

# A Fully-Automated Solver for Multiple Square Jigsaw Puzzles Using Hierarchical Clustering

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- ▶ First jigsaw puzzle introduced in the 1760s. Modern jigsaw puzzles were introduced in the 1930s.
- ▶ First computation jigsaw puzzle solver introduced in 1964.
- ▶ Solving a jigsaw puzzle is NP Complete [1, 2]



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- ▶ Solving a jigsaw puzzle is NP Complete [1, 2]
- ▶ **Example Applications:** DNA fragment reassembly, shredded document reconstruction, speech descrambling, and image editing.



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- ▶ First computation jigsaw puzzle solver introduced in 1964.
- ▶ Solving a jigsaw puzzle is NP Complete [1, 2]
- ▶ **Example Applications:** DNA fragment reassembly, shredded document reconstruction, speech descrambling, and image editing.
  - ▶ In most cases, the original, “ground-truth” image is unknown.



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## Jigsaw Puzzles – Variant of the traditional jig saw puzzle

- ▶ All pieces are equal-sized squares
- ▶ Substantially more difficult



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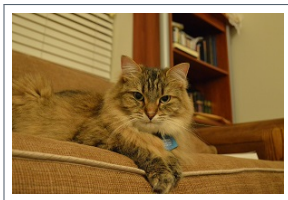
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Ground-Truth Image



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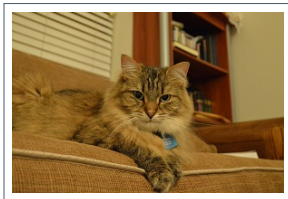
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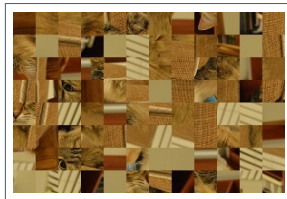
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**Jigsaw Puzzles** – Variant of the traditional jig saw puzzle

- ▶ All pieces are equal-sized squares
- ▶ Substantially more difficult



Ground-Truth Image



Randomized Jig Swap Puzzle



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There are four primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is unknown.

- **Type 1:** Puzzle dimension and piece rotation are known. One or more “anchor” pieces are fixed in their correct location.



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- ▶ **Type 1:** Puzzle dimension and piece rotation are known. One or more “anchor” pieces are fixed in their correct location.
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.



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- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
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- ▶ **Type 1:** Puzzle dimension and piece rotation are known. One or more “anchor” pieces are fixed in their correct location.
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
- ▶ **Type 3:** All piece locations are known. Only rotation is unknown.
- ▶ **Mixed-Bag:** Pieces come from multiple puzzles.



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There are four primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is unknown.

- ▶ **Type 1:** Puzzle dimension and piece rotation are known. One or more “anchor” pieces are fixed in their correct location.
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
- ▶ **Type 3:** All piece locations are known. Only rotation is unknown.
- ▶ **Mixed-Bag:** Pieces come from multiple puzzles.

Mixed-Bag Puzzles are the focus of this thesis.



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## Best Buddies

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- ▶ **Basis of all Modern Jig Swap Solvers:** The more compatible two pieces are on their respective sides, the more likely they are to be adjacent.
- ▶ **Best Buddies:** Any pair of puzzles pieces that are more compatible with each other on their respective sides than they are to any other piece [4]

$$\forall p_k \forall s_z, C(p_i, s_x, p_j, s_y) \geq C(p_i, s_x, p_k, s_z)$$

and (1)

$$\forall p_k \forall s_z, C(p_j, s_y, p_i, s_x) \geq C(p_j, s_y, p_k, s_z)$$

- ▶ **Importance of Best Buddies:** Strong indicator of piece adjacency



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- ▶ **Cho *et al.* [5]** – Introduced the first Modern Jig Swap Puzzle Solver Introduced
  - ▶ Graphical model-based Type 1 solver
  - ▶ Puzzle dimensions are known
  - ▶ Used one or more anchor pieces
  - ▶ Defined quality metrics for Type 1 and Type 2 puzzles
  - ▶ Established the standard comparative test conditions



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- ▶ **Cho *et al.* [5]** – Introduced the first Modern Jig Swap Puzzle Solver Introduced
  - ▶ Graphical model-based Type 1 solver
  - ▶ Puzzle dimensions are known
  - ▶ Used one or more anchor pieces
  - ▶ Defined quality metrics for Type 1 and Type 2 puzzles
  - ▶ Established the standard comparative test conditions
- ▶ **Pomeranz *et al.* [4]** – Iterative, greedy Type 1 puzzle solver
  - ▶ Eliminated the use of anchor pieces
  - ▶ Created multiple solver benchmarks of various sizes
  - ▶ Introduced the concept of “best buddies”

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## Paikin & Tal [6] – Current State of the Art

- ▶ Greedy, kernel growing solver
- ▶ Supports Type 1, Type 2, and Mixed-Bag puzzles
- ▶ Immune to missing pieces



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Paikin & Tal

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## Paikin & Tal [6] – Current State of the Art

- ▶ Greedy, kernel growing solver
- ▶ Supports Type 1, Type 2, and Mixed-Bag puzzles
- ▶ Immune to missing pieces

## Limitations

- ▶ **Poor Seed Selection:** All decisions are made at runtime using as few as 13 pieces
- ▶ **Externally Supplied Information:** The solver must be told the number of input puzzles

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**Primary Goal:** Develop a puzzle solver for Mixed-Bag jig swap puzzles with performance that exceeds the state of the art.



