Homework04

- 姓名：郭灵
- 学号：3170105370
- 专业：计算机科学与技术

Circle by Cubic Rational Bezier Curves

Solution 1

- This solution can draw a cricle only using four points within one knot. For implementing that, we need to do generation for the bezier cubic function.

Generation of Bezier-Like Cubic Function

$$
\begin{aligned}
&B_0(u) = (1-u)^2(1+(2-m)u)\\
&B_1(u) = m(1-u)^2u\\
&B_2(u) = m(1-u)u^2\\
&B_3(u) = u^2(1+(2-m)(1-u))\\
&where \quad u \in [0,1]
\end{aligned}
$$

- when m=3, the basis function will be Bernstein Bezier basis function
- when m=4,the basis function will be Timmer cubic basis function, which will actually be used to draw circle

The rational cubic Bezier-like curve with single parameter m will be :

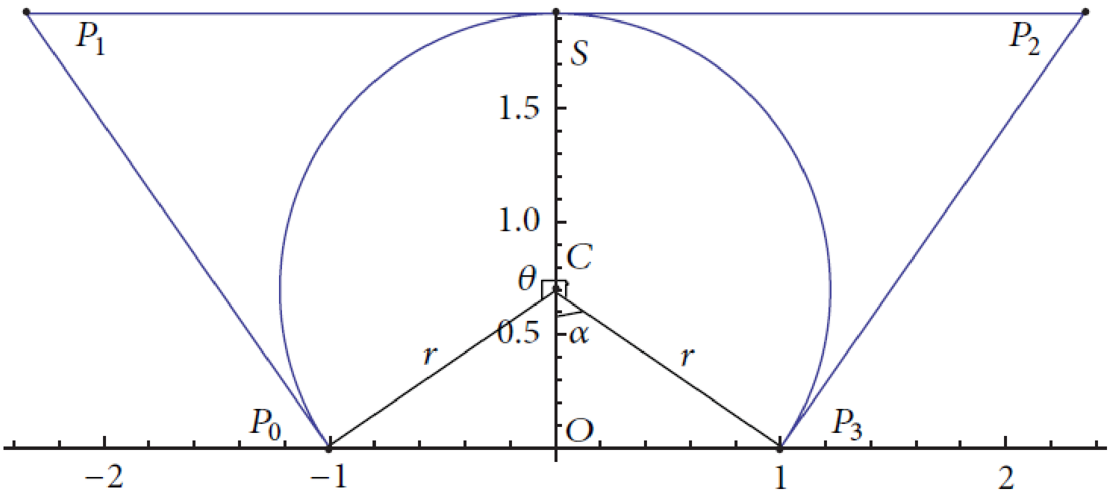
$$
r(u) = \frac{\sum_{i=0}^{3} w_i P_i B_i(u)}{\sum_{i=0}^{3} w_i B_i(u)}, \quad u \in [0,1] \tag{1}
$$

- This equation will still hold all corresponding properties.

How to set weight?

A cubic function will need four points, and when we draw a nearly circle, four points can be symmetric.

We set a trapezoidal to draw a circle as followed:



- The order of points chain is $P_0 \to P_1 \to P_2 \to P3$
- The start point is P_0
- The end point is P_3
- To simplify the explantion, just make weight of P_i is w_i
- For easily programming, just make
  - $w_0 = 1$
  - $w_1 = w$
  - $w_2 = w$
  - $w_3 = 1$
- Towards parameter m, it's produced by calculation according to known variables, which are the coordinates of P_0, P_3 and angle of θ, α. The equation is as followed:

$$w = \left(\frac{m-2}{m}\right)\frac{\overrightarrow{P_3P_0} \cdot \overrightarrow{P_2P_1}}{\overrightarrow{P_2P_1} \cdot \overrightarrow{P_2P_1}} \tag{2}$$

- The basic idea is that we need to ensure the equation (1) is quadratic about u for a 2D circle.
  - Since middle points P_1, P_2 have the same weight, so we can ensure the dominator is quadratic.
  - What we need to do left is that parameter ensure the coefficient of u^3 is 0. According to $B_i(u)$, we can get:

$$(2-m)(P_0 - P_3) + m(P_1 - P_2)w = 0 \tag{3}$$

  - From equation (3) and $\overrightarrow{P_2P_1} || \overrightarrow{P_3P_0}$, we can get equation (2) easily

How to get X-coordinate and Y-coordinate?

We still use the former picture to illustrate the parameters' definition.

