# Convolutional operators and their structure

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## Output size

• Input:  $\mathbf{X} \in \mathbb{R}^{H \times W \times C_1}$ 

• Kernel:  $\mathbf{w} \in \mathbb{R}^{h \times w \times C_1 \times C_2}$ 

• Output:  $\mathbf{Z} \in \mathbb{R}^{(H-h+1)\times (W-w+1)\times C_2}$ 



\*

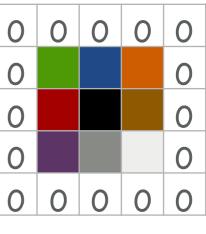
a	b
С	d





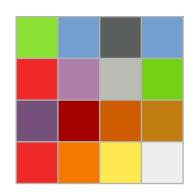
## Padding

- Add  $P_w, P_h$  zeros in each dimension
- Input:  $\mathbf{X} \in \mathbb{R}^{H \times W \times C_1}$
- Kernel:  $\mathbf{w} \in \mathbb{R}^{h \times w \times C_1 \times C_2}$
- Output:  $\mathbf{Z} \in \mathbb{R}^{(H-h+2p_h+1)\times(W-w+2p_w+1)\times C_2}$



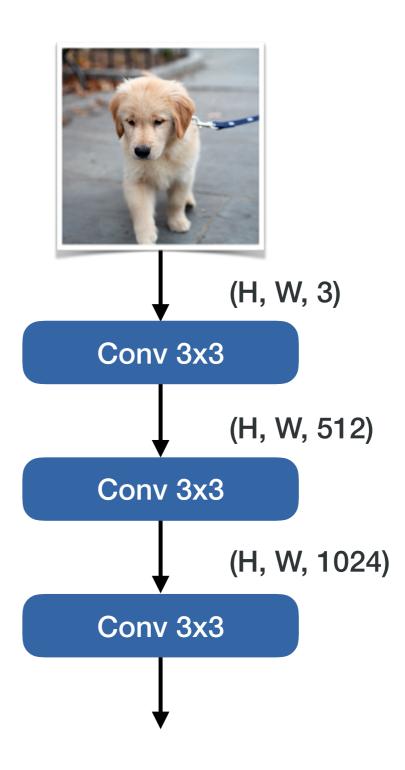
\*

a	b
С	d



## Output resolution

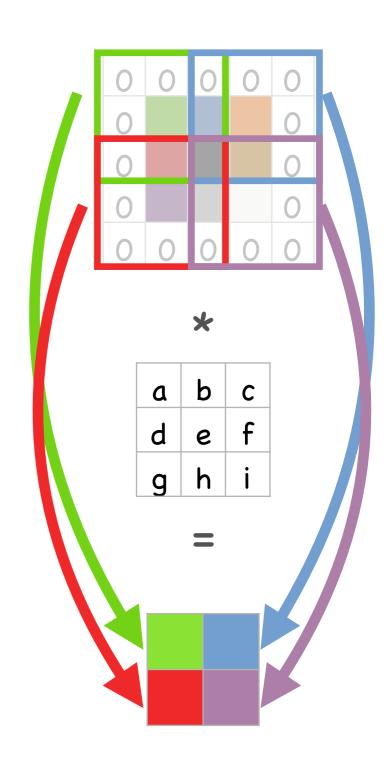
- High output resolution
  - Slow computation



