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Square Jigsaw Puzzle Solver Literature Review

Prepared by: Zayd Hammoudeh
(zayd.hammoudeh@sjsu.edu)

Introduction

- "Jigsaw Puzzle Problem"
  - **Problem Statement:** Reconstruct an image from a set of image patches
  - **Problem Complexity:** NP-Complete (via the set partition problem) when pairwise affinity of pieces is unreliable [1]

- **Problem Formulation**: Set of square, non-overlapping patches
  - This specific type of puzzle is known as "*jig swap*" [7] or "*Type 1*" puzzles [19]
  - *Type 2 Puzzles*: Allow/disallow patch rotation [19]

- **A Key Difference with Standard Jigsaw Puzzle Solving:** The source image you are trying to reconstruct is unknown.

Square Jigsaw Puzzle Example

- Source image (left) is divided into 81 (9x9) uniform, square patches (center). The goal is to organize the patches to reconstruct the source image (right).

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Jigsaw Puzzle Solver Applicability

- Possible and existing applications of the jigsaw puzzle problem include:
  - **Computer Forensics**: Reconstructing deleted JPEG, block-based images [2]
  - **Document Investigation**: Reconstruct shredded documents [3]
  - **Bioinformatics**: DNA/RNA modelling and reconstruction [4]
  - **Archeology**: Reconstruction of damaged relics [5]
  - **Audio Processing**: Voice descrambling [6]

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Additional Variants of Problem

[9] proposes a list of additional variants to the jigsaw puzzle problem including:

- Missing piece(s)
- Extra piece(s)
- Unknown piece orientation (type 2 puzzle)
- Three dimensional puzzles
- Unknown puzzle dimension

Pairwise Affinity

- **Definition:** Quantifies the similarity/compatibility between two patches.

- Between two pieces x_i and x_j, there are 4 pairwise affinity values when rotation is not allowed and 16 when rotation is allowed.

- Metrics of particular interest in the literature are divided into two categories.
  - Boundary/Edge Based:
    - Normalized and Unnormalized Dissimilarity-based Compatibility
    - Prediction-based Compatibility

  - Statistical based using the entire patch and its statistical properties [14]

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Dissimilarity-Based Compatibility

- Proposed in Cho *et. al.* [7]

- Uses the LAB (lightness, and a/b color opponent dimensions), which is three (3) dimensions.

- Given two patches x_i and x_j that are size K pixels by K pixels, then left-right (LR) dissimilarity (where x_j is to the right of x_i) is:

$$D_{LR}(x_i, x_j) = \sum_{l=1}^{K} \sum_{d=1}^{3} \left(x_i(l, K, d) - x_j(l, 1, d)\right)^2$$

Where $x_m(r, c, d)$ is value for the pixel in row r and column c of patch x_m at dimension d.

- **Disadvantage of this Approach:**
  - Severely penalizes boundary differences between patches which *do* occur in actual images [10].
  - It is common that actual image does **not** the minimum dissimilarity. Hence, this "*better than perfect score*" where the solved solution has a lower score than the original is a type of overfitting [9].

$\left(L_p\right)^q$ Dissimilarity-Based Compatibility

- Proposed by Pomeranz *et. al*. in [10]. *Generalizes* the dissimilarity metric from [7] with the L_p norm.

$$D_{p,q}(x_i, x_j) = \left(\sum_{l=1}^{K} \sum_{d=1}^{3} \left| x_i(l, K, d) - x_j(l, 1, d) \right|^p \right)^{\frac{q}{p}}$$

Hence, [7]'s metric is essentially the $(L_2)^2$ norm.

- While q has no effect on the patch pairwise classification accuracy, [10] observed it had an effect on their solver's performance

Prediction-Based Compatibility

- The dissimilarity based approach measured the difference between two pieces.
  - Prediction based attempts to predict the boundary pixel value of the neighboring patch.

