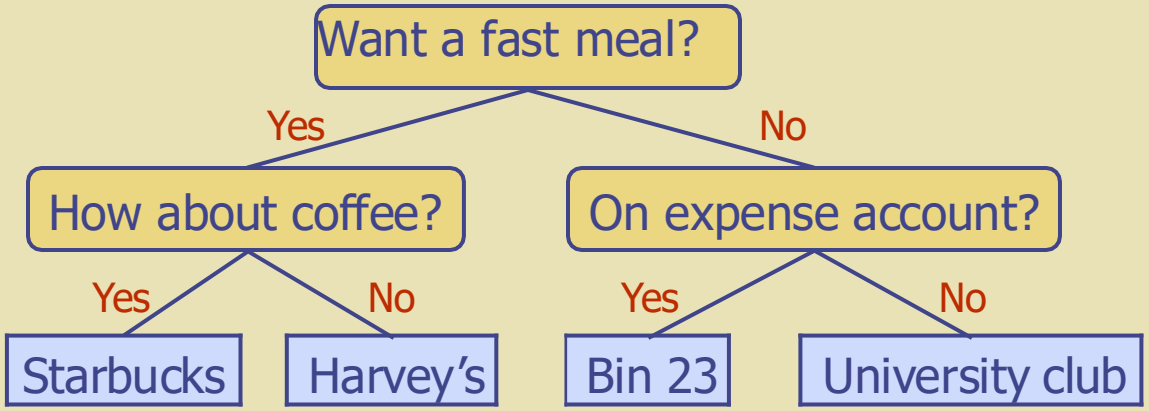
- Question: What is the arithmetic expression the following tree represent?



Expression tree

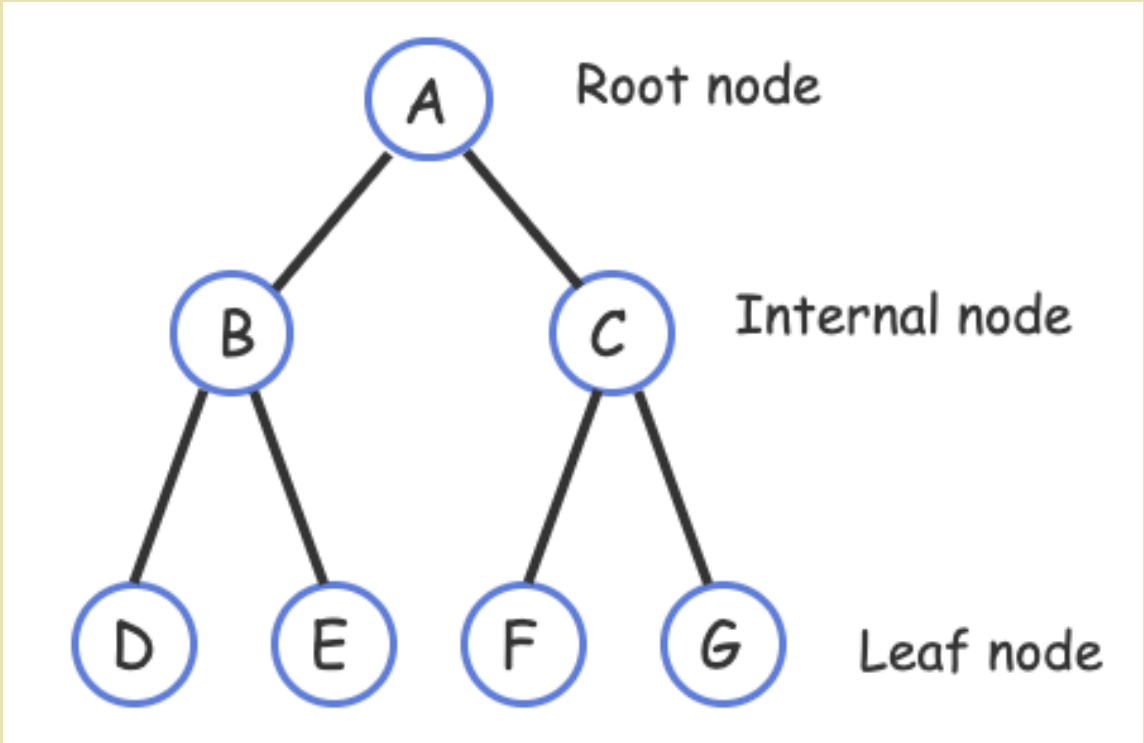
Decision Tree

- Binary tree associated with a decision process
- internal nodes: questions with yes/no answer
- external nodes: decisions
- Example



Binary Tree Traversal

- Traversal -- the process of visiting each node in the tree exactly once in a systematic way
- Discussion -- How would you traverse this binary tree?

