

# Optimal Vehicle Routing and Crew Allocation for White Gloves Service Problem

*Final Year Project Report*

*submitted in partial fulfilment of the requirements for the degree of  
Bachelor of Technology in Mechanical Engineering*

*by*

Soorya Sriram  
(Roll No: MDM18B049)



DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF INFORMATION TECHNOLOGY,  
DESIGN AND MANUFACTURING, KANCHEEPURAM

May 2022

# Certificate

I, **Soorya Sriram**, with Roll No: **MDM18B049** hereby declare that the material presented in the Internship Report titled **Optimal Vehicle Routing and Crew Allocation for White Gloves Service Problem** represents original work carried out by me in the **Department of Mechanical Engineering** at the **Indian Institute of Information Technology, Design and Manufacturing, Kancheepuram** during the year **2022**. With my signature, I certify that:

- I have not manipulated any of the data or results.
- I have not committed any plagiarism of intellectual property. I have clearly indicated and referenced the contributions of others.
- I have explicitly acknowledged all collaborative research and discussions.
- I have understood that any false claim will result in severe disciplinary action.
- I have understood that the work may be screened for any form of academic misconduct.

Date: 10 - 05 - 2022

Student's Signature: Soorya Sriram

In my capacity as supervisor of the above-mentioned work, I certify that the work presented in this Report is carried out under my supervision, and is worthy of consideration for the requirements of internship work during the period May 2021 to October 2021.

Advisor's Name: Dr Kalpana P

Advisor's Signature

# *Abstract*

Variants of the vehicle routing problem have been studied for the last few decades. The pickup and delivery vehicle routing problem is a very popular and pertinent problem to address in today's post pandemic world where personalising the experience of the customers is of utmost priority. This novel variant proposed involves installation and crew allocation with specific skill sets to the pickup and delivery vehicle routing problem with time windows. This problem was undertaken due to the large increase in demand of large and fragile goods during the course of the pandemic. It is impractical to send two separate fleets to first deliver the goods and to install/ fix up such goods later. The 'White Gloves Service' which is an elite personalised service comprising of skilled labourers are employed by many retail outlets where its important to minimise the total transportation cost using minimum number of vehicles assigning appropriate crews to the corresponding vehicles maximising the load transported from the depot. A closed vehicle routing problem is addressed where there is fixed service time for the products under consideration for every product and there is a fixed location for the pick and drop of goods. The crew is allotted to a specific customer by matching the product procured and the skill set part of the crew.

## *Acknowledgements*

I would like to express my sincerest gratitude to my academic supervisor Dr Kalpana P, Assistant Professor of the Department of the Mechanical Engineering. I thank her for giving me the opportunity to work under her guidance and providing me with the flexibility in allowing to pursue the topic and project of my choice with utmost freedom. Her constant encouragement was the main driving force for me to complete the project within the time provided. I would like to thank her for her patience and efforts put in to take up doubts at odd hours of the day.

I would like to thank all the review members of the panel, Dr.Kishor Kumar Gajrani, Dr. Venkata Timmaraju Mallina and External Industry Panelist for their valuable feedback regarding the project.

I would like to thank Surya N, a MTech Student, PHD Scholars Subin Sahayam and Cyriac Antony for their time to evaluate my project and providing valuable advice.

I would like to thank my fellow friends for their continuous support and different point of view to tackle the problem at hand. Last but not the least, I would like to thank my family for their love and support throughout my studies.

# Contents

|  |             |
|--|-------------|
| <b>Certificate</b>   | <b>i</b>    |
| <b>Abstract</b>  | <b>ii</b>   |
| <b>Acknowledgements</b>  | <b>iii</b>  |
| <b>Contents</b>  | <b>iv</b>   |
| <b>List of Figures</b>   | <b>vi</b>   |
| <b>List of Tables</b>  | <b>viii</b> |
| <b>1 Introduction</b>  | <b>1</b>    |
| 1.1 Background . . . . .   | 1           |
| 1.2 Motivation . . . . .   | 2           |
| 1.3 Objectives and Work Done . . . . .                                 | 2           |
| <b>2 Literature Review</b>   | <b>4</b>    |
| 2.1 Vehicle Routing Problem and its Variants . . . . .                 | 4           |
| 2.1.1 Travelling Salesman Problem . . . . .                            | 5           |
| 2.1.2 Capacitated Vehicle Routing Problem . . . . .                    | 5           |
| 2.1.3 Vehicle Routing Problem with Time Windows . . . . .              | 6           |
| 2.1.4 Vehicle Routing Problem with Pickup and Delivery . . . . .       | 6           |
| 2.1.5 Fleet Management . . . . .                                       | 7           |
| 2.1.6 Vehicle Routing Problem with Delivery and Installation . . . . . | 8           |
| 2.1.7 Data Driven Optimisation . . . . .                               | 8           |
| <b>3 Keywords and Definitions</b>                                      | <b>10</b>   |
| 3.1 Problem Exploration . . . . .                                      | 10          |
| 3.1.1 Last Mile Delivery . . . . .                                     | 11          |
| 3.1.2 White Gloves Service . . . . .                                   | 11          |
| <b>4 Problem Description</b>   | <b>12</b>   |
| 4.1 Problem Definition . . . . .                                       | 12          |

---

|          |  |           |
|----------|--|-----------|
| 4.1.1    | Haversine Formula . . . . .                    | 13        |
| 4.1.2    | Branch and Bound Algorithm . . . . .           | 14        |
| <b>5</b> | <b>Mathematical Formulation</b>                | <b>15</b> |
| 5.1      | Mathematical Model . . . . .                   | 15        |
| 5.1.1    | Decision Variables . . . . .                   | 15        |
| 5.1.2    | Parameters . . . . .                           | 15        |
| 5.1.3    | Objective Function . . . . .                   | 16        |
| 5.1.4    | Constraints . . . . .                          | 17        |
| <b>6</b> | <b>Experiments and Results</b>                 | <b>19</b> |
| 6.1      | Standard Parameters and Output . . . . .       | 19        |
| 6.2      | Change in Product Request . . . . .            | 20        |
| 6.3      | Change in Product Number . . . . .             | 23        |
| 6.4      | Change in Product Capacity . . . . .           | 26        |
| 6.5      | Change in Crew . . . . .                       | 27        |
| 6.6      | Change in Installation Time . . . . .          | 28        |
| 6.7      | Change in Time Window . . . . .                | 31        |
| 6.8      | Change in other parameters . . . . .           | 33        |
| <b>7</b> | <b>Inferences, Conclusion and Future Scope</b> | <b>43</b> |
| 7.1      | Inferences . . . . .                           | 43        |
| 7.2      | Conclusion . . . . .                           | 46        |
| 7.3      | Future Scope . . . . .                         | 46        |
| <b>A</b> | <b>Results of Variants</b>                     | <b>47</b> |

# List of Figures

|      |   |    |
|------|---|----|
| 2.1  | Depot and Warehouse Mapping [1]                   | 4  |
| 2.2  | TSP Data  | 5  |
| 4.1  | Branch and Bound Formula [1]                      | 14 |
| 6.1  | Standard Map used for testing                     | 21 |
| 6.2  | Network Map for Product Requests (1)              | 22 |
| 6.3  | Network Map for Product Requests (2)              | 22 |
| 6.4  | Network Map for Product Requests (3)              | 24 |
| 6.5  | Network Map for Product Number (1)                | 25 |
| 6.6  | Network Map for Product Number (2)                | 25 |
| 6.7  | Network Map for Product Number (3)                | 28 |
| 6.8  | Network Map for Product Capacity (1)              | 29 |
| 6.9  | Network Map for Product Capacity (2)              | 30 |
| 6.10 | Network Map for Product Capacity (3)              | 30 |
| 6.11 | Network Map for Crew Change (1)                   | 32 |
| 6.12 | Network Map for Crew Change (2)                   | 33 |
| 6.13 | Network Map for Crew Change (3)                   | 34 |
| 6.14 | Network Map for Crew Installation Time Change (1) | 35 |
| 6.15 | Network Map for Crew Installation Time Change (2) | 36 |
| 6.16 | Network Map for Crew Installation Time Change (3) | 37 |
| 6.17 | Network Map for Time Window Change (1)            | 38 |
| 6.18 | Network Map for Time Window Change (2)            | 39 |
| 6.19 | Network Map for Time Window Change (3)            | 40 |
| 6.20 | Network Map for Crew Cost Change                  | 41 |
| 6.21 | Network Map for Service Time Change               | 42 |
| 7.1  | Inference for Product Request                     | 44 |
| 7.2  | Inference for Product Number                      | 44 |
| 7.3  | Inference for Product Capacity                    | 44 |
| 7.4  | Inference for Crew Change                         | 45 |
| 7.5  | Inference for Time Window                         | 45 |
| A.1  | TSP Model   | 47 |
| A.2  | TSP   | 48 |
| A.3  | VRP Model   | 48 |

---

|      |             |    |
|------|-------------|----|
| A.4  | VRP (1)     | 49 |
| A.5  | VRP (2)     | 49 |
| A.6  | VRP (3)     | 50 |
| A.7  | VRPTW Model | 50 |
| A.8  | VRPTW       | 50 |
| A.9  | PDVRP Model | 51 |
| A.10 | PDVRP       | 51 |



# List of Tables

|      |  |    |
|------|--|----|
| 6.1  | Coordinates of all the Points and their time windows for all experiments . . | 20 |
| 6.2  | Product Requests for the standard experiment . . . . .                       | 20 |
| 6.3  | Crew Skill Set Matrix standard . . . . .                                     | 20 |
| 6.4  | Crew Installation Time Matrix standard . . . . .                             | 20 |
| 6.5  | Standard Value Experiment . . . . .  | 21 |
| 6.6  | Product Requests (1) . . . . .   | 21 |
| 6.7  | Product Requests (1) Experiment . . . . .                                    | 23 |
| 6.8  | Product Requests (2) . . . . .   | 23 |
| 6.9  | Product Requests (2) Experiment . . . . .                                    | 23 |
| 6.10 | Product Requests (3) . . . . .   | 24 |
| 6.11 | Product Requests (3) Experiment . . . . .                                    | 24 |
| 6.12 | Product Number (1) . . . . .   | 26 |
| 6.13 | Product Number (1) Experiment . . . . .                                      | 26 |
| 6.14 | Product Number (2) . . . . .   | 26 |
| 6.15 | Product Number (2) Experiment . . . . .                                      | 27 |
| 6.16 | Product Number (3) . . . . .   | 27 |
| 6.17 | Product Number (3) Experiment . . . . .                                      | 28 |
| 6.18 | Product Capacity (1) Experiment . . . . .                                    | 29 |
| 6.19 | Product Capacity (2) Experiment . . . . .                                    | 31 |
| 6.20 | Product Capacity (3) Experiment . . . . .                                    | 31 |
| 6.21 | Crew Skill Set Change (1) . . . . .  | 31 |
| 6.22 | Crew Change (1) Experiment . . . . .   | 32 |
| 6.23 | Crew Skill Set Change (2) . . . . .  | 32 |
| 6.24 | Crew Change (2) Experiment . . . . .   | 33 |
| 6.25 | Crew Skill Set Change (3) . . . . .  | 33 |
| 6.26 | Crew Change (3) Experiment . . . . .   | 34 |
| 6.27 | Crew Installation Time Matrix (1) . . . . .                                  | 34 |
| 6.28 | Installation Time Change (1) Experiment . . . . .                            | 35 |
| 6.29 | Crew Installation Time Matrix (2) . . . . .                                  | 35 |
| 6.30 | Installation Time Change (2) Experiment . . . . .                            | 36 |
| 6.31 | Crew Installation Time Matrix (3) . . . . .                                  | 36 |
| 6.32 | Installation Time Change (3) Experiment . . . . .                            | 37 |
| 6.33 | Time Window Change (1) . . . . .   | 37 |
| 6.34 | Change in Time Window (1) Experiment . . . . .                               | 38 |
| 6.35 | Time Window Change (2) . . . . .   | 38 |

---

|   |    |
|---|----|
| 6.36 Change in Time Window (2) Experiment . . . . . | 39 |
| 6.37 Time Window Change (3) . . . . .               | 39 |
| 6.38 Change in Time Window (3) Experiment . . . . . | 40 |
| 6.39 Crew Cost Values Change . . . . .              | 40 |
| 6.40 Change in Crew Cost Experiment . . . . .       | 41 |
| 6.41 Change in values of Service time . . . . .     | 41 |
| 6.42 Change in Service Time Experiment . . . . .    | 42 |

# Chapter 1

## Introduction

The Indian E - Commerce industry, one of the biggest consumer market is expected to triple by 2025 as compared to 2020. The industry comprises of the first, middle and last mile delivery issues with the final mile being the most tumultuous rid with problems. The Last Mile delivery is the process in the supply chain where the goods have to be transported from fulfillment centres and warehouses to the customer. Every company spends a substantial amount of its expenditure making sure the Last Mile is effective as it is a highly volatile sector. The goods might vary from perishable to non perishable, bulky or fragile and "White Gloves Service" has become the differentiating factor. The 'White Gloves Service' is provided as an exclusive service to customers who would like a tailored, customised service.

### 1.1 Background

The valuation for White Gloves service is in billions in the global e-commerce market for furniture and appliances accounting to roughly 17 percent of the existing e-commerce market. With more than 30% of revenue being generated via online sales, there has been an increase in the sales of bulkier items that require installation making it of utmost importance to explore further.