- **First-Order Example**:
  - Use the last two pixels of each patch to predict the neighboring piece's value.
  - Gradient between two right edge pixels for patch x_i in row l for dimension d:

$$x_i(l, K, d) - x_i(l, K - 1, d)$$

  - Gradient between two left edge pixels for patch x_j row l for dimension d:

$$x_j(l, 1, d) - x_i(l, 2, d)$$

Prediction-Based Compatibility (Continued)

- The two pixel gradient can be combined with the dissimilarity-based compatibility as shown below for patch x_i's right edge:

$$\big(x_i(l,K,d) - x_j(l,1,d)\big) + \big(x_i(l,K,d) - x_i(l,K-1,d)\big)$$

which is equivalent to:

$$\big(2 * x_i(l,K,d) - x_i(l,K-1,d)\big) - x_j(l,1,d)$$

- If the $\left(L_p\right)^q$ dissimilarity is used, the entire prediction based compatibility for the left-right boundary of x_i and x_j is:

$$\sum_{l=1}^{K}\sum_{d=1}^{3}\left(\left|\big(2 * x_i(l,K,d) - x_i(l,K-1,d)\big) - x_j(l,1,d)\right|^p + \left|\big(2 * x_j(l,1,d) - x_j(l,2,d)\big) - x_i(l,K,d)\right|^p\right)^{\frac{q}{p}}$$

- **Advantage of this Approach:** Incorporates a predictor of the pairwise change which may help detect expected pixel pairwise differences.

Accuracy Comparison of the Compatibility Metrics

- Pomeranz *et. al.* in [10] compared the accuracy of the three compatibility metrics on 20 images in a test dataset.

- Using the $(L_p)^q$ norm resulted in a 7% to 10% improvement in selecting the correct neighbor.

- The impact of using the prediction-technique varied from no change up to a 3% improvement.

Puzzle Size	Dissimilarity-Based	$(L_{3/10})^{1/16}$	Prediction-Based
432 Patches	78%	**86%**	**86%**
540 Patches	76%	85%	**88%**
805 Patches	74%	84%	**86%**

Comparison of Pairwise Similarity Metric Accuracy

Asymmetric Dissimilarity

- The previous definitions of pairwise affinity have been symmetrically similar such that:

$$D\left(p_i, p_j, right\right) = D(p_j, p_i, left)$$

- [20] proposes using an asymmetric dissimilarity such that equality in the above equation does not hold.

- One sided, L_1 version of Pomeranz *et. al.*'s prediction based distance as shown below:

$$D\left(x_i, x_j, right\right) = \sum_{l=1}^{K} \sum_{d=1}^{3} \left\| \left(2 * x_i(l, K, d) - x_i(l, K-1, d)\right) - x_j(l, 1, d) \right\|$$

Benefits of Asymmetric Dissimilarity

- Three times faster due to the elimination of the exponent (80% of runtime is in distance calculations)
  - Additional speedup can be gained if when the asymmetric dissimilarity is sufficiently large (i.e. no chance of a pairing), the calculation is stopped and the distance set to infinity.
- Number of correct "best buddies" increased
- Number of incorrect decreased
- Using the benchmark in [17], the number of correctly solved puzzles increased from 25 to 37.

Performance Metrics

- **Summary:** Evaluate the performance of a jigsaw puzzle solver against the original (correct) image.

- Cho *et. al.* proposed three performance metrics:
  - *Direct Comparison Method:* Most naïve approach. The ratio of the number of patches in their correct locations versus the total number of patches.
    - *Disadvantage:* Susceptible to shifts
  - *Neighbor Comparison Method:* For each patch, calculate the fraction of the four neighbors that are correct. The total accuracy is the average neighbor accuracy of all patches.

"Best Buddies"

Definition: Two patches are *best buddies* if they are more similar to each other on their respective sides than they are two any other pieces [10].

Hence, two patches, x_i and x_j, are said to be "best buddies" for a spatial relationship R. if and only if, two conditions hold:

$$\forall x_k \in \{Patches\}, C\left(x_i, x_j, R_1\right) \geq C(x_i, x_k, R_1)$$

$$\forall x_k \in \{Patches\}, C\left(x_j, x_i, R_2\right) \geq C(x_j, x_k, R_2)$$

Where:

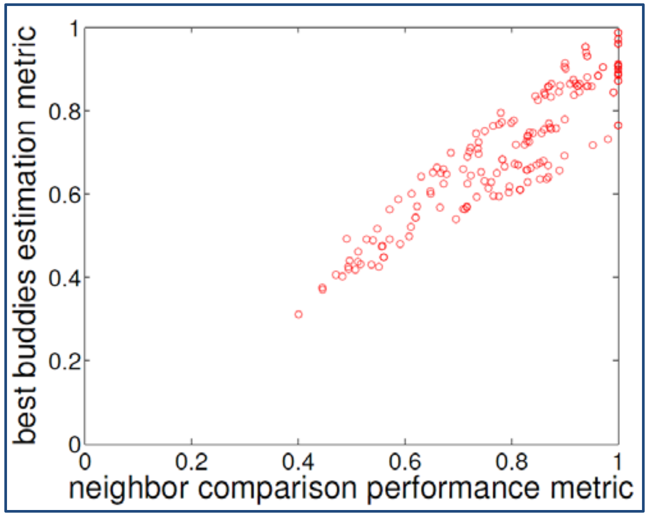
- $C(x_i, x_j, R_1)$ – Compatibility between patches x_i and x_j on side R_i of x_i
- $\{Patches\}$ – Set of all patches in the puzzle
- R_1 – One of the four sides (e.g. top, bottom, left, right) of x_i where x_j will be placed assuming no rotation.
- R_2 - Given x_i and R_1, this represents the complementary side of x_j. For example if R_1 is "left", then R_2 would be "right"

"Best Buddies" Estimation Metric

- **Definition:** Ratio of the number of neighbors who are said to be "best buddies" to the total number of neighbors [10].

- Correlation between the "Best Buddies" Estimation Metric and Cho *et. al.*'s two performance metrics:
  - *Direct Comparison Metric*: Little to no correlation since direct comparison method is not based on pairwise accuracy.

  - *Neighbor Comparison Metric*: Stronger correlation Graph below is for 20 images tested 10 times each (for 200 total points)



Scatterplot of "Best Buddy" Metric versus Neighbor Comparison Metric

Existing Square Jigsaw Puzzle Approaches

- Dynamic Programming and the "Hungarian" Procedure [13]
- Patch Transform using a Low Resolution "Solution Image" [8]
- "Dense and Noisy" or "Sparse and Accurate" with Loopy Belief Propagation [7]
- Particle Filter-Based Solver [11]
- Greedy Algorithm [10]
- Genetic Algorithm [9]

- **Variations of the Problem**
  - Unknown orientation (i.e. rotation) and puzzle dimensions [12]

Patch Transform

- Introduced by Cho *et. al.* in [8]

- **Overview of the Patch Transform**: Segment a source image into a set of non-overlapping "patches" and rearrange these patches and reorganize the image in the "patch" domain.
  - *Intended Usage*: Image editing

- **"Inverse" Patch Transform**: Reconstruct an image from a set of patches. This requires two components:
  - A patch compatibility function
  - An algorithm that places all patches

- Uses a provided low resolution image as part of the patch placement algorithm.

Markov Random Field

- Use a Markov Random Field (MRF) to enforce three rules:
  - Adjacent pieces should fit plausibly together
  - A patch should "*never*" (or in the loosened case "*seldomly*") be reused.
  - User constraints (e.g. board size) on patch placement.

- Consider each possible patch location as a node in the MRF. The key notation definitions:
  - x_i – Undetermined state for the node i^{th} in the MRF.
  - $\psi_{i,j}(k, l)$ – Compatibility between patches k and l at adjacent MRF locations i and j
  - X – Vector of N determined patch indices, x_i
  - Y – Low resolution version of the original image.

Maximizing the Patch Assignment Probability

For a given patch assignment X, the probability of that assignment is defined as:

$$P(X) = \frac{1}{Z}\prod_{i}\phi_i(x_i)\prod_{j\in\zeta(i)}\left(\psi_{ij}\left(x_i, x_j\right) * E(x)\right)$$

- i : i^{th} node in the MRF/board
- N : Number of nodes in the MRF/board.
- $\phi_i(x_i)$: User constraints (e.g. board size)
- $\psi_{ij}\left(x_i, x_j\right)$: Patch to patch compatibility
- $\zeta(i)$: Markov blanket of node i
- $E(X)$: Exclusion term that discourages patches being used more than once.
- Z : Normalization term to ensure $\int P(X)\, dX = 1$

Loopy Belief Propagation Solver

- Maximizes the preceding probability function using loopy belief propagation.
- Susceptible to local maxima so random restarts may be performed.
- **Question:** What if I do not have access to a low resolution version of the original image? Can I make one or use a substitute?

“Dense and Noisy” Estimation

- Proposed by Cho *et. al.* in [7] in 2010.

- **Review:** In Cho *et. al.*’s work in [8], they assumed access to a correct, low resolution version of the original image.
  – In many real world applications, such a low resolution image is not available.

- **Solution:** Estimate a low resolution image from a “bag of patches.” The simplified procedure is:
  – Creating a histogram of the bag of patches
  – “Estimate” a low resolution version by comparing the histogram to a set of K centroids with predefined low resolution images.