Preparation

- $P_0 = (-1, 0)\ and\ P_1 = (1, 0)$

- Except for that, the other input parameter is angle α, which actually is angle OCP_3
  - point O is origin
  - point C is the center of circle
    - $C(0, cot\alpha)$
  - point P_3 is the end point
- From the geometric principle, we can calculate the radius r and exact coordinates of P_1, $P2$
  - $r = csc\alpha$
  - $P_1(-rcot\frac{\alpha}{2}, r + rcos\alpha)$
  - $P_2(rcot\frac{\alpha}{2}, r + rcos\alpha)$
- Finally, we can get the X-coordinate and Y-coordinate:

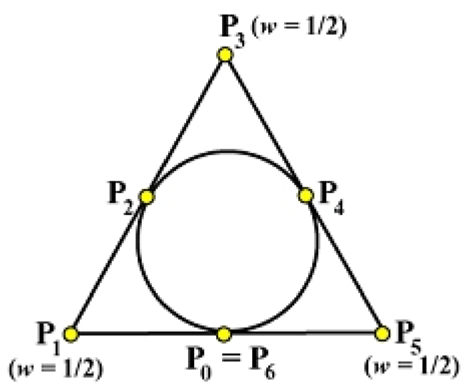
$$
\begin{aligned}
r(u) &= (x, y) \\
x(u) &= \frac{2u-1}{1+2(u-1)u+(m-2)(u-1)ucos\alpha} \\
y(u) &= \frac{(m-2)(u-1)usin\alpha}{1+2(u-1)u+(m-2)(u-1)ucos\alpha}
\end{aligned}
$$

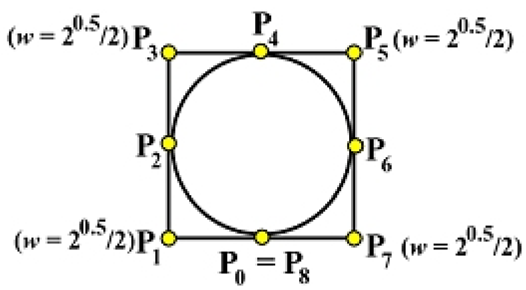
- We can use equation $x^2(u) + (y(u) - cot\alpha)^2 = csc^2\alpha$ to prove the correctness of $x(u)$ and $y(u)$

How to draw?

- When using OpenGL to implement drawing a circle, I just use the parametric coordinates to draw.
- All input parameter is angle α, and P_0, P_1 is fixed, while all other can be calculated by equation.
- By adjusting the angle to limit, we can get a nearly circle, but still exists a very small gap, so the perfect solution still needs to draw twice, and one is for , the other is for

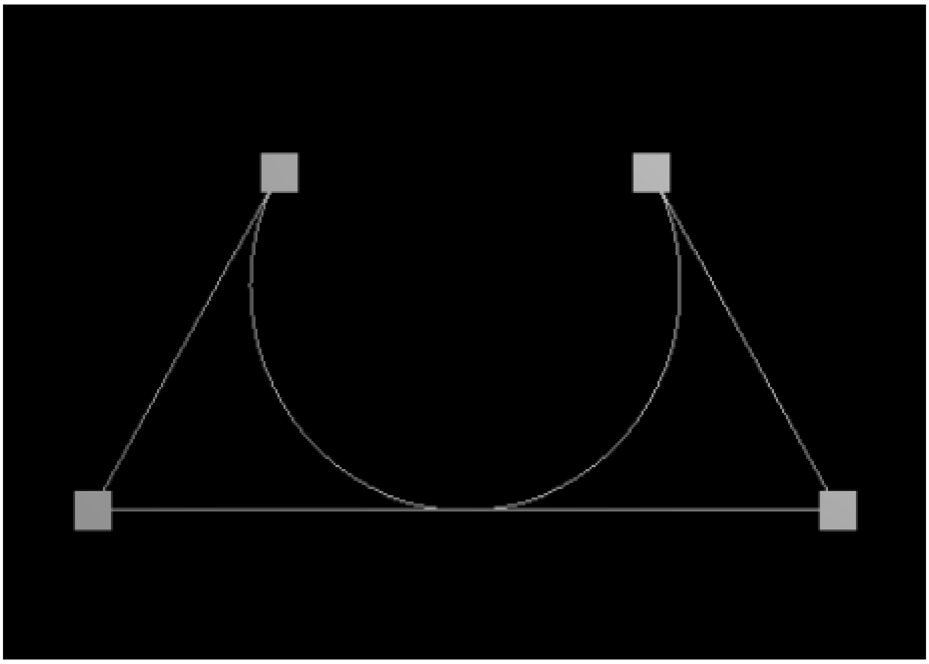
Related:



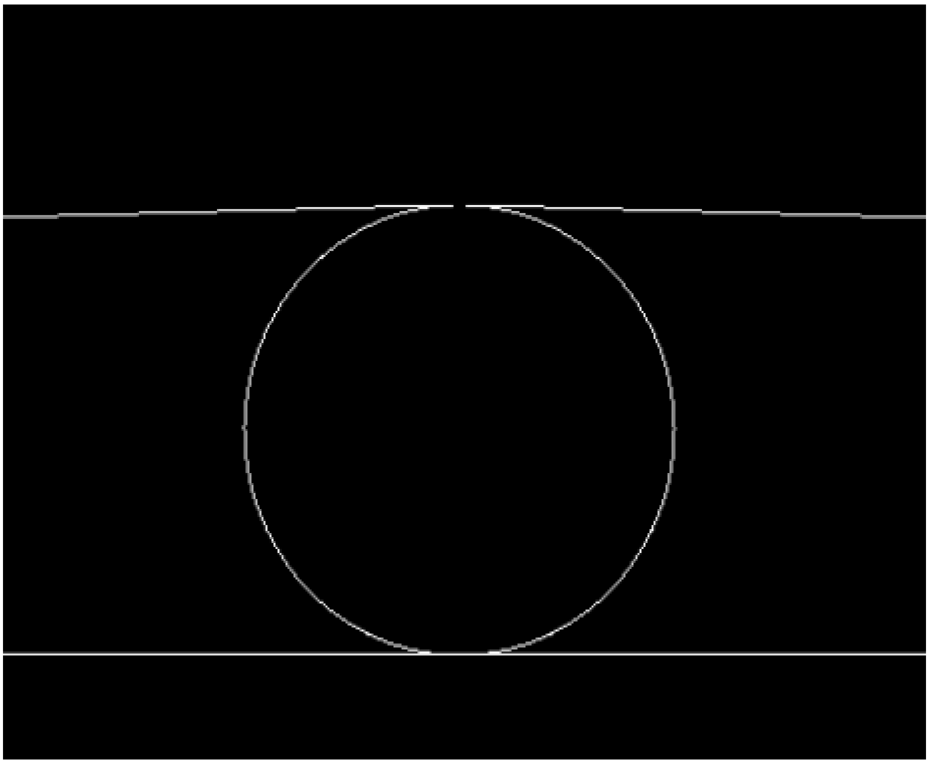


- I think the quadratic bezier curve is more easy to implement, and the weight is easy to calculate.

Info about program



- Four points
- Line is attachment of points
- current α is 60°



```
radius: 114.593
control point0:-1,0
control point1:-26262.6,-229.182
control point2:26262.6,-229.182
control point3:1,0
current weight: 3.80755e-05
```

- current α is 1°
- It's almost a circle
- In fact， I think using two semicircles is better choice, which mean we need to draw two quadratic bezier curves. But using one quadratic bezier curves can still get a nearly circle

Surface Revolution by NURBS

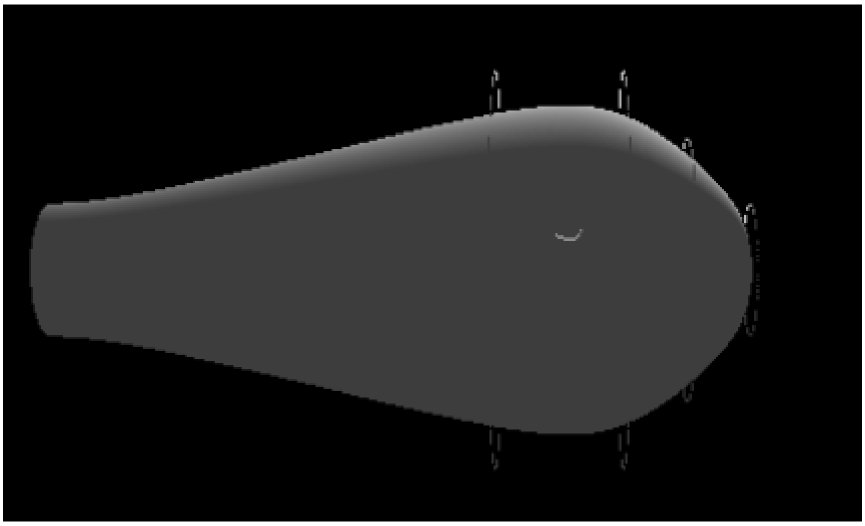
- For bonus, I don't know the meaning of it, because bezier curve is one special case of NURBS, and to get this bonus, I just need to **rorate one bezier curve around some axis**, then I can get a surface of revolution.
  - There are 11 control points to draw a half a curve
  - using the equation following to calculate every sample points