Many companies unable to provide support in-house are now looking at third party logistic companies. Customers have become more vigilant and are seeking transparency throughout which includes unpacking of items to overnight of fragile items and also the

transportation of the order up or down stairs and finally installing the product upon delivery.

The white Gloves service is often described as a premium delivery. It involves special attention to goods that require extra effort during transportation. In addition to careful handling, it also includes delivery and installation where servicemen with specific skillsets are assigned to the customer.

The high level of visibility in white Gloves Service is what makes it stand out as compared to the other third party services available. Optimal routes are to be generated providing real time scheduling and tracking and also have the capability to handle shipments being returned.

## 1.2 Motivation

With the advent of online shopping and the volatility in customer requests, it is pertinent to address the “Last-Mile” element of the home-delivery model. The uncountable number of problems that may arise from traffic jams to inaccessibility of remote areas has made it a highly unpredictable endeavour. Practically, incorrect drop details, very small time windows when customers are available to receive the good all add cost, time and inconvenience to an already marginal activity.

The retail and e-commerce market has witnessed a complete revolution in the sense where customers are of utmost priority. Consumers are willing to pay more for a personalized experience where convenience, speed and hassle - free experiences have become the major criteria with which consumers differentiate service providers.

## 1.3 Objectives and Work Done

1. A novel closed Vehicle Routing Problem has been defined which involves multiple pickup, delivery and installation of goods where a homogeneous set of vehicles with various crews with specific skill sets start from a single depot start and return to the starting location at the end of service satisfying constraints such working hour, time window, capacity, precedence and installation demand.
2. The objective function involves the minimisation of the total cost of distance travelled optimally assigning the crews with skill sets required for installation to customer requests.

3. A three index arc decision variable, and two index load and time decision variable have been defined.
4. Known parameters such as skill set matrix, installation time window, customer time windows, vehicle capacity, distance and time matrices have been defined trying to capture the intricacies of the problem.
5. Travelling Salesman Problem was understood, modelled and executed with a random data set and the ordered route was found.
6. The Capacitated VRP, a variant of the TSP was studied and modelled, executed using a sample test data with 3 vehicles with a homogeneous capacity of 100kg and routes were generated
7. Vehicle Routing Problem with time windows was executed and its validity was tested against the standard benchmark data set of Solomon's 1987 test case.
8. The Pickup and Delivery VRP was then worked on where time windows and load capacity was included. The model was validated with a small data set with 2 pickup and 2 customer location.
9. A novel VRP with multiple pickup, delivery and installation with crew allocation based on skill sets and product request. 3 crews were considered and homogeneous vehicle capacity with hard time window constraints and working hour constraint for every crew.
10. Possible extension would be to use heuristics such as genetic algorithm or simulated annealing to solve a larger scale industry data set for heterogeneous vehicles.

## Chapter 2

# Literature Review

### 2.1 Vehicle Routing Problem and its Variants

Vehicle routing problem (VRP) is an issue of much interest in the area of transportation logistics and was first introduced by Dantzig and Ramser.

A popular variant of the VRP is the pickup and delivery problem which can be described as the process of determining optimal routes from the depot to various consumers located to at different locations where goods are picked up from different pickup locations.

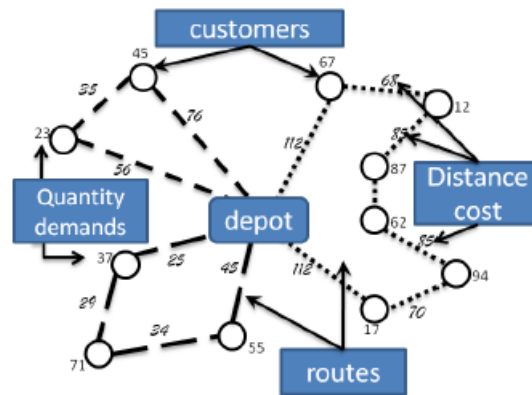


FIGURE 2.1: Depot and Warehouse Mapping [1]

### 2.1.1 Travelling Salesman Problem

TSP, also commonly known as the travelling salesman problem has been of interest for researchers for quite sometime and the first mathematical model was proposed by Kulkarni - Bhavne [2] in 1985. The M - TSP is a generalisation of the TSP which was derived from the former for M salesman where the objective was to minimise the total distance travelled and to reach the depot or home city. The sub-tour elimination was introduced and the Mixed Integer Problem was solved using Branch and Cut algorithm. The traditional Travelling Salesman problem was executed using 100 data points randomly generated.

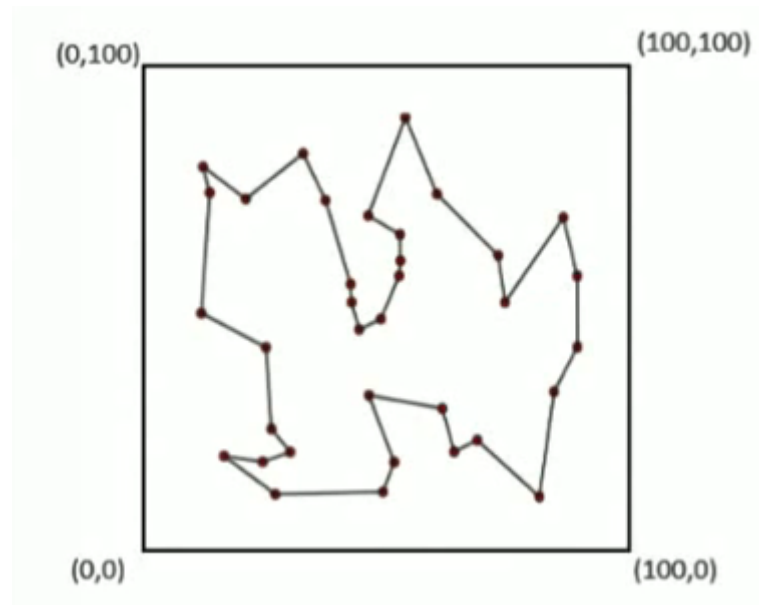


FIGURE 2.2: TSP Data

### 2.1.2 Capacitated Vehicle Routing Problem

[3]The capacitated VRP was modelled by Achutan and Caccetta in 1991 where they solved using MILP. A set of  $N-1$  customers where the locations and requirements for some commodity is to be supplied from a single depot using a set of homogeneous delivery vehicles each with a prescribed capacity. The problem is to determine the delivery routes, one for each vehicle, which minimize the total distance travelled by all the vehicles and which satisfies the following restrictions:

- (i) each customer appears on exactly one route
- (ii) the maximum number of customers serviced by a vehicle is  $L$

(iii) the total requirement of customers appearing in each vehicle route cannot exceed the capacity  $W$

(iv) the total distance travelled by each vehicle cannot exceed  $T$

The heterogeneous CVRP uses vehicles of different capacities to deliver commodities from the depot to consumers

The CVRP was modelled using a sample test data set which contains the latitude, longitude and demand at each location. All the vehicles were modelled to have a capacity of 60kg and 3 homogeneous vehicles were used to test the model.

### 2.1.3 Vehicle Routing Problem with Time Windows

The Vehicle Routing Problem with Time Windows (VRPTW) is an extension of the Capacitated Vehicle Routing Problem (CVRP) where the service at each customer must start within an associated time interval, called a time window. Time windows may be hard or soft. In case of hard time windows, a vehicle that arrives too early at a customer must wait until the customer is ready to begin service. In general, waiting before the start of a time window incurs no cost. In the case of soft time windows, every time window can be violated barring a penalty cost.

[4] Author uses branch and cut algorithm for exact solution. Each route starts and ends at the depot within a given scheduling horizon and the cumulative demand of the customers visited on a route does not exceed vehicle capacity. Each customer is served by exactly one vehicle within its given time window.

### 2.1.4 Vehicle Routing Problem with Pickup and Delivery

[5] The review on PDVRP refers to 2 different types of models, the commodity flow model where the amount of pickup and delivery commodity along each arc while the vehicle flow model only specify vehicle routes. The models use 2 or 3 index variable to define the arc of the graph where the usage of heterogeneous vehicles adds to the third index. Tabu Search, a local search heuristic is suggested as the most commonly used method to solve the PDVRP.

[6] The authors propose a multi depot pickup and delivery model where they use the concept that the load for a pickup node is positive and load is negative for a delivery node.



Furthermore, to set the precedence constraint the vehicle is to visit the pickup node before the time window as compared to the delivery node such that departure time of pickup node should be before the delivery node

### 2.1.5 Fleet Management

[7] Exploring on Fleet management for vehicle routing problem, the authors proposed a modification to the traditional VRP with fixed time slots by introducing delivery options to ship to alternate locations with time windows. For any customer it is necessary that only one delivery option is designated and that they differ from one customer to the other.

Branch and cut algorithm was used by the authors. They defined their objectives to be minimisation of overall cost ensuring minimum customer satisfaction level and not violating location capacity constraints. They approach this by arriving at the least cost set of feasible routes covering exactly one option taking into consideration fleet size, location capacity and required service level. They have introduced the their problem to be static and deterministic.

[8] The authors explored the impact of multi parcel CL (crowdsourcing logistics) compared to the traditional -by - van LMD. An analytical model has been developed to compute costs for both CL and LMD depending on customer demand and a sensitivity analysis has been performed on parameters.

Sensitivity analysis performed on the following factors: Demand, rider's wage, duration of working shifts, number of riders, failure delivery rate

Conclusions:

- (i) Increased demand reduces average CL delivery Cost/parcel - distance between 2 consecutive destination reduces
- (ii) Cheaper than traditional LMD however availability and higher rider wage has not been compared
- (iii) Remuneration for entire delivery period however lower average CL delivery cost/parcel
- (iv) Number of riders - 100 tours 135 riders were taken varying the availability was evaluated. Increase in fleet size, higher the probability of finding a match between duration of rider and delivery tour

(v) Failure Delivery Rate: 18% for traditional LMD and 10% for CL in the evenings has been modelled initially. CL was found to be more profitable

### 2.1.6 Vehicle Routing Problem with Delivery and Installation

[9] The total costs are reduced by separating and synchronizing delivery from installation/assembly and visiting customers twice, once for delivery and once for assembly.

Problem Description: Served by exactly one delivery vehicle, Each customer requires an installation ( binary variable), Service time for delivery and installation, Delivery with service window, Include waiting time if before time window, After delivery is done, installation within time window,

In summary:

1. The first one includes large trucks and performs deliveries (capable of installations as well)
2. A faster method is proposed by having a second fleet complete installation after delivery for certain customers.
3. The target is to integrate the installation with delivery if it has not been performed by the deliverymen.
4. Sensitivity analysis: Effect of instance parameters on the utilization of fleets, Effect of delivery men efficiency on synchronization constraints

The Adaptive Large Neighborhood Search (algorithm) is used to find the optimal solution.

### 2.1.7 Data Driven Optimisation

[10] The authors focus on a Data driven optimization approach combining Machine Learning methods for capacitated vehicle routing problems. They use a predict then optimise framework where prediction objective is constructed by decision error, mini batching gradient and heuristics for joint order assignment and vehicle routing for Last mile delivery.

Their objectives were total travel time cost, total Operating Cost, Customers to be visited precisely once in a route and to Eliminate of Illegal Routes. The problem involves Drivers with limited capacity, a Radius threshold and Driver total travel Time Threshold making

it a capacitated Vehicle Routing Problem. Predictive Analytics to show demand of White Gloves Service in the E - Commerce Industry was understood and the following methods are suggested by [11]. The authors considering the impacts of Covid -19 on healthcare operations have developed machine learning models for predictive analytics for forecasting to improve the healthcare operations and resilience.

Parameters used to Evaluate the proposed models for better training for Univariate Time Series are: Mean Square Error, Root Mean Square Error, Mean Absolute Error and Mean Absolute Percentage Error.

Univariate Time series analysis:

- (i) Statistical: Use of Arima (Auto regressive integrated moving average models and Box Jenkin Models for identifying linear relationship of changing variable with time
- (ii) Machine Learning Model: In Specific, the Extreme Learning machine which is single feed forward neural network with no backpropagation and use Moore Penrose pseudoinverse to generate weights

FeedForward Multi Layer Perceptron with time lag window with more than one hidden layer using a non linear activation function

- (iii) LSTM: Long Short Term Memory uses Recurrent Neural networks to predict non linear time series data using forget gates

Multivariate Time Series Analysis:

- (i) MARIMA, VAR - vector autoregression, a extension of univariate autoregression method, P - lag vector auto regressive model model where p is the number of lagged observations and pi representing the relationship between lagged relationships
- (ii) LSTM - very accurate for short term predictions observing long term dependencies, author uses bivariate time analysis of new and old cases and analyses in parallel simultaneously.
- (iii) ELM with its single layered network with batch and incremental learning, MLP with multi-layered ANN generalised the data set well.

## Chapter 3

# Keywords and Definitions

### 3.1 Problem Exploration

Two different fleet are generally sent for the Last Mile Delivery Problem when installation is involved. The delivery process starts at the warehouse where orders are accepted and information is sent to both the sender and recipient. The goods are then dispatched to specific fleet and the optimal route is calculated minimizing the total distance traveled. The package arrives at the final delivery destination and delivery is confirmed.

