



# DESIGN FOR QUALITY AND RELIABILITY (DES303T)

## ASSIGNMENT 6: PROBABILITY DISTRIBUTION FUNCTIONS AND ITS APPLICATION

**AUTOMATION IN HIGH RISE WINDOW CLEANING  
AND OTHER EXTENDED ACTIVITIES**

**GROUP NO. : B2-43**

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# About our product

Manual facade cleaning of buildings is susceptible to accident due to the irregular shape of the building, tangled rope, sudden gust, crash against the building, breaking of wire, unfit posture, lousy equipment, heavy weight and many other parameters. Hence incorporation of Robots is an obvious need to reduce the interference of labor for extremely unsafe and dangerous jobs. The major stumbling block is that the existing climbing robots are not well suited to complex structures as most systems are mounted to the building and are very expensive to develop, and the payback often takes a long time. Our double arm window cleaning robot (Heaven's Roar) proves to be cost effective robotic solution in the this industry with market driving differentiating factors like bio mimicked movement of the robot using vacuum suction technology, weather independent cleaning to maximum extent which is more than current robotic technologies ,monitored cleaning solution using dirt density index identification(AI), recycling water systems for increased sustainability etc.

## Product Reliability

A product's most important characteristic is its ability to sustain over a period of time and perform its intended function as expected under specified conditions. Product Reliability is defined as the probability that a device will perform its required function, subjected to stated conditions, for a specific period of time. Product Reliability is quantified as MTBF (Mean Time Between Failures) for repairable product and MTTF (Mean Time To Failure) for non-repairable product.

## How to Improve the Reliability?

- (i) More precise and perfect components should be used for assembly of product.
- (ii) Standard quality input materials procured from reliable sources should be utilized
- (iii) Plant/machinery must be properly repaired and maintained.
- (iv) Suitable and standard equipment/machines should be used.
- (v) Machines or tools should be replaced before they become unsuitable.

## Repairable and Non Repairable Systems

### Repairable Systems:

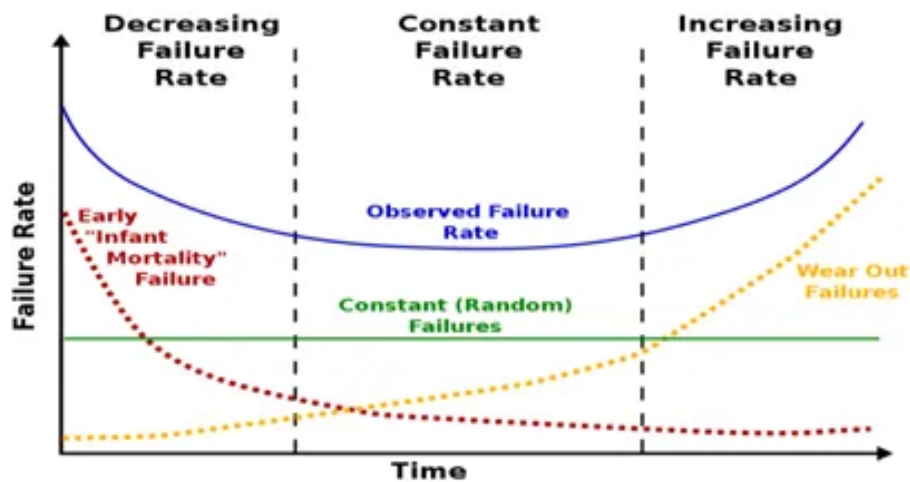
- (i) Restored to operating conditions without replacing entire system
- (ii) Lifetime is age of system or total hours of operation
- (iii) Random variables of interest are times between failure and number of failures at particular age.
- (iv) Failure rate is rate of occurrence of failures (ROCOF) – a property of a sequence of failure times

### Non Repairable Systems:

- (i) Discarded upon failure
- (ii) Lifetime is random variable described by single time to failure
- (iii) Group of systems – lifetime assumed independent & identically distributed
- (iv) Failure rate is rate of occurrence of failures (ROCOF) – a property of a sequence of failure times

# Probability Distribution Functions

## Bath Tub Curve



**Infant Mortality** – This is the part of the curve where failures happen at the very beginning of a product's life cycle. This part accounts for things such as DOA or dead on arrival products, manufacturing errors and material flaws.

**Normal Life** – The second piece of the bend are failures that happen inside the ordinary working time frame or lifespan of the gadget.

**End of Life Wear-Out** – The last piece of the bend is the finish of life for the item. This is the place you will see the bend rise steeply as the device components just reach the point where they will fail due to simple age or wear and tear.

