Probability, likelihood, sampling and expectation

Overview

- Probabilities and likelihood
- Sampling
- Linearity of expectation

What is a (discrete) distribution?

- Informal: bunch of positive numbers that some to one
- Distribution: $P : a \rightarrow [0,1]$
- Event: $a \in [0,\ldots,n-1]$
- Probability: $P(a)$
  - Chance of a occurring

What is a conditional probability?

- Conditional probability:

  $P(a|\theta)$

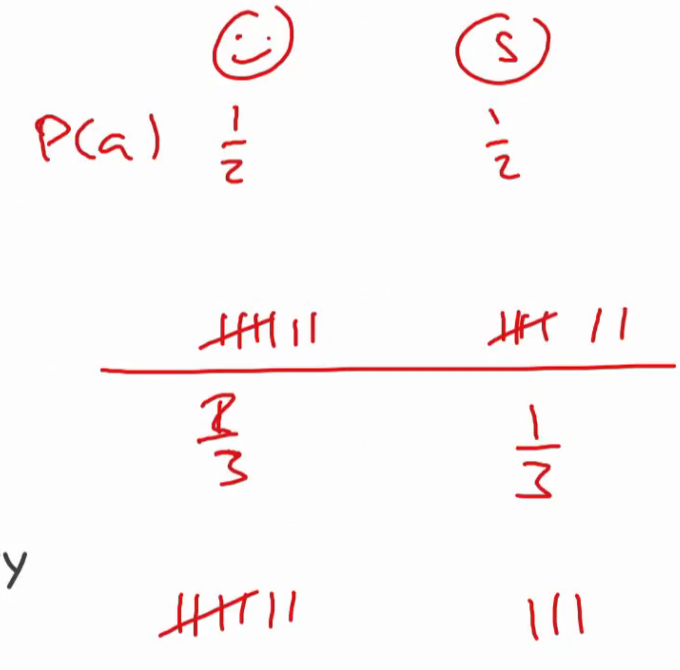
  - Chance of a occurring given θ
- Example θ's:
  - Other event
  - Model parameters

What is the likelihood?

- Informal: same as probability
- Formal: a function of parameter θ that describes the probability of observing data x^n given θ.
- Definition: $L(\theta) \equiv L(\theta; x^n) = P(x^n \mid \theta)$
- Usually refers to past events

Sampling

- Definition: $a \sim P$
  - Produce events a following $P(a)$
- Sampling bias
  - Empirical probability of samples != $P(a)$



Expectation

- Definition:
  - $\mathbb{E}_{a\sim P}[f(a)]$
- For any function f
  - $\sum_a P(a)f(a)$
  - $\frac{1}{N}\sum_{a\sim P} f(a)$

Linearity of expectation

- $\mathbb{E}_{a\sim P}[\alpha f(a)] = \alpha \mathbb{E}_{a\sim P}[f(a)]$
- $\mathbb{E}_{a\sim P}[f(a) + g(a)] = \mathbb{E}_{a\sim P}[f(a)] + \mathbb{E}_{a\sim P}[g(a)]$
- $\mathbb{E}_{a\sim P}[f(a)g(a)] \neq \mathbb{E}_{a\sim P}[f(a)]\mathbb{E}_{a\sim P}[g(a)]$

Summary

- Event: $a \in [0,\ldots,n-1]$
- Distribution: $P : a \rightarrow [0,1]$
- Probability: $P(a)$
- Sampling: $a \sim P$
- Expectation: $\mathbb{E}_{a \sim P}[f(a)]$

[Introduction to Probability, Bertsekas and Tsitsiklis 2002]
[All of Statistics, Wasserman 2004]