To track the order in the supply chain, a centralized system is used to process the orders providing the platform for senders and recipients the ability to track where the order is.

There are mainly 2 ways in which the good is transported from the warehouse to the consumer. One is the usage of internal fleets or 'owned fleets of a specific company and the other is the usage of third party logistic providers and shipping carriers.

The depot location for local delivery plays an integral part to improve the efficiency of delivery especially in times of high volume and for fast on demand delivery. Keeping inventory levels for fast-moving items high in local fulfillment centers. Slower moving inventory, or products which are not ordered for on-demand delivery, can be kept in a warehouse further off.

### 3.1.1 Last Mile Delivery

Challenges of LMD: (i) Outdated Technology (ii) Lack Of Transparency (iii) Improper route optimization (iv) Unpredictable Factors (v) High Delivery Cost (vi) Friction (vii) Speed

Delivery Exceptions:

(i) An order of an out-of-stock item (ii) Improper timing of good and service (iii) Change in delivery time and address (iv) Delay of delivery to previous delays (v) A failed or missed delivery (vi) Rejection of good due to faulty or missing part

Possible Solutions of LMD:

(i) Dynamic Route Planning (ii) Tracking Consignments and drivers in Real Time (iii) Control tower management (iv) Optimizing first-mile operations, locations of warehouses/distribution centers, reducing sorting times at fulfillment centers and last-mile distribution centers (v) Automatically Prioritize Delivery Time of Customers (vi) Auto allocate the consignments to the right vehicles (vii) Fitting more deliveries in a day by minimizing driving hours with more efficient routes

### 3.1.2 White Gloves Service

White Gloves Requirements:

(i) Special Packaging (ii) Attention to Detail (iii) Time Precision (iv) Real-Time Freight Tracking (v) Hassle-Free Set-Up (vi) Premium Customer Service

Challenges of White Gloves Service:

(i) Provide Ample Delivery Location Details (ii) Provide Customer Contact Details (iii) Detailed Explanation of Needs (iv) Less Than Truckload Shipping

Local terminals serve as the spokes with the main central terminal functioning as the hub is identified as a hub and spoke model in shipping. The spokes function as freight reception centres and then transferred to the hub terminal where further sorting, consolidation and final delivery is carried out.

## Chapter 4

# Problem Description

### 4.1 Problem Definition

Objective of this undertaking is to minimize the total transportation cost which is essentially trying to minimise the total distance travelled by all the crews and finding the optimal routes for all the crews satisfying all the constraints and visiting each node only once.

The proposed Vehicle Routing Problem is a NP - Hard Problem and it is not possible to arrive at an accurate solution in a short period of time. A novel problem has been proposed where in addition to pickup and delivery of goods by deliverymen, the issue of installation of goods that require skilled men is being addressed within a 8hr working constraint.

- (i) Optimizing Truck capacity
- (ii) Optimizing Route with minimum travelling cost
- (iii) Precedence of Pickup before Delivery/Installation
- (iv) Crew Allocation with specific skill set required for installation
- (v) Working Hour Constraint
- (vi) Time Window constraint
- (vii) Arrival time at next customer node should be after the departure time of previous customer
- (viii) All the routes start from a single depot

- (ix) Homogeneous fleet capacity
- (x) Heterogeneous Goods (Different Capacities)

Mixed Integer Linear Programming solved using Branch and Cut algorithm is to be used where the entire problem is to be modelled mathematically addressing various objective and constraints.

Before executing the mathematical model for the problem, it is essential to design a small experiment and identify the possible routes from the experiment before proceeding with the actual implementation.

The implementation was done using optimisation solver ILOG Cplex and the model was coded in JAVA.

This closed VRP is complex where the servicemen have to return to the initial depot within their working hours as well as satisfy the time windows during which customers are available. Proper fleet management is to be done where a combination of deliverymen and skilled personnel are sent according to the type of good being ordered and clustering concept is to be used to reduce the number of pickup and drop locations.

#### 4.1.1 Haversine Formula

With the help of Haversine Formula, one can find the great - circle distance which is the Shortest Distance between two points on the surface of the sphere. Prof. James Inman in 1835 is responsible for coining the term Haversine and if the (latitude, longitude) of the points are known, it is possible to calculate the distance between the two points using the Haversine formula.

$$a = \sin^2 \left( \frac{\Delta\Phi}{2} \right) + \cos \Phi_1 \cdot \cos \Phi_2 \cdot \sin^2 \left( \frac{\Delta\lambda}{2} \right)$$

$$c = 2 \cdot \text{atan2}(\sqrt{a}, \sqrt{1-a})$$

$$d = R \cdot c$$

where  $\Delta$  is latitude,  $\lambda$  is longitude, R is earth's radius (mean radius 6371km)

### 4.1.2 Branch and Bound Algorithm

Branch and Bound algorithm is used for Solving Combinatorial Mathematical Optimization Problems. A systematic approach to find candidate solutions using tree method with the full set at the root. Each branch is checked against upper and lower estimated bounds on the optimal solution. Solution is discarded if it cannot produce a better solution than the best one found so far by the algorithm.

The Branch and Cut algorithm is a faster method than the branch and bound algorithm. It uses Gomory Cutting plane method in addition to Branch and Bound to reduce the feasible solution space where all solutions for the unknowns are integer values Branch and Cut algorithm can be used to find the lower bounds even if optimal solution can not be found. It can be used with Heuristics like GA to find near optimal solutions

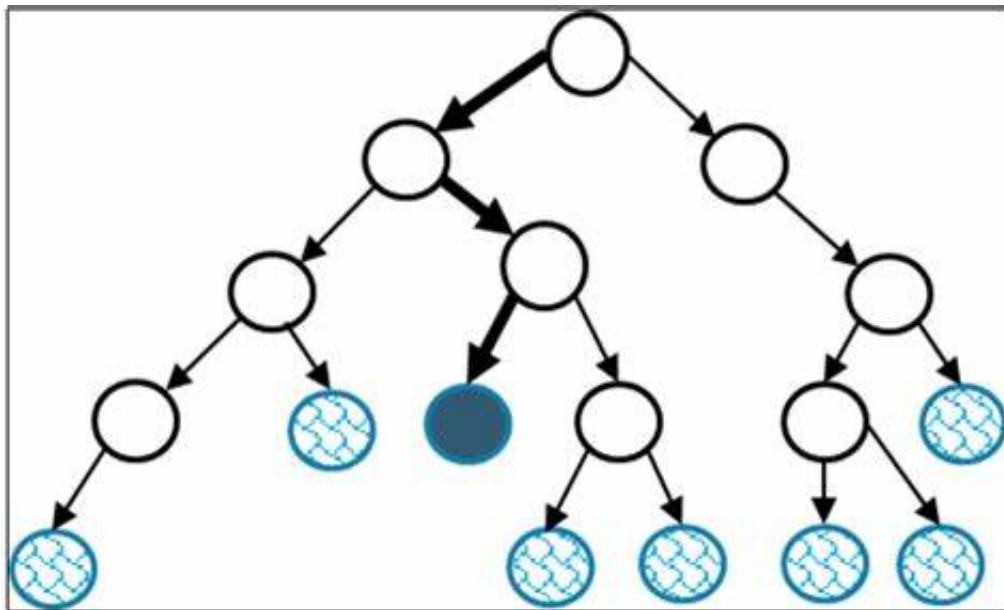


FIGURE 4.1: Branch and Bound Formula [1]



## Chapter 5

# Mathematical Formulation

### 5.1 Mathematical Model

#### 5.1.1 Decision Variables

The following are the Decision Variables:

- (i) Location i to j using with crew c (3 index variable) - a binary variable with the value 1 if crew c travels from node i to node j and is zero otherwise -  $x_{ijc}$
- (ii) The load being carried by crew c from location i to j -  $l_{ijc}$
- (iii) The time taken by crew c to deliver and install from depot to location i to j and back to depot -  $t_{ijc}$

#### 5.1.2 Parameters

1. Number of Pickup Locations -  $pick_{num}$
2. Number of Drop Locations -  $drop_{num}$
3. Number of Vertex Locations -  $vertex_{num}$
4. Number of Crews -  $crew_{num}$
5. Product ID -  $prod_{id}$
6. Vertex Names -  $vertex_{name}$

7. Pickup Nodes -  $pickup_{node}$
8. Drop Nodes -  $drop_{node}$
9. Earliest Time -  $ET$
10. Latest Time -  $LT$
11. Vehicle Capacity - 200
12. Start of time window -  $a$
13. End of time window -  $b$
14. Service time at each node -  $s$
15. Graph arcs -  $arcs$
16. Distance between each node -  $dist$
17. Cost of travelling from 1 node to another by a crew -  $cost$
18. Time taken to travel from 1 node to another -  $time$
19. Installation time for each -  $zt$
20. Crew Skillset -  $crewskills$
21. Installation or only Delivery Required defined by binary variable -  $z$

### 5.1.3 Objective Function

The objective function is defined to minimise the total transportation cost.

$$Minimize \sum_{c=1}^C \sum_{i=1}^{vertex_{num}} \sum_{j=1}^{vertex_{num}} cost_{i,j,c} * x_{i,j,c} \quad (5.1)$$

### 5.1.4 Constraints

The following are the soft and hard constraints that have to be taken into consideration to make the problem as realistic as possible:

$$\sum_{c=0}^C \sum_{j=1}^{vertex_{num}} x_{i,j,c} = 1 \quad \forall i \in pick_{num} \quad (5.2)$$

$$\sum_{j=1}^{vertex_{num}} x_{0,j,c} = 1 \quad \forall c \in crew_{num} \quad (5.3)$$

$$\sum_{j=0}^{vertex_{num}} x_{i,j,c} - \sum_{j=1}^{vertex_{num}} x_{j,i+drop_{num},c} = 0 \quad \forall c \in crew_{num}, i \in pick_{num} \quad (5.4)$$

$$\sum_{i=0}^{vertex_{num}-1} x_{i,j,c} - \sum_{i=1}^{vertex_{num}} x_{j,i,c} = 0 \quad \forall c \in crew_{num}, j \in pick_{num} + drop_{num} \quad (5.5)$$

$$\sum_{i=1}^{vertex_{num}} x_{i,vertex_{num},c} = 1 \quad \forall c \in crew_{num} \quad (5.6)$$

$$t_{i,c} + s_i + time_{i,j} - t_{j,c} + z * (zt_{c,prod_{id}} \leq M * (1 - 1 * x_{i,j,c})) \quad \forall c \in crew_{num}, j \in vertex_{num}, i \in vertex_{num} \quad (5.7)$$

$$a_i \leq t_{i,c} \leq b_i \quad \forall c \in crew_{num}, j \in vertex_{num}, i \in vertex_{num} \quad (5.8)$$

$$ET \leq t_{0,c} \leq LT \quad \forall c \in crew_{num}, i \in 0 \quad (5.9)$$

$$ET \leq t_{vertex_{num},c} \leq LT \quad \forall c \in crew_{num}, i \in vertex_{num} \quad (5.10)$$

$$t_{i,c} + time_{pick_{num}+i,c} \leq t_{i+drop_{num},c} \quad \forall c \in crew_{num}, i \in pick_{num} \quad (5.11)$$

$$load_i \leq l_{i,c} \leq vehicle_{capacity} \quad \forall c \in crew_{num}, i \in pick_{num} \quad (5.12)$$

$$0 \leq l_{pick_{num}+i,c} \leq vehicle_{capacity} + load_{drop_{num}+i} \quad \forall c \in crew_{num}, i \in drop_{num} \quad (5.13)$$

$$l_{0,c} = 1 \quad \forall c \in crew_{num} \quad (5.14)$$

$$\sum_{i=1}^{vertex_{num}-1} load_i * \sum_{i=1}^{vertex_{num}-1} \sum_{j=0}^{vertex_{num}} x_{j,i,c} \leq vehicle_{capacity}, c \in crew_{num} \quad (5.15)$$

$$\sum_{i=0}^{vertex_{num}} \sum_{j=0}^{vertex_{num}} z * crewskills_{c,prod_{id}} x_{i,j,c} = 1 \quad \forall c \in crew_{num} \quad (5.16)$$

Constraints are described as follows:

Each pickup customer is allocated to only one crew - (5.2)

Each request is served exactly once by a crew from the depot or starting point - (5.3)

Each pickup and delivery/installation is visited only once by the same crew - (5.4)

No return to previous node, starts from the depot and ends at depot - (5.5)

Closed VRP where every crew returns back to the depot - (5.6)

Service Time at customer  $i$  should be lesser than customer  $j$  - (5.7)

Start of Service time after start of customer time window - (5.8)

8hr working hr constraint for closed VRP - (5.9, 5.10)

Precedence of time at pickup node should be less than drop node - (5.11)

Load at every pickup node should be less than vehicle capacity - (5.12)

Load at drop node should be lesser than vehicle capacity and greater than 0 - (5.13)

Load at depot for every crew should be zero - (5.14)

Vehicle Capacity Constraint - the load of a vehicle never exceeds its capacity - (5.15)

Matching Installation skill set required with the skill set of servicemen in crew - (5.16)

## Chapter 6

# Experiments and Results

### 6.1 Standard Parameters and Output

To validate the model proposed, a small dataset of 1 depot, 2 pickup and 2 drop locations were considered in Latitude and Longitude.

Appropriate time window and service time were considered for each node. A product request of 5 requests were created to test the constraints with delivery and installation requirements.