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**Primary Goal:** Develop a puzzle solver for Mixed-Bag jig swap puzzles with performance that exceeds the state of the art.

### Additional Goals:

- ▶ Develop new metrics for quantifying the performance of Mixed-Bag puzzles
- ▶ Design new best buddy and solver output visualizations

# The Mixed-Bag Solver





# Mixed-Bag Solver

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## Paikin & Tal's Algorithm

- ▶ Begin each puzzle with a single piece
- ▶ Place all pieces around the expanding core



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## Paikin & Tal's Algorithm

- ▶ Begin each puzzle with a single piece
- ▶ Place all pieces around the expanding core

## Alternate Jigsaw Puzzle Solving Strategy

- ▶ Correctly assemble small puzzle regions (i.e., segments)
- ▶ Iteratively merge smaller regions to form large ones



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## Paikin & Tal's Algorithm

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- ▶ Place all pieces around the expanding core

## Alternate Jigsaw Puzzle Solving Strategy

- ▶ Correctly assemble small puzzle regions (i.e., segments)
- ▶ Iteratively merge smaller regions to form large ones
- ▶ **Advantages of this Approach:**
  - ▶ Reduces the size of the problem
  - ▶ Provides structure to the unordered set of puzzle pieces.





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## Alternate Jigsaw Puzzle Solving Strategy

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- ▶ Iteratively merge smaller regions to form large ones
- ▶ **Advantages of this Approach:**
  - ▶ Reduces the size of the problem
  - ▶ Provides structure to the unordered set of puzzle pieces.

The latter strategy is the basis of the **Mixed-Bag Solver**.



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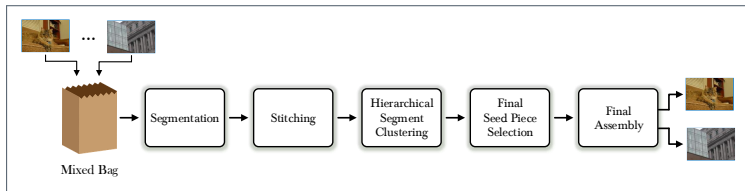
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- ▶ The Mixed-Bag Solver is fully-automated. It makes no assumptions concerning the piece orientation, puzzle dimensions, or number of puzzles.
  - ▶ **Input:** Set of puzzle pieces
  - ▶ **Output:** One or more disjoint, solved puzzles.
- ▶ The Mixed-Bag Solver consists of five distinct stages:



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# Assembler

## Mixed-Bag Solver Component

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- **Role:** Place the individual pieces in the solved puzzle.
  - Mixed-Bag Solver is independent of the assembler used, giving the solver significant upgradability and flexibility.

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- ▶ **Role:** Place the individual pieces in the solved puzzle.
  - ▶ Mixed-Bag Solver is independent of the assembler used, giving the solver significant upgradability and flexibility.
- ▶ **Assembler Used in this Thesis:** Paikin & Tal
  - ▶ Current state of the art
  - ▶ Allows for more direct comparison of performance
  - ▶ Natively supports Mixed-Bag puzzles
- ▶ **Implementation:** Assembler re-implemented as part of this thesis based off the description in [6]
  - ▶ Written in the Python language and fully open source.



# Segmentation

## Mixed-Bag Solver Stage #1

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- ▶ **Segment:** Partial puzzle assembly where this is a high degree of confidence pieces are placed correctly.
- ▶ **Role of Segmentation:** Provide structure to the set of puzzle pieces by partitioning them into disjoint segments
  - ▶ **Input:** Set of puzzle pieces
  - ▶ **Output:** Set of saved segments
- ▶ **Relationship between Puzzle Pieces and Segments:**
  - ▶ Pieces from a single ground-truth input may be separated into multiple segments
  - ▶ A piece can be assigned to at most one segment

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- ▶ Iterative process consisting of one or more rounds
- ▶ In each round, all pieces not yet assigned to a segment are assembled as if they are all from the same input image.
- ▶ Segments of sufficient size are saved to be used in future Mixed-Bag Solver stages
- ▶ Pieces in a saved segment are not placed in future rounds.
- ▶ Segmentation terminates if all pieces are assigned to a saved segment or no segment is larger than the minimum allowed size.