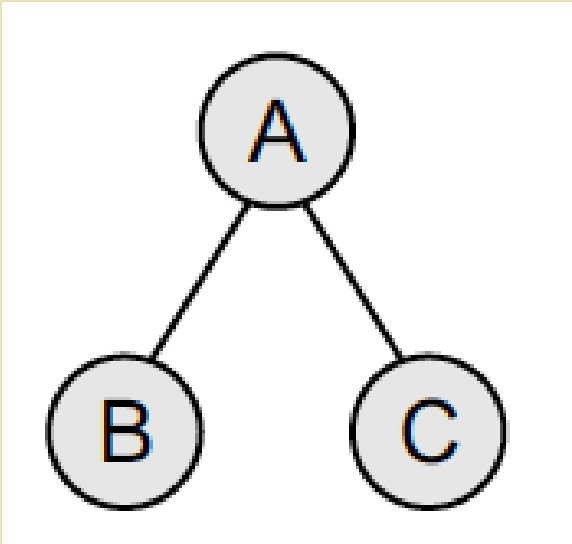


Binary Tree Traversal

- Algorithms differ in the order in which the nodes are visited.
- Pre-order
- In-order
- Post-order
- Level-order

Traversal – Pre-order

1. Visiting the root node,
2. Traversing the left sub-tree, and finally
3. Traversing the right sub-tree

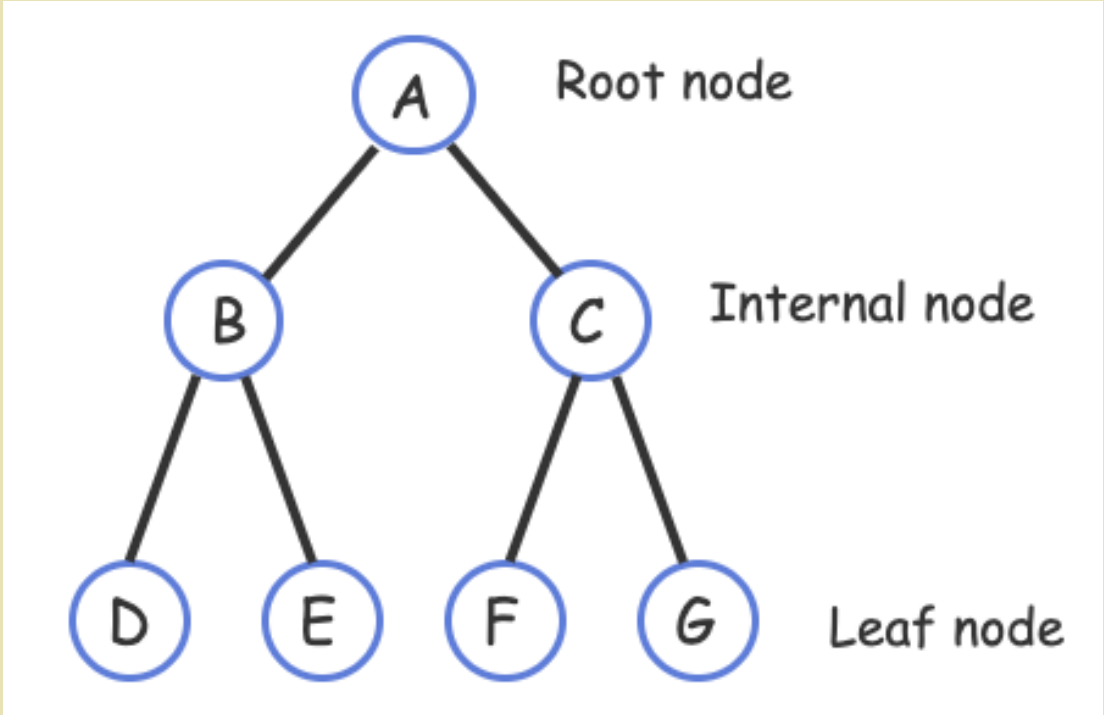


Order A, B, C

- Pre-order traversal is also called as depth-first traversal.