Three large products such as Air Conditioner, Bed and Washing Machine were considered of product weight of 30kg, 60kg and 50kg.

The Standard vehicle capacity was taken to be 200kg with an average speed of 50km/hr to create the time matrix between 2 nodes using the distance matrix generated using Haversine Formula.

Three crews were considered having 2 alternate skillsets comprising of AC Mechanic, Carpenter and Plumber with a standard delivery and service time for each crew. Each crew is assigned a crew cost as well.

The objective function was found to be 9745.39 and the CPLEX Time was 1.1422656s.

| ID | Location | Latitude  | Longitude | StartTW | EndTW | ServiceTime |
|----|----------|-----------|-----------|---------|-------|-------------|
| 1  | Depot    | 12° 50' N | 80° 8' E  | 0       | 480   | 0           |
| 2  | P1       | 13° 2' N  | 80° 13' E | 30      | 92    | 10          |
| 3  | P2       | 13° 4' N  | 80° 11' E | 15      | 67    | 10          |
| 4  | C1       | 13° 5' N  | 80° 9' E  | 169     | 224   | 30          |
| 5  | C2       | 12° 58' N | 80° 7' E  | 166     | 235   | 30          |

TABLE 6.1: Coordinates of all the Points and their time windows for all experiments

| ID | PickupID | CustomerID | ProductID | Del/Ins |
|----|----------|------------|-----------|---------|
| 1  | 3        | 4          | 1         | 1       |
| 2  | 3        | 4          | 2         | 0       |
| 3  | 3        | 4          | 3         | 1       |
| 4  | 2        | 5          | 2         | 1       |
| 5  | 2        | 5          | 3         | 0       |

TABLE 6.2: Product Requests for the standard experiment

| CrewID | S1 | S2 | S3 | Crew Cost |
|--------|----|----|----|-----------|
| 1      | 1  | 0  | 1  | 100       |
| 2      | 1  | 1  | 0  | 100       |
| 3      | 0  | 1  | 1  | 100       |

TABLE 6.3: Crew Skill Set Matrix standard

| CrewID | ST1 | ST2 | ST3 |
|--------|-----|-----|-----|
| 1      | 30  | 0   | 30  |
| 2      | 30  | 30  | 0   |
| 3      | 0   | 30  | 30  |

TABLE 6.4: Crew Installation Time Matrix standard

## 6.2 Change in Product Request

Three experiments were conducted to validate the model. The product requests were changed that is the pickup and customer nodes with corresponding products and their requirement for installation was varied.

Experiment 1 Product Request: The objective cost was 9427.66 and the CPLEX Time was found to be 1.3613241s.

Experiment 2 Product Request: The objective cost was 9427.66 and the CPLEX Time was found to be 1.48381211s.

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0,2,1,7,6,0       | 0.0, 31.8, 67.0, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 30.0, -60.0, -30.0, 0.0 |
| 2      | 0, 4, 5, 9, 10, 0 | 0.0, 30.0, 92.0, 166.0, 226.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |
| 3      | 0, 3, 8, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 50.0, -50.0, 0.0              |

TABLE 6.5: Standard Value Experiment

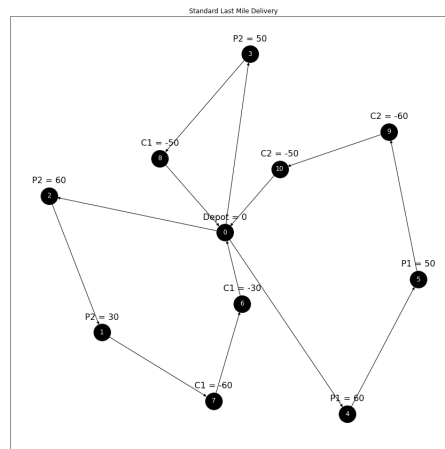


FIGURE 6.1: Standard Map used for testing

Experiment 3 Product Request: The objective cost was 9427.664250206275 and the CPLEX Time was found to be 1.3613241s.

| ID | PickupID | CustomerID | ProductID | Del/Ins |
|----|----------|------------|-----------|---------|
| 1  | 3        | 4          | 3         | 1       |
| 2  | 3        | 4          | 2         | 0       |
| 3  | 2        | 5          | 1         | 1       |
| 4  | 2        | 5          | 2         | 1       |
| 5  | 2        | 5          | 3         | 0       |

TABLE 6.6: Product Requests (1)

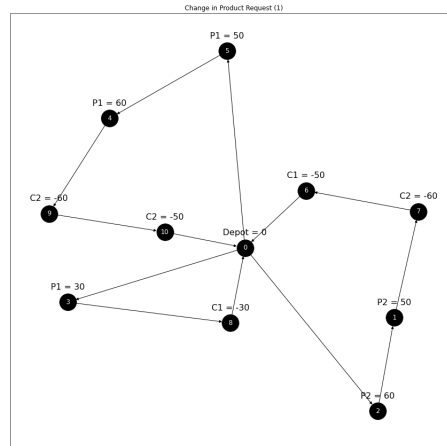


FIGURE 6.2: Network Map for Product Requests (1)

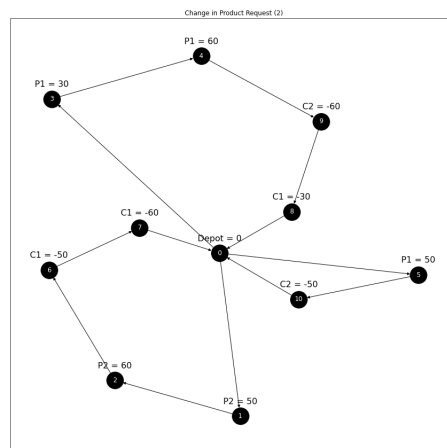


FIGURE 6.3: Network Map for Product Requests (2)



| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 5, 4, 9, 10, 0 | 0.0, 30.0, 92.0, 166.0, 226.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 50.0, 60.0, -60.0, -50.0, 0.0 |
| 2      | 0, 2, 1, 7, 6, 0  | 0.0, 31.8, 67.0, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |
| 3      | 0, 3, 8, 0        | 0.0, 30.0, 166.0, 480.0              | Depot, P1, C2, Depot         | 0.0, 30.0, -30.0, 0.0              |

TABLE 6.7: Product Requests (1) Experiment

| ID | PickupID | CustomerID | ProductID | Del/Ins |
|----|----------|------------|-----------|---------|
| 1  | 3        | 4          | 3         | 0       |
| 2  | 3        | 4          | 2         | 1       |
| 3  | 2        | 5          | 1         | 1       |
| 4  | 2        | 5          | 2         | 0       |
| 5  | 2        | 5          | 3         | 1       |

TABLE 6.8: Product Requests (2)

| CrewID | Routes           | Service Time                         | Route Locations              | Loads                              |
|--------|------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 2, 6, 7, 0 | 0.0, 31.8, 67.0, 169.0, 224.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 50.0, 60.0, -60.0, -50.0, 0.0 |
| 2      | 0, 5, 10, 0      | 0.0, 30.0, 166.0, 480.0              | Depot, P1, C2, Depot         | 0.0, 50.0, -50.0, 0.0              |
| 3      | 0, 3, 4, 9, 8, 0 | 0.0, 30.0, 92.0, 166.0, 196.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 30.0, 60.0, -60.0, -30.0, 0.0 |

TABLE 6.9: Product Requests (2) Experiment

### 6.3 Change in Product Number

Three experiments were conducted to validate the model. The product numbers were changed.

Experiment 1 Product Number: The objective cost was 9545.39 and the CPLEX Time was found to be 0.0437274s.

Experiment 2 Product Number: The objective cost was 10365.55 and the CPLEX Time was found to be 0.3044732s.

| ID | PickupID | CustomerID | ProductID | Del/Ins |
|----|----------|------------|-----------|---------|
| 1  | 3        | 4          | 3         | 0       |
| 2  | 3        | 4          | 2         | 1       |
| 3  | 2        | 4          | 1         | 1       |
| 4  | 2        | 5          | 2         | 0       |
| 5  | 2        | 5          | 3         | 1       |

TABLE 6.10: Product Requests (3)

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 2, 1, 6, 7, 0  | 0.0, 31.8, 41.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 2      | 0, 5, 4, 9, 10, 0 | 0.0, 30.0, 40.0, 166.0, 196.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 50.0, 60.0, -60.0, -50.0, 0.0 |
| 3      | 0, 3, 8, 0        | 0.0, 30.0, 224.0, 480.0              | Depot, P1, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |

TABLE 6.11: Product Requests (3) Experiment

Experiment 3 Product Number: The objective cost was 10538.34 and the CPLEX Time was found to be 0.2105646s.

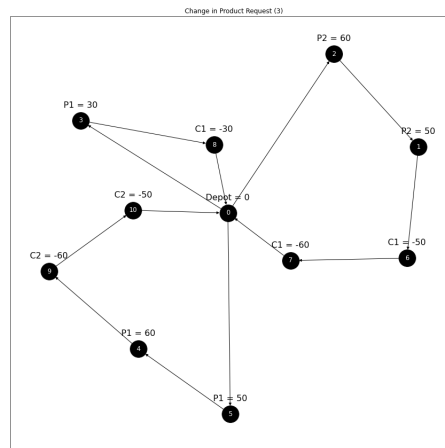


FIGURE 6.4: Network Map for Product Requests (3)

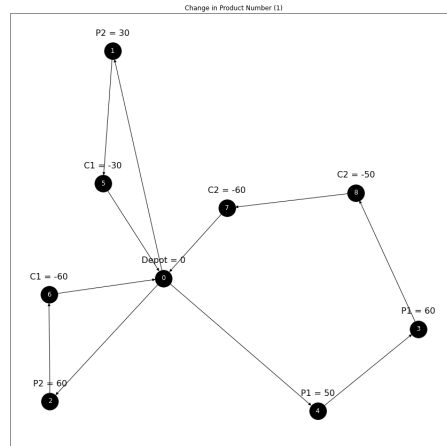


FIGURE 6.5: Network Map for Product Number (1)

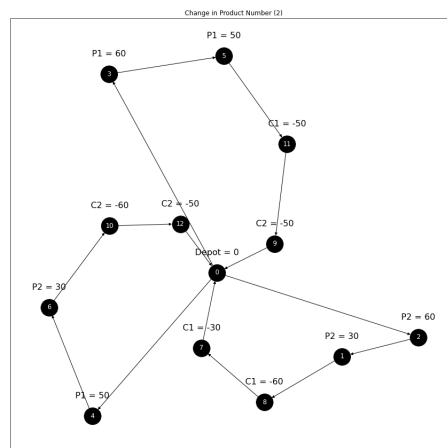


FIGURE 6.6: Network Map for Product Number (2)

| ID | PickupID | CustomerID | ProductID | Del/Ins |
|----|----------|------------|-----------|---------|
| 1  | 3        | 4          | 1         | 1       |
| 2  | 3        | 4          | 2         | 1       |
| 3  | 2        | 5          | 2         | 0       |
| 4  | 2        | 5          | 3         | 1       |

TABLE 6.12: Product Number (1)

| CrewID | Routes           | Service Time                         | Route Locations              | Loads                              |
|--------|------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 2, 6, 0       | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 60.0, -60.0, 0.0              |
| 2      | 0, 4, 3, 8, 7, 0 | 0.0, 30.0, 92.0, 166.0, 226.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 50.0, 60.0, -50.0, -60.0, 0.0 |
| 3      | 0, 1, 5, 0       | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |

TABLE 6.13: Product Number (1) Experiment

| ID | PickupID | CustomerID | ProductID | Del/Ins |
|----|----------|------------|-----------|---------|
| 1  | 3        | 4          | 1         | 1       |
| 2  | 3        | 4          | 2         | 0       |
| 3  | 2        | 5          | 2         | 1       |
| 4  | 2        | 5          | 3         | 1       |
| 5  | 2        | 5          | 3         | 0       |
| 6  | 2        | 5          | 3         | 0       |

TABLE 6.14: Product Number (2)

## 6.4 Change in Product Capacity

Three experiments were conducted to validate the model. The product capacity were changed.

1. Three large products such as Air Conditioner, Bed and Washing Machine were considered of product weight of 100kg, 70kg and 60kg.

Experiment 1 Product Capacity: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0597761s.

2. Three large products such as Air Conditioner, Bed and Washing Machine were considered of product weight of 80kg, 40kg and 90kg.

| CrewID | Routes             | Service Time                         | Route Locations              | Loads                              |
|--------|--------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 2, 1, 8, 7, 0   | 0.0, 31.8, 41.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 30.0, -60.0, -30.0, 0.0 |
| 2      | 0, 3, 5, 11, 9, 0  | 0.0, 30.0, 40.0, 188.8, 235.0, 480.0 | Depot, P1, P1, C1, C1, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 3      | 0, 4, 6, 10, 12, 0 | 0.0, 30.0, 46.2, 166.0, 226.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 50.0, 30.0, -50.0, -30.0, 0.0 |

TABLE 6.15: Product Number (2) Experiment

| ID | PickupID | CustomerID | ProductID | Del/Ins |
|----|----------|------------|-----------|---------|
| 1  | 3        | 4          | 1         | 1       |
| 2  | 3        | 4          | 2         | 0       |
| 3  | 2        | 5          | 2         | 1       |
| 4  | 2        | 5          | 3         | 0       |
| 5  | 2        | 4          | 3         | 1       |
| 6  | 3        | 5          | 1         | 0       |
| 7  | 2        | 4          | 1         | 0       |

TABLE 6.16: Product Number (3)

Experiment 2 Product Capacity: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0375777s.