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PLACEHOLDER



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## Example – Input Images

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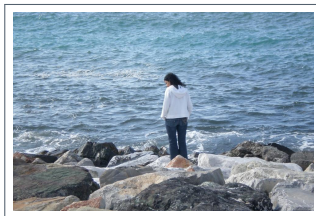


Image (a) – 805 Pieces [7]

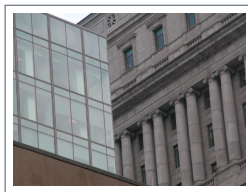


Image (b) – 540 Pieces [8]





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Example – First Segmentation Round Output Image

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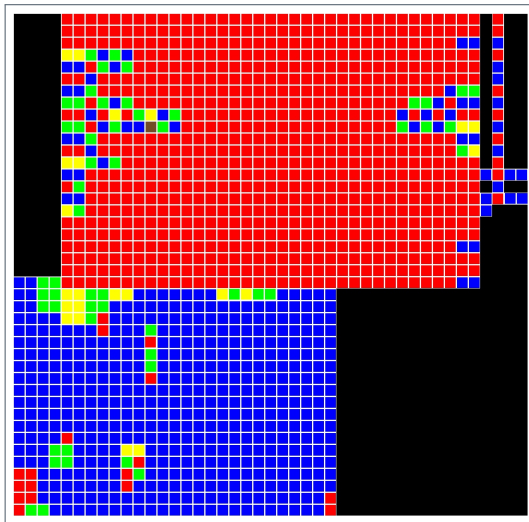
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# Stitching

## Mixed-Bag Solver Stage #2

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- ▶ **Role of Stitching:** Quantify the extent that any pair of segments is related.
  - ▶ **Input:** All puzzle pieces and the set of saved segments
  - ▶ **Output:** Segment overlap matrix



# Stitching

## Example – Input Image

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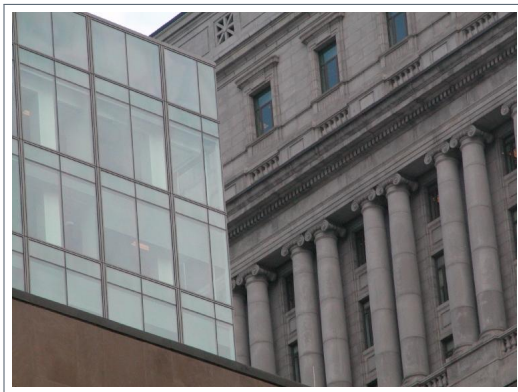
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## Example – Two Segment Images

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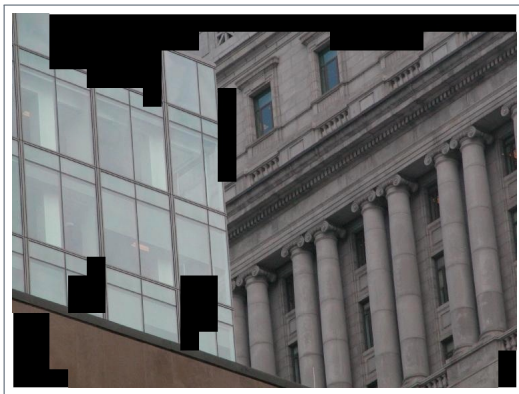
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Segment #1



Segment #2



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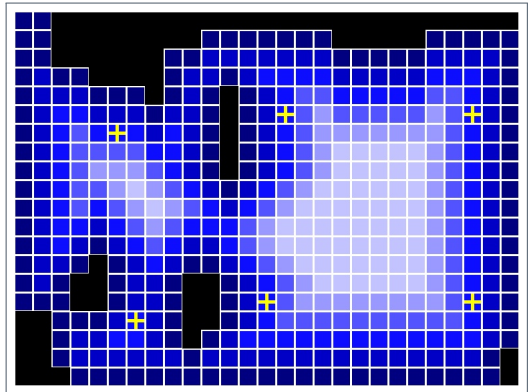
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Segment #1



Segment #2



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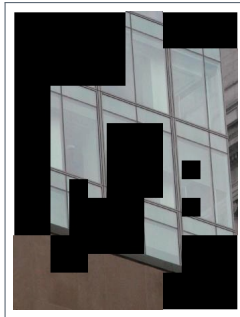
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Stitching Result from Segment #1

**Segment Overlap:**

$$Overlap_{\phi_1, \phi_2} = 0.83$$

(2)



# Hierarchical Segment Clustering

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# Final Seed Piece Selection

## Mixed-Bag Solver Stage #4

### Paikin & Tal

- All puzzle seeds are selected greedily at run time, which often leads to poor decisions.

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# Final Seed Piece Selection

## Mixed-Bag Solver Stage #4

### Paikin & Tal

- ▶ All puzzle seeds are selected greedily at run time, which often leads to poor decisions.