Traversal – Pre-order

- Exercise: Write down the order of the nodes visited in the tree using Pre-order traversal



Traversal – Pre-order

- Pre-order traversal algorithms are used to extract a prefix notation from an expression tree.

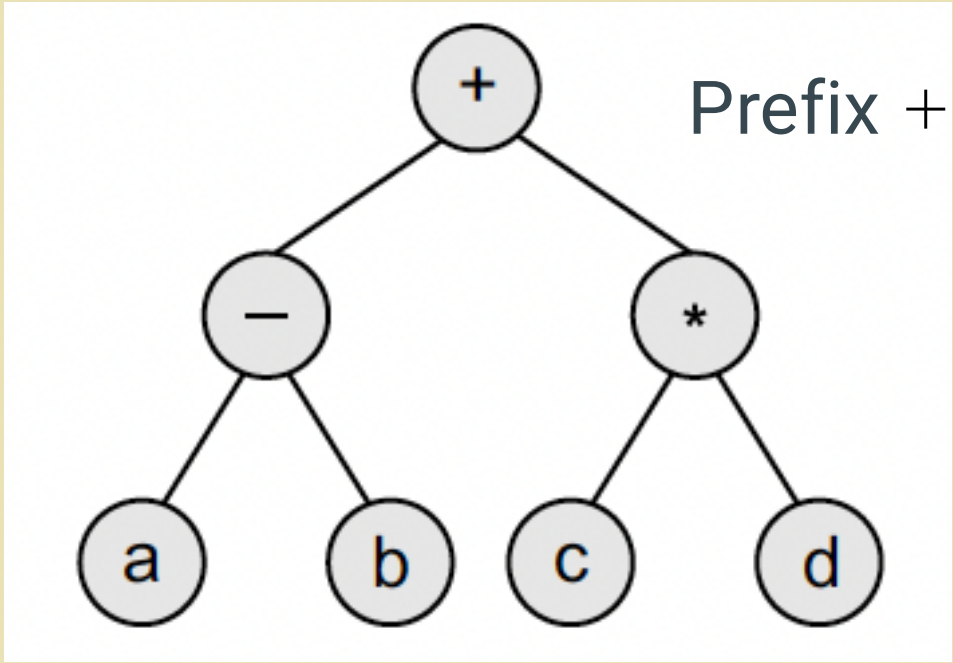
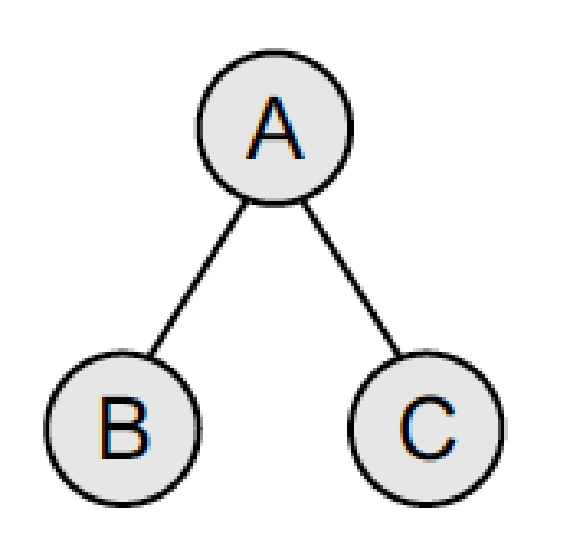


Figure from book Data Structures Using C, 2nd edition (Chapter 9, Figure 9.13), by Reema Thareja

