Comparison of Exponential Distribution against Central Limit Theorem

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1. Overview

The objective of this project is to investigate the Exponential Distribution and compare it against Central Limit Theorem.

Central Limit Theorem: In probability theory, the central limit theorem (CLT) states that, given certain conditions, the arithmetic mean of a sufficiently large number of iterates of independent random variables, each with a well-defined expected value and well-defined variance, will be approximately normally distributed, regardless of the underlying distribution

The exponential distribution will be simulated in R with rexp(n, lambda) where lambda is the rate parameter. In theory the mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda.

We will set n = 40 and set lambda = 0.2 for all of the simulations and compare the resulting distribution of the smaple averages to the theory for 1000 simulations.

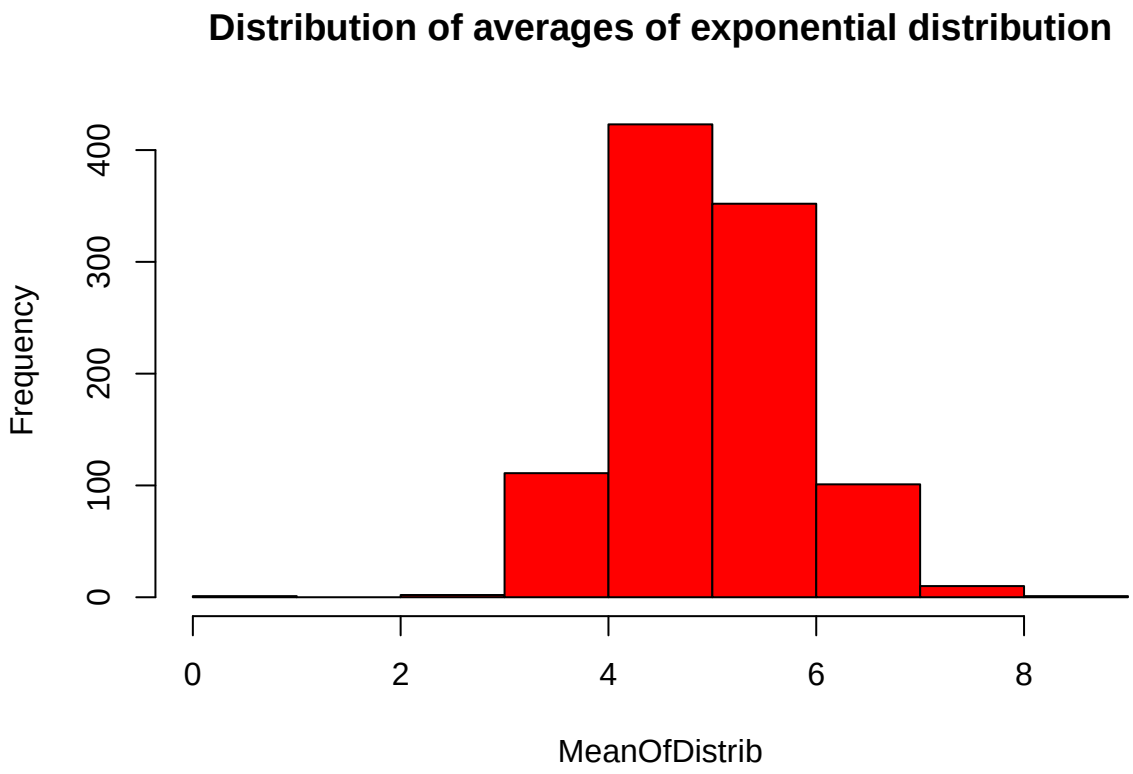
```
library(ggplot2)
library(psych)
```

```
set.seed(1000)
lambda <- 0.2 ## Set lambda as per instructions

## Distibution size
DisSize <- 40

## number of simulations
NoOfSim <- 1000

MeanOfDistrib <- 0
for (i in 1 : NoOfSim)
  MeanOfDistrib <- c(MeanOfDistrib, mean(rexp(DisSize,lambda)))
hist(MeanOfDistrib,col="red",main="Distribution of averages of exponential distribution")
```




```
varxp <- ((1/lambda)^2)/DisSize ## theoretical variance
varmean <- var(MeanOfDistrib) ## variance of the means
Mean_theoritical <- 1/lambda
```

```
## [1] "The theoretical mean is 5.000000 while mean of means is 4.981981"
```

```
## [1] "The theoretical variance is 0.625000 while variance of means 0.678534"
```

```
head(MeanOfDistrib)
```

```
## [1] 0.000000 4.514222 5.050788 3.252216 3.916899 4.898008
```

```
summary(MeanOfDistrib)
```