### Mixed-Bag Solver

- ▶ **Role of Final Seed Selection:** Determine the pieces that will be used as the seed for the final output puzzles.
  - ▶ **Input:** Set of cluster segments
  - ▶ **Output:** Final seed pieces
- ▶ A single seed piece is selected from each segment cluster
- ▶ Each seed piece is “distinctive” as defined by Paikin & Tal

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# Final Assembly Stage

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- ▶ **Role of Final Assembly:** Generate the solved puzzles that will be returned by the solver. by placing all pieces around the previously selected seeds.
  - ▶ **Input:** Set of puzzle pieces with the seeds marked
  - ▶ **Output:** Final solved puzzles
- ▶ All pieces are placed around the seeds selected in the previous stage.
- ▶ Assembly proceeds in this stage normally without any custom modifications.

# Quantifying Solver Quality





# Quantifying Solver Quality

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- ▶ Jigsaw puzzle solvers are not yet able to always correctly reconstruct the input puzzle(s)
  - ▶ Metrics compare the quality of solved outputs
- ▶ **Two Most Common Quality Metrics:**
  - ▶ Direct Accuracy
  - ▶ Neighbor Accuracy

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- ▶ Jigsaw puzzle solvers are not yet able to always correctly reconstruct the input puzzle(s)
  - ▶ Metrics compare the quality of solved outputs
- ▶ **Two Most Common Quality Metrics:**
  - ▶ Direct Accuracy
  - ▶ Neighbor Accuracy
- ▶ **Disadvantages of Current Metrics:** Neither account for:
  - ▶ Pieces misplaced in different puzzles
  - ▶ Extra pieces from other puzzles
- ▶ **Goal:** Define new Mixed-Bag puzzle quality metrics.



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Standard and Enhanced Direct Accuracy

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- **Standard Direct Accuracy:** Fraction of pieces ( $c$ ) placed in the same location in both the ground-truth and solved image versus the total number of pieces ( $n$ )

$$DA = \frac{c}{n} \quad (3)$$

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- **Standard Direct Accuracy:** Fraction of pieces ( $c$ ) placed in the same location in both the ground-truth and solved image versus the total number of pieces ( $n$ )

$$DA = \frac{c}{n} \quad (3)$$

- **Enhanced Direct Accuracy Score (EDAS):** Modified direct accuracy that accounts for missing and extra pieces.

$$EDAS_{P_i} = \arg \max_{S_j \in S} \frac{c_{i,j}}{n_i + \sum_{k \neq i} (m_{k,j})} \quad (4)$$

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- **Standard Direct Accuracy:** Fraction of pieces ( $c$ ) placed in the same location in both the ground-truth and solved image versus the total number of pieces ( $n$ )

$$DA = \frac{c}{n} \quad (3)$$

- **Enhanced Direct Accuracy Score (EDAS):** Modified direct accuracy that accounts for missing and extra pieces.

$$EDAS_{P_i} = \arg \max_{S_j \in S} \frac{c_{i,j}}{n_i + \sum_{k \neq i} (m_{k,j})} \quad (4)$$

- **Direct Accuracy Range:** 0 to 1
- **Perfectly Reconstructed Image:** All pieces are placed in their original location ( $DA = EDAS = 1$ )



# Direct Accuracy

Effect of Shifts

**Problem:** Direct accuracy is highly vulnerable to shifts, in particular when puzzle dimensions are not fixed

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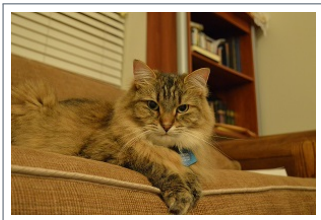
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**Problem:** Direct accuracy is highly vulnerable to shifts, in particular when puzzle dimensions are not fixed



Ground-Truth Image



# Direct Accuracy

## Effect of Shifts

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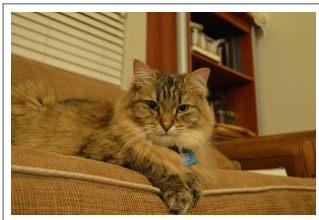
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**Problem:** Direct accuracy is highly vulnerable to shifts, in particular when puzzle dimensions are not fixed



Ground-Truth Image



Solver Output



# Direct Accuracy

## Effect of Shifts

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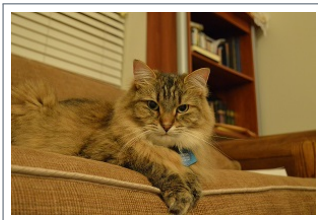
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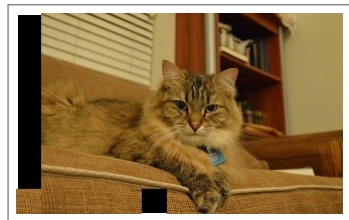
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**Problem:** Direct accuracy is highly vulnerable to shifts, in particular when puzzle dimensions are not fixed