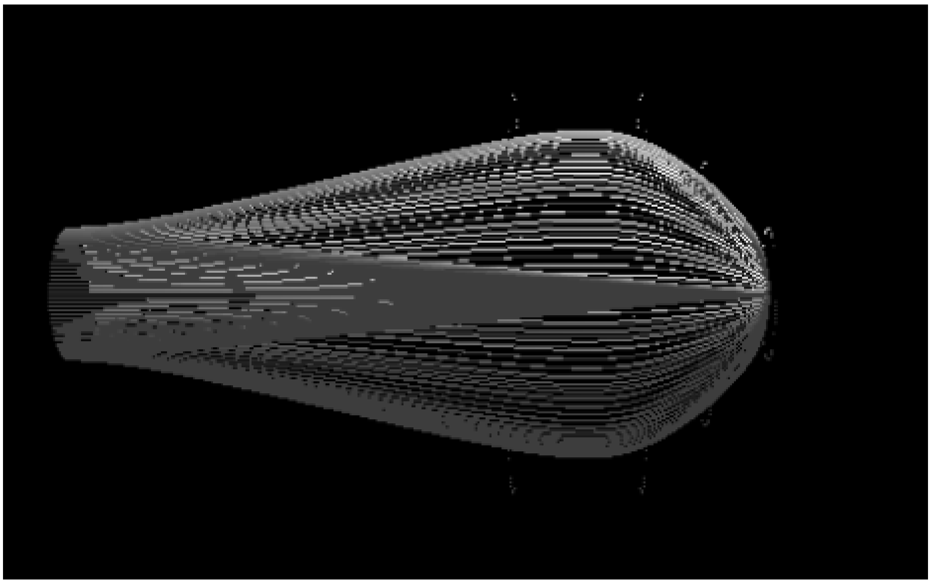
$$R(t) = \frac{\sum_{i=0}^{n} B_{i,n}(t) w_i P_i}{\sum_{i=0}^{n} B_{i,n}(t) w_i} \tag{4}$$

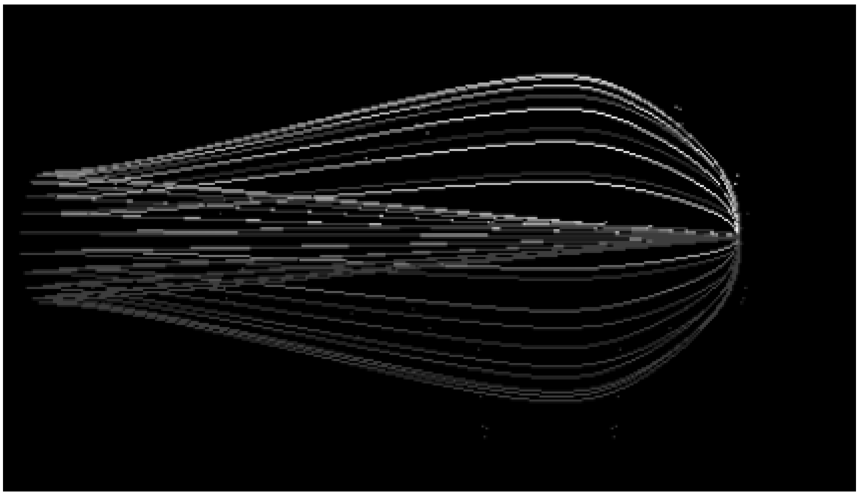
$$B_{i,n}(t) = C_n^i t^i (1-t)^{n-i}$$

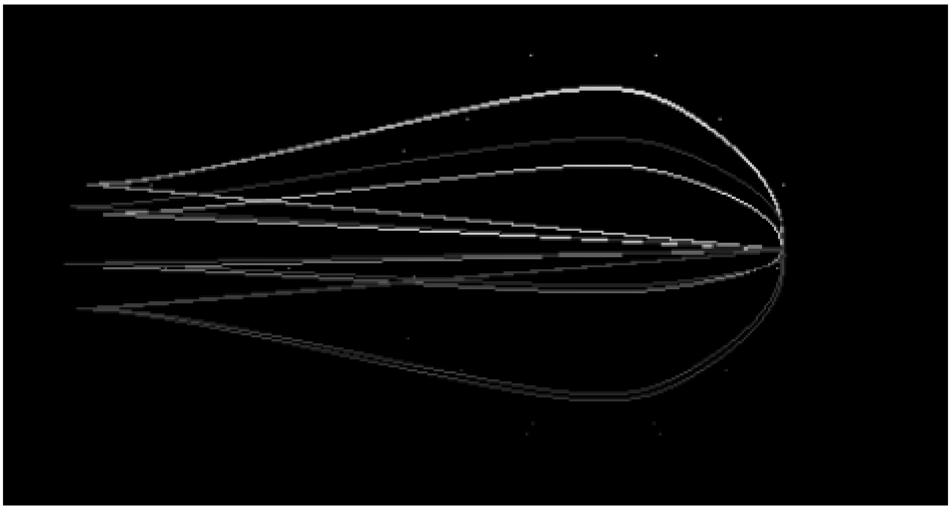
$$t \in [0, 1]$$

  - The demo as followed:









- So besides drawing a simple surface, I find a program can calculate NURBS surface directly, and it's very beutiful and more efficient.