3. Three large products such as Air Conditioner, Bed and Washing Machine were considered of product weight of 50kg, 50kg and 50kg.

Experiment 3 Product Capacity: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0344872s.

## 6.5 Change in Crew

Three experiments were conducted to validate the model. The crew skill sets were changed.

Experiment 1 Crew Change: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0157105s.

Experiment 2 Crew Change: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0193994s.

| CrewID | Routes                       | Service Time  | Route Locations                               | Loads  |
|--------|------------------------------|---|---|--|
| 1      | 0, 3, 4, 6,<br>13, 11, 10, 0 | 0.0, 30.0,<br>40.0, 56.2,<br>166.0,<br>196.0,<br>226.0, 480.0 | Depot, P1,<br>P1, P2, C2,<br>C2, C2,<br>Depot | 0.0, 60.0,<br>50.0, 30.0,<br>-30.0, -50.0,<br>-60.0, 0.0 |
| 2      | 0, 5, 7, 14,<br>12, 0        | 0.0, 30.0,<br>40.0, 166.0,<br>196.0, 480.0                    | Depot, P1,<br>P1, C1, C1,<br>Depot            | 0.0, 50.0,<br>30.0, -30.0,<br>-50.0, 0.0                 |
| 3      | 0, 2, 1, 9, 8,<br>0          | 0.0, 31.8,<br>41.8, 169.0,<br>224.0, 480.0                    | Depot, P2,<br>P2, C1, C1,<br>Depot            | 0.0, 60.0,<br>30.0, -60.0,<br>-30.0, 0.0                 |

TABLE 6.17: Product Number (3) Experiment

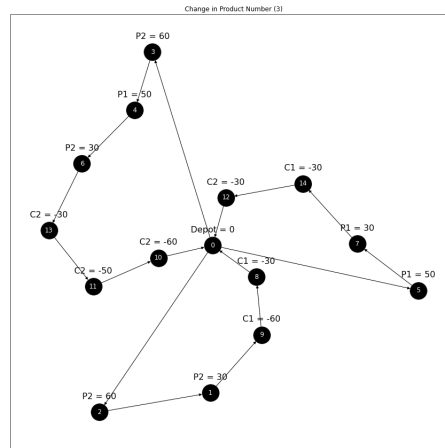


FIGURE 6.7: Network Map for Product Number (3)

Experiment 3 Crew Change: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0363798s.

## 6.6 Change in Installation Time

Three experiments were conducted to validate the model. The crew skill installation time matrix were changed.

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 100.0, -100.0, 0.0            |
| 2      | 0, 4, 5, 9, 10, 0 | 0.0, 30.0, 39.9, 166.0, 226.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 70.0, 60.0, -70.0, -60.0, 0.0 |
| 3      | 0, 3, 2, 7, 8, 0  | 0.0, 31.8, 41.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 70.0, -70.0, -60.0, 0.0 |

TABLE 6.18: Product Capacity (1) Experiment

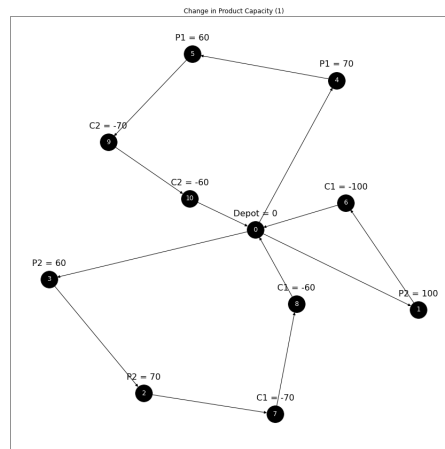


FIGURE 6.8: Network Map for Product Capacity (1)

Experiment 1 Installation Time: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0804898s.

Experiment 2 Installation Time: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0637389s.

Experiment 3 Installation Time: The objective cost was 9745.39 and the CPLEX Time was found to be 0.1155746s.

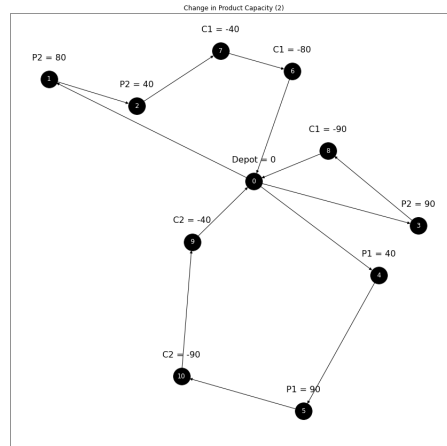


FIGURE 6.9: Network Map for Product Capacity (2)

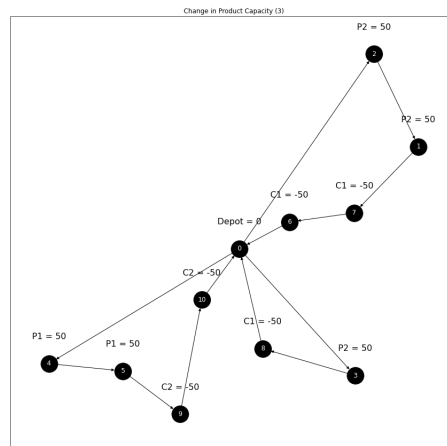


FIGURE 6.10: Network Map for Product Capacity (3)



| CrewID | Routes            | Service Time                          | Route Locations              | Loads                              |
|--------|-------------------|---------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 2, 7, 6, 0  | 0.0, 31.80, 67.0, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 80.0, 40.0, -40.0, -80.0, 0.0 |
| 2      | 0, 4, 5, 10, 9, 0 | 0.0, 30.0, 92.0, 166.0, 235.0, 480.0  | Depot, P1, P1, C2, C2, Depot | 0.0, 40.0, 90.0, -90.0, -40.0, 0.0 |
| 3      | 0, 3, 8, 0        | 0.0, 31.8, 224.0, 480.0               | Depot, P2, C1, Depot         | 0.0, 90.0, -90.0, 0.0              |

TABLE 6.19: Product Capacity (2) Experiment

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 2, 1, 7, 6, 0  | 0.0, 31.8, 67.0, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 50.0, 50.0, -50.0, -50.0, 0.0 |
| 2      | 0, 4, 5, 9, 10, 0 | 0.0, 30.0, 92.0, 166.0, 226.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 50.0, 50.0, -50.0, -50.0, 0.0 |
| 3      | 0, 3, 8, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 50.0, -50.0, 0.0              |

TABLE 6.20: Product Capacity (3) Experiment

| CrewID | S1 | S2 | S3 | Crew Cost |
|--------|----|----|----|-----------|
| 1      | 0  | 1  | 0  | 100       |
| 2      | 1  | 0  | 0  | 100       |
| 3      | 0  | 0  | 1  | 100       |

TABLE 6.21: Crew Skill Set Change (1)

## 6.7 Change in Time Window

Three experiments were conducted to validate the model. The crew skill installation time matrix were changed.

Experiment 1 Time Window: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0722041s.

Experiment 2 Time Window: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0526633s.

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 4, 5, 10, 9, 0 | 0.0, 30.0, 92.0, 166.0, 235.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 2      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |
| 3      | 0, 2, 3, 7, 8, 0  | 0.0, 31.8, 67.0, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |

TABLE 6.22: Crew Change (1) Experiment

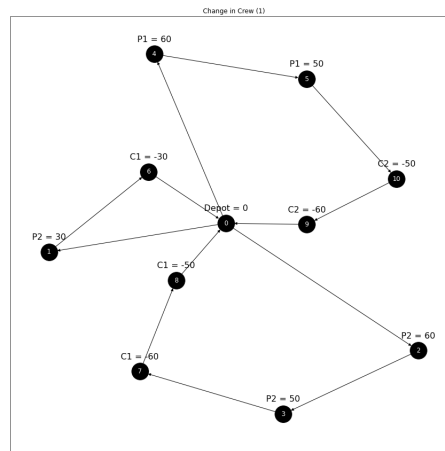


FIGURE 6.11: Network Map for Crew Change (1)

| CrewID | S1 | S2 | S3 | Crew Cost |
|--------|----|----|----|-----------|
| 1      | 0  | 0  | 1  | 100       |
| 2      | 0  | 1  | 0  | 100       |
| 3      | 1  | 0  | 0  | 100       |

TABLE 6.23: Crew Skill Set Change (2)

Experiment 3 Time Window: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0689425s.

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 3, 2, 7, 8, 0  | 0.0, 31.8, 41.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 50.0, 60.0, -60.0, -50.0, 0.0 |
| 2      | 0, 4, 5, 9, 10, 0 | 0.0, 30.0, 40.0, 166.0, 226.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |
| 3      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |

TABLE 6.24: Crew Change (2) Experiment

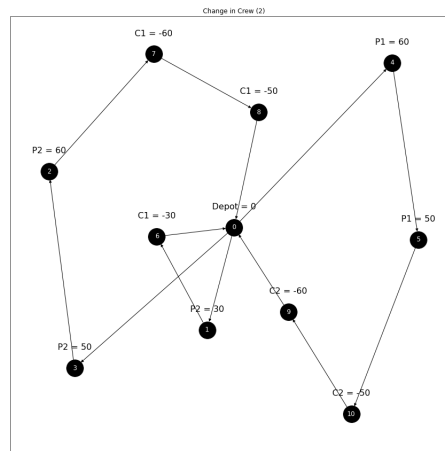


FIGURE 6.12: Network Map for Crew Change (2)

| CrewID | S1 | S2 | S3 | Crew Cost |
|--------|----|----|----|-----------|
| 1      | 1  | 0  | 1  | 100       |
| 2      | 0  | 1  | 1  | 100       |
| 3      | 1  | 1  | 0  | 100       |

TABLE 6.25: Crew Skill Set Change (3)

## 6.8 Change in other parameters

Three experiments were conducted to validate the model. The crew skill installation time matrix were changed.

1. Vehicle Capacity was kept as 50 and 100. Problem could not be solved due to violation of vehicle capacity.

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |
| 2      | 0, 2, 3, 7, 8, 0  | 0.0, 31.8, 67.0, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |
| 3      | 0, 4, 5, 10, 9, 0 | 0.0, 30.0, 40.0, 166.0, 235.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |

TABLE 6.26: Crew Change (3) Experiment

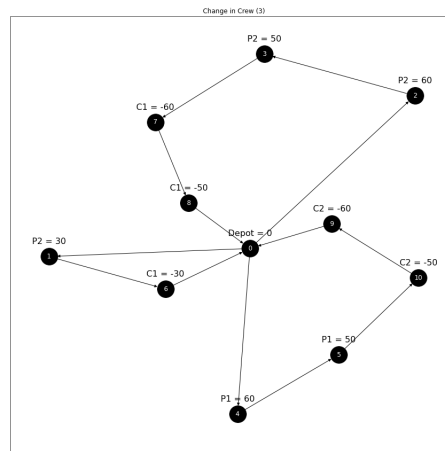


FIGURE 6.13: Network Map for Crew Change (3)

| CrewID | ST1 | ST2 | ST3 |
|--------|-----|-----|-----|
| 1      | 50  | 0   | 60  |
| 2      | 70  | 40  | 0   |
| 3      | 0   | 30  | 80  |

TABLE 6.27: Crew Installation Time Matrix (1)

2. Change in Crew Cost was experimented with.

Experiment Crew Cost: The objective cost was 9745.39 and the CPLEX Time was found to be 0.033917301s.

3. Change in Service time at each node.

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 4, 5, 10, 9, 0 | 0.0, 30.0, 40.0, 166.0, 235.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 2      | 0, 2, 3, 7, 8, 0  | 0.0, 31.8, 41.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |
| 3      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |

TABLE 6.28: Installation Time Change (1) Experiment

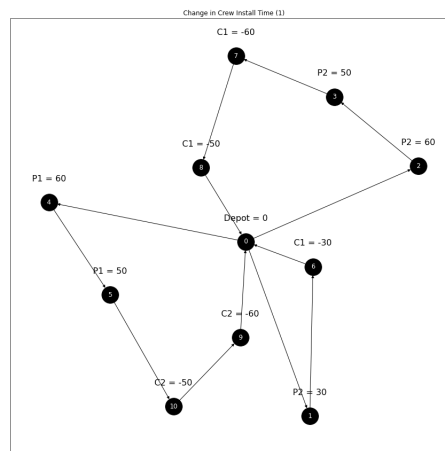


FIGURE 6.14: Network Map for Crew Installation Time Change (1)

| CrewID | ST1 | ST2 | ST3 |
|--------|-----|-----|-----|
| 1      | 45  | 0   | 45  |
| 2      | 25  | 65  | 0   |
| 3      | 0   | 55  | 30  |

TABLE 6.29: Crew Installation Time Matrix (2)

Experiment Service Time: The objective cost was 9745.39 and the CPLEX Time was found to be 0.0774731s.