Traversal – In-order

1. Traversing the left sub-tree,
2. Visiting the root node, and finally
3. Traversing the right sub-tree

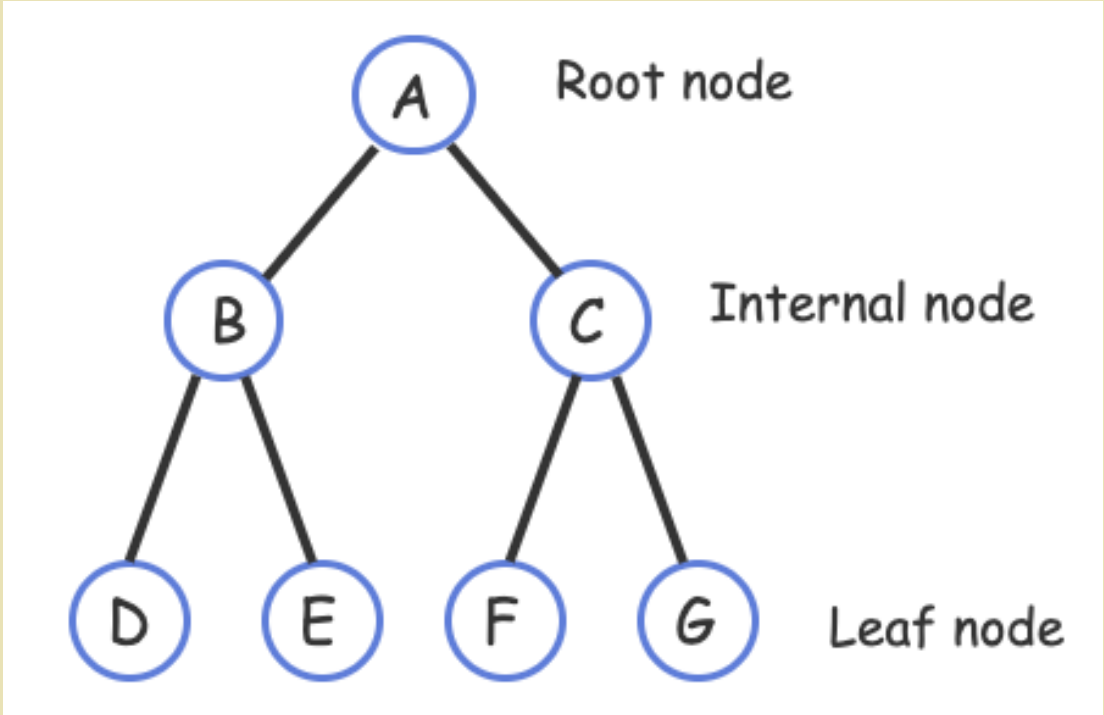


Order B, A, C

- In-order traversal is also called as symmetric traversal.

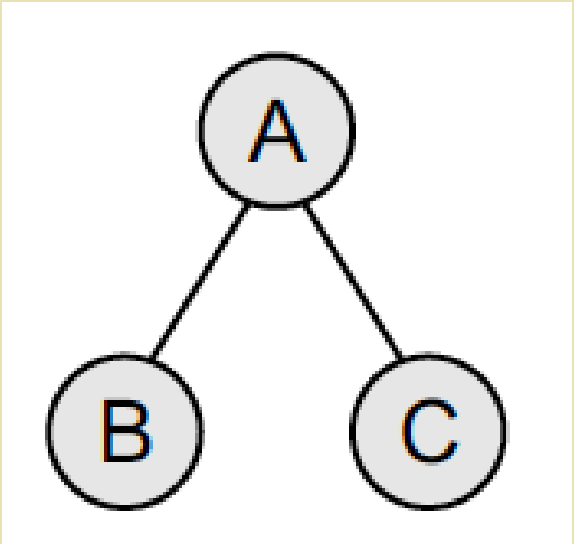
Traversal – In-order

- Exercise: Write down the order of the nodes visited in the tree using in-order traversal