Ground-Truth Image



Solver Output

**Conclusion:** Direct accuracy can be overly punitive.



# Direct Accuracy

Shiftable Enhanced Direct Accuracy Score (SEDAS)

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- ▶ **Solution:** Allow the reference point for direct accuracy to shift beyond the upper left corner of the image
- ▶ **Shiftable Enhanced Direct Accuracy Score (SEDAS):** Select the reference point,  $l$ , within radius  $d_{min}$  of the upper left corner of the solved puzzle
  - ▶  $d_{min}$  - Manhattan distance between the upper left corner of the solved image and the nearest puzzle piece

## ▶ Formal Definition of SEDAS:

$$SEDAS_{P_i} = \arg \max_{l \in L} \left( \arg \max_{s_j \in S} \frac{c_{i,j,l}}{n_i + \sum_{k \neq i} (m_{k,j})} \right) \quad (5)$$

- ▶ **SEDAS Range:** 0 to 1



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- **Standard Neighbor Accuracy:** Ratio of puzzle piece sides adjacent in both the original and solved images ( $a$ ) versus the total number of sides ( $n \cdot q$ )

$$NA = \frac{a}{n \cdot q} \quad (6)$$

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- **Standard Neighbor Accuracy:** Ratio of puzzle piece sides adjacent in both the original and solved images ( $a$ ) versus the total number of sides ( $n \cdot q$ )

$$NA = \frac{a}{n \cdot q} \quad (6)$$

- **Enhanced Neighbor Accuracy Score (ENAS):** Modified neighbor accuracy that accounts for missing and extra pieces.

$$ENAS_{P_i} = \arg \max_{S_j \in S} \frac{a_{i,j}}{q(n_i + \sum_{k \neq i} (m_{k,j}))} \quad (7)$$

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- ▶ **Standard Neighbor Accuracy:** Ratio of puzzle piece sides adjacent in both the original and solved images ( $a$ ) versus the total number of sides ( $n \cdot q$ )

$$NA = \frac{a}{n \cdot q} \quad (6)$$

- ▶ **Enhanced Neighbor Accuracy Score (ENAS):** Modified neighbor accuracy that accounts for missing and extra pieces.

$$ENAS_{P_i} = \arg \max_{S_j \in S} \frac{a_{i,j}}{q(n_i + \sum_{k \neq i} (m_{k,j}))} \quad (7)$$

- ▶ **Neighbor Accuracy Range:** 0 to 1
- ▶ **Advantage of Neighbor Accuracy:** Less vulnerable to shifts than direct accuracy



# Quantifying Solver Quality

## Visualizing Accuracy

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- ▶ The thesis includes visualization standards for direct and neighbor accuracy.
- ▶ They are not reviewed here due to limited time.

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# Quantifying Solver Quality

## Best Buddy Density

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- ▶ **Best Buddies Review:** Any pair of puzzles pieces that are more compatible with each other on their respective sides than they are to any other piece

- ▶ **Note:** Not all puzzle pieces will have a best buddy.

- ▶ **Best Buddy Density (BBD):** A metric for quantifying the best buddy profile of an image that is independent of image size.

$$BBD = \frac{b}{n \cdot q} \quad (8)$$

- ▶ A greater BBD means the pieces are more differentiated making the puzzle easier to solve.



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## Visualizing Best Buddy Density

- ▶ Transform each puzzle piece into a square consisting of four isosceles triangles.
- ▶ Color each triangle according to whether the adjacent piece is a best buddy. The scheme used in this thesis:

No Best Buddy	Non-Adjacent Best Buddy	Adjacent Best Buddy	No Piece Present

- ▶ Areas with higher best buddy density will have more green triangles.



# Best Buddy Density

## Visualization Example

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(a) Original Image [9]

Figure: Visualization of Best Buddy Density

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# Best Buddy Density

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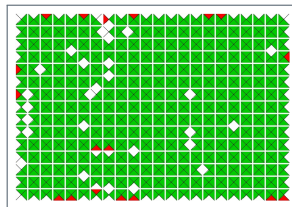
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(a) Original Image [9]



(b) Best Buddy  
Visualization

Figure: Visualization of Best Buddy Density

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# Experimental Results

- Paikin & Tal's algorithm is the current state of the art and was used as the basis for all performance comparisons.

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- ▶ Paikin & Tal's algorithm is the current state of the art and was used as the basis for all performance comparisons.
- ▶ **Standard Test Conditions**
  - ▶ **Puzzle Type:** 2
  - ▶ **Dimensions Fixed:** No
  - ▶ **Piece Width:** 28 pixels
  - ▶ **Benchmark:** Twenty 805 piece images [7]
  - ▶ **Image Encoding:** LAB colorspace

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- ▶ Paikin & Tal's algorithm is the current state of the art and was used as the basis for all performance comparisons.
- ▶ **Standard Test Conditions**
  - ▶ **Puzzle Type:** 2
  - ▶ **Dimensions Fixed:** No
  - ▶ **Piece Width:** 28 pixels
  - ▶ **Benchmark:** Twenty 805 piece images [7]
  - ▶ **Image Encoding:** LAB colorspace
- ▶ **Number of Ground-Truth Inputs:** 1 to 5

# Puzzles	1	2	3	4	5
# Iterations	20	55	25	8	5

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- ▶ Paikin & Tal's algorithm is the current state of the art and was used as the basis for all performance comparisons.

