11_19_2023_12_Trigonometry_04

$1+\tan^2\alpha = \sec^2\alpha$

6) $\cos\alpha \, \operatorname{cosec}\alpha \sqrt{\sec^2\alpha - 1} = 1$

$L.H.S = \cos\alpha \cdot \frac{1}{\sin\alpha} \cdot \sqrt{\sec^2\alpha - (\underbrace{\sec^2\alpha - \tan^2\alpha}))}$ $\leftarrow \frac{\sin\alpha}{\cos\alpha}$

$= \frac{\cos\alpha}{\sin\alpha} \cdot \tan\alpha$

$= 1$

$= R.H.S //$

7) $(1-\underline{\cos^2 A})(1+\cot^2 A) = 1$

$L.H.S = (\sin^2 A + \cancel{\cos^2 A} - \cancel{\cos^2 A})(\operatorname{cosec}^2 A)$

$= \sin^2 A \cdot \operatorname{cosec}^2 A$

$= \sin^2 A \cdot \left(\frac{1}{\sin A}\right)^2$

$= 1$

$= R.H.S //$

8) $\sin^2\theta \cdot \cot^2\theta + \sin^2\theta = 1$

$L.H.S = \cancel{\sin^2}\theta \cdot \left(\frac{\cos\theta}{\cancel{\sin\theta}}\right)^2 + \sin^2\theta$

(M1)

$= \cos^2\theta + \sin^2\theta$

$= 1$

$= R.H.S$

(M2) $L.H.S = \sin^2\theta(\cot^2\theta + 1)$

$= \sin^2\theta \cdot \operatorname{cosec}^2\theta$

$= \sin^2\theta \cdot \left(\frac{1}{\sin\theta}\right)^2$

$= 1$

$= R.H.S$

9) $(1+\tan^2\theta)(1-\sin^2\theta) = 1$

$L.H.S = \sec^2\theta \cdot \cos^2\theta$

$= \left(\frac{1}{\cos\theta}\right)^2 \cdot \cos^2\theta$

$= 1$

$= R.H.S //$

10) $\tan\theta + \cot\theta = \overbrace{\sec\theta}^{1/\cos\theta} \cdot \overbrace{\operatorname{cosec}\theta}^{1/\sin\theta}$

$L.H.S = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\cos\theta \cdot \sin\theta}$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$= \sec\theta \cdot \operatorname{cosec}\theta$

$= R.H.S //$

11) $\frac{1+\tan A}{1+\cot A} = \tan A$

$L.H.S = \frac{1+\tan A}{1+1/\tan A}$

$= \frac{1+\tan A}{\frac{(\tan A+1)}{\tan A}}$

$= \tan A \frac{(1+\tan A)}{(\tan A+1)}$

$= \tan A$

$= R.H.S //$

12) $\frac{1}{\sec A+\tan A} = \sec A - \tan A$

$L.H.S = \frac{\sec^2 A - \tan^2 A}{\sec A + \tan A}$

$a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sec A - \tan A)\cancel{(\sec A + \tan A)}}{\cancel{(\sec A + \tan A)}}$

$= \sec A - \tan A$

$= R.H.S //$

13) $\sin^4\theta - \cos^4\theta = 2\underline{\sin^2}\theta - 1$

$L.H.S = \sin^4\theta - \cos^4\theta$

$= (\underbrace{\sin^2\theta}_{x})^2 - (\underbrace{\cos^2\theta}_{y})^2$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$= \boxed{\sin^2\theta - \boxed{\cos^2\theta} \cdot 1}$

$= (\sin^2\theta - [1-\sin^2\theta])$

$= 2\sin^2\theta - 1$

$= R.H.S //$

$a^4 - b^4$

$(a^2)^2 - (b^2)^2$

$(a^2-b^2)(a^2+b^2)$

$(a-b)(a+b)(a^2+b^2)$

$a^2 - b^2 = (a+b)(a-b)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

- $a^2 - b^2$
- $a^3 - b^3$
- $a^3 + b^3$
- $(a+b)^3$
- $(a-b)^3$