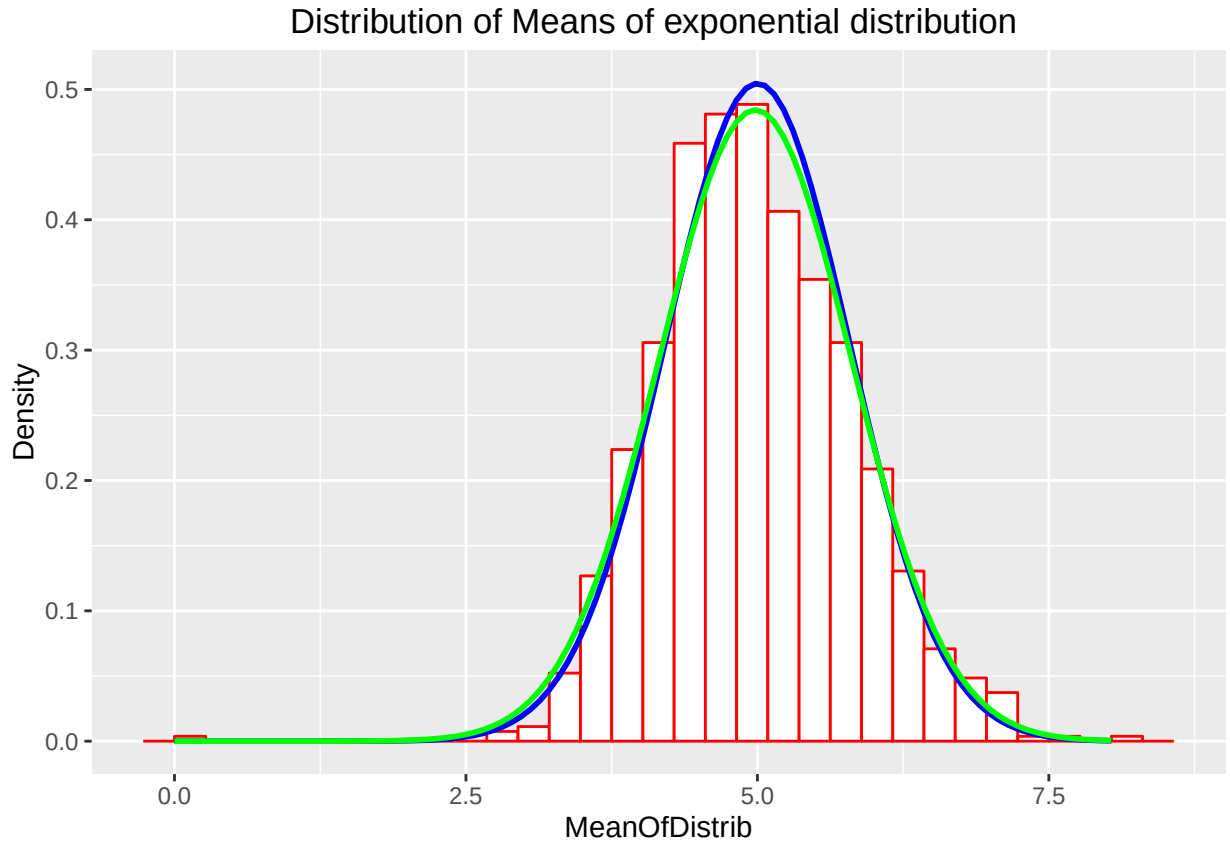
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   4.413   4.902   4.982   5.533   8.036
```

```
describe(MeanOfDistrib)
```

```
##   vars    n mean   sd median trimmed  mad min  max range skew kurtosis
## 1    1 1001 4.98 0.82    4.9    4.96 0.83   0 8.04  8.04 0.09     1.07
##     se
## 1 0.03
```