- ▶ **Standard Test Conditions**

- ▶ **Puzzle Type:** 2
- ▶ **Dimensions Fixed:** No
- ▶ **Piece Width:** 28 pixels
- ▶ **Benchmark:** Twenty 805 piece images [7]
- ▶ **Image Encoding:** LAB colorspace

- ▶ **Number of Ground-Truth Inputs:** 1 to 5

# Puzzles	1	2	3	4	5
# Iterations	20	55	25	8	5

- ▶ **Test Condition Variation:** Only Paikin & Tal's algorithm was provided the number of input puzzles.

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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the number of input puzzles
  - ▶ **Importance** – The Mixed-Bag Solver must estimate this accurately to provide meaningful outputs.



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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the number of input puzzles
  - ▶ **Importance** – The Mixed-Bag Solver must estimate this accurately to provide meaningful outputs.
- ▶ **Two Subexperiments:**
  - ▶ **Single Puzzle Accuracy** – This represents the solver's performance ceiling
  - ▶ **Multiple Puzzle Accuracy** – A more general estimate of the solver's performance

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# Determining Input Puzzle Count

## Single Input Puzzle Results

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- ▶ **Summary:** 17 out of the 20 images were correctly identified as a single ground-truth input
- ▶ **Misclassified Images:** 3 out of the 20 images misclassified as if they were two images.
  - ▶ All three images have large areas with little variation (e.g., blue sky, smooth water)
  - ▶ The solver's performance on these puzzles is due to the assembler as noted in [6]

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- ▶ **Summary:** 17 out of the 20 images were correctly identified as a single ground-truth input
- ▶ **Misclassified Images:** 3 out of the 20 images misclassified as if they were two images.
  - ▶ All three images have large areas with little variation (e.g., blue sky, smooth water)
  - ▶ The solver's performance on these puzzles is due to the assembler as noted in [6]
- ▶ **Note:** 85% (17/20) represents the maximum accuracy when solving multiple puzzles.

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## Comparison of Best Buddy Density for Misclassified Images

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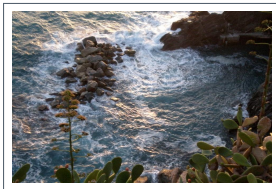
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Perfectly Reconstructed  
Image (a)



Misclassified Image (b)

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# Determining Input Puzzle Count

## Comparison of Best Buddy Density for Misclassified Images

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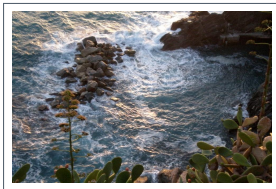
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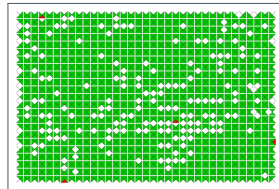
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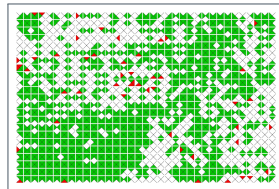
Perfectly Reconstructed  
Image (a)



Best Buddy Visualization (a)



Misclassified Image (b)



Best Buddy Visualization (b)



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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the input puzzle count for multiple images
- ▶ **Procedure:** Randomly select the specified number of images (between 2 and 5) from the 20 image data set.

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# Determining Input Puzzle Count

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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the input puzzle count for multiple images
- ▶ **Procedure:** Randomly select the specified number of images (between 2 and 5) from the 20 image data set.
- ▶ **Input Puzzle Count Error:** Difference between the actual number of input puzzles and that found by the Mixed-Bag Solver.
  - ▶ **Example:** If 3 images were supplied to the solver, but it determined there were 4, the error would be 1.