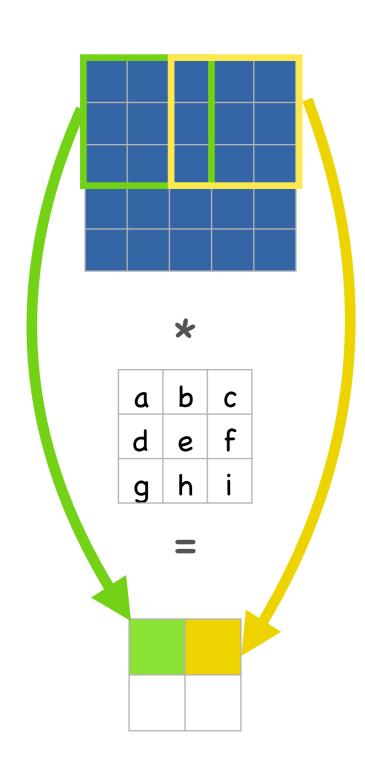
## Striding

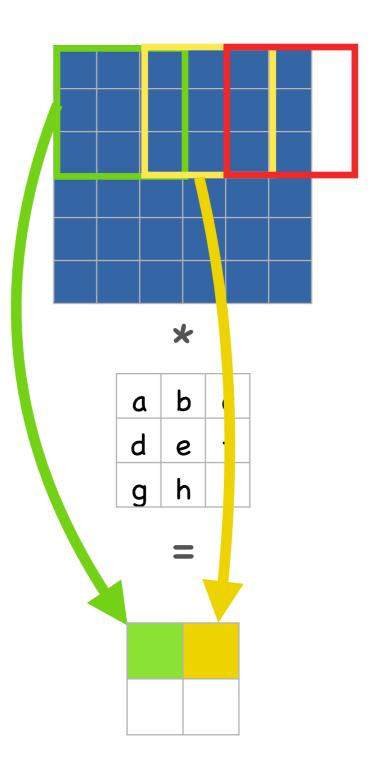
- Only compute every nth output:  $S_w$ ,  $S_h$
- Input:  $\mathbf{X} \in \mathbb{R}^{H \times W \times C_1}$
- Kernel:  $\mathbf{w} \in \mathbb{R}^{h \times w \times C_1 \times C_2}$
- Output:

$$\mathbf{Z} \in \mathbb{R}^{\left(\frac{H-h+2p_h}{s_h}+1\right)\times\left(\frac{W-w+2p_w}{s_w}+1\right)\times C_2}$$



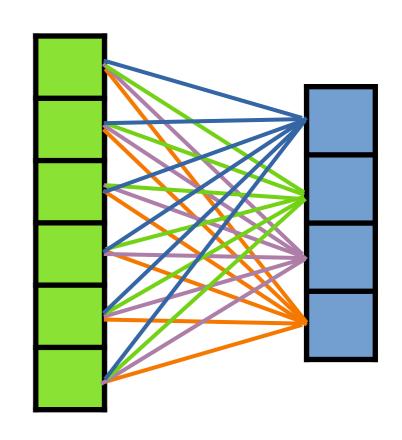
## Output size with striding





### Parameters

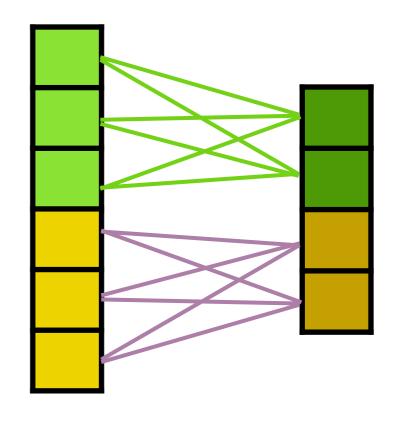
• Every input channel  $C_1$  is connected to every output channel  $C_2$ 



$$C_1 = 6$$

## Grouping

- Split channels into g groups
- Reduce parameters and computation by factor g



$$C_1 = 6$$

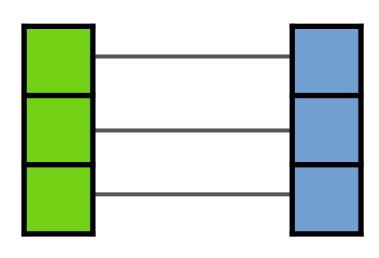
$$C_2 = 4$$

### Depthwise convolution

#### Special grouping

$$\bullet \quad C_1 = g$$

• 
$$C_2 = g$$



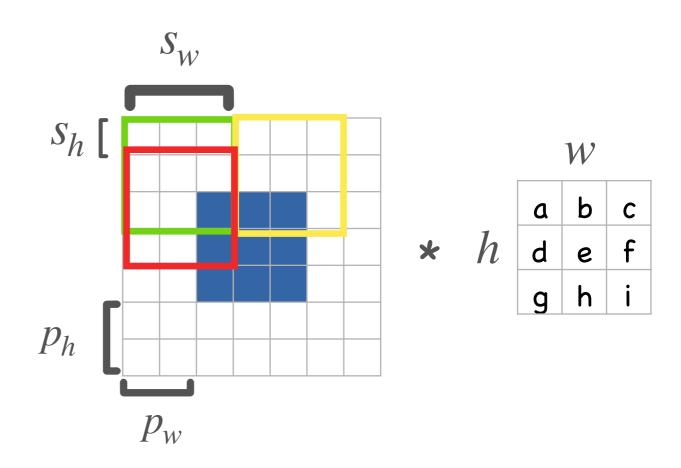
$$C_1 = 3$$
  $C_2 = 3$ 

## Hyper-parameters of convolutions

• Kernel size:  $w \times h$ 

• Padding:  $p_w$ ,  $p_h$ 

• Stride:  $S_w$ ,  $S_h$ 



## Convolutional operators

• Run arbitrary operation  $f(\mathbf{x})$  "over" image





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a	b	С
d	e	f
g	h	i

