

Elimination

$$\begin{aligned} 7x + 6y &= 241 \quad \text{--- (1)} & x &\rightarrow 52 \\ 5x + 5y &= 241 \quad \text{--- (2)} & x &\rightarrow 63 \end{aligned}$$

$5 \times 63 = 315 \Rightarrow$   
 $63 \times 63 + 63 \times y = 241 \times 63$   
 $-(52 \times 63 + 52 \times y = 241 \times 52)$   
 $(63 - 52)y = 241 \times 63 - 241 \times 52$

Simple Method

$$\begin{aligned} (1) + (2) \Rightarrow \\ 115x + 115y &= 460 \\ 115(x + y) &= 460 \\ x + y &= 4 \quad \text{--- (3)} \\ (1) - (2) \Rightarrow -11x + 11y &= -22 \\ x - y &= -2 \quad \text{--- (4)} \\ (3) + (4) \Rightarrow \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

$x = 3, y = 1$

$$\begin{aligned} 2) \frac{x+y+5}{x+2y} &= \frac{4}{3}, \quad \frac{x+5y}{7y-x} = \frac{5}{3} \\ 3x+3y+15 &= 4x+8y & 3x+15y &= 35y-5x \\ x+5y &= 15 \quad \text{--- (1)} & 8x-20y &= 0 \\ 3x &= 15 & 2x &= 5y \quad \text{--- (2)} \\ x &= 5, y = 2 \end{aligned}$$

$$\begin{aligned} 3) \frac{x-1}{2} &= \frac{y+2}{3} = \frac{2x+2y}{9} \\ a=b=c \Rightarrow \begin{matrix} a=b \\ b=c \end{matrix} \\ \frac{x-1}{2} &= \frac{y+2}{3} \quad \text{--- (1)} \Rightarrow 3x-3 = 2y+4 \\ \frac{x-1}{2} &= \frac{2x+2y}{9} \quad \text{--- (2)} \Rightarrow 9x-9 = 4x+4y \\ 5x-4y &= 9 \quad \text{--- (3)} \\ 10x-8y &= 18 \\ -10y+12y &= 35-27=8 \\ y &= 4, x=5 \end{aligned}$$

$$\begin{aligned} 4) \begin{cases} x-2y+z = -1 \quad \text{--- (1)} \\ 3x+4y-z = -4 \quad \text{--- (2)} \\ 2x-y+z = 2 \quad \text{--- (3)} \end{cases} \\ (1) + (2) \Rightarrow 7x+2y = -5 \quad \text{--- (4)} \\ (1) - (3) \Rightarrow 2x-y = -3 \quad \text{--- (5)} \\ (4) + 5 \times (5) \Rightarrow 11x = -5-6 = -11 \\ x = -1, y = 1, z = 5 \end{aligned}$$

$$\begin{aligned} 5) \begin{cases} x+y+z = 1 \quad \text{--- (1)} \\ y+z = 2a \quad \text{--- (2)} \\ x+z = 2b \quad \text{--- (3)} \end{cases} \\ (1) + (2) + (3) \Rightarrow \\ x+y+z = a+b+c \quad \text{--- (4)} \end{aligned}$$

$$\begin{aligned} 6) \frac{x}{x^2+1} + \frac{x^2+1}{x} &= \frac{5}{2} \quad \{1, 1\} \\ t = \frac{x}{x^2+1} \Rightarrow t + \frac{1}{t} &= 2 + \frac{1}{2} \\ t &= 2, \frac{1}{2} \\ \frac{x}{x^2+1} &= 2 \quad \text{or} \quad \frac{x}{x^2+1} = \frac{1}{2} \\ 2x^2+2 &= x & x^2-2x+1 &= 0 \\ 2x^2-x+2 &= 0 & (x-1)^2 &= 0 \\ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}, a=2, b=-1, c=2 & x=1, 1 \\ x &= \frac{1 \pm \sqrt{1-8}}{4} \\ x &= \frac{1 \pm \sqrt{-7}}{4} \\ \sqrt{-7} &= 2.3... \\ \sqrt{-7} &= \text{spurious} \\ 7) 2x-3y &= 1 \quad \text{--- (1)} \Rightarrow x = \frac{(1+3y)}{2} \quad \text{--- (2)} \\ 2x^2+3x-3y^2 &= 38 \quad \text{--- (3)} \\ (2) \text{ in } (3) \Rightarrow \end{aligned}$$