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# Determining Input Puzzle Count

## Multiple Input Puzzles – Results

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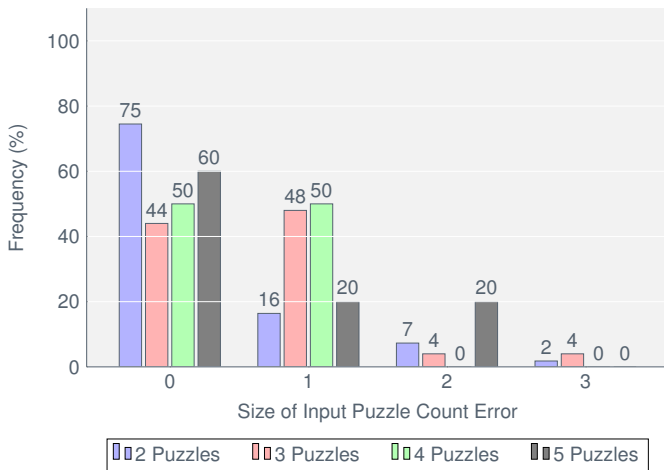
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### Mixed-Bag Solver's Input Puzzle Count Error Frequency





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- ▶ **Overall Accuracy: 65%**
- ▶ **Iterations with Error Greater than One: 8%**
- ▶ Accuracy did not significantly degrade as the number of input puzzles increased.

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# Determining Input Puzzle Count

## Multiple Input Puzzles – Results Summary

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- ▶ **Overall Accuracy:** 65%
- ▶ **Iterations with Error Greater than One:** 8%
- ▶ Accuracy did not significantly degrade as the number of input puzzles increased.
- ▶ **Over-Rejection of Cluster Merges:** The Mixed-Bag Solver never underestimated the number of input puzzles.
  - ▶ Performance may be improved by reducing the minimum clustering similarity threshold or minimum segment size

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- ▶ **Goal:** Compare the performance of the Mixed-Bag Solver and Paikin & Tal's algorithm
- ▶ **Procedure:** Randomly select a specified number of images and input them into both solvers.
- ▶ **Quality Metrics Used:**
  - ▶ Shiftable Enhanced Direct Accuracy Score (SEDAS)
  - ▶ Enhanced Neighbor Accuracy Score (ENAS)
  - ▶ Perfect Reconstruction Percentage

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- ▶ **Goal:** Compare the performance of the Mixed-Bag Solver and Paikin & Tal's algorithm
- ▶ **Procedure:** Randomly select a specified number of images and input them into both solvers.
- ▶ **Quality Metrics Used:**
  - ▶ Shiftable Enhanced Direct Accuracy Score (SEDAS)
  - ▶ Enhanced Neighbor Accuracy Score (ENAS)
  - ▶ Perfect Reconstruction Percentage
- ▶ **Note:** The results include the Mixed-Bag Solver's performance when it correctly estimated the puzzle count.
  - ▶ This represents the performance ceiling for optimal hierarchical clustering.

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# Performance on Multiple Input Puzzles

## Shiftable Enhanced Direct Accuracy Score (SEDAS)

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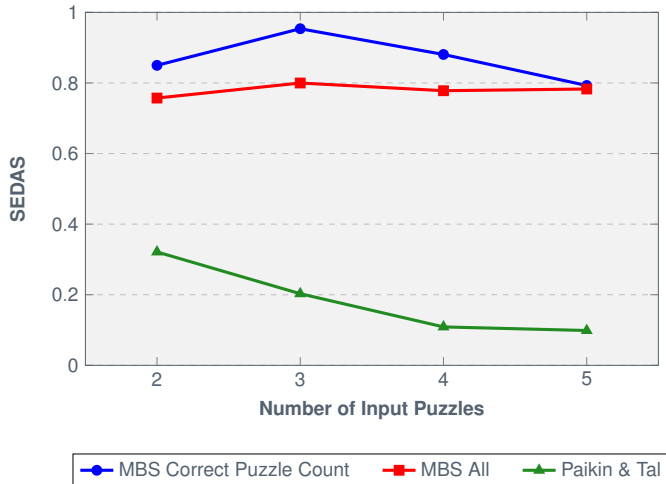
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### Effect of the Number of Input Puzzles on SEDAS





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## Enhanced Neighbor Accuracy Score (ENAS)

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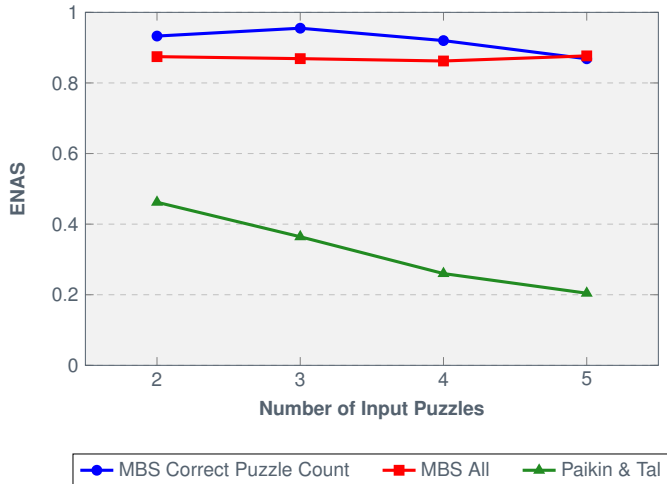
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## Perfect Reconstruction Percentage