“Dense and Noisy” Clustering and Histogram Generation

- **Training Set:** 8.5M patches from 15,000 images.
  - *Patch Size:* 7px by 7px by 3 (LAB) for 147 total, original dimensions. This dimensionality is reduced via PCA.

- **Clustering the Patches**
  - *Step #1:* Cluster each image’s patches into L (e.g. 20) centroids.
  - *Step #2*: Re-cluster L centroids from all images into N (e.g. 200) centroids.

- **Creating the Histogram:** For a given image, assign each patch to its closest centroid.

"Dense and Noisy" – Generating the Low Res. Image

- **Theoretical Motivation:** Different colors are more likely to be at different places in an image.
  - *Example*: Blue (sky) is more likely to be towards the top of the image while brown (soil) tends to be in the image foreground.

- **Mapping Bins to the Image**: Use the training set to generate probability density maps for each histogram bin.

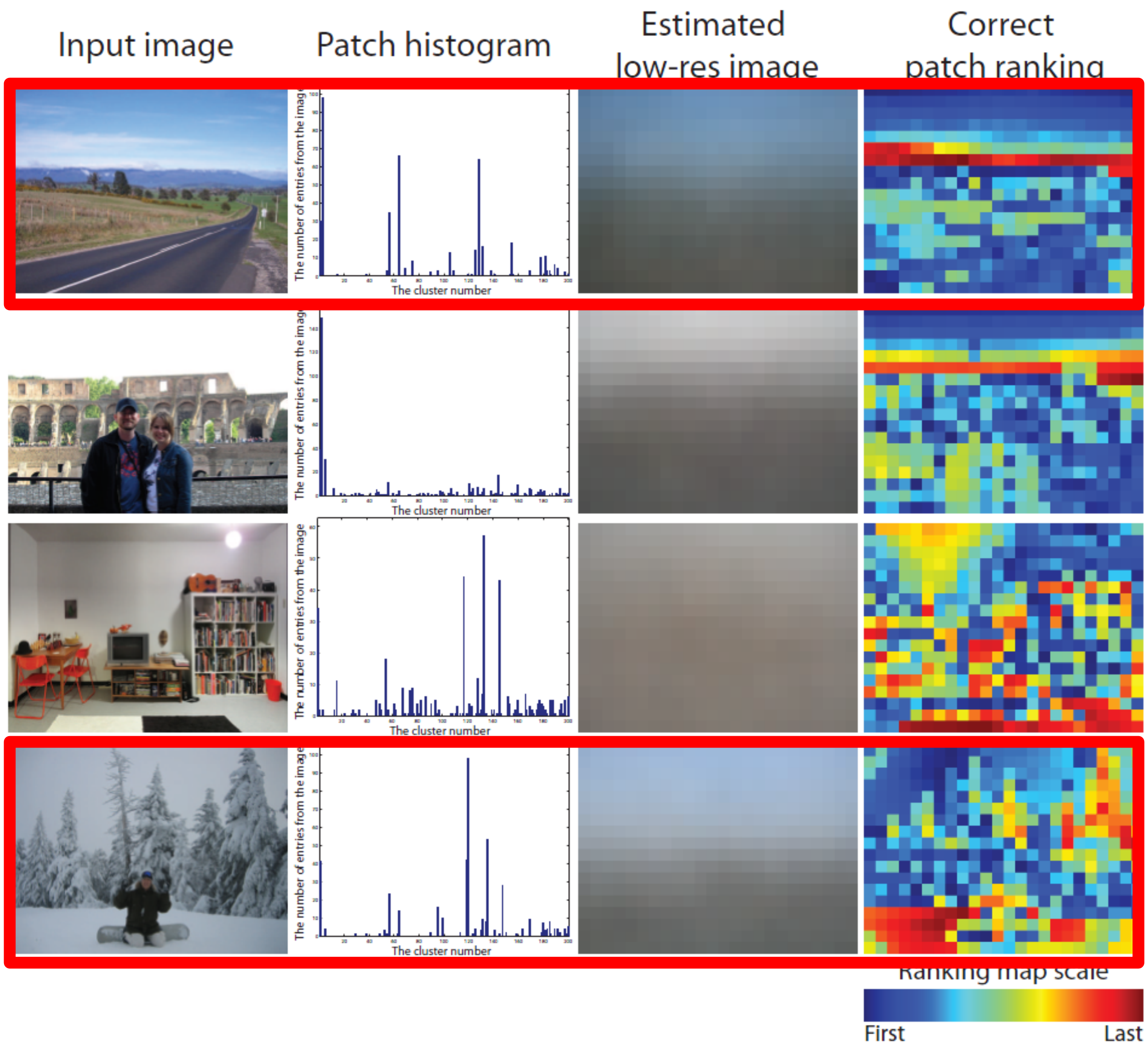
- **Using the Histogram to Create the Low Resolution Image**: Use a trained, linear regression function to map a bag of pieces histogram to the training images (i.e. use prior knowledge).