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |
| 2      | 0, 4, 5, 10, 9, 0 | 0.0, 30.0, 40.0, 166.0, 235.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 3      | 0, 3, 2, 7, 8, 0  | 0.0, 31.8, 41.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 50.0, 60.0, -60.0, -50.0, 0.0 |

TABLE 6.30: Installation Time Change (2) Experiment

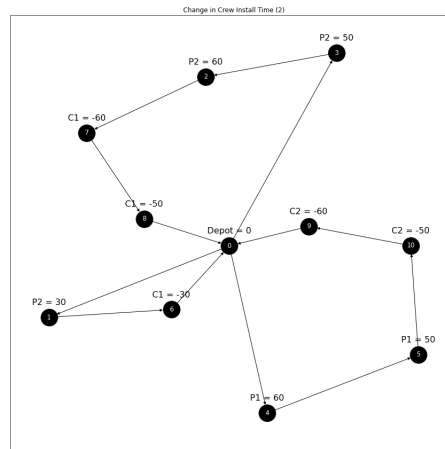


FIGURE 6.15: Network Map for Crew Installation Time Change (2)

| CrewID | ST1 | ST2 | ST3 |
|--------|-----|-----|-----|
| 1      | 50  | 0   | 150 |
| 2      | 150 | 50  | 0   |
| 3      | 0   | 150 | 50  |

TABLE 6.31: Crew Installation Time Matrix (3)

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 4, 5, 10, 9, 0 | 0.0, 30.0, 40.0, 166.0, 235.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 2      | 0, 2, 3, 7, 8, 0  | 0.0, 31.8, 41.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |
| 3      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |

TABLE 6.32: Installation Time Change (3) Experiment

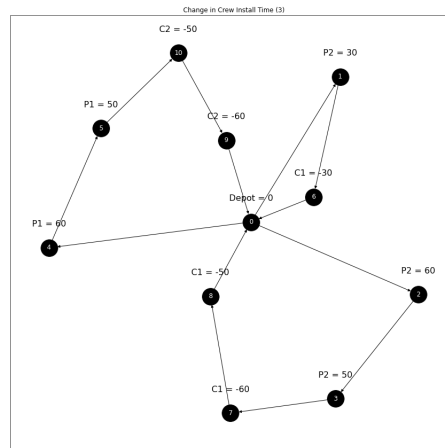


FIGURE 6.16: Network Map for Crew Installation Time Change (3)

| ID | Location | Latitude  | Longitude | StartTW | EndTW | ServiceTime |
|----|----------|-----------|-----------|---------|-------|-------------|
| 1  | Depot    | 12° 50' N | 80° 8' E  | 0       | 480   | 0           |
| 2  | P1       | 13° 2' N  | 80° 13' E | 99      | 148   | 10          |
| 3  | P2       | 13° 4' N  | 80° 11' E | 68      | 149   | 10          |
| 4  | C1       | 13° 5' N  | 80° 9' E  | 261     | 316   | 30          |
| 5  | C2       | 12° 58' N | 80° 7' E  | 278     | 345   | 30          |

TABLE 6.33: Time Window Change (1)

| CrewID | Routes            | Service Time                          | Route Locations              | Loads                              |
|--------|-------------------|---------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 4, 5, 10, 9, 0 | 0.0, 99.0, 148.0, 278.0, 308.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 2      | 0, 3, 2, 7, 8, 0  | 0.0, 68.0, 149.0, 261.0, 291.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 50.0, 60.0, -60.0, -50.0, 0.0 |
| 3      | 0, 1, 6, 0        | 0.0, 68.0, 260.9, 480.0               | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |

TABLE 6.34: Change in Time Window (1) Experiment

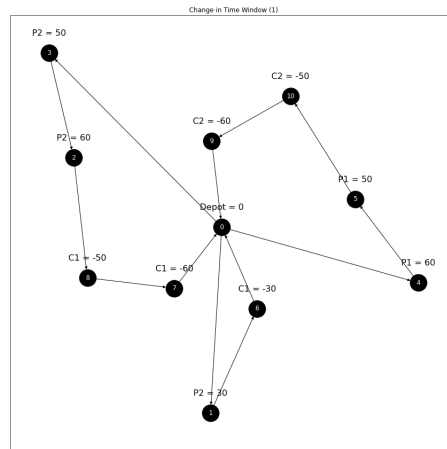


FIGURE 6.17: Network Map for Time Window Change (1)

| ID | Location | Latitude  | Longitude | StartTW | EndTW | ServiceTime |
|----|----------|-----------|-----------|---------|-------|-------------|
| 1  | Depot    | 12° 50' N | 80° 8' E  | 0       | 480   | 0           |
| 2  | P1       | 13° 2' N  | 80° 13' E | 31      | 100   | 10          |
| 3  | P2       | 13° 4' N  | 80° 11' E | 87      | 158   | 10          |
| 4  | C1       | 13° 5' N  | 80° 9' E  | 261     | 316   | 30          |
| 5  | C2       | 12° 58' N | 80° 7' E  | 265     | 338   | 30          |

TABLE 6.35: Time Window Change (2)



| CrewID | Routes            | Service Time                          | Route Locations              | Loads                              |
|--------|-------------------|---------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 2, 7, 6, 0  | 0.0, 87.0, 97.0, 261.0, 291.0, 480.0  | Depot, P2, P2, C1, C1, Depot | 0.0, 30.0, 60.0, -60.0, -30.0, 0.0 |
| 2      | 0, 5, 4, 9, 10, 0 | 0.0, 31.0, 100.0, 265.0, 325.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 50.0, 60.0, -60.0, -50.0, 0.0 |
| 3      | 0, 3, 8, 0        | 0.0, 87.0, 260.99, 480.0              | Depot, P2, C1, Depot         | 0.0, 50.0, -50.0, 0.0              |

TABLE 6.36: Change in Time Window (2) Experiment

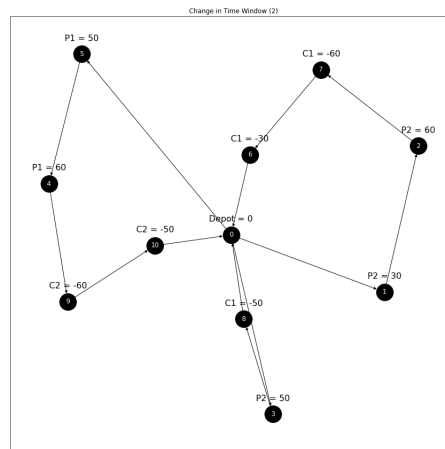


FIGURE 6.18: Network Map for Time Window Change (2)

| ID | Location | Latitude  | Longitude | StartTW | EndTW | ServiceTime |
|----|----------|-----------|-----------|---------|-------|-------------|
| 1  | Depot    | 12° 50' N | 80° 8' E  | 0       | 480   | 0           |
| 2  | P1       | 13° 2' N  | 80° 13' E | 31      | 100   | 10          |
| 3  | P2       | 13° 4' N  | 80° 11' E | 12      | 77    | 10          |
| 4  | C1       | 13° 5' N  | 80° 9' E  | 186     | 257   | 30          |
| 5  | C2       | 12° 58' N | 80° 7' E  | 357     | 410   | 30          |

TABLE 6.37: Time Window Change (3)

| CrewID | Routes            | Service Time                          | Route Locations              | Loads                              |
|--------|-------------------|---------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 6, 0        | 0.0, 31.8, 186.0, 480.0               | Depot, P2, P2, C1, C1, Depot | 0.0, 30.0, -30.0, 0.0              |
| 2      | 0, 4, 5, 10, 9, 0 | 0.0, 31.0, 41.0, 357.0, 402.07, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |
| 3      | 0, 2, 3, 8, 7, 0  | 0.0, 31.8, 41.8, 186.00, 245.9, 480.0 | Depot, P2, C1, Depot         | 0.0, 60.0, 50.0, -50.0, -60.0, 0.0 |

TABLE 6.38: Change in Time Window (3) Experiment

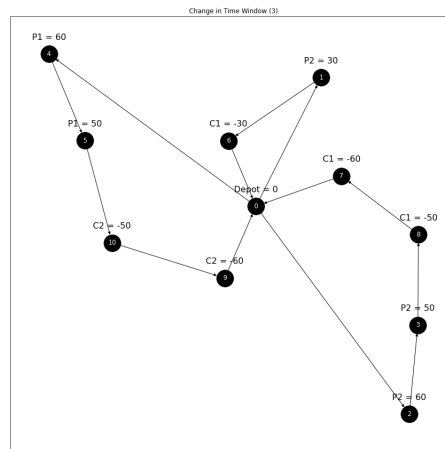


FIGURE 6.19: Network Map for Time Window Change (3)

| CrewID | S1 | S2 | S3 | Crew Cost |
|--------|----|----|----|-----------|
| 1      | 0  | 1  | 0  | 500       |
| 2      | 1  | 0  | 0  | 200       |
| 3      | 0  | 0  | 1  | 350       |

TABLE 6.39: Crew Cost Values Change

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 1, 6, 0        | 0.0, 31.8, 224.0, 480.0              | Depot, P2, C1, Depot         | 0.0, 30.0, -30.0, 0.0              |
| 2      | 0, 5, 4, 10, 9, 0 | 0.0, 30.0, 92.0, 166.0, 186.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 50.0, 60.0, -50.0, -60.0, 0.0 |
| 3      | 0, 2, 3, 7, 8, 0  | 0.0, 31.8, 46.8, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |

TABLE 6.40: Change in Crew Cost Experiment

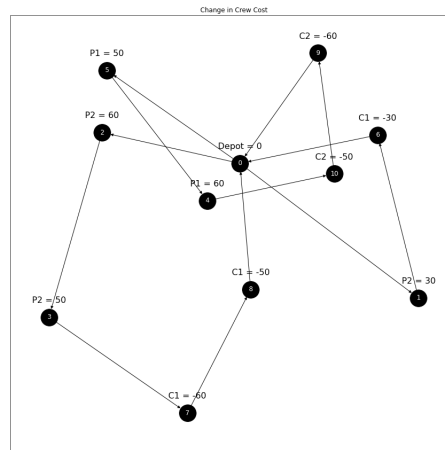


FIGURE 6.20: Network Map for Crew Cost Change

| ID | Location | Latitude  | Longitude | StartTW | EndTW | ServiceTime |
|----|----------|-----------|-----------|---------|-------|-------------|
| 1  | Depot    | 12° 50' N | 80° 8' E  | 0       | 480   | 0           |
| 2  | P1       | 13° 2' N  | 80° 13' E | 30      | 92    | 20          |
| 3  | P2       | 13° 4' N  | 80° 11' E | 15      | 67    | 15          |
| 4  | C1       | 13° 5' N  | 80° 9' E  | 169     | 224   | 40          |
| 5  | C2       | 12° 58' N | 80° 7' E  | 166     | 235   | 20          |

TABLE 6.41: Change in values of Service time

| CrewID | Routes            | Service Time                         | Route Locations              | Loads                              |
|--------|-------------------|--------------------------------------|------------------------------|------------------------------------|
| 1      | 0, 2, 1, 7, 6, 0  | 0.0, 31.8, 67.0, 169.0, 224.0, 480.0 | Depot, P2, P2, C1, C1, Depot | 0.0, 60.0, 30.0, -60.0, -30.0, 0.0 |
| 2      | 0, 4, 5, 9, 10, 0 | 0.0, 30.0, 92.0, 166.0, 186.0, 480.0 | Depot, P1, P1, C2, C2, Depot | 0.0, 60.0, 50.0, -60.0, -50.0, 0.0 |
| 3      | 0, 3, 8, 0        | 0.0, 31.8, 46.8, 169.0, 224.0, 480.0 | Depot, P2, C1, Depot         | 0.0, 50.0, -50.0, 0.0              |

TABLE 6.42: Change in Service Time Experiment

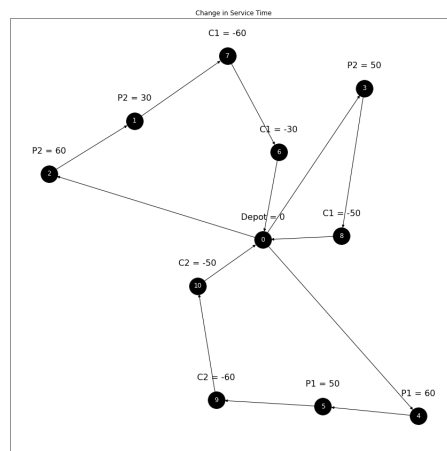


FIGURE 6.21: Network Map for Service Time Change

## Chapter 7

# Inferences, Conclusion and Future Scope

### 7.1 Inferences

(1) The proposed new Mathematical model for Multiple Pickup, Delivery and Installation Vehicle Routing Problem with time window and vehicle capacity constraints has been validated using a small dataset. The standard experiment was conducted with standard set of values for all inputs and a network map was generated for each crew with the optimal route.

(2) Change in Product Request indicates the model's ability to handle different requests. For different product requests, the objective function is minimised for all cases. It can be witnessed in the third trial the objective function is almost 400 more than the first trial. This indicates that when the product request varies, the objective varies optimally.

(3) Change in Product Number indicates the model's ability to handle larger number of requests. For different product numbers, the objective function is minimised for all cases and optimal route is found for each fleet. A linear increase in objective function was found where as the product numbers increased from 4 to 7 in the third trial, the demand and the number of nodes increased in the network for each crew.

(4) Change in Product Capacity indicates that as the volume of load increase, the time taken by the solver to obtain the same objective function increases as it indicates that vehicle capacity needs to be checked for its violation. The objective function is independent of the product capacity and the optimal routes generated.

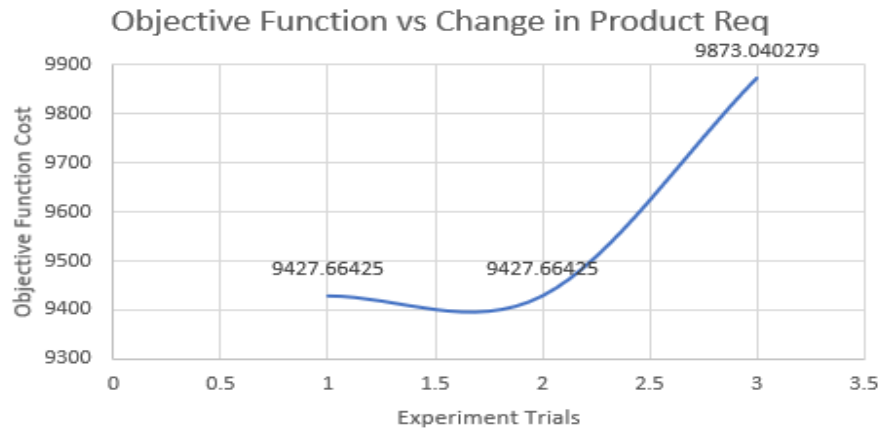


FIGURE 7.1: Inference for Product Request

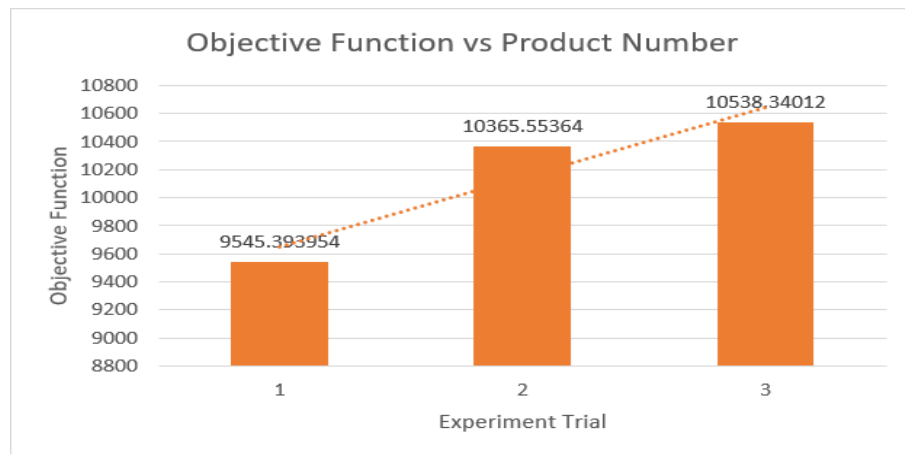


FIGURE 7.2: Inference for Product Number

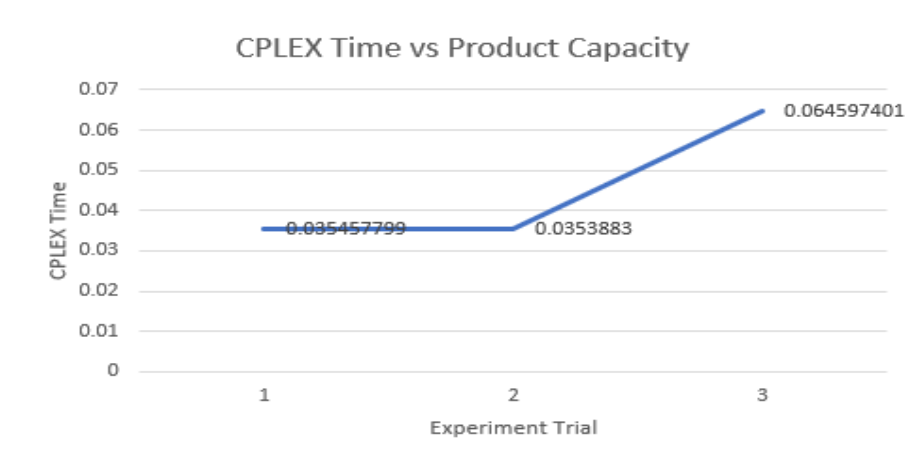


FIGURE 7.3: Inference for Product Capacity

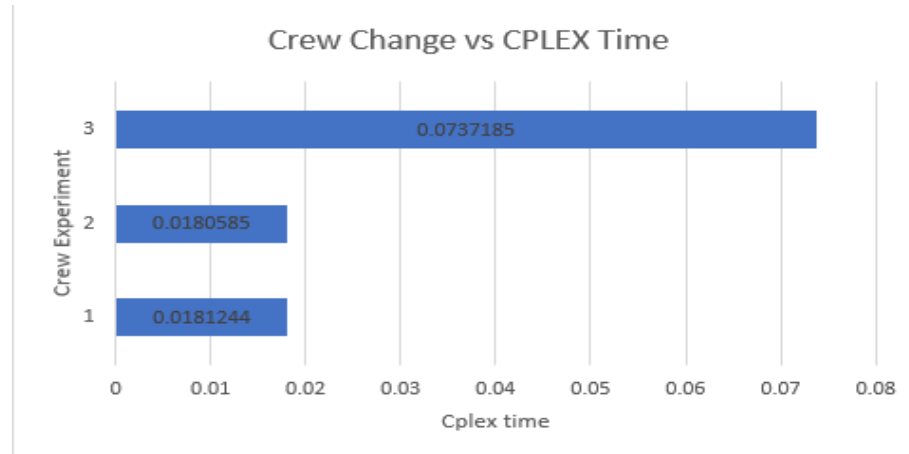


FIGURE 7.4: Inference for Crew Change

Change in Time Window vs CPLEX Time

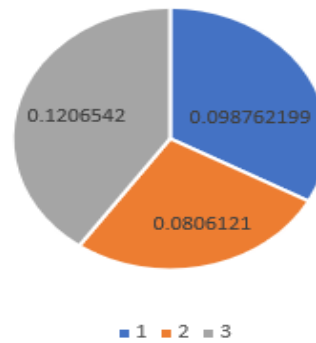


FIGURE 7.5: Inference for Time Window

(5) Change in Crew and their skill sets changes the time taken by the solver to obtain the same objective function increases especially in the case when a single crew has multiple skill sets. The optimal value is reached earlier when the skill set for crews is lesser. As the complexity increases and crews have 2 skill sets each, the optimal value is reached substantially later for the third trial

(6) Change in Time Window changes the time taken by the solver to obtain the same objective function increases especially in the case when the time windows are close by. The solving time is independent of the time window values however the optimal routes generated are different for each case.

(7) Change in Installation Time of each crew does not change the objective function however the service time changes for each crew and the route calculated is within the time windows defined.

- (8) Change in Vehicle capacity indicated the problem could not be solved when the vehicle capacity was less than the load given as input. Two cases were tested where the vehicle capacity can be lesser than all the good capacity weights else the summation of loads does not satisfy the total vehicle capacity.
- (9) Change in Crew Cost where crews with different crew cost will change the optimal route generated for each crew with a greater number of nodes assigned to crews with lesser crew cost so to minimise the total crew cost.
- (10) Change in service time indicated that when the time window was violated, the optimal routes generated varied for each crew.

## 7.2 Conclusion

A novel mathematics model was proposed for the multi pickup, delivery and installation vehicle routing problem involving time window and capacity constraints. The PDIVRP also included crew allocation using a single fleet based on the product demand and skill set required for such a service. The Branch and Bound algorithm was used to find the accurate solution for a small dataset to verify the validity of the model proposed. TSP, VRP, VRPTW, PDVRP were also understood and executed before Installation using crew allocation was understood and implemented. Various experiments to check the model's output was conducted and was found to versatile in terms of product request and skillset of the crew, yet optimal for larger product numbers and smaller time windows. For easy visualisation of the routes, a network map was created where the load picked up and dropped are indicated for each crew for each case.

## 7.3 Future Scope

The model can be extended to larger industry datasets which is currently unavailable and near optimal solutions can be achieved by using heuristic techniques like simulated annealing and genetic algorithm. TABLEAU and other visualisation tools could have been used for better visualisation of the routes generated.



## Appendix A

### Results of Variants

$$\begin{aligned} &A = \{1, 2, 3, \dots, n\} \text{ is the set of all cities} \\ &x_{ij} = \begin{cases} 1 & \text{the path goes from city } i \text{ to city } j \\ 0 & \text{otherwise} \end{cases} \\ &\min \sum_{i \in A} \sum_{j \neq i, j \in A} c_{ij} x_{ij} \\ &\text{s.t.} \quad \sum_{i \in A, i \neq j} x_{ij} = 1 \quad j \in A \\ &\quad \sum_{j \in A, j \neq i} x_{ij} = 1 \quad i \in A \\ &\quad u_i - u_j + (n-1)x_{ij} \leq n-2 \quad i \in A \setminus \{1\}, j \in A \setminus \{1\}, i \neq j \end{aligned}$$

FIGURE A.1: TSP Model

| Route         |   |
|---------------|---|
| 0             | 3 |
| 1             | 8 |
| 2             | 0 |
| 3             | 1 |
| 4             | 9 |
| 5             | 2 |
| 6             | 4 |
| 7             | 5 |
| 8             | 6 |
| 9             | 7 |
| Ordered Route |   |
| 0             |   |
| 3             |   |
| 1             |   |
| 8             |   |
| 6             |   |
| 4             |   |
| 9             |   |
| 7             |   |
| 5             |   |
| 2             |   |
| 0             |   |

FIGURE A.2: TSP

Parameters:  $n$  = number of points (1 - depot, 2, ...,  $n$  - clients)

$d_{ij}$  = distance from point  $i$  to point  $j$

$D_i$  = demand of client  $i$

$C$  = capacity of each truck

Variables:  $x_{ij}$  = 1 if a truck goes from node  $i$  to node  $j$  (binary)

$f_{ij}$  = number of units in a truck going from node  $i$  to node  $j$

$$\begin{aligned}
 & \min \sum_{i=1}^n \sum_{j=1}^n d_{ij} x_{ij} \\
 & \sum_{j=1}^n x_{ij} = 1 \quad \forall i = 2, \dots, n \\
 & \sum_{j=1}^n x_{ji} = 1 \quad \forall i = 2, \dots, n \\
 & \sum_{j=1}^n f_{ji} - \sum_{j=1}^n f_{ij} = D_i \quad \forall i = 2, \dots, n \\
 & 0 \leq f_{ij} \leq C x_{ij} \quad \forall i, j = 1, \dots, n \\
 & x_{ij} \in \{0, 1\} \quad \forall i, j = 1, \dots, n
 \end{aligned}$$

FIGURE A.3: VRP Model

| Route |    |
|-------|----|
| 0     | 12 |
| 0     | 13 |
| 0     | 14 |
| 1     | 18 |
| 2     | 6  |
| 3     | 5  |
| 4     | 19 |
| 5     | 9  |
| 6     | 4  |
| 7     | 17 |
| 8     | 3  |
| 9     | 11 |
| 10    | 0  |
| 11    | 0  |
| 12    | 20 |
| 13    | 1  |
| 14    | 16 |
| 15    | 7  |
| 16    | 15 |
| 17    | 2  |
| 18    | 0  |
| 19    | 10 |
| 20    | 8  |

FIGURE A.4: VRP (1)

```
Number of Routes : 3

Ordered Route

Route : 1
0 - Madras (Chennai)
12 - Krishnagiri
20 - Yercaud
8 - Salem
3 - Namakkal
5 - Karur
9 - Tiruchchirappalli
11 - Thanjavur (Tanjore)
0 - Madras (Chennai)

Route : 2
0 - Madras (Chennai)
13 - Vriddhachalam
1 - Kanchipuram
18 - Tiruvallur
0 - Madras (Chennai)
```

FIGURE A.5: VRP (2)

```

Route : 3
0 - Madras (Chennai)
14 - Tiruvannamalai
16 - Tiruchengodu
15 - Tiruppur
7 - Satyamangalam
17 - Mettupalaiyam
2 - Coimbatore
6 - Palni Hills
4 - Madurai
19 - Kilakarai
10 - Pudukkottai
0 - Madras (Chennai)

```

FIGURE A.6: VRP (3)

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} \quad (1)$$

$$\sum_{k \in K} \sum_{j \in \Delta^+(i)} x_{ijk} = 1 \quad \forall i \in N \quad (2)$$

$$\sum_{j \in \Delta^+(0)} x_{0jk} = 1 \quad \forall k \in K \quad (3)$$

$$\sum_{i \in \Delta^-(j)} x_{ijk} - \sum_{i \in \Delta^+(j)} x_{jik} = 0 \quad \forall k \in K, j \in N \quad (4)$$

$$\sum_{i \in \Delta^-(n+1)} x_{i,n+1,k} = 1 \quad \forall k \in K \quad (5)$$

$$w_{ik} + s_i + t_{ij} - w_{jk} \leq (1 - x_{ijk}) M_{ij} \quad \forall k \in K, (i,j) \in A \quad (6)$$

$$a_i \sum_{j \in \Delta^+(i)} x_{ijk} \leq w_{ik} \leq b_i \sum_{j \in \Delta^+(i)} x_{ijk} \quad \forall k \in K, i \in N \quad (7)$$

$$E \leq w_{ik} \leq L \quad \forall k \in K, i \in \{0, n+1\} \quad (8)$$

$$\sum_{i \in N} d_i \sum_{j \in \Delta^+(i)} x_{ijk} \leq C \quad \forall k \in K \quad (9)$$

$$x_{ijk} \geq 0 \quad \forall k \in K, (i,j) \in A \quad (10)$$

$$x_{ijk} \in \{0,1\} \quad \forall k \in K, (i,j) \in A \quad (11)$$

FIGURE A.7: VRPTW Model

```

Routes=[[0, 5, 3, 7, 8, 10, 11, 9, 6, 4, 2, 1, 75, 101],
[0, 81, 78, 76, 71, 70, 73, 77, 79, 80, 101],
[0, 98, 96, 95, 94, 92, 93, 97, 100, 99, 101],
[0, 43, 42, 41, 40, 44, 46, 45, 48, 51, 50, 52, 49, 47, 101],
[0, 101], [0, 13, 17, 18, 19, 15, 16, 14, 12, 101],
[0, 101],
[0, 57, 55, 54, 53, 56, 58, 60, 59, 101],
[0, 32, 33, 31, 35, 37, 38, 39, 36, 34, 101],
[0, 67, 65, 63, 62, 74, 72, 61, 64, 68, 66, 69, 101],
[0, 90, 87, 86, 83, 82, 84, 85, 88, 89, 91, 101],
[0, 20, 24, 25, 27, 29, 30, 28, 26, 23, 22, 21, 101]]
Cplex_Time 83.5109963 Best Cost 827.3

```

FIGURE A.8: VRPTW

$$\begin{aligned}
& \min \sum_{k \in K} \sum_{(i,j) \in A_k} c_{ijk} x_{ijk} \\
& \sum_{k \in K} \sum_{j \in N_k \cup \{d(k)\}} x_{ijk} = 1 \quad \forall i \in P, \\
& \sum_{j \in N_k} x_{ijk} - \sum_{j \in N_k} x_{j,n+i,k} = 0 \quad \forall k \in K, i \in P_k, \\
& \sum_{j \in P_k \cup \{d(k)\}} x_{o(k),j,k} = 1 \quad \forall k \in K, \\
& \sum_{i \in N_k \cup \{o(k)\}} x_{ijk} - \sum_{i \in N_k \cup \{d(k)\}} x_{jik} = 0 \quad \forall k \in K, j \in N_k, \\
& \sum_{i \in D_k \cup \{o(k)\}} x_{i,d(k),k} = 1 \quad \forall k \in K, \\
& x_{ijk}(T_{ik} + s_i + t_{ijk} - T_{jk}) \leq 0 \quad \forall k \in K, (i,j) \in A_k, \\
& a_i \leq T_{ik} \leq b_i \quad \forall k \in K, i \in V_k, \\
& T_{ik} + t_{i,n+i,k} \leq T_{n+i,k} \quad \forall k \in K, i \in P_k, \\
& x_{ijk}(L_{ik} + \ell_j - L_{jk}) = 0 \quad \forall k \in K, (i,j) \in A_k, \\
& \ell_i \leq L_{ik} \leq C_k \quad \forall k \in K, i \in P_k, \\
& 0 \leq L_{n+i,k} \leq C_k - \ell_i \quad \forall k \in K, n+i \in D_k, \\
& L_{o(k),k} = 0 \quad \forall k \in K, \\
& x_{ijk} \geq 0 \quad \forall k \in K, (i,j) \in A_k, \\
& x_{ijk} \text{ binary} \quad \forall k \in K, (i,j) \in A_k.
\end{aligned}$$

FIGURE A.9: PDVRP Model

```

Vehicles = [[1], [2], [3]]
Routes = [[0, 2, 1, 6, 7, 11],
[0, 3, 8, 11], [0, 5, 4, 10, 9, 11]]
Servicetime = [[0.0, 31.806258416181663, 67.0, 169.0, 199.0, 480.0],
[0.0, 31.806258416181663, 169.0, 480.0],
[0.0, 30.0, 40.0, 166.0, 196.0, 480.0]]
Route Locations = [[Depot, P2, P2, C1, C1, Depot],
[Depot, P2, C1, Depot], [Depot, P1, P1, C2, C2, Depot]]
Loads = [[0.0, 60.0, 30.0, -30.0, -60.0, 0.0], [0.0, 50.0, -50.0, 0.0],
[0.0, 50.0, 60.0, -50.0, -60.0, 0.0]]
Cplex_Time 0.3271329 Best Cost 8445.393954328178

```

FIGURE A.10: PDVRP

# Bibliography

- [1] A. Konstantinidis, S. Pericleous, and C. Charalambous, “Adaptive evolutionary algorithm for a multi-objective vrp,” *International Journal on Engineering Intelligent Systems*, vol. 22, pp. 145–162, 09 2014.
- [2] R. Kulkarni and P. Bhawe, “Integer programming formulations of vehicle routing problems,” *European Journal of Operational Research*, vol. 20, no. 1, pp. 58–67, 1985.
- [3] N. Achuthan and L. Caccetta, “Integer linear programming formulation for a vehicle routing problem,” *European Journal of Operational Research*, vol. 52, no. 1, pp. 86–89, 1991.
- [4] I. Harbaoui Dridi, E. Alaïa, P. Borne, and H. Bouchriha, “Optimisation of the multi-depots pick-up and delivery problems with time windows and multi-vehicles using pso algorithm,” *International Journal of Production Research*, vol. 58, pp. 1–14, 09 2019.
- [5] Çağrı Koç, G. Laporte, and İlknur Tükenmez, “A review of vehicle routing with simultaneous pickup and delivery,” *Computers & Operations Research*, vol. 122, p. 104987, 2020.
- [6] G. Desaulniers, J. Desrosiers, A. Erdmann, M. Solomon, and F. Soumis, *VRP with Pickup and Delivery*, 01 2002, pp. 225–242.
- [7] C. Tilk, K. Olkis, and S. Irnich, “The last-mile vehicle routing problem with delivery options,” *OR Spectrum*, vol. 43, no. 4, pp. 877–904, Dec 2021.
- [8] A. Seghezzi and R. Mangiaracina, “Investigating multi-parcel crowdsourcing logistics for b2c e-commerce last-mile deliveries,” *International Journal of Logistics Research and Applications*, pp. 1–18, 02 2021.
- [9] O. Ali, J.-F. Côté, and L. Coelho, “The delivery and installation routing problem,” 09 2019.

- 
- [10] H. Chu, W. Zhang, P. Bai, and Y. Chen, “Data-driven optimization for last-mile delivery,” *Complex & Intelligent Systems*, 02 2021.
  - [11] J. P. Devarajan, A. Manimuthu, and V. R. Sreedharan, “Healthcare operations and black swan event for covid-19 pandemic: A predictive analytics,” *IEEE Transactions on Engineering Management*, pp. 1–15, 2021.
  - [12] S. Shokry, S. Tanaka, F. Nakamura, R. Ariyoshi, and S. Miura, “Bandwidth maximization approach for displaced left-turn crossovers coordination under heterogeneous traffic conditions,” *Journal of Traffic and Transportation Engineering*, vol. 6, pp. 183–196, 09 2018.

# Final Year Project

---

## ORIGINALITY REPORT

---

11%

SIMILARITY INDEX

9%

INTERNET SOURCES

6%

PUBLICATIONS

7%

STUDENT PAPERS

---

## PRIMARY SOURCES

---

|   |   |     |
|---|---|-----|
| 1 | Submitted to Indian Institute of Information Technology, Design and Manufacturing - Kancheepuram<br>Student Paper   | 1%  |
| 2 | Submitted to Universiti Tunku Abdul Rahman<br>Student Paper   | 1%  |
| 3 | epubs.siam.org<br>Internet Source   | 1%  |
| 4 | coek.info<br>Internet Source  | 1%  |
| 5 | www.igi-global.com<br>Internet Source   | <1% |
| 6 | www.coursehero.com<br>Internet Source   | <1% |
| 7 | N.R. Achuthan, L. Caccetta. "Integer linear programming formulation for a vehicle routing problem", European Journal of Operational Research, 1991<br>Publication | <1% |

---



|    |  |      |
|----|--|------|
| 8  | Ousmane Ali, Jean-François Côté, Leandro C. Coelho. "Models and algorithms for the delivery and installation routing problem", European Journal of Operational Research, 2021<br>Publication | <1 % |
| 9  | <a href="http://tede.ufam.edu.br">tede.ufam.edu.br</a><br>Internet Source  | <1 % |
| 10 | <a href="http://fedetd.mis.nsysu.edu.tw">fedetd.mis.nsysu.edu.tw</a><br>Internet Source  | <1 % |
| 11 | <a href="http://www.bringg.com">www.bringg.com</a><br>Internet Source  | <1 % |
| 12 | <a href="http://www.supplychain247.com">www.supplychain247.com</a><br>Internet Source  | <1 % |
| 13 | <a href="http://link.springer.com">link.springer.com</a><br>Internet Source  | <1 % |
| 14 | <a href="http://onlineresource.ucsy.edu.mm">onlineresource.ucsy.edu.mm</a><br>Internet Source  | <1 % |
| 15 | <a href="http://hdl.handle.net">hdl.handle.net</a><br>Internet Source  | <1 % |
| 16 | <a href="http://library.binus.ac.id">library.binus.ac.id</a><br>Internet Source  | <1 % |
| 17 | Jinil Persis Devarajan, Manimuthu A, V Raja Sreedharan. "Healthcare Operations and Black Swan Event for COVID-19 Pandemic: A   | <1 % |

# Predictive Analytics", IEEE Transactions on Engineering Management, 2021

Publication

18

[www.tandfonline.com](http://www.tandfonline.com)

Internet Source

<1 %

19

[digi.library.tu.ac.th](http://digi.library.tu.ac.th)

Internet Source

<1 %

20

Submitted to Southern New Hampshire University - Continuing Education

Student Paper

<1 %

21

Arianna Seghezzi, Riccardo Mangiaracina. "Investigating multi-parcel crowdsourcing logistics for B2C e-commerce last-mile deliveries", International Journal of Logistics Research and Applications, 2021

Publication

<1 %

22

[documents.tips](http://documents.tips)

Internet Source

<1 %

23

[orbit.dtu.dk](http://orbit.dtu.dk)

Internet Source

<1 %

24

Submitted to University of Southampton

Student Paper

<1 %

25

[www.conscientiabeam.com](http://www.conscientiabeam.com)

Internet Source

<1 %

26

Submitted to Technical University of Cluj-Napoca

<1 %

27

[ir.lib.uwo.ca](http://ir.lib.uwo.ca)

Internet Source

<1 %

28

Martin Grunow. "Transportation Planning/Vehicle Scheduling (TP/VS)", Advanced Planning in Supply Chains, 2012

Publication

<1 %

29

T Iswari, A M S Asih. "Comparing genetic algorithm and particle swarm optimization for solving capacitated vehicle routing problem", IOP Conference Series: Materials Science and Engineering, 2018

Publication

<1 %

30

Submitted to University of Portsmouth

Student Paper

<1 %

31

[arxiv.org](http://arxiv.org)

Internet Source

<1 %

32

[www.koreascience.or.kr](http://www.koreascience.or.kr)

Internet Source

<1 %

33

Mir Ehsan Hesam Sadati, Vahid Akbari, Bülent Çatay. "Electric vehicle routing problem with flexible deliveries", International Journal of Production Research, 2022

Publication

<1 %

34

[eiao.net](http://eiao.net)

Internet Source

<1 %

35

[www.hindawi.com](http://www.hindawi.com)

Internet Source

&lt;1 %

36

C. Chen, C. Lee, C. McGillem. "Task assignment and load balancing of autonomous vehicles in a flexible manufacturing system", Proceedings. 1987 IEEE International Conference on Robotics and Automation, 1987

Publication

&lt;1 %

37

Dondo, R.. "Optimal management of logistic activities in multi-site environments", Computers and Chemical Engineering, 20081124

Publication

&lt;1 %

38

Jean-François Cordeau. "A branch-and-cut algorithm for the pickup and delivery traveling salesman problem with LIFO loading", Networks, 2009

Publication

&lt;1 %

39

[docplayer.net](http://docplayer.net)

Internet Source

&lt;1 %

40

[estudogeral.sib.uc.pt](http://estudogeral.sib.uc.pt)

Internet Source

&lt;1 %

41

[mafiadoc.com](http://mafiadoc.com)

Internet Source

&lt;1 %

42

Weiqiang Pan, Zhilong Shan, Ting Chen, Fangjiong Chen, Jing Feng. "Optimal pilot design for OFDM systems with non-contiguous subcarriers based on semi-definite programming", Telecommunication Systems, 2015

Publication

&lt;1 %

43

repub.eur.nl

Internet Source

&lt;1 %

44

Dorian Dumez, Christian Tilk, Stefan Irnich, Fabien Lehuédé, Olivier Péton. "Hybridizing large neighborhood search and exact methods for generalized vehicle routing problems with time windows", EURO Journal on Transportation and Logistics, 2021

Publication

&lt;1 %

45

Douglas Moura Miranda, Samuel Vieira Conceição. "The vehicle routing problem with hard time windows and stochastic travel and service time", Expert Systems with Applications, 2016

Publication

&lt;1 %

46

Submitted to The University of Manchester

Student Paper

&lt;1 %