## Histogram

Histograms are used when you have continuous measurements and want to understand the distribution of values and look for outliers. These graphs take your continuous measurements and place them into ranges of values known as bins. Each bin has a bar that represents the count or percentage of observations that fall within that bin.

Failure Rate of a product can be identified by the increasing and decreasing bins size with which the product lifetime can be predicted.

## Survival Curve

Survival analysis is a statistical procedure for data analysis in which the outcome variable of interest is the time until an event occurs.

Goals of Survival Analysis:

Survival analysis has three goals to be addressed:

1. To estimate and interpret survivor and/or, hazard functions from survival data
2. To compare survivor and/or, hazard function
3. To assess the relationship of explanatory variables to survival time

# Mean Time Between Failures

Mean time between failure (MTBF) refers to the average amount of time that a device or product functions before failing. This unit of measurement includes only operational time between failures and does not include repair times, assuming the item is repaired and begins functioning again. MTBF can be calculated as the arithmetic mean (average) time between failures of a system.

# Mean Time To Failures

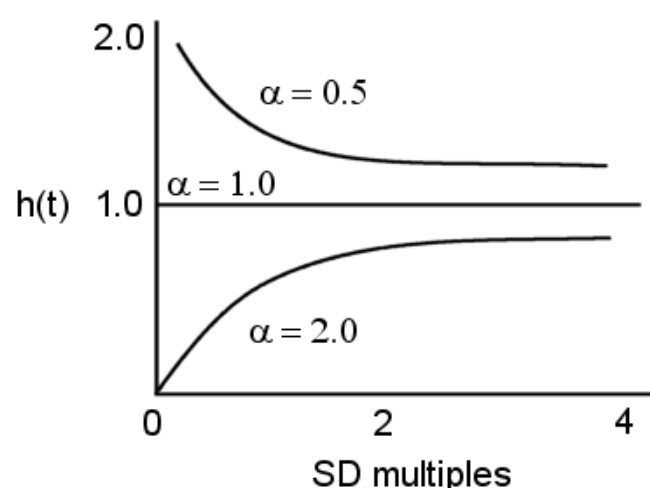
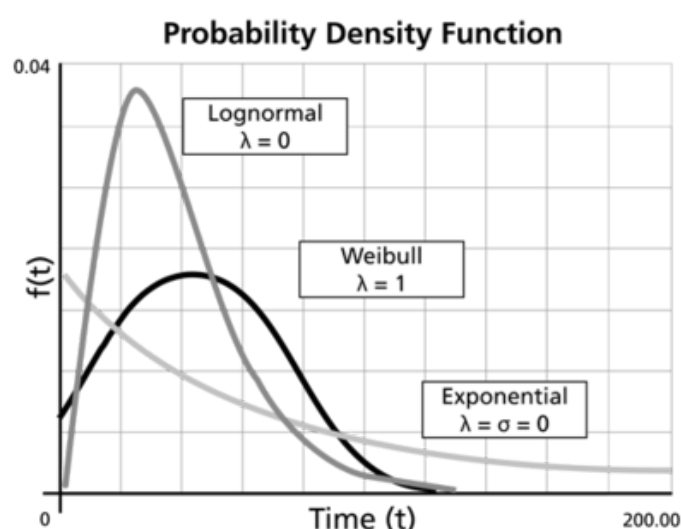
Mean time to failure (MTTF) is a maintenance metric that measures the average amount of time a non-repairable asset operates before it fails. Because MTTF is relevant only for assets and equipment that cannot or should not be repaired, MTTF can also be thought of as the average lifespan of an asset.

# Failure Rate

Failure rate can be defined as the anticipated number of times that an item fails in a specified period of time. It is a calculated value that provides a measure of reliability for a product. The failure rate is the Rate of Occurrence Of Failures (ROCOF) or simply the Recurrence Rate (RR) for repairable systems and Hazard Rate for non repairable systems.

## (i) Gamma Distribution Functions

The **gamma distribution** is used in **reliability** analysis for cases where partial failures can exist, i.e., when a given number of partial failures must occur before an item fails (e.g., redundant systems) or the time to second failure when the time to failure is exponentially **distributed**. The failure density function is for  $t > 0$



$$f(x) = \lambda / T(a) * (\lambda x)^{a-1} e^{-\lambda x} \text{ For } x \geq 0 \text{ and } 0 \text{ for } x < 0$$
$$\text{mean } \mu = a / \lambda ; \sigma = a^{1/2} / \lambda ; \text{ note } T(a) = (a - 1)!$$

## (ii) Chi Square Distribution Functions

The chi-squared distribution (chi-square or  $\chi^2$  - distribution) with degrees of freedom,  $k$  is the distribution of a sum of the squares of  $k$  independent standard normal random variables. It is one of the most widely used probability distributions in statistics. It is a special case of the gamma distribution.

Chi-squared distribution is widely used by statisticians to compute the following:

1. Estimation of Confidence interval for a population standard deviation of a normal distribution using a sample standard deviation.
2. To check independence of two criteria of classification of multiple qualitative variables.
3. To check the relationships between categorical variables.
4. To study the sample variance where the underlying distribution is normal.
5. To test deviations of differences between expected and observed frequencies.
6. To conduct a The chi-square test (a goodness of fit test).

## (iii) Exponential Distribution Functions

The exponential distribution is related to the Poisson distribution. For example, if a random variable,  $x$ , is exponentially distributed then the reciprocal follows a Poisson distribution. If  $x = 1 / y$  then  $y$  is a Poisson distribution. The exponential distribution is a reasonable model for the mean time between failures, such as failure arrivals, whereas the Poisson distribution is used to model the number of occurrences in an interval.

### Probability Density Function (PDF):

The exponential distribution PDF is similar to a histogram view of the data and expressed as

$$f(x) = \frac{1}{\theta} e^{-x/\theta} = \lambda e^{-\lambda x}$$

Where,  $\lambda$  is the failure rate and  $\theta$  is the mean

$$\lambda = 1/\theta$$

### Cumulative Density Function (CDF):

The CDF is the integral of the PDF and expressed as

$$F(x) = 1 - e^{-\lambda x} = 1 - e^{-\frac{x}{\theta}}$$

And represents the probability of failure over  $x$  which is commonly a duration. Of course,  $x \geq 0$ .

### Reliability Function:

The reliability or probably of success over a duration,  $x$ , is

$$R(x) = e^{-\lambda x} = e^{-\frac{x}{\theta}}$$

And  $x \geq 0$  applies here as well.

### Hazard Function:

The exponential hazard function is determined via the ration of the PDF and Reliability functions

$$h(x) = \frac{f(x)}{R(x)} = \frac{\lambda e^{-\lambda x}}{e^{-\lambda x}} = \lambda$$



## (iv) Lognormal Distribution Functions

The **lognormal distribution** is a flexible **distribution** that is closely related to the normal **distribution**. The continuous probability distribution of a random variable whose logarithm is normally distributed is called a lognormal distribution. A random variable of lognormal distribution takes only positive real values. It is skewed towards the right. The value of probability distribution function starts at zero, increases and then decreases. If increases for a given , then the degree of skewness will increase. For the same , if increases, then the probability distribution function's skewness will also increase.

$$f_x(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left[-\frac{\left\{\log_e\left(\frac{x}{m}\right)\right\}^2}{2\sigma^2}\right] \quad P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

A random variable,  $X$ , whose natural logarithm has a normal distribution, has a Lognormal distribution

( $m, \sigma$  are the mean and standard deviation of  $\log_e x$ )

Since logarithms of negative values do not exist,  $X > 0$

the mean value of  $X$  is equal to  $m \exp(\sigma^2/2)$

the variance of  $X$  is equal to  $m^2 \exp(\sigma^2) [\exp(\sigma^2) - 1]$

the skewness of  $X$  is equal to  $[\exp(\sigma^2) + 2][\exp(\sigma^2) - 1]^{1/2}$  (positive)

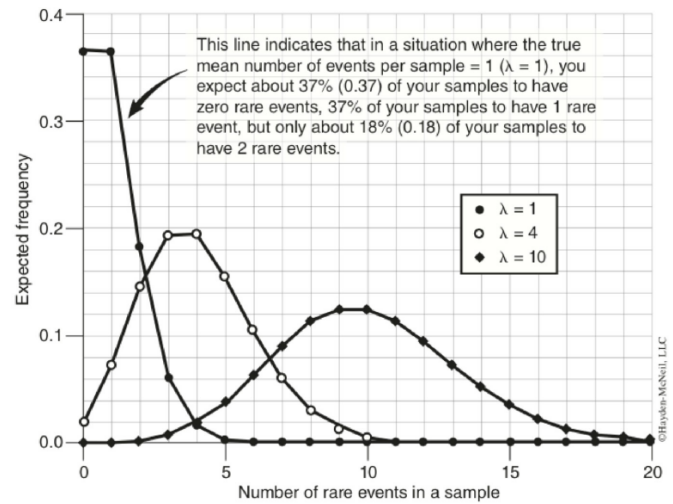
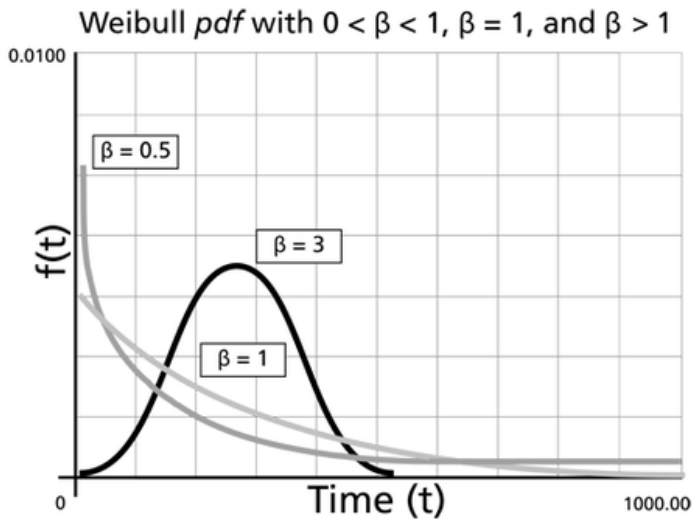
## (v) Weibull Distribution Functions

The **Weibull Distribution** is a continuous probability distribution used to analyse life data, model failure times and assess product reliability. The Weibull distribution is a general-purpose reliability distribution used to model material strength, times-to-failure of electronic and mechanical components. The Weibull shape parameter,  $\beta$ , is also known as the Weibull slope. This is because the value of  $\beta$  is equal to the slope of the line in a probability plot.

Another parameter that is used is B-Life, which is the time by which 3% of population can be expected to fail.

The Weibull plot tells about:

1. Failure modes: Infant mortality / early life failures
2. Chance failure during the active life, or wear out at the end of the product life-cycle
3. About characteristic life
4. About Goodness of fit of the life data to the curve



### The 3-Parameter Weibull

The 3-parameter Weibull pdf is given by:

$$f(t) = \beta \eta (t - \gamma)^{\beta-1} e^{-(t-\gamma)\eta}$$

where:

$$f(t) \geq 0, t \geq \gamma$$

$$\beta > 0$$

$$\eta > 0$$

$$-\infty < \gamma < +\infty$$

and:

$\eta$  = scale parameter, or characteristic life

$\beta$  = shape parameter (or slope)

$\gamma$  = location parameter (or failure free life)

### The 2-Parameter Weibull

The 2-parameter Weibull pdf is obtained by setting  $\gamma=0$ , and is given by:

$$f(t) = \beta \eta (t)^{\beta-1} e^{-(t)\eta}$$

### The 1-Parameter Weibull

The 1-parameter Weibull pdf is obtained by again setting  $\gamma=0$  and assuming  $\beta=C$ =Constant assumed value or:

$$f(t) = C \eta (t)^{C-1} e^{-(t)\eta}$$

where the only unknown parameter is the scale parameter,  $\eta$ .

Note that in the formulation of the 1-parameter Weibull, we assume that the shape parameter  $\beta$  is known a priori from past experience with identical or similar products. The advantage of doing this is that data sets with few or no failures can be analyzed.

## (vi) Poisson Distribution Functions

The Poisson distribution is a discrete probability function that means the variable can only take specific values in a given list of numbers, probably infinite. A Poisson distribution measures how many times an event is likely to occur within “x” period of time. A Poisson experiment is a statistical experiment that classifies the experiment into two categories, such as success or failure. Poisson distribution is a limiting process of the binomial distribution. A Poisson random variable “x” defines the number of successes in the experiment.

If events are Poisson distributed, they occur at a constant average rate and the number of events occurring in any time interval are independent of the number of events occurring in any other time interval. For example, the number of failures in a given time would be given by:

$$f(x) = \frac{a^x e^{-a}}{x!}$$

where x is the number of failures and a is the expected number of failures. For the purpose of reliability analysis, this becomes:

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$$f(x; \lambda, t) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

where:

$\lambda$  = failure rate

t = length of time being considered

x = number of failures

The reliability function, R(t), or the probability of zero failures in time t is given by:

$$R(t) = \frac{(\lambda t)^0 e^{-\lambda t}}{0!} = e^{-\lambda t}$$

In the case of redundant equipments, the R(t) might be desired in terms of the probability of r or fewer failures in time t. For that case

$$R(t) = \sum_{x=0}^r \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$



## **PART B : Suitable probability distribution for our product**

Since we use a collective model of robotic system in the field and each system has  $n$  number of sub domains and our product is mostly dominated by electromechanical systems which enables the product to fail anytime possible hence continuous assessment of the robotic system is required. Also all the data generated by system might not be reliable hence small samples of data are used to predict the reliability analysis which is accurately predicted using weibull probability distribution. Also since our product is expensive to manufacture, reliability analysis must be done at low cost using efficient tool which jotted down to weibull distribution. Also it provides associated predictive risks in terms of confidence bands.