“Dense and Noisy” Results

- **Summary:** Patch histogram can “coarsely predict” a low resolution of the original image.
  - *Possible Explanation*: There is enough “structural regularity” in images that a bag of patches proves spatial information.

- **Patch Rank Map:** For each pixel in the low resolution images, patches are ranked from least likely to most likely to reside in that location.
  - *Ideal Case*: The set of patches that map to the low resolution will have the best rank (i.e. 1)
  - *Worst Case*: The matching set of patches will have rank N, where N is the number of patches in the image.

"Dense and Noisy" End to End Example



Best Results

Worst Results

Confused snow for sky

“Sparse and Accurate”

- Proposed by Cho *et. al.* in [7]

- **Common Human Approach to Solving Puzzles**: “Outside-in”
  - Find the puzzle’s four corner pieces.
  - Build from the corner pieces until all four sections converge.

- “Sparse and accurate” is based off the “outside-in” technique.
  - **Definition on an “Anchor Patch”**: A puzzle patch that is placed in its correct location and orientation.
  - **Summary of the Approach:** Place a set of N anchor patches and then solve the puzzle.

- Two most important criteria of anchor patches
  - Quantity
  - Uniform Spatial Distribution

Generalized Greedy Algorithm

- Proposed by Pomeranz *et. al.* in [10] in 2011.
- **Goal:** Provide a computational framework for handling square jigsaw puzzles in reasonable time that does not rely on any prior knowledge or human intervention.
- Solver divides the puzzle reconstruction into three subproblems:
  - *Placement*: Given a single patch or partially-placed set of patches, place the remaining patches.
  - *Segmentation*: Given a fully-placed board, segment the board into subcomponents that are *believed* to be placed correctly.
  - *Shifting*: Given a set of trusted segments, relocate entire segments and individual patches to improve solution quality.

Overview of the Greedy Placement Phase

- Given a partially assembled board (either a single patches or set of patches), continue applying the greedy choice until all patches are placed.
- **Overview of the Greedy Choice**:
  - Board dimensions are known in advance and fixed
  - Board locations with a higher number of occupied neighbors are preferred as the choice of the next piece is more informed.
  - Patch selection criteria:
    - *Primary Criteria*: Prefer a “best buddy” first.
    - *Secondary Criteria*: If no or multiple patches satisfy the primary criteria, select the patch with the highest compatibility score.
- **Question:** Why is a placer not enough?
- **Answer:** It works solely on local information. To get the best results, we must also look at the entire global solution.

Segmenter Phase

- **Definition of “Segments”**: Areas of the puzzle that are (or “are believed to be”) assembled correctly.

- **Procedure**: Using random seeds and a segmentation predicate based on the “best buddies” metric, grow the segments via “region growing segmentation algorithm” described in [15].

- **Accuracy of the Segmenter**: 99.7%

Pomeranz's Complete Algorithm

- ***Step #1***: Select a single puzzle patch as the seed to placement phase.
- ***Step #2***: Perform the placement phase around the seed.
- ***Step #3***: Use the segmenter to partition the board.
- ***Step #4***: Calculate the "best buddies" ratio. If you are at a local maximum, stop.
- ***Step #5***: Select the largest segment from step #3 and use it as the seed of the placement phase. Return to step #2.
  - Performing this step is similar to shifting the largest segment.

Genetic Algorithm (GA) Solver

- Proposed by Sholomon *et. al.* in 2013 [9].
  - A genetic algorithm puzzle solver was first proposed in [16] in 2002.

- **Genetic Algorithm Review**
  - Based off the biological theory of natural selection.
  - Divided into a series of stages
    - Random generation of initial population
    - Successor selection
    - Reproduction
    - Mutation
  - Requires a "fitness function" that measures solution quality.