```
plotdata <- data.frame(MeanOfDistrib)
plot1 <- ggplot(plotdata,aes(x = MeanOfDistrib))
plot1 <- plot1 +geom_histogram(aes(y=..density..), colour="red",fill="white")
plot1<-plot1+labs(title="Distribution of Means of exponential distribution ", y="Density")
plot1<-plot1 +stat_function(fun=dnorm,args=list( mean=1/lambda, sd=sqrt(varxp)),color = "blue", size = 
plot1<-plot1 +stat_function(fun=dnorm,args=list( mean=mean(MeanOfDistrib), sd=sqrt(varmean)),color = "g
print(plot1)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

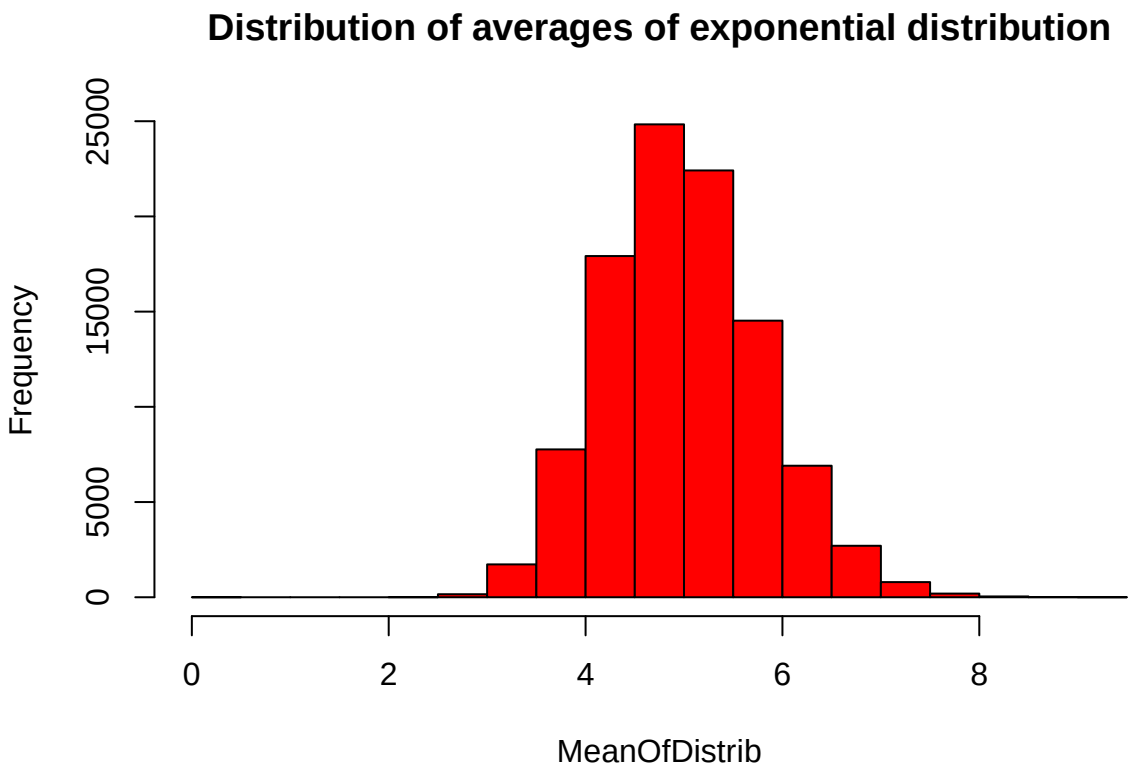



```
set.seed(1000)
lambda <- 0.2 ## Set lambda as per instructions

## Distibution size
DisSize <- 40

## number of simulations
NoOfSim <- 100000

MeanOfDistrib <- 0
for (i in 1 : NoOfSim)
  MeanOfDistrib <- c(MeanOfDistrib, mean(rexp(DisSize,lambda)))
hist(MeanOfDistrib,col="red",main="Distribution of averages of exponential distribution")
```




```
varxp <- ((1/lambda)^2)/DisSize ## theoretical variance
varmean <- var(MeanOfDistrib) ## variance of the means
Mean_theoritical <- 1/lambda
```

```
## [1] "The theoretical mean is 5.000000 while mean of means is 4.995464"
```

```
## [1] "The theoretical variance is 0.625000 while variance of means 0.624027"
```

```
head(MeanOfDistrib)
```

```
## [1] 0.000000 4.514222 5.050788 3.252216 3.916899 4.898008
```

```
summary(MeanOfDistrib)
```