Traversal – Post-order

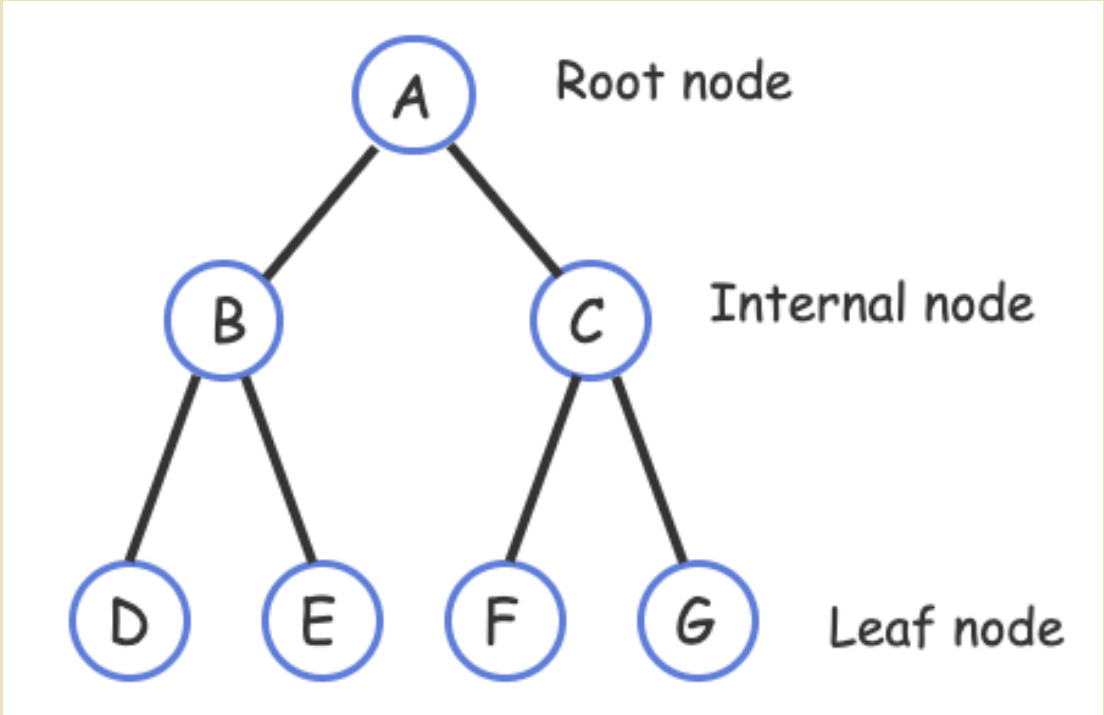
1. Traversing the left sub-tree,
2. Traversing the right sub-tree, and finally
3. Visiting the root node



Order B, C, A

Traversal – Post-order

- Exercise: Write down the order of the nodes visited in the tree using post-order traversal



Traversal -- Post-order

- Post-order traversal algorithms are used to extract a postfix notation from an expression tree.

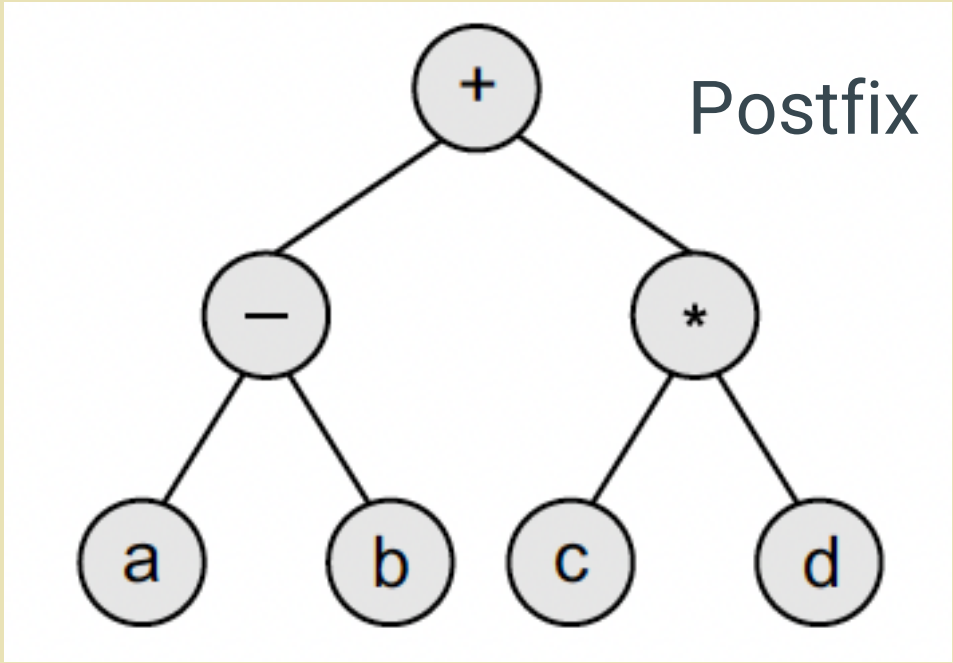
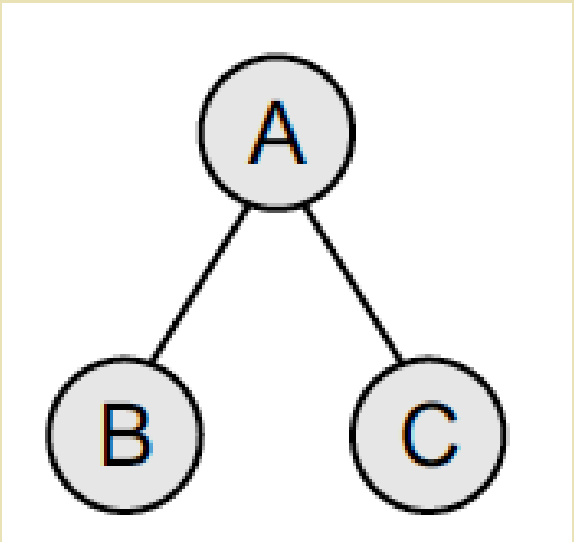