Sholomon's GA Implementation

- **Puzzle Type:** 1 (patches have known orientation)
- **Chromosome (Solution) Representation:** N by M matrix where each cell represents one patchin the puzzle.
- **Population Size:** 1,000
- **Number of Generations:** 100
- **Number of Restarts:** 10
- **Successor Selection Algorithm:** Roulette Wheel
- ***Elitism***: Always pass *four* best solutions to the next generation
- **Culling:** None
- **Mutation Rate:** 5%
- **Fitness Function:** Sum of the L_2 of all pieces in the puzzle
- **Color Space:** LAB

GA Crossover

- Takes two "*highly fit*" parents and returns one child.
  - Non-trivial as the crossover must ensure there are no duplicate/missing patches in the solution.

- Correctly assembled segments may be at incorrect absolute locations. Hence, the crossover must allow for "*position independence*", which is the ability to shift segments.

- **Sholomon *et. al.*'s Approach:** Kernel-growing.

Sholomon’s Kernel Growing Algorithm

- Start with a single puzzle patch that is “floating” in the board such that the puzzle can grow in any direction.
  - Boundary size (i.e. length by width) is fixed and known.
- **Patch Placement Algorithm:** When deciding on the next patch to place, the algorithm iterates through up to three phases.
  - *Phase #1*: In an available boundary location, place the piece where both parents agree on the neighbor.
  - *Phase #2*: Place a “best buddy” that *exists in one of the parents*.
  - *Phase #3*: Select a location randomly and pick the piece with the best pairwise affinity.
  - If in any phase there is a tie, the tie is broken randomly.
  - After a piece is placed, the placement algorithm returns to phase #1 for the next piece.
  - Once a piece is placed, it can never be reused.

Kernel Growing with Mutation

- Mutations in genetic algorithms are used to improve the quality of the final solution via increased population diversity.

- **Sholomon's Mutation Strategy:** During the first and third phase of placement, place a piece at random with some low probability (e.g. 5%)

A Possible Benchmark

- Sholomon *et. al.* provide three large puzzle datasets as well as their results for comparative benchmarking [17].
  - *Dataset Puzzle Sizes*: 5,015, 10,375, and 22,834

- Unfortunately the website seems to no longer exist. I will separately send an email to the authors about why the removed the content.

- Used as a benchmark in [20].

Measuring Solution Quality

- **Problem Statement:** There is no uniform technique for grading the final output of a square jigsaw puzzle solver.

- **Two Divergent Approaches:**
  - *Performance Metrics:* Use the original image to grade solution quality.
    - Direct Comparison [7]
    - Neighbor Comparison [7]
  - *Estimation Metrics:* Evaluates the quality of a solution without reference to the original image [10].
    - “Best Buddies” Ratio

Algorithm Runtime Comparison

To improve execution time, Sholomon *et. al.* precompute and store all pairwise dissimilarity values.

# of Pieces	Sholomon *et. al.*	Pomeranz *et. al.*
432	48.3s	1.2min
540	64.1s	1.9min
805	116.2s	5.1min
2,360	17.60min	N/A
3,300	30.24min	N/A
5,015	61.06min	N/A
10,375	3.21hr	N/A
22,834	13.19hr	N/A

Comparison of the Algorithm Execution Time
for Sholomon *et. al.* and Pomeranz *et. al.*

Managing Missing Pieces and Multiple Puzzles

- Proposed by Paikin and Tal in [20].

- Inspired by Pomeranz *et. al.*'s greedy algorithm [10] with **three additional requirements**:
  - *New Requirement #1*: A modified compatibility function
  - *New Requirement #2*: Superior initial seed selection.
  - *New Requirement #3*: Rather than making the "best"/ "closest matching" selection at each round, make the selection with the lowest chance of erring regardless of location.
    - This makes their algorithm deterministic eliminating the need for restarts.

- **Accuracy:** 97.7% on dataset in [17]

Puzzle Problem Requirements

Paikan's & Tal's jigsaw puzzle problem definition is the most difficult presented to date. It is described below:

- Size of the puzzle is unknown and may be different
- Orientation of the patches is unknown
- Patches may missing
- Input may contain pieces from multiple puzzles

Comparison of Patch Sizes

Reference	Patch Size
Cho *et. al.* (2010)	7px by 7px
Pomeranz *et. al.* (2010)	28px by 28px
Sholomon *et. al.* (2013)	28px by 28px
Wu (SJSU Thesis) [20]	25px by 25px

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