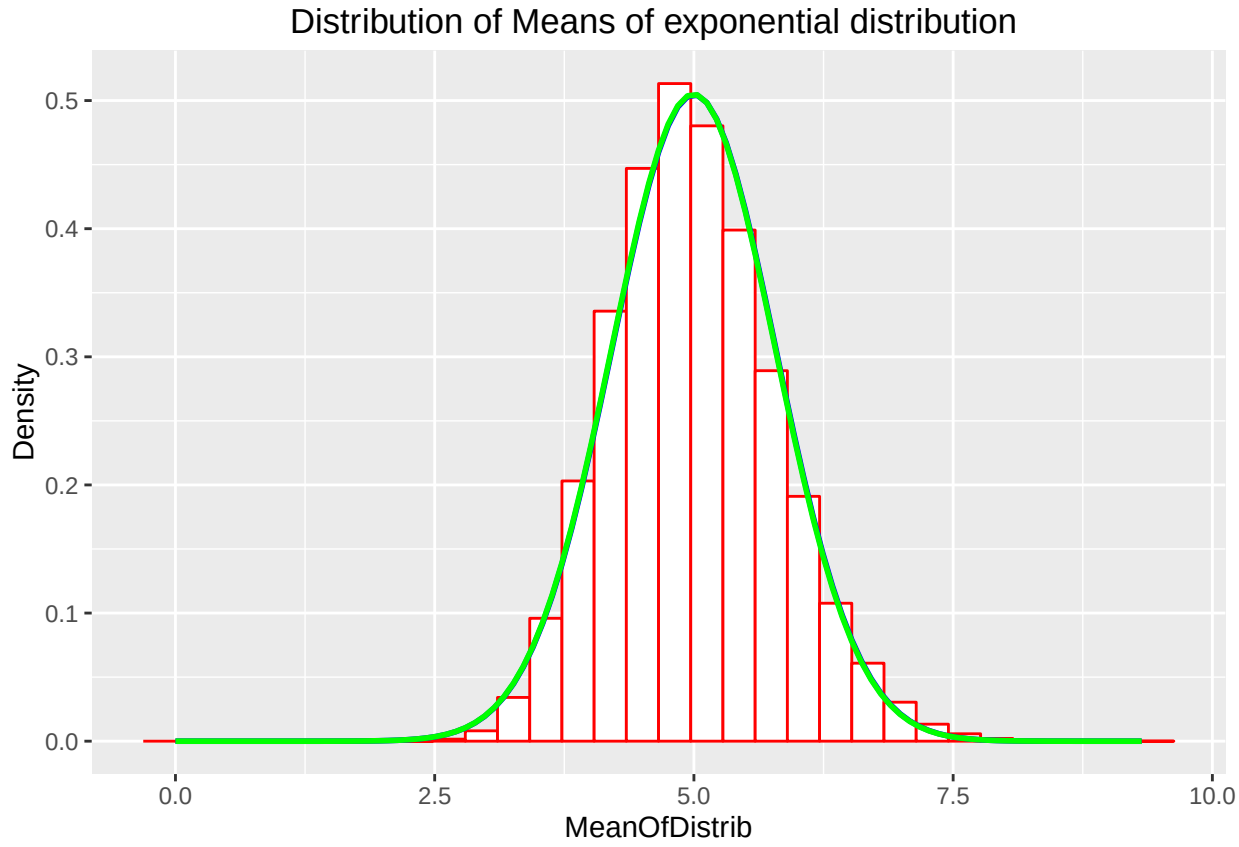
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   4.444   4.952   4.995   5.504   9.318
```

```
describe(MeanOfDistrib)
```

```
##   vars      n mean   sd median trimmed  mad min  max range skew kurtosis
## 1    1 100001    5 0.79   4.95    4.97 0.78   0 9.32  9.32 0.32     0.16
##   se
## 1  0
```

```r
plotdata <- data.frame(MeanOfDistrib)
plot1 <- ggplot(plotdata,aes(x = MeanOfDistrib))
plot1 <- plot1 +geom_histogram(aes(y=..density..), colour="red",fill="white")
plot1<-plot1+labs(title="Distribution of Means of exponential distribution ", y="Density")
plot1<-plot1 +stat_function(fun=dnorm,args=list( mean=1/lambda, sd=sqrt(varxp)),color = "blue", size = 
plot1<-plot1 +stat_function(fun=dnorm,args=list( mean=mean(MeanOfDistrib), sd=sqrt(varmean)),color = "g
print(plot1)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.