Figure from book Data Structures Using C, 2nd edition (Chapter 9, Figure 9.13), by Reema Thareja

Traversal – Level-order

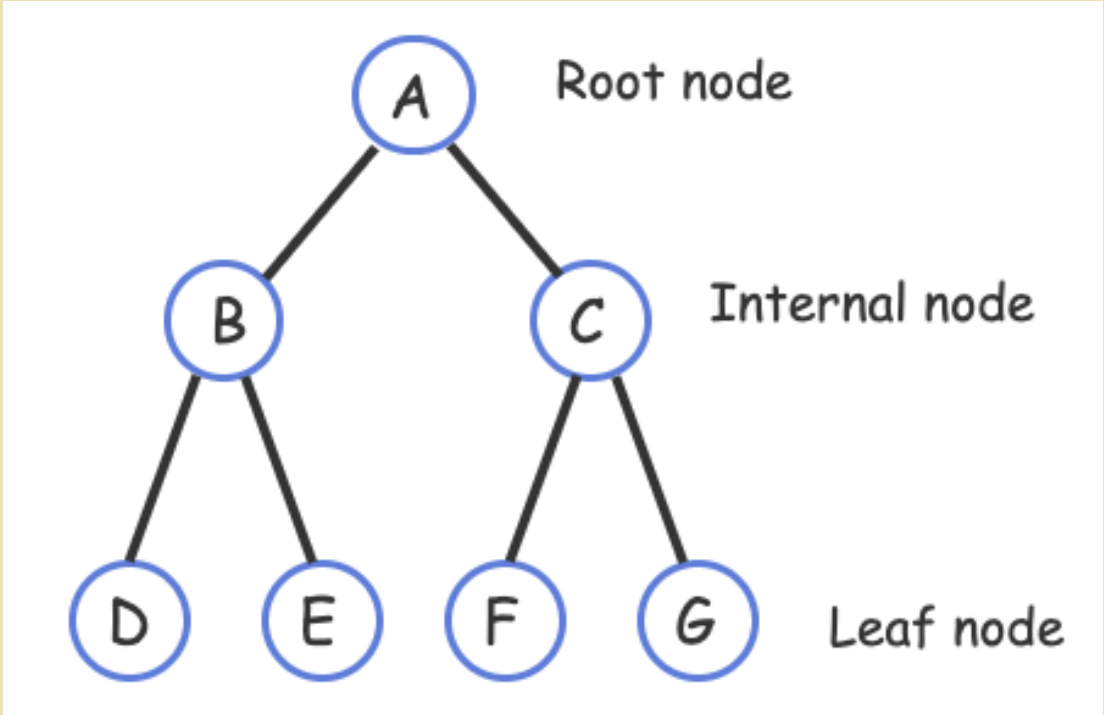
- All the nodes at a level are accessed before going to the next level.
- Also called as the breadth-first traversal algorithm.
- (within the same level, usually from left to right)



Order A, B, C

Traversal – Level-order

- Exercise: Write down the order of the nodes visited in the tree using level-order traversal



References and Useful Resources

- Data Structures Using C, 2nd edition (Chapter 9, Figure 9.13), by Reema Thareja
- Depth-First Search (DFS) Traversal of a Tree https://www.geeksforgeeks.org/dfs-traversal-of-a-tree-using-recursion/
- Level Order Traversal (Breadth First Search or BFS) of Binary Tree https://www.geeksforgeeks.org/level-order-tree-traversal/

That's about this lecture!