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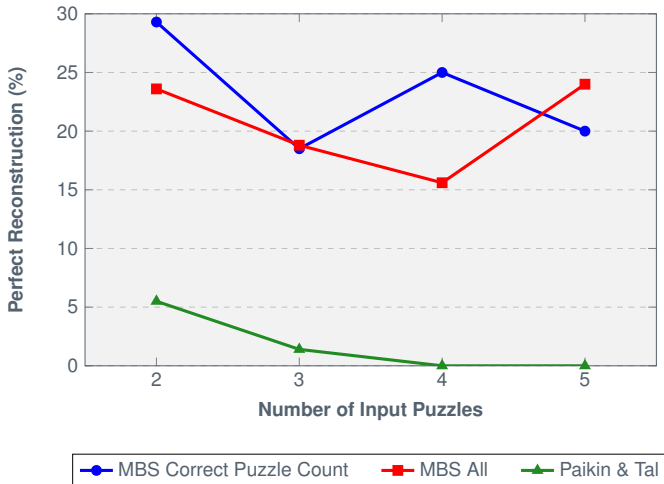
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- ▶ **Summary:** The Mixed-Bag Solver significantly outperformed Paikin & Tal's algorithm across all metrics.
  - ▶ This is notwithstanding that only their algorithm was supplied with the number of input puzzles.

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- ▶ **Summary:** The Mixed-Bag Solver significantly outperformed Paikin & Tal's algorithm across all metrics.
  - ▶ This is notwithstanding that only their algorithm was supplied with the number of input puzzles.
- ▶ **Puzzle Input Count:** Unlike Paikin & Tal's algorithm, the Mixed-Bag Solver saw no significant decrease in performance with additional input puzzles

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- ▶ **Summary:** The Mixed-Bag Solver significantly outperformed Paikin & Tal's algorithm across all metrics.
  - ▶ This is notwithstanding that only their algorithm was supplied with the number of input puzzles.
- ▶ **Puzzle Input Count:** Unlike Paikin & Tal's algorithm, the Mixed-Bag Solver saw no significant decrease in performance with additional input puzzles
- ▶ **Effect of Clustering Errors:** Performance only decreased slightly when incorrectly estimated input puzzle count.
  - ▶ Many of the extra puzzles were relatively insignificant in size

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- ▶ This thesis presented a fully-automated solver for Mixed-Bag jig swap puzzles.
- ▶ Mixed-Bag Solver significantly outperforms the current state of the art while receiving no externally supplied information.
- ▶ Introduced the first set of solver quality metrics for Mixed-Bag puzzles.

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- Improved Assembler
  - Prioritize placement using multiple best buddies



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- ▶ Improved Assembler
  - ▶ Prioritize placement using multiple best buddies
  - ▶ Address placement performance in regions with low best buddy density
- ▶ Dynamic determination of the segment clustering threshold



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- ▶ Improved Assembler
  - ▶ Prioritize placement using multiple best buddies
  - ▶ Address placement performance in regions with low best buddy density
- ▶ Dynamic determination of the segment clustering threshold
- ▶ Expanded stitching piece selection

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## Solving More than Five Puzzles

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- ▶ As the number of puzzles increases, the difficulty of simultaneously reconstructing them also increases.



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- ▶ As the number of puzzles increases, the difficulty of simultaneously reconstructing them also increases.
- ▶ **Current State of the Art:** Paikin & Tal [6] solved up to five puzzles simultaneously.





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- ▶ As the number of puzzles increases, the difficulty of simultaneously reconstructing them also increases.
- ▶ **Current State of the Art:** Paikin & Tal [6] solved up to five puzzles simultaneously.
- ▶ **Goal:** Compare the performance of the Mixed-Bag Solver and Paikin & Tal's algorithm on 10 puzzles.



# Ten Puzzle Results

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### ► Paikin & Tal

- Seed of nine images came from just three input images
- SEDAS and EDAS greater than 0.9 for only one image
- No perfectly reconstructed images



# Ten Puzzle Results

## Summary

A Fully-Automated  
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### ► Paikin & Tal

- Seed of nine images came from just three input images
- SEDAS and EDAS greater than 0.9 for only one image
- No perfectly reconstructed images

### ► Mixed-Bag Solver

- SEDAS and EDAS greater than 0.9 for all images
- Four images perfectly reconstructed
- Results comparable to Paikin & Tal's algorithm solving each puzzle individually



# Ten Puzzle Results

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### ► Paikin & Tal

- Seed of nine images came from just three input images
- SEDAS and EDAS greater than 0.9 for only one image
- No perfectly reconstructed images

### ► Mixed-Bag Solver

- SEDAS and EDAS greater than 0.9 for all images
- Four images perfectly reconstructed
- Results comparable to Paikin & Tal's algorithm solving each puzzle individually

- **Conclusion:** The performance difference between the Mixed-Bag Solver and Paikin & Tal's algorithm is even starker with more input puzzles.



# List of References I

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