

## A.L Combined Maths - 07th October 2023 - Part 1

$\frac{1}{x} = 4 \Rightarrow \text{multiply all } x \rightarrow$   
 $8(4^2 + 2) - 42t + 27 = 0$   
 $8t^2 - 42t + 45 = 0; a = 8$   
 $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad b = -42 \quad c = 45$   
 $t = \frac{42 \pm \sqrt{1764 - 4 \cdot 8 \cdot 45}}{16}$   
 $t = \frac{42 \pm \sqrt{1764 - 1440}}{16}$   
 $t = \frac{42 \pm \sqrt{324}}{16} \quad 42^2 \rightarrow 40^2 = 1600$   
 $t = \frac{42 \pm \sqrt{18^2}}{16} \quad 1^2 \rightarrow 20^2$   
 $t = \frac{42 \pm 18}{16}$   
 $t = \frac{60}{16} \quad \text{or} \quad t = \frac{24}{16}$   
 $t = \frac{15}{4} \quad \text{or} \quad t = \frac{3}{2}$

$x = \frac{1}{x} = t$   
 $x - \frac{1}{x} = \frac{15}{4} \quad \swarrow$   
 $x^2 - 1 = \frac{15x}{4}$   
 $4x^2 - 4 = 15x$   
 $4x^2 - 15x - 4 = 0; a = 4$   
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad b = -15 \quad c = -4$   
 $x = \frac{15 \pm \sqrt{225 + 64}}{8}$   
 $x = \frac{15 \pm \sqrt{289}}{8} \quad \cdot 15 \pm 17$   
 $x = \frac{15 + 17}{8} \quad \text{or} \quad x = \frac{-2}{8} = -\frac{1}{4}$

$$t = \frac{3}{\sqrt{2}}$$
$$\lambda - \frac{\lambda}{x} = \frac{3}{2}$$
$$2x^2 - 2 + 3x$$
$$2x^2 - 2x - 2 = 0; a=2$$
$$X = \frac{3 \pm \sqrt{9+16}}{4}; b=-3; c=-2$$
$$\lambda = \frac{3+\underline{-5}}{4}$$
$$x = 2 // \text{ or } -\frac{1}{2} //$$
$$2^{nd} \text{ eqn}, \text{ let } y=x_2 = \left\{ 2, -\frac{1}{2}, 7, -\frac{1}{4} \right\} //$$
$$d) x^4 + 2x^3 - x^2 + 2x + 1 = 0$$
$$(x^2+1)^2 + 2(x^3+x^2) - x^2 = 0$$
$$(x^2+1)x = 2(x+\frac{1}{x})-1 = 0$$
$$(x+\frac{1}{x}=t) \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$
$$(t^2 - 2) + 2t - 1 = 0$$
$$\begin{matrix} t^2 + 2t - 3 = 0; & a=1 \\ & b=2 \\ & c=-3 \end{matrix}$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-2 \pm \sqrt{4+12}}{2} = -1 \pm 2 = 2 \text{ or } -1$$
$$t = -3 \quad \left| \begin{array}{l} t = 1 \\ x + \frac{1}{x} = 1 \end{array} \right.$$
$$x + \frac{1}{x} = -3$$
$$x^2 + 3x + 1 = 0; a=1$$
$$x = \frac{-3 \pm \sqrt{9-4}}{2}; b=3; c=1$$
$$c=1; x^2 - 2x + 1 = 0$$
$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$
$$x = \frac{-3 \pm \sqrt{5}}{2}; -1 \pm \sqrt{5}$$
$$x = \frac{-3}{2}, -\frac{3-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$$

$\gamma(x+1)^2(x+2) = 72$   
 $(x+1)^2 = x^2 + 2x + 1$   
 $x(x+1)^2 = x(x^2 + 2x + 1) = 72$  or  $x(x^2 + 2x + 1) = 72$   
 $x^3 + 2x^2 + x = 72$   
 $(x^3 + 2x^2 + x)(x+2) = 72 \cdot (x+2)$   
 $x(x+2)(x+1)^2 = 72$   
 $(x^2 + 2x)(x^2 + 2x + 1) = 72$   
 $x^2 + 2x = 6$  or  $x^2 + 2x - 6 = 0$   
 $x = \frac{-2 \pm \sqrt{4 + 24}}{2} = \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$

$$\begin{aligned} 8) \quad & \left(x + \frac{2}{x} - 1\right) \left(x + \frac{2}{x} + 4\right) = 6 \\ & x + \frac{2}{x} = t \quad \left| \begin{array}{l} \left(\frac{2}{x} + 2 - 1\right) \cdot \frac{1}{(x+2)} \cdot \frac{1}{(x+2)} \\ + x^2 \left(\frac{2}{x} + 1\right) \left(\frac{2}{x} + 4\right) = 6x \\ + x \left(x + \frac{2}{x} - 1\right) x \left(x + \frac{2}{x} + 4\right) = 6x^2 \end{array} \right. \\ & x + \frac{2}{x} = t + 1 \\ & (t-1)(t+4) = 6 \\ & t^2 + 3t - 4 = 0 \\ & t^2 + 3t - 10 = 0 \quad \omega = 1 \\ & (t+5)(t-2) = 0 \quad b = 3 \\ & \quad \quad \quad \omega = -2 